

Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with $\mu = 45$ minutes and $\sigma = 8$ minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
- A. 0.3875
B. 0.2676
C. 0.5
D. 0.6987

Solution:- B. 0.2676

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean $\mu = 38$ and Standard deviation $\sigma = 6$. For each statement below, please specify True/False. If false, briefly explain why.

A. More employees at the processing center are older than 44 than between 38 and 44.

Solution:- False

Around 70% of the data falls within one standard deviation of the mean

B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Solution:- True

$P(x < 30)$

$$Z = (X - \mu) / \sigma = (30 - 38) / 6 = -1.3333333$$

$$P = 0.09121121972586788$$

$$\text{Expected count is } 0.09121121972586788 * 400 = 36.48448789034$$

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *id* normal random variables, then what is the difference between $2X_1$ and $X_1 + X_2$? Discuss both their distributions and parameters.

Solution:- $2X_1$ will be greater scale version than $X_1 + X_2$. If X_1 and X_2 are normally distributed then the sum of the random sample will be exactly same.

4. Let $X \sim N(100, 20^2)$. Find two values, a and b , symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

- A. 90.5, 105.9
- B. 80.2, 119.8
- C. 22, 78
- D. 48.5, 151.5
- E. 90.1, 109.9

Solution:- D)

```
stats.norm.interval(0.99,100,20)
```

```
(48.48341392902199, 151.516586070978)
```

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $\text{Profit}_1 \sim N(5, 3^2)$ and $\text{Profit}_2 \sim N(7, 4^2)$ respectively. Both the profits are in \$ Million. Answer the following questions about the total profit of the company in Rupees. Assume that \$1 = Rs. 45

- A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
- B. Specify the 5th percentile of profit (in Rupees) for the company
- C. Which of the two divisions has a larger probability of making a loss in a given year?

Solution:- A) import numpy as np

```
from scipy import stats
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```
from scipy.stats import norm
```

```
mean = 5+7
```

```
print("Mean profit is Rs",mean*45,"million")
```

Mean profit is Rs 540 million

```
SD=np.sqrt(9+16)
```

```
print("Standard deviation is Rs",SD*45,"million")
```

Standard deviation is Rs 225.0 million

```
print("Range is Rs", stats.norm.interval(0.95,540,225),"million")
```

Range is Rs (99.00810347848784, 980.9918965215122) million

B) stats.norm.ppf(0.05)

-1.6448536269514729

```
X=540-(1.64485*225)
```

```
print("5th percentile of the profit is",X,"million")
```

5th percentile of the profit is Rs.170 million

C) stats.norm.cdf(0,5,3)

0.0477903522728147

Stats.norm.cdf(0,7,4)

0.040059156863817086

Profit2 has a larger probability of making a loss in a given year.