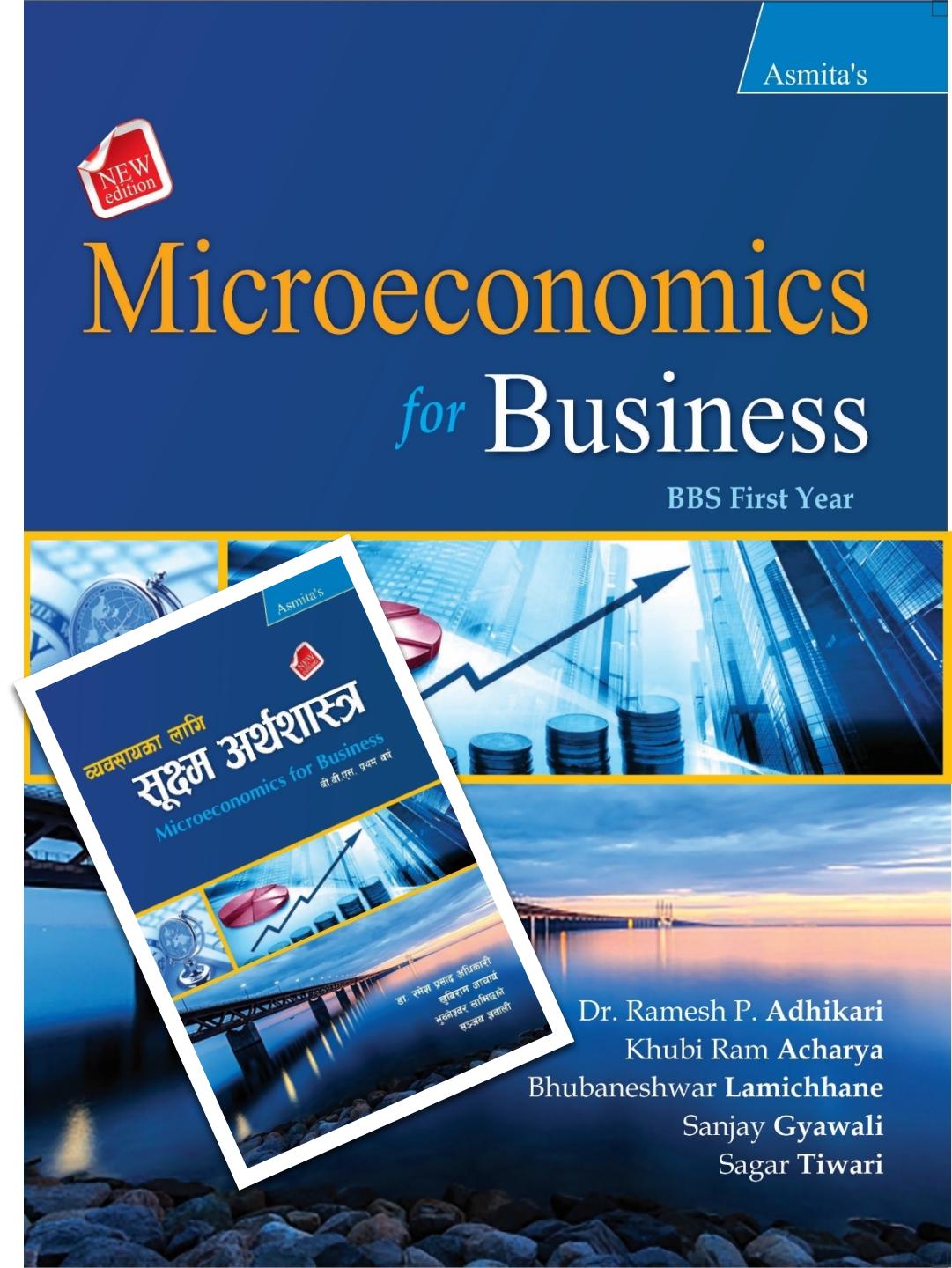
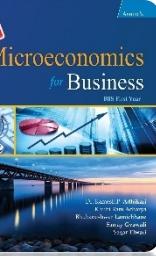


# Analysis of Consumer Behaviour

Unit 4



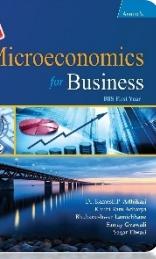
Dr. Ramesh P. Adhikari  
Khubi Ram Acharya  
Bhubaneshwar Lamichhane  
Sanjay Gyawali  
Sagar Tiwari



# Learning Objectives

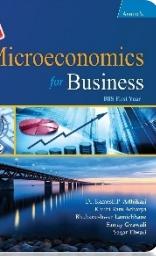
**On completion of this unit, students will be able to:**

- describe the concept of cardinal and ordinal utility approaches
- explain the consumer's equilibrium by using cardinal approach and derive demand curve
- define indifference curve and explain its properties
- describe marginal rate of substitution
- explain the consumer's equilibrium by using indifference curve
- explain the price effect, income effect and substitution effect
- explain the derivation of demand curve for normal goods.



# Introduction

- In this unit, we are concerned with the questions:
  - (i) How does a consumer decide how much of a commodity to buy at a given price?
  - (ii) How does the consumer respond to change in price of the commodity, given his/her income, and price of the related goods?
- These questions take us to the theory of consumer behaviour.
- The theory of consumer behaviour is based on a fundamental assumption that consumers always seek to maximise their total utility or satisfaction, under certain given conditions.



# Concept of Utility Analysis

- Utility is the want satisfying power of a commodity. This concept was introduced by English Philosopher, **Jeremy Bentham** in 1789 to social thought and by William **Stanley Jevons** in 1871, **Walras** in 1874 and **Carl Menger** in 1871 to economic thought.
- The concept of utility is used to analyse the consumer's tastes which is a crucial step in determining how a consumer maximizes satisfaction by spending his/her limited income.
- In order to analyze utility obtained from the consumption of goods and services, there are two basic approaches, i.e. cardinal and ordinal.

# Cardinal Utility Approach

Initially, the concept of cardinal utility analysis was developed by **Hermann Heinrich Gossen** and popularized by famous neoclassical economist **Alfred Marshall**. According to **Marshall**, utility is a subjective phenomenon and it can be quantitatively measured by means of money as a measuring rod. Neoclassical economists believed first that utility can be measured cardinally.

## Assumptions

1. Rational consumers
2. Cardinal measurement
3. Constant marginal utility of money
4. Diminishing marginal utility
5. Additivity of utility

# Concept of Total Utility and Marginal Utility

## Total Utility (TU)

- Total utility is the sum of the utility derived from the consumption of given units of a commodity.
- In other words, it is a sum of marginal utility.
- For example, if a consumer consumes n units, then his/her total utility from n units may be expressed as

$$TU = MU_1 + MU_2 + \dots + MU_n$$

where

TU = Total Utility

MU = Marginal utility of the commodity

# Concept of Total Utility and Marginal Utility Contd.

## Marginal Utility (MU)

- Marginal utility is the change in the total utility resulting from the consumption of one additional unit.

$$MU = \frac{\Delta TU}{\Delta N}$$

where

$\Delta TU$  = Change in total utility

$\Delta N$  = Change in units of consumption

- In other words, marginal utility is the addition to the total utility derived from the consumption of an additional unit of a commodity.

$$MU = TU_n - TU_{n-1}$$

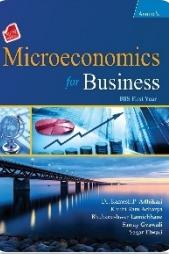
where

$TU_n$  = Total utility derived from the consumption of  $n^{\text{th}}$  unit of a commodity.

$TU_{n-1}$  = Total utility derived from the consumption of  $(n - 1)^{\text{th}}$  unit of a commodity.

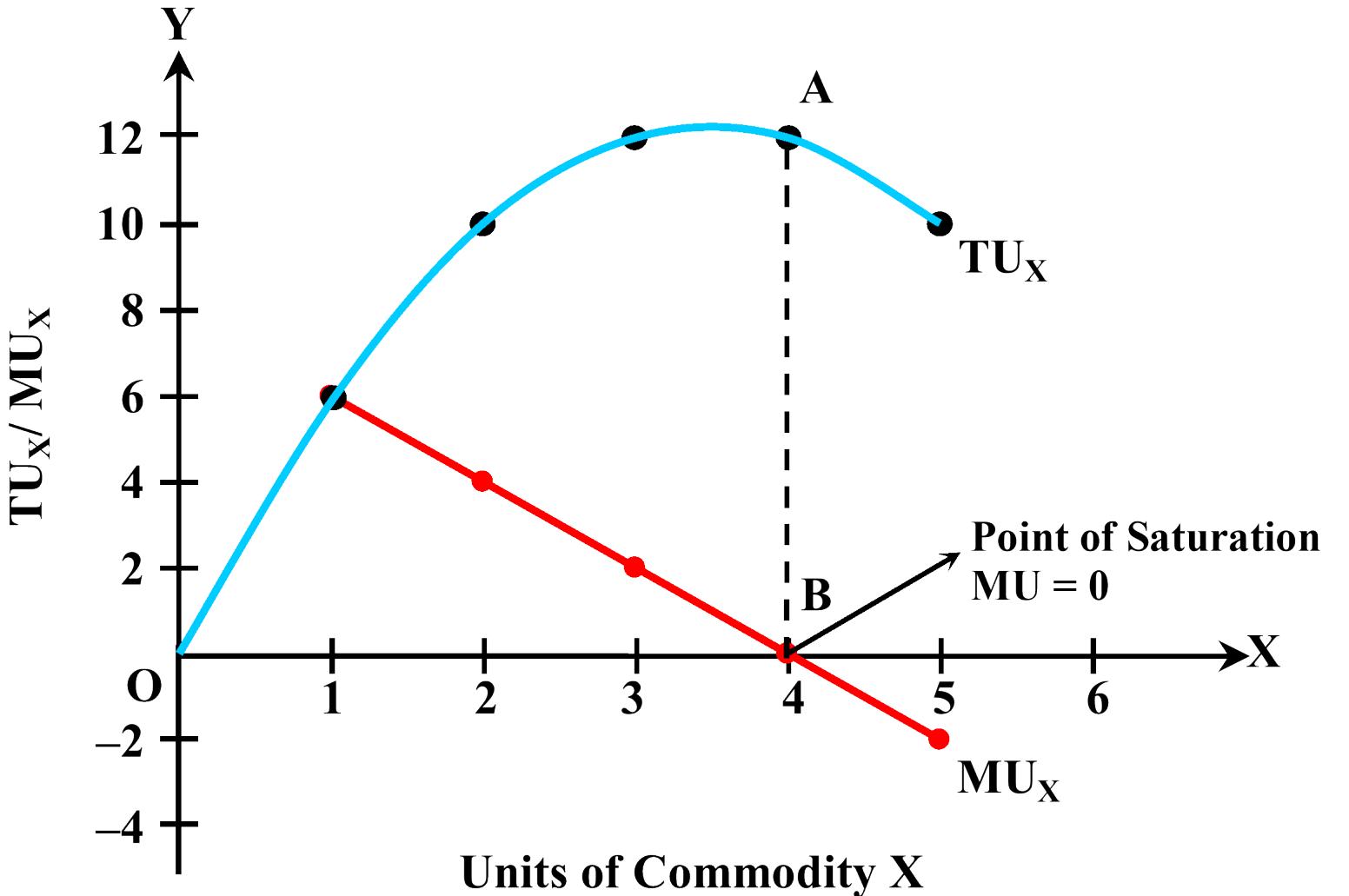
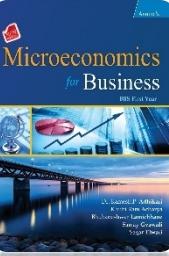
# Concept of Total Utility and Marginal Utility

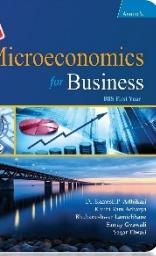
## Contd.



Units of Commodity X	TU <sub>X</sub>	MU <sub>X</sub>
1	6	6
2	10	4
3	12	2
4	12	0
5	10	-2

# Concept of Total Utility and Marginal Utility Contd.





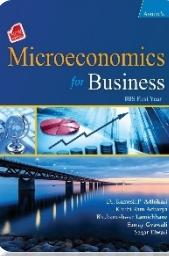
# Consumer's Equilibrium under Cardinal Utility Approach

- A consumer is in equilibrium position when s/he maximizes his/her total utility with given money income and market prices of goods consumed.
- At the equilibrium point, the consumer spends all his income among different goods and services and derives maximum satisfaction.

There are two approaches of consumer's equilibrium under cardinal utility:

- Consumer's Equilibrium: One Commodity Model
- Consumer's Equilibrium: Two Commodity Model

# Consumer's Equilibrium: One Commodity



## Model

- In order to get utility / satisfaction, a consumer consumes goods and services.
- In cardinal utility approach, the sum of the utility derived from the consumption of given units of a commodity is called total utility (TU).
- The additional utility derived from the consumption of an additional unit of a commodity is known as marginal utility.
- In this model, the consumer is in equilibrium when the marginal utility of the commodity is equated to its market price.
- Suppose that the consumer consumes X commodity.
- In this situation, the consumer is in equilibrium when the marginal utility of X commodity is equated to its market price ( $P_x$ ).

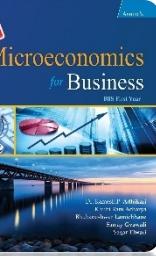
$$\frac{MU_x}{P_x} = MUm$$

where

MUm = Marginal utility of money

$P_x$  = Price of X commodity

$MU_x$  = Marginal utility of X commodity



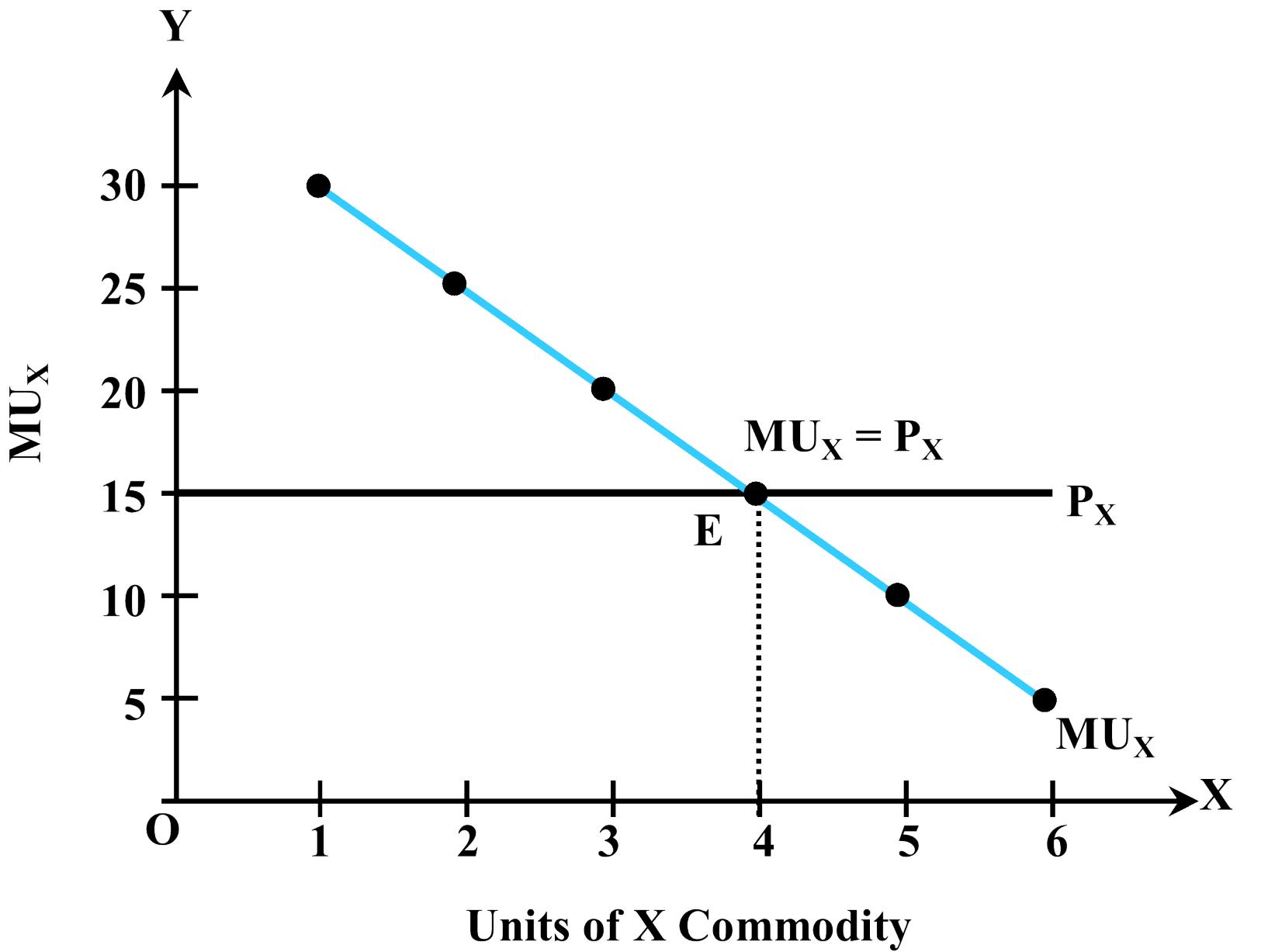
# Consumer's Equilibrium: One Commodity Model Contd.

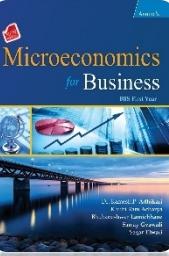
## Assumptions

Consumer's equilibrium under one commodity model is based on the following assumptions:

- The consumer is rational.
- Cardinal measurement of utility is possible.
- Marginal utility of money remains constant.
- The law of diminishing marginal utility operates.
- Prices of commodities are given or remain constant.

<b>Unit of X</b>	<b><math>MU_X</math> (in Rs.)</b>	<b><math>P_X</math> (in Rs.)</b>
1	30	15
2	25	15
3	20	15
4	15	15 $MU_X = P_X$
5	10	15
6	5	15





# Consumer's Equilibrium: Two Commodity Model

- A consumer is said to be in equilibrium when the ratio of marginal utility and price of each commodity is equivalent to marginal utility of money.
- It means that consumer always tries to get equal marginal utility by consuming a commodity which is equal to money spent on that commodity.
- This is called law of equi-marginal utility or law of substitution.
- According to this law, consumer is in equilibrium position when the following condition is fulfilled:

$$\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y} = MU_M$$

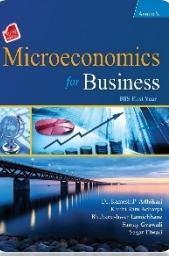
where

$MU_X$  = Marginal Utility of X Commodity

$MU_Y$  = Marginal Utility of Y Commodity

$P_X$  = Price of X Commodity

$P_Y$  = Price of Y Commodity



# Consumer's Equilibrium: Two Commodity Model

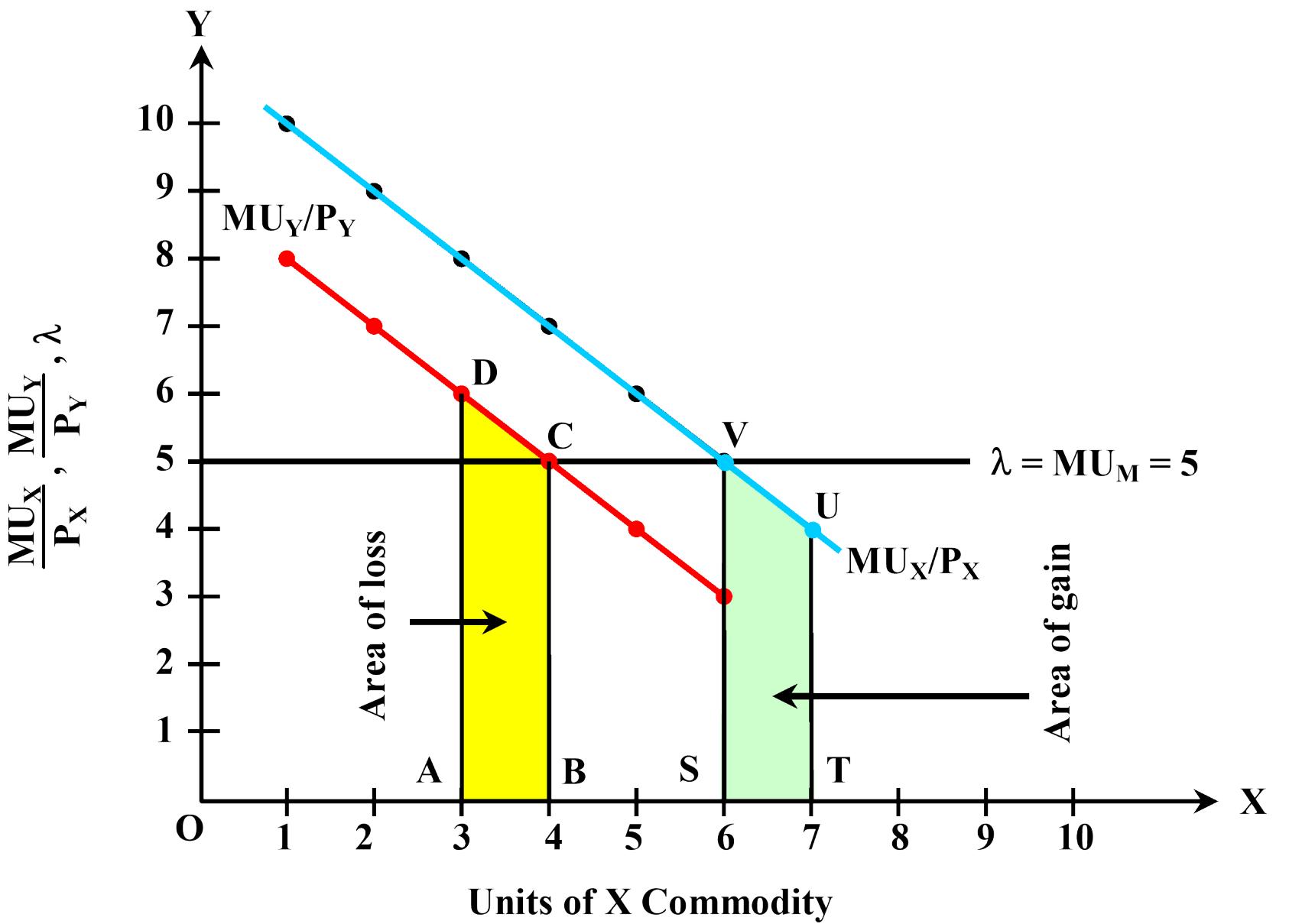
## Contd.

### Assumptions

Consumer's equilibrium under two commodity model is based on the following assumptions:

- The consumer is rational.
- Cardinal measurement of utility is possible.
- Marginal utility of money remains constant.
- The law of diminishing marginal utility operates.
- Prices of commodities and income of the consumer are given.
- Consumer spends his income in two goods.

Units	MU <sub>X</sub> (Utility)	MU <sub>Y</sub> (Utility)	MU <sub>X</sub> / P <sub>X</sub>	MU <sub>Y</sub> / P <sub>Y</sub>
1	20	24	10	8
2	18	21	9	7
3	16	18	8	6
4	14	15	7	5
5	12	12	6	4
6	10	9	5	3



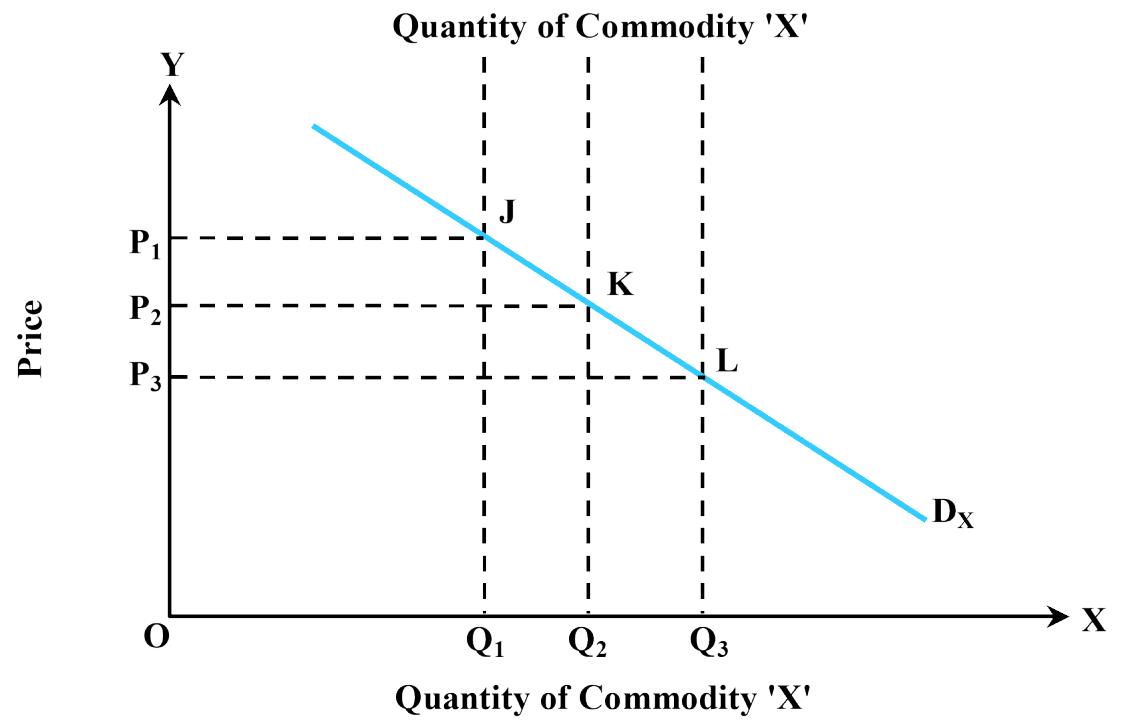
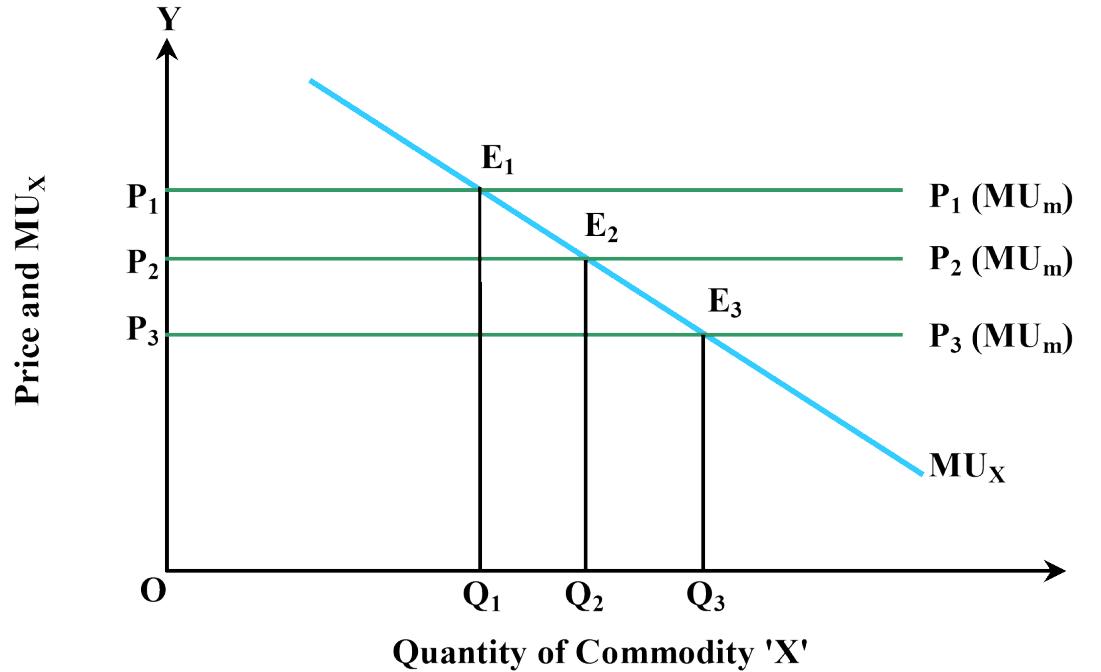
# Derivation of Demand Curve

## 1. One Commodity Model

- In Cardinal utility approach, a consumer will be in equilibrium when marginal utility of a commodity equals to price of the commodity or marginal utility of money.

$$MU_x = P_x / MU_m$$

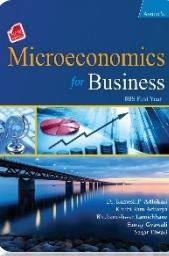
- According to the law of diminishing marginal utility, as the quantity of the commodity increases with a consumer, marginal utility diminishes. Therefore, marginal utility curve is downward sloping.
- It implies that demand curve of the commodity is also downward sloping, i.e. as the price of the commodity falls, more of it will be bought to attain equilibrium.

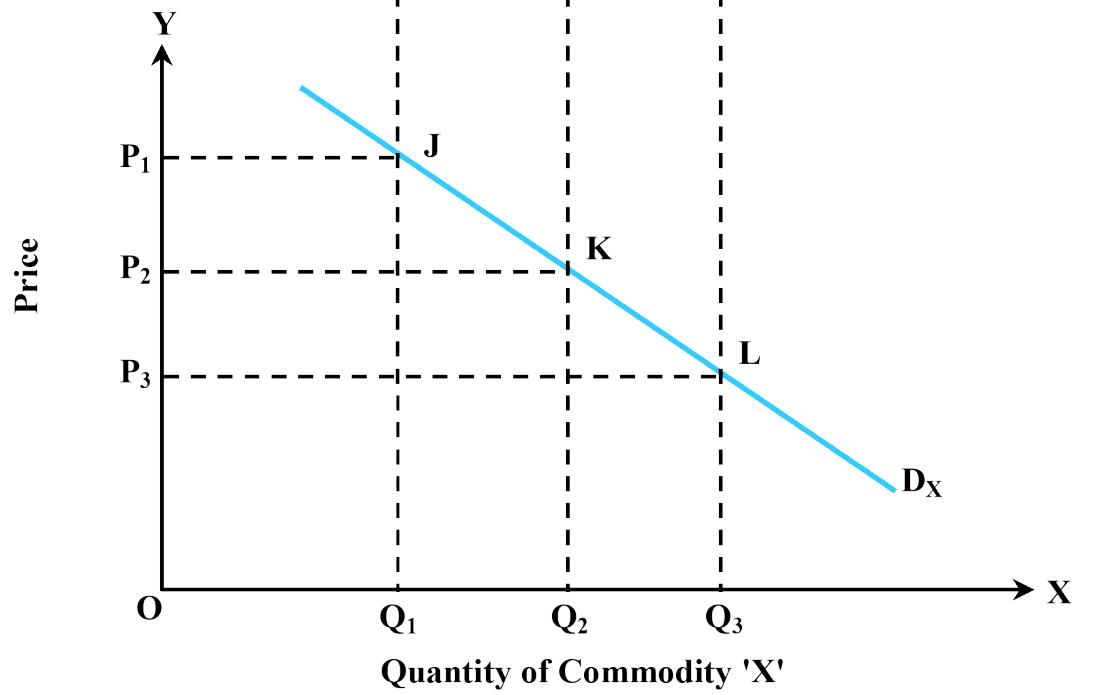
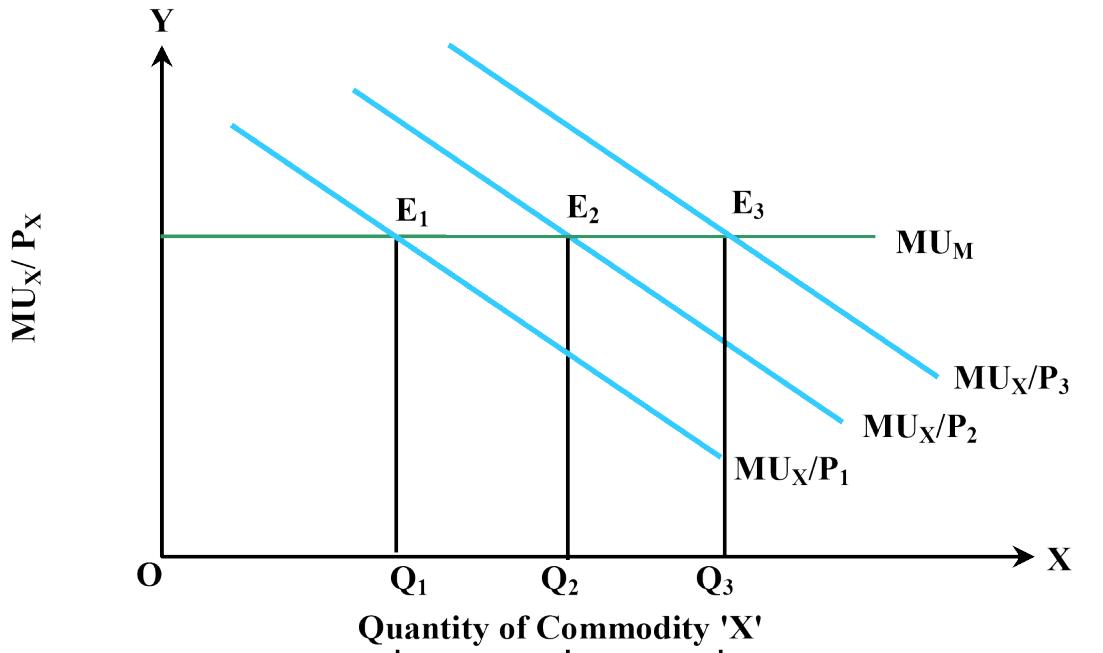


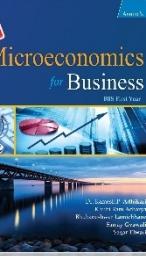
# Derivation of Demand Curve

## 2. Two Commodity Model

- The demand curve for two commodity can be derived with the help of the law of equi-marginal utility.
- It can be expressed as  $\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y} = MU_m$ .
- Let us suppose that the price of commodity X falls whereas price of Y-commodity and income of the consumer are constant.
- $\frac{MU_X}{P_X}$  will become greater than  $\frac{MU_Y}{P_Y}$  due to fall in price of X commodity.
- To restore the equilibrium, marginal utility of commodity X must be reduced.
- It is possible only by consuming more units of the commodity X.
- So, when price of the commodity falls, the consumer must purchase more units of that commodity to attain the equilibrium. Hence, demand curve is downward sloping.

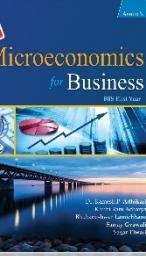






# Criticisms (Limitations) of Cardinal Utility Approach

1. Cardinal measurement of utility is not practical
2. Marginal utility of money may not be constant
3. Diminishing marginal utility is not valid for all type of goods
4. Utilities are dependent
5. No classification of goods
6. No analysis of price effect
7. Less work more assumptions



# Ordinal Utility Analysis Contd.

## Concept of Ordinal Utility

- The ordinal utility analysis believes that, as a subjective phenomena, utility only can be ranked and put in order. It is impossible to measure utility in cardinal number. It is only an expression of the consumer's preference for one commodity over another or for one basket of goods over another.
- Ordinal utility approach uses indifference curve to analyze consumers' behavior.

## Assumptions

1. The consumer must be rational
2. Ordinal measurement of utility
3. Diminishing marginal rate of substitution
4. Transitivity and consistency of choice
5. Non-satiety

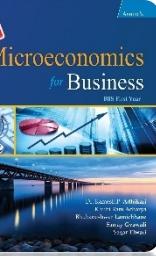
# Indifference Curve

## Definition

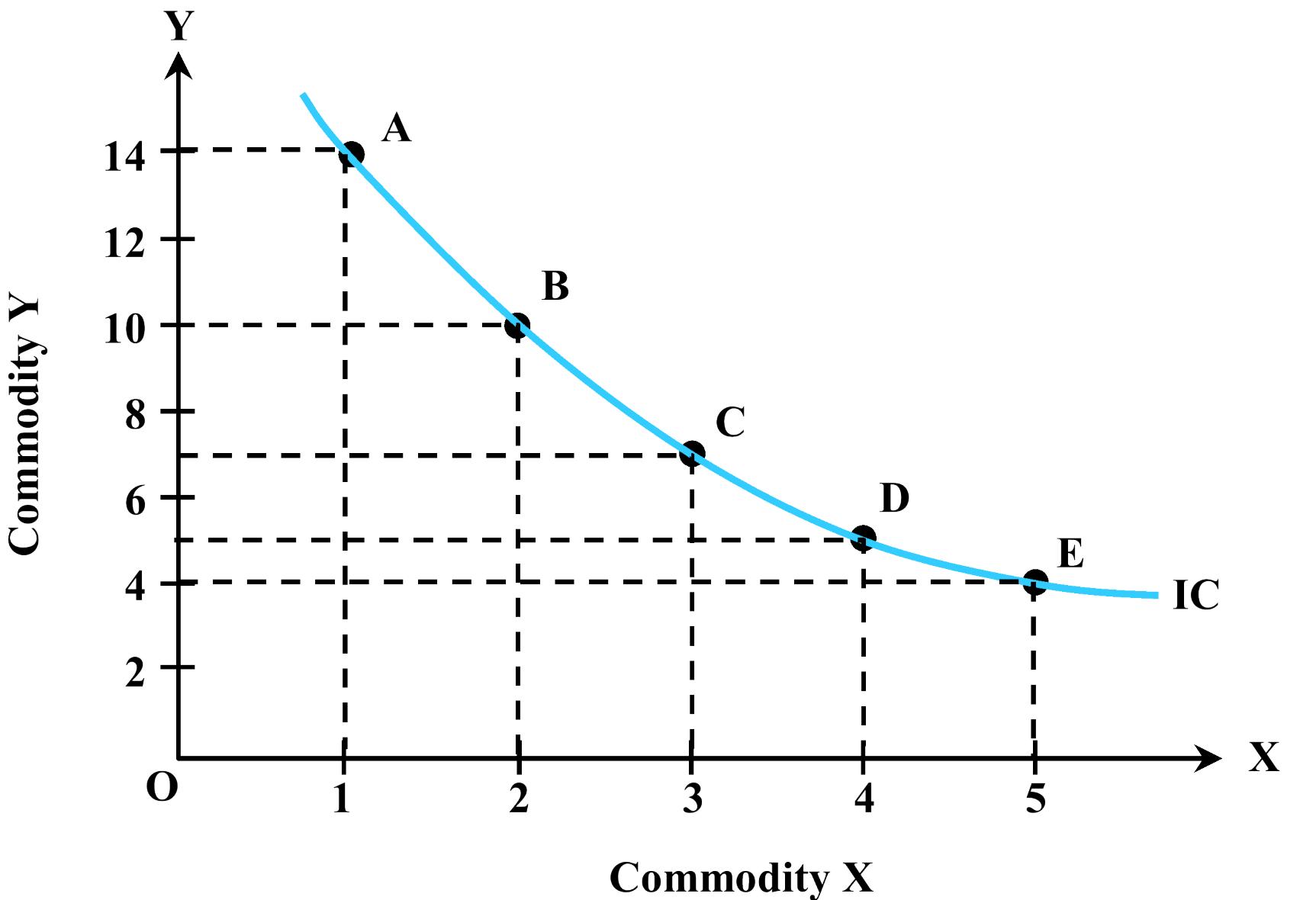
An indifference curve is the locus of all those combinations of two goods which yields the same level of utility/ satisfaction to the consumer, so that consumer is indifferent to purchase the combination s/he selects. It explains consumer's behaviour in terms of his/her preferences of ranking for different combination of two goods.

## Assumptions

1. The consumer must be rational
2. Ordinal measurement of utility
3. Diminishing marginal rate of substitution
4. Transitivity and consistency of choice
5. Non-satiety

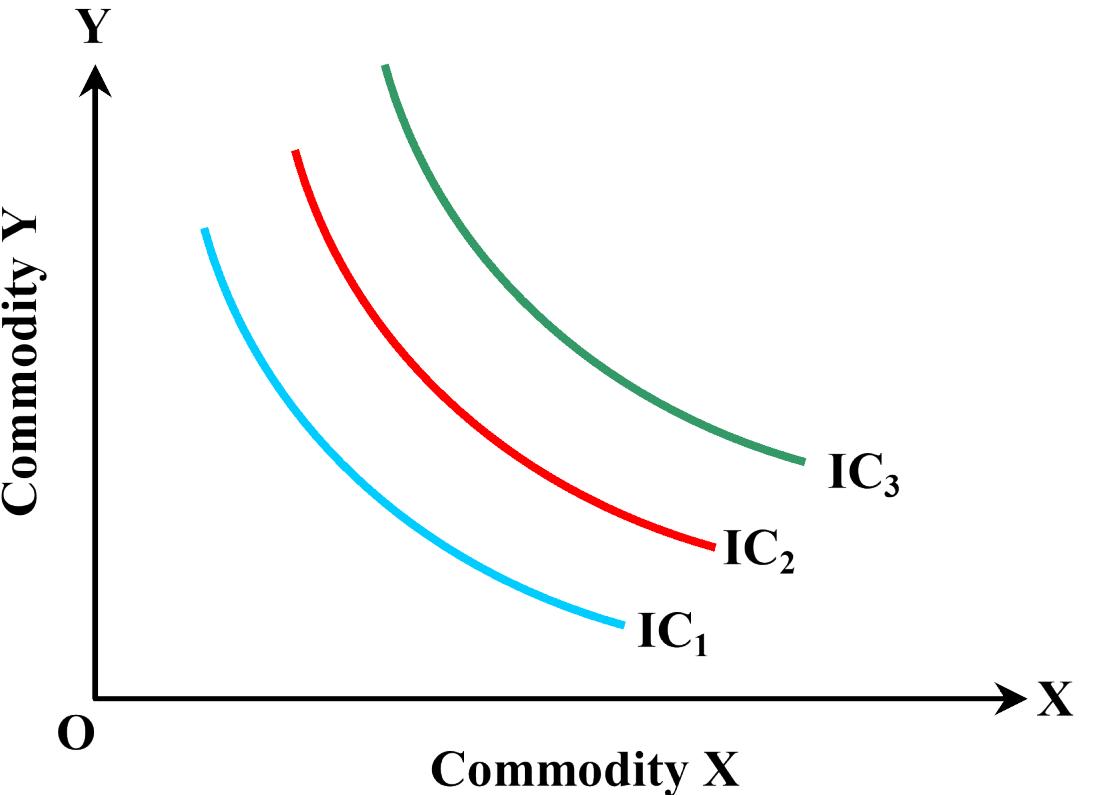


Combinations	Commodity X	Commodity Y
A	1	14
B	2	10
C	3	7
D	4	5
E	5	4



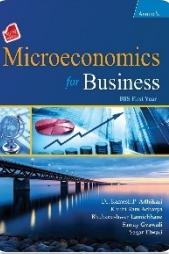
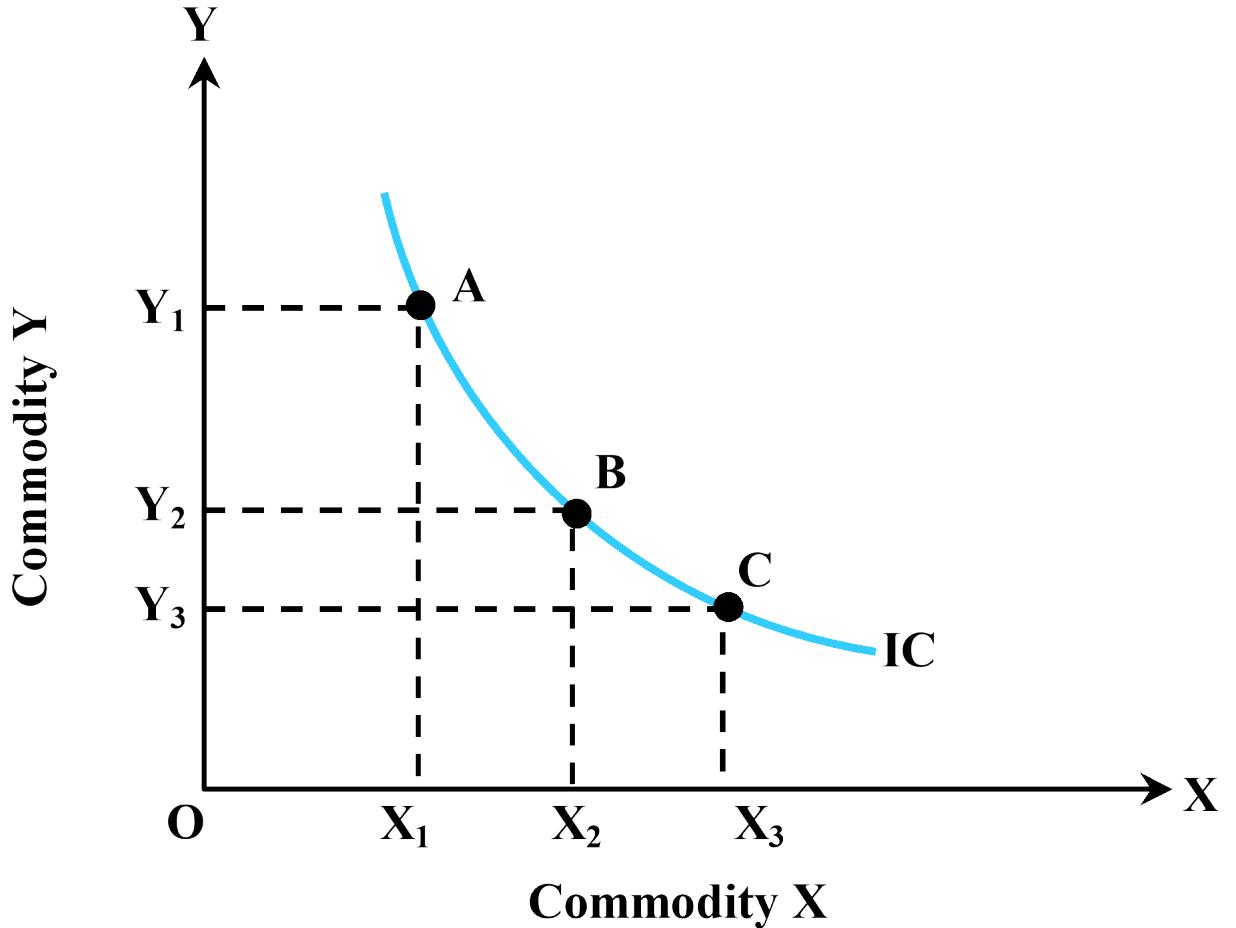
# Indifference Map

- A set of indifference curves is called indifference map.
- An indifference map shows different indifference curves which rank the preference of the consumer.



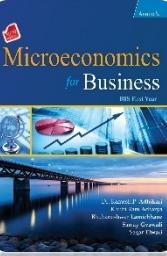
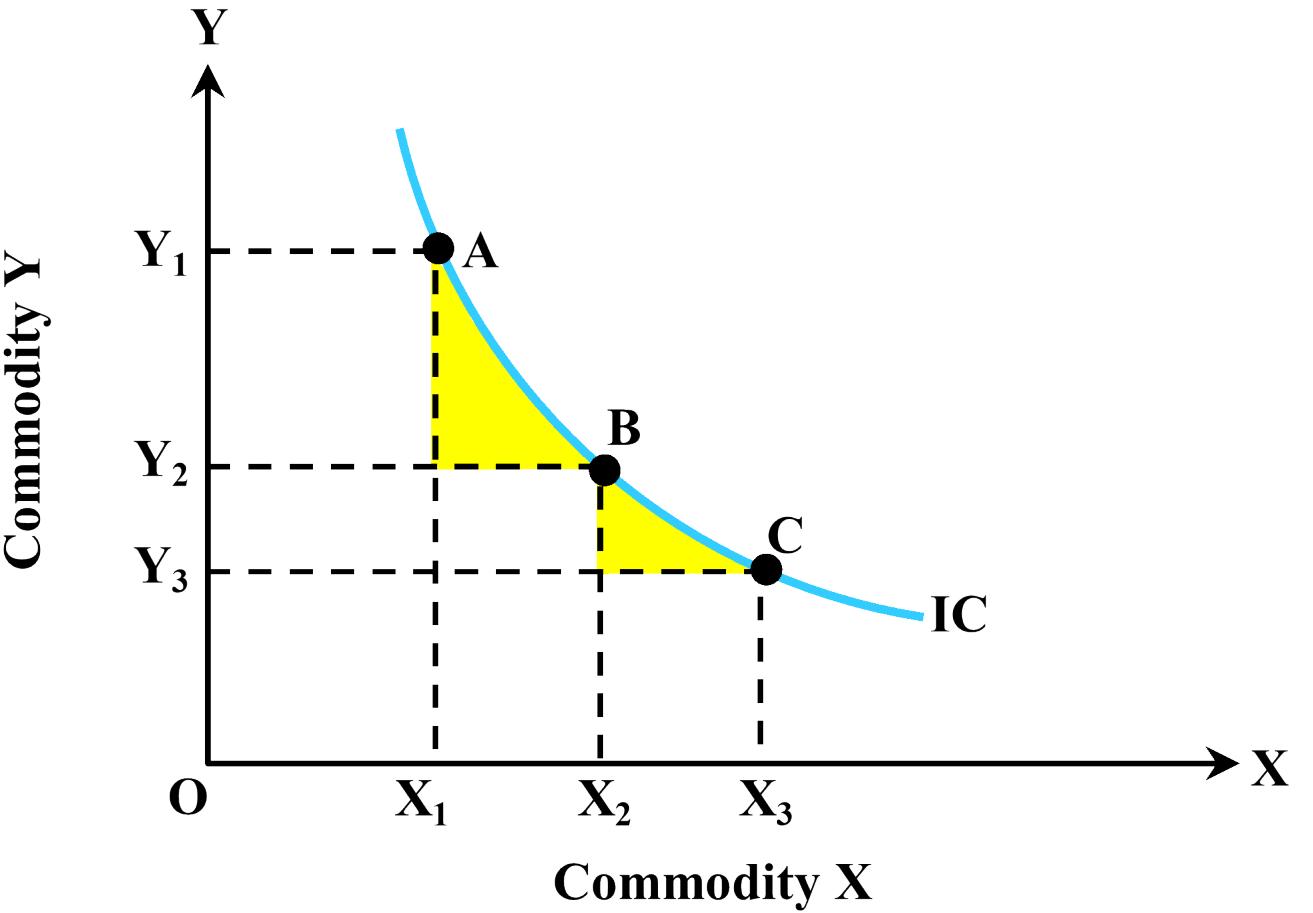
# Properties/ Characteristics of Indifference Curve

1. Indifference Curve has a negative slope



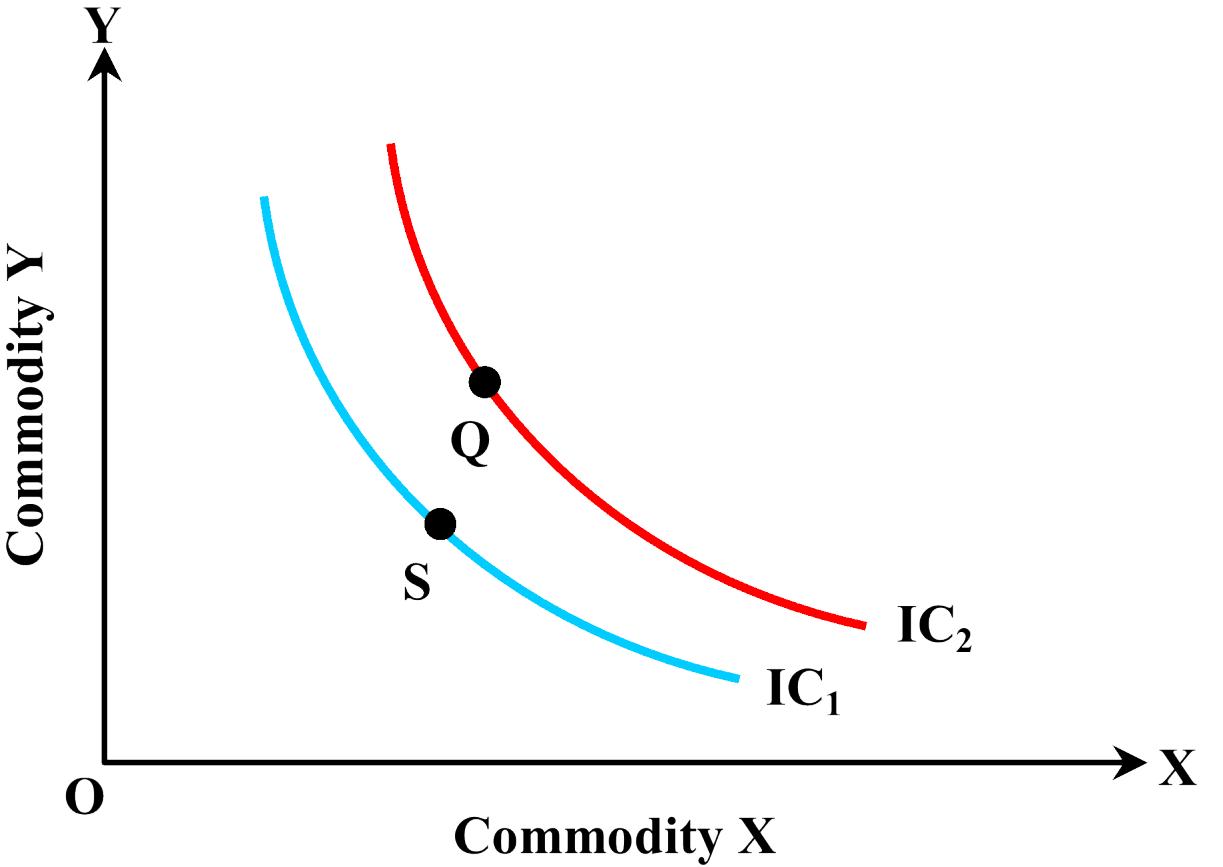
# Properties/ Characteristics of Indifference Curve Contd.

## 2. Convex to the origin



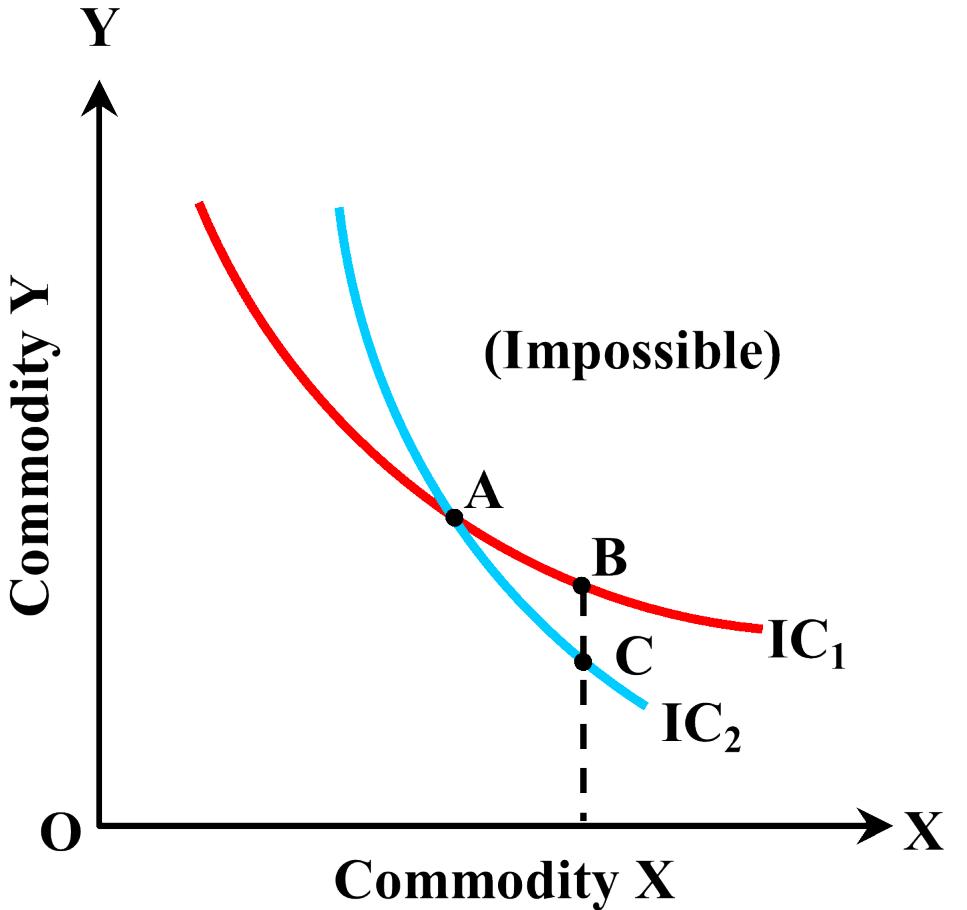
# Properties/ Characteristics of Indifference Curve Contd.

3. Two indifference curves never intersect to each other



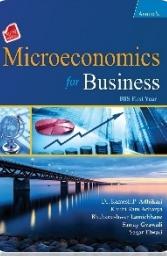
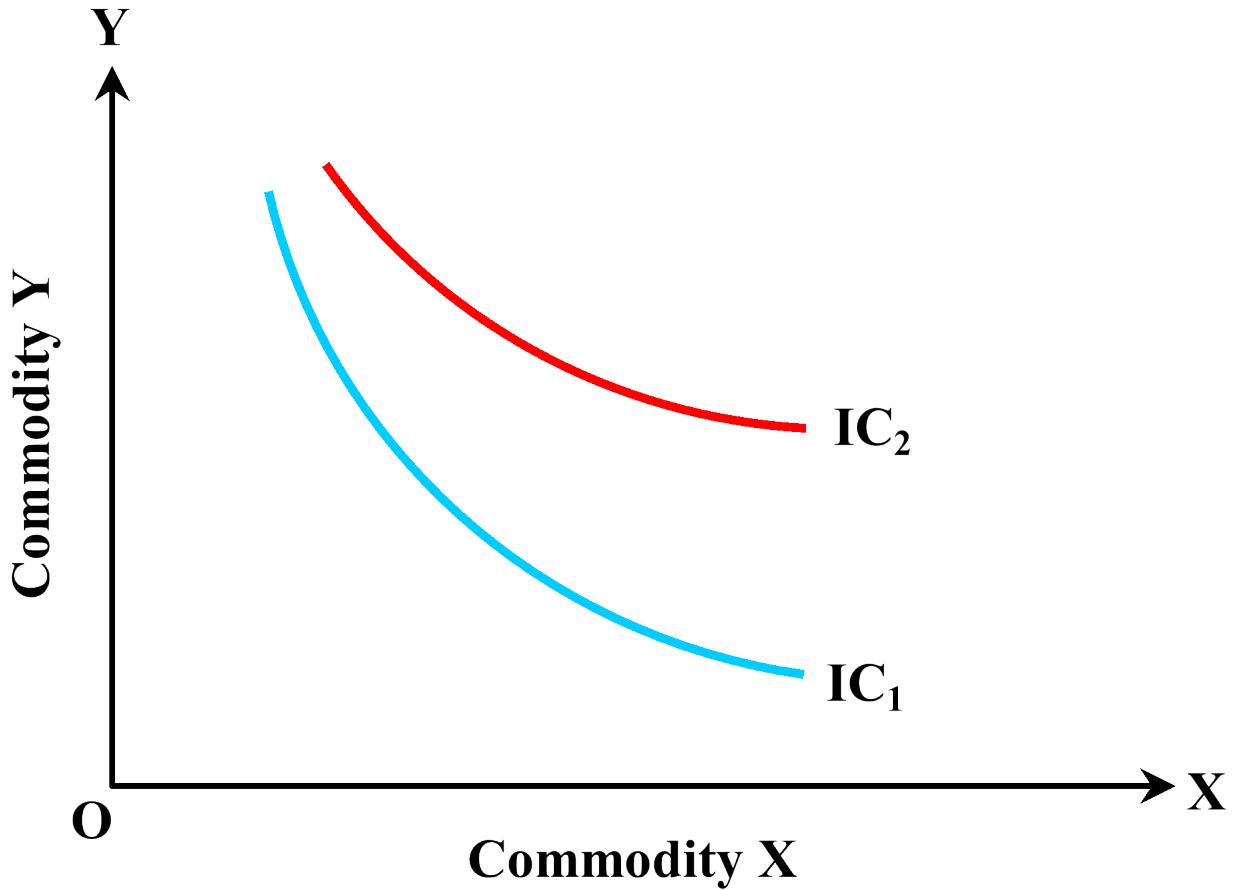
# Properties/ Characteristics of Indifference Curve Contd.

- Higher indifference curve represents higher level of satisfaction than the lower ones



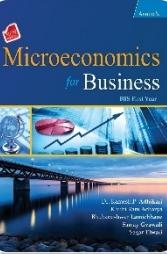
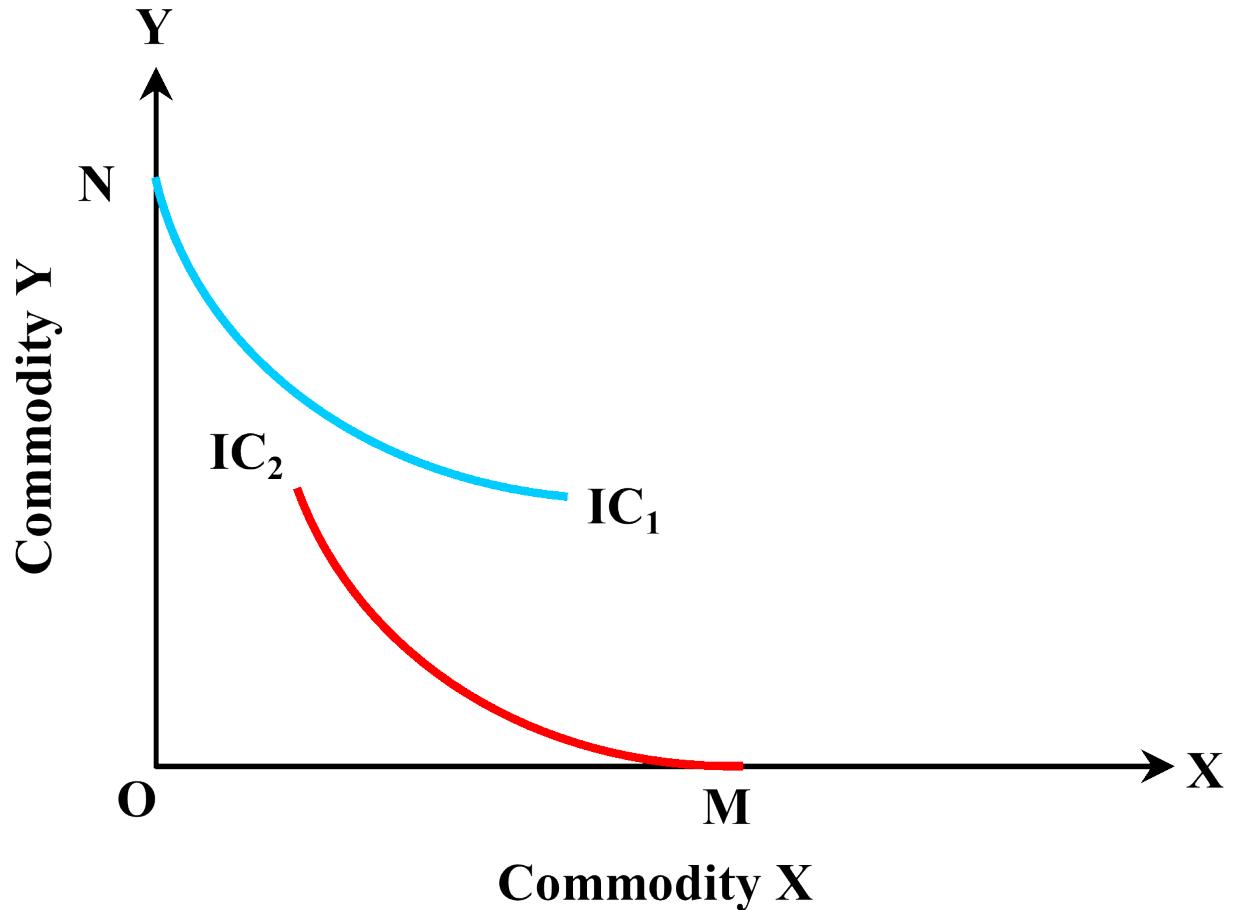
# Properties/ Characteristics of Indifference Curve Contd.

- Indifference curves are not necessarily parallel



# Properties/ Characteristics of Indifference Curve Contd.

- Indifference curve does not touch either axis



# The Marginal Rate of Substitution (MRS)

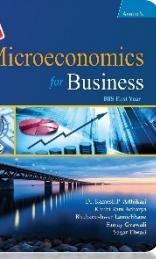
- The marginal rate of substitution is the rate at which one commodity can be substituted for another without affecting total satisfaction. is given by the slope of the Indifference curve.
- The utility function of the consumer is given as

$$U = f(X, Y)$$

where X and Y are substitutes

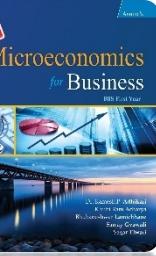
$$MRS_{XY} = - \frac{\Delta Y}{\Delta X} = \frac{MU_X}{MU_Y}$$

$$MRS_{YX} = - \frac{\Delta X}{\Delta Y} = \frac{MU_Y}{MU_X}$$

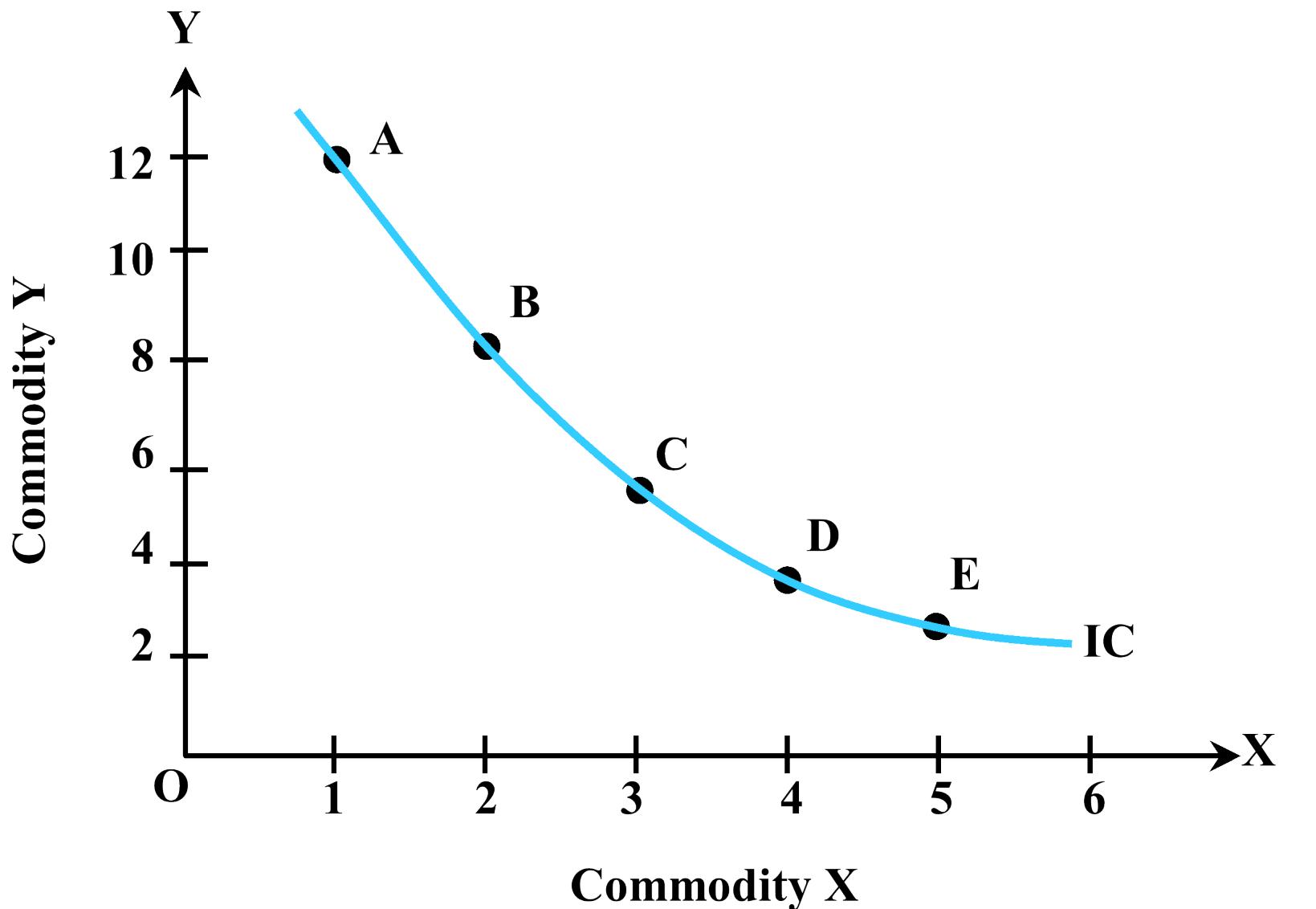


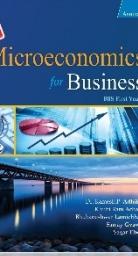
# Principle of Diminishing MRS

- The marginal rate of substitution is the amount of one good that an individual is willing to give up for an additional unit of another good without affecting the level of satisfaction.
- For example, the marginal rate of substitution of good X for good Y ( $MRS_{XY}$ ) is the amount of Y that an individual is willing to exchange per unit of X by maintain the same level of satisfaction.
- Marginal rate of substitution of X for Y diminishes as more and more units of good X is substituted for Y.
- In other words, as a consumer has more and more units of goods X he is willing to forgo less and less units of good Y.



Combination	Good X	Good Y	$MRS_{XY}$
A	1	12	—
B	2	8	4
C	3	5	3
D	4	3	2
E	5	2	1

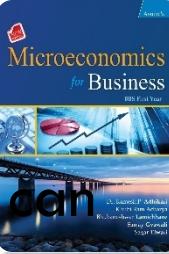




# Why does MRS Diminish?

1. The particular want is satiable.
2. Goods are not perfect substitute for each other.
3. Increase in the quantity of one good does not increase the want satisfying of the other.

# Budget Line (Price Line)



- Budget line is the locus of different combinations of two goods which a consumer purchase by spending his/her given money income and market prices.
- A consumer can purchase any combination of two goods that lies on the budget line with his/her given money income and market prices of goods.
- The budget constraint may be expressed as

$$P_X \cdot Q_X + P_Y \cdot Q_Y = M \dots (i)$$

where

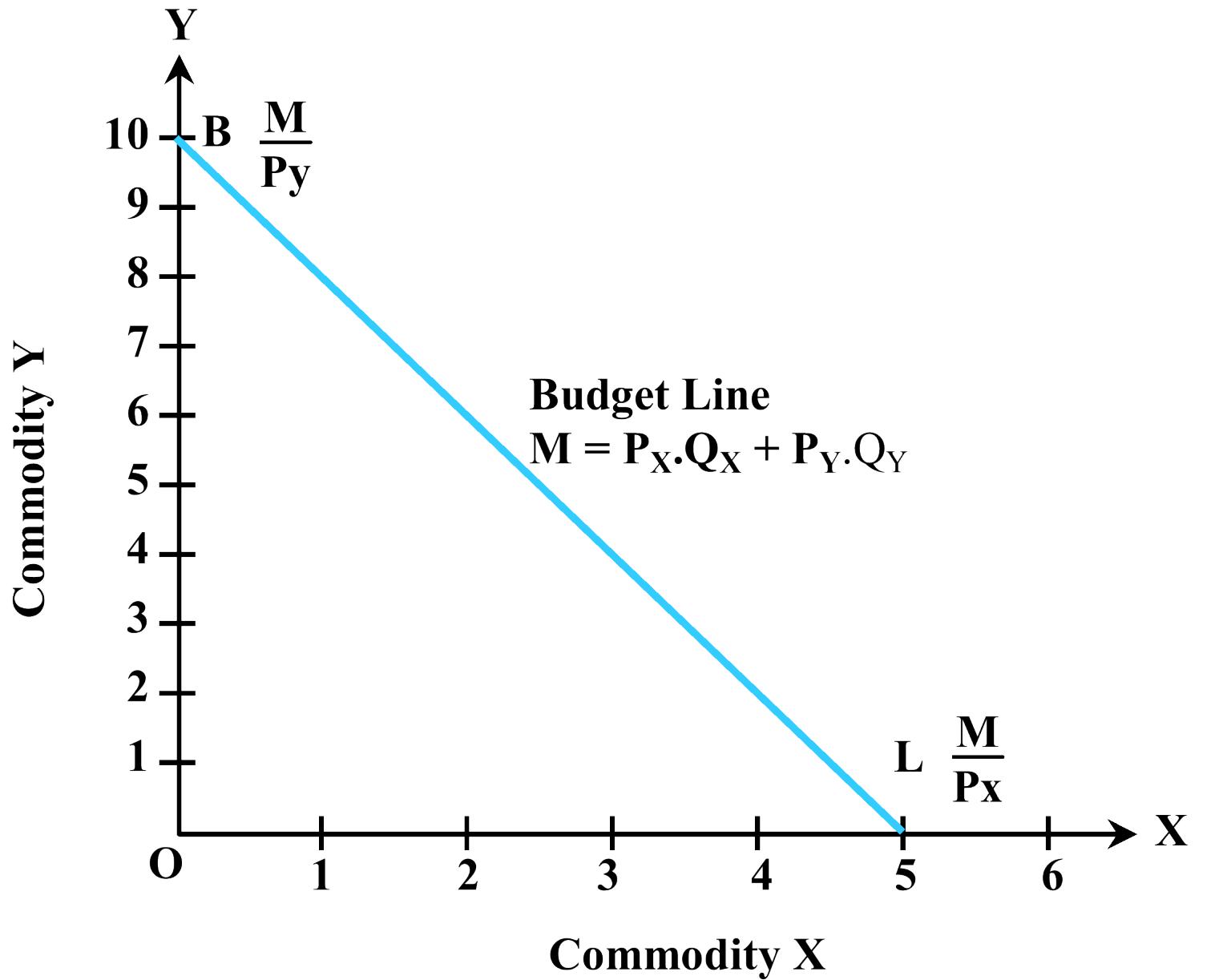
$P_X$  = Price of X commodity

$P_Y$  = Price of Y commodity

$M$  = Money income (Budget)

$Q_X$  = Quantity of X commodity

$Q_Y$  = Quantity of Y commodity



# Slope of Price Line (Budget Line)

- In order to obtain the slope of budget line, let's rewrite equation (i) as

$$Q_Y = \frac{M}{P_Y} - \frac{P_X}{P_Y} Q_X \quad \dots \text{(ii)}$$

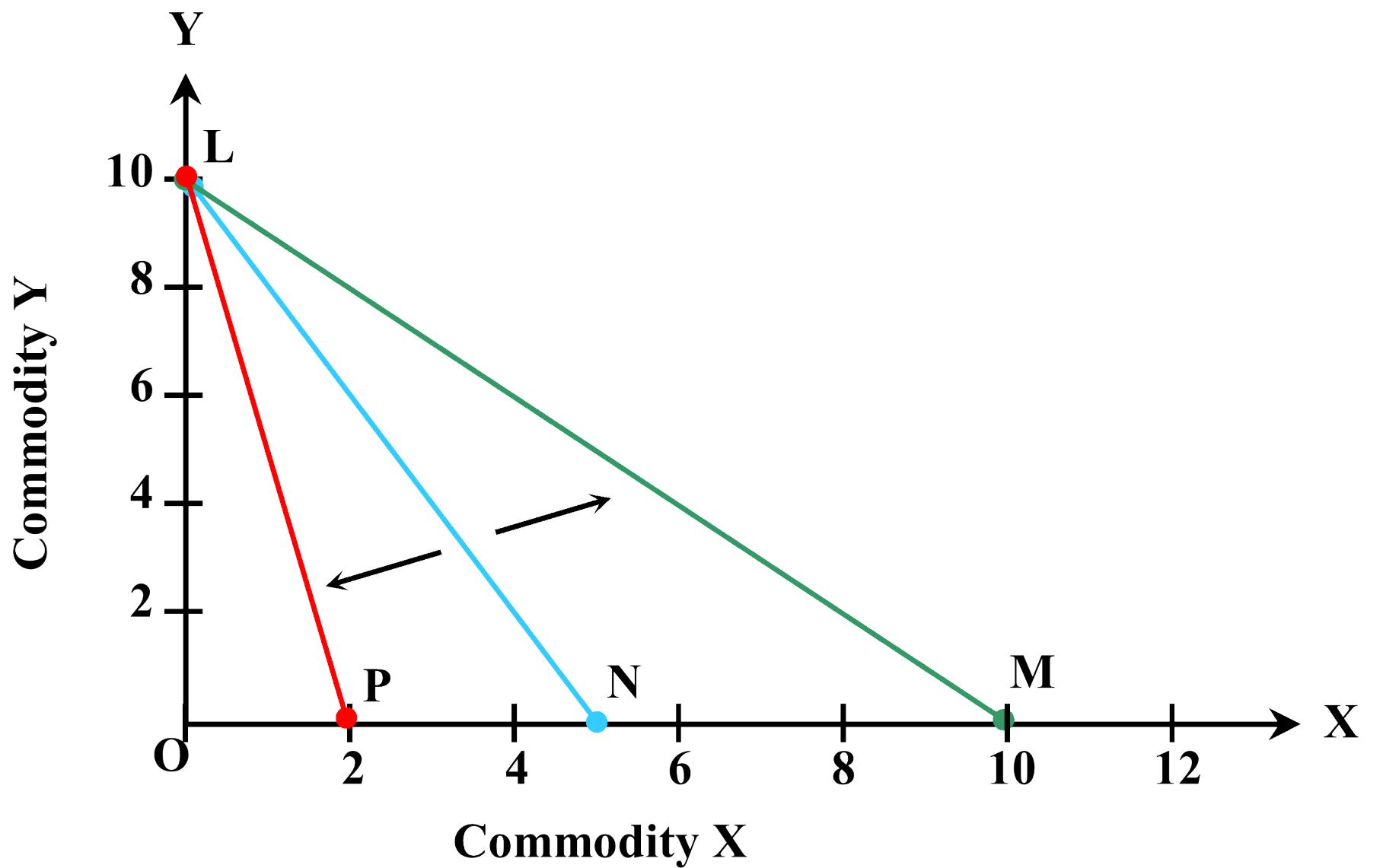
- Differentiating equation (ii) with respect to X partially, we get the slope of budget line.

$$\frac{\partial Q_Y}{\partial Q_X} = -\frac{P_X}{P_Y}$$

- Since, prices are always positive, the slope of the budget line is negative.
- Negative slope of the budget line indicates that if the consumer wants to spend more on one commodity he must decrease the money expenditure for another commodity because of the budget constant.

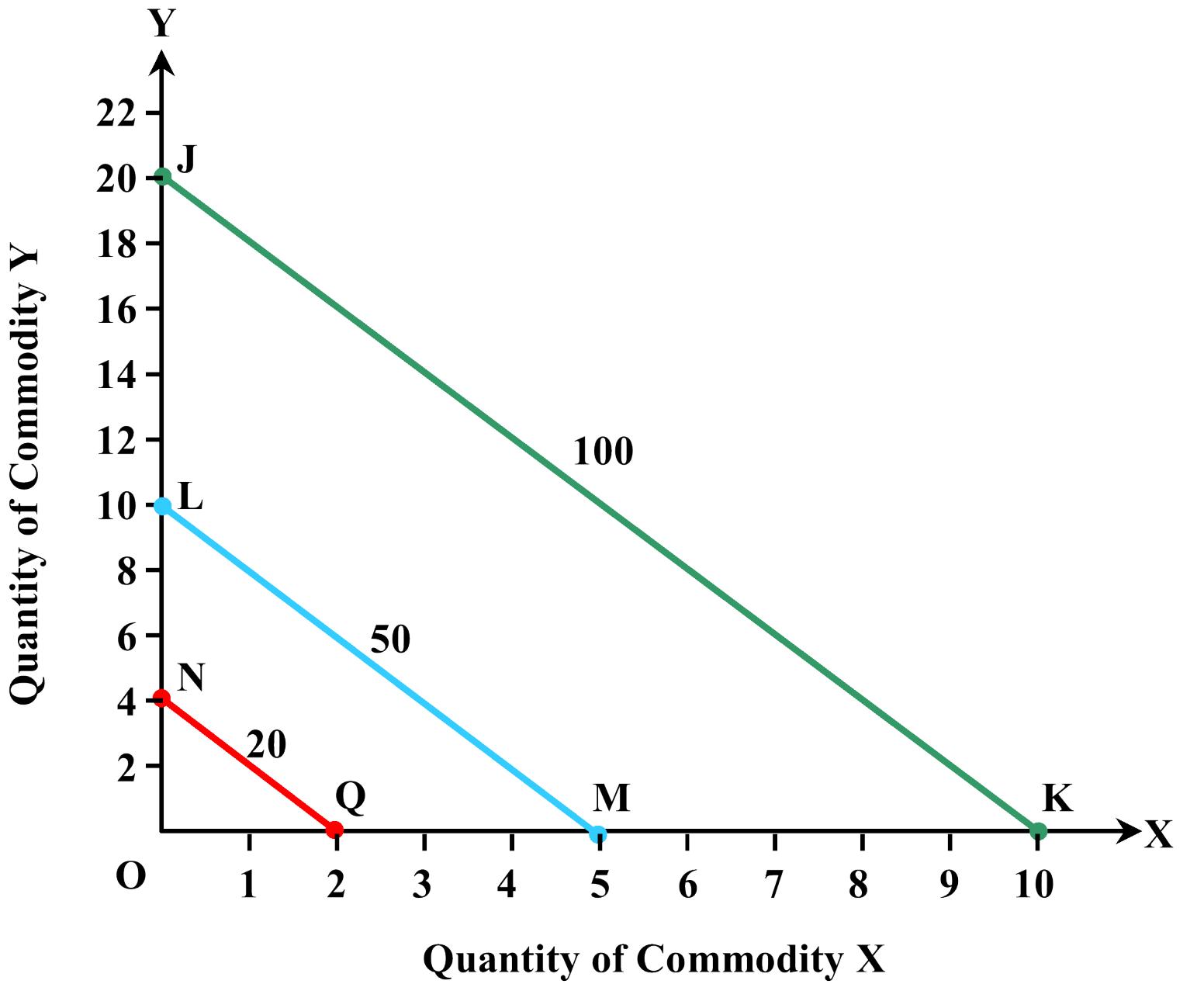
# Swing in Price Line (Budget Line)

- Swing in budget line is the process of change in budget line where one point is constant and another point moves up and down.
- Other things remaining the same, if price of the particular commodity changes then budget line will swing upward or downward.
- If price of the commodity decreases then purchasing capacity of consumer will increase, which causes the upward swing in price line.
- Similarly, when price increases then purchasing capacity of a consumer will decrease, which causes downward swing in price line.



# Shift in Price Line (Budget Line)

- Shift is the process of change in position of budget line on the both axes.
- Other things remaining the same, when income of consumer changes, there is shift in budget or price line.
- When income increases, budget line will shift rightward, where consumer can consume more units of both X and Y commodity.
- On the other hand, if income decreases, consumption level decreases, which causes leftward shift in price line.

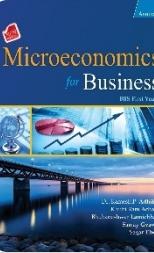


# Consumer's Equilibrium under Ordinal Utility Approach

- A consumer is said be in equilibrium when s/he maximizes his/her utility (satisfaction) given his/her income and market prices of the goods and services consumed.
- Equilibrium is the point where consumer maximized the satisfaction by consuming two goods with given money income and market prices.

## Assumptions

- The consumer has an indifference map showing his scale of preferences for various combinations of two goods. This scale of preferences remains unchanged throughout the analysis.
- He has given or constant amount of money to spend on the goods.
- Prices of the goods in the market are given and constant.
- Goods are substitutable.
- The consumer is rational.
- Marginal rate of substitution must be diminishing.



# Consumer's Equilibrium under Ordinal Utility Approach Contd.

## Conditions for Equilibrium

**1. Necessary Condition (First Order Condition):** The budget line should be tangent to the indifference curve. In other words, slope of indifference curve should be equal to the slope of budget line.

$$\text{Slope of Indifference Curve} = \text{Slope of Budget Line}$$

$$\text{or, } (-MRS_{X,Y}) = \left(-\frac{P_X}{P_Y}\right)$$

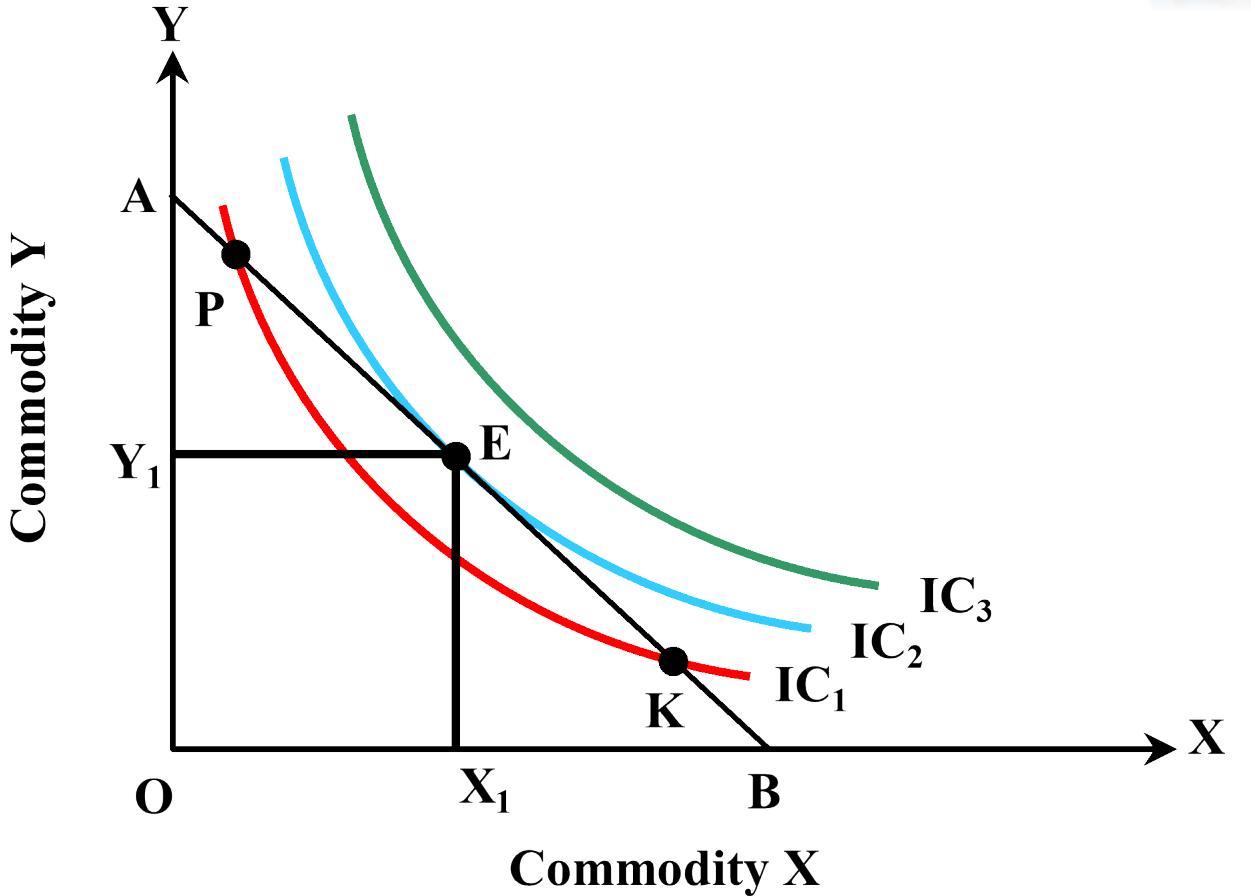
$$\therefore MRS_{X,Y} = \frac{P_X}{P_Y}$$

**2. Sufficient Condition (Second Order Condition):** Indifference curve should be convex to the origin. In other words, marginal rate of substitution must be diminishing.

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# Consumer's Equilibrium under Ordinal Utility Approach Contd.

At point E, both conditions of equilibrium are fulfilled. Therefore, this is the equilibrium point. The consumer derives maximum satisfaction by consuming  $OX_1$  quantities of good X and  $OY_1$  quantities of good Y.

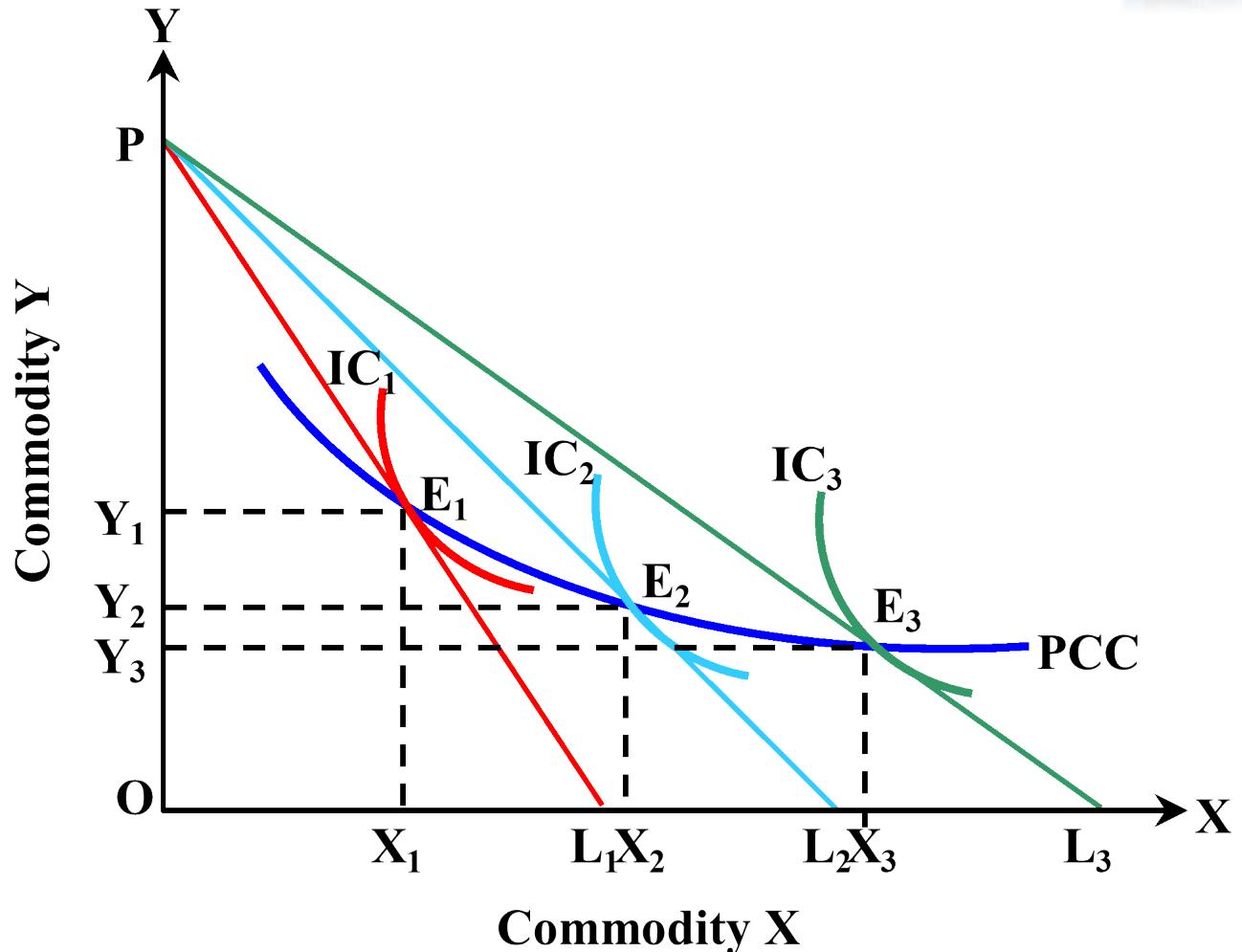


# Price Effect

- Price effect explains how the consumer reacts to change in the price of a commodity, other things remaining the same.
- When the price of the commodity changes, consumer's equilibrium position would lie in higher or lower indifference curve accordingly to fall or rise in price respectively.

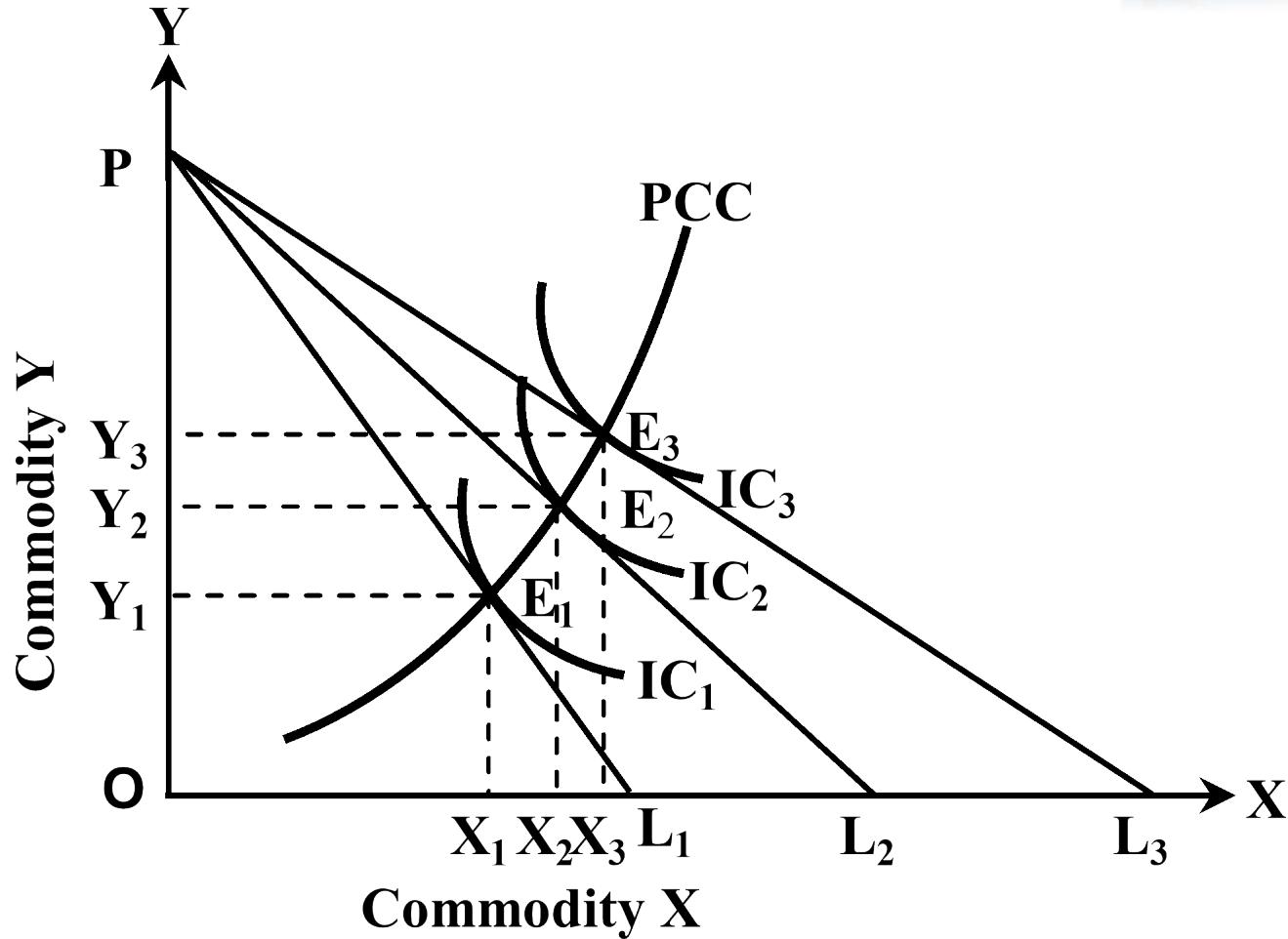
# Price Effect on Substitute Goods

- Those goods are substitute goods which can be used in place of each other to satisfy a particular want.
- In case of substitute goods, there is positive relationship between price of a commodity and demand for related commodity.
- For example, let us suppose, X and Y are substitute goods.
- If price of good X falls, the demand for good Y decreases and vice versa.
- Thus, in case of substitute goods, price consumption curve is downward sloping.



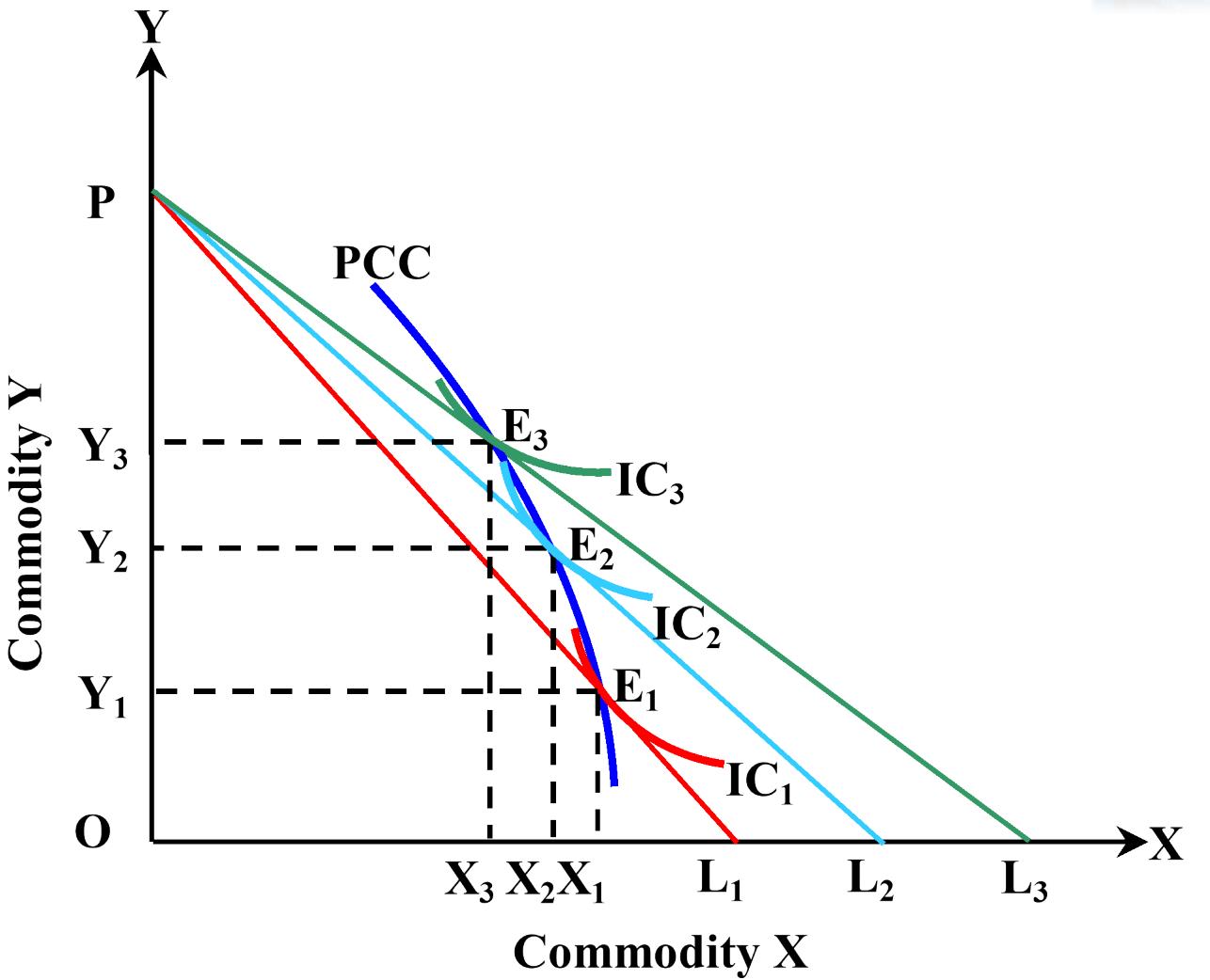
# Price Effect on Complementary Goods

- Those goods are complementary goods which are jointly used to satisfy a particular want.
- In case of complementary goods, there is inverse relationship between price of a good and demand for related good. Therefore, if the price of one good fall then the demand for related goods will increase and vice-versa.
- For example; let us suppose, X and Y are complementary goods. If price of good X falls, the demand for good Y increases and vice-versa. Therefore, in case of complementary goods, price consumption curve is upward sloping from left to right.



# Price Effect on Giffen Goods

- Those goods are Giffen goods which have positive price effect.
- It means that price and demand for Giffen goods are positively related.
- For example, let us suppose that X is a Giffen good. Therefore, as the price of X falls, the purchase of X decreases but the purchase of Y increases.
- Hence, PCC slopes backward in case of Giffen goods.

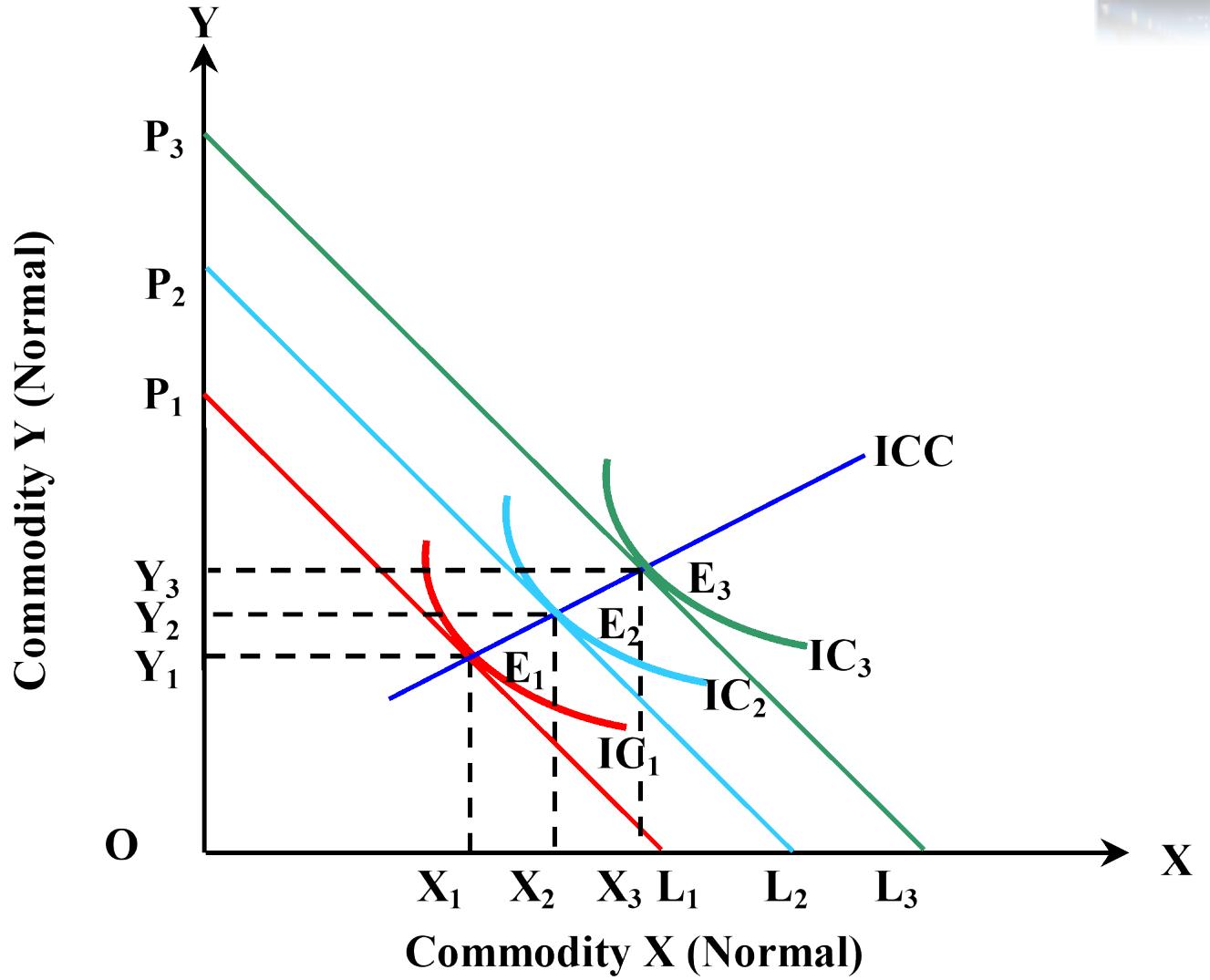


# Income Effect

- Income effect is defined as the change in quantity purchased due to the change in money income, other things remaining the same.
- If income of a consumer changes then original equilibrium point will also change.
- If money income increases, budget line shift upward where consumer will be in equilibrium at higher Indifference Curve.

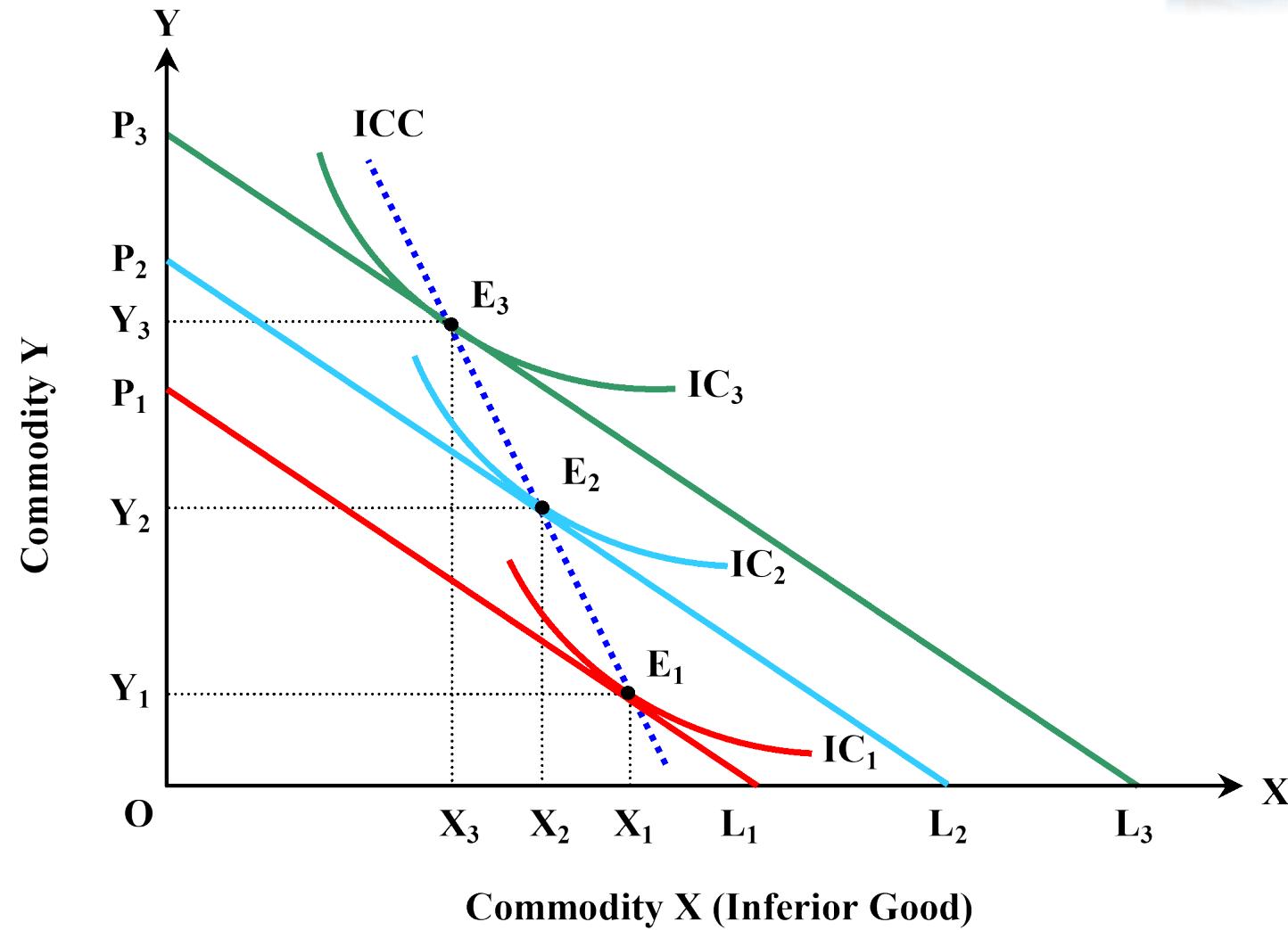
# Income Effect on Normal Goods

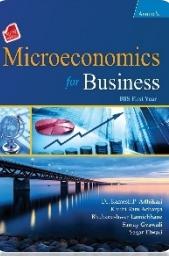
- Those goods are normal goods whose income effect is positive.
- It means that when income of a consumer increases, the quantity purchase also increases and vice-versa. Therefore, in case of normal goods, income consumption curve (ICC) is upward sloping.
- Income consumption curve is the locus of equilibrium points at different level of income.



# Income Effect on Inferior Goods

- Those goods are inferior goods, whose income effect is negative.
- It means that with the increase in income, the quantity purchase of inferior goods decreases because the consumer substitutes the inferior goods by superior goods.
- In case of inferior goods, the income consumption curve slopes backward to the left or downward to the right.



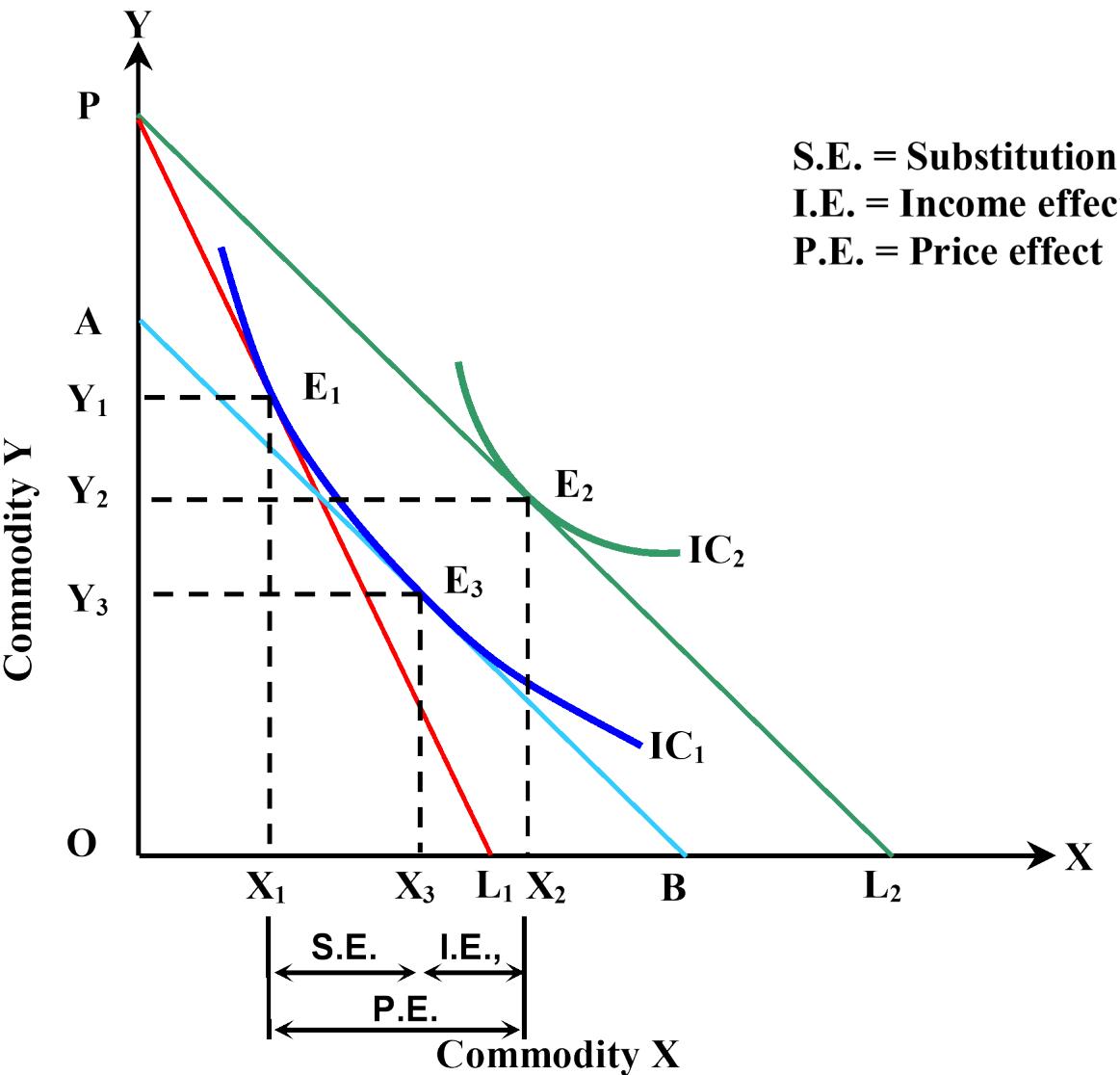


# Substitution Effect (Decomposition of Price Effect into Income and Substitution Effect)

- Substitution effect is defined as the change in purchase of a commodity as a result of a change in relative prices alone, money income remaining constant.
- If the price of a commodity changes, the real income or purchasing power of a consumer also changes.
- If there is price effect, income and substitution effects simultaneously occur.
- To find out the substitution effect, we must decompose price effect into income and substitution effect.
- For this purpose, it is necessary to keep real income of the consumer constant, price change is compensated by a simultaneous change in money income, but how?
- We have two different approaches suggested by two different economists
  - (i) Hicks and (ii) Slutsky.

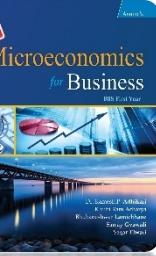
# Hicksian Approach

- In Hicksian substitution effect, money income of the consumer is changed by an amount which keeps the consumer in the initial level of satisfaction at the same indifference curve on which he was in equilibrium before the change in price.
- The amount by which the money income of the consumer is changed so that consumer stays on the same indifference curve is called compensating variation in income.
- In other words, compensating variation in income is a change in the income of the consumer which is just sufficient to compensate the consumer for a change in the price of a good.
- To find out the substitution effect, we must draw one hypothetical budget line which must be parallel to new budget line at the same time it must tangent with initial indifference curve.



$$\text{Price effect} = \text{Substitution effect} + \text{Income effect}$$

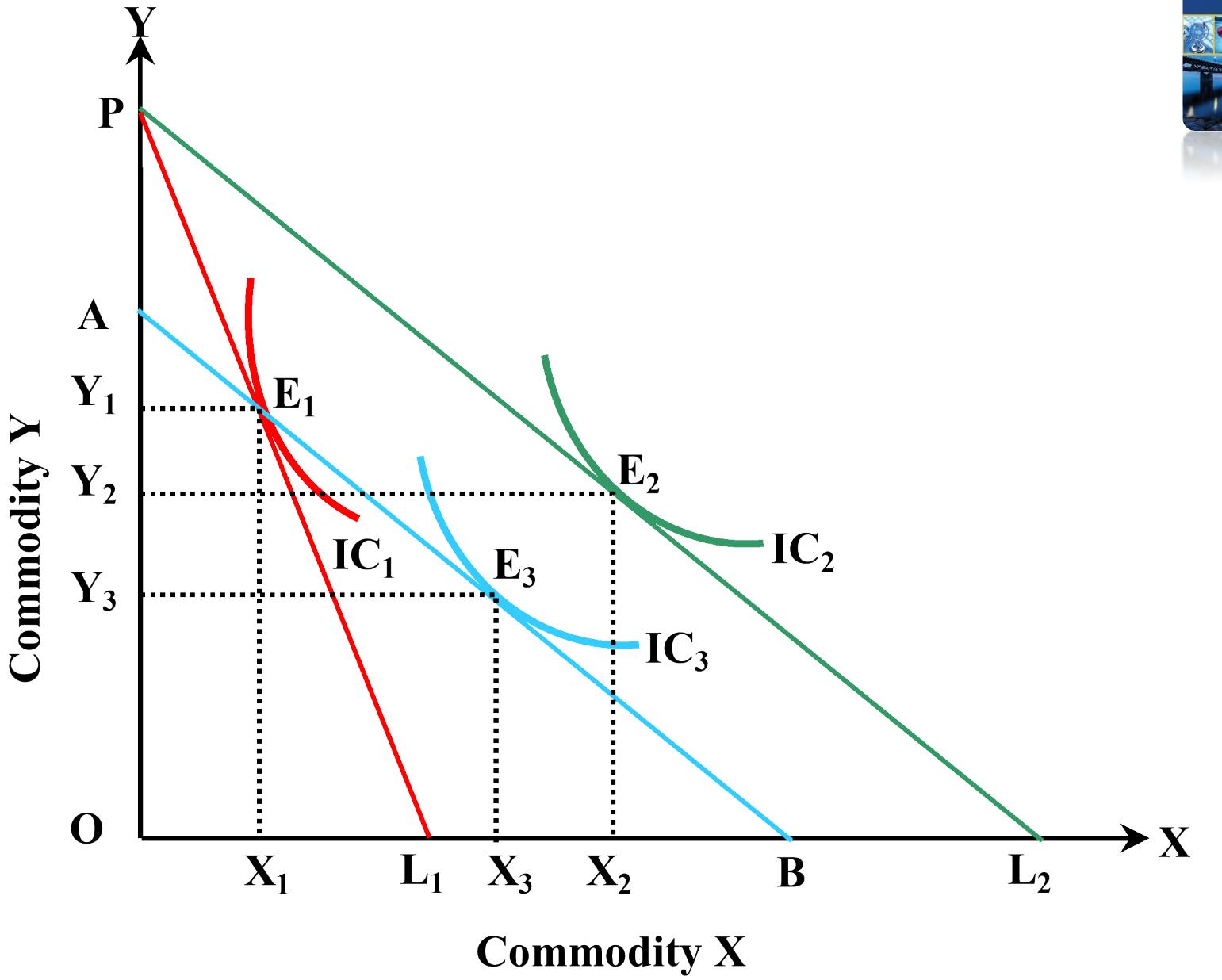
$$X_1 X_2 = X_1 X_3 + X_2 X_3.$$

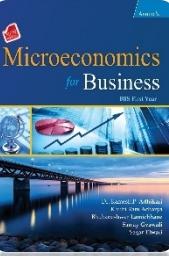


# Slutskian Approach

- In Slutsky's approach, consumer's income must be so changed that the consumer returns not only to his/her original indifference curve but also to the original equilibrium point.
- In other words, the income of the consumer is changed in such a way that the consumer can purchase the original combination of the two goods if he so desires.
- Consumer's income-adjusted budget line must pass through the initial equilibrium point on the original indifference curve.

$$\begin{aligned}
 \text{S.E.} &= \text{P.E.} - \text{I.E.} \\
 &= X_1 X_2 - X_2 X_3 \\
 \therefore \text{S.E.} &= X_1 X_3
 \end{aligned}$$



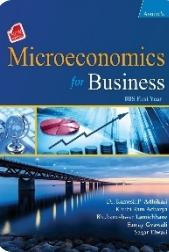
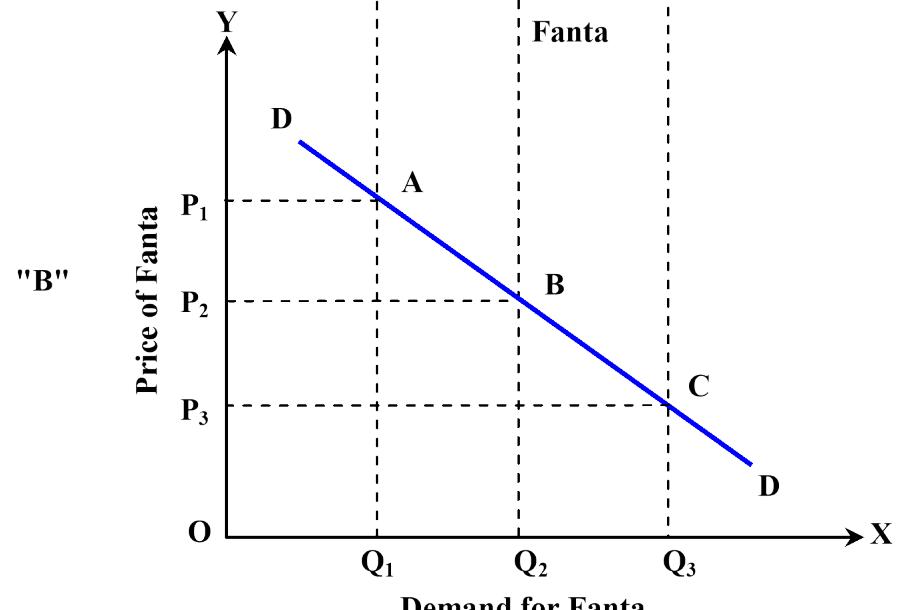
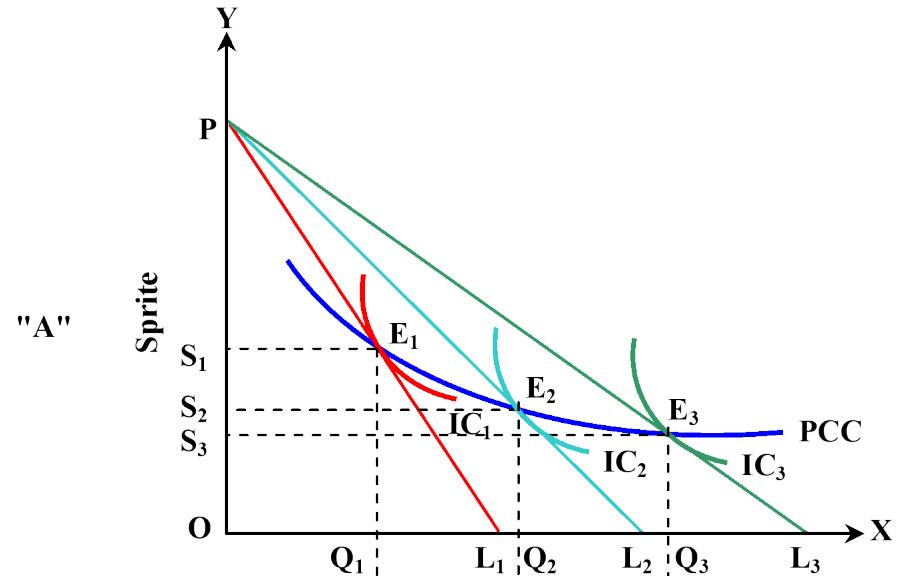


# Derivation of Demand Curve for Normal Goods

- Those goods are normal goods whose price effect is negative.
- In case of normal goods, when price of a commodity falls, the quantity purchased of that commodity will increase and vice-versa.
- Therefore, demand curve for normal goods is downward sloping from left to right. But normal goods are two types.
- They are substitute goods and complementary goods.
- We separately explain the method of derivation of demand for substitute goods and complementary goods.

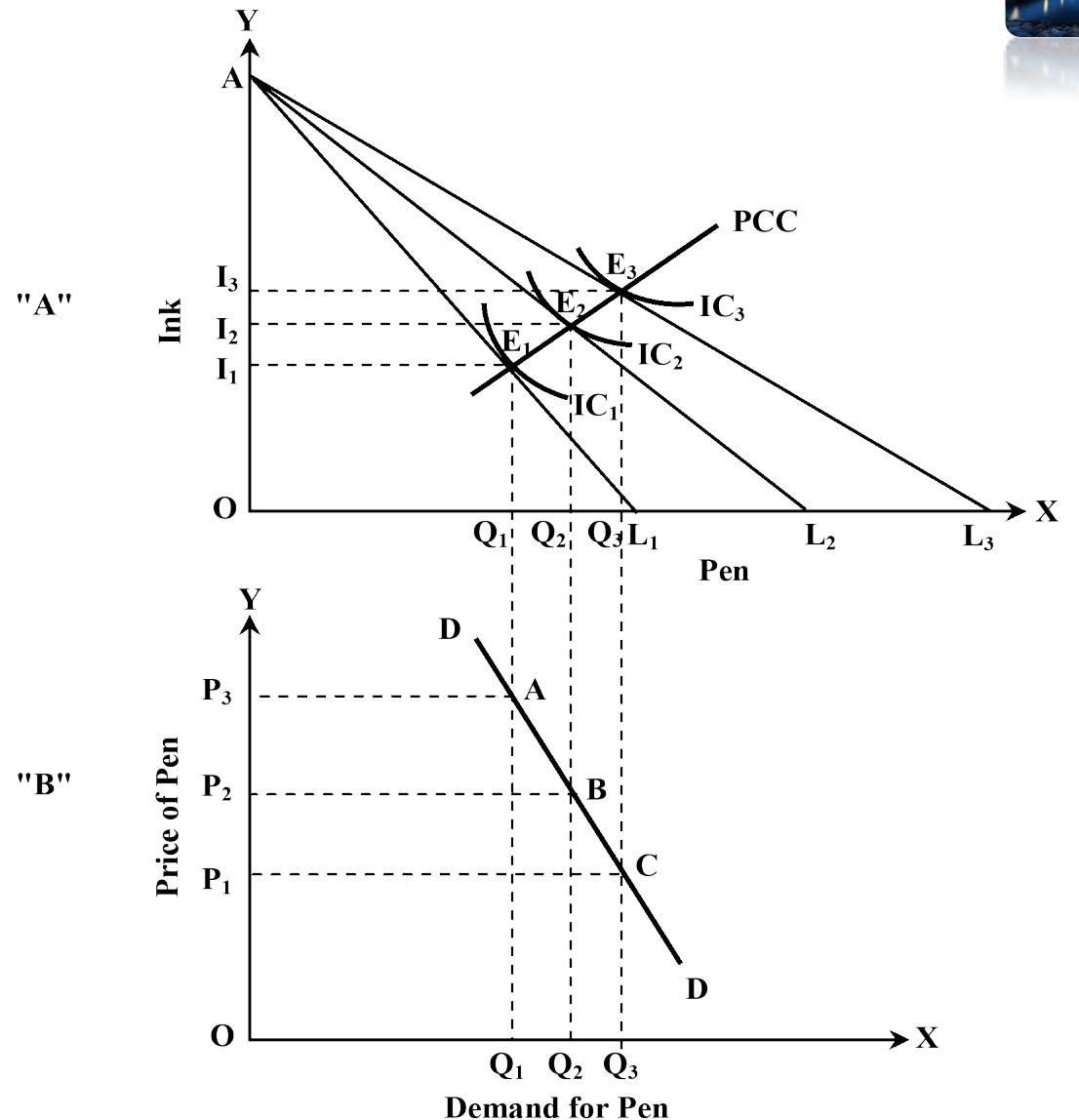
# Derivation of Demand Curve for Substitute Goods

- Those goods are substitute goods which can be used in the absence of other goods to satisfy a particular want. In case of substitute goods, there is positive relationship between price of a commodity and demand for related commodity.
- For example, let us suppose that Fanta and Sprite are substitute goods.
- If the price of Fanta is decrease, the demand for Sprite is also decrease and vice-versa.
- Therefore, price consumption curve (PCC) is downward sloping from left to right and demand curve for Fanta is also downward sloping.



# Derivation of Demand Curve for Complementary Goods

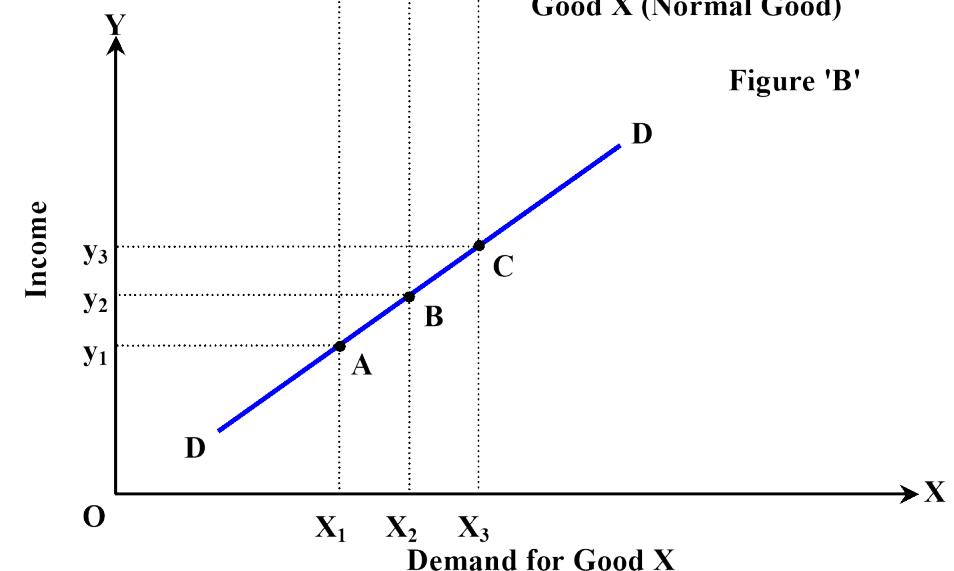
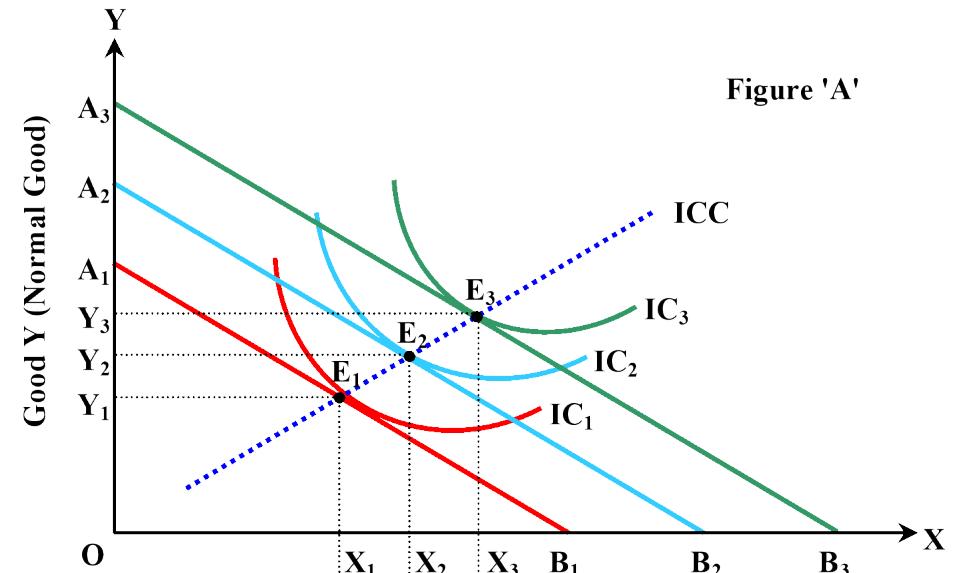
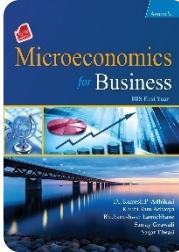
- Those goods are complementary goods which are jointly used to satisfy a particular want.
- In case of complementary goods, there is inverse relationship between price of a good and demand for related good.
- For example, let us suppose pen and ink are complementary goods. If the price of pen falls, the demand for ink is increases and vice-versa.
- Therefore, price consumption curve is upward sloping from left to right and the demand curve is downward sloping.

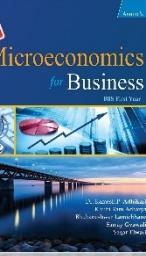


# Derivation of Income Demand Curve or Engel's Curve for Normal Goods

Normal goods are those goods whose demand or consumption increases with increase in consumer's income and vice-versa.

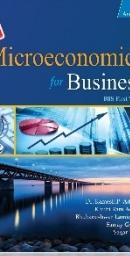
It means that in case of normal goods income effect is positive and income consumption curve (ICC) slopes upward from left to right.





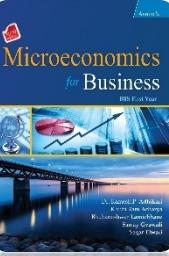
# Criticism of Indifference Curve

1. Two commodity model
2. Wrong assumption of rational consumer
3. No newness
4. Income, preference and habit changed
5. Cannot explain about uncertainty
6. Unrealistic assumption of perfect competition



# Difference between Ordinal and Cardinal Utility Analysis

1. The concept of cardinal utility analysis was developed by **H.H. Gossen**, and popularized by Alfred Marshall and other economists. According to cardinal economists, utility can be measured in number in term of money.  
The concept of ordinal utility analysis was developed by the economists **J.R. Hicks** and **R.G.D. Allen**. They have expressed that utility is only psychological or a subjective factor. So, this can be felt but not measured in a numerical form.
2. The cardinalists assumed that marginal utility of money is constant. But the ordinalists opposed this assumption. In practical life marginal utility of money cannot be constant. It may change with the change in amount of money. Rather indifference technique analyzes the income effect when the income of the consumer changes.
3. As pointed by cardinalists, utility can be measured in terms of number. But the ordinalists rejected this assumption. The ordinal approach doesn't say that utility has to be measured by using cardinal number. It says that utility cannot be measured quantitatively.



# Difference between Ordinal and Cardinal Utility Analysis Contd.

4. The ordinal approach explains price effect into income effect and substitution effect which is not possible under cardinal utility analysis. It discusses the income effect when the consumer's income changes and the price effect when price of a good changes. Thus, it discusses dual effect in the form of the income and substitution effect. But cardinal approach cannot explain dual effect.
5. Ordinal approach explains that the two commodity model which discuss about the consumer's behavior in the case of substitutes, complementary and unrelated goods. But cardinal approach explains that single commodity model.
6. The ordinal analysis explains the law of demand more realistically by considering inferior and different goods as well. This makes the IC technique definitely superior to the cardinal utility analysis.

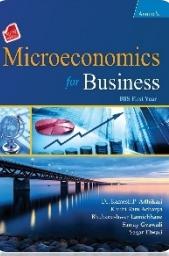
# Similarities between Ordinal and Cardinal Utility Analysis

1. Both approaches assume the rational behavior of the consumer that he seeks to attain an equilibrium position by maximizing satisfaction.
2. Both techniques used the same proportionality rule for the consumer to maximise satisfaction or to reach in equilibrium position.

According to cardinal utility analysis, the equilibrium condition is:

$$\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y} \quad \dots \text{(i)}$$

According to ordinal utility analysis, consumer is in equilibrium when his MRS is equal to price ratio, i.e.



# Similarities between Ordinal and Cardinal Utility Analysis Contd.

$$MRS_{XY} = \frac{P_X}{P_Y}$$

$$\text{But } MRS_{XY} = \frac{MU_X}{MU_Y}$$

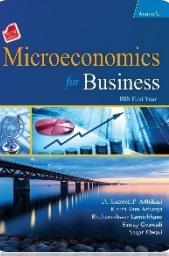
Therefore,

$$\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$$

$$\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y}$$

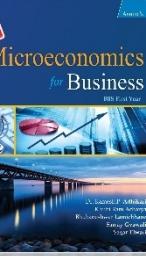
which is same with (i)

3. Both approaches assume diminishing utility, i.e. diminishing marginal utility in one case and diminishing marginal rate of substitution in another case.
4. Both approaches apply the psychological or introspective method. The law of diminishing utility which is psychological in nature lies at the bottom of law of demand being based on introspection. The indifference curve technique too, is based on introspection. Thus, both approaches are introspective.



# Superiority of Ordinal Utility (Indifference Curve Approach)

1. More realistic measurement of utility
2. No assumptions of constant of marginal utility of money
3. Study of combination of two goods
4. Less restrictive
5. More general theory of demand
6. Closer analysis of price effect



## Numerical Examples 1

Given a consumer's money income ( $M$ ), prices of  $X$  and  $Y$  are  $P_X$  and  $P_Y$  respectively.

- a. Find out quantity of  $X$  if he/she spent all his/her income on  $X$  commodity.
- b. Find out quantity of  $Y$  if he/she spent all his/her income on  $Y$  commodity.
- c. Find slope of budget line in terms of  $P_X$  and  $P_Y$ .

## SOLUTION

Given,

Price of  $X$ -Commodity =  $P_X$

Price of  $Y$ -Commodity =  $P_Y$

Budget or Income =  $M$

- a. If consumer spent his all income in X and Y- Commodities. Then we get an equation

$$P_x \cdot Q_x + P_Y \cdot Q_Y = M \quad \dots (i)$$

Where,

$P_x$  = Price of X- commodity

Quantity of X

$$P_x \cdot Q_x + P_Y \cdot Q_Y = M$$

$$\text{or } P_x \cdot Q_x = M - P_Y \cdot Q_Y$$

$$\text{or } Q_x = \frac{M}{P_x} - \frac{P_Y}{P_x} \cdot Q_Y$$

When,  $Q_Y = 0$ , then,

$$Q_x = \frac{M}{P_x} - \frac{P_Y}{P_x} \times 0$$

$$\text{or, } Q_x = \frac{M}{P_x}$$

b. Quantity of Y:

$$P_X \cdot Q_X + P_Y \cdot Q_Y = M$$

$$\text{or } P_Y Q_Y = M - P_X Q_X$$

$$\therefore Q_Y = \frac{M}{P_Y} - \frac{P_X}{P_Y} \cdot Q_X$$

When,

$$Q_X = 0, \text{ then,}$$

$$Q_Y = \frac{M}{P_Y} - \frac{P_X}{P_Y} \times 0$$

$$\text{or, } Q_Y = \frac{M}{P_Y}$$

- c. In order to obtain the slope of the budget line from equation (i)

$$P_Y \cdot Q_Y = M - P_X \cdot Q_X$$

$$Q_Y = \frac{M}{P_Y} - \frac{P_X}{P_Y} \cdot Q_X \dots \text{(ii)}$$

Differentiating equation second with respect to X partially, we get the slope of budget line as,

$$\frac{\partial Q_Y}{\partial Q_X} = \left( \frac{-P_X}{P_Y} \right) \text{ (Since, } M \text{ is constant)}$$

$$\therefore \text{Slope of budget line} = \left( \frac{-P_X}{P_Y} \right)$$

## Numerical Examples 2

Given the following marginal utility schedule of X good and Y good for the individual and given the price of X and price of Y is Re. 1 and the individual spends all income Rs. 7 on X and Y.

$Q_X$	1	2	3	4	5	6	7
$MU_X$	15	11	9	6	5	3	1
$MU_Y$	12	9	6	5	3	2	1

- a. Indicate how much of X and Y the individual should purchase to maximize utility.
- b. Show that the condition for constrained utility maximization is satisfied when the individual is at his or her optimum condition.
- c. Determine how much total utility the individual receives when he or she maximizes utility? How much utility would the individual get if he or she spent all income on X and Y?

## SOLUTION

- a. For utility maximization individual should purchase 4 units of X and 3 units of Y commodity.
- b. From answer a value of  $Q_X = 4$  and value of  $Q_Y = 3$ . Similarly  $P_X = 1$ ,  $P_Y = 1$  and  $M = 7$  is given.

Put all the values in equation (i)

$$P_X \cdot Q_X + P_Y \cdot Q_Y = M$$

$$\text{or } 1 \times 4 + 1 \times 3 = 7$$

$$\text{or } 4 + 3 = 7$$

$$\therefore 7 = 7$$

- c. We find from answer a that consumer maximized the utility when he consume 4 units of X and 3 units of Y– commodity

$$TU_X \text{ from 4 units} = 15 + 11 + 9 + 6 = 41 \text{ utils}$$

$$TU_Y \text{ from 3 units} = 12 + 9 + 6 = 27 \text{ utils}$$

$$TU = TU_X + TU_Y$$

$$TU = 41 + 27$$

$$\therefore TU = 68 \text{ utils}$$

If he consumes only good X,  $TU_X = 50$  utils

If he consumes only good Y,  $TU_Y = 38$  utils

## Numerical Examples 3

Suppose price of commodity 'X' is Rs. 100 and price of commodity 'Y' is Rs. 50 and a consumer has Rs. 2000 to spend per month on goods X and Y.

- a. Sketch the consumer's budget constraint.
- b. Assume that he splits his income equally between X and Y.  
Show where the consumer ends up on the budget constraints.
- c. Suppose that income rises from Rs. 2000 to Rs. 4000. Sketch the new budget constraint.
- d. Assume that he again splits total budget equally on two goods.  
Show where the consumer ends up on the new budget constraint.

## SOLUTION

Given,

$$\text{Budget (B)} = \text{Rs. } 2000$$

$$\text{Price of the commodity X (P}_X) = \text{Rs. } 100$$

$$\text{Price of the commodity Y (P}_Y) = \text{Rs. } 50$$

- a. Consumer's Budget constraint

The budget equation:

$$B = Q_X \cdot P_X + Q_Y \cdot P_Y$$

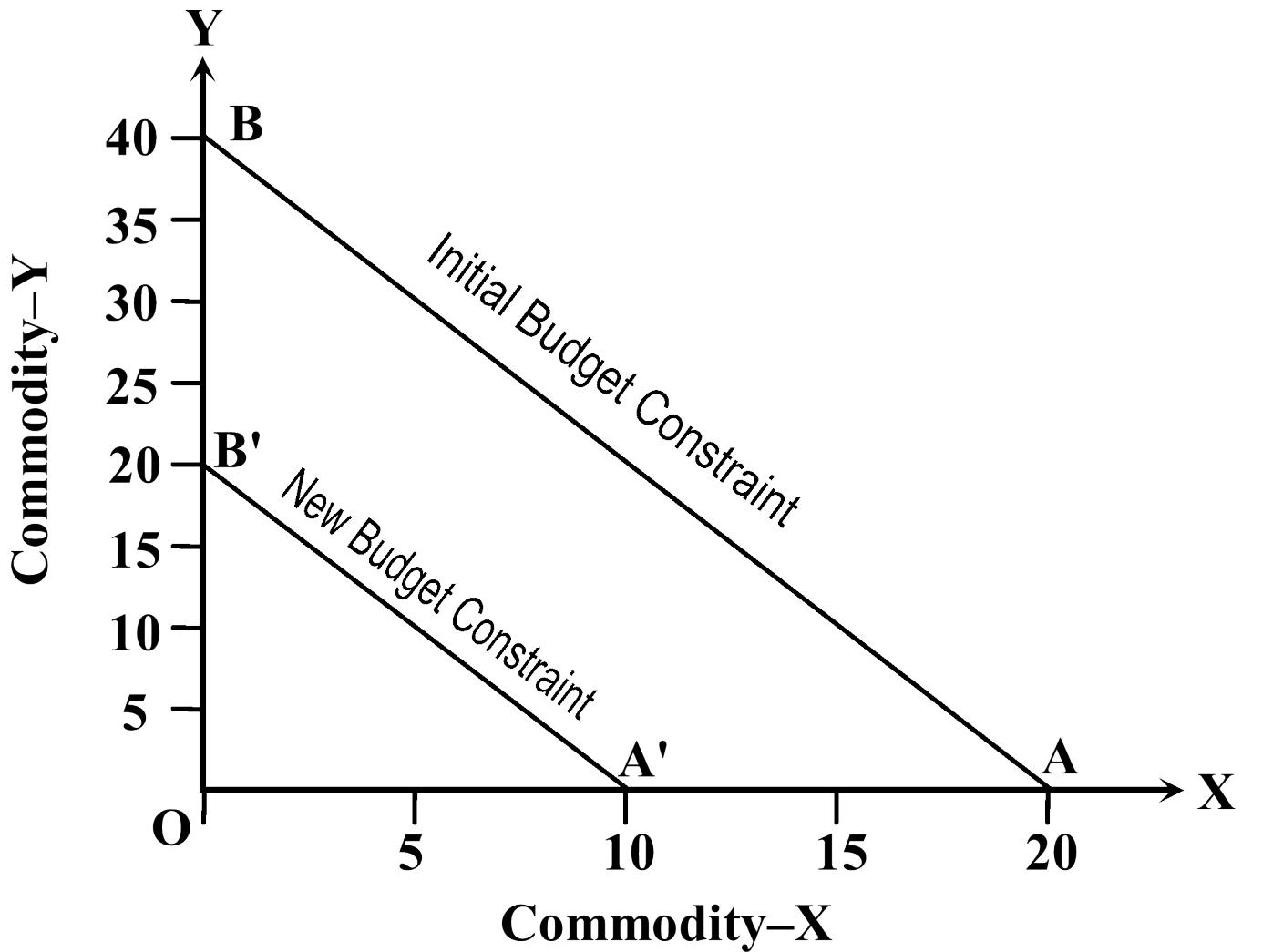
If consumer spends total budget on commodity X,

$$Q_X = \frac{B}{P_X} = \frac{2000}{100} = 20 \quad \text{Hence, A(20, 0).}$$

If consumer spends total budget on commodity Y,

$$Q_Y = \frac{B}{P_Y} = \frac{2000}{50} = 40 \quad \text{Hence, B(0, 40).}$$

The budget line is  $20 P_X + 40 P_Y = 2000$



- b. If the consumer equally divides or splits his income equally, then,

$$Q_X = \frac{B}{P_X} = \frac{1000}{100} = 10 \quad \text{Hence, A}(10, 0).$$

$$Q_Y = \frac{B}{P_Y} = \frac{1000}{50} = 20 \quad \text{Hence, B}(0, 20).$$

The budget equation:  $10P_X + 20P_Y = 1000$

The new budget constraint is A, B, in the above figure

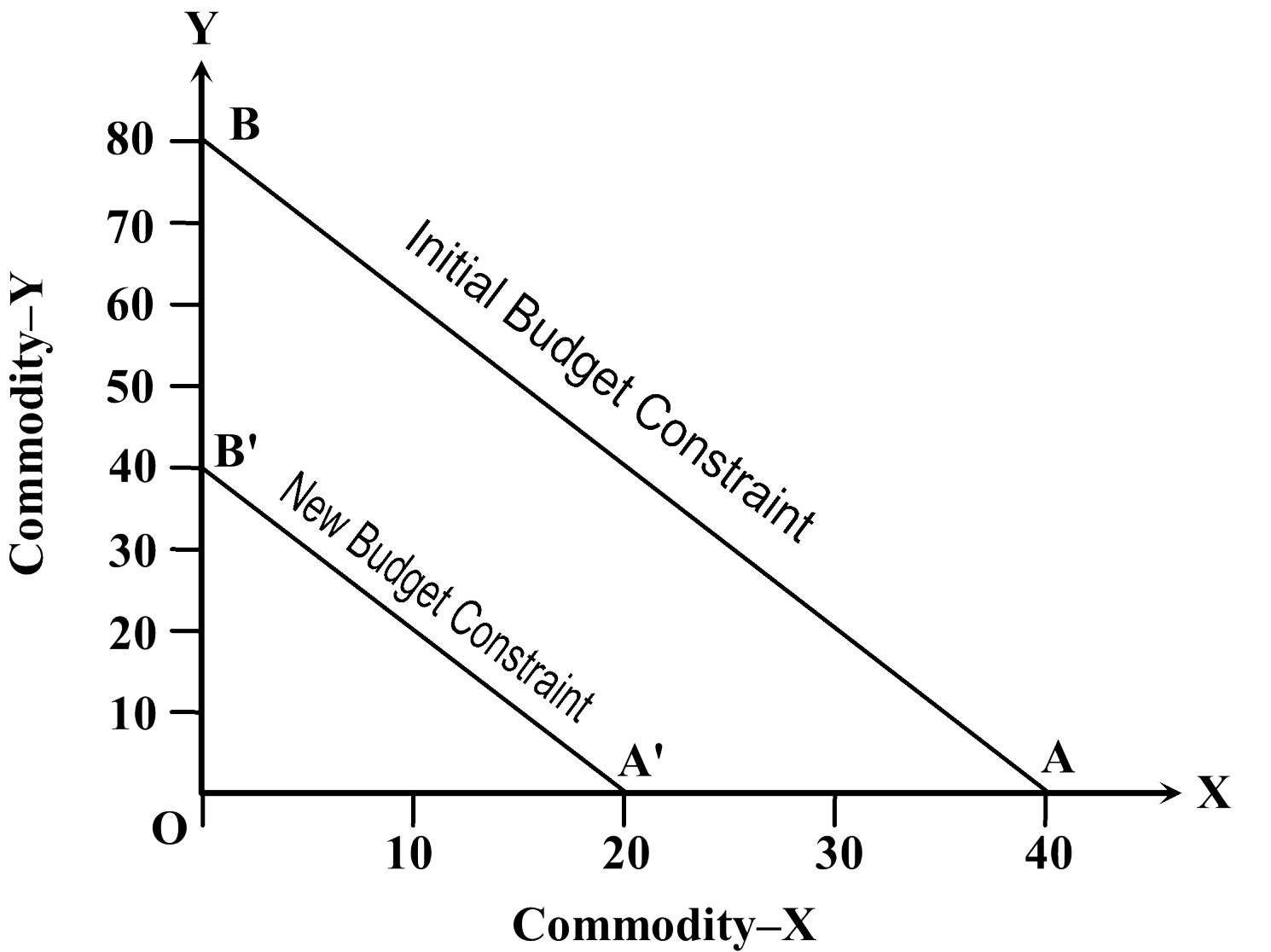
- c. New Budget/ Income (B) = Rs. 4000

Then,

$$\text{If } Q_Y = 0, \text{ then } Q_X = \frac{B}{P_X} = \frac{4000}{100} = 40. \text{ Hence, A}(40, 0).$$

$$\text{If } Q_X = 0, \text{ then } Q_Y = \frac{B}{P_Y} = \frac{4000}{50} = 80. \text{ Hence, B}(0, 80).$$

Budget equation:  $40P_X + 80P_Y = 4000$



- d. If Consumer equally splits his budget on two goods, then,

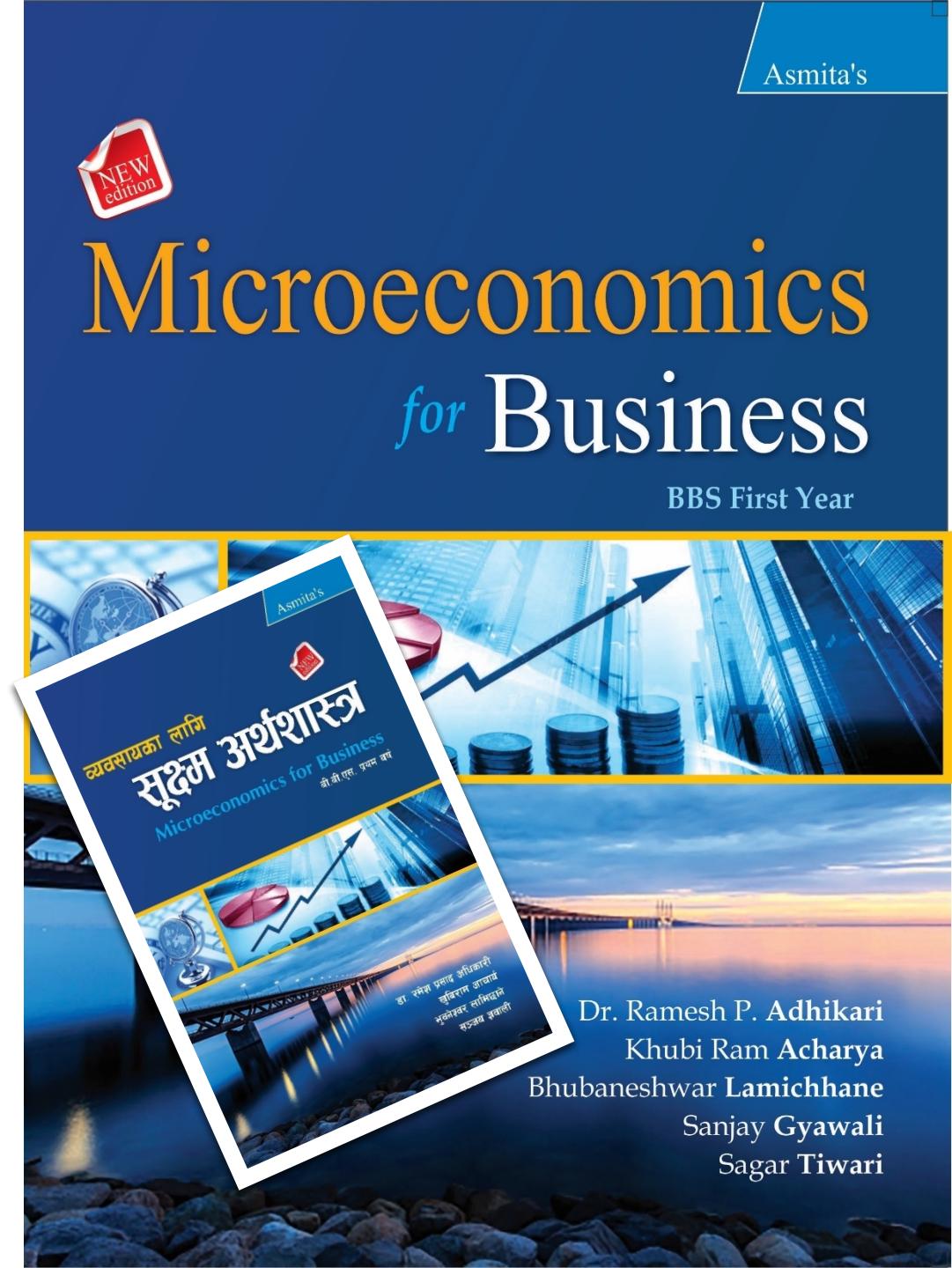
$$Q_X = \frac{B}{P_X} = \frac{2000}{100} = 20 \quad \text{Hence, } A(20, 0).$$

$$Q_Y = \frac{B}{P_Y} = \frac{2000}{50} = 40 \quad \text{Hence, } B(0, 40).$$

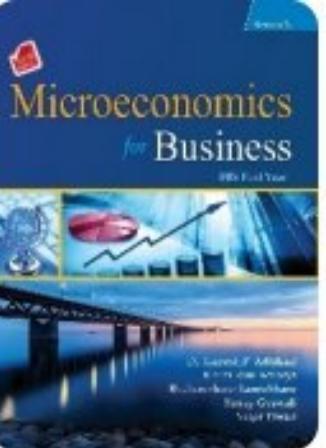
The new budget constraint is AB in the above figure

The budget equation:  $20P_X + 40 P_Y = 4000$

# Thank You

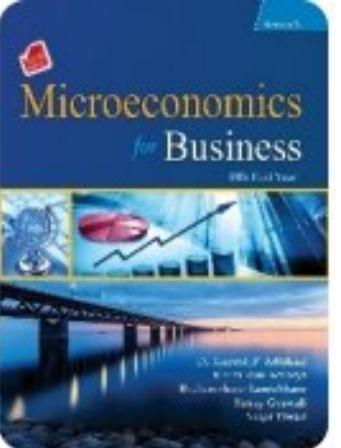


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# Concept of Utility Analysis

- Utility is the want satisfying power of a commodity. This concept was introduced by English Philosopher, **Jeremy Bentham** in 1789 to social thought and by William **Stanley Jevon** in 1871, **Walras** in 1874 and **Carl Mengar** in 1871 to economic thought.
- The concept of utility is used to analyse the consumer's tastes which is a crucial step in determining how a consumer maximizes satisfaction by spending his/her limited income.
- In order to analyze utility obtained from the consumption of goods and services, there are two basic approaches, i.e. cardinal and ordinal.

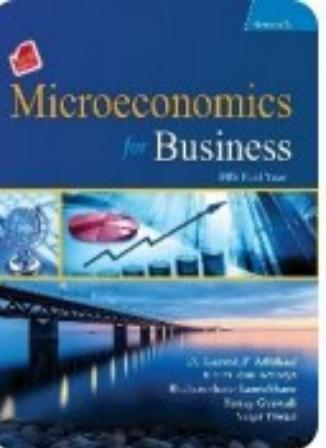


# Cardinal Utility Approach

Initially, the concept of cardinal utility analysis was developed by **Hermann Heinrich Gossen** and popularized by famous neoclassical economist **Alfred Marshall**. According to **Marshall**, utility is a subjective phenomenon and it can be quantitatively measured by means of money as a measuring rod. Neoclassical economists believed first that utility can be measured cardinally.

## Assumptions

1. Rational consumers
2. Cardinal measurement
3. Constant marginal utility of money
4. Diminishing marginal utility
5. Additivity of utility



# Concept of Total Utility and Marginal Utility

## Total Utility (TU)

- Total utility is the sum of the utility derived from the consumption of given units of a commodity.
- In other words, it is a sum of marginal utility.
- For example, if a consumer consumes  $n$  units, then his/her total utility from  $n$  units may be expressed as

$$TU = MU_1 + MU_2 + \dots + MU_n$$

where

TU = Total Utility

MU = Marginal utility of the commodity

# Concept of Total Utility and Marginal Utility Contd.

## Marginal Utility (MU)

- Marginal utility is the change in the total utility resulting from the consumption of one additional unit.

$$MU = \frac{\Delta TU}{\Delta N}$$

where

$\Delta TU$  = Change in total utility

$\Delta N$  = Change in units of consumption

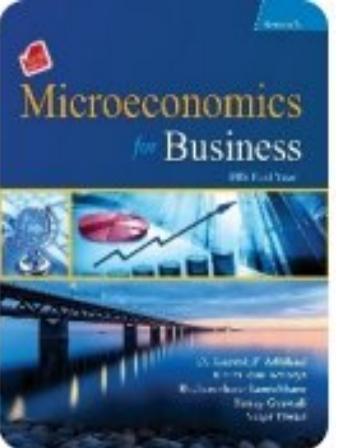
- In other words, marginal utility is the addition to the total utility derived from the consumption of an additional unit of a commodity.

$$MU = TU_n - TU_{n-1}$$

where

$TU_n$  = Total utility derived from the consumption of  $n^{\text{th}}$  unit of a commodity.

$TU_{n-1}$  = Total utility derived from the consumption of  $(n - 1)^{\text{th}}$  unit of a commodity.



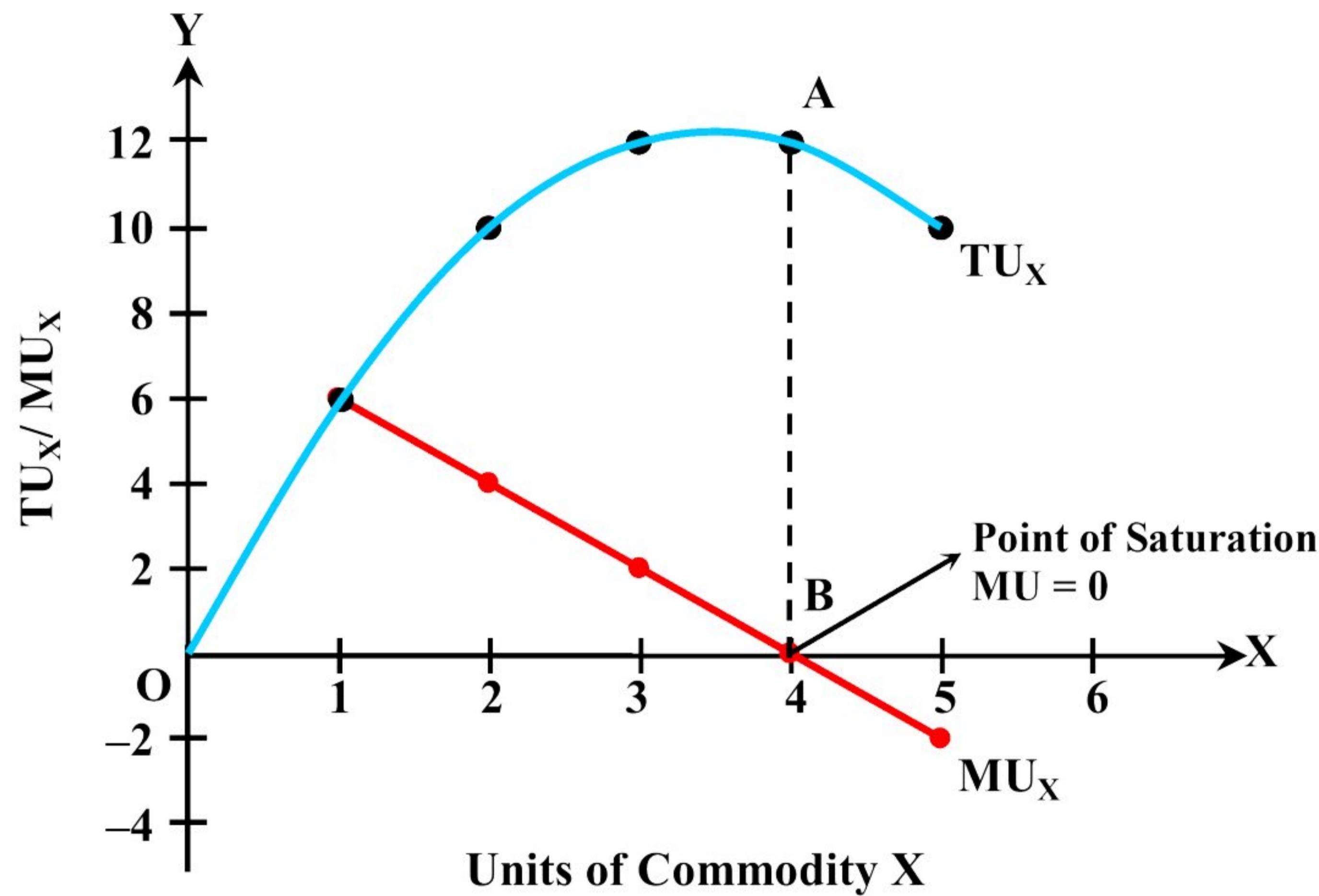
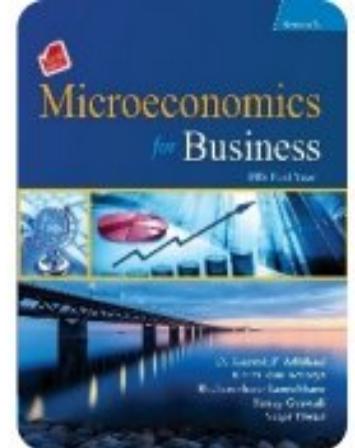
# Concept of Total Utility and Marginal Utility

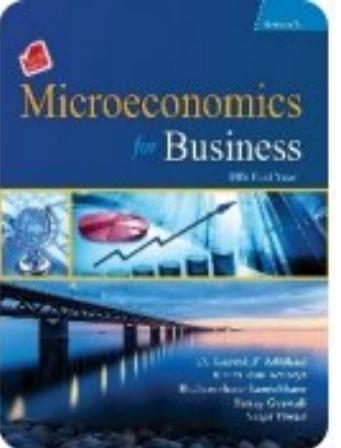
## Contd.

Units of Commodity X	TU <sub>X</sub>	MU <sub>X</sub>
1	6	6
2	10	4
3	12	2
4	12	0
5	10	-2

# Concept of Total Utility and Marginal Utility

## Contd.



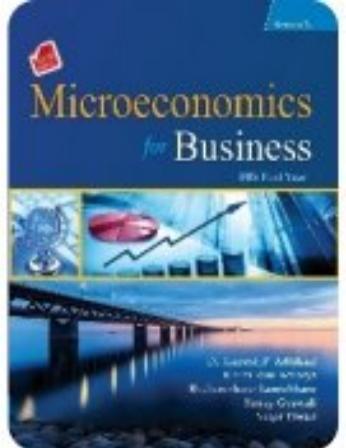


# Consumer's Equilibrium under Cardinal Utility Approach

- A consumer is in equilibrium position when s/he maximizes his/her total utility with given money income and market prices of goods consumed.
- At the equilibrium point, the consumer spends all his income among different goods and services and derives maximum satisfaction.

There are two approaches of consumer's equilibrium under cardinal utility:

- Consumer's Equilibrium: One Commodity Model
- Consumer's Equilibrium: Two Commodity Model



# Consumer's Equilibrium: One Commodity Model

- In order to get utility/ satisfaction, a consumer consumes goods and services.
- In cardinal utility approach, the sum of the utility derived from the consumption of given units of a commodity is called total utility (TU).
- The additional utility derived from the consumption of an additional unit of a commodity is known as marginal utility.
- In this model, the consumer is in equilibrium when the marginal utility of the commodity is equated to its market price.
- Suppose that the consumer consumes X commodity.
- In this situation, the consumer is in equilibrium when the marginal utility of X commodity is equated to its market price ( $P_x$ ).

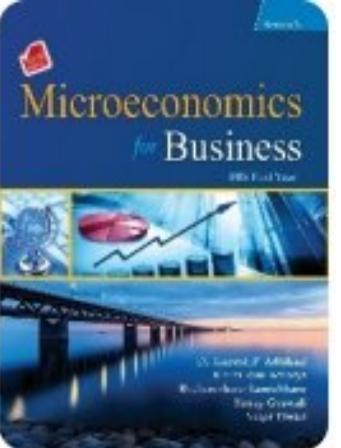
$$\frac{MU_x}{P_x} = MUm$$

where

MUm = Marginal utility of money

$P_x$  = Price of X commodity

$MU_x$  = Marginal utility of X commodity



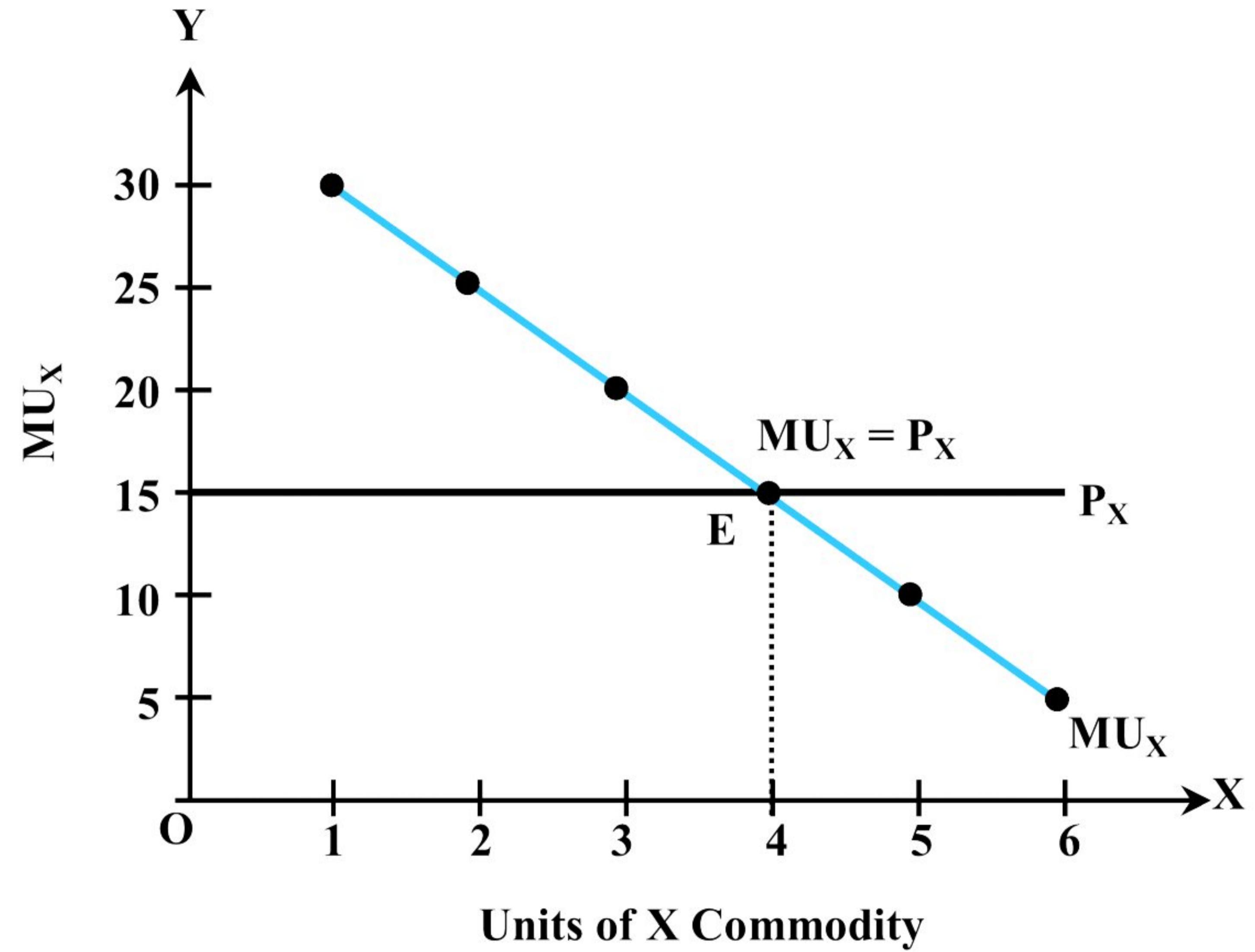
# Consumer's Equilibrium: One Commodity Model Contd.

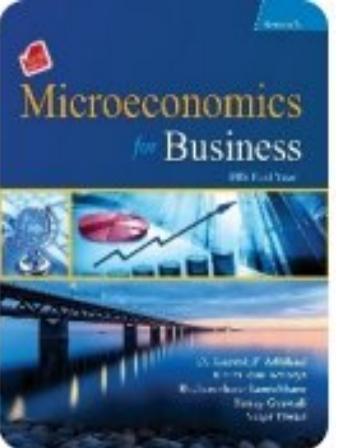
## Assumptions

Consumer's equilibrium under one commodity model is based on the following assumptions:

- The consumer is rational.
- Cardinal measurement of utility is possible.
- Marginal utility of money remains constant.
- The law of diminishing marginal utility operates.
- Prices of commodities are given or remain constant.

Unit of X	$MU_x$ (in Rs.)	$P_x$ (in Rs.)
1	30	15
2	25	15
3	20	15
4	15	15 $MU_x = P_x$
5	10	15
6	5	15





# Consumer's Equilibrium: Two Commodity Model

- A consumer is said to be in equilibrium when the ratio of marginal utility and price of each commodity is equivalent to marginal utility of money.
- It means that consumer always tries to get equal marginal utility by consuming a commodity which is equal to money spent on that commodity.
- This is called law of equi-marginal utility or law of substitution.
- According to this law, consumer is in equilibrium position when the following condition is fulfilled:

$$\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y} = MU_M$$

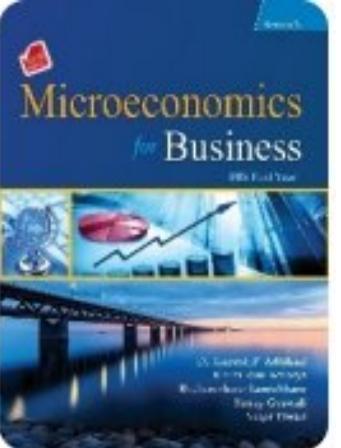
where

$MU_X$  = Marginal Utility of X Commodity

$MU_Y$  = Marginal Utility of Y Commodity

$P_X$  = Price of X Commodity

$P_Y$  = Price of Y Commodity



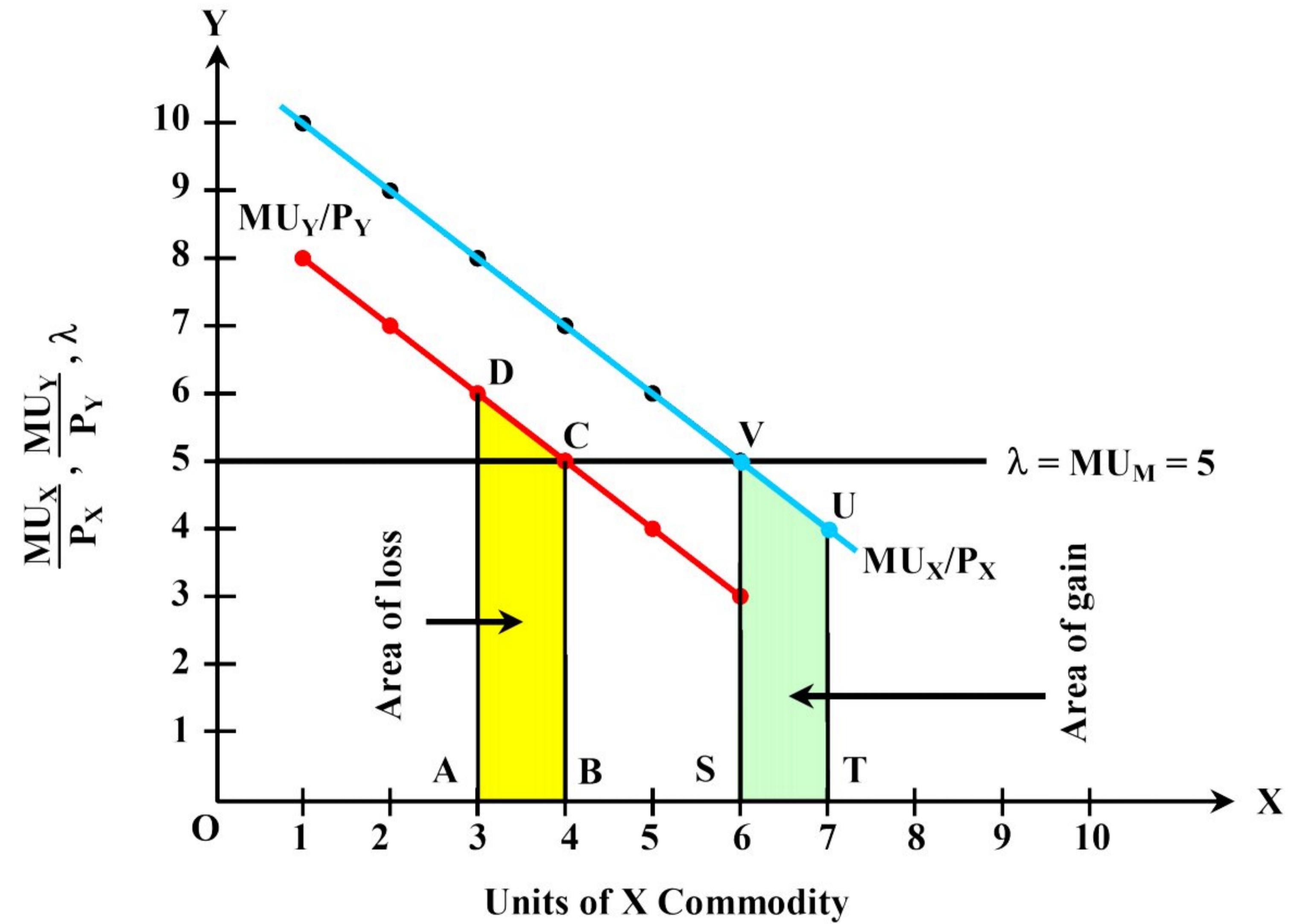
# Consumer's Equilibrium: Two Commodity Model Contd.

## Assumptions

Consumer's equilibrium under two commodity model is based on the following assumptions:

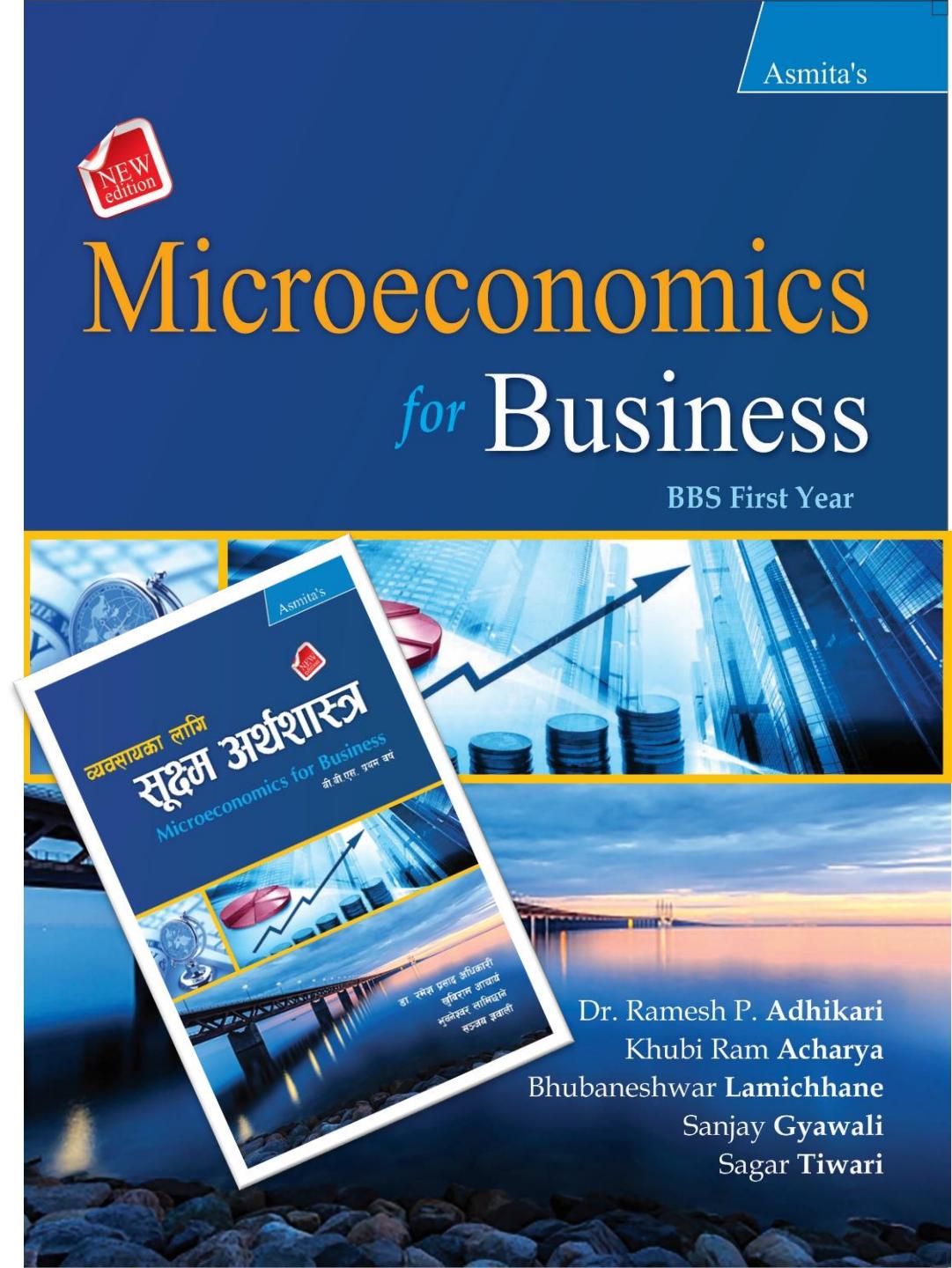
- The consumer is rational.
- Cardinal measurement of utility is possible.
- Marginal utility of money remains constant.
- The law of diminishing marginal utility operates.
- Prices of commodities and income of the consumer are given.
- Consumer spends his income in two goods.

Units	MU <sub>X</sub> (Utility)	MU <sub>Y</sub> (Utility)	MU <sub>X</sub> / P <sub>X</sub>	MU <sub>Y</sub> / P <sub>Y</sub>
1	20	24	10	8
2	18	21	9	7
3	16	18	8	6
4	14	15	7	5
5	12	12	6	4
6	10	9	5	3

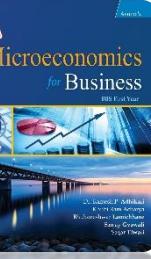


# Theory of Production

Unit 5



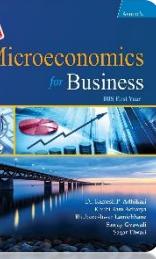
Dr. Ramesh P. Adhikari  
Khubi Ram Acharya  
Bhubaneshwar Lamichhane  
Sanjay Gyawali  
Sagar Tiwari



# Learning Objectives

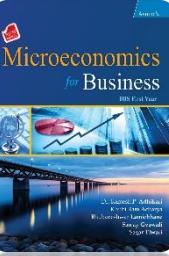
**On completion of this unit, students will be able to:**

- define TP, AP and MP
- define the production function and describe its types
- explain the concept of Cobb-Douglas production function
- explain the law of variable proportions
- describe the concept of isoquant and its properties
- explain the marginal rate of technical substitution and iso-cost line
- explain the optimal combination of inputs
- describe the laws of returns to scale.



# Introduction

- Production is the process of transforming inputs into outputs or goods and services.
- Traditionally, production is also defined as the process of creating utility.
- Production is regarded as the mother of all economic activities because there will be no other economic activities, i.e. consumption, exchange and distribution in absence of production.
- The aim of all the firm is to maximize profit or sales revenue.
- The profit maximization or sales revenue maximization is possible only through efficient production of goods and services.
- The efficient production of goods and services is possible only through optimum combination of inputs or factors of production.



# Concept of Total Product (TP), Average Product (AP), and Marginal Product (MP)

## Total Product (TP)

- Total product is defined as the total quantity of output produced by the producer employing all the available units of inputs in the given period of time.
- TP is the sum of marginal product, i.e.

$$TP = MP_1 + MP_2 + \dots + MP_n = SMP$$

where

TP = Total product

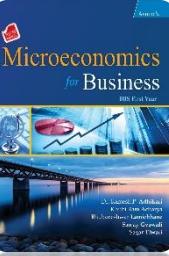
MP = Marginal product

TP can also be obtained by multiplying average product (AP) by units of labour or any variable factor used in production process, i.e.

$$TP = AP \times L$$

where

L = Labour or variable input



# Concept of Total Product (TP), Average Product (AP), and Marginal Product (MP) Contd...

## Average Product (AP)

- Average product is obtained by dividing total product by number of variable input or factor used in production process.
- In other words, it is the output per unit variable factor. Thus,

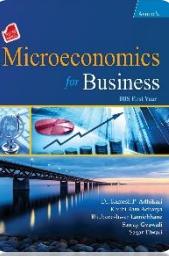
$$AP = \frac{TP}{L}$$

where

AP = Average product

TP = Total product

L = Units of labour or variable input or factor



# Concept of Total Product (TP), Average Product (AP), and Marginal Product (MP) Contd...

## Marginal Product (MP)

- Marginal product is defined as the addition in total product as a result of an additional unit of available factor, i.e. labour.
- In other words, it is the ratio of change in total product and change in units of labour or any variable input. Thus,

$$MP = TP_n - TP_{n-1}$$

$$= \frac{\Delta TP}{\Delta L}$$

where

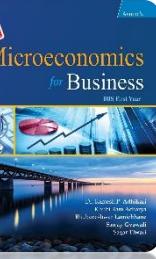
MP = Marginal product

$\Delta TP$  = Change in total product

$TP_n$  = Total product of 'n' units of labour

$\Delta L$  = Change in variable input or labour

$TP_{n-1}$  = Total product of 'n - 1' units of labour



# Production Function

- Production function is defined as functional relationship between physical inputs and physical output of a firm.
- In other words, production function shows the maximum possible output which can be produced by the given quantities of inputs.

$$Q = f(L_d, L, K, O, T) \dots (i)$$

where

$Q$  = Output

$L_d$  = Land

$L$  = Labour

$K$  = Capital

$O$  = Organization

$T$  = Technology

the general equation of this simple production function is expressed symbolically as

$$Q = f(L, K) \dots (ii)$$

# Types of Production Function

## 1. Short Run Production Function

- Short run production function is the technical or functional relationship between inputs and output where quantities of some inputs are kept constant and quantities of some inputs are varied.
- The input and output relationship in the short run is studied under the law of variable proportions.

$$Q = f(L, \bar{K})$$

... (iii)

where

f = Function

Q = Output

L = Labour which is variable factor

$\bar{K}$  = Capital, which is fixed factor

- The short-run production function is also expressed as

$$Q = f(N_{vt}, \bar{K})$$

where

$N_{vt}$  = Units of variable input

# Types of Production Function Contd...

## 2. Long Run Production Function

- Long run production function is defined as the production function in which all inputs are variable.
- In other words, long run production function is the technical or functional relationship between inputs and output when quantities of all inputs are variable.

$$Q = f(L, K) \dots \text{(iv)}$$

where

$Q$  = Quantity of output

$L$  = Units of labour

$K$  = Units of capital

# Cobb-Douglas Production Function

- Cobb–Douglas production function is a specific production function which is most widely used in economic analysis.
- The Cobb–Douglas production function was first proposed by **Kunt Wicksell** (1851–1926), a leading Swedish economist. Later, it was tested empirically by two American originators, **C.W. Cobb** (a mathematician) and **P.H. Douglas** (an economist) in 1928 AD. Therefore, it is known as the Cobb–Douglas production function.
- Cobb–Douglas production function:

$$Q = A K^\alpha L^\beta \quad \dots \text{(i)}$$

where

$Q$  = Output

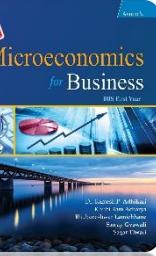
$A$  = Efficiency parameter, positive constant

$K$  = Capital

$L$  = Labour

$\alpha$  = Elasticity of output with respect to capital     $\alpha$  and  $\beta$  = Positive constants

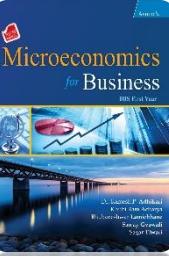
$\beta$  = Elasticity of output with respect of labour =  $1 - \alpha$



# Properties/ Features of Cobb-Douglas Production Function

1. **Returns to scale:** The Cobb-Douglas production function can be used to calculate nature of returns to scale from the value of  $(\alpha + \beta)$ .  
If  $\alpha + \beta = 1$ , it shows constant returns to scale.  
If  $\alpha + \beta > 1$ , it shows increasing returns to scale.  
If  $\alpha + \beta < 1$ , it shows decreasing returns to scale.
2. **Marginal product of an input can be expressed in terms of its average product:** Cobb-Douglas production function shows that marginal product of a factor can be expressed in terms of its average product. It is given as follows:

# Properties/ Features of Cobb-Douglas Production Function Contd.



## Marginal product of labour

$$\begin{aligned}
 MP_L &= \frac{\partial Q}{\partial L} \\
 &= \frac{\partial(A K^\alpha L^\beta)}{\partial L} \\
 &= A K^\alpha \cdot \frac{\partial(L^\beta)}{\partial L} \\
 &= A K^\alpha \cdot \beta \cdot L^{\beta-1} \\
 &= A K^\alpha \cdot \beta \cdot \frac{L^\beta}{L} \\
 &= \beta \frac{(A K^\alpha L^\beta)}{L} \\
 &= \beta \cdot \frac{Q}{L} \\
 &= \beta \cdot AP_L \left[ \because AP_L = \frac{Q}{L} \right]
 \end{aligned}$$

## Marginal product of capital

$$\begin{aligned}
 MP_K &= \frac{\partial Q}{\partial K} \\
 &= \frac{\partial(A K^\alpha L^\beta)}{\partial K} \\
 &= A L^\beta \cdot \frac{\partial(K^\alpha)}{\partial K} \\
 &= A L^\beta \cdot \alpha \cdot K^{\alpha-1} \\
 &= A L^\beta \cdot \alpha \cdot \frac{K^\alpha}{K} \\
 &= \alpha \frac{(A L^\beta K^\alpha)}{K} \\
 &= \alpha \cdot \frac{Q}{K} \\
 &= \alpha \cdot AP_K \left[ \because AP_K = \frac{Q}{K} \right]
 \end{aligned}$$

# Properties/ Features of Cobb-Douglas Production Function Contd.

3. **Marginal rate of technical substitution:** The marginal rate of technical substitution can be expressed in terms of ratio between labour and capital.

$$MRTS_{L,K} = \frac{dK}{dL} = \frac{MP_L}{MP_K} = \frac{\beta \left( \frac{Q}{L} \right)}{\alpha \left( \frac{Q}{K} \right)} = \frac{\beta}{\alpha} \left( \frac{K}{L} \right)$$

$$MRTS_{K,L} = \frac{dL}{dK} = \frac{MP_K}{MP_L} = \frac{\alpha \left( \frac{Q}{K} \right)}{\beta \left( \frac{Q}{L} \right)} = \frac{\alpha}{\beta} \left( \frac{L}{K} \right)$$

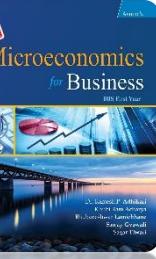
# Properties/ Features of Cobb-Douglas Production Function Contd.

## 4. Measurement of elasticity of factor substitution is equal to unity:

The elasticity of substitution ( $\sigma$ ) of Cobb-Douglas production function is equal to unity. The elasticity of substitution is defined as the percentage change in capital-labour ratio divided by percentage change in marginal rate of technical substitution.

$$\sigma = \frac{\text{Percentage change in } \frac{K}{L}}{\text{Percentage change in MRTS}}$$

$$= \frac{\frac{d(K/L)}{(K/L)}}{\frac{d(MRTS_K)}{(MRTS_K)}} = \frac{\frac{d(K/L)}{(K/L)}}{\frac{d\left(\frac{\beta \cdot K}{\alpha \cdot L}\right)}{\left(\frac{\beta \cdot K}{\alpha \cdot L}\right)}} = \frac{\frac{d(K/L)}{(K/L)}}{\frac{\beta}{\alpha} d\left(\frac{K}{L}\right)} = \frac{\frac{\beta}{\alpha} d\left(\frac{K}{L}\right) \cdot \left(\frac{K}{L}\right)}{\frac{\beta}{\alpha} d\left(\frac{K}{L}\right) \cdot \left(\frac{K}{L}\right)} = 1$$



# Properties/ Features of Cobb-Douglas Production Function Contd.

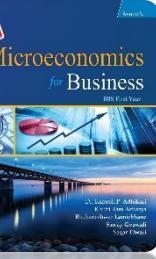
5. **Factor intensity:** In the Cobb–Douglas production function, factor intensity is measured by the ratio between  $\alpha$  and  $\beta$ .

If  $\frac{\alpha}{\beta} > 1$ , there is use of capital intensive technique in production.

If  $\frac{\alpha}{\beta} < 1$ , there is use of labour intensive technique in production.

If  $\frac{\alpha}{\beta} = 1$ , there is equal factor intensive technique of production, i.e. equal proportion of labour and capital is used in production.

6. **The efficiency of production:** The efficiency in the organization of the factors of production is measured by the coefficient A. Higher the value of A, higher will be degree of efficiency of production and vice-versa.



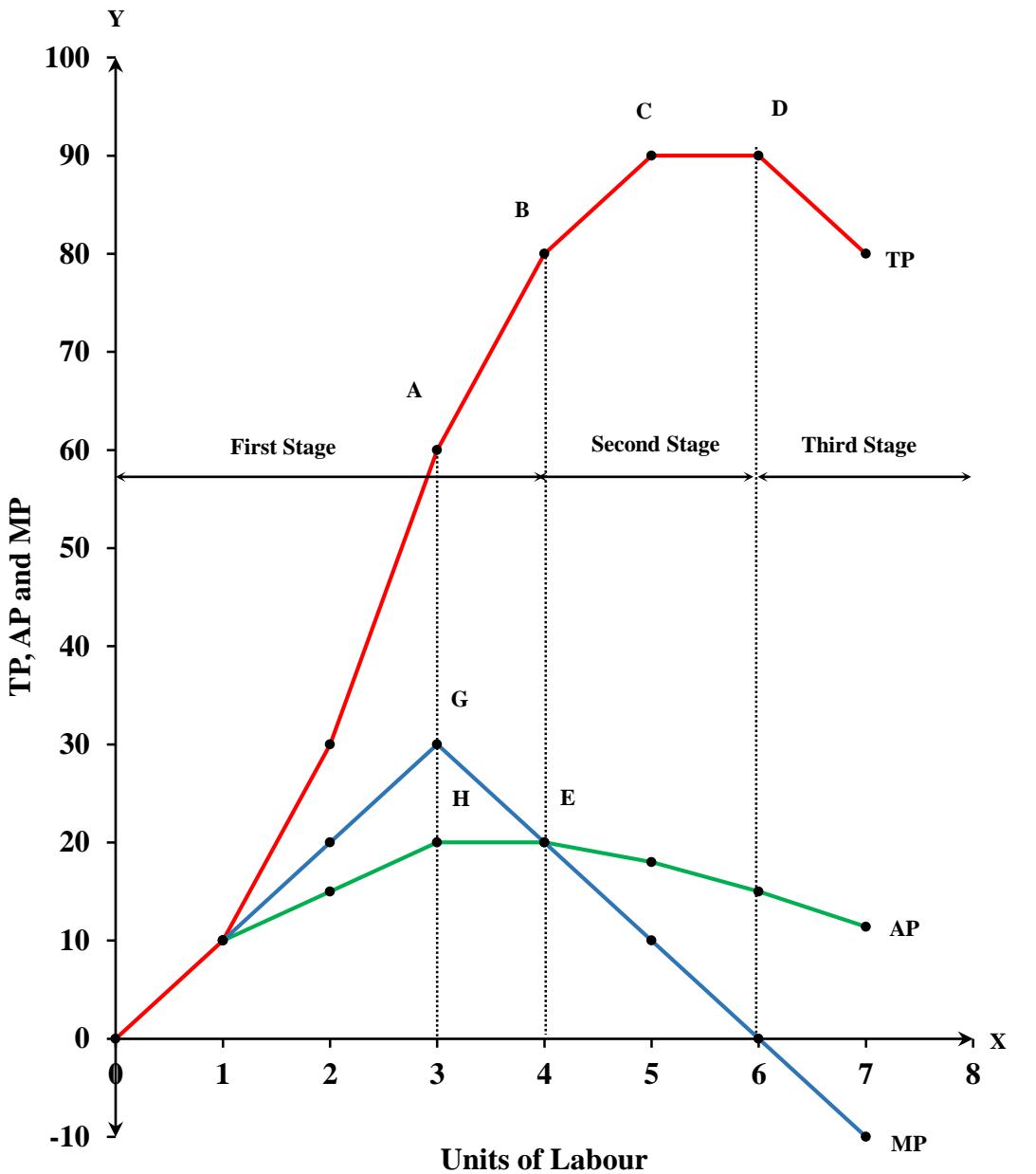
# Law of Variable Proportions

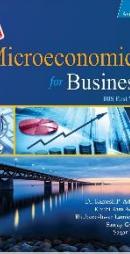
The law of variable proportions is concerned with the short run production function. It examines production with single variable factor keeping quantities of other factors constant. This law was propounded by the economists like **Joan Robinson, Alfred Marshall, P.A. Samuelson, etc.** This law is also known as the law of diminishing returns.

## Assumptions

- There is no change in technology.
- At least, one factor of production is fixed.
- There must be possibility of varying the proportion of factors of production.
- Labour is only a variable factor.
- All units of labour are homogeneous.

Land (in Ropanies)	Units of Labour	TP	AP	MP	Stage of Production
10	0	0	0	0	First stage
10	1	10	10	10	
10	2	30	15	20	
10	3	60	20	30	
10	4	80	20	20	
10	5	90	18	10	Second Stage
10	6	90	15	0	
10	7	80	11.4	-10	Third Stage

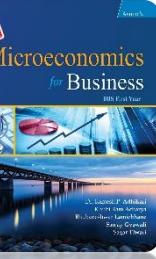




# Law of Variable Proportions Contd.

## Causes of Operation of Stages

1. **First Stage (Stage of Increasing Returns)**
  - i. Increase in efficiency of fixed factor
  - ii. Increase in the efficiency of variable factor
2. **Second Stage (Stage of Decreasing Returns)**
  - i. Scarcity of fixed factor
  - ii. Indivisibility of fixed factor
  - iii. Imperfect substitutability of the factor
3. **Third Stage (Stage of Negative Returns)**
  - i. Inefficient utilization of variable factor
  - ii. Over utilization of fixed inputs
  - iii. Complexity of management
  - iv. Over utilization of fixed inputs

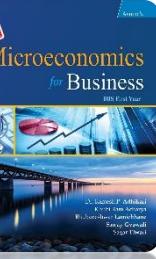


# Law of Variable Proportions Contd.

## Stage of Operation

(Which stage of production does a rational producer choose?)

- ❖ A rational producer does not choose first and third stage.
- ❖ In the first stage, TP increases at the increasing rate and MP of the variable factor also increases; and there is no full utilization of fixed factors of production. Thus, there is opportunity of increasing production by increasing quantity of variable factor.
- ❖ In the third stage, TP declines, AP also declines and MP becomes negative.
- ❖ Thus, the rational producer will choose second stage where both AP and MP of variable factors are diminishing; and there is full utilization of fixed factor.
- ❖ At which particular point of this stage, the producer will choose to produce depends upon the prices of factors.



# Law of Variable Proportions Contd.

## Application of the Law of Variable Proportions

The law of variable proportions specially applies to the agriculture. There are some reasons why agriculture is subject to this law, which are as follows:

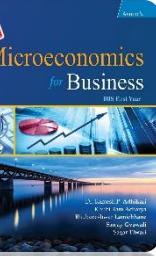
- The agricultural operations spread out over a wide area. Therefore, it cannot be effectively supervised.
- Scope for the use of specialized machinery is also very limited in the agricultural sector.
- Agricultural operations are affected by rain fall and climate change.

# Isoquant

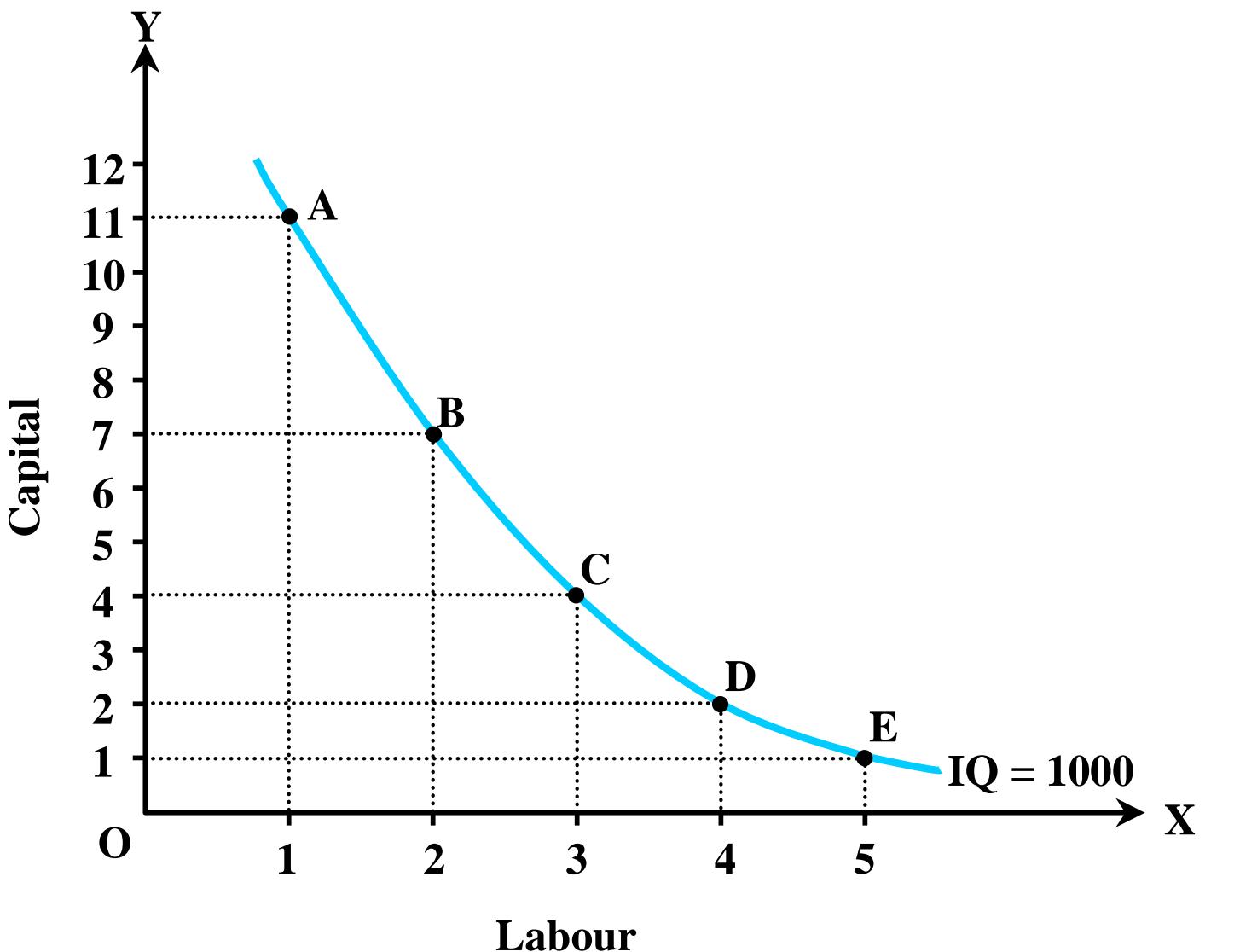
- Isoquant is defined as the locus of different combinations of any two inputs (labour and capital) which yield same level of output.
- This term 'isoquant' has been derived from a Greek word '*iso*' meaning equal and a Latin word '*quant*' meaning quantity.
- Therefore, the isoquant curve is also known as the *equal product curve* or *production indifference curve*.

## Assumptions

- The two inputs are imperfect substitute.
- Labour and capital can be substituted for one another only up to a certain limit.
- Production function is continuous, i.e. labour and capital are perfectly divisible and can be substituted in any small quantity.

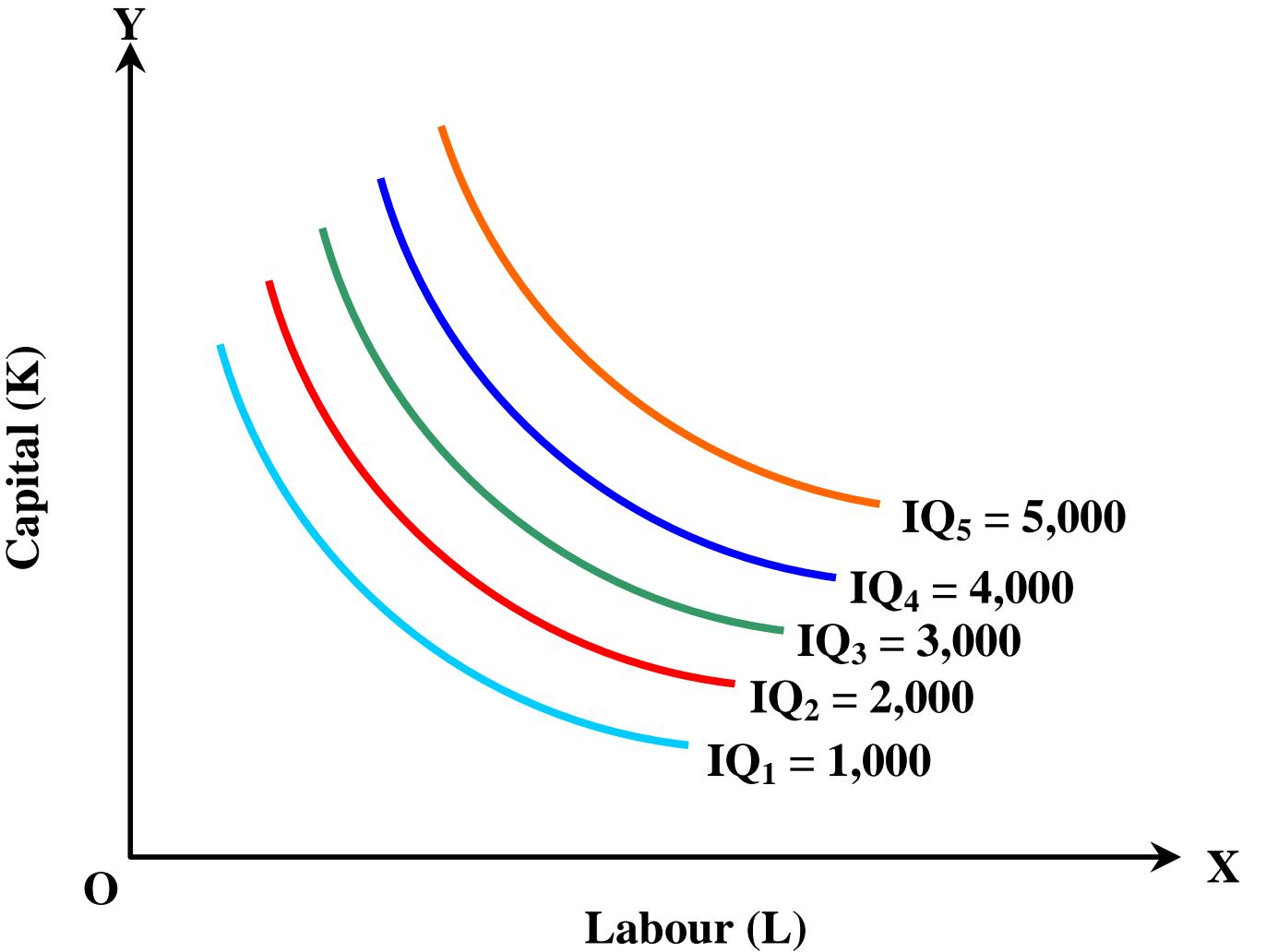


Combinations	Labours	Capital	Output
A	1	11	1,000
B	2	7	1,000
C	3	4	1,000
D	4	2	1,000
E	5	1	1,000



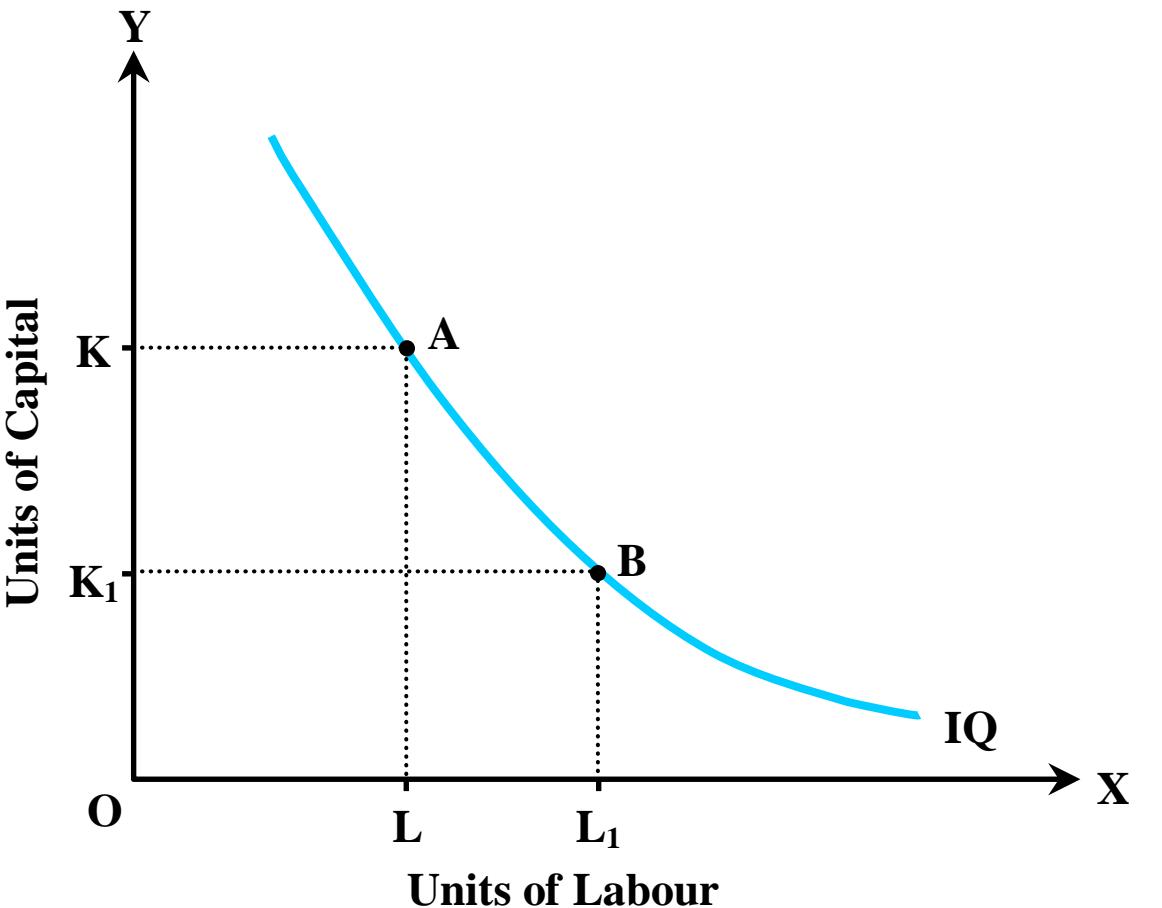
# Isoquant Map

The set of isoquants is called isoquant map. A higher isoquant represents higher level of output and lower isoquant represents a lower level of output.



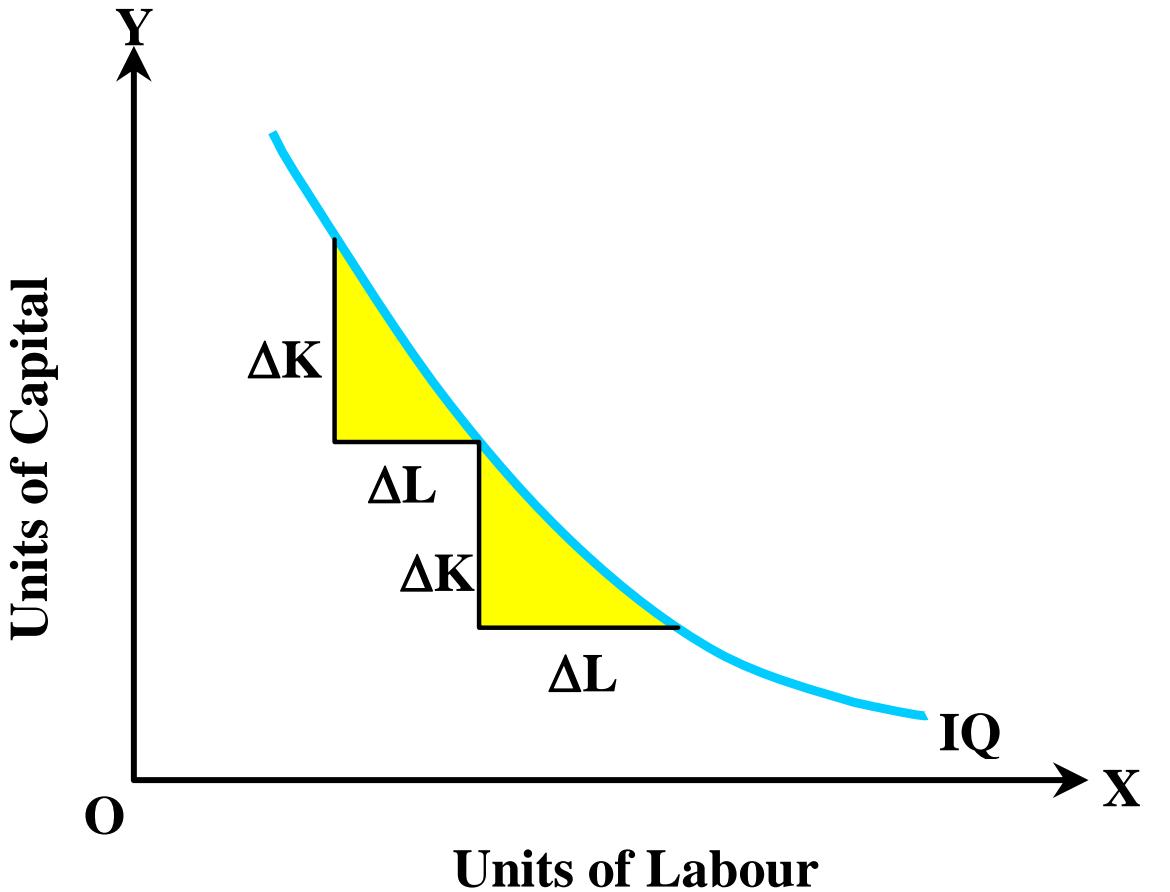
# Properties of Isoquant

1. Isoquant has negative slope.



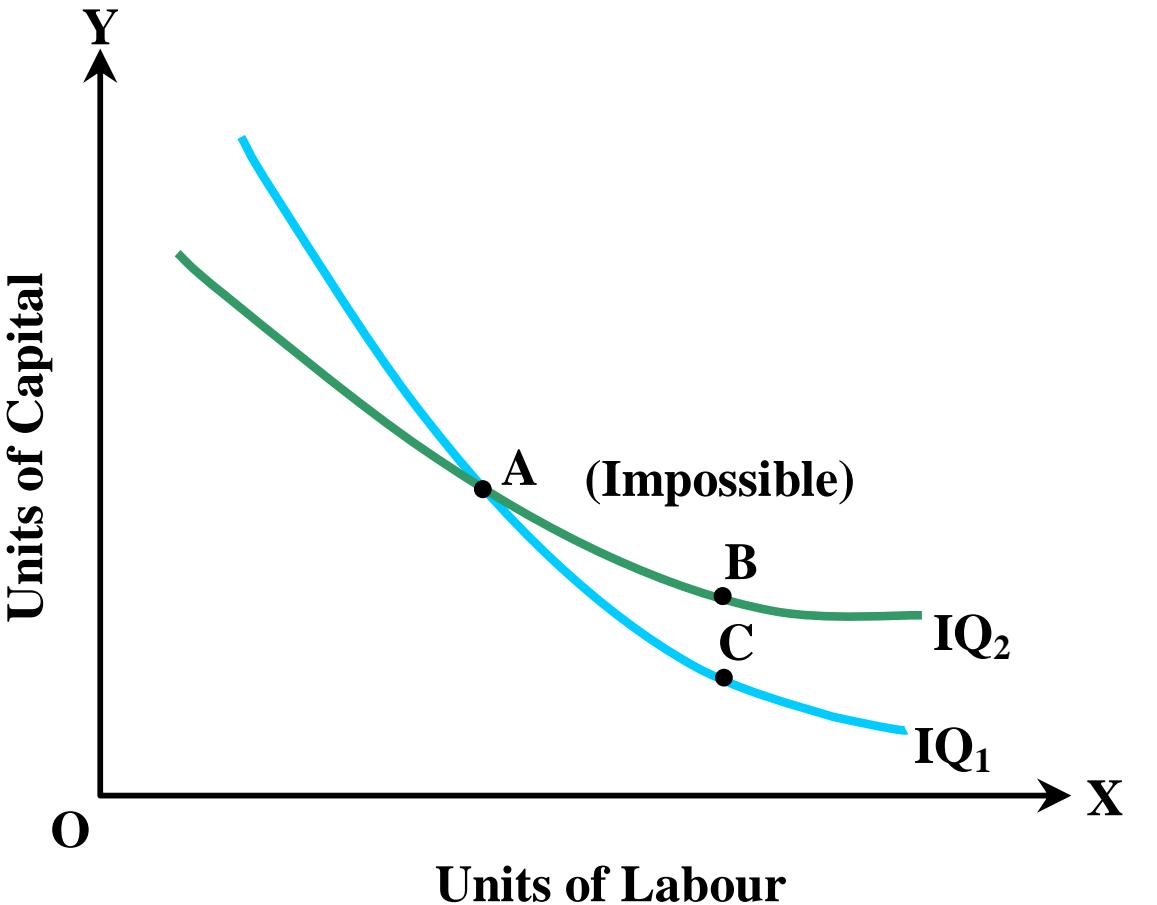
# Properties of Isoquant Contd.

- Isoquant is convex to the origin.



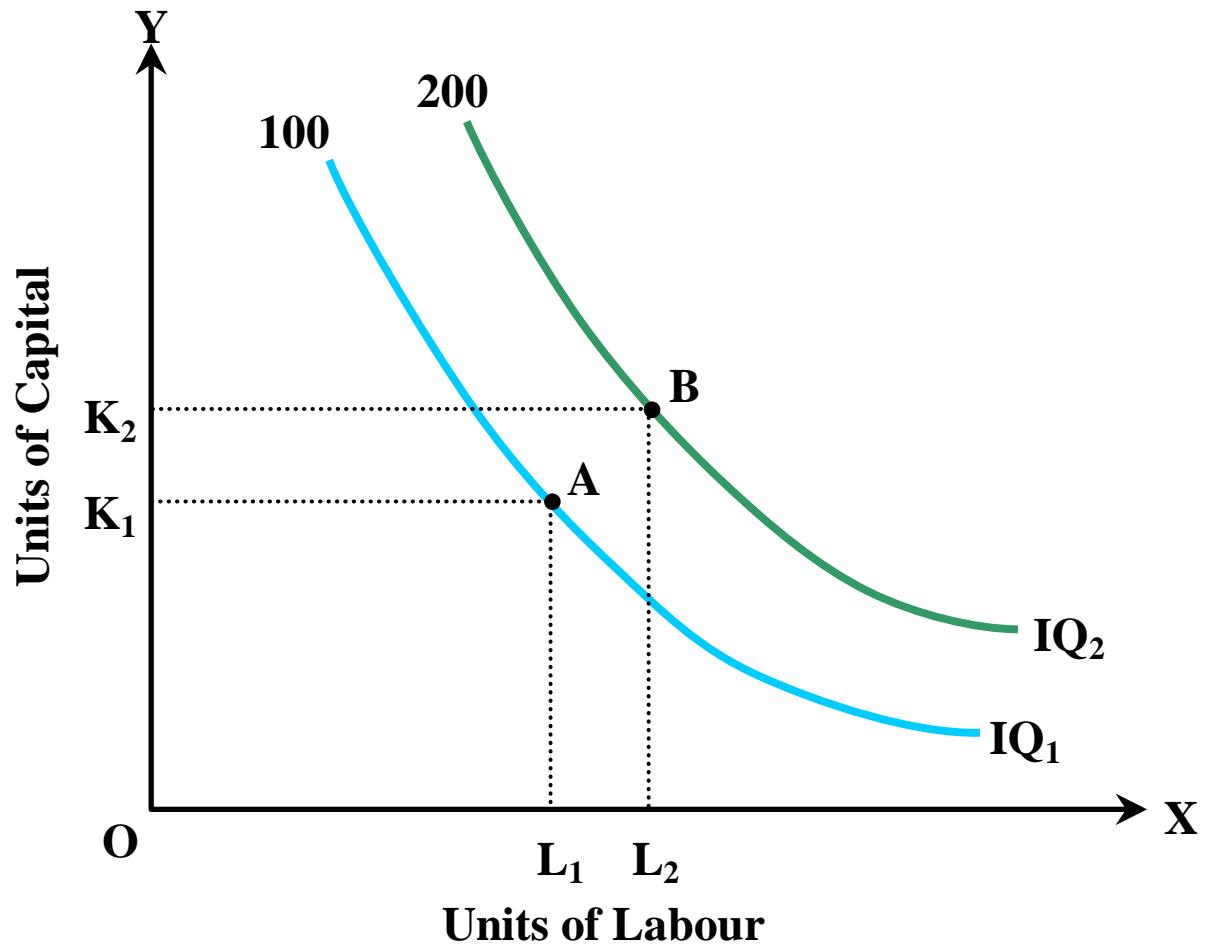
# Properties of Isoquant Contd.

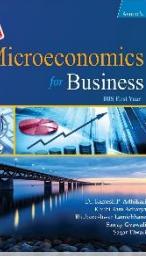
- Isoquants never intersect with each other.



# Properties of Isoquant Contd.

- Higher the isoquant, higher will be output.





# Marginal Rate of Technical Substitution (MRTS)

The marginal rate of technical substitution is defined as the rate at which one input can be substituted for another output remaining constant. In other words, marginal rate of technical substitution of labour for capital can be defined as the number of units of capital which can be replaced by one unit of labour keeping the level of output constant. Marginal rate of technical substitution is slope of the isoquant.

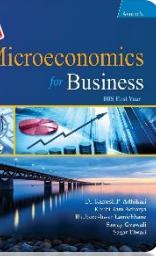
$$MRTS_{L, K} = - \frac{dK}{dL} = \frac{MP_L}{MP_K}$$

where

$MRTS_{L, K}$  = Marginal rate of technical substitution of labour for capital

$MP_L$  = Marginal productivity of labour

$MP_K$  = Marginal productivity of capital



Factor Combinations	Units of labor	Units of Capital	$MRTS_{L,K} = -\frac{\Delta K}{\Delta L}$
A	1	11	-
B	2	7	4
C	3	4	3
D	4	2	2
E	5	1	1

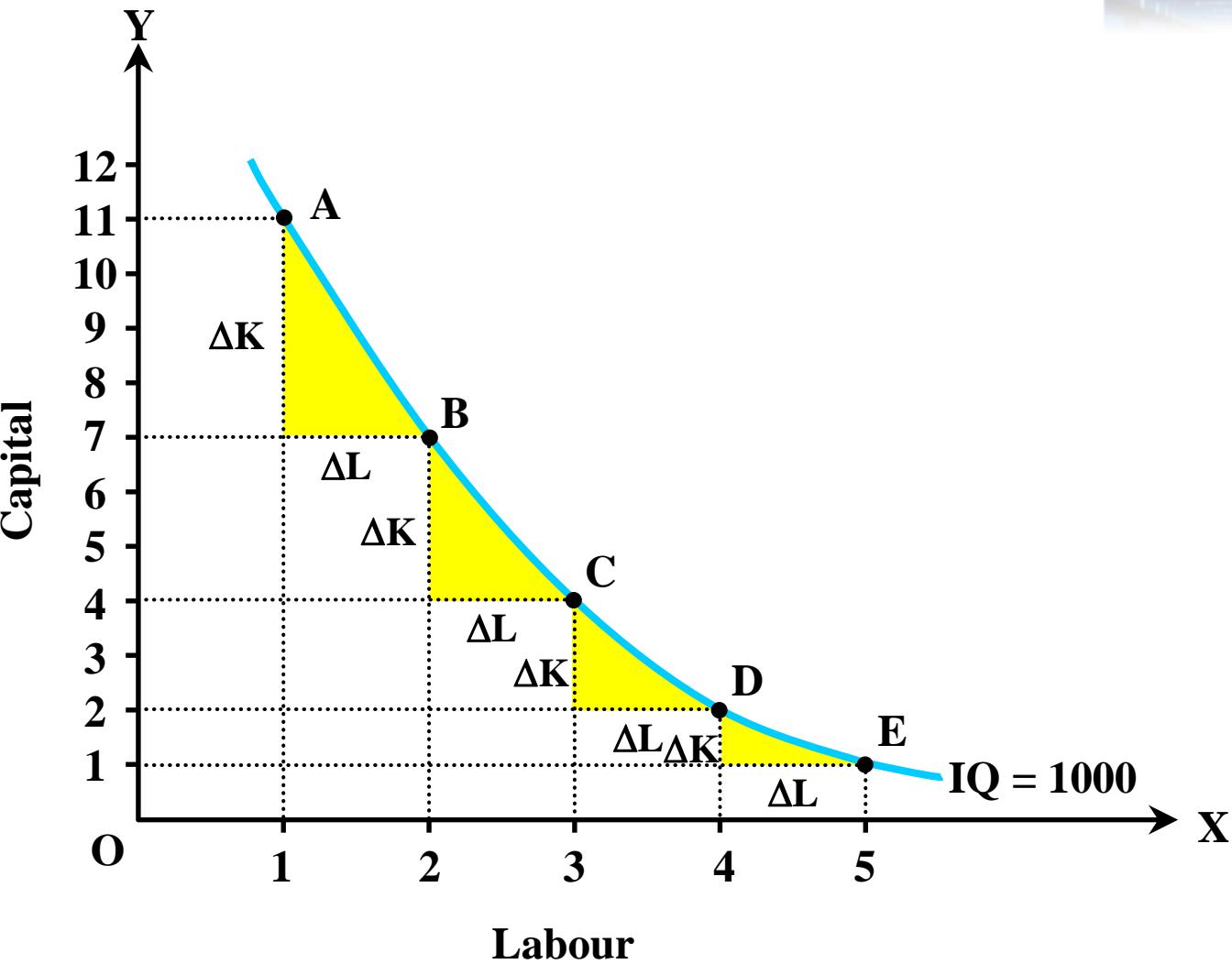
$$\text{Slope of the Isoquant} = MRTS_{L,K} = -\frac{dK}{dL} = -\frac{MP_L}{MP_K}$$

where

$MRTS_{L,K}$  = Marginal rate of technical substitution labour of capital

$MP_L$  = Marginal productivity of labour

$MP_K$  = Marginal productivity of capital



# Isocost Line

Isocost line is defined as the locus of various combinations of any two inputs which the producer can get for a certain amount of money at a given prices of the factors of production or inputs. The concept of isocost line is based on the assumptions of two inputs, i.e. labour and capital; and given total cost or money outlay. Total cost or outlay is the sum of total expenditure made to purchase labour and capital. Thus,

**Total outlay (C) = Total expenditure on labour + Total expenditure on capital**

$$\text{or, } C = P_L \cdot L + P_K \cdot K$$

$$\therefore C = w \cdot L + r \cdot K \quad \dots (\text{i})$$

where

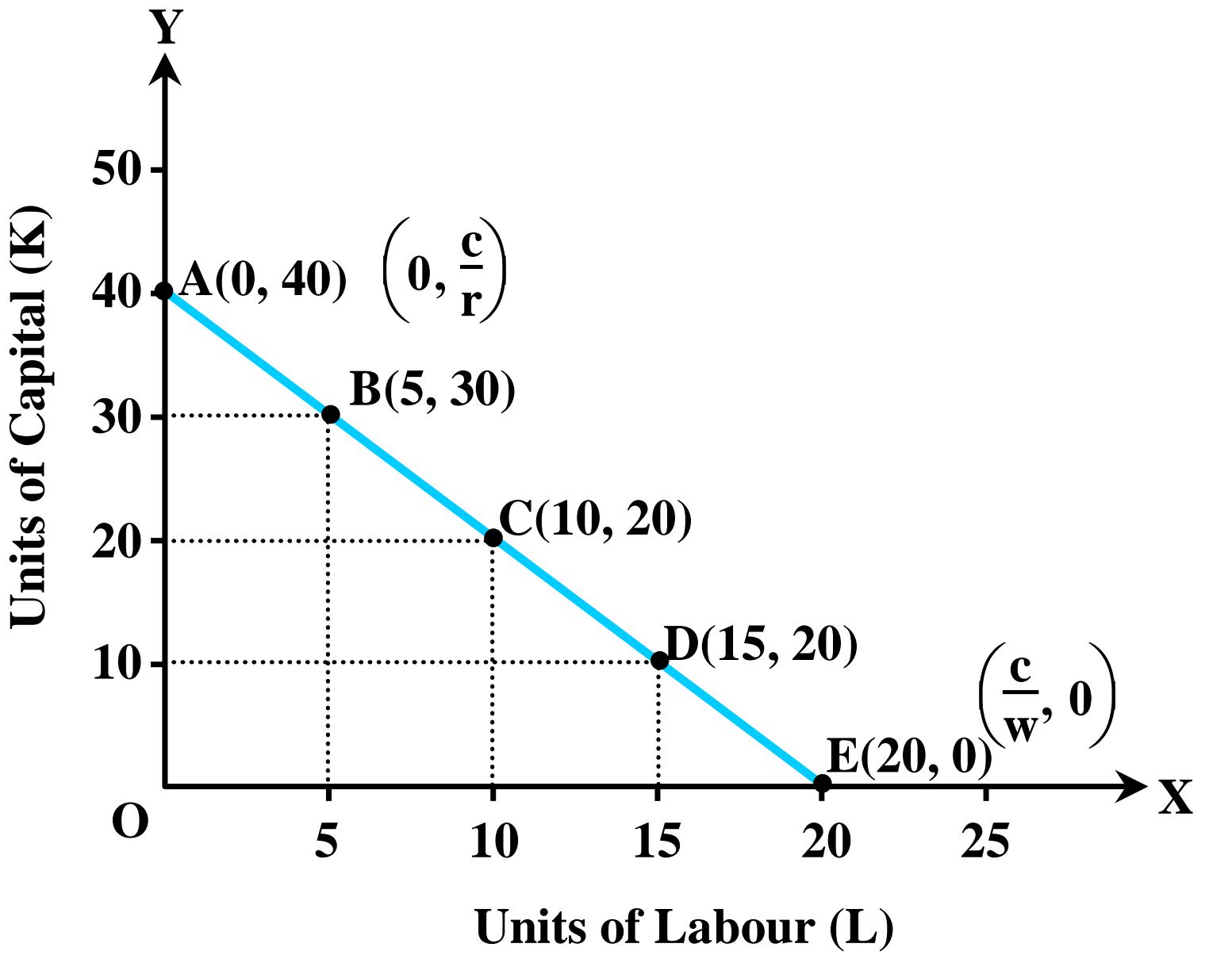
C = Total cost or outlay

w = Wage rate (Price of labour) r = Rate of interest (Price of capital)

L = Units of labour

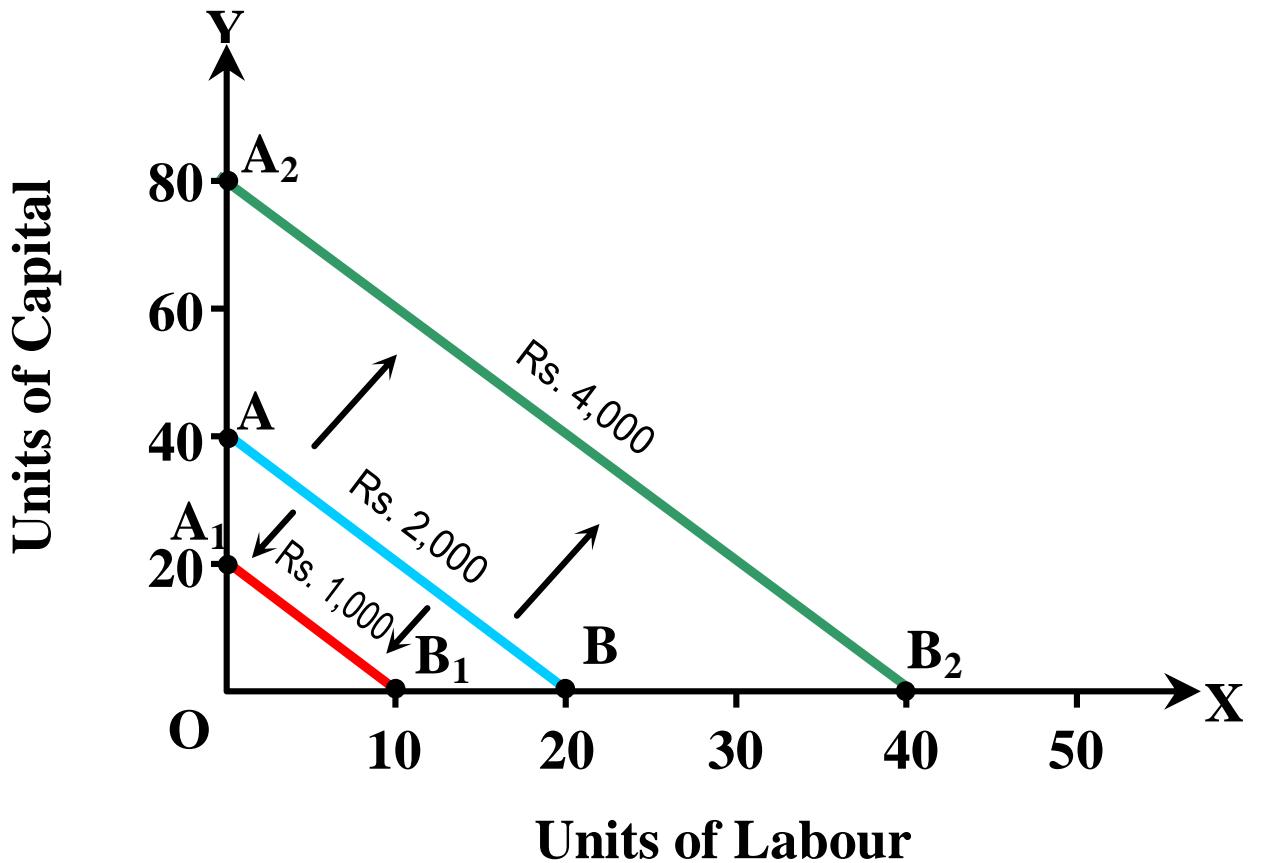
K = Units of capital

Combinations	Price of Labour (w)	Units of Labour (L)	Price of Capital (r)	Units of Capital (K)	Cost Outlay ( $C = wL + rK$ )
A	Rs. 100	0	Rs. 50	40	$100 \times 0 + 50 \times 40 = \text{Rs. } 2,000$
B	Rs. 100	5	Rs. 50	30	$100 \times 5 + 50 \times 30 = \text{Rs. } 2,000$
C	Rs. 100	10	Rs. 50	20	$100 \times 10 + 50 \times 20 = \text{Rs. } 2,000$
D	Rs. 100	15	Rs. 50	10	$100 \times 15 + 50 \times 10 = \text{Rs. } 2,000$
E	Rs. 100	20	Rs. 50	0	$100 \times 20 + 50 \times 0 = \text{Rs. } 2,000$



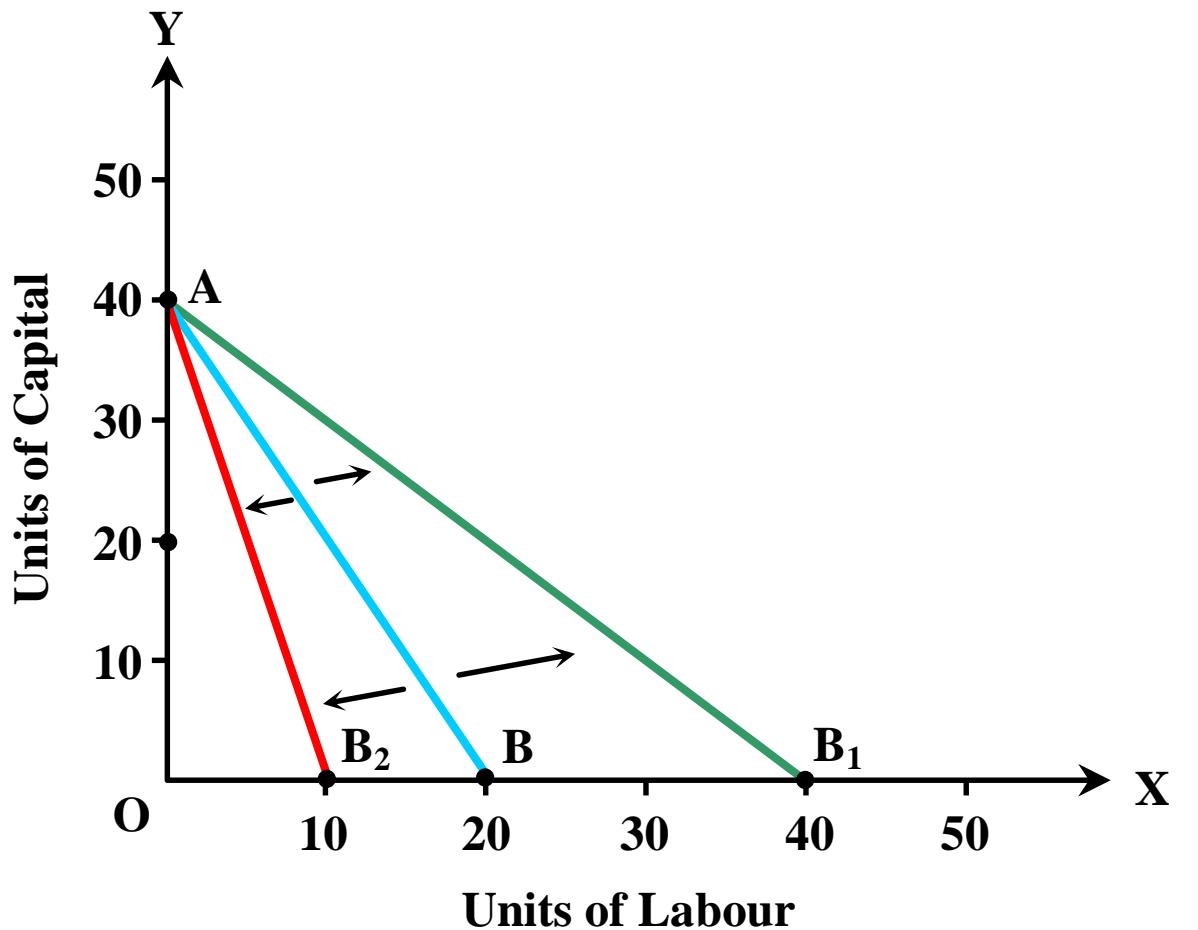
# Change in Isocost Line

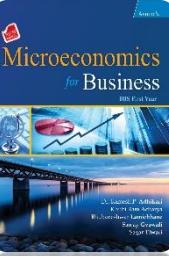
- Effect of change in total outlay/ shift in isocost line



# Change in Isocost Line Contd.

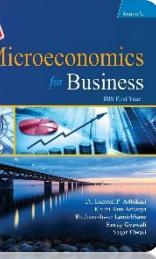
2. Effect of change in price of factors of production or inputs/ swing in isocost line





# Optimum Employment of Inputs (Least Cost Combination of Two Inputs)

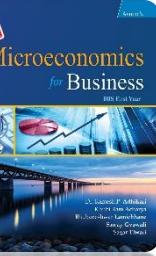
- Optimum employment of two inputs is also known as the least cost combination of two inputs or producer's equilibrium.
- It is assumed that a rational firm or producer always seeks to maximize profit.
- For profit maximization, the firm seeks to minimize cost of production for producing a given quantity of output or maximize output for the given level of cost outlay.
- The choice of particular combination of factors or inputs depends upon the technical possibilities of production and prices of factors of production or inputs used for the production of the particular product.
- The technical possibilities of production are represented by the isoquant map and prices of inputs used for the production of the particular products is represented by the isocost line.



# Optimum Employment of Inputs (Least Cost Combination of Two Inputs) Contd.

## Assumptions

- The producer is rational, i.e. he/she seeks to maximize profit.
- The producer uses two inputs: labour and capital.
- The price of both inputs (labour and capital) is fixed or constant.
- All units of inputs are homogeneous.
- The total cost or money outlay is given.
- There is existence of perfect competition in the factor market.
- Marginal rate of technical substitution must diminish.
- There exists isoquant map in case of output maximization and family of isocost line in case of cost minimization.



# Optimum Employment of Inputs (Least Cost Combination of Two Inputs) Contd.

## Conditions for Equilibrium

1. **First order condition (Necessary condition):** Isoquant must be tangent to the isocost line. In other words, the slope of isoquant should be equal to slope of isocost line.

$$\text{Slope of isoquant} = \text{Slope of isocost line}$$

$$\text{or, } MRTS_{L, K} = \left( -\frac{w}{r} \right)$$

$$\text{or, } \left( -\frac{MP_L}{MP_K} \right) = \left( -\frac{w}{r} \right)$$

$$\text{or, } \frac{MP_L}{MP_K} = \frac{w}{r}$$

$$\therefore \frac{MP_L}{w} = \frac{MP_K}{r}$$

where

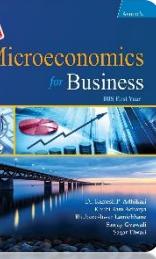
$MRTS_{L, K}$  = Marginal rate of technical substitution of labour for capital

$w$  = Wage rate or price of labour

$MP_L$  = Marginal productivity of labour

$r$  = Interest rate or price of capital

$MP_K$  = Marginal productivity of capital

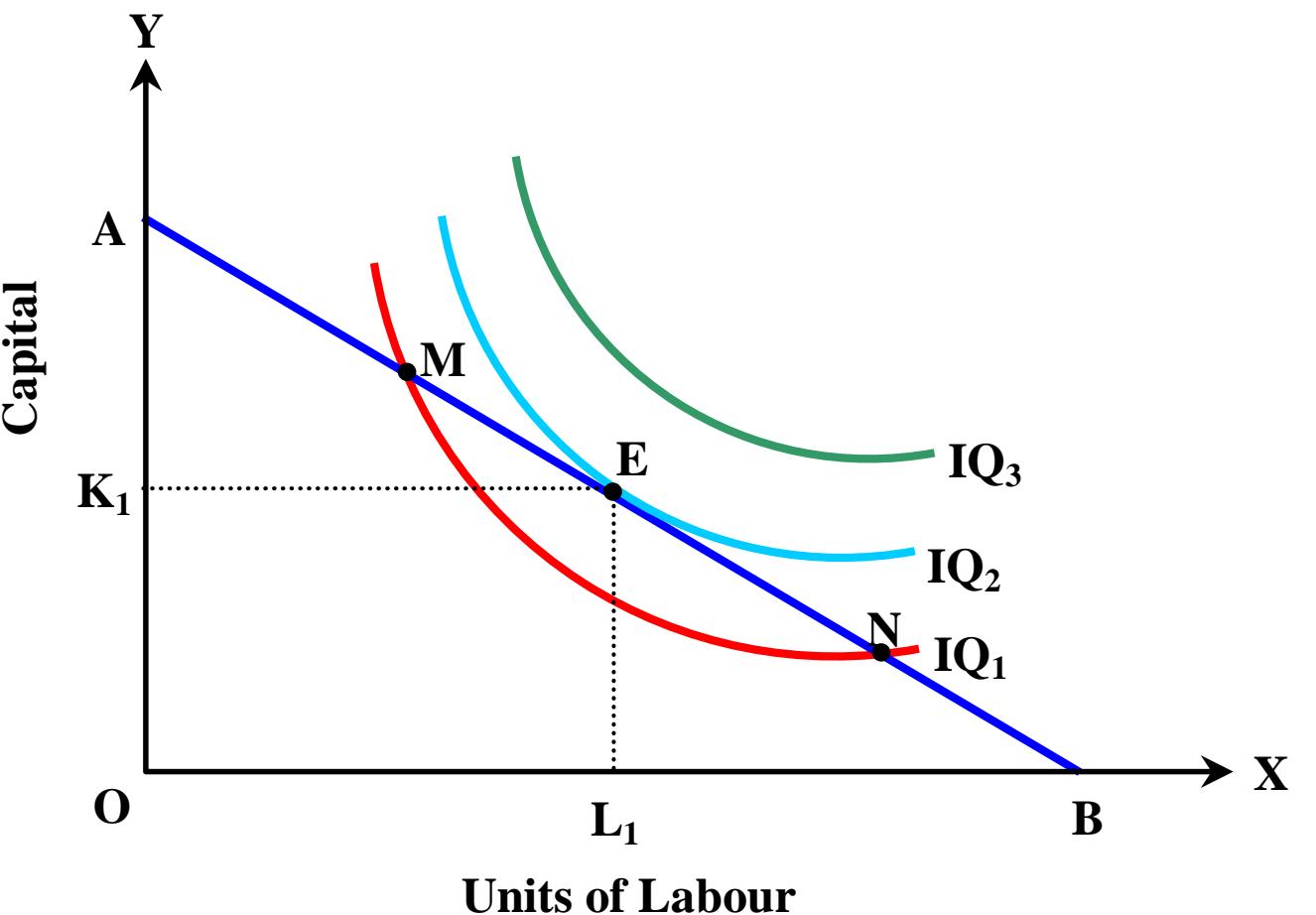


# Optimum Employment of Inputs (Least Cost Combination of Two Inputs) Contd.

2. **Second order condition (Sufficient condition):** Isoquant must be convex to the origin at the point of tangency.

# Approaches of Optimum Employment of Inputs

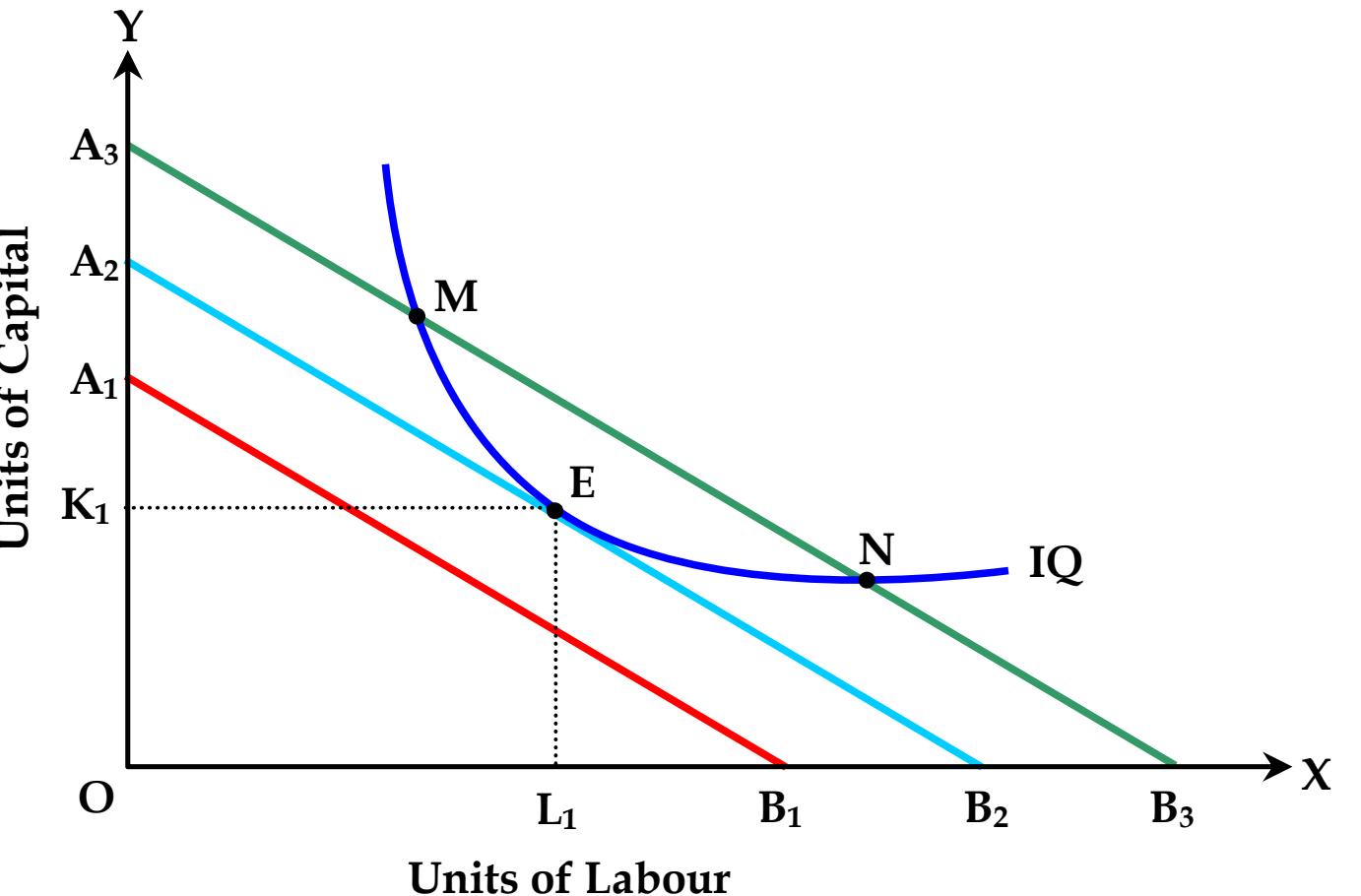
1. **Maximization of output for the given cost outlay (Output maximization subject to cost constraint or financial constraint):** A rational firm or producer seeks to maximize output at the given cost outlay. This is the situation in which the firm or producer is fixed with the resource constraint and seeks to maximize the output.

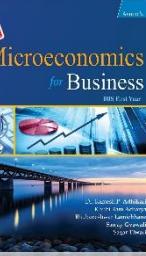


# Optimum Employment of Inputs (Least Cost Combination of Two Inputs) Contd.

## 2. Minimization of cost for the given level of output (Cost minimization subject to output constraint):

A rational firm or producer seeks to minimize cost at the given level of output. This is the situation in which producer or firm is faced with output constraint.





# Laws of Returns to Scale

- Laws of returns to scale refers to long run input output relationship which explains how output changes when all inputs are varied in the equal proportions.
- In the returns to scale all factors of production are varied simultaneously at the same proportion.

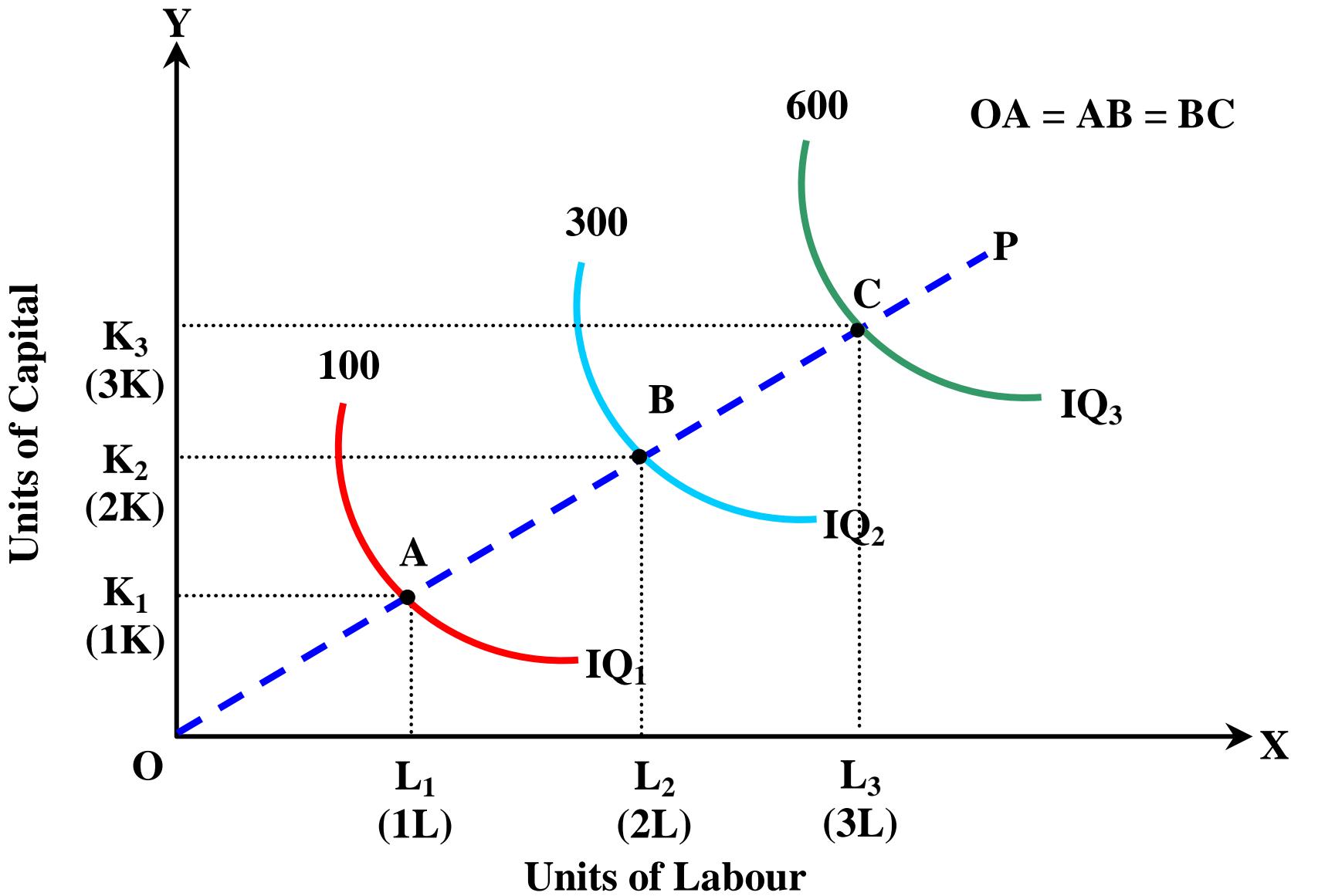
## Types of Laws of Returns to Scale

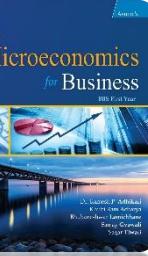
1. Increasing Returns to Scale
2. Constant Returns to Scale
3. Decreasing Returns to Scale

# Increasing Returns to Scale

- Increasing returns to scale refers to the increase in output at a greater proportion (percentage) than the proportionate or percentage increase in inputs.
- It means that if inputs are doubled, output will be more than double and if inputs are tripled, output will be more than triple.

Combinations	Labors (L)	Capital (K)	Total Product (TP)
A	1	1	100
B	2	2	300
C	3	3	600

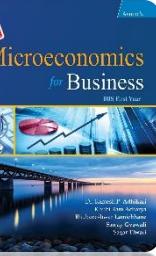




# Increasing Returns to Scale Contd.

## Causes of Increasing Returns to Scale

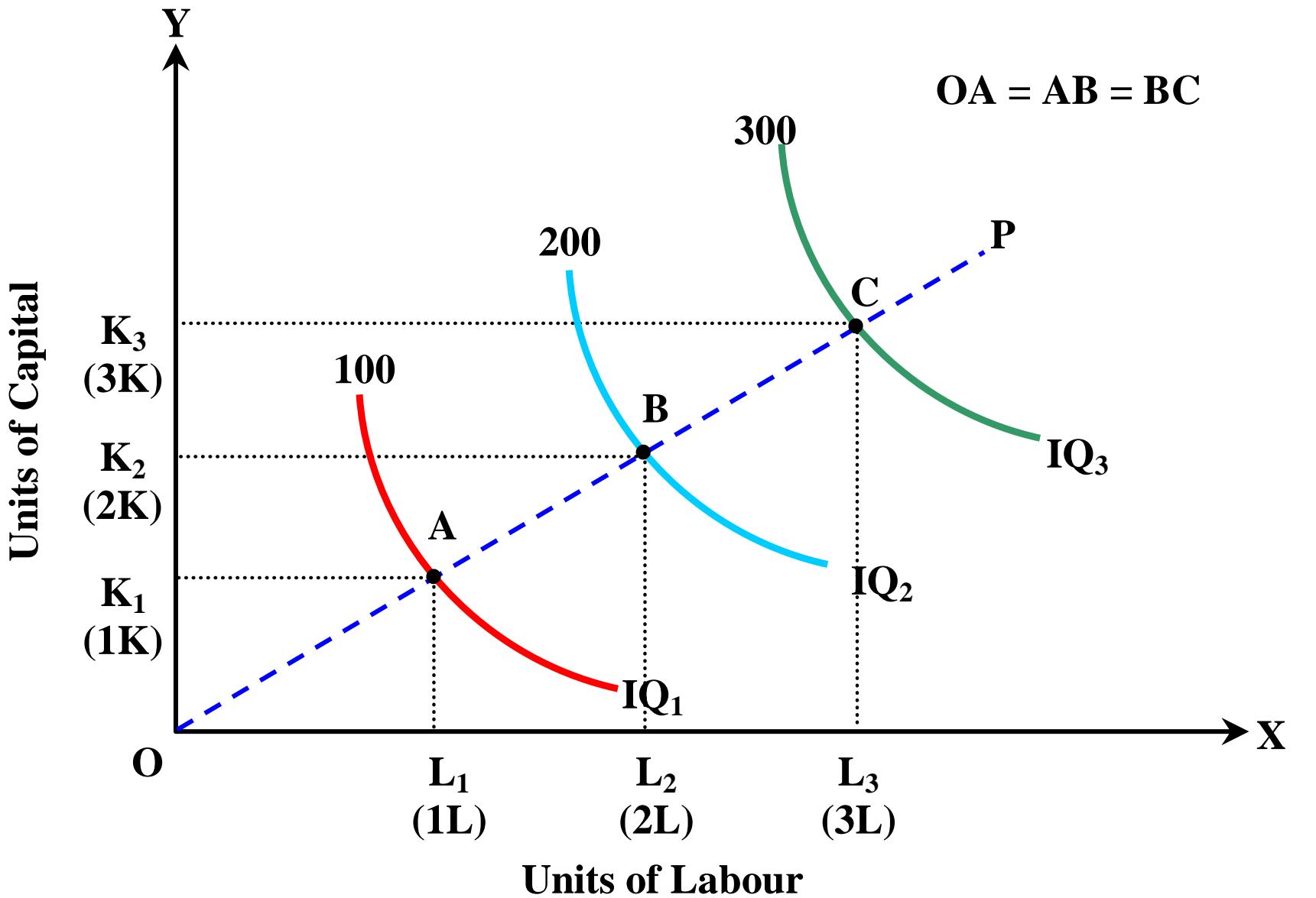
1. Technical and managerial indivisibilities
2. Higher degree of specialization
3. Dimensional relations

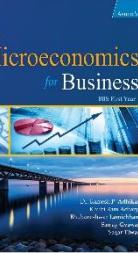


# Constant Returns to Scale

- Constant returns to scale refers to the equal proportionate or percentage change in output and inputs.
- It means that if inputs are doubled, output will be also double and if inputs are tripled, output will be also triple and so on.

Combinations	Labors (L)	Capital (K)	Total Product (TP)
A	1	1	100
B	2	2	200
C	3	3	300





# Constant Returns to Scale Contd.

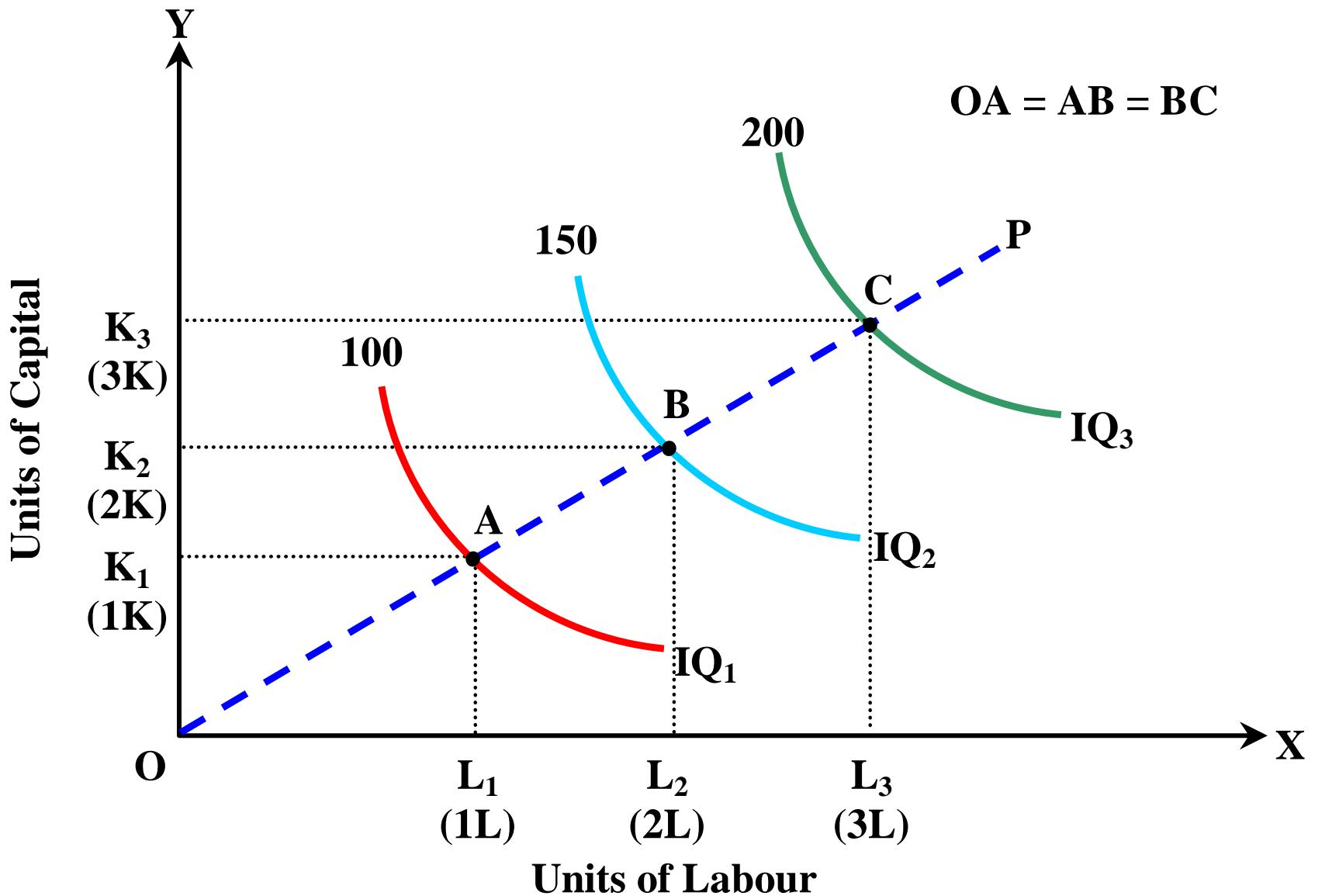
## Causes of Constant Returns to Scale

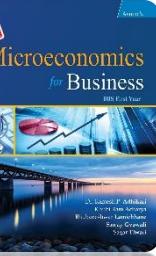
1. Limitations of economies of scale
2. Divisibility of inputs

# Decreasing Returns to Scale

- Decreasing returns to scale refers to increase in output at a smaller proportion or percentage than the proportionate or percentage increase in inputs.
- It means that if inputs are doubled, output will be less than double and if inputs are tripled, output will be less than triple and so on.

Combinations	Labors (L)	Capital (K)	Total Product (TP)
A	1	1	100
B	2	2	150
C	3	3	200

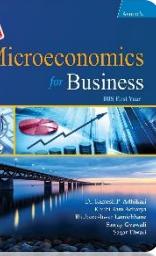




# Decreasing Returns to Scale Contd.

## Causes of Decreasing Returns to Scale

1. Managerial diseconomies
2. Limitedness of the natural resources
3. Labour diseconomies
4. Entrepreneurship as a fixed factor



## Numerical Examples 1

Consider the following data

No. of Labour (L)	1	2	3	4	5	6	7	8
Total Output	40	100	180	240	280	300	310	300

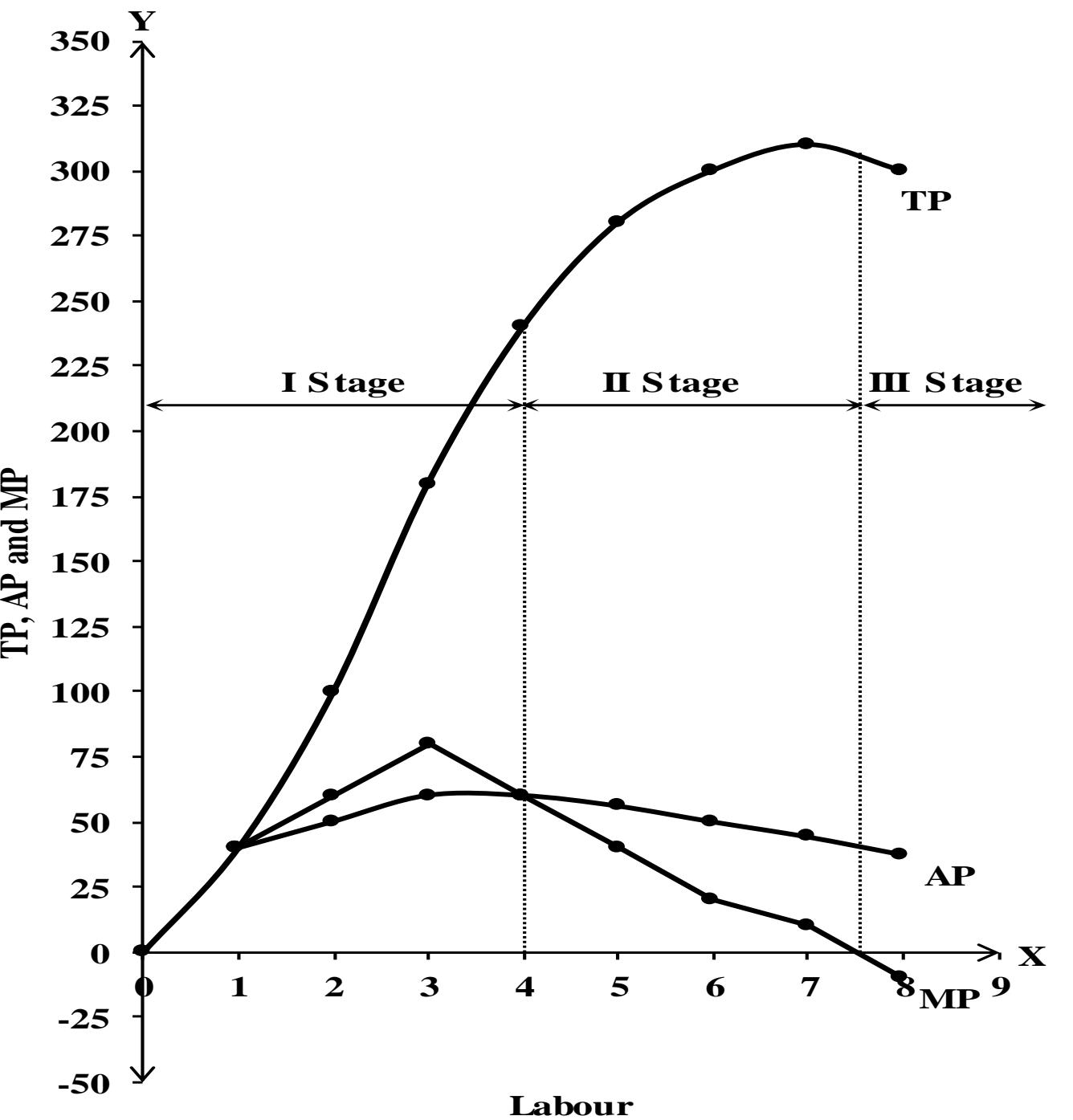
- a. Compute AP and MP.
- b. Graph TP, AP and MP and explain their relationship in reference to law of variable proportions.
- c. Using schedule, explain the relationship between (i) TP and MP and (ii) AP and MP.

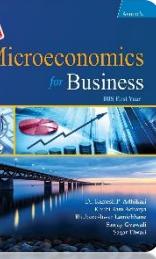
# SOLUTION

## a. Computation of AP and MP

Labour (Units)	TP	AP	MP
0	0	-	-
1	40	40.0	40
2	100	50.0	60
3	180	60.0	80
4	240	60.0	60
5	280	56.0	40
6	300	50.0	20
7	310	44.3	10
8	300	37.5	-10

b. Graphical Representation of TP, AP and MP.



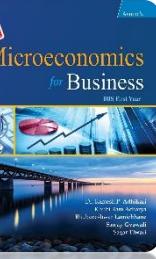


There are three stages of the law of variable proportions which are explained below:

**Stage I:** In this stage, TP first increases at an increasing rate up to the 3<sup>rd</sup> unit of labour and increases at a diminishing rate up to the 4<sup>th</sup> unit of labour. AP is increasing throughout the stage. MP first increases and after reaching its maximum starts falling. This stage ends at the point where AP = MP. AP and MP are equal at 4<sup>th</sup> unit of output.

**Stage II:** In this stage, TP increases at a diminishing rate. AP and MP both are decreasing. This stage ends at the point where MP = 0 or TP is the maximum.

**Stage III:** In this stage, TP is decreasing. Both AP and MP are decreasing. AP remains positive but MP is negative.



- c. i. The relationship between AP and MP are as follows:
  - When  $AP < MP$ , AP increases
  - When  $AP = MP$ , AP is the maximum
  - When  $AP > MP$ , AP is decreasing.
- ii. The relationship between TP and MP are as follows:
  - When  $MP > 0$ , TP is increasing.
  - When  $MP = 0$ , TP is at its maximum.
  - When  $MP < 0$ , TP is decreasing.

## Numerical Examples 2

Using the production function,  $Q = 16L + 8L^2 - L^3$ , answer the following:

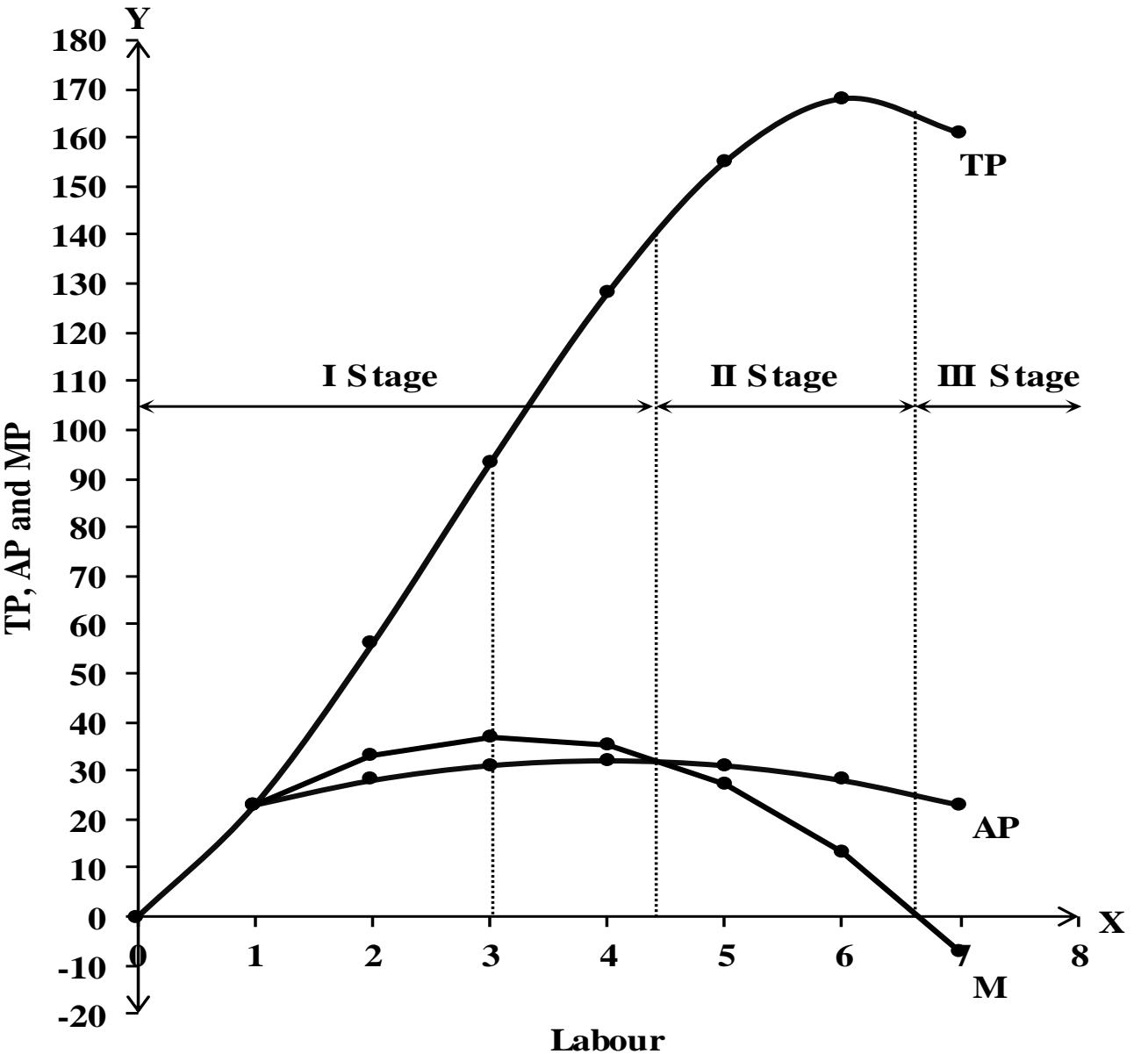
- a. Compute TP, AP and MP schedules.
- b. Draw TP, AP and MP and explain the three stages of production or law of variable properties.
- c. Using production schedule, explain the relationship between AP and MP.

## SOLUTION

a. TP, AP and MP schedules has been computed as follows:

Labour (Units)	$TP = Q = 16L + 8L^2 - L^3$	AP	MP	Stages of Production
0	$16 \times 0 + 8 \times 0^2 - 0^3 = 0$	-	-	I Stage
1	$16 \times 1 + 8 \times 1^2 - 1^3 = 23$	23	23	
2	$16 \times 2 + 8 \times 2^2 - 2^3 = 56$	28	33	
3	$16 \times 3 + 8 \times 3^2 - 3^3 = 93$	31	37	
4	$16 \times 4 + 8 \times 4^2 - 4^3 = 128$	32	35	II Stage
5	$16 \times 5 + 8 \times 5^2 - 5^3 = 155$	31	27	
6	$16 \times 6 + 8 \times 6^2 - 6^3 = 168$	28	13	
7	$16 \times 7 + 8 \times 7^2 - 7^3 = 161$	23	-7	III Stage

b. Based on the above schedule, TP, AP and MP curves can be drawn as follows:



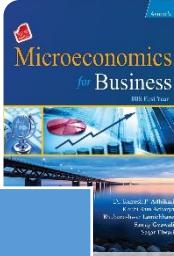
Based on the above table and schedule, three stages of production can be explained as follows:

- i. **First stage (Stage of increasing returns):** In this stage, TP increases at an increasing rate up to 3<sup>rd</sup> unit of labour and increases at the decreasing rate with increase in units of labour. AP increases throughout the stage. MP increases up to 3<sup>rd</sup> unit of labour and thereafter, it declines. This stage ends when AP = MP.
- ii. **Second stage (Stage of diminishing returns):** In this stage, TP increases at the diminishing rate. At 6<sup>th</sup> unit of labour, TP is maximum. Both AP and MP are decreasing. At the end of the stage, when TP is maximum, MP = 0.
- iii. **Third stage (Stage of negative returns):** In this stage, TP is continuously decreasing. AP is also continuously decreasing but never becomes zero and negative. MP is negative.

- c. The relationship between AP and MP is as follows:
- When  $AP > MP$  up to 3<sup>rd</sup> unit of output, AP is increasing.
  - At the labour range of 3<sup>rd</sup> unit to 4<sup>th</sup> unit, AP increasing but MP is decreasing.
  - At the labour range of 4<sup>th</sup> unit to 7<sup>th</sup> unit, both AP and MP are declining. MP is negative at 7<sup>th</sup> unit of labour.

## Numerical Examples 3

Consider the following three production preference schedules:



Schedule I				Schedule II				Schedule III			
Combinations	K	L	Out-put	Combinations	K	L	Out-put	Combinations	K	L	Out-put
A	1	20	1000	E	1	22	1200	M	1	27	1500
B	2	16	1000	F	2	17	1200	N	2	22	1500
C	3	13	1000	G	3	14	1200	O	3	18	1500
D	4	12	1000	H	4	13	1200	P	4	17	1500

Suppose, a producer has fixed total cost outlay equal to Rs. 2000. Prices of labour per units and capital per unit are Rs. 100 and Rs. 200 respectively.

- Compute total cost for each combinations containing in each production preference schedule and identify least cost combination which maximize output at given total cost outlay.
- Sketch an iso-cost line and IQ map and identify that which combination of capital and labour will put the producer at an optimum point.

## SOLUTION

Given

Total cost outlay ( $C$ ) = 2000

Price of labour ( $P_L$  or  $w$ ) = Rs. 100

Price of capital ( $P_K$  or  $r$ ) = Rs. 200

## a. Calculation of Total Cost

Schedule	Combination	K	P <sub>K</sub>	L	P <sub>L</sub>	Total Outlay (P <sub>K</sub> . K + P <sub>L</sub> . L = C)
i.	A	1	200	20	100	$200 \times 1 + 100 \times 20 = 2200$
	B	2	200	16	100	$200 \times 2 + 100 \times 16 = 2000$
	C	3	200	13	100	$200 \times 3 + 100 \times 13 = 1900$
	D	4	200	12	100	$200 \times 4 + 100 \times 12 = 2000$
ii.	E	1	200	22	100	$200 \times 1 + 100 \times 22 = 2400$
	F	2	200	17	100	$200 \times 2 + 100 \times 17 = 2100$
	G	3	200	14	100	$200 \times 3 + 100 \times 14 = 2000$
	H	4	200	13	100	$200 \times 4 + 100 \times 13 = 2100$
iii.	M	1	200	27	100	$200 \times 1 + 100 \times 27 = 2900$
	N	2	200	22	100	$200 \times 2 + 100 \times 22 = 2600$
	O	3	200	18	100	$200 \times 3 + 100 \times 18 = 2400$
	P	4	200	17	100	$200 \times 4 + 100 \times 17 = 2500$

As shown in the above schedule least cost combinations of two inputs are C, G and O respectively. However, given the total outlay combinations G is the optimal combination. It is the highest possible combinations producing 1200 units at the given prices of two inputs that is  $P_K = \text{Rs. } 200$  and  $P_L = \text{Rs. } 100$ .

b. If  $L = 0, K = \frac{C}{P_K} = \frac{2,000}{200} = 10 \text{ units}$

Hence, A(0, 10)

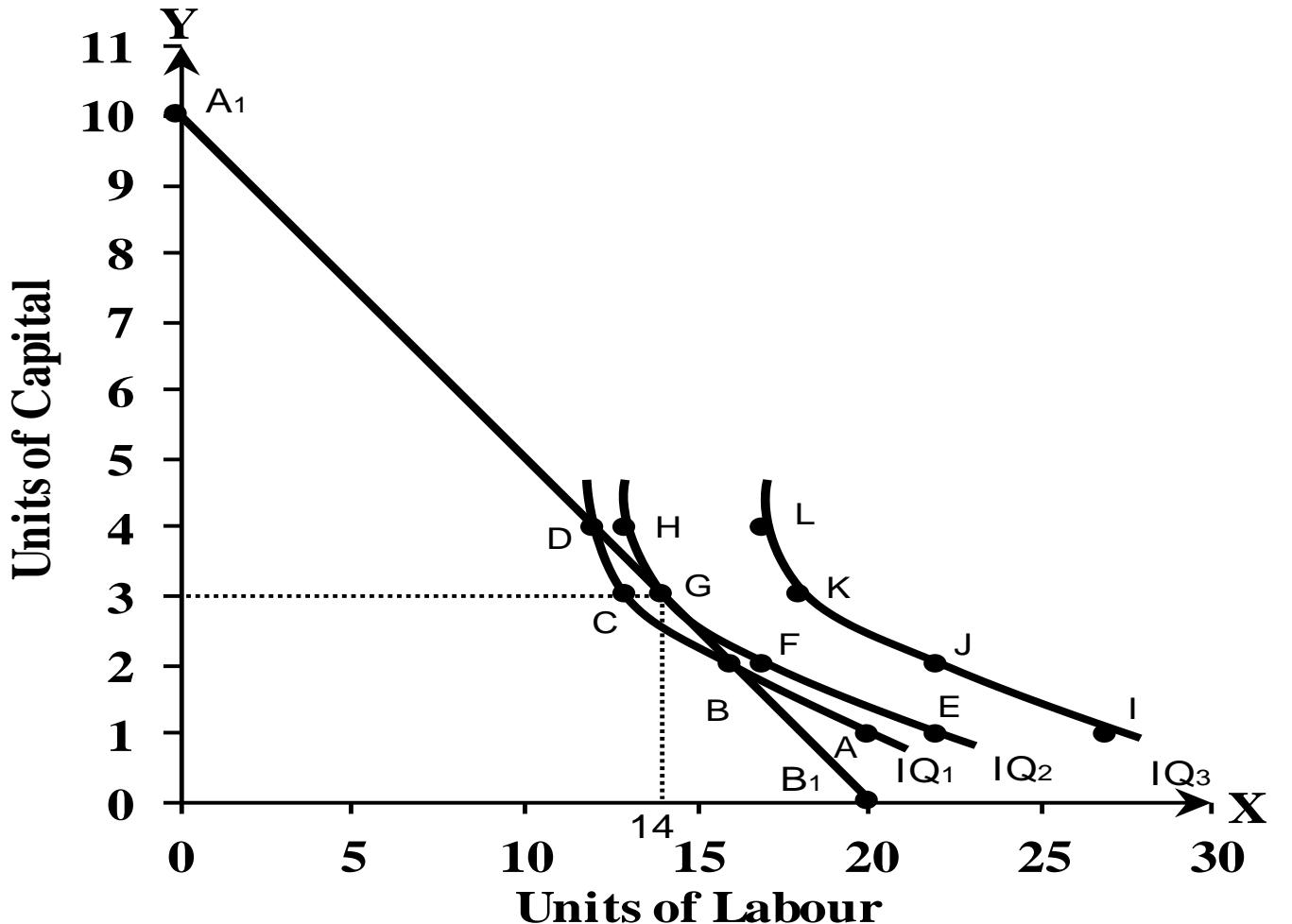
If  $K = 0, L = \frac{C}{P_L} = \frac{2,000}{100} = 20 \text{ units}$

Hence, B(20, 0)

This gives iso-cost cost line  $A_1B_1$ , which is shown in the following figure.:

Now, plotting these points and the given production schedules, we get equilibrium point as show in the following figure.

In the figure, a producer is in equilibrium at point F. In this situation he produces 1200 units of output by employing 3 units of capital and 14 units of labour.



# Thank You

