

# Evolution of Quadrilateral Meshing Techniques

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## Abstract:

*This reports presents various research techniques of quadrilateral meshing that have been evolved over recent years. Starting with a quadrilateral mesh simplification (QMS) technique which presents a global method designed to improve any quad mesh structure, Another coarsening method has been presented which is a localized approach addressing the limitations of previous global method. Semi regular meshing scheme also has been discussed which has introduced a geometry processing algorithm to sub divide quad only meshes, This scheme implements mesh simplification using adaptive sampling of the base domain. In the recent years example based meshing techniques have been introduced which are data driven and interactive. Two such advanced mesh scheme were also presented. Research has also been done predominantly focusing on editing quad dominant meshes. This paper illustrates one such framework of connectivity editing to formulate pure quad meshes. All these techniques were illustrated in detail and various experiments and results have been demonstrated.*

## 1. Introduction

Mesh generation and optimization is the primary step which will be serving as a fundamental step for later high geometry processing steps like compression and rendering. Many meshing techniques have been introduced over the period of time and

still the research is happening till date to address the challenges in this area. In the initial years researchers have started introducing simple meshing schemes which are based on triangle shape polygons. In the later years it has been realized that triangle meshes are not just sufficient to maintain the connectivity and geometry structure of the meshes. Quad meshes have been then introduced in which meshes are only composed of polygons such as quadrilateral elements. Other high polygon structural meshes like tetrahedral and hexahedral meshes have been also introduced but this report is confined only to the evolution of quad meshing schemes.

Many quadrilateral meshing techniques have been evolved based on collapse methods. Edge or vertex collapsing methods are always simpler in case of triangle based meshes due to the valency three. Geometry processing of quad meshes is always challenging where the valency is four. The challenge with quadrilateral meshes is that not every model can be represented with quadrilateral elements whereas with triangle elements it is always possible. Addressing this challenge researchers have come up with mesh conversion methods maintaining the connectivity in mesh. Many new global collapse methods like poly chord and quadrilateral edge collapse methods have been introduced.

Initially mesh simplification methods were confined to automatic without any user intervention. New methods have been introduced focusing on user defined maps and thresholds. In the later years Template

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based methods have been introduced which are flexible enough to produce semi regular quad only meshes aligning well with the surface features. These template based methods are always complex and challenging to integrate on subjective decisions. New interactive and data driven quadrangulation methods have been introduced. These methods presents user and interactive way of giving inputs and applying the decision scheme on collected data and meshing is done with patches.

## 2. Related Work

Though many papers have been proposed in the past, [1, 2] introduced advanced methods for edge collapsing in quadrilateral meshes. These two papers have used dual representation of mesh as a primary model and performed deletion operations on the dual in a controlled manner. [1] presented a global method for deletion called as polychord collapse or ring collapse which is efficient in deleting multiple edges in one go. This global deletion operator introduced a challenge where important features might be lost during the mesh simplification. Addressing this challenge [2] proposed a localized coarsening algorithm where a new deletion operation called quadrilateral edge merge has been introduced which on repeated application gives similar results as polychord. Thus a localized operator works on mesh to simplify and results are near or better with that of QMS in [1].

Different meshing techniques have been evolved based on the surface geometry. It is complex to perform surface construction on

irregular meshes. [3] has described one such meshing scheme using adaptive sampling on simplified based domains. Template based meshing schemes are then introduced in which there is no user intervention for inputs and logical method for automatic detection of alignment features. [4] presents one such flexible and robust scheme with highly desired results. Though many algorithms have been introduced dealing with irregular meshes was a big challenge. Different delete operations applied on meshes during simplification always ended up in forming quad-dominant meshes. Quad dominant meshes are the meshes in which the majority of elements are quads but not all polygons. Many papers have been published which addressed this challenge by using Catmull-Clark subdivision method for conversion of triangle and quad dominant meshes to quadrilaterals. [6] presented a new framework to translate these irregular quad dominant meshes to pure quadrilateral meshes.

Other quadrilateral mesh simplification methods which involved user for making subjective decisions have been introduced.[5,7] introduced example based quadrangulation methods where user has been provided with advanced interactive techniques like boundary drawing and curve input provision. These methods are also data driven where a data base with patches of quadrilateral surfaces has been maintained and queried on demand when user queries with inputs. These quadrangulation methods builds complete meshes from identified patches. [7] is a most recent paper which presents a sketch-based user interface.

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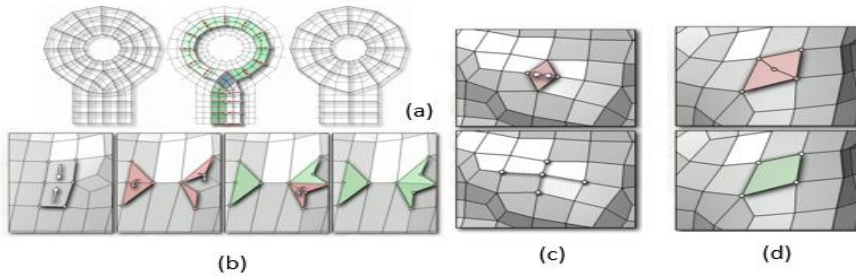


Figure 1: a) poly-chord collapse b) quadrilateral edge merge c) quadrilateral vertex merge d) doublet collapse

## 3. Overview

This report presents a detail overview of each meshing scheme and illustration deletion operations, surface reconstruction methods and various algorithms involved.

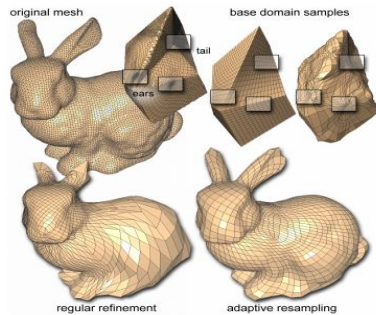


Figure 2: semi-regular meshing from simplified base domains

## 3.1 Global and localized coarsening methods

Dual representation of a mesh is modified in a controlled manner in these global and localized meshing schemes. poly chord collapse method is a global deletion operator and quadrilateral edge merge, quadrilateral vertex merge and doublet collapse methods are the local deletion operators implemented for these algorithms. Fig.1 describes each of these simplification operators. These methods have also described a prioritization technique suggesting an order of application

of the local and global deletion operations on duals for desired results. Both these methods focused on remeshing without loss of level of details.

## 3.2 Semi-Regular meshing from simplified Base domains

Semi-regular quadrilateral meshes are the structures which have extraordinary vertices in limited number. This method illustrates an approach of saving important levels of hierarchy to form a simplified base domain. These domains are again simplified using adaptive sampling. In adaptive sampling base domains are sub divided to backward project vertices that are newly introduced. Based on the approximation error identified and the mapped surface area weighted centroids are formed. Fig.2 describes this meshing scheme of semi-regular structures. Based on the mapped surface area adaptive resampling is done uniformly resampling the original surface and these points are projected through key frame meshes. Backward projection is done on sampled points. Key meshes store the details of important levels when changes in the model are tracked. Arbitrary optimization method can be supported by using Ray-casting approach. Thus this simplifying scheme can be

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generalized for arbitrary polygonal remeshing.

## 3.3 Template Based Quadrilateral Meshing

This method of quadrilateral meshing introduces an automatic refinement without any user intervention for input. Template based remeshing is done in three stages namely base triangle construction, template based meshing and mesh quality improvement. Base triangle construction involves feature based cell decomposition where feature point and the necessary curves are defined from the triangle surface inputs. In the second stage base triangle faces are mapped to regular and singular templates which are canonical quad faces served as templates. As a result of these template mapping quad only mesh is obtained. Fig.3 describes the three stages of the template based meshing scheme. In the last stage the quad mesh is then refined using normal vertex deletion operations mapping to the templates. This method of deletion during simplification maintains the element alignment as well as mesh structure. These template based meshing methods has paved a way for animation in mesh specific configuration.

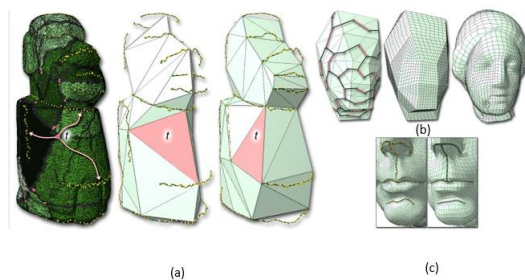


Figure 2: semi-regular meshing from simplified base domains

## 3.4 Connectivity Editing for Quad-Dominant Meshes

A mesh is said to be regular if the degree of each face in it is four and the valency of every vertex is also four. Quad-Dominant mesh has majority part regular with few irregular vertices and faces. CC method has been introduced based on Discrete Gauss-Bonnet Theorem which involves basic Euler characteristics to align the quad structures and get the quad only meshes. Fundamental editing operations like moving vertices, edges and faces is applied on different domains such as primal, dual and CC. At each editing the structure relations are maintained across the domains. Joining operations like irregular face or vertex merge is then done to form only regular components in the mesh. T- Junction movement and cancellation is done adjusting the structure of mesh. Fig.4 shows various operations of CC editing scheme. Lastly type-change operation is applied where irregular elements types i.e. valency and degree is modified to four without introducing new elements.

## 3.5 Pattern Based Quadrangulation Methods

Pattern based remeshing techniques are evolved which rely on basic models

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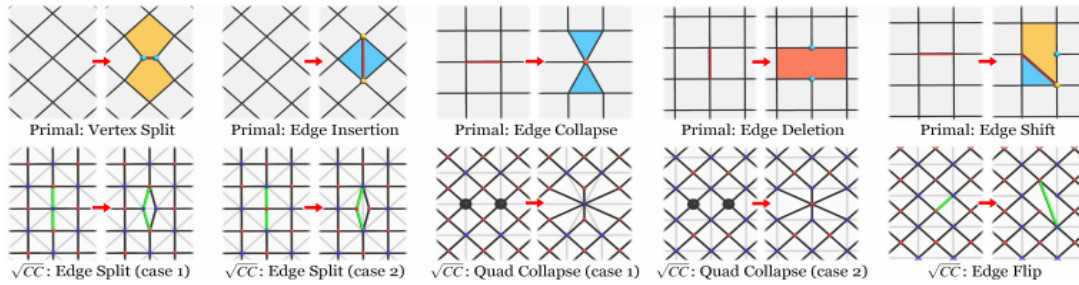


Figure 4: connectivity editing operations in quad dominant meshes

are collected and stored in database as say corpus. These example models are later queried dynamically in an interactive way. Interactive querying allows user to work on the basic models and to provide details of patches to be looked up. Data driven interactive quadrangulation method has introduced sketch based interaction where a user can draw strokes indicating boundaries for polygon meshes also curves which can map poly chords or rings on a patch. Cross-parametrization algorithms are then applied for lookup and results are updated dynamically. Once lookup is done and patches are chosen the topology is reconstructed for surfaces maintaining intrinsic reflexive symmetries driven by other topological primitives. These pattern based methods are always helps for fast remeshing. In some pattern based methods the patches are sorted based on the geometry which is basically a ranking solution resulting in quality quad meshes. Other methods have also performed laplacian editing on the extracted patches while lookup to provide a effective criteria for sorting. Fig.5 shows some examples of pattern based methods applied on various basic models for decomposition and surface reconstruction. Since pattern based methods focused on connectivity of geometry localizing the large

mesh models into patches, The data driven approach speeded up the remeshing with efficient topology preservation.

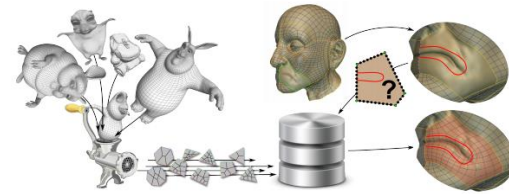


Figure 5: interactive data driven quadrangulation

## 4. Results and Comparison

The efficiency of remeshing and mesh simplification schemes is determined by the quality of meshes and number of quadrilateral elements that optimized mesh contains. All the different mesh decimation techniques discussed in this paper are focused on maintaining high level of details and connectivity of the quad elements thus resulting in quality meshes. Global and localized coarsening algorithms have similar simplification operators hence resulted in same number of quad elements in experiment models. QMS was always performing better on regular models without loss of any details from mesh. Fig.6 shows the results comparison for QMS and qCoarsen meshing methods. Other models have proved that both



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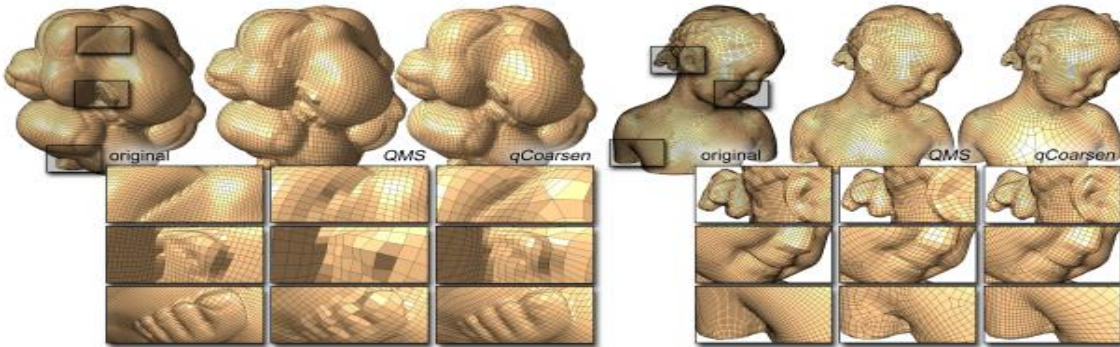


Figure 6: comparison of qms vs qCoarsen with model results

techniques are efficient enough in giving similar results. Semi regular meshing algorithm has employed a variety of deletion operations and the approach is tested on various mesh models including triangle surface areas and quad dominant meshes. Since this algorithm support global based operations even the irregular meshes have given efficient results. This meshing technique is also supported by fine coarse based domains in general. Extraordinary vertices and more faces of base domains have given the flexibility for improvement of quality in remeshing. Template based meshing scheme discussed in this paper is dependent on the base feature points identified from the triangular surfaces. An efficient automatic feature detection scheme helped in giving better results were accurate regions are been mapped to base triangulation surfaces. Experiments conducted using this scheme has revealed that methods in which user intervention is present to provide inputs computing subjective decision is always risky and does not result in guaranteed results. The interactive framework for editing quad

dominant meshes discussed in this paper is easy to use tool and the editing strategies discusses are basic operations which drive in providing fine results. This method also demonstrated that pure quad meshes always ensures planarity and smooth mesh lines are obtained from irregular mesh faces. Sharp features are preserved with T-junction movement and cancellation. Simple mathematical mapping tecqniques have been experimented to use regular vertices and editing of various quad dominant mesh models. Pattern based quadrangulation methods involved interactive querying from user and different cross map algorithms have been experimented for lookup of patches. Data sets used for map models has different patterns usually observed by the humans. Also complex input mesh has given a comparison criteria for accuracy of techniques and results derived. Results in pattern based meshing schemes are dependent on similarity seen between the example models and input. Mapping can be done iteratively for further refinement of meshes and dynamic update of patches in the composed output mesh models. Memory

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occupancy is another concern in these meshing schemes which needed caching mechanism and efficient querying systems. These methods have employed a huge data repository for the storage of sketch based patterns. It is highly complex task to obtain various patterns to map the basic decomposition models. The sophisticated algorithms are used in pattern based meshing schemes to compress connectivity keeping the querying and lookup system efficient.

## 5. Conclusion

In this paper different meshing schemes like localized, global, template based, example based and pattern based meshing schemes have been discussed and all these methods are performant and results driven. These meshing schemes evolved over the recent years have the primary focus on connectivity of quad elements and level of detail preservation while surface construction for the mesh topology. Various state of art algorithms have been evolved for different needs in the area of quad meshing and the scope of research in this area of geometry processing is always exciting and challenging. It is clearly observed that improvement has been observed in the results and experiments carried out various meshing schemes. Simple and easy to use algorithms have been employed. Many algorithms have shown how to use the dual structure of a mesh in refinement process with various local and global deletion operations and different prioritization techniques have also been implemented. Though there are limitations presented in these meshing schemes over the years the limitations have been addressed and

the improvements are clearly discussed with simplicity and robustness of the results provided.

## 6. Acknowledgements

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