# Enhanced Multiuser Scheduling Using Modified SLNR Metric with Outdated Partial CSI

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***Abstract*—In this paper, we study the problem of multiuser (MU) scheduling in a MU-MIMO system with imperfect channel state information (CSI) feedback. The CSI feedback for scheduling is considered as a channel direction quantization with feedback delay effect. Inspired by the conventional zero-forcing (ZF) based user scheduling, we present an improved user selection scheme by using a modified signal-to-leakage-and-noise ratio (SLNR) based metric under only quantized CSI feedback with a channel quality information (CQI) indicator. Different from the traditional ZF-based beamforming(BF), we find that a simple channel-norm based CQI feedback will be sufficient for our modified SLNR-based scheduling. Numerical results show the effectiveness of our proposed scheduling under outdated and quantized CSI feedback.**

***Index Terms*—Multiple-input multiple-output (MIMO), multiuser scheduling, limited feedback, signal-to-leakage-and-noise ratio (SLNR), feedback delay.**

## I. INTRODUCTION

Multiple-input multiple-output (MIMO) techniques present a significant amelioration in achieving high capacity and reliability. With multiple transmit antennas, multiple users can be served simultaneously for multiplexing. To achieve the full capacity of a MU-MIMO system, a nonlinear space-division multiple access scheme, namely dirty paper coding (DPC) [1], can be used. Though DPC is optimal in performance, the implementation of DPC is too complex to be practical so that efficient linear precoding/beamforming(BF) methods are put forward [2].

With linear beamforming techniques, suboptimal performance can be achieved with reduced complexity. However, since it is generally a nonconvex problem to optimize the beamformers via maximizing the signal-to-interference-and-noise ratio (SINR) [3], researchers focused on some efficient strategies including zero-forcing beamforming (ZFBF) [4], minimum mean square error beamforming (MMSEBF) [5], and the signal-to-leakage-and-noise ratio (SLNR) based beamforming design [6].

Consider systems which have more users than antennas, multiuser scheduling is utilized to choose a subset of users for simultaneously transmission. In order to select users and calculate the beamformers, it is assumed that perfect channel state information is available at the transmitter (CSIT). With perfect CSIT, semi-orthogonal user selection (SUS) in [7] and greedy user selection (GUS) in [8] have been proposed. However, perfect CSIT is usually hard to achieve in practical applications. With only partial CSIT, [9] presented a multiuser broadcast channel with finite-rate feedback techniques, where each user feeds back limited bits to quantize channel direction information (CDI). User selection and calculation of beamformers are based on those imperfect CSIT at base stations (BSs). To further reduce the required CSI feedback, a multiuser scheduling scheme which exploits statistic channel-correlation information has been proposed in [10]. Meanwhile, another type of two-phase scheduling scheme has been put forward in [11] with better performance. More theoretically, in order to achieve the asymptotically optimal performance of a MU-MISO system, a user selection strategy based on CDI quantization and CQI in [7] has been proved to achieve asymptotically optimal capacity.

It has been revealed in [7] that the SINR-based CQI feedback for scheduling with ZFBF is able to achieve the asymptotical upper bound of a MU-MIMO system under limited feedback. However, it is known that ZFBF suffers a performance degradation due to the effect of noise-amplification especially at low SNR regime. Hence, it is natural to consider the scheduling with an improved beamforming strategy, i.e., SLNR-based beamforming [6]. However, when limited feedback is utilized, the SINR can not be easily estimated with SLNR beamforming at the user terminal side, and thus the expression of CQI feedback is still unknown for the limited feedback SLNR-based multiuser scheduling. Moreover, since CSI feedback is utilized, there should exist some CSI mismatch due to the feedback delay effect which may affect the overall scheduling performance of the system.

In this paper, we consider the problem of user scheduling under limited feedback MU-MIMO systems using SLNR-based beamforming. For practical considerations, the CSI mismatch due to feedback delay is also taken into account for the scheduling design. Based on our observation, different from scheduling with limited feedback ZFBF, a proper CQI feedback for scheduling with SLNR is in the form of channel norms instead of the estimated SINR. With the information of channel norms at the BS, we present an expression for evaluating the expected SLNR and then accordingly develop a multiuser scheduling based on the idea of semi-orthogonal user selection strategy.

The rest of this paper is organized as follows. In section

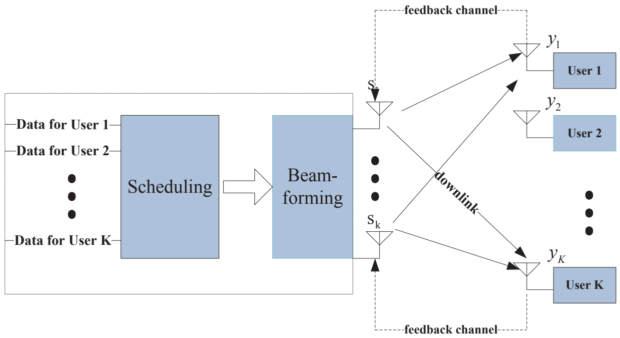


Fig. 1. System model of multiuser MISO downlink with finite rate feedback.

II, a finite rate feedback system model and the channel model are introduced. Section III focuses on presenting an algorithm of our user selection strategy based on SLNR precoding. In section IV, numerical results and comparisons are shown before concluding remarks in Section V.

*Notations:* In this paper, we use uppercase boldface letters for matrices and lowercase boldface for vectors. $\mathbf{x}^{\mathrm{T}}$, $\mathbf{x}^{H}$ and $\|\mathbf{x}\|$ denote the transpose, the conjugate transpose and the Euclidean norm of vector $\mathbf{x}$. $\zeta_m(\mathbf{A})$ returns the eigenvector corresponding to the maximum eigenvalue of matrix $\mathbf{A}$ and $\mathbb{E}\{\cdot\}$ stands for the statistical expectation of a random variable. $\mathbf{I}_M$ denotes the $M \times M$ identity matrix, $eig(\mathbf{A},\mathbf{B})$ represents the generalized eigenvector of matrixes $\mathbf{A}$ and $\mathbf{B}$. $eigval(\mathbf{A},\mathbf{B})$ represents the generalized eigenvalue of matrixes $\mathbf{A}$ and $\mathbf{B}$.

## II. System Model

Consider a MU-MIMO system with an $M$-antenna BS and $K$ single-antenna users. Let $\mathbf{h}_k \in \mathbb{C}^{M\times 1}$ denote the channel vector for user $K$, whose entries are i.i.d. and complex Gaussian distributed with zero-mean and unit variance. Then, the received symbol at user $k$ is

$$y_k = \mathbf{h}_k^H \mathbf{f}_k d_k + \mathbf{h}_k^H \sum_{i=1, i\neq k}^{K} \mathbf{f}_i d_i + n_k, \quad k = 1, \cdots, K \quad (1)$$

where $\mathbf{f}_i \in \mathbb{C}^{M\times 1}$ and $d_i \in \mathbb{C}$ are respectively the beamforming vector and symbol for user $i, i = 1, 2, \cdots, K$, and $n_k$ is the zero-mean complex Gaussian noise with its variance $\mathbb{E}\left\{\|n_k\|^2\right\} = N_0$. We normalize the transmit energy of symbol $d_k$ for each user, i.e., $\mathbb{E}\left\{\|d_k\|^2\right\} = 1$, so that the power at the BS is constrained by $\mathbf{f}_k^H \mathbf{f}_k = P_k, k = 1, 2, \cdots, K$.

### A. Outdated CSI Quantization Feedback

We assume that each user has the full information of $\mathbf{h}_k$. In limited feedback systems, user $k$ normalizes $\mathbf{h}_k$ into $\tilde{\mathbf{h}}_k = \mathbf{h}_k / \|\mathbf{h}_k\|$ as CDI. Then, it quantizes the CDI by using a random vector quantized codebook $\mathbf{C}_k \in \mathbb{C}^{M\times 2^B}$ which is known at both the BS and the user $k$ side. Finally, user $k$ feeds back the index of CDI $\hat{\mathbf{h}}_k$ to BS, and BS recovers it by using the codebook. However, with the impact of channel delay, the $\hat{\mathbf{h}}_k$ BS received is an outdated quantized version of CDI at the transmission time-slot.

According to the procedure we discussed above, and by assuming unit symbol feedback delay without loss of generality, at time slot $[n]$, the BS receives the feedback quantized version of CDI of time slot $[n-1]$. As in [9], the relationship between downlink CDI $\tilde{\mathbf{h}}_k[n-1] = \mathbf{h}_k[n-1] / \|\mathbf{h}_k[n-1]\|$at slot $[n-1]$ and its quantized version $\hat{\mathbf{h}}_k[n-1]$ can be characterized by:

$$\begin{aligned}\tilde{\mathbf{h}}_k[n-1] = \sqrt{q_k[n-1]}\mathbf{x}_k[n-1] \\ + \sqrt{1-q_k[n-1]}\hat{\mathbf{h}}_k[n-1]\end{aligned} \quad (2)$$

where $q_k[n-1] = 1 - |\tilde{\mathbf{h}}_k^H[n-1]\hat{\mathbf{h}}_k[n-1]|^2$, and $\mathbb{E}\{q_k[n-1]\} = \frac{M-1}{M}\delta$ with $\delta = 2^{-B/(M-1)}$ [9]. $\mathbf{x}_k[n-1] \in \mathbb{C}^{M\times 1}$ represents the vector of quantization error, which is zero-mean and unit-variance, and is independent of $q_k[n-1]$.

Moreover, from [12], we assume that the cross-correlation matrix between $\mathbf{h}_k[n]$ and $\mathbf{h}_k[n-1]$ is diagonal, which means that each channel vector $\mathbf{h}_k$ has a diagonal covariance matrix. Hence, the relationship between the CSI $\mathbf{h}_k[n-1]$ at time slot $[n-1]$ and the CSI $\mathbf{h}_k[n]$ at time slot $[n]$ can be expressed by:

$$\mathbf{h}_k[n] = \gamma_k \|\mathbf{h}_k[n-1]\| \tilde{\mathbf{h}}_k[n-1] + \mathbf{z}_k[n] \quad (3)$$

where $\gamma_k = J_0(2\pi f_{dk} T_k)$ is the correlation factor, where $J_0(\cdot)$ is the zero-th order Bessel function of the first kind, $f_{dk}$ represents Doppler spread of user $k$, and $T_k$ is the length of the transmitted symbol. $\mathbf{z}_k[n]$ is the vector of channel error which is independent of $\mathbf{h}_k[n]$.

Based on equations (2) and (3), we have our channel model which not only takes the quantization error into consideration but also reflects the impact of channel delay. It gives

$$\begin{aligned}\mathbf{h}_k[n] =& \gamma_k \sqrt{1-q_k[n-1]} \|\mathbf{h}_k[n-1]\| \hat{\mathbf{h}}_k[n-1] \\ &+ \gamma_k \sqrt{q_k[n-1]} \|\mathbf{h}_k[n-1]\| \mathbf{x}_k[n-1] + \mathbf{z}_k[n].\end{aligned} \quad (4)$$

### B. SLNR Precoding Scheme

We first review the traditional SLNR metric defined by

$$\mathrm{SLNR}_k[n] = \frac{\mathbf{f}_k^H \mathbf{h}_k[n] \mathbf{h}_k^H[n] \mathbf{f}_k}{\mathbf{f}_k^H \left(\sum_{i=1, i\neq k}^{K} \mathbf{h}_i[n] \mathbf{h}_i^H[n]\right) \mathbf{f}_k + N_0}. \quad (5)$$

To optimize the beamformer $\mathbf{f}_k$ via maximizing the SLNR metric, it has been proven to be a generalized Rayleigh quotient problem [12]. The optimal beamformer admits a closed-form expression as an eigen-vector, given by

$$\mathbf{f}_k = eig\left(\mathbf{h}_k[n]\mathbf{h}_k^H[n], \left(\sum_{i=1, i\neq k}^{K} \mathbf{h}_i[n]\mathbf{h}_i^H[n]\right) + (N_0/P_k)\mathbf{I}_M\right).$$

Due to the finite rate feedback, BS can receive only the imperfect information of channel $\hat{\mathbf{h}}_k[n]$ rather than the full CSI, the beamforming optimization strategy should turn into maximizing the expectation of $\mathrm{SLNR}_k[n]$ based on $\hat{\mathbf{h}}_k[n]$. It yields

$$\begin{aligned} \max_{\mathbf{f}_k} \quad & \mathbb{E}_{\mathbf{h}_k[n]|\hat{\mathbf{h}}_k[n-1]}\{\mathrm{SLNR}_k[n]\} \\ \mathrm{s.t.} \quad & \mathbf{f}_k^H \mathbf{f}_k = P_k. \end{aligned} \tag{6}$$

In the following sections, we will present a user scheduling strategy which considers the user selection combined with the modified SLNR-beamforming design problem in (6).

## III. Limited Feedback Multiuser Scheduling

In a MU-MIMO system with perfect CSIT, the optimal sum capacity is proven to grow with $M \log\log K$ with user scheduling [7]. For another case with imperfect CSI, the optimal capacity scaling law is achieved under some conditions on partial CSI feedback. It is inadequate to achieve the optimal capacity law with only CDI feedback to the BS when using ZFBF and user scheduling. In order to approach the optimal performance, as revealed in [7], one needs to let user terminals send back both the CDI and a proper scalar form of CQI. When ZFBF is exploited at the BS, the CQI for achieving optimal capacity law is said to be SINR-based instead of a direct channel norm-based CQI indicator.

### *A. SLNR-based Multiuser Scheduling with CDI*

In this section, we consider the user scheduling for a MU-MIMO system using an enhanced SLNR-based beamforming. Since SLNR-based beamforming is utilized, the user terminals are unable to estimate its achieved SINR for CQI feedback to support user selection. This is because when SLNR-based beamforming is utilized, the derived beamforming is a generalized eigenvector of a given matrix which does not admit a simple closed-form solution for SINR estimation at user terminals. Without a proper CQI feedback for SLNR-beamforming, we will first introduce a semi-orthogonal user scheduling algorithm by recalling the conventional scheduling scheme for ZFBF in [7] with some modifications with respect to the user selection metric, namely $\overline{\mathrm{SLNR}}_u$ for user $u$ in the following Algorithm.

The SLNR-based semi-orthogonal user selection algorithm is presented to sequentially chose a serving user from a user subset semi-orthogonal to the group of selected users. A proper SLNR-based metric is utilized for user selection from the subset with only CDI feedback from users. A detailed description of the SLNR-based user scheduling algorithm is presented in **Algorithm 1**.[1]

From the above Algorithm, it is important to note that, to implement the scheduling algorithm, the metric $\overline{\mathrm{SLNR}}_u$ should be estimated at the BS. In the following, we will first concentrate on calculating the metric $\overline{\mathrm{SLNR}}_u$ by using only outdated CDI at the BS. Inspired by our previous result in [13], we here derived an estimated SLNR-based user scheduling method on only CDI information from users for scheduling design. Note that from the following derivations, we will further develop another SLNR estimator with both CDI and channel magnitude information. The later SLNR estimator will be utilized for designing an improved user scheduling algorithm with both CDI and CQI feedback. The additional CQI feedback will help the scheduler to achieve better performance approaching the optimal user scheduling scaling law as illustrated in [7].

[1]In Algorithm 1, $\alpha$ represents the semi-orthogonal factor in [7].

**Algorithm 1** : SLNR-based Multiuser Scheduling with CDI

*User side (feedback) at* $[n-1]$

**for** $k = 1$ to $K$ **do**
  Send CDI $\hat{\mathbf{h}}[n-1]$ back to BS
**end for**

*BS side (Scheduling) at* $[n]$

Initialize user group $\mathcal{U}_1 = \{1, 2, \cdots, K\}$
Select the 1st user $s_1 = \arg \max_{u\in\mathcal{U}_1} \overline{\mathrm{SLNR}}_u$
Initialize selected user group $\mathcal{S} = \{s_1\}$
Initialize user sequence number $i = 1$
**while** $|\mathcal{U}_i| > 0$ and $i < M$ **do**
  Calculate sum rate $R_i$ of $\mathcal{S}$ using SLNR-BF in [13]
  Find user group $\mathcal{U}_{i+1}$ semi-orthogonal to $s_i$ according to:

$$\mathcal{U}_{i+1} = \left\{ u \in \mathcal{U}_i, u \neq s_i \middle| \left\| \hat{\mathbf{h}}_u^H \hat{\mathbf{h}}_{s_i} \right\| < \alpha \right\}. \tag{7}$$

  If $|\mathcal{U}_{i+1}| > 0$, select $s_{i+1} = \arg \max_{u\in\mathcal{U}_{i+1}} \overline{\mathrm{SLNR}}_u$
  Calculate $R_{i+1}$ with SLNR-beamforming
  If $R_{i+1} > R_i$, then $\mathcal{S} = \mathcal{S} \cup \{s_{i+1}\}$, $i = i+1$. Else, stop
**end while**

With only quantized channel information feedback at the BS, from (5), neither the SLNR beamformer nor the finalized SLNR value of each user can be obtained. Therefore, we resort to designing the SLNR via maximizing an average metric over the unknown channel information. According to an upper bound of the average SLNR in (5), which we have derived in our previous work in [13], the $\overline{\mathrm{SLNR}}_u$ metric for user $u$ is given by

$$\overline{\mathrm{SLNR}}_u = \frac{\mathbf{f}_u^H \mathbb{E}\left\{\mathbf{h}_u[n]\mathbf{h}_u^H[n]\right\}\mathbf{f}_u}{\mathbf{f}_u^H \left(\sum_{i\in\mathcal{S}, i\neq u} \mathbb{E}\left\{\mathbf{h}_i[n]\mathbf{h}_i^H[n]\right\}\right)\mathbf{f}_u + N_0} \tag{8}$$

where $\mathcal{S}$ is the selected user group in Algorithm 1, and the expectations $\mathbb{E}\left\{\mathbf{h}_k[n]\mathbf{h}_k^H[n]\right\}$, $k = 1, \cdots, K$ is derived with closed-form expressions as follows. By using (4), the channel auto-correlation equals

$$\begin{aligned} &\mathbb{E}\left\{\mathbf{h}_k[n]\mathbf{h}_k^H[n]\right\} \\ &= \mathbb{E}\left\{\mathbf{z}_k[n]\mathbf{z}_k^H[n]\right\} \\ &\quad + \mathbb{E}\left\{\gamma_k^2 \|\mathbf{h}_k[n-1]\|^2 \tilde{\mathbf{h}}_k[n-1]\tilde{\mathbf{h}}_k^H[n-1]\right\} \\ &\quad + \mathbb{E}\Bigg\{\gamma_k \|\mathbf{h}_k[n-1]\| \\ &\quad \times \left(\tilde{\mathbf{h}}_k[n-1]\mathbf{z}_k^H[n] + \mathbf{z}_k[n]\tilde{\mathbf{h}}_k^H[n-1]\right)\Bigg\} \qquad (9) \\ &= \left(1-\gamma_k^2\right)\mathbf{I}_M + \mathbb{E}\left\{\gamma_k^2 \|\mathbf{h}_k[n-1]\|^2 \tilde{\mathbf{h}}_k[n-1]\tilde{\mathbf{h}}_k^H[n-1]\right\} \qquad (10) \end{aligned}$$

where we use the facts $\mathbb{E}\left\{\mathbf{z}_k[n]\mathbf{z}_k^H[n]\right\} = \left(1-\gamma_k^2\right)\mathbf{I}_M$

[13] and the third summation term in (9) equals to zero. For notational simplify, we assume that each user has the same Doppler spread $f_{dk} = f$ and length of the transmitted symbol $T_k = T$, which results in the same correlation factor $\gamma_k = \gamma$. Then, by further considering the 2nd term in (10), it is simplified to

$$\begin{aligned}
&\mathbb{E}\left\{\gamma_k^2 \|\mathbf{h}_k[n-1]\|^2 \tilde{\mathbf{h}}_k[n-1]\tilde{\mathbf{h}}_k^H[n-1]\right\} \\
=&\gamma^2 \mathbb{E}\left\{\|\mathbf{h}_k\|^2 \tilde{\mathbf{h}}_k \tilde{\mathbf{h}}_k^H\right\} \\
=&\gamma^2 \mathbb{E}\left\{\|\mathbf{h}_k\|^2\right\} \Bigg( \mathbb{E}\left\{1-q_k\right\} \hat{\mathbf{h}}_k \hat{\mathbf{h}}_k^H + \mathbb{E}\left\{q_k \mathbf{x}_k \mathbf{x}_k^H\right\} \\
&\qquad + \mathbb{E}\left\{\sqrt{1-q_k}\sqrt{q_k}\left(\hat{\mathbf{h}}_k \mathbf{x}_k^H + \mathbf{x}_k \hat{\mathbf{h}}_k^H\right)\right\} \Bigg) \\
=&\gamma^2 \mathbb{E}\left\{\|\mathbf{h}_k\|^2\right\} \Bigg( \mathbb{E}\left\{1-q_k\right\} \hat{\mathbf{h}}_k \hat{\mathbf{h}}_k^H + \mathbb{E}\left\{q_k\right\} \mathbb{E}\left\{\mathbf{x}_k \mathbf{x}_k^H\right\} \\
&\qquad + \mathbb{E}\left\{\sqrt{1-q_k}\sqrt{q_k}\right\}\left(\hat{\mathbf{h}}_k \mathbb{E}\left\{\mathbf{x}_k^H\right\} + \mathbb{E}\left\{\mathbf{x}_k\right\} \hat{\mathbf{h}}_k^H\right) \Bigg) \\
=&\gamma^2 \mathbb{E}\left\{\|\mathbf{h}_k\|^2\right\} \Bigg( \left(1 - \frac{M-1}{M}\delta\right) \hat{\mathbf{h}}_k \hat{\mathbf{h}}_k^H \\
&\qquad + \frac{M-1}{M}\delta \left[\frac{1}{M-1}\left(\mathbf{I}_M - \hat{\mathbf{h}}_k \hat{\mathbf{h}}_k^H\right)\right] \Bigg) \\
=&\gamma^2 \mathbb{E}\left\{\|\mathbf{h}_k\|^2\right\} \left( (1-\delta) \hat{\mathbf{h}}_k \hat{\mathbf{h}}_k^H + \frac{\delta}{M}\mathbf{I}_M \right). \qquad (11)
\end{aligned}$$

The above equations are based on the following facts: The norm of channel $\|\mathbf{h}_k\|$ is independent of CDI $\tilde{\mathbf{h}}_k$, and the vector of quantization error $\mathbf{x}_k$ is independent of $q_k$. We also use $\mathbb{E}\{q_k\} = \frac{M-1}{M}\delta$, $\mathbb{E}\{x_k\} = 0$, and $\mathbb{E}\left\{\mathbf{x}_k \mathbf{x}_k^H\right\} = \frac{1}{M-1}\left(\mathbf{I}_M - \hat{\mathbf{h}}_k \hat{\mathbf{h}}_k^H\right)$ in [13].

Note that since the information of $\|\mathbf{h}_k\|$ is not available at the BS, we substitute the result $\mathbb{E}\left\{\|\mathbf{h}_k\|^2\right\} = M$ and obtain the final closed-form expression of $\overline{\mathrm{SLNR}}_u$ by substituting (10), (11) in (8). Now, with explicit closed-form expression of the SLNR in (8), the solution of the optimization problem (6) can be expressed as

$$\mathbf{f}_k = \zeta_m(\mathbf{V}_k) \times \sqrt{P_k} \qquad (12)$$

where

$$\mathbf{V}_k = \left( A_1 \sum_{i=1, i\neq k}^{M} \hat{\mathbf{h}}_i \hat{\mathbf{h}}_i^H + A_3 \mathbf{I}_M \right)^{-1} \left( A_1 \hat{\mathbf{h}}_k \hat{\mathbf{h}}_k^H + A_2 \mathbf{I}_M \right) \qquad (13)$$

where $A_1 = \gamma^2 M (1-\delta)$, $A_2 = 1 - (1-\delta)\gamma^2$, $A_3 = (i-1)A_2 + N_0/P_u$. Accordingly, the maximum $\overline{\mathrm{SLNR}}_u$ is given by

$$\overline{\mathrm{SLNR}}_u = \max eigval \left( \left( A_1 \hat{\mathbf{h}}_u \hat{\mathbf{h}}_u^H + A_2 \mathbf{I}_M \right), \left( A_1 \sum_{i\in\mathcal{S}, i\neq u} \hat{\mathbf{h}}_i \hat{\mathbf{h}}_i^H + A_3 \mathbf{I}_M \right) \right) \qquad (14)$$

Thus far, the user scheduling illustrated in Algorithm 1 can be implemented with the SLNR estimator $\overline{\mathrm{SLNR}}_u$ in (14).

### B. SLNR-based Multiuser Scheduling with CDI and CQI

As we know, user scheduling with limited feedback can hardly achieve the optimal performance even asymptotically without any CQI feedback available at the BS. Different from ZFBF with limited feedback in [7], the SINR-based CQI feedback can not be easily estimated at the user side when SLNR-based beamforming is utilized. Therefore, we need to develop a proper CQI feedback metric for SLNR beamforming. Fortunately, from the estimated SLNR derived in (14), we find that channel norm based CQI feedback from users is really helpful for the BS to evaluate an SLNR estimator. As observed from (11), if the channel norm $\|\mathbf{h}_k\|^2$ is available for different users at the BS, the estimated SLNR accuracy can be improved by replacing $\mathbb{E}\left\{\|\mathbf{h}_k\|^2\right\}$ in (11) by its accurate value $\|\mathbf{h}_k\|^2$. Hence, we define the new SLNR-estimator with both CDI and channel-norm based CQI by $\widetilde{\mathrm{SLNR}}_u$. From (8), (11), and with the value of $\|\mathbf{h}_k\|$, we have

$$\widetilde{\mathrm{SLNR}}_u = \frac{\mathbf{f}_u^H \left( A_4 \hat{\mathbf{h}}_u \hat{\mathbf{h}}_u^H + A_5 \mathbf{I}_M \right) \mathbf{f}_u}{\mathbf{f}_u^H \left( A_4 \sum_{i\in\mathcal{S}, i\neq u} \hat{\mathbf{h}}_i \hat{\mathbf{h}}_i^H + (i-1) A_5 \mathbf{I}_M \right) \mathbf{f}_u + N_0} \qquad (15)$$

where $A_4 = \gamma^2 \|\mathbf{h}_u\|^2 (1-\delta)$, $A_5 = 1 - \left(1 - \frac{\delta \|\mathbf{h}_u\|^2}{M}\right)\gamma^2$. Accordingly, the maximum value of $\widetilde{\mathrm{SLNR}}_u$ is:

$$\widetilde{\mathrm{SLNR}}_u = \max eigval \left( \left( A_4 \hat{\mathbf{h}}_u \hat{\mathbf{h}}_u^H + A_5 \mathbf{I}_M \right), \left( A_6 \sum_{i\in\mathcal{S}, i\neq u} \hat{\mathbf{h}}_i \hat{\mathbf{h}}_i^H + A_7 \mathbf{I}_M \right) \right) \qquad (16)$$

where $A_6 = \gamma^2 \sum_{i\in\mathcal{S}, i\neq u} \|\mathbf{h}_i\|^2 (1-\delta)$. and $A_7 = (i-1) A_5 + N_0/P_u$.

At this step with (16), given the SLNR estimator based on outdated CDI and channel-norm-based CQI, an performance enhanced user scheduling algorithm based on both CDI and CQI feedback from users can be directly obtained by replacing the metric $\overline{\mathrm{SLNR}}_u$ by $\widetilde{\mathrm{SLNR}}_u$. In detail, the algorithm of CQI feedback SLNR-based scheduling is described in **Algorithm 2**.

Note that it is later observed by numerical results the enhanced user scheduling with both CDI and CQI under SLNR beamforming significantly outperforms the scheduling performance with ZFBF at the same CSI feedback load, which shows the same asymptotically optimal performance.

## IV. Numerical Results

In this section, we present numerical results to compare the performance of traditional ZFBF-based user selection in [7] and our proposed SLNR-based multiuser scheduling. All these simulations are under channels which have both quantization error and outdated error. We use $M = 4$ and $K = 100$ systems throughout this section. The semi-orthogonal factor $\alpha = 0.4$

For comparison, we provide comparison results of four user selection algorithms, as follows

**Algorithm 2** : SLNR-based Multiuser Scheduling with CDI and CQI

*User side (feedback) at* $[n-1]$

**for** $k=1$ to $K$ **do**

Send CDI $\hat{\mathbf{h}}_k[n-1]$ and CQI $\|\mathbf{h}_k[n-1]\|$ back to BS

**end for**

*BS side (Scheduling) at* $[n]$

Initialize $\mathcal{U}_1=\{1,2,\cdots,K\}$

Select $s_1=\arg\ \max_{u\in\mathcal{U}_1}\ \widetilde{\mathrm{SLNR}_u}$

Initialize $\mathcal{S}=\{s_1\}$

Set $i=1$

**while** $|\mathcal{U}_i|>0$ and $i<M$ **do**

Calculate sum rate $R_i$ of $\mathcal{S}$ using SLNR-BF in [13]

Find user group $\mathcal{U}_{i+1}$ semi-orthogonal to $s_i$ according to equation (7).

If $|\mathcal{U}_{i+1}|>0$, select $s_{i+1}=\arg\ \max_{u\in\mathcal{U}_{i+1}}\ \widetilde{\mathrm{SLNR}_u}$

Calculate $R_{i+1}$ with SLNR-beamforming

If $R_{i+1}>R_i$, then $\mathcal{S}=\mathcal{S}\cup\{s_{i+1}\}$, $i=i+1$. Else, stop

**end while**

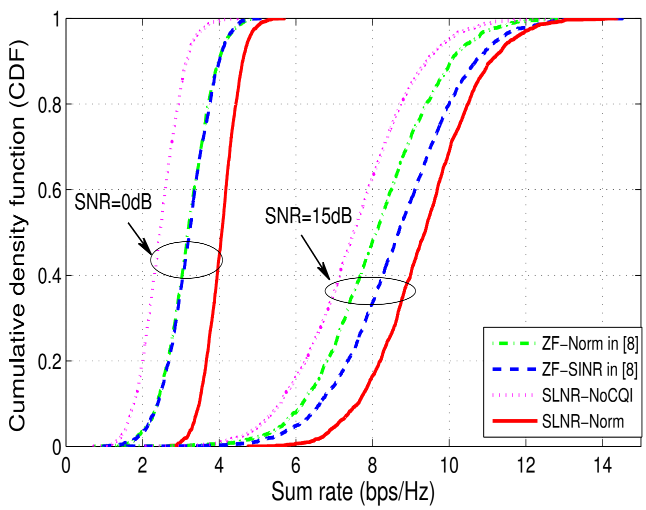


Fig. 2. CDF of achievable sum rate figure under $M=4, K=100, B=8$ bits, and $fT=0.1$ .

- ZF-Norm represents the ZFBF-based scheduling in [7] with channel norms as CQI .
- ZF-SINR represents the ZFBF-based scheduling in [7] with SINR norms as CQI.
- SLNR-NoCQI represents our proposed scheduling in Algorithm 1 with only quantized CDI feedback.
- SLNR-Norm represents the proposed scheduling in Algorithm 2 with CDI and channel norms as CQI feedback.

In Fig. 2, we compare the ZFBF-based user selection and SLNR-based multiuser scheduling at different SNRs. Simulations are under channels which have both outdated and quantization error. The number of feedback bits per user for CDI feedback is 8, and the doppler spread $fT=0.1$. From the figure, we find that at low SNR regime, the performance gap

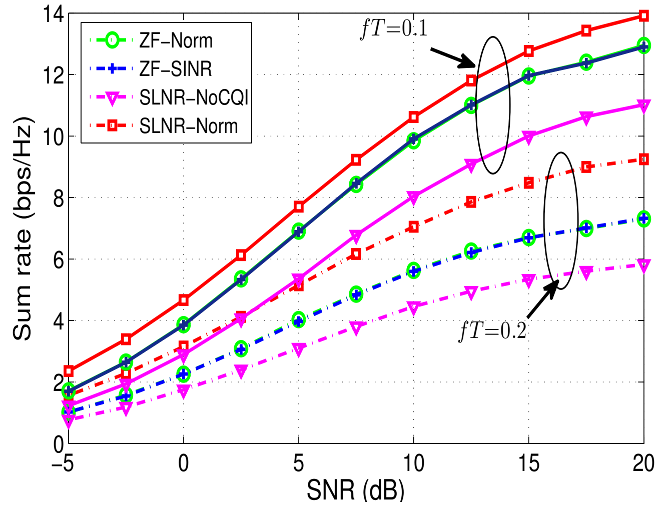


Fig. 3. Sum rate v.s. SNR under $M=4, K=100$, and $B=16$ bits.

between ZFBF-based user selection with norms and SINR as CQI is so marginal that can be ignore. But at high SNR regime, the SINR-feedback ZFBF scheduling outperforms the norm-feedback one. Meanwhile, the SLNR-based user selection with norms feedback has much better performance than ZFBF-based scheduling at both low and high SNR regimes. More specifically, at low SNR regime, over sixty percent of the simulated channels which use SLNR-based scheduling can achieve the sum rate of 4 bps/Hz. However, only ten percent of the ZFBF-based channels can achieve that sum rate. Note that at high SNR regime, over thirty percent of the simulated channels which use SLNR-based scheduling can achieve the sum rate of 1 bps/Hz. However, only twenty percent of the ZFBF-based channels can achieve that sum rate. The performance between those two scheduling at high SNR regime is not as noticeable as at low SNR regime. That is because ZFBF-based beamforming suffers a performance degradation due to the effect of noise-amplification especially at low SNR regime, and SLNR-based beamforming can avoid that degradation. So that it can be concluded that the outage capacity of SLNR-based scheduling with norms feedback is more than ZFBF-based user selection, especially at low SNR regime.

In Fig. 3, we compare those different user selection algorithms in different SNR under channels with both quantization errors and delayed errors. It can be easily concluded that SLNR-based user selection with norms feedback have much more sum rate than ZFBF-based strategies. Meanwhile, considering the impact of channel delay, we simulate those user selection methods under different doppler spread $fT$. The results show that when $fT$ is large, which means the impact of delayed channel cannot be neglected, the performance of ZFBF-based user selection suffers a degradation and our SLNR-based strategies have much better performance at severe delayed regime.

Moreover, let us compare Fig. 3 and Fig. 2 to discuss the impact of feedback bits. In Fig. 3, users feedback 16 bits to BS and in Fig. 2, there are only 8 bits. It can be observed that when the number of feedback bits is large, the difference between ZFBF-based user selection and SLNR-based scheduling with norms feedback is not as significant as at fewer bits regime. That is because when there is a large number of bits feedback to express CDI, the quantization error becomes small so that ZFBF beamforming can achieve a relative good performance. So that it can be concluded that SLNR-based scheduling has more advantages than the ZFBF-based method at few feedback bits and severe channel delay regime.

## V. Conclusion

To achieve optimal capacity in MU-MIMO systems, ZFBF-based user selection methods with CQI have been proposed in [7]. However, due to the degradation of ZFBF beamforming in limited feedback systems and the impact of channel delay, we propose a new multiuser scheduling algorithm based on SLNR and requesting channel magnitudes as CQI. We consider both quantization errors and outdated errors in our algorithm. Numerical results show that our proposed scheduling scheme outperforms the conventional methods, especially at low SNRs and severe channel delay.

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