

Quiz 07 - 1905304

$$u_i = w_j = (1, x, x^2, x^3, x^4)$$

$$\therefore u_i' = w_j' = \left( \frac{\sqrt{2}}{2}, \frac{\sqrt{6}x}{2}, \frac{\sqrt{10}}{4}(3x^2-1), \frac{\sqrt{14}x}{4}(5x^2-3), \frac{3\sqrt{2}}{16}(35x^4-30x^2+3) \right)$$

$$\text{inner product } \langle f, g \rangle = \int_{-1}^1 f(x)g(x)dx \quad (\text{from quiz 06})$$

$$T(u_i') = (w_j')A'$$

$$\frac{\sqrt{2}}{2} = (1)\left(\frac{\sqrt{2}}{2}\right) + (x)0 + (x^2)0 + (x^3)0 + (x^4)0 = (1, x, x^2, x^3, x^4) \begin{pmatrix} \sqrt{2}/2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\frac{\sqrt{6}x}{2} = (1)0 + (x)\left(\frac{\sqrt{6}}{2}\right) + (x^2)0 + (x^3)0 + (x^4)0 = (1, x, x^2, x^3, x^4) \begin{pmatrix} 0 \\ \sqrt{6}/2 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\frac{\sqrt{10}}{4}(3x^2-1) = (1)\left(\frac{-\sqrt{10}}{4}\right) + (x)0 + (x^2)\left(\frac{3\sqrt{10}}{4}\right) + (x^3)0 + (x^4)0 = (1, x, x^2, x^3, x^4) \begin{pmatrix} -\sqrt{10}/4 \\ 0 \\ 3\sqrt{10}/4 \\ 0 \\ 0 \end{pmatrix}$$

$$\frac{\sqrt{14}x}{4}(5x^2-3) = (1)0 + x\left(\frac{-3\sqrt{14}}{4}\right) + (x^2)0 + (x^3)\left(\frac{5\sqrt{14}}{4}\right) + (x^4)0 = (1, x, x^2, x^3, x^4) \begin{pmatrix} 0 \\ -3\sqrt{14}/4 \\ 0 \\ 5\sqrt{14}/4 \\ 0 \end{pmatrix}$$

$$\frac{3\sqrt{2}}{16}(35x^4-30x^2+3) = (1)\left(\frac{9\sqrt{2}}{16}\right) + (x)0 + (x^2)\left(\frac{-90\sqrt{2}}{16}\right) + (x^3)0 + (x^4)\left(\frac{105\sqrt{2}}{16}\right)$$

$$= (1, x, x^2, x^3, x^4) \begin{pmatrix} 9\sqrt{2}/16 \\ 0 \\ -90\sqrt{2}/16 \\ 0 \\ 105\sqrt{2}/16 \end{pmatrix}$$



$$\left[ \frac{\sqrt{3}}{4}, \frac{\sqrt{6}n}{2}, \frac{\sqrt{10}}{4}(2n^2-1), \frac{\sqrt{14}}{4}n(5n^2-3), \frac{3\sqrt{2}}{16}(25n^4-30n^2+3) \right]$$

$$= (1, n, n^2, n^3, n^4) \begin{bmatrix} \sqrt{3}/2 & 0 & \sqrt{10}/4 & 0 & 9\sqrt{2}/16 \\ 0 & \sqrt{6}/2 & 0 & -3\sqrt{14}/4 & 0 \\ 0 & 0 & 3\sqrt{10}/4 & 0 & -90\sqrt{2}/16 \\ 0 & 0 & 0 & 5\sqrt{14}/4 & 0 \\ 0 & 0 & 0 & 0 & 105\sqrt{2}/16 \end{bmatrix}$$

$P$

$$u_i' = (u_i)P \rightarrow \textcircled{1}$$

$$w_j' = (w_j)P \rightarrow \textcircled{2}$$

$$|P| = \frac{1575\sqrt{910}}{256} \neq 0; \quad P \text{ is invertible.}$$

$$T(u_i') = (w_j')A' \rightarrow \textcircled{3}$$

$$\textcircled{1} \rightarrow T(u_i') = [T(u_i)]P \rightarrow \textcircled{1}'$$

$$\textcircled{1}' \text{ and } \textcircled{3} \rightarrow [T(u_i)]P = (w_j')A'$$

$$\textcircled{2} \rightarrow (w_j)AP = (w_j)PA'$$

$$w_j (AP - PA') = \underline{0}; \quad w_j = \{1, n, n^2, n^3, n^4\} \text{ (basis).}$$

$\therefore w_j$  is linearly independent.

$$AP - PA' = \underline{0}$$

$$\Rightarrow AP = PA'$$

$$\Rightarrow A' = P^{-1}(AP)$$



$$\therefore A^* = \begin{bmatrix} \sqrt{1/2} & 0 & \sqrt{10}/4 & 0 & 9\sqrt{1}/16 \\ 0 & \sqrt{1/2} & 0 & -\frac{3\sqrt{14}}{4} & 0 \\ 0 & 0 & \frac{3\sqrt{10}}{4} & 0 & -\frac{90\sqrt{1}}{16} \\ 0 & 0 & 0 & \frac{5\sqrt{14}}{4} & 0 \\ 0 & 0 & 0 & 0 & \frac{105\sqrt{1}}{16} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} \sqrt{1/2} & 0 & \sqrt{10}/4 & 0 & 9\sqrt{1}/16 \\ 0 & \sqrt{1/2} & 0 & -\frac{3\sqrt{14}}{4} & 0 \\ 0 & 0 & \frac{3\sqrt{10}}{4} & 0 & -\frac{90\sqrt{1}}{16} \\ 0 & 0 & 0 & \frac{5\sqrt{14}}{4} & 0 \\ 0 & 0 & 0 & 0 & \frac{105\sqrt{1}}{16} \end{bmatrix}$$

$$A' = \begin{bmatrix} \sqrt{1/2} & 0 & \sqrt{10}/4 & 0 & 9\sqrt{1}/16 \\ 0 & \sqrt{1/2} & 0 & -\frac{3\sqrt{14}}{4} & 0 \\ 0 & 0 & \frac{3\sqrt{10}}{4} & 0 & -\frac{90\sqrt{1}}{16} \\ 0 & 0 & 0 & \frac{5\sqrt{14}}{4} & 0 \\ 0 & 0 & 0 & 0 & \frac{105\sqrt{1}}{16} \end{bmatrix} \begin{bmatrix} \sqrt{1/2} & 0 & -\sqrt{10}/4 & 0 & 9\sqrt{1}/16 \\ 0 & \sqrt{1} & 0 & -\frac{3\sqrt{14}}{2} & 0 \\ 0 & 0 & \frac{9\sqrt{10}}{4} & 0 & -\frac{270\sqrt{1}}{16} \\ 0 & 0 & 0 & 5\sqrt{14} & 0 \\ 0 & 0 & 0 & 0 & 525\sqrt{1}/16 \end{bmatrix}$$

$$A^* = \begin{bmatrix} \sqrt{1/2} & 0 & \sqrt{10}/4 & 0 & 9\sqrt{1}/16 \\ 0 & \sqrt{1/2} & 0 & -\frac{3\sqrt{14}}{4} & 0 \\ 0 & 0 & \frac{3\sqrt{10}}{4} & 0 & -\frac{90\sqrt{1}}{16} \\ 0 & 0 & 0 & \frac{5\sqrt{14}}{4} & 0 \\ 0 & 0 & 0 & 0 & \frac{105\sqrt{1}}{16} \end{bmatrix} \begin{bmatrix} \sqrt{1/2} & 0 & -\sqrt{10}/4 & 0 & 9\sqrt{1}/16 \\ 0 & \sqrt{1} & 0 & -\frac{3\sqrt{14}}{2} & 0 \\ 0 & 0 & \frac{9\sqrt{10}}{4} & 0 & -\frac{270\sqrt{1}}{16} \\ 0 & 0 & 0 & 5\sqrt{14} & 0 \\ 0 & 0 & 0 & 0 & 525\sqrt{1}/16 \end{bmatrix}$$

$$A' = \begin{bmatrix} 1 & 0 & \sqrt{5} & 0 & 3 \\ 0 & 2 & 0 & \sqrt{1} & 0 \\ 0 & 0 & 3 & 0 & 3\sqrt{5} \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$