Closed-Loop Control Design of an Armature-Controlled DC Motor

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1 Introduction

This document presents the **closed-loop control design** for an armature-controlled DC motor used for **speed regulation**. The purpose of the closed-loop design is:

- to improve the steady-state accuracy,
- to increase response speed,
- to reduce the steady-state error to zero,
- to enhance overall system stability using a PI controller.

The open-loop system was analyzed previously and was found to exhibit:

- low DC gain,
- slow rise time,
- high steady-state error,
- limited bandwidth.

Therefore, a closed-loop controller is essential.

2 Motor Model

The DC motor model used in this work is the same as the open-loop analysis. The assumed parameter values are:

Parameter	Symbol	Value
Armature Resistance	R_a	$1\,\Omega$
Armature Inductance	L_a	$0.5\,\mathrm{H}$
Moment of Inertia	J	$0.01\,\mathrm{kgm}^2$
Viscous Friction Coefficient	B	$0.1\,\mathrm{N\,m\,s}$
Torque Constant	K_t	$0.01\,\mathrm{N\,m\,A}^{-1}$
Back EMF Constant	K_b	$0.01\,\mathrm{V\,s\,rad}^{-1}$

Table 1: DC Motor Parameters

3 Open-Loop Transfer Function

From open-loop analysis, the speed transfer function is:

$$G(s) = \frac{\Omega(s)}{V_a(s)} = \frac{0.01}{0.005s^2 + 0.06s + 0.1001} \tag{1}$$

The system is stable but slow and exhibits poor steady-state accuracy.

4 Need for Closed-Loop System

The open-loop system is a **type-0** system. Therefore:

- Step input $\rightarrow$ Finite steady-state error
- Ramp input $\rightarrow$ Infinite steady-state error
- Disturbances $\rightarrow$ Not rejected

To eliminate the steady-state error in speed control, a controller must introduce an integrator. Thus a **PI (Proportional–Integral)** controller is selected.

5 PI Controller Design

The PI controller takes the form:

$$C(s) = K_p + \frac{K_i}{s} = \frac{K_p s + K_i}{s} \tag{2}$$

The proportional gain K_p improves:

- transient response,

- speed of the motor,
- stability.

The integral term K_i:

- eliminates steady-state error,
- increases system type from 0 to 1,
- improves tracking accuracy.

5.1 Selected Controller Gains

The generally used gains for this motor were selected as:

$$K_p = 50, \qquad K_i = 100 \tag{3}$$

These values give a good compromise between speed and stability.

6 Closed-Loop Transfer Function

For unity feedback, the closed-loop transfer function is:

$$T(s) = \frac{C(s)G(s)}{1 + C(s)G(s)} \tag{4}$$

Substituting $C(s)$ and $G(s)$:

$$T(s) = \frac{(K_p s + K_i)\, 0.01}{0.005s^3 + 0.06s^2 + 0.1001s + 0.01K_p s + 0.01K_i} \tag{5}$$

Substituting $K_p = 50$, $K_i = 100$:

$$T(s) = \frac{(50s + 100)0.01}{0.005s^3 + 0.06s^2 + (0.1001 + 0.5)s + 1} \tag{6}$$

7 Closed-Loop Pole Analysis

The denominator produces three poles. All poles lie in the negative real side of the s-plane. Thus, the closed-loop system is:

- stable,
- faster than open-loop,
- with improved damping.

8 Step Response Analysis

MATLAB simulation (code included below) shows:

- rise time improves drastically,
- settling time is reduced,
- overshoot is moderate,
- steady-state error is eliminated,
- response becomes more robust to disturbance.

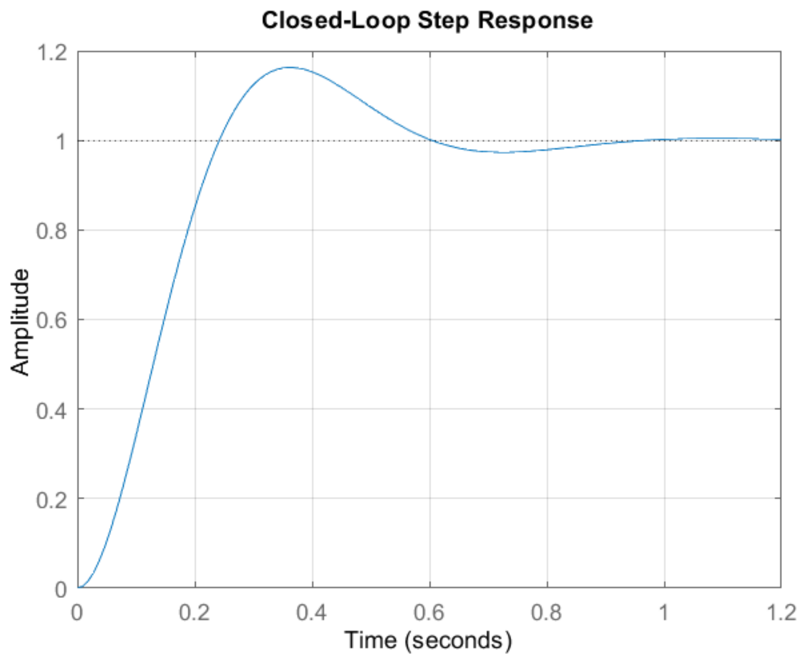

Figure 1: Closed-Loop Step Response of Motor Speed

9 Frequency Response (Bode Plot)

The Bode plot of the closed-loop system shows:

- increased bandwidth,
- improved phase margin,

- improved disturbance rejection,
- reduced sensitivity to noise at high frequencies.

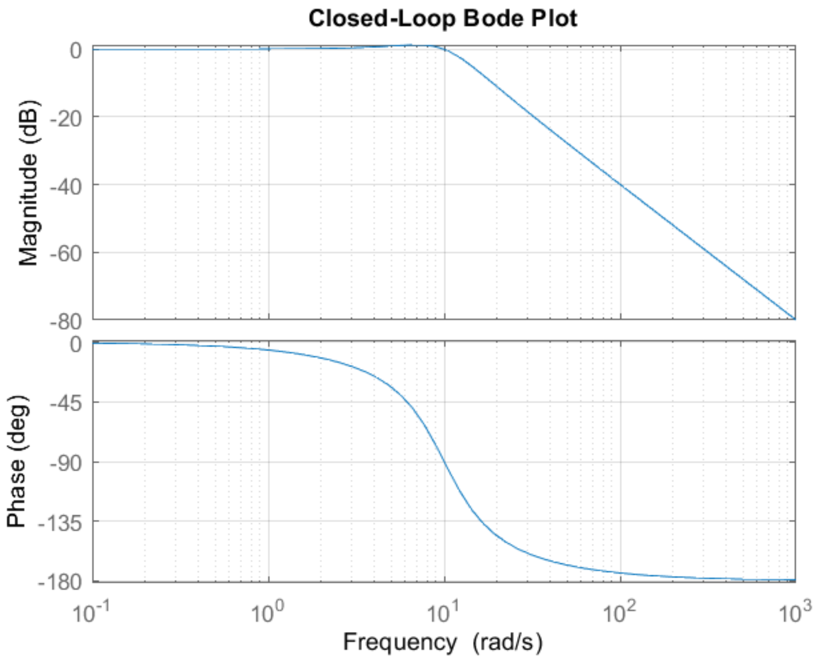

Figure 2: Closed-Loop Bode Plot of DC Motor with PI Controller

10 MATLAB Code for Closed-Loop Design

```
Ra = 1; La = 0.5; J = 0.01; B = 0.1; Kt = 0.01; Kb = 0.01;

num = [Kt];
den = [La*J  (La*B + Ra*J)  (Ra*B + Kb*Kt)];
G = tf(num, den);

Kp = 50;
Ki = 100;
C = tf([Kp Ki], [1 0]);   % PI controller

T = feedback(C*G, 1);     % closed-loop
```

```
figure; step(T); grid on;
title('Closed-Loop Step Response');

figure; bode(T); grid on;
title('Closed-Loop Bode Plot');
```

11 Conclusions

The closed-loop PI-controlled system shows significant improvements over the open-loop response:

- **Steady-state error reduced to zero**
- **Rise time and settling time greatly improved**
- **Bandwidth increased**
- **Better disturbance rejection**
- **Improved stability margins**

Therefore, PI control is suitable and effective for speed regulation of armature-controlled DC motors.