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Evaluation of the General Motors based car-following models and a proposed fuzzy inference model

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Abstract

This paper evaluates the properties of the General Motors (GM) based car-following models, identifies their characteristics, and proposes a fuzzy inference logic based model that can overcome some of the shortcomings of the GM based models. This process involves developing a framework for evaluating a car-following model and comparing the behavior predicted by the GM models with the behavior observed under the real world situation. For this purpose, an instrumented vehicle was used to collect data on the headway and speeds of two consecutive vehicles under actual traffic conditions. Shortcomings of the existing GM based models are identified, in particular, the stability conditions were analyzed in detail. A fuzzy-inference based model of car-following is developed to represent the approximate nature of stimulus–response process during driving. This model is evaluated using the same evaluation framework used for the GM models and the data obtained by the instrumented test vehicle. Comparison between the performance of the two models show that the proposed fuzzy inference model can overcome many shortcomings of the GM based car-following models, and can be useful for developing the algorithm for the adaptive cruise control for automated highway system (AHS). © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Car-following; Fuzzy inference system; Simulation; Stability analysis

1. Introduction

Models that represent the interaction between two consecutive vehicles in a reasonably congested traffic stream are generally referred to as the car-following model. The driver behavior under car-following is an important topic in the study of vehicular traffic flow. In particular, this

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subject has become essential in developing the adaptive cruise control strategies for automated highway system (AHS) in recent years. Edie (1974) describes the study of car-following behavior as "fundamental to the theory of traffic flow" and Herman (1992) describes them as one of the foundations of the "science of vehicular traffic". Over the years various models of car-following behavior have been developed (Chandler et al., 1958; Forbes et al., 1958; Forbes, 1963; Gazis et al., 1959, 1961; Herman et al., 1959; Herman and Potts, 1959; Pipes, 1953, 1967; Rockwell et al., 1968).

This paper evaluates the existing General Motors (GM) based models both from the theoretical and also from the practical standpoints. A vehicle equipped with sophisticated electronic instruments collected data on driver behavior on regular roadways (as opposed to the test tracks). The paper then proposes a new model that uses the fuzzy inference logic and presents how this model overcomes some of the shortcomings of the existing models.

The paper is divided into eight sections of which this is the first. Section 2 examines car-following behavior and develops a framework against which the existing models as well as the proposed models will be evaluated. Section 3 evaluates some of the existing models using the proposed framework. Section 4 presents the data on car-following behavior collected by the test vehicle. Section 5 analyzes the existing models in light of the data. Section 6 presents the proposed fuzzy inference model. Section 7 evaluates the proposed model based on the framework and the data collected. Section 8 provides a summary of the findings of the present study.

2. Car-following behavior

Car-following is a control process in which the driver of the following vehicle attempts to maintain a safe distance between his/her car and the vehicle ahead by accelerating or decelerating in response to the actions of the vehicle ahead. It is essential to point out here that the controller is human. We propose a set of five distinct features of car-following behavior as a framework for evaluating the different models of car-following. These features are:

1. The car-following behavior is *approximate in nature*.
2. Response to stimuli in car-following is *asymmetric*.
3. Phenomena of *closing-in* and *shying-away* are observed in car-following.
4. Phenomenon of *drift* is observed in car-following.
5. Car-following is *locally* and *asymptotically stable*.

2.1. Approximate nature

The basic motivation of studying car-following behavior, in the words of Herman (1992) is to conceive "a theoretical basis for describing the dynamical behavior of vehicles in terms of human behavior" in an attempt to "explore vehicular traffic, i.e., how human beings interact through a machine known as the automobile (Herman, 1991)". The reasons for laying such importance to humans is the realization of the fact that in car-following a driver perceives the current situation, infers (using his/her knowledge and experience of driving) the appropriate action and responds.

One of the main features of human decision making (or reasoning) and response processes is their inherent approximate nature. This can be attested in many forms and has been long recognized (Glucksberg, 1988; Kruse, 1991; Nickerson, 1986). Vagueness exists in the perception of stimuli, in the rules one applies (to obtain the appropriate actions), and in the decision (action) one arrives at. Given this nature of human decision making process and the understanding that car-following is a human decision making process, it is not surprising that Ceder (1976) makes the following observation: "drivers do not completely follow any deterministic behavior."

This view on approximation of stimulus–response pattern can also be supported by recent studies on the speed–density (*u–k*) relationship that cast doubts on representing *u–k* relations as deterministic relations. For example, Ross (1987) states, "the idea that there is a deterministic relation between speed and density, be it straight line or curve is simply untenable. The most obvious problem is that speed–density observations always have much more scatter than can be explained by any reasonable amount of experimental error". Similar concerns are found elsewhere too (Ross, 1988; Gilchrist and Hall, 1989). These views enforce our belief that the behaviors of individual drivers are inherently based on the approximate inference rather than a deterministic causality.

These findings on *u–k* relationships, the preceding discussion, and the general understanding of the human reasoning and response process leaves little doubt about the approximate nature of the car-following behavior, and a need for an approach that accounts for the approximate human decision process.

2.2. Asymmetric response

Drivers react differently when decelerating and when accelerating. The reaction of a driver to positive relative speed of a given magnitude is not the same in magnitude as to a negative relative speed of the same magnitude. In other words, the driver's response is asymmetric with respect to relative speed. Leutzbach (1988) suggests that this asymmetry is due to the fact that "drivers pay closer attention to decreases in spacing (decrements) than to increases in spacing (increments) simply on the basis of their own safety".

2.3. Closing-in and shying away

It is often seen that when the following vehicle (FV) is reasonably behind the leading vehicle (LV), the FV accelerates even if LV is decelerating (a negative relative speed). Such behavior is seen when the FV intends to close-in, situations in which the relative speed is necessarily negative. Similarly, if the FV finds itself too close to the LV then the FV decelerates and shies away from the LV; then the FV keeps decelerating even though the LV accelerates (a positive relative speed). The driver's desired speed and headway combination dictates such behavior.

2.4. Drift

The distance headway at which the LV–FV pair stabilizes does not remain constant but oscillates (drifts) around what might be termed as the stable distance headway. This happens because drivers can neither judge the speed of the LV accurately nor can they maintain their own speed precisely. Further discussions on this phenomenon of drift may be found in Leutzbach (1988).

2.5. Stability

The fact that the distance headway between LV and FV reaches a particular value after a perturbation (to the distance headway) caused by the actions of LV is referred to as *stability* in car-following behavior. Note that maintaining a particular distance headway implies that the relative speed is maintained at zero. (One should note here, that in light of the discussion on drift, stability is to be understood only in the approximate sense.)

There are two important aspects to stability in car-following, namely local stability and asymptotic stability. The stability in the distance headway and velocities of a pair of vehicles (which is reached over time) is referred to as local stability. Whereas, asymptotic stability refers to stability in a platoon of vehicles. In other words, *local stability* studies how perturbations in distance headway and speed (introduced due to the actions of LV) propagate over time, and *asymptotic stability* studies how the perturbations propagate over a platoon of vehicles. Car-following behavior is generally locally and asymptotically stable; that is, the perturbations die down over time (locally stable), and also reduce as they propagate down the platoon of vehicles (asymptotically stable).

Another related and important point that should be mentioned here with regard to the GM based models is that the distance headway at which a pair of vehicles stabilize (henceforth referred to as stable distance headway, SDH) is *only a function of the speed at which these vehicles stabilize* (henceforth referred to as stable speed, SS), and it is independent of the initial condition.

3. The GM models and their properties

Among the various models of car-following developed over the years, a series of models developed by Chandler et al. (1958), Herman et al. (1958), Herman and Potts (1959) and Gazis et al. (1959, 1961) at the GM Research Laboratories have received the maximum attention over the years. Of these models, the generalized model of car-following (referred to as the GM model) is the most important model. In fact most of the other models of car-following, for example, Pipes (1953, 1967), Forbes et al. (1958), and Forbes (1963), are also special cases of the GM model.

The GM model is a stimulus–response model of car-following. It assumes that the FV (the $n+1$th vehicle) responds to non-zero relative speed (which is the only stimulus) between itself and the LV (the nth vehicle) by accelerating or decelerating. The model also hypothesizes that the degree to which a given stimulus affects the response is a function of the distance headway between the LV and FV and the speed of FV. Mathematically the model can be represented as:

$$\ddot{x}_{n+1}(t+\Delta t) = \left\{ \frac{\alpha(\ell, m)(\dot{x}_{n+1}(t+\Delta t))^m}{(x_n(t) - x_{n+1}(t))^\ell} \right\} \left[\dot{x}_n(t) - \dot{x}_{n+1}(t) \right], \tag{1}$$

where $x(t)$, $\dot{x}_n(t)$, $\ddot{x}_n(t)$ are the distance (from some arbitrary point), speed and acceleration/deceleration of the nth vehicle at time t, respectively; Δt is the perception reaction time; ℓ and m are exponents; and $\alpha(\ell, m)$ is a constant whose dimensions are dependent on the exponents. In the above model the term inside the braces is called the sensitivity term, and the term within the brackets is the stimulus.

The following properties (referred to as the *general properties*) of the GM model become amply clear from Eq. (1):

1. It is a deterministic stimulus–response model; it assumes that the stimulus as well as the distance headway can be perceived precisely and that the driver can tune the response precisely.
2. Its result is symmetric with respect to the relative speed; that is, with other things remaining the same the magnitude of the response is only dependent on the magnitude of the stimulus. For example, if FV responds by accelerating at a m/s^2 when the relative speed is $+r$ m/s then under similar conditions of distance headway and speed the response to a relative speed of $-r$ m/s will be $-a$ m/s^2.
3. The response of the driver is based on only one stimulus, namely, the relative speed. Once the relative speed is zero the FV neither accelerates nor decelerates irrespective of the distance headway between the vehicles.

In the following section, the GM model is evaluated in the light of the analysis framework discussed and on the results of certain stability analysis.

4. Evaluation of the GM models

In evaluating the GM model the following procedure is followed: (1) a comparison is made between the *general properties* of the GM model enumerated in the previous section and the relevant properties of actual car-following behavior, and (2) local stability analysis with the GM model are performed and the outcome of the analysis (referred to as the *stability properties*) are compared with the relevant stability properties of actual car-following behavior.

4.1. The general properties

1. The car-following behavior is approximate in nature. However, the GM model is deterministic thereby ignoring the inherent vagueness of the car-following process.
2. The GM model is symmetric, whereas the car-following behavior is thought to be asymmetric.
3. Owing to the single stimulus nature of the GM model, it fails to explain behavior such as closing-in and shying-away. For example, consider the scenario where a vehicle V_1 joins a platoon of vehicles ahead of vehicle V_2 at the same speed at which V_2 is traveling and at a distance headway of D between the two. If the driver of V_2 finds D to be too short then he/she will decelerate first to increase the distance to a desirable value and then accelerate to reduce the relative speed between the two. This is the behavior of shying-away. However, according to the GM model vehicle V_2 will not take any corrective action and will continue to travel at its original speed albeit at a short distance headway, because relative speed is the only relevant stimulus and it is 0 (zero) in this simulation.

4.2. The stability properties

The car-following behavior of FV within an LV–FV pair of vehicles as predicted by the GM models are analyzed under (i) different initial conditions of speed and distance headway, and (ii) different perturbation patterns (i.e., the acceleration and deceleration patterns of the LV).

Further, the study is carried out for different values of m and ℓ. For the purposes of brevity, results from two such combinations are presented. It should be understood, however, that the properties suggested by the results are found to be generic to the GM model and not dependent on the m and ℓ values.

Also note that the value of the proportionality constant $\alpha(\ell, m)$ for a given ℓ and m is computed by fitting the corresponding speed–density relation to a real world speed–density data set obtained from the Queen Elizabeth Way freeway in Canada. The value of $\alpha(1, 0)$ is obtained as 29.72 ft/s, and the value of $\alpha(2, 1)$ is obtained as 69 ft. These values have been used here for the purpose of stability analysis.

Figs. 1 and 2 present the distance headway variations with time for a pair of vehicles for $(\ell = 1, m = 0)$ and $(\ell = 2, m = 1)$, respectively, and also under the following base conditions:

Initial distance headway for Case 1 (Line 1)	140 ft
Initial distance headway for Case 2 (Line 2)	120 ft
Initial speeds of LV and FV for both cases	44 ft/s
Final speeds of LV and FV for both cases	44 ft/s
Acceleration pattern (acceleration every second) for LV for both cases	-4 ft/s^2, -2 ft/s^2, 2 ft/s^2, and 4 ft/s^2

The figures show that the GM model achieves local stability (i.e., the perturbations die down with time). However, it can be seen from the figures that the distance headway at which the FV stabilizes are different in the two cases. A close examination shows that the two cases vary only in their initial distance headway. *This implies that the stable distance headway* (*SDH*) *obtained using the GM model is sensitive to the initial distance headway* (*IDH*). Also note that the distance headway returns to its initial value (SDH = IDH) as the speed returns to its initial value (final speed = initial speed). *This implies that the GM model implicitly assumes that the initial condition* (*of speed and distance headway*) *is a stable condition* (*i.e., the initial distance headway is the SDH for the initial speed*).

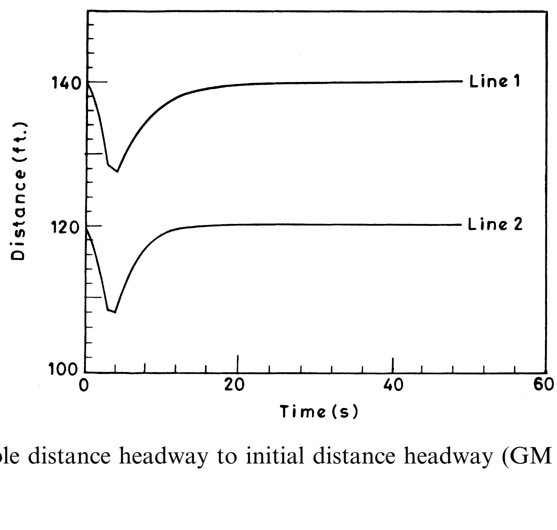

Fig. 1. Sensitivity of stable distance headway to initial distance headway (GM model with $m = 0, \ell = 1$).

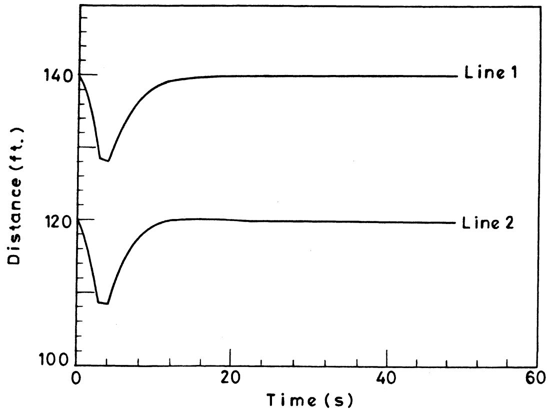

Fig. 2. Sensitivity of stable distance headway to initial distance headway (GM model with $m = 1$, $\ell = 2$).

In the following, two more studies are presented in order to illustrate certain other properties of the SDH obtained using the GM model. Figs. 3 and 4 present the distance headway variations with time for a pair of vehicles for $\ell = 1$, $m = 0$, combination and for $\ell = 2$, $m = 1$, combination, respectively, and under the following conditions:

Case 1 (Line 1 in the figures)	Initial velocity of LV and FV	44 ft/s
	Final velocity of LV and FV	28 ft/s
	IDH	183 ft
	LV's deceleration pattern	8 ft/s^2 for 2 s
Case 2 (Line 2 in the figures)	Initial velocity of LV and FV	44 ft/s
	Final velocity of LV and FV	28 ft/s
	IDH	133 ft
	LV's deceleration pattern	8 ft/s^2 for 2 s
Case 3 (Line 3 in the figures)	Initial velocity of LV and FV	55 ft/s
	Final velocity of LV and FV	28 ft/s
	IDH	133 ft
	LV's deceleration pattern	6.75 ft/s^2 for 3 s

In these figures the final speed is the same for all the cases, however, unlike the earlier cases the final speed is not equal to the initial speed. The following observations can be made from Figs. 3 and 4: (a) from Cases 1 and 2 it can be seen that the *SDH is dependent on the initial distance headway* (as was noted earlier), (b) from Cases 2 and 3 it can be seen that the *SDH is dependent on LV's perturbation* (*in the form of deceleration, in this case*) *pattern and initial velocity*.

In summary, the GM model does achieve local and asymptotic stability (Herman et al., 1959); however;

1. The observations on traffic flow suggest that the distance headway at which people drive is only dependent on the speed at which they are driving and not on how they arrived at that speed.

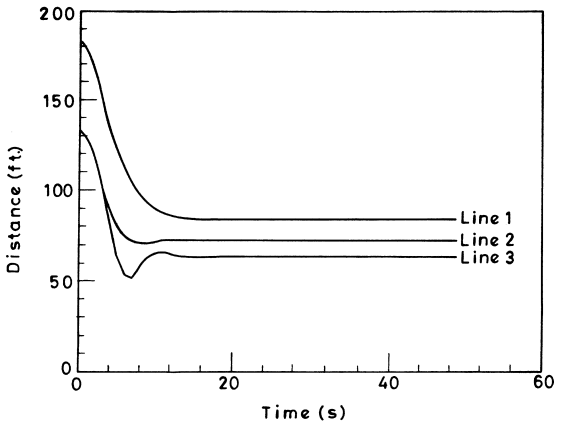

Fig. 3. Sensitivity of stable distance headway to actions of LV and initial speed (GM model with $m = 0$, $\ell = 1$).

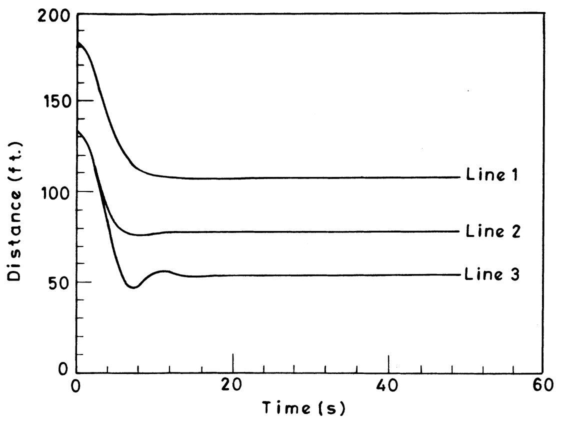

Fig. 4. Sensitivity of stable distance headway to actions of LV and initial speed (GM model with $m = 1$, $\ell = 2$).

That is, the *SDH is only dependent on the final speed* (*or stable speed*) *and not on anything else*. The SDH obtained using the GM model, however, does not possess this property, as it is dependent on initial distance headway, initial speed and the perturbation pattern of the LV, in other words, how the driver reaches the speed.

2. The GM models implicitly assume that the initial condition is a stable condition. This, however, leads to a serious problem because such an assumption makes it impossible to describe how the FV reaches a stable condition in the first place.
3. The GM model has no limit on the acceleration and deceleration rates which, in reality, is dictated by the dynamic performance of the vehicle. Considering the fact that in real world unstable conditions do not last long but instead tend to settle quickly, limiting values of acceleration and deceleration should be incorporated.

5. Field data on car-following

Extensive data on car-following behavior is collected in the field using a test vehicle. The processes of data collection, data processing and the equipment are discussed here.

5.1. Description of data and the experimental procedure

The test vehicle is capable of measuring the distance headway between the LV and the FV and the speed of the FV continuously at every 50 milliseconds (ms). The distance measuring equipment is a laser radar. Given the speed of FV and distance headway, speed of FV, relative speed, acceleration/deceleration rate of FV and acceleration/deceleration rate of LV are computed at every 50 ms.

A two person team (the driver and the analyst) collected the data. The driver is instructed to drive in a lane of his/her choice, and in the natural course of events the driver is in a car-following situation. The analyst then activates the data collection device without the knowledge of the driver. The experiments were conducted on arterial roads in and around Newark, Delaware. The leading vehicle (target) used for measurement was always a passenger car.

5.2. Data processing

The raw data is processed in three steps for it to be used in evaluating the car-following behavior. The steps, in the order in which they are executed, are as follows:

1. Conversion of the data from volts to distance headway in feet and speed of FV in ft/s using the calibration functions provided by the manufacturer Fig. 5 (a) and (b) shows typical data obtained on headway (in ft) and speed (in ft/s), respectively.
2. Smoothing of the data; in this process filters are used to smooth the data on speed and headway. The smoothed version of the data presented in Fig. 5(a) and (b) are shown in Fig. 6(a) and (b).
3. Quantities of relative speed, acceleration/deceleration rates of FV, speed of the LV, and acceleration/deceleration rates of the LV, are computed by noting that the change in headway

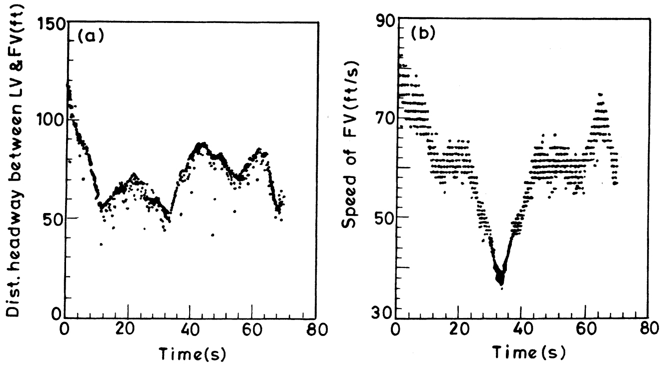

Fig. 5. Raw data on: (a) distance headway; (b) speed of FV collected using the test vehicle.

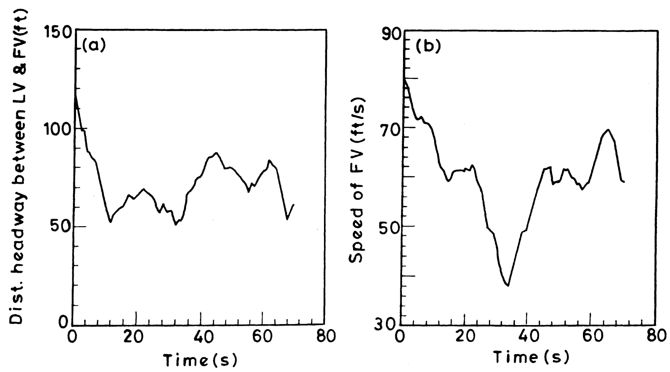

Fig. 6. Smoothed plot for: (a) distance headway; (b) speed of FV obtained from the raw data presented in Fig. 5.

per unit time is equal to the relative speed and speed of LV is equal to the algebraic sum of the speed of FV and the relative speed.

5.3. Sample data sets on car-following behavior

The data sets used in this paper are presented in Figs. 7–12. Each figure consists of data on two relationships: (1) speeds of LV and FV over time, and (2) distance headway between LV and FV over time. Figs. 7 and 8 refer to data obtained using Subject 1 as the driver of FV, Figs. 9 and 10 to those using Subject 2 and Figs. 11 and 12 to those using Subject 3.

6. Comparison between car-following data and the GM models

The observed data are plotted with acceleration/deceleration rate at time t seconds as the ordinate and the relative speed at time $t-1$ s as the abscissa. Figs. 13–15 show these plots for the car-following data obtained using three subjects. It is noted that the nature of the figures do not change even if the abscissa is the relative speed at some $t-a$ s (where a is some number around 1 s).

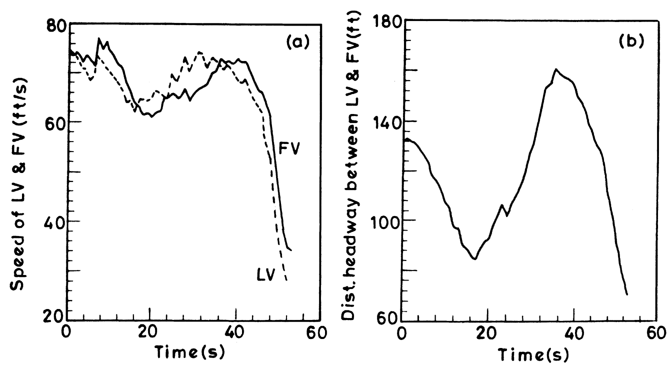

Fig. 7. Car-following data on: (a) speed variation; (b) distance headway variation from Experiment 1 using Subject 1.

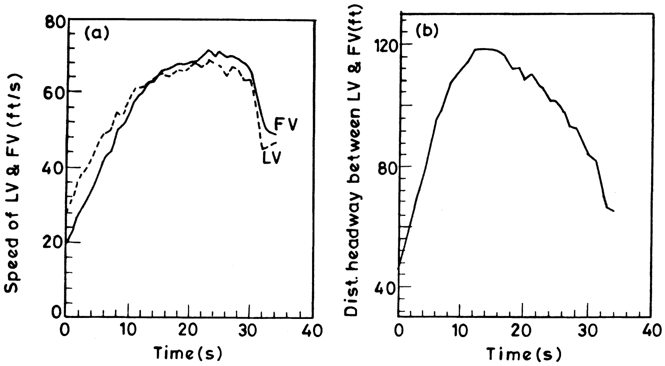

Fig. 8. Car-following data on: (a) speed variation; (b) distance headway variation from Experiment 2 using Subject 1.

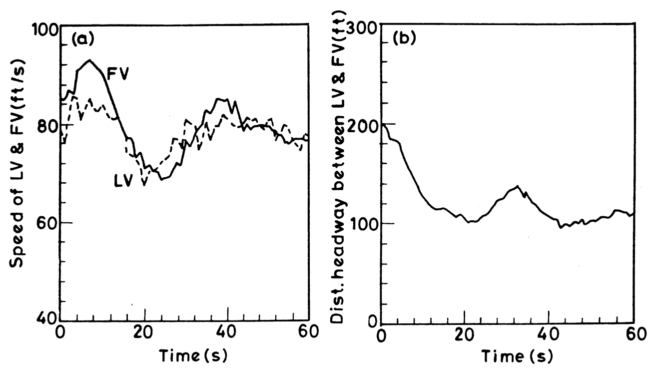

Fig. 9. Car-following data on: (a) speed variation; (b) distance headway variation from Experiment 1 using Subject 2.

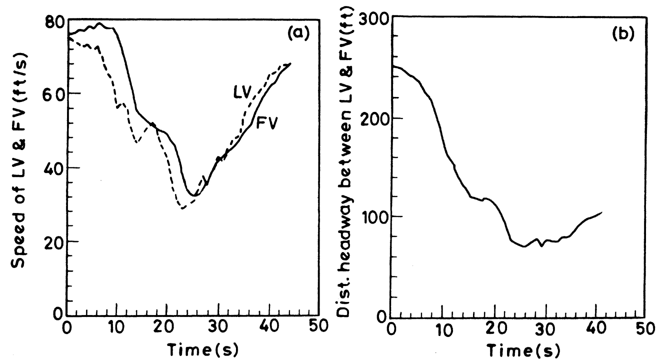

Fig. 10. Car-following data on: (a) speed variation; (b) distance headway variation from Experiment 2 using Subject 2.

In Figs. 13, 14 and 15, about 20% of the points are found in the second and fourth quadrants (quadrants of upper left and lower right in the figures). According to the GM model in Eq. (1), FV will always accelerate if the relative speed is positive and decelerate if the relative speed is negative

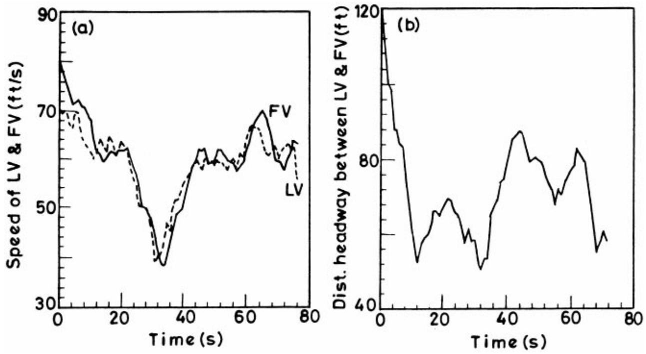

Fig. 11. Car-following data on: (a) speed variation; (b) distance headway variation from Experiment 1 using Subject 3.

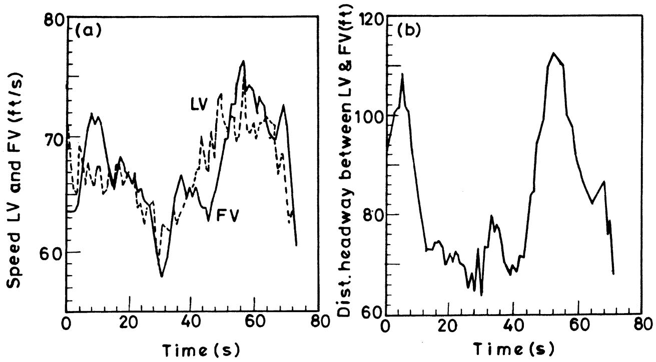

Fig. 12. Car-following data on: (a) speed variation; (b) distance headway variation from Experiment 2 using Subject 3.

and hence according to the model, the second and the fourth quadrants should not have any points. On the other hand, presence of 20% of the points in these "infeasible quadrants" (according to the GM model) cannot be discarded as the random error. This discrepancy between the viewpoint in the GM model (that relative speed is the only stimulus and FV's actions are proportional to it) and the data suggests that the assumptions of single stimulus and proportionality are not reasonable.

The above data sets are also used to calibrate different versions of the GM model (i.e., with different m and ℓ combinations). Even after calibration, major discrepancies exist between the values computed by the GM model and those of the actual data. Figs. 16–18 show the comparisons. The abscissa in the figures refers to the observed response and the ordinate refers to the predicted response. The observed responses in Figs. 16–18 are from Subjects 1–3, respectively (these responses are the same as those shown in Figs. 13–15). The predicted responses in each of the figures are obtained from the GM model with $m=1$ and $\ell=2$ after calibrating it using the observed data shown in that figure. The band in the figures is a tolerance band and represents an acceptable difference between the predicted and the observed responses. The tolerance band is obtained as *observed acceleration/deceleration value* ± 1 ft/s^2. It must be noted that the compari-

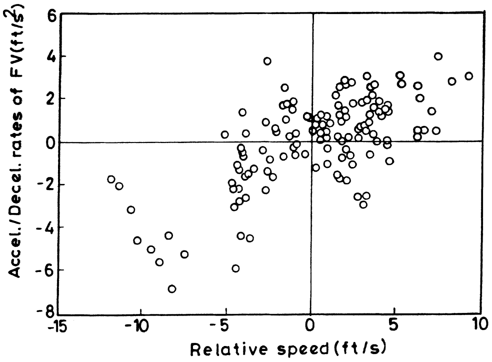

Fig. 13. Acceleration/Declaration rate of FV at time t versus relative speed between LV and FV at time $t-1$ (for Subject 1).

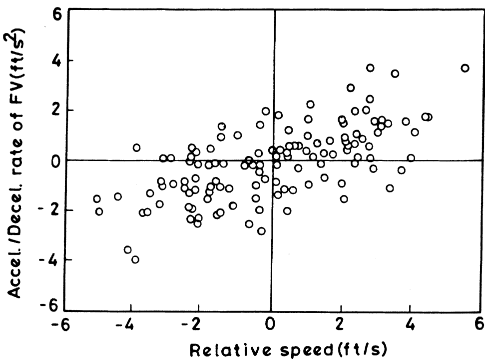

Fig. 14. Acceleration/Declaration rate of FV at time t versus relative speed between LV and FV at time $t-1$ (for Subject 2).

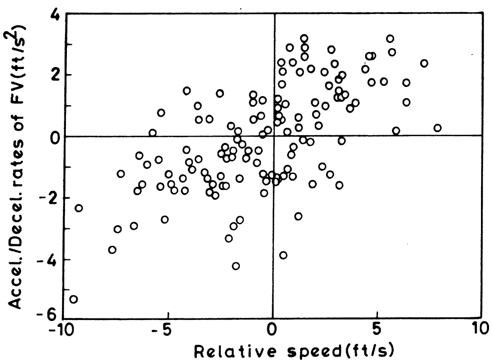

Fig. 15. Acceleration/Declaration rate of FV at time t versus relative speed between LV and FV at time $t-1$ (for Subject 3).

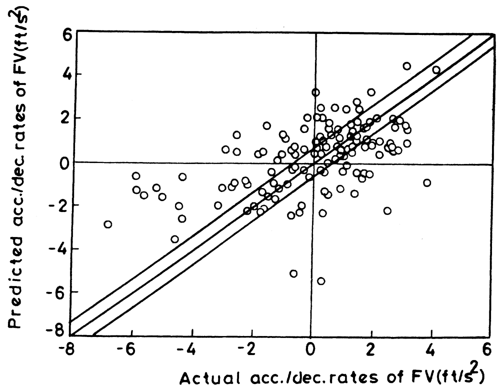

Fig. 16. Predicted actions of FV (using GM model with $m = 1,\ \ell = 2$) versus observed actions of FV (using Subject 1).

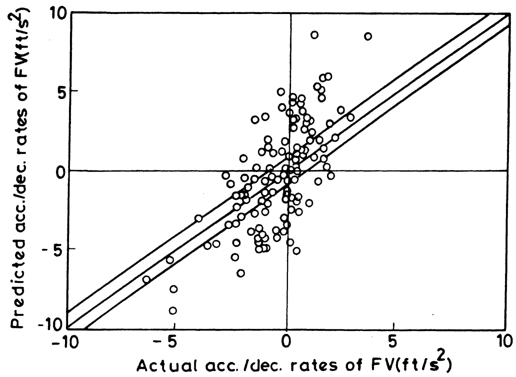

Fig. 17. Predicted actions of FV (using GM model with $m = 1,\ \ell = 2$) versus observed actions of FV (using Subject 2).

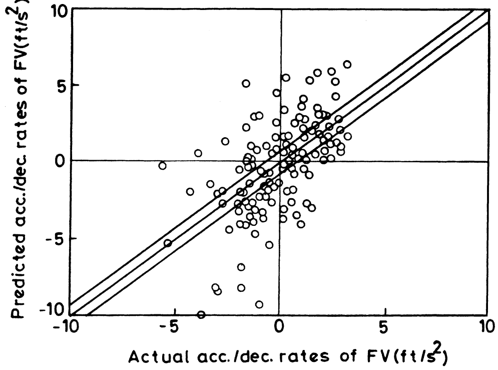

Fig. 18. Predicted actions of FV (using GM model with $m = 1,\ \ell = 2$) versus observed actions of FV (using Subject 3).

sons using the (m, ℓ) combination shown here, are the best among all the different combinations of m and ℓ values that are used.

Ideally, in the above figures, the following two features should be present: (a) all or most of the points should lie inside the tolerance band, and (b) the general orientation of the points (whether inside the band or outside) should be close to the orientation of the band. However, as seen from the figures, the general orientation of the points is different from the orientation of the band and that the number of points that lie within the tolerance band is small. For example, approximately 35% of the points lie within the tolerance band in Figs. 16, and 17 and approximately 23% lie inside the band in Fig. 18.

The above discussions show that the single stimulus assumption of the GM model is not sufficient to represent the reality; at least for the range of the observed distance-headway and speed. Herman and Potts (1959) possibly refer to these drawbacks of the model when they write "they [meaning the outcome from some of their experiments] do suggest that in the complicated functional form of the true stimulus–response equation, different parts of the functional [the current GM model being just one part of the functional] operate under different condition". This statement of Herman and Potts seem to be motivated by their observation that "in certain runs a quite different [different from the GM model] stimulus–response law seemed to be obeyed (Herman and Potts, 1959)". Unfortunately, the data that prompted the authors to make this statement does not appear in the literature. The phrases inside the brackets in the above sentences have been added to explain the context in which the quotes appear in the original paper.

7. The proposed model: a fuzzy inference model

This section describes a new approach to model the car-following behavior. The reader may refer to an earlier paper by Kikuchi and Chakroborty (1992) for more details.

7.1. The rationale and the model structure

This approach considers that human perceives the environment, uses his/her knowledge and experience to infer possible actions, and responds in an approximate manner. A fuzzy inference model is introduced in order to reflect such an inherent imprecision in human perception and reasoning process.

In the proposed model, it is assumed that the stimulus (like relative speed) is perceived only linguistically by the driver who then utilizes a set of approximate driving rules (which is the outcome of a person's driving experience and attitude) to infer an approximate response. The response of the driver is expressed in terms of acceleration or deceleration. Further the model assumes that the driver's response depends on three stimuli, namely, relative speed, distance headway and acceleration/deceleration of the LV. The structure of the model is schematically represented in Fig. 19.

The basic features of the model are as follows:

1. The model consists of a set of fuzzy inference rules that relate a particular driving environment (described through the existing relative speed, distance headway and actions of the LV) at time t to an appropriate action by FV at time $t + \Delta t$. The rules are of the form:

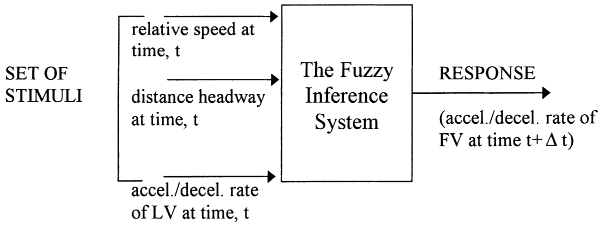

Fig. 19. Schematic representation of the proposed model.

If (at time t)	Distance Headway (DS) is $\mathbf{A}_i$	AND
	Relative Speed (RS) is$\mathbf{B}_j$	AND
	Acceleration of LV (ALV) is $\mathbf{C}$	

Then (at time $t + \Delta t$) Accel./decel. of FV (AFV) should be $\mathbf{D}_\ell$.

2. The antecedent (the part which follows "If" and precedes "Then") of the rule consists of three fuzzy propositions consisting of fuzzy sets, $\mathbf{A}_i$, $\mathbf{B}_j$, and $\mathbf{C}_k$. They refer to certain linguistically described conditions of the variables DS, RS, and ALV, respectively. For example, $\mathbf{A}_i$, may be a fuzzy set representing the concept, ADEQUATE; $\mathbf{B}_j$ may be a fuzzy set representing the concept, LARGE POSITIVE, and $\mathbf{C}_k$ may be a fuzzy set representing, NONE or VERY MILD Deceleration.
3. The consequent (the part which follows "Then") of each rule consists of a single proposition stating the appropriate action (acceleration or deceleration rate) of the FV. The linguistic description of the appropriate action is represented using a fuzzy set. For example, $\mathbf{D}_\ell$, may be a fuzzy set for the concept POSITIVE MILD;
4. The fuzzy sets $\mathbf{A}_i$, $\mathbf{B}_j$, $\mathbf{C}_k$ and $\mathbf{D}_\ell$ are represented by using the triangular or trapezoidal shape membership functions.
5. Relative speed (RS) is grouped into six linguistic classes. Distance headway (DS) is grouped into six linguistic classes (however, the linguistic class to which the distance headway at some instant belongs depends on the speed of FV at that instant; for example a DS of 60 ft may be considered ADEQUATE when FV's speed is 35 ft/s whereas the same DS (i.e., 60 ft) may be considered VERY SMALL if FV's speed is 80 ft/s). Accel./Decel. rate of LV (ALV) is grouped into eleven classes. The entire rule base consists of 396 ($= 6 \times 6 \times 11$) rules; each rule represents a particular combination of RS, DS, and ALV class.
   This fuzzy inference model is based on the generalized podos ponens of the following form:

Input	DS is x and RS is y and ALV is z
Rule 1	if DS is A_1 and RS is B_1 and ALV is C_1, then AFV is D_1 OR
Rule 2	if DS is A_2 and RS is B_2 and ALV is C_2, then AFV is D_2 OR
Conclusion	AFV is D'

where x, y, z are the observed values of RS, DS and ALV at time t. Each rule is a fuzzy relation. The conclusion is the predicted action of AFV expressed in fuzzy set, D'. In general, the degree that input matches the antecedent of each rule is the truth of the consequent of the rule. Because antecedents of many rules have some match with the input, different consequents with different degree of truth values are obtained. These different consequents are aggregated to form a single conclusion expressed in fuzzy set, D'. For practical application a single value is chosen from the fuzzy set, D', by a process called defuzzification.

In fuzzy logic, the truth of a proposition, "x is $Z1$", can be any real number in the range between 0 and 1, and when x is given, it is measured by the membership degree of the value of x in the fuzzy set $Z1$. The membership degree is specified by a membership function, $h_{Z1}(x)$. Further, the truth of a compound proposition like "x is $Z1$ AND y is $Z2$" can be obtained by taking the minimum of the truths of the two constituent propositions as the two propositions are connected by AND. For example, the degree of truth, T, of the proposition "DS is A_i and RS is B_j and ALV is C_k" for a specific value of x, y and z for DS, RS and ALV, respectively, can be obtained as follows:.

$$T = \min\{h_{A_i}(x), h_{B_j}(y), h_{C_k}(z)\}, \tag{2}$$

where $h_{A_i}(x), h_{B_j}(y), h_{C_k}(z)$, each represents the truth that x, y, and z is compatible with the notion of A_i, B_j and C_k, respectively. In the proposed model we use the minimum operator (a popular operator for fuzzy control application) to compute the truth-value, because elements of the antecedents are connected by AND.

As mentioned above the truth of the antecedent of a rule obtained as above is taken as the truth of the consequent suggested by the rule. In the following a step-by-step procedure for executing the proposed fuzzy inference logic is given.

- Calculate the truth value of the antecedent of each rule.
- Aggregate the consequents from different rules to obtain ALV.
- Defuzzify AFV to obtain the action of FV.

Calculate the truth-value of the antecedent of a rule. Given the current (at time t) conditions (values) of RS, DS, and ALV, the truth of the antecedent of the rule is evaluated by the degree of match between (x, y, z), the current values of DS, RS and ALV, and the antecedent, as given in Eq. (2). The truth of the antecedent of a rule indicates the degree to which the consequent of the rule is applicable under the current condition. Thus the process is mathematically expressed as:

$$h_{D_{\ell'}}(w) = \min\{h_{A_i}(x), h_{B_j}(y), h_{C_k}(z), h_{D_\ell}(w)\}, \tag{3}$$

where $h_{D_{\ell''}}(w)$ is the membership function of notion of consequent ($D_{\ell'}$) after the input is given.

Aggregate the consequents of different rules. Because the antecedents of the rules consist of propositions that are fuzzy, the current condition (the values of DS, RS and ALV) can match the antecedent of more than one rule. Thus, more than one consequent can be derived for the same set of current values. The degree of truth of each consequent is computed by Eq. (2). Some consequents may indeed be in conflict with one another. Nevertheless, a set of consequents that suggests the action of FV is derived.

The action of the following vehicle (AFV) for a given condition is obtained by taking the aggregate of the consequents from the different rules that are derived under the given condition.

$$h_{D_\ell}(w) = \max\{h_{D_{\ell'}}(w), h_{D_{\ell''}}(w), h_{D_{\ell'''}}(w), \ldots\}, \tag{4}$$

where $h_{D_{\ell'}}(w)$, $h_{D_{\ell''}}(w)$, $h_{D_{\ell'''}}(w)$ are the membership functions of consequents of different applicable rules. The max operator is used because the rules are connected by OR. $h_{D_\ell}(w)$ is the membership function of the conclusion.

"*Defuzzify*" *the conclusion to obtain the action of FV*. The consequent obtained (either in aggregate form or singular form) is still a fuzzy set as defined in Eq. (4). It is "defuzzified" to obtain a specific acceleration or deceleration rate of FV. In our model, the value of w that corresponds to the highest value of the membership function of the conclusion is used for the defuzzified value. This value represents the highest (or some pre-assigned) *possibility*, $\max_w h_{D_\ell}(w)$. The value of w, either positive or negative, is the acceleration (or deceleration) rate of the FV at time $t + \Delta t$.

7.2. The general properties of the proposed model

Most of the properties stated here follow directly from the above descriptions. Wherever the property stated is not obvious, results and discussions that prove the presence of the property are added.

1. The proposed model represents an *approximate* behavior. It does not rely on a precise idealization of the stimulus–response pattern. The model represents the natural language based "rules-of-thumb" of driving which is believed to be reasonable; and this action is translated into a specific value of acceleration or deceleration rate through the process of defuzzification.

2. The proposed model uses the compromise of more than one rule of behavior. Different rules apply when the stimuli received are different, hence it is not a single function based model. This also allows for incorporating *asymmetry in response*. For example, a rule which applies for large positive relative speed may suggest a response of *medium* acceleration; whereas, a rule which applies for similar conditions of headway and actions of the LV but for large negative relative speed may suggest *large* deceleration.

3. The proposed model assumes that there are more than one stimulus which affects the actions of FV. This property allows the proposed model to explain behavior such as *closing-in and shying-away*. This is illustrated by the following examples obtained from the proposed model. Figs. 20 and 21 respectively show the variation of FV's speed, and the distance headway (between FV and LV) with respect to time for an LV–FV pair under the following conditions:

Case 1 (Line 1 in the figures)	Initial velocity of LV and FV	50 ft/s
	initial distance headway	180 ft
	LV neither accelerates nor decelerates.	
Case 2 (Line 2 in the figures)	Initial velocity of LV and FV	50 ft/s
	initial distance headway	30 ft
	LV neither accelerates nor decelerates.	

Fig. 20 also shows the speed variation plot obtained using the GM model; in this case, there is no variation in the speed since the initial relative speed is zero. However, according to the proposed model, when initial distance headway is large (Case 1), the FV accelerates first and then decelerates in order to reduce the distance headway. Whereas, when the initial distance headway is small (Case 2), the FV decelerates first and then accelerates in order to increase the distance

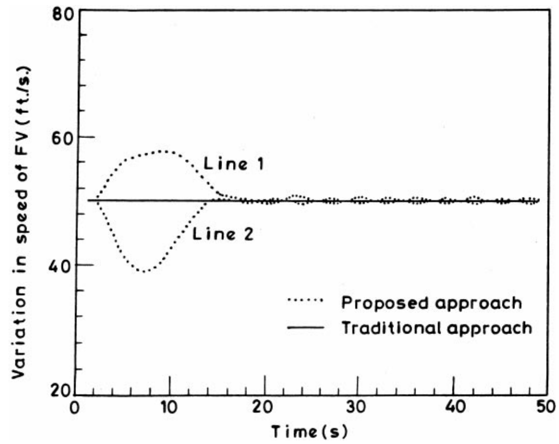

Fig. 20. Variation in speed of FV (obtained using the proposed model) illustrating the properties of closing-in and shying-away.

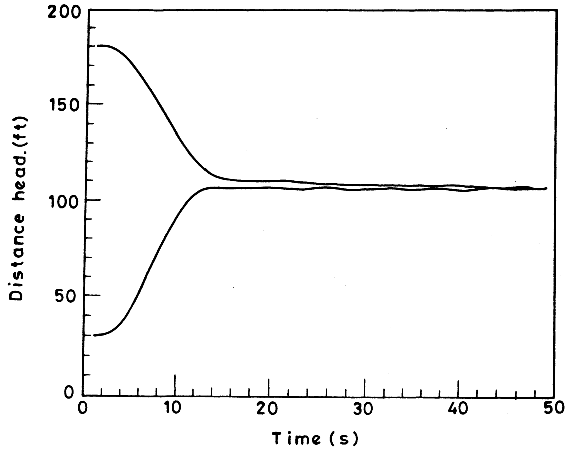

Fig. 21. Variation in distance headway between FV and LV (obtained using the proposed model) illustrating the properties of closing-in and shying-away.

headway to a safe value. The corresponding variations in distance headway are shown in Fig. 21. Thus, even though the relative speed is initially zero and the LV does not accelerate or decelerate, the FV does take corrective actions in order to attain the stable (safe) distance headway.

4. Owing to the representation of multi-rule behavior, the stable condition does not mean absolutely no variations in FV's speed. Rather, the speed of FV at the stable condition, oscillate (drift) around zero. This feature of the proposed model can be seen from Fig. 20.

7.3. Calibration of the proposed model

One of the key issues related to the proposed model is how to calibrate the fuzzy inference based model (i.e., modify the membership functions of the fuzzy sets in the rules) using the real world car-following data. The authors have developed a generalized Parallel Distributed Processing (PDP) representation of the fuzzy inference based model that can be calibrated. The authors have further proved that using the back-propagation learning algorithm developed by Rumelhart and McClelland (1986) on this representation the shapes of the membership functions for the antecedents and the consequents can be tuned in the presence of data on the stimuli and the response of the driver of FV. A detailed discussion of the PDP representation and the associated proofs are not provided here because the primary thrust of this paper is to evaluate the performance of the GM based models and the proposed model. The reader may refer to the work done by Chakroborty (1993) and Kikuchi and Chakroborty (1995) for more details on this. Here only the before and after calibration are compared and presented (Fig. 22) in order to show the effectiveness of the process used.

7.4. The stability properties: local stability

Fig. 23 presents the patterns of variations in distance headway over time, obtained using the proposed model under the following initial distance headway and acceleration of LV (note, three different cases are presented):

The following conclusions can be drawn from the figure: (a) the distance headway in all the cases settles to a particular value; *this shows that local stability is achieved*, and (b) in all the three

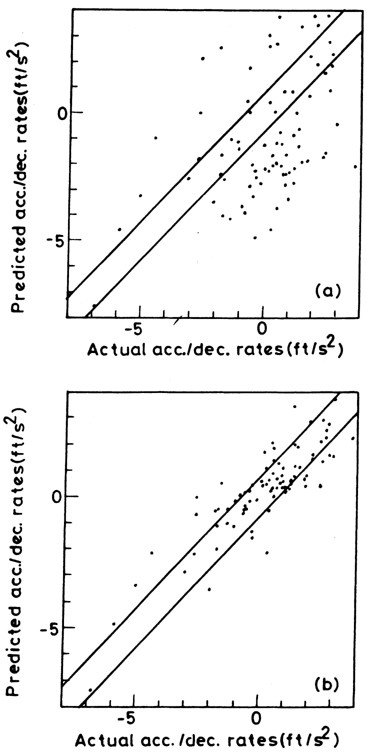

Fig. 22. Predicted actions of FV versus the observed actions of FV: (a) before; (b) after calibration of the proposed model.

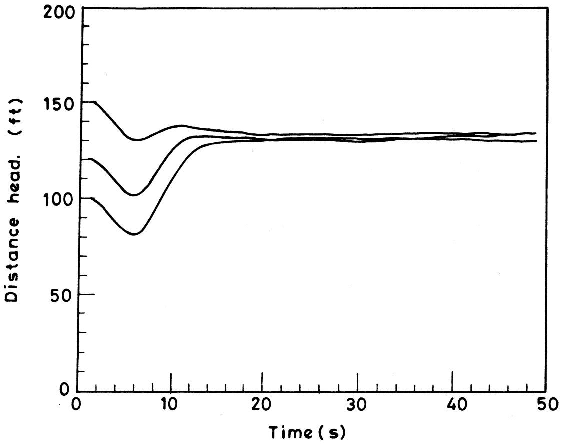

Fig. 23. Independence of SDH to IDH (using the proposed model).

Initial distance headway for Case 1 (Line 1)	150 ft
Initial distance headway for Case 2 (Line 2)	120 ft
Initial distance headway for Case 3 (Line 3)	100 ft
Initial speeds of LV and FV for all the cases	80 ft/s
Final speeds of LV and FV for all the cases	60 ft/s
Acceleration pattern of LV for all the cases	-5 ft/s^2 for 4 s

cases the stable distance headway is the same (note that the final speed is the same in all the cases); this shows that *the stable distance headway* (*obtained using the proposed model*) *is independent of the initial distance headway*.

Fig. 24 shows results obtained using the proposed model for 12 different cases. The 12 cases are classified into four groups. Each group represents a different value of final speed (of LV and FV).

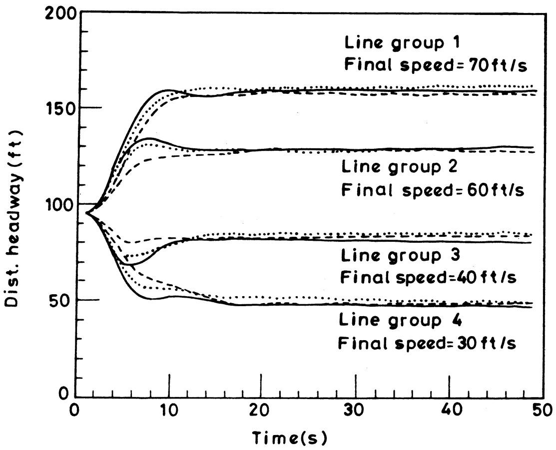

Fig. 24. Independence of SDH to actions of LV and initial speeds, and the dependence of SDH only on final speeds (using the proposed model).

Group I (*Final speed of FV and LV* = 70 *ft/s*)		
Case 1 (continuous line)	Initial speed of FV and LV	50 ft/s
	LV's acceleration pattern	6.66 ft/s^2 for 3 s
Case 2 (dotted line)	Initial speed of FV and LV	55 ft/s
	LV's acceleration pattern	5 ft/s^2 for 3 s
Case 3 (dashed line)	Initial speed of FV and LV	60 ft/s
	LV's acceleration pattern	3.33 ft/s^2 for 3 s
Group II (*Final speed of FV and LV* = 60 *ft/s*)		
Case 4 (continuous line)	Initial speed of FV and LV	40 ft/s
	LV's acceleration pattern	6.66 ft/s^2 for 3 s
Case 5 (dotted line)	Initial speed of FV and LV	45 ft/s
	LV's acceleration pattern	5 ft/s^2 for 3 s
Case 6 (dashed line)	Initial speed of FV and LV	50 ft/s
	LV's acceleration pattern	3.33 ft/s^2 for 3 s
Group III (*Final speed of FV and LV* = 40 *ft/s*)		
Case 7 (continuous line)	Initial speed of FV and LV	60 ft/s
	LV's acceleration pattern	−6.66 ft/s^2 for 3 s
Case 8 (dotted line)	Initial speed of FV and LV	55 ft/s
	LV's acceleration pattern	−5 ft/s^2 for 3 s
Case 9 (dashed line)	Initial speed of FV and LV	50 ft/s
	LV's acceleration pattern	−3.33 ft/s^2 for 3 s
Group IV (*Final speed of FV and LV* = 30 *ft/s*)		
Case 10 (continuous line)	Initial speed of FV and LV	50 ft/s
	LV's acceleration pattern	−6.6667 ft/s^2 for 3 s
Case 11 (dotted line)	Initial speed of FV and LV	45 ft/s
	LV's acceleration pattern	−5 ft/s^2 for 3 s
Case 12 (dashed line)	Initial speed of FV and LV	40 ft/s
	LV's acceleration pattern	−3.33 ft/s^2 for 3 s

Further, each case within a group considers a different combination of initial speeds (of LV and FV) and perturbation pattern (i.e., actions of LV). The detailed specifications of all the cases are as follows (note that in all the cases the initial distance headway is 95 ft):

From Fig. 24 the following conclusions can be drawn: (a) *in all the cases local stability is reached*, and (b) for the cases within each group the stable distance headway is the same; further, the stable distance headway is different for different groups. These show that *the stable distance headway is independent of initial speed*, *distance headway*, *and the actions of LV*; *it is only dependent on the final speed.*

7.5. The stability properties: asymptotic stability

The behavior of a five-car platoon is simulated using the proposed model in order to study asymptotic stability. Fig. 25 shows the distance headway variations with time for all the pairs of

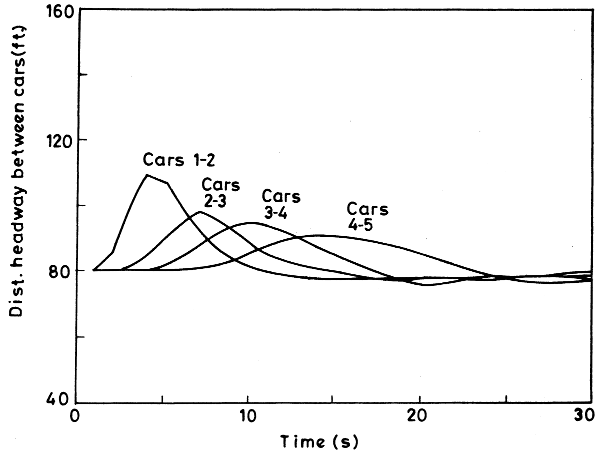

Fig. 25. Variations in distance headway in a five-car platoon illustrating asymptotic stability (using the proposed model).

two consecutive cars in this platoon. The initial conditions and the actions of the leader of the platoon are as follows:

IDH between each pair of vehicles	80 ft
Initial speeds of all the vehicles	40 ft/s
Final speeds of all the vehicles	40 ft/s
Acceleration pattern of the leader of the platoon	10 ft/s^2 for 2 s followed by -10 ft/s^2 for another 2 s

It can be seen that the variations in the distance headway reduce as one proceeds down the platoon. That is, the variations (or perturbations) in distance headway is much larger for Car 1–Car 2 pair than for the Car 4–Car 5 pair. This shows that the proposed model does achieve asymptotic stability.

7.6. Real world car-following data and the predictions by the proposed model

The predicted responses obtained using the proposed model (calibrated with only a part of the observed data) are compared with the observed responses shown in Figs. 13–15. (Recall that these observations were also used for the calibration of the GM model and for the purposes of comparing the predictions obtained using the GM model). Figs. 26–28 compare the responses predicted by the proposed model and those observed. The abscissas in all of these figures refer to the observed response of the drivers and the ordinate refers to the predicted response. The observed responses in Figs. 26–28 are from Subjects 1–3, respectively. The band in the figures is a tolerance band and represents an acceptable difference between the predicted and the observed responses (this band is the same as that which was in Figs. 16–18).

The figures show that the general orientation of the points is similar to that of the band and that the number of points that lie within the tolerance band is large. For example, approximately

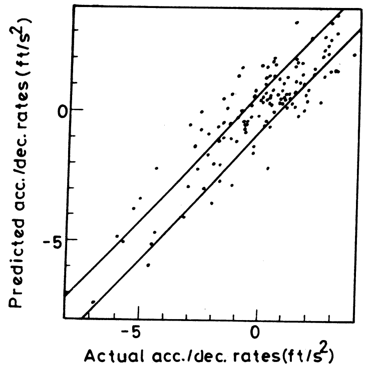

Fig. 26. Predicted actions of FV versus observed actions of FV (using the proposed model for Subject 1).

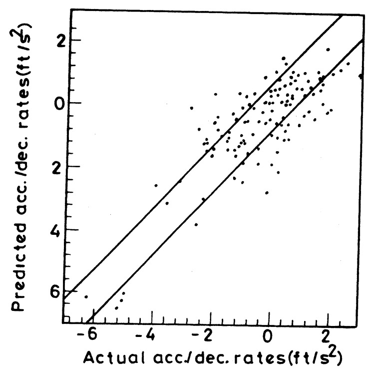

Fig. 27. Predicted actions of FV versus observed actions of FV (using the proposed model for Subject 2).

64% of the points lie within the tolerance band in Figs. 26, and 27 (as compared to approximately 35% for the GM model) and approximately 60% lie inside the band in Fig. 28 (as compared to 23% for the GM model).

8. Summary and conclusions

The purpose of this paper is to study the properties of the GM based car-following models and compare the performance with the real world data. A framework for evaluating the car-following models is developed. Real world data on car-following is collected using an instrumented vehicle traveling on regular roads recording the responses of the following vehicle under different car-following situations. The theoretical properties of the GM models and the observed behavior are analyzed. It is found that the GM based models fall short of representing the theoretical and actual expectations. A model that is based on fuzzy inference logic is proposed for representing the car-following behavior. This model is found to possess many of the features that are desirable

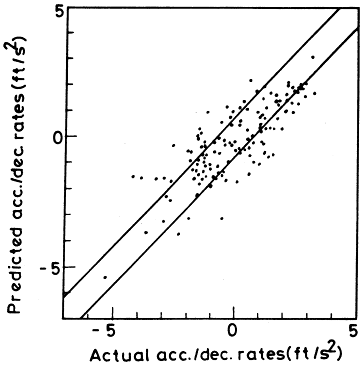

Fig. 28. Predicted actions of FV versus observed actions of FV (using the proposed model for Subject 3).

in a model of car-following but are not available in the GM models. The properties of the GM based models and the proposed model are compared and summarized in Table 1. The fuzzy inference based model may become useful for simulating the flow of AHS vehicles equipped with the fuzzy inference based adaptive cruise control.

Table 1
Summary of comparisons between the GM models and the proposed model

Basis for comparison	Outcome of the comparison using	
	The GM model	The proposed model
General properties of car-following behavior		
1. Approximate nature	Deterministic model; ignores the approximate nature	Fuzzy inference based model; accounts for the appropriate nature
2. Closing-in and shying-away	Cannot represent/explain these properties	Can represent/explain these properties
3. Drift	Cannot represent/explain this property	Can represent/explain this properties
4. Asymmetric response	Assumes symmetric response	Assumes asymmetric response
Stability properties of car-following behavior		
5. Local stability	Achieves local stability	Achieves local stability
6. Asymptotic stability	Achieves asymptotic stability	Achieves asymptotic stability
7. Stable distance headway (dependence only on final speed)	Stable distance headway is dependent on number of factors	Stable distance headway is only dependent on final speed
Data on car-following behavior		
1. Macroscopic view (presence of points on all quadrants of the Cartesian product space of response and relative speed)	Does not allow points to be on all the four quadrants	Considers points on all four quadrants as feasible
12. Individual responses	The predicted responses do not match well with the individual responses	The predicted responses do match well with the individual responses

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