CONVEX HULL

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Universitat Politècnica de Catalunya

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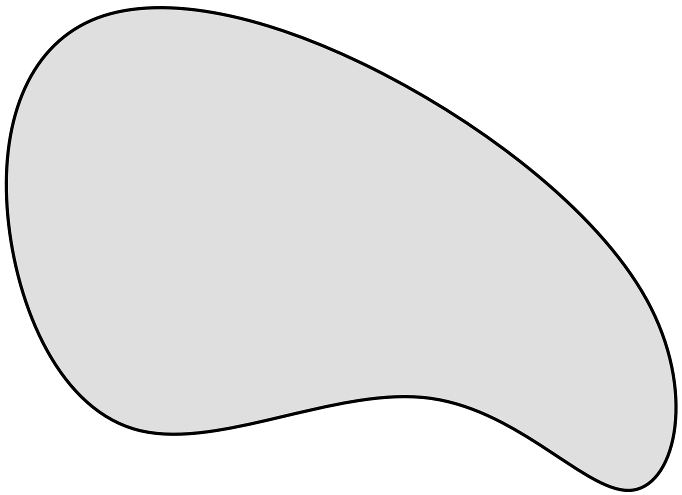

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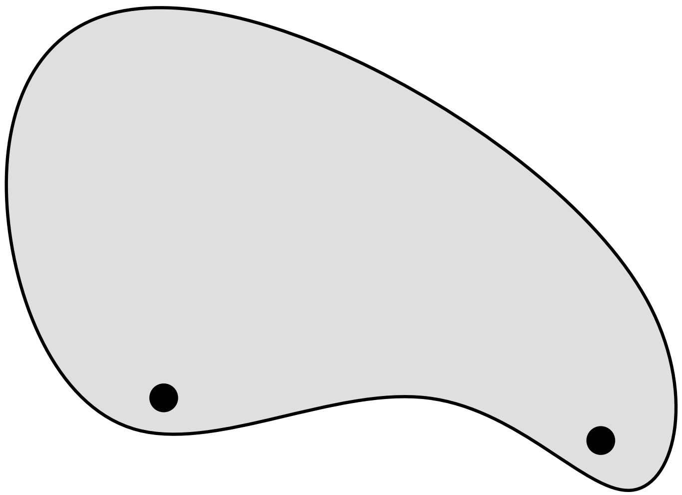

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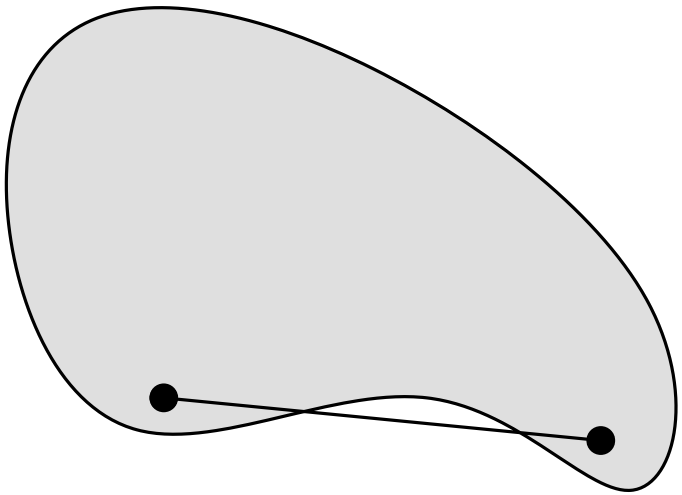

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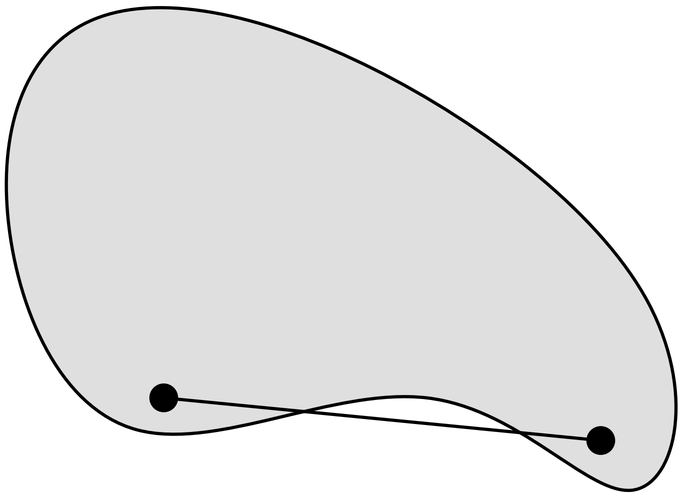

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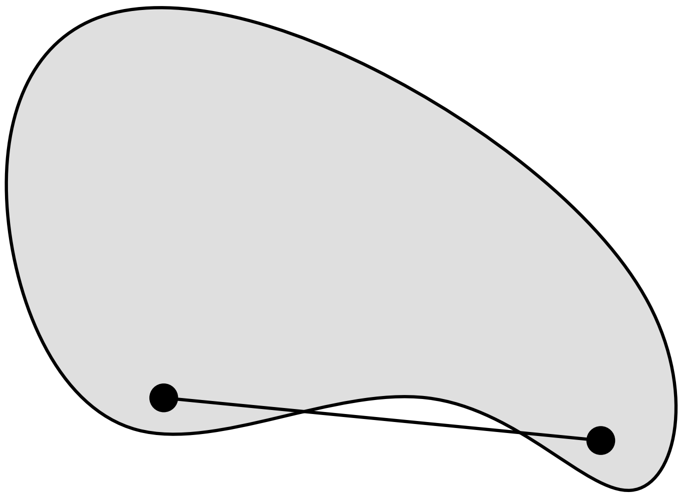

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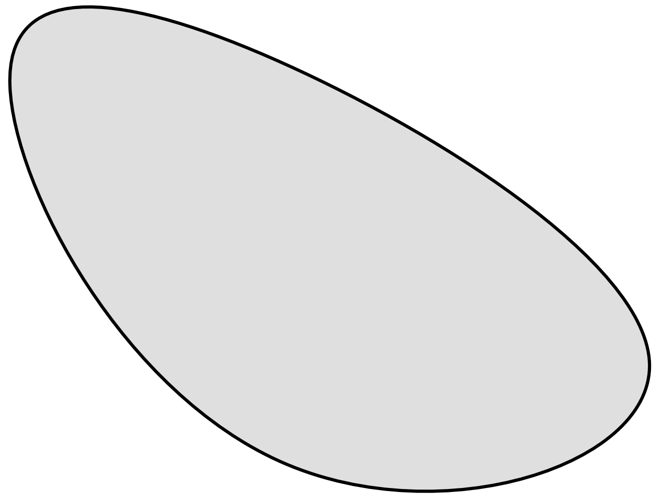

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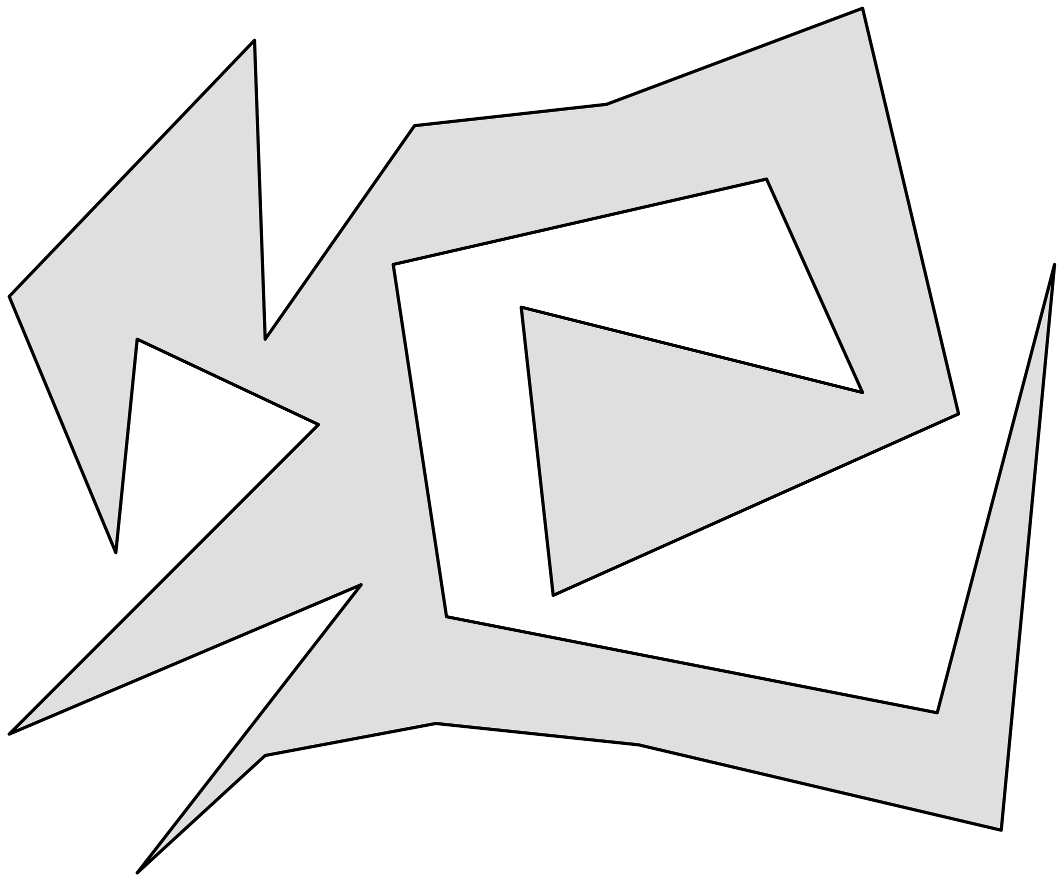

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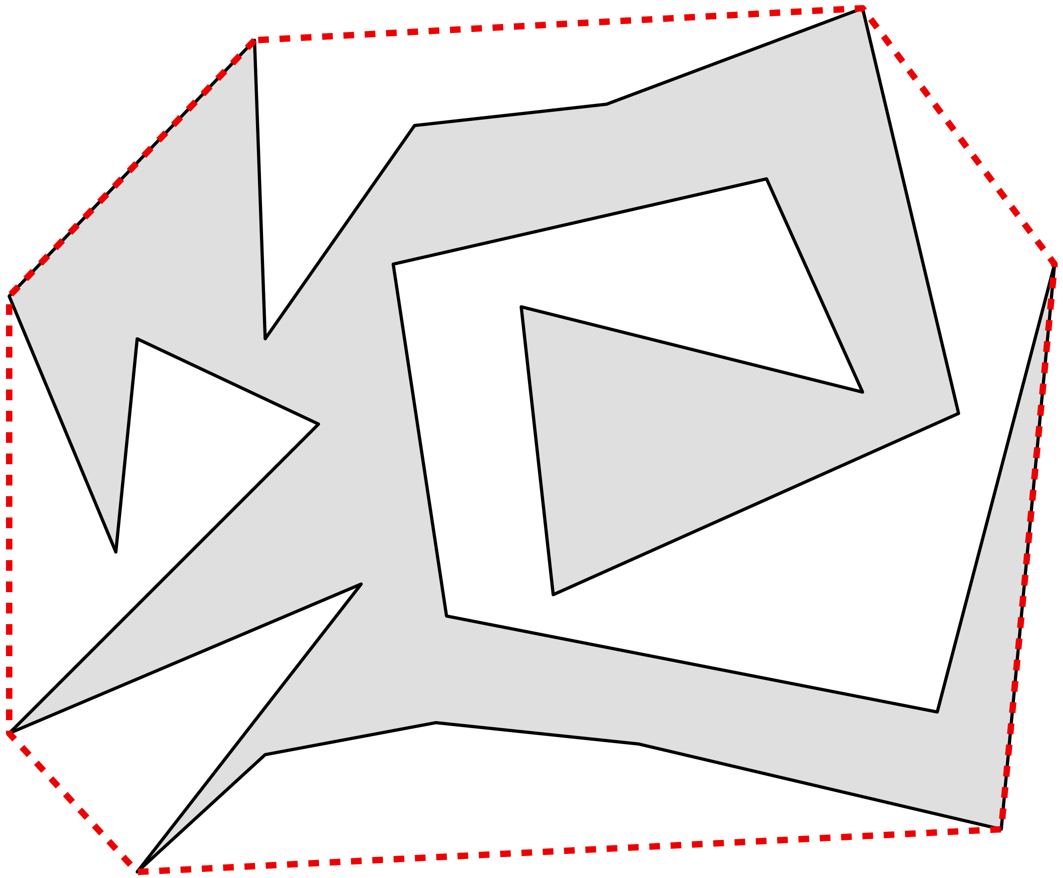

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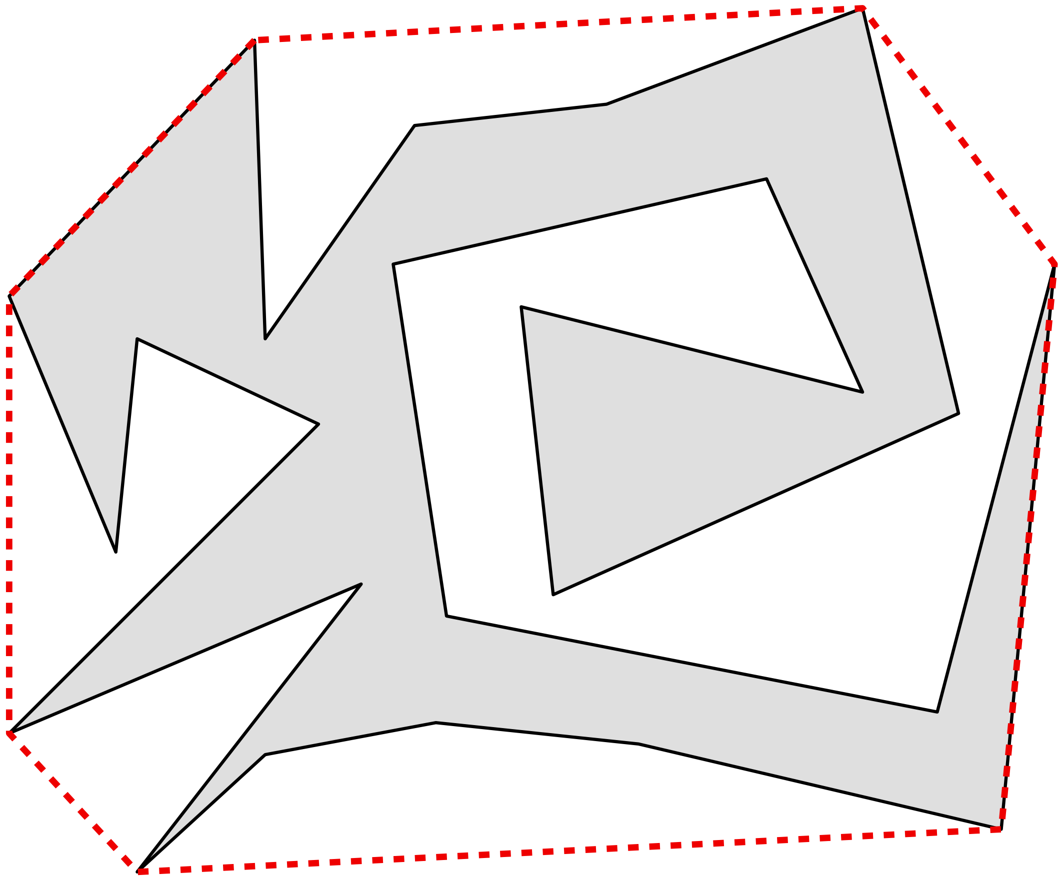

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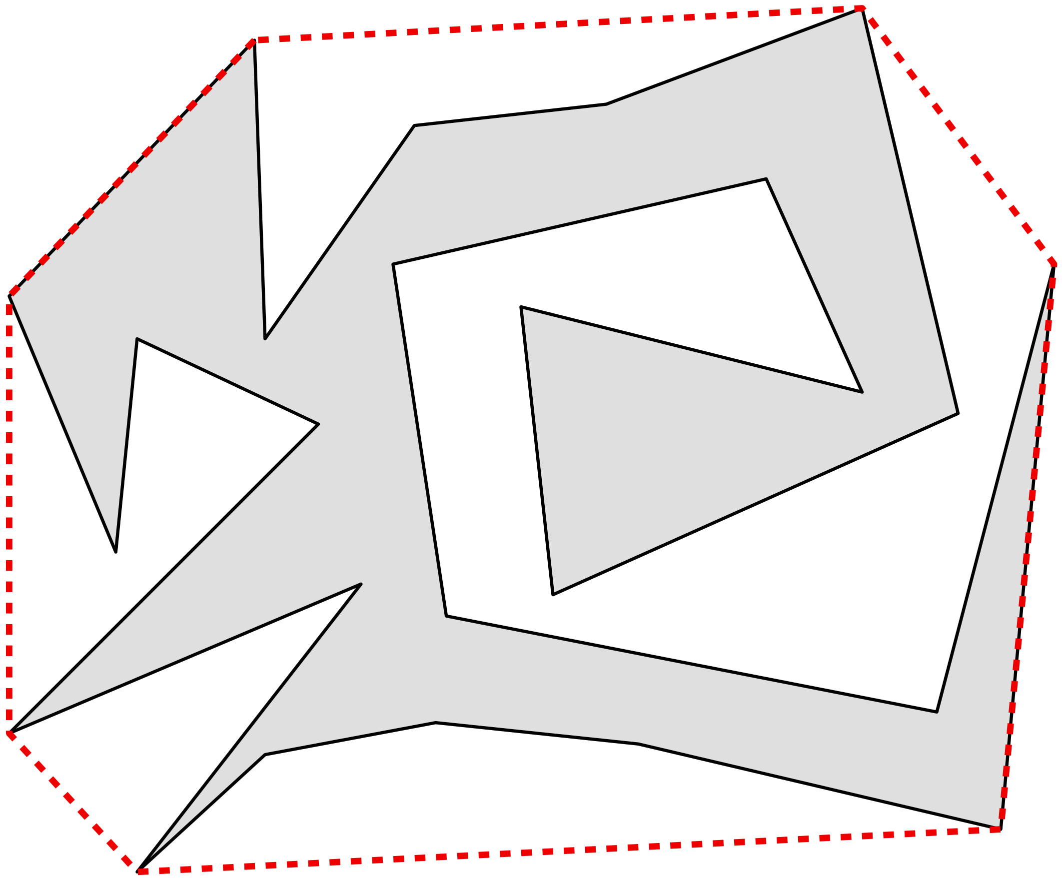

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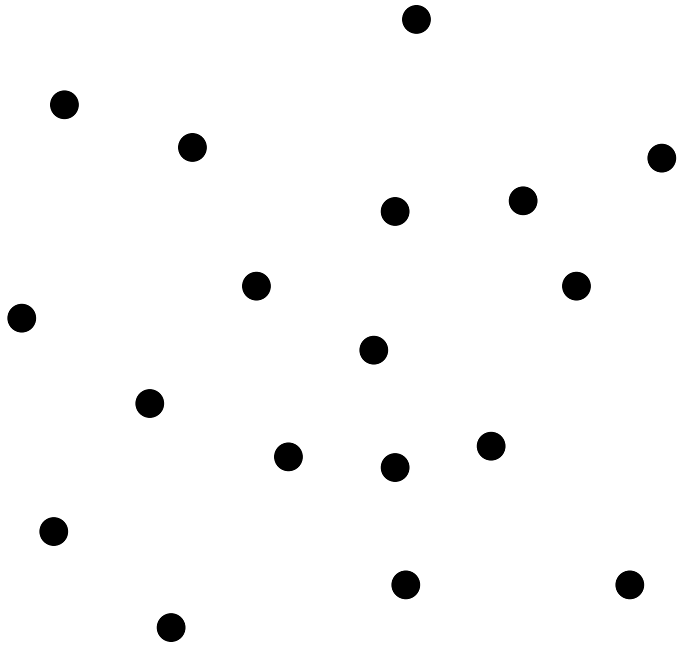

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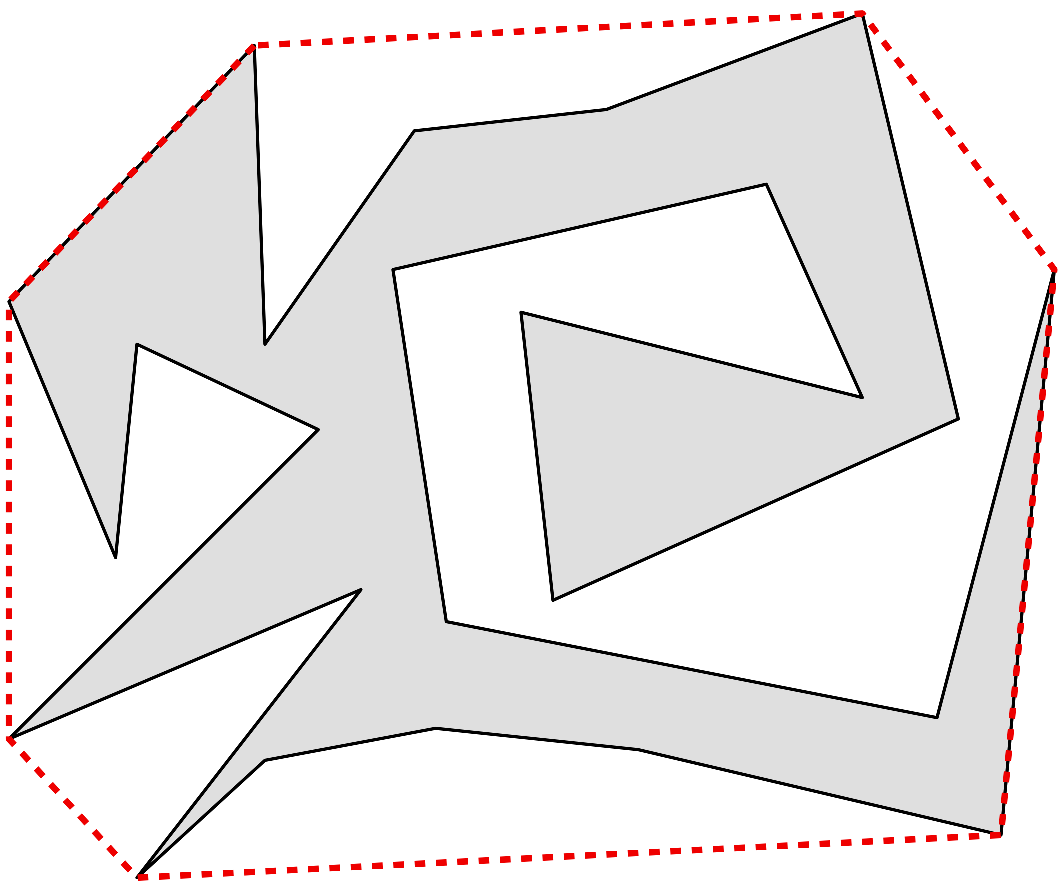

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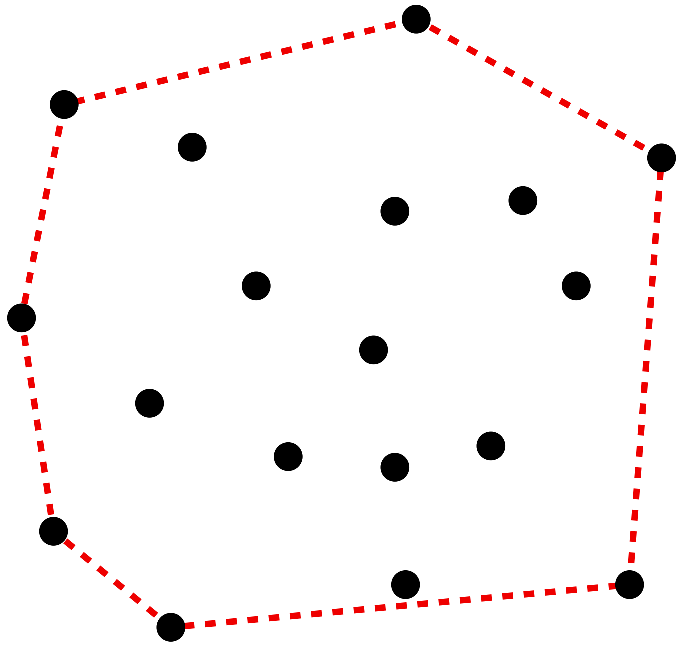

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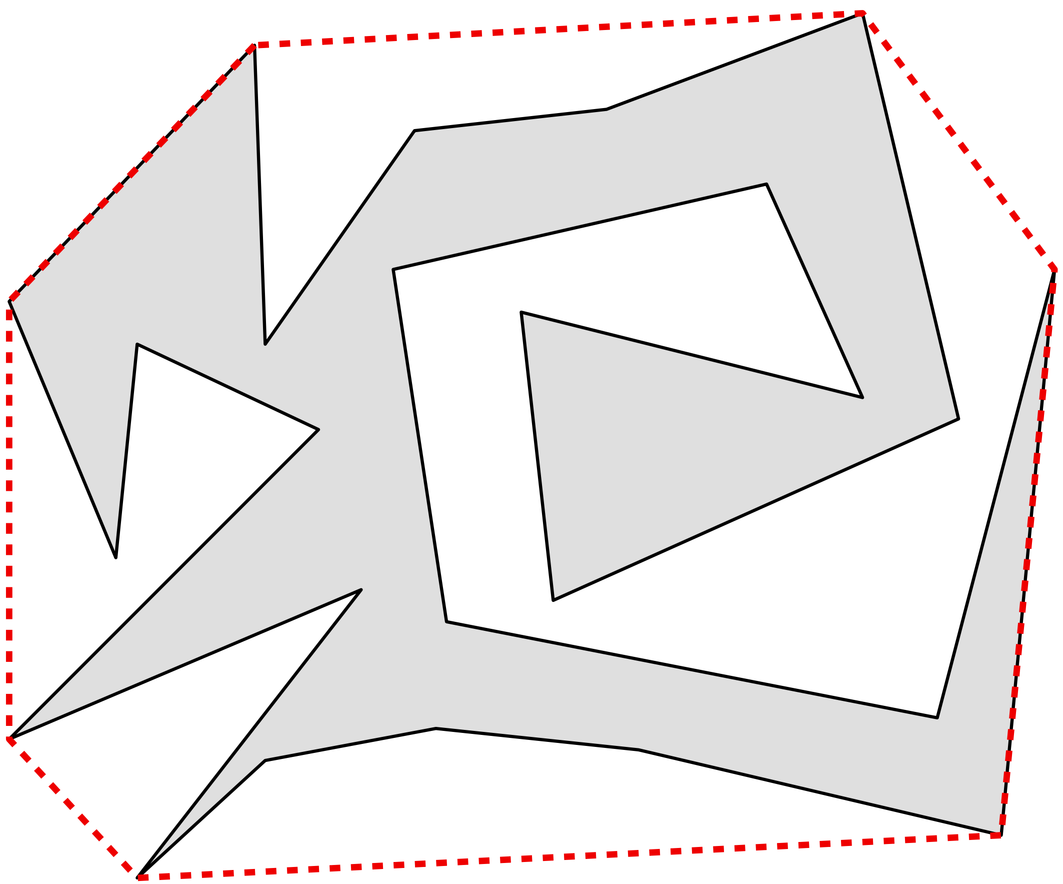

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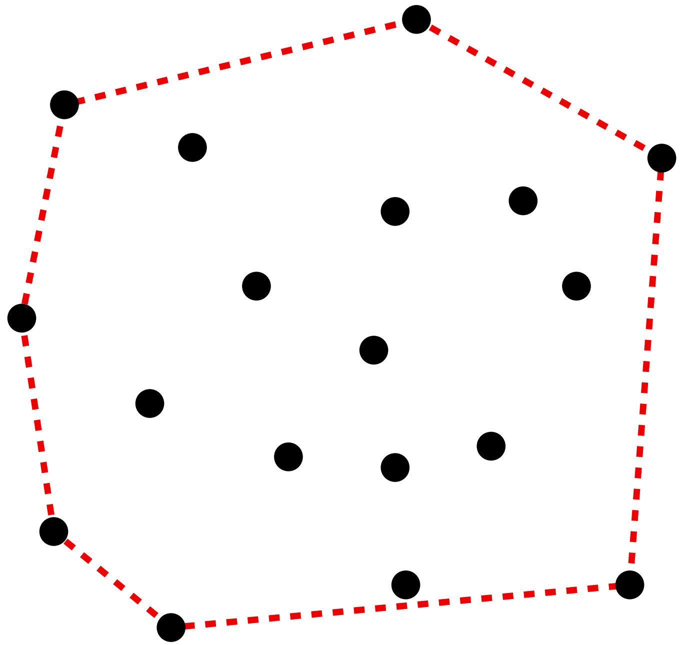

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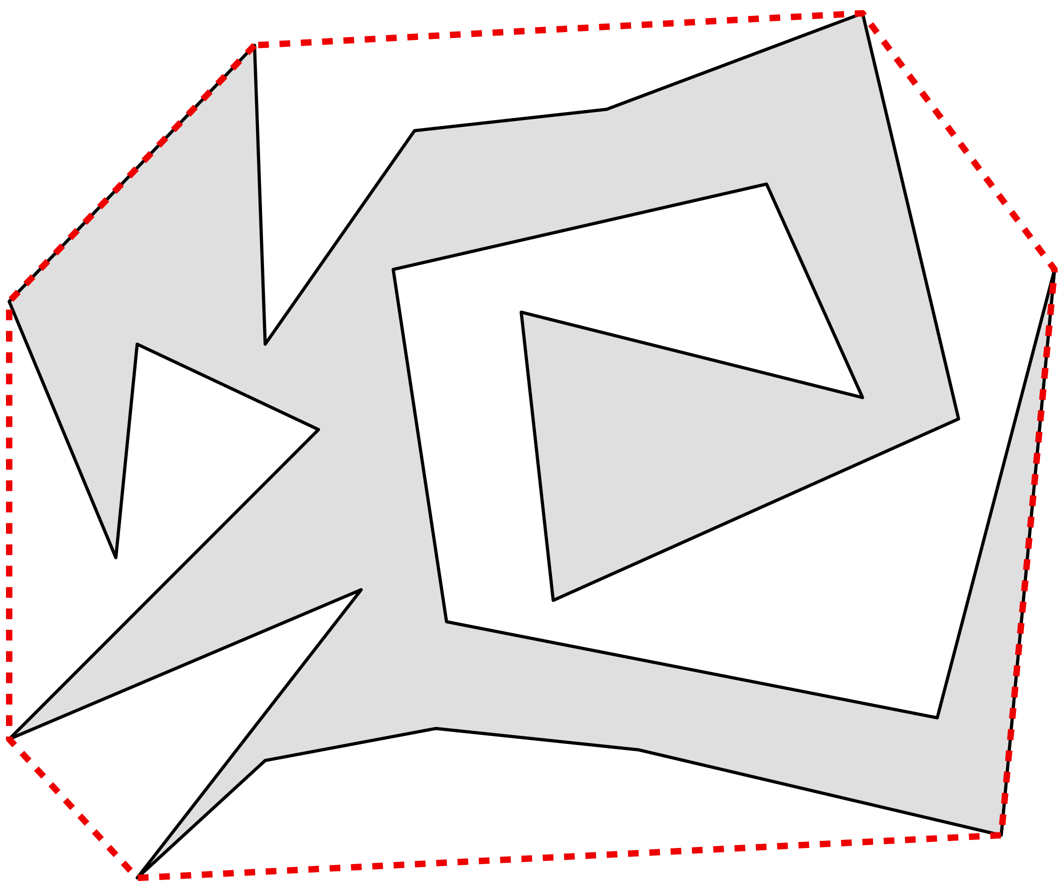

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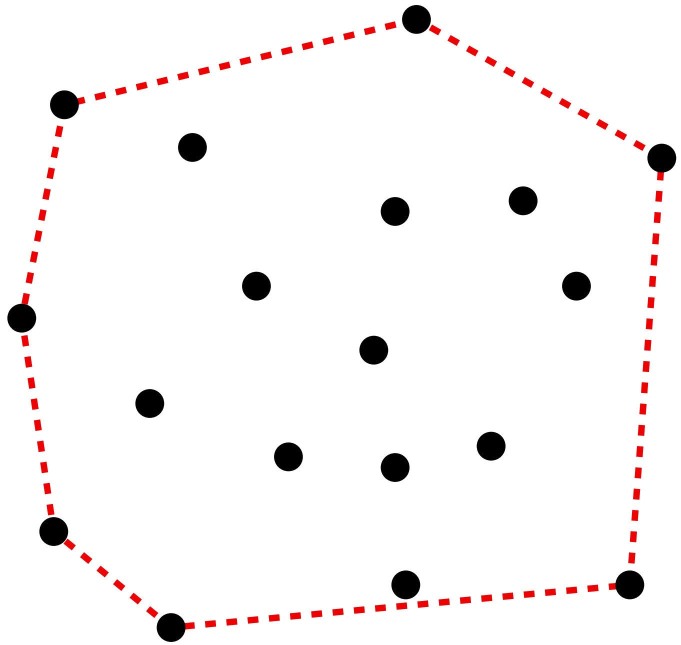

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In both cases, the vertices of $ch(X)$ are points of X.

Computing the extreme points

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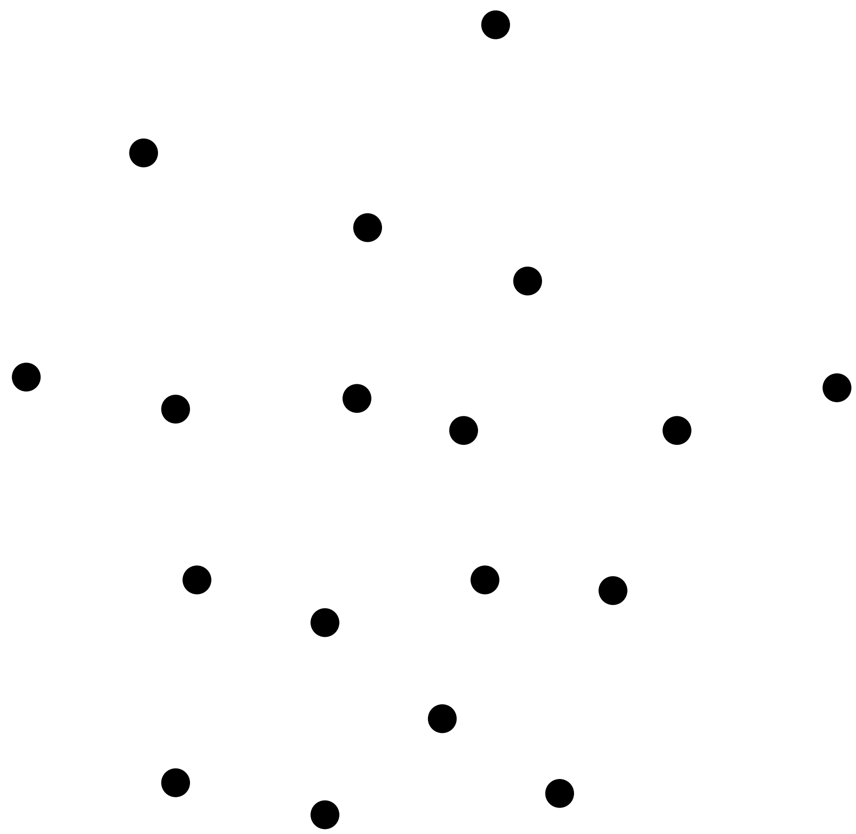

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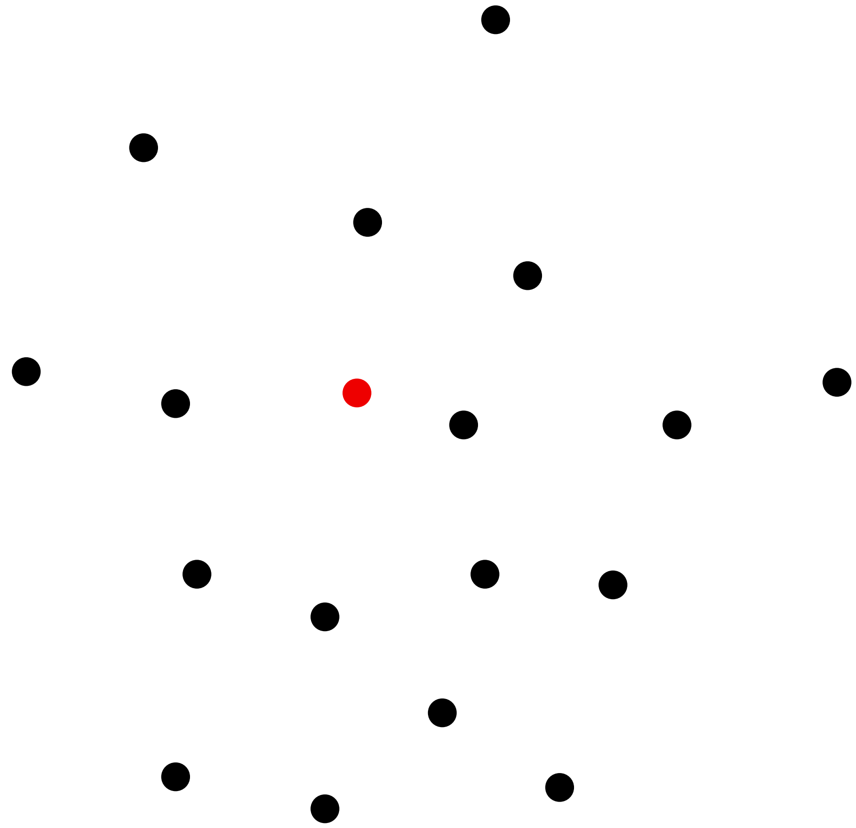

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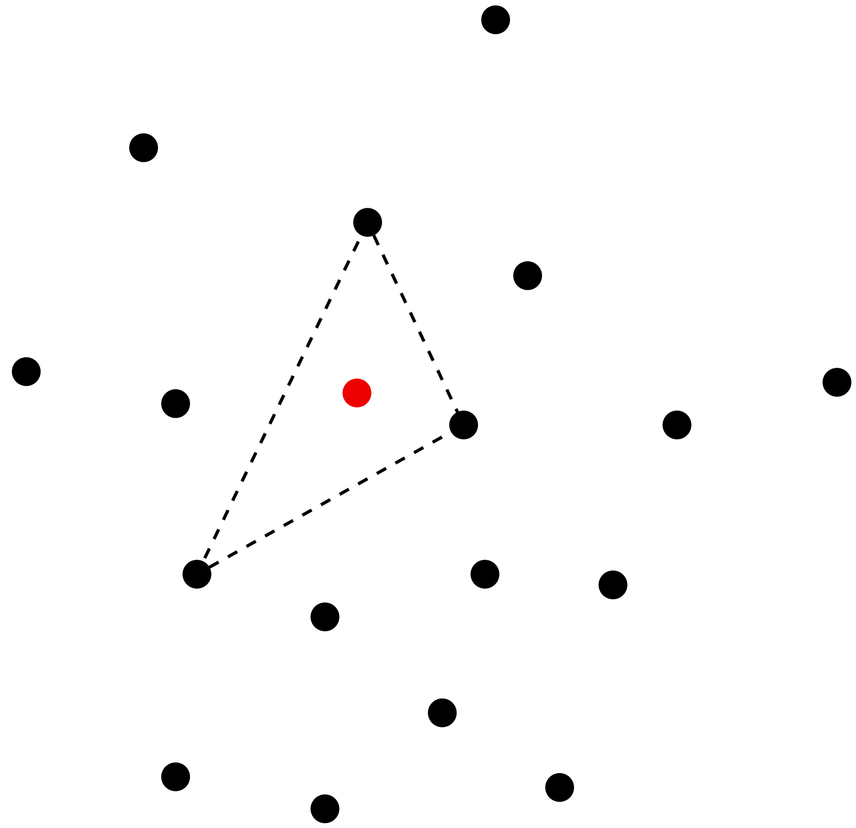

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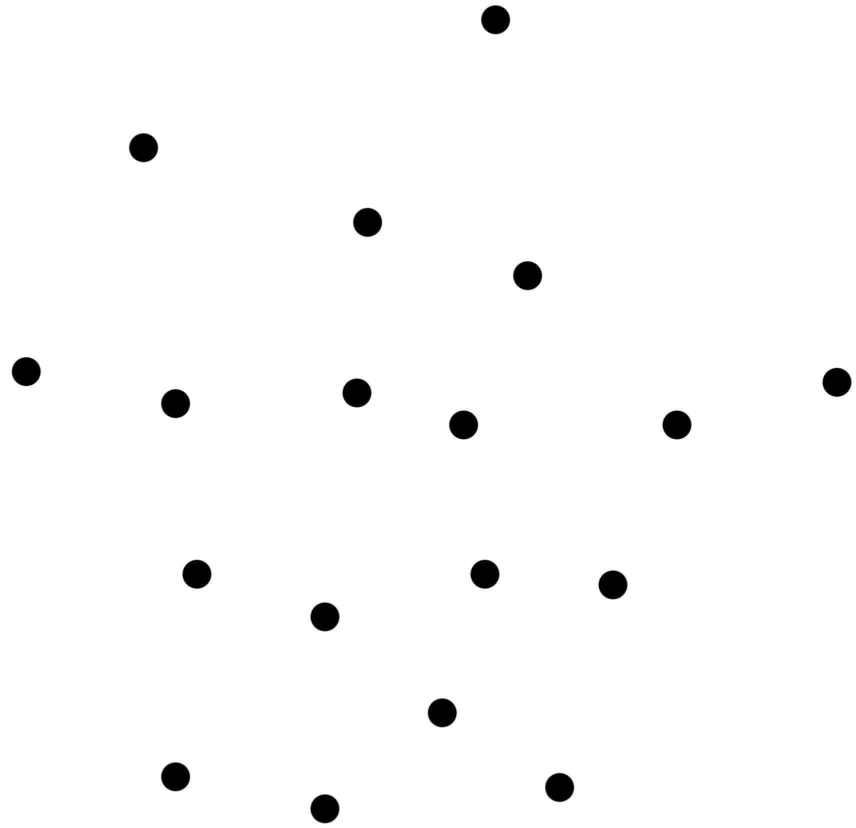

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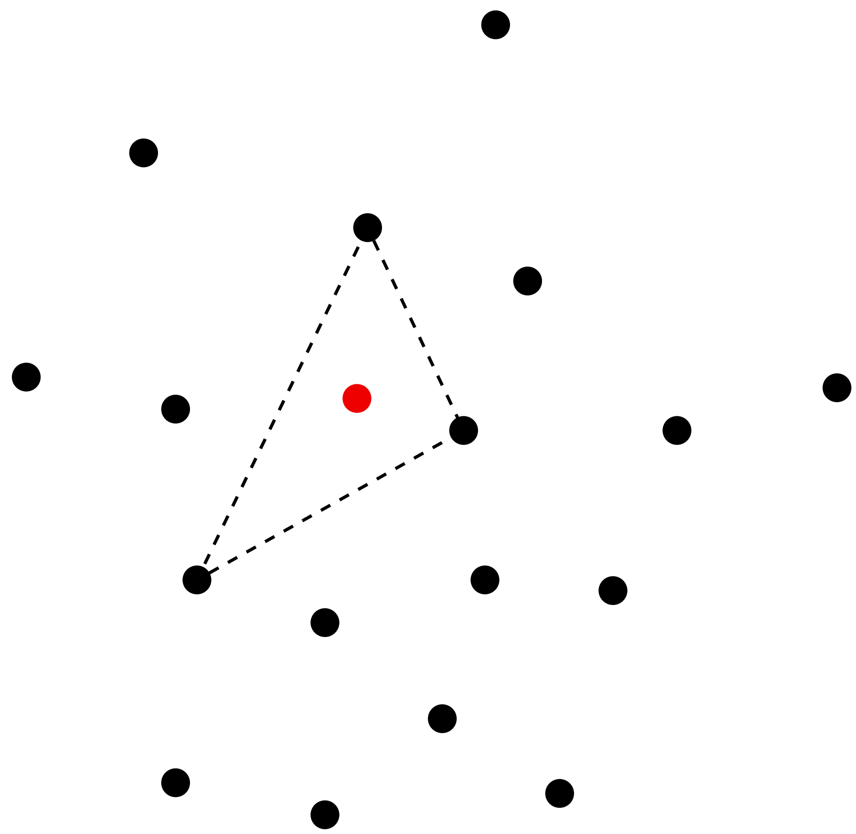

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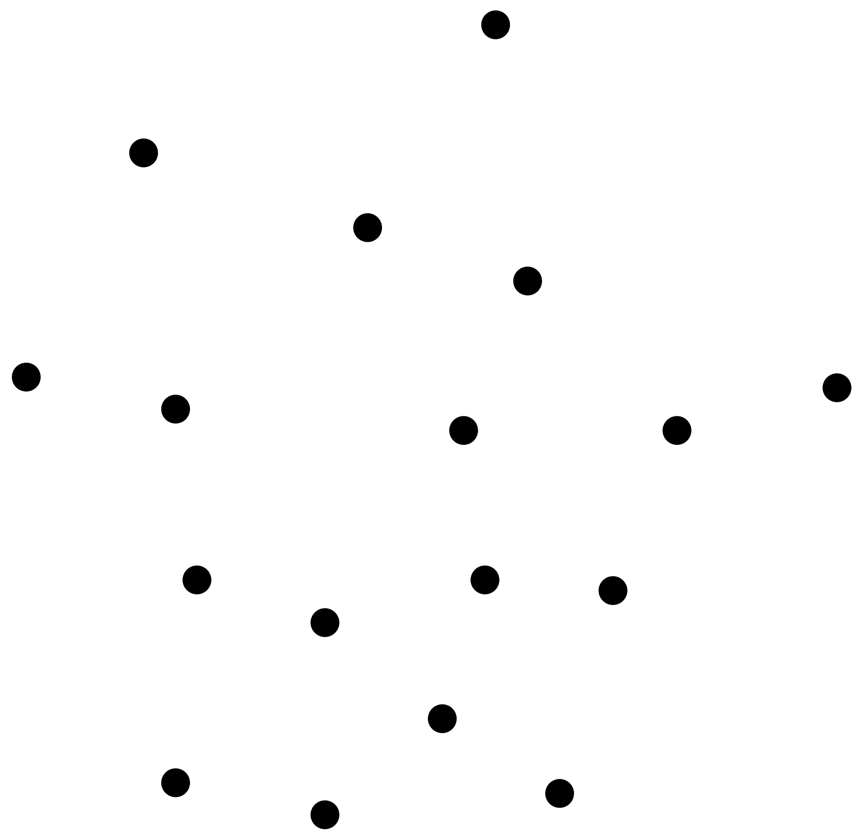

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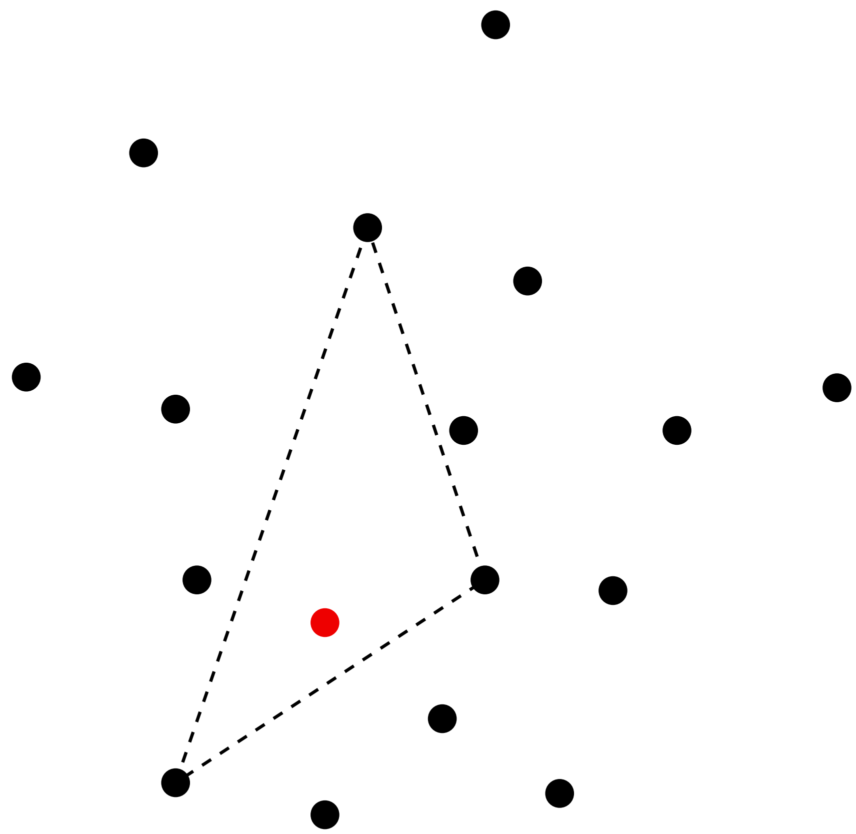

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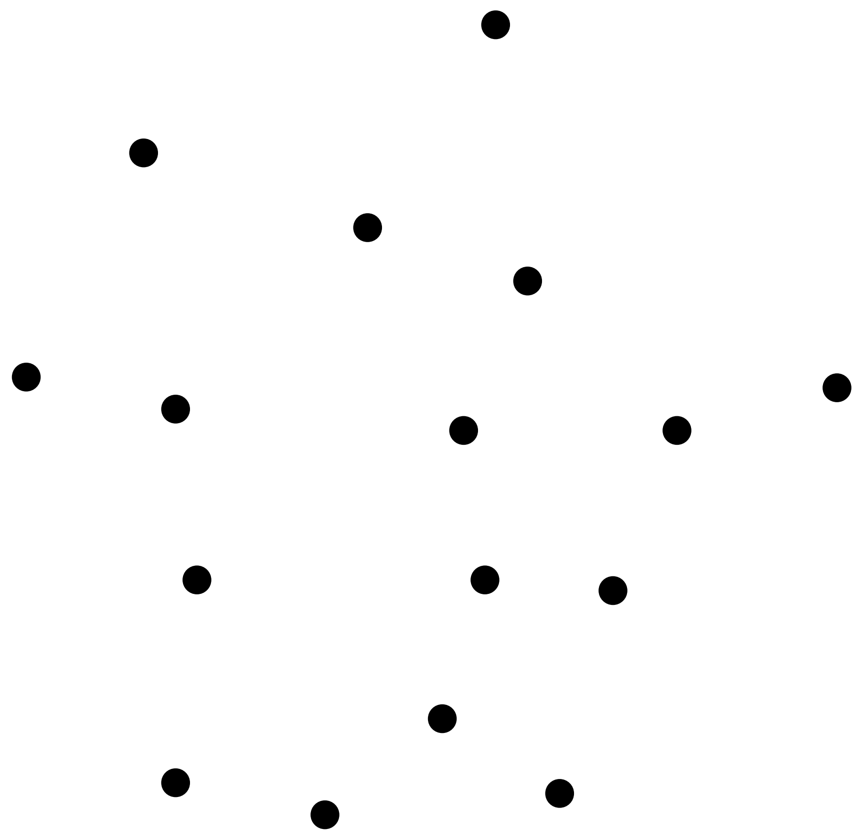

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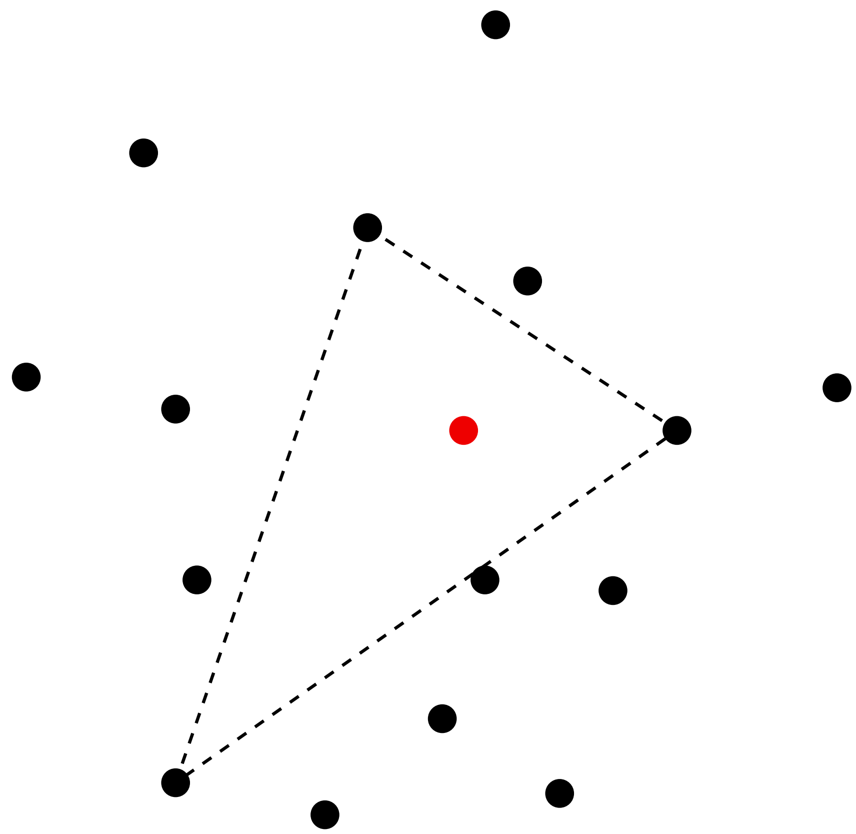

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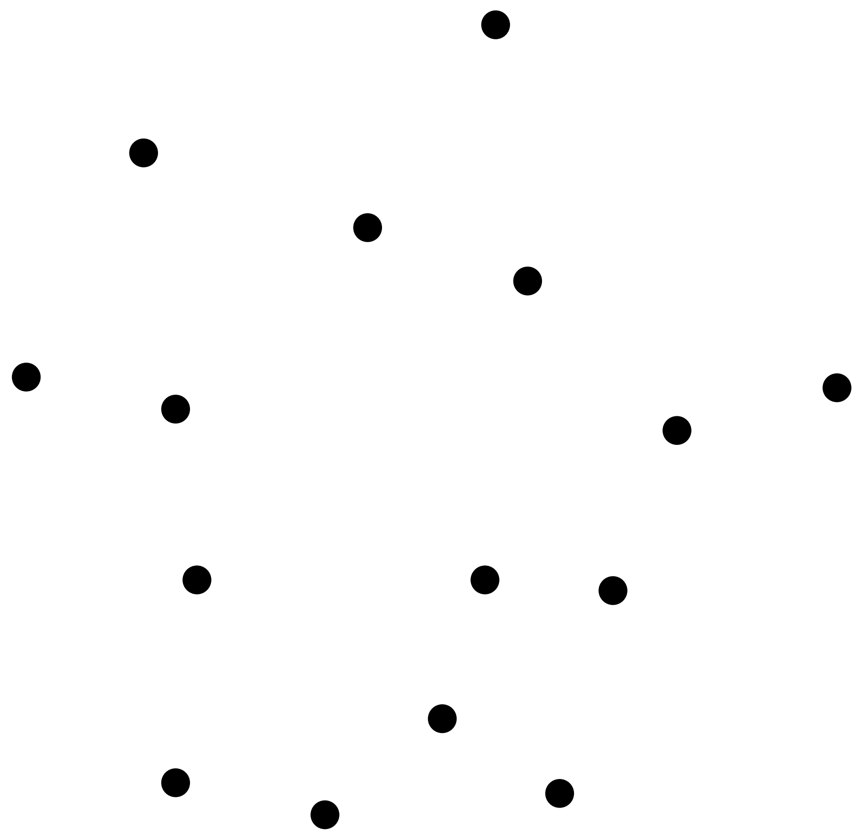

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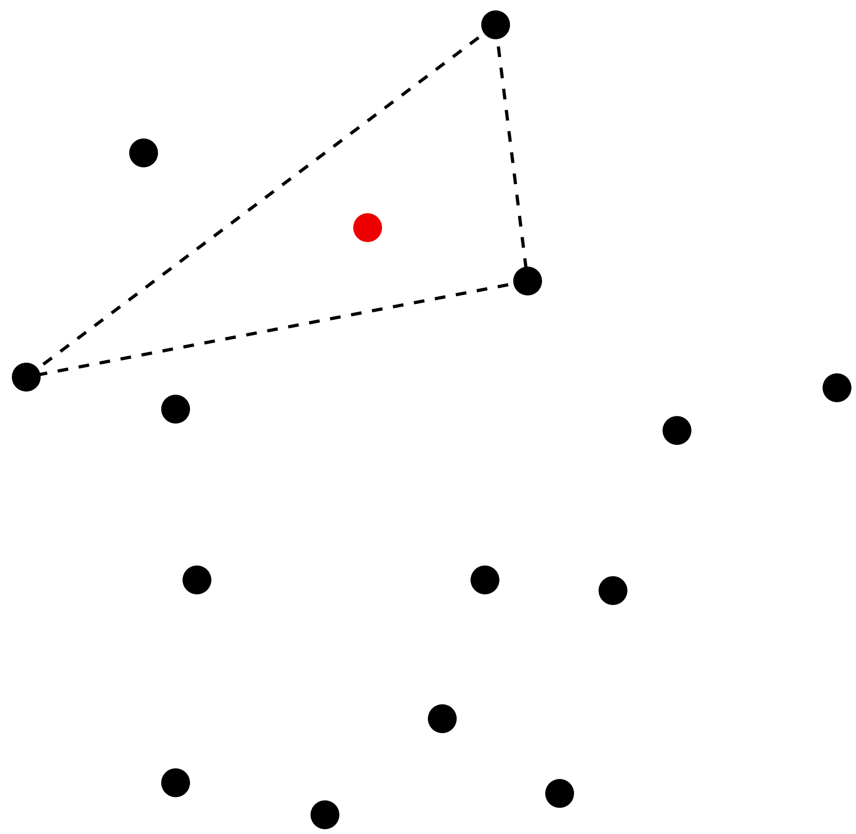

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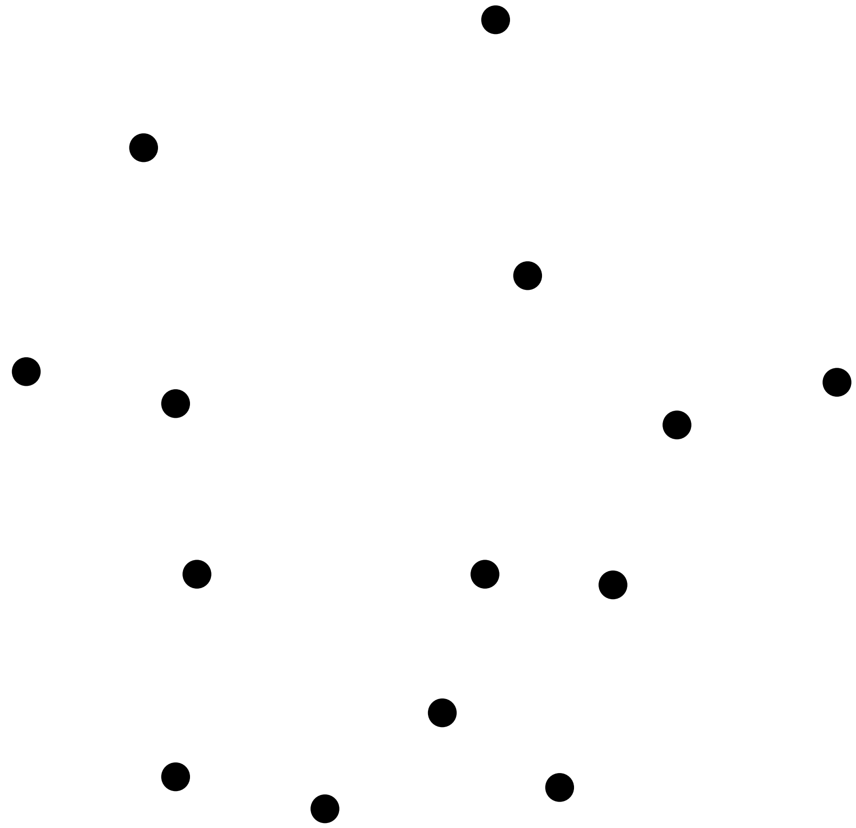

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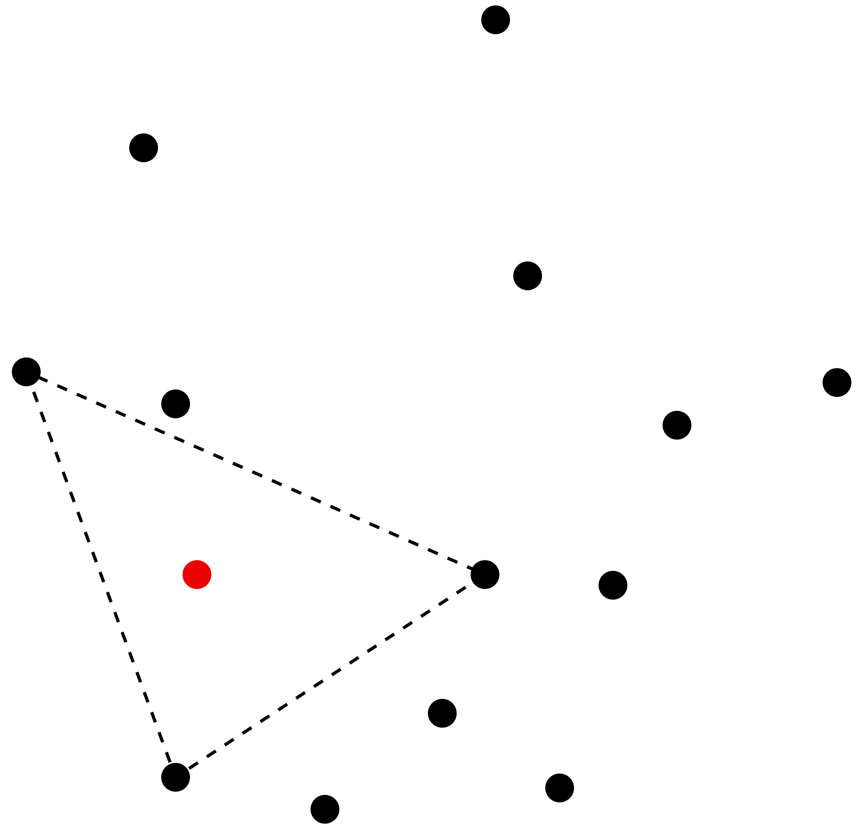

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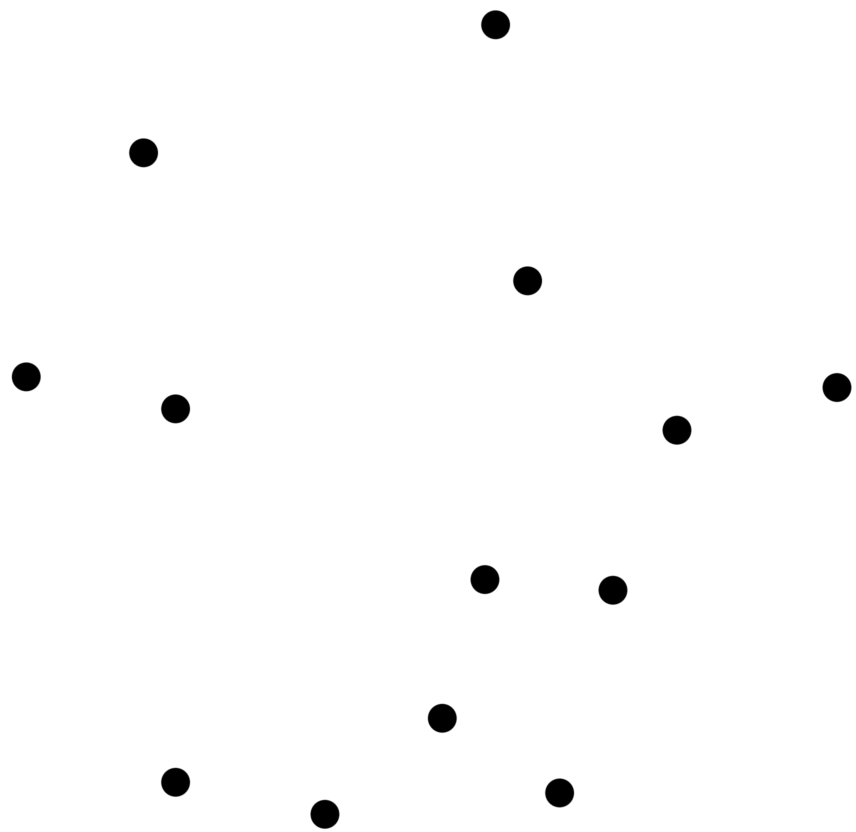

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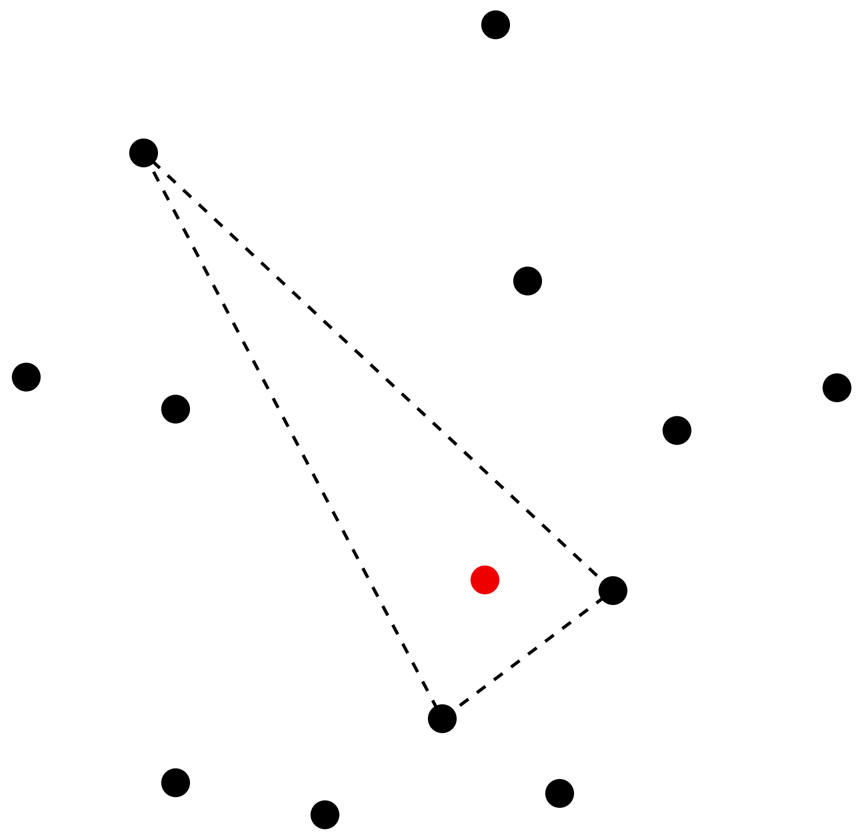

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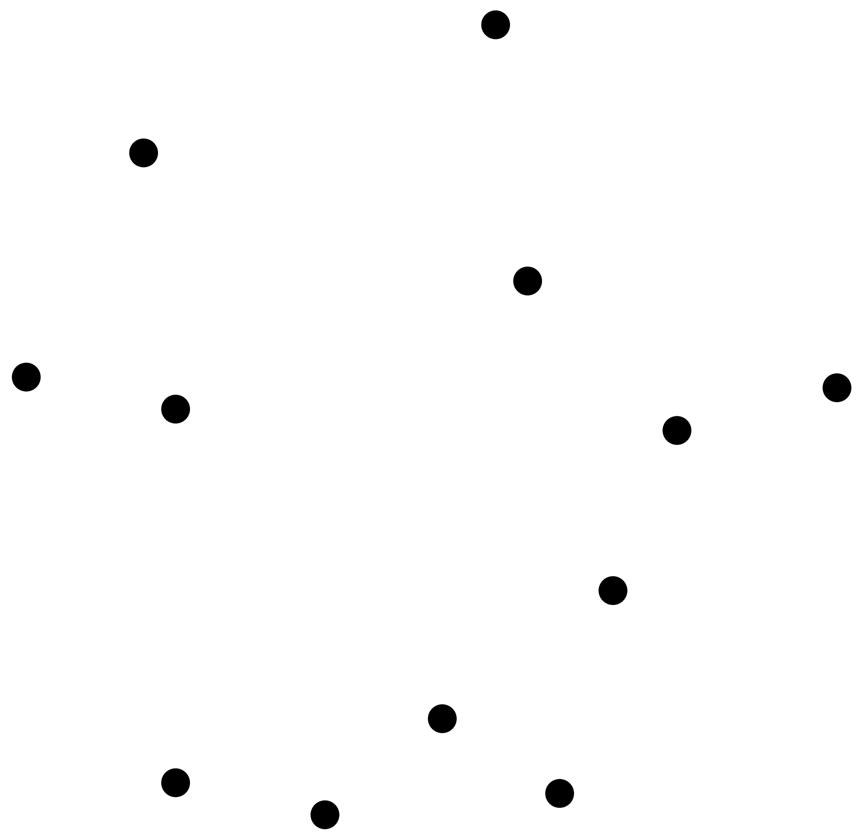

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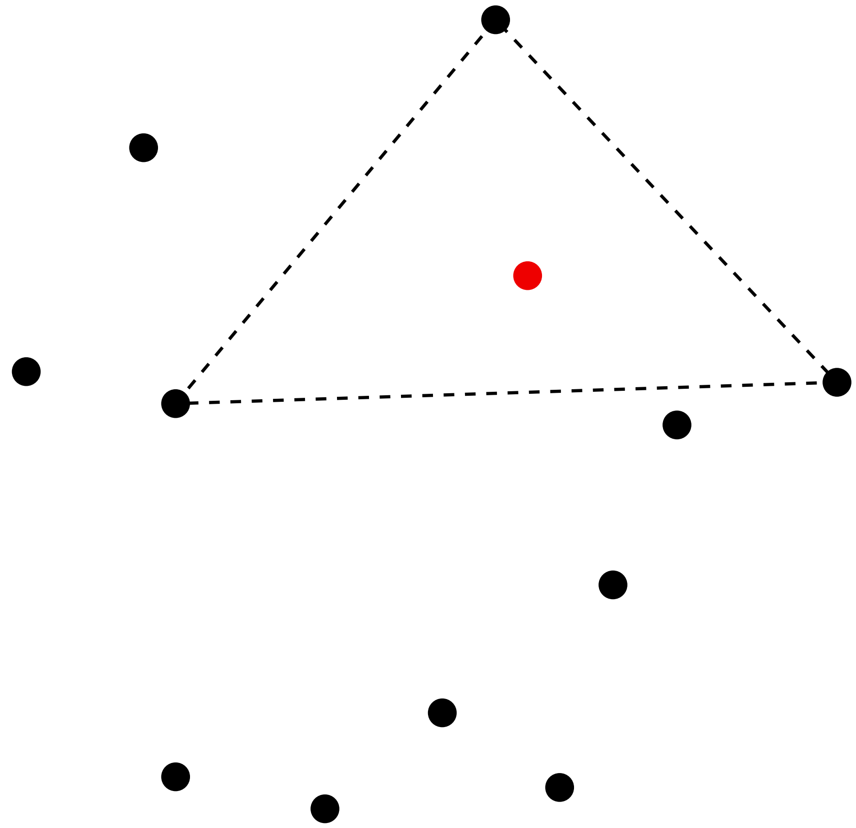

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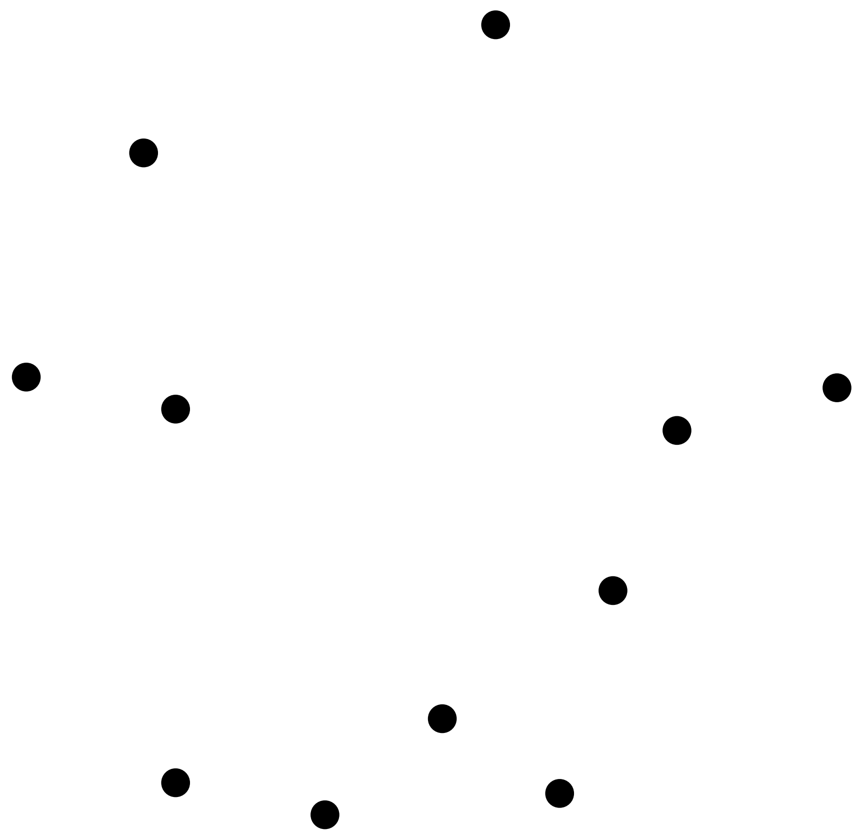

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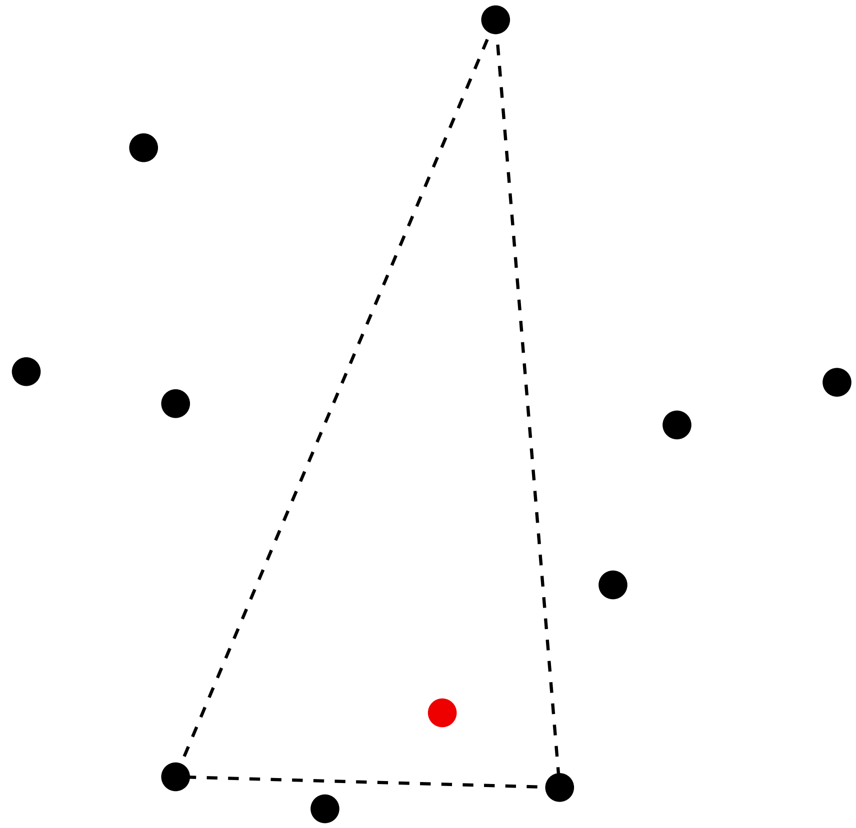

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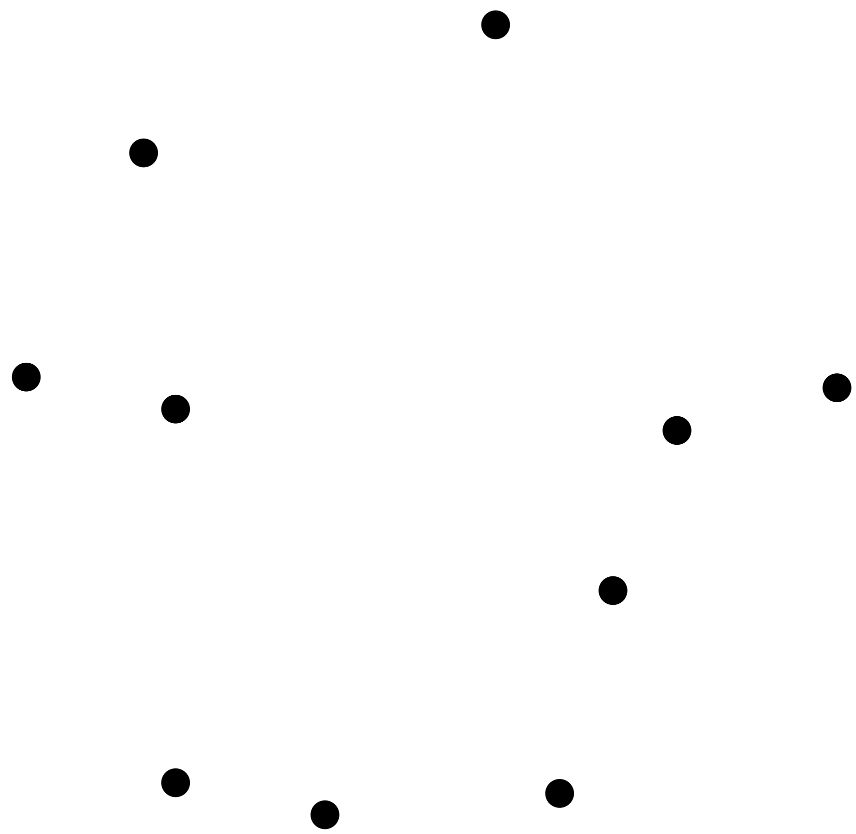

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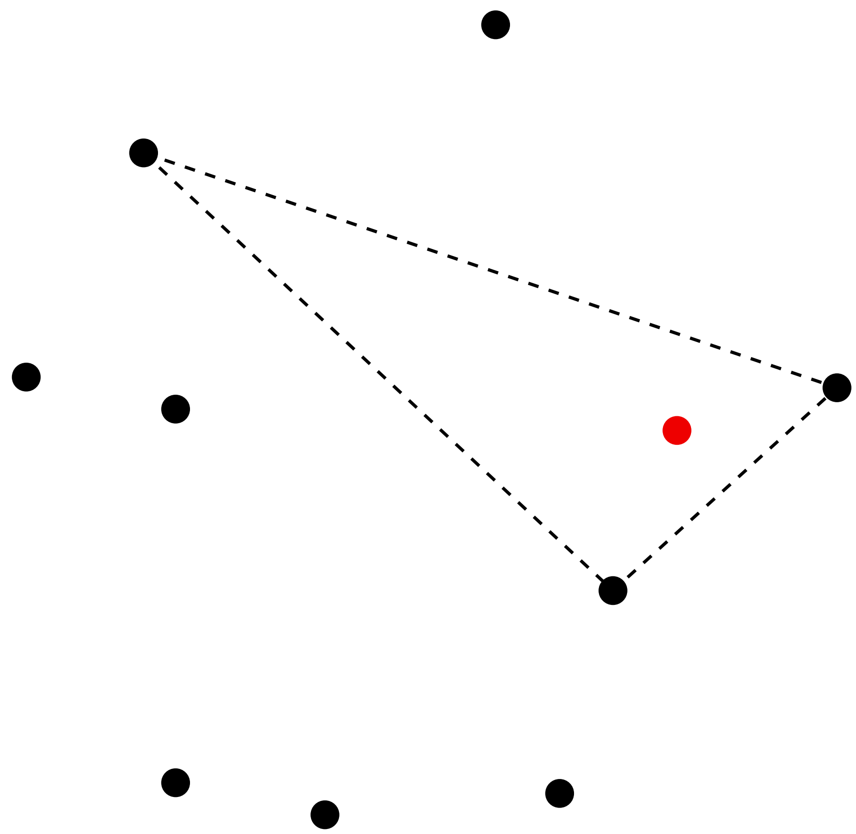

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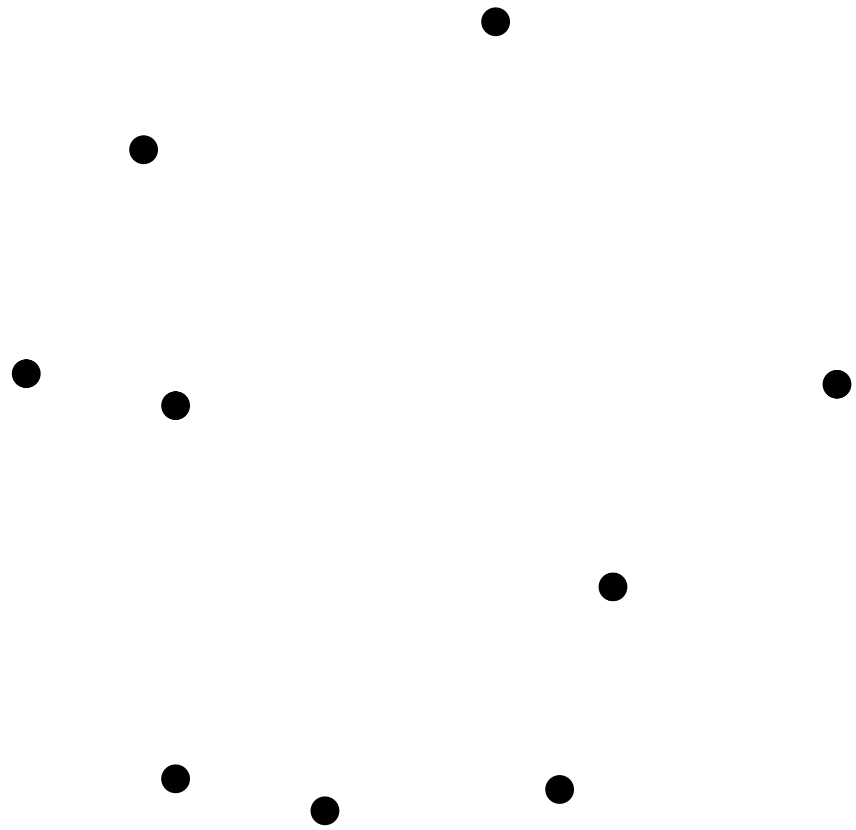

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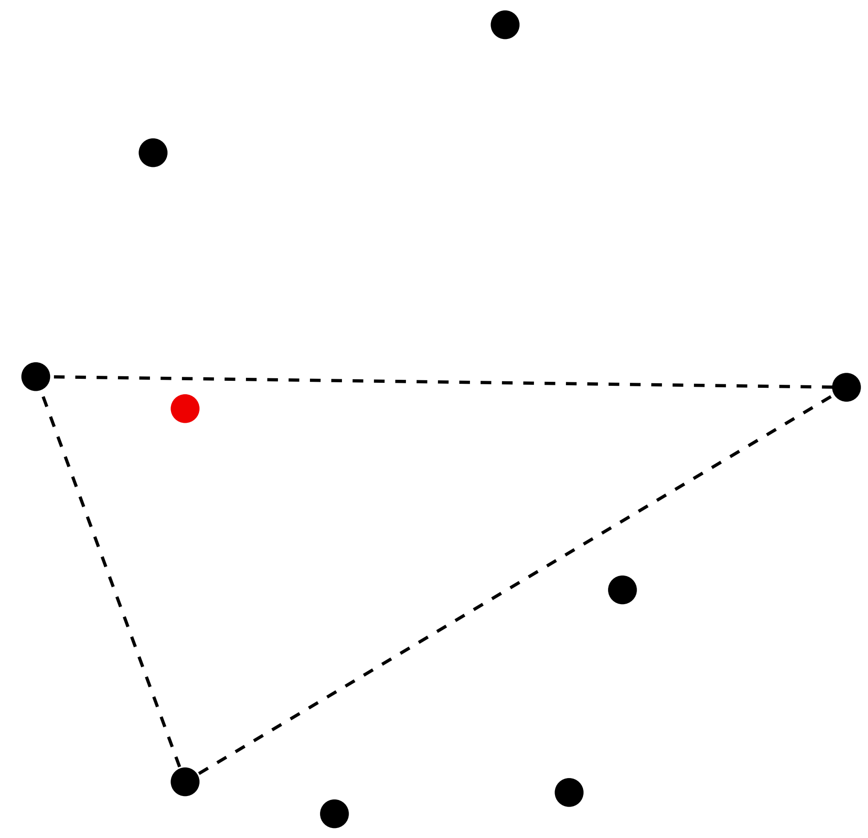
Given $X = \{p_1, \ldots, p_n\}$, the point p_i belongs to the boundary of the convex hull of X if and only if p_i does not lie in any of the triangles $p_j p_k p_l$.

Algorithm

Input: $p_1, \ldots, p_n$
Output: set of the extreme points

Procedure:
  For each i,
    For each $j, k, l \neq i$,
    If p_i lies in the triangle p_j, p_k, p_l, eliminate p_i.
  Return the set of surviving p_i's.



CONVEX HULL

Computing the extreme points

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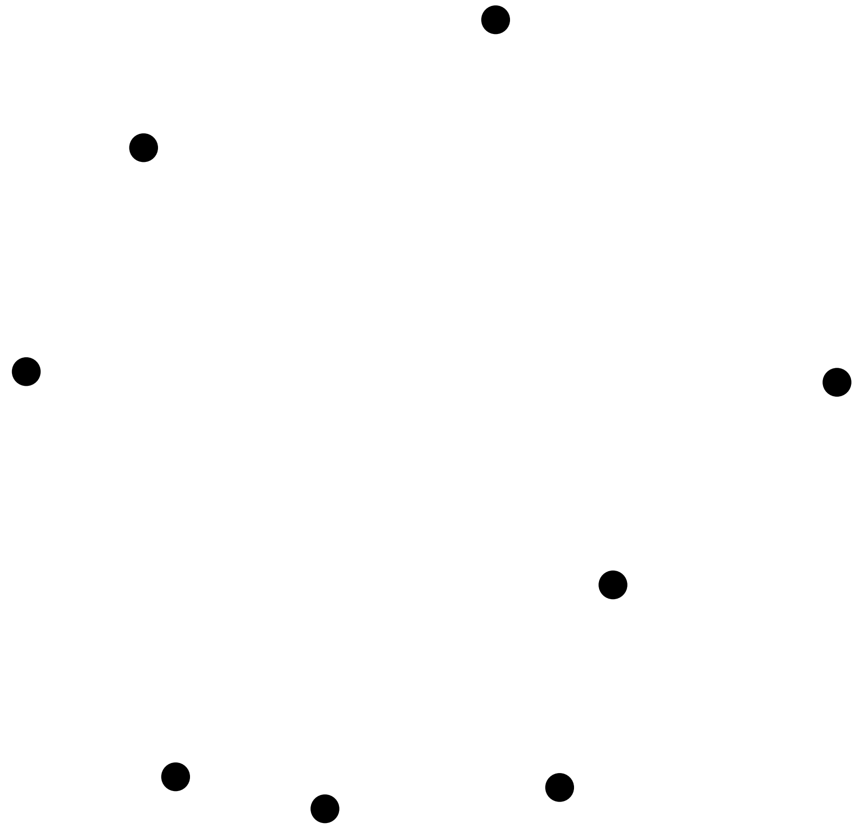

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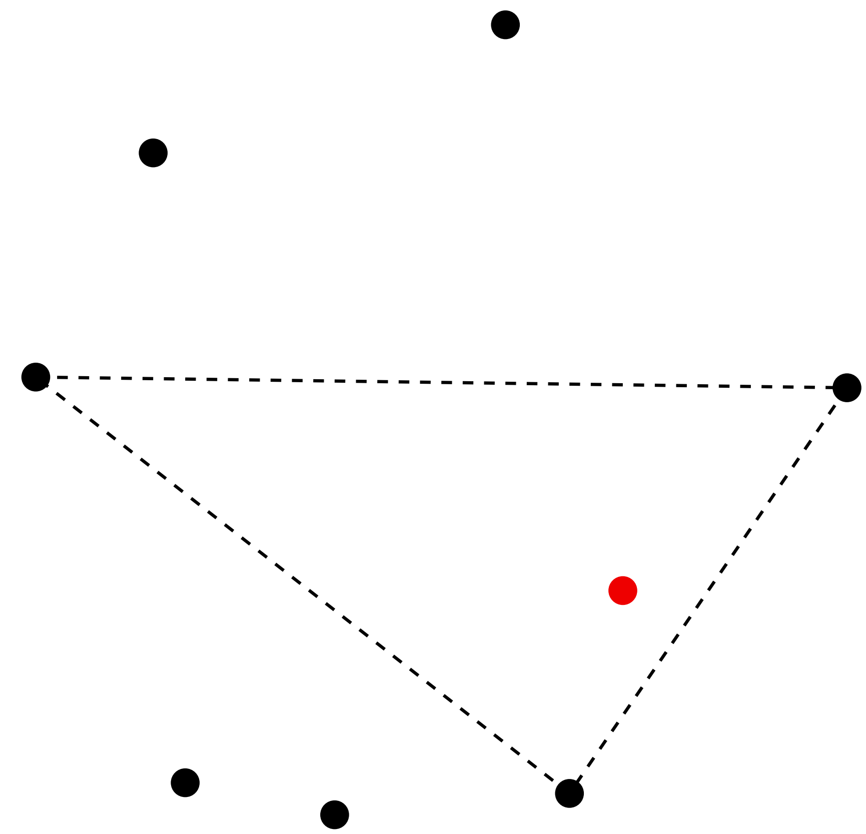

CONVEX HULL

Computing the extreme points

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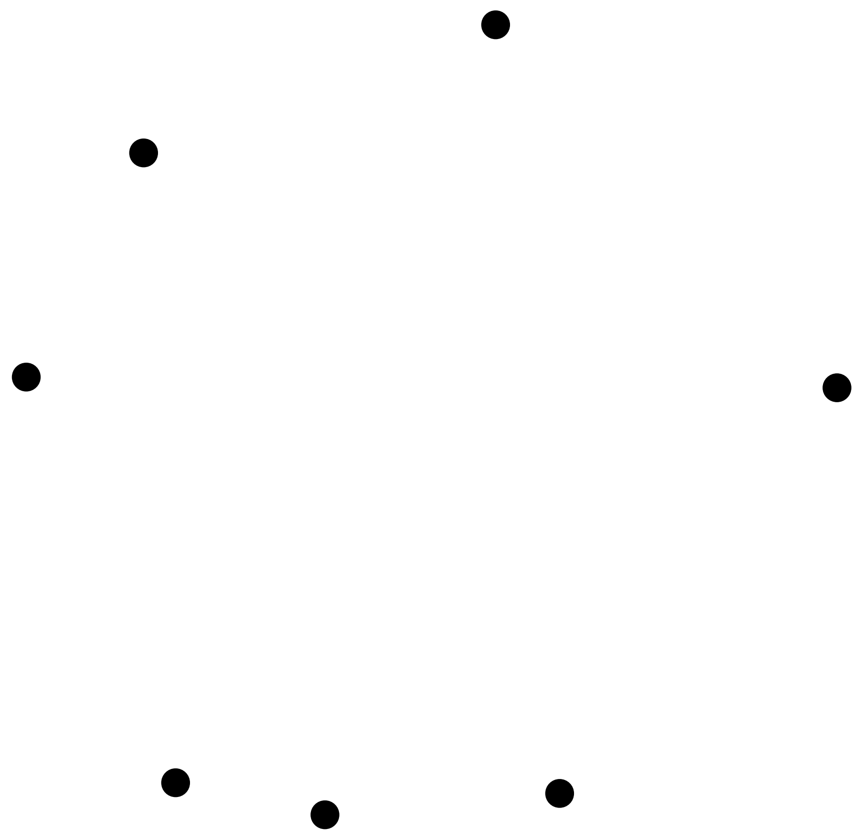

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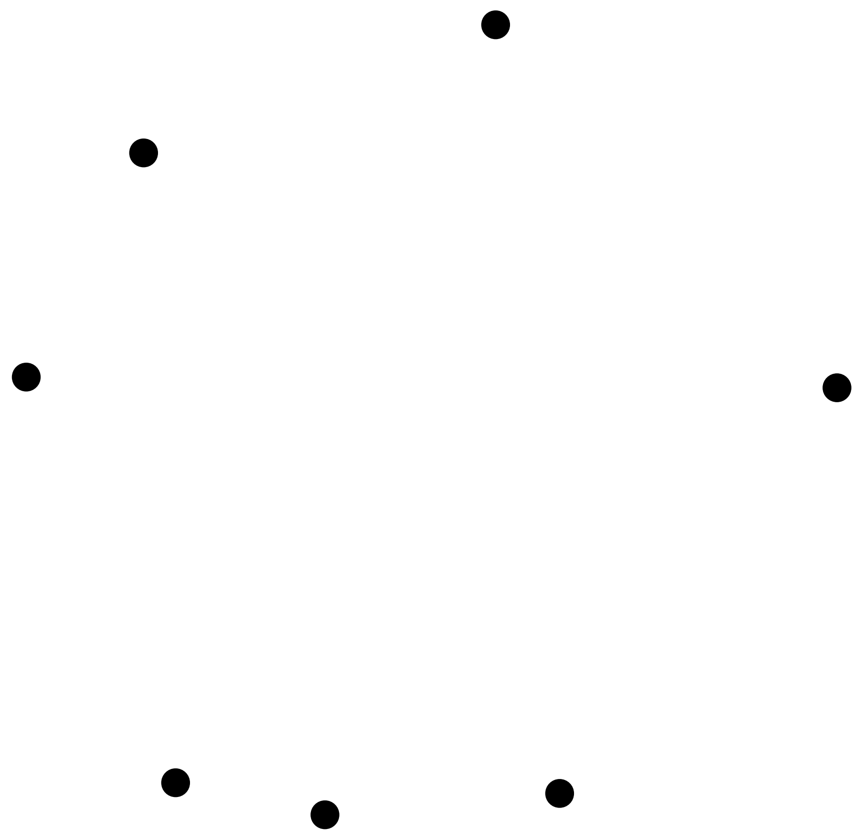
Running time: $\Theta(n^4)$



Computing the extreme segments

Computing the extreme segments

Characterization

Given $X = \{p_1, \ldots, p_n\}$, the segment $p_i p_j$ is an extreme segment if and only if all the points p_k with $k \neq i, j$ lie in the same halfplane defined by the line $p_i p_j$.

Computing the extreme segments

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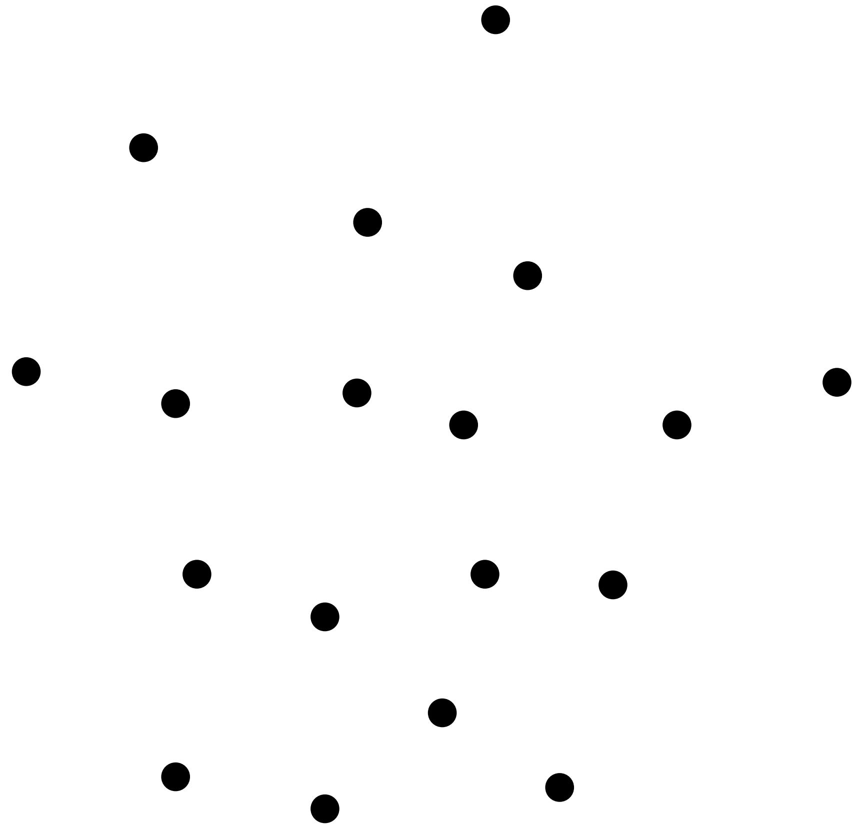
Given $X = \{p_1, \ldots, p_n\}$, the segment $p_i p_j$ is an extreme segment if and only if all the points p_k with $k \neq i, j$ lie in the same halfplane defined by the line $p_i p_j$.



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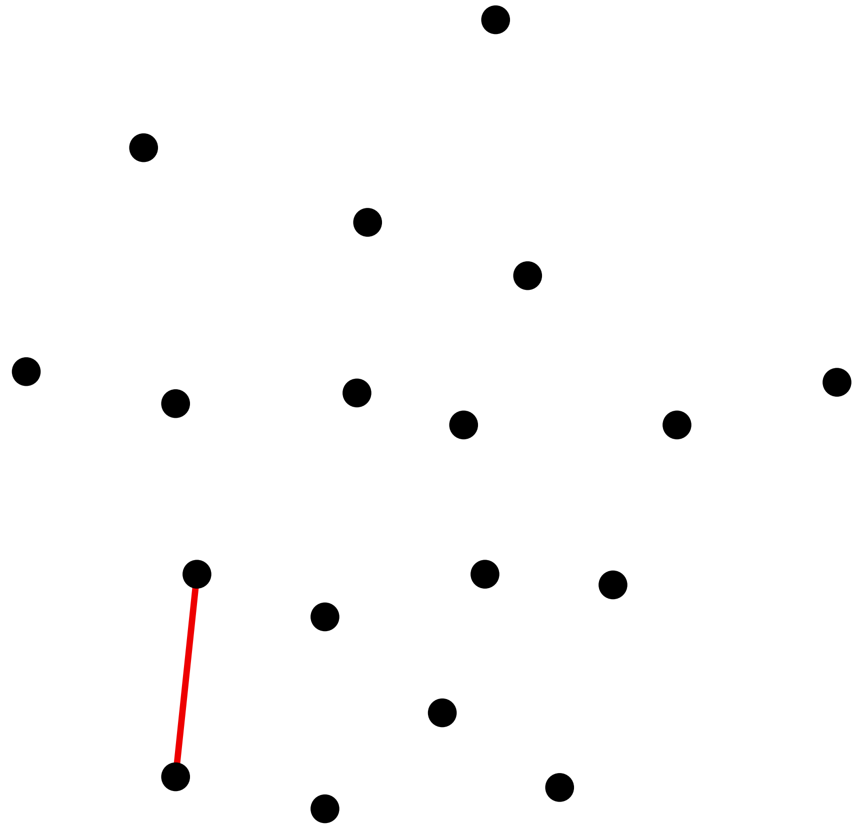
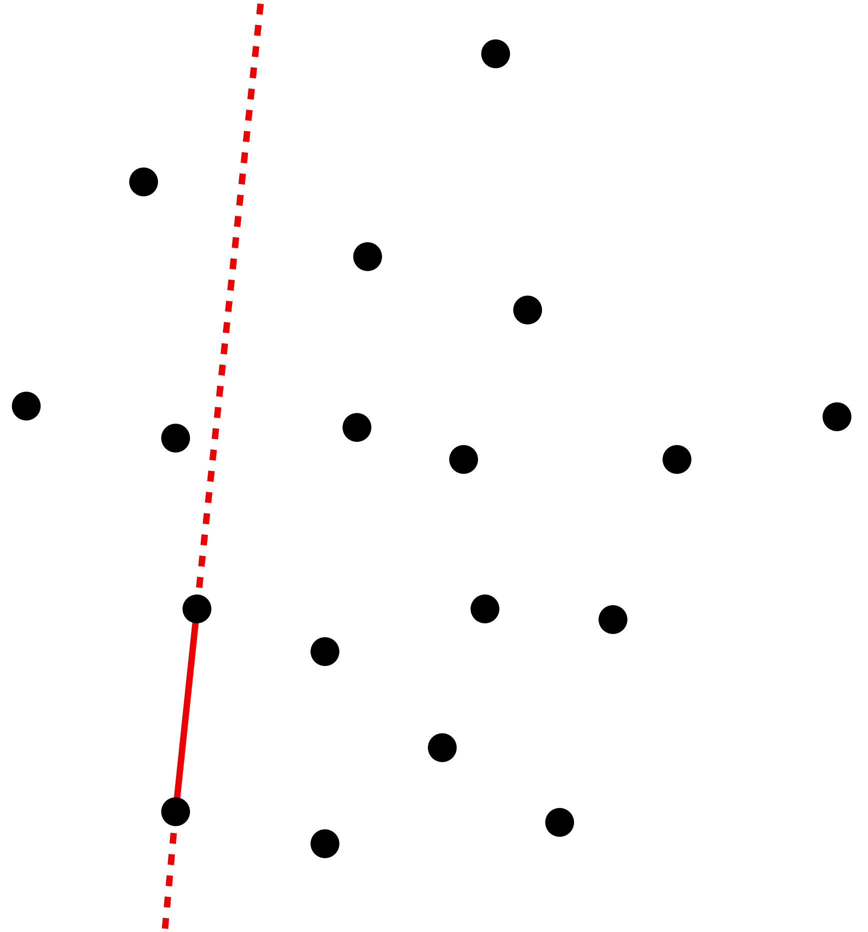


Computing the extreme segments

Characterization

Given $X = \{p_1, \ldots, p_n\}$, the segment p_ip_j is an extreme segment if and only if all the points p_k with $k \neq i, j$ lie in the same halfplane defined by the line p_ip_j.



Computing the extreme segments

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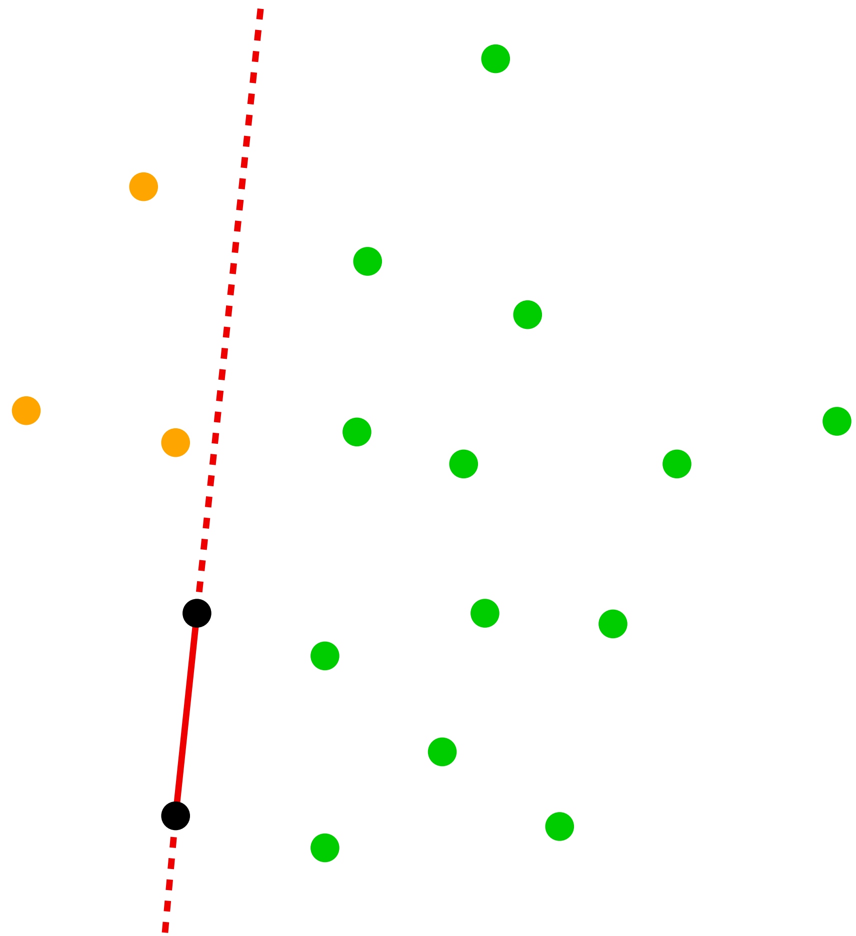

Computing the extreme segments

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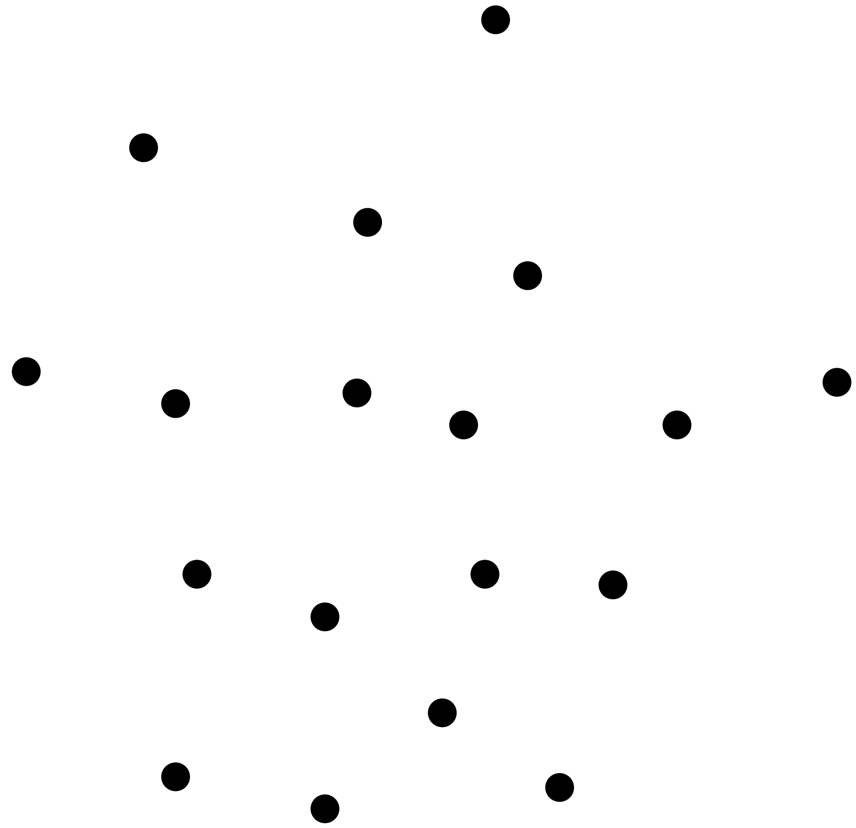

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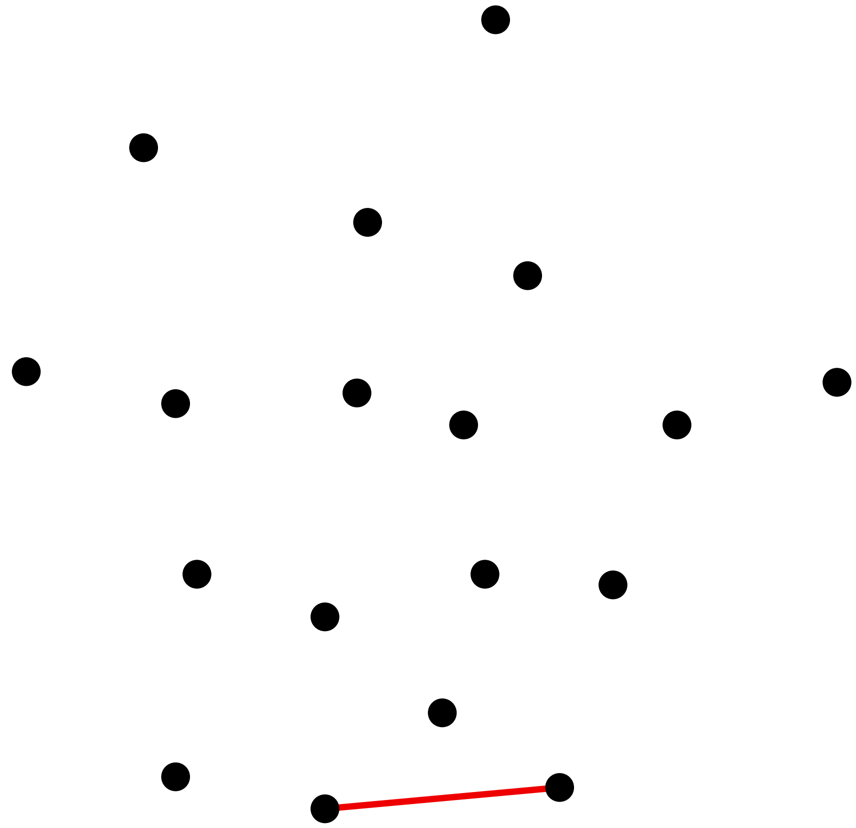

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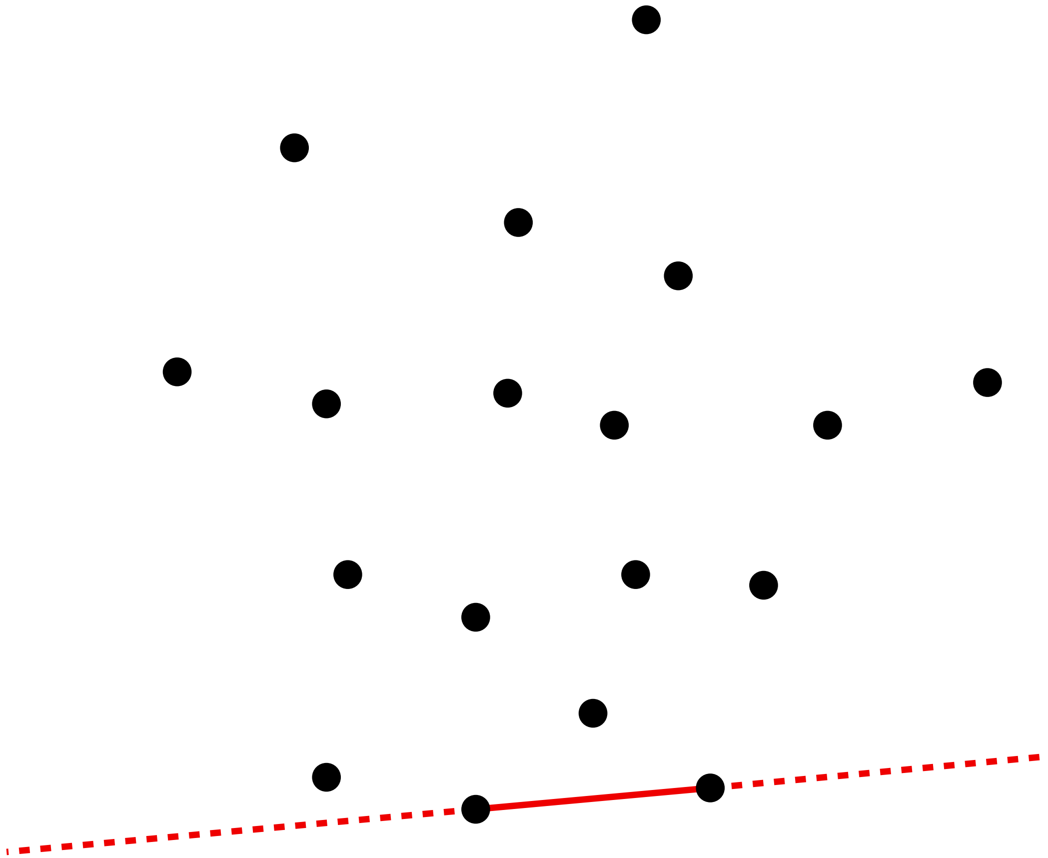

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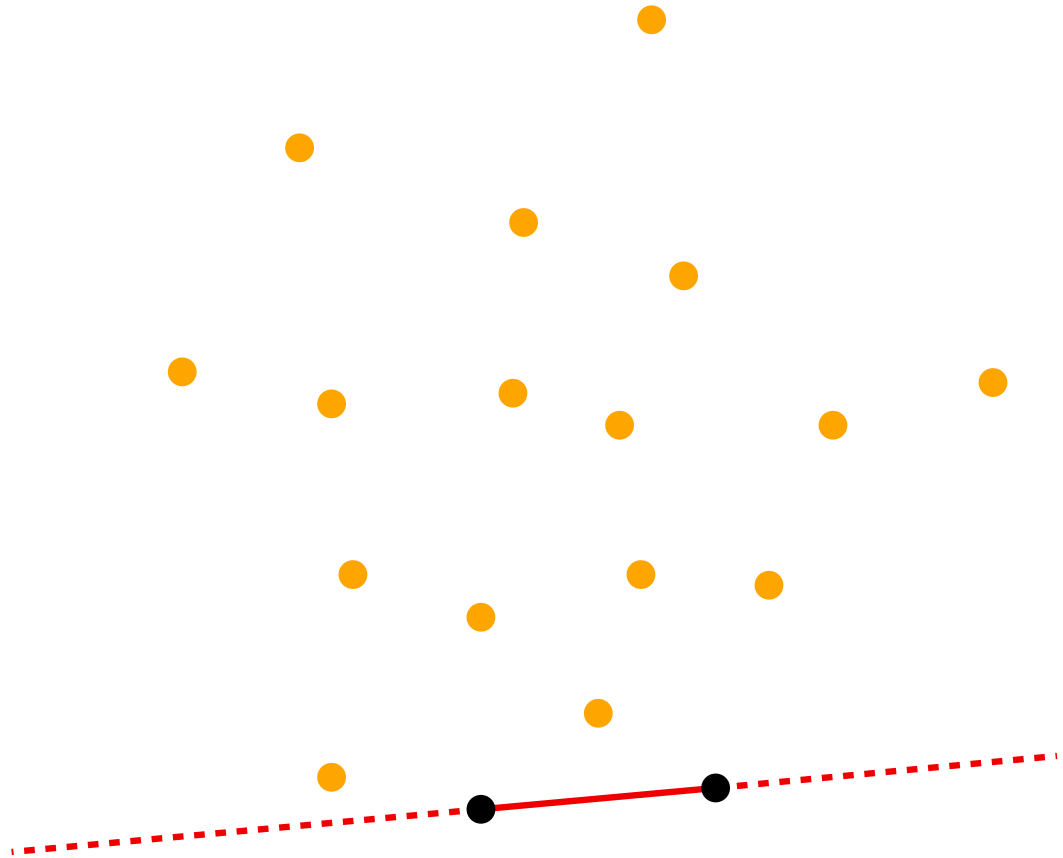

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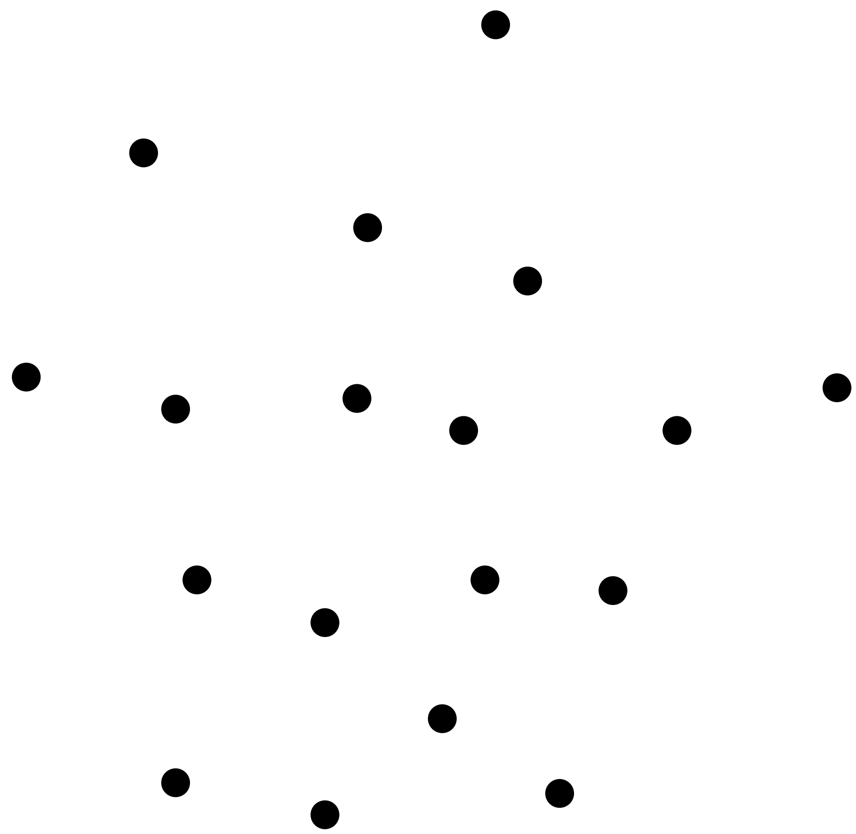
Algorithm

Input: $p_1, \ldots, p_n$
Output: set of the extreme segments

Procedure:
For each i, j,
Check whether all p_k with $k \neq i, j$ lie in the same halfplane defined by $p_i p_j$.
In the affirmative, return the segment $p_i p_j$.



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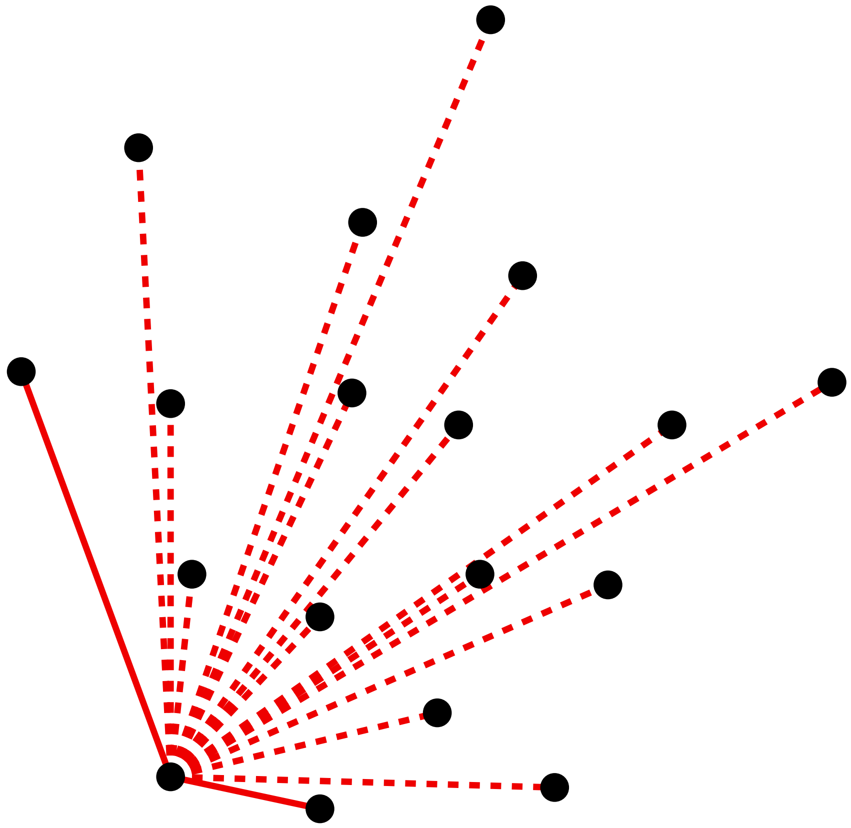

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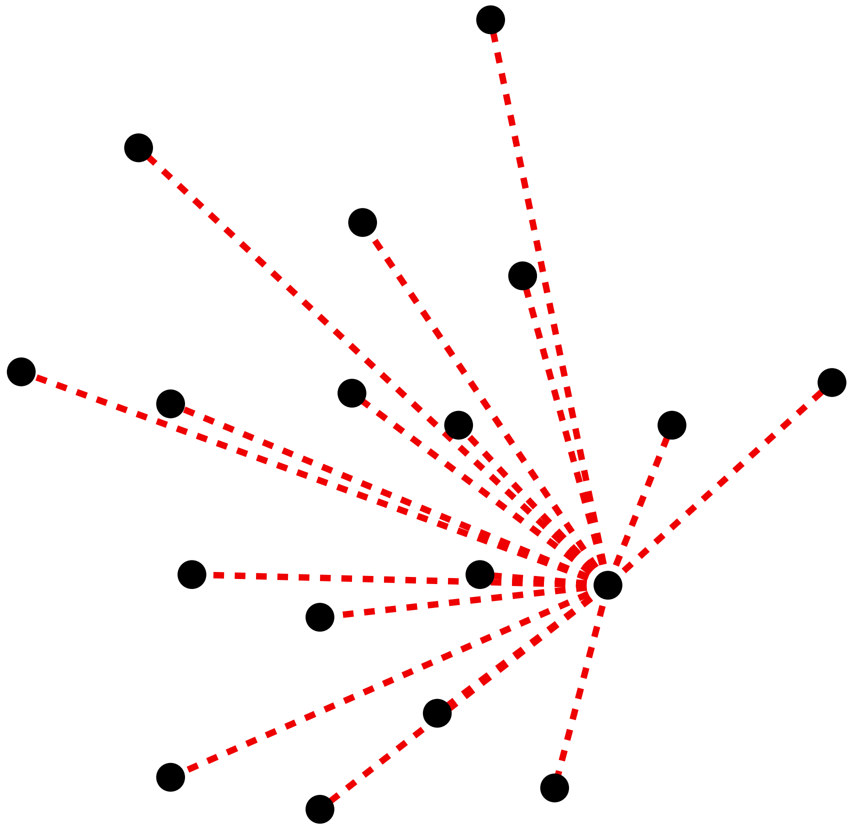

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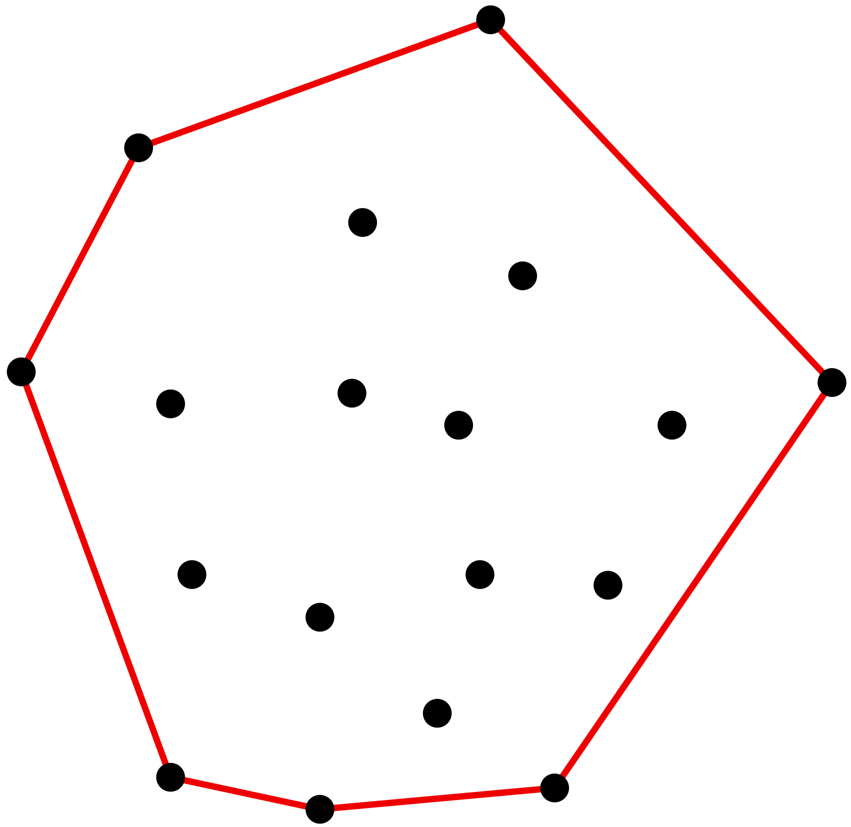

Computing the extreme segments

Characterization

Given $X = \{p_1, \ldots, p_n\}$, the segment $p_i p_j$ is an extreme segment if and only if all the points p_k with $k \neq i, j$ lie in the same halfplane defined by the line $p_i p_j$.

Algorithm

Input: $p_1, \ldots, p_n$
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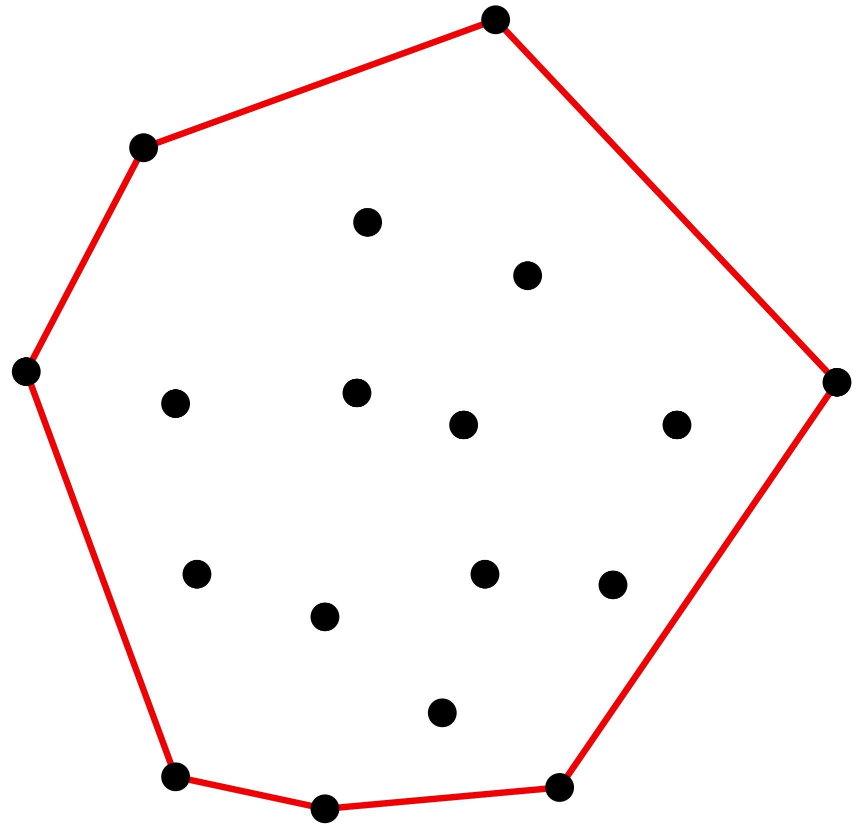
Running time: $\Theta(n^3)$



Computing the convex hull

Computing the convex hull **(sorted list of its vertices)**

Computing the convex hull

Input:

$P = \{p_1, \ldots, p_n\} \subset \mathbb{R}^2$ a set of n points in the plane

Output:

l, the list of the vertices of $ch(P)$ sorted in counterclockwise order

Computing the convex hull

Input:
$P = \{p_1, \ldots, p_n\} \subset \mathbb{R}^2$ a set of n points in the plane

Output:
l, the list of the vertices of $ch(P)$ sorted in counterclockwise order

Characterization
Given $X = \{p_1, \ldots, p_n\}$, the segment $p_i p_j$ is an edge of the convex hull of X if and only if all the points p_k with $k \neq i, j$ lie to the left of the oriented line $p_i p_j$.

Jarvis march

Jarvis march

1. Find a vertex of $ch(P)$ (for example, the lexicographically smaller point $p_i \in P$) and add it to l
2. While $v = \mathrm{Last}(l) \neq \mathrm{First}(l)$, do:
   (a) Detect the angularly rightmost point $p_j \in P$ with respect to v.
   (b) Add p_j to l
3. Return l

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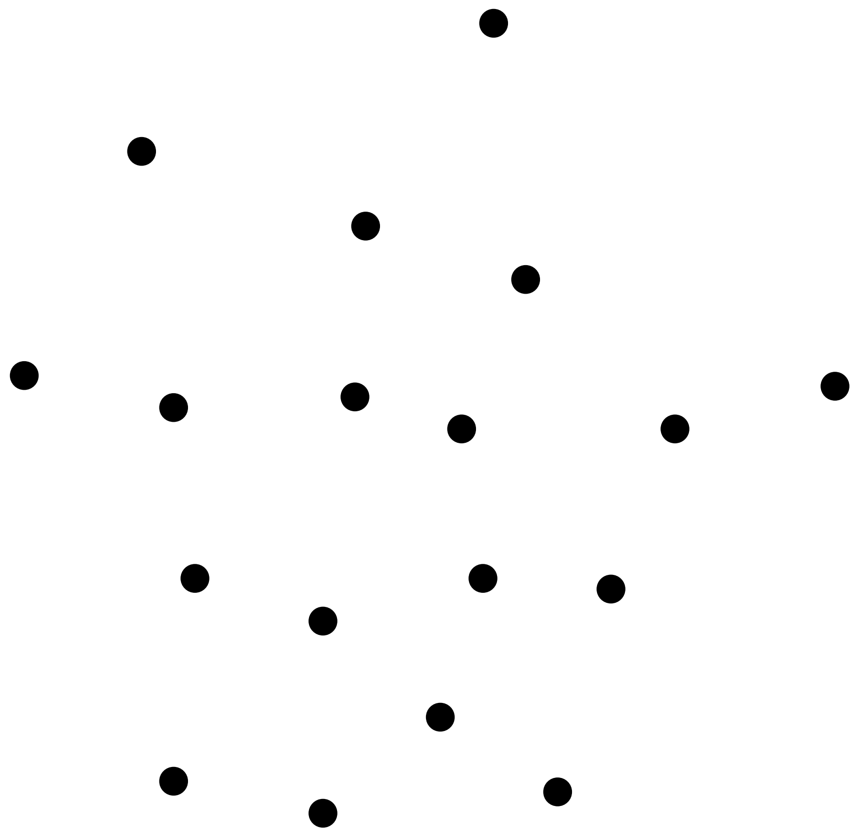

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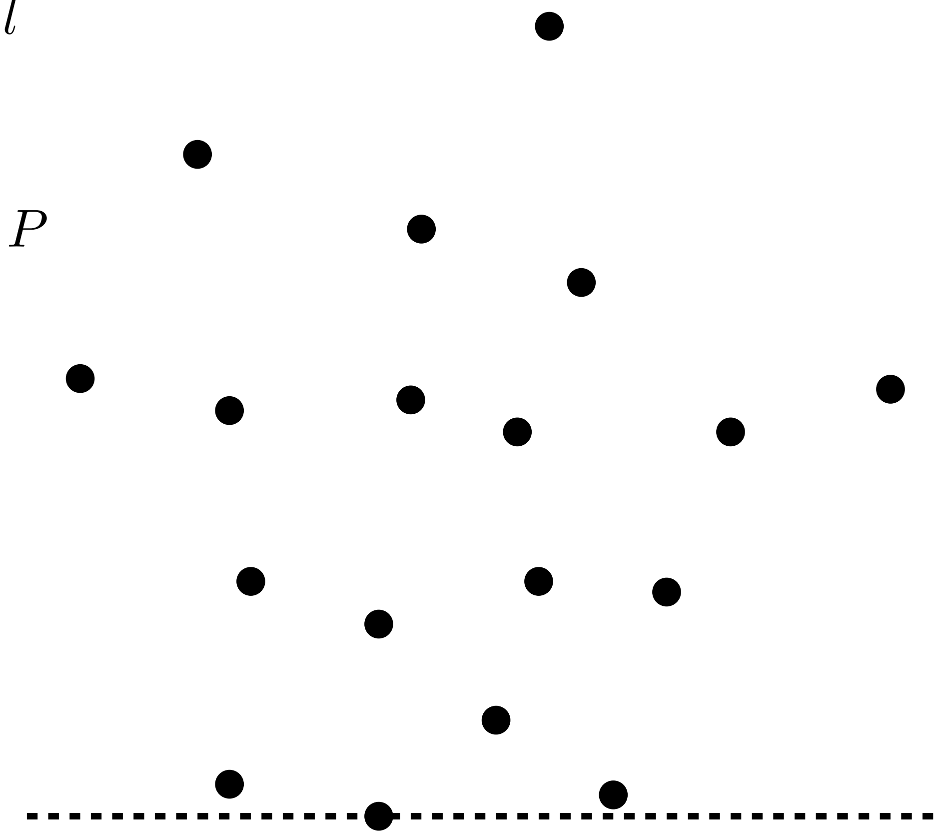

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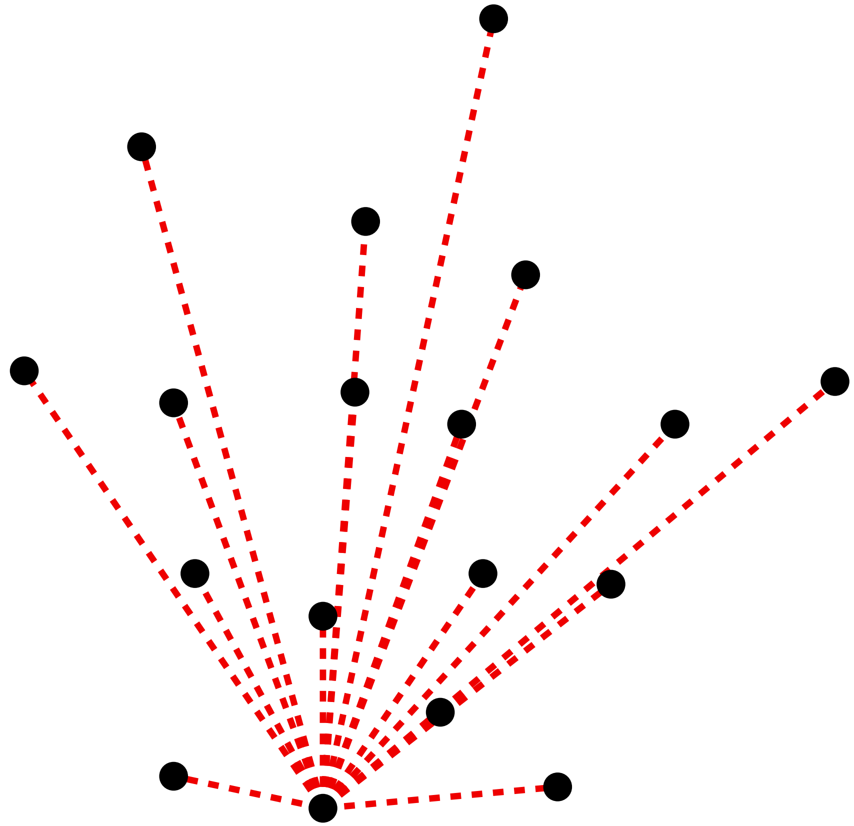

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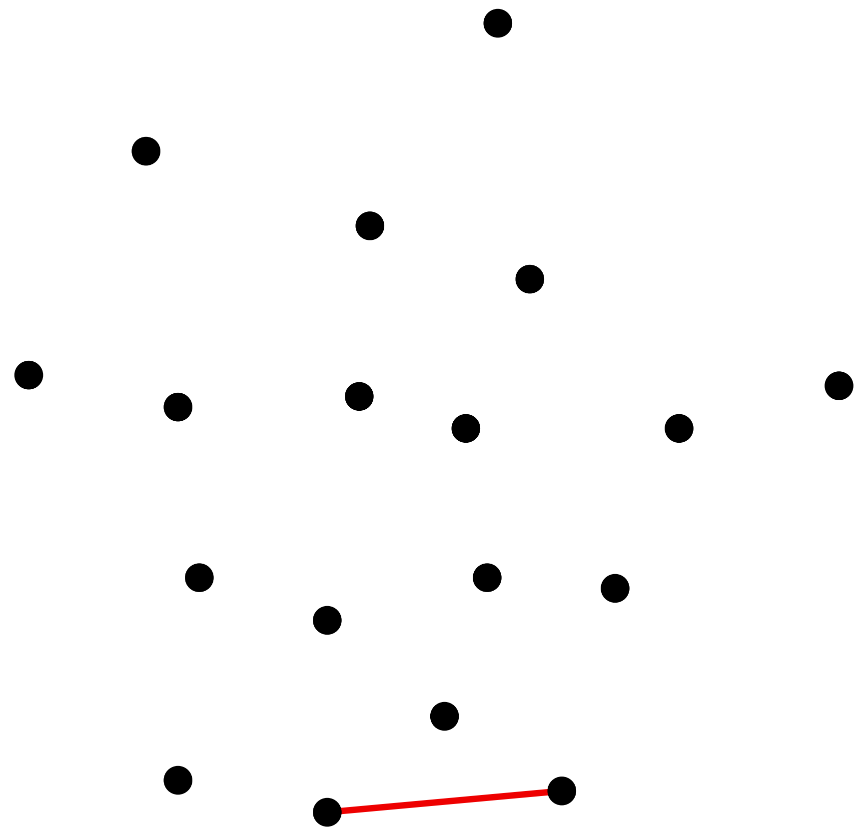

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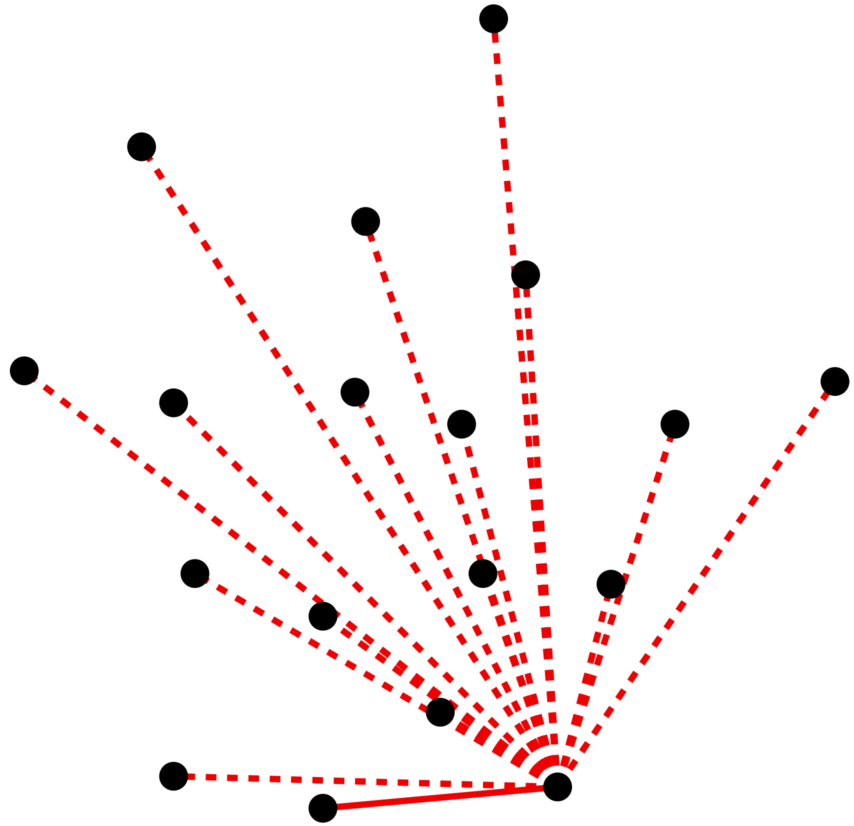

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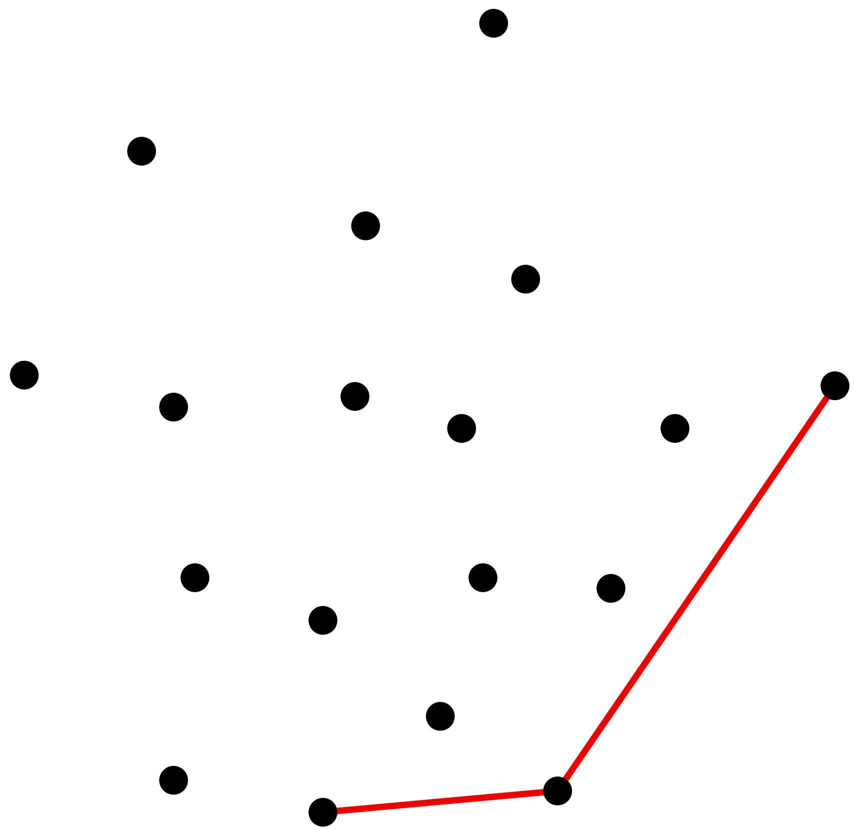

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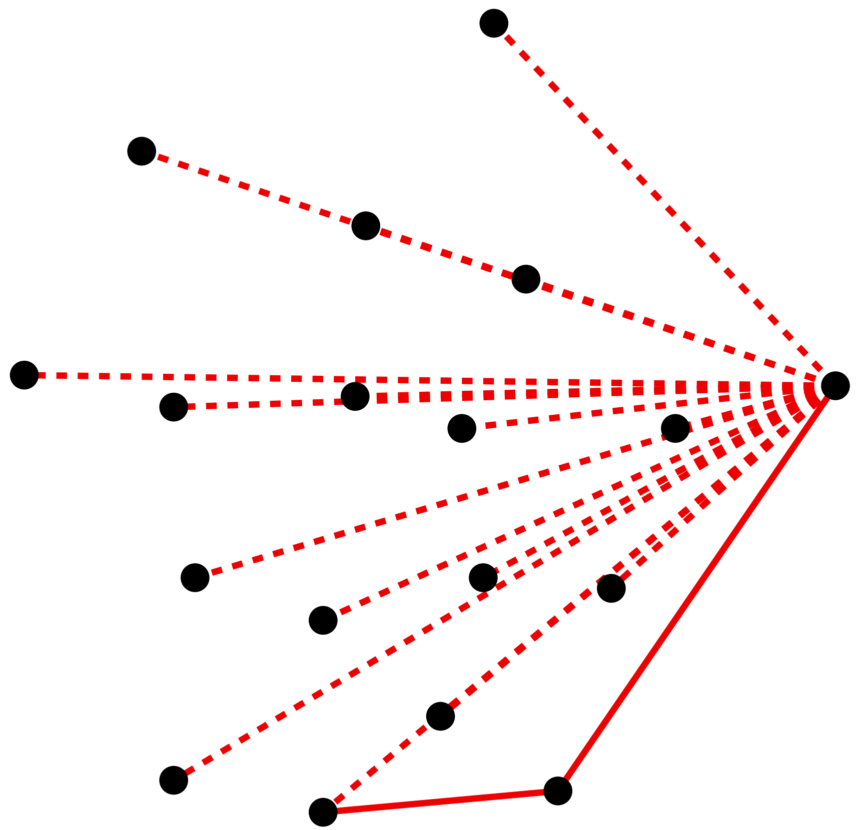

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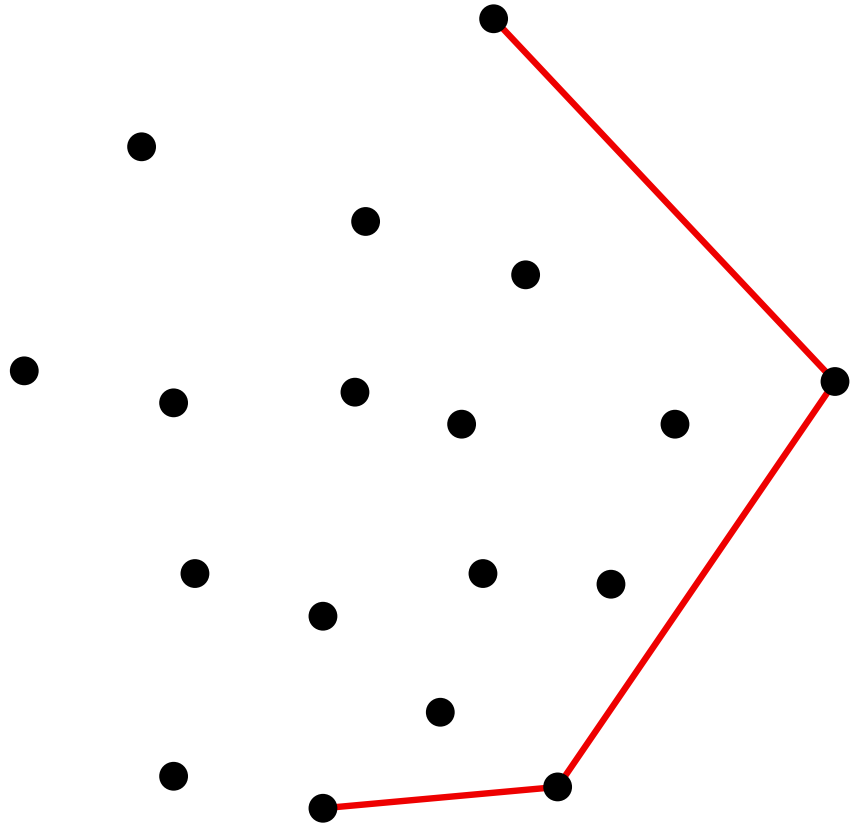

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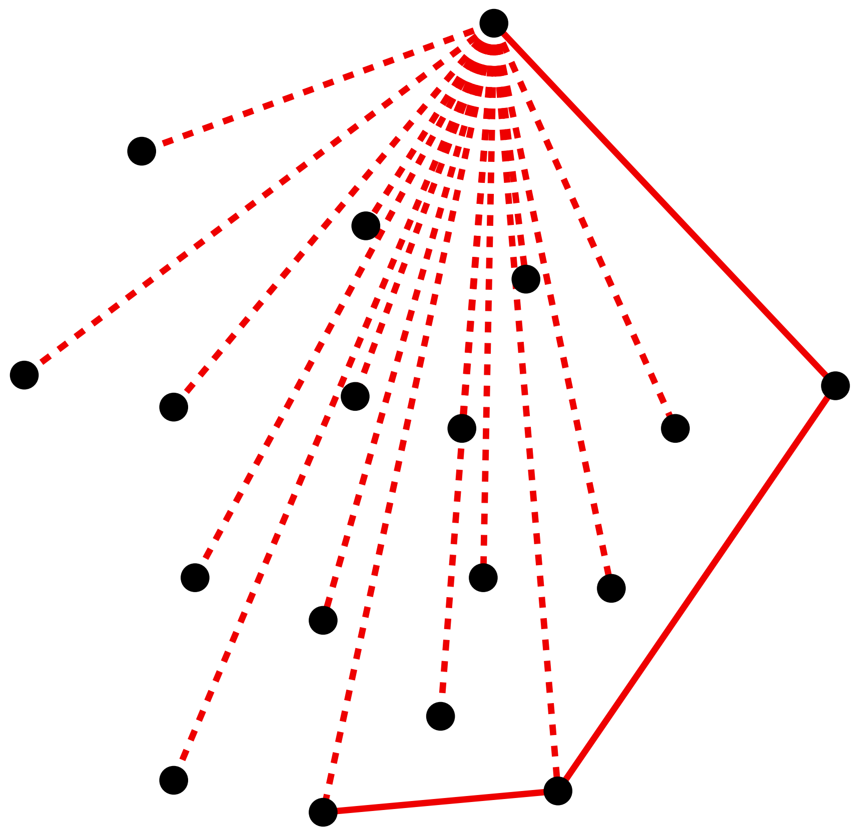

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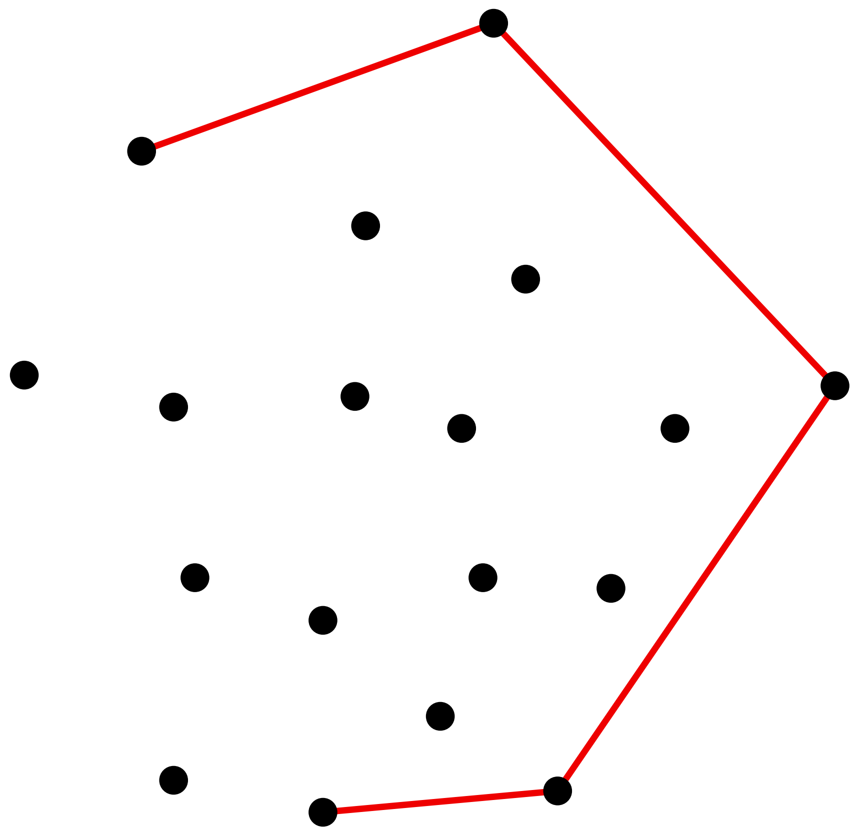

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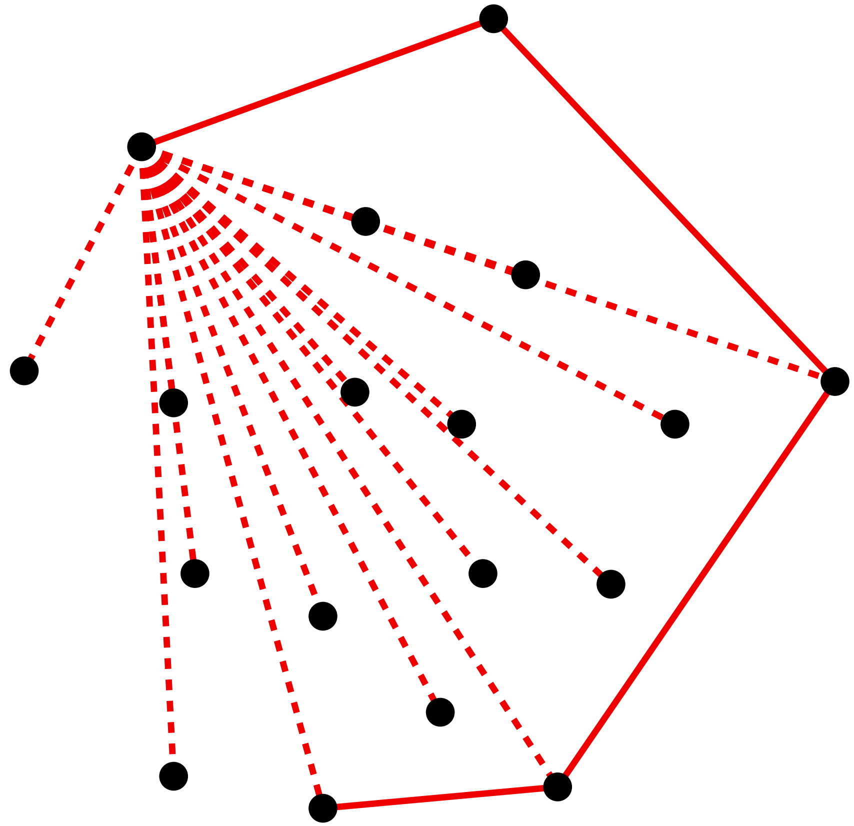

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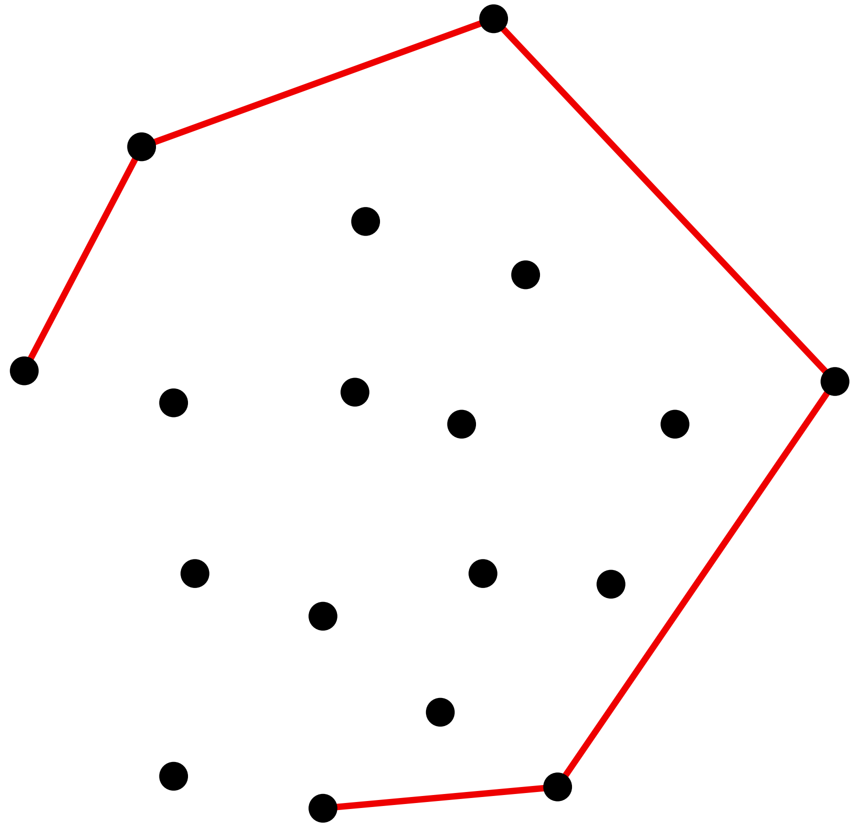

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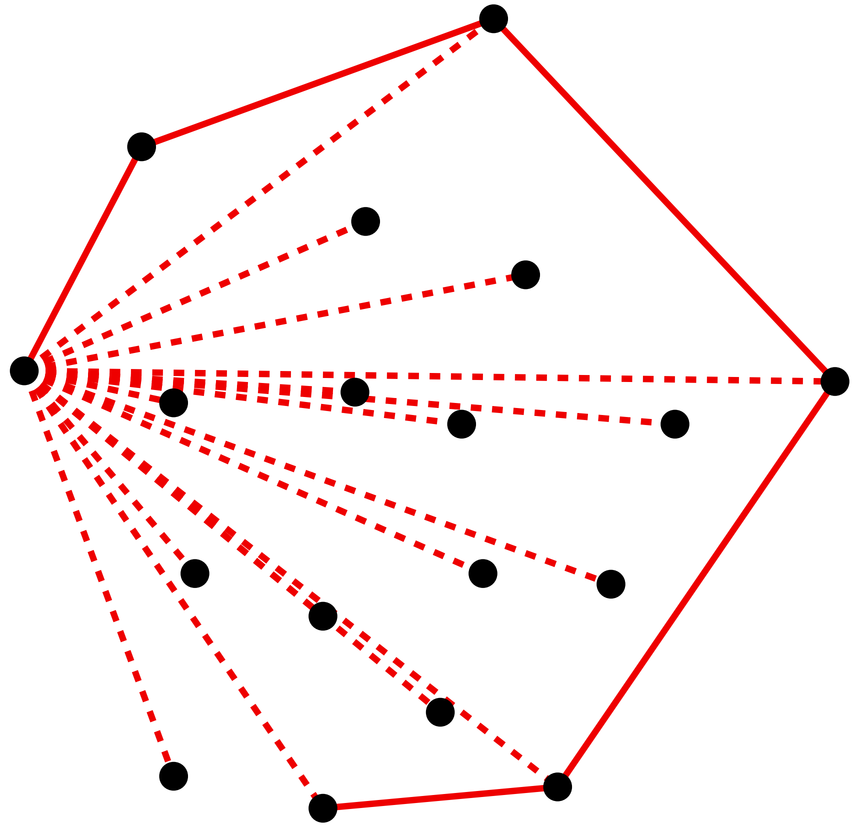

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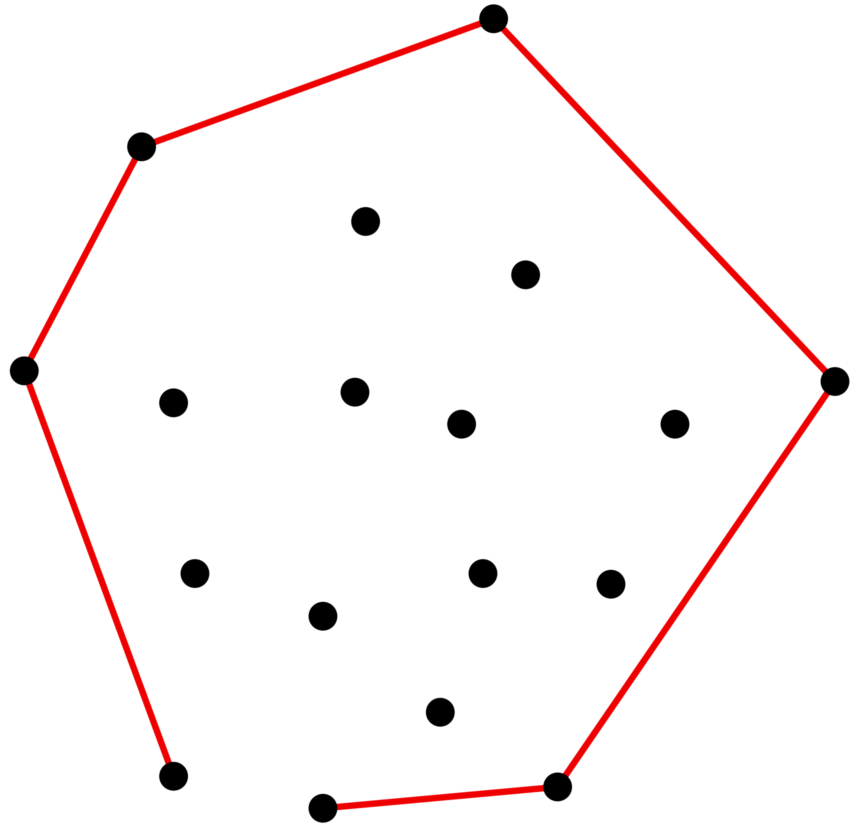

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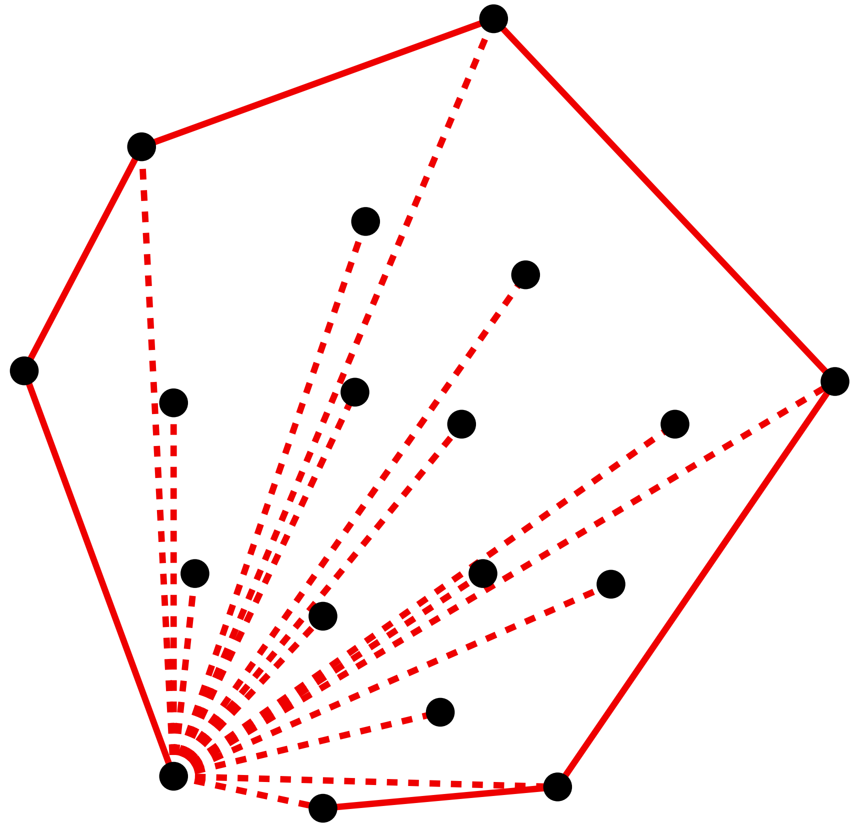

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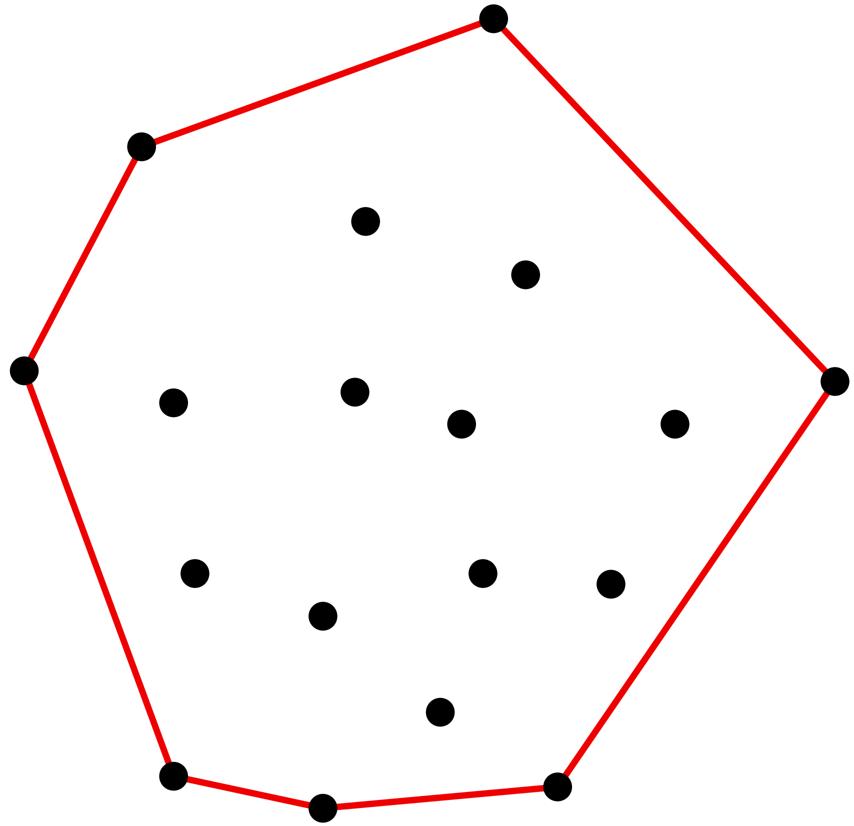
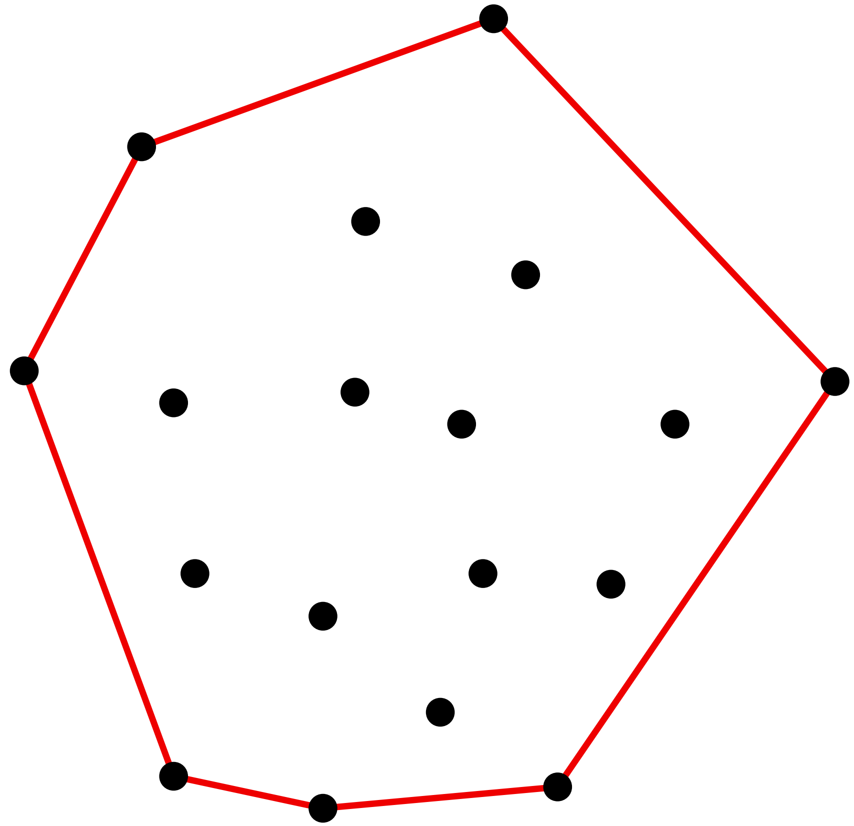


CONVEX HULL

Jarvis march

1. Find a vertex of $ch(P)$ (for example, the lexicographically smaller point $p_i \in P$) and add it to l
2. While $v = \text{Last}(l) \neq \text{First}(l)$, do:
   (a) Detect the angularly rightmost point $p_j \in P$ with respect to v.
   (b) Add p_j to l
3. Return l



Time cost: $\Theta(hn) = O(n^2)$

QuickHull algorithm (by prune-and-search)

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Initialization

1. Find the extreme points in the horizontal and vertical directions.
2. Compute the convex hull of these (at most four) points.
3. Test all the remaining points, and classify them according to their position (NE, SE, SW, NW) or eliminate them if they lie in the interior.

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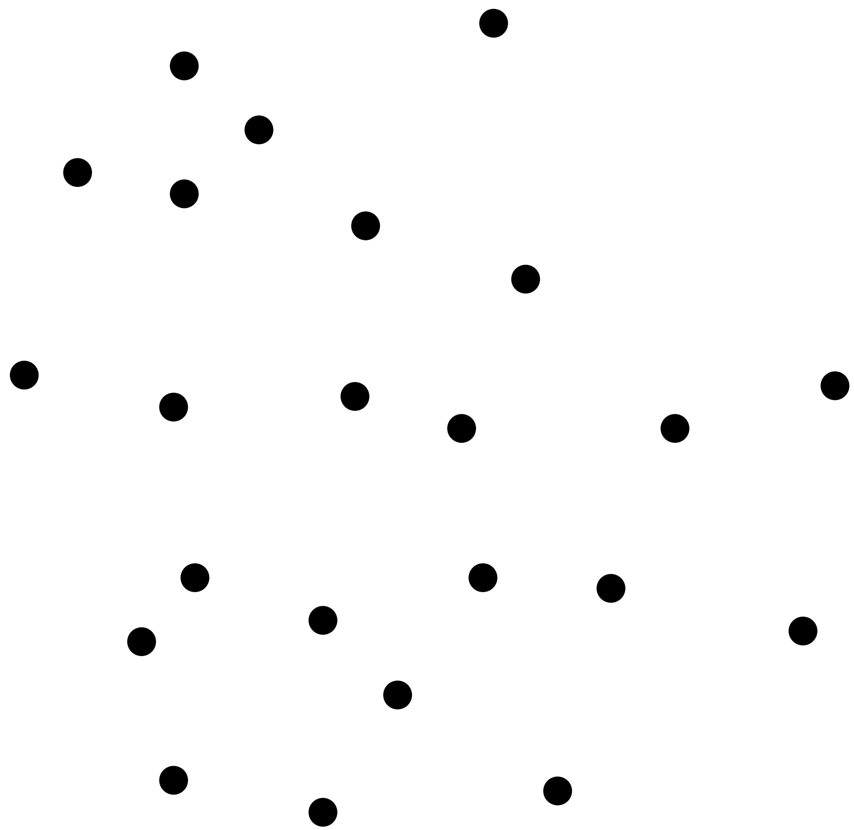

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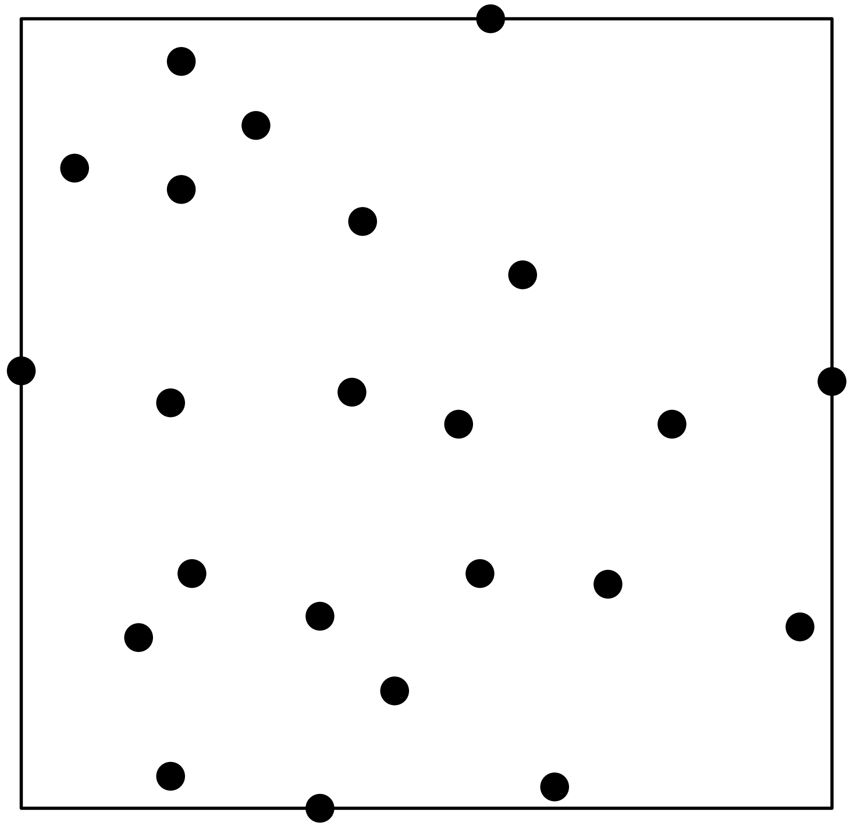

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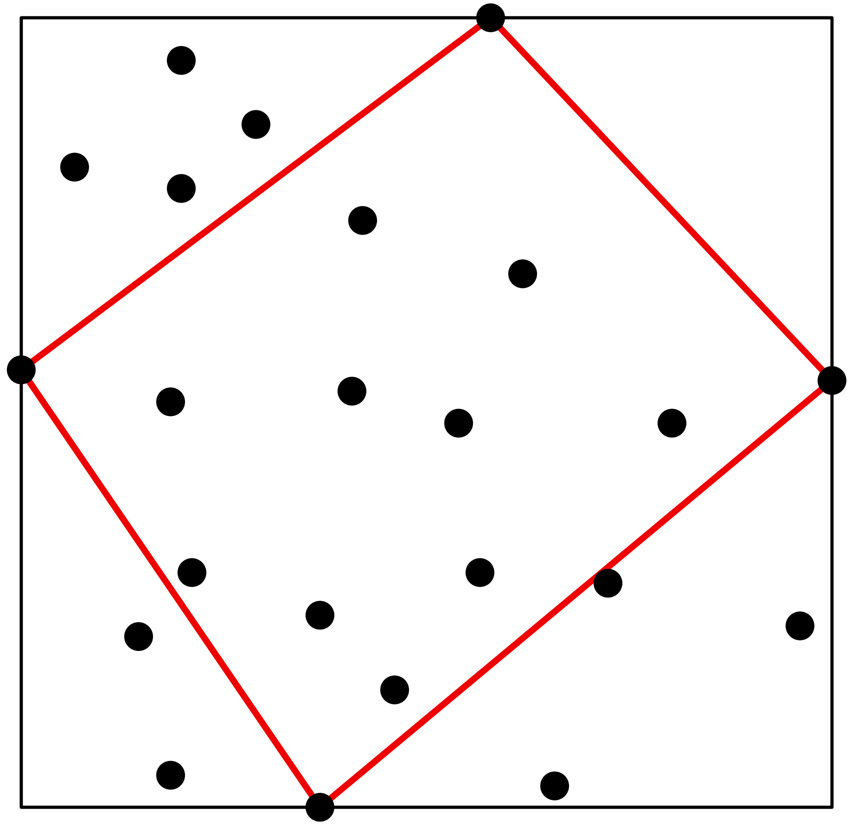

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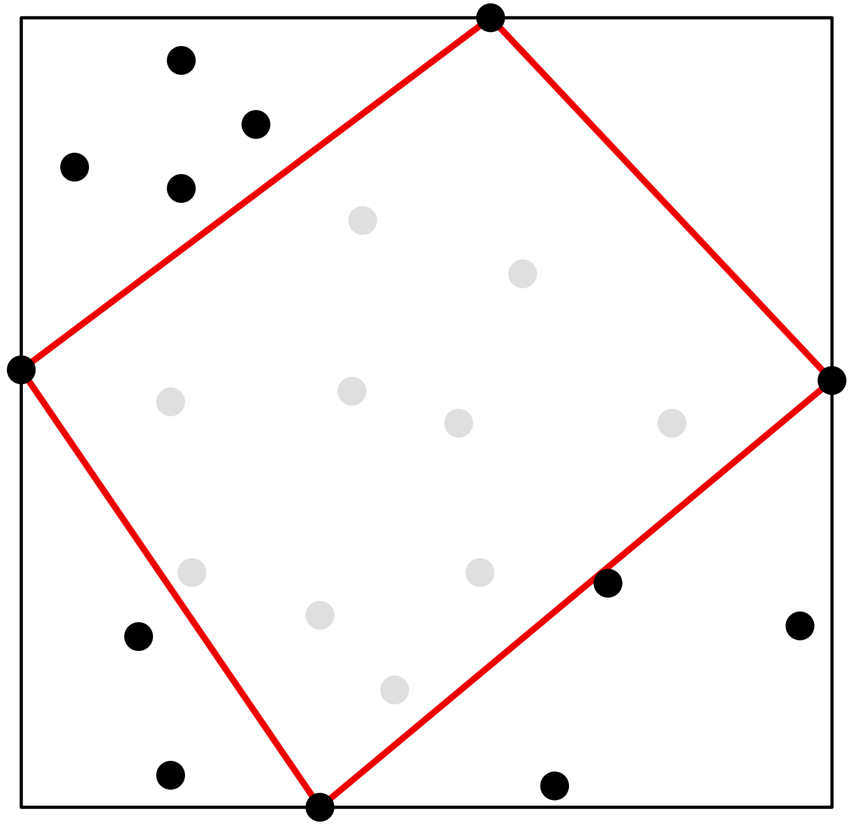

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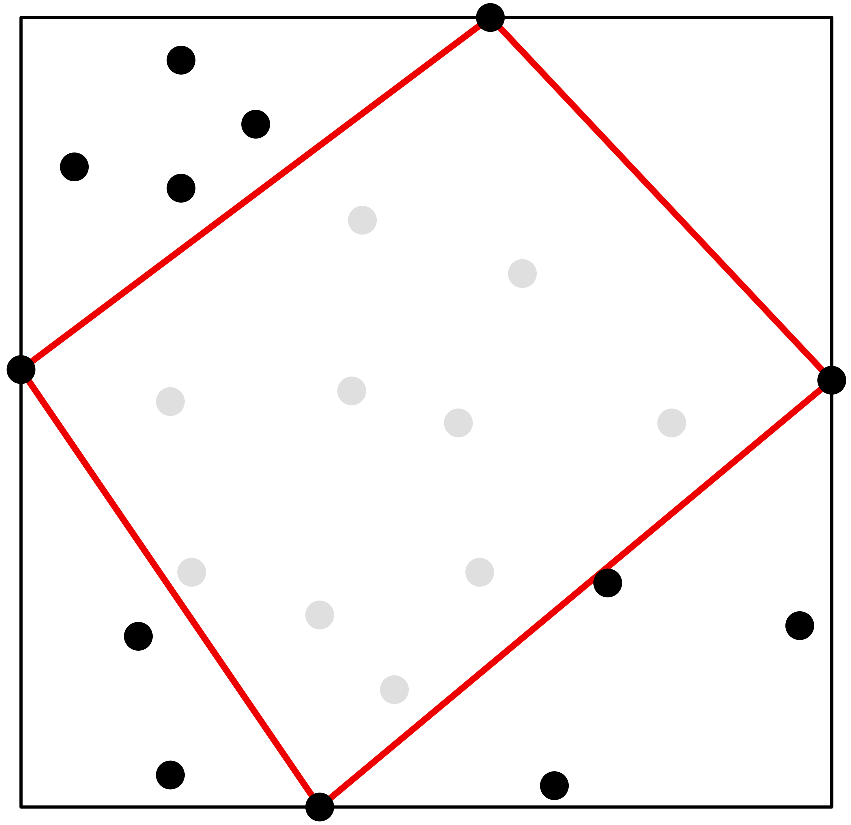

Running time of this step: $O(n)$

QuickHull algorithm (by prune-and-search)

Advance

Recursively, do:

1. Among all points lying in each region, find the extreme point in the direction orthogonal to the edge that determines the region.
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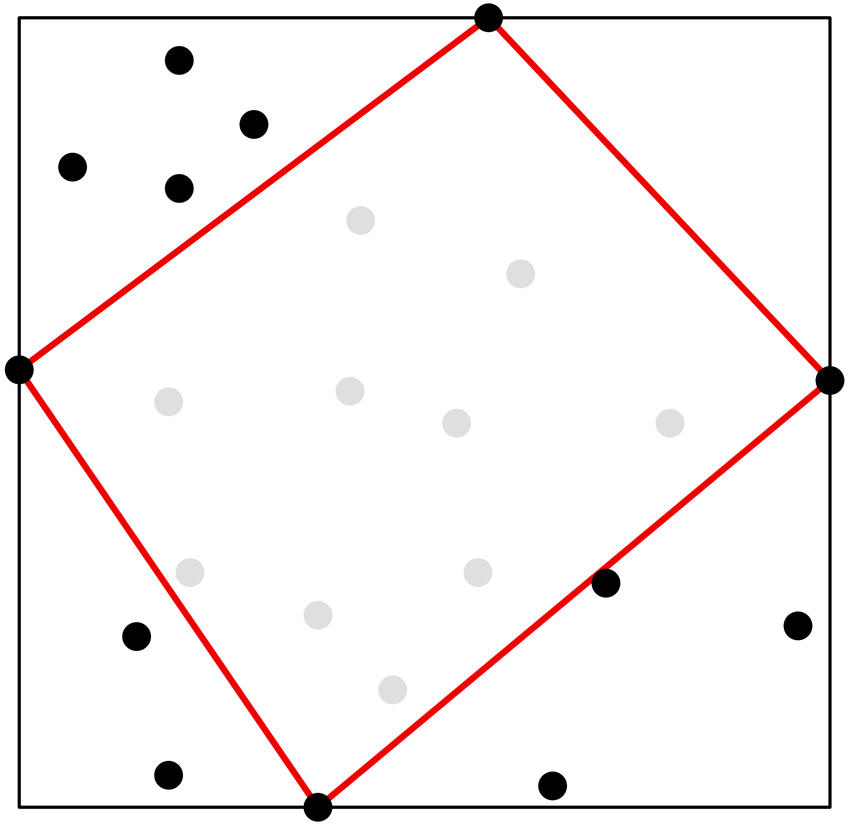
3. Test all the remaining points of each region, and classify them according to their position (left or right) or eliminate them if they lie in the interior of the newly created triangle.



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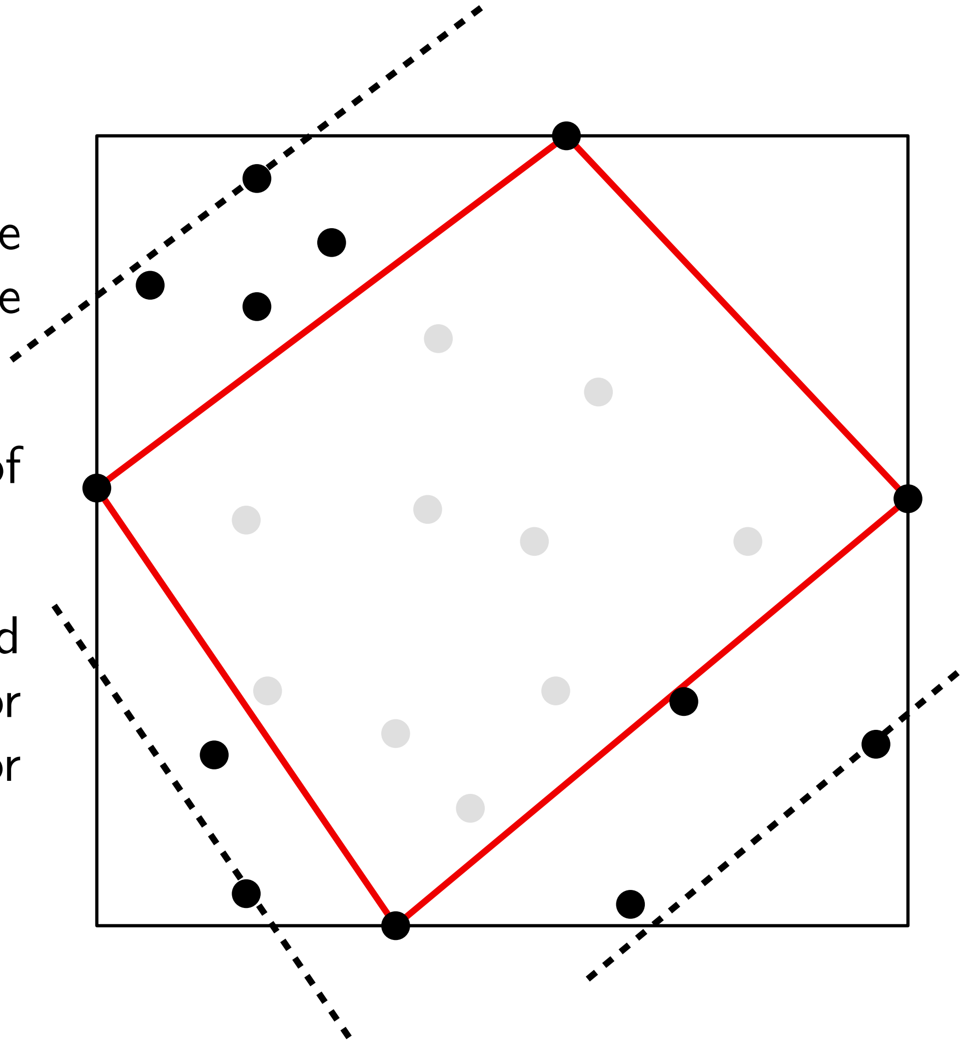

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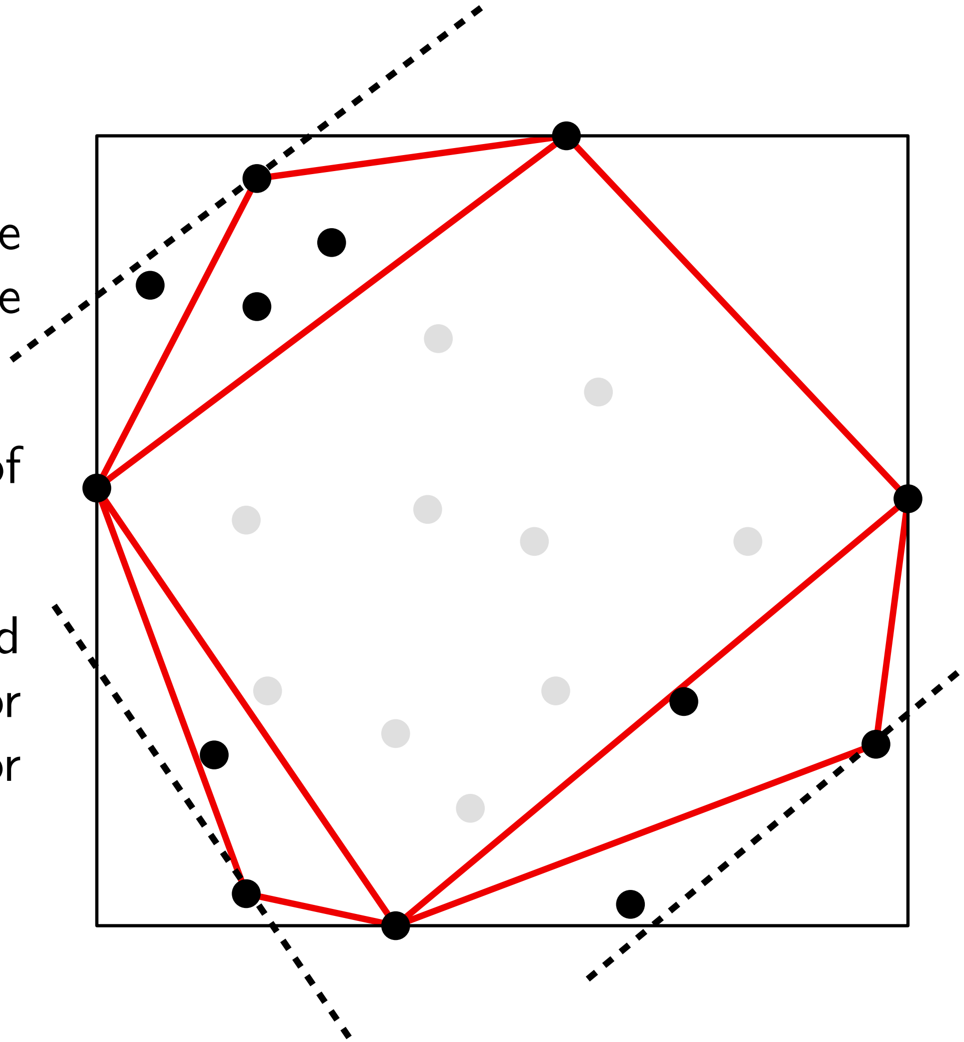

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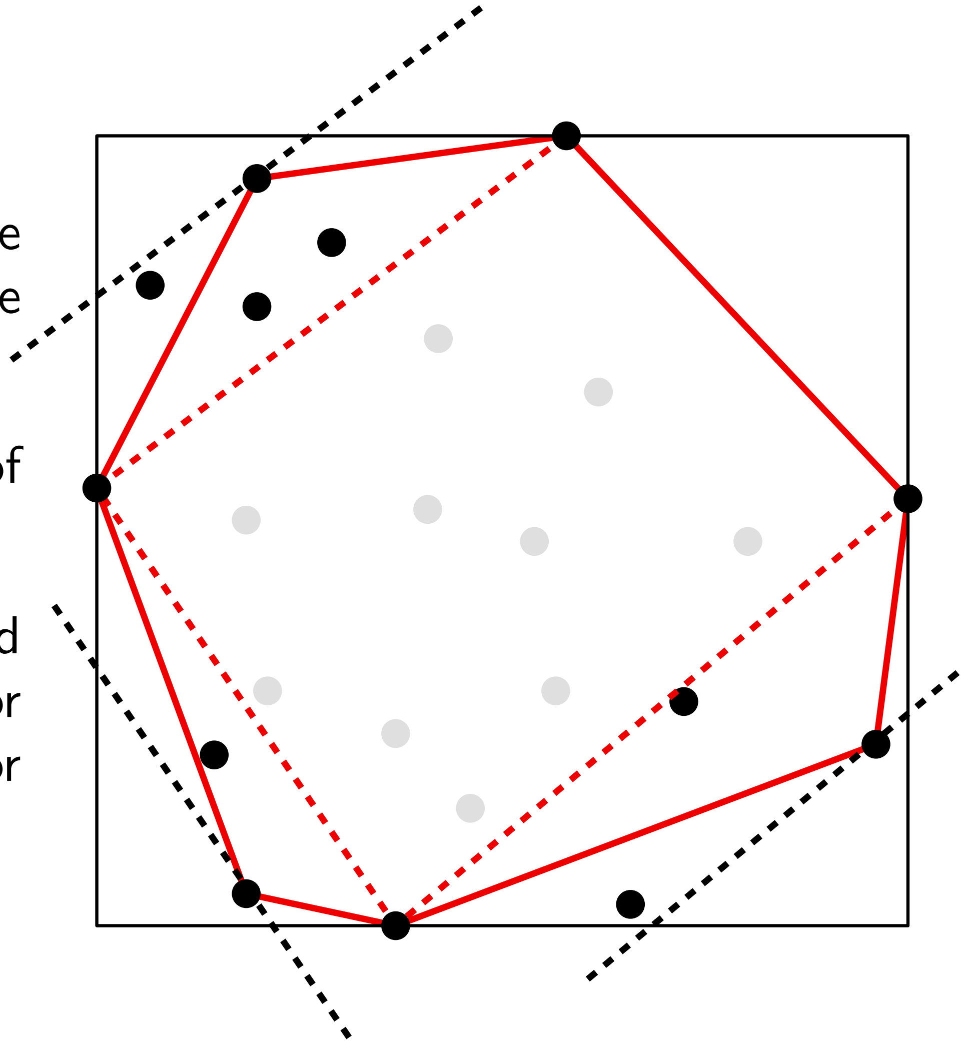

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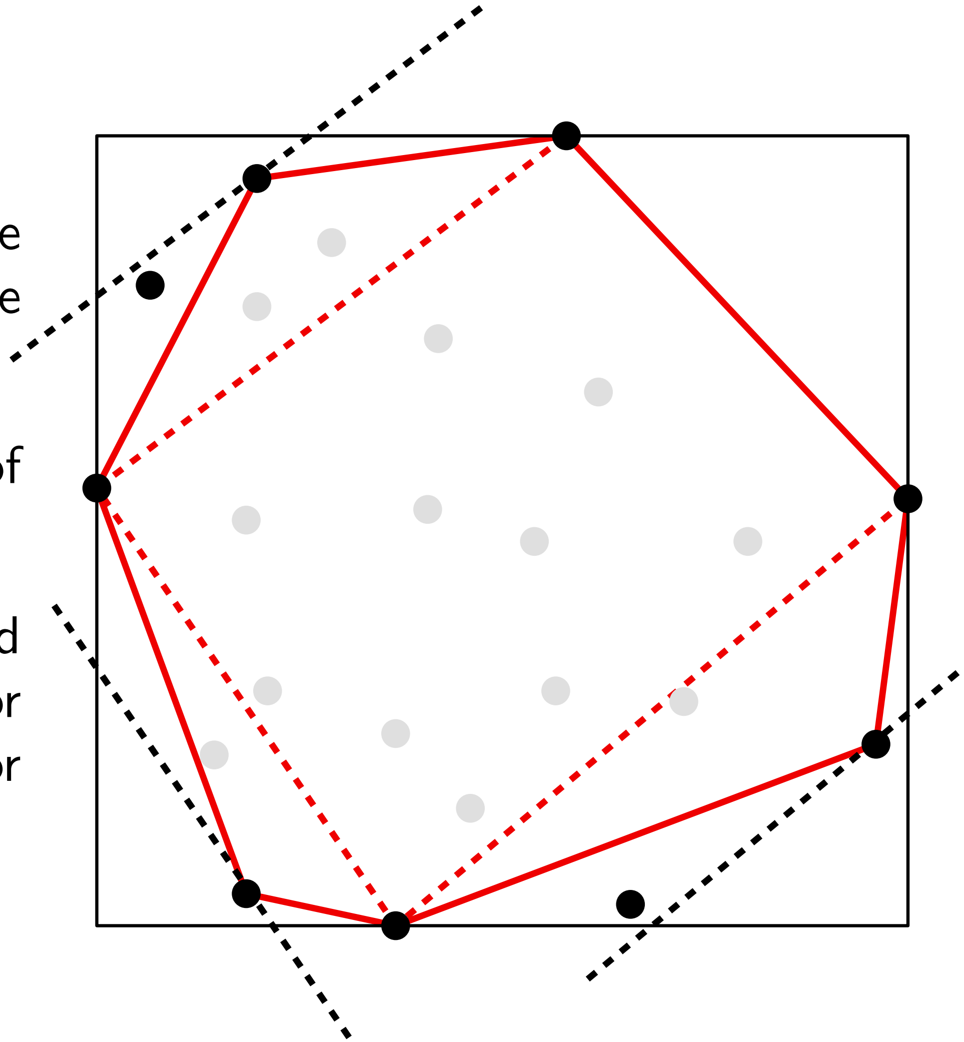

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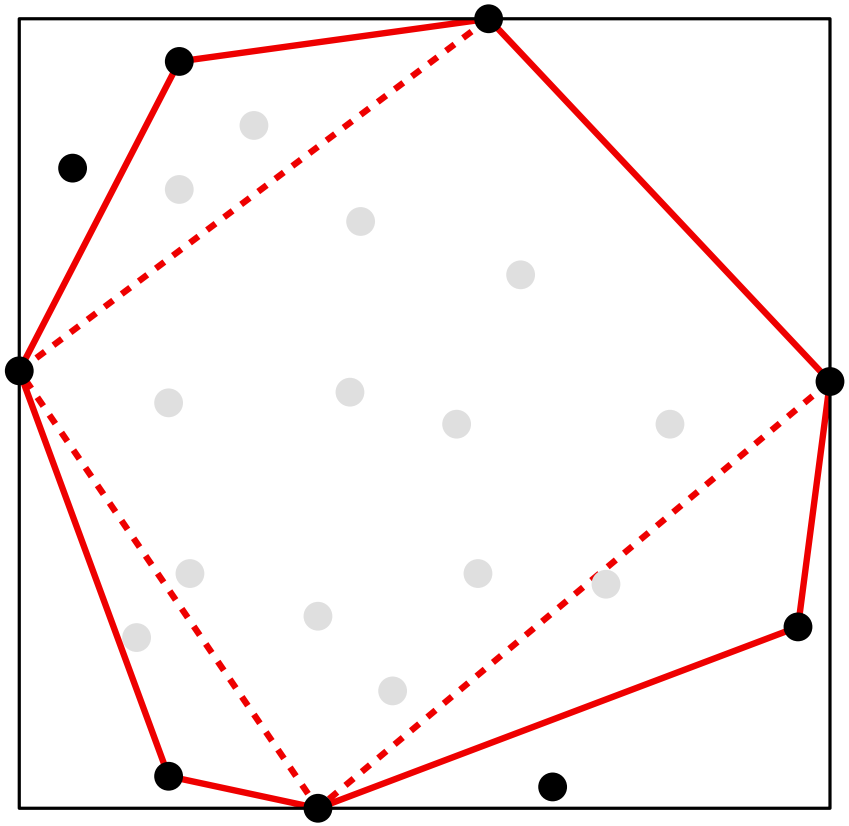

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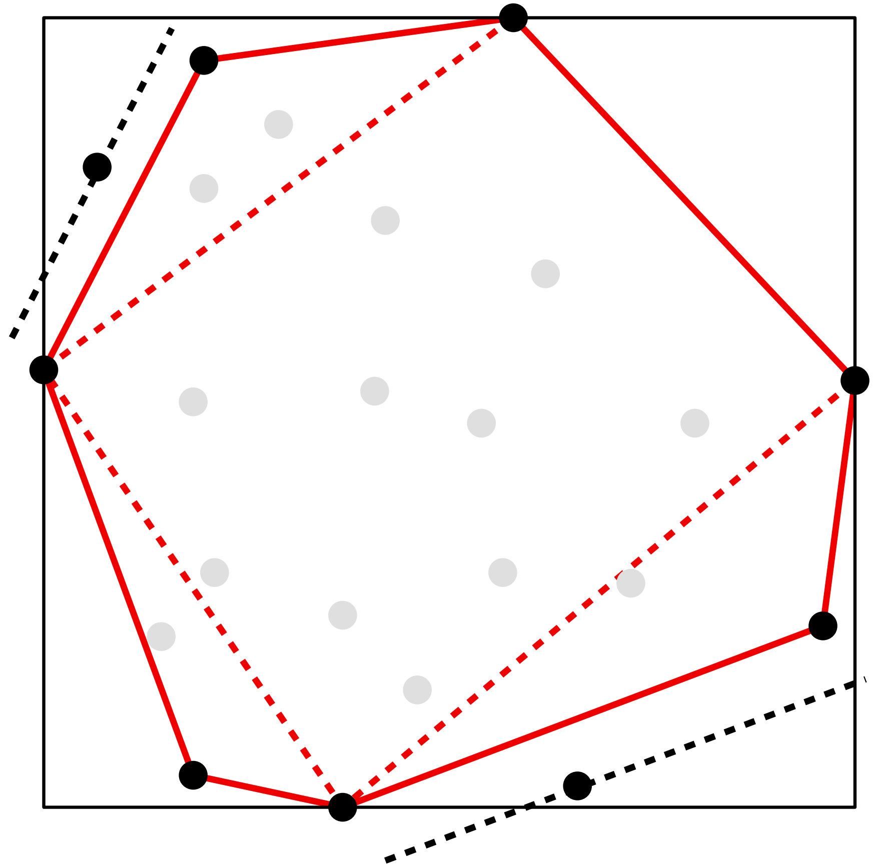

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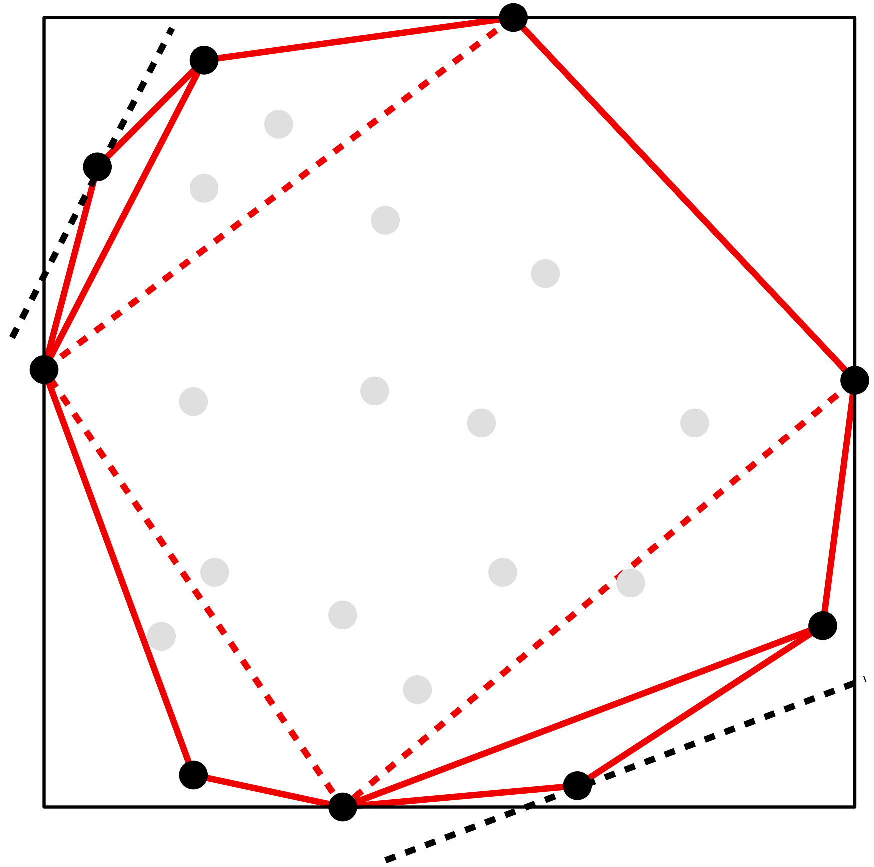

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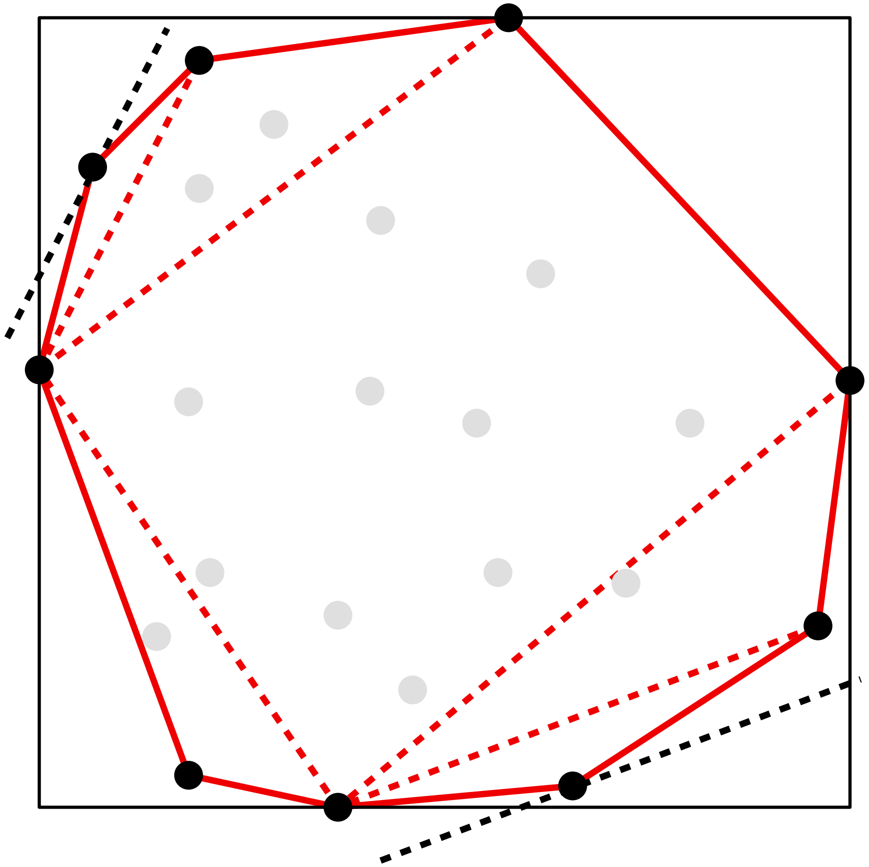

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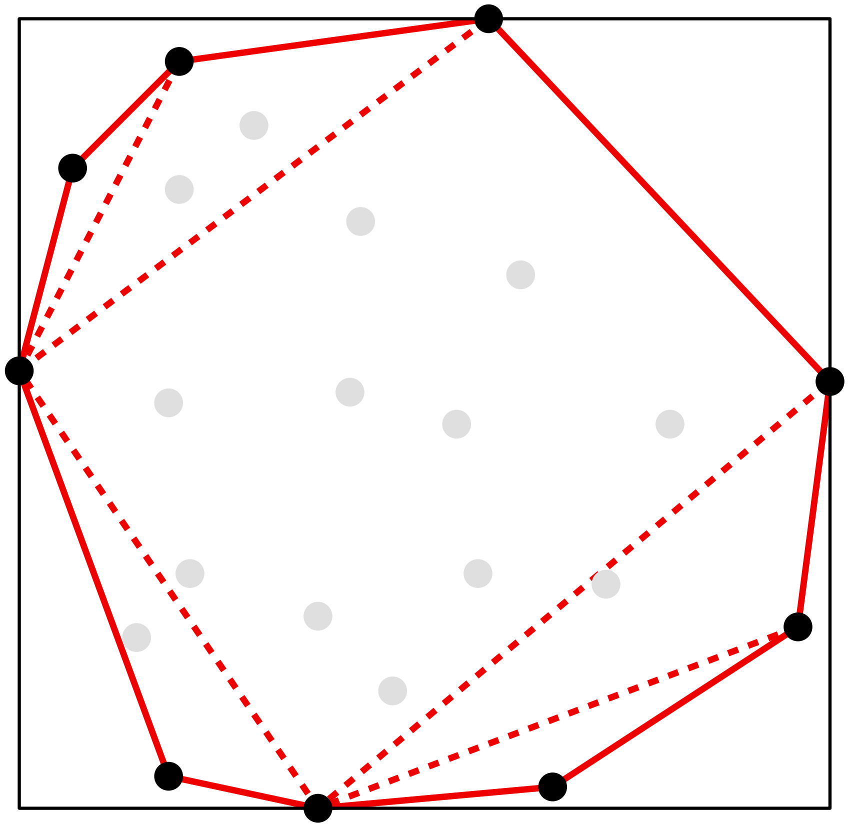

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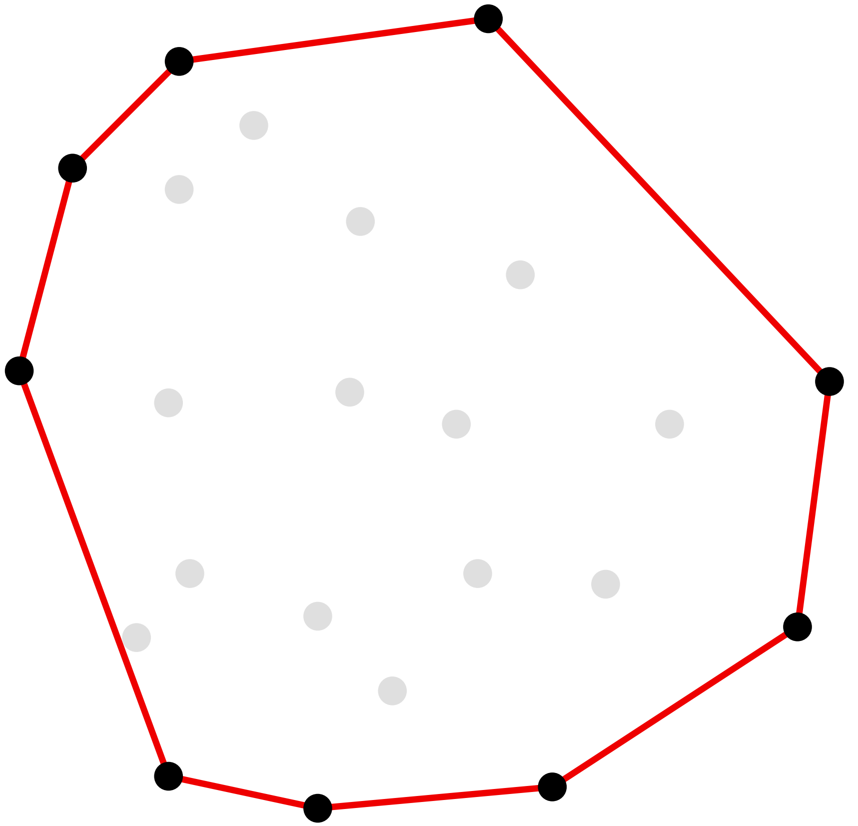

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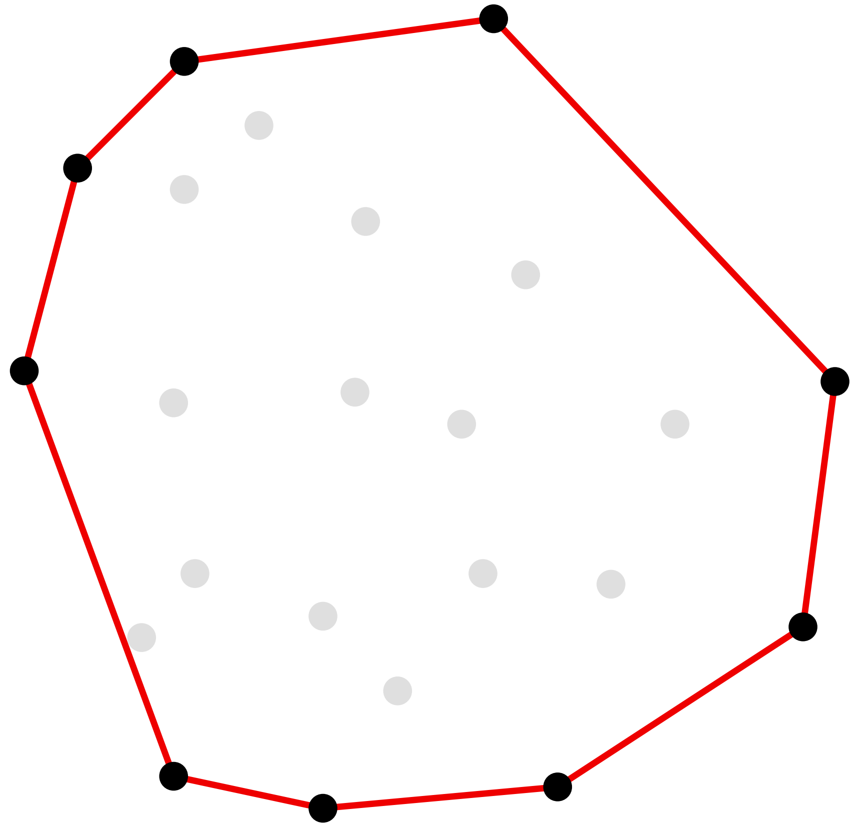
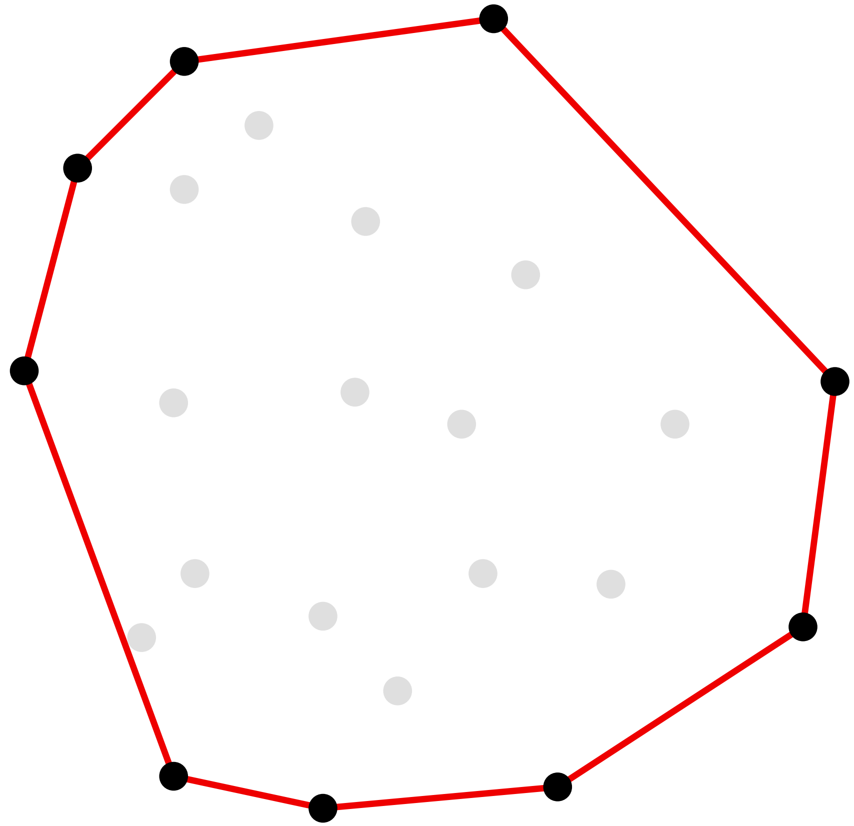


Running time of this step: $O(n^2)$

CONVEX HULL

QuickHull algorithm (by prune-and-search)

Overall running time: $O(n^2)$

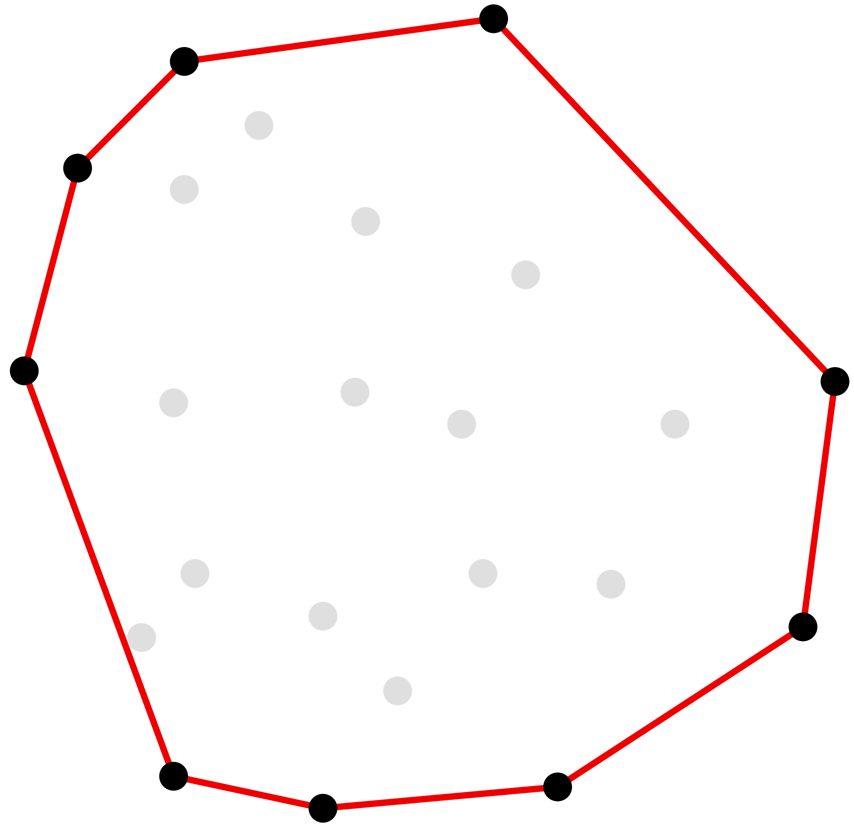


QuickHull algorithm (by prune-and-search)

Overall running time: $O(n^2)$

Nevertheless, the running time of this algorithm depends on the position of the input points.
For example:

- If the input points are in convex position, the running time is $\Theta(n^2)$.
- If the points are such that each prune step eliminates half of the current points, then the algorithm runs in $\Theta(n \log n)$ time.
- If the convex hull is triangular (or constant size), the algorithm runs in $\Theta(n)$ time.



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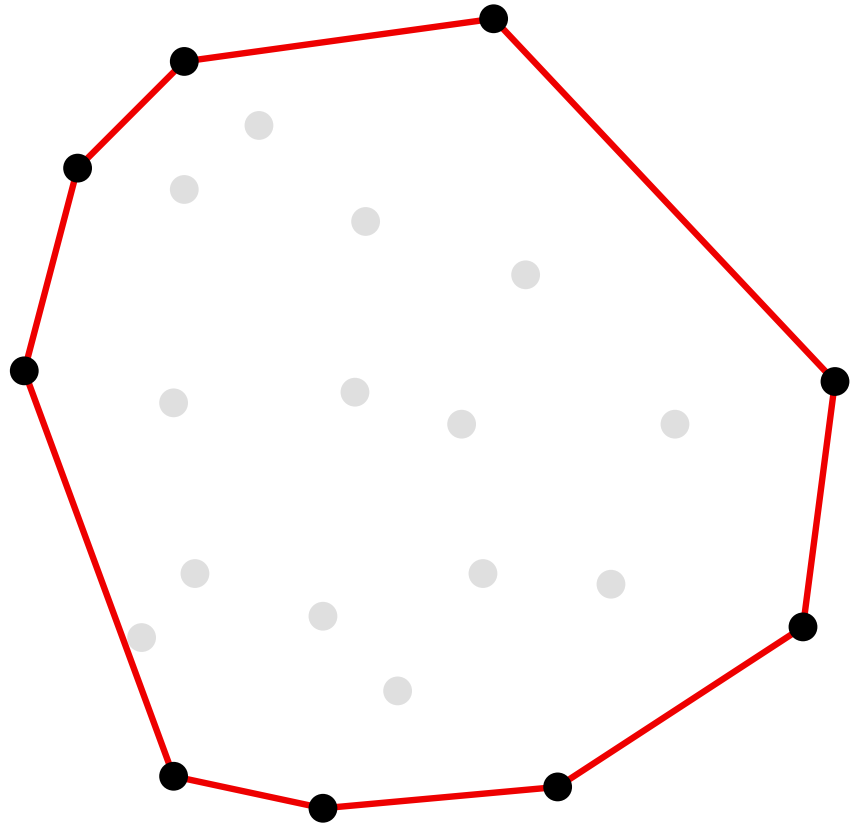

In fact, this algorithm is also **output sensitive**.

Graham's algorithm

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete if from P

Advance

While there exist points $p_i \in P$ to be explored, do:

$p = \mathrm{top}(l)$

$p^- = \mathrm{previous}(\mathrm{top}(l))$

- If $p^- p p_i$ is a left turn:
  - Push p_i in l
  - Advance i
- Else:
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Return l

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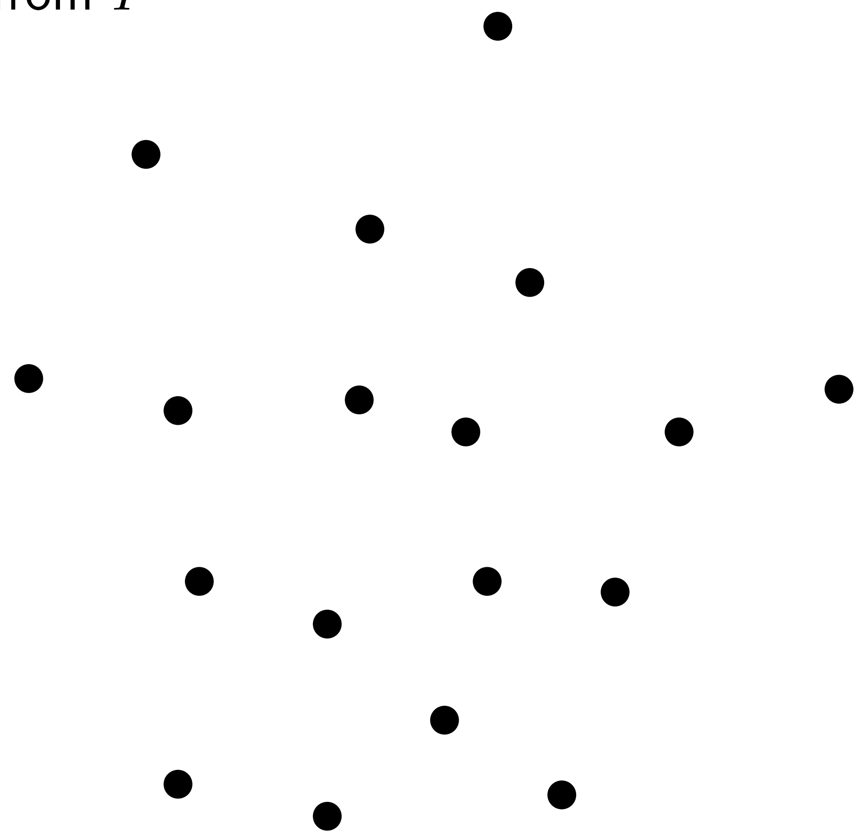

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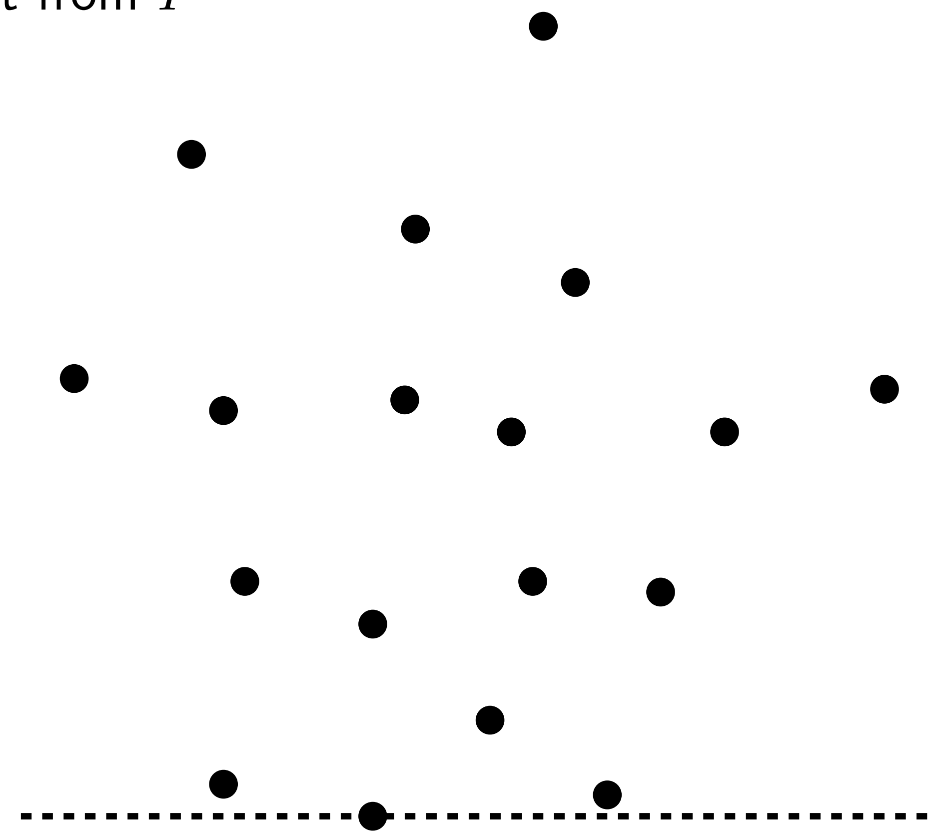

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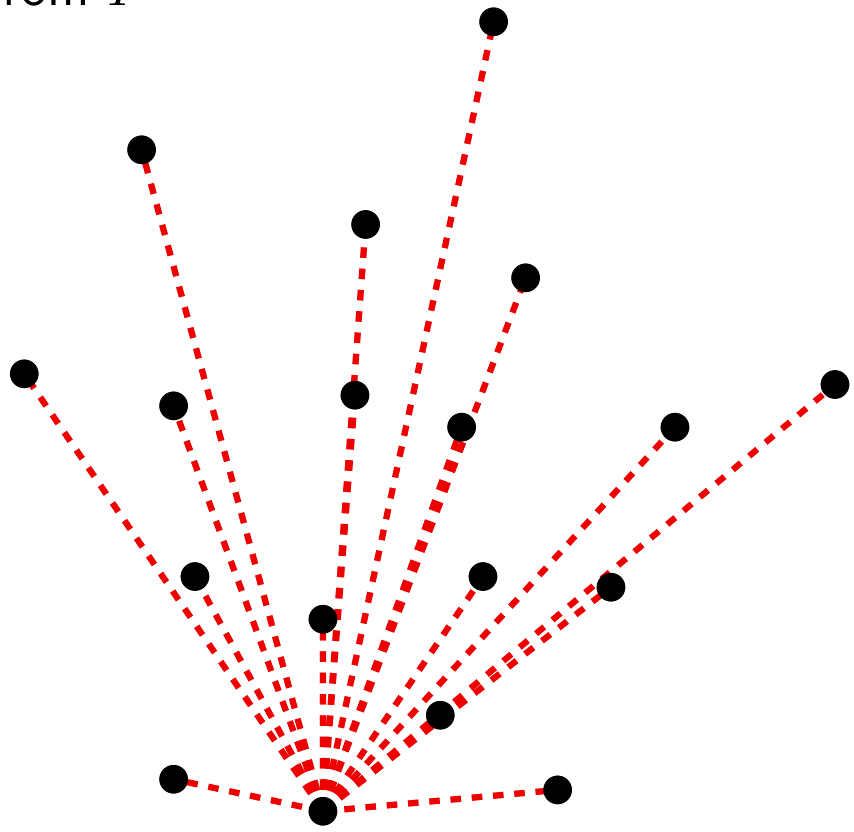

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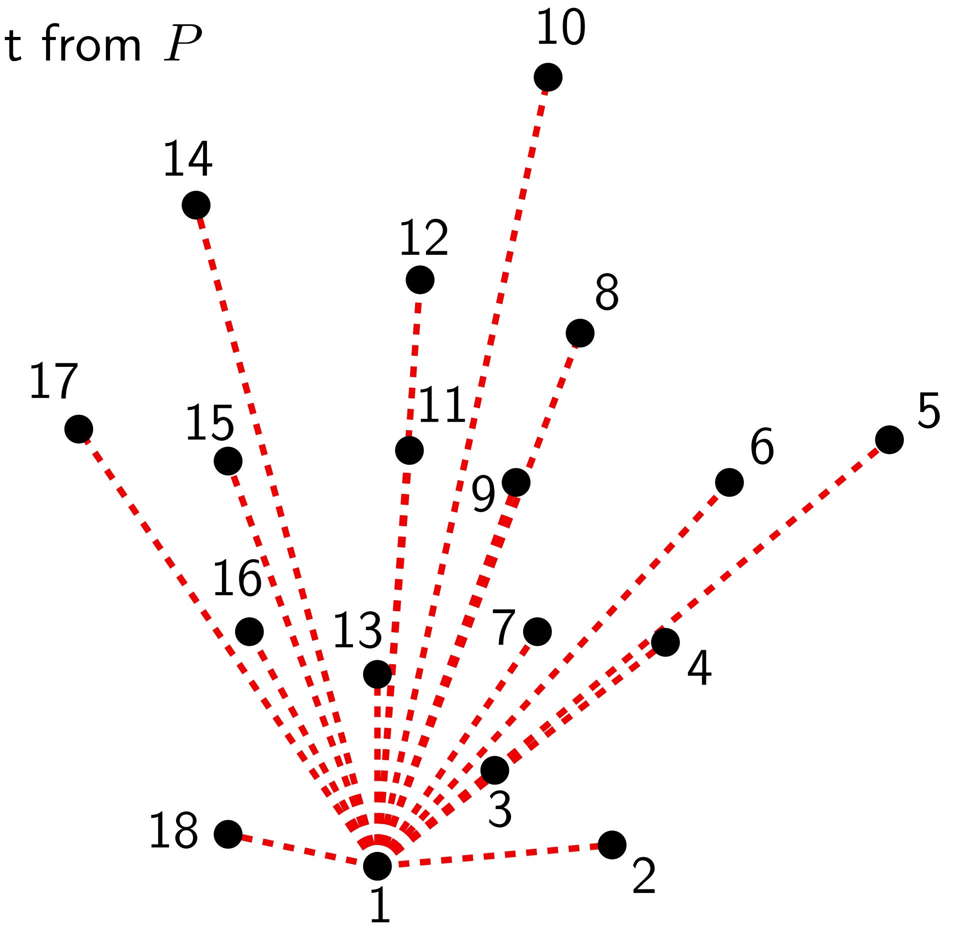

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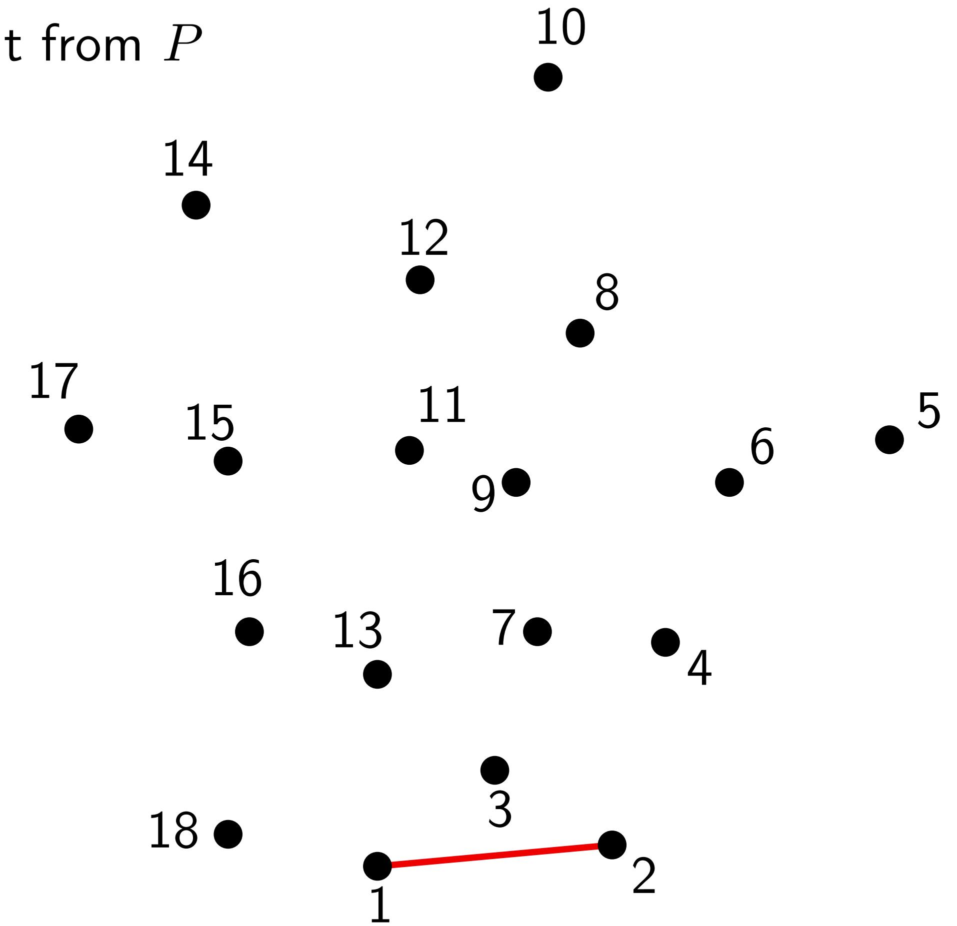

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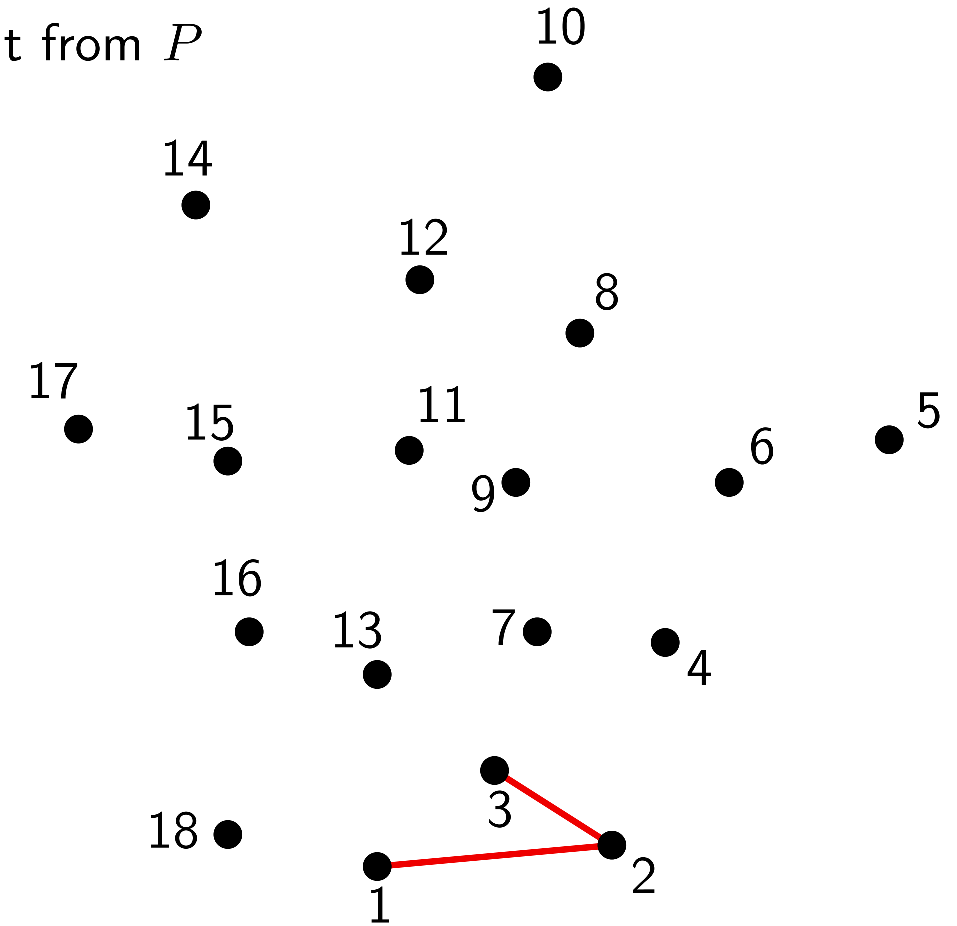

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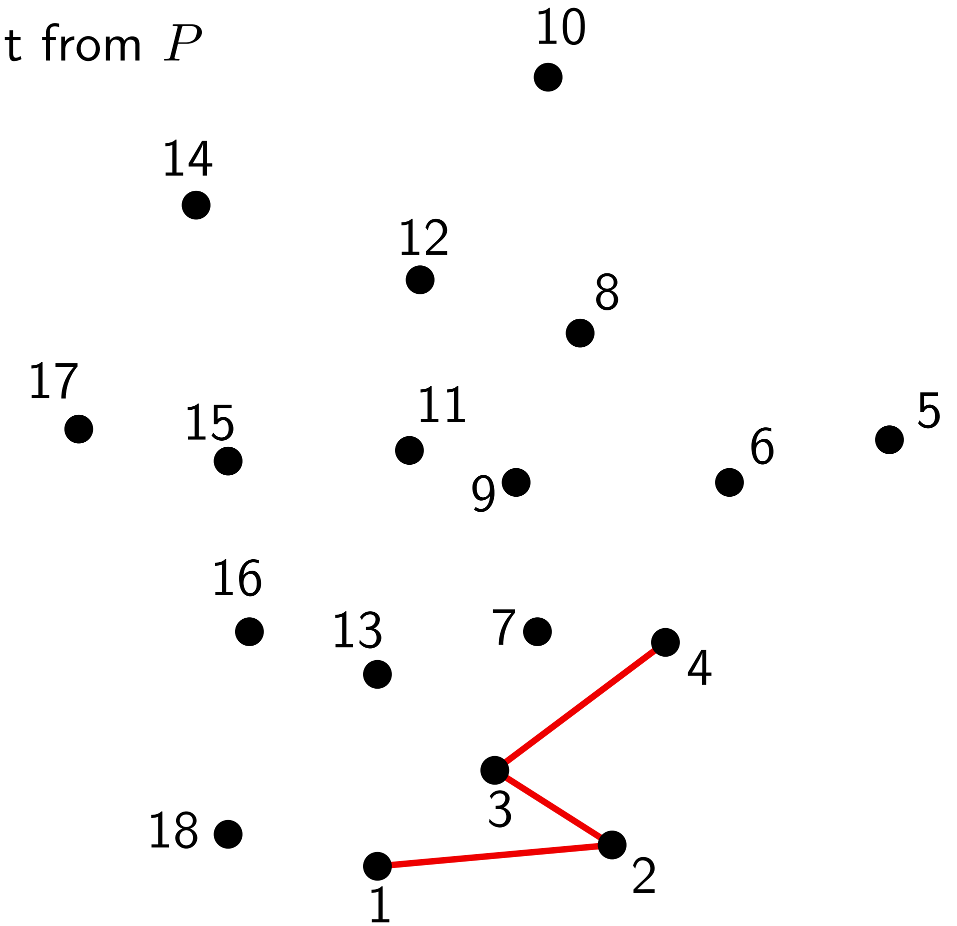

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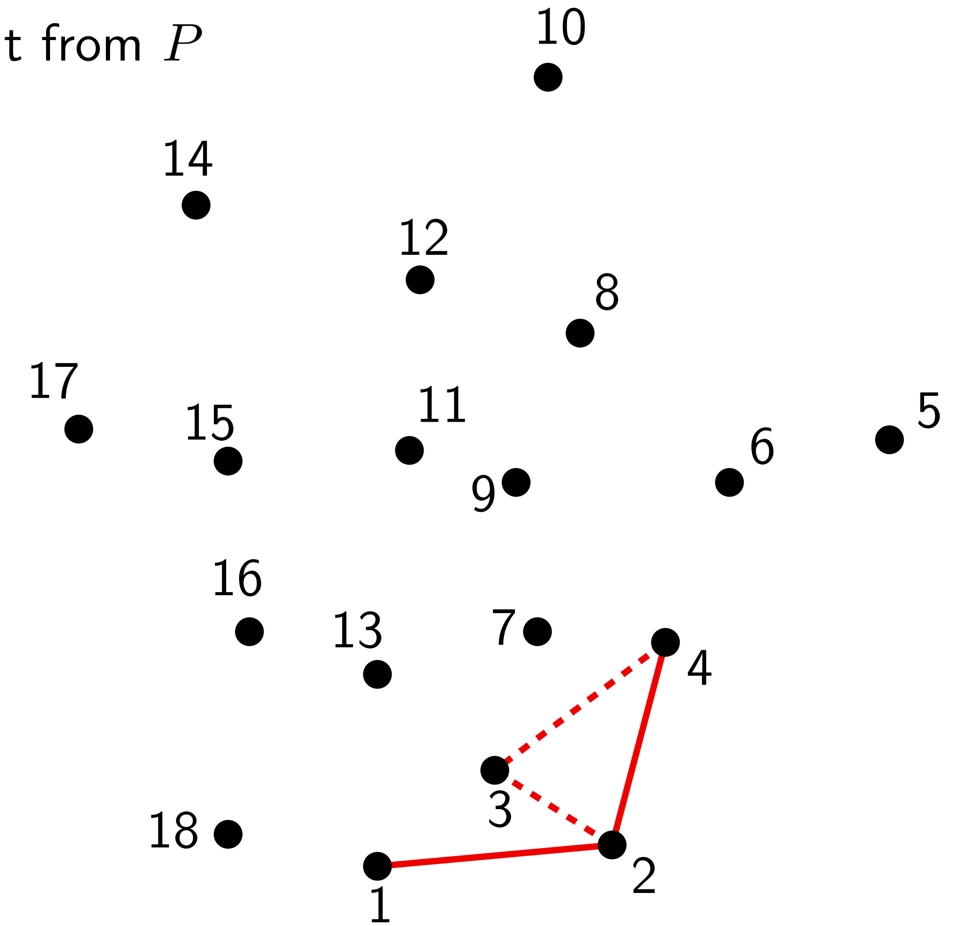

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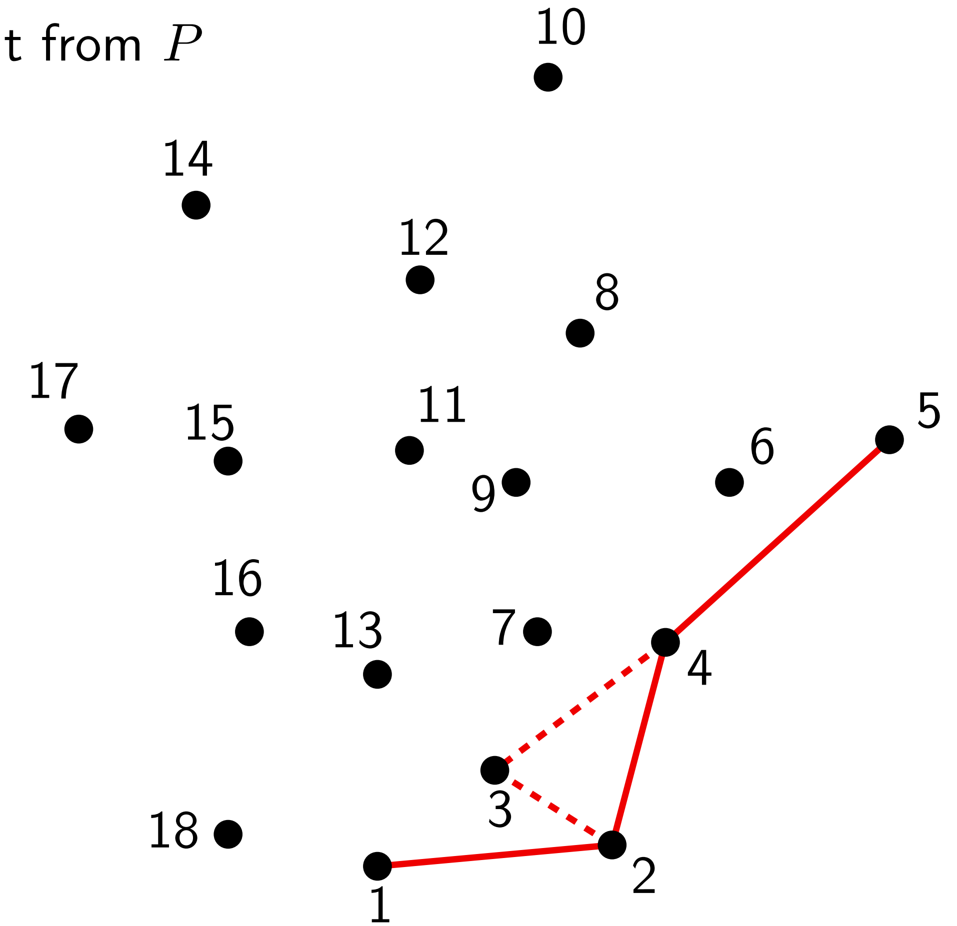

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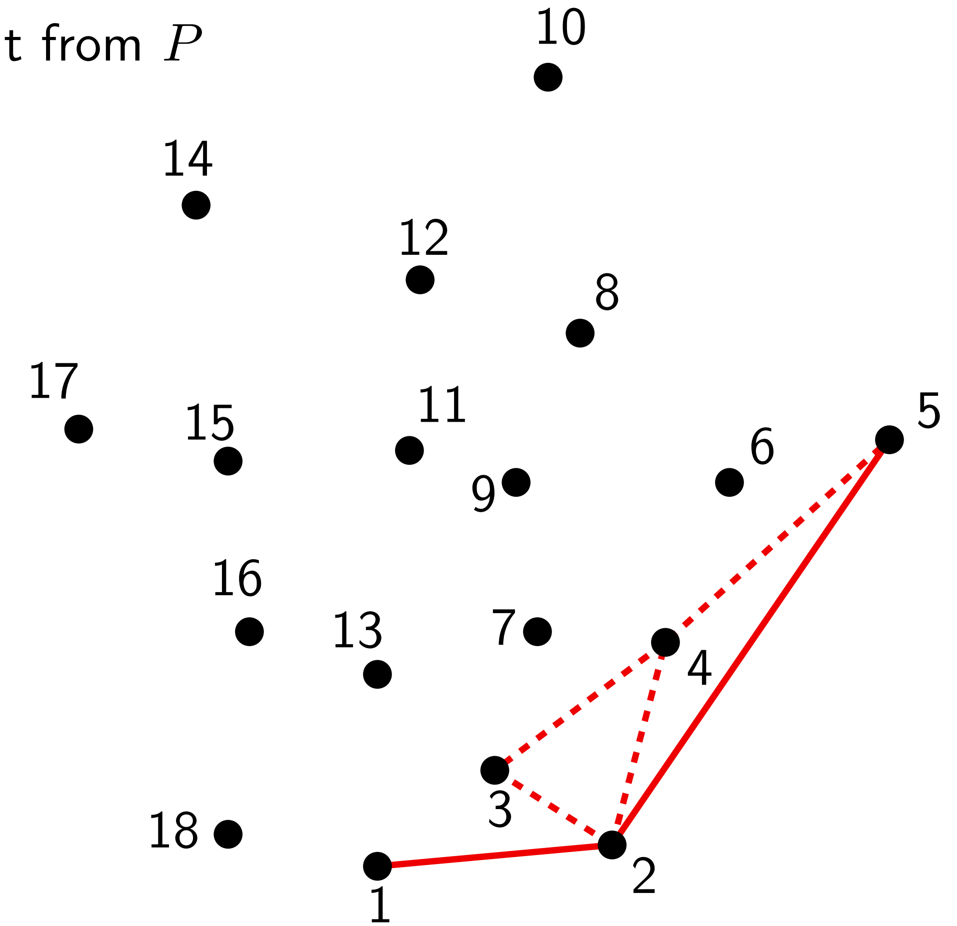

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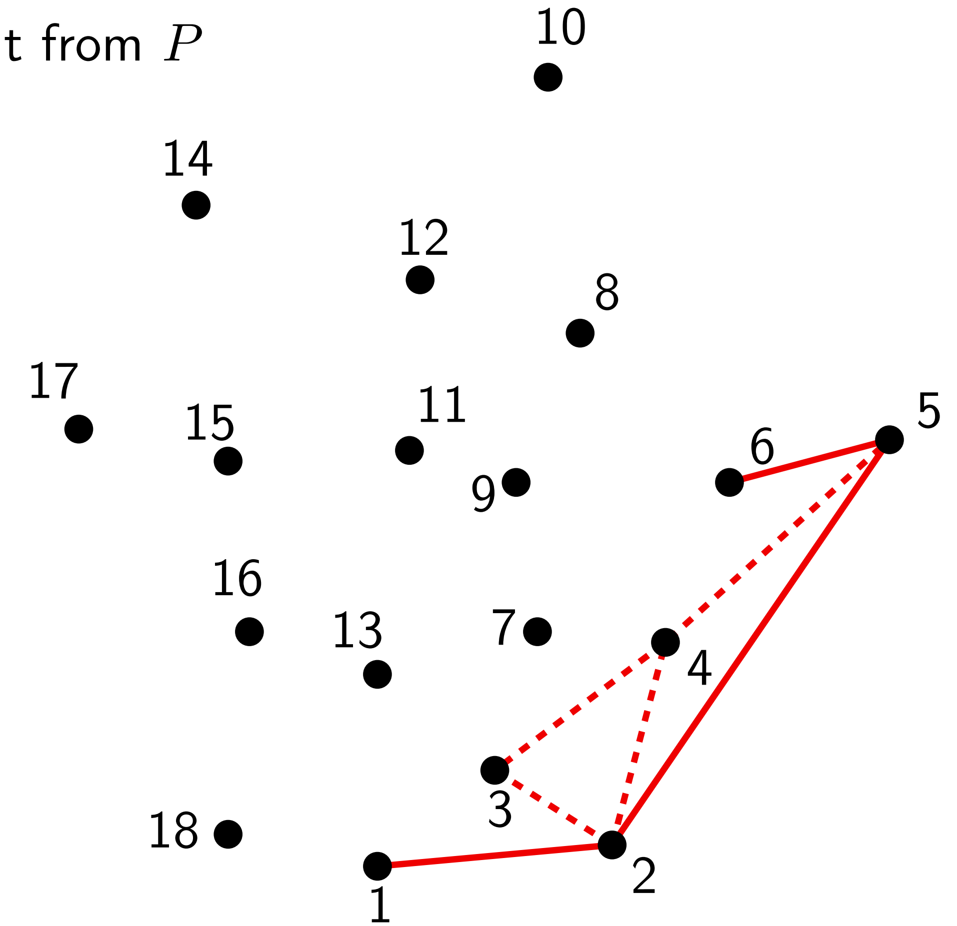

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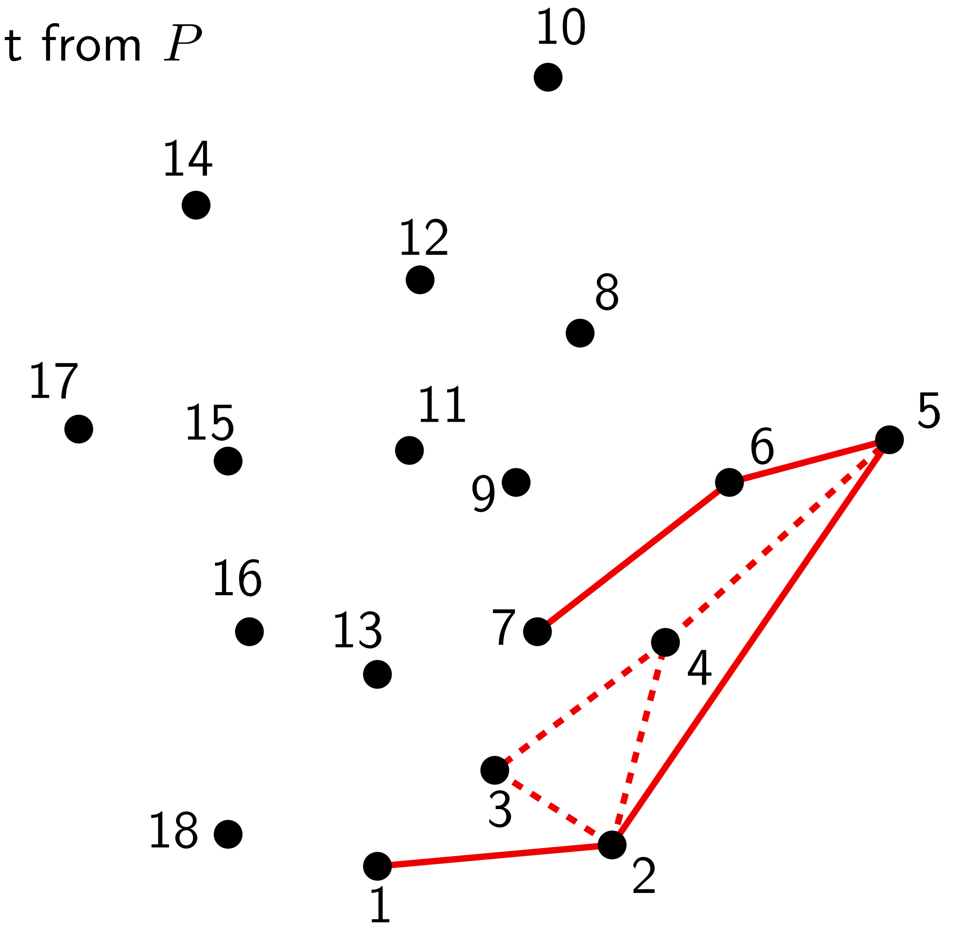

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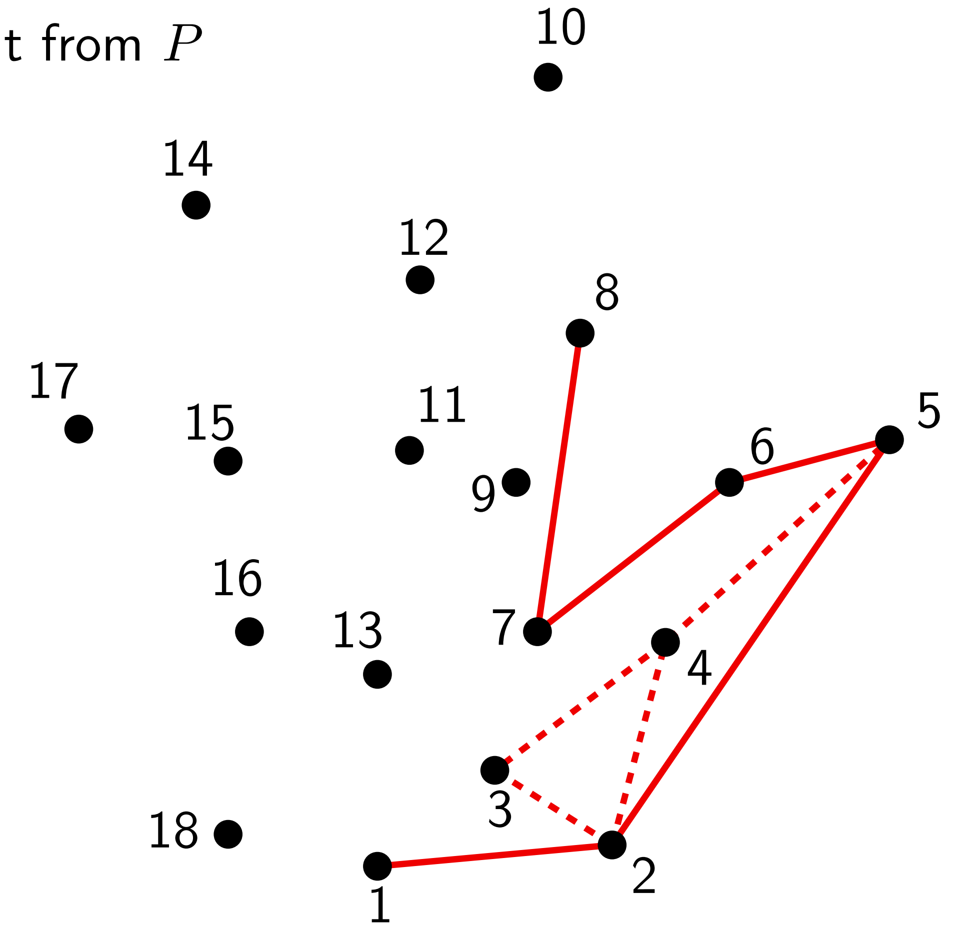
Return l



Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete if from P

Advance

While there exist points $p_i \in P$ to be explored, do:

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$p^- = \mathrm{previous}(\mathrm{top}(l))$

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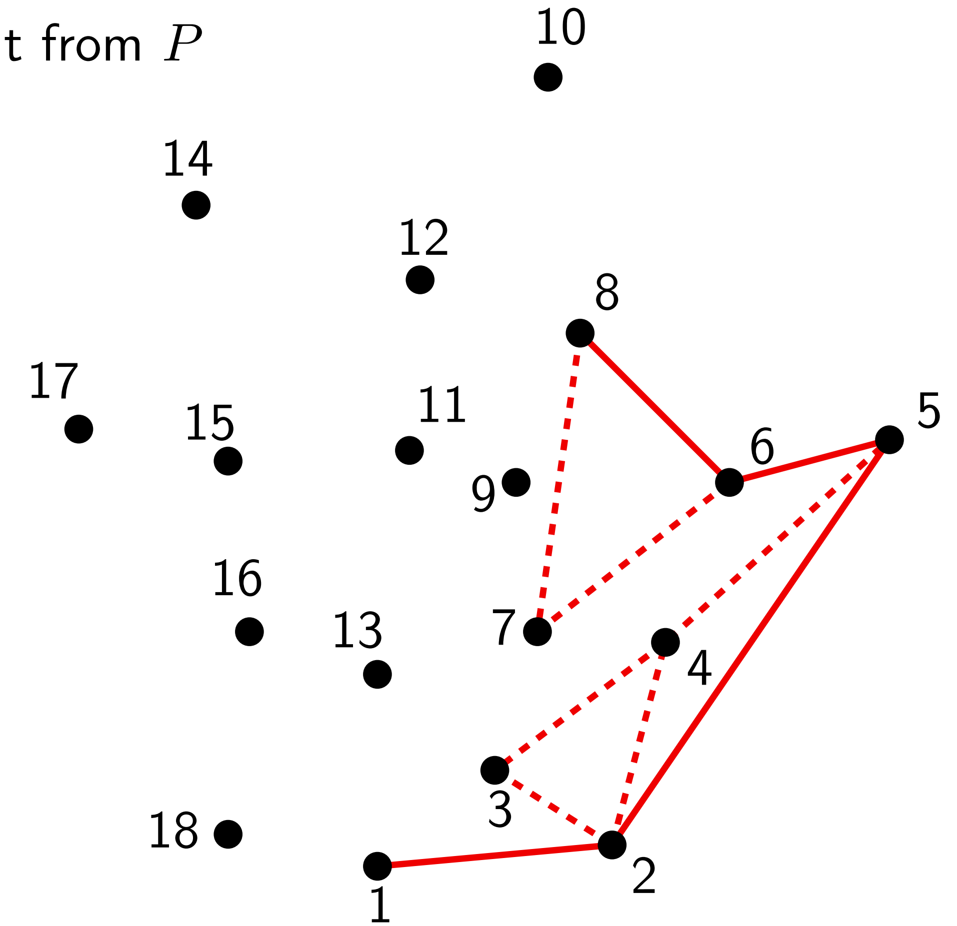

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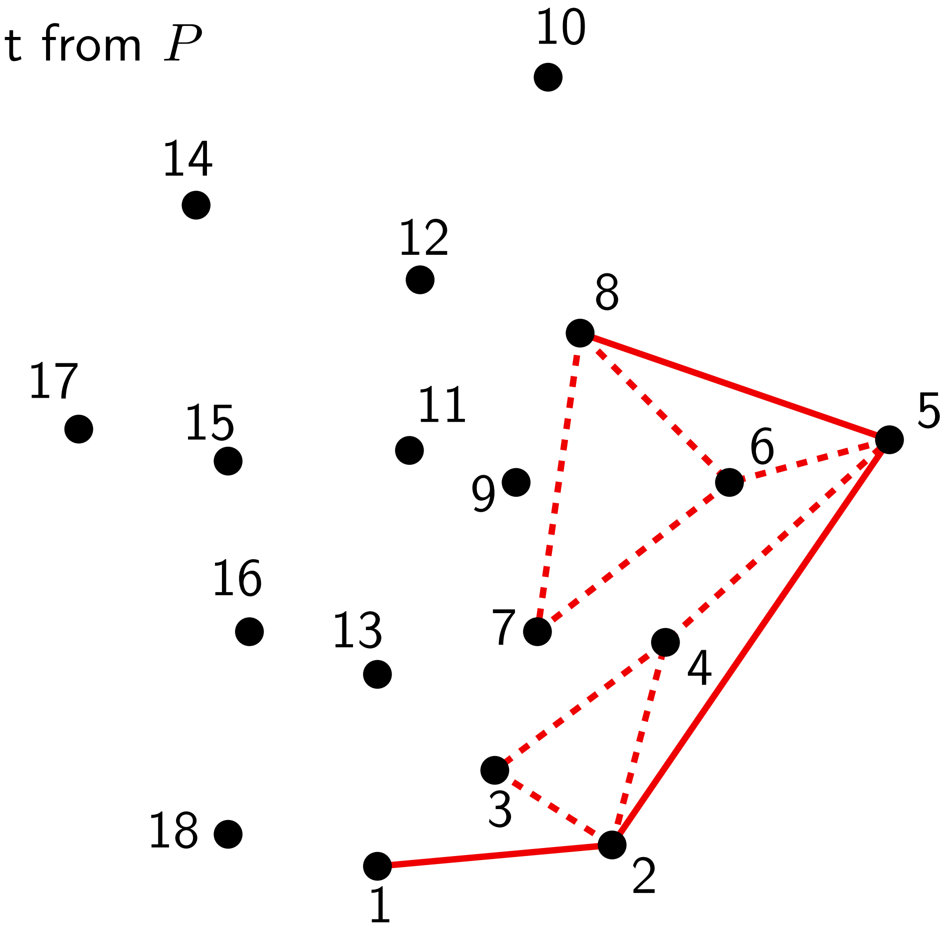

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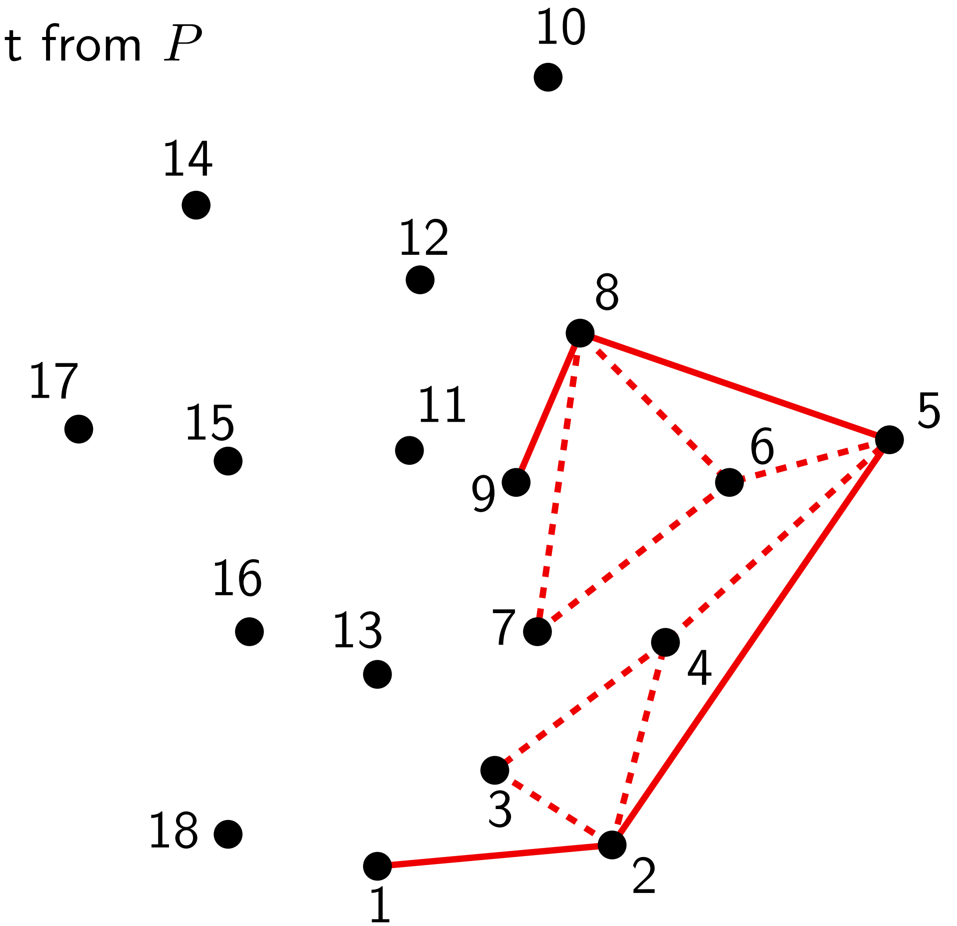

Graham's algorithm

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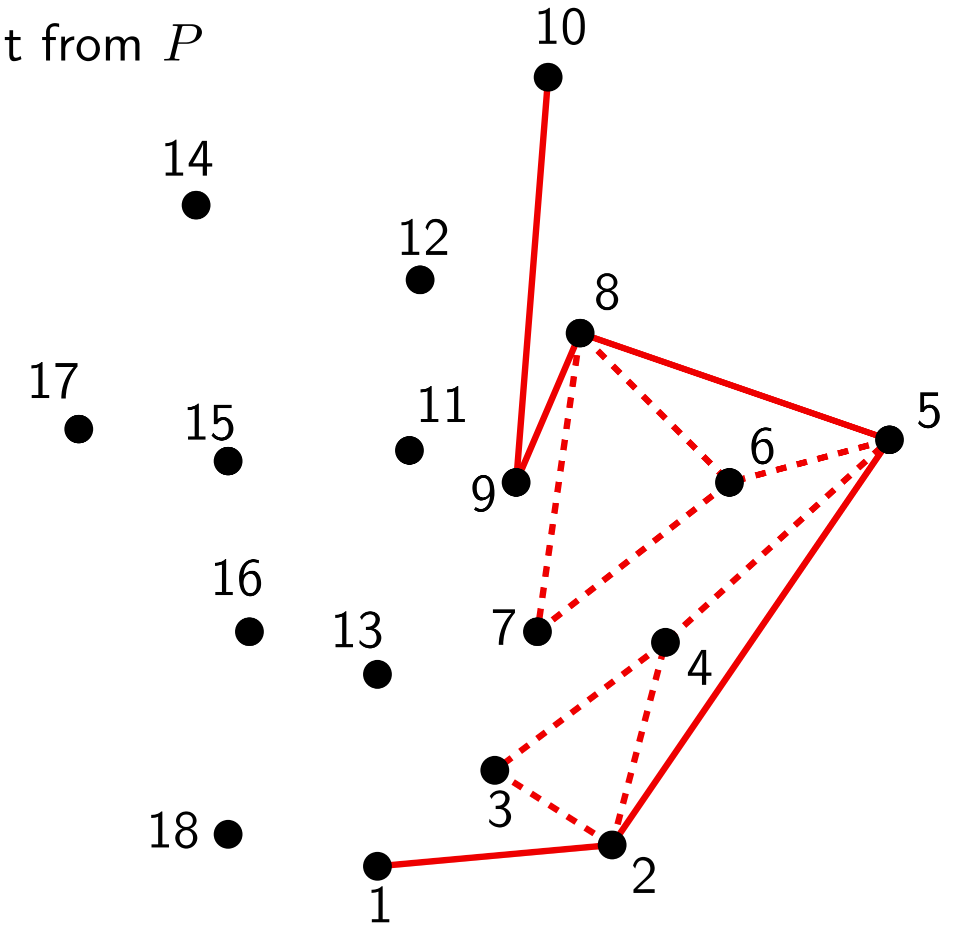

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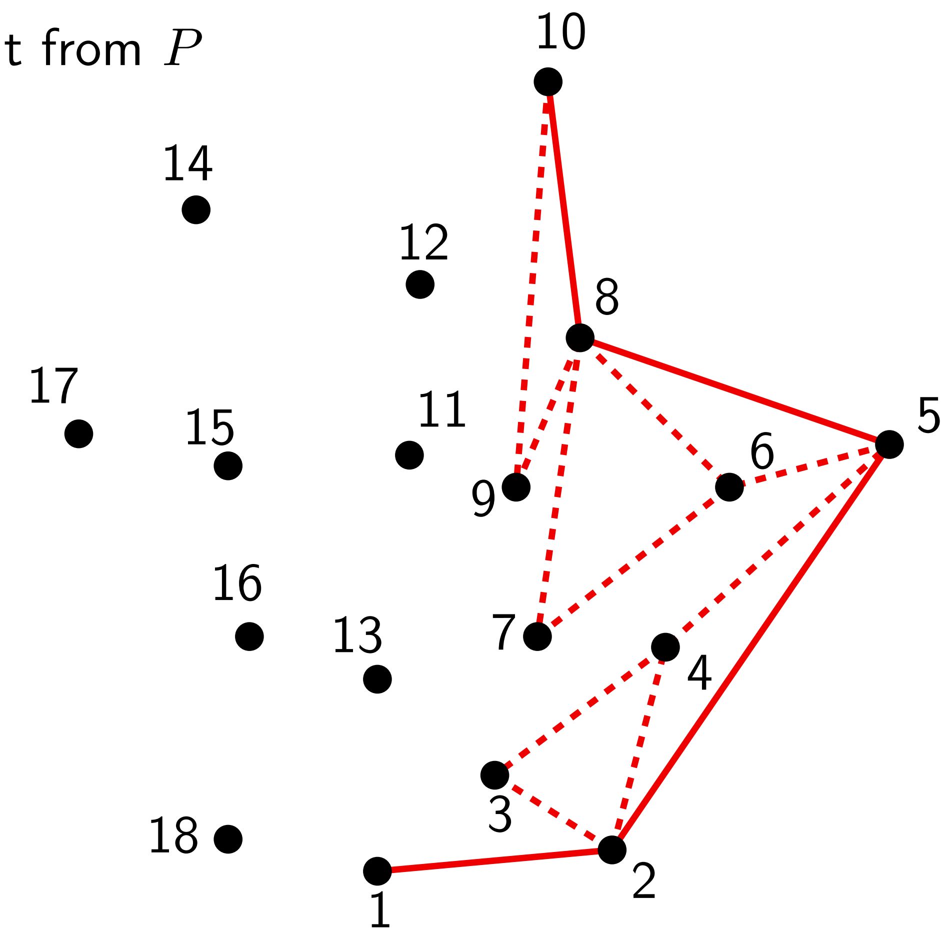

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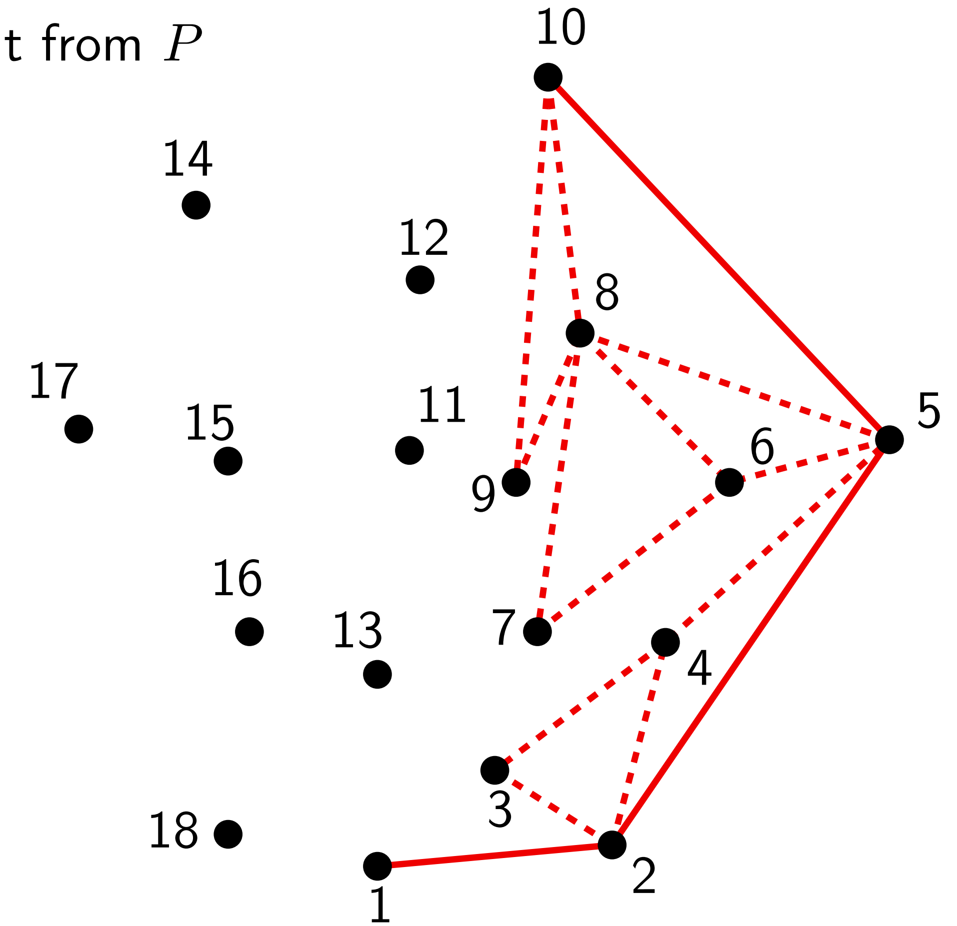

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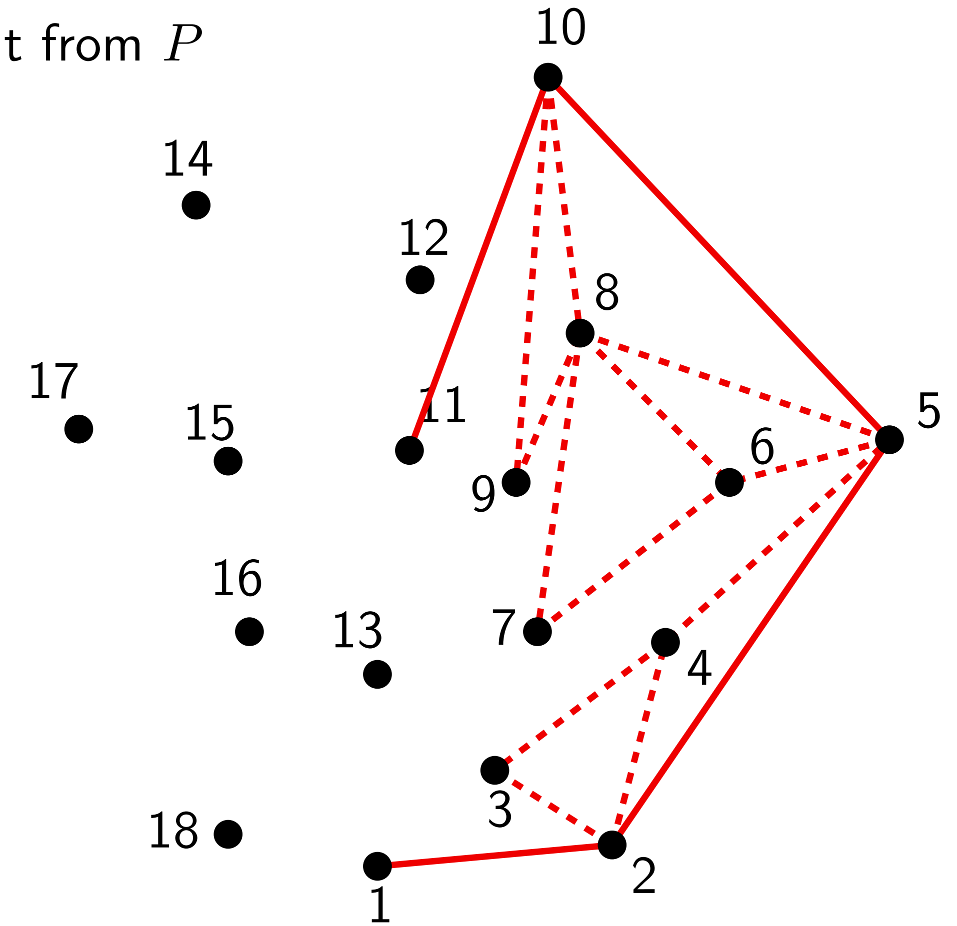

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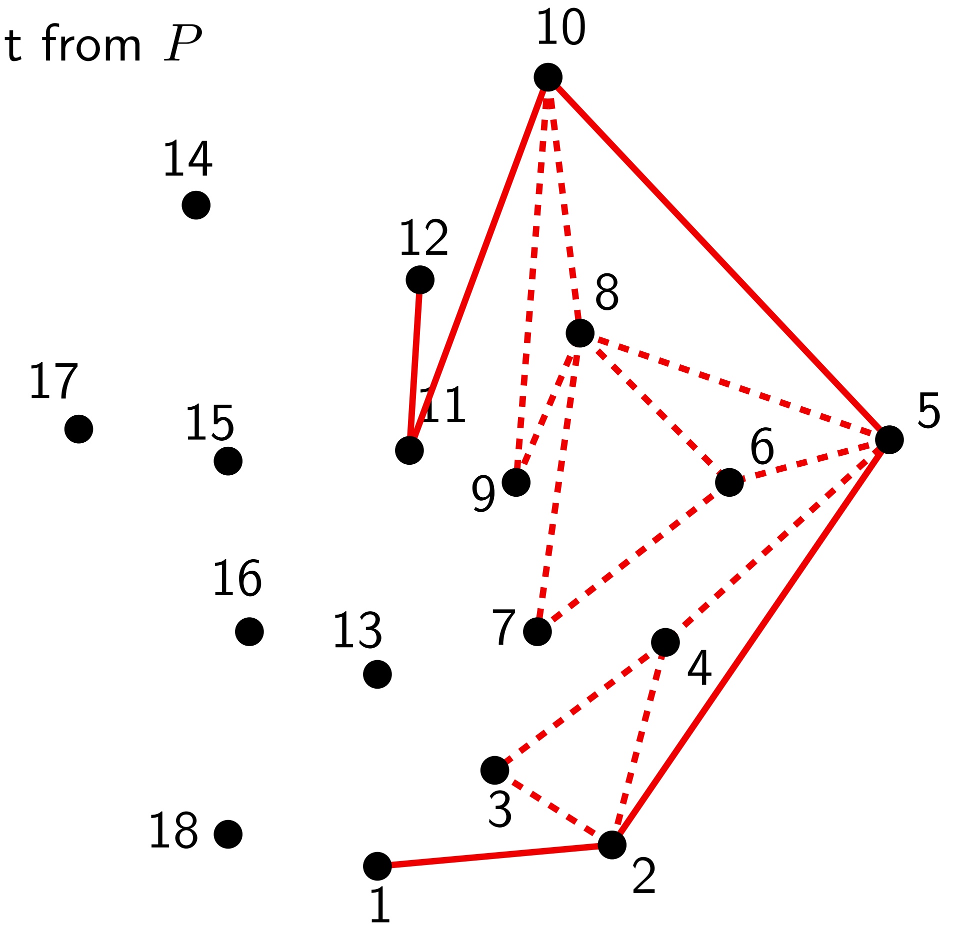

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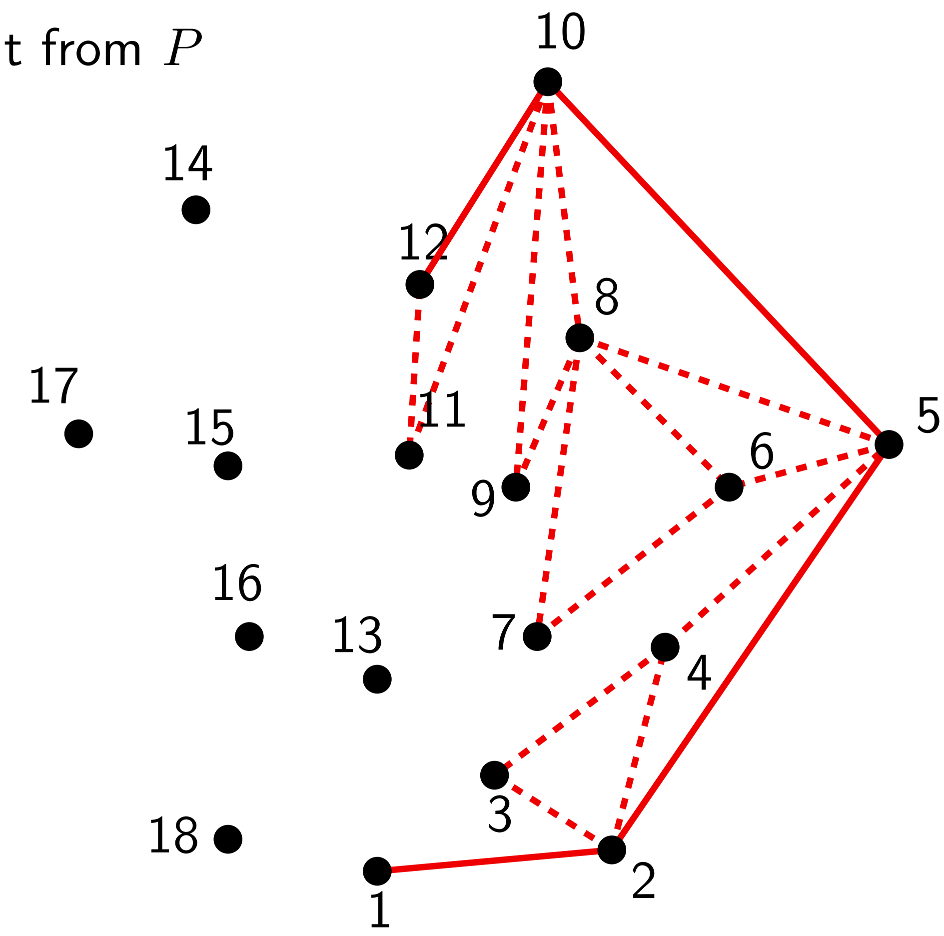

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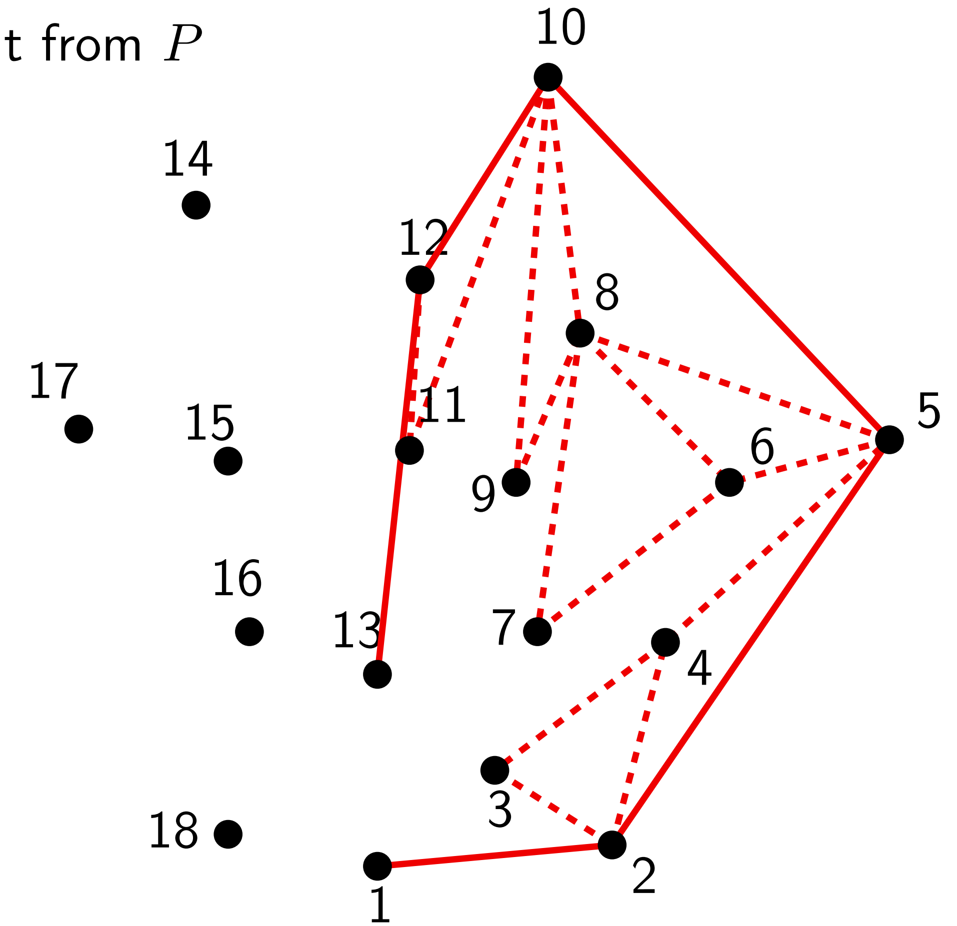

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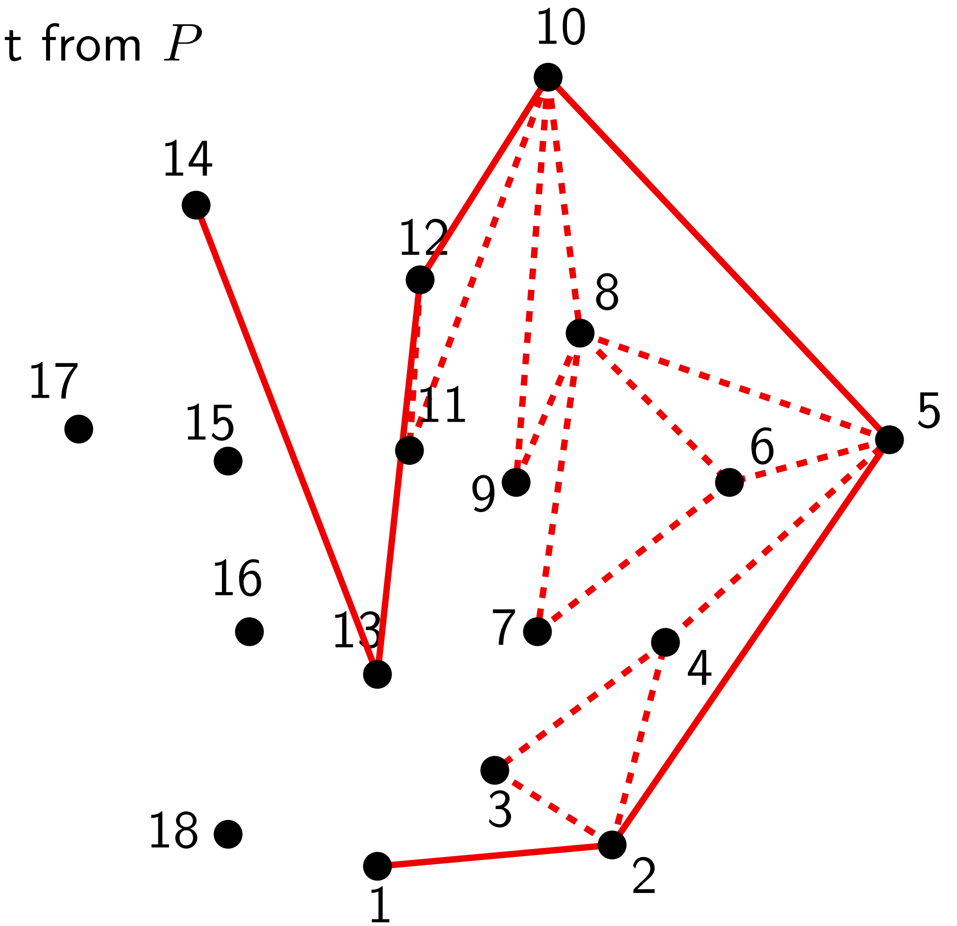

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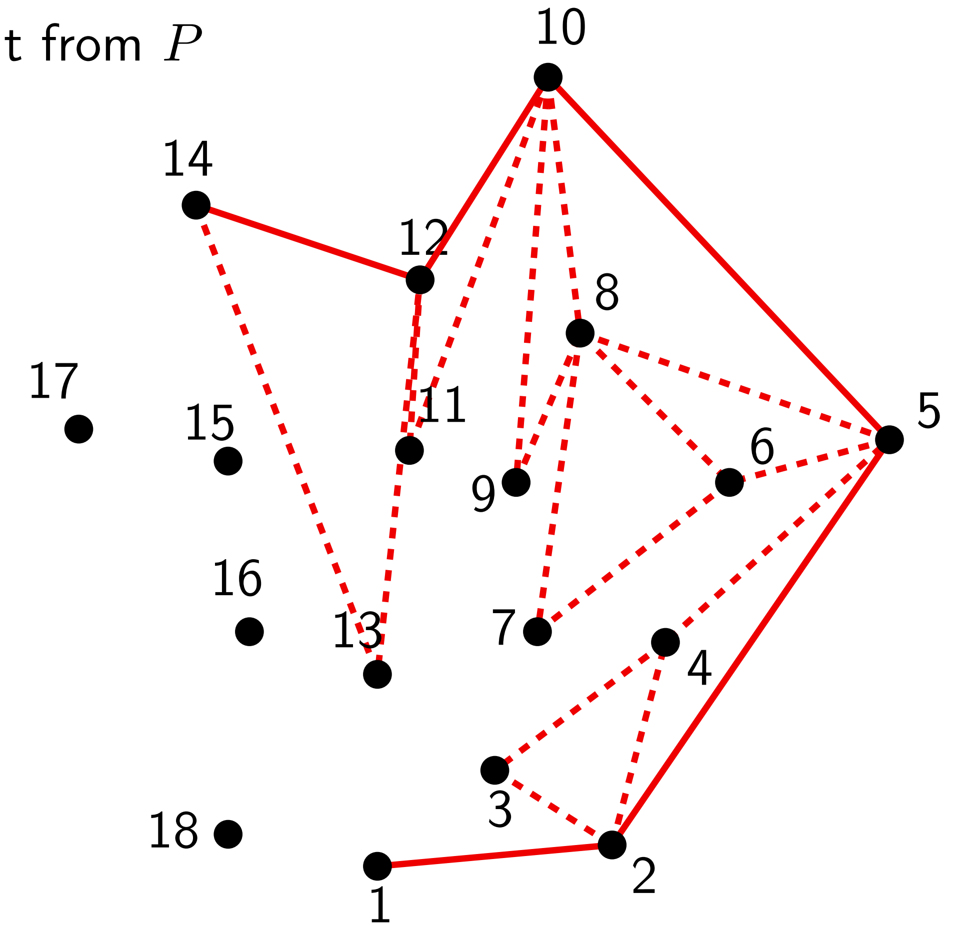

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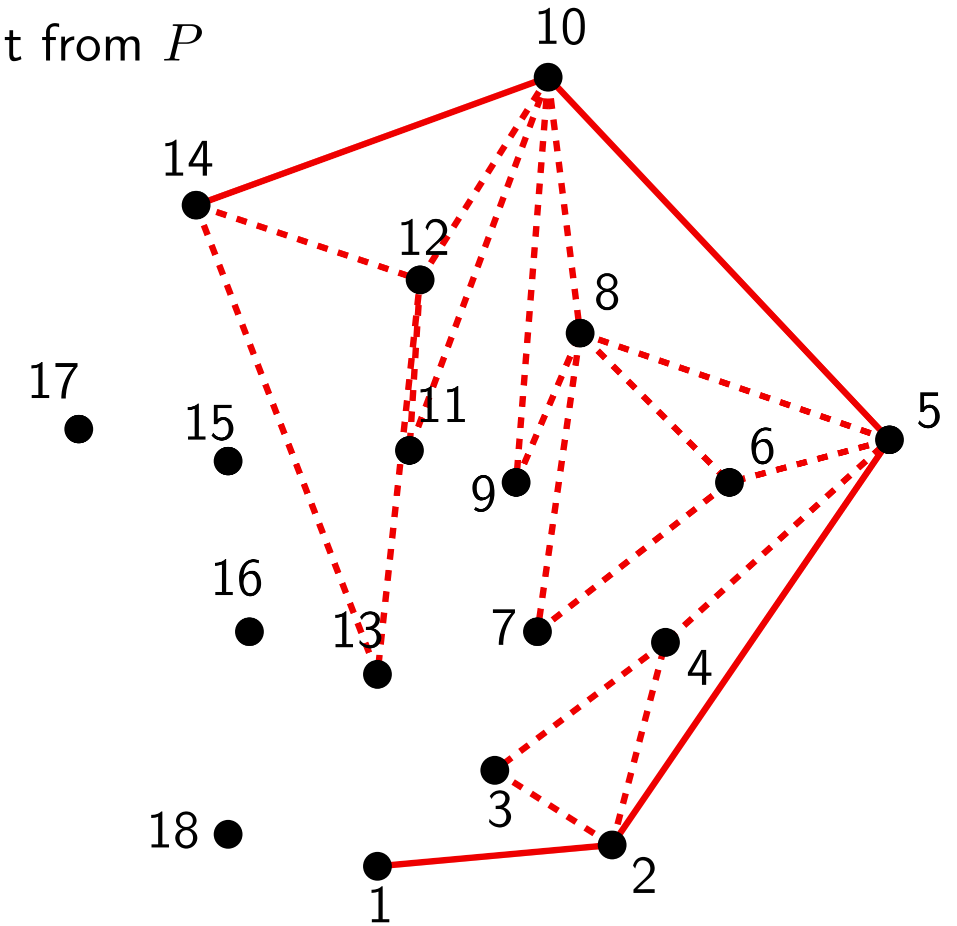

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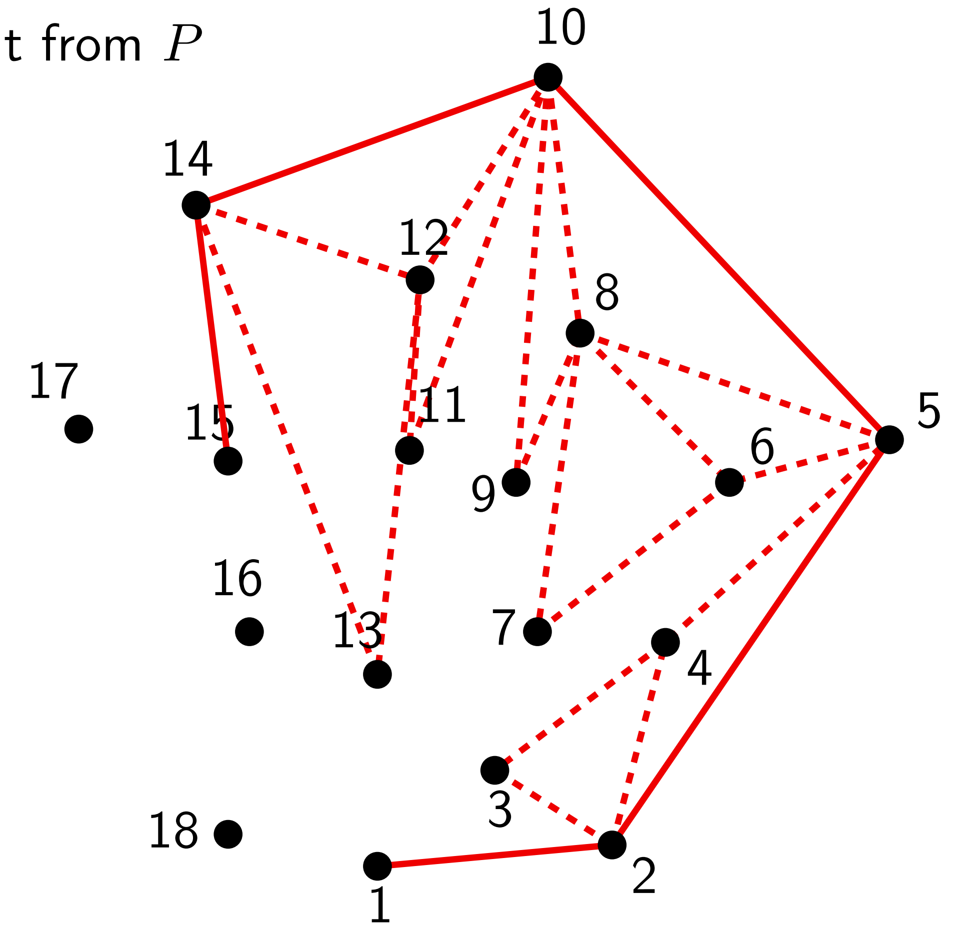

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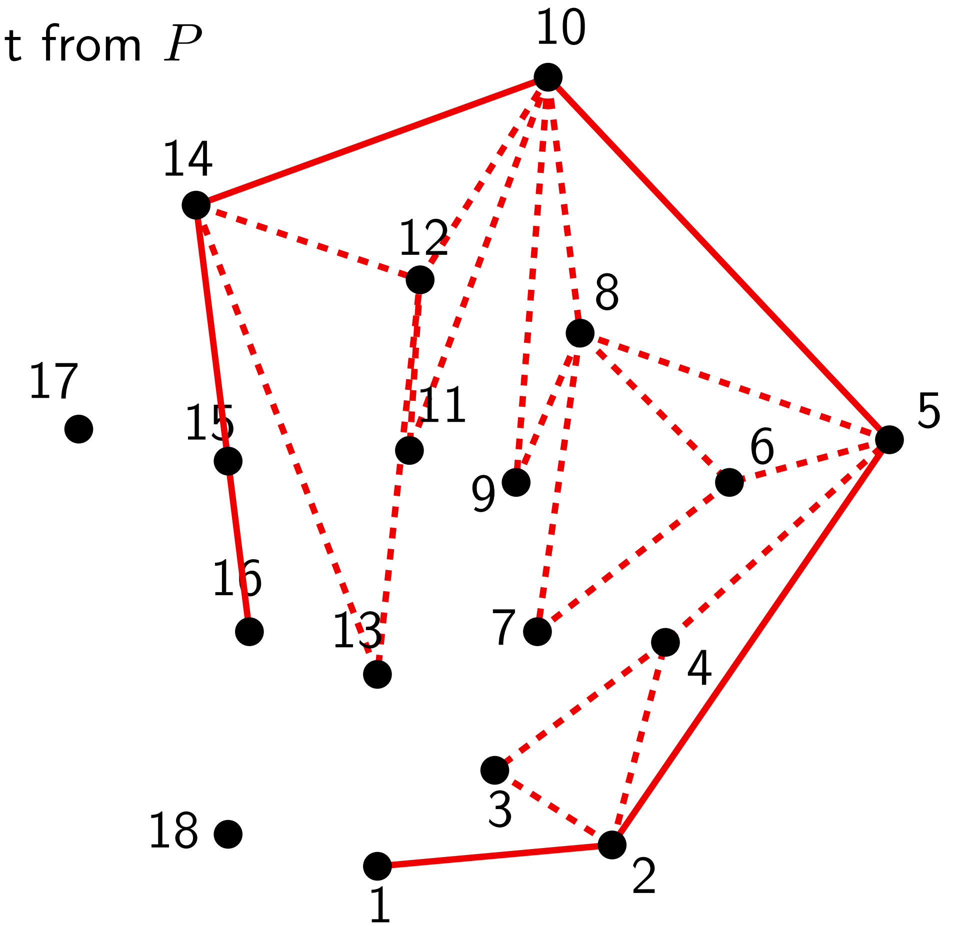

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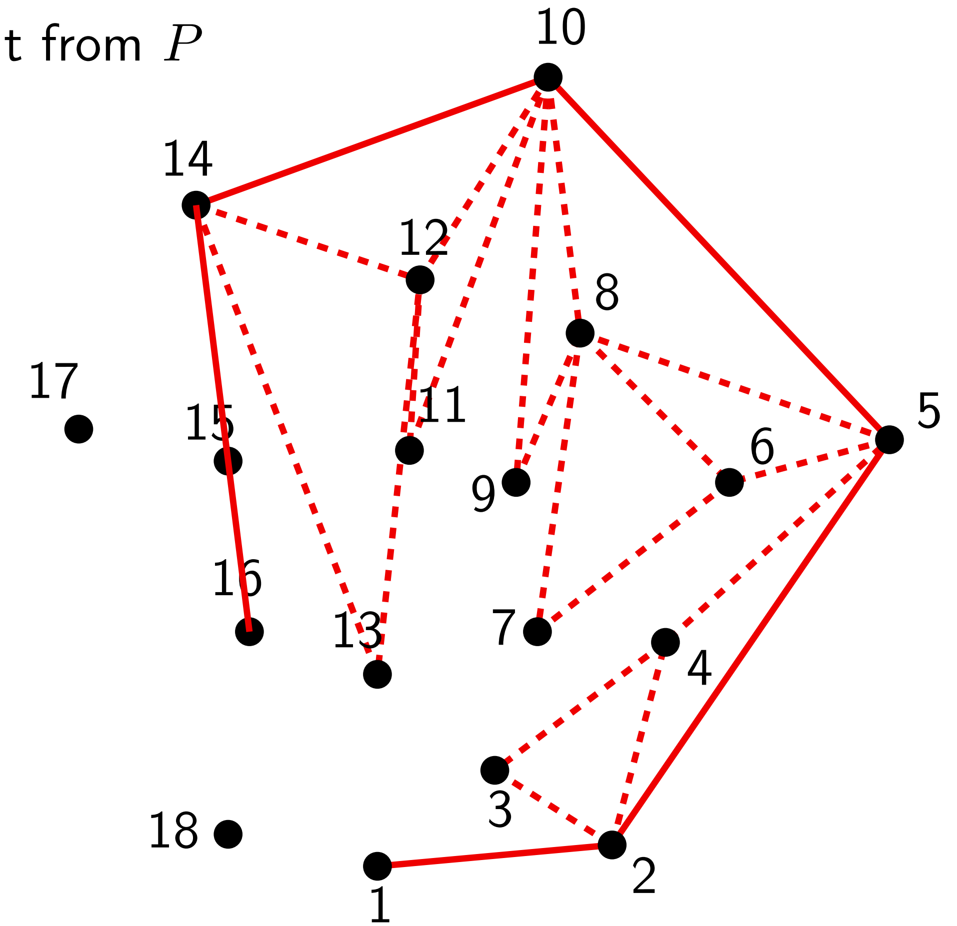

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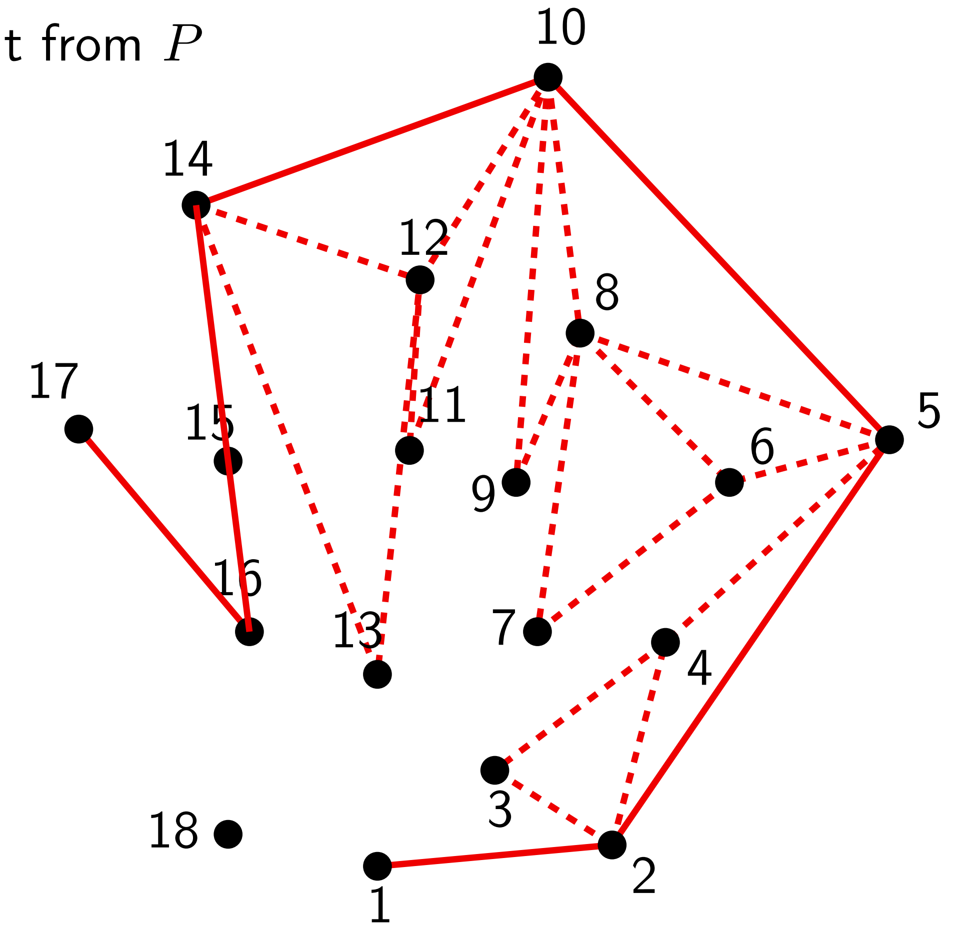

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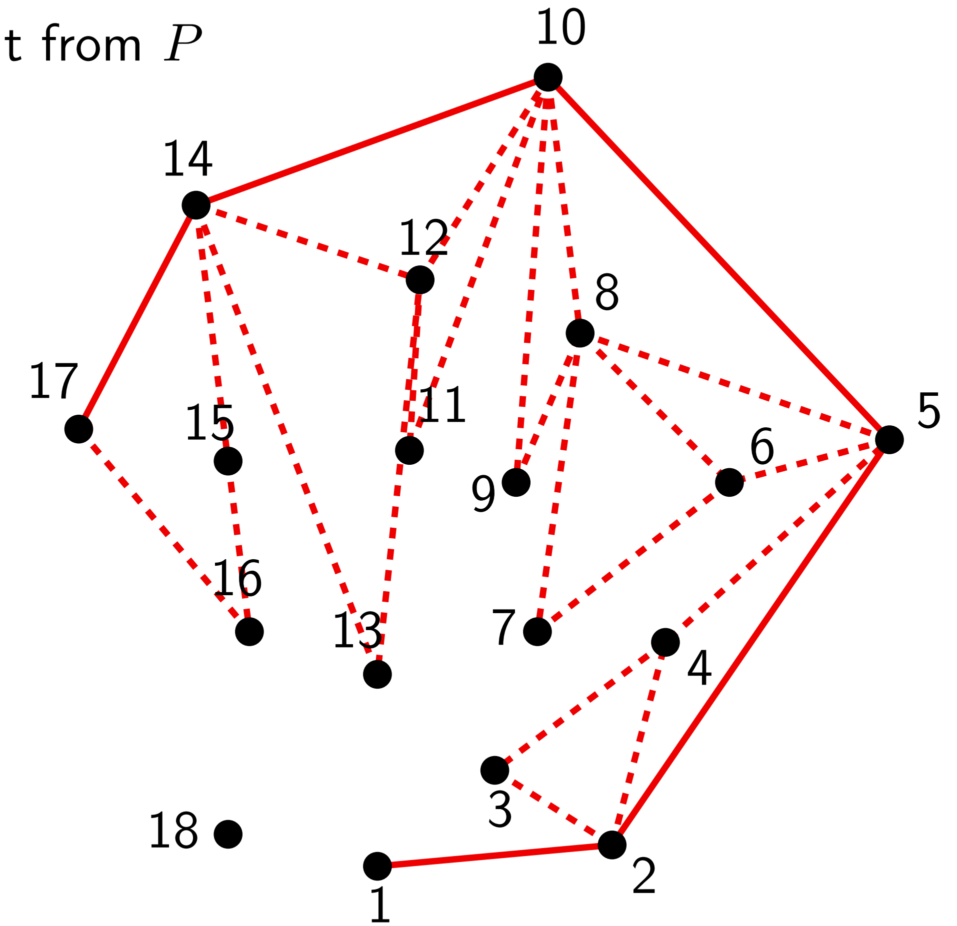

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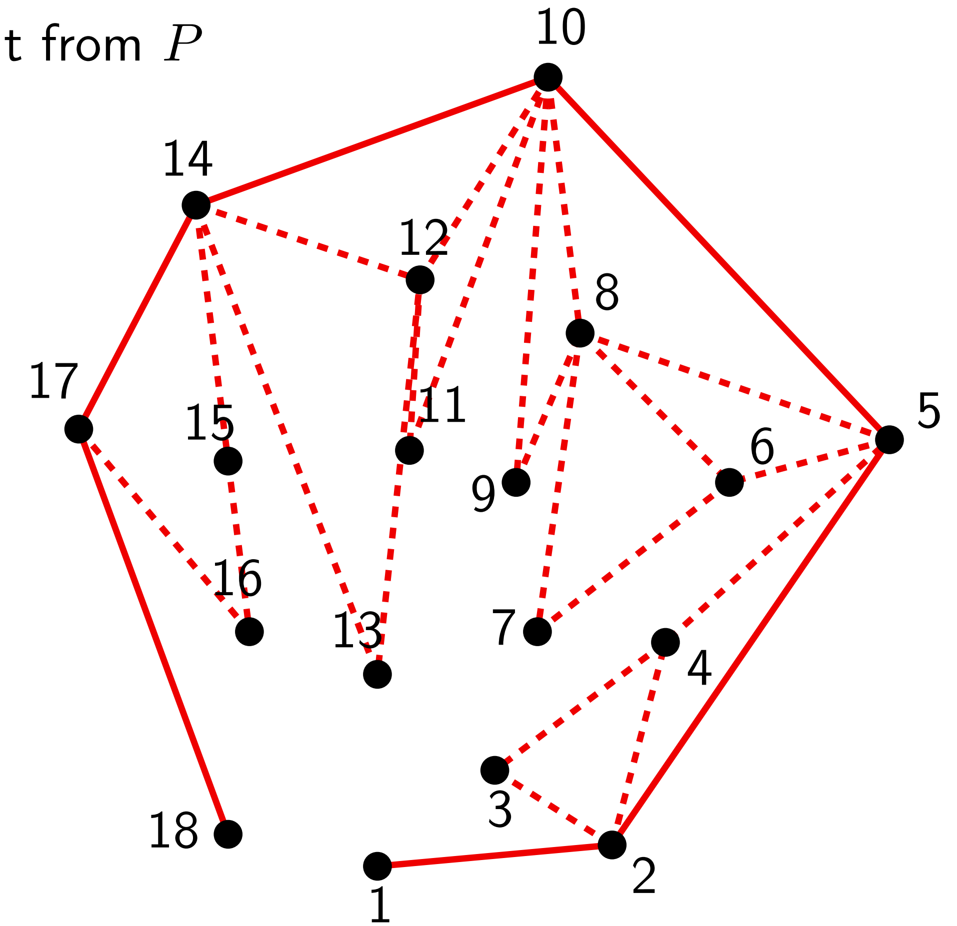

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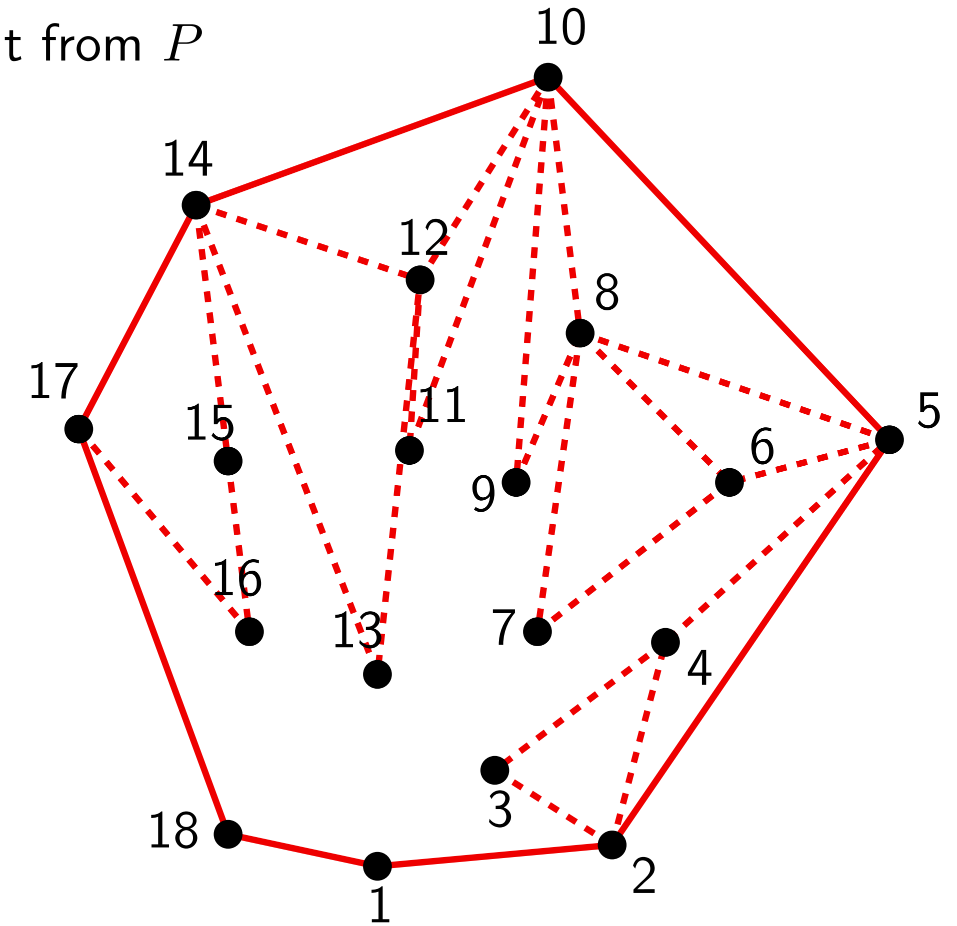

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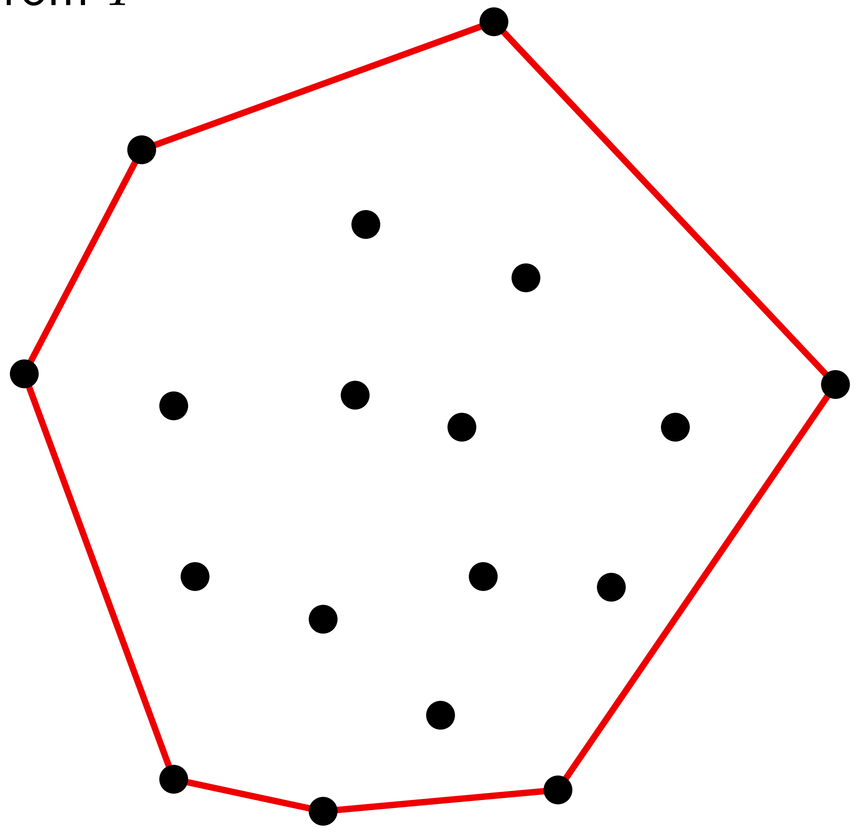

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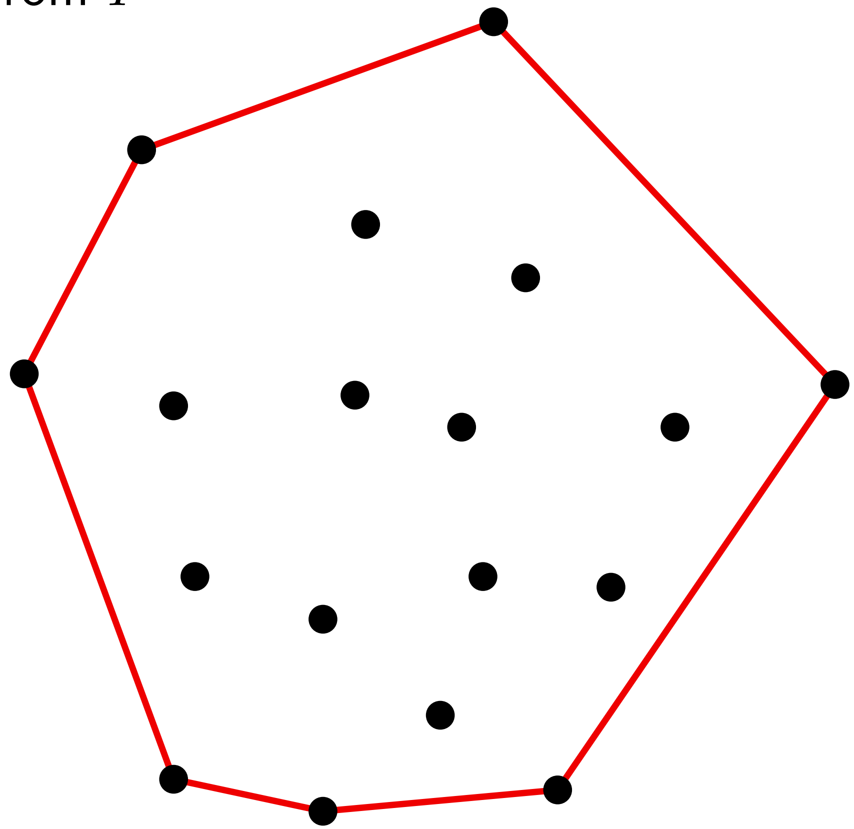

Running time: $O(n \log n)$

Incremental algorithm

Incremental algorithm

Initialization

$l = p_1, p_2, p_3$

Advance

From $i = 4$ to n, do:

If p_i lies in the exterior of the polygon defined by l:

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain $p_l, \ldots, p_r$ in l with the chain p_l, p_i, p_r

Return l

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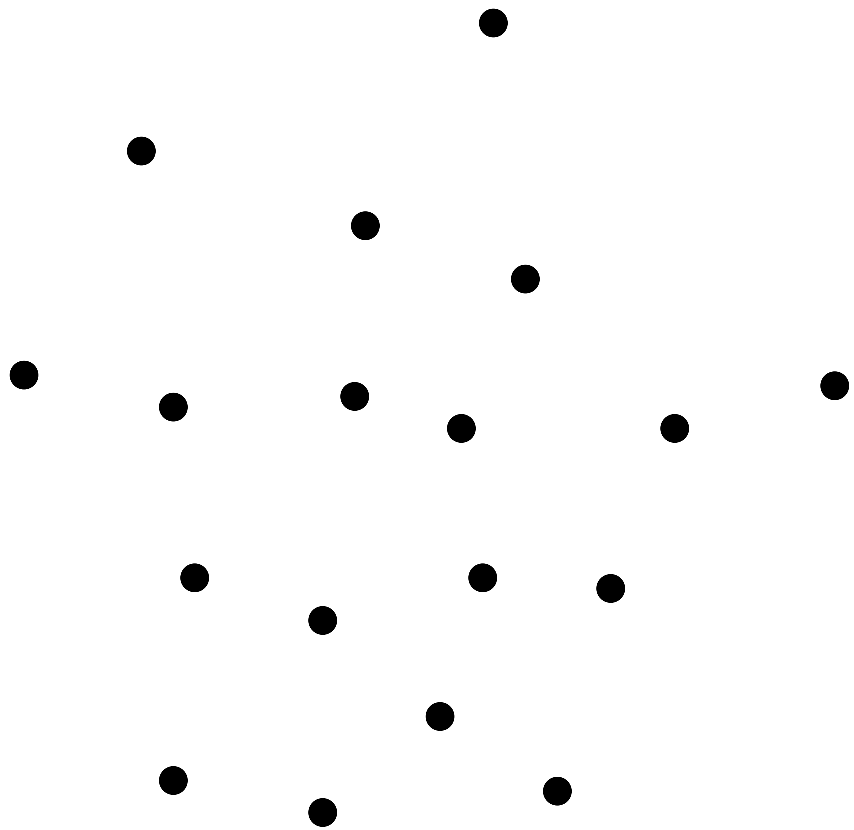

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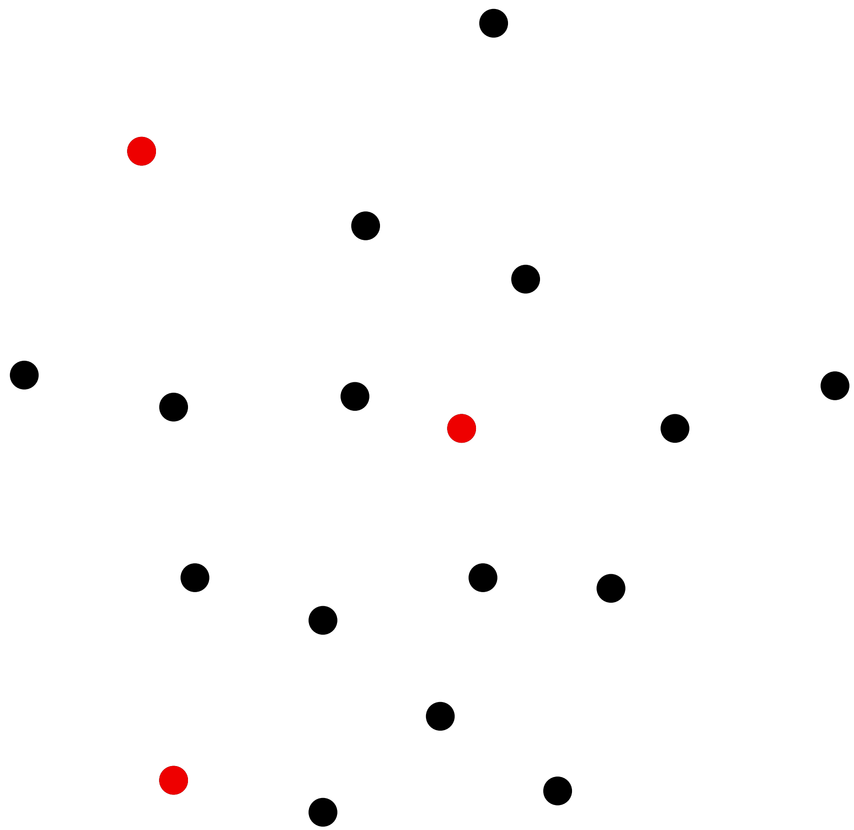

CONVEX HULL

Incremental algorithm

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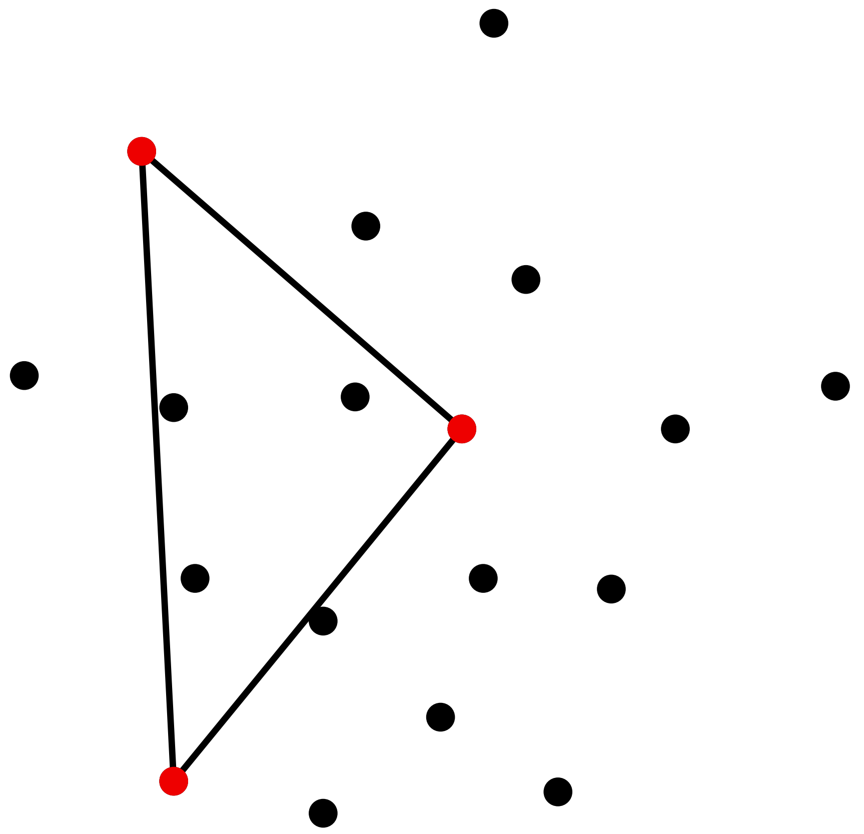

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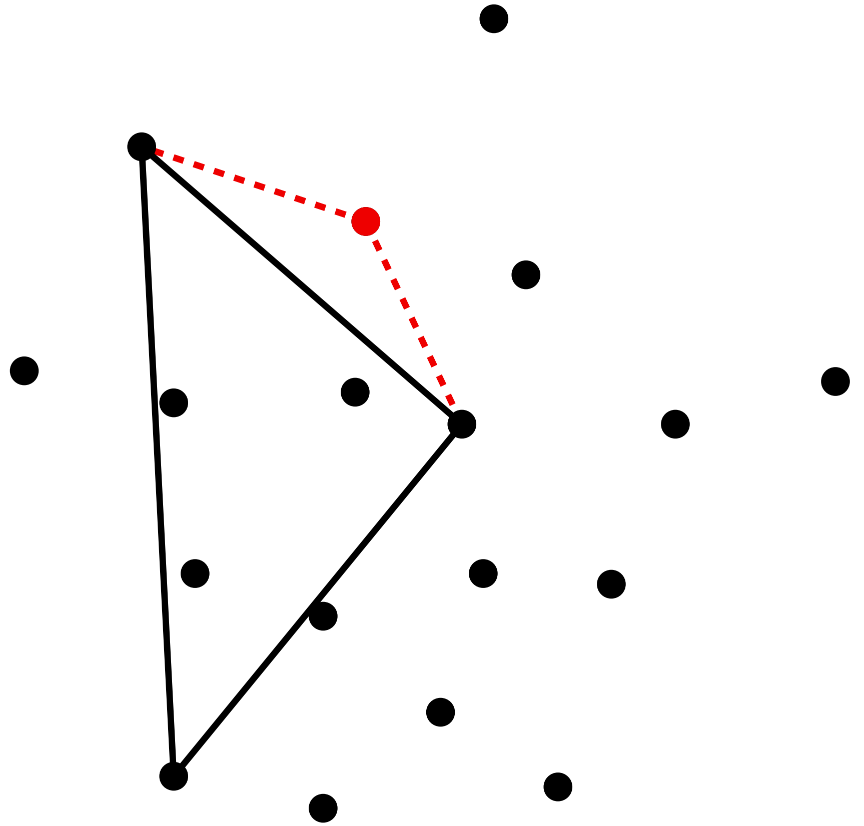

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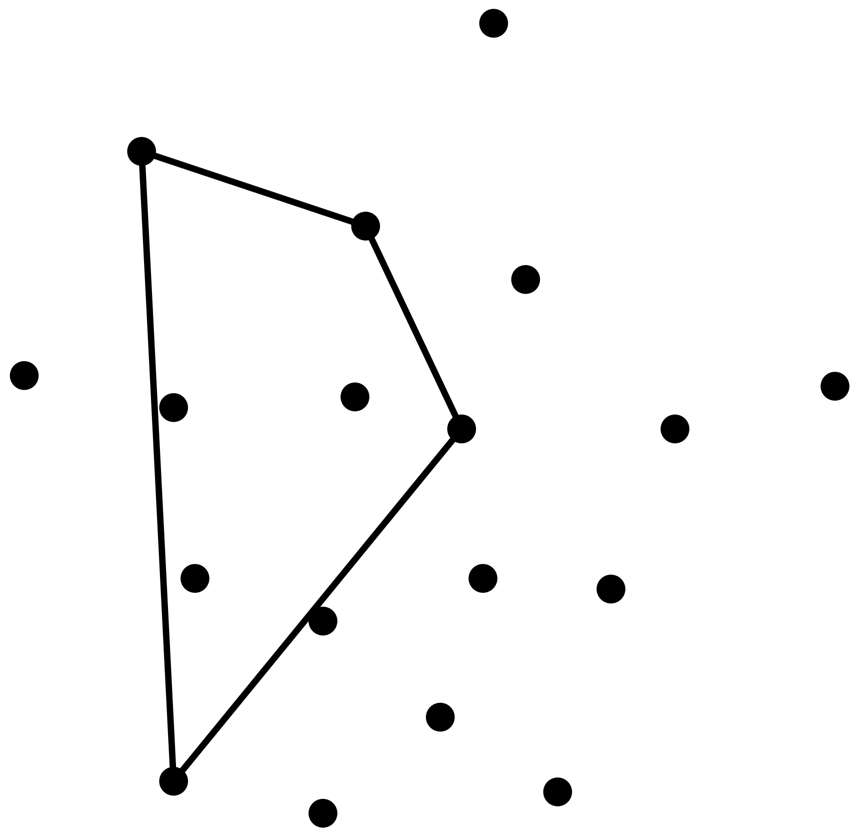

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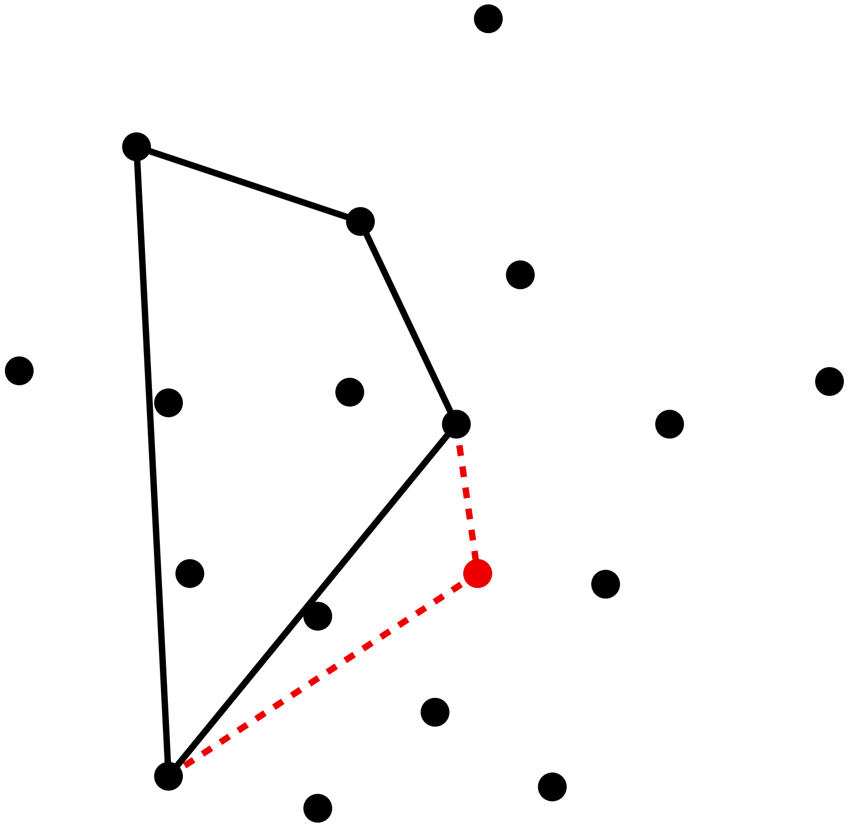

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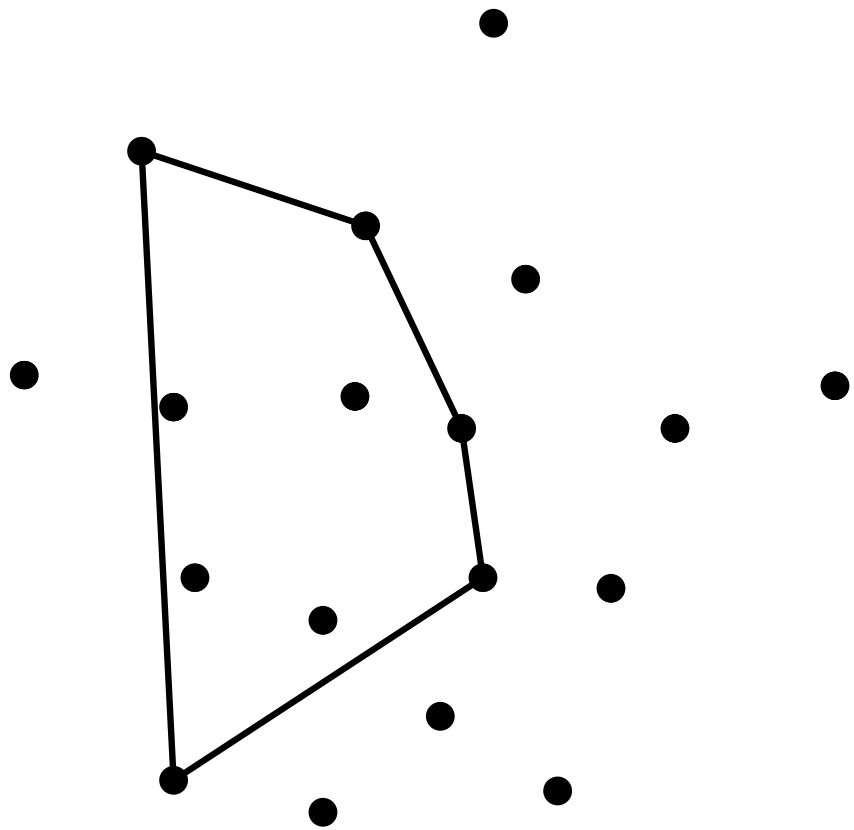

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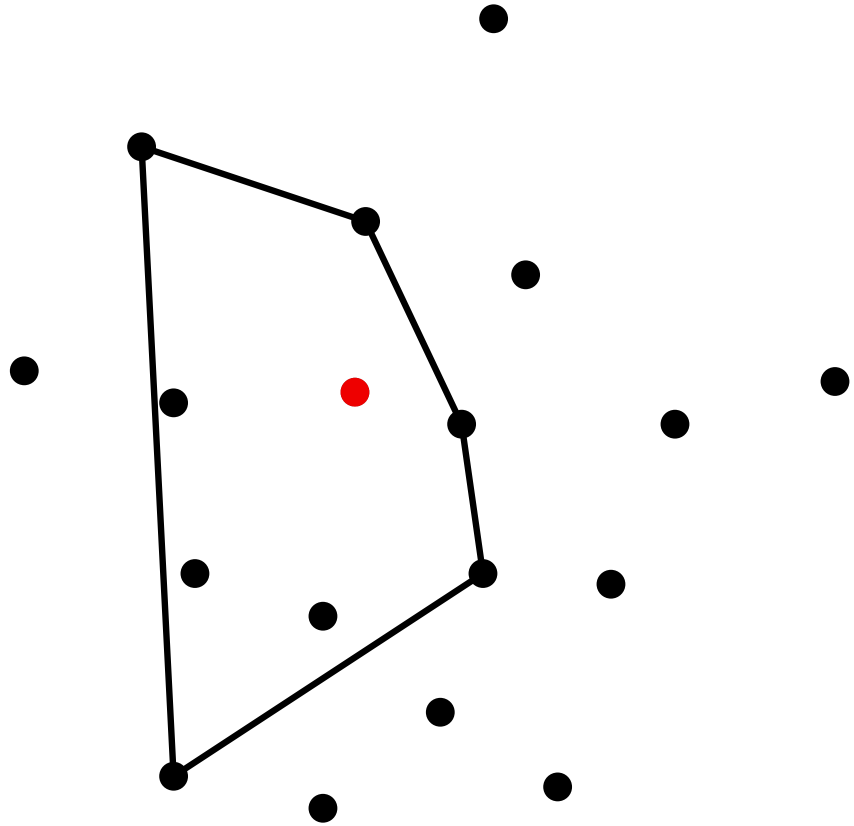

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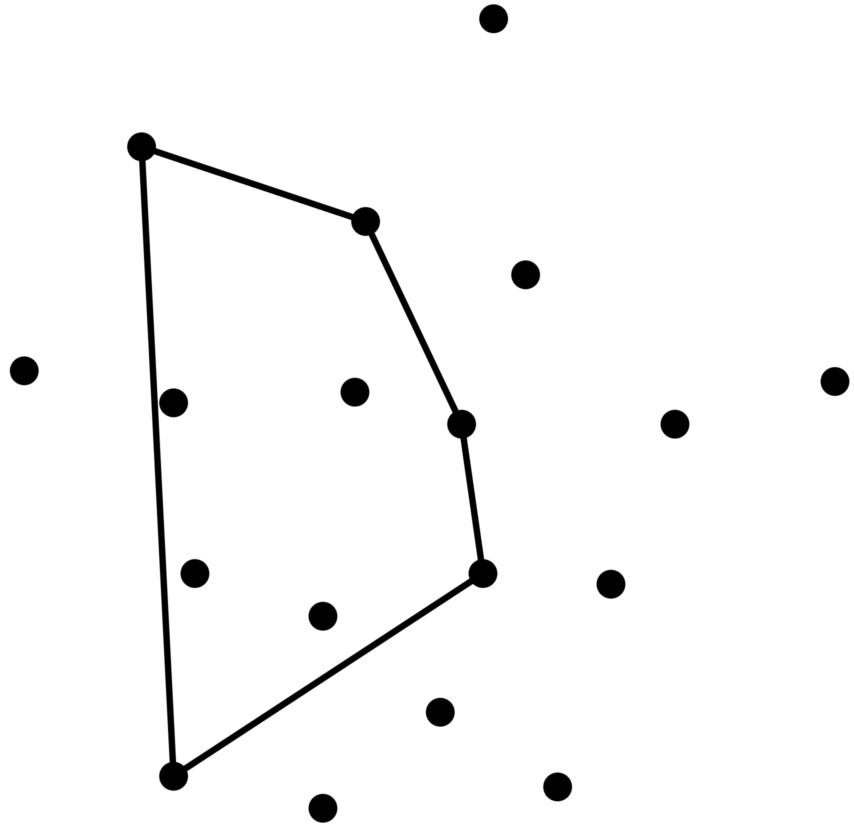

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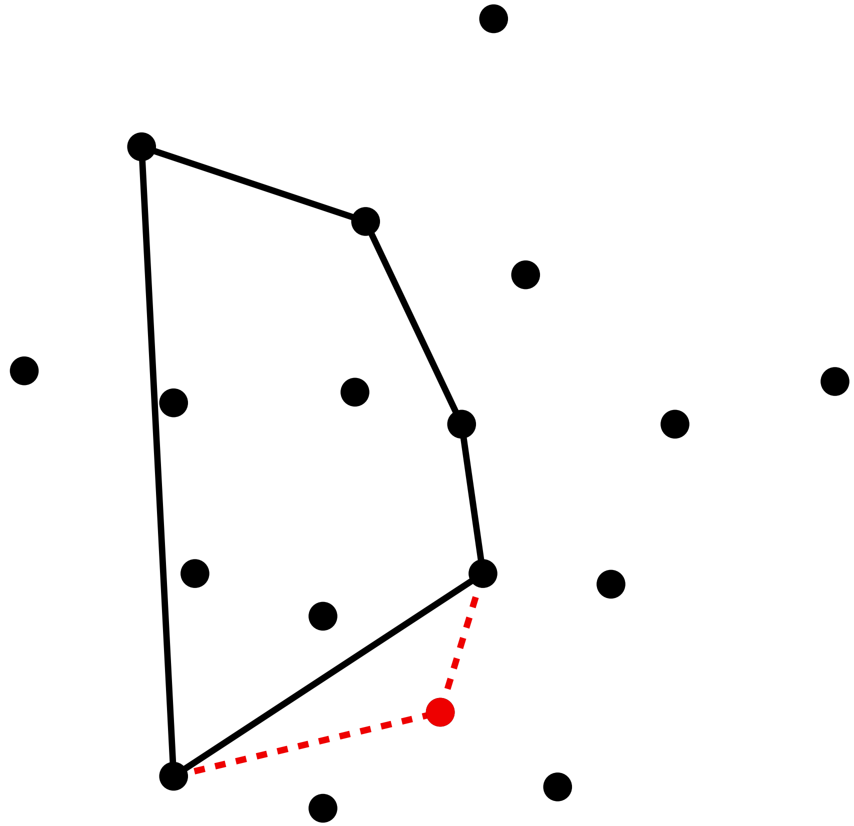

Incremental algorithm

Initialization

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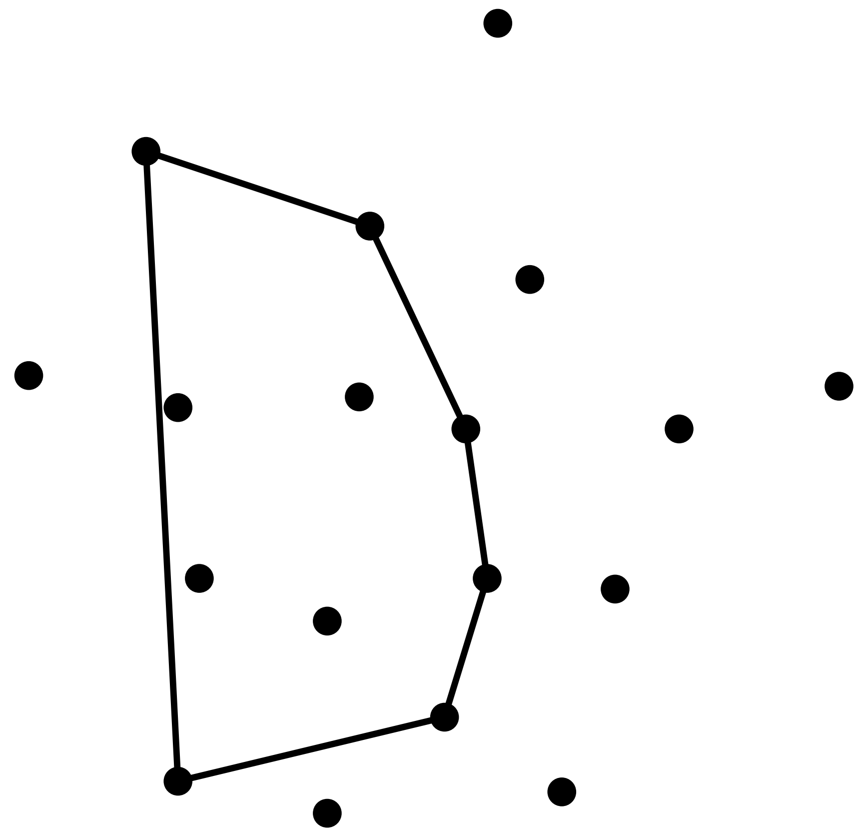

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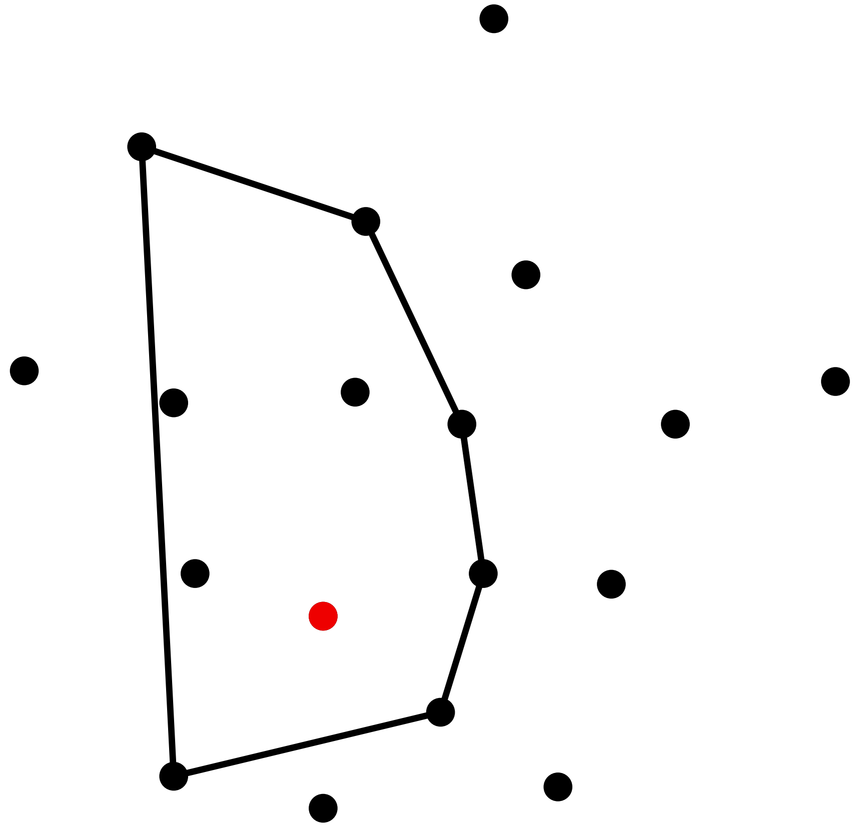

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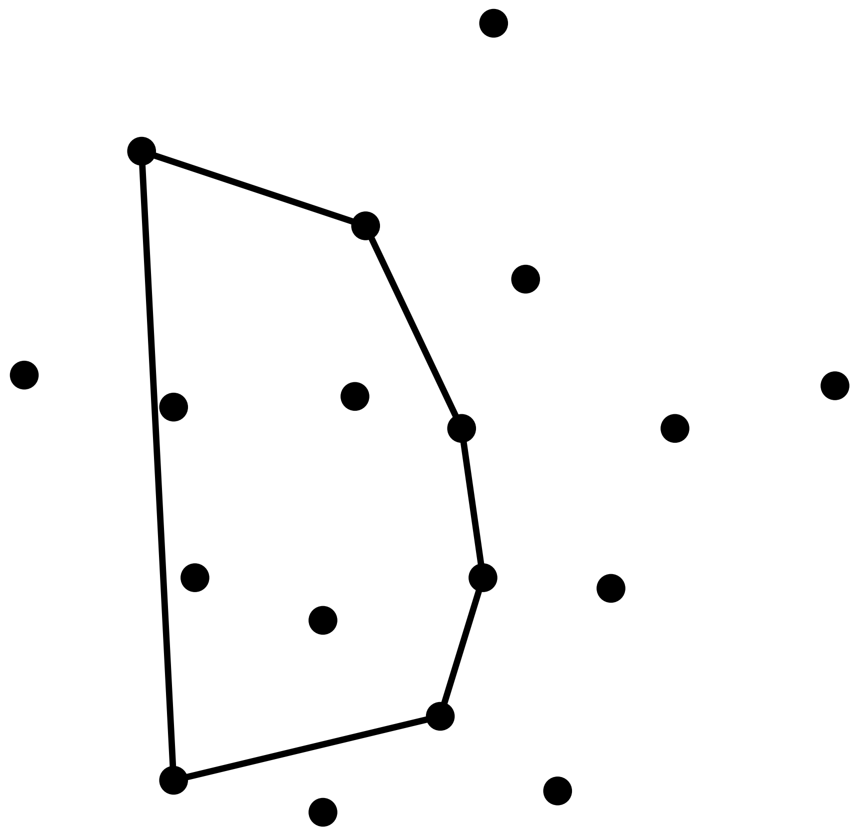

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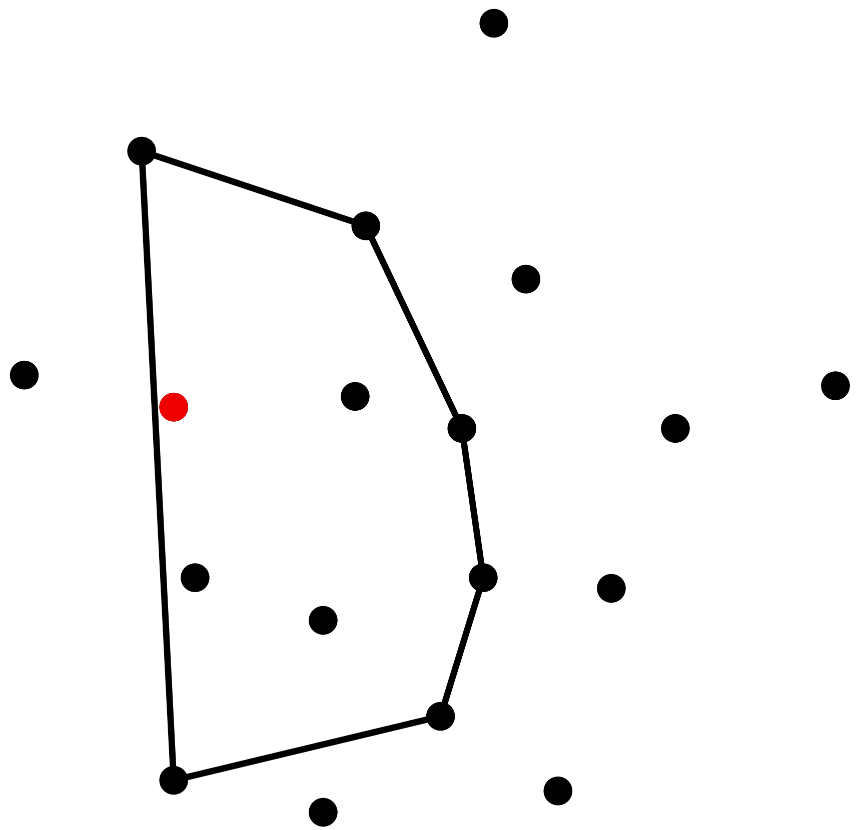

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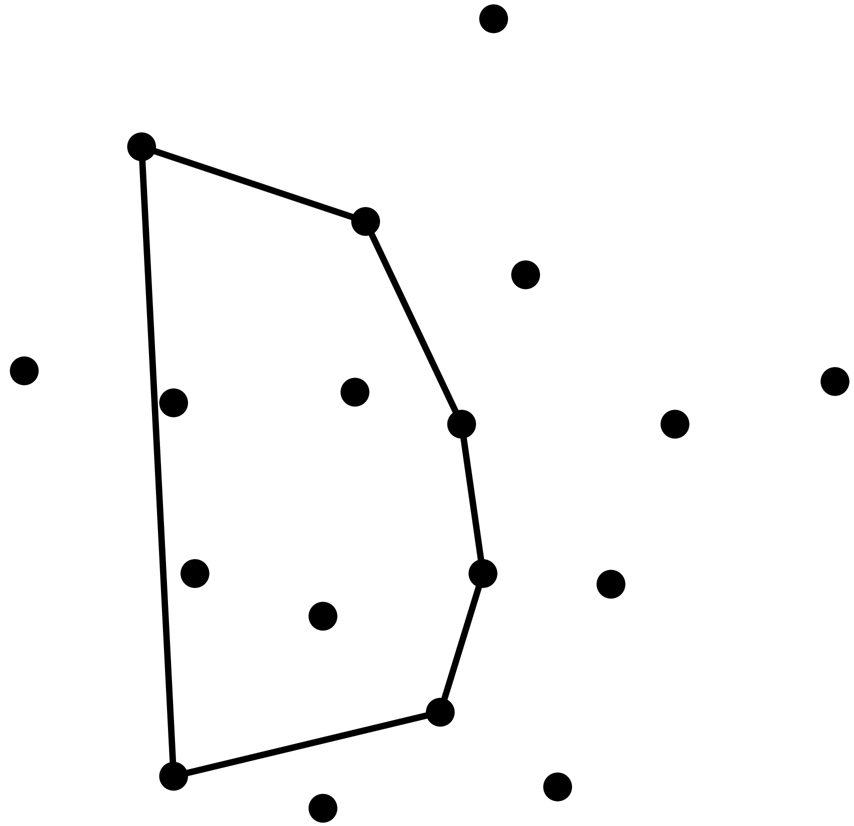

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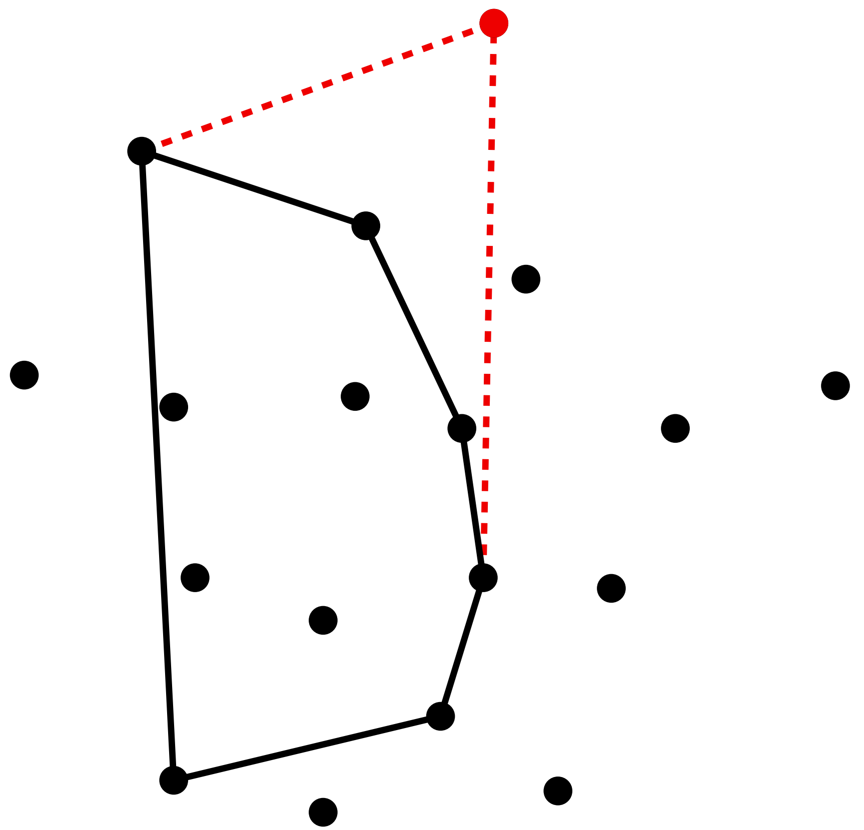

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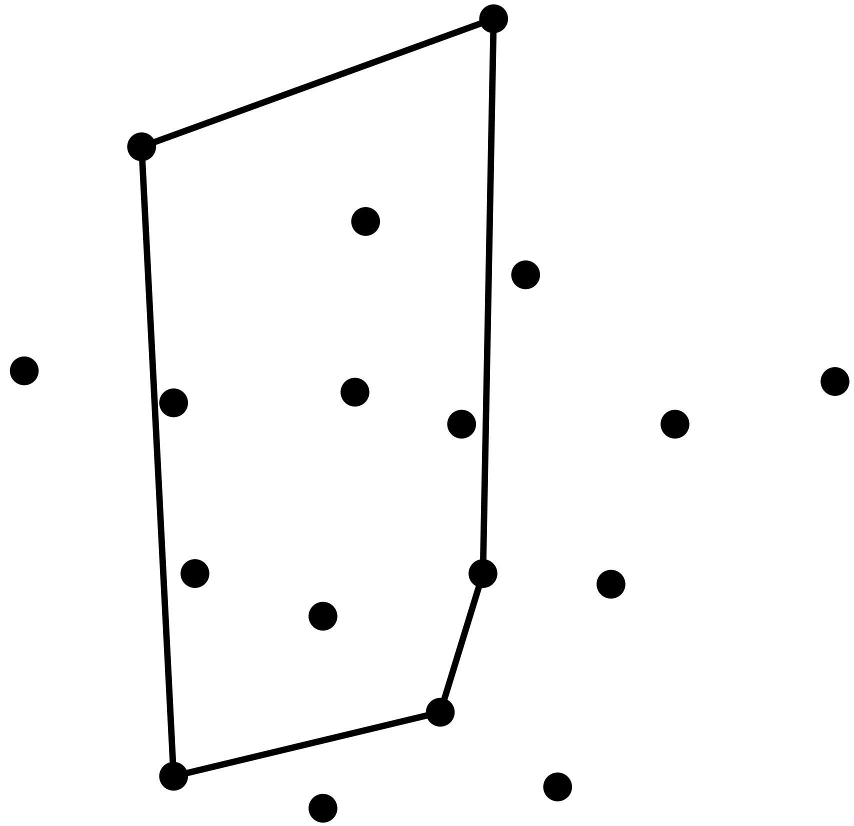

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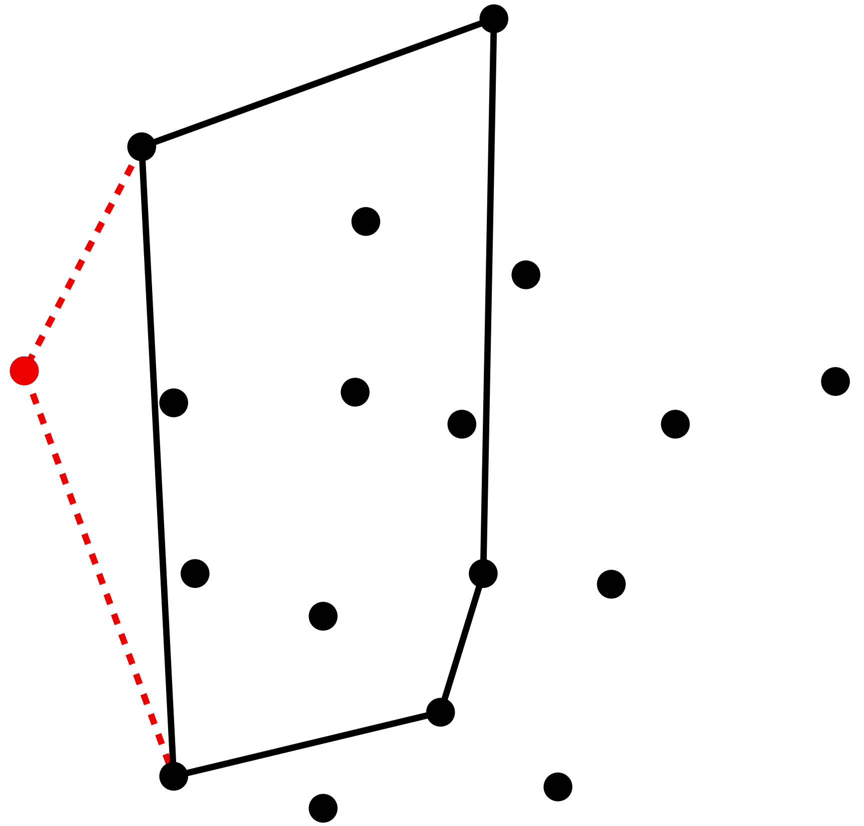

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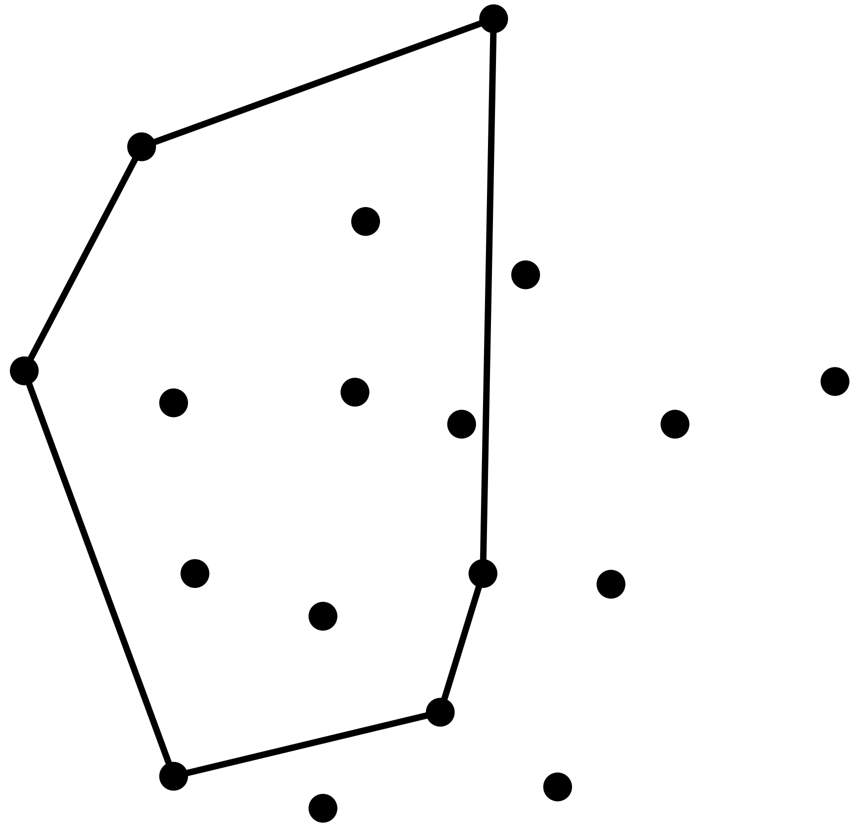

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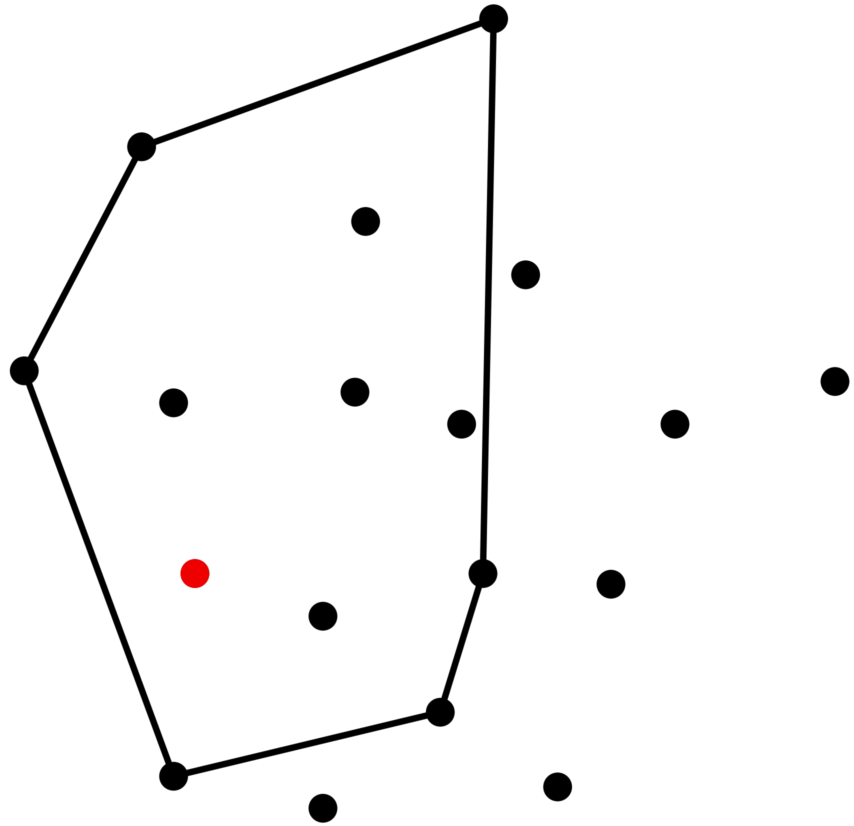

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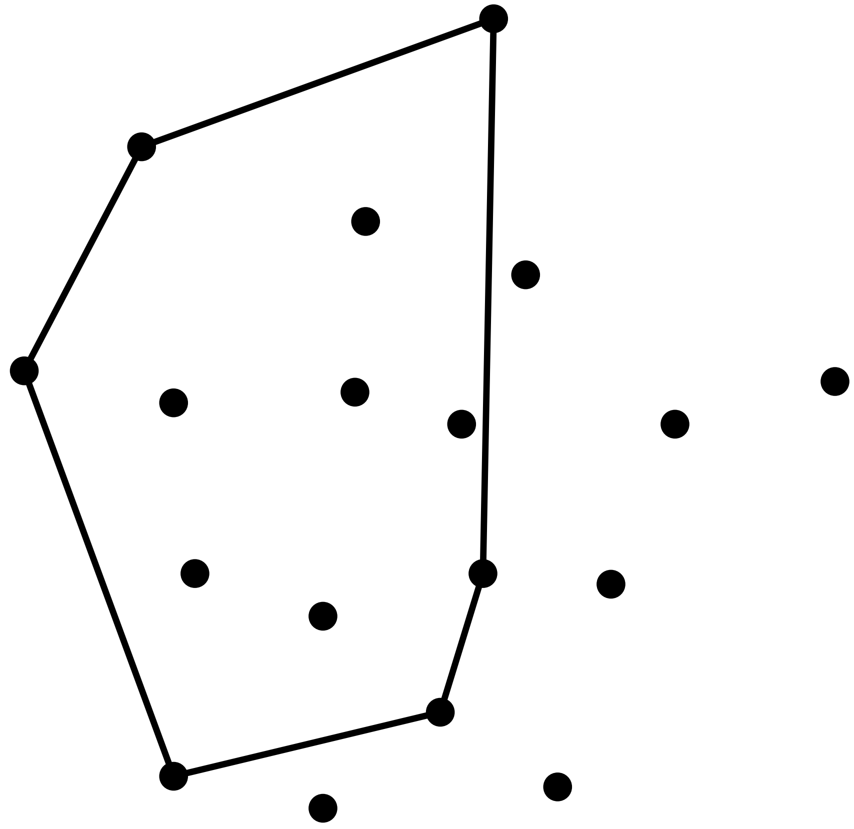

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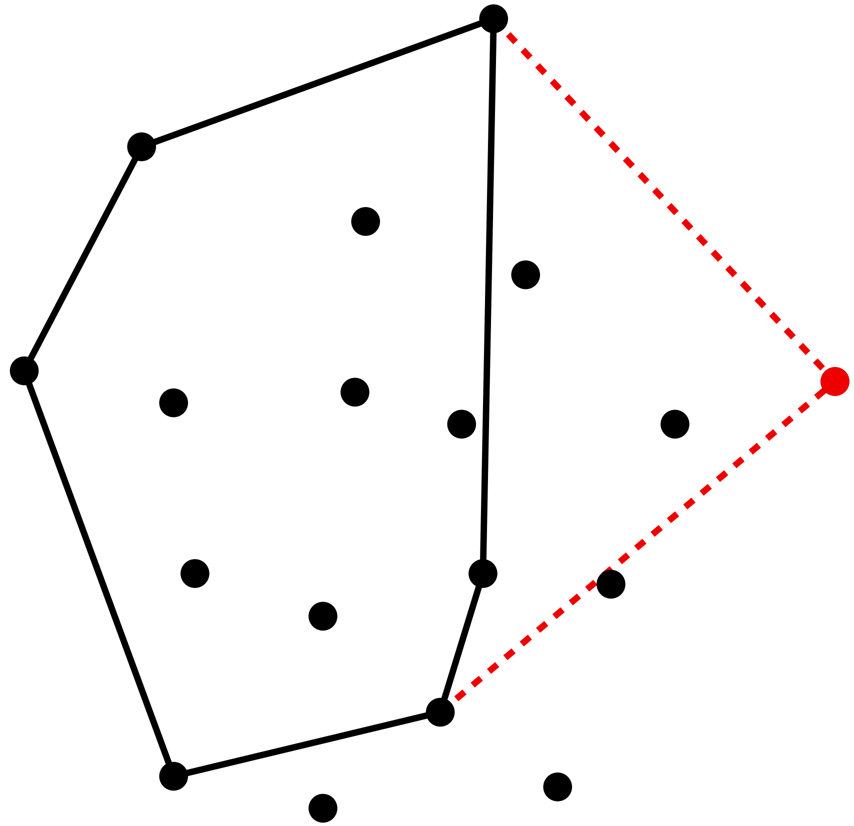

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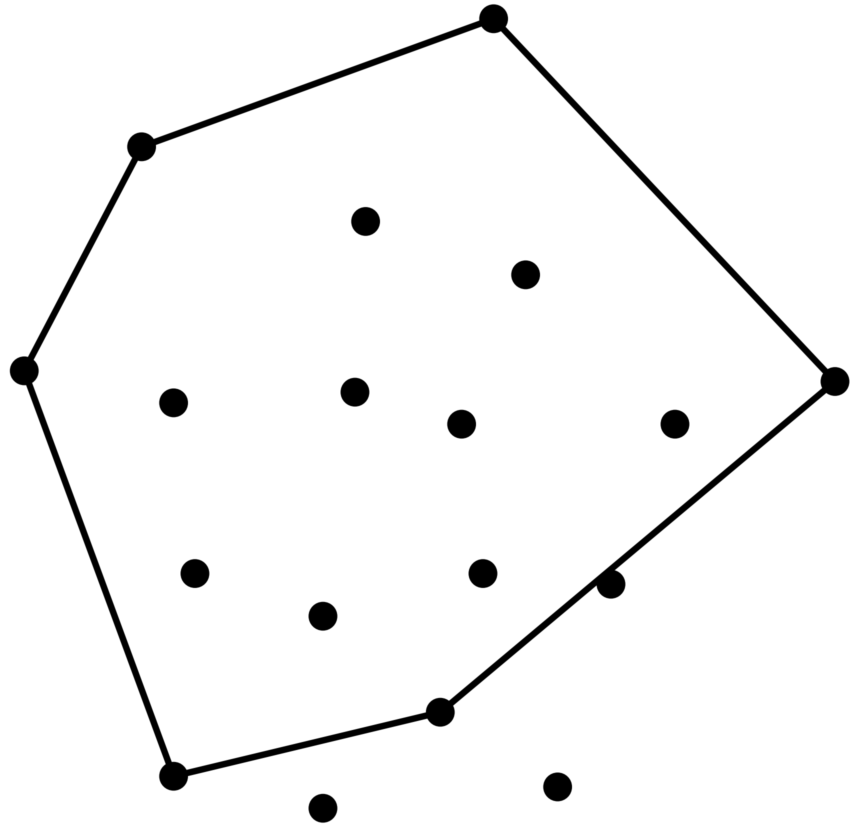

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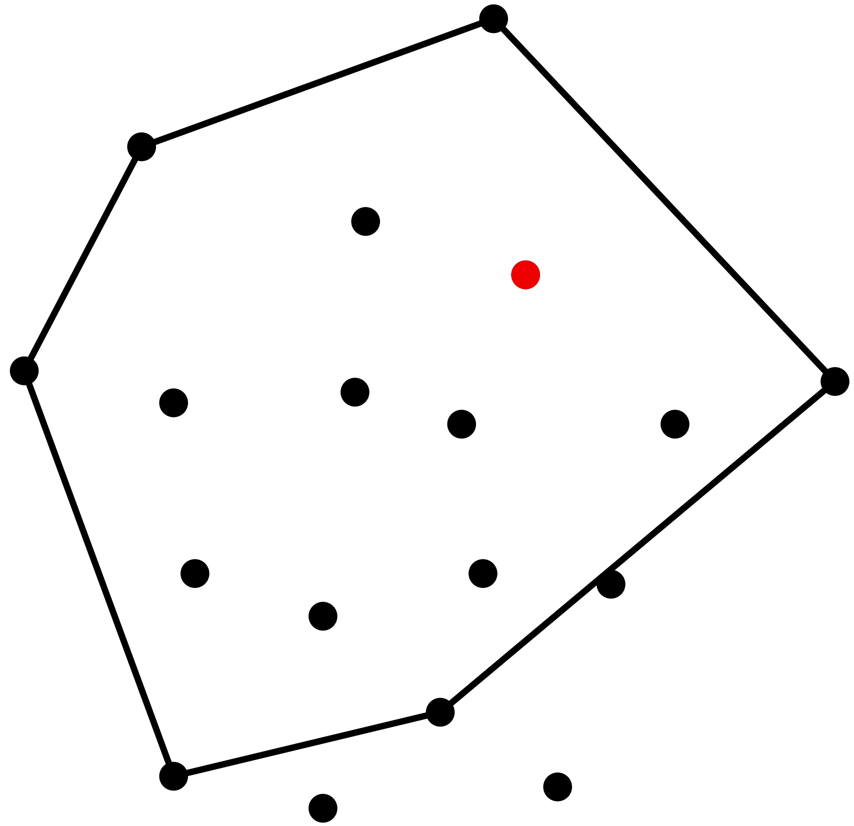

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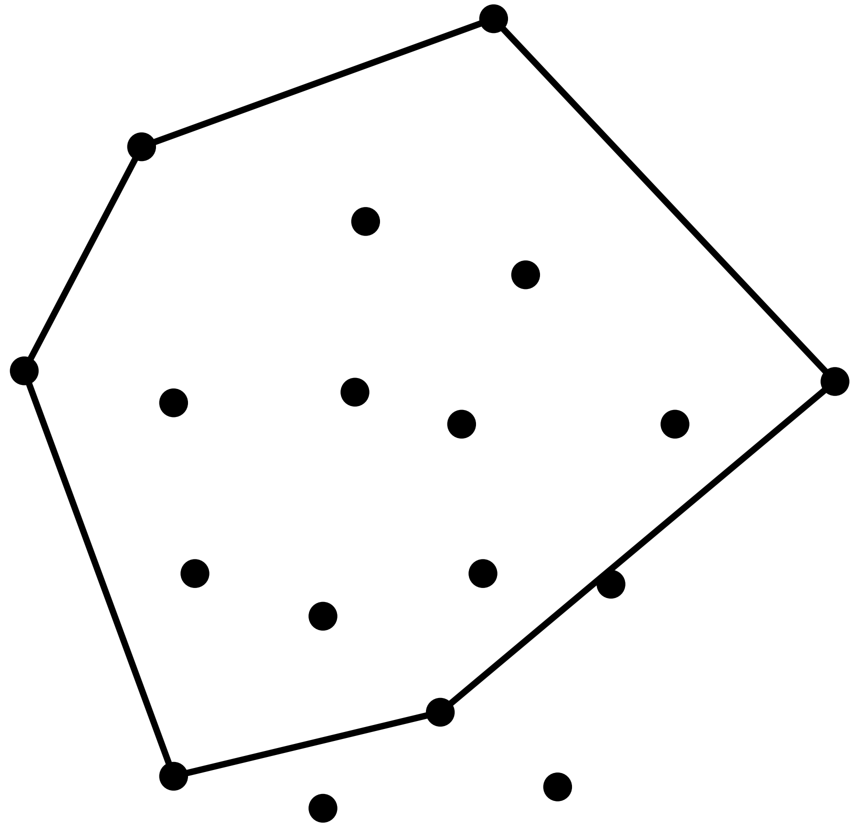

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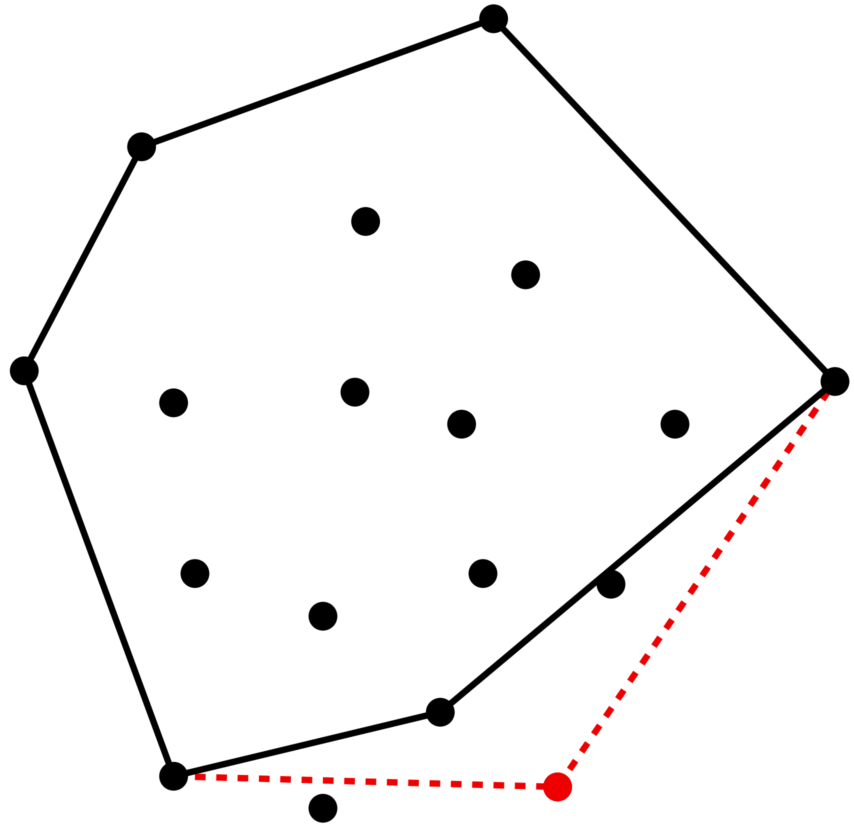

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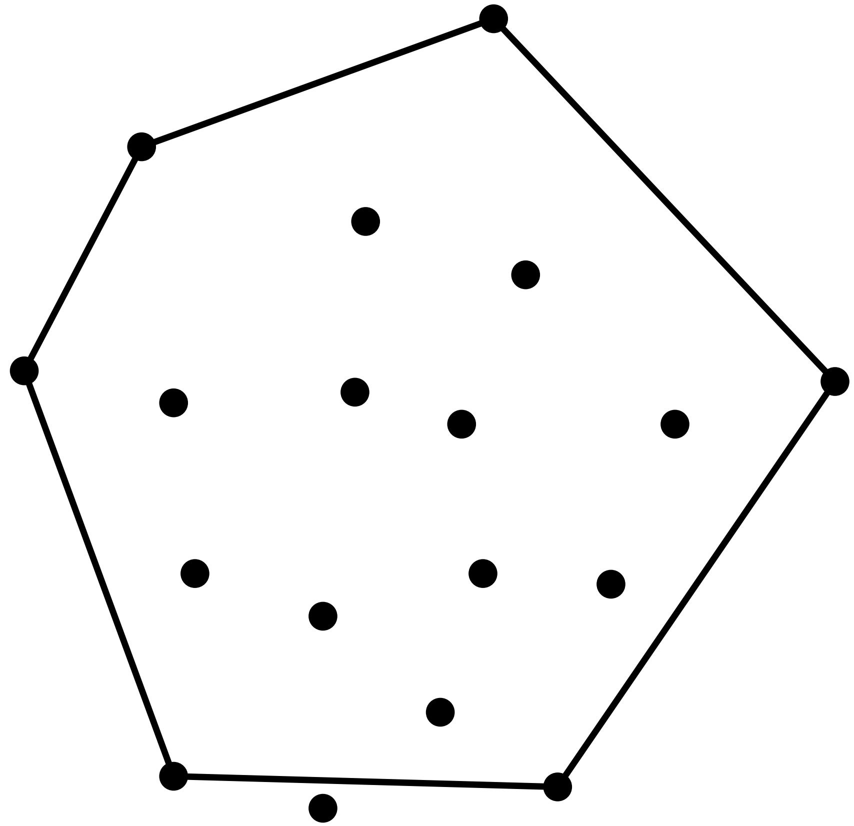

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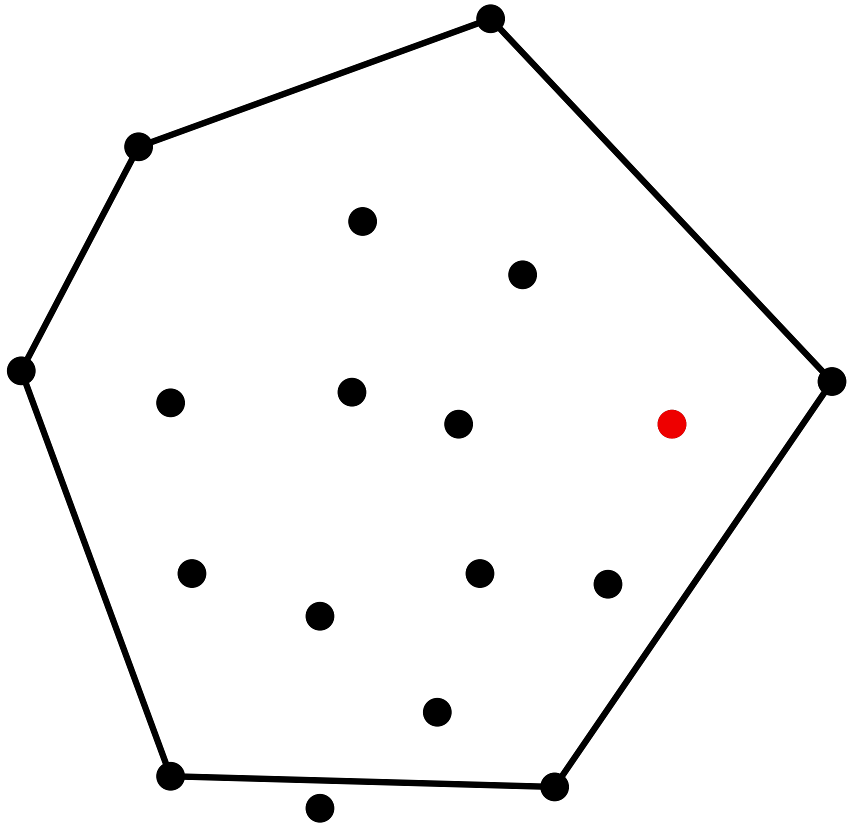

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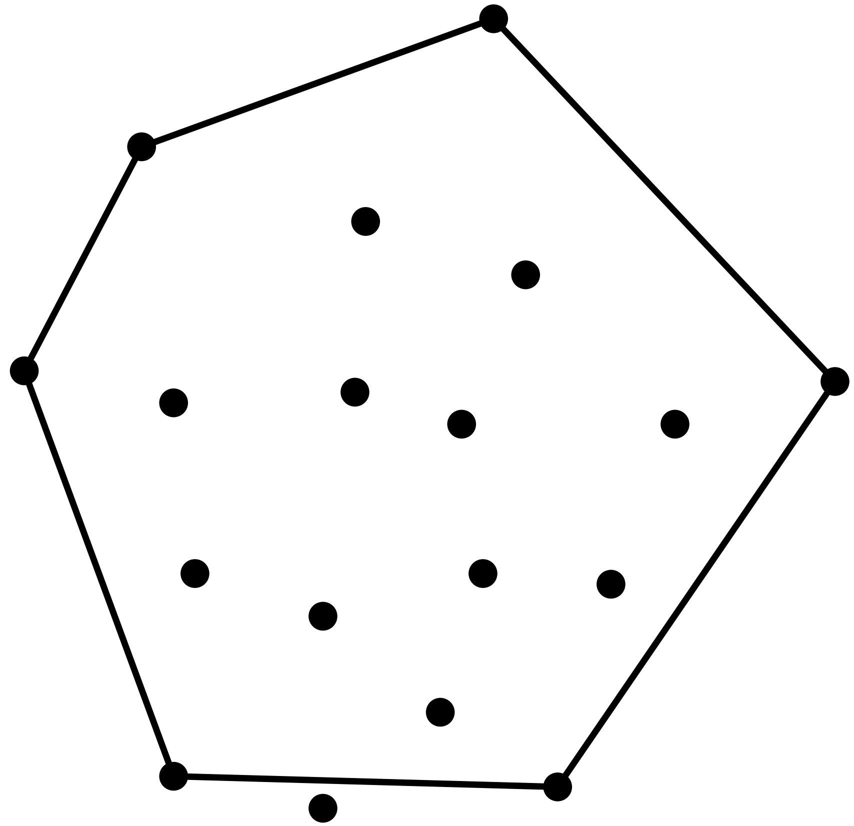

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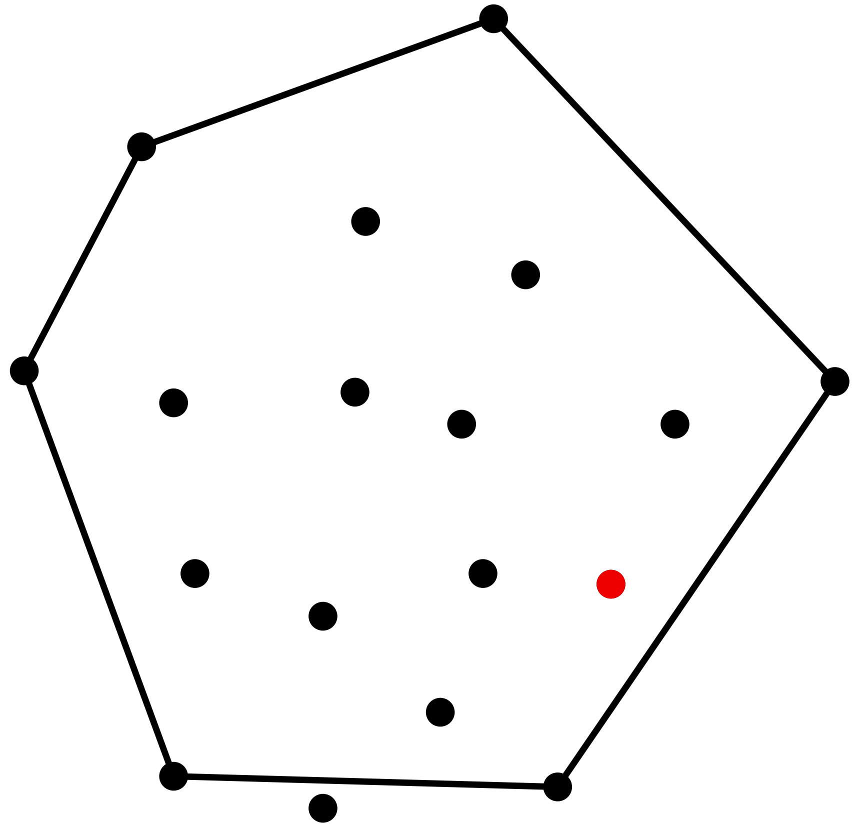

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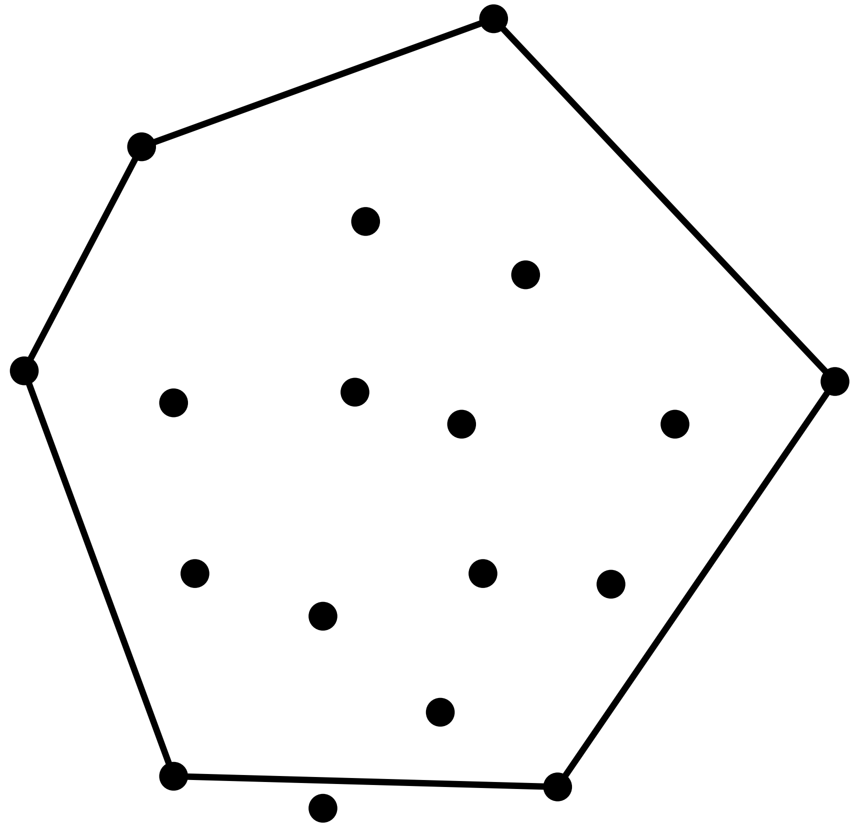

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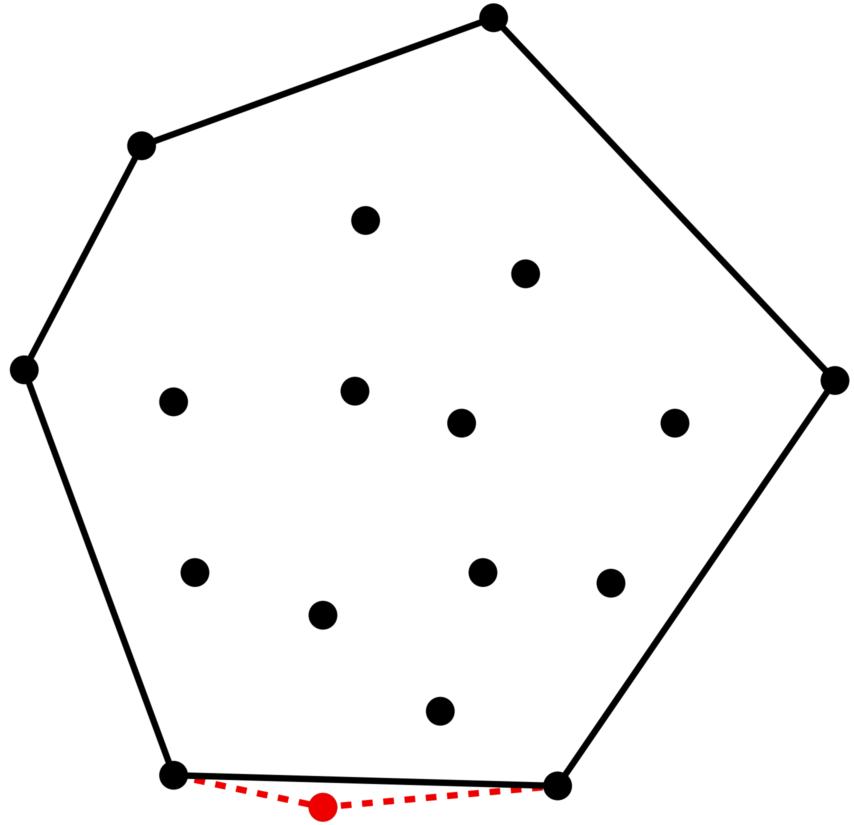

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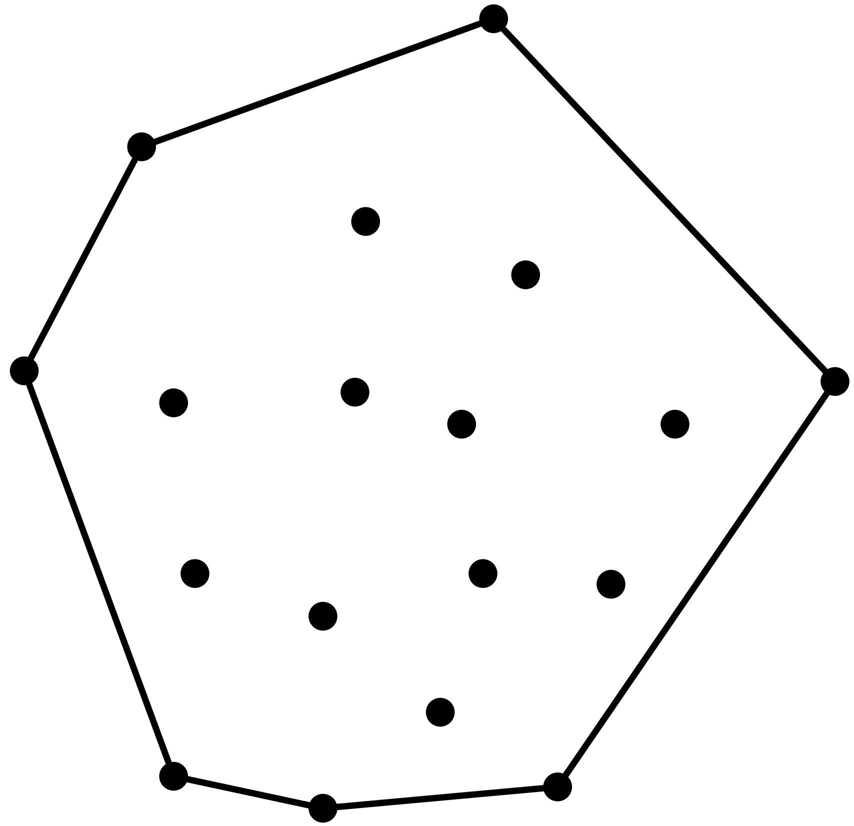

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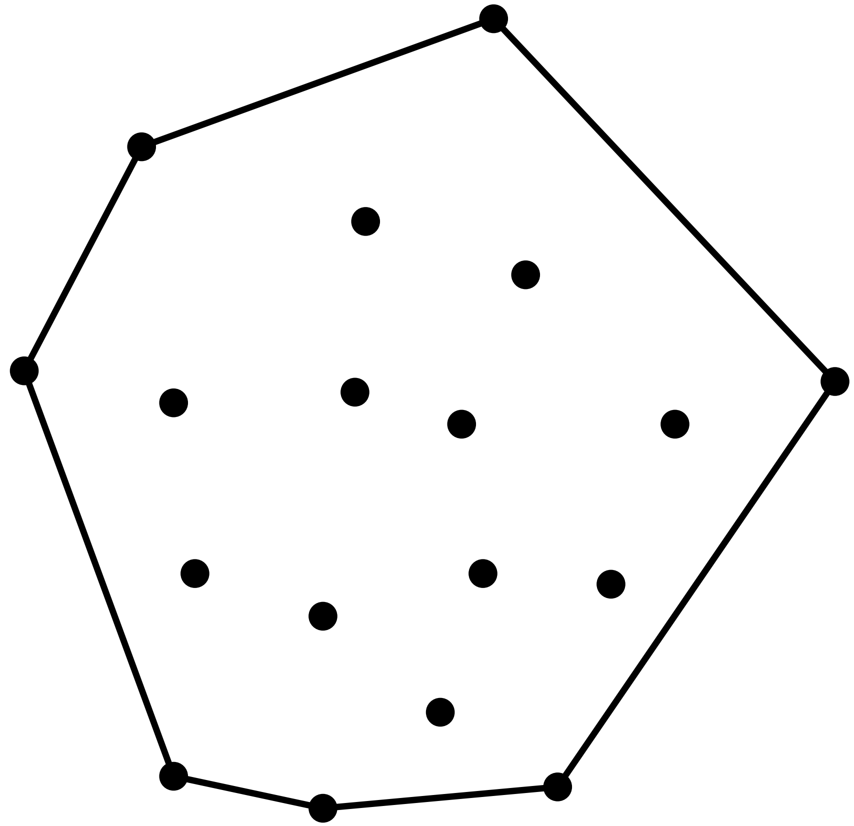

Running time: $O(n \log n)$

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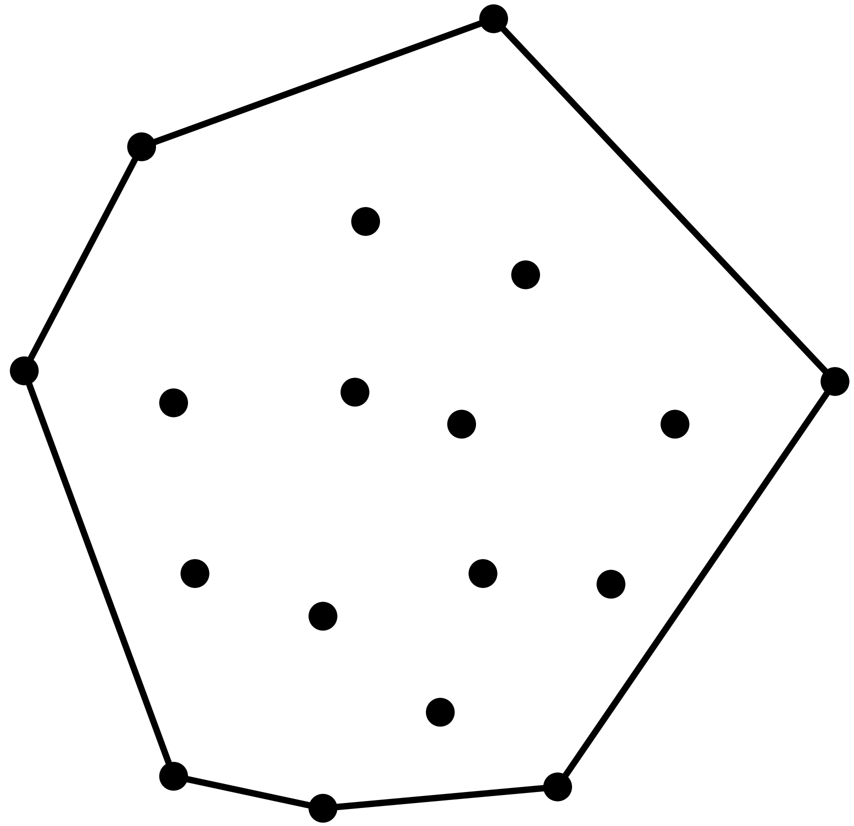

Running time: $O(n \log n)$

By storing l in a structure allowing binary search and updates (insertions and deletions) in $O(\log n)$ time.

Divide-and-conquer algorithm

Divide-and-conquer algorithm

Initialization

1. Sort the points by abscissae

Divide-and-conquer algorithm

Initialization

1. Sort the points by abscissae

Division

1. Divide the points (x_i, y_i) into two subsets, wrt the median value of the abscissae

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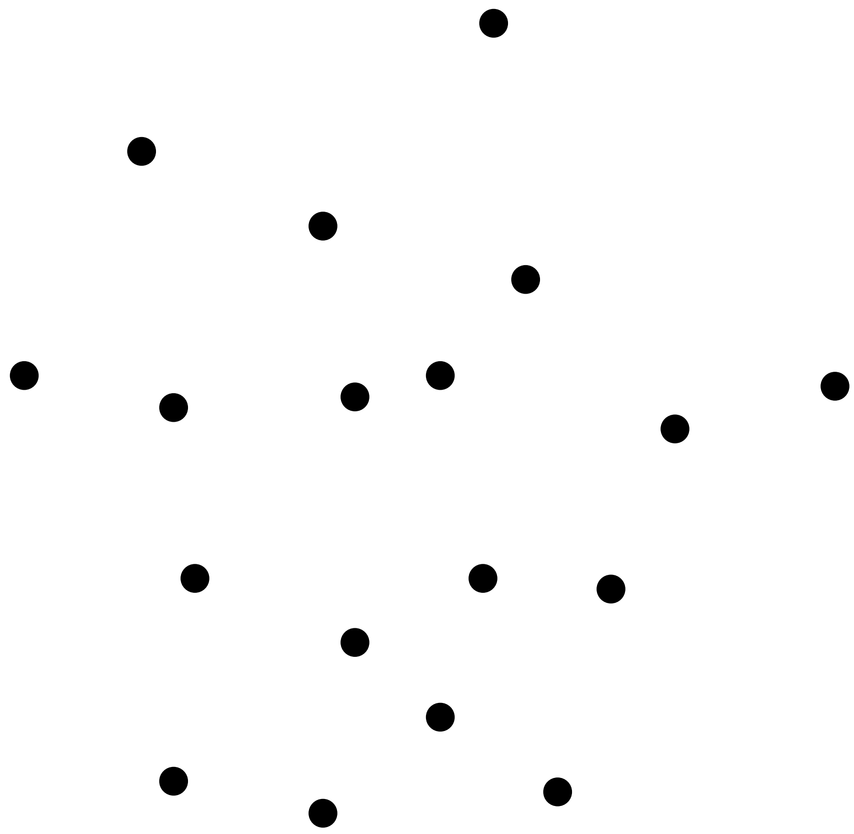

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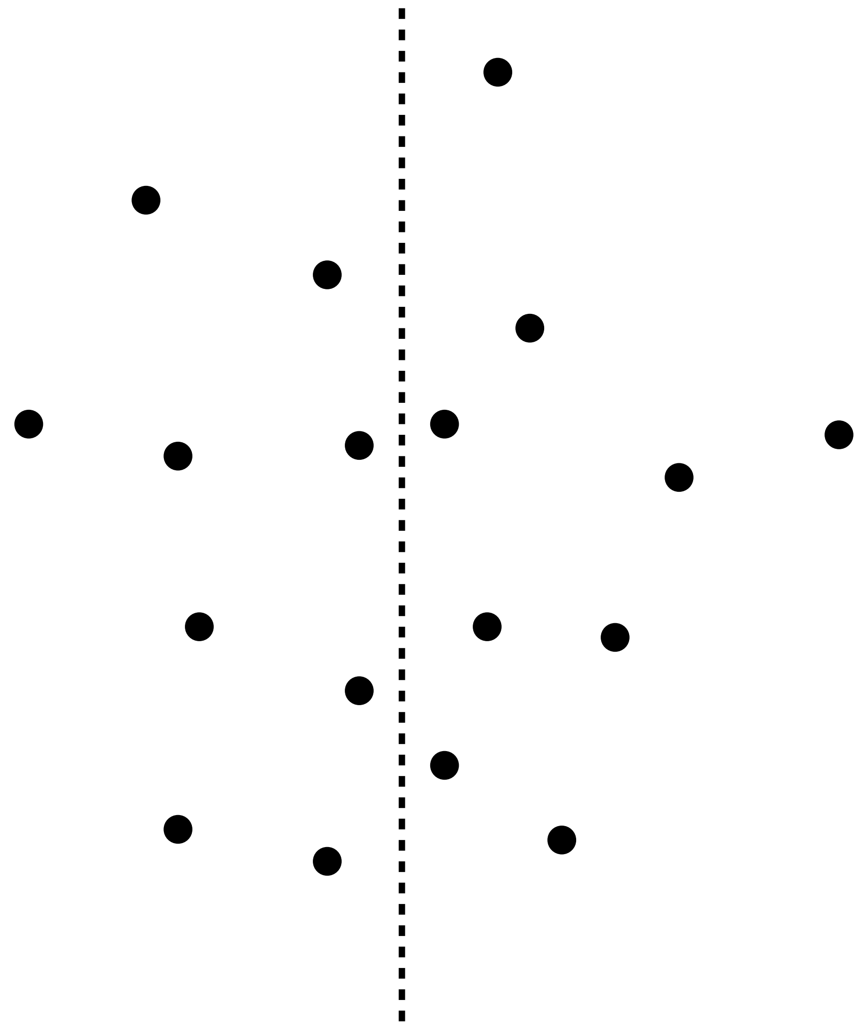

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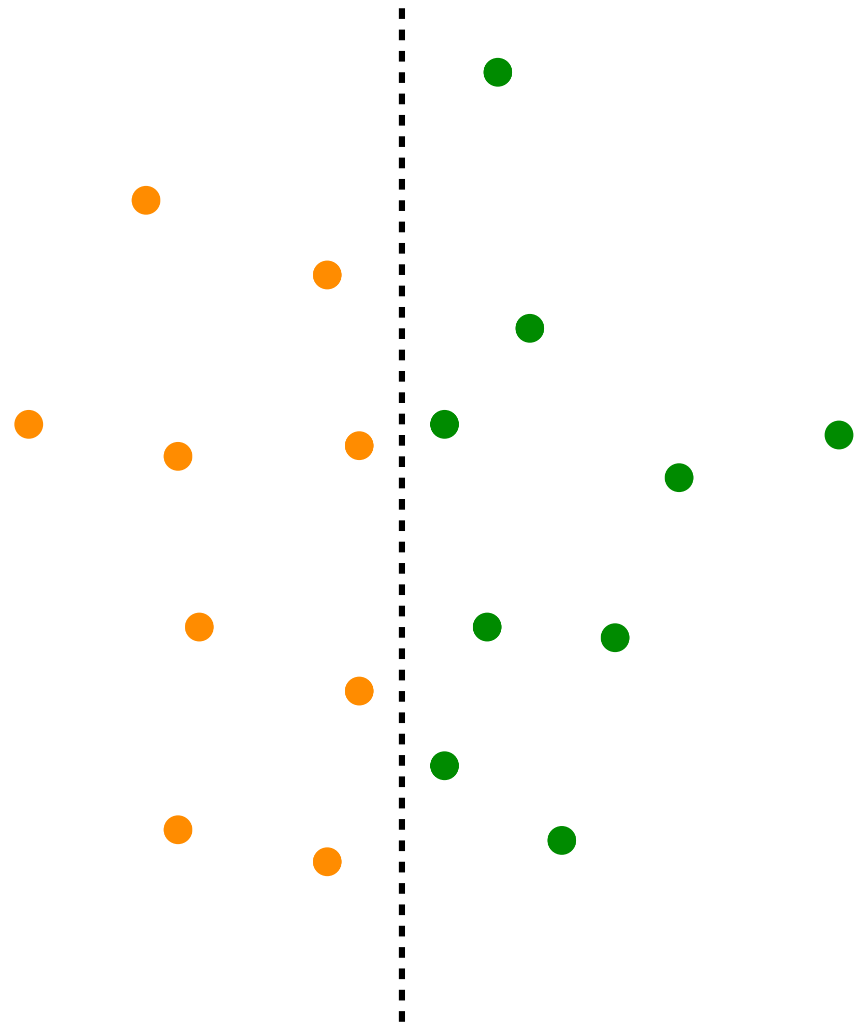

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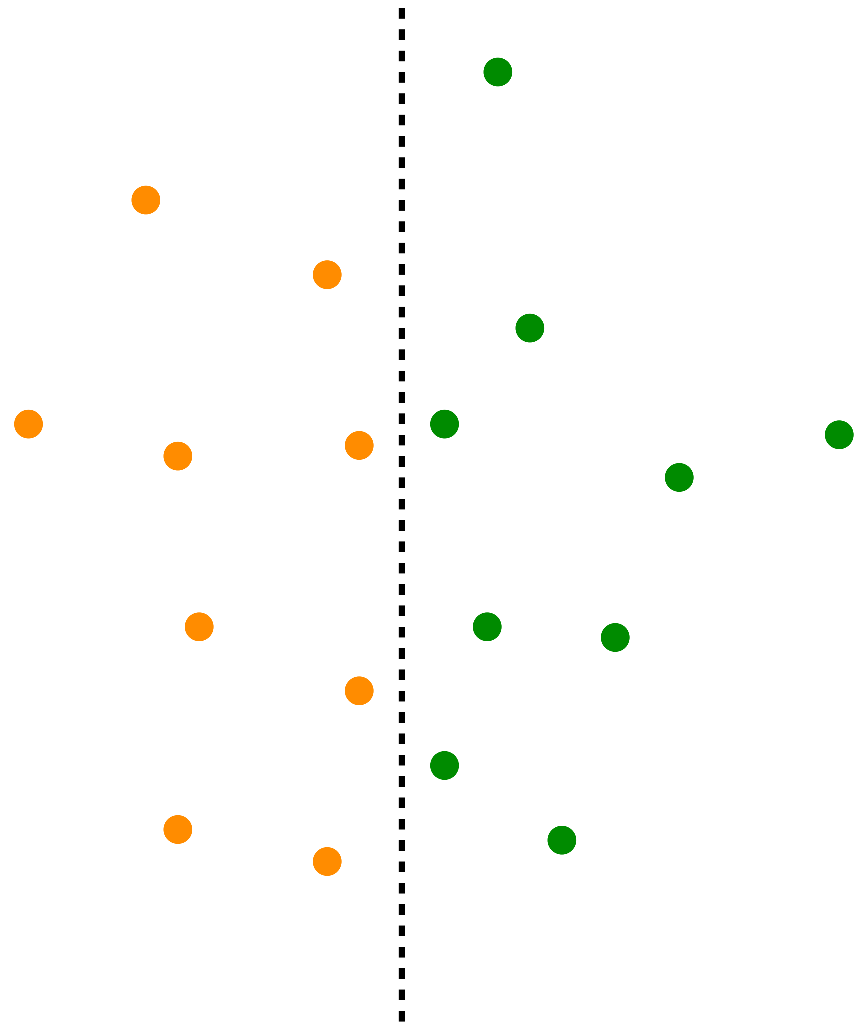
Recursion

1. Recursively compute the convex hull of the two subsets



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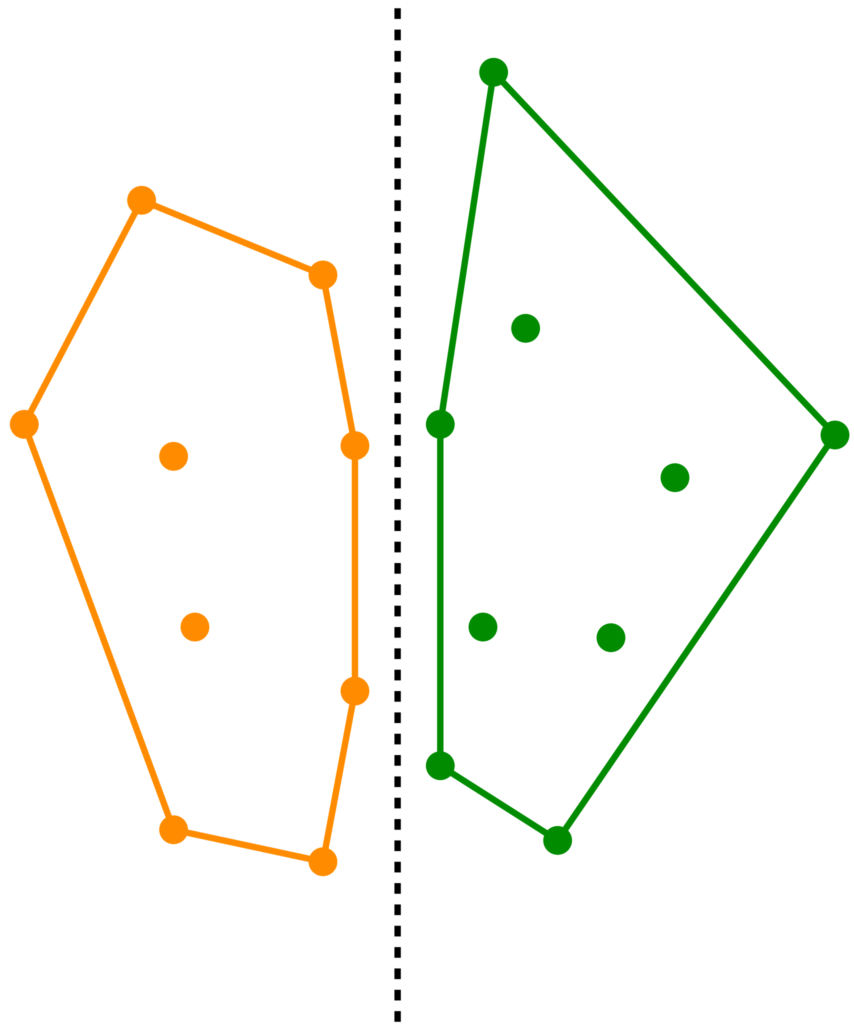

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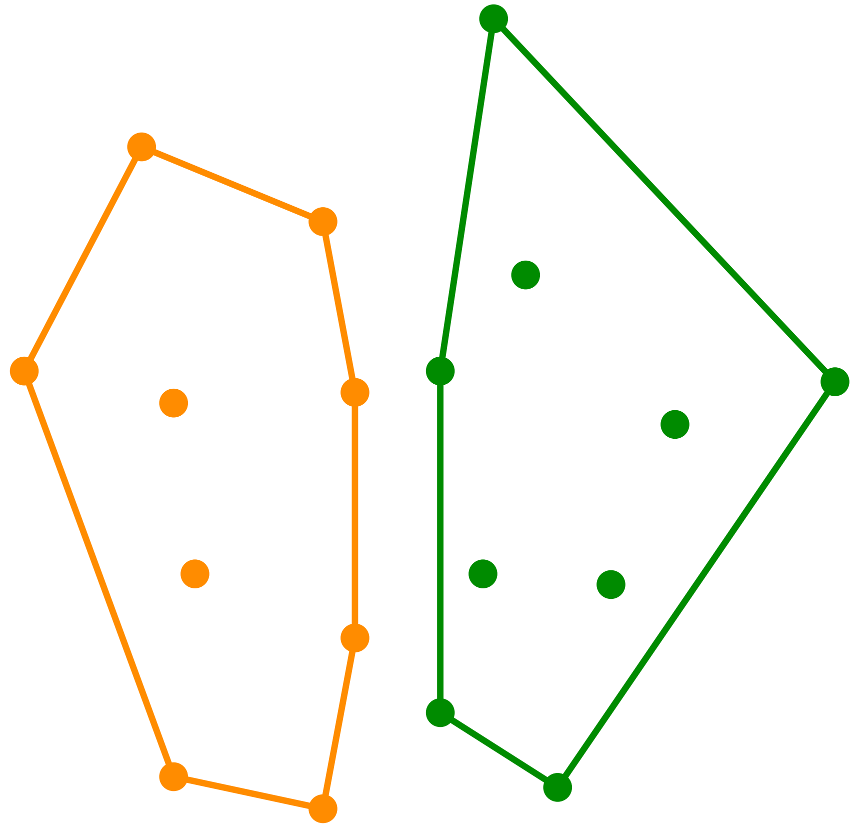

CONVEX HULL

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Recursion

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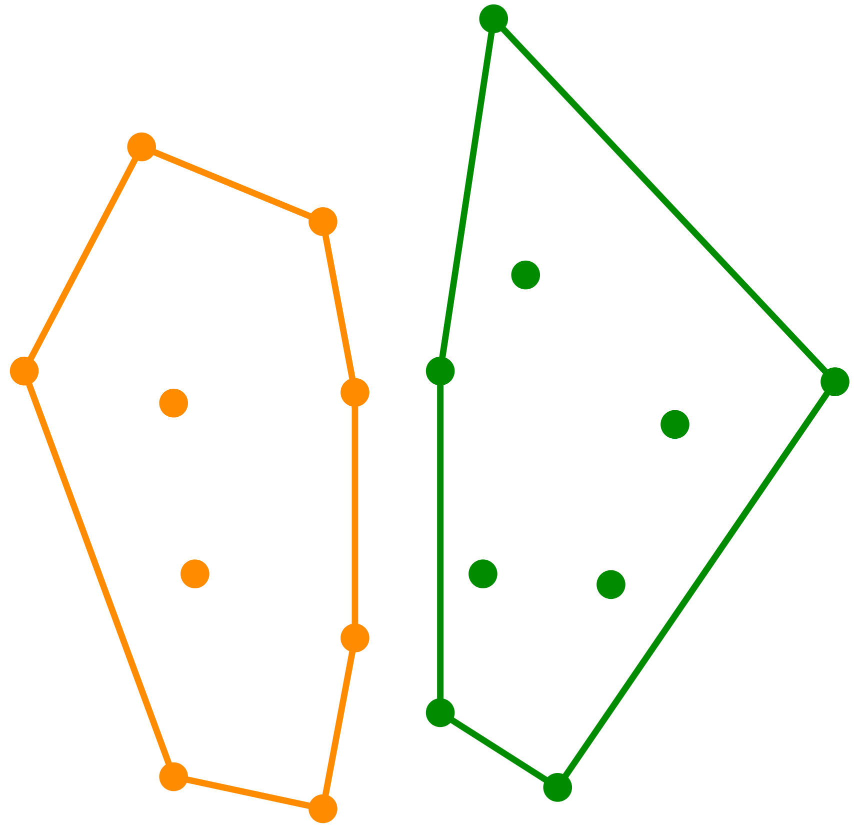
Merge

1. Compute the external common tangents of the two convex polygons
2. Delete the interior chains of the two polygons and join the external chains through the supporting segments



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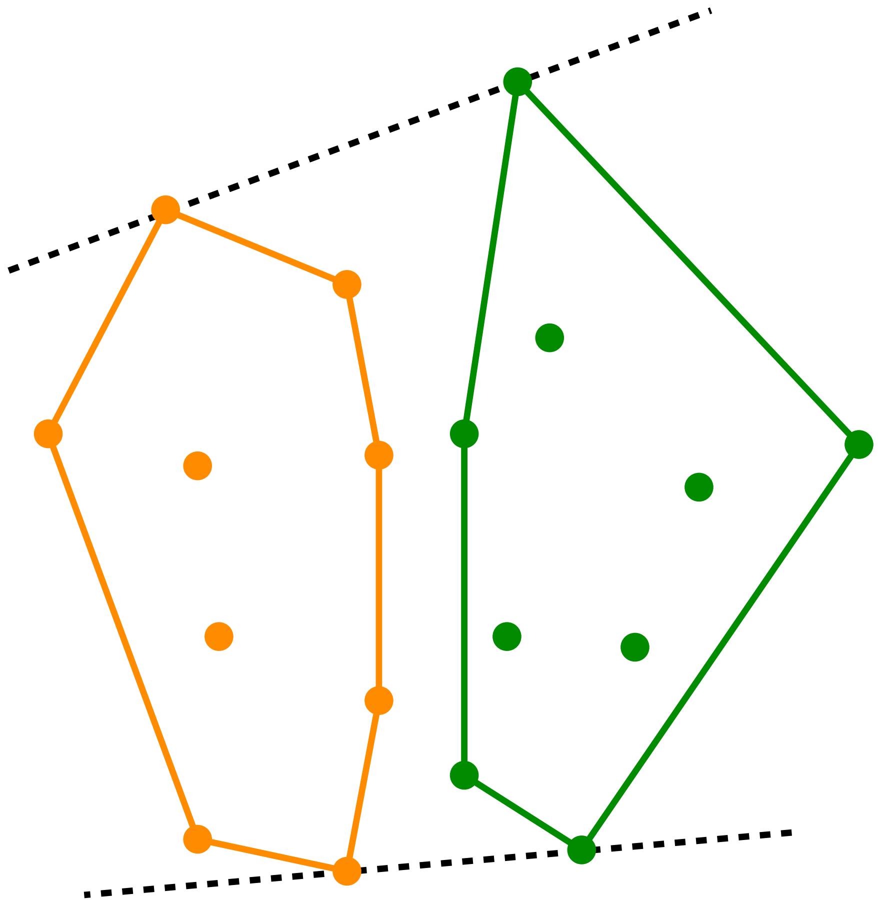

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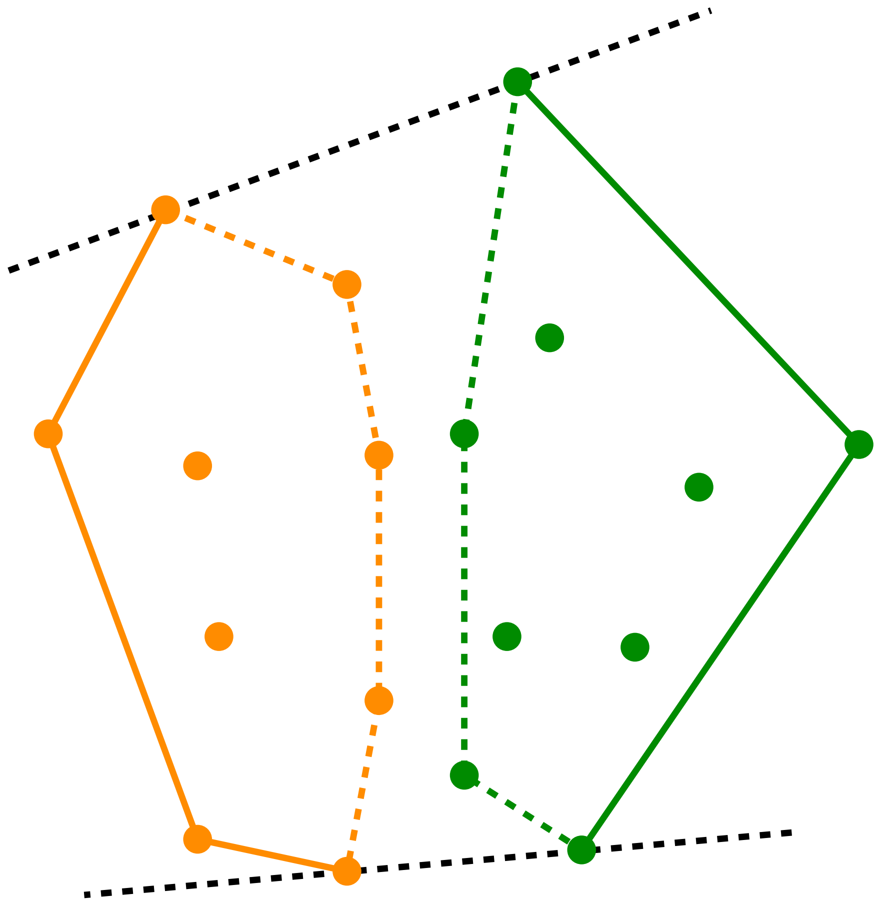

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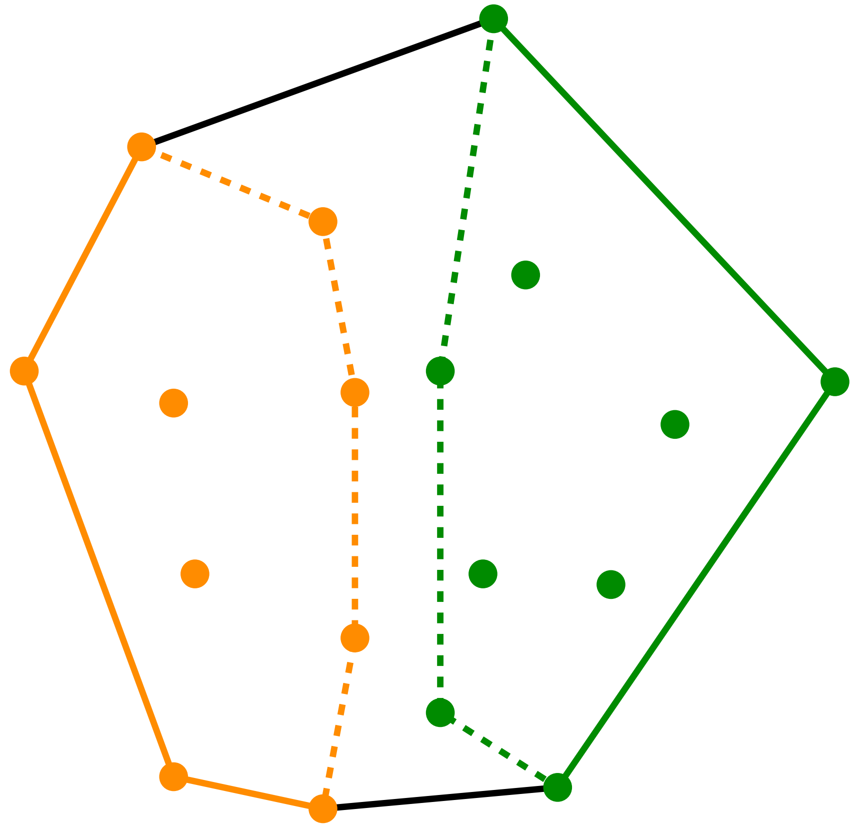

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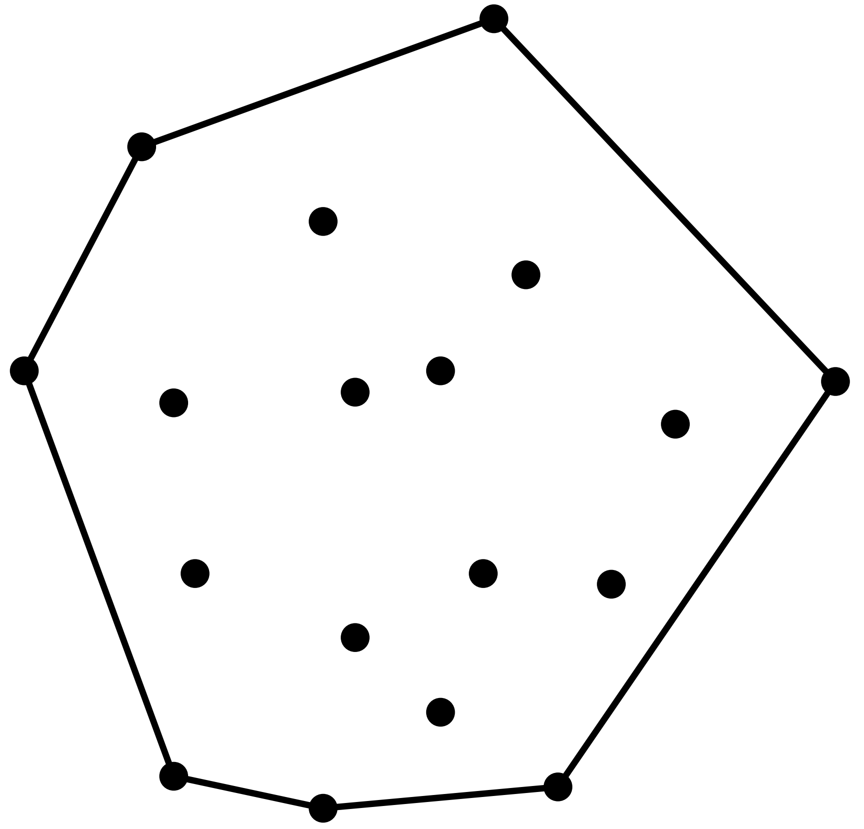
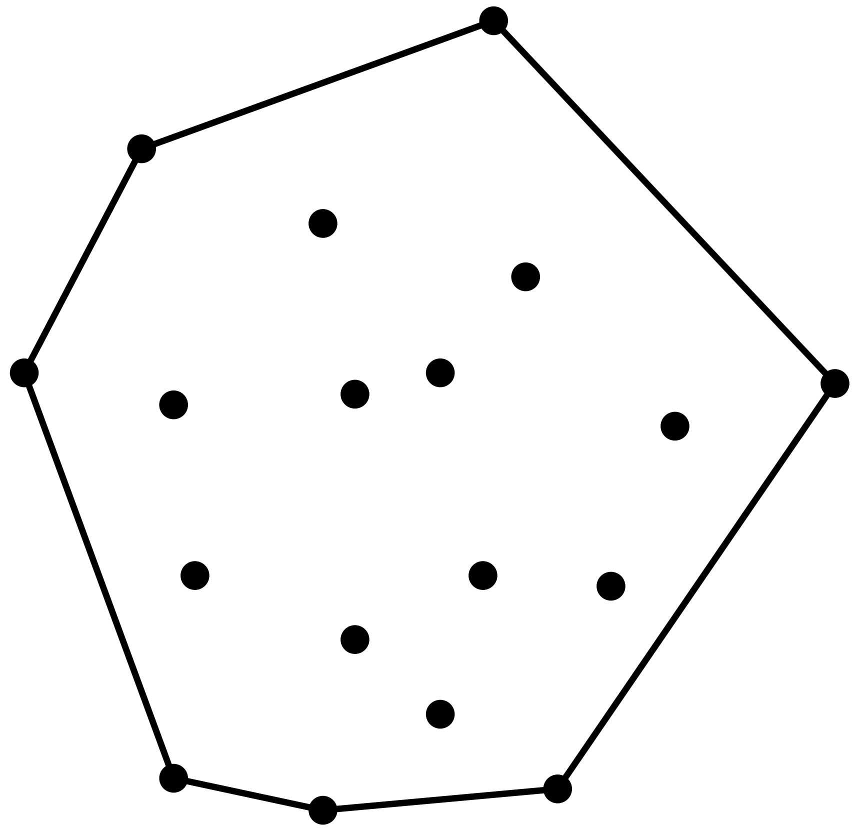


Divide-and-conquer algorithm

Running time

Initialization: $O(n \log n)$ (only once)



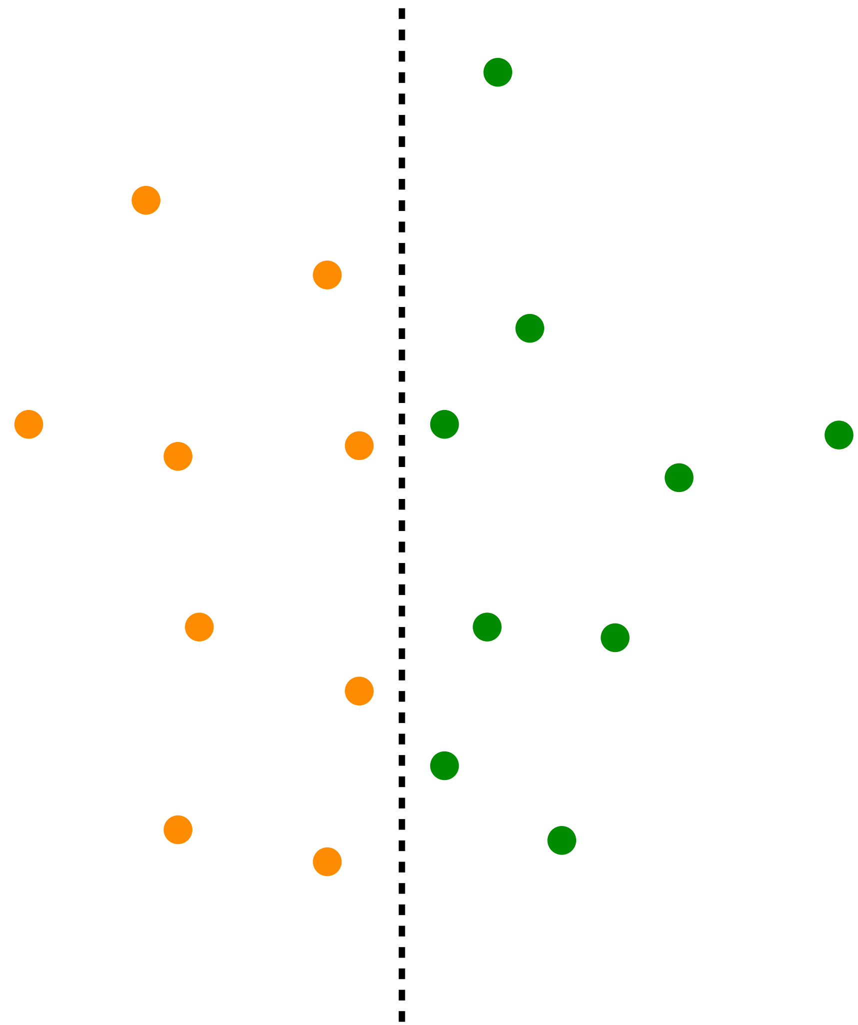
CONVEX HULL

Divide-and-conquer algorithm

Running time

Initialization: $O(n \log n)$ (only once)

Division: $O(n)$



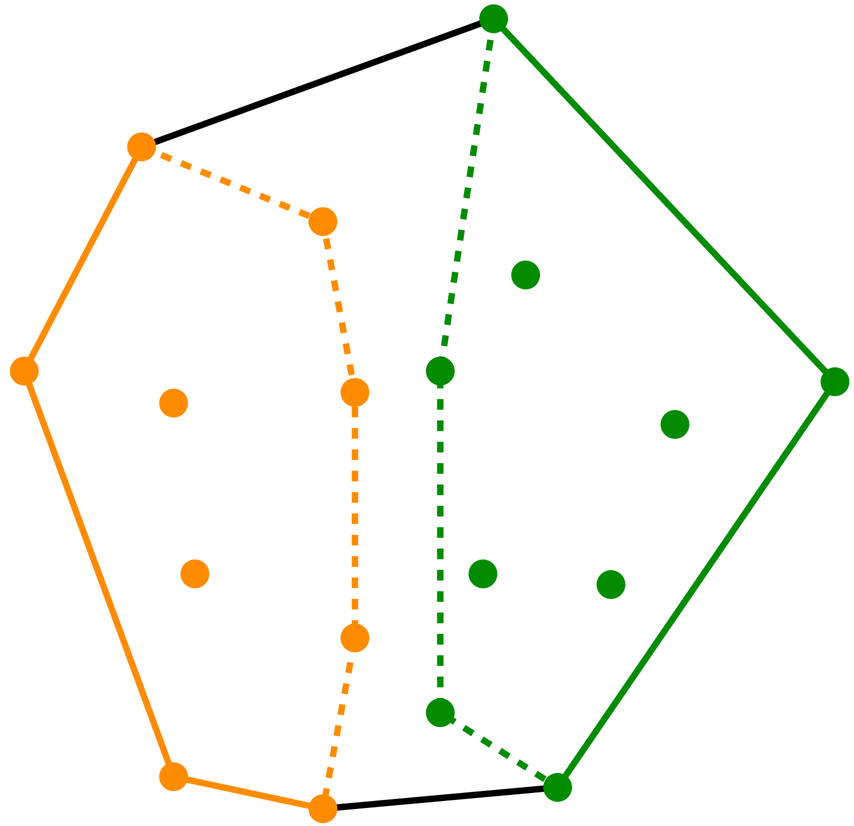
Divide-and-conquer algorithm

Running time

Initialization: $O(n \log n)$ (only once)

Division: $O(n)$

Merge: $O(n)$



Divide-and-conquer algorithm

Running time

Initialization: $O(n \log n)$ (only once)

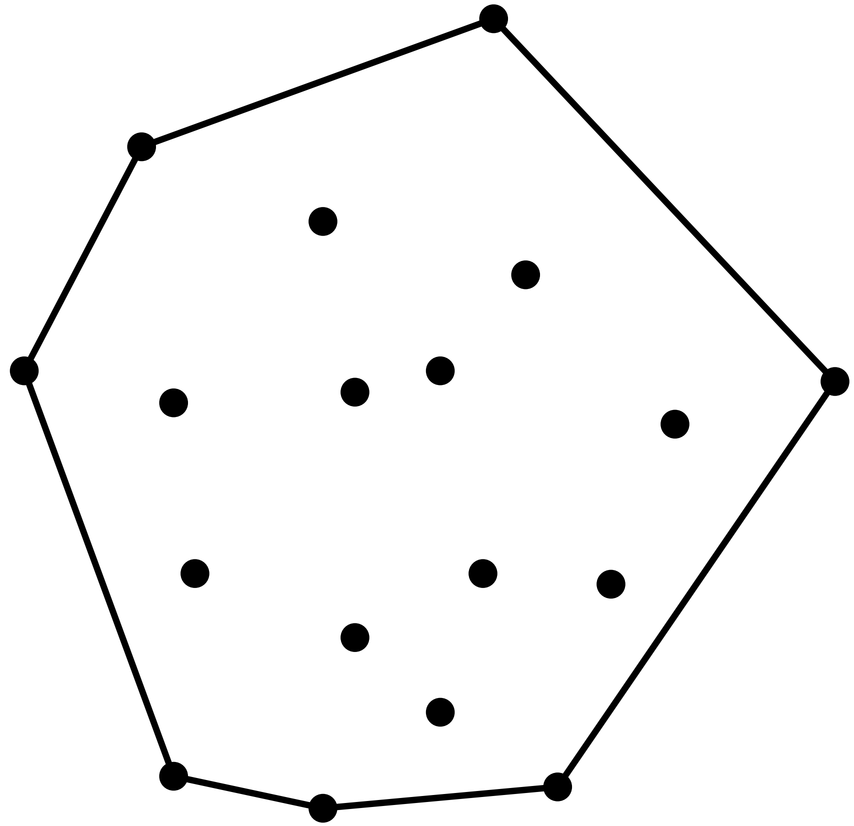
Division: $O(n)$

Merge: $O(n)$

Advance:

$$T(n) = 2\,T\left(\frac{n}{2}\right) + O(n) = O(n \log n)$$

Overall: $O(n \log n)$



Lower bound

Lower bound

Input: n real numbers

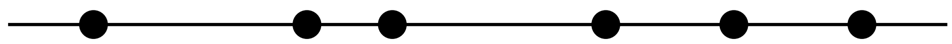
$x_1, \ldots, x_n$ real numbers

CONVEX HULL

Lower bound

Input: n real numbers

$x_1, \ldots, x_n$ real numbers



CONVEX HULL

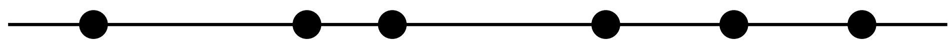
Lower bound

Input: n **real numbers**

$x_1, \ldots, x_n$ real numbers

Input: n **points**

$p_1, \ldots, p_n$, with $p_i = (x_i, x_i^2)$



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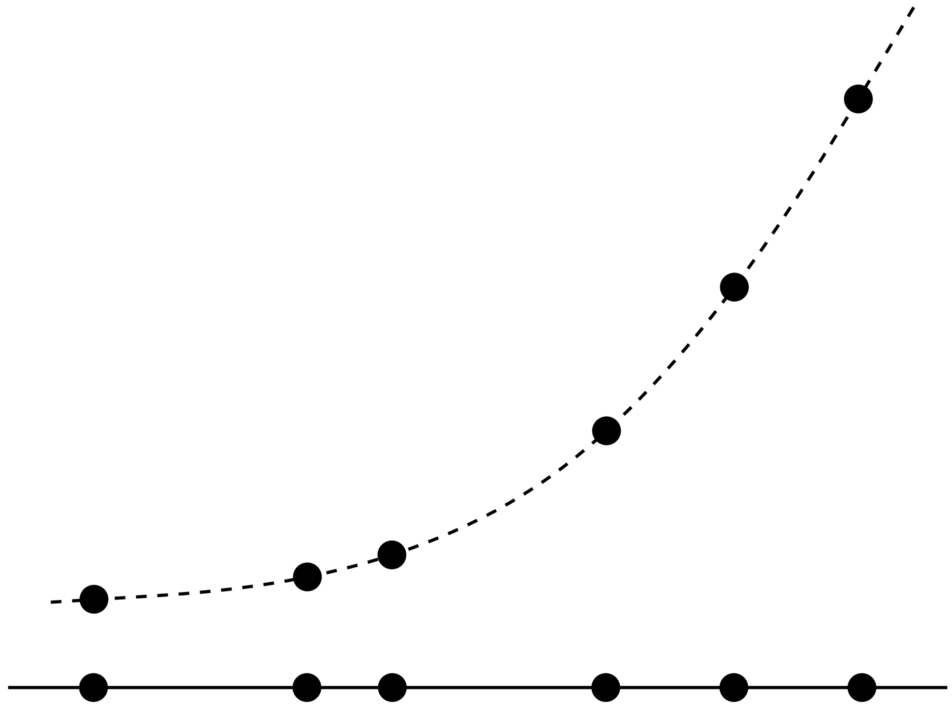

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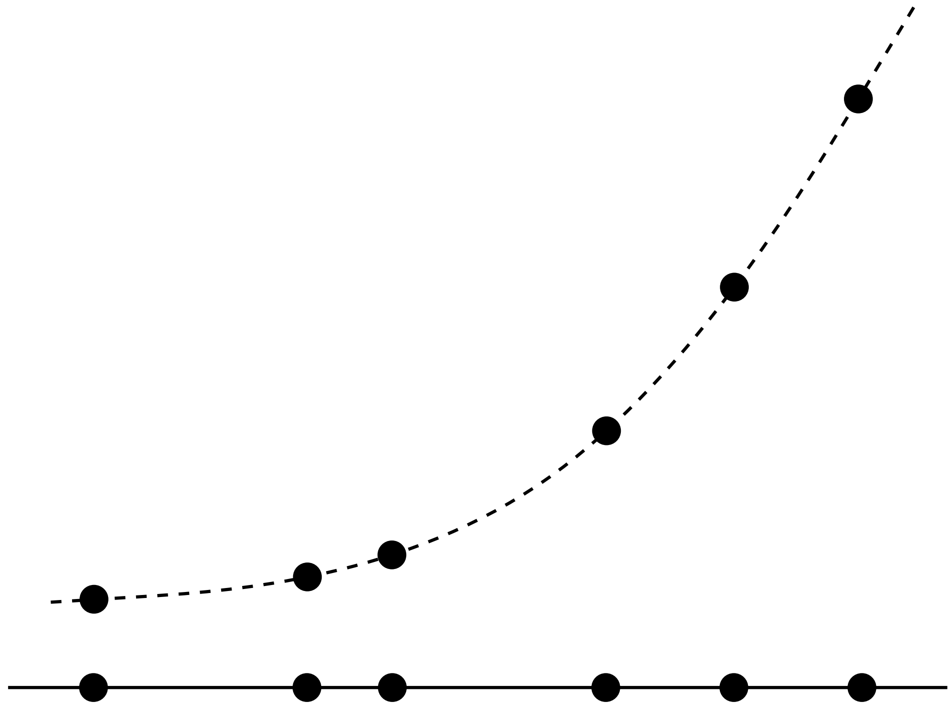
Output: convex hull of the points
Sorted list of the vertices of the convex hull



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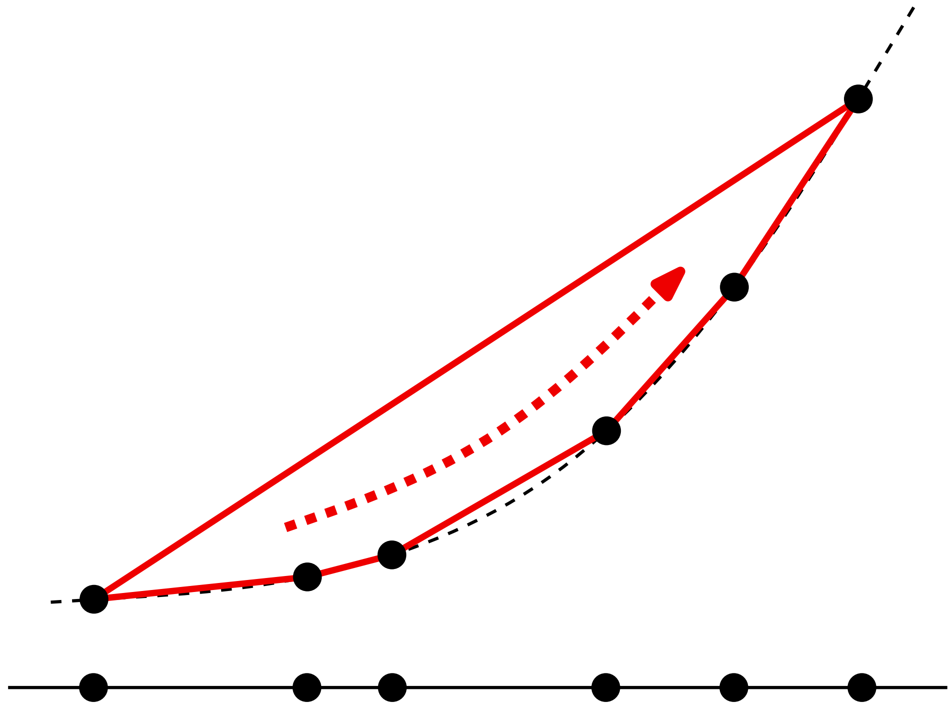

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Input: n points
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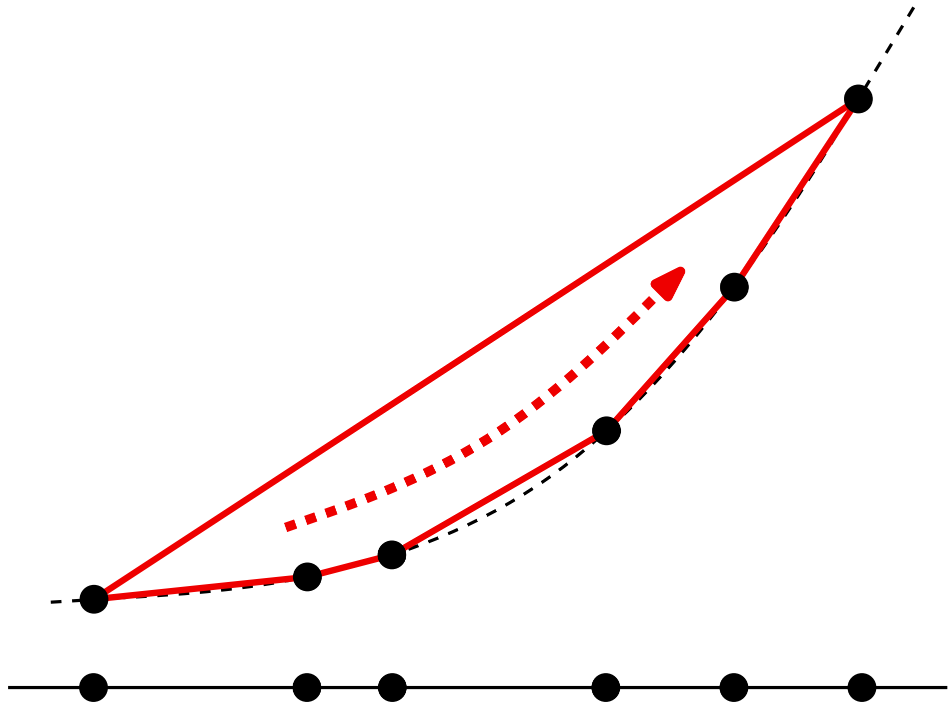
Output: convex hull of the points
Sorted list of the vertices of the convex hull

Output: sorting the numbers
Sorted list of the numbers $x_1, \dots, x_n$



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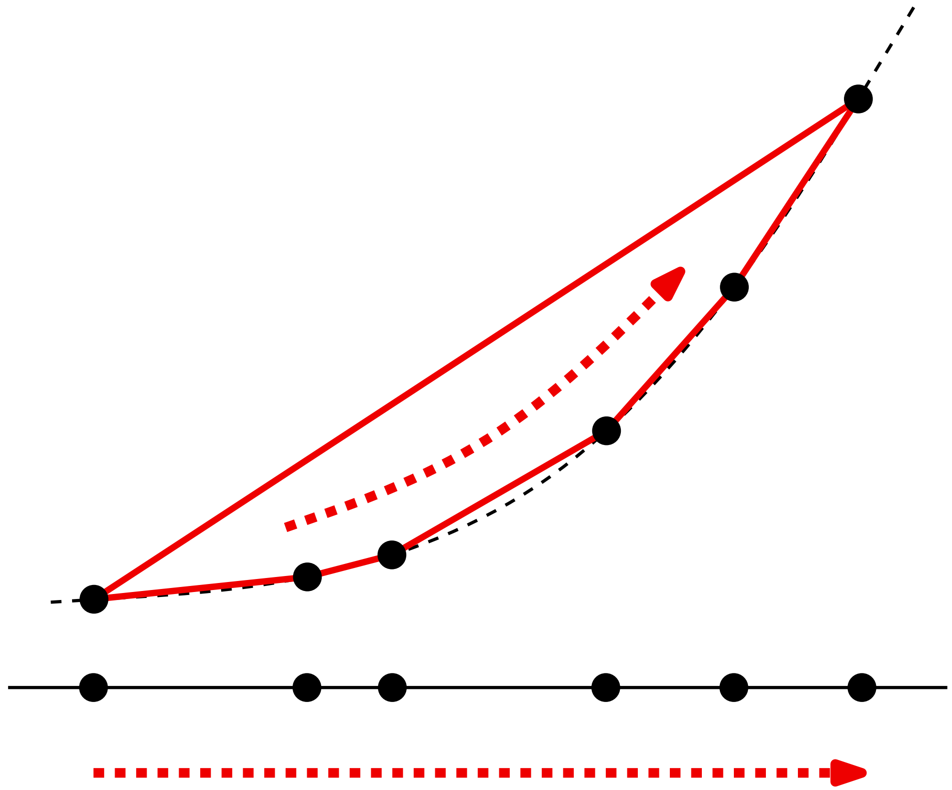

CONVEX HULL

Lower bound

Input: n real numbers
$x_1, \ldots, x_n$ real numbers

$\Big\downarrow O(n)$

Input: n points
$p_1, \ldots, p_n$, with $p_i = (x_i, x_i^2)$

$\Big\downarrow$

Output: convex hull of the points
Sorted list of the vertices of the convex hull

$\Big\downarrow O(n)$

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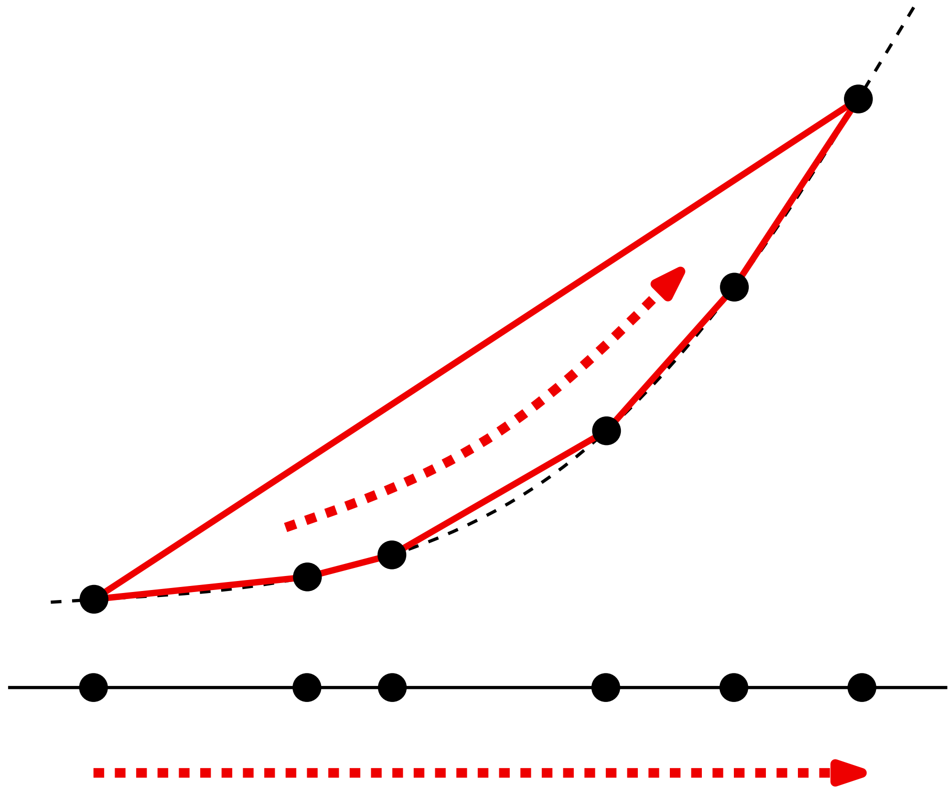

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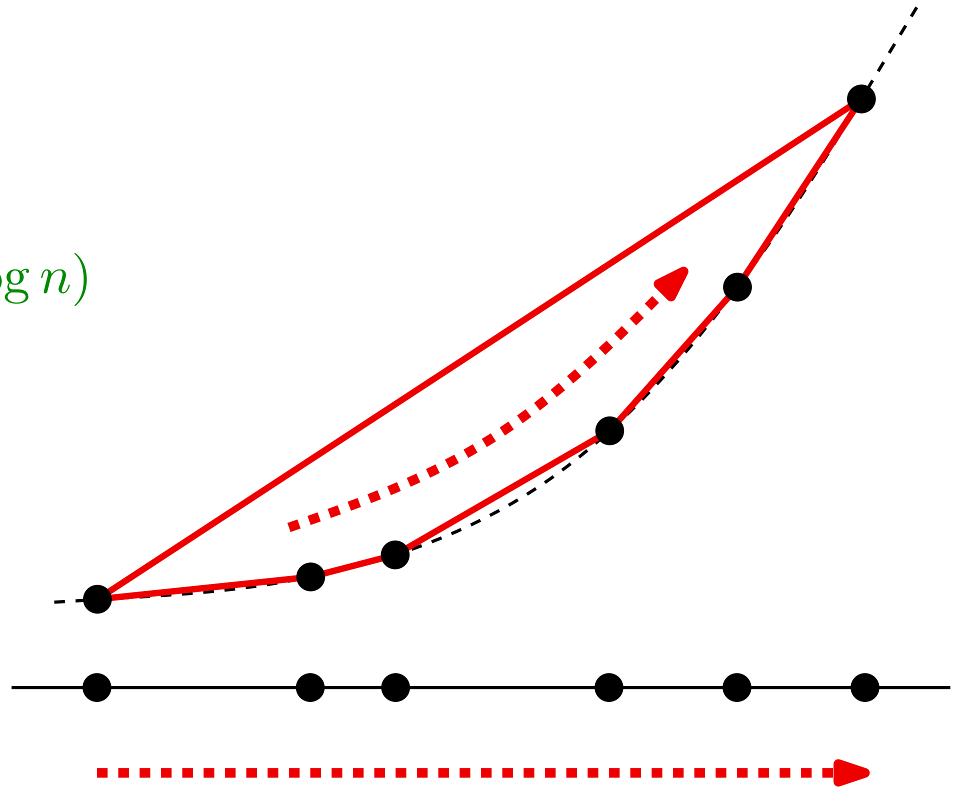
$\Omega(n \log n)$



CONVEX HULL

Lower bound

Input: n real numbers
$x_1, \ldots, x_n$ real numbers

$\downarrow O(n)$

Input: n points
$p_1, \ldots, p_n$, with $p_i = (x_i, x_i^2)$

$\downarrow \boxed{\Omega(n \log n)}$

Output: convex hull of the points
Sorted list of the vertices of the convex hull

$\downarrow O(n)$

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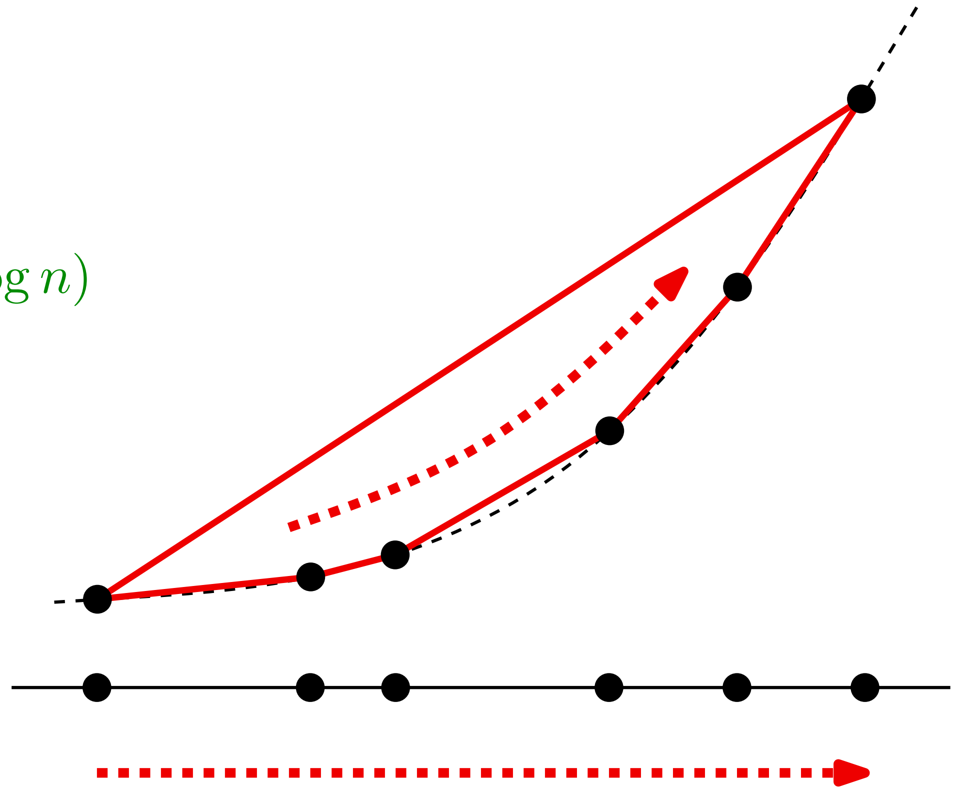
$\Omega(n \log n)$



Extensions

Extensions

- Convex hull of a set of n points in 3D

  (proposed for a theory presentation)

  - Gift wrapping
  - Divide-and-conquer
  - Incremental

- Convex hull of a simple polygon

  (proposed for a theory presentation)

  - Is it possible to design an $o(n \log n)$ time algorithm by exploiting the order of the vertices of the polygon?
  - Is it possible, for example, to apply Graham's algorithm using the order of the vertices of the polygon?

SOME LINKS TO PLAY
WITH THE CONSTRUCTION OF CONVEX HULLS

In 2D:

http://www.dma.fi.upm.es/docencia/segundociclo/geomcomp/convex.html

http://www.geometrylab.de/ConvexHull/

In 3D:

http://www.cse.unsw.edu.au/∼lambert/java/3d/hull.html

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TO LEARN MORE

- J. O'Rourke, **Computational Geometry in C (2nd ed.)**, Cambridge University Press, 1998.
- F. Preparata, M. Shamos, **Computational Geometry: An introduction (revised ed.)**, Springer, 1993.