Open-Loop Analysis of Armature-Controlled DC Motor Speed System

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1 Introduction

The purpose of this analysis is to study the **open-loop behavior** of an armature-controlled DC motor for **speed control**. The open-loop analysis provides insight into:

- System stability
- Natural response
- Steady-state behavior
- Justification for using a feedback controller

2 Motor Parameters

The DC motor used in this project has the following assumed parameters:

Parameter	Symbol	Value
Armature resistance	R_a	$1\,\Omega$
Armature inductance	L_a	$0.5\,\mathrm{H}$
Moment of inertia	J	$0.01\,\mathrm{kgm}^2$
Friction coefficient	B	$0.1\,\mathrm{N\,m\,s}$
Torque constant	K_t	$0.01\,\mathrm{N\,m\,A^{-1}}$
Back EMF constant	K_b	$0.01\,\mathrm{V\,s\,rad^{-1}}$

Table 1: Assumed DC Motor Parameters

3 Mathematical Model

3.1 Electrical Equation

The armature circuit of the DC motor is modeled as:

$$v_a(t) = R_a i(t) + L_a \frac{di(t)}{dt} + K_b \omega(t) \tag{1}$$

3.2 Mechanical Equation

The mechanical equation of the motor is:

$$J \frac{d\omega(t)}{dt} + B\omega(t) = K_t i(t) \tag{2}$$

3.3 Laplace Transform

Applying Laplace transform assuming zero initial conditions:

$$V_a(s) = (R_a + L_a s)I(s) + K_b \Omega(s) \tag{3}$$

$$(Js + B)\Omega(s) = K_t I(s) \tag{4}$$

3.4 Open-Loop Transfer Function

Solving for the transfer function $\frac{\Omega(s)}{V_a(s)}$:

$$I(s) = \frac{(Js + B)\Omega(s)}{K_t} \tag{5}$$

$$V_a(s) = (R_a + L_a s)\frac{(Js + B)\Omega(s)}{K_t} + K_b \Omega(s) \tag{6}$$

$$\frac{\Omega(s)}{V_a(s)} = \frac{K_t}{L_a J s^2 + (L_a B + R_a J)s + (R_a B + K_b K_t)} \tag{7}$$

Substituting the assumed numerical values:

$$\begin{aligned} L_a J &= 0.5 \times 0.01 = 0.005 \\ L_a B + R_a J &= 0.5 \times 0.1 + 1 \times 0.01 = 0.06 \\ R_a B + K_b K_t &= 1 \times 0.1 + 0.01 \times 0.01 = 0.1001 \\ K_t &= 0.01 \end{aligned}$$

Hence, the open-loop transfer function becomes:

$$G(s) = \frac{\Omega(s)}{V_a(s)} = \frac{0.01}{0.005 s^2 + 0.06 s + 0.1001} \tag{8}$$

4 DC Gain

The DC gain of the system is the steady-state value of speed for a step voltage input:

$$G(0) = \lim_{s \to 0} \frac{\Omega(s)}{V_a(s)} \tag{9}$$
$$= \frac{K_t}{R_a B + K_b K_t} \tag{10}$$
$$= \frac{0.01}{0.1001} \approx 0.0999 \tag{11}$$

Interpretation: For a 1 V input, the steady-state speed of the motor is approximately 0.1 rad/s, which is very low, indicating the need for a feedback controller.

5 Step Response Analysis

The open-loop step response was simulated in MATLAB. Observations include:

- Slow rise time (0.5 s–1.5 s, depending on numerical tuning)
- No oscillation (overdamped)
- Low final speed due to small DC gain

Step response characteristics can be extracted using: `stepinfo(G)` in MATLAB.

6 Poles and Stability

The poles of the open-loop system are the roots of the denominator:

$$0.005s^2 + 0.06s + 0.1001 = 0 \tag{12}$$

Solving:

$$s = \frac{-0.06 \pm \sqrt{0.06^2 - 4 \cdot 0.005 \cdot 0.1001}}{2 \cdot 0.005}$$
$$\approx -6 \text{ and } -3.34$$

Conclusion: Both poles are in the left-half plane, confirming the system is **stable** but has a slow response.

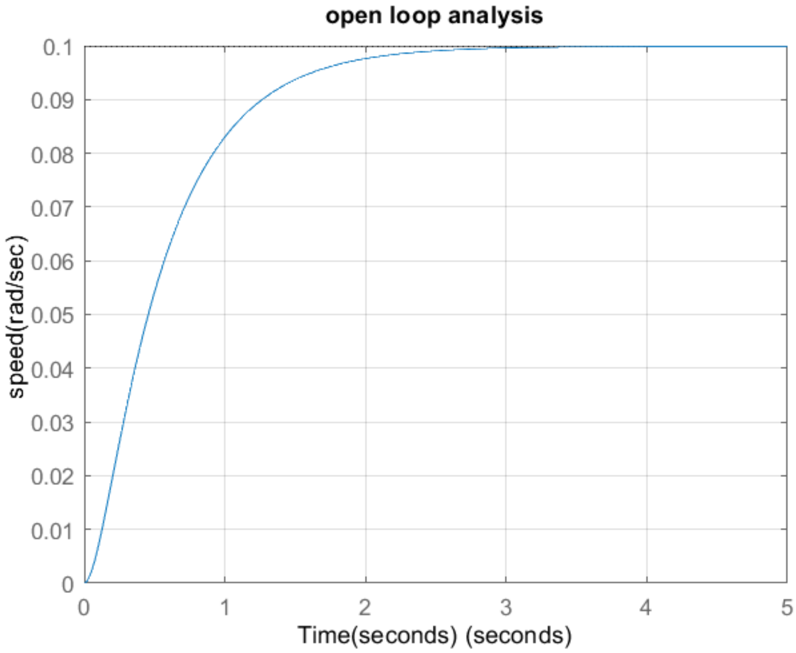

Figure 1: Open-loop Step Response of the DC Motor Speed

7 Frequency Response

A Bode plot shows:

- Low bandwidth
- Small magnitude at low frequencies
- Confirms the system is **not suitable for accurate speed tracking** without feedback

8 Steady-State Error Analysis

For a unity-feedback system (hypothetical):

- System type = 0 (no integrators in open-loop) - Step input → finite steady-state error - Ramp input → infinite error

Implication: Feedback and integral control are required to achieve zero steady-state error for speed regulation.

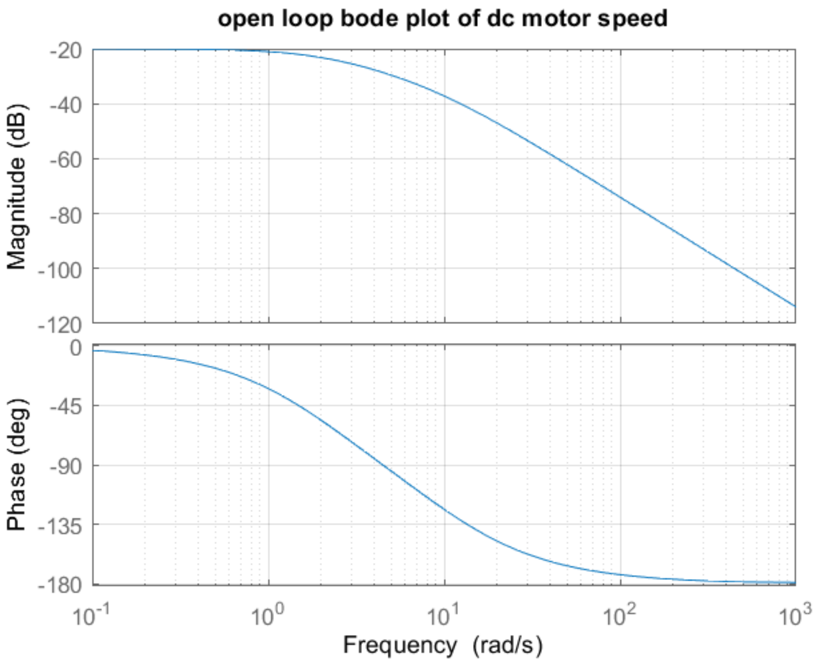

Figure 2: Bode Plot of Open-Loop DC Motor System

9 Conclusions

The open-loop analysis of the armature-controlled DC motor shows:

- The system is stable (poles in LHP)
- Low DC gain results in very slow steady-state speed
- Step response is sluggish, indicating poor transient performance
- Frequency response shows low bandwidth
- System requires a PI controller for acceptable speed regulation and zero steady-state error