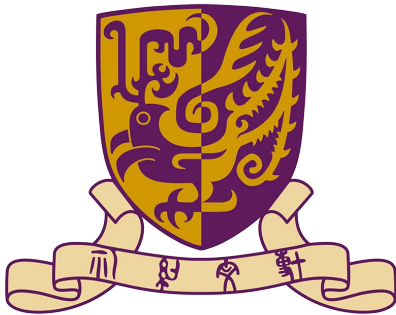


EIE4512 - Digital Image Processing

Histogram Operations and Spatial Filtering



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Image Compression

- The goal of image compression is to reduce the amount of data required to represent a digital image.

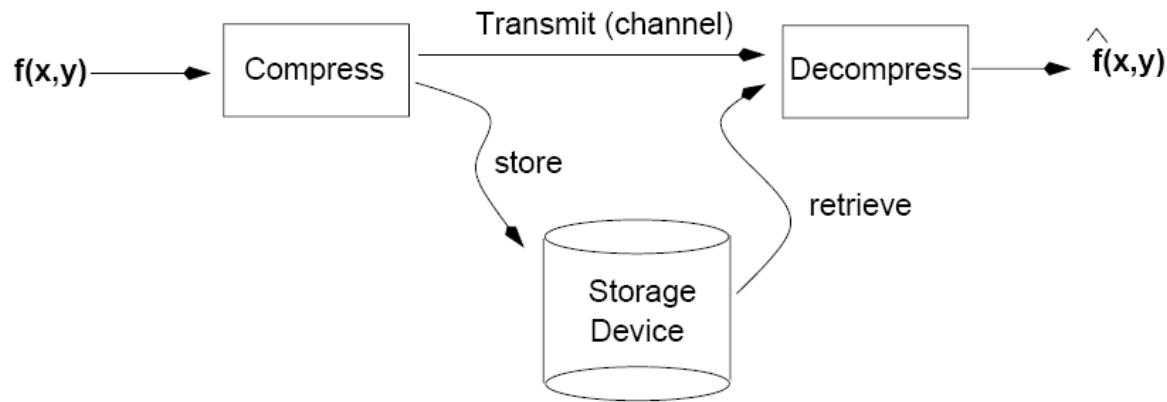


Image Compression (cont'd)

- Lossless

- Information preserving
- Low compression ratios

- Lossy

- Information loss
- High compression ratios

Trade-off: information loss **vs** compression ratio

Data \neq Information

- Data and information are not synonymous terms!
- **Data** is the means by which **information** is conveyed.
- Data compression aims to reduce the amount of data while preserving as much information as possible.

Data vs Information (cont'd)

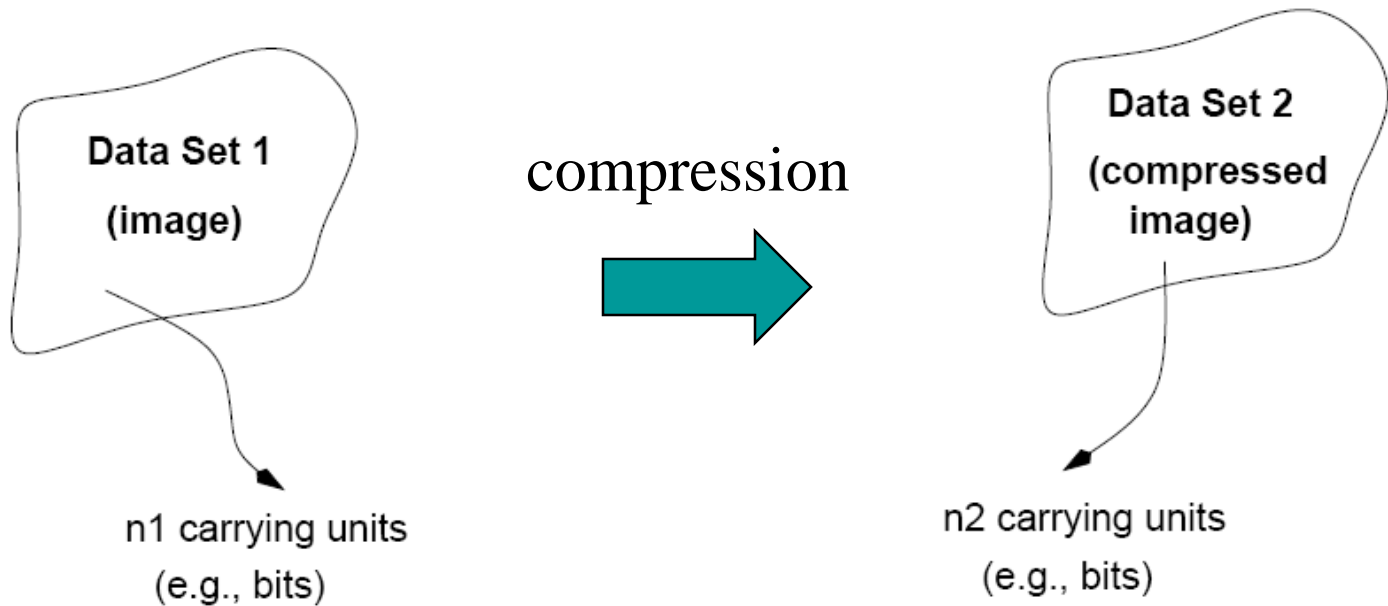
- The same **information** can be represented by different amount of **data** – for example:

Ex1: *Your wife, Helen, will meet you at Logan Airport in Boston at 5 minutes past 6:00 pm tomorrow night*

Ex2: *Your wife will meet you at Logan Airport at 5 minutes past 6:00 pm tomorrow night*

Ex3: *Helen will meet you at Logan at 6:05 pm tomorrow night.*

Compression Ratio



Compression ratio: $C_R = \frac{n_1}{n_2}$

Relevant Data Redundancy

$$R_D = 1 - \frac{1}{C_R}$$

Example:

If $C_R = \frac{10}{1}$, then $R_D = 1 - \frac{1}{10} = 0.9$

(90% of the data in dataset 1 is redundant)

if $n_2 = n_1$, then $C_R=1$, $R_D=0$

if $n_2 \ll n_1$, then $C_R \rightarrow \infty$, $R_D \rightarrow 1$

Types of Data Redundancy

- (1) Coding Redundancy
- (2) Interpixel Redundancy
- (3) Psychovisual Redundancy

- Data compression attempts to reduce one or more of these redundancy types.

Coding - Definitions

- **Code:** a list of symbols (letters, numbers, bits etc.)
- **Code word:** a sequence of symbols used to represent some information (e.g., gray levels).
- **Code word length:** number of symbols in a code word.

Example: (binary code, symbols: 0,1, length: 3)

0: 000	4: 100
1: 001	5: 101
2: 010	6: 110
3: 011	7: 111

Coding - Definitions (cont'd)

N x M image

r_k: k-th gray level

l(r_k): # of bits for r_k

P(r_k): probability of r_k

Average # of bits: $L_{avg} = E(l(r_k)) = \sum_{k=0}^{L-1} l(r_k)P(r_k)$

Total # of bits: NML_{avg}

Expected value: $E(X) = \sum_x xP(X = x)$

Coding Redundancy

- Case 1: $l(r_k) = \text{constant length}$

Example:

r_k	$p_r(r_k)$	Code 1	$l_1(r_k)$
$r_0 = 0$	0.19	000	3
$r_1 = 1/7$	0.25	001	3
$r_2 = 2/7$	0.21	010	3
$r_3 = 3/7$	0.16	011	3
$r_4 = 4/7$	0.08	100	3
$r_5 = 5/7$	0.06	101	3
$r_6 = 6/7$	0.03	110	3
$r_7 = 1$	0.02	111	3

Assume an image with $L = 8$

$$\text{Assume } l(r_k) = 3, \quad L_{avg} = \sum_{k=0}^7 3P(r_k) = 3 \sum_{k=0}^7 P(r_k) = 3 \text{ bits}$$

Total number of bits: $3NM$

Coding Redundancy (cont'd)

- Case 2: $l(r_k) = \text{variable length}$

Table 6.1 Variable-Length Coding Example variable length

r_k	$p_i(r_k)$	Code 1	$l_1(r_k)$	Code 2	$l_2(r_k)$
$r_0 = 0$	0.19	000	3	11	2
$r_1 = 1/7$	0.25	001	3	01	2
$r_2 = 2/7$	0.21	010	3	10	2
$r_3 = 3/7$	0.16	011	3	001	3
$r_4 = 4/7$	0.08	100	3	0001	4
$r_5 = 5/7$	0.06	101	3	00001	5
$r_6 = 6/7$	0.03	110	3	000001	6
$r_7 = 1$	0.02	111	3	000000	6

$$L_{avg} = \sum_{k=0}^7 l(r_k)P(r_k) = 2.7 \text{ bits}$$

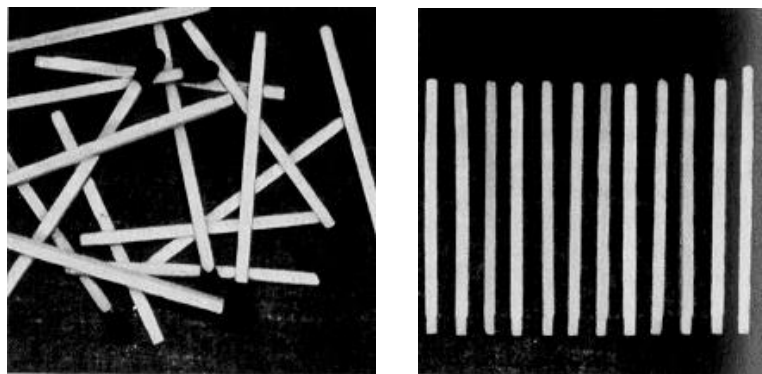
$$C_R = \frac{3}{2.7} = 1.11 \text{ (about 10\%)}$$

Total number of bits: $2.7NM$

$$R_D = 1 - \frac{1}{1.11} = 0.099$$

Interpixel redundancy

- Interpixel redundancy implies that pixel values are correlated (i.e., a pixel value can be reasonably predicted by its neighbors).



$$f(x) \circ g(x) = \int_{-\infty}^{\infty} f(x)g(x+a)da$$

auto-correlation: $f(x)=g(x)$

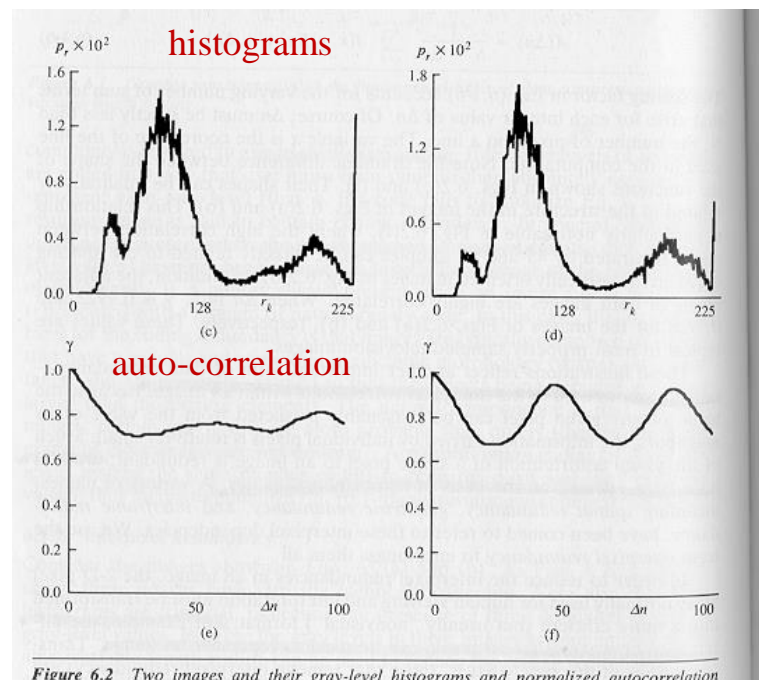


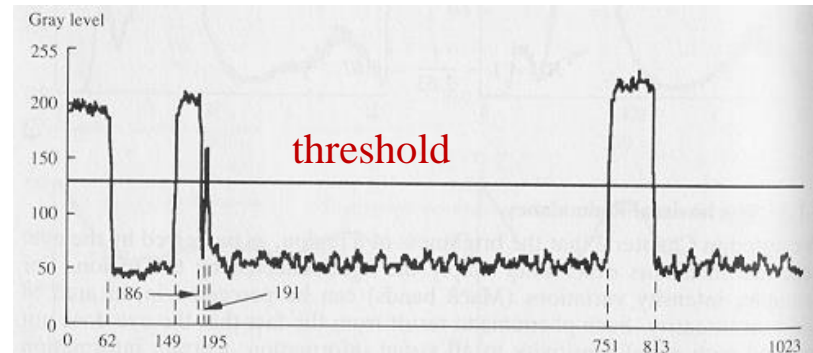
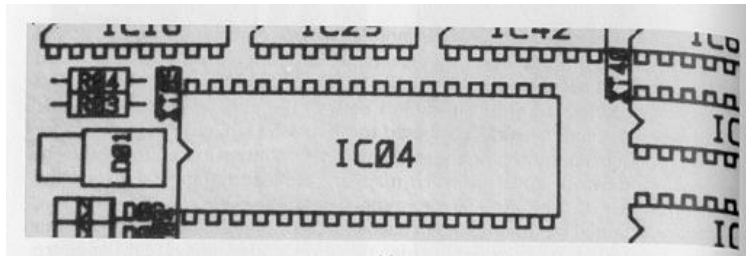
Figure 6.2 Two images and their gray-level histograms and normalized autocorrelation

Interpixel redundancy (cont'd)

- To reduce interpixel redundancy, some kind of transformation must be applied on the data (e.g., thresholding, DFT, DWT)

Example:

original



110000.....11.....000.....

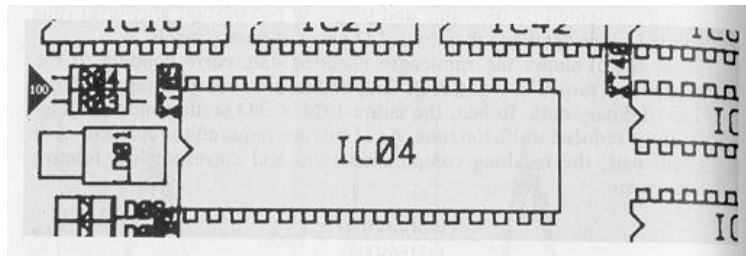
Run-length encoding:

(1,63) (0,87) (1,37) (0,5) (1,4) (0, 556) (1,62) (0,210)

Using 11 bits/pair: (1+10) bits/pair

88 bits are required (compared to 1024 !!)

thresholded



Psychovisual redundancy

- The human eye is more sensitive to the **lower** frequencies than to the **higher** frequencies in the visual spectrum.
- Idea: discard data that is perceptually insignificant!

Psychovisual redundancy (cont'd)

Example: quantization

256 gray levels



16 gray levels



16 gray levels + random noise



$$C=8/4 = 2:1$$

add a small pseudo-random number
to each pixel prior to quantization

Measuring Information

- A key question in image compression is:
 - “What is the minimum **amount of data** that is sufficient to describe completely an image without **loss of information**?”
- How do we measure the **information content** of an image?

Measuring Information (cont'd)

- We assume that **information generation** is a probabilistic process.
- Idea: associate information with probability!

A random event E with probability $P(E)$ contains:

$$I(E) = \log\left(\frac{1}{P(E)}\right) = -\log(P(E)) \text{ units of information}$$

Note: $I(E)=0$ when $P(E)=1$

How much information does a pixel value contain?

- Suppose that gray level values are generated by a random process, then r_k contains:

$$I(r_k) = -\log(P(r_k)) \quad \text{units of information!}$$

(assume statistically independent random events)

How much information does an image contain?

- Average information content of an image:

$$E = \sum_{k=0}^{L-1} I(r_k) P(r_k)$$

using $I(r_k) = -\log(P(r_k))$

Entropy: $H = - \sum_{k=0}^{L-1} P(r_k) \log(P(r_k))$ units/pixel
(e.g., bits/pixel)

Redundancy

- **Redundancy:** $R = L_{avg} - H$
(data vs info)

where:
$$L_{avg} = E(l(r_k)) = \sum_{k=0}^{L-1} l(r_k)P(r_k)$$

Note: if $L_{avg} = H$, then $R=0$ (no redundancy)

Entropy Estimation

- It is not easy to estimate H reliably!

image

21	21	21	95	169	243	243	243
21	21	21	95	169	243	243	243
21	21	21	95	169	243	243	243
21	21	21	95	169	243	243	243

<i>Gray Level</i>	<i>Count</i>	<i>Probability</i>
21	12	3/8
95	4	1/8
169	4	1/8
243	12	3/8

Entropy Estimation (cont'd)

- First order estimate of H:

$$H = - \sum_{k=0}^3 P(r_k) \log(P(r_k)) = 1.81 \text{ bits/pixel}$$

$$L_{\text{avg}} = 8 \text{ bits/pixel} \quad R = L_{\text{avg}} - H$$

The first-order estimate provides only a lower-bound on the compression that can be achieved.

Estimating Entropy (cont'd)

- **Second order** estimate of H:
 - Use relative frequencies of pixel blocks :

image

21	21	21	95	169	243	243	243
21	21	21	95	169	243	243	243
21	21	21	95	169	243	243	243
21	21	21	95	169	243	243	243

<i>Gray Level Pair</i>	<i>Count</i>	<i>Probability</i>
(21, 21)	8	1/4
(21, 95)	4	1/8
(95, 169)	4	1/8
(169, 243)	4	1/8
(243, 243)	8	1/4
(243, 21)	4	1/8

$$H = 2.5/2 = 1.25 \text{ bits/pixel}$$

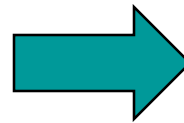
Differences in Entropy Estimates

- Differences between higher-order estimates of entropy and the first-order estimate indicate the presence of **interpixel redundancy**!
- Need to apply some **transformation** to deal with interpixel redundancy!

Differences in Entropy Estimates (cont'd)

- **Example:** consider pixel differences

21	21	21	95	169	243	243	243		21	0	0	74	74	74	0	0
21	21	21	95	169	243	243	243		21	0	0	74	74	74	0	0
21	21	21	95	169	243	243	243		21	0	0	74	74	74	0	0
21	21	21	95	169	243	243	243		21	0	0	74	74	74	0	0



<i>Gray Level or Difference</i>	<i>Count</i>	<i>Probability</i>
0	16	1/2
21	4	1/8
74	12	3/8

Differences in Entropy Estimates (cont'd)

- What is the entropy of the pixel differences image?

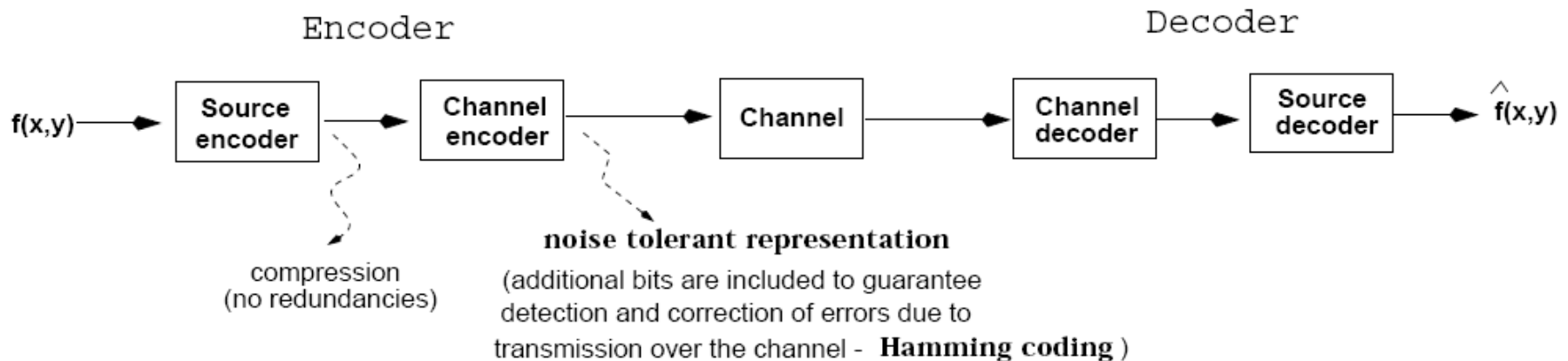
$$H = - \sum_{k=0}^2 P(r_k) \log(P(r_k)) = 1.41 \text{ bits/pixel}$$

(better than the entropy of the original image $H=1.81$)

- An even better transformation should be possible since the second order entropy estimate is lower:

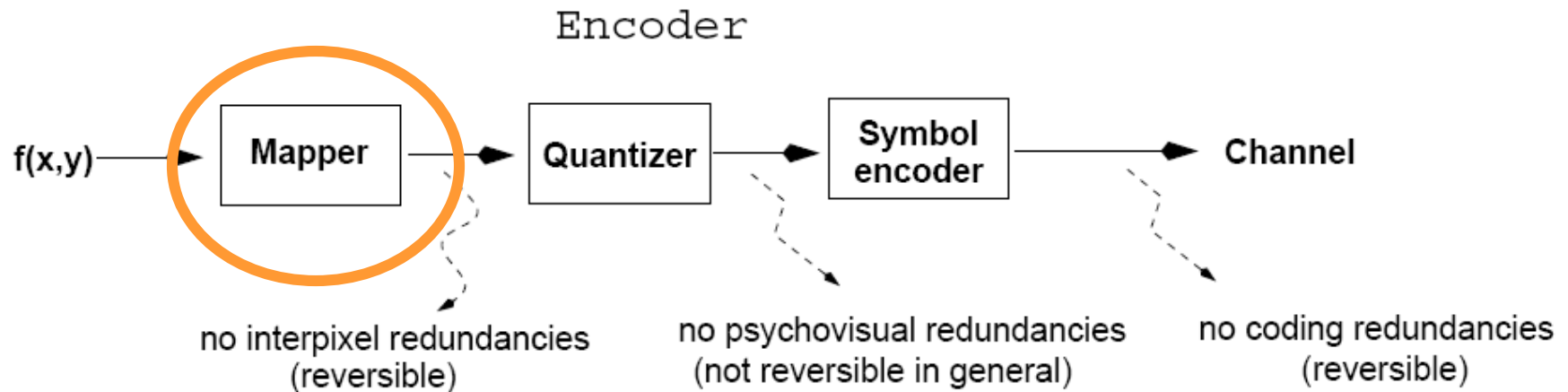
$$1.41 \text{ bits/pixel} > 1.25 \text{ bits/pixel}$$

Image Compression Model



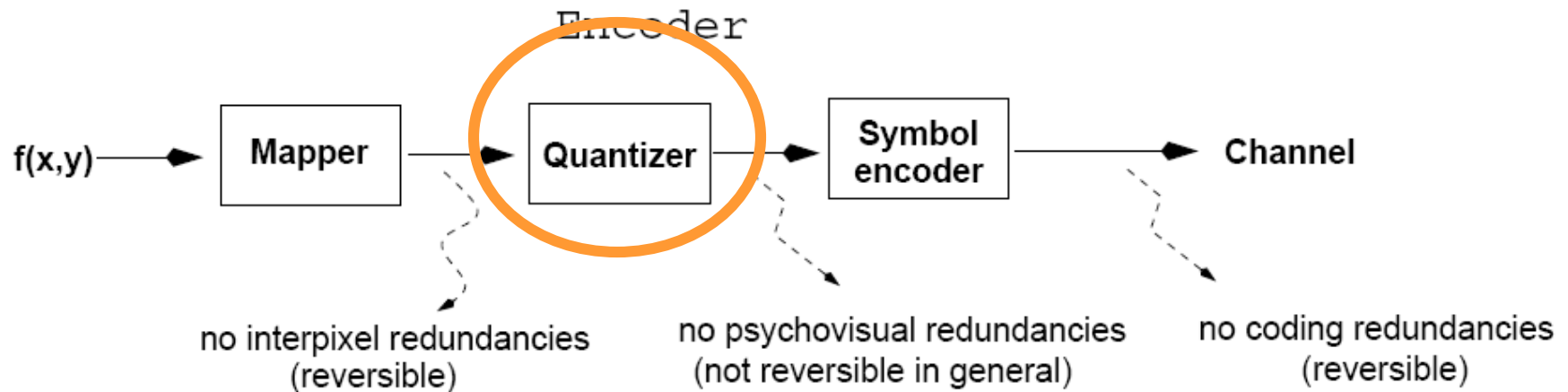
We will focus on the **Source** Encoder/Decoder only.

Image Compression Model (cont'd)



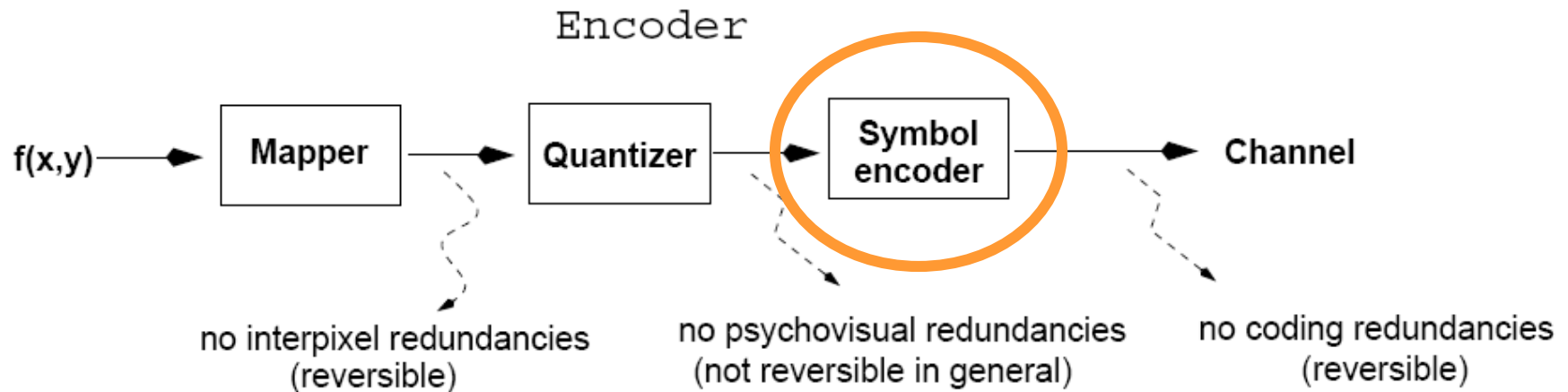
- **Mapper:** transforms data to account for interpixel redundancies.

Image Compression Model (cont'd)



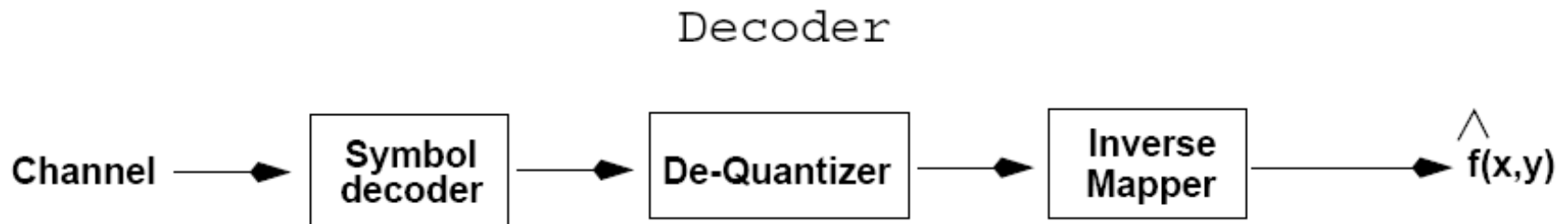
- **Quantizer:** quantizes the data to account for psychovisual redundancies.

Image Compression Model (cont'd)



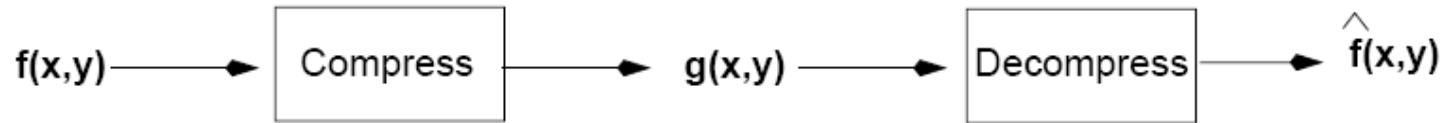
- **Symbol encoder:** encodes the data to account for coding redundancies.

Image Compression Models (cont'd)



- The decoder applies the inverse steps.
- Note that quantization is **irreversible** in general.

Fidelity Criteria



- How close is $f(x, y)$ to $\hat{f}(x, y)$?
- Criteria
 - Subjective: based on human observers
 - Objective: mathematically defined criteria

Subjective Fidelity Criteria

Value	Rating	Description
1	Excellent	An image of extremely high quality, as good as you could desire.
2	Fine	An image of high quality, providing enjoyable viewing. Interference is not objectionable.
3	Passable	An image of acceptable quality. Interference is not objectionable.
4	Marginal	An image of poor quality; you wish you could improve it. Interference is somewhat objectionable.
5	Inferior	A very poor image, but you could watch it. Objectionable interference is definitely present.
6	Unusable	An image so bad that you could not watch it.

Objective Fidelity Criteria

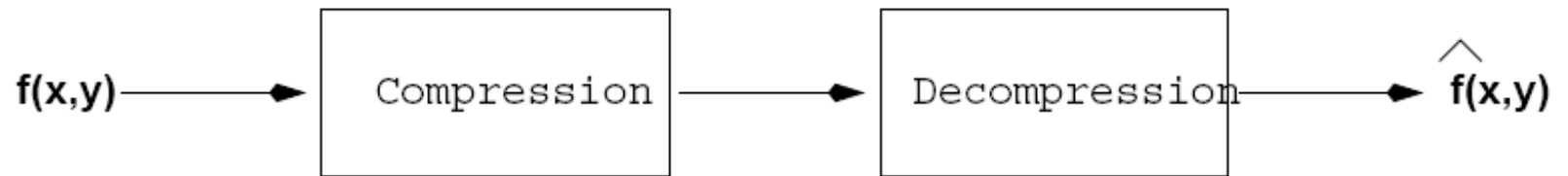
- Root mean square error (RMS)

$$e_{rms} = \sqrt{\frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} (\hat{f}(x, y) - f(x, y))^2}$$

- Mean-square signal-to-noise ratio (SNR)

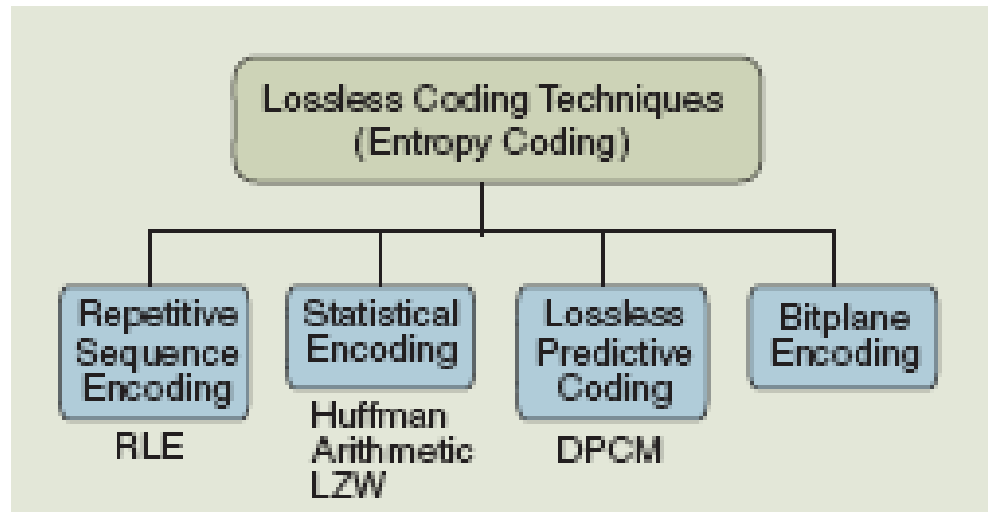
$$SNR_{ms} = \frac{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} (\hat{f}(x, y))^2}{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} (\hat{f}(x, y) - f(x, y))^2}$$

Lossless Compression



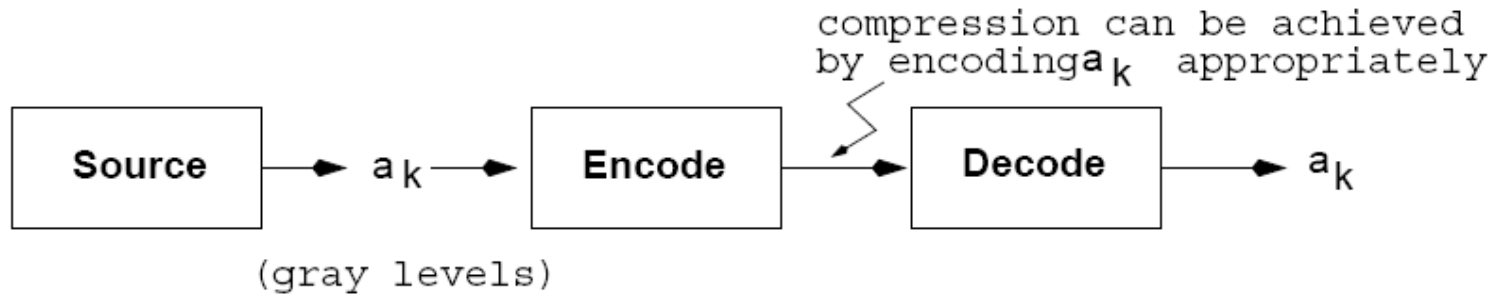
$$e(x, y) = \hat{f}(x, y) - f(x, y) = 0$$

Taxonomy of Lossless Methods



Huffman Coding

(addresses coding redundancy)



- A **variable-length coding** technique.
- Source symbols are encoded **one** at a time!
 - There is a **one-to-one correspondence** between source symbols and code words.
- **Optimal code** - minimizes code word length per source symbol.

Huffman Coding (cont'd)

- Forward Pass

1. Sort probabilities per symbol
2. Combine the lowest two probabilities
3. Repeat *Step2* until only two probabilities remain.

Original source		Source reduction			
Symbol	Probability	1	2	3	4
a_2	0.4	0.4	0.4	0.4	0.6 0.4
a_6	0.3	0.3	0.3	0.3	
a_1	0.1	0.1	0.2	0.3	
a_4	0.1	0.1			
a_3	0.06	0.1	0.1		
a_5	0.04				

Huffman Coding (cont'd)

- Backward Pass

Assign code symbols going backwards

Original source			Source reduction			
Sym.	Prob.	Code	1	2	3	4
a_2	0.4	1	0.4 1	0.4 1	0.4 1	0.6 0
a_6	0.3	00	0.3 00	0.3 00	0.3 00	0.4 1
a_1	0.1	011	0.1 011	0.2 010	0.3 01	
a_4	0.1	0100	0.1 0100	0.1 011		
a_3	0.06	01010	0.1 0101			
a_5	0.04	01011				

Huffman Coding (cont'd)

- L_{avg} assuming **Huffman coding**:

$$L_{avg} = E(l(a_k)) = \sum_{k=1}^6 l(a_k)P(a_k) =$$

$$3 \times 0.1 + 1 \times 0.4 + 5 \times 0.06 + 4 \times 0.1 + 5 \times 0.04 + 2 \times 0.3 = 2.2 \text{ bits/symbol}$$

- L_{avg} assuming **binary coding**:

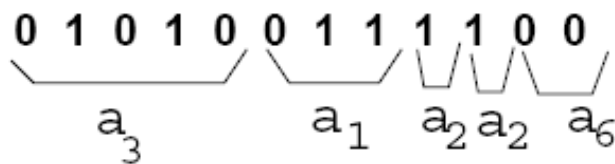
6 symbols, we need a 3-bit code

$(a_1: 000, a_2: 001, a_3: 010, a_4: 011, a_5: 100, a_6: 101)$

$$L_{avg} = \sum_{k=1}^6 l(a_k)P(a_k) = \sum_{k=1}^6 3P(a_k) = 3 \sum_{k=1}^6 P(a_k) = 3 \text{ bits/symbol}$$

Huffman Coding/Decoding

- Coding/Decoding can be implemented using a **look-up table**.
- Decoding can be done unambiguously.



Original source		
Sym.	Prob.	Code
a_2	0.4	1
a_6	0.3	00
a_1	0.1	011
a_4	0.1	0100
a_3	0.06	01010
a_5	0.04	01011

Arithmetic (or Range) Coding

(addresses coding redundancy)

- Huffman coding encodes source symbols **one** at a time.
- Arithmetic coding encodes **sequences** of source symbols.
 - Slower than Huffman coding but can achieve better compression.
 - There is **no** one-to-one correspondence between source symbols and code words.

Arithmetic Coding (cont'd)

- Represent a sequence of source symbols by a sub-interval in $[0,1)$ which can be encoded using an arithmetic code.

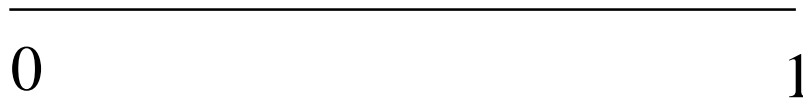


- Start with the interval $[0, 1)$
- As more symbols are encoded, a sub-interval is chosen to represent the message which keeps shrinking as the message increases.

Arithmetic Coding (cont'd)

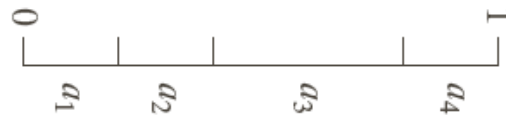
Suppose there are four symbols $\alpha_1 \alpha_2 \alpha_3 \alpha_4$

1) Start with interval $[0, 1)$



Source Symbol	Probability
a_1	0.2
a_2	0.2
a_3	0.4
a_4	0.2

2) Subdivide $[0, 1)$ based on the probabilities of α_i



Initial Subinterval
$[0.0, 0.2)$
$[0.2, 0.4)$
$[0.4, 0.8)$
$[0.8, 1.0)$

3) Update interval by processing message

Example

Encode

$a_1 a_2 a_3 a_3 a_4$



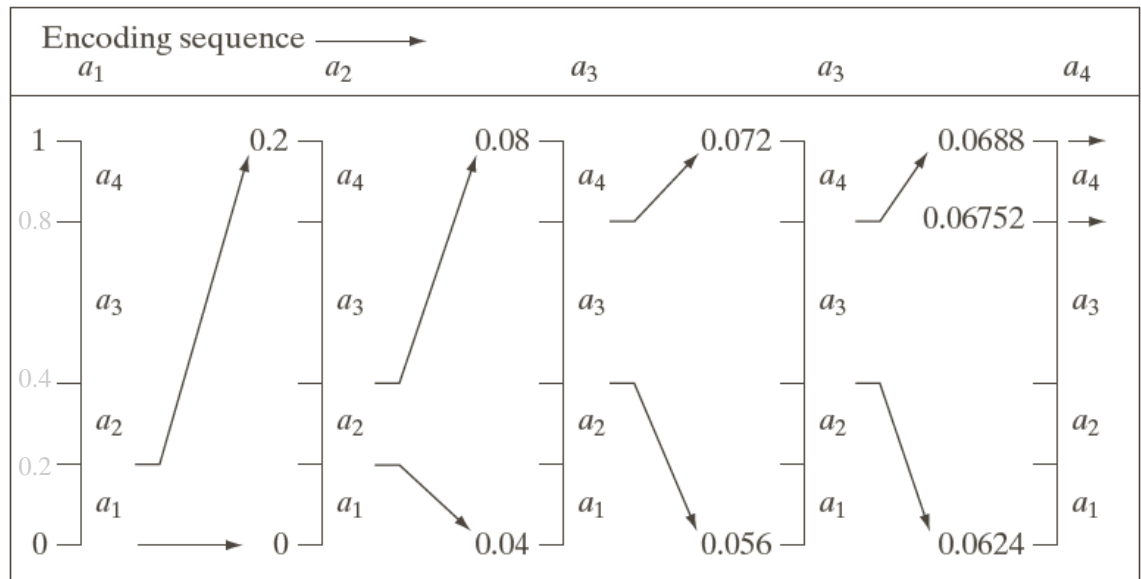
$[0.06752, 0.0688)$



code: 0.068

(must be inside sub-interval)

Source Symbol	Probability	Initial Subinterval
a_1	0.2	$[0.0, 0.2)$
a_2	0.2	$[0.2, 0.4)$
a_3	0.4	$[0.4, 0.8)$
a_4	0.2	$[0.8, 1.0)$



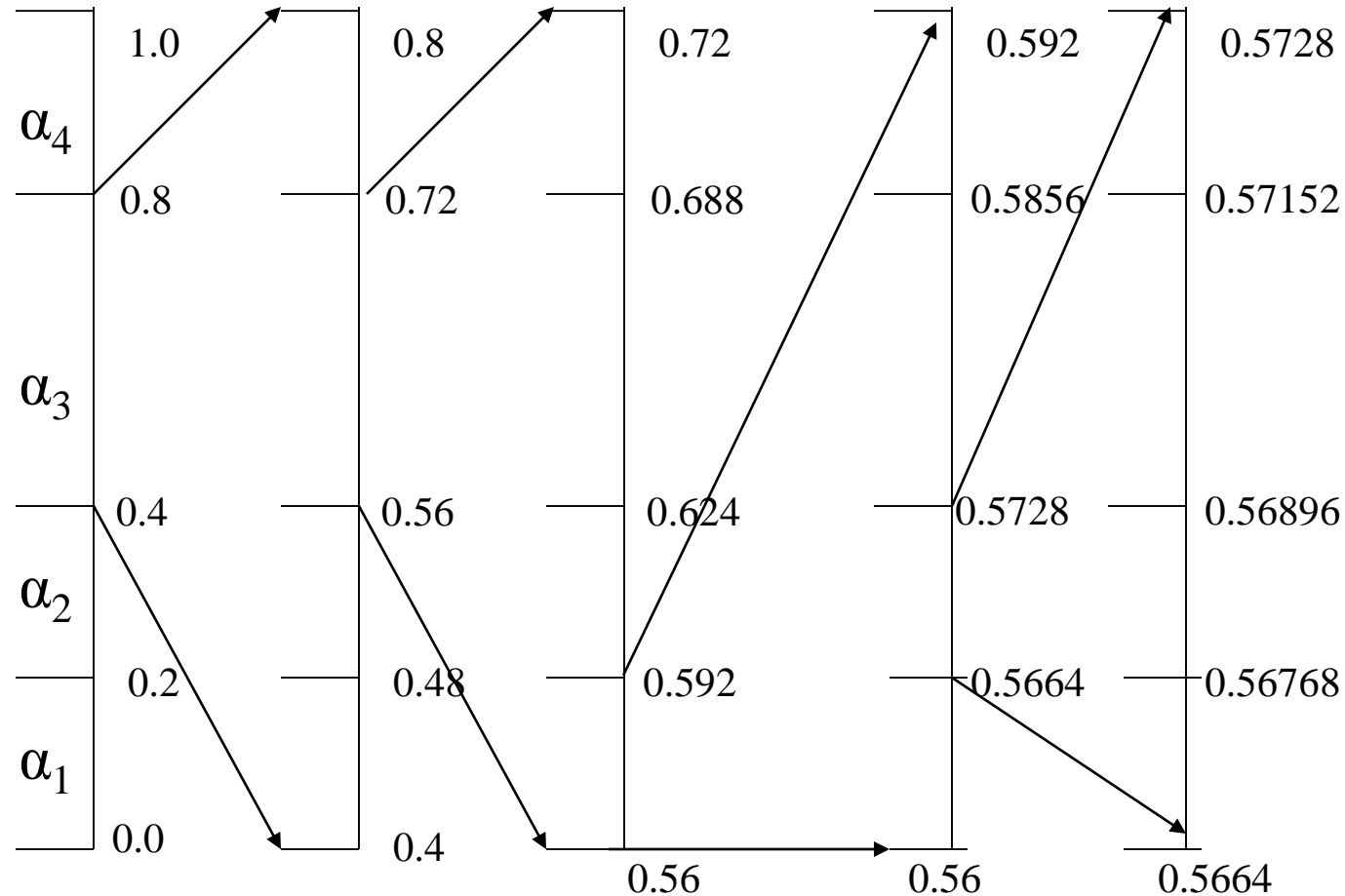
Example (cont'd)

- The message $\alpha_1 \alpha_2 \alpha_3 \alpha_3 \alpha_4$ is encoded using 3 decimal digits or $3/5 = 0.6$ decimal digits per source symbol.
- The entropy of this message is: $H = - \sum_{k=0}^3 P(r_k) \log(P(r_k))$

$$-(3 \times 0.2 \log_{10}(0.2) + 0.4 \log_{10}(0.4)) = 0.5786 \text{ digits/symbol}$$

Note: finite precision arithmetic might cause problems due to truncations!

Arithmetic Decoding



Decode 0.572
(sequence length=5)



$\alpha_3 \alpha_3 \alpha_1 \alpha_2 \alpha_4$

LZW Coding

(addresses interpixel redundancy)

- Requires no prior knowledge of symbol probabilities.
- Assigns **fixed length** code words to **variable length** symbol sequences.
 - There is **no** one-to-one correspondence between source symbols and code words.
- Included in GIF, TIFF and PDF file formats

LZW Coding

- A **codebook** (or **dictionary**) needs to be constructed.
- Initially, the first 256 entries of the dictionary are assigned to the gray levels 0,1,2,...,255 (i.e., assuming 8 bits/pixel)

Initial Dictionary

Dictionary Location	Entry
0	0
1	1
·	·
255	255
256	-
511	-

LZW Coding (cont'd)

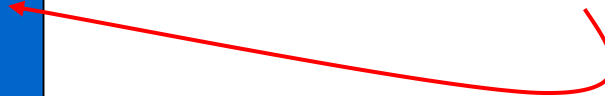
Example:

39 39 126 126
39 39 126 126
39 39 126 126
39 39 126 126

As the encoder examines image pixels, gray level sequences (i.e., blocks) that are not in the dictionary are assigned to a new entry.

Dictionary Location	Entry
0	0
1	1
.	.
255	255
256	39-39
511	-

- Is 39 in the dictionary.....Yes
- What about 39-39.....No
 - * Add 39-39 at location 256



Example

39 39 126 126
 39 39 126 126
 39 39 126 126
 39 39 126 126

Concatenated Sequence: $CS = CR + P$

(CR) (P)

Currently Recognized Sequence	Pixel Being Processed	Encoded Output	Dictionary Location (Code Word)	Dictionary Entry
	39			
39	39	39	256	39-39
39	126	39	257	39-126
126	126	126	258	126-126
126	39	126	259	126-39
39	39			
39-39	126	256	260	39-39-126
126	126			
126-126	39	258	261	126-126-39
39	39			
39-39	126			
39-39-126	126	260	262	39-39-126-126
126	39			
126-39	39	259	263	126-39-39
39	126			
39-126	126	257	264	39-126-126
126		126		

CR = empty

repeat

P=next pixel

CS=CR + P

If CS is found:

(1) No Output

(2) CR=CS

else:

(1) Output D(CR)

(2) Add CS to D

(3) CR=P

Decoding LZW

- Use the dictionary for decoding the “encoded output” sequence.
- The dictionary need not be sent with the encoded output.
- Can be built on the “fly” by the decoder as it reads the received code words.

Run-length coding (RLC)

(addresses interpixel redundancy)

- Reduce the size of a repeating string of symbols (i.e., runs):

1 1 1 1 1 0 0 0 0 0 0 1 \rightarrow (1,5) (0, 6) (1, 1)

a a a b b b b b b c c \rightarrow (a,3) (b, 6) (c, 2)

- Encodes a run of symbols into two bytes: **(symbol, count)**
- Can compress any type of data but cannot achieve high compression ratios compared to other compression methods.

Combining Huffman Coding with Run-length Coding

- Assuming that a message has been encoded using Huffman coding, additional compression can be achieved using run-length coding.

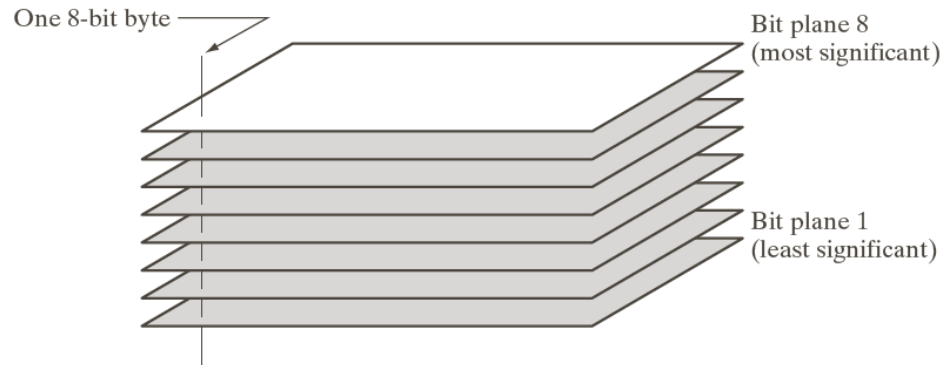
0 1 0 1 0 0 1 1 1 1 0 0

e.g., (0,1)(1,1)(0,1)(1,0)(0,2)(1,4)(0,2)

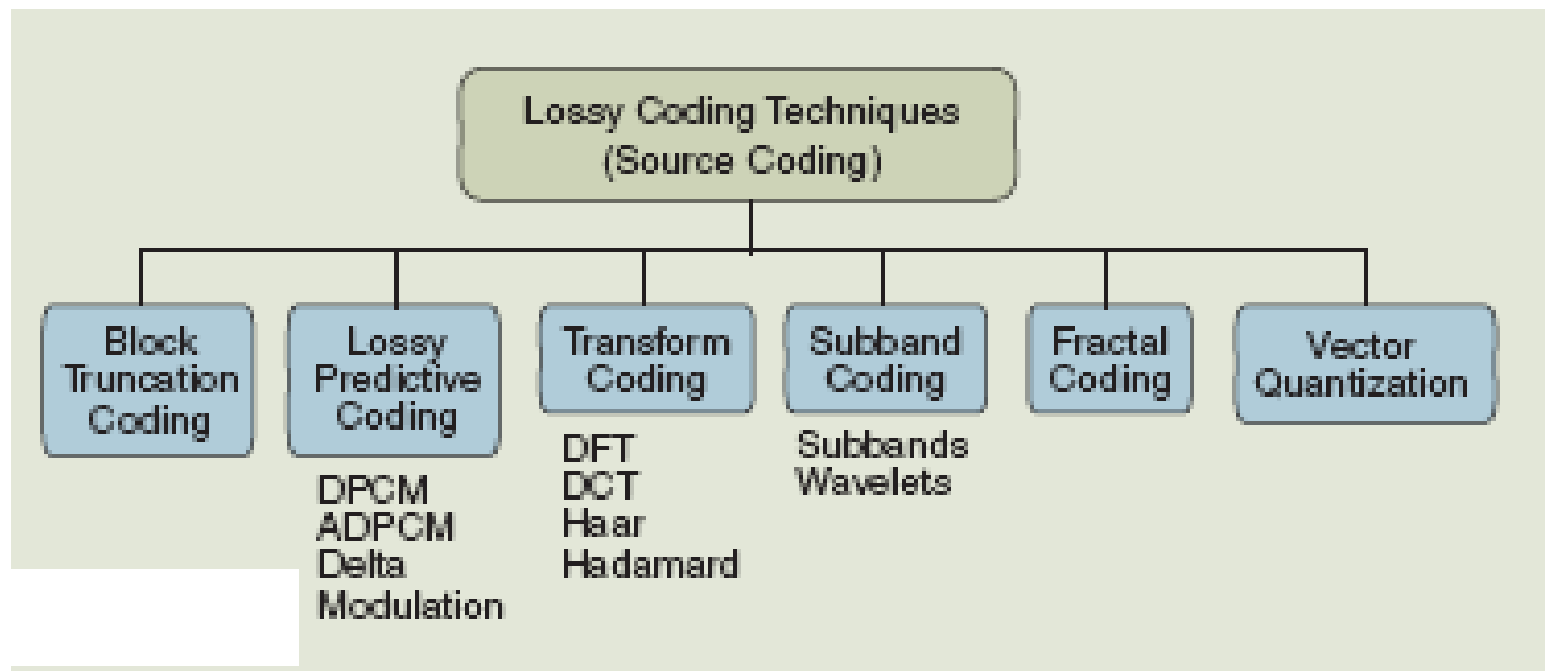
Bit-plane coding

(addresses interpixel redundancy)

- Process each **bit plane** individually.
- (1) Decompose an image into a series of binary images.
 - (2) Compress each binary image (e.g., using run-length coding)

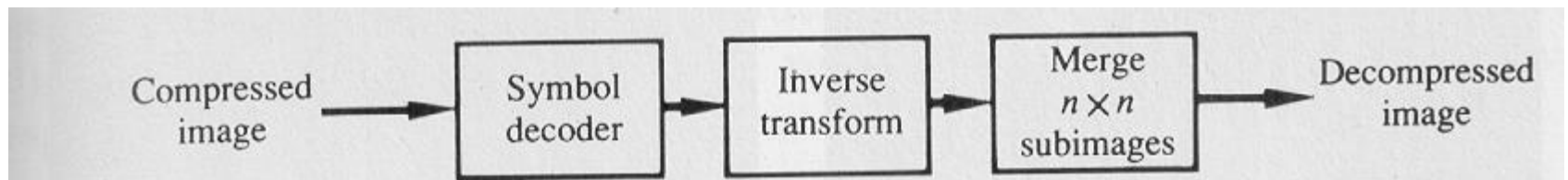
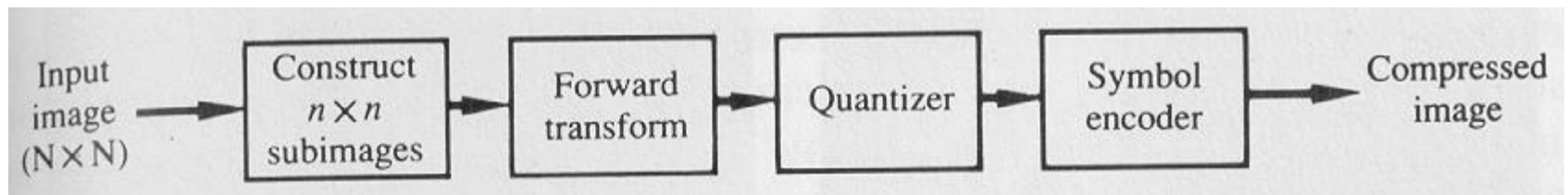


Lossy Methods - Taxonomy



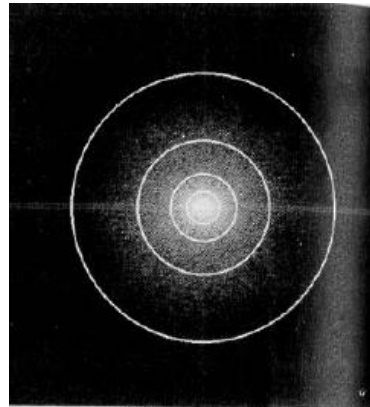
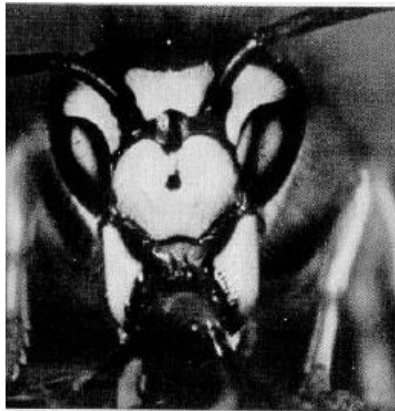
Lossy Compression

- Transform the image into some other domain to reduce interpixel redundancy.



Example: Fourier Transform

$$f(x, y) = \frac{1}{N} \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} F(u, v) e^{\frac{j2\pi(ux+vy)}{N}}, \quad x, y=0, 1, \dots, N-1$$



Note that the magnitude of the FT decreases, as u, v increase!

$$K \ll N$$

$$\hat{f}(x, y) = \frac{1}{N} \sum_{u=0}^{K-1} \sum_{v=0}^{K-1} F(u, v) e^{\frac{j2\pi(ux+vy)}{N}}, \quad x, y=0, 1, \dots, N-1$$

$\sum_{x,y} (\hat{f}(x, y) - f(x, y))^2$ is very small !!

Transform Selection

$$f(x, y) = \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} T(u, v)h(x, y, u, v)$$

- $T(u, v)$ can be computed using various transformations, for example:
 - DFT
 - DCT (Discrete Cosine Transform)
 - KLT (Karhunen-Loeve Transformation) or Principal Component Analysis (PCA)
- JPEG uses DCT for handling interpixel redundancy.

DCT (Discrete Cosine Transform)

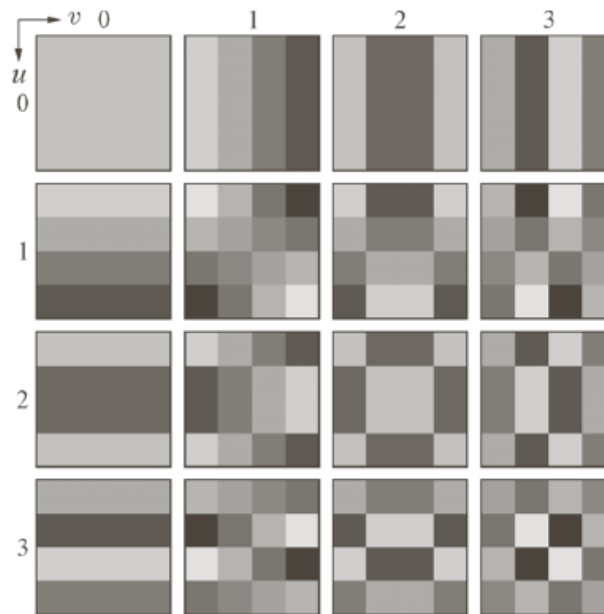
Forward:
$$C(u, v) = \alpha(u)\alpha(v) \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x, y) \cos\left(\frac{(2x+1)u\pi}{2N}\right) \cos\left(\frac{(2y+1)v\pi}{2N}\right),$$
$$u, v=0, 1, \dots, N-1$$

Inverse:
$$f(x, y) = \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} \alpha(u)\alpha(v) C(u, v) \cos\left(\frac{(2x+1)u\pi}{2N}\right) \cos\left(\frac{(2y+1)v\pi}{2N}\right),$$
$$x, y=0, 1, \dots, N-1$$

$$\alpha(u) = \begin{cases} \sqrt{1/N} & \text{if } u=0 \\ \sqrt{2/N} & \text{if } u>0 \end{cases} \quad \alpha(v) = \begin{cases} \sqrt{1/N} & \text{if } v=0 \\ \sqrt{2/N} & \text{if } v>0 \end{cases}$$

DCT (cont'd)

- Basis functions for a 4x4 image (i.e., cosines of different frequencies).



DCT (cont'd)

Using
8 x 8 sub-images
yields 64 coefficients
per sub-image.

Reconstructed images
by **truncating**
50% of the
coefficients

DCT is a more **compact**
transformation!

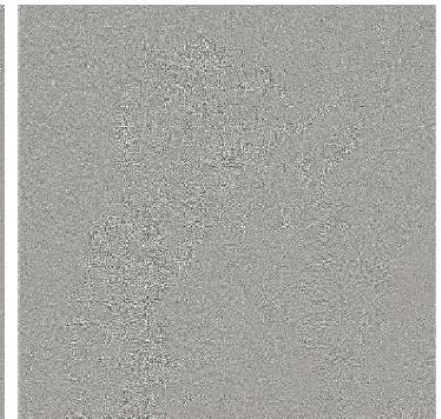
DFT



WHT



DCT



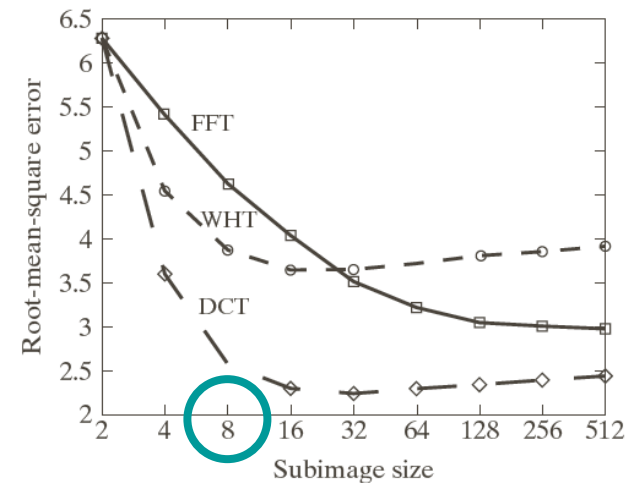
RMS error: 2.32

1.78

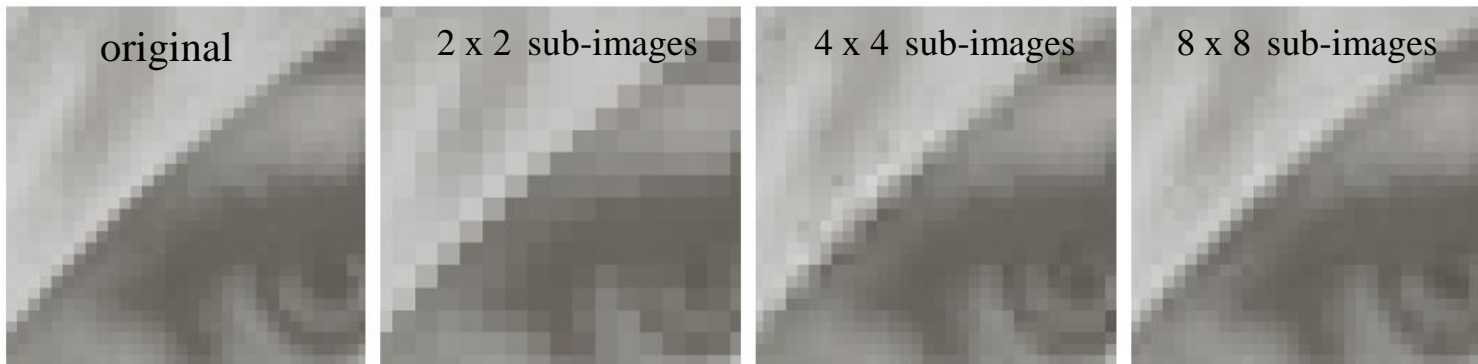
1.13

DCT (cont'd)

- Sub-image size selection:



Reconstructions

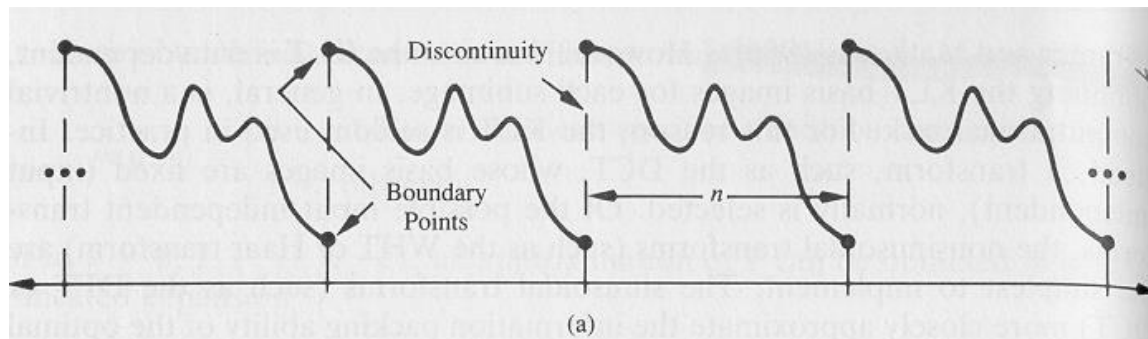


DCT (cont'd)

- DCT minimizes "blocking artifacts" (i.e., boundaries between subimages do not become very visible).

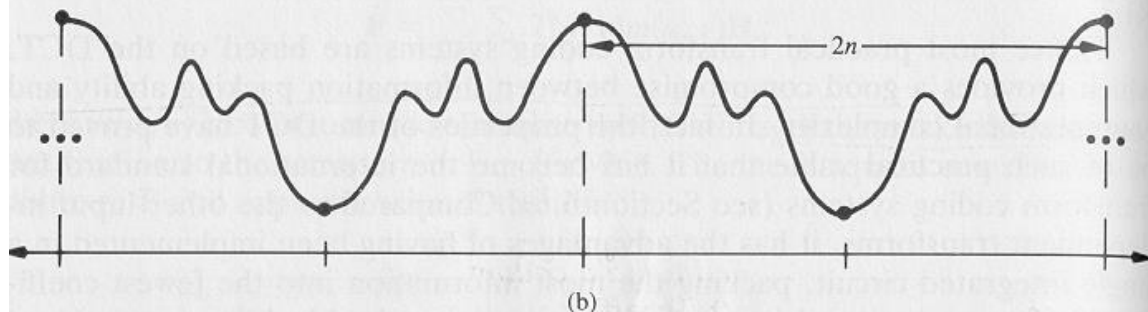
DFT

has n -point periodicity



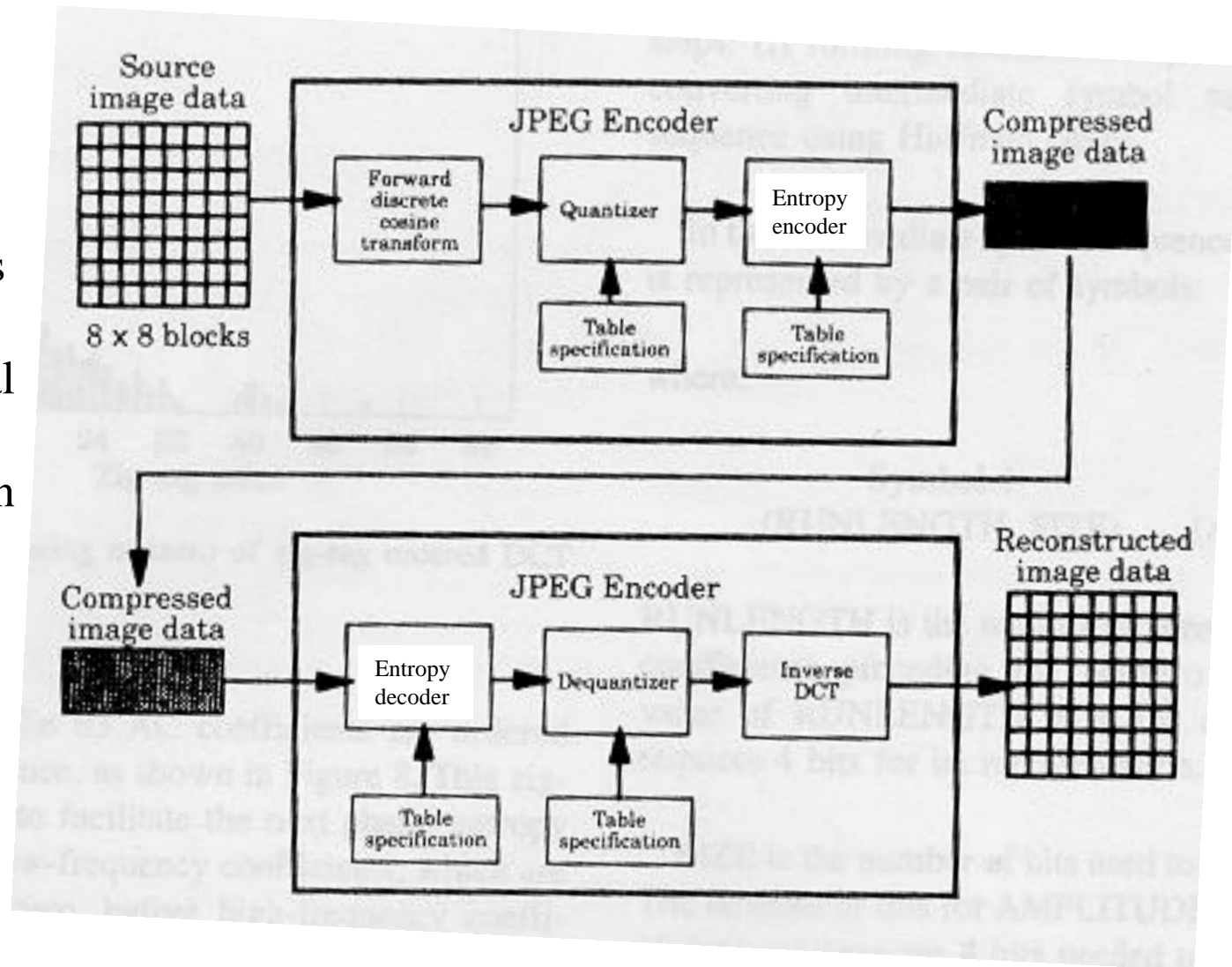
DCT

has $2n$ -point periodicity



JPEG Compression

Accepted as an international image compression standard in 1992.



JPEG - Steps

1. Divide image into 8x8 subimages.

For each subimage do:

2. Shift the gray-levels in the range $[-128, 127]$

3. Apply DCT \rightarrow 64 coefficients

1 **DC coefficient**: $F(0,0)$

63 **AC coefficients**: $F(u,v)$

Example

(a) Original 8 8 block	(b) Shifted block	(c) Block after FDCT Eqn. (5)
140 144 147 1140 140 155 179 175 144 152 140 147 140 148 167 179 152 155 136 167 163 162 152 172 168 145 156 160 152 155 136 160 162 148 156 148 140 136 147 162 147 167 140 155 155 140 136 162 136 156 123 167 162 144 140 147 148 155 136 155 152 147 147 136	12 16 19 12 11 27 51 47 16 24 12 19 12 20 39 51 24 27 8 39 35 34 24 44 40 17 28 32 24 27 8 32 34 20 28 20 12 8 19 34 19 39 12 27 27 12 8 34 8 28 -5 39 34 16 12 19 20 27 8 27 24 19 19 8	185 -17 14 -8 23 -9 -13 -18 20 -34 26 -9 -10 10 13 6 -10 -23 -1 6 -18 3 -20 0 -8 -5 14 -14 -8 -2 -3 8 -3 9 7 1 -11 17 18 15 3 -2 -18 8 8 -3 0 -6 8 0 -2 3 -1 -7 -1 -1 0 -7 -2 1 1 4 -6 0

$[-128, 127]$

(DCT - non-centered spectrum)

JPEG Steps

4. Quantize the coefficients (i.e., reduce the amplitude of coefficients that do not contribute a lot).

$$C_q(u, v) = \text{Round}\left[\frac{C(u, v)}{Q(u, v)}\right]$$



$Q(u, v)$: quantization table

Example

- Quantization Table $Q[i][j]$

```
for i=0 to n;  
  for j=0 to n;  
     $Q[i,j] = 1 + (1+i+j) * \text{quality}$ ;  
  end j;  
end i;
```

(d) Quantization table
(quality = 2)

3	5	7	9	11	13	15	17
5	7	9	11	13	15	17	19
7	9	11	13	15	17	19	21
9	11	13	15	17	19	21	23
11	13	15	17	19	21	23	25
13	15	17	19	21	23	25	27
15	17	19	21	23	25	27	29
17	19	21	23	25	27	29	31

$$1 \leq \text{quality} \leq 25$$

←
(best - low compression)

→
(worst - high compression)

Example (cont'd)

(c) Block after FDCT
Eqn. (5)

185	-17	14	-8	23	-9	-13	-18
20	-34	26	-9	-10	10	13	6
-10	-23	-1	6	-18	3	-20	0
-8	-5	14	-14	-8	-2	-3	8
-3	9	7	1	-11	17	18	15
3	-2	-18	8	8	-3	0	-6
8	0	-2	3	-1	-7	-1	-1
0	-7	-2	1	1	4	-6	0

(d) Quantization table
(quality = 2)

3	5	7	9	11	13	15	17
5	7	9	11	13	15	17	19
7	9	11	13	15	17	19	21
9	11	13	15	17	19	21	23
11	13	15	17	19	21	23	25
13	15	17	19	21	23	25	27
15	17	19	21	23	25	27	29
17	19	21	23	25	27	29	31

Quantization



(e) Block after quantization
Eqn. (6)

61	-3	2	0	2	0	0	-1
4	-4	2	0	0	0	0	0
-1	-2	0	0	-1	0	-1	0
0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	-1	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

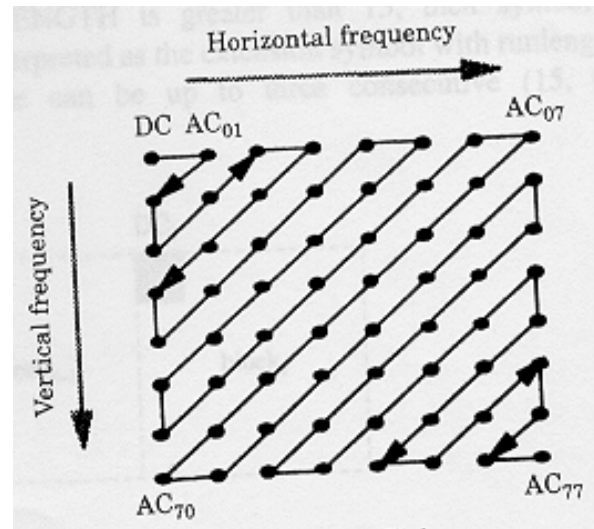
JPEG Steps (cont'd)

5. Order the coefficients using **zig-zag** ordering

- Creates long runs of zeros (i.e., ideal for run-length encoding)

(e) Block after quantization
Eqn. (6)

61	-3	2	0	2	0	0	-1
4	-4	2	0	0	0	0	0
-1	-2	0	0	-1	0	-1	0
0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	-1	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0



(f) Zig-zag sequence

61,-3,4,-1,-4,2,0,2,-2,0,0,0,0,0,2,0,0,0,1,0,0,0,0,0,0,-1,0,0,-1,0,0,
0,0,-1,0,0,0,0,0,0,0,-1,0

JPEG Steps (cont'd)

6. Encode coefficients:

6.1 Form “intermediate” symbol sequence.

6.2 Encode “intermediate” symbol sequence into a binary sequence.

Intermediate Symbol Sequence – DC coeff

(f) Zig-zag sequence

61,-3,4,-1,-4,2,0,2,-2,0,0,0,0,0,2,0,0,0,1,0,0,0,0,0,0,-1,0,0,-1,0,0,
0,0,-1,0,0,0,0,0,0,0,-1,0



(g) Intermediate symbol sequence

(6)(61), (0,2)(-3), (0,3)(4), (0,1)(-1), (0,3)(-4), (0,2)(2), (1,2)(2), (0,2)(-2),
 (5,2)(2), (3,1)(1), (6,1)(-1), (2,1)(-1), (4,1)(-1), (7,1)(-1), (0,0)

symbol_1 (SIZE)

symbol_2 (AMPLITUDE)

DC

(6)

(61)

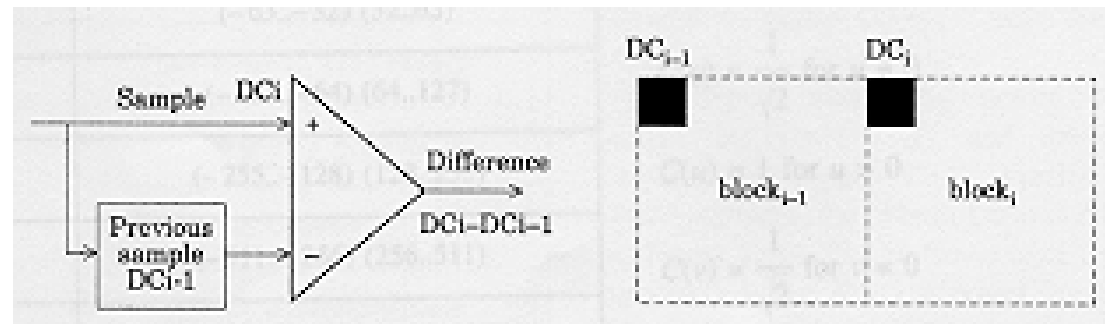
SIZE: # bits need to encode the coefficient

DC Coefficient Encoding

symbol_1
(SIZE)

symbol_2
(AMPLITUDE)

predictive
coding:



Intermediate Symbol Sequence – AC coeff

(f) Zig-zag sequence

61, -3, 4, -1, -4, 2, 0, 2, -2, 0, 0, 0, 0, 0, 2, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, -1, 0, 0, -1, 0, 0,
0, 0, -1, 0, 0, 0, 0, 0, 0, 0, -1, 0



(g) Intermediate symbol sequence

(6)(61), (0,2)(-3), (0,3)(4), (0,1)(-1), (0,3)(-4), (0,2)(2), (1,2)(2), (0,2)(-2),
 (5,2)(2), (3,1)(1), (6,1)(-1), (2,1)(-1), (4,1)(-1), (7,1)(-1), (0,0)

symbol_1 (RUN-LENGTH, SIZE)

symbol_2 (AMPLITUDE)

end of block

AC

 $(0, 2)$

(-3)

RUN-LENGTH: run of zeros preceding coefficient

SIZE: # bits for encoding the amplitude of coefficient

Note: If RUN-LENGTH > 15 , use symbol (15,0) ,

Example: AC Coefficients Encoding

Symbol_1

(Variable Length Code (VLC))

(Runlength, size)	Code word
(0,0) EOB	1010
(0,1)	00
(0,2)	01
(0,3)	100
(1,2)	11011
(2,1)	11100
(3,1)	111010
(4,1)	111011
(5,2)	1111110111
(6,1)	1111011
(7,1)	11111010

Symbol_2

(Variable Length Integer (VLI))

# bits	Amplitude range
1	(-1, 1)
2	(-3, -2) (2,3)
3	(-7..-4) (4..7)
4	(-15..-8) (8..15)
5	(-31..-16) (16..31)
6	(-63..-32) (32..63)
7	(-127..-64) (64..127)
8	(-255..-128) (128..255)
9	(-511..-256) (256..511)
10	(-1023..-512) (512..1023)

(1,4) (12) → (111110110 1100)

VLC

VLI

Final Symbol Sequence

(g) Intermediate symbol sequence

(6)(61),(0,2)(-3),(0,3)(4),(0,1)(-1),(0,3)(-4),(0,2)(2),(1,2)(2),(0,2)(-2),
██████████ (5,2)(2),(3,1)(1),(6,1)(-1),(2,1)(-1),(4,1)(-1),(7,1)(-1),(0,0)



(e) Encoded bit sequence (total 98 bits)

1110111101001001000001000110110110111001011111111011
11011101011111011011100011101101111101001010

What is the effect of the “Quality” parameter?



(58k bytes)



(21k bytes)



(8k bytes)

lower compression

higher compression

$$1 \leq \textit{quality} \leq 25$$

What is the effect of the “Quality” parameter? (cont’d)

Table 6. Results of JPEG Compression for Grayscale Image ‘Lisa’ (320 ×240 pixels)

Quality factors	Original number of bits	Compressed number of bits	Compression ratio (Cr)	Bits/pixel (Nb)	RMS error
1	512,000	48,021	10.66	0.75	2.25
2	512,000	30,490	16.79	0.48	2.75
4	512,000	20,264	25.27	0.32	3.43
8	512,000	14,162	36.14	0.22	4.24
15	512,000	10,479	48.85	0.16	5.36
25	512,000	9,034	56.64	0.14	6.40

Effect of Quantization: homogeneous 8 x 8 block



An 8×8 block from the Y image of 'Lena'

200	202	189	188	189	175	175	175
200	203	198	188	189	182	178	175
203	200	200	195	200	187	185	175
200	200	200	200	197	187	187	187
200	205	200	200	195	188	187	175
200	200	200	200	200	190	187	175
205	200	199	200	191	187	187	175
210	200	200	200	188	185	187	186

$f(i, j)$

515	65	-12	4	1	2	-8	5
-16	3	2	0	0	-11	-2	3
-12	6	11	-1	3	0	1	-2
-8	3	-4	2	-2	-3	-5	-2
0	-2	7	-5	4	0	-1	-4
0	-3	-1	0	4	1	-1	0
3	-2	-3	3	3	-1	-1	3
-2	5	-2	4	-2	2	-3	0

$F(u, v)$

Fig. 9.2: JPEG compression for a smooth image block.

Effect of Quantization: homogeneous 8 x 8 block (cont'd)

Quantized

32	6	-1	0	0	0	0	0
-1	0	0	0	0	0	0	0
-1	0	1	0	0	0	0	0
-1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

$\hat{F}(u, v)$

De-quantized

512	66	-10	0	0	0	0	0
-12	0	0	0	0	0	0	0
-14	0	16	0	0	0	0	0
-14	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

$\tilde{F}(u, v)$

Effect of Quantization: homogeneous 8 x 8 block (cont'd)

Reconstructed

```
199 196 191 186 182 178 177 176
201 199 196 192 188 183 180 178
203 203 202 200 195 189 183 180
202 203 204 203 198 191 183 179
200 201 202 201 196 189 182 177
200 200 199 197 192 186 181 177
204 202 199 195 190 186 183 181
207 204 200 194 190 187 185 184
```

Original

```
200 202 189 188 189 175 175 175
200 203 198 188 189 182 178 175
203 200 200 195 200 187 185 175
200 200 200 200 197 187 187 187
200 205 200 200 195 188 187 175
200 200 200 200 200 190 187 175
205 200 199 200 191 187 187 175
210 200 200 200 188 185 187 186
```

Error is low!

```
1  6 -2  2  7 -3 -2 -1
-1  4  2 -4  1 -1 -2 -3
0 -3 -2 -5  5 -2  2 -5
-2 -3 -4 -3 -1 -4  4  8
0  4 -2 -1 -1 -1  5 -2
0  0  1  3  8  4  6 -2
1 -2  0  5  1  1  4 -6
3 -4  0  6 -2 -2  2  2
```



Effect of Quantization: non-homogeneous 8 x 8 block



Another 8×8 block from the Y image of 'Lena'

```

70 70 100 70 87 87 150 187
85 100 96 79 87 154 87 113
100 85 116 79 70 87 86 196
136 69 87 200 79 71 117 96
161 70 87 200 103 71 96 113
161 123 147 133 113 113 85 161
146 147 175 100 103 103 163 187
156 146 189 70 113 161 163 197

```

$f(i, j)$

```

-80 -40 89 -73 44 32 53 -3
-135 -59 -26 6 14 -3 -13 -28
47 -76 66 -3 -108 -78 33 59
-2 10 -18 0 33 11 -21 1
-1 -9 -22 8 32 65 -36 -1
5 -20 28 -46 3 24 -30 24
6 -20 37 -28 12 -35 33 17
-5 -23 33 -30 17 -5 -4 20

```

$F(u, v)$

Effect of Quantization: non-homogeneous 8 x 8 block (cont'd)

Quantized

-5	-4	9	-5	2	1	1	0
-11	-5	-2	0	1	0	0	-1
3	-6	4	0	-3	-1	0	1
0	1	-1	0	1	0	0	0
0	0	-1	0	0	1	0	0
0	-1	1	-1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

$\hat{F}(u, v)$

De-quantized

-80	-44	90	-80	48	40	51	0
-132	-60	-28	0	26	0	0	-55
42	-78	64	0	-120	-57	0	56
0	17	-22	0	51	0	0	0
0	0	-37	0	0	109	0	0
0	-35	55	-64	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

$\tilde{F}(u, v)$

Effect of Quantization: non-homogeneous 8 x 8 block (cont'd)

Reconstructed

```
70 60 106 94 62 103 146 176
85 101 85 75 102 127 93 144
98 99 92 102 74 98 89 167
132 53 111 180 55 70 106 145
173 57 114 207 111 89 84 90
164 123 131 135 133 92 85 162
141 159 169 73 106 101 149 224
150 141 195 79 107 147 210 153
```

Original:

```
70 70 100 70 87 87 150 187
85 100 96 79 87 154 87 113
100 85 116 79 70 87 86 196
136 69 87 200 79 71 117 96
161 70 87 200 103 71 96 113
161 123 147 133 113 113 85 161
146 147 175 100 103 103 163 187
156 146 189 70 113 161 163 197
```

Error is high!



```
0 10 -6 -24 25 -16 4 11
0 -1 11 4 -15 27 -6 -31
2 -14 24 -23 -4 -11 -3 29
4 16 -24 20 24 1 11 -49
-12 13 -27 -7 -8 -18 12 23
-3 0 16 -2 -20 21 0 -1
5 -12 6 27 -3 2 14 -37
6 5 -6 -9 6 14 -47 44
```

Case Study: Fingerprint Compression

- FBI is digitizing fingerprints at 500 dots per inch with 8 bits of grayscale resolution.
- A single fingerprint card turns into about 10 MB of data!



A sample fingerprint image

768 x 768 pixels = 589,824 bytes

WSQ Fingerprint Compression

- An image coding standard for digitized fingerprints employing the **Discrete Wavelet Transform** (*Wavelet/Scalar Quantization or WSQ*).
- Developed and maintained by:
 - FBI
 - Los Alamos National Lab (LANL)
 - National Institute for Standards and Technology (NIST)

Need to Preserve Fingerprint Details



The "white" spots in the middle of the black ridges are *sweat pores* and they are admissible points of identification in court.

These details are just a couple pixels wide!

What compression scheme should be used?

- Lossless or lossy compression?
- In practice lossless compression methods haven't done better than 2:1 on fingerprints!
- Does JPEG work well for fingerprint compression?

Results using JPEG compression

file size 45853 bytes

compression ratio: 12.9



Fine details have been lost.

Image has an artificial “**blocky**” pattern superimposed on it.

Artifacts will affect the performance of fingerprint recognition.

Results using WSQ compression

file size 45621 bytes

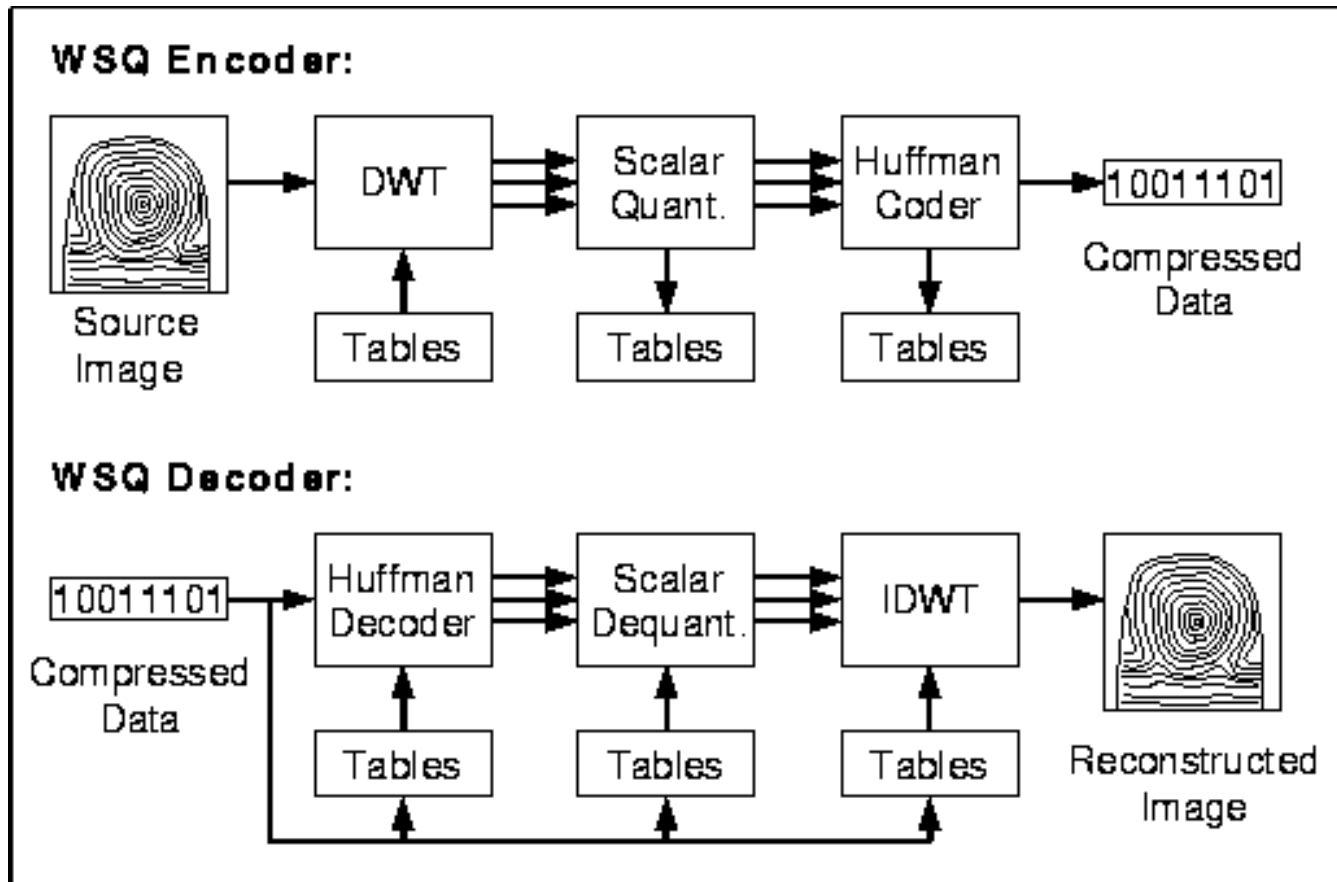
compression ratio: 12.9



Fine details are better preserved.

No “blocky” artifacts.

WSQ Algorithm



Compression ratio

- FBI's target bit rate is around 0.75 bits per pixel (bpp)
- This corresponds to a compression ratio of $8/0.75=10.7$
- Target bit rate can set via a parameter, similar to the "quality" parameter in JPEG.

Varying compression ratio (cont'd)

Original image 768 x 768 pixels (589824 bytes)



Varying compression ratio (cont'd)

0.9 bpp compression

WSQ image, file size 47619 bytes,
compression ratio 12.4



JPEG image, file size 49658 bytes,
compression ratio 11.9



Varying compression ratio (cont'd)

0.75 bpp compression

WSQ image, file size 39270 bytes
compression ratio 15.0



JPEG image, file size 40780 bytes,
compression ratio 14.5



Varying compression ratio (cont'd)

0.6 bpp compression

WSQ image, file size 30987 bytes,
compression ratio 19.0



JPEG image, file size 30081 bytes,
compression ratio 19.6



JPEG Modes

- JPEG supports several different modes
 - Sequential Mode
 - Progressive Mode
 - Hierarchical Mode
 - Lossless Mode
- The default mode is “sequential”
 - Image is encoded in a **single scan** (left-to-right, top-to-bottom).

Progressive JPEG

- Image is encoded in **multiple scans**, in order to produce a quick, rough decoded image when transmission time is long.

Sequential



Progressive



Progressive JPEG (cont'd)

- Each scan encodes a subset of DCT coefficients.
- We'll examine the following algorithms:
 - (1) Progressive spectral selection algorithm
 - (2) Progressive successive approximation algorithm
 - (3) Combined progressive algorithm

Progressive JPEG (cont'd)

(1) Progressive spectral selection algorithm

- Group DCT coefficients into several spectral bands
- Send low-frequency DCT coefficients first
- Send higher-frequency DCT coefficients next

Band 1: DC coefficient only

Band 2: AC_1 and AC_2 coefficients

Band 3: AC_3 , AC_4 , AC_5 , AC_6 , coefficients

Band 4: $AC_7 \dots AC_{63}$, coefficients

Example

Table 8. Progressive spectral selection JPEG. (Image 'Cheetah': 320×240 pixels \rightarrow 512,000 bits)

Scan number	Bits transmitted	Compression ratio	Bits/pixel	RMS error
1	29,005	17.65	0.45	19.97
2	37,237	7.73	1.04	13.67
3	71,259	3.72	2.15	7.90
4	32,489	3.01	2.66	4.59
Sequential JPEG	172,117	2.97	2.69	4.59

Progressive JPEG (cont'd)

(2) Progressive successive approximation algorithm

- Send all DCT coefficients but with lower precision.
- Refine DCT coefficients in later scans.

Band 1: All DCT coefficients (divided by four)

Band 2: All DCT coefficients (divided by two)

Band 3: All DCT coefficients (full resolution)

Example

Table 9. Progressive successive approximation JPEG. (Image 'Cheetah': 320×240 pixels \rightarrow 512,000 bits)

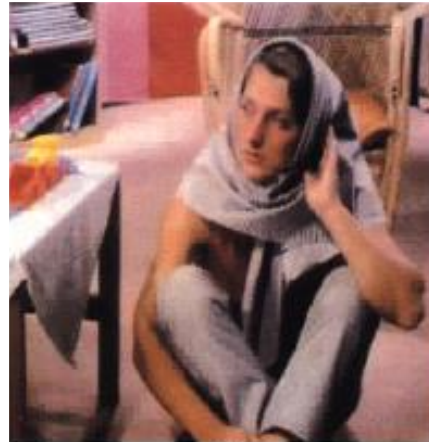
Scan number	Bits transmitted	Compression ratio	Bits/pixel	RMS error
1	26,215	19.53	0.41	22.48
2	34,506	8.43	0.95	12.75
3	63,792	4.11	1.95	7.56
4	95,267	2.33	2.43	4.59
Sequential JPEG	172,117	2.97	2.69	4.59

Example

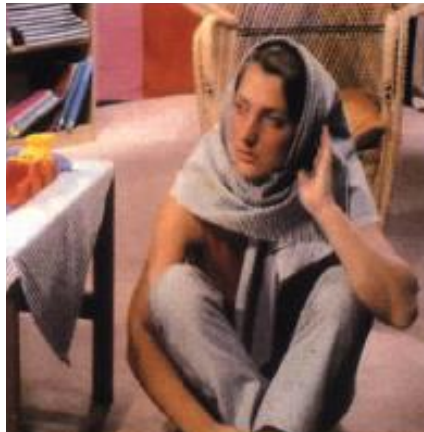
after 0.9s



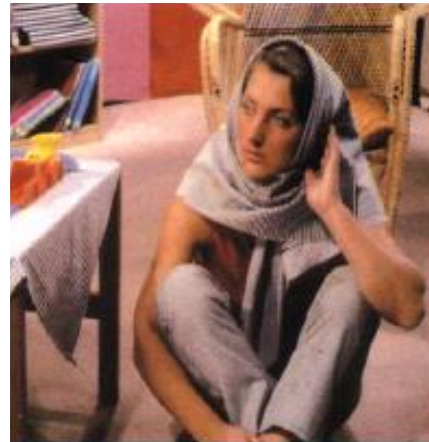
after 1.6s



after 3.6s



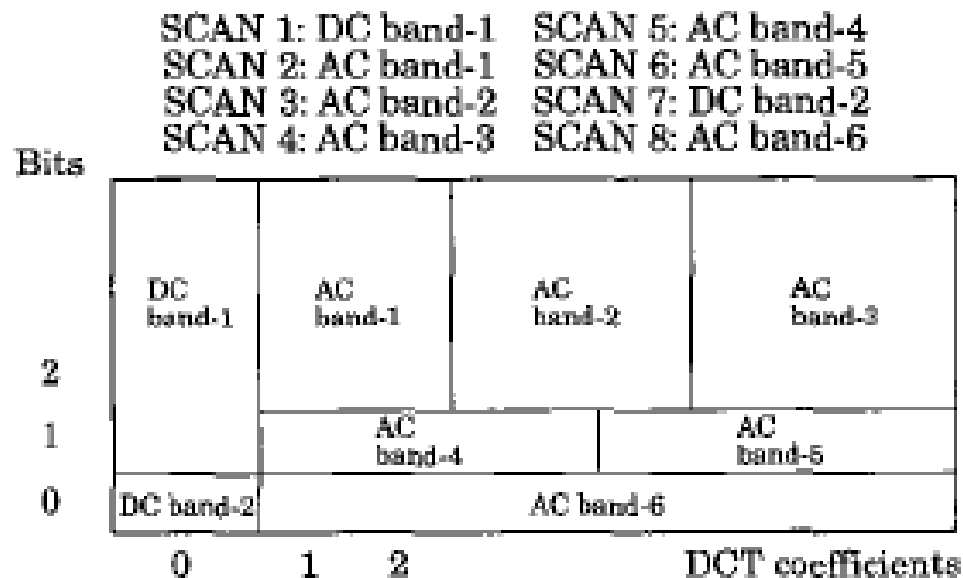
after 7.0s



Progressive JPEG (cont'd)

(3) Combined progressive algorithm

- Combines spectral selection and successive approximation.



Hierarchical JPEG

- Hierarchical mode encodes the image at different resolutions.
- Image is transmitted in multiple passes with increased resolution at each pass.

Hierarchical JPEG (cont'd)

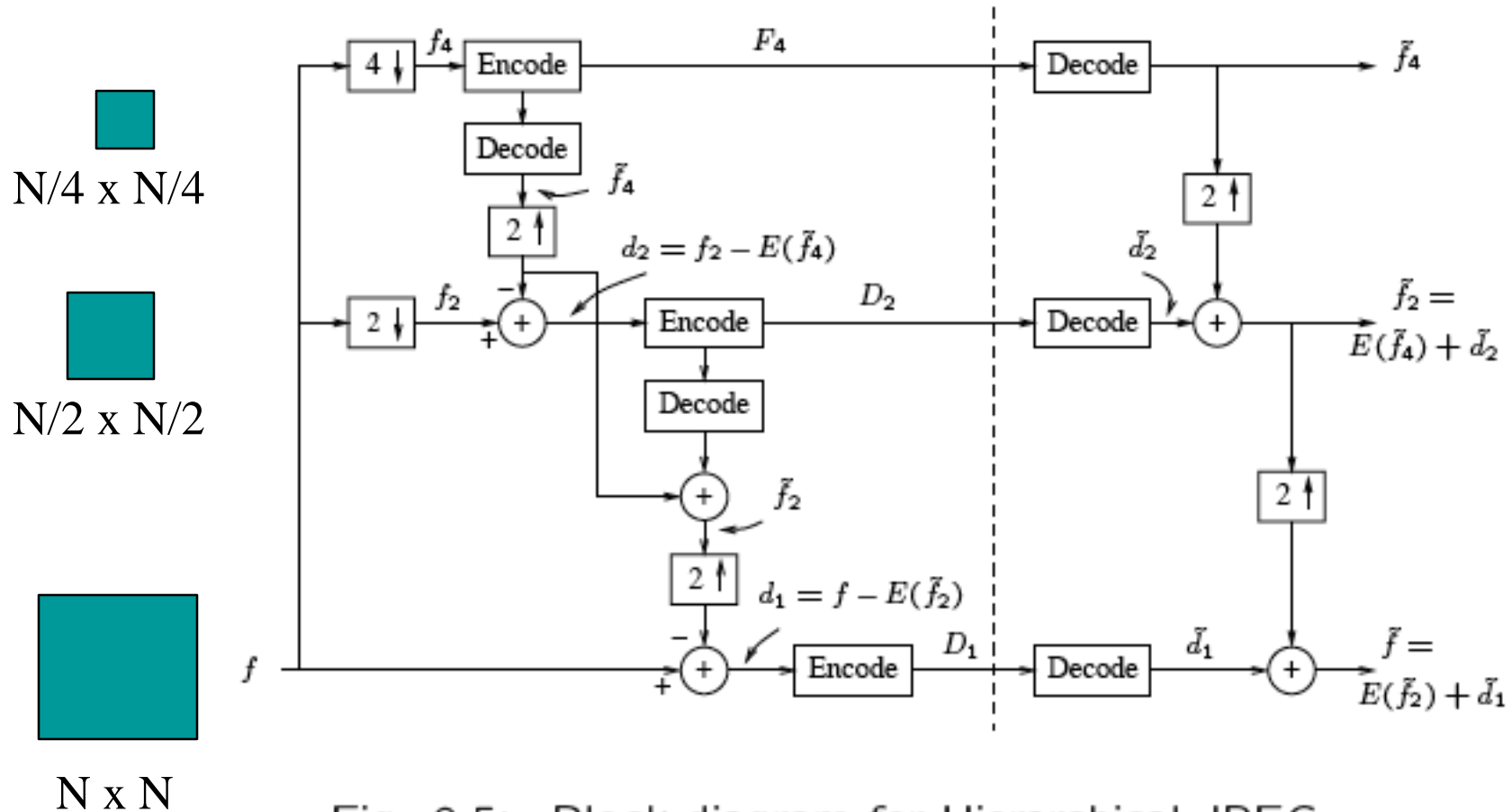


Fig. 9.5: Block diagram for Hierarchical JPEG.

More Methods ...

- See “Image Compression Techniques”, IEEE Potentials, February/March 2001