Learning Based MPC

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Documentation of `LBmpcTP` template class

Implementation of primal-dual infeasible start interior point method (PD IIPM)

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December 20, 2011

Introduction

This report introduces the `LBmpcTP` template class which implements the primal-dual infeasible interior point method (**PD IIPM**) based on Mehrotra's predictor-corrector algorithm [1]. Our solver is tailored to learning-based model predictive control (LBMPC) framework [2] for the special case where the costs are quadratic and the involved dynamics affine.

The report is structured as follows: First, LBMPC for the case where all dynamics are affine and quadratic cost is introduced. Second, the interface to the `LBmpcTP` class is presented. In the third section, advanced parameters for configuration and fine-tuning are described.

1 The Learning-Based MPC Model

We consider a special case of LBMPC [2], in which all dynamics are linear and the cost quadratic. Furthermore, we consider the situation where the feasible sets are convex polyhedron.

This instance of LBMPC is given by the following optimization problem:

$$\begin{aligned}
\min_{c[\cdot],\theta} \quad & (\tilde{x}[m+N|m]-x^\star[m+N|m])^T\tilde{Q}_f(\tilde{x}[m+N|m]-x^\star[m+N|m])+ \\
& \sum_{i=0}^{N-1}\{(\tilde{x}[m+i|m]-x^\star[m+i|m])^T\tilde{Q}(\tilde{x}[m+i|m]-x^\star[m+i|m])+ \\
& (\check{u}[m+i|m]-u^\star[m+i|m])^T R(\check{u}[m+i|m]-u^\star[m+i|m])\}
\end{aligned} \tag{1}$$

$$\begin{aligned}
\text{s.t.} \quad & \tilde{x}[m|m]=\hat{x}[m], \quad \bar{x}[m|m]=\hat{x}[m] \\
& \tilde{x}[m+i|m]=A\tilde{x}[m+i-1|m]+B\check{u}[m+i-1|m]+s+\mathcal{O}_m(\tilde{x}[m+i-1|m],\check{u}[m+i-1|m]), \\
& \mathcal{O}_m(\tilde{x}[m+i-1|m],\check{u}[m+i-1|m])=L_m\tilde{x}[m+i-1|m]+M_m\check{u}[m+i-1|m]+t_m, \\
& \bar{x}[m+i|m]=A\bar{x}[m+i-1|m]+B\check{u}[m+i-1|m]+s, \\
& \check{u}[m+i-1|m]=K\bar{x}[m+i-1|m]+c[m+i-1|m], \\
& F_{\bar{x},i}\bar{x}[m+i|m]\le f_{\bar{x},i}, \quad F_{\check{u},i}\check{u}[m+i|m]\le f_{\check{u},i}, \\
& F_{x\theta}\bar{x}[m+j|m]+F_\theta\theta\le f_{x\theta}, \quad \text{for some } j\in\{1,\dots,N\}
\end{aligned}$$

We assume that the conditions given in Table 1 hold.

Table 1: We make the following assumptions on the matrices in (1).

$\tilde{Q} \in \mathbb{R}^{n \times n}$	positive definite
$\tilde{Q}_f \in \mathbb{R}^{n \times n}$	positive definite
$R \in \mathbb{R}^{m \times m}$	positive semidefinite
$F_{\bar{x},i} \in \mathbb{R}^{\texttt{nSt} \times n} \quad \forall i$	full rank
$F_{\bar{u},i} \in \mathbb{R}^{\texttt{nInp} \times m} \quad \forall i$	full rank
$F_\theta \in \mathbb{R}^{\texttt{nF\_xTheta} \times m}$	full rank
$K \in \mathbb{R}^{m \times n}$	$A + BK$ Schur

For the purpose of solving this LBMPC problem, it is useful to consider the optimization problem at each time step as belonging to a general class of problems, in this case a convex quadratic program (QP-LBMPC):

$$\begin{aligned} \min_{z} \quad & z_m^T H z_m + g_m^T z_m \\ \text{s.t.} \quad & C_m z_m = b_m \\ & P z_m \leq h_m, \end{aligned} \tag{2}$$

where z is the stacked vector:

$$z_m = \begin{pmatrix} c[m]^T & \tilde{x}[m+1]^T & \bar{x}[m+1]^T & \cdots & c[m+N-1]^T & \tilde{x}[m+N]^T & \bar{x}[m+N]^T & \theta^T \end{pmatrix}^T.$$

For the sake of clarity, we omit the index m in (2) in all following calculations.

It can be shown that the above QP-LBMPC is convex. Furthermore, we assume that Slater's condition (or any other constraint qualification) holds [3, Ch. 2]. Hence, the KKT-conditions for optimality are necessary and sufficient. They state that for every optimal z_{opt} there exists vectors $\lambda_{\text{opt}}, \nu_{\text{opt}}$ and t_{opt} such that at the optimal point $(z, \lambda, \nu, t) = (z_{\text{opt}}, \lambda_{\text{opt}}, \nu_{\text{opt}}, t_{\text{opt}})$ the following equations are satisfied [4]:

$$\mathcal{F}(z, \lambda, \nu, t) \triangleq \begin{pmatrix} r_H \\ r_C \\ r_P \\ r_T \end{pmatrix} \triangleq \begin{pmatrix} 2Hz + g + P^T\lambda + C^T\nu \\ Cz - b \\ Pz - h + t \\ T\Lambda\mathbf{1} \end{pmatrix} = 0, \qquad (\lambda, t) \geq 0 \tag{3}$$

where t is the slack variable associated with the inequality in (2), $T \triangleq \text{diag}(t)$, $\Lambda \triangleq \text{diag}(\lambda)$ and $\mathbf{1}$ is the all-one vector. Our PD IIPM algorithm generates sequences $(z^i, \lambda^i, \nu^i, t^i)$ with $(\lambda^i, t^i) > 0$ that approach the optimality condition (3).

Because the PD IIPM algorithm is not guaranteed to generate feasible iterates (except in the limit as the algorithm converges), a duality gap cannot be defined. Instead, the complementary measure μ is used to measure the optimality of the point (z, λ, ν, t):

$$\mu \triangleq \frac{\lambda^T t}{m_P}, \tag{4}$$

where m_P is the number of inequality equations, i.e. the number of rows in the Matrix P.

2 Using the LBmpcTP template class

The LBmpcTP template class is typically called in two seperate steps:

1. Definition of matrices A, B, s, $\tilde{Q}$, $\tilde{Q}_f$, R, K, $\{F_{\bar{x},i}\}_{i=1}^{N}$, $\{f_{\bar{x},i}\}_{i=1}^{N}$, $\{F_{\check{u},i}\}_{i=0}^{N-1}$, $\{f_{\check{u},i}\}_{i=0}^{N-1}$, $F_{x\theta}$, F_θ, $f_{x\theta}$, scalars n_{iter}, ϵ_{reg}, ϵ_{primal}, ϵ_{dual}, ϵ_μ and string `fileName` in MATLAB file `Init.m`, see Table 2.
2. The main C++-file (e.g. `mainLBmpcTP.cpp`) instantiates `LBmpcTP`-object and calls the solver routine, i.e.:
   (a) Instantiation of an object, e.g. `myObj`:
   `LBmpcTP<double, _n, _m, _N, _nSt, _nInp, _nF_xTheta, _pos_omega> myObj( fileName, verbose)`
   (b) It calls the step-function `myObj.step(.)` which computes the optimal input and returns a status flag. Each call of `myObj.step` requires the following set of (updated) parameters: L_m, M_m, t_m, $\hat{x}$, $\{x^\star[m+i]\}_i$.
   ```
   status = myObj.step( Lm, Mm, tm, x_hat, x_star );
   u_opt = myObj.u_opt;
   ```

The C++ code is then compiled using a suitable compiler. In case of the gcc-compiler, the compilation may look like this:
`g++ -I /usr/local/include/eigen3/ -O3 mainLBmpcTP.cpp -o mainLBmpcTP`.

In the following sections, steps 1. and 2. are described in more detail.

2.1 MATLAB: `Init.m`

`Init.m` defines the parameters required for the instantiation of the `LBmpcTP` object and consists of the following two parts:

- User specifies the parameters given in Table 2.
- The MATLAB script `writeParam.m` writes the parameters to the binary file specified by `fileName` (default: `ConstrParam.bin`).

Appendix 4.1 shows a typical implementation of `Init.m`.

Remarks:

- The number of state constraints is assumed to be constant, i.e. the number of rows in $\texttt{Fx}\{i\}$ is constant for all i, and denoted by `_nSt`.
- The number of input constraints is assumed to be constant, i.e. the number of rows in $\texttt{Fu}\{i\}$ is constant for all i, and denoted by `_nInp`.
- The number of constraints with θ in (1) is assumed to be `_nF_xTheta`.

2.2 C++: `mainLBmpcTP.cpp`

This file is the main control routine that manipulates `LBmpcTP` objects. An example file is provided in Appendix 4.3. In the following, we outline the main structure of `mainLBmpcTP.cpp`:

1. **SPECIFY parameters:** `_N`, `_m`, `_n`, `_nSt`, `_nInp`, `_nF_xTheta`, `_pos_omega` (see Table 3).
2. **SPECIFY binary source file name and verbose-flag:** `fileName`, `verbose`:
   ```
   const char fileName[] = "ConstrParam.bin";
   bool verbose = 0;    // 0 = shut up
   ```
3. Call the constructor and instantiate an object (e.g. `myObj`):
   `LBmpcTP<double,_n,_m,_N,_nSt,_nInp,_nF_xTheta,_pos_omega> myObj(fileName,verbose);`

Table 2: Key parameters which are to be defined in `Init.m`

MATLAB variable	description	typical range/value
`N` $\in \mathbb{N}$	length of MPC horizon	
`m` $\in \mathbb{N}$	number of inputs	
`n` $\in \mathbb{N}$	number of states	
`fileName`	name of binary file that stores the matrices and scalars below.	
`A` $\in \mathbb{R}^{n\times n}$	linear dynamics matrix: $\bar{x}^+ = A\bar{x} + B\check{u} + s$	
`B` $\in \mathbb{R}^{n\times m}$	input-state dynamics matrix: $\bar{x}^+ = A\bar{x} + B\check{u} + s$	
`s` $\in \mathbb{R}^{n}$	affine offset in state dynamics: $\bar{x}^+ = A\bar{x} + B\check{u} + s$	
`K` $\in \mathbb{R}^{m\times n}$	feedback gain matrix, $\check{u} = K\bar{x} + c$, $A + BK$ is stable	
`Q_tilde` $\in \mathbb{R}^{n\times n}$	p.d. weight matrix for state	
`Q_tilde_f` $\in \mathbb{R}^{n\times n}$	p.d. weight matrix for final state	
`R` $\in \mathbb{R}^{m\times m}$	p.s.d. weight matrix on input	
`Fx`$\{i\} \in \mathbb{R}^{$`nSt`$\times n}$	$F_{\bar{x},i}\bar{x}[m+i\|m] \le f_{\bar{x},i}$, full-rank, $i = 1,\dots,N$	
`fx`$\{i\} \in \mathbb{R}^{$`nSt`$}$	$F_{\bar{x},i}\bar{x}[m+i\|m] \le f_{\bar{x},i}$, $i = 1,\dots,N$	
`Fu`$\{i\} \in \mathbb{R}^{$`nInp`$\times m}$	$F_{\check{u},i}\check{u}[m+i\|m] \le f_{\check{u},i}$, full-rank, $i = 0,\dots,N-1$	
`fu`$\{i\} \in \mathbb{R}^{$`nInp`$}$	$F_{\check{u},i}\check{u}[m+i\|m] \le f_{\check{u},i}$, $i = 0,\dots,N-1$	
`F_xTheta` $\in \mathbb{R}^{$`nF_xTheta`$\times n}$	$F_{x\theta}\bar{x}[m+j\|m] + F_\theta\theta \le f_{x\theta}$, $j \in \{1,\dots,N\}$	
`F_theta` $\in \mathbb{R}^{$`nF_xTheta`$\times m}$	$F_{x\theta}\bar{x}[m+j\|m] + F_\theta\theta \le f_{x\theta}$, full-rank, $j \in \{1,\dots,N\}$	
`f_xTheta` $\in \mathbb{R}^{$`nF_xTheta`$}$	$F_{x\theta}\bar{x}[m+j\|m] + F_\theta\theta \le f_{x\theta}$, $j \in \{1,\dots,N\}$	
`n_iter` $\in \mathbb{N}$	max. number of Newton steps to solve (2)	100
`reg` $\in \mathbb{R}$	regularization coefficient to render Matrix H (2) positive definite	$[0, 0.1]$, depends on H
`eps_primal` $\in \mathbb{R}$	ϵ_{primal}, necessary stopping criteria, see (5)	0.1
`eps_dual` $\in \mathbb{R}$	ϵ_{dual}, necessary stopping criteria, see (5)	0.1
`eps_mu` $\in \mathbb{R}$	ϵ_μ, necessary stopping criteria, see (5)	0.1

Table 3: The following template parameters are required to instantiate a `LBmpcTP`-object.

variable	description	default
`Type`	only `double` is supported	`double`
`_N`	length of MPC horizon	
`_m`	number of inputs	
`_n`	number of states	
`_nSt`	number of state constraints (constant over the horizon)	
`_nInp`	number of input constraints (constant over the horizon)	
`_nF_xTheta`	number of constraints involving θ in (1)	
`_pos_Omega`	index $j \in \{1,\dots,N\}$ in $F_{x\theta}\bar{x}[m+j\|m] + F_\theta\theta \le f_{x\theta}$	

Table 4: These arguments must be updated before each `step()`-call.

variable	description
`Lm`	oracle matrix, i.e. $\mathcal{O}_m(\tilde{x}[m+i\|m], \check{u}[m+i\|m]) = L_m\tilde{x}[m+i\|m] + M_m\check{u}[m+i\|m] + t_m$
`Mm`	oracle matrix, i.e. $\mathcal{O}_m(\tilde{x}[m+i\|m], \check{u}[m+i\|m]) = L_m\tilde{x}[m+i\|m] + M_m\check{u}[m+i\|m] + t_m$
`tm`	oracle matrix, i.e. $\mathcal{O}_m(\tilde{x}[m+i\|m], \check{u}[m+i\|m]) = L_m\tilde{x}[m+i\|m] + M_m\check{u}[m+i\|m] + t_m$
`x_hat`	current state estimate, i.e. $\tilde{x}[m] = \hat{x}[m], \quad \bar{x}[m] = \hat{x}[m]$
`x_star[_N]`	states our system wants to track

4. Update the variables needed for step()-function: `Lm`, `Mm`, `tm`, `x_hat`, `x_star[_N]` (Table 4). Note: these values are not provided by `LBmpcTP`.
5. Call the step-function to solve the optimization problem:
   `status = myObj.step( Lm, Mm, tm, x_hat, x_star );`
   Table 5 lists the possible status codes together with their meaning.

Table 5: The meaning of the status-flags returned by `step()`-function.

status flag	meaning
0	success
1	problem (possibly) primal infeasible
2	problem (possibly) dual infeasible
3	stopping criterion $\mu \leq \epsilon_\mu$ not satisfied
4	`nan`
5	other error

6. The optimal input can be accessed by: `u_opt = myObj.u_opt;`

Note that the parameters in `Init.m` and `mainLBmpcTP.cpp` have to be consistent with each other.

2.3 C++ template class: `LBmpcTP.h`

Here, we give a rough overview of `LBmpcTP`'s internal structure. As mentioned, basic access to the class is granted through two methods (constructor and `step(.)`-method) as well as the public member variable `u_opt`. Details on the implementation and underlying mathematics can be found in [5, 6] [1, 4, 7, 8]. The constructor initializes the private variables discussed in the previous sections. The `step(.)`-method performs the following tasks:

- It recursively computes the sequence $\{u^\star[m+i|m]\}_i$ from the given desired state sequence $\{x^\star[m+i|m]\}_i$ by solving

  $$x^\star[m+i|m] = (A+L_m)x^\star[m+i-1|m] + (B+M_m)u^\star[m+i-1|m] + (s+t_m), \quad x^\star[m|m] = \hat{x}$$

  and taking the least-squares solution (SVD).
- Casts (1) into (2).
- Finally, it computes the optimal control input and stores it in the public member variable `u_opt`.

3 Advanced Topics

Some parameters in Tab. 2 can be used to tweak `LBmpcTP`:

- The problem cannot be solved with the default parameters. It either does not converge (number of iterations exceeds `num_iter`) or the program returns `nan` (i.e. error code 4).
- Convergence is too slow for the desired purpose, i.e. the optimization step needs too many Newton iterations.
- The exact solution is not necessary and an approximate solution suffices to speed up algorithm.

The goal of this section is to describe some implementation details.

Table 6 lists the tuning parameters from Tab. 2 and describes their role and influence in greater detail.

Table 6: Tuning parameters defined in `Init.m`

tuning variable	influence	typical range/value
`n_iter`	Can be used to limit the number of Newton iterations or for early termination to obtain an inaccurate solution of (2). This can be useful if the computational time is limited	100
`reg`	regularization coefficient to render Matrix H in (2) positive definite, see (7)	$[0, 0.1]$, depends on H
`eps_primal`	ϵ_{primal}, necessary stopping criteria, see (5)	0.1
`eps_dual`	ϵ_{dual}, necessary stopping criteria, see (5)	0.1
`eps_mu`	ϵ_{μ}, necessary stopping criteria on μ, see (5)	0.1

3.1 Compiling

The EIGEN* library has to be installed to use `LBmpcTP`.

Furthermore, some compilers provide the option to generate optimized executable codes. For example, the gcc compiler allows the user to add the `-O3` option which reduces the size of the executable file and increases the performance of the generated code:
`g++ -I /usr/local/include/eigen3/ -O3 mainLBmpcTP.cpp -o mainLBmpcTP`

Also, it is advised to use the latest compiler for compatibility and performance reasons. `LBmpcTP` is tested to work with gcc versions 4.2 and 4.6.

3.2 Stopping Criterion and Regularization

This section addresses the termination criterion. `LBmpcTP` uses a heuristic stopping criterion adapted from [9, 10] and consists of the following three conditions:

$$\begin{aligned} \frac{\| \begin{pmatrix} r_C^T & r_P^T \end{pmatrix}^T \|}{\| \begin{pmatrix} h^T & b^T \end{pmatrix}^T \| + 1} &\leq \epsilon_{\text{primal}} \\ \frac{\|r_H\|}{\|g\| + 1} &\leq \epsilon_{\text{dual}} \\ \mu &\leq \epsilon_{\mu}. \end{aligned} \tag{5}$$

The algorithm terminates successfully if and only if all three conditions are satisfied. It should be mentioned at this place that the smaller we choose $\epsilon_{\text{primal}}, \epsilon_{\text{dual}}, \epsilon_{\mu}$ to be, the ill-conditioned our problem becomes. Because the quadratic cost matrix H in (1) is not strictly convex, numerical issues arise as we try to push the residuals (3) towards zero: the linear equation becomes poorly-conditioned, posing challenges when computing the Cholesky decomposition numerically. This issue has been widely discussed, e.g. in [11, 12]. In `LBmpcTP`, the regularization parameter ϵ_{reg} (`reg` in Table 2) is introduced that regularizes the coefficient matrix in the so-called "augmented form", i.e. instead of solving a linear equation with the coefficient matrix

$$\begin{pmatrix} \Phi & C^T \\ C & 0 \end{pmatrix}, \tag{6}$$

where $\Phi > 0$ is ill-conditioned, we solve with the regularized coefficient matrix

$$\begin{pmatrix} \Phi & C^T \\ C & 0 \end{pmatrix} + \epsilon_{\text{reg}} \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix} \tag{7}$$

*`http://eigen.tuxfamily.org/`

where $\mathbb{I}$ is the identity matrix. As $\epsilon \to 0$, the solution of (7) approaches the solution of (6) because $\Phi > 0$.

3.3 Troubleshooting

In this section, some common errors are described. Possible sources for these errors are given and solutions are proposed. We assume that the posed problem has a solution, i.e. that it is primal and dual feasible.

1. **We only want to approximately solve (2).**
   Solutions:
   - This can be achieved by bounding the permitted number of Newton iterations by choosing a small `n_iter`. Depending on the problem, numbers as few as 3 iterations are enough to produce satisfying results.
2. **Obtained result is a `nan`-vector (not a number).**
   Solutions:
   - This problem is directly related to the ill-conditioning described in Section 3.2. The way to handle this in `LBmpcTP` is by choosing ϵ_{reg} such that the program does not produce `nan` (not a number).

3.4 Additional Remarks

- So far, the algorithm only works for a minimum prediction horizon of 3.
- There are more parameters and class methods in `LBmpcTP.h` that can be tuned to improve the performance of the solver. These include
  - How to implement the regularization in class method `compPhi()`.
  - Initialization of starting point $(z^0, \lambda^0, \nu^0, t^0)$ using a different heuristic in `compInitPoints()`.
  - How to choose the final step size:
    1. Use Mehrotra's heuristic as it is done in class method `compAlpha_corrector()` and choose the parameter $0 <$ `gamma_f` $\ll 1$.
    2. Do it using method `compAlpha_affine()` and some predefined damping factor $0 <$ `damp` $\ll 1$
  - Use another kind of stopping criterion.
  - Use another heuristic to detect primal and dual infeasibility.

  However, it usually suffices to tune the parameters given in Table (6).
- If the prediction horizon N is large, then the variable `Nhoriz` in the class file `LBmpcTP.h` has to be increased. The reason is that some arrays are preallocated with length `Nhoriz`, which is due to a limitation in EIGEN.
- When an `LBmpcTP` object is instantiated, the class variable `z` is initialized. Between the time steps, `z` can take the role of "warm start". However, when the `LBmpcTP` is instantiated, it set as the 0-vector. If a priori information is available, then a more suitable `z` can be chosen.

4 Example Files

4.1 Init.m

```
%% Init.m
% Writes relevant data to binary file.
% author: Xiaojing ZHANG
% date: November 10, 2011


clc;
clear all;
format('short');

%% MPC parameters:
N = 10;      % MPC horizon
m = 2;       % # input
n = 5;       % # states

%% binary file name
fileName = 'ConstrParam.bin';

%% Parameters for constructor

n_iter = 100; % maximum number of Newton iterations
reg = 1e-3;  % regularization Term
eps = 0.1;
eps_primal = eps;
eps_dual = eps;
eps_mu = eps;

%% System dynamic parameters

A = [1 0 1.2 1.3 1
    0.5 2.1 1 1 -0.3
    1 1 .2 1 -2
    0 1 0.3 1.4 -2
    0.4 -0.9 2 1.2 -.4];

B = [1 0
    1.3 1
    0 1.2
    -0.1 1
    0.2 -1];

s = [0 ; 2 ; 1.4 ; 2 ; 1];

K = -[  -0.687725010189527   1.970370349984470  -0.865901978685416  -3.069636538756281  2.096473307971948
   0.181027584433678   1.040671203681152  -0.344287251091615   0.362844179335401  -1.109614558033092];

%% cost and constraint matrices
Q_tilde = 1*eye(n);
Q_tilde_f = Q_tilde+1;

R = 1*eye(m);

% constraint matrices: constrained on
H = eye(n); k = 1000*ones(n,1);
Fx{1} = [H ; -H];
Fx{2} = [H ; -H];
Fx{3} = [H ; -H];
Fx{4} = [H ; -H];
Fx{5} = [H ; -H];
Fx{6} = [H ; -H];
```

```
Fx{7} = [H ; -H];
Fx{8} = [H ; -H];
Fx{9} = [H ; -H];
Fx{10} = [H ; -H];
fx{1} = [k ; k]-3;
fx{2} = [k ; k]-0;
fx{3} = [k ; k];
fx{4} = [k ; k];
fx{5} = [k ; k]-2;
fx{6} = [k ; k]+3;
fx{7} = [k ; k]+4;
fx{8} = [k ; k]+2;
fx{9} = [k ; k]-10;
fx{10} = [k ; k]+10;


H = eye(m); k = 100*ones(m,1);
Fu{1} = [H ; -H];
Fu{2} = [H ; -H];
Fu{3} = [H ; -H];
Fu{4} = [H ; -H];
Fu{5} = [H ; -H];
Fu{6} = [H ; -H];
Fu{7} = [H ; -H];
Fu{8} = [H ; -H];
Fu{9} = [H ; -H];
Fu{10} = [H ; -H];
fu{1} = [k ; k+20]+1;
fu{2} = [k+4 ; k]-0;
fu{3} = [k ; k]+5;
fu{4} = [k ; k]+10;
fu{5} = [k ; k-10];
fu{6} = [k ; k]+2;
fu{7} = [k ; k]-7;
fu{8} = [k ; k]+10;
fu{9} = [k ; k]-10;
fu{10} = [k+10 ; k];


F_xTheta = [ 1  1  1  0  0
            -1 -1 -1  0  0
             0  0  0  1  1
             0  0  0 -1 -1
             1  0  1  0  0
            -1  0 -1  0  0
             0  0  0  1  0
             0  0  0 -1  0
             0 0   -1  1  1
             0  0  1 -1  -1];
F_theta = [ 1  0
           -1  0
            0  1
            0 -1
            0  1
            0 -1
            1  0
           -1  0
            0  1
            0 -1];

f_xTheta = 100*[20 20 20 20 30 30 40 40 50 50]';

%% write data for constructor arguments into file
% ConstrParam.bin

writeParam;      % call writeParam.m
```

```
127
128 disp(['new parameters written to binary file']);
```

4.2 writeParam.m

```
1  %% Init.m
2  % Writes relevant data to binary file.
3  % author: Xiaojing ZHANG
4  % date: November 10, 2011
5
6
7  clc;
8  clear all;
9  format('short');
10
11 %% MPC parameters:
12 N = 10;      % MPC horizon
13 m = 2;      % # input
14 n = 5;      % # states
15
16 %% binary file name
17 fileName = 'ConstrParam.bin';
18
19 %% Parameters for constructor
20
21 n_iter = 100; % maximum number of Newton iterations
22 reg = 1e-3;  % regularization Term
23 eps = 0.1;
24 eps_primal = eps;
25 eps_dual = eps;
26 eps_mu = eps;
27
28 %% System dynamic parameters
29
30 A = [1 0 1.2 1.3 1
31     0.5 2.1 1 1 -0.3
32     1 1 .2 1 -2
33     0 1 0.3 1.4 -2
34     0.4 -0.9 2 1.2 -.4];
35
36 B = [1 0
37     1.3 1
38     0 1.2
39     -0.1 1
40     0.2 -1];
41
42 s = [0 ; 2 ; 1.4 ; 2 ; 1];
43
44 K = -[  -0.687725010189527   1.970370349984470  -0.865901978685416  -3.069636538756281  2.096473307971948
45    0.181027584433678   1.040671203681152  -0.344287251091615   0.362844179335401  -1.109614558033092];
46
47 %% cost and constraint matrices
48 Q_tilde = 1*eye(n);
49 Q_tilde_f = Q_tilde+1;
50
51 R = 1*eye(m);
52
53 % constraint matrices: constrained on
54 H = eye(n); k = 1000*ones(n,1);
55 Fx{1} = [H ; -H];
56 Fx{2} = [H ; -H];
57 Fx{3} = [H ; -H];
58 Fx{4} = [H ; -H];
59 Fx{5} = [H ; -H];
60 Fx{6} = [H ; -H];
```

```
Fx{7} = [H ; -H];
Fx{8} = [H ; -H];
Fx{9} = [H ; -H];
Fx{10} = [H ; -H];
fx{1} = [k ; k]-3;
fx{2} = [k ; k]-0;
fx{3} = [k ; k];
fx{4} = [k ; k];
fx{5} = [k ; k]-2;
fx{6} = [k ; k]+3;
fx{7} = [k ; k]+4;
fx{8} = [k ; k]+2;
fx{9} = [k ; k]-10;
fx{10} = [k ; k]+10;


H = eye(m); k = 100*ones(m,1);
Fu{1} = [H ; -H];
Fu{2} = [H ; -H];
Fu{3} = [H ; -H];
Fu{4} = [H ; -H];
Fu{5} = [H ; -H];
Fu{6} = [H ; -H];
Fu{7} = [H ; -H];
Fu{8} = [H ; -H];
Fu{9} = [H ; -H];
Fu{10} = [H ; -H];
fu{1} = [k ; k+20]+1;
fu{2} = [k+4 ; k]-0;
fu{3} = [k ; k]+5;
fu{4} = [k ; k]+10;
fu{5} = [k ; k-10];
fu{6} = [k ; k]+2;
fu{7} = [k ; k]-7;
fu{8} = [k ; k]+10;
fu{9} = [k ; k]-10;
fu{10} = [k+10 ; k];


F_xTheta = [ 1  1  1  0  0
            -1 -1 -1  0  0
             0  0  0  1  1
             0  0  0 -1 -1
             1  0  1  0  0
            -1  0 -1  0  0
             0  0  0  1  0
             0  0  0 -1  0
             0 0   -1  1  1
             0  0  1 -1  -1];
F_theta = [ 1  0
           -1  0
            0  1
            0 -1
            0  1
            0 -1
            1  0
           -1  0
            0  1
            0 -1];

f_xTheta = 100*[20 20 20 20 30 30 40 40 50 50]';

%% write data for constructor arguments into file
% ConstrParam.bin

writeParam;      % call writeParam.m
```

```
127
128 disp(['new parameters written to binary file']);
```

4.3 mainLBmpcTP.cpp

```cpp
// mainLBmpcTP.cpp
// example file to test simple examples
// date: November 08, 2011
// author: Xiaojing ZHANG

// matrices are imported from binary file created by MATLAB


#include <iostream>     // I-O
#include <Eigen/Dense>  // matrix computation
#include "LBmpcTP.h"    // class template
#include <sys/time.h>

using namespace Eigen;
using namespace std;

int main()
{
    // ------ SPECIFY parameters -------
    const int _N = 10;      // MPC horizon
    const int _m = 2;       // #input
    const int _n = 5;       // #states
    const int _nSt = 10;    // # state constraints
    const int _nInp = 4;    // # state constraints
    const int _nF_xTheta = 10;  // # Omega constraints
    const int _pos_omega = 1;   // ≤ _N
    const char fileName[] = "ConstrParam.bin";
    bool verbose = 1;   // '0' if it should shut up
    int status;     // stores status code
    int iterations = 1;


    // -------- object instantiation -----------
    // fileName contains name of file with parameters
    // bool verbose: 1 or 0
    LBmpcTP<double, _n, _m, _N, _nSt, _nInp, _nF_xTheta, _pos_omega> myObj( fileName, verbose);


    // ----------- SPECIFY arguments for step() ---------------
    // those matrices are computed externely
    // -----------  sizes of matrices ---------------
        Matrix<double, _n, _n> Lm;      // n x n
        Matrix<double, _n, _m> Mm;      // n x m
        Matrix<double, _n, 1> tm;       // n x 1
        Matrix<double, _n, 1> x_hat;        // n x 1, state estimate
        Matrix<double, _n, 1> x_star[_N];   // n x 1, [_N], tracking
        Matrix<double, _m, 1> u_opt;            // m x 1, optimal input is saved there

    // -------- they are updated at each time step ------------
    Lm << 1,  2,  0,  1,  2,
          -2,  1.3,  1,  2,  2.3,
           1,  2, -1,  0,  -1,
           1,  2,  2,  -2.3,  1,
          0, 0,  2,  1.4, -2;

    Mm << 1,  1.4,
       2,  -1,
       1,   2,
       0,   0,
       2, -1;

    tm << -1, 2, 1, 1, 2;
    x_hat << 3, 3, -2, 3, 4;
```

```
64
65      for(int i = 0; i ≤ _N-1; i++)
66      {
67          x_star[i].setZero();
68      }
69
70      struct timeval start;
71      struct timeval end;
72      double elapsedTime = 0;
73      double timeTmp;
74      int newtonSteps = 0;
75      gettimeofday(&start, NULL);
76      for (int i = 0; i < iterations; i++)
77      {
78          status = myObj.step( Lm, Mm, tm, x_hat, x_star );
79          newtonSteps += myObj.n_iter_last;
80          if (status)
81          {
82              cerr << "status error at iteration " << i << ": " << status << endl;
83              return 1;
84          }
85      }
86      gettimeofday(&end, NULL);
87      timeTmp =   (end.tv_sec*1000.0 + end.tv_usec/1000.0) - (start.tv_sec*1000.0 + start.tv_usec/1000.0);
88      elapsedTime += timeTmp;
89
90      cout << "elapsedTime with " << iterations << " iterations: " << elapsedTime << " [ms]" << endl;
91      cout << "needed " << newtonSteps << " Newton steps" << endl;
92
93      return 0;
94  }
```

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