

1.) X_2 and X_6 are independent

From the graph X_9 is closed. Thus X_2 cannot get to X_6

- If X_7, X_{11} known,

New path: $X_2 \rightarrow X_3 \rightarrow X_9 \rightarrow X_5 \rightarrow X_8 \rightarrow X_9 \rightarrow X_{10} \rightarrow X_{11}$
 \downarrow
 $X_6 \leftarrow X_9 \leftarrow X_{10}$

Thus, it is now an open path. Therefore, X_2 and X_6 are dependent.

2.)

State	No of free parameter (Before)	Δ (After)
$X_1 =$	1 (Before)	4 (After)
$X_1 - X_2 =$	2	20
$X_2 \rightarrow X_3 =$	2	10
$X_3 \rightarrow X_4 =$	2	12
$X_4 \rightarrow X_5 =$	2	20
$X_6 =$	1	4
$X_5 \rightarrow X_7 =$	2	20
$X_5 \rightarrow X_8 =$	2	20
$X_6, X_7, X_8 - X_9 =$	8	250
$X_9 \rightarrow X_{10} =$	2	12
$X_{10} \rightarrow X_{11} =$	2	20
total	<u>26 free param</u>	<u>392</u>

Before = 26 free parameters

After = 392 free parameters

39.)

$$P(X_3 = 1) = 0.3$$

$$P(X_4 = 1) = 0.1 \times 0.3 + 0.5 \times 0.7 = 0.38$$

$$P(X_4 = 1 | X_3 = 1) = 0.1$$

$$P(X_3 = 1 | X_4 = 1)$$

$$= \frac{P(X_4 = 1 | X_3 = 1) \times P(X_3 = 1)}{P(X_4 = 1)}$$

$$P(X_4 = 1)$$

$$= \frac{0.3 \times 0.1}{0.38} = 0.0789$$

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b) ~~X_1~~ independent of X_5

X_3 independent of X_5

X_{11} independent of X_5 (X_{10} is independent)

$$P(X_5 = 2 | X_3 = 2, X_{11} = 2, X_1 = 2)$$

$$\# P(X_3 = 2, X_{11} = 2, X_1 = 2 | X_5 = 2) \times P(X_5 = 2)$$

$$= \frac{P(X_3 = 2, X_{11} = 2, X_1 = 2)}{P(X_5 = 2)}$$

$$\# P(X_3 = 2) \times P(X_{11} = 2) \times P(X_1 = 2) \times P(X_5 = 2)$$

$$= \frac{P(X_3 = 2) \times P(X_{11} = 2) \times P(X_1 = 2)}{P(X_3 = 2) \times P(X_{11} = 2) \times P(X_1 = 2)}$$

$$= P(X_5 = 2)$$

$$P(X_5 = 2)$$

$$= P(X_5 = 2 | X_4 = 1) \times P(X_4 = 1 | X_3 = 2)$$

$$+ P(X_5 = 2 | X_4 = 2) \times P(X_4 = 2 | X_3 = 2)$$

$$= \left(\frac{1}{2} \times \frac{1}{2}\right) + (0.4 \times 0.5)$$

$$= 0.45$$

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