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2015 MID

① False. $W = \{(a,b,c) \in \mathbb{R}^3, a+b+c=1\}$

$x, y \in W$ $x+y \in W$ closure for vector addition is violated.

eg:- $(1,0,0) \in W$ & $(0,1,0) \in W$.

$(1,0,0)+(0,1,0) = (1,1,0)$.

but $1+1+0 \neq 1$

$\therefore (1,0,0)+(0,1,0) = (1,1,0) \notin W$.

② ~~False~~ True.

Theorem:- let V vector space with n no of basis any subset of V having $m (>n)$ no of elements is linear independent

in this case if given set is a L.D. ~~basis~~ this is True.

if given set is not L.D then its a set having less no of element and being L.I. so T should be L.D.

③ ~~True~~. False

Basis for $P_n = \{1, x^1, x^2, \ldots x^n\}$.

no of elements $(n+1)$ not n

④ False. atleast $\underline{0}$ should be in set to be valid conditions

⑤ True.

⑥ True.

⑦ $B = \{\underline{0}\}$ is L.D ~~basis~~. So False

$a_k \underline{0} = \underline{0}$

a_k can be any value need not to be 0

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⑧. False.

⑨. $U+W=\{u+w \mid u\in U \wedge w\in W\}$

⑩. i) B is L.I. (i.e. $\forall a_k \in F$ $\sum a_i x_i = \underline{0} \Rightarrow a_k = 0$)

ii). Span $B = V$.
Span $B = \{\sum a_k x_k ; x_k \in B, a_k \in F\}$.

⑪. Basis for
$V=\{(x,y,z,t) \mid x+y=z, x-y=t\}$

let
$\therefore v\in V \Rightarrow$
$v=(x,y,x+y,x-y) = x(1,0,1,1)+y(0,1,1,-1)$

$\therefore$ Basis $=\{(1,0,1,1),(0,1,1,-1)\}$ //

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⑫. $V=\{(x,y,z) \mid x-y+2z=0\}$ $W=\{(x,y,z) \mid 3x+2y+z=0\}$

any $v\in V$
$v=(x,y,z)=(x,\cancel{x+}y,\frac{y-x}{2})$
$=x(1,0,-\frac{1}{2})+$
$y(0,1,\frac{1}{2})$

any $w\in W$
$w=(x,y,z)=(x,y,-3x-2y)$
$=x(1,0,-3)+y(0,1,-2)$

$x-y+2z=0$
$3x+2y+z=0$

$2x+4z+3x+z=0$
$x+z=0$
$z=-x$

$x-y-2x=0$
$y=-x$

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$\therefore$ any elemen $u \in V \cap W$

$$u = (x,y,z) = (x,-x,-x) = x\,(1,-1,-1)$$

$\therefore$ Basis for $V \cap W = \{(1,-1,-1)\}$ //

(13). $W = \{(x,y) \in \mathbb{R}^2 \mid xy = 0\}$ is not a subspace.

dot product

$(\mathbb{R},+,\cdot)$ is a vector space over $(\mathbb{R},+,\cdot)$

vector addition is not closure
Countor example for that

$1.0 = 0$ $(1,0) \in \mathbb{R}^2$ $\therefore$ $(1,0) \in W$

$0.1 = 0$ $(0,1) \in \mathbb{R}^2$ $\therefore$ $(0,1) \in W$

$(1,0) + (0,1) = (1,1) \in \mathbb{R}^2$ but $1.1 \neq 0$ $\therefore$ $(1,1) \notin W$.

(2018 MID TEST)

(91). $T^2(v) = T(T(v))$ $v = (v_1, v_2)$

a). $T(\underline{v}) = (\underline{v}) + (1,1)$

For any $v \in V$

$$T^2(v) = T(T(v)) = T(\underline{v} + (1,1)) = T(\underline{v}) + T(1,1)$$
$$= T(\underline{v}) + (1,1) + (1,1)$$
$$= T(\underline{v}) + (2,2)$$
$$= \underline{v} + (2,2)$$

let $v_1, v_2 \in V$

$$T^2(v_1) = T(v_1) + (2,2) = v_1 + (2,2)$$
$$T^2(v_2) = T(v_2) + (2,2) = v_2 + (2,2)$$
$$T^2(v_1 + v_2) = T(v_1 + v_2) + (2,2)$$
$$= (v_1 + v_2) + (3,3).$$
$$\neq T^2(v_1) + T^2(v_2)$$

$\therefore$ not a L.T.

b). $T(v) = -v$

$T(T(v)) = T(-v) = -(-v) = v$

$\therefore$ $T^2(v) = v$ is a Linear Transformation.

~~Proof;~~ any $v_1, v_2 \in V$ and any $a \in F$

$T^2(v_1+v_2) = (v_1+v_2)$

$T^2(v_1) = v_1$ $T^2(v_2) = v_2$

$\forall v_1, v_2 \in V$ $\therefore T^2(v_1+v_2) = T^2(v_1) + T^2(v_2)$ //

$\forall a \in F$ $T^2(av_1) = av = a \cdot T^2(v_1)$ //
$\forall v_1 \in V$

$\therefore$ $T^2(v)$ is a L.T.

c). take any v.

$T^2(v) = T\left(\left(\frac{v_1+v_2}{2}, \frac{v_1+v_2}{2}\right)\right) = \left(\cancel{2} \times \frac{\left(\frac{v_1+v_2}{2}\right)}{\cancel{2}}, \frac{v_1+v_2}{2}\right)$
(v_1, v_2)

L.T. can be proved.

d). $T((v_1, v_2)) = (-v_2, v_1)$

$T(T(v_1, v_2)) = T((-v_2, v_1)) = (-v_1, -v_2)$
$= -(v_1, v_2)$

L.T.

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II).

$$\|(1,2)\|^2 = \langle (1,2),(1,2)\rangle$$

$$= 2\times1\times1 + 1\times2 + 2\times1 + 2\times2\times2$$

$$= 2+2+2+8 = 14$$

$\therefore \|(1,2)\| = \sqrt{14}$

[Ans c]

III).

$$U = \text{span}\left\{\begin{pmatrix}x\\y\\z\end{pmatrix} \mid x-y+2z=0\right\}$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}x\\x+2z\\z\end{pmatrix} = x\begin{pmatrix}1\\1\\0\end{pmatrix} + z\begin{pmatrix}0\\2\\1\end{pmatrix}$$

$\therefore$ Basis for $U = \left\{\begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}0\\2\\1\end{pmatrix}\right\}$. Dim $(U) = 2$

$$W = \text{span}\left\{\begin{pmatrix}x\\y\\z\end{pmatrix} \mid 3x+2y+z=0\right\}$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}x\\y\\-3x-2y\end{pmatrix} = x\begin{pmatrix}1\\0\\-3\end{pmatrix} + y\begin{pmatrix}0\\1\\-2\end{pmatrix}$$

$\therefore$ Basis for $W = \left\{\begin{pmatrix}1\\0\\-3\end{pmatrix}, \begin{pmatrix}0\\1\\-2\end{pmatrix}\right\}$. Dim $(W) = 2$

any $\begin{pmatrix}x\\y\\z\end{pmatrix} = a\begin{pmatrix}1\\1\\0\end{pmatrix} + b\begin{pmatrix}0\\2\\1\end{pmatrix} + c\begin{pmatrix}1\\0\\-3\end{pmatrix}$.

$$\left.\begin{aligned}x &= a+c\\ y &= a+2b\\ z &= b-3c\end{aligned}\right\} \text{ theor are solution}$$

$\therefore W+U = \mathbb{R}^3$.

$\therefore$ Dim $(W+U) = 3$.

2 equation
1 variable
$\therefore$ 1 ~~element~~ element in basis

Dim $(U \cap W) = 1$

[Ans f) & i are correct] //

(Q2) I). True.

II). Finding L.I. element for given set

$(x,y,z)\cdot(0,1,3)=0$

$y+3z=0$ — ①

$2x+4y=0$ — ②

$x+y-3z=0$ — ③

② − ③ × 2 $\quad 2y+6z=0$ — ④

$\therefore$ 3 is L.D To ① & ②

$\therefore \{(0,1,3),(2,4,0)\}$

~~2, 5, 3~~

let perpendicular vect (x,y,z)

Used theorem on orthogonal vectors. Orthogonal ⟺ L.I

$y+3z=0 \Rightarrow y=-3z$

$2x+4y=0 \qquad x=-2y=-2(-3z)=6z$

$$\therefore \begin{pmatrix}6z\\-3z\\z\end{pmatrix}=z\begin{pmatrix}6\\-3\\1\end{pmatrix} \leftarrow \text{perpendicular vector}$$

$\therefore$ equat. $\quad 6x-3y+z=d.$

point $(1,1,-3)$

$6-3+(-3)=0=d$

$\therefore 6x-3y+z=0 \leftarrow$ plane //

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(Q3) :— $T: \mathbb{R}^3 \to \mathbb{R}^2$.

$$T(x_1, x_2, x_3) = (x_1 + x_2, 2x_3 - x_1)$$

I). prove that Linear Transformation.

any (x_1, x_2, x_3), $(y_1, y_2, y_3) \in \mathbb{R}$.

$$T((x_1, x_2, x_3) + (y_1, y_2, y_3)) = T(x_1+y_1, x_2+y_2, x_3+y_3)$$

$$= (x_1+y_1+x_2+y_2, 2(x_3+y_3) - (x_1+y_1))$$

$$= (x_1 + x_2, 2x_3 - x_1) + (y_1 + y_2, 2y_3 - y_1)$$

$$= T(x_1, x_2, x_3) + T(y_1, y_2, y_3)$$

any $a \in \mathbb{R}$ $(x_1, x_2, x_3) \in \mathbb{R}^3$.

$$T(a(x_1, x_2, x_3)) = T(ax_1, ax_2, ax_3)$$

$$= (ax_1 + ax_2, 3ax_3 - ax_1)$$

$$= a(x_1 + x_2, 3x_3 - x_1)$$

$$= aT(x_1, x_2, x_3).$$

$\therefore$ T is a L.T.

II).

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II). $B = \{(1,0,-1), (1,1,1), (1,0,0)\}$ $B' = \{(0,1), (1,0)\}$

$T: \mathbb{R}3 \longrightarrow \mathbb{R}2$

$T_{B',B}$.

$T(1,0,-1) = (1,-3) = (0,1)\times(-3) + (1,0)\times 1$

$T(1,1,1) = (2,1) = (0,1)\times 1 + (1,0)\times 2$

$T(1,0,0) = (1,-1) = (0,1)\times(-1) + (1,0)\times 1$

$$\left(T\begin{pmatrix}1\\0\\-1\end{pmatrix} T\begin{pmatrix}1\\1\\1\end{pmatrix} T\begin{pmatrix}1\\0\\0\end{pmatrix}\right) = \left(\begin{pmatrix}0\\1\end{pmatrix} \begin{pmatrix}1\\0\end{pmatrix}\right)\underbrace{\begin{pmatrix}-3 & 1 & -1\\1 & 2 & 1\end{pmatrix}}_{T_{B'/B}}$$

III done privously AB (F, j)
(Same Same Qustions answered earlier)

III $S_1 = \{(1,-2,3), (0,1,-1)\}$ $S_2 = \{(2,-3,5)\}$.

$a(1,-2,3) + b(0,1,-1) + c(2,-3,5) = (0,0,0)$

$a + 2c = 0$ — ①
$-2a + b - 3c = 0$ — ②
$3a - b + 5c = 0$ — ③
② − ① = ③.

∴ $S_1 \cup S_2$ are have 2 dim basis

But $\mathrm{Dim}(S_1) = 2$ $\mathrm{Dim}[S_2] = 1$

$\mathrm{Dim}(S_1 \cup S_2) = 2$

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(Q2) I).

$\underline{u} = (u_1, u_2, \ldots u_n)$
$\underline{v} = (v_1, v_2, \ldots, v_n)$

a). $\langle \underline{u}, \underline{v} \rangle = \underbrace{2u_1v_1}_{\in \mathbb{R}} + \underbrace{u_2v_2}_{\in \mathbb{R}} \in \mathbb{R} = F$

i). $\underline{W} = (w_1, w_2 \ldots v_n)$
$\forall u, w, v \in V = \mathbb{R}^n.$

$\langle \underline{u}+\underline{w}, \underline{v} \rangle = 2(u_1+w_1)\times v_1 + (u_2+w_2)v_2$
$= (2u_1v_1 + u_2v_2) + 2w_1v_1 + w_2v_2$
$= \langle u, v \rangle + \langle w, v \rangle$

ii) $\forall a \in \mathbb{R}$
$\langle au, \underline{v} \rangle = 2(au_1)v_1 + (au_2)v_2$
$= a[2u_1v_1 + u_2v_2]$
$= a \langle u, v \rangle$ //

iii). $\forall u \in \mathbb{R}^n$
$\langle u, u \rangle = 2u_1 \cdot u_1 + u_2v_2 = 2u_1^2 + u_2^2 \geq 0$

$\langle u, u \rangle = 0 \Leftrightarrow 2u_1^2 + u_2^2 = 0$
$\Rightarrow u_1 = u_2 = 0$
but $\underline{u}$ don't want to be $\underline{0}$ other terms $(u_3, u_4, \ldots u_n$ don't want to be 0

$\therefore$ not a inner product.

b). this inner product fail same as above