

1. X2 and X6 are independent of each other. From the graph it can be seen that X9 is closed, hence there is no dependence relation between X2 and X6.

If X7 and X11 are known, then there is now a path from X2 → X3 → X4 → X5 → X8 → X9 → X10 → X11 → X10 → X9 → X6 that is open, and hence there is now a dependence relation between X2 and X6

2. No. of free params from:

x1: 1  
x1-x2: 2  
x2-x3: 2  
x3-x4: 2  
x4-x5: 2  
x6: 1  
x5-x7: 2  
x5-x8: 2  
x6,x7,8-x9: 8  
x9-x10: 2  
x10-x11: 2  
Total: 26 free parameters

After change:

x1: 4  
x1-x2: 20  
x2-x3: 10  
x3-x4: 12  
x4-x5: 20  
x6: 4  
x5-x7: 20  
x5-x8: 20  
x6,x7,8-x9:  $125 \times 2 = 250$   
x9-x10: 12  
x10-x11: 20  
Total: 392 free parameters

3(a)

$$\begin{aligned}P(X3=1) &= 0.3 \\P(X4=1) &= 0.1 \times 0.3 + 0.5 \times 0.7 = 0.38 \\P(X4=1|X3=1) &= 0.1\end{aligned}$$

$$\begin{aligned}P(X3=1|X4=1) &= \frac{[P(X4=1|X3=1) \times P(X3=1)]}{P(X4=1)} \\&= \frac{0.3 \times 0.1}{0.38} \\&= 0.0789\end{aligned}$$

3(b)

P(X1) is independent of X5  
P(X3) is independent of X5  
P(X11) is independent of X5 since X10 is independent

$$\begin{aligned}P(X5=2|X3=2, X11=2, X1=2) &= \frac{[P(X3=2, X11=2, X1=2|X5=2) \times P(X5=2)]}{P(X3=2, X11=2, X1=2)} \\&= \frac{[P(X3=2, X11=2, X1=2|X5=2) \times P(X5=2)]}{[P(X3=2) \times P(X11=2) \times P(X1=2)]} \\&= \frac{[P(X11=2) \times P(X1=2) \times P(X3=2) \times P(X5=2)]}{[P(X3=2) \times P(X11=2) \times P(X1=2)]} \\&= P(X5=2) \\&= P(X5=2|X4=1) \times P(X4=1|X3=2) + P(X5=2|X4=2) \times P(X4=2|X3=2) \\&= (0.5 \times 0.5) + (0.4 \times 0.5) \\&= 0.45\end{aligned}$$