The Failure of Formalism: From the Artworld to the Mathworld

Word Count: 9368

Abstract

Mathematics and art are often treated as opposites: the former a paradigm of rigor, the latter of creativity. Yet both have long been theorized through formalism—the view that what matters is intrinsic structure, whether the "significant form" of an artwork or the logical form of a proof. I argue that formalism fails in both domains, revealing the limits of purely formal criteria across human practices. Artificial intelligence sharpens this critique: systems can generate formally correct works or proofs that nonetheless lack meaning, insight, or understanding. Following Arthur Danto's "artworld," I sketch a parallel "mathworld," in which proofs gain their status through uptake within evolving mathematical practices. The lesson extends beyond art and mathematics: formal criteria alone cannot capture the significance of human practices.

Introduction

It is tempting to think of mathematics and art as opposites: one the austere province of rigor and truth, the other the free realm of creativity. Yet both share a similar theoretical impulse: an attempt to locate their essence in form.

In aesthetics, formalism holds that the value of an artwork lies in intrinsic properties — relations of line, color, rhythm, or structure — that Clive Bell called its "significant form."[1] This idea extends across the arts, from music to literature to architecture. In mathematics, formalism likewise treats the validity of a proof as a matter of its logical form: a sequence of symbol manipulations carried out according to fixed rules.[2] Despite their differences, both promise clarity and objectivity. Both, I will argue, ultimately fail.

The failure is not confined to art and mathematics. It is symptomatic of a wider problem with attempts to define human practices by formal criteria alone. Formalism isolates what can be mechanized — but in doing so, misses what practitioners prize: insight, context, creativity, and meaning. This point becomes strikingly clear when we consider artificial intelligence. AI can generate images, music, or proofs that meet the most rigorous formal standards, yet these outputs lack the qualities that give human work significance.

I develop the argument in five stages. First, I trace the structural analogy between art and mathematics, showing how formalism promised objective standards of value. Second, I consider the most influential critiques of formalism in each domain, highlighting their persistence and force. Third, I transpose a critique from art into the mathematical domain, revealing its structural implications. Fourth, AI provides an empirical test: it can realize the formalist ideal with precision, yet its outputs illustrate the hollowness of purely formal achievement. Fifth, I sketch a "mathworld," inspired by Danto's "artworld," in which a proof's status depends on uptake within evolving mathematical practices, rather than formal derivation alone.[3] The paper concludes by reflecting on what this analogy reveals about the limits of formalism and the broader conditions under which human practices attain value.

[1] See Bell [1914].
[2] See Bourbaki [1939].
[3] See Danto [1964].

The lesson is general: human practices cannot be captured by form alone. Art and mathematics, unusual companions though they are, together reveal broader truths about creativity, knowledge, and human achievement.

Drawing the Analogy: Formalism in Art and Math

At first glance, art and mathematics could not be further apart. One is saturated with history, culture, and emotion; the other is the paradigm of abstract rationality. Yet formalism reveals a strikingly deep structural similarity between them. In both cases, the entire project of the discipline has been taken, at least by formalists, to be the production of objects that satisfy a certain set of formal criteria.

In the case of art, the formalist outlook was canonically developed by figures such as Clive Bell and Roger Fry in the early twentieth century.[4] On their view, the aim of the artist is to produce objects, processes, or events that instantiate "significant form." What matters are the intrinsic, structural features of the work — its lines, colors, rhythm, unity, and other formal qualities — regardless of who made it, when it was made, or what cultural and historical meanings attach to it. This view was applied broadly, not only to painting but to sculpture, music, and other creative practices, as formalists sought to identify the common, aesthetically relevant thread across different media.

To see what is meant, consider Picasso's Guernica (1937).[5] On the formalist account, what makes the painting valuable is not its reference to the bombing of the Basque town, its place in the history of modernist protest art, or Picasso's political commitments, but the orchestration of forms: the fractured planes, the almost monochrome palette, the lattice of diagonals, the pyramidal massing,

[4] See Bell [1914] and Fry [1920].

[5] See Chipp, ed., [1988] for an excellent discussion of the context of Guernica.

the harsh alternation of light and shadow that binds a sprawling surface into a unified whole. The painting's power would lie, on this view, in the "significant form" itself — the relations among lines, shapes, and tonal contrasts — which, in principle, could be appreciated by a viewer who knew nothing of the Spanish Civil War or of Picasso's biography.

The attraction of this view is clear and multifaceted. By abstracting away from context, intention, and interpretive content, formalism appears to solve several persistent problems in aesthetics. It offers a unified account of the diverse objects we call "art," grounding aesthetic value in features that can, at least in principle, be analyzed and compared systematically. It promises universality: a work can be judged as aesthetically significant independent of cultural or temporal contingencies. It also offers a clear criterion for criticism, which seems to allow evaluation to aspire to objectivity rather than subjective taste. In this sense, formalism presents itself as a tractable, elegant framework for understanding artistic value: it identifies intrinsic properties as central, and relegates artist, meaning, and context to the periphery.

Formalism was, for a time, the dominant current in twentieth-century art theory. Though later overshadowed by interpretive and contextualist frameworks, it has not vanished. Its influence persists in contemporary debates, particularly in computational aesthetics and the evaluation of AI-generated art, where the prospect of quantifying value in terms of structural or visual properties is especially attractive. In this way, formalism remains a live option: not the ruling paradigm, but a recurring response to the challenge of explaining aesthetic value in universal, objective terms.[6]

In mathematics, an analogous outlook was championed in the late nineteenth and early twentieth centuries by Hilbert and later codified in the programmatic

[6]See Harrison and Wood eds., [2002] for discussion.

work of Bourbaki.[7] On this picture, just as the aim of art is to produce works with significant form, the aim of mathematics is to produce proofs that have a certain log9cal form. In outline, the idea is that mathematics, and informal mathematical proof, reduces to symbol manipulation within a fixed formal system — typically classical first-order logic with Zermelo–Fraenkel set theory (ZFC).

To see what this means in practice, consider Euclid's celebrated proof of the infinitude of primes. The proof as Euclid gives it is short and striking: assume finitely many primes, multiply them together and add one, and show that the resulting number is either prime itself or divisible by a new prime. On the formalist account, this elegant argument is merely a sketch. To make it fully rigorous, each inference must be spelled out in first-order logic with ZFC: defining primality in set-theoretic terms, formally proving that multiplication and addition preserve certain properties, explicitly deriving the existence of a divisor, and so on. What once appeared as a luminous flash of mathematical insight — a single, decisive stroke — is here dissolved into a protracted march of symbols. The drama of Euclid's proof, its sense of sudden inevitability, is flattened into the patient mechanics of formal derivation.

Accordingly, the narrative, explanatory, or aesthetic aspects of a proof are deemed extraneous. A proof is, in principle, nothing more than a finite sequence of formulas, each following mechanically from its predecessors by the application of formal rules. On this account, broader context — historical development, intuitive reasoning, elegance, or the "story" of the proof — is largely irrelevant. What matters is the formal structure, its internal coherence, and its adherence to syntactic rules.

The attractions of this view are, if anything, even more pronounced than in the artistic case. Formalism promises objectivity, together with a strikingly

[7]See Hilbert and Bernays [1934-1939] and Bourbaki [1939].

simple and tractable epistemology, semantics, and ontology. Mathematics, on this account, is not about an external realm of abstract objects; it is, rather, a game of manipulating symbols. Mathematical claims are known because they are derivable within the system, not because they correspond to some external reality. There is no truth or meaning beyond derivability. The ontology of mathematics is exhausted by the sentences that can be derived and those that cannot.

Moreover, the view illuminates a striking sociological fact: mathematicians across cultures and eras overwhelmingly converge on what counts as a proof. Just as formalism in art explains how diverse objects can be united under a single conception of "art" by appeal to form, formalism in mathematics explains convergence by appeal to derivation. By grounding validity in formal procedure, it provides an elegant explanation of mathematical consensus, while also offering a seemingly unified, systematic, and neutral framework for evaluating mathematical work. Just as formalism in art abstracts from artist, context, and meaning, formalism in mathematics abstracts from intuition, narrative, and explanatory depth, locating significance solely in formal structure.

Unlike its artistic counterpart, however, mathematical formalism has never waned. The Hilbertian conception that all proofs could, in principle, be reduced to finite formal derivations in first-order ZFC continues to shape the foundations of the discipline and remains the background assumption of much contemporary philosophy of mathematics. Even where mathematicians do not explicitly embrace formalism in their daily practice, the formalist picture remains the silent default: the horizon against which alternative philosophies of mathematics must position themselves.

Seen this way, the analogy is not superficial but structural. Art, on the formalist view, is an enterprise whose goal is the creation of aesthetically adequate objects; mathematics, on the formalist view, is an enterprise whose goal is the creation of formally valid derivations. The difference lies not in the structure of the project, but in its register: art is cast in aesthetic terms, while mathematics is cast in epistemic terms. Both, however, are governed by the same underlying impulse: to reduce a human practice to a domain of form, stripped of history, context, and intention.

Yet it is precisely these features — the promise of objectivity, universality, and tractability — that will later reveal the tension at the heart of formalism. By privileging form above all else, both artistic and mathematical formalism risks ignoring dimensions of practice that are essential to what art and mathematics are in lived experience. The very traits that make the formalist project so elegant also make it vulnerable, opening the door to a critique that will extend across both domains. And it sets the stage for a more general critique: if formalism falters in one domain, that failure may illuminate why it falters in the other.

The Analogy at Work: Three Common Critiques

Formalism in both art and mathematics has long attracted criticism, but three interrelated critiques are particularly revealing. First, both domains are essentially open-ended, resisting capture by any fixed set of formal criteria. Second, formalist approaches are exclusionary, privileging certain methods while marginalizing others. Third, formalism strips away context, meaning, and interpretive depth, reducing complex practices to surface-level structures. Taken together, these critiques suggest that formalism's appeal—its neutrality, rigor, and generality—is inseparable from the very features that make it inadequate.

The first critique, openness, highlights the inherent expansiveness of both fields. In art, Weitz argued that the concept of art is "open," resisting definitions in terms of necessary and sufficient conditions.[8] Artistic practice is historically embedded and evolves as new forms emerge. Attempts at closed definitions either exclude works that audiences accept or rely on covert reference to the very concept they seek to define. Later innovations, such as John Cage's 4'33" or Robert Rauschenberg's Erased de Kooning Drawing, could not have been anticipated by prior formal definition s —they reshape the category itself. Mathematics exhibits a parallel phenomenon. Hilbert's program sought to formalize all mathematics through a complete, consistent axiomatic system, but Gödel's incompleteness theorems decisively undermined this ambition: any system capable of expressing arithmetic leaves some truths unprovable within it.[9] Later independence results, such as Cohen's work on the Continuum Hypothesis, show that even central questions cannot be settled within standard axioms.[10] One might hope to circumvent this by moving to richer or alternative frameworks. But here we encounter an even deeper problem, stressed by Hamkins and other pluralists: the alternative frameworks themselves are often mutually incompatible.[11] For example, the continuum hypothesis may hold in one framework and fail in another, with no neutral standpoint from which to adjudicate. Thus, mathematical practice does not merely overflow any one formal system; it cannot be encompassed by a family of mutually consistent systems either. Mathematics, like art, is essentially open-ended, unfolding in ways no formal scheme can anticipate or contain.

The second critique concerns exclusion. Formalism does not merely fail to capture everything; it actively prioritizes certain methods while marginalizing others. In art, formalist aesthetics often elevate painting and sculpture judged by line, color, or proportion, sidelining practices rooted in performance, politics,

[8]See Weitz [1956].
[9]See Godel [1931].
[10]See Godel [1940] and Cohen [1963]
[11]See Hamkins [2012].

or community.[12] In mathematics, privileging ZFC and first-order logic sidelines constructivist, intuitionist, categorical, and paraconsistent traditions, each offering distinct conceptions of mathematical practice and value. Formalism, in seeking universality, functions as intellectual gatekeeping, enforcing orthodoxy while suppressing genuine diversity.

The third critique addresses meaning. Formalism systematically removes semantic, contextual, and interpretive dimensions, yet these are precisely what practitioners regard as essential. In art, works like Duchamp's readymades or Warhol's silkscreens cannot be understood merely as configurations of visual elements; their significance depends on semantic, historical, and cultural positioning. Similarly, in mathematics, proofs are valued not simply as formal derivations but as meaningful arguments about numbers, sets, functions, or manifolds. Their significance derives from the understanding, explanation, unification, and heuristic insight they provide. While formal derivation may suffice for verification, it is the semantic and explanatory dimensions that confer the virtues mathematicians prize: elegance, depth, and productive insight.

Across these three critiques, a common pattern emerges. Formalism promises clarity and universality, yet its rigor inevitably strips away openness, diversity, and meaning. The analogy between art and mathematics demonstrates that this is not an isolated problem but a structural limitation: practices that matter to humans — whether aesthetic or mathematical — cannot be captured fully by fixed formal criteria. Attempts to reduce living practices to formal properties inevitably leave out what makes them significant.

[12] See Nochlin [1971] and Elkins [1996].

The Analogy at Work: A Transposed Critique

A striking, novel philosophical payoff of the art/math analogy is that it permits a targeted transposition of a familiar critique from aesthetics into the foundations of mathematics, producing an objection to formalism that is both sharp and structurally revealing. The worry runs as follows. Bell hoped to ground aesthetic significance in a formal property ("significant form"); Weitz and others showed that this project is circular because the putative formal property is fixed and vindicated by the very aesthetic judgments it was supposed to explain. If we transpose that diagnosis to mathematics we obtain an analogous (and, I will argue, decisive) objection to Hilbertian formalism: the claim that proofs are epistemically grounded by their reducibility to formal derivations (say, in first-order ZFC) is parasitic on the informal mathematical judgments formalism purports to replace. In short: the formalist promise of explanatory independence collapses into circularity. Let's now examine this in some detail.

Bell's basic thesis is simple: aesthetic value supervenes on structural, formal properties of artworks—relations of line, colour, rhythm and so on—what he called "significant form." If one could specify a formal predicate S that unambiguously identified those configurations that confer aesthetic worth, then aesthetic judgment would rest on a non-interpretive, objective basis.

Two tasks face anyone who attempts to make this program concrete. One must (i) fix the extension of S (which objects instantiate S?) and (ii) show that instantiation of S suffices to warrant the evaluative verdicts we care about. In practice both steps are covertly driven by antecedent aesthetic judgment. The typical procedure is abductive: one inspects paradigmatic works, abstracts common structural features, and treats those features as marking significance. But which works count as paradigms is itself a value judgment. If we change the canon—if, for instance, we include socially engaged murals or community performance pieces whose value is not primarily formal—then the common formal thread we extract will differ. Thus the extension of S is effectively fixed by prior aesthetic verdicts.

A concrete instance helps. Take Cézanne's *Mont Sainte-Victoire* paintings. A candidate formalist predicate might highlight faceted planes, distributed color masses, and a balance between contour and modulation. Suppose a technically expert copyist produces near-duplicates that preserve all coarse compositional ratios and local line–colour relations that a workable S would register. To insist that Cézanne's originals have S while the copies lack it, the formalist must either (a) refine S to pick out micro-regularities that, by fiat, track our verdicts (thus building the verdicts into the predicate), or (b) rely on the very aesthetic sensibility that the theory purported to replace in order to show why the originals matter. Either way the explanatory independence promised by S evaporates: the theory relabels rather than grounds our judgments. This is the circularity that Weitz emphasized and that subsequent literature has probed in various ways.[13]

At first glance the mathematical case seems different in kind because formalism in mathematics is articulated with exceptional precision: proofs can, in principle, be reduced to finite derivations in a precisely specified first-order system (most commonly first-order logic with Zermelo–Fraenkel set theory, ZFC). The formalist says: informal mathematical arguments are epistemically secure because they can be regimented into such derivations; the derivation, checked step by step, guarantees that the conclusion follows from the axioms. That response, however, depends on a chain of justificatory moves that import the very informal judgments formalism sought to make dispensable. I'll now unpack this.

[13] For the canonical statements of these views see Bell [1914] and Weitz [1956]. For modern elaborations and criticisms see especially Danto [1964] and Dickie [1974].

Working mathematicians routinely employ inferential moves that are not manifestly first-order syntactic steps: quantification over properties or functions, appeals to uniqueness or universal characterization, naturality arguments in category theory, transfinite constructions, and heuristic compressions that treat families of analogous steps as a single conceptual move. These higher-level moves are not mere rhetorical flourishes; they are the inferential forms mathematicians rely on when judging an argument correct and explanatory. So, suppose that a theorem ϕ is established informally using such inferences.

Proofs involving such steps are clearly not formal derivations. But the formalist replies that such higher-level reasoning can be simulated inside a first-order framework. Properties and functions are coded as sets; second-order quantification is represented by quantifying over sets; category-theoretic structure can be encoded by set-theoretic proxies; induction and recursion are unrolled into schemas or defined by set-theoretic constructions. Thus one can, in many cases, produce a first-order derivation D in ZFC whose final formula corresponds, under a regimenting translation, to the informal conclusion ϕ.

The idea then is that the justification flows up from the formal derivation D in the system F to the informal proof of ϕ. However, why should we think that D is truth-preserving in the pertinent sense? Why should D justify ϕ? The standard account invokes (i) metatheoretic results (soundness, conservativity, relative consistency) about F – the formal system in which D occurs, (ii) informal semantic reasoning that shows the coding faithfully represents the informal content, and (iii) informal mathematical and philosophical reasoning that the rules F are truth-preserving, and the set-theoretic axioms of F faithfully mimic truth-preserving second-order inferences.

Clearly, each stage of the reduction relies on essential informal judgment. Soundness proofs, for example, are carried out in the meta-theory: one proves by induction on the length or structure of derivations that every inference rule preserves truth in all models, and hence that if $\Gamma \vdash \phi$ then $\Gamma \models \phi$. That induction, the appeal to model-theoretic semantics, and the background set-theoretic reasoning that makes the argument go are not performed inside the object theory whose reliability is at issue; they depend on ordinary mathematical practice and informal understanding of truth and model.

Likewise, assessing the faithfulness of an encoding cannot be settled by syntax alone. When a second-order idea (e.g., "for all properties" or a categorical universal property) is represented in first-order set theory, the semantics change: full second-order semantics are not recursively axiomatizable, and first-order "codings" typically correspond to Henkin-style or set-theoretic surrogates. Determining that such a surrogate captures the same content is a conceptual, semantic judgment: one must argue that the representation preserves the essential ideas (invariance, universality, explanatory role), even if the resulting formal proof looks very different. Those are precisely the kinds of reasons that cannot be fully internalized as further first-order derivations.

Finally, the choice of formal framework itself is justified informally. Opting for classical ZFC, constructive CZF, predicative Kripke–Platek, or a type-theoretic foundation such as HoTT is not a neutral, purely formal move; each embodies substantive commitments about logic (classical vs. intuitionistic), ontology (power sets, large cardinals, univalence), and admissible forms of inference (impredicativity, choice, excluded middle). We adopt a framework because it better fits our explanatory aims, integrates with existing theory, supports the proofs we care about, or yields fruitful new results. Those are mathematical and philosophical reasons, not theorems of any one system. In short, the supposedly "formal" foundation rests on prior informal evaluation at every critical juncture — of soundness, of representational adequacy, and of which system to trust —

so formalism cannot escape the very judgments it seeks to mechanize.

Moreover, there is also a precise technical obstacle to the formalist aspiration that the system should, in itself, deliver final assurance. Gödel's Second Incompleteness Theorem shows that a sufficiently strong, consistent formal system (one that, roughly, is able to represent a fair amount of arithmetic) cannot prove its own consistency. Since internal proof of consistency is blocked, any assurance that derivations in F will not lead to falsehoods must be provided by some external, meta-theoretic argument—again, by the kind of informal reasoning the formalist hoped to supersede. More generally, relative consistency and proof-theoretic analysis typically show that the acceptability of one formal system is justified only by appeal to principles (often of stronger proof-theoretic strength) that lie outside the system itself.

The formalist reduction—informal proof $\longrightarrow$ formal derivation within a fixed system—does not, in fact, provide an independent foundation. The reliability of the derivation, the adequacy of the encoding, the soundness proof, and the legitimacy of the underlying axioms all ultimately rest on informal mathematical judgments. The explanatory arrow is therefore reversed: derivations are trusted because practice-sensitive, informal arguments show them to be reliable, not the other way around.

The structural parallel with aesthetics is now clear:

- In aesthetics: the putative predicate S ("significant form") cannot be fixed or justified independently of prior aesthetic judgments. Any attempt to do so collapses into stipulation or presupposition. The formal predicate merely glosses over pre-theoretic verdicts rather than grounding them.

- In foundations: claims that formal derivability confers epistemic authority rely on informal metamathematical judgments — soundness arguments, choices of axioms, adequacy of encodings — that cannot themselves be formalized within the system. The formal derivation is authenticated by the very informal practices that formalism sought to underwrite.

The circularity identified in the aesthetic literature thus maps directly onto a precise epistemic circularity in mathematics. This is more than a rhetorical resemblance. In both domains, formalism promises autonomy from human judgment but, on inspection, must appeal to those judgments to define and validate the very formal criteria it proposes. The ideal of an independent, self-standing formal foundation is therefore illusory.

The Analogy at Work: The Case of AI

Thus far we have argued, by conceptual means and analogy, that formalism in art and mathematics systematically omits dimensions of practice that practitioners regard as essential. Contemporary artificial intelligence provides a particularly vivid empirical test of that diagnosis. AI systems instantiate, in differing ways, the formalist ideal: they manipulate formal patterns and statistical regularities at scale and without the kinds of contextual sensitivity that human agents bring to their work. Examining how AI outputs are received in both domains therefore helps us see — in practice rather than merely in principle — what a strictly formal account leaves out.

Consider first the artistic case. Contemporary image generators can produce images that satisfy many of the formal features foregrounded by aesthetic formalism—rhythm, balance, composition, and even convincing mimicry of stylistic signatures. Yet the reception of much AI-generated imagery is revealingly mixed: critics, curators, and many viewers report a feeling of emptiness, inauthenticity, or "thinness." The problem is not mere novelty but the lack of embedding in traditions of practice, intention, and history: AI images do not arise from situated practices, do not participate in dialogues of interpretation, and do not bear the mark of an artist's struggle or deliberate theoretical stance.[14] In short, they instantiate form without the interpretive ecology that, by Danto's lights, helps make artworks meaningful.

A parallel phenomenon appears in mathematics. Instances such as the Appel–Haken computer proof of the Four Color Theorem (and its later formal verifications) show that a result can be established by computation in a way that many mathematicians find epistemically unsatisfying: correct, but opaque and explanatorily thin.[15] Proof assistants and automated provers routinely produce derivations that meet the formal criteria one could hope for; yet those derivations often fail to supply the explanatory, heuristic, and aesthetic virtues that mathematicians prize. Proofs are not merely certificates that a theorem follows from certain axioms; they are vehicles of understanding: they explain why a result holds, suggest generalizations, reveal structure, and provide insight that can be transported to other problems. A purely formal derivation typically affords few of these virtues.

Two points should be emphasized. First, the failure in both domains is not a failure of formal competence. AI systems are strikingly good at the formal tasks formalism elevates; indeed, their prowess makes the point. The more readily machines can satisfy formal criteria, the clearer it becomes that formal criteria alone are thin measures of the values that practitioners prize. Second, this is not to denigrate the instrumental value of machine derivations or generated images. Formal verification is indispensable in domains (software verification, cryptographic proofs, some areas of combinatorics) where syntactic correctness matters enormously. Likewise, generative systems have already become important tools in design and artistic practice. The philosophical claim is narrower and stronger: success at formal tasks does not imply success at the

[14] See for instance, Cramer [2024] and Cheng et al. [2023].

[15] See Apple & Haken [1989] for the proof, and and Tymoczko [1979] for a discussion of the response.

broader cognitive and evaluative tasks that underwrite human appreciation and mathematical understanding.

In short, AI provides an empirical instantiation of the broader theoretical diagnosis of the paper. Generative systems and automated provers perfectly enact the formalist ideal, and in doing so they make visible what that ideal leaves out: context, narrative, interpretive depth, and the pedagogical and communal processes through which human appreciation and understanding arise. The lesson is constructive as well as critical. If we accept that formal derivations and generated images are valuable but incomplete, then the natural institutional response is to develop the interpretive and translational practices that will make machine outputs intelligible, explanatory, and integrated into our communal standards.

From the *Artworld* to the *Mathworld*

One virtue of the analogy developed so far is that it not only diagnoses formalism's limits but points toward a constructive alternative. In aesthetics, the most influential corrective to formalist reductionism is the idea that art's status is not determined solely by intrinsic perceptual features but by the broader social and interpretive matrix in which objects are embedded. Danto's notion of the artworld — the horizon of theory, practice, institutions, and interpretive frameworks that makes an object count as an artwork — captures that corrective with considerable explanatory force.[16] If Danto's lesson rescues artworks from a narrow ontology by making status a function of situated practices and uptake, then a parallel move suggests itself for mathematics: a mathworld view according to which what makes an argument a bona fide mathematical proof is not the possibility of formal derivation but its acceptance, interpretation, and

[16]See Danto [1964] and [1981].

uptake within an evolving network of practices, institutions, and standards.

The View

Core Claim: An argument is a mathematical proof in virtue of being accepted-as-proof by the mathematical community: that is, its status as a proof is constituted by its having been taken, interpreted, and integrated as such within the community's practices, institutions, and standards.

Reducibility to a formal derivation is neither the constitutive condition for proof-status nor what makes an argument a proof; at best it is an instrumental resource that may assist the process by which acceptance is achieved. Rather, an argument typically becomes a proof only by traversing three interlocking stages. These are not mere descriptions of sociological background; they are the procedural steps by which an argument acquires the status "proof."

1. Institutional vetting (public exposure and technical scrutiny). A proposed argument must be brought into public channels where it can be examined: preprints, seminar talks, referee reports, correspondence, and the archival apparatus of journals and conference proceedings. These institutional loci perform epistemic work: they reveal hidden assumptions, expose gaps, solicit alternative routes, and generate the technical conversation within which faults are detected or confidence accumulates.

2. Interpretive translation (exposition and conceptual rendering). Technical correctness alone does not make an argument usable. The raw sequence of inferences must be translated—explicated, compressed into schemata, paraphrased, diagrammed, and rephrased in heuristics and metaphors—so that other practitioners can grasp the idea, reproduce the reasoning, and apply it elsewhere. This interpretive labour creates the explanatory profile

of the argument: it is what converts a correct chain of steps into an intelligible and generative mathematical account.

3. Uptake and integration (pedagogy, application, and canonicalization). Finally, the argument must be assimilated into the community's operative stock: it must survive teaching, be cited and used, enable generalizations, and become a tool in subsequent research. Uptake is a temporally extended process—through expository articles, graduate courses, further theorems, and applied uses—by which the argument is stabilized as part of the discipline's working corpus.

These stages are sequential in the ordinary course: institutional vetting typically precedes broad interpretive translation, which in turn is necessary for deep uptake. An argument that skips these stages—formally derivable but never publicly vetted, never interpreted so others understand it, and never integrated into practice—fails to achieve proof-status even if a formal derivation exists.

Formal derivations and mechanized proofs (Lean, Coq, etc.) are important instruments within this process: they can serve as certificates used during vetting, as archives later mined during expository work, or as tools that reveal hidden steps. But they do not by themselves constitute acceptance-as-proof. A machine-checked derivation may strengthen confidence (and sometimes accelerate uptake), but constitutive status is conferred only through the communal processes above.

A Case Study

Wiles's proof of Fermat's Last Theorem provides a vivid, concrete case of how an argument becomes a proof only through the institutional, interpretive, and uptake work I have been describing. The strategy that made the result accessible to communal scrutiny was itself strikingly conceptual: rather than attacking Fermat's equation directly, the argument linked two previously distinct realms of mathematics by means of the (then-conjectural) modularity relation between elliptic curves and modular forms — a special instance of the broader Langlands-style reciprocity that connects arithmetic geometry and automorphic forms. Frey had earlier observed that a putative counterexample to Fermat would produce an elliptic curve with anomalous properties; Ribet showed that if the relevant elliptic curve were modular then this anomaly could not occur.[17] Thus the problem of Fermat was reduced to proving modularity for a class of elliptic curves. This reduction is precisely the kind of conceptual unification that gives a proof explanatory power: it reframes a Diophantine problem as a question about the deep interaction of Galois representations, modular forms, and arithmetic geometry.

Wiles's contribution lay in supplying the technical bridge — a modularity-lifting argument built from the deformation theory of Galois representations together with sophisticated commutative-algebra "patching" techniques. When he first announced his result in the early 1990s the community response exemplified institutional vetting: the argument circulated in preprints and talks, and specialists subjected the manuscript to intense technical scrutiny. That scrutiny revealed a substantive gap in a key deformation/patching step. The gap itself is an instructive instance of how institutions do epistemic work: peers detected a precise mathematical deficiency that would have been invisible without communal examination.

But discovery of the gap did not terminate the process; it initiated collaborative repair and, crucially, a wave of interpretive translation. Wiles, together with collaborators, revised the argument and introduced modifications that closed the difficulty; contemporaneously, survey articles, lecture series, and expository accounts reframed the main ideas — modularity lifting, the role of Galois de-

[17] See Frey [1986] and Ribet [1989]

formation spaces, and the algebraic patching device — in ways that made the methods intelligible and teachable.[18] These expository renderings are not cosmetic: they are the channel by which the argument acquires the explanatory profile that other mathematicians can reuse and build upon.

Finally, uptake completed the transition from argument to proof. In the years after the repair and publication, the modularity techniques were taught in graduate courses, incorporated into surveys and textbooks, and deployed in subsequent research programs that extended modularity results to broader classes of curves and influenced related advances in the Langlands program.[19] Only after this cycle of critique, repair, exposition, and pedagogical absorption did the community treat the corrected argument as a canonical proof in the full, practice-sensitive sense: not merely as a formally correct derivation, but as an explanatory, generative achievement that changed the field's conceptual toolkit.

Two lessons follow. First, technical correctness — even when eventually secured — is only one node in the process: institutional vetting discovers errors and sharpens standards; interpretive work makes the argument intelligible; uptake embeds it in practice. Second, the proof's significance lies more in the conceptual unification it effected (bringing arithmetic geometry and modular forms into close dialogue) than in any particular sequence of formal steps. That conceptual unity is what other mathematicians learn, teach, and exploit — and this is precisely what a world-oriented account aims to capture.

[18] For instance, Taylor's expository account of the proof (*Taylor 1995*) and Diamond–Im's graduate-level notes (*Diamond and Im 1995*) translated highly technical steps into accessible formulations; Cornell, Silverman, and Stevens's edited volume (*Cornell, Silverman, and Stevens 1997*) provided a broad survey of the ideas and their place in number theory.

[19] For example, the proof's methods were systematized in expository works such as *Diamond and Shurman 2005*, became part of graduate pedagogy in modular forms and elliptic curves (e.g. *Stein 2007*), and were extended in major advances like the proof of modularity for all elliptic curves over $\mathbb{Q}$ (*Breuil, Conrad, Diamond, and Taylor 2001*).

The Mathworld at Work

First, consider open-endedness. The formalist hope that a fixed system could capture all of mathematics founders both on technical facts—Gödelian incompleteness, independence results—and on the lived reality that new frameworks continually arise: category theory, homotopy type theory, forcing axioms, large cardinal hypotheses. From a mathworld perspective, open-endedness is not a problem but a constitutive feature of mathematical practice. Standards and methods expand as new problems and concepts emerge, and this expansion is disciplined by communal norms. Open-endedness thus appears not as a definitional failure but as the expression of a living, evolving practice.

Second, consider circularity. Bell's attempt to ground aesthetic judgment in "significant form" faltered because justification was supposed to come from a property that could not bear that normative weight. Hilbertian formalism faces a parallel difficulty: the idea that informal proofs gain their epistemic status simply because they can, in principle, be made into derivations. In the mathworld view, by contrast, justification is distributed across the network of practices—peer critique, pedagogy, explanatory integration, and interpretive labor. What confers justificatory authority is not an abstract formal property but the interlocking of these practices, which are themselves revisable and testable. The vicious circularity thus dissolves.

Third, consider exclusion. By insisting on a single canonical framework (ZFC derivations, say), formalism marginalizes alternative approaches: constructive mathematics, intuitionism, category theory, and non-classical logics are all sidelined. The mathworld, however, is naturally pluralistic. Different subcommunities may adopt distinct but overlapping criteria for acceptance. Category theory, for example, gained legitimacy as the community adopted its interpretive grammar, while intuitionistic analysis is regarded as legitimate within communities that prize constructive methods. Through institutional pluralism and cross-fertilization—seminars, journals, collaborations—the practice remains fertile rather than ossified.

Finally, consider meaning and aesthetic value. Mathematicians value proofs for reasons beyond correctness: explanatory depth, conceptual unification, heuristic fertility, and aesthetic qualities such as elegance. These values are exactly those exercised in processes of uptake and transmission. A formally correct but inscrutable derivation may be archived for verification purposes, but it will not occupy the same imaginative or pedagogical role as an argument that has been interpreted, explained, and integrated. The mathworld account, uniquely among the theories considered, captures this dimension of proof.

The mathworld conception has several further interlocking advantages that recommend it as a realistic and philosophically illuminating account of mathematical practice. It explains why mathematicians can rationally rely on community-endorsed proofs even when they do not inspect every formal step: trust is grounded in publicly accessible practices of scrutiny and exposition. It accommodates both the remarkable convergence of standards and principled, technical disagreements (foundational pluralism) without collapsing into arbitrariness. It integrates formal and computational tools without mistaking them for the whole of mathematical value. And it yields concrete norms: if proofs gain status through uptake, then institutional reforms—stronger incentives for expository translation of machine derivations, pluralist editorial policies, and support for replication and pedagogical synthesis—are direct levers to strengthen mathematical epistemic practices.

Objections and Replies

Relativism

A natural worry about the mathworld proposal is that it collapses into relativism. If a putative argument counts as a proof insofar as it is accepted, interpreted, and taken up within the mathematical community, might what counts as a proof vary capriciously with time (an eighteenth-century argument ceasing to qualify), with local subcommunities (classical vs constructive), or with transient fashions in the seminar room? Could a piece of reasoning be a proof in one era, then cease to be one, and later be reinstated purely because communal attitudes shift? If so, then mathematical justification would reduce to mere to social contingency on the view.

The short answer is: it does not. Acceptance in the mathworld is not mere popularity or whim; it is uptake measured against a dense lattice of disciplinary constraints and tests. The historical record exhibits remarkable continuity of result and long-run convergence of standards. When standards change, the dominant pattern is refinement or regimentation of earlier practice rather than wholesale overturning of established theorems[20]; and when subcommunities adopt different standards, their disagreements are structured, technical, and subject to translation and cross-checking[21]. Taken together, these features support a moderate, constrained pluralism, not relativism.

What the relativist must show, in order to succeed, is stronger than mere

[20]For example, the early proofs of results in calculus by Newton and Leibniz often relied on heuristic appeals to infinitesimals; over the nineteenth century, these arguments were regimented by the ϵ–δ limit definition of continuity and differentiability, yielding results that both preserved the earlier insights and eliminated vagueness. See Carl B. Boyer, The Concepts of the Calculus: A Critical and Historical Discussion of the Derivative and the Integral (New York: Dover, 1949), chs. 6–8.

[21]A well-studied case is the reinterpretation of classical theorems within intuitionistic mathematics: for instance, Brouwer's rejection of the law of excluded middle required reworking existence proofs, yet many classical results were shown to have constructive counterparts or double-negation translations. See Michael Dummett, Elements of Intuitionism (Oxford: Clarendon Press, 1977), esp. ch. 5.

methodological diversity or historical change. She must establish that (i) acceptance is unconstrained by standards of reason and evidence and (ii) mathematics' truth-judging mechanisms are essentially arbitrary or capricious. The evidence points the other way. Mathematical practice is disciplinarily constrained: proposed proofs are exposed to counterexamples, attempts at generalization, and the demands of explanatory integration; institutional mechanisms (refereeing, seminars, replication) and pedagogical practices (textbook reworking, expository synthesis) subject claims to public, repeatable scrutiny. These are not the practices of a fashion-driven guild but the practices that produce intersubjective reliability.

Two recurring historical patterns are instructive. First, continuity: many propositions accepted long ago remain accepted today, albeit often after their presentations have been reworked to meet newer standards. Euclid's theorems, Archimedean results, and elementary arithmetic survive successive refinements of rigor; when older arguments are found defective, the usual response is regimentation — filling gaps and making implicit steps explicit — rather than rejecting the result.[22] Second, correction: when genuine errors arise, communal procedures tend to expose and repair them rather than transmute them into permanent disagreement. As noted above, Wiles's initial gap in his proof of Fermat's Last Theorem and its subsequent repair exemplify how communal scrutiny produces durable consensus by corrigible means.[23]

A further and important datum is methodological accumulation. Mathemat-

[22]A classic example is the Fundamental Theorem of Algebra. Early proofs, such as those by d'Alembert (1746) and even Gauss's first attempt (1799), contained serious gaps. Over time, these arguments were repaired and regimented using complex analysis and topology, yielding rigorous proofs while preserving the original insight that every non-constant polynomial has a complex root. See Jean Dieudonné, History of Algebraic and Differential Topology, 1900–1960 (Boston: Birkhäuser, 1989), ch. 1.

[23]For instance, Kempe's 1879 proof of the Four Color Theorem was long accepted but later shown to be flawed (Heawood 1890). Rather than leaving mathematicians divided, the error was acknowledged, and the theorem was eventually established by Appel and Haken's computer-assisted proof in 1976. See Robin Wilson, Four Colors Suffice: How the Map Problem Was Solved (Princeton: Princeton University Press, 2002), chs. 3–6.

ics typically grows by adding sanctioned methods and conceptual resources (category theory, forcing, homotopy type theory, large-cardinal hypotheses) rather than by repudiating earlier tools wholesale. Consequently, proofs that were once acceptable are in many cases interpretable within newer frameworks, or else failures are traceable to principled limits of those older methods. This cumulative enlargement makes plural standards possible without collapse: different projects can legitimately operate under different norms while translations, comparisons, and cross-checks secure coherence across the discipline.

Contemporary subcommunities with differing standards (classical versus constructive mathematics, for example) might seem to instantiate relativism, but their disagreements are neither anarchic nor unprincipled. Constructivists reject certain nonconstructive inferences (notably unrestricted uses of the law of excluded middle) for principled philosophical reasons; these are substantive methodological disputes. At the same time, there exist many systematic devices — negative translations, realizability interpretations, proof mining — that allow classical proofs to be reinterpreted constructively or to isolate precisely which nonconstructive principles are used. Such technical bridges show that pluralism can be both rigorous and constrained. Stewart Shapiro and others defend variants of this moderate pluralism: multiple, partially overlapping standards of acceptability coexist, yet they are themselves subject to evaluation by evidence, applicability, and explanatory payoff.[24]

Put concisely: the mathworld locates justificatory weight in structured communal practices that are institutionally supported (refereeing, education, seminars), normatively demanding (clarity, explanatory integration, reproducibility), corrigible by counterexample and repair, and capable of cross-community translation and verification. Acceptance is therefore constrained by objective-looking tests (counterexamples, transferability, applications); it yields long-run

[24]See Shapiro [2014].

convergence rather than chaotic fluctuation; and it permits pluralism without relativism because standards can be assessed against one another through technical translation and comparison.

The mathworld view thus reframes the simplistic dichotomy between "socially determined" and "metaphysically fixed." Mathematical justification is social in that it is mediated by communal norms and institutions, but it is far from arbitrary because those norms are disciplined by exacting tests and by the cumulative project of the discipline itself. The unconstrained, whim-driven variability required by the relativist simply does not track the historical or contemporary evidence. The view preserves the epistemic seriousness of mathematics while offering a more accurate account of how mathematical reliability is in fact secured.

Truth-Tracking

A different, and often sharper, worry than relativism targets the epistemic credentials of the mathworld proposal. Even if communal standards are disciplined rather than arbitrary, why suppose those standards track mind-independent mathematical truth (if there is any such truth to be tracked) rather than merely producing a coherent, useful, but ultimately conventional edifice? In short: the mathworld view may explain how we come to accept theorems, but does it explain why those theorems are likely to be true in anything like an objective sense?

The reply rests on two linked moves. First, this is primarily an epistemological account: it describes the practices by which mathematicians come to know. It does not settle metaphysics. Platonists can read communal acceptance as the most reliable epistemic route to an independently existing realm of mathematical facts; anti-realists can treat the success of practice as the primary datum. The philosophical burden, however, is not to force a metaphysical verdict at the level of epistemology, but to show that the social and institutional procedures that deliver acceptance are themselves reliable—that they generate indicators one should expect to correlate with truth. Second, and more positively, objectivity is here best understood as intersubjective reliability: the capacity of community procedures to generate stable, corrigible, and convergent bodies of belief. The remainder of the reply aims to show why the mechanisms of mathworld make this kind of reliability plausible.

Those mechanisms come in several mutually reinforcing varieties. First, independent convergence: important mathematical results are frequently reached by distinct methods—analytic, geometric, algebraic, combinatorial—that are, in practice, theoretically independent.[25] That a theorem can be obtained by diverse routes makes it improbable that its acceptance is an artifact of a single methodological prejudice. Second, explanatory unification and depth: theorems that unify disparate phenomena do explanatory work; their capacity to bring together previously disconnected problems is exactly the sort of success that picks out robust structure rather than mere convention.[26] Third, cross-domain corroboration: when abstract mathematics later finds independent application (differential geometry in general relativity; group theory in particle physics; number theory in cryptography), this furnishes a non-social check—an external

[25]A standard example is the Prime Number Theorem, proved independently by Hadamard and de la Vallée Poussin in 1896 using complex analysis, and later given elementary proofs by Erdős and Selberg (1949). The convergence of analytic and elementary approaches provides strong evidence of the theorem's robustness. See G. H. Hardy and E. M. Wright, An Introduction to the Theory of Numbers, 6th ed. (Oxford: Oxford University Press, 2008), ch. 22.

[26]Examples include the Langlands program, which unifies number theory and representation theory by conjecturing correspondences between automorphic forms and Galois representations. See Robert P. Langlands, "Problems in the Theory of Automorphic Forms," in Lectures in Modern Analysis and Applications, III, ed. C. T. Taam (Berlin: Springer, 1970), 18–86. Another case is the use of category-theoretic dualities (e.g. Stone duality, Pontryagin duality), which reveal deep structural connections across algebra, topology, and logic. See Saunders Mac Lane, Categories for the Working Mathematician, 2nd ed. (New York: Springer, 1998), ch. 5.

constraint—on the legitimacy of the mathematical structure. Fourth, robustness under reformulation: theorems that survive translation into different formal frameworks (for example, those that admit both constructive and classical renderings, or that can be expressed both categorically and set-theoretically) display a kind of conceptual stability that is evidence of their non-accidental character. Fifth, institutional error-correction: refereeing, seminars, attempted generalizations, counterexamples, and the production of multiple expositions (textbooks, surveys, formalizations) generate a high-fidelity feedback loop that weeds out mistakes and pathologies. Finally, methodological pluralism is itself an epistemic virtue: overlapping subcommunities with different standards (classical, constructive, categorical, etc.) place reciprocal constraints on one another; convergence across these divides is particularly persuasive.

These features together help explain why communal acceptance is plausibly truth-tracking rather than merely conventional on the view. They also provide a ready rejoinder to natural sceptical retorts. Suppose one worries that communal procedures could generate a stable but false mathematical edifice through systematic bias. That would require evading (i) internal tests of mathematics (counterexamples, failed generalizations, contradiction), (ii) cross-method convergence, and (iii) external corroboration via applicability. Historical experience suggests such a coordinated failure is difficult to sustain: entrenched but vulnerable views have been overturned when they could not withstand these pressures.[27] Another rejoinder is that applicability might be a post hoc rationalization: yes, applicability is not decisive in isolation, but the repeated, independent appearance of the same abstract structures across very different empirical

[27]Classic examples include the 19th-century abandonment of naive infinitesimals in favor of rigorous ϵ–δ analysis after Berkeley's criticisms in The Analyst (1734) and the subsequent development of Cauchy–Weierstrass methods (see Judith V. Grabiner, The Origins of Cauchy's Rigorous Calculus (Cambridge, MA: MIT Press, 1981)). Another case is the rejection of naive set theory after Russell's paradox, which prompted the creation of axiomatic systems like Zermelo–Fraenkel set theory (see Gregory H. Moore, Zermelo's Axiom of Choice: Its Origins, Development, and Influence (New York: Springer, 1982)).

domains makes mere after-the-fact rationalization an implausible general account. A final rejoinder notes that none of this constitutes a proof of platonism; and rightly so. The view does not aim to establish metaphysical Realism. Its goal is epistemic: to show why the social, institutional, and methodological practices of mathematics produce results that are remarkably stable, corrigible, and convergent—properties one should reasonably expect of a truth-tracking enterprise.

In short, the challenge of truth-tracking is best met not by insisting on a metaphysically literal account of how mathematics is tethered to a separate realm, but by showing that the communal practices the mathworld idea describes are precisely those that, historically and pragmatically, have produced reliable knowledge. Whether one interprets that reliability as evidence of a Platonic realm, of structural objectivity, or of an exceptionally successful human practice is a metaphysical gloss one may impose subsequently; but the epistemic core remains.

Conclusion

This paper began with a simple but, I hope, provocative claim: the attractive philosophical project of reducing significant human practices to formal criteria is fundamentally inadequate. Formalism has appeared in two very different domains precisely because it promises clarity, neutrality, and tractability: in aesthetics as a theory that locates value in intrinsic "significant form," and in the foundations of mathematics as a program that locates epistemic authority in formal derivability (typically within a system such as first-order ZFC). I have argued that these two manifestations of the same philosophical impulse fail for closely related reasons. When we put the failures side by side and exploit the analogy between them, three things become evident.

First, despite their very different vocabularies, art and mathematics share structural features — historical contingency, institutional embedding, and an interpretive ecology — that resist capture by fixed, purely formal criteria. The classic criticisms of each domain are not merely domain-specific curiosities; they are instances of a single, deeper problem: the attempt to make a living, open-ended human practice exhaustible by a single formal schema.

Second, transposing arguments between the domains yields fresh philosophical payoffs. Aesthetic objections to Bell-style formalism — its circularity, its indeterminacy, its exclusion of context and agency — have structural analogues in the mathematical case. The Hilbertian move to ground informal proofs in regimentable derivations imports a parallel circularity: the justificatory force of a formal derivation itself presupposes informal metamathematical judgments. Bringing the two literatures into contact clarifies that the core problem is not merely imprecision in the aesthetic case, nor merely technical incompleteness in the mathematical case, but the deeper mistake of expecting formal structure alone to carry the normative and evaluative burden that, in practice, resides with interpretive communities.

Third, the contemporary AI landscape provides a vivid empirical stress-test of formalism. Generative image models and automated theorem provers instantiate the formalist ideal: they manipulate form with extraordinary competence. The widespread sense that many AI images are aesthetically hollow, or that machine-generated derivations often lack the explanatory and aesthetic virtues that mathematicians prize, is not a merely sentimental reaction. Instead, it is diagnostic: the very success of machines at the formalist task highlights what formalism leaves out — meaning, interpretive embedding, narrative, pedagogy, and the conditions for communal uptake.

These conclusions motivate a constructive alternative: a world-oriented account of practice. Drawing on Danto's insight in aesthetics and on institutional ideas in philosophy of science, I sketched a mathworld model according to which proofs acquire their status through practices of acceptance, interpretation, exposition, and uptake within a community. The proposal is modest in method but far-reaching in consequence. It preserves the epistemic seriousness of mathematics — its standards of clarity, rigor, and truth-seeking — while explaining why formal derivability cannot by itself supply justification, meaning, or aesthetic value. It explains how mathematics can be both disciplined and open, how it can integrate new methods without collapsing into relativism, and how automated tools can be epistemically powerful while remaining instruments rather than replacements for communal judgment.

This proposal has evident limits. Any world-oriented account must confront two central challenges: the charge of relativism and the demand for truth-tracking. My responses are necessarily provisional. Acceptance within a community is not mere arbitrariness; it is structured by rigorous intersubjective standards, and these standards are themselves subject to scrutiny, reform, and diversification. It is also reasonable to expect that practices disciplined in this way will converge on a body of claims that reliably track truth, since the very criteria by which standards are evaluated—consistency, explanatory power, fecundity, coherence with other well-established results — are themselves indicators of epistemic reliability. These matters are thus not purely philosophical abstractions, but empirical, institutional, and normative questions, requiring philosophical analysis to be integrated with historical, sociological, and ethnographic inquiry.

To return to the paper's opening image: when a machine faithfully satisfies the formal criteria of a practice, that success is illuminating precisely because it shows where the formalist diagnosis was thin. Machines do well at what formalism prizes; they do poorly at what formalism ignores. The lesson is not Luddite nostalgia for human uniqueness, but a sober recognition of the richness of human intellectual life. Art and mathematics are not, in the end, paradigms of pure form alone; they are practices constituted by histories, communities, narratives, and standards of intelligibility. Any adequate philosophical account must attend to that complexity.

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