Set Theory, An Introduction!

Introduction

$a \in A \Rightarrow$ a is an element in the set A.

$a \notin A \Rightarrow$ a is not an element of the set A.

ex

$A = \{1, 2, \pi, 3\}$ ie A has 4 elements.

$2 \in A$, $3 \in A$, but $4 \notin A$

Certain Common Sets:

(i) $N = \{0, 1, 2, 3, \ldots\}$ = set of Natural nos.

(ii) $Z = \{\ldots, -3, -2, -1, 0, 1, 2, 3 \ldots\}$ = set of Integers

(iii) Q = set of rational nos.

(iv) R = set of real nos.

Basic definitions:

Let A, B be 2 sets:

1. $A = B$, both A and B have equal no. of elements
2. $A \subseteq B$, A is a subset of B

   ie every element of A is also an element of B
3. $A \subset B$, A is a proper subset of B

   ie, $A \subseteq B$ & $A \neq B$
4. ϕ, empty set (no elements)
5. A, B are <u>disjoint</u> when they do not have elements in Common

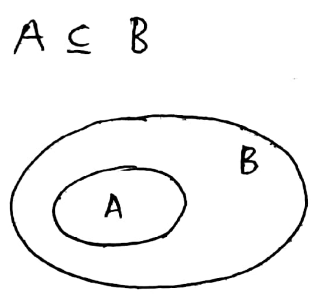
Thus, $A \subseteq A$, $\phi \subseteq A$ for any set A.

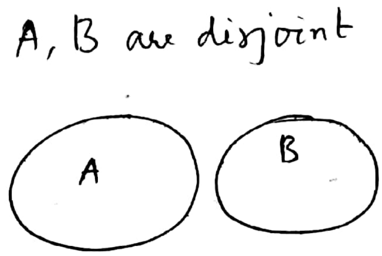
<u>Question</u> Why $\phi \subseteq A$?

<u>Proof</u> : Let us suppose, $\phi \nsubseteq A$ ie there must exist $x \in \phi$ such that $x \notin A$. As there is no such x st $x \in \phi$ ($\because \phi$ is an empty set), we arrive at a contradiction.

Thus, $\phi \subseteq A$ ▨

<u>Venn Diagrams</u> : (for Visualization)





Defn <u>Property</u> : A property is a statement that asserts something about one or more variables

eg. $\{x \in R \text{ and } x \notin N\}$ is a property of variable x

Q. <u>How to construct a subset ?</u>

One way is by the method of <u>Separation</u>.

Let A be a set

Given a <u>property</u> $P(x)$ about variable x, We can construct a set of objects $x \in A$ that satisfy the property $P(x)$

ie, we form the truth set : $\{x \in A ; P(x)\} \subseteq A$

eg: $A = \{ x \in N ;\ 3 < x < 11 \}$

Here $A = \{4, 5, 6, 7, 8, 9, 10\} \subseteq N$

Def: Interval : An interval is a set too.

1. (a, b) : Open interval $\Rightarrow (a, b) = \{ x \in R : a < x < b \}$
2. $[a, b]$: Closed interval $\Rightarrow [a, b] = \{ x \in R : a \le x \le b \}$
3. $(a, b]$: right closed interval $\Rightarrow (a, b] = \{ x \in R : a < x \le b \}$
4. $[a, b)$: left closed interval $\Rightarrow [a, b) = \{ x \in R : a \le x < b \}$

Def: Power Set :

Let A be a set.

$P(A)$ is the power set of A ; $P(A) = \{ X : X \subseteq A \}$

ie, Power set of A is the set whose elements are all the subsets of A.

Let us say A has n elements ie $A = \{a_1, a_2, \dots a_n\}$

Then $P(A)$, the power set of A, will have 2^n elements

eg: $A = \{1, 2, 3\}$

$P(A) = \{ \phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\} \}$

= ie, $2^3 = 8$ elements

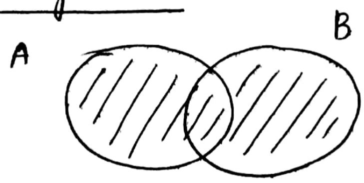
Def: Let A, B be 2 sets.

$$A \cup B = \{x : x \in A \text{ or } x \in B\} \Rightarrow \text{Union}$$

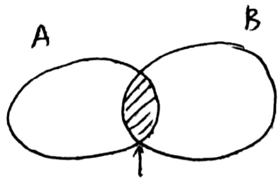
$$A \cap B = \{x : x \in A \text{ and } x \in B\} \Rightarrow \text{intersection}$$

$$A \setminus B = \{x : x \in A \text{ and } x \notin B\} \Rightarrow \text{set difference}$$

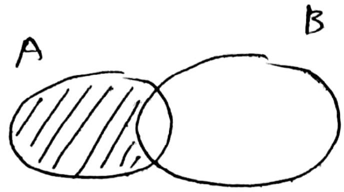
Venn Diagrams



$A \cup B$



$A \cap B$



$A \setminus B$

Use Venn diagrams to understand the below results:

1. $A \setminus B \subseteq C$ then $A \setminus C \subseteq B$.
2. if $A \subseteq B$ then $C \setminus B \subseteq C \setminus A$
3. if $A \subseteq B$, $A \subseteq C$ then $A \subseteq (B \cap C)$
4. $A \setminus (B \setminus C) \subseteq (A \setminus B) \cup C$