Optimal Networked Control Systems with State-dependent Markov Channels

Bin Hu and Tua A. Tamba

***Abstract*—This paper considers a co-design problem for industrial networked control systems to ensure both system stability and resources efficiency. Solving such a co-design problem is challenging because wireless communications in industrial environments are subject to shadow fading, and are stochastically correlated with their surrounding environments. To address such challenges, this paper first introduces a novel state-dependent Markov channel model that explicitly captures the *state-dependent features* of the wireless communications by correlating model's transition probabilities with environment states. Under the proposed channel model, sufficient conditions on *maximum allowable transmission interval* are presented to ensure *almost sure asymptotic stability* for a nonlinear networked control system. With the stability constraints, the co-design problem is then formulated as constrained optimization problems, which can be efficiently solved by Semidefinite programs for a two-state Markov channel. Simulation results are provided to demonstrate the efficacy of the proposed co-design scheme.**

I. INTRODUCTION

Over the past decades, wireless communication technologies, such as WirelessHart and WiMAX [1], have been successfully implemented in various industrial applications with the goals of building efficient, safe, and reconfigurable industrial automation systems. Building a safe and efficient industrial networked control system, however, is fairly challenging due to the fact that wireless communication channels in industrial environments are inherently unreliable. The quality degradation in communication links inevitably compromises system stability and performance.

Recent studies have shown that the radio communication in industrial environments often exhibits *shadow fading* that is statistically dependent on various environment states, such as large metal objects, moving machines and vehicles [2]–[4]. Such *state-dependent features* prevent conventional modeling formalisms, such as Markov chain or identically distributed independent process (i.i.d.), from being applicable to complex industrial environments [5], [6]. Thus, recent works have focused on developing channel models that correlate temporal variations of channel conditions with the external environment states in different industrial settings, e.g., [3]–[7]. The proposed *state-dependent Markov channel model* differs from existing models in two aspects. First, the works in [3], [4] model the external environment (a moving vehicle) as a (semi)-Markov chain assuming that the moving vehicle cannot be controlled. This paper removes the uncontrollable assumptions and model the external environment as a Markov Decision Process (MDP). Secondly, channel models adopted in [3], [4], [7] are restricted in considering only random packet dropouts and ignoring the quantization effects, while this paper considers a more generalized state-dependent Markov model.

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Under the *state-dependent Markov model*, this paper adopts a joint design scheme of transmission power and control policy to ensure both system stability and efficiency for the whole industrial networked system. The traditional methods to solve this joint-design problem are to use separation principle under which the optimal design of communication and control strategies can be decoupled by leveraging the independent assumption in both systems. Numerous co-design results were built upon the separation principle to address control and state estimation problems in networked control systems [3], [4], [8], [9]. The separation principle, however, cannot be simply applied in complex industrial process environments where communication and control systems are tightly coupled.

The contributions of this paper are three fold. First, a novel *state-dependent Markov channel model* is proposed to explicitly capture the stochastic correlation between data rates, external environments, and the transmission power. Secondly, this paper further presents sufficient conditions on Maximum Allowable Transmission Interval (MATI) that assure *almost sure asymptotic stability* for the nonlinear networked control system. Thirdly, this paper formulates the co-design framework as a novel constrained optimization problem with the stability conditions as hard constraints. The solutions to the constrained optimization problem represent optimal control and transmission power policies that minimize an average joint costs for both communication and control systems. Moreover, the constrained optimization problem can be efficiently solved by a SDP for a two-state Markov channel.

Notations: Throughout the paper, let $\mathbb{R}$, $\mathbb{Z}$ denote the real and integer number respectively, and $\mathbb{R}_{\geq 0}, \mathbb{Z}_{\geq 0}$ denote their non-negative counterparts. Let $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional real vector space and matrix with the dimension of $n \times m$, respectively. $\forall x \in \mathbb{R}^n$, the infinity norm of the vector is denoted by $|x| = \max_i |x_i|, 1 \leq i \leq n$ with x_i being the i^{th} element of the vector. For notation simplicity, let $|A| := \|A\|_\infty = \max_{1\leq i\leq m} \sum_{j=1}^{n} |a_{ij}|$ denote the infinity norm for a matrix $A \in \mathbb{R}^{n\times m}$.

II. SYSTEM FRAMEWORK AND PROBLEM FORMULATION

Fig. 1 depicts a system framework considered in this paper, which consists of a *nonlinear plant*, a *state-dependent Markov* *channel* $\mathcal{M}_c$, a *remote controller* and the *external environment* that is modeled by a MDP $\mathcal{M}$.

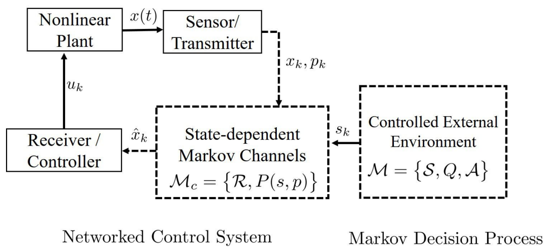

Fig. 1: Nonlinear Networked Control System with State-dependent Markov Channels

1) Nonlinear Plant: The dynamics of the nonlinear plant satisfies the following ODE

$$\dot{x} = f(x, u) \tag{1}$$

where $x \in \mathbb{R}^{n_x}$ represents the system states, $u \in \mathbb{R}^{n_u}$ is the input to the system that is generated by a remote controller. The vector field $f : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \to \mathbb{R}^{n_x}$ is a nonlinear function that is locally Lipschitz with respect to x.

2) Sensor and Transmitter: The state x is first sampled and encoded by a *sensor/transmitter* module before the transmission. Specifically, let $\mathcal{I} = \{t_k\}_{k=0}^{\infty}$ denote a sequence of sampling/transmission time instants with $t_k < t_{k+1}$, and $x_k := x(t_k)$ denote the sampled state at time instant t_k. The sampled state $x(t_k)$ is then encoded by the index of one of the symbols that are constructed using a dynamic quantization scheme [10]. Let $R \in \mathbb{N}$ denote the number of bits used to construct the symbol, and the sequence of the symbols can then labeled as $\mathcal{S} = \{1, 2, \ldots, 2^R\}$ with a total number of 2^R. A dynamic quantizer is then constructed to track the evolution of the $x(t_k)$ at each transmission instant. The quantizer is defined as a tuple $\mathcal{Q} = (\mathcal{S}, q(\cdot), \xi)$ where $q(\cdot) : \mathbb{R}^{n_x} \to \mathcal{S}$ is a quantization function that maps the system state into the symbol and $\xi \in \mathbb{R}_{\geq 0}$ is an anxiliary variable defining the size of the quantization regions. Consider a box-based dynamic quantizer, a hypercubic box is constructed at time instant t_k with $\hat{x}(t_k)$ representing the center of the box and $2\xi(t_k)$ being its size. With R number of bits, the box is divided equally into 2^R smaller sub-boxes with each sub-box labeled as one of the symbols $\mathcal{S}$. Among all the symbols, let $q(x) \in \mathcal{S}$ denote the symbol (sub-box) that contains the state information x. The center of that sub-box, $\hat{x}(t_k^+)$, is an estimate of the state information x at time instant t_k. The hyper-cubic box is then updated with a new center $\hat{x}(t_k^+)$ and a new size $\xi(t_k^+) = \xi(t_k)/2^R$. The symbol representing this updated hyber-cubic is transmitted through the wireless communication channel. The following equations are used to characterize the dynamics of the dynamic quantizer

$$\hat{x}(t_k^+) = h(k, q(x(t_k)), \hat{x}(t_k), \xi(t_k), R_k) \tag{2a}$$

$$\xi(t_k^+) = \frac{\xi(t_k)}{2^{R_k}} \tag{2b}$$

where R_k is the number of bits received at time instant t_k, and is a time varying variable that depends on wireless channel conditions in real time. As discussed in prior work [10], [11], within each time interval $[t_k, t_{k+1}), \forall k \in \mathbb{Z}_{\geq 0}$, the size of the hyper-cubic box needs to be propagated to ensure that the constructed box captures the actual state x. The following differential equation characterizes the evolution of the size within time interval $[t_k, t_{k+1})$

$$\dot{\xi}(t) = g_\xi(\xi), \forall t \in [t_k, t_{k+1}) \tag{3}$$

3) Remote Controller: Under the dynamic quantizer $\mathcal{Q}$, a model-based remote controller that maintains a "copy" of the plant dynamics is constructed as below,

$$\begin{aligned} \dot{\hat{x}} &= f(\hat{x}, u), \\ u &= \kappa(\hat{x}), \forall t \in [t_k, t_{k+1}) \end{aligned} \tag{4}$$

with the initial state $\hat{x}(t_k) = \hat{x}(t_k^+)$. The control function $\kappa(\cdot) : \mathbb{R}^{n_x} \to \mathbb{R}^{n_u}$ is a "nominal" controller that is selected to stabilize the dynamic system in (4) without considering the effect of the network.

With the definitions of system dynamics in (1), dynamic quantizer in (2) and (3), and remote controller in (4), the closed loop system can be characterized as a stochastic hybrid system as below,

$$\dot{x} = \tilde{f}(x, e) \tag{5a}$$

$$\dot{e} = g_e(x, e) \tag{5b}$$

$$\dot{\xi} = g_\xi(\xi), \forall t \in (t_k, t_{k+1}) \tag{5c}$$

and

$$e(t_k^+) = J_e(k, x(t_k), e(t_k), \xi(t_k), R_k) \tag{6a}$$

$$\xi(t_k^+) = J_\xi(\xi(t_k), R_k), \forall k \in \mathbb{Z}_{\geq 0} \tag{6b}$$

where $e := x - \hat{x}$ is the estimation error and $\tilde{f}(x, e) = f(x, \kappa(x - e))$, $g_e(x, e) = f(x, \kappa(x - e)) - f(x - e, \kappa(x - e))$, $J_e(k, x(t_k), e(t_k), \xi(t_k), R_k) = x(t_k) - h(k, q(x(t_k)), x(t_k) - e(t_k), \xi(t_k), R_k)$ and $J_\xi = \xi(t_k) 2^{-R_k}$. The equations (5) above characterize the continuous dynamics of the closed loop system while the equations (6) describe the stochastic jump behavior of the system whose dynamics are governed by the time varying data rate R_k.

4) Controlled Environments & State-dependent Markov Channel: The external environments in industrial settings, e.g., moving vehicles or machines, are modeled by a MDP $\mathcal{M}_{env} = \{S, s_0, A, Q\}$ where $S = \{s_i\}_{i=1}^{M_s}$ is a finite set of environment states, s_0 is an initial state, $A = \{a_i\}_{i=1}^{M_a}$ is a finite set of actions, and $Q = \{q(s|s', a)\}_{s, s' \in S, a \in A}$ is a transition matrix. Taking a forklift vehicle operating in an industrial factory as an example, the state set S in MDP represents a group of partitions for the regions in the factory floor. By taking an action $a \in A$, the vehicle moves from one region s' to another s following the transition probabilities $q(s|s', a)$.

Under the MDP model, the quality of wireless communication links is affected by which state/region the vehicle is located. The link quality is measured by a time varying data rate set $\mathcal{R} = \{r_1, r_2, \ldots, r_{M_R}\}$. Let a random variable $R_k \in \mathcal{R}$ denote the data rate at time instant t_k, and $\{R_k\}_{k=0}^{\infty}$ denote a random process characterizing stochastic variations on the channel conditions over time sequence $\mathcal{I} = \{t_k\}_{k=0}^{\infty}$. At each time instant t_k, the communication system can adjust its transmission power level to send the data through a wireless communication channel. Let $\Omega_p = \{1, 2, \ldots, M_p\}$ denote a finite set of transmission power levels with $i \in \Omega_p$ representing the power level i. The transmission power set is sorted in an ascending order such that larger number represents higher power level, i.e., $r_i < r_j, \forall i < j$. Let $p_k := p(t_k) \in \Omega_p$ denote the power level selected at time instant t_k. The *state-dependent Markov channel* is defined as below,

Definition 2.1: Given a power set Ω_p, a MDP $\mathcal{M}_{env}$ and a finite data rate set $\mathcal{R} = \{r_1, r_2, \ldots, r_{M_R}\}$, a wireless communication channel is a *state-dependent Markov channel* if $\forall s \in S, p \in \Omega_p$ and $\forall r_i, r_j \in \mathcal{R}$

$$\mathbb{P}\{R_{k+1} = r_i | R_k = r_j, s_k = s, p_k = p\} = P_{ij}(s, p) \tag{7}$$

where $P_{ij}(s, p)$ is a transition probability from data rate r_j to r_i given the transmission power p and environment state s.

The *state-dependent Markov channel* model in (7) can be viewed as a generalization of traditional Markov channel by taking into account the impact of environment state and transmission power [12], [13].

Given the closed-loop networked control system represented by (5), (6) and (7), the stability and co-design problems are formally defined as below,

Problem 2.2 (Stability Problem): The stability problem considered in this paper is to find the MATI T_{MATI} such that $t_{k+1} - t_k \leq T_{MATI}, \forall k \in \mathbb{Z}_{\geq 0}$. Under the MATI, the networked control system defined in (5) and (6) with *state-dependent Markov channel* is *almost surely asymptotically stable* (ASAS), i.e., $\forall \epsilon, t' > 0$, such that:

$$\mathbb{P}\{\lim_{t' \to \infty} \sup_{t \geq t'} |x(t)| \geq \epsilon\} = 0 \tag{8}$$

Problem 2.3 (Co-design Problem): Let μ_m and μ_p denote the control policy and transmission power policy respectively, and for a given joint cost function $\{c(s, p, r)\}_{s \in S, p \in \Omega_p, r \in \mathcal{R}}$, the co-design problem is to find an optimal jointly policy $\mu^\star := (\mu_m^\star, \mu_p^\star)$ such that the following average expected costs are minimized under the stability conditions obtained by solving Problem 2.2,

$$\begin{aligned} \min_{\mu_m, \mu_p} \quad & \lim_{\ell \to \infty} \frac{1}{\ell} \mathbb{E}_{s_0, R_0}^{\mu_m, \mu_p} \sum_{k=0}^{\ell} c(s_k, p_k, R_k) \\ \text{s.t.} \quad & \text{Stability conditions ensuring (8)} \end{aligned} \tag{9}$$

where s_0 and R_0 are initial states for the MDP system and Markov channel respectively.

III. Main Results

The following assumptions are needed to derive the main results.

Assumption 3.1 ([14], [15]): Consider the networked control system in (5) and (6), let $\overline{e} := [e; \xi]$ denote an augmented vector for the error states e and the size of the dynamic quantizer ξ. Suppose there exist a function $W : \mathbb{Z}_{\geq 0} \times \mathbb{R}^{n_e+1} \to \mathbb{R}_{\geq 0}$ that is locally Lipschitz with respect to $\overline{e}$, a locally Lipschitz, positive definite, radially unbounded function $V : \mathbb{R}^{n_x} \to \mathbb{R}_{\geq 0}$, a continuous function $H : \mathbb{R}^{n_x} \to \mathbb{R}_{\geq 0}$, a finite set of constants $\{\lambda_i\}_{i=1}^{M_R}$, real numbers $L \geq 0, \zeta > 0$, positive constants $\underline{\alpha}_W, \overline{\alpha}_W, \underline{\alpha}_V, \overline{\alpha}_V, \varrho > 0$ such that

1): $\forall k \in \mathbb{N}$, $\overline{e} \in \mathbb{R}^{n_x+1}$ and $r_i \in \mathcal{R} = \{r_1, r_2, \ldots, r_{M_R}\}$

$$\underline{\alpha}_W |\overline{e}|^2 \leq W(k, \overline{e}) \leq \overline{\alpha}_W |\overline{e}|^2 \tag{10a}$$

$$W(k+1, \overline{J}(k, \overline{e}, r_i)) \leq \lambda_i W(k, \overline{e}) \tag{10b}$$

where $\overline{J}(k, \overline{e}, r_i) = [J_e; J_\xi]$ with J_e and J_ξ defined in (6).

2): $\forall k \in \mathbb{N}, x \in \mathbb{R}^{n_x}$ and for almost all $\overline{e} \in \mathbb{R}^{n_x+1}$, such that

$$\left\langle \frac{\partial W(k, \overline{e})}{\partial \overline{e}}, \overline{g}(x, \overline{e}) \right\rangle \leq LW(k, \overline{e}) + H(x) \tag{11}$$

where $\overline{g}(x, \overline{e}) = [g_e; g_\xi]$ with g_e and g_ξ defined in (5).

3): $\forall x \in \mathbb{R}^{n_x}$

$$\underline{\alpha}_V |x|^2 \leq V(x) \leq \overline{\alpha}_V |x|^2 \tag{12}$$

and $\forall k \in \mathbb{N}, \overline{e} \in \mathbb{R}^{n_x+1}$, and for almost all $x \in \mathbb{R}^{n_x}$

$$\begin{aligned} \langle \nabla V(x), \tilde{f}(x, e) \rangle \leq \varrho |x|^2 &- \varrho W(k, \overline{e}) - H^2(x) \\ &+ \zeta^2 W^2(k, \overline{e}) \end{aligned} \tag{13}$$

Remark 3.2: Similar to Assumption 1 in [15], the inequalities (10) in part 1) characterize the bounds on the error function $W(\overline{e})$ as well as its growths at discrete time instants. Specifically, a positive real λ_i for each data rate $r_i \in \mathcal{R}$ bounds the growth of the error function from the above. The inequality (11) of part 2) assumes a linear growth of the error function in the continuous time domain. The inequalities (12) and (13) of part 3) characterize the growth rate of the Lyapunov function with respect to the state x in the continuous time domain. The parameters in this assumption will be used in Theorem 3.3 to derive the MATI bound that ensures almost sure stability.

Theorem 3.3: Suppose Assumption 3.1 holds, for a given joint policy $\mu = (\mu_m, \mu_p)$, the networked control system in (5) and (6) under SD-MC in (7) is *almost surely asymptotically stable* if the MATI T_{MATI} satisfies

$$T_{MATI} \leq \begin{cases} \frac{1}{L\eta} \arctan\left(\frac{\eta(1-\overline{\lambda})}{2\frac{\overline{\lambda}}{1+\overline{\lambda}}(\frac{\zeta}{L}-1)+1+\overline{\lambda}} \right) & \zeta > L \\ \frac{1}{L} \frac{1-\overline{\lambda}}{1+\overline{\lambda}} & \zeta = L \\ \frac{1}{L\eta} \operatorname{arctanh}\left(\frac{\eta(1-\overline{\lambda})}{2\frac{\overline{\lambda}}{1+\overline{\lambda}}(\frac{\zeta}{L}-1)+1+\overline{\lambda}} \right) & \zeta < L \end{cases} \tag{14}$$

with $\eta = \sqrt{\left|(\frac{\zeta}{L})^2 - 1\right|}$ for some constant $\overline{\lambda}$ that satisfies

$$\overline{\lambda} > \sqrt{\| \operatorname{diag}(\lambda_i^2) \overline{P}(\mu) \|} \tag{15}$$

where $\overline{P}(\mu) = \left[\overline{P}_{ij}(\mu)\right]_{1 \leq i,j \leq M_R}$ with $\overline{P}_{ij}(\mu) = \sum_{p \in \Omega_p, s \in S} \mathbb{P}(r_i | r_j, s, p) \mathbb{P}(s, p | r_j)$.

Remark 3.4: The MATI bounds shown in (14) are functions of the parameters ξ and L defined in Assumption 3.1, and $\overline{\lambda}$

that depends on the parameters of the SD-MC model in (7). The proposed MATI bounds differ from the existing results of [7], [14]–[16] in two aspects. First, the MATI bounds in (14) generalizes the deterministic results in [14], [15] to stochastic cases via the parameter $\overline{\lambda}$. As shown in (15), the parameter $\overline{\lambda}$ can be viewed as the impact from both SD-MC and joint-policy μ. Second, the MATI bounds extend our prior work in [7], [16] by considering a less conservative assumption on the system structure and a more general SD-MC.

With the stability condition derived in (15), Theorem 3.5 shows that if stationary policies are considered, the co-design Problem 2.3 can be reformulated as a polynomial constrained program with a linear objective function.

Theorem 3.5: Consider the following polynomial constrained optimization problem, for given sets of MDP state S, transmission power Ω_p and data rate $\mathcal{R}$, and $\forall 1 \leq i \leq M_R$, let $X(s,r,p) \geq 0, \forall s,r,p$ denote the decision variables for the following optimization problem

$$\min_{\{X(s,r,p)\}} \sum_{s\in S, p\in\Omega_p, r\in\mathcal{R}} c(r,s,p)X(r,s,p) \tag{16a}$$

$$\text{s.t.} \quad \sum_{s,p} X(s,r_i,p) - \sum_{p,s,r_j} P_{ij}(s,p)X(r_j,s,p) = 0, \quad \forall r_i \in \mathcal{R} \tag{16b}$$

$$\sum_{s,p,r} X(s,r,p) = 1, \tag{16c}$$

$$\sum_{j=1}^{M_R}\sum_{s,p} P_{ij}(s,p)X(s,p)\prod_{\ell\neq j} X(r_\ell) - \theta_i^2 \prod_{j=1}^{M_R} X(r_j) \leq 0 \tag{16d}$$

where $X(s,p) = \sum_{j=1}^{M_R} X(r,s,p)$, $X(r) = \sum_{s,p} X(r,s,p)$, $\theta_i = \overline{\lambda}/\lambda_i$, and $P_{ij}(s,p)$ is the transition probability of the SD-MC channel defined in (7). The optimal stationary power policy $\mu_p^\star = \{\mathbb{P}(p|r)\}_{p\in\Omega_p, r\in\mathcal{R}}$ and optimal probability distribution $\pi^\star = \{\mathbb{P}(s)\}_{s\in S}$ for the MDP states can then be represented as below

$$\mathbb{P}(p|r) = \frac{\sum_{s\in S} X^*(s,r,p)}{\sum_{p\in\Omega_p, s\in S} X^*(s,r,p)} \tag{17}$$

$$\mathbb{P}(s) = \sum_{p\in\Omega_p, r\in\mathcal{R}} X^*(s,r,p) \tag{18}$$

where $\{X^*(s,r,p)\}_{s\in S, r\in\mathcal{R}, p\in\Omega_p}$ are the solutions to the polynomial constrained program in (16).

The stability conditions in (16d) are polynomial constraints where the order of the polynomial functions depends on the number of states (data rate) in the SD-MC model. If a two-state SD-MC model is considered, it can be shown that the polynomial constraints can be reduced to quadratic constraints, which can be efficiently solved by SDP programs if the matrices associated with the constraints are positive semi-definite [17]. The two-state SD-MC can be considered as a generalization of the well known bursty erasure channel [12].

With the optimal power policy $\mu_p^\star = \{\mathbb{P}(p|r)\}_{p\in\Omega_p, r\in\mathcal{R}}$ and optimal stationary distribution for the MDP states $\pi^\star = \{\mathbb{P}(s)\}_{s\in S}$ obtained from Theorem 3.5, the next step is to find a control policy $\mu_m = \{\mathbb{P}(a|s)\}_{a\in A(s), s\in S}$ for MDP to achieve the optimal stationary distribution $\pi^\star$. Let $\{c_m(s,a)\}_{s\in S, a\in A}$ denote the cost function for each state-action pair of the MDP process. The optimal control policy $\mu_m^\star$ is obtained by solving the following optimization problem

$$\min_{\mu_m} \lim_{T\to\infty} \frac{1}{T}\mathbb{E}_{s_0}^{\mu_m} \sum_{i=0}^{T} c_m(s_k, a_k) \tag{19a}$$

$$\text{s.t.} \quad Q(\mu_m)\pi^\star = \pi^\star \tag{19b}$$

where $Q(\mu_m)$ is the transition matrix of the induced Markov chain under the control policy μ_m.

The following Theorem 3.6 shows that the optimization problem in (19) can be efficiently solved by a linear program.

Theorem 3.6: Consider an ergodic MDP process $\mathcal{M}_{env} = \{S, s_0, A, Q\}$ with the associated cost function $\{c_m(s,a)\}_{s\in S, a\in A}$, for a given stationary distribution $\pi^\star = [\pi^\star(s_1), \ldots, \pi^\star(s_{M_s})]^T$ with $\pi^\star(s)$ representing the probability distribution for the state $s \in S$, let $\{Y(s,a)\}_{s\in S, a\in A}$ denote the decision variables for the following LP problem,

$$\min_{\{Y(s,a)\}} \sum_{s\in S, a\in A} c_m(s,a)Y(s,a) \tag{20a}$$

$$\text{s.t.} \quad \sum_a Y(s,a) - \sum_{s',a} q(s|s',a)Y(s',a) = 0, \forall s \in S \tag{20b}$$

$$\sum_{s\in S, a\in A} Y(s,a) = 1, \quad Y(s,a) \geq 0, \forall s,a \tag{20c}$$

$$\sum_{a\in A} Y(s,a) = \pi^\star(s), \forall s \in S. \tag{20d}$$

Then, the optimization problem in (19) can be solved by the LP formulated in (20) and the corresponding optimal control policy $\mu_m^\star$ can be obtained by $\mathbb{P}(a|s) = \frac{Y^*(s,a)}{\sum_{a\in A} Y^*(s,a)}, \forall s \in S, a \in A$. where $\{Y^*(s,a)\}$ is the solution to the LP (20).

Proof: The proof follows straightforwardly from the LP representation for constrained MDP (See equation (4.3) in Chapter 4 of [18]). The equation (20d) is equivalent to the constraint of the stationary distribution imposed in (19b). ■

IV. Simulation Results

In the simulation, a linear batch reactor process described in [19] is used for the networked control system part. The state-space model for the unstable linear batch reactor process is $\dot{x} = Ax + Bu$ where

$$A = \begin{bmatrix} 1.38 & -0.2077 & 6.715 & -5.676 \\ -0.5814 & -4.29 & 0 & 0.675 \\ 1.067 & 4.273 & -6.654 & 5.893 \\ 0.048 & 4.273 & 1.343 & -2.104 \end{bmatrix}$$

and $B = [0, 0; 5.679, 0; 1.136, -3.146; 1.136, 0]$. A remote controller is constructed as $\hat{x} = A\hat{x} + Bu, u = K\hat{x}$ with the state feedback controller gain selected to be

$$K = \begin{bmatrix} 0.6961 & 0.8133 & 0.5639 & -1.8492 \\ 2.6908 & 1.1764 & -1.2762 & 0.9968 \end{bmatrix}$$

The controlled external environment is a moving vehicle modeled by a two-state MDP with a state set of $\{s_1, s_2\}$ and an action set of $\{\text{Go}, \text{Stay}\}$. The states represent the partitions of regions in the factory floor. Let the state s_1 denote the region that causes shadow fading if it is occupied by the vehicle, and s_2 denote the non-shadow fading region. The transition probabilities Q of the states under each action are $q(s_1|s_1, \text{“}stay\text{”}) = 0.9, q(s_1|s_1, \text{“}go\text{”}) = 0.1, q(s_1|s_2, \text{“}stay\text{”}) = 0.1$ and $q(s_1|s_2, \text{“}go\text{”}) = 0.9$. The corresponding costs are $c_m(s_1, \text{“}stay\text{”}) = c_m(s_1, \text{“}go\text{”}) = 0.4$ and $c_m(s_2, \text{“}stay\text{”}) = c_m(s_2, \text{“}go\text{”}) = 0.6$. Under the two-state MDP for external environment, a two-state SD-MC model with the data rates of $r_1 = 0$ and $r_2 = 2$ is used to simulate the *state-dependent* fading channel for the networked batch reactor process. The communication system chooses either high transmission power H or low transmission power L to transmit the information over the communication channel. The state-dependent transition probabilities are $P_{11}(s_1, L) = P_{21}(s_1, L) = 0.8, P_{11}(s_1, H) = P_{21}(s_1, H) = 0.6, P_{11}(s_2, L) = 0.4, P_{11}(s_2, H) = P_{21}(s_2, H) = 0.1, P_{21}(s_2, L) = 0.5$. The power costs are $c_p(H) = 0.6, c_p(L) = 0.4$ and the costs for the data rates are $c_r(r_1) = 0.6, c_r(r_2) = 0.4$.

Consider $W(\bar{e}) = |\bar{e}|^2$, and the parameters $L = 17.8870, \zeta = 26.5415, \lambda_0 = 1, \lambda_1 = 0.5$ are selected to satisfy the Assumption 3.1. From equation (14) in Theorem 3.3, the MATI can be determined as $MATI = 0.0104s$ for $\overline{\lambda} > \sqrt{\| \operatorname{diag}(\lambda_i^2)\overline{P}(\mu)\|} = 0.6325$ with $\overline{P}(\mu) = [0.2, 0.2; 0.8, 0.8]$, and then $T = 0.01s \leq MATI$. Fig. 2 shows that the maximum (blue dashed lines) and minimum (red dash-dot lines) trajectories evaluated over 1000 runs under $T = 0.01s$, asymptotically converge to the origin, which implies almost sure asymptotic stability.

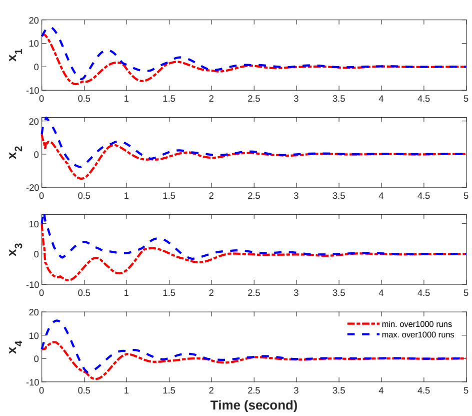

Fig. 2: Maximum and minimum value of state trajectories for the batch reactor process under the transmission time interval $T = 0.01s$.

Besides the system stability, the system performance under the proposed co-design strategy and the separation design method are compared to demonstrate the benefits and robustness of our approach over a wide range of shadow fading levels ($\mathbb{P}(0 \mid 0, s_1, H)$). In the separation design method, the optimal power policy is designed to minimize only the communication costs, i.e, $\min_{\mu_p} \lim_{\ell \to +\infty} \frac{1}{\ell} \mathbb{E} \sum_{k=0}^{\ell} [c_p(p_k) + c_r(R_k)]$ while respecting the stability constraint. The optimal control policy is determined to minimize the costs $\min_{\mu_m} \lim_{\ell \to +\infty} \frac{1}{\ell} \mathbb{E} \sum_{k=0}^{\ell} [c_m(s_k, a_k)]$. Fig. 3 shows the comparison results of optimal joint costs generated by the separation design method (marked by black dash-dot lines) and the co-design strategy (marked by red dash lines). As shown by the plots, the co-design method leads to lower costs across the whole range of the shadow fading than that under the separation design. More interestingly, the co-design strategy is more robust in the high shadow fading regime (i.e., the region between 0.3 and 0.55) than the separation design.

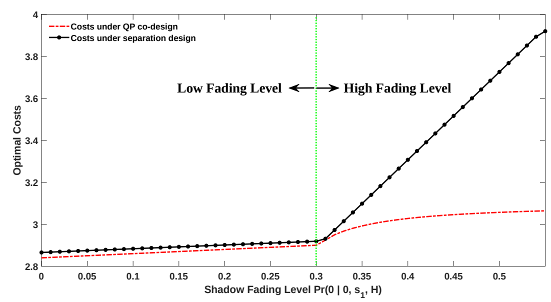

Fig. 3: Performance comparison of the proposed co-design method against the separation method under a wide range of channel conditions ranging from 0 to 0.55.

V. Conclusion

This paper presented a co-design paradigm to ensure both stability and performance for industrial networked control systems under *state-dependent fading channels*. A novel SD-MC model was proposed to characterize the correlation between channel conditions and external environments. The proposed channel model was used to derive sufficient conditions on MATI under which the networked control system is *almost surely asymptotically stable*. The stability conditions are then imposed as hard constraints in the co-design problem whose optimal solutions can be found by solving constrained optimization problems. Numerical results were provided to demonstrate the benefits of the proposed co-design approach.

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VI. Appendices

Proof of Theorem 3.3: Define $\overline{x} := [x^T, \overline{e}, \tau, k]^T$ and $F(\overline{x}) := [\tilde{f}(x,e)^T, \overline{g}(x,\overline{e})^T, 1, 0]^T$, and consider a candidate Lyapunov function $U(\overline{x}) := V(x) + \zeta\phi(\tau)W^2(k,\overline{e})$ where the function $\phi(\tau) : [0, T_{MATI}] \to \mathbb{R}$ is the solution to a nonlinear differential equation $\dot{\phi} = -2L\phi - \zeta(\phi^2+1)$ with $\phi(0) = \overline{\lambda}^{-1}$. Following the proof of Theorem 1 in [15], for all τ and k, $\langle \nabla U(\overline{x}), F(\overline{x})\rangle \le -\varrho|x|^2 - \varrho W(k,\overline{e}) \le -\varrho|x|^2 - \varrho\overline{\alpha}_W|\overline{e}|^2 \le -(\varrho + \varrho\overline{\alpha}_W)(|x|^2 + |\overline{e}|^2) = -\tilde{\varrho}|\overline{x}|^2$ with $\tilde{\varrho} = \varrho + \varrho\overline{\alpha}_W$. By the conditions in (10) and (12), it is straightforward to show that the $\underline{\alpha}_U|\overline{x}|^2 \le U(\overline{x}) \le \overline{\alpha}_U|\overline{x}|^2$ with the positive constants $\underline{\alpha}_U = \underline{\alpha}_V + \zeta\overline{\lambda}\underline{\alpha}_W$ and $\overline{\alpha}_U = \overline{\alpha}_V + \zeta\overline{\lambda}^{-1}\overline{\alpha}_W$ where $\overline{\lambda}$ is defined in (15). Then, one can show that $U(\overline{x}(t,k)) \le \exp\left(-\frac{\tilde{\varrho}}{\overline{\alpha}_U}(t-t_k)\right) U(\overline{x}(t_k,k)), \forall k \in \mathbb{N}, t-t_k \in [0, T_{MATI}]$. Let $\mathbb{1}_A$ denote an indicator function for a set A. Given a data rate set $\mathcal{R} = \{r_i\}_{i=1}^{M_R}$, let $V_{k+1} := V(x(t_{k+1})), V_{k+1}^+ := V(x(t_{k+1}^+)), U_{k+1} := U(\overline{x}(t_{k+1}^+))$ and $\overline{U}_{k+1} := \left[\mathbb{E}[U_{k+1}\mathbb{1}_{r_1}], \ldots, \mathbb{E}[U_{k+1}\mathbb{1}_{r_i}], \ldots, \mathbb{E}[U_{k+1}\mathbb{1}_{r_{M_R}}]\right]^T$ where $\mathbb{E}[U_{k+1}\mathbb{1}_{r_i}] = \mathbb{E}[V_{k+1}^+\mathbb{1}_{r_i}] + \zeta\phi(\tau^+)\mathbb{E}[W^2(k+1, \overline{J}(k,\overline{e},r))\mathbb{1}_{r_i}]$. By Assumption 3.1 and SD-MC in (7), one has $\mathbb{E}[U_{k+1}\mathbb{1}_{r_i}] = V_{k+1} + \zeta\overline{\lambda}^{-1}\lambda_i^2\sum_{j=1}^{M_R}\sum_{s\in S, p\in\Omega_p} P_{ij}(s,p)\mathbb{P}(s,p|r_j)\mathbb{E}[W^2(k,\overline{e})\mathbb{1}_{r_j}]$. Then, $\overline{U}_{k+1} \le V(x(t_{k+1}))\mathbf{e}_{M_R} + \zeta\overline{\lambda}^{-1}\operatorname{diag}(\lambda_i^2)\overline{P}(\mu)\overline{W^2}(k,\overline{e})$ where $\overline{W^2}(k,\overline{e}) = \left[\mathbb{E}[W^2(k,\overline{e})\mathbb{1}_{r_1}], \ldots, \mathbb{E}[W^2(k,\overline{e})\mathbb{1}_{r_{M_R}}]\right]^T$, $\mathbf{e}_{M_R} := [1, 1, \ldots, 1]^T$ is a column vector with M_R of 1, and the transition probability $\overline{P}_{ij}(\mu) = \sum_{s\in S, p\in\Omega_p} P_{ij}(s,p)\mathbb{P}(s,p|r_j)$, $\forall i, j$. Taking the infinity norm on both sides leads to $|\overline{U}_{k+1}| \overset{(d)}{\le} V(x(t_{k+1})) + \zeta\overline{\lambda}^{-1}\|\operatorname{diag}(\lambda_i^2\mathbb{I}_{M_S})\overline{P}^T(\mu^m, \mu^p)\||\overline{W^2}(k,\overline{e})| \overset{(e)}{\le} V(x(t_{k+1})) + \zeta\overline{\lambda}^{-1}\overline{\lambda}^2|\overline{W^2}(k,\overline{e})| \overset{(f)}{\le} |\overline{U}_k|$. The inequality (d) holds due to the norm condition $|y| \le |Ax| \le \|A\||x|$, $\forall x \in \mathbb{R}^n, y \in \mathbb{R}^m$ and $A \in \mathbb{R}^{n\times m}$. The inequality (e) holds due to the condition in (15). The inequality (f) holds because $|\overline{U}(\overline{x})| = V(x) + \zeta\phi(\tau)|\overline{W^2}|$. Thus, the expected value of the Lyapunov function is non-increasing at the discrete states. By combining continuous and discrete dynamics, one can show that $\mathbb{E}[U(\overline{x}(t,k))] \le \exp\left(-\frac{\tilde{\varrho}}{\overline{\alpha}_U}t\right)U(\overline{x}(0,0))$ which implies that $\mathbb{E}[|\overline{x}(t,k)|^2] \le \exp\left(-\frac{\tilde{\varrho}}{\overline{\alpha}_U}t\right)\frac{\overline{\alpha}_U}{\underline{\alpha}_U}|\overline{x}(0,0)|^2$. The *almost sure asymptotic stability* can then be proved by applying the Borel-Cantelli Lemma [7], [16]. ■

Proof of Theorem 3.5: The proof is based on the occupation method used in [18]. For any stationary control and power policy, let $X(r,s,p) := \mathbb{P}(r,s,p)$, the objective function can be equivalently represented as $\lim_{\ell\to\infty}\frac{1}{\ell}\mathbb{E}\sum_{k=0}^{\ell} c(s_k,p_k,R_k) = \sum_{\substack{s\in S, r\in\mathcal{R}\\ p\in\Omega_p}}\left[\lim_{\ell\to\infty}\frac{1}{\ell}\sum_{k=0}^{\ell}\mathbb{P}(s,p,r)c(s,p,r)\right] = \sum_{s\in S, r\in\mathcal{R}, p\in\Omega_p} X(s,p,r)c(s,p,r)$. The conditions in (16b) and (16c) are due to the Markov property and the probability definition. Since $\|\operatorname{diag}(\lambda_i^2)\overline{P}\| \le \overline{\lambda}^2 \implies \lambda_i^2\sum_{j=1}^{M_R}\overline{P}_{ij} \le \overline{\lambda}^2 \implies \sum_{j=1}^{M_R}\sum_{s,p} P_{ij}(s,p)\mathbb{P}(s,p\mid r_j) \le \overline{\lambda}^2/\lambda_i^2 \triangleq \theta_i^2$, replacing $\mathbb{P}(s,p\mid r_j)$ with $\sum_{r_j} X(s,p,r_j)/\sum_{s,p} X(s,p,r_j)$ leads to $\forall 1 \le i \le M_R$

$$\sum_{j=1}^{M_R}\sum_{s,p} P_{ij}(s,p)\frac{\sum_{r_j} X(s,p,r_j)}{\sum_{s,p} X(s,p,r_j)} \le \theta_i^2 \tag{21}$$

Let $X(r_j) := \sum_{s,p} X(s,p,r_j) > 0$ and $X(s,p) := \sum_{r_j} X(s,p,r_j)$, then multiplying $\prod_{j=1}^{M_R} X(r_j) > 0$ on both sides of inequality (21) leads to

$$\sum_{j=1}^{M_R}\sum_{s,p} P_{ij}(s,p)X(s,p)\prod_{\ell\neq j} X(r_\ell) \le \theta_i^2\prod_{j=1}^{M_R} X(r_j).$$

The proof is complete. ■