Model Predictive Control for Urban Traffic Flows

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Abstract—**A problem of optimal urban traffic flows control is considered. A mathematical model of control by the traffic lights at intersections using the controlled networks theory is given. It is a system of nonlinear finite-differential equations. To present a large scale road networks the model contains the connection matrices that describe interactions between input and output roads in subnetworks. The traffic flow control is performed by the coordination of active phases of traffic lights. A control goal is to minimize the difference between the total input flow and total output flow for all subnetworks. In this paper, a neural network approach for traffic road network parameters adjustment is presented. A simulation is conducted under a microscopic traffic simulation software CTraf. Results demonstrate that neural network reinforcement training obtained good parameters of the network model.**

I. INTRODUCTION

Despite multiple efforts the scientists still have not obtained a full adequate mathematical models of urban traffic flow control systems. The model should determine quantitative parameters of traffic flows depending on the control values. Nowadays traffic flows are widely controlled by traffic lights. If we have an accurate mathematical model of traffic flows control then we can find optimal durations of traffic lights signals to guaranty maximum throughput.

A new trend to replace traffic lights control by velocity control of individual vehicle still requires accurate mathematical models that are supposed to be even more complex, because of larger number of control modes and richer information from multiple sensors and sensor technologies.

Accurate mathematical model of traffic flow control in prediction mode will provide estimation of optimal control values and right decision making in emergency situations.

Known mathematical models of traffic flow control [1]-[8], [13] estimate only mean traffic flow parameters, but not the number of vehicles on the road at given moment.

In this paper we try to estimate the exact number of cars at every moment on any road of the road network. We consider traffic on the roads at different moments in given time interval. A number of vehicles on all roads at some moment we determine as a current traffic flow state. It is evident, that the current number of vehicles on the road at some moment depends on the number of traffic on this road at the previous moment, as well as the number of traffic that left this road, and the number of traffic that come on this road during the last time interval. The traffic flow state changes depending on the traffic that left one roads and came on the other.

To derive a mathematical model that presents the traffic flow state, it is necessary to know how the roads are connected, maneuver throughputs between connected roads and the routes of vehicles that perform maneuvers. We also need to know which maneuvers are allowed and which are prohibited by certain traffic lights phases. It is supposed that if we know all this information we can obtain mathematical model of traffic flow at given road network.

We view this traffic flow control as the control system, in which we have the plant and therefore the control theory methods can be applied to it. Benefits of addressing traffic issue as the controlled plant are discussed in [14].

Here to derive a mathematical model of traffic flow we use a model, based on the controlled network theory [9-12]. The model describes a road network, its changes, depending on the traffic lights signals, and it allows estimating traffic flow state at each moment. In works [11], [12] we have further developed the model by introducing subnetworks that can be assembled in the whole network. The main challenge that we have faced at that point was that application of this model needed accurate values of maneuvers throughputs and the routes of groups of vehicles. To determine these parameters it is proposed to use artificial neural networks (ANNs) with supervised learning [7, 8]. ANNs can refine necessary parameters by accurate values of traffic flow state.

In this paper an example of traffic control for complex intersection in Moscow, Russia is presented.

II. MATHEMATICAL MODEL OF TRAFFIC FLOWS BASED ON THE CONTROLLED NETWORKS APPROACH

A. Model for one network

To construct a mathematical model of traffic flow control we use a controlled networks approach [9-12].

Let the network consists of M intersections and L parts of the road. Maneuvers are performed at intersections. The traffic lights phases at intersections prohibit certain maneuvers. Nodes of the graph are the roads between intersections. The edges of the graph are maneuvers

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between the roads. As a result we obtain a directed graph of flexible configuration. To present the graph we use the following matrices:

- an adjacency matrix of basis network graph

$$\mathbf{A}=[a_{i,j}],\ a_{i,j}\in\{0,1\},\ i,j=1,\dots,L; \quad (1)$$

- a capacity matrix

$$\mathbf{B}=[b_{i,j}],\ b_{i,j}\in\mathbf{R}_{+}^{1}\cup\{0\}, \quad (2)$$

where $b_{i,j}$ estimates the flow from road i to road j for some time interval;

- a control matrix

$$\mathbf{C}=[c_{i,j}],\ c_{i,j}\in\{1,\dots,M\}, \quad (3)$$

where $c_{i,j}$ is a number of the intersection at which maneuver from road i to road j is performed;

- a distribution matrix

$$\mathbf{D}=[d_{i,j}],\ d_{i,j}\in[0;1], \quad (4)$$

where $d_{i,j}$ indicates the part of the traffic flow on road i that performs maneuver to road j. For all parts of the road (5) should be satisfied

$$\sum_{j=1}^{L} d_{i,j}=1,\ i=1,\dots,L, \quad (5)$$

- a traffic light phases matrix

$$\mathbf{F}=[\mathrm{F}_{i,j}],\ i,j=1,\dots,L, \quad (6)$$

where $\mathrm{F}_{i,j}=\{f_{i,j,1},\dots,f_{i,j,k(c_{i,j})}\}$, $f_{i,j,r}\in\{0,u_{c_{i,j}}^{+}\}$, $1\le r\le k(c_{i,j})$, $u_{c_{i,j}}^{+}$ is a maximal number of active phases at intersection $c_{i,j}$, $k(c_{i,j})$ is a maximal quantity of traffic light phases that permits maneuver from road i to road j at intersection $c_{i,j}$, $\mathrm{F}_{i,j}$ is a set of indices of the phases that permit maneuver from road i to road j. All matrices have identical structure: $b_{i,j}>0$, $c_{i,j}>0$, $d_{i,j}>0$, $\mathrm{F}_{i,j}\ne\varnothing$, if $a_{i,j}=1$, otherwise $b_{i,j}=0$, $c_{i,j}=0$, $d_{i,j}=0$, $\mathrm{F}_{i,j}=0$.

To describe the flexibility of the network configuration we introduce the control vector

$$\mathbf{u}=[u_1\dots u_M]^T,\ u_i\in\{0,\dots,u_i^{+}\}, \quad (7)$$

where u_i is a number of the phase of traffic light at intersection i, u_i^{+} is a maximal number of active phases at intersection i, $i=1,\dots,M$.

A network configuration change is described by a configuration matrix that is also an adjacency matrix of a partial subgraph

$$\mathbf{A}(\mathbf{u})=[a_{i,j}(\mathbf{u})],$$

$$a_{i,j}(\mathbf{u})=\begin{cases}1, & \text{if } a_{i,j}=1,\ u_{c_{i,j}}\in\{\mathrm{F}_{i,j}\}\\ 0, & \text{otherwise}\end{cases}. \quad (8)$$

A configuration matrix influences the structures of all other matrices.

To describe the parameters of the traffic flow let us introduce a time interval, Δt. We assume that the durations of all phases are estimated in integer value, Δt. We also assume that all traffic lights are synchronized so that the count of integer values of time intervals for all traffic lights in the network is done simultaneously. To obtain the quantitative characteristics of the traffic flow for each part of the road we use a flow state vector

$$\mathbf{x}(t_k)=[x_1(t_k)\ \dots\ x_L(t_k)]^T, \quad (9)$$

where $x_j(t_k)$ is a number of cars on the road j at time t_k, $x_j(t_k)\in\mathbf{R}^1$, $j=1,\dots,L$, $k=0,\dots,N$, N is a number of control timing periods.

Further on rewrite

$$x_j(t_k)=x_j(k). \quad (10)$$

Flow state $\mathbf{x}(k)$ depends on the road network configuration and the values of flow state vector at the previous moment of time, $\mathbf{x}(k-1)$.

Suppose that all cars perform maneuvers at one timing period simultaneously. Maneuvers are performed in two steps. At the first step the cars leave the road where they have been to perform a maneuver. At the second step they finish the maneuver and go to other roads.

At the first step the number of cars is decreased by the number of cars that performed the maneuver as

$$\mathbf{x}(k-1/2)=\mathbf{x}(k-1)-\\ -\min\{\Delta\mathbf{x}'(k-1/2),\Delta\mathbf{x}''(k-1/2)\}, \quad (11)$$

where $\Delta\mathbf{x}'(k-1/2)$ is a number of cars that needs to perform a maneuver, $\Delta\mathbf{x}''(k-1/2)$ is a number of cars that can perform the maneuvers for one timing period according to the throuthputof the road,

$$\Delta\mathbf{x}'(k-1/2)=((\mathbf{x}(k-1)\mathbf{1}_L^T)\odot\mathbf{D}\odot\mathbf{A}(\mathbf{u}(k))\mathbf{1}_L, \quad (12)$$

$$\Delta\mathbf{x}''(k-1/2)=(\mathbf{A}(\mathbf{u}(k))\odot\mathbf{B})\mathbf{1}_L, \quad (13)$$

$\mathbf{1}_L^T=\underbrace{[1\dots1]}_{L}$, $\odot$ is an Hadamard product of matrices,

$\mathbf{x}(k-1/2)=[x_1(k-1/2)\dots x_L(k-1/2)]^T$,

$\Delta\mathbf{x}'(k)=[\Delta x_1'\dots\Delta x_L']^T$, $\Delta\mathbf{x}''(k)=[\Delta x_1''\dots\Delta x_L'']^T$.

Present (11) as

$$\mathbf{x}(k-1/2)=\mathbf{x}(k-1)-\\ -(\Delta\mathbf{x}'(k-1/2)-(\Delta\mathbf{x}'(k-1/2)\dot{-}\Delta\mathbf{x}''(k-1/2))),$$

where

$$a\dot{-}b=\begin{cases}a-b, & \text{if } a>b,\\ 0, & \text{othewise.}\end{cases}$$

At the second step the change of the traffic flow state is described as

$$\mathbf{x}(k)=\mathbf{x}(k-1/2)+\min\{\Delta\mathbf{x}'(k),\Delta\mathbf{x}''(k)\}, \quad (14)$$

or

$$\mathbf{x}(k)=\mathbf{x}(k-1/2)+\Delta\mathbf{x}'(k)-(\Delta\mathbf{x}'(k)\dot{-}\Delta\mathbf{x}''(k)), \quad (15)$$

where

$$\Delta\mathbf{x}'(k)=((\mathbf{x}(k-1)\mathbf{1}_L^T)\odot\mathbf{D}\odot\mathbf{A}(\mathbf{u}(k))^T\mathbf{1}_L, \quad (16)$$

$$\Delta \mathbf{x}''(k) = (\mathbf{A}(\mathbf{u}(k)) \odot \mathbf{B})^T \mathbf{1}_L . \qquad (17)$$

As a result, we obtain the following model of traffic flow control

$$\begin{aligned}
\mathbf{x}(k) = \mathbf{x}(k-1) - ((\mathbf{x}(k-1)\mathbf{1}_L^T) \odot \mathbf{A}(\mathbf{u}(k)) \odot \mathbf{D} - \\
-((\mathbf{x}(k-1)\mathbf{1}_L^T) \odot \mathbf{A}(\mathbf{u}(k)) \odot \mathbf{D} \dot{-} \mathbf{A}(\mathbf{u}(k)) \odot \mathbf{B}))\mathbf{1}_L + \\
+((\mathbf{x}(k-1)\mathbf{1}_L^T) \odot \mathbf{A}(\mathbf{u}(k)) \odot \mathbf{D} - \\
-((\mathbf{x}(k-1)\mathbf{1}_L^T) \odot \mathbf{A}(\mathbf{u}(k)) \odot \mathbf{D} \dot{-} \\
\dot{-} \mathbf{A}(\mathbf{u}(k)) \odot \mathbf{B}))^T \mathbf{1}_L + \boldsymbol{\delta}(k) , \qquad (18)
\end{aligned}$$

where $\boldsymbol{\delta}(k) = [\delta_1(k) \ \ldots \ \delta_L(k)]^T$, $\delta_i(k)$ is the value of input flow on road i, $i = 1, \ldots, L$, depending from some random factor.

B. Subnetworks

Consider a large road network of K subnetworks. The models of subnetworks are presented as: $(\mathbf{A}^l(\mathbf{u}(k)), \mathbf{B}^l, \mathbf{C}^l, \mathbf{D}^l, \mathbf{F}^l : l = 1, \ldots, K)$.

To connect the models of all subnetworks let us introduce a connection matrix for each subnetwork

$$\mathbf{R}^l = [r_{i,j}^l] ,\ i = 1, \ldots, n_1^l ,\ j, l = 1, K , \qquad (19)$$

where $r_{i,j}^l$ is an index of element in the input roads vector for the road j, n_1^l is a number of output roads in the subnetwork l.

For each part of the road the model should include vectors of input and output roads

$$\mathbf{v}^l = [v_1^l \ \ldots \ v_{n_0^l}^l]^T , \qquad (20)$$

$$\mathbf{w}^l = [w_1^l \ \ldots \ w_{n_1^l}^l]^T , \qquad (21)$$

where v_i^l is an index of an input road in subnetwork l, w_i^l is an index of an output road in subnetwork l, n_0^l is a number of input roads in subnetwork l.

Using connection matrices we can simulate the flow dynamics in all subnetworks simultaneously. At each time interval Δt we recalculate the flow vector in accordance with connection matrix

$$\forall r_{i,j}^l = \gamma \neq 0 ,\ x_\alpha^j(k) = x_\beta^l(k) , \qquad (22)$$

where $i = 1, \ldots, n_1^l$, $j, l = 1, \ldots, K$, $\alpha = v_\gamma^j$, $\beta = w_i^l$.

III. Artificial Neural Networks for Model Parameters Definition

Mathematical model of traffic flow control system (18) includes parameters that have to be obtained experimentally for each intersection. These parameters are elements of capacity matrix $\mathbf{B}$ and distribution matrix $\mathbf{D}$. Maneuver throughput $b_{i,j}$ from road i to road j shows the number of vehicles that perform this maneuver for one time interval. Maneuver throughput $b_{i,j}$ depends on geometrical configuration of intersection, velocity of vehicle and number of vehicles on roads i and j.

Elements $d_{i,j}$ of distribution matrix $\mathbf{D}$ contain the fractions of vehicles that perform maneuver from road i to road j. The sum of these fractions equals the total flow on road i. The values of elements $d_{i,j}$ depend on the routes of groups of vehicles and can vary from day time, days of the week and season.

To determine the parameters of model (18) we use ANN. The learning scheme to determine the parameters of model is presented on Fig.1.

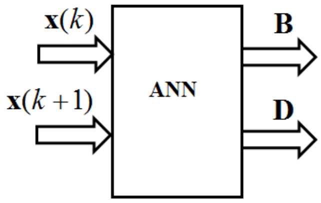

Figure 1. Model parameters definition by ANNs

Model (18) can cope with any parts of the road and can describe traffic flow dynamics on intersection. Vectors $\mathbf{x}(k)$ and $\mathbf{x}(k+1)$ show traffic flow on all parts of the road at intersection in times k and $k+1$.

Let for given intersection the traffic lights control law $\mathbf{u}^*(\cdot)$, input traffic flows $\boldsymbol{\delta}^*(\cdot)$ and current traffic flow $\mathbf{x}^*(k)$ are known. Then we can calculate the traffic flow on the next time step by using (18)

$$\mathbf{x}(k+1) = f(\mathbf{x}^*(k), \mathbf{u}^*(k), \boldsymbol{\delta}^*(k+1), \mathbf{B}, \mathbf{D}) . \qquad (23)$$

Let $\mathbf{x}^*(k+1)$ is a real traffic flow state at moment $k+1$, measured by sensors for example. Then to define model parameters we use norm between real traffic flow state and the one, calculated by (18)

$$\| \mathbf{x}^*(k+1) - \mathbf{x}(k+1) \| > \varepsilon , \qquad (24)$$

where ε is a given small positive value.

Equation (24) is a criterion for defining parameters of (18).

IV. Optimal Control of Traffic Flows

The traffic flow control is performed by the change of active phase durations at controlled intersections in the given network. It should be taken into consideration that the phases are switched in the certain order, and the duration of each phase is limited. A phase duration can be estimated in the number of time intervals Δt. Let us introduce a vector of sets of minimal phase durations

$$\mathrm{Q} = \begin{bmatrix} \left(q_{1,1}, \ldots, q_{1,u_1^+} \right) \\ \ldots \\ \left(q_{M,1}, \ldots, q_{M,u_M^+} \right) \end{bmatrix} , \qquad (23)$$

where $q_{j,i}$ is a duration of active phase i at intersection j, $i = \overline{1, u_i^+}$, $j = \overline{1, M}$.

Suppose that at the moment k the phase i is active at intersection j, then

$$u_j(k) = i. \tag{24}$$

A phase switch condition is formulated as

$$s_j(k) = \begin{cases} s_j(k-1)+1, \text{ if } s_j(k-1) < q_{j,i} \\ 0, \text{ if } i = (i-1) \bmod u_j^{+} + 1 \end{cases}, . \tag{25}$$

where $s_j(k)$ is a duration of active phase $i = u_j(k)$ at the moment k at intersection j.

To solve the optimal control problem for the traffic flows it is necessary to calculate the active phases durations of the traffic lights at intersections

$$q_{j,i}^{-} \le q_{j,i} \le q_{j,i}^{+},\ i = \overline{1, u_j^{+}},\ j = \overline{1, M}, \tag{26}$$

where $q_{j,i}^{-}$ and $q_{j,i}^{+}$ are given.

The control should minimize the functional

$$J_1 = \sum_{i \in I_1} x_i(N) - \sum_{j \in I_2} x_j(N) \to \min, \tag{27}$$

where I_1 is a set of input roads indices, I_2 is a set of output roads indices.

All values of flows should be constrained by

$$x_i(k) \le x_i^{+},\ i = \overline{1, L},\ k = \overline{1, N}. \tag{28}$$

Let us add the constraints to the functional

$$J_1 = \sum_{i \in I_1} x_i(N) - \sum_{j \in I_2} x_j(N) + \\ + p \sum_{k=1}^{N} \sum_{i=1}^{L} \left(\left(\frac{x_i(k)}{x_i^{+}} - 1 \right) + \left| \frac{x_i(k)}{x_i^{+}} - 1 \right| \right) x_i^{+} \to \min, \tag{29}$$

where p is a penalty coefficient.

In our case (29) allows coping with the input and output roads where the number of cars is not limited. If the number of cars on the road is limited then $x_i^{+} \ge 0$. If the number of cars does not exceed limitation $x_i(k) \le x_i^{+}$, then

$$\left(\frac{x_i(k)}{x_i^{+}} - 1 \right) + \left| \frac{x_i(k)}{x_i^{+}} - 1 \right| = 0. \tag{30}$$

If the limitation is exceeded then

$$\left(\frac{x_i(k)}{x_i^{+}} - 1 \right) + \left| \frac{x_i(k)}{x_i^{+}} - 1 \right| > 0. \tag{31}$$

For the roads without limitations we have $x_i^{+} = -1$. In this case for any $x_i(k)$ we get (30).

Usually input and output roads have no limitations. According to (29) we assume

$$x_i^{+} = -1, \text{ if } i \in I_1 \cup I_2 \text{ or } x_i^{+} = \infty. \tag{32}$$

V. The Synthesis of Intelligent control

To solve the problem of control synthesis it is necessary to find the dependence between the active phase durations and the number of cars on the roads

$$q_{i,j} = g_{i,j}(\mathbf{x}),\ i = \overline{1, u_i^{+}},\ j = \overline{1, M}, \tag{33}$$

where $g_{i,j}(\mathbf{x})$ is a multidimensional function that describes dependence between the flow state on all roads and duration of phase i at intersection j.

In our case we are more interested in the value that shows how occupied the road is rather than in the absolute value of flow $\mathbf{x}$ in it. We can estimate the occupancy of the road by discrete values from 0 to z^{+}. To estimate the traffic flows let us introduce an integer grid

$$\mathrm{Z} = \{0, 1, \ldots, z^{+}\}. \tag{34}$$

To estimate the flows on internal roads we take into consideration the limitations on these roads. The more the value $\frac{x_i}{x_i^{+}}$, the more is its estimation by the grid (34).

To avoid having too long phase durations let us introduce an integer grid for phases durations from 0 to y^{+}

$$\mathrm{Y} = \{0, 1, \ldots, y^{+}\}. \tag{35}$$

To obtain a phase duration we use a given increment $\Delta q_{i,j}$.

The phase duration is found from

$$q_{i,j} = q_{i,j}^{-} + y_{i,j} \Delta q_{i,j}, \tag{36}$$

where $y_{i,j} \in \mathrm{Y}$.

Thus to solve the control synthesis problem it is necessary to determine the discrete value of phase duration by the discrete value of flow. As a result, we obtain a multidimensional function

$$\mathbf{Y} = G(\mathbf{z}), \tag{37}$$

where $\mathbf{Y} = [y_{i,j}]$, $y_{i,j} \in \mathrm{Y}$, $i = \overline{1, u_i^{+}}$, $j = \overline{1, M}$, $z = [z_1 \ \ldots \ z_L]^T$, $z_k \in \mathrm{Z}$, $k = \overline{1, L}$.

To solve the problem of control system synthesis namely finding $G(\mathbf{z})$ we use the logical network operator method.

VI. An Example

Consider a road network that consists of 6 intersections. Graphs of intersections are shown on Fig. 2 – 4.

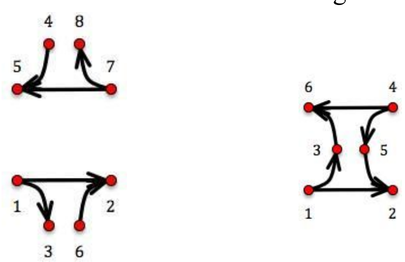

Figure 2. Intersections 1 (left) and 2 (right).

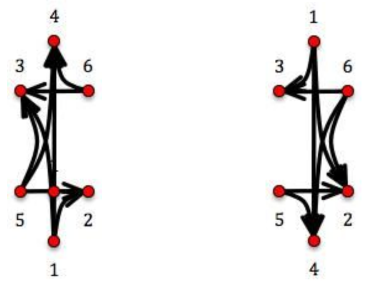

Figure 3. Intersections 3 (left) and 4 (right).

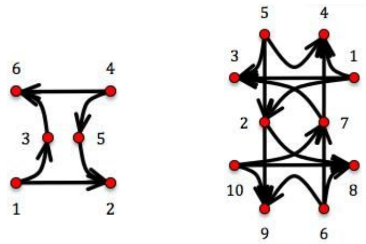

Figure 4. Intersections 3 (left) and 4 (right).

Adjacency matrices of graphs are the following:

$$\mathbf{A}^1 = \begin{bmatrix} 0&1&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&1 \\ 0&0&0&0&0&0&0&0 \end{bmatrix}, \ \mathbf{A}^2 = \begin{bmatrix} 0&1&1&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&1 \\ 0&0&0&0&1&1 \\ 0&1&0&0&0&0 \\ 0&0&0&0&0&0 \end{bmatrix},$$

$$\mathbf{A}^3 = \begin{bmatrix} 0&1&1&1&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&1&0&1&0&0 \\ 0&1&1&1&0&0 \end{bmatrix}, \ \mathbf{A}^4 = \begin{bmatrix} 0&1&1&1&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&1&1&1&0&0 \\ 0&0&1&1&0&0 \end{bmatrix},$$

$$\mathbf{A}^5 = \begin{bmatrix} 0&1&1&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&1 \\ 0&0&0&0&1&1 \\ 0&1&0&0&0&0 \\ 0&0&0&0&0&0 \end{bmatrix}, \ \mathbf{A}^6 = \begin{bmatrix} 0&1&1&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&1&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&1&1&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&1&1&0 \\ 0&0&1&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&1&1&0 \end{bmatrix}.$$

The intersections are connected through vectors of input roads and connection matrices [12]. Vectors of output roads are:

$\mathbf{w}^1 = [2 \ 3 \ 5 \ 8]^T$, $\mathbf{w}^2 = [2 \ 6]^T$, $\mathbf{w}^3 = [2 \ 3 \ 4]^T$, $\mathbf{w}^4 = [2 \ 3 \ 4]^T$, $\mathbf{w}^5 = [2 \ 6]^T$, $\mathbf{w}^6 = [3 \ 4 \ 8 \ 9]^T$.

Connection matrices are:

$$\mathbf{R}^1 = \begin{bmatrix} 0&1&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \end{bmatrix}, \ \mathbf{R}^2 = \begin{bmatrix} 7&0&0&0&0&0 \\ 0&0&5&0&0&0 \end{bmatrix},$$

$$\mathbf{R}^3 = \begin{bmatrix} 0&0&0&5&0&0 \\ 0&4&0&0&0&0 \\ 0&0&0&0&0&0 \end{bmatrix}, \ \mathbf{R}^4 = \begin{bmatrix} 0&0&0&0&1&0 \\ 0&0&6&0&0&0 \\ 0&0&0&0&0&0 \end{bmatrix},$$

$$\mathbf{R}^5 = \begin{bmatrix} 0&0&0&0&0&10 \\ 0&0&0&6&0&0 \end{bmatrix}, \ \mathbf{R}^6 = \begin{bmatrix} 0&0&0&0&4&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \end{bmatrix}.F$$

Inequality in connection matrix $r_{i,j}^l \neq *$ means that output w_i^l is connected to road $r_{i,j}^l$ at intersection j. The optimal control problem with criteria (29) was solved. The results for intersection 6 are given on Fig. 5 – 12.

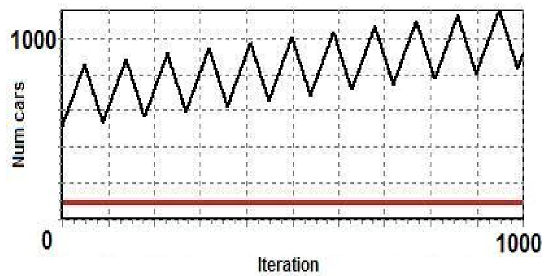

Figure 5. Intersection 6, node 0, initial control

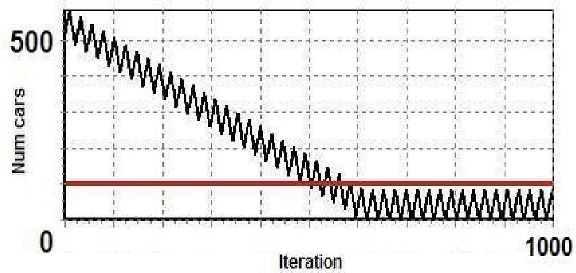

Figure 6. Intersection 6, node 0, optimal control

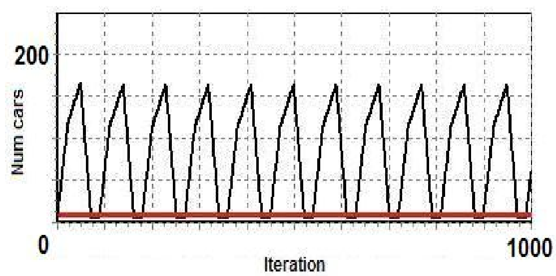

Figure 7. Intersection 6, node 1, initial control

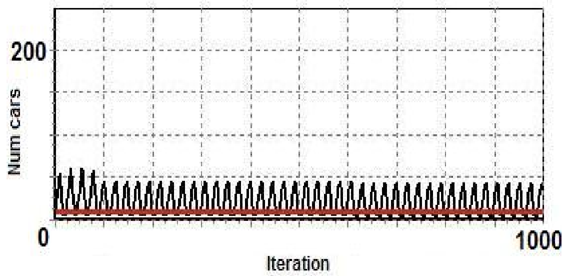

Figure 8. Intersection 6, node 1, optimal control

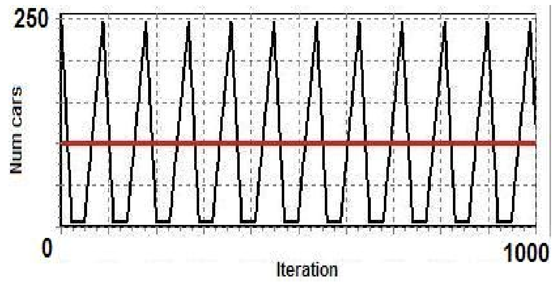

Figure 9. Intersection 6, node 2, initial control

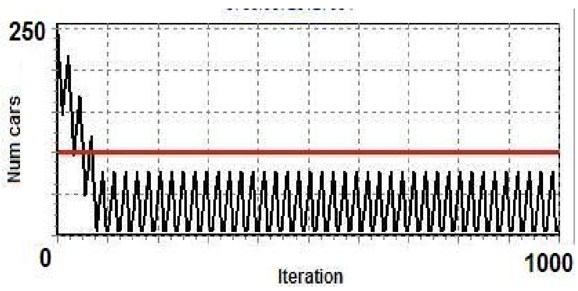

Figure 10. Intersection 6, node 2, optimal control

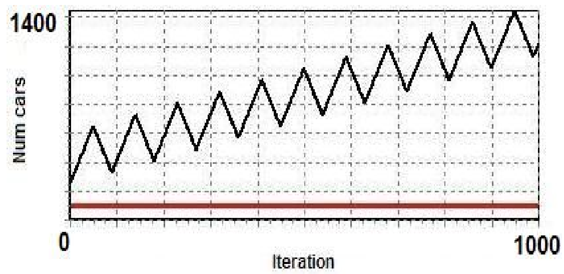

Figure 11. Intersection 6, node 3, initial control

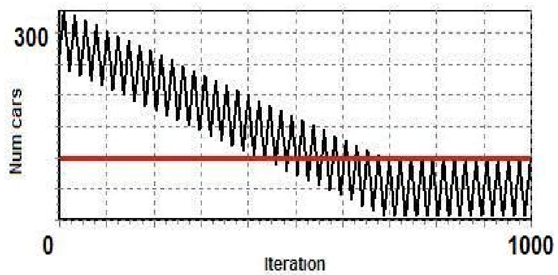

Figure 12. Intersection 6, node 3, optimal control

Simulation showed that the obtained optimal control lead to significant decrease of congestions. The criteria (29) changing through algorithm's iterations is on Fig. 13.

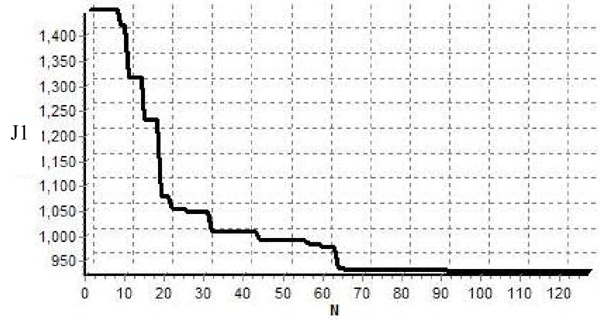

Figure 13. Criteria per generations

For intersection 6 the parameters of capacity matrix have been adjusted by double-layer ANN. ANN had 10 inputs and 15 outputs for each nonzero element in $\mathbf{B}$. Three values of each element with step 1/3 from nominal value were considered. Learning time was 1000 iterations.

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