Ordinary Differential Equation © BinaryPhi

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Sec. 1: Separable Differential Equations (Solution and 2 Examples)

Sec. 2: Homogeneous Equations (Solution and 1 Example)

Sec. 1: Separable Differential Equations

Form

$$\frac{dy}{dx} = f(x)g(y) \,. \tag{1}$$

Solution

1) When $g(y) \neq 0$,

$$\frac{dy}{g(y)} = f(x)dx \,,$$

$$\int \frac{dy}{g(y)} = \int f(x)dx + C \,.$$

This is the General Solution to this type of differential equations.

2) When $g(y) = 0$, of all functions of y, if exists y_0, as a constant, such that $g(y) = g(y_0) = 0$, where y_0 has the property that

$$\frac{dy}{dx} = 0.$$

We can say that $y = y_0$ is also one of the solutions to the differential equation.

Example

$$\tan(x)\,\frac{dy}{dx} = 1 + y\,.$$

[Solution] First, make this equation neat:

$$\frac{dy}{dx} = \frac{1}{\tan(x)}\,(1+y)\,,$$

which is the form of the separable differential equation.

Then assume the function of y to be not equal to 0:

$$\frac{1}{1+y}\,dy = \frac{1}{\tan(x)}\,dx\,,$$

$$\frac{1}{1+y}\,dy = \frac{\cos(x)}{\sin(x)}\,dx\,,$$

$$\frac{1}{1+y}\,dy = \frac{1}{\sin(x)}\,d\left(\sin(x)\right),$$

$$\int \frac{1}{1+y}\,dy = \int \frac{1}{\sin(x)}\,d\left(\sin(x)\right) + c\,,$$

$$\ln|1+y| = \ln|\sin(x)| + c$$

$$= \ln\left|e^c \cdot \sin(x)\right|,$$

$$|1+y| = \left|e^c \cdot \sin(x)\right|,$$

$$1 + y = \pm\, e^c \cdot \sin(x),\, C := \pm\, e^c$$

$$y = C \cdot \sin(x) - 1\,,$$

where C is not equal to 0. Now, consider function $1+y$ to be equal to 0:

$$1 + y = 0 \Longrightarrow y = -1\,.$$

$y = -1$ is, in fact, a solution to this equation that $\tan(x)\,0 = 0$. In addition, the assumption of $y = -1$ matches the circumstance when $C = 0$.

Therefore, the general solution of this differential equation is

$$\boxed{y = C \cdot \sin(x) - 1,\, \forall C \in \mathbb{R}\,.}$$

$$dx + xydy = y^2dx + ydy.$$

[Solution] First, make this equation neat: