Let $B = \{x_1, x_2, \dots, x_n\} \subseteq V$, $x_1, x_2, \dots, x_n \in V$.

Assume B is linearly dependent. then,

$\exists\, x_j \in B$; $\quad x_j = \sum_{k=1, k\neq j}^{n} b_k x_k$. where $b_k \in F$, $x_k \in B - \{x_j\}$.

Let P be a proper subset of B, s.t

$$P = B - \{x_j\} = \{x_1, x_2, \dots, x_{j-1}, x_{j+1}, \dots, x_n\}$$

$$\text{Span } P = \left\{ \sum_{k=1, k\neq j}^{n} p_k x_k \;\middle|\; p_k \in F,\ x_k \in P \right\}.$$

Let $p_k = (a_k + a_j b_k) \in F$.

Then,

$$\text{Span } P = \left\{ \sum_{k=1, k\neq j}^{n} (a_k + a_j b_k) x_k \;\middle|\; a_k, b_k \in F,\ x_k \in P \right\}.$$

$$\text{Span } P = \left\{ \sum_{k=1, k\neq j}^{n} a_k x_k + a_j \sum_{k=1, k\neq j}^{n} b_k x_k \;\middle|\; a_k, b_k \in F,\ x_k \in P \right\}.$$

$$\text{span } P = \left\{ \sum_{k=1}^{n} a_k x_k \;\middle|\; a_k \in F,\ x_k \in B \right\}, \text{ because } a_j \sum_{k=1, k\neq j}^{n} b_k x_k = a_j x_j$$

$\text{span } P = \text{span } B$.

$\therefore\ \text{Span } P = V \Rightarrow P$ spans V.

but, since no proper subset of B should not spans V, this is a contradiction. Assumption is incorrect.

B is linearly independent.

Since B also spans V, B is a basis of V //.

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