MODULE 1: NUMERICAL METHODS

Numerical solution of ordinary differential equations of first order and first degree, Numerical solution of algebraic and transcendental equations

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1 Numerical solution of ordinary differential equations of first order and first degree

1.1 Taylor's series method

Method 1.1. Consider a differential equation

$$\frac{dy}{dx} = f(x, y), \quad y(x_0) = y_0 \tag{1}$$

The Taylor's Series Solution is:

$$y = y(x_0) + (x - x_0)y^{'}(x_0) + \frac{(x - x_0)^2}{2!}y^{''}(x_0) + \frac{(x - x_0)^3}{3!}y^{'''}(x_0) + \cdots \tag{2}$$

Problem 1.1.1. Solve the following differential equation using Taylor's Series Method at $x = 0.1$:

$$\frac{dy}{dx} = x - y^2, \quad y(0) = 1$$

Solution

Sol. Given that: $y^{'} = x - y^2, \quad x_0 = 0, \quad y_0 = 1$
$\therefore y^{'} = x - y^2, \quad y^{'}(0) = -1$
$\Rightarrow y^{''} = 1 - 2yy^{'}, \quad y^{''}(0) = 1$
$\Rightarrow y^{'''} = -2y^{'}y^{'} - 2yy^{''}, \quad y^{'''}(0) = -2$
$\Rightarrow y^{IV} = -2y^{'}y^{''} - 2y^{'}y^{''} - 2yy^{'''} - 2y^{'}y^{''}, \quad y^{IV}(0) = -4$
By Taylor's Series Method:
$y = y_0 + (x - x_0)y^{'}(x_0) + \frac{(x-x_0)^2}{2!}y^{''}(x_0) + \frac{(x-x_0)^3}{3!}y^{'''}(x_0) + \cdots$
$\Rightarrow y = 1 + (x - 0)(-1) + \frac{(x-0)^2}{2!}(1) + \frac{(x-0)^3}{3!}(-2) + \frac{(x-0)^4}{4!}(-4)$
$\Rightarrow y = 1 - x + \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{4}$
at $x = 0.1$
$\Rightarrow y = 1 - 0.1 + \frac{0.1^2}{2} - \frac{0.1^3}{3} + \frac{0.1^4}{4}$
$\Rightarrow y = 0.904$

1.2 modified Euler's method

2 Examples

Method 2.1. This is a theorem.

proposition 2.2. This is a proposition.

Principle 2.3. This is a principle.

2.1 Pictures

Figure 1: Sydney, NSW

2.2 Citation

This is a citation[**?**].

References