第四章 正弦稳态电路的分析

4-1. 写出下列正弦量的相量，列出有效值和初相位，分别画出各自的相量图。

（1）$i=-10\cos(\omega t-60^{\circ})A$

（2）$u=-10\cos 2\pi\times10^{6}(t-0.2\times10^{-6})V$

（3）$u=\cos 2\pi f(t+0.15T)mV$

（4）$u=7.5\cos 2\pi/T(t-0.15T)V$

解：（1）$i=-10\cos(\omega t-60^{\circ})A=10\cos(\omega t-60^{\circ}+180^{\circ})A=10\cos(\omega t+120^{\circ})A$

$\dot{\boldsymbol{I}}_{\boldsymbol{m}}=10\angle120^{\circ}\boldsymbol{A}$ 或 $\dot{\boldsymbol{I}}=5\sqrt{2}\angle120^{\circ}\boldsymbol{A}$

有效值 $\boldsymbol{I}=5\sqrt{2}\boldsymbol{A}$，初相位 $\boldsymbol{\psi_i}=120^{\circ}$

（2）$u=-10\cos2\pi\times10^{6}(t-0.2\times10^{-6})V=10\cos[2\pi\times10^{6}t-0.4\pi+\pi]V=10\cos[2\pi\times10^{6}t+0.6\pi]V$

$\dot{\boldsymbol{U}}_{\boldsymbol{m}}=10\angle108^{\circ}\boldsymbol{V}$ 或 $\dot{\boldsymbol{U}}=5\sqrt{2}\angle108^{\circ}\boldsymbol{A}$

有效值 $\boldsymbol{U}=5\sqrt{2}\boldsymbol{A}$，初相位 $\boldsymbol{\psi_u}=108^{\circ}$

（3）$u=\cos 2\pi f(t+0.15T)mV=\cos[2\pi ft+0.3\pi]mV$

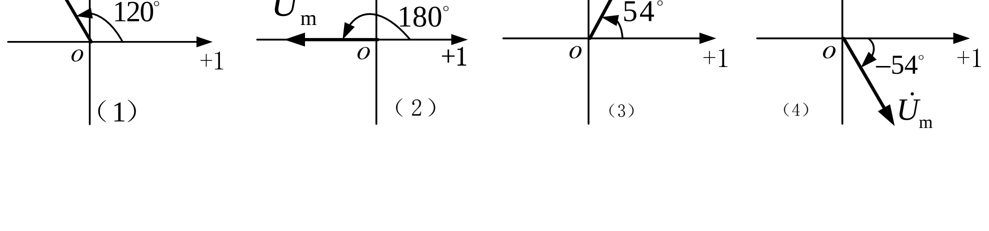
$\dot{\boldsymbol{U}}_{\boldsymbol{m}}=1\angle54^{\circ}\boldsymbol{mV}$ 或 $\dot{\boldsymbol{U}}=\dfrac{\sqrt{2}}{2}\angle54^{\circ}\boldsymbol{mV}$

有效值 $\boldsymbol{U}=0.707\boldsymbol{mV}$，初相位 $\boldsymbol{\psi_u}=54^{\circ}$

（4）$u=7.5\cos 2\pi/T(t-0.15T)V=7.5\cos[2\pi t/T-0.3\pi)V$

$\dot{\boldsymbol{U}}_{\boldsymbol{m}}=7.5\angle-54^{\circ}\boldsymbol{V}$ 或 $\dot{\boldsymbol{U}}=\dfrac{7.5}{\sqrt{2}}\angle-54^{\circ}\boldsymbol{A}$

有效值 $\boldsymbol{U}=\dfrac{7.5}{\sqrt{2}}\boldsymbol{V}$，初相位 $\boldsymbol{\psi_u}=-54^{\circ}$



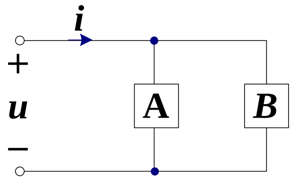
4-2. 已知$u(t)=5\cos\left(\omega t+60^{\circ}\right)V$，请写出该电压$u(t)$的相量形式$\dot{U}_m$。

解：$\dot{\boldsymbol{U}}_{\boldsymbol{m}}=5\angle 60^{\circ}\boldsymbol{V}$

4-3. 已知电流$i(t)$的相量形式为$\dot{I}=6+\mathrm{j}8=10\angle 53.1^{\circ}$，请写出电流的时域表达式。

解：$\dot{I}_m=\sqrt{2}\dot{I}=10\sqrt{2}\angle 53.1^{\circ}$，所以$\boldsymbol{i}(\boldsymbol{t})=10\sqrt{2}\cos(\boldsymbol{\omega t}+53.1^{\circ})\boldsymbol{A}$

4-4. 题图 4-1 所示正弦交流电路，已知$u=10\cos(10t+30^{\circ})V$，$i=10\cos(10t+75^{\circ})A$，则图中 A、B 为何元件，其值多少？



题图 4-1

解：$u=10\cos(10t+30^{\circ})V\Rightarrow\dot{U}_m=10\angle 30^{\circ}$

$i=10\cos(10t+75^{\circ})A\Rightarrow\dot{I}_m=10\angle 75^{\circ}$

则导纳$Y=\dfrac{1}{Z}=\dfrac{\dot{I}_m}{\dot{U}_m}=\dfrac{10}{10}\angle(75^{\circ}-30^{\circ})=1\angle 45^{\circ}=\dfrac{\sqrt{2}}{2}+j\dfrac{\sqrt{2}}{2}$

因为导纳角为-45°电阻，所以是容性导纳。

$$Y_R=\frac{1}{R}=\frac{\sqrt{2}}{2}\Rightarrow R=\sqrt{2}\Omega$$

$$Y_C=j\omega C=j\frac{\sqrt{2}}{2},\omega=10\Rightarrow C=\frac{\sqrt{2}}{20}=0.0707F$$

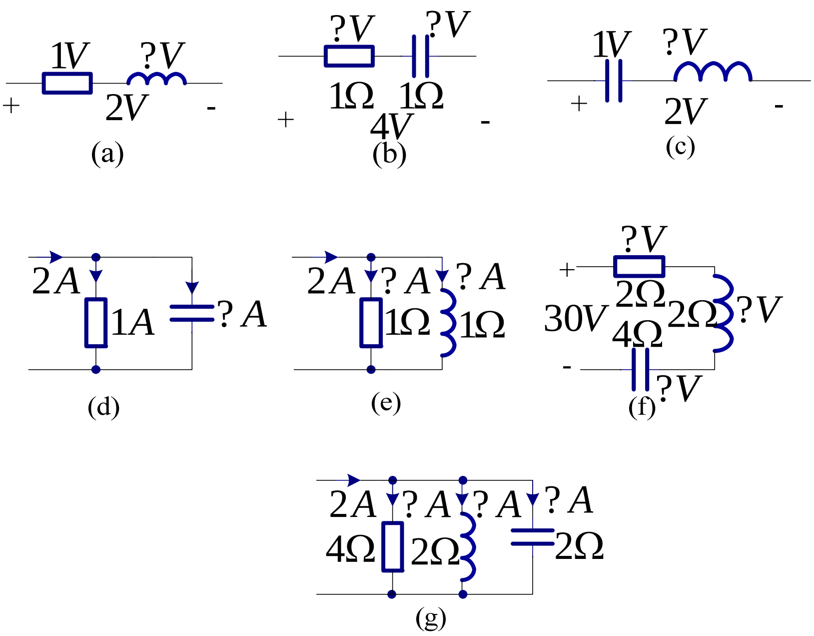
A 为$\sqrt{2}\Omega$电阻，B 为$0.0707\boldsymbol{F}$ 电容；或 A 为$0.0707\boldsymbol{F}$ 电容，B 为$\sqrt{2}\Omega$电阻。

4-5. 已知$i(t)=2\sqrt{2}\sin(\omega t+\dfrac{\pi}{6})A$请写出电流的相量形式$\dot{I}$。

解：$i(t)=2\sqrt{2}\sin(\omega t+\dfrac{\pi}{6})A=2\sqrt{2}\cos(\omega t+\dfrac{\pi}{3})A$

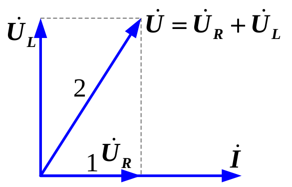
$\dot{\boldsymbol{I}}=2\angle\dfrac{\boldsymbol{\pi}}{3}\boldsymbol{A}$

4-6. 试对题图 4-2 中各个电路的问题做出答案（可借助于相量图），图中给出的电压、电流皆为有效值，待求的也是相应的有效值。



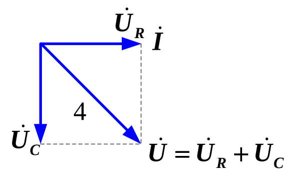
题图 4-2

解：（a）$\dot{U}=\dot{U}_R+\dot{U}_L=R\dot{I}+j\omega L\dot{I}$



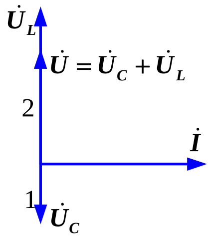
所以有$U_L=\sqrt{2^2-1^2}=\sqrt{3}V$

（b）$\dot{U}=\dot{U}_R+\dot{U}_C=R\dot{I}+\dfrac{1}{j\omega C}\dot{I}=\dot{I}-j\dot{I}$



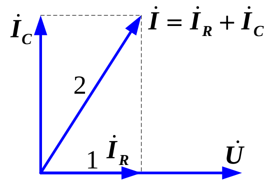
所以有$U_C=\dfrac{\sqrt{2}}{2}\times4=2\sqrt{2}V$， $U_R=\dfrac{\sqrt{2}}{2}\times4=2\sqrt{2}V$

（c）$\dot{U}=\dot{U}_C+\dot{U}_L=\dfrac{1}{j\omega C}\dot{I}+j\omega L\dot{I}$



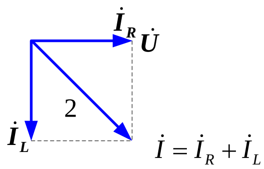
所以有$U_L = 2 + 1 = 3V$

（d）$\dot{I} = \dot{I}_R + \dot{I}_C = \dfrac{\dot{U}}{R} + \dfrac{\dot{U}}{1/j\omega C} = \dfrac{\dot{U}}{R} + j\omega C\dot{U}$



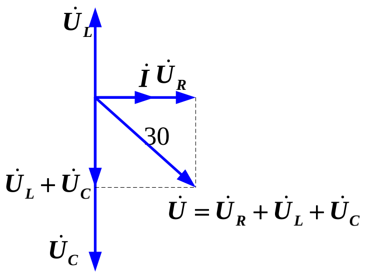
所以有$I_C = \sqrt{2^2 - 1^2} = \sqrt{3}A$

（e）$\dot{I} = \dot{I}_R + \dot{I}_L = \dfrac{\dot{U}}{R} + \dfrac{\dot{U}}{j\omega L} = \dot{U} - j\dot{U}$



所以有$I_L = \dfrac{\sqrt{2}}{2} \times 2 = \sqrt{2}A$， $I_R = \dfrac{\sqrt{2}}{2} \times 2 = \sqrt{2}A$

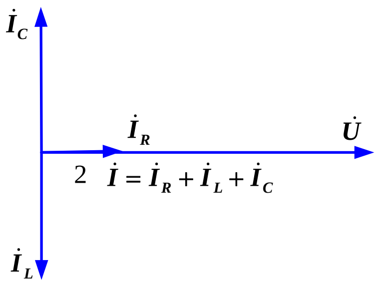
（f）$\dot{U} = \dot{U}_R + \dot{U}_L + \dot{U}_C = R\dot{I} + j\omega L\dot{I} + \dfrac{1}{j\omega C}\dot{I} = 2\dot{I} + j2\dot{I} - j4\dot{I}$



所以有$U_R = \dfrac{\sqrt{2}}{2} \times 30 = 15\sqrt{2}V$， $U_L = \dfrac{\sqrt{2}}{2} \times 30 = 15\sqrt{2}V$，

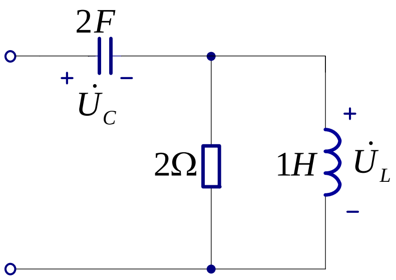
$$U_L = \frac{\sqrt{2}}{2} \times 30 \times 2 = 30\sqrt{2} V$$

（g） $\dot{I} = \dot{I}_R + \dot{I}_C + \dot{I}_L = \dfrac{\dot{U}}{R} + \dfrac{\dot{U}}{1/j\omega C} + \dfrac{\dot{U}}{j\omega L} = \dfrac{\dot{U}}{4} + j\dfrac{1}{2}\dot{U} - j\dfrac{\dot{U}}{2}$



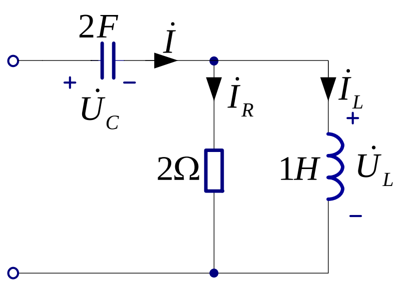
所以有 $I_R = 2A$， $I_C = 4A$， $I_L = 4A$

4-7. 电路如题图 4-3 所示，已知 $\dot{U}_L = 2\angle 0^\circ V$， $\omega = 4rad/s$，求 $\dot{U}_C$ 与 $\dot{U}_L$ 的相位差角。



题图 4-3

解：



$$\dot{I}_L = \frac{\dot{U}_L}{j\omega L} = \frac{\dot{U}_L}{j4\times 1} = -j\frac{\dot{U}_L}{4}, \quad \dot{I}_R = \frac{\dot{U}_L}{R} = \frac{\dot{U}_L}{2}$$

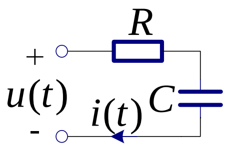
$$\dot{I} = \dot{I}_R + \dot{I}_L = \frac{\dot{U}_L}{2} - j\frac{\dot{U}_L}{4} = (\frac{1}{2} - j\frac{1}{4})\dot{U}_L$$

$$\dot{U}_C=\frac{1}{j\omega C}\dot{I}=\frac{1}{j4\times 2}\dot{I}=\frac{1}{j8}(\frac{1}{2}-j\frac{1}{4})\dot{U}_L$$

$$\frac{\dot{U}_C}{\dot{U}_L}=\frac{1}{j8}(\frac{1}{2}-j\frac{1}{4})=\frac{1}{32}(-1-j2)=\frac{\sqrt{5}}{32}(-\frac{1}{\sqrt{5}}-j\frac{2}{\sqrt{5}})=\frac{\sqrt{5}}{32}\angle -116.5^\circ$$

$\dot{U}_C$ 与 $\dot{U}_L$ 的相位差角为 -116.5°。

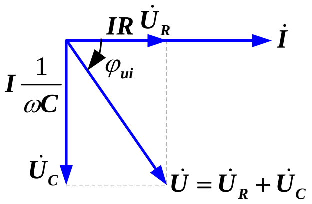
4-8. 题图 4-4 所示电路处于正弦稳态中，请判断电压 u 与电流 i 的相位超前滞后关系。



题图 4-4

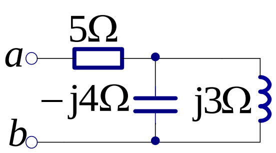
解：$\dot{U}=\dot{U}_R+\dot{U}_C=R\dot{I}+\frac{1}{j\omega C}\dot{I}=(R+\frac{1}{j\omega C})\dot{I}=(R-j\frac{1}{\omega C})\dot{I}$

$$\frac{\dot{U}}{\dot{I}}=R-j\frac{1}{\omega C}=\frac{U}{I}\angle\psi_u-\psi_i=\frac{U}{I}\angle\varphi_{ui},\quad \varphi_{ui}=\arctan\frac{-1/\omega C}{R}=\arctan\frac{-1}{\omega CR}$$



所以电压 $\dot{U}$ 滞后 $\dot{I}$ $\arctan\frac{1}{\omega CR}$

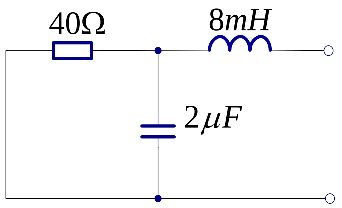
4-9. 题图 4-5 所示电路，求单口网络的输入阻抗 Z_{ab}。



题图 4-5

解：$Z_{ab}=5+\frac{-j3\times j4}{j3-j4}=(5+j12)\Omega$

4-10. 如题图 4-6 所示，$\omega = 10^4 rad/s$，求单口网络的输入阻抗 Z_{ab}。

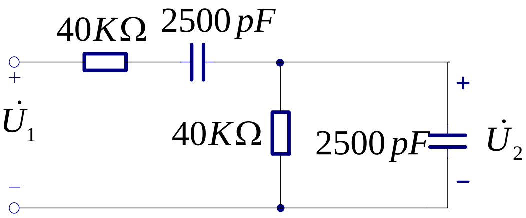


题图 4-6

解：$\boldsymbol{Z_L} = \boldsymbol{j\omega L} = \boldsymbol{j}\times 10^4 \times 8\times 10^{-3} = \boldsymbol{j}80\Omega$，$\boldsymbol{Z_C} = \dfrac{1}{\boldsymbol{j\omega C}} = \dfrac{1}{\boldsymbol{j}\times 10^4 \times 2\times 10^{-6}} = -\boldsymbol{j}50\Omega$

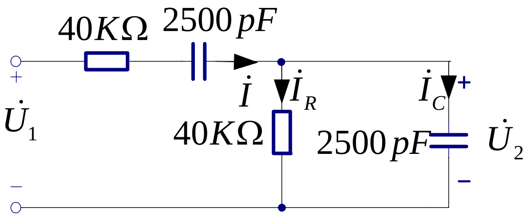
$\boldsymbol{Z_R} = 40\Omega$，所以有 $\boldsymbol{Z_{ab}} = \boldsymbol{Z_L} + \dfrac{\boldsymbol{Z_R Z_C}}{\boldsymbol{Z_R} + \boldsymbol{Z_C}} = \boldsymbol{j}80 + \dfrac{40\times(-\boldsymbol{j}50)}{40+(-\boldsymbol{j}50)} = \dfrac{1}{41}(1000+\boldsymbol{j}2480)\Omega$

4-11. 电路如题图 4-7 所示，求当电压频率 f 为多少时，电压 $\dot{U}_1$ 和 $\dot{U}_2$ 同相。



题图 4-7

解：



$\dot{I}_C = \dfrac{\dot{U}_2}{1/j\omega C} = j\omega C\dot{U}_2$， $\dot{I}_R = \dfrac{\dot{U}_2}{R}$， $\dot{I} = \dot{I}_R + \dot{I}_C = \dfrac{\dot{U}_2}{R} + j\omega C\dot{U}_2 = (\dfrac{1}{R} + j\omega C)\dot{U}_2$

$$\dot{U}_1 = R\dot{I} + \frac{1}{j\omega C}\dot{I} + \dot{U}_2 = [(R + \frac{1}{j\omega C})(\frac{1}{R} + j\omega C) + 1]\dot{U}_2$$

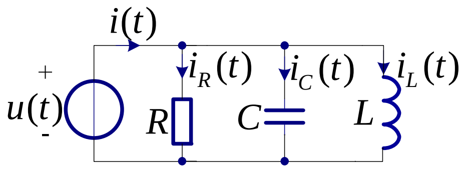
$$\dot{U}_1 = [3 + j\omega RC + \frac{1}{j\omega RC}]\dot{U}_2 = [3 + j(\omega RC - \frac{1}{\omega RC})]\dot{U}_2$$

电压$\dot{U}_1$和$\dot{U}_2$同相，所以有$\omega RC-\dfrac{1}{\omega RC}=0$

即$\omega=\dfrac{1}{RC}=\dfrac{1}{40\times10^3\times2500\times10^{-12}}=10^4 rad/s$

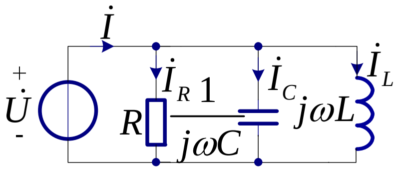
$\omega=2\pi f\Rightarrow f=\dfrac{\omega}{2\pi}=\dfrac{10^4}{2\pi}=1592Hz$

4-12. 如 题 图 4-8 所 示 ， 已 知 $R=10\Omega, C=20\mu F, L=30mH$ ，$u(t)=30\cos(1000t+45^\circ)$。求电路中电流$i(t)$。



题图 4-8

解：$u(t)=30\cos(\omega t+45^\circ)\Rightarrow\dot{U}=15\sqrt{2}\angle45^\circ$



$\dot{I}_R=\dfrac{\dot{U}}{R}$，$\dot{I}_L=\dfrac{\dot{U}}{j\omega L}=-j\dfrac{\dot{U}}{\omega L}$，$\dot{I}_C=\dfrac{\dot{U}}{1/j\omega C}=j\omega C\dot{U}$

$$\dot{I}=\dot{I}_R+\dot{I}_C+\dot{I}_L=\frac{\dot{U}}{R}+j\omega C\dot{U}-j\frac{\dot{U}}{\omega L}=[\frac{1}{R}+j(\omega C-\frac{1}{\omega L})]\dot{U}$$

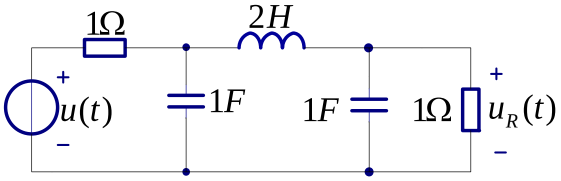
$R=10\Omega, C=20\mu F, L=30mH$，$\omega=1000rad/s$

$$\dot{I}=[\frac{1}{10}+j(1000\times20\times10^{-6}-\frac{1}{1000\times30\times10^{-3}})]\dot{U}=(\frac{1}{10}-j\frac{1}{75})\dot{U}$$

$$\dot{I}=(\frac{1}{10}-j\frac{1}{75})\dot{U}=0.1\angle-8^\circ\times15\sqrt{2}\angle45^\circ=1.5\sqrt{2}\angle37^\circ$$

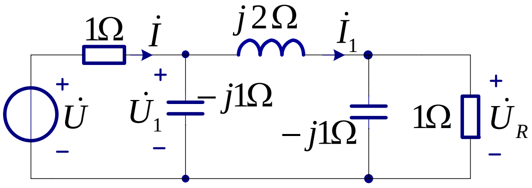
$$i(t)=1.5\sqrt{2}\times\sqrt{2}\cos(\omega t+37^\circ)=3\cos(\omega t+37^\circ)A$$

4-13. 电路如题图 4-9 所示，已知$u_R(t)=\cos\omega t\ V$， $\omega=1rad/s$， 求$u(t)$。



题图 4-9

解：$u_R(t)=\cos\omega t \Rightarrow \dot{U}_R=\dfrac{1}{\sqrt{2}}\angle 0^\circ$， $j\omega L=j2\Omega$， $\dfrac{1}{j\omega C}=-j1\Omega$



$$\dot{I}_1=\frac{\dot{U}_R}{\dfrac{-j1\times 1}{-j1+1}}=\dot{U}_R(1+j)=\frac{1}{\sqrt{2}}\angle 0^\circ\times(1+j)=\frac{1}{\sqrt{2}}\angle 0^\circ\times\sqrt{2}\angle 45^\circ=1\angle 45^\circ$$

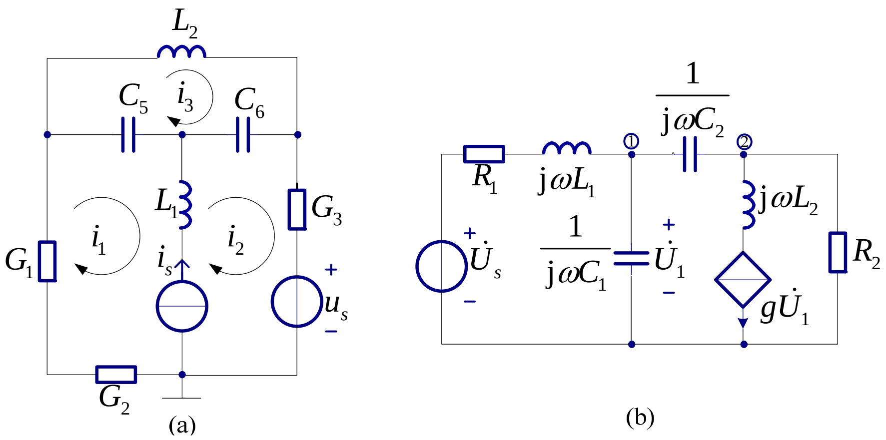
$$\dot{U}_1=(j2+\frac{-j1\times 1}{-j1+1})\dot{I}_1=(\frac{1}{2}+j\frac{3}{2})\dot{I}_1=\frac{\sqrt{10}}{2}\angle 72^\circ\times\dot{I}_1=\frac{\sqrt{10}}{2}\angle 72^\circ\times 1\angle 45^\circ=\frac{\sqrt{10}}{2}\angle 117^\circ$$

$$\dot{I}=\frac{\dot{U}_1}{\dfrac{-j1\times(\frac{1}{2}+j\frac{3}{2})}{-j1+(\frac{1}{2}+j\frac{3}{2})}}=\frac{-j1+(\frac{1}{2}+j\frac{3}{2})}{-j1\times(\frac{1}{2}+j\frac{3}{2})}\dot{U}_1=\frac{1+j}{3-j}\dot{U}_1=\frac{\sqrt{2}\angle 45^\circ}{\sqrt{10}\angle -18^\circ}\times\frac{\sqrt{10}}{2}\angle 117^\circ=\frac{\sqrt{2}}{2}\angle 144^\circ$$

$$\dot{U}=(1+\frac{-j1\times(\frac{1}{2}+j\frac{3}{2})}{-j1+(\frac{1}{2}+j\frac{3}{2})})\dot{I}=(1+\frac{3-j}{1+j})\dot{I}=\frac{4}{1+j}\dot{I}=\frac{4}{\sqrt{2}\angle 45^\circ}\times\frac{\sqrt{2}}{2}\angle 144^\circ=2\angle 99^\circ$$

$$u(t)=2\sqrt{2}\cos(\omega t+99^\circ)V$$

4-14. 正弦稳态电路如题图 4-10（a）、（b）所示，列写图（a）电路的网孔电流方程，图（b）电路的节点电压方程。



(a) (b)

题图 4-10

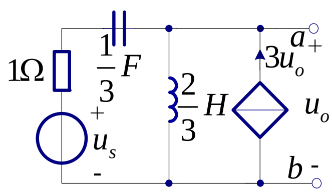
解：（a）网孔电流方程：

$$
\begin{cases}
(\dfrac{1}{G_1}+\dfrac{1}{j\omega C_5}+j\omega L_1+\dfrac{1}{G_2})\dot{I}_1-j\omega L_1\dot{I}_2-\dfrac{1}{j\omega C_5}\dot{I}_3=-\dot{U} \\
-j\omega L_1\dot{I}_1+(\dfrac{1}{G_3}+\dfrac{1}{j\omega C_6}+j\omega L_1)\dot{I}_2-\dfrac{1}{j\omega C_6}\dot{I}_3=-\dot{U}_s+\dot{U} \\
-\dfrac{1}{j\omega C_5}\dot{I}_1-\dfrac{1}{j\omega C_6}\dot{I}_2+(\dfrac{1}{j\omega C_5}+\dfrac{1}{j\omega C_6}+j\omega L_2)\dot{I}_3=0 \\
\dot{I}_2-\dot{I}_1=\dot{I}_s
\end{cases}
$$

（b）节点电压方程

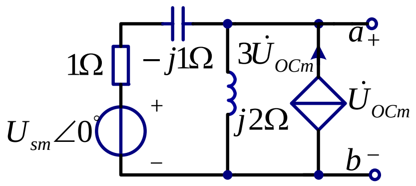
$$
\begin{cases}
(\dfrac{1}{R_1+j\omega L_1}+j\omega C_1+j\omega C_2)\dot{U}_1-j\omega C_2\dot{U}_2=\dfrac{\dot{U}_s}{R_1+j\omega L_1} \\
-j\omega C_2\dot{U}_1+(\dfrac{1}{R_2}+j\omega C_2)\dot{U}_2=-g\dot{U}_1
\end{cases}
$$

4-15. 电路如题图 4-11 所示，已知$u_s=U_{sm}\cos 3t$，求ab端的戴维南等效电路。



题图 4-11

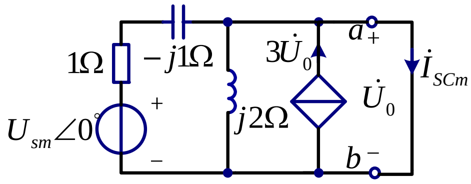
解：求$\dot{U}_{OCm}$。首先画出相量模型。



$$\dot{\boldsymbol{U}}_{OCm}=(\frac{\dot{\boldsymbol{U}}_{sm}-\dot{\boldsymbol{U}}_{OCm}}{1-\boldsymbol{j}1}+3\dot{\boldsymbol{U}}_{OCm})\times\boldsymbol{j}2$$

$$\dot{\boldsymbol{U}}_{OCm}=\frac{\boldsymbol{j}2}{-5-\boldsymbol{j}5}\dot{\boldsymbol{U}}_{sm}=\frac{2\angle 90^{\circ}}{5\sqrt{2}\angle -45^{\circ}}\times\boldsymbol{U}_{sm}\angle 0^{\circ}=\frac{\sqrt{2}\boldsymbol{U}_{sm}}{5}\angle 135^{\circ}$$

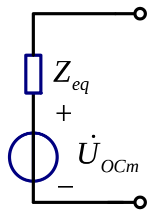
求等效内阻:



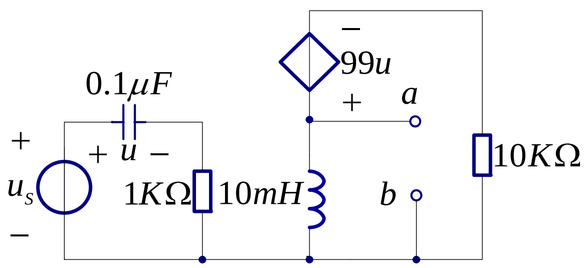
$$\dot{\boldsymbol{I}}_{SCm}=\frac{\dot{\boldsymbol{U}}_{sm}}{1-\boldsymbol{j}},\quad \dot{\boldsymbol{U}}_{OCm}=\frac{\boldsymbol{j}2}{-5-\boldsymbol{j}5}\dot{\boldsymbol{U}}_{sm}$$

所以 $\boldsymbol{Z}_{eq}=\frac{\dot{\boldsymbol{U}}_{SCm}}{\dot{\boldsymbol{I}}_{SCm}}=\frac{\boldsymbol{j}2}{-5-\boldsymbol{j}5}\times(1-\boldsymbol{j})=-\frac{2}{5}\Omega$

由此得到戴维南等效电路的相量模型：



4-16. 电路如题图 4-12 所示，已知 $u_S=\sqrt{2}\cos 10^4 t$ V。求 a b 端的戴维南等效电路。



题图 4-12

解：求开路电压：

$u_S(t)=\sqrt{2}\cos 10^4 t \Rightarrow \dot{U}_S=1\angle 0^\circ$， $\omega=10^4 rad/s$

$j\omega L=j10^4\times 10\times 10^{-3}=j100\Omega$， $\frac{1}{j\omega C}=-j\frac{1}{10^4\times 0.1\times 10^{-6}}=-j1000\Omega$

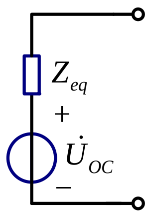
$$\dot{U}=\frac{-j1000}{-j1000+1000}\dot{U}_S=\frac{1000\angle -90^\circ}{1000\sqrt{2}\angle -45^\circ}\times 1\angle 0^\circ=\frac{1}{\sqrt{2}}\angle -45^\circ$$

$$\dot{U}_{OC}=\dot{U}_{ab}=\frac{j100}{j100+10000}\times 99\dot{U}=\frac{100\angle 90^\circ}{10000.5\angle 0.6^\circ}\times 99\times\frac{1}{\sqrt{2}}\angle -45^\circ\approx\frac{\sqrt{2}}{2}\angle 45^\circ V$$

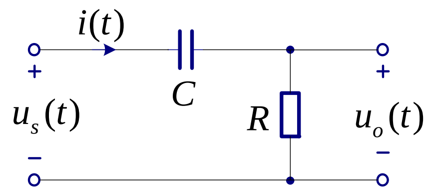
求等效电阻：

$$\boldsymbol{Z_{eq}}=\frac{\boldsymbol{j}100\times 10000}{\boldsymbol{j}100+10000}\approx \boldsymbol{j}100\Omega$$

由此得到戴维南等效电路的相量模型：



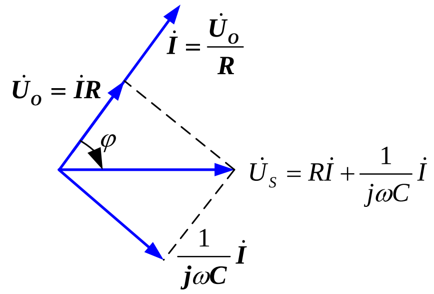
4-17. 已知电路如题图 4-13 所示，输入电压$u_s=2\cos(\omega t)$，请用相量图表示输入电压$u_s(t)$与输出电压$u_o(t)$之间的相位关系



题图 4-13

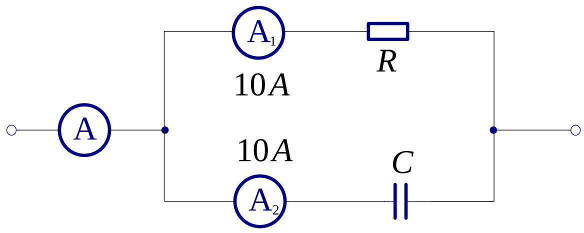
解：$\dot{I}=\frac{\dot{U}_O}{R}$， $\dot{U}_S=R\dot{I}+\frac{1}{j\omega C}\dot{I}$

$$\dot{U}_O=\dot{I}R=\frac{R}{R+\frac{1}{j\omega C}}\dot{U}_S=\frac{R}{R-j\frac{1}{\omega C}}\dot{U}_S$$



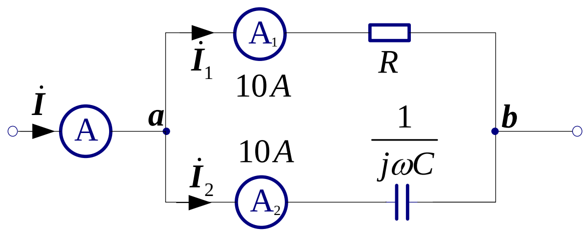
所以电压$\dot{U}_O$超前$\dot{U}_O$ $\arctan\frac{1}{\omega CR}$

4-18. 已知电路如题图 4-14 所示，求电流表 A 的读数。



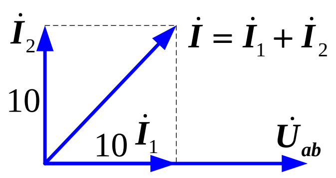
题图 4-14

解：



$$\dot{I}_1=\frac{\dot{U}_{ab}}{R}，\quad \dot{I}_2=\frac{\dot{U}_{ab}}{1/j\omega C}=j\omega C\dot{U}_{ab}$$

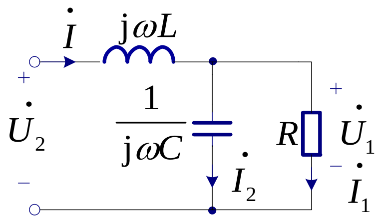
$$\dot{I}=\dot{I}_1+\dot{I}_2=\frac{1}{R}\dot{U}_{ab}+j\omega C\dot{U}_{ab}$$



$\therefore \boldsymbol{I}=\sqrt{10^2+10^2}=10\sqrt{2}\boldsymbol{A}$，即电流表 A 的读数为$10\sqrt{2}\boldsymbol{A}$。

4-19. 正弦稳态电路如题图 4-15 所示，$R=2K\Omega$，$I_2/I_1=\sqrt{3}$，试求以$\dot{U}_1$作为参

考向量，使$\dot{U}_2$超前$\dot{U}_1$ 45°时的感抗ωL的值。



题图 4-15

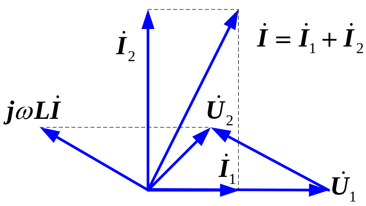
解：$\dot{I}=\dot{I}_1+\dot{I}_2=\dfrac{\dot{U}_1}{R}+\dfrac{\dot{U}_1}{1/j\omega C}=\dfrac{\dot{U}_1}{R}+j\omega C\dot{U}_1$

因为$I_2/I_1=\sqrt{3}$，所以有$\dot{I}=\dot{I}_1+\dot{I}_2=\dfrac{\dot{U}_1}{R}+j\dfrac{\sqrt{3}}{R}\dot{U}_1=\dfrac{2}{R}\dot{U}_1\angle 60^\circ$

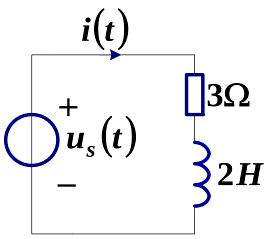
$$\dot{U}_2=j\omega L\dot{I}+\dot{U}_1=j\omega L\times\frac{2}{R}\dot{U}_1\angle 60^\circ+\dot{U}_1=\frac{2\omega L}{R}\dot{U}_1\angle 150^\circ+\dot{U}_1$$

$$\frac{\dot{U}_2}{\dot{U}_1}=\frac{2\omega L}{R}\angle 150^\circ+1=\frac{2\omega L}{R}(\cos 150^\circ+j\sin 150^\circ)+1=1-\sqrt{3}\frac{\omega L}{R}+j\frac{\omega L}{R}$$

$\dot{U}_2$超前$\dot{U}_1$ 45°时，$1-\sqrt{3}\dfrac{\omega L}{R}=\dfrac{\omega L}{R}$， $\omega L=\dfrac{R}{1+\sqrt{3}}=\dfrac{2}{1+\sqrt{3}}k\Omega$



4-20. 题图 4-16 所示电路中，已知$u_s(t)=\cos t+\cos 2t\ V$，求电流$i(t)$以及电路吸收的功率。



题图 4-16

解：$\dot{I}_m=\dfrac{\dot{U}_{sm}}{R+j\omega L}$

激励为 cost： $\omega=1$， $\dot{I}_m=\dfrac{1\angle 0^{\circ}}{3+j2}=0.277\angle-33.7^{\circ}$

激励为 cos2t： $\omega=2$， $\dot{I}_m=\dfrac{1\angle 0^{\circ}}{3+j4}=0.2\angle 53.1^{\circ}$

$i(t)=[0.277\cos(t-33.7^{\circ})+0.2\cos(2t-53.1^{\circ})]A$

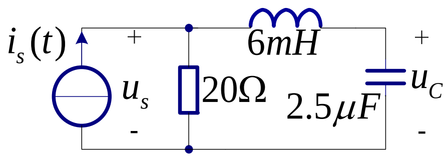
由于 $\int_0^T\cos t\cos 2tdt=0$， $\int_0^T\sin t\cos 2tdt=0$， $\int_0^T\cos t\sin 2tdt=0$， $\int_0^T\sin t\sin 2tdt=0$

$$P=\frac{1}{T}\int_0^T u(t)i(t)dt=\frac{1}{T}\int_0^T 0.277\cos t\cos(t-33.7^{\circ})dt+\frac{1}{T}\int_0^T 0.\cos 2t\cos(2t-53.1^{\circ})dt$$

对于单一频率： $P=UI\cos\varphi_{ui}=\dfrac{U_m}{\sqrt{2}}\dfrac{I_m}{\sqrt{2}}\cos\varphi_{ui}$

$$P=\frac{1}{\sqrt{2}}\times\frac{0.277}{\sqrt{2}}\cos 33.7^{\circ}+\frac{1}{\sqrt{2}}\times\frac{0.2}{\sqrt{2}}\cos 53.1^{\circ}=0.176W$$

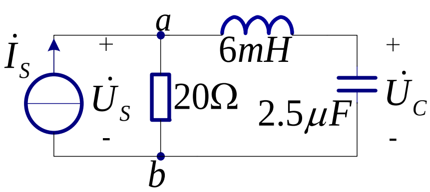
4-21. 电路如题图 4-17 所示， 已知 $i_s(t)=5\sin(10^4t-20^{\circ})A$ 。试求（1）电路的输入阻抗 Z_{ab} 并说明电路的性质，（2） $\dot{U}_S$ 及 $u_s(t)$，（3） $\dot{U}_C$ 及 $u_C(t)$，（4）电路吸收的平均功率 P 。



题图 4-17

解： $i_s(t)=5\sin(10^4t-20^{\circ})A=5\cos(10^4t-110^{\circ})A\Rightarrow\dot{I}_S=\dfrac{5}{\sqrt{2}}\angle-110^{\circ}$， $\omega=10^4 rad/s$

$$j\omega L=j10^4\times6\times10^{-3}=j60\Omega,\quad \frac{1}{j\omega C}=-j\frac{1}{10^4\times2.5\times10^{-6}}=-j40\Omega$$



（1）电路的输入阻抗 $Z_{ab}=\dfrac{20\times(j60-j40)}{20+(j60-j40)}=\dfrac{j20}{1+j}=10\sqrt{2}\angle45^{\circ}\Omega$， 为感性阻抗。

（2）$\dot{U}_S = Z_{ab}\dot{I}_S = 10\sqrt{2}\angle 45^\circ \times \frac{5}{\sqrt{2}}\angle -110^\circ = 50\angle -65^\circ V$

$$u_s(t) = 50\sqrt{2}\cos(10^4 t - 65^\circ)V$$

（3）$\dot{U}_C = \frac{-j40}{j60 - j40}\dot{U}_S = \frac{-j40}{j20}\dot{U}_S = -2\dot{U}_S = 2\angle 180^\circ \times 50\angle -65^\circ = 100\angle 115^\circ V$

$$u_C(t) = 100\sqrt{2}\cos(10^4 t + 115^\circ)V$$

（4）$\frac{\dot{U}_S}{\dot{I}_S} = Z_{ab} = 10\sqrt{2}\angle 45^\circ \Rightarrow \varphi_{ui} = 45^\circ$

$$\boldsymbol{P = U_S I_S} \cos\varphi_{\boldsymbol{ui}} = 50 \times \frac{5}{\sqrt{2}} \times \cos 45^\circ = 50 \times \frac{5}{\sqrt{2}} \times \frac{\sqrt{2}}{2} = 125\boldsymbol{W}$$

4-22. 已知某电路的瞬时功率为 $p = 10 + 8\sin(300t + 45^\circ)W$，求最大瞬时功率、最小瞬时功率和平均功率。

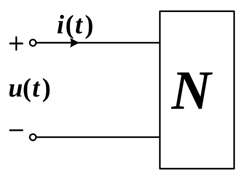
解：$\boldsymbol{p(t) = UI}\cos\varphi_{\boldsymbol{ui}} + \boldsymbol{UI}\cos(2\omega \boldsymbol{t} + \psi_{\boldsymbol{u}} + \psi_{\boldsymbol{i}})$

平均功率 $\boldsymbol{P = UI}\cos\varphi_{\boldsymbol{ui}} = 10\boldsymbol{W}$

最大瞬时功率 $\boldsymbol{p}_{\max}(\boldsymbol{t}) = 18\boldsymbol{W}$

最小瞬时功率 $\boldsymbol{p}_{\min}(\boldsymbol{t}) = 2\boldsymbol{W}$

4-23. 题图 4-18 所示二端网络 N，已知 $u(t) = 110\cos(\omega t + 45^\circ)V$，$i(t) = 10\cos(\omega t + 15^\circ)A$，求网络N吸收的平均功率 P，无功功率 Q，视在功率 S。



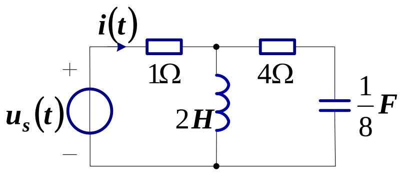
题图 4-18

解：$P = UI\cos\varphi_{ui} = \frac{110}{\sqrt{2}}\frac{10}{\sqrt{2}}\cos 30^\circ = 275\sqrt{3} = 476.3\text{W}$

$$Q = UI\sin\varphi_{ui} = \frac{110}{\sqrt{2}}\frac{10}{\sqrt{2}}\sin 30^{\circ} = 275\text{VAR}$$

$$S = UI = \frac{110}{\sqrt{2}}\frac{10}{\sqrt{2}} = 550\text{VA}$$

4-24. 电路题图 4-19 所示，已知$u_s(t) = 10\sqrt{2}\cos 2t\ V$，试求电流$i(t)$、电源供出的有功功率$P$和无功功率$Q$。



题图 4-19

解： $j\omega L = j4\Omega$， $\dfrac{1}{j\omega C} = -j4\Omega$

$$\dot{I} = \frac{\dot{U}}{Z} = \frac{10\angle 0^{\circ}}{1 + \dfrac{j4\times(4 - j4)}{j4 + (4 - j4)}} = \frac{10\angle 0^{\circ}}{5 + j4} = 1.56\angle -38.66^{\circ}$$

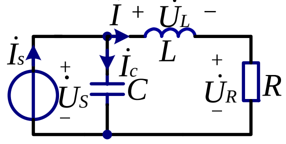
$$i(t) = 1.56\sqrt{2}\cos(2t - 38.66^{\circ})A$$

$$P = UI\cos\varphi_{ui} = 10\times 1.56\cos 38.66^{\circ} = 12.18\text{W}$$

$$Q = UI\sin\varphi_{ui} = 10\times 1.56\sin 38.66^{\circ} = 9.7\text{VAR}$$

4-25. 已知某单口网络当负载功率为 30kW 时，功率因数为 0.6（感性），负载电压为 220V，若使得负载功率因数提高到 0.9，若电源频率为 100Hz，求并联电容为多大？

解：



负载功率为$P = UI\cos\varphi_{ui} = 220I\times 0.6 = 30\times 10^3 W$，所以$I = \dfrac{2500}{11}A$

设$\dot{U}_S = 220\angle 0^\circ V$， $\dot{I} = \frac{2500}{11}\angle -53^\circ A$， $\dot{I}_C = \frac{\dot{U}_S}{1/j\omega C} = j\omega C\dot{U}_S = 220\omega C\angle 90^\circ A$

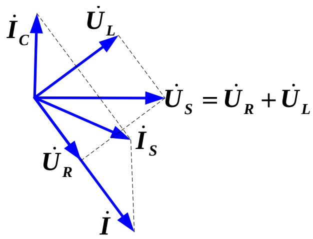
$$\dot{I}_S = \dot{I} + \dot{I}_C = \frac{2500}{11}\angle -53^\circ + 220\omega C\angle 90^\circ = \frac{2500}{11}\cos(-53^\circ) + j\frac{2500}{11}\sin(-53^\circ) + j220\omega C$$

$$\dot{I}_S = \frac{2500}{11}\times 0.6 - j\frac{2500}{11}\times 0.8 + j220\omega C = \frac{1500}{11} + j(-\frac{2000}{11} + 220\omega C)$$

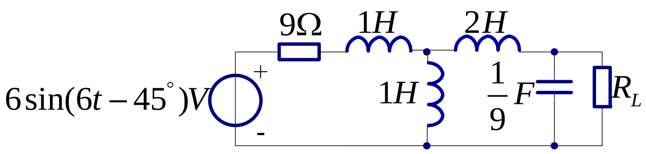
$$\frac{\dot{U}_S}{\dot{I}_S} = \frac{220\angle 0^\circ}{\frac{1500}{11} + j(-\frac{2000}{11} + 220\omega C)} = \frac{U_S}{I_S}\angle \arctan\frac{\frac{2000}{11} - 220\omega C}{\frac{1500}{11}} = \frac{U_S}{I_S}\angle \arctan\frac{100 - 121\omega C}{75}$$

$$\cos[\arctan\frac{100 - 121\omega C}{75}] = 0.9 \Rightarrow \frac{100 - 121\omega C}{75} = \frac{\sqrt{1 - 0.9^2}}{0.9} \Rightarrow \omega C = 0.562$$

$$\omega C = 2\pi fC = 0.562 \Rightarrow C = \frac{0.562}{2\pi f} = \frac{0.562}{2\pi \times 100} = 894\mu F$$



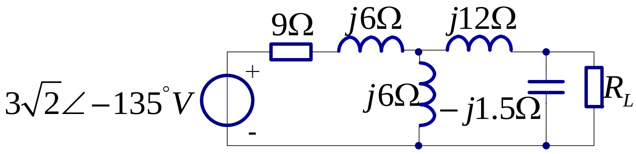
4-26. 已知电路如题图 4-20 所示，试求电阻 R_L 为多大值能够获取最大功率，最大功率是多少？

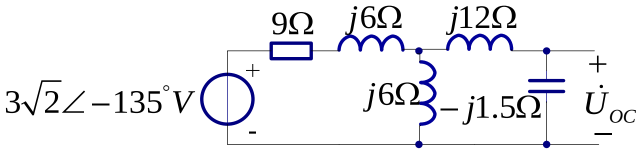


题图 4-20

解：$6\sin(6t - 45^\circ) = 6\cos(6t - 135^\circ) = 3\sqrt{2}\angle -135^\circ, \omega = 6rad/s$，

$j\omega L_1 = j6\Omega$， $j\omega L_1 = j12\Omega$， $\frac{1}{j\omega C} = \frac{1}{j6\times\frac{1}{9}} = -j1.5\Omega$





$$\dot{U}_{OC}=\frac{\dfrac{j6\times(j12-j1.5)}{j6+(j12-j1.5)}}{9+j6+\dfrac{j6\times(j12-j1.5)}{j6+(j12-j1.5)}}\times\frac{-j1.5}{j12-j1.5}\times3\sqrt{2}\angle-135^{\circ}=\frac{-j2}{11+j12}\times\sqrt{2}\angle-135^{\circ}$$

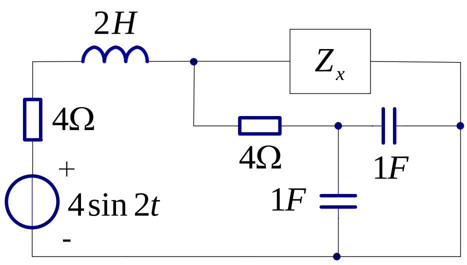
$$=\frac{-j2}{11+j12}\times\sqrt{2}\angle-135^{\circ}=0.12\sqrt{2}\angle87.5^{\circ}$$

$$Z_{eq}=\frac{-j1.5\times(j12+\dfrac{j6\times(9+j6)}{j6+(9+j6)})}{-j1.5+(j12+\dfrac{j6\times(9+j6)}{j6+(9+j6)})}=\frac{1-j109.5}{66.25}\Omega$$

$Z_x=\dfrac{1+j109.5}{66.25}\Omega$时获得最大功率，最大功率为：

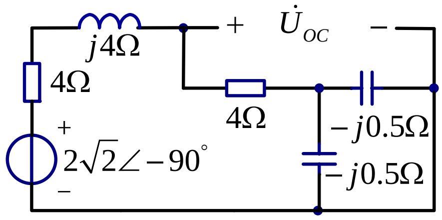
$$P_{L\max}=\frac{U_{OC}^{2}}{4R_x}=\frac{(0.12\sqrt{2})^2}{4\times\dfrac{1}{66.25}}=0.477W$$

4-27. 电路如题图 4-21 所示，试求负载Z_x为多大值能够获得最大功率，最大功率是多少？



题图 4-21

解： $j\omega L=j4\Omega$， $\dfrac{1}{j\omega C}=-j0.5\Omega$



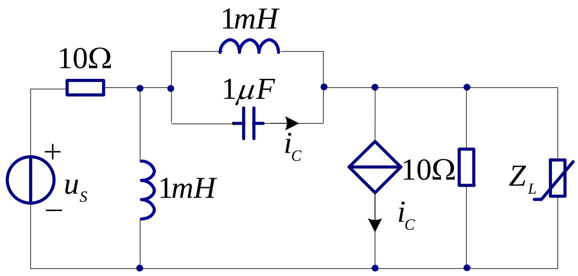
$$\dot{U}_{OC}=\frac{(4-j0.25)\times 2\sqrt{2}\angle-90^\circ}{4+j4+4-j0.25}=1.28\angle-118.7^\circ V$$

$$Z_{eq}=\frac{(4+j4)\times(4-j0.25)}{4+j4+4-j0.25}=(2.463+j0.72)\Omega$$

$Z_x=(2.463-j0.72)\Omega$时获得最大功率，最大功率为：

$$P_{L\max}=\frac{U_{OC}^2}{4R_x}=\frac{1.28^2}{4\times 2.463}=0.166W$$

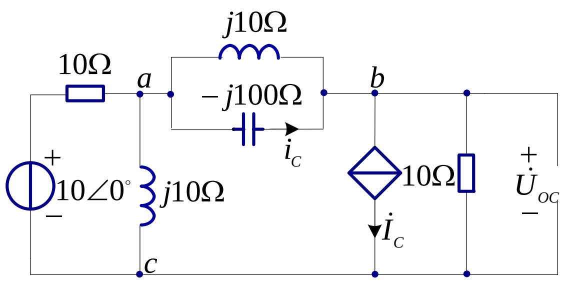
4-28. 题图 4-22 所示电路中，$u_s(t)=10\sqrt{2}\cos 10^4 t\ V$ ，若负载 Z 的实部和虚部均可调，求负载 Z 获得的最大功率。



题图 4-22

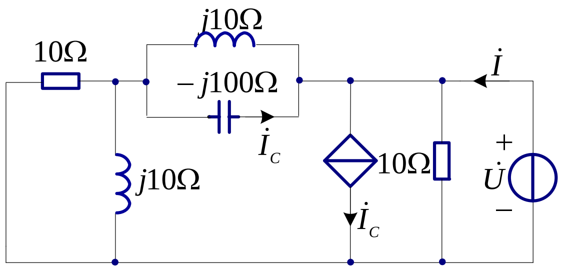
解：$u_s(t)=10\sqrt{2}\cos 10^4 t \Rightarrow \dot{U}_S=10\angle 0^\circ, \omega=10^4 rad/s$

$$j\omega L=j10^4\times 10^{-3}=j10\Omega，\quad \frac{1}{j10^4\times 10^{-6}}=-j100\Omega$$



设 c 为参考节点，节点 a 和 b 的节点电压为$\dot{U}_a,\dot{U}_b$，节点电压方程为：

$$\begin{cases}(\frac{1}{10}+\frac{1}{j10}+\frac{1}{j10}+\frac{1}{-j100})\dot{U}_a-(\frac{1}{j10}+\frac{1}{-j100})\dot{U}_b=\frac{10\angle 0^\circ}{10}\\ -(\frac{1}{j10}+\frac{1}{-j100})\dot{U}_a+(\frac{1}{j10}+\frac{1}{-j100}+\frac{1}{10})\dot{U}_b=-\dot{I}_C\\ \frac{\dot{U}_a-\dot{U}_b}{-j100}=\dot{I}_C\end{cases}\Rightarrow\begin{cases}\dot{U}_{OC}=\dot{U}_b=\frac{100\angle 0^\circ}{29}V\\ \dot{U}_a=\frac{100\sqrt{2}\angle 45^\circ}{29}V\end{cases}$$



$$\dot{I}=\frac{\dot{U}}{10}+\dot{I}_C+\frac{\dot{U}}{\frac{10\times j10}{10+j10}+\frac{j10\times(-j100)}{j10-j100}}$$

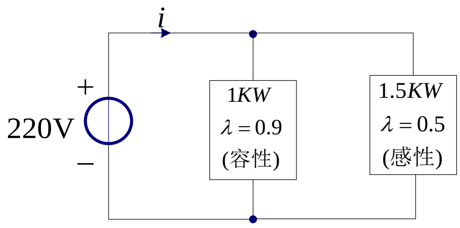
$$\dot{I}_C=-\frac{\dot{U}}{\frac{10\times j10}{10+j10}+\frac{j10\times(-j100)}{j10-j100}}\times\frac{j10}{j10-j100}$$

$$\Rightarrow Z_{eq}=\frac{\dot{U}}{\dot{I}}=\frac{190+j100}{29}\Omega$$

$Z_x=\dfrac{190-j100}{29}\Omega$时获得最大功率，最大功率为：

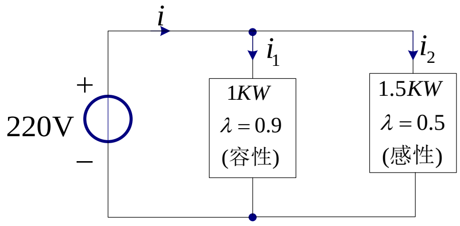
$$P_{L\max}=\frac{U_{OC}^2}{4R_x}=\frac{(\frac{100}{29})^2}{4\times\frac{190}{29}}=0.454W$$

4-29. 电路如题图 4-23 所示，试求电路中输入电流和总功率因数。



题图 4-23

解：



容性： $P = UI\cos\varphi_{ui} = 1KW$， $\cos\varphi_{ui} = 0.9 \Rightarrow \varphi_{ui} = -25.84^{\circ}$

$$I_1 = \frac{1000}{220\times 0.9} = 5.05A$$

感性： $P = UI\cos\varphi_{ui} = 1.5KW$， $\cos\varphi_{ui} = 0.5 \Rightarrow \varphi_{ui} = 60^{\circ}$

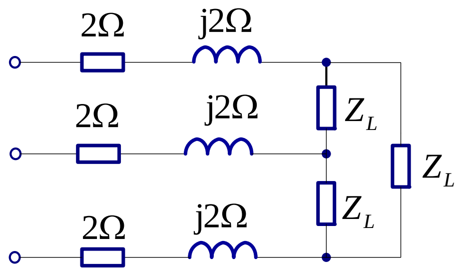
$$I_2 = \frac{1500}{220\times 0.5} = 13.64A$$

所以 $\dot{I} = 5.05\angle 25.84^{\circ} + 13.64\angle -60^{\circ} = 11.365 - j9.61 = 14.88\angle -40.22^{\circ}A$

$\varphi_{ui} = 40.22^{\circ} > 0$（感性）

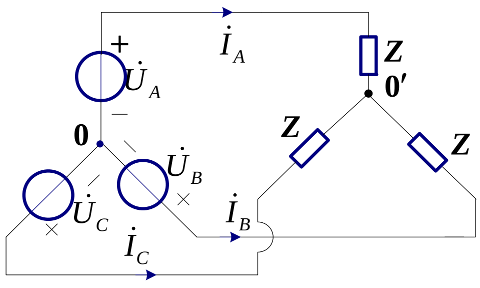
$\lambda = \cos\varphi_{ui} = \cos 40.22^{\circ} = 0.764$

4-30. 题图 4-24 所示对称三相电路，负载阻抗 $Z_L = (60 + j60)\Omega$，负载端的线电压为 $380V$，求电源端线电压。



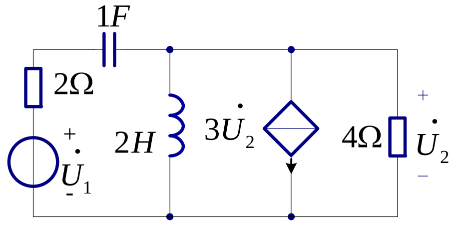
题图 4-24

4-31. 题图 4-25 所示对称三相电路中，已知线电压 $U_c = 380V$，负载 $Z = 20 + j15\Omega$，求线电流 $\dot{I}_A$， $\dot{I}_B$ 和 $\dot{I}_C$ 及负载吸收总功率 $P_{总}$。



题图 4-25

4-32. 求题图 4-26 所示电路的网络系统函数 $H(j\omega)=\dot{U}_2/\dot{U}_1$。

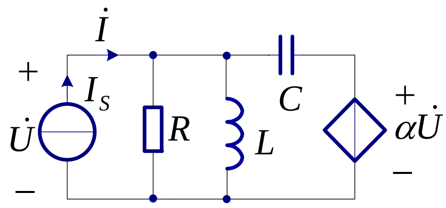


题图 4-26

解：节点电压法：$\left(\dfrac{1}{2+\dfrac{1}{j\omega}}+\dfrac{1}{j2\omega}+\dfrac{1}{4}\right)\dot{U}_2=\dfrac{\dot{U}_1}{2+\dfrac{1}{j\omega}}-3\dot{U}_2$

$$H(j\omega)=\frac{\dot{U}_2}{\dot{U}_1}=\frac{1}{2+\dfrac{1}{j\omega}}\times\frac{1}{\left(\dfrac{1}{2+\dfrac{1}{j\omega}}+\dfrac{1}{j2\omega}+\dfrac{1}{4}+3\right)}=\frac{4\omega^2}{26\omega^2-j17\omega-2}$$

4-33. 如题图 4-27 所示正弦电路，求电路的谐振角频率，设 $\alpha<1$。



题图 4-27

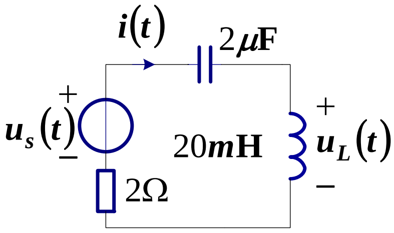
解：求 Z_{ab}： $\dot{I}=\dfrac{\dot{U}}{R}+\dfrac{\dot{U}}{j\omega L}+\dfrac{\dot{U}-\alpha\dot{U}}{\dfrac{1}{j\omega C}}=\left[\dfrac{1}{R}+\dfrac{1}{j\omega L}+j\omega C(1-\alpha)\right]\dot{U}$

$$Z_{ab}=\frac{\dot{U}}{\dot{I}}=\frac{1}{\dfrac{1}{R}+\dfrac{1}{j\omega L}+j\omega C(1-\alpha)}=\frac{1}{\dfrac{1}{R}+j\left[-\dfrac{1}{\omega L}+\omega C(1-\alpha)\right]}$$

电路发生谐振，呈纯电阻性： $-\dfrac{1}{\omega L}+\omega C(1-\alpha)=0$

所以电路的谐振角频率为： $\omega=\dfrac{1}{\sqrt{LC(1-\alpha)}}$

4-34．如题图 4-28 所示串联电路，已知$u_s(t)=4\cos\omega t\ mV$，求该电路的谐振频率，谐振时的电流$i(t)$和电感电压$u_L(t)$。



题图 4-28

解：$Z=R+j\omega L+\dfrac{1}{j\omega C}$

该电路的谐振频率为：$\omega L=\dfrac{1}{\omega C}\Rightarrow\omega=\dfrac{1}{\sqrt{LC}}=\dfrac{1}{\sqrt{2\times10^{-6}\times20\times10^{-3}}}=5000rad/s$

此时$Z=R=2\Omega$，$i(t)=\dfrac{u_s(t)}{2}=(2\cos5000t)\text{mA}$

$\dot{U}_{Lm}=j\omega L\dot{I}_m=j5000\times20\times10^{-3}\times2\angle0^{\circ}=200\angle90^{\circ}$

$u_L(t)=200\cos(5000t+90^{\circ})\text{mV}$