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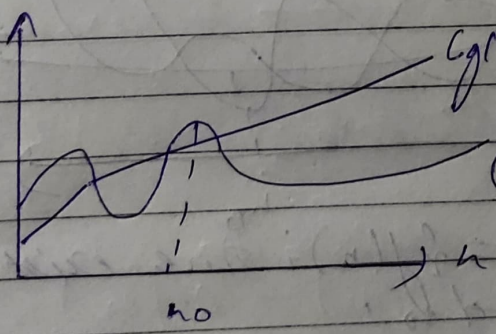
## Assignment - 1

Ans-1 Asymptotic notation are used to represent the complexities of algorithms for asymptotic analysis.

These notation are used for very large input.

(1) Big-Oh (O) -

It gives upper bound for a function $f(n)$ to within a constant factor

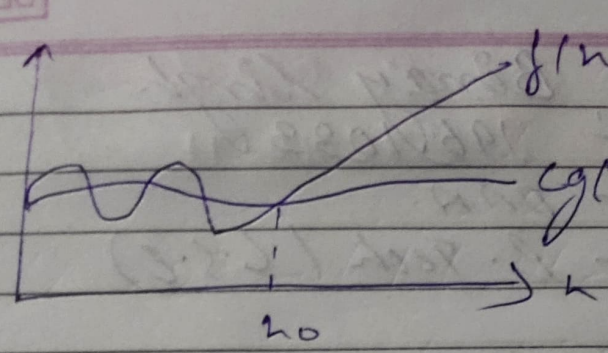



$f(n) \leq c \cdot g(n)$ $\forall$ $n \geq n_0$ , $c > 0$

eg- $o(n^2 + 3n) = O(n^2)$

(2) Big-omega Notation ($\Omega$)

Big-omega ($\Omega$) notation gives a lower bound for $f(n)$ to within a constant factor

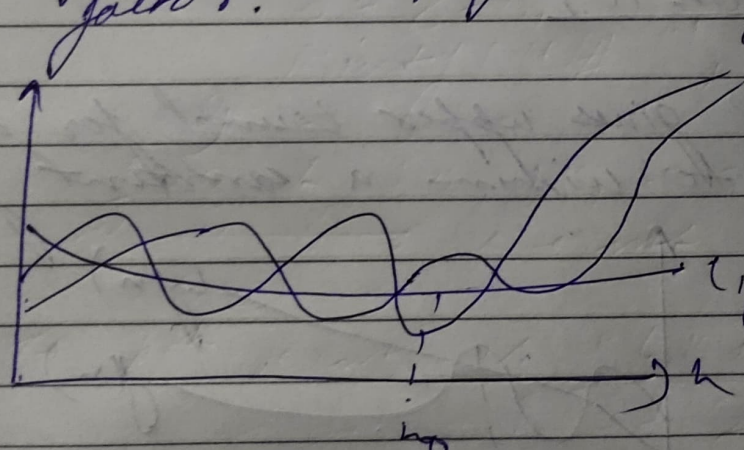



$\Omega(g(n)) = \{ f(n) \}$ ; There exist the constant c and $n_0$ such that $0 \leq cg(n) \leq f(n)$ $\forall n \geq n_0$ }

eg $= n(n \log n)$

(B) Big theta notation ($\Theta$)

It gives bound of function within a constant factor.




$\Theta(g(n)) = \{ f(n)$ ; There exist +ve constant $c_1, c_2$ and $n_0$ such that $0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n), \forall n \geq$

eg $= \Theta(n^2)$

Ans 2

```
for(i=1 to n)
{
    for(j=i+2);
}
```

$1, 2, 4, 8, \ldots\ldots n$

$T(n) = O(\log_2 n)$

Ans-3 $T(n) = \begin{cases} 3T(n-1) & n > 0 \\ 1 & n = 0 \end{cases}$

$T(n) = 3T(n-1) \quad -(1)$

$T(n-1) = 3T(n-2)$

$T(n) = 9T(n-2) \quad -(2)$

$T(n) = 3^3 T(n-3) \quad -(3)$

$T(n) = 3^3 T(n-k) \quad -(4)$

for $T(n-k) = T(0)$

$n - k = 0$

$n = k$

$T(n) = 3^n T(0)$

$T(n) = 3^n$

$T(n) = O(3^n)$ ,,

Ans-4 $T(n) = \begin{cases} 2T(n-1) - 1 & , n > 0 \\ 1 & , n = 0 \end{cases}$

$T(n) = 2T(n-1) - 1$

$T(n-1) = 2T(n-2) - 1$

$T(n) = 4T(n-2) - 1 - 2 \quad -(2)$

$T(n) = 8T(n-3) - (1+2+4) \quad -(3)$

$T(n) = 2^n T(n-k) - \underbrace{(1+2+4+\cdots 2^{k-1})}_{k \text{ terms}}$

$T(n-k) = T(0)$

$n = k$

$T(n) = 2^n T(0) - (1+2+4+\cdots\cdots)$

$k$ terms

Th $a C_2 \cdot 1^0$

$a = 1$

$r = 2$

$T(n) = 2^n - \frac{(1(2^{n-1}-1))}{2-1}$

$T(n) = 2^n - 2^n + 1$

$T(n) = 1$

$T(n) = O(1)$

Ans-5

```
int i = 1, s = 1;
while (s <= n) {
    i++;
    s = s + i;
    printf("#");
}
```

1, 3, 6, 10, 15, ---- n

<- k terms ->

If $k^{th}$ terms is $\frac{k(k+1)}{2} = n$

$k = \sqrt{n}$

$T(n) = O(\sqrt{n})$

Ans-6

```
void function (int n)
{
    int i, count = 0;
    for (int i = 1; i*i < n; i++)
        count ++
}
```

$T(n) = O(\sqrt{n})$

Ans-7 $T(n) = O(n * \log_2 n + \log_2 n)$

$T(n) = O(n * \log_2 n)^2$

$T(n) = O(n \log n)^2$

Ans-8 function (int n) & $T(n)$

if (n == 1) return;

for (i=1 to n) & $n^2$

for (j=1 to n) }

print f("*"); }

}

function (n-3); $T(n-3)$

}

$T(n) = T(n-3) + n^2$ — (1)

$T(n-1) = T(n-4) + (n-1)^2$

$T(n) = T(n-4) + n^2 + (n-1)^2$

$T(n) = T(n-5) + n^2 + (n-1)^2 + (n-2)^2$

$T(n) = T(n-k) + (n^2 + (n-1)^2 + (n-2)^2 + \cdots$ (k+2) terms

for $T(n-k) = 1$

$k = n-1$

$T(n) = T(1) + (n^2 + (n-1)^2 + (n-2)^2 + \cdots$

$(n-3)$ terms.

$T(n) = T(1) + (4^2 + 5^2 + \cdots n^2)$

$T(n) = T(1) + \left(\frac{(n-3)(n-2)(2n-5)}{6}\right)$

$T(n) = 1 + \left(\frac{2n^3 + \cdots}{6}\right)$

$T(n) = n^3$

$T(n) = O(n^3)$

Ans-9 void function (int n) {
for (i=1 to n) {
for (j=1 j<=n], j=j*i)
printf ("*");
}

i = 1 n times
i=2 1, 3, 5 --- n n
i=3 1, 4, 7 --- n n/2
;
;
i=n 0

$$T(n) = \left( n + \frac{n}{2} + \frac{n}{3} - - - \right)$$

$$T(n) = O(n \log n)$$

Ans-10 for the functions $n^k$ and $a^n$, what is the relation.

$k \geq 1$ and $a > 1$
relation is $n^k = O(a^n)$

Ans-11
```
void fun (int n)
{
  int j=1, i=0;
  while (i<n)
  {
    i = i+j;
    j = ++j;
  }
}
```

$\underbrace{0, 3, 6, 10, 15, \ldots\ldots n}_{k \text{ terms}}$

to for this series, is

$k^{th}$ terms is $k\frac{(k+1)}{2}$

$$n = \frac{k^2+k}{2}$$

$$n \approx \sqrt{n}$$

$$T = O(\sqrt{n})$$

Ans-12 Recurrence relation of fabonacci series

$$T(n) = \begin{cases} 1 & \\ T(n-1)+T(n-2)+1 & \end{cases}$$

$$T(n) = 2T(n-2)+1$$

$$T(n) = 4T(n-4)+3$$
$$T(n) = 8T(n-6)+7$$
$$T(n) = 16T(n-8)+15$$

$$T(n) = 2^k T(n-2k) + (2^k - 1)$$

for $T(n-2k) = T(0)$

$$n = 2k$$

$$k = \frac{n}{2}$$

$$T(n) = 2^{n/2} T(0) + (2^{n/2} - 1)$$
$$T(n) = 2^n - 1$$
$$T(n) = O(2^n)$$

Hence space complexity of fabinacci series is $O(n)$ as it depends on height of recursive tree it is equal to $n$ in fabinacci series.

Ans-15

```
for (int i to n)
{
    for(int j=1, j<n ; j+=i)
    {
        //O(1)
    }
}
```

$n, \frac{n}{2}, \frac{n}{3}, \frac{n}{4}, \frac{n}{5}, \dots\dots 1$

(k times)

$k = \log_2 n$.

$n\left(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \dots\dots \frac{1}{n}\right)$

$(n(\log n))$

$T(n) = O(n \log n)$

Ans-16

```
for (int i=2 ; i<=n ; i= pow(i, k)
{
    //O(1)
}
```

$2, 2^k, 2^{k^2}, 2^{k^3} \dots\dots n$

It G.P $a = 2$

$r = 2^k$

$k^m k \times m = a r^{k-1}$

$n = 2(2k)^{k-1}$

let $k(k-1) = x$

$k \log_2 k = \log x$

$k = \log x - (1)$

$k = 2^x$

$\log_2 k = x \log_2 2$

$x = \log_2 k$

$\log x = \log(\log k)$

from (1)

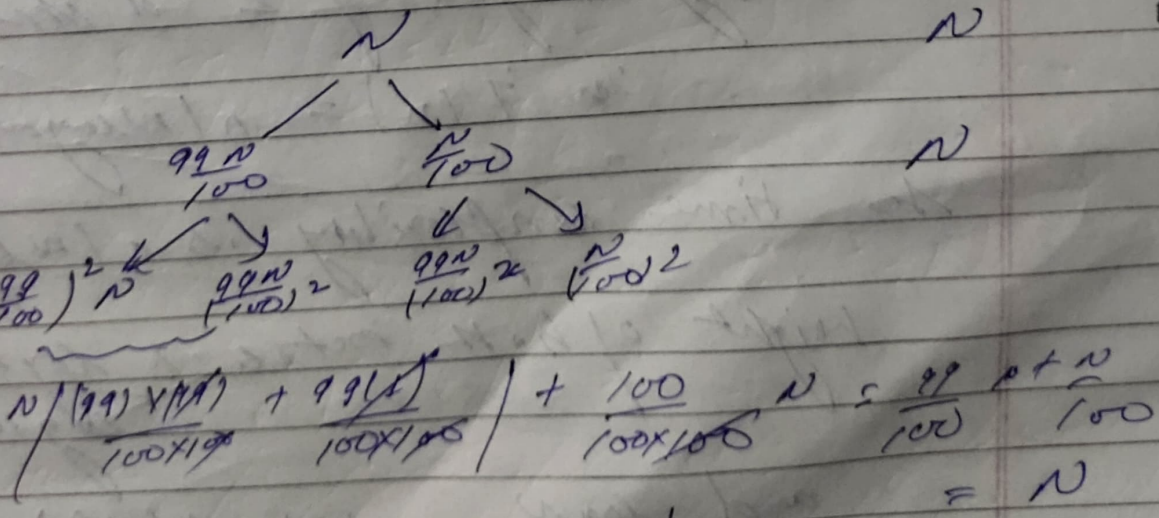
$k = \log(\log(n))$

$T(n) = O(\log(\log(n)))$

Ans-17 hence pivot is divided in 99% and 1% so.

$$T(n) = T\left(\frac{99}{100} n\right) + T\left(\frac{n}{100}\right) + n$$

Now as we can use 2 extreme of a tree.
where starting point is n



to cost of each level is N only.

Total cost of ~~each level is~~
= height * cost of each level.

to for k levels = $N, \frac{99N}{100}, \left(\frac{99}{100}\right)^2 N$ ----

$$\left(\frac{99}{100}\right)^{k-1} N = 1$$

$$\left(\frac{99}{100}\right)^{k-1} = \frac{1}{N}$$

$$N = \left(\frac{100}{99}\right)^{k-1}$$

$$\log N = k \log\left(\frac{1}{k}\right)$$

$$k = \log N \text{ of } \gamma$$

$$k = \frac{\log N}{\log(100/99)} + 1$$

similarly height of 2nd term -

$\therefore T(n) = O(N \log N)$,,

to time complexity is $O(N \log N)$

height of both octree is $\frac{\log N}{\log 100} + 1 \text{ of } \left(\frac{1}{100}\right)$

and $\frac{\log N}{\log\left(\frac{100}{99}\right)} + 1 \text{ of } \left(\frac{99}{100}\right)$

to we can conclude that if division is done more than height of tree will be boost and when division ratio is less than height is less.

Ans-19

```
void linear search (int arr[ ], int n, int key)
{
    for (i=0 to i=n)
        if arr[i] == key)
            cout << "found";
        else
            continue
}
```

Ans-20

## Iterative Insertion sort

```
Void Insertion sort (arr, n) {
    int i, temp, j
    for i=1 to n
    {
        temp = arr[i]
        j = i-1
        while j>0 && arr[j]>temp
        {
            arr[j+1] = arr[j]
            j--
        }
        arr[j+1] = temp
    }
}
```

## Recursive Insertion sort

```
insertion sort (arr, n)
{
  if n<=1
    return;
  insertion sort (arr, n-1);
  last = arr[n-1]
  j = n-2

  while (j>=0 and arr[j] > last)
  {
    arr[j+1] = arr[j]
    j--
  }
  arr[j+1] = last;
}
```

Insertion sort is called online sorting because it don't know the whole input it might make decision that later turn out to be not optimal.

Other algorithm are off-line algorithm not one discussed in lectures.

Ans-21)

| | Time complexity | | | Space |
|---|---|---|---|---|
| | Best | Avg | worst | |
| Bubble sort | $O(n^2)$ | $O(n^2)$ | $O(n^2)$ | $O(1)$ |
| Selection sort | $O(n^2)$ | $O(n^2)$ | $O(n^2)$ | $O(1)$ |
| Insertion sort | $(n)$ | $O(n^2)$ | $O(n^2)$ | $O(1)$ |
| Merge sort | $O(n \log n)$ | $O(n \log n)$ | $O(n \log n)$ | $O(n)$ & due to recursion |
| Quick sort | $O(n \log n)$ | $O(n \log n)$ | $O(n^2)$ | $O(n)$ |
| Heap sort | $O(n \log n)$ | $O(n \log n)$ | $O(n \log n)$ | $O(1)$ |

Ans-22

| | inplace | stable | online (1) |
|---|---|---|---|
| Bubble sort | Yes | Yes | NO |
| Selection sort | Yes | NO | NO |
| Insertion sort | Yes | Yes | Yes |
| Merge sort | NO | Yes | NO |
| Quick sort | Yes | NO | NO |
| Heap sort | Yes | NO | NO |

Ans-23

```
Binary search (arr, int n, key)
{
    beg = 0
    end = n - 1
    while (beg <= end)
    {
        mid = (beg + end) / 2
        if (arr[mid] == key)
            found
        else if arr[mid] < key
            beg = mid + 1
        else
            end = mid - 1
    }
}
```

Time complexity of linear search - O(n)

Space complexity of linear search - O(1)

Time complexity of Binary search = O(log n)

Space complexity of Binary search = O(n)

Ans-24

$$T(n) = T\left(\frac{n}{2}\right) + 1$$