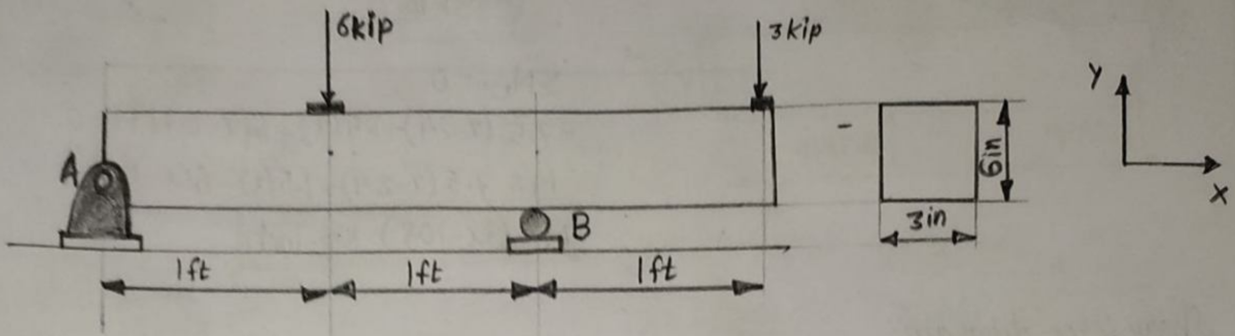
GROUP-22 FEA ANALYSIS ASSIGNMEN-01



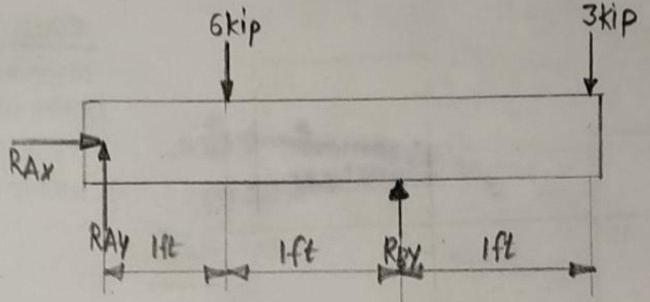
Applying Static conditions:

$\Sigma F_y = 0$

$\therefore R_{AY} + R_{BY} = 9$ kip

$\Sigma F_x = 0$

$\therefore R_{AX} = 0$

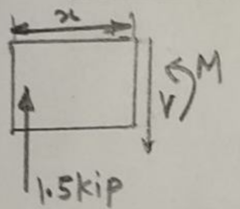


$\Sigma M_{A_z} = 0$

$-6 \times 12 + R_{BY} \times 24 - 3 \times 36 = 0$

$\boxed{R_{BY} = 7.5\text{kip} \quad R_{AY} = 1.5\text{ kip}}$

I]



$\Sigma F_y = 0$

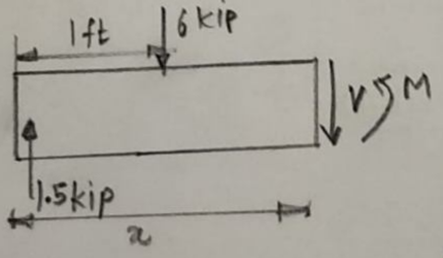
$\boxed{V = 1.5\text{kip}}$

$\Sigma M_{xz} = 0$

$M - 1.5x = 0$

$\boxed{M = (1.5x)\text{ kip-inch}}$

II]



$\Sigma F_y = 0$

$1.5 - 6 - V = 0$

$\boxed{V = -4.5\text{kip}}$

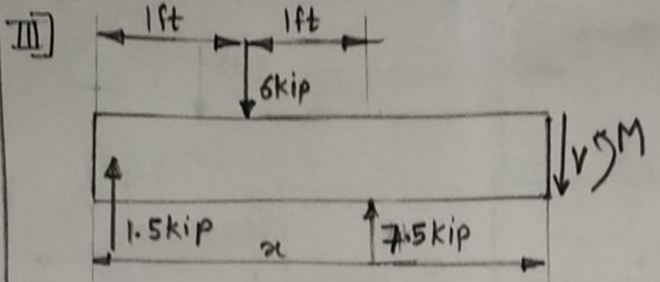
$\Sigma M_{xz} = 0$

$M + 6(x-12) - 1.5(x) = 0$

$M = 6(12-x) + 1.5x$

$\boxed{M = (72 - 4.5x)\text{ kip-inch}}$

III)



$\Sigma F_y = 0$

$\boxed{V = 3\text{ kip}}$

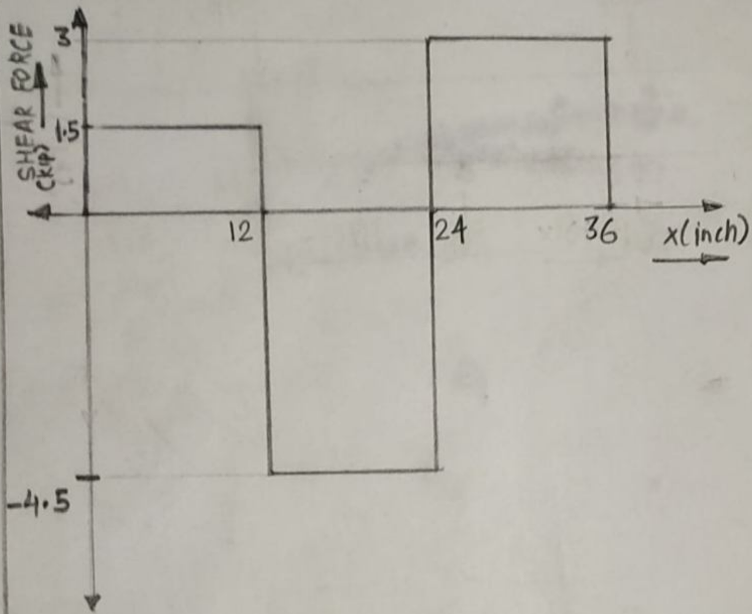
$\Sigma M_{xz} = 0$

$-7.5(x-24) - 1.5(x) + 6(x-12) + M = 0$

$M = 7.5(x-24) + 1.5(x) - 6(x-12)$

$\boxed{M = (3x-108)\text{ kip-inch}}$

Shear force diagram:

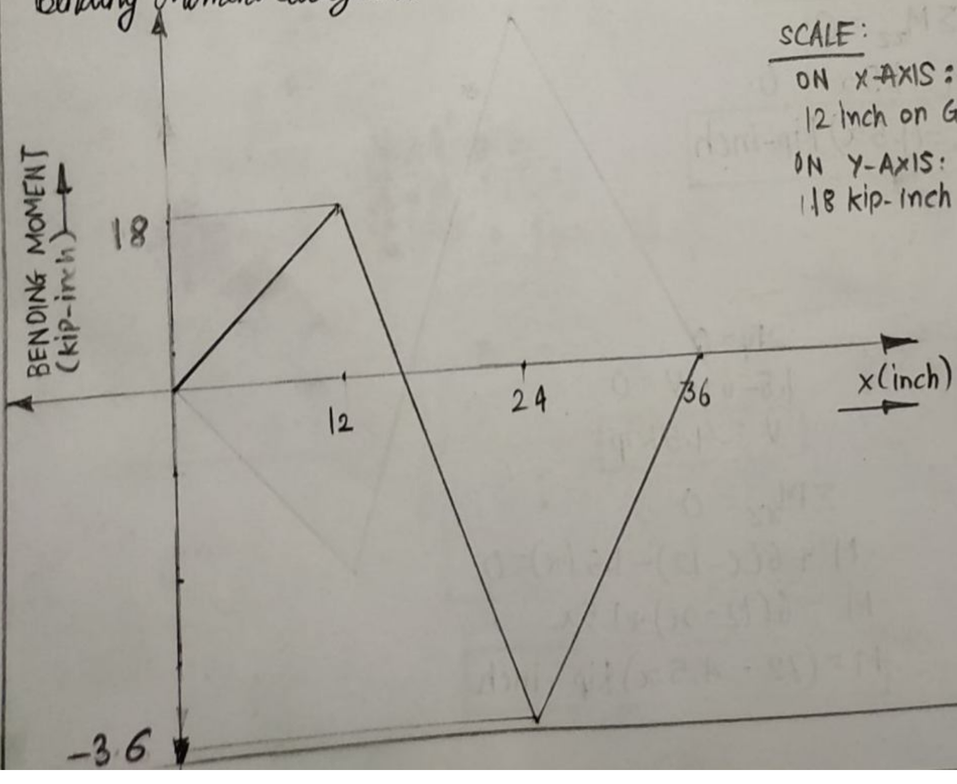


SCALE:

ON X-AXIS;
12 inch on GRAPH = 1 inch

ON Y-AXIS:
1.5 kip ON GRAPH = 0.5 inch

Bending Moment diagram



SCALE:

ON X-AXIS:
12 inch on GRAH = 1 inch

ON Y-AXIS:
18 kip-inch on GRAH = 1 inch

Deflection:

I) $\frac{d^2v_1}{dx^2} = \frac{M}{EI}$ for $0 \leq x \leq 12$ inch

$EI\frac{d^2v_1}{dx^2} = M$

$\therefore EI\frac{d^2v_1}{dx^2} = 1.5x$

$EI\frac{dv_1}{dx} = \frac{1.5x^2}{2} + C_1$

$EI(v_1) = \frac{1.5x^3}{6} + C_1x + C_2$

$$\boxed{v_1 = \frac{1}{EI}\left(\frac{x^3}{4} + C_1x + C_2\right)}$$

II) $\frac{d^2v_2}{dx^2} = \frac{M}{EI}$ for $12 \leq x < 24$ inch

$EI\frac{d^2v_2}{dx^2} = M$

$EI\frac{d^2v_2}{dx^2} = (72 - 4.5x)$

$EI\frac{dv_2}{dx} = 72x - \frac{4.5x^2}{2} + C_3$

$EI\frac{dv_2}{dx} = 72x - \frac{45x^2}{20} + C_3$

$EI\ v_2 = \frac{72x^2}{2} - \frac{4.5x^3}{6} + C_3x + C_4$

$$\boxed{v_2 = \frac{1}{EI}\left(36x^2 - \frac{3}{4}x^3 + C_3x + C_4\right)}$$

III) $\frac{d^2v_3}{dx^2} = \frac{M}{EI}$

$EI\left(\frac{d^2v_3}{dx^2}\right) = M$

$EI\frac{d^2v_3}{dx^2} = 3x - 108$

$EI\frac{dv_3}{dx} = \frac{3x^2}{2} - 108x + C_5$

$EI\ v_3 = \frac{3x^3}{6} - \frac{108x^2}{2} + C_5x + C_6$

$$\boxed{v_3 = \frac{1}{EI}\left(\frac{x^3}{2} - 54x^2 + C_5x + C_6\right)}$$

Solving Equations I, II

Boundary conditions:

At $x = 0$; $v_1 = 0$ — ①

At $x = 24$; $v_2 = 0$ — ②

At $x = 12$; $v_1 = v_2$ — ③

At $x = 12$; $\frac{dv_1}{dx} = \frac{dv_2}{dx}$ — ④

We get;
on applying ①;

$0 = \frac{1}{EI}(0 + 0 + C_2)$

$\boxed{C_2 = 0}$ — (i)

We get;
on applying ②;

$0 = 36(24)^2 - \frac{3}{4}(24)^3 + C_3(24) + C_4$

$\boxed{\therefore 24C_3 + C_4 = 10368}$ — (ii)

Second moment of Inertia:

$$I = \int y^2 dA$$
$$= 2 \times \int_0^3 y^2 dA$$
$$= 2 \times 3 \times \int_0^3 y^2 dy$$
$$= 6 \times \left[\frac{y^3}{3}\right]_0^3$$
$$I = 54 \text{ inch}^4$$

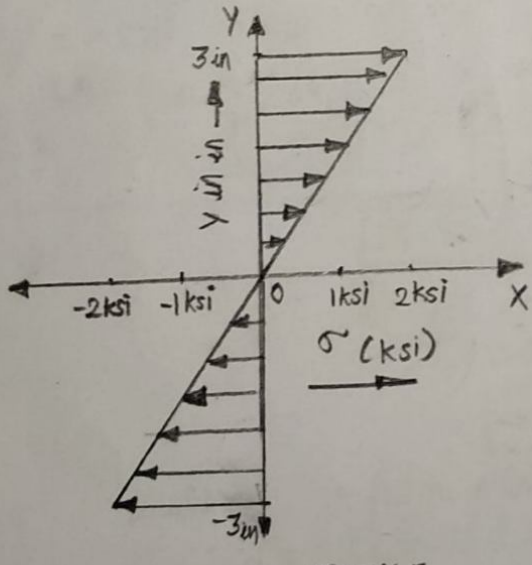
$$E = 200 \text{ GPa}$$
$$= 29007.5 \text{ ksi}$$

$$Q = \int_y^3 y\,dA$$
$$= 3 \times \int_y^3 y\,dy$$
$$= 3 \times \left[\frac{y^2}{2}\right]_y^3$$
$$= \frac{3}{2}\left[9 - y^2\right]$$

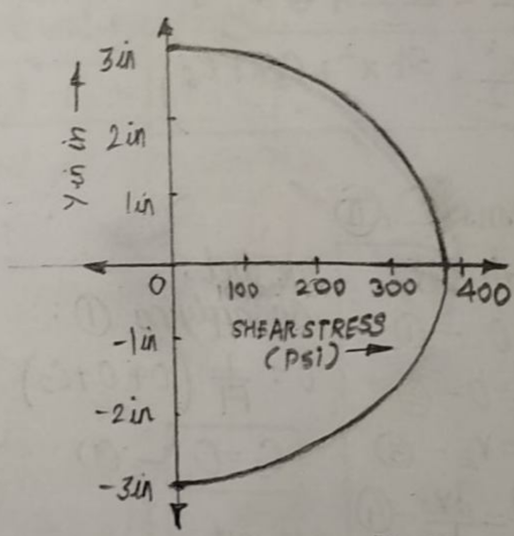
At $y = 0$; $Q = Q_{max} = 13.5 \text{ inch}^3$

$$\sigma = \frac{-My}{I}$$
$$\sigma_{max} = \frac{-M_{max} \times 3}{I}$$
$$= \frac{+36 \times 3}{54}$$
$$\sigma_{max} = 2 \text{ ksi}$$

$$\tau_{max} = \left(\frac{V \times Q}{I \times b}\right)_{max}$$
$$= \frac{V_{max} \times Q}{I \times t}$$
$$= \frac{4.5 \times 13.5}{54 \times 3}$$
$$\tau_{max} = 0.375 \text{ ksi}$$



NORMAL STRESS PROFILE
AT X = 24 inch



SHEAR STRESS PROFILE
AT 12 ≤ x < 24 in

We get on applying ③;

$$\frac{(12)^3}{4} + C_1(12) = 36(12)^2 - \frac{3}{4}(12)^3 + C_3(12) + C_4$$

$$\boxed{12C_1 = 3456 + 12C_3 + C_4} \quad \text{— (iii)}$$

We get on applying ④;

$$\frac{1.5}{2}(12)^2 + C_1 = 72(12) - \frac{4.5}{2}(12)^2 + C_3$$

$$108 + C_1 = C_3 + 540$$

$$\boxed{C_1 = 432 + C_3} \quad \text{— (iv)}$$

Solving (i), (ii), (iii), (iv)

$$\boxed{\begin{aligned} C_1 &= -72 \\ C_2 &= 0 \\ C_3 &= -504 \\ C_4 &= 1728 \end{aligned}}$$

Solving Ⓘ, Ⓘ

Boundary conditions:

At $x = 24$; $v_2 = 0$ — ⑤

At $x = 24$; $v_3 = 0$ — ⑥

At $x = 24$; $v_2 = v_3$ — ⑦

At $x = 24$; $\frac{dv_2}{dx} = \frac{dv_3}{dx}$ — ⑧

We get on applying ⑤;

$$\boxed{24C_3 + C_4 = -10368}$$

We get on applying ⑥;

$$\boxed{24C_5 + C_6 = 24192}$$

We get on applying ⑦;

$$\boxed{24C_5 + C_6 = 24C_3 + C_4 + 34560}$$

We get on applying ⑧;

$$432 + C_3 = -1728 + C_5$$

$$\boxed{C_3 = C_5 - 2160}$$

Using C_3, C_4 from previous results;

$$C_5 = C_3 + 2160 = -504 + 2160 = 1656$$

$$C_6 = 24192 - 24 \times C_5 = -15552$$

$$\boxed{\begin{aligned} C_5 &= 1656 \\ C_6 &= -15552 \end{aligned}}$$

We get;

$$v_1 = \frac{1}{EI}\left(\frac{x^3}{4} - 72x\right) \quad 0 \leqslant x < 12 \text{ in}$$

$$v_2 = \frac{1}{EI}\left(36x^2 - \frac{3}{4}x^3 - 504x + 1728\right) \quad 12 \leqslant x < 24 \text{ in}$$

$$v_3 = \frac{1}{EI}\left(\frac{x^3}{2} - 54x^2 + 1656x - 15552\right) \quad 24 \leqslant x < 36 \text{ in}$$

Deflection at $x = 12$ in;

$$v_1 = \frac{+10^3}{EI}(432 - 864) \text{ in}$$

$$= -2.757 \times 10^{-4} \text{ in}$$

Deflection at $x = 24$ in

$$v_2 = 0 \text{ in}$$

Deflection at $x = 36$ in

$$v_3 = -1.655 \times 10^{-3} \text{ in}$$