Equations Reducible to Quadratics

Q1 Solve :

$$\boxed{9x^4 - 148x^2 + 64 = 0}$$

Technique : $\boxed{x^2 = y}$

$$9(x^2)^2 - 148x^2 + 64 = 0$$

$$\Rightarrow 9y^2 - 148y + 64 = 0$$

[α, β = roots] $\alpha + \beta = \frac{148}{9} \Rightarrow \alpha = \frac{74}{9} + u \; ; \; \beta = \frac{74}{9} - u$

$$\alpha\beta = \frac{64}{9} = \left(\frac{74}{9}\right)^2 - u^2$$

$$\Rightarrow u^2 = \left(\frac{74}{9}\right)^2 - \frac{64}{9} = \left(\frac{74}{9}\right)^2 - \left(\frac{8}{3}\right)^2$$

$$\Rightarrow u = \sqrt{\frac{98 \times 50}{81}} = \frac{7 \times 10}{9} = \left(\frac{70}{9}\right)$$

$$\alpha = \frac{74}{9} + \frac{70}{9} = \frac{144}{9}$$

$$\beta = \frac{74}{9} - \frac{70}{9} = \frac{4}{9}$$

So,

$$x^2 = \frac{144}{9}$$

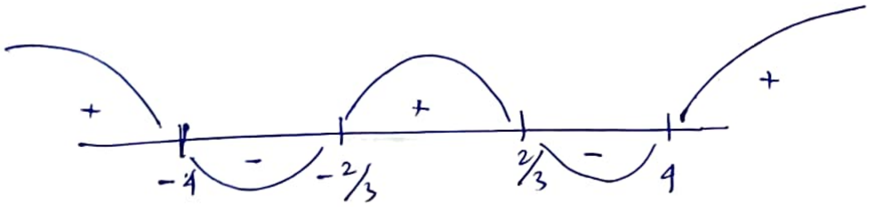
$$x = \pm\sqrt{\frac{144}{9}} = \pm\frac{12}{3}$$

$$\boxed{x = \pm 4}$$

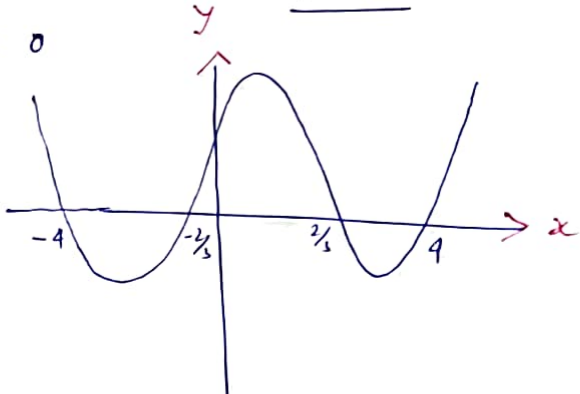
$$x^2 = \frac{4}{9}$$

$$\boxed{x = \pm\frac{2}{3}}$$

$$(x+4)(x-4)(x+2/3)(x-2/3) = 0$$



Graph



~~Q2.~~ 2. Solve. $\boxed{8x^{3/2} - 8x^{-3/2} = 63}$.

Consider : $8x^{3/2} - \dfrac{8}{x^{3/2}} = 63$

$\Rightarrow 8x^3 - 8 = 63x^{3/2}$ ———— eq. 1.

Consider $x^{3/2} = y$,

$8y^2 - 8 = 63y$

$8y^2 - 63y - 8 = 0$

$\uparrow$

$-64 + 1 \Rightarrow \left.\begin{array}{l} \alpha = \dfrac{64}{8} = 8 \\ \beta = -\dfrac{1}{8} \end{array}\right\}$

$x^{3/2} = 8$

$x = 8^{2/3}$

$x = (\sqrt[3]{8})^2 = 4$

$x^{3/2} = -\dfrac{1}{8}$

$x = \left(-\dfrac{1}{8}\right)^{2/3} = \left(\sqrt[3]{-\dfrac{1}{8}}\right)^2 = \dfrac{1}{4}$

So, $\boxed{x = \frac{1}{4} \text{ or } 4}$

~~3~~. B.3. Solve: $\boxed{\sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} = 2\frac{1}{6}}$.

An: Consider $y = \sqrt{\frac{x}{1-x}}$

$$y + \frac{1}{y} = 2\frac{1}{6}$$

$$\Rightarrow y^2 + 1 = \frac{13}{6}y$$

$$\Rightarrow 6y^2 - 13y + 6 = 0$$

$$-9 - 4 \Rightarrow \left.\begin{array}{l} \alpha = \frac{9}{6} = \frac{3}{2} \\ \beta = \frac{4}{6} = \frac{2}{3} \end{array}\right\}$$

So,

$$\sqrt{\frac{x}{1-x}} = \frac{3}{2}$$

$$\frac{x}{1-x} = \frac{9}{4}$$

$$4x = 9 - 9x$$

$$13x = 9$$

$$\boxed{x = \frac{9}{13}}$$

$$\sqrt{\frac{x}{1-x}} = \frac{2}{3}$$

$$\Rightarrow \frac{x}{1-x} = \frac{4}{9}$$

$$\Rightarrow 9x = 4 - 4x$$

$$\Rightarrow \boxed{x = \frac{4}{13}}$$

~~4.~~ 34. Solve:

$$\boxed{(x+2)(x+4)(x+6)(x+8) = 105}$$

Technique

$$f(x) = (x+2)(x+\boxed{4})(x+\boxed{6})(x+8) - 105 = 0$$

10

10

$$= \left[(x+2)(x+8)\right]\left[(x+4)(x+6)\right] - 105 = 0.$$

$$\Rightarrow (x^2+10x+16)(x^2+10x+24) - 105 = 0$$

Consider, $x^2+10x = y$,

$$(y+16)(y+24) = 105$$

$$\Rightarrow y^2 + 40y + (320 + 64 - 105) = 0$$

$$\Rightarrow y^2 + 40y + 279 = 0$$

$\uparrow$

31, 9

$\left.\begin{aligned}\alpha &= -31\\ \beta &= -9\end{aligned}\right\}$

So, $x^2+10x+31 = 0$

roots: $-5 \pm \sqrt{6}\, i$

$x^2+10x+9 = 0$

$\uparrow$

9+1

$\left.\begin{aligned}\alpha' &= -9\\ \beta' &= -1\end{aligned}\right\}$

So, $\boxed{x = -5 \pm i\sqrt{6},\ -1,\ -9}$

Q.5
imp

5. Solve: $\boxed{x^2 - 12x + \sqrt{x^2 - 12x + 81} = 9}$

Ans: Technique

$$x^2 - 12x + \sqrt{\boxed{x^2 - 12x + 81}} = 9$$

Sol: $\Rightarrow (x^2 - 12x + 81) + \sqrt{x^2 - 12x + 81} - 81 - 9 = 0$

$\Rightarrow$ Let $\sqrt{x^2 - 12x + 81} = y$

$\Rightarrow \quad y^2 + y - 90 = 0$

$\alpha + \beta = -1 \quad \Rightarrow \quad \alpha = -\frac{1}{2} + u \qquad \beta = -\frac{1}{2} - u$

$\alpha\beta = -90$

$\Rightarrow \quad \frac{1}{4} - u^2 = -90$

$\Rightarrow \quad u^2 = \left(90 + \frac{1}{4}\right) = \left(\frac{361}{4}\right)$

$\alpha, \beta = -\frac{1}{2} \pm \sqrt{\frac{361}{4}} \quad \Rightarrow \quad \alpha = \sqrt{\frac{361}{4}} - \frac{1}{2} = \left(\frac{19}{2} - \frac{1}{2}\right)$

$= 9$

$\beta = -10$

Hence, $x^2 - 12x + \cancel{81} = \cancel{81}$ or $x^2 - 12x + 81 = 100$

$\Rightarrow x(x - 12) = 0$

$$\boxed{x = 0, 12}$$

$x^2 - 12x - 19 = 0$

$r_1 + r_2 = 12 \quad (6 + u,\ 6 - u)$

$36 - u^2 = -19 \Rightarrow u = \sqrt{55}$

$$\boxed{x = 6 \pm \sqrt{55}}$$

~~6~~. 86

Solve: $\boxed{x^4 - 3x^3 + 3x + 1 = 0}$ Form: $a x^4 + b x^3 + c.x^2 + b.x + a = 0$

Soln:

$$x^4 - 3x^3 + 3x + 1 = 0$$

$$\Rightarrow x^2 + \frac{3}{x} - 3x + \frac{1}{x^2} = 0 \quad (\text{dividing by } x^2)$$

$$\Rightarrow \left(x^2 + \frac{1}{x^2}\right) - 3\left(x - \frac{1}{x}\right) = 0$$

$$\Rightarrow \left\{\left(x - \frac{1}{x}\right)^2 + 2\right\} - 3\left(x - \frac{1}{x}\right) = 0$$

Consider, $x - \frac{1}{x} = y$,

$$(y^2 + 2) - 3y = 0$$

$$\Rightarrow y^2 - 3y + 2 = 0$$

$$-2, -1 \Rightarrow \left.\begin{aligned} \alpha &= 2 \\ \beta &= 1 \end{aligned}\right\}$$

So, $x - \frac{1}{x} = 2$ | $x - \frac{1}{x} = 1$

$x^2 - 2\cancel{y}x - 1 = 0$

$\alpha + \beta = 2$ $\alpha = 1 + u$, $\beta = 1 - u$

$1 - u^2 = -1$

$u^2 = 2$

$u = \pm\sqrt{2}$

$$\boxed{x = 1 \pm \sqrt{2}}$$

$x^2 - x - 1 = 0$

$\alpha + \beta = 1$ $\alpha\beta = -1$

$\alpha = \frac{1}{2} + u$, $\beta = \frac{1}{2} - u$

$\frac{1}{4} - u^2 = -1$

$\Rightarrow u^2 = \frac{5}{4}$

$\Rightarrow u = \pm\frac{\sqrt{5}}{2}$

$$\boxed{x = \frac{1}{2} \pm \frac{\sqrt{5}}{2}}$$

~~Q.7.~~

~~7~~. Solve: $\boxed{2\sqrt{x-3} + \sqrt{2x+1} = \sqrt{10x+1}}$

Ans:

Squaring both sides:

$$4(x-3) + (2x+1) + 4\sqrt{(x-3)(2x+1)} = 10x+1$$

$$4x - 12 + 2x + \cancel{1} + 4\sqrt{(x-3)(2x+1)} = 10x + \cancel{1}$$

$$4\sqrt{(x-3)(2x+1)} = 4x + 12$$

$$\sqrt{(x-3)(2x+1)} = x+3$$

Squaring again,

$$(x-3)(2x+1) = x^2 + 9 + 6x$$

$$\Rightarrow 2x^2 - 5x - 3 = x^2 + 9 + 6x$$

$$\Rightarrow x^2 - 11x - 12 = 0$$

$\uparrow$

$$-12, +1 \Longrightarrow \left.\begin{matrix} \alpha = 12 \\ \beta = -1 \end{matrix}\right\}$$

$$\boxed{\begin{matrix} x = 12 \\ x = -1 \end{matrix}}$$

Q.8

8 Solve:

$$\boxed{\sqrt{x^2+4x-21} + \sqrt{x^2-x-6} = \sqrt{6x^2-5x-39}}$$

Sol: ~~Technique~~ ; I see that $x^2+4x-21=0$ has 2 factors -7 and 3. Look if other 2 equations also have a factor 3. or -7.

$x^2+4x-21=0 \Rightarrow \alpha = -7 \Rightarrow (x+7)(x-3)$ ——— (1)

$+7, -3$ $\quad \beta = 3$

$x^2-x-6=0 \Rightarrow \alpha = 6 \Rightarrow (x-6)(x+1)$ ——— (2)

$-6, 1$ $\quad \beta = -1$

$6x^2-5x-39=0$

$\alpha+\beta = \frac{5}{6} \Rightarrow \alpha = \frac{5}{12} + \frac{31}{12} \Rightarrow (x-3)(x+\frac{13}{6})$ ——— (1)

$\alpha\beta = -\frac{39}{6}$ $\qquad = \frac{36}{12} = 3$

$\alpha = \frac{5}{12}+u \quad \beta = \frac{5}{12}-u$ $\qquad \beta = \frac{5}{12} - \frac{31}{12}$

$\frac{25}{144} - u^2 = -\frac{39}{6}$ $\qquad = \frac{-26}{12}$

$u^2 = \left(\frac{25}{144} + \frac{39}{6}\right)$ $\qquad = -\frac{13}{6}$

$= \frac{25+39\times 24}{144}$

$= \frac{961}{144}$

$u = \sqrt{\frac{961}{144}} = \frac{31}{12}$

$$\Rightarrow \left\{ \sqrt{x+7} + \sqrt{x+2} - \sqrt{6x+13} \right\} \sqrt{x-3} = 0$$

sol.

$$\Rightarrow \text{ either } \boxed{x = 3}$$

$$\text{or } \sqrt{x+7} + \sqrt{x+2} = \sqrt{6x+13}$$

Squaring, $x+7+x+2+2\sqrt{(x+7)(x+2)} = 6x+13$

$$\Rightarrow 4x+4 = 2\sqrt{(x+7)(x+2)}$$

$$\Rightarrow 2x+2 = \sqrt{(x+7)(x+2)}$$

$$\Rightarrow 4x^2+4+8x = x^2+9x+14$$

$$\Rightarrow 3x^2 - x - 10 = 0$$

$$\uparrow$$

$$-6, 5 \Longrightarrow \alpha = \frac{6}{3} = 2$$

$$\beta = -\frac{5}{3}$$

sol.

So, $\boxed{x = 2 \text{ or } -\frac{5}{3}}$

~~Q.9~~

Q. Solve

$$\boxed{\sqrt{x^2 - 3x + 16} - \sqrt{x^2 - 3x + 9} = 1}$$

Ans:

$$(x^2 - 3x + 16) - (x^2 - 3x + 9) = 16 - 9 = 7.$$

So,
$$\sqrt{x^2 - 3x + 16} + \sqrt{x^2 - 3x + 9} = 7$$

&
$$\sqrt{x^2 - 3x + 16} - \sqrt{x^2 - 3x + 9} = 1$$

$$2\sqrt{x^2 - 3x + 9} = 6$$

$$x^2 - 3x + \cancel{9} = \cancel{9}$$

$$x^2 - 3x + 0 = 0$$

$$\uparrow$$

$+3, 0$

So, Sol: $\boxed{x = 0, \quad x = 3}$