PROGRAMMING LOGIC AND TECHNIQUES

Problem solving

Unit Price: Definition

☐ The first step of cross multiplication is to calculate the **unit price** from a package price.

Example:

If12 eggs cost $1.80, how much does 1 egg cost?

The cost of 1 egg is 12 times less than the cost of 12 eggs.

So divide $1.80 by 12 eggs.

$$1.80 \div 12 = 0.15$$

One egg costs $0.15

The best-value purchase

- To know which of two products is the better-value purchase, calculate the unit price of each item and compare them.

- Whichever has the lowest unit price is the most economical.

The best-value purchase

☐ Example: Which is the better-value purchase: a box of 200 ml of syrup for $4.79, or a box of 500 ml for $9.99?

Calculate the unit price of each product:

The first is: $4.79 ÷ 200ml = 0.025 $/ml

The second is: $9.99 ÷ 500ml = 0.019 $/ml

These values are the cost of each product per millilitre.

So the most economical buy is the 500 ml box at $9.99.

Exercises

- What is the best-value purchase in each of the following situations:
  - Ice cream: 3 litres for $3.45 or 2 litres for $2.58
  - Cream of tomato: 3 boxes for $1.00 or 2 boxes for $0.78
  - Oranges: 12 for $2.52 or 8 for $1.84

Exercises

☐ What is the best-value purchase in each of the following situations:

■ Ice cream: 3 litres for $3.45 or 2 litres for $2.58

3 litres for $3.45

■ Cream of tomato: 3 boxes for $1.00 or 2 boxes for $0.78

3 boxes for $1.00

■ Oranges: 12 for $2.52 or 8 for $1.84

12 for $2.52

Exercises

- What is the best-value purchase in each of the following situations:

  - Pencils: 10 for $2.09 or 12 for $2.55

  - Sandwich bags: 20 for $0.99 or 75 for $1.79

  - Garbage bags:10 for $0.99 or 12 for $1.08

Exercises

- What is the best-value purchase in each of the following situations:

  - Pencils: 10 for $2.09 or 12 for $2.55

    10 for $2.09

  - Sandwich bags: 20 for $0.99 or 75 for $1.79

    75 for $1.79

  - Garbage bags:10 for $0.99 or 12 for $1.08

    12 for $1.08

Directly Proportional Situations

- All of the previous exercises are examples of **directly proportional** situations.

- A directly proportional situation is a relationship involving two quantities where if one quantity increases at a constant rate, then so does the other quantity.

- Examples:
  - If I buy more apples, I will pay more
  - If I rent a car for longer, it will cost more
  - If I have a longer distance to travel, it will take more time

Cross-multiplication

- Cross-multiplication allows us to calculate an unknown value in a directly proportional situation.

  Example: It costs \$1.20 to buy 6 apples. How much will 10 apples cost?

  We must first find the cost of 1 apple to then calculate the price of 10 apples.

  1) Calculate the cost of 1 apple: \$1.20 ÷ 6 = \$0.20

  2) Multiply by 10 apples: \$0.20 × 10 = \$2.00

  As is demonstrated here, we must first find the unit cost and then calculate the total.

Cross-multiplication

a) The method for calculating a cross-multiplication

　　1) Calculate the unit price of an object

　　　2) Multiply the unit price by the number of objects desired

Using the previous example, we get the following calculation:

$$\frac{1.20}{6} \times 10 = \frac{1.20 \times 10}{6} = 2.00$$

The cost of 10 apples is $2.00.

The best-value purchase

Example: The price of 12 pencils is $4.26. How much do 10 pencils cost?

$$12 \text{ pencils} \Rightarrow \$4.26$$

$$10 \text{ pencils} \Rightarrow \$X$$

$$X = \frac{4.26}{12} \times 10 = \frac{4.26 \times 10}{12} = 3.55$$

10 pencils cost $3.55

Cross-multiplication

b) The cross-multiplication formula

    1) Write out the values of the problem, taking care to vertically align items of the same type.

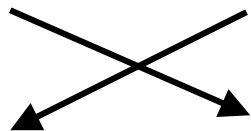   Example: 12 pencils $\Rightarrow$ \$4.26

                        10 pencils $\Rightarrow$ \$**X** (unknown amount)

Cross-multiplication

2) Cross-multiply the values; that is, multiply the values that are diagonally adjacent.

Example:

12 pencils $\longrightarrow$ $4.26

10 pencils $\longrightarrow$ $**x** (unknown amount)

Cross-multiplication: 12 * **X** = 10 * 4.26

Cross-multiplication

3) Divide the one product by the coefficient of the **x**, making sure that **x** is isolated.

$$X = \frac{10 \times 4.26}{12} = 3.55$$

Exercises

1- Calculate the amount of litres required to travel 845 km, if you can travel 65 km on 3 litres (to the nearest hundredth).

2- A lumberjack cuts 36 pine trees in 4 hours. How many will be cut in 6 hours (to the unit)?

3- Louise travels 160 km in 2 hours. How far will she travel in 5 hours (to the unit)?

Exercises

1- Calculate the amount of litres required to travel 845 km, if you can travel 65 km on 3 litres (to the nearest hundredth).

39 L

2- A lumberjack cuts 36 pine trees in 4 hours. How many will be cut in 6 hours (to the unit)?

54 pine trees

3- Louise travels 160 km in 2 hours. How far will she travel in 5 hours (to the unit)?

400 km

Inverse proportionality

- So far, the problems we have seen have been **directly proportional** problems: both values increase at the same rate.

- There are also **inversely proportional** problems: as one value increases, the other decreases.

- The most common situation is one of a fixed amount work. For instance, the more workers we have, the less time a job will take. In such cases, it is necessary to invert the cross-multiplication.

Inverse proportionality

- Example 1: If 2 painters take 13 hours to paint a house, how many hours will it take 1 painter?

- Since there is 1 fewer painter, which is half the painters that there were, it will take twice as long.

$$13 \times 2 = 26$$

26 hours.

Inverse proportionality

- In general, inversely proportional situations involve a **fixed amount of work**. This is given by the formula:
  - Work = People * Hours
- So in the previous example, the fixed amount of work to be done was:
  - Work = 2 people * 13 hours
  - Work = 26 work hours
- If we then want to know the amount of time 1 person would take to complete the job, we have:
  - 26 = 1 person * **X hours**
  - So **X** = 26 ÷ 1 = 26 hours

Inverse proportionality

□ Example 2: It took 3 workers 30 days to build a house. How many days would it have taken 5 workers?

Step 1: 3 workers $\Rightarrow$ 30 days

5 workers $\Rightarrow$ **X** days

Step 2: $3 \times 30 = 5 \times \mathbf{X}$

Step 3: $\mathbf{X} = \dfrac{3 \times 30}{5} = 18$

It would have taken 18 days.

Exercises

1- My father and I took 45 minutes to wash the dishes. If we had been 3 people washing these dishes, how long would it have taken us (to the minute)?

2- A job can be done in 18 hours by 4 workers. How many workers would be required to do it in 8 hours (to the unit)?

3- After a snowstorm, the City of Montréal hired 1200 employees for 3 days to clear the streets. How many employees would have been necessary to do this job in 2 days (to the unit)?

Exercises

1- My father and I took 45 minutes to wash the dishes. If we had been 3 people washing these dishes, how long would it have taken us (to the minute)?

30 minutes

2- A job can be done in 18 hours by 4 workers. How many workers would be required to do it in 8 hours (to the unit)?

9 workers

3- After a snowstorm, the City of Montréal hired 1200 employees for 3 days to clear the streets. How many employees would have been necessary to do this job in 2 days (to the unit)?

1800 employees

Exercises

Challenge

It takes John 30 minutes to get to work when he is travelling at a speed of 45 km/h. How long will the journey take him if he travels at a speed of 55 km/h?

Percentages

- The percentage of a quantity always corresponds to a fraction whose denominator is 100.

Example: 50% of $20

$$50\% = \frac{50}{100} = \frac{1}{2}$$

$$\text{So, } 50\% \text{ of } 20 = 20 \times \frac{1}{2} = 10$$

50% of $20 is equal to $10

Percentages and fractions

□ So, to turn a percentage into a fraction, you must write the percentage in the form of a fraction, and then reduce it to its lowest terms.

Examples: $75\% = \dfrac{75}{100} = \dfrac{3}{4}$

$$20\% = \dfrac{20}{100} = \dfrac{1}{5}$$

Percentages and decimals

□ In addition to percentages corresponding to fractions, they also correspond to a decimal number

$$\text{Examples: } 75\% = \frac{75}{100} = 0.75$$

$$20\% = \frac{20}{100} = 0.2$$

Calculating percentages

□ To find the value equal to the percentage of a quantity, it is necessary to:

  □ Step 1- Convert the percentage into a fraction or into a decimal number.

  □ Step 2- Multiply this fraction or number by the given quantity.

Calculating percentages

☐ Example:

A pair of running shoes normally sells for $22.50. The vendor is offering a discount of 40%. What is the value of the discount?

$$\text{Step } 1 - 40\% = \frac{40}{100} = \frac{2}{5} \text{ or } 40\% = \frac{40}{100} = 0.4$$

$$\text{Step } 2 - 22.50 \times \frac{2}{5} = 9 \text{ or } 22.50 \times 0.4 = 9$$

The discount is thus worth $9

Calculating percentages

In the case of a discount, to find the final amount we must subtract the calculated percentage amount from the initial amount

Example: $22.50 - 9 = 13.50$

In the previous example, $13.50 is the final amount.

Calculating percentages

- In the case of a tax, add the percentage amount to the initial amount

  Example:

  The price for four tires is $249.55. If a 7% tax is applied to this price, what is the total price of this purchase?

  Tax calculation: $0.07 \times 249.55 = 17.47$

  So, $249.55 + 17.47 = 267.02$

  The total price is $267.02.

Cross-multiplication and percentages

In short, percentage always corresponds to a ratio of a part to a whole (the 100%)

So to calculate the percentage of one quantity with respect to another, we can use cross-multiplication.

Cross-multiplication and percentages

Example:

You received a score of 27 out of 30 on your exam. What is your score expressed as a percentage?

$$30 \Rightarrow 100\%$$

$$27 \Rightarrow \textbf{X}\%$$

$$\textbf{X} = \frac{27}{30} = 0.9 \Rightarrow 90\%$$

You got a score of 90% on your exam.

Exercises

Your neighbour is a real estate broker. When he sells a house, he receives 1.5% of the amount of the sale. How much will he receive for having sold a $152,000 house?

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$2280

Exercises

You ordered a $6.65 meal at a restaurant. You left a tip equal to 15% of this amount. How much did the waiter receive as a tip?

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You ordered a $6.65 meal at a restaurant. You left a tip equal to 15% of this amount. How much did the waiter receive as a tip?

$0.99

Exercises

Challenge

A jacket is on sale for $140 after applying a 28% discount. What was the original price of this jacket?