
# Probability

# Understanding Probability

Consider the following statements. How do you interpret “probability” in each one of those? And how is it computed?

- Coin Toss – Probability of Head

- Weather – Probability of thunderstorm tomorrow is 25 %

# Probability vs Statistics

Probability – Predict the likelihood of a future event.

Statistics – Analyze the past events

Probability – What will happen in a given ideal world ?

Statistics – How ideal is the world ?

# Applications

- Gamming Industry – Establish charges and payoffs
- Manufacturing – Prevent Major breakdowns
- Business – Deciding on a business proposal based on probability of success vs cost
- Risk Evaluation

# Computing Probability

- Classical Method – ***A priori or Theoretical***

*Probability* can be determined prior to conducting any

experiment.

$$P(E) \;=\; \frac{No.of\;\; outcomes\; in\; which\; the\; even\; occures}{total\; possible\; no.of\;\; outcomes}$$

*Eg. Tossing a fair dice.*

# Computing Probability

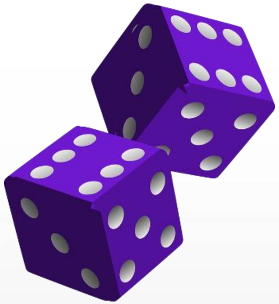
P(H) = ?


$$\frac{\text{\# of possibilities that meet my conditions}}{\text{\# of equal likely possibility}}$$

= 1 / 2 = 50 %

# Computing Probability

- If I roll a die, what are the possible outcomes I can get ?
  - Exactly "1"
  - Exactly "2"
  - Exactly "3"
  - Exactly "4"
  - Exactly "5"
  - Exactly "6"



- Only "Six" outcomes
- P( 1 ) = 1/ 6
- P (2 or 6) = 2/6
- P(2 and 6 ) = 0 /6

# Computing Probability

Find the probability of pulling a yellow marble from a bag with 3 yellow, 2 red, 3 green and 1 blue.

P (yellow) = ?
P (red) = ?
P (green) = ?
P(blue) = ?

# Computing Probability

We have a bag with 9 red marbles, 2 blue marbles, and 3 green marbles in it. What is the probability of randomly selecting a **non**-blue marble from bag ?

$$P(nB) = \frac{12}{14}$$

$$1 - P(B) = 1 - \frac{2}{14}$$

# Computing Probability

The circumference of a circle is 36π. Contained in that circle is a smaller circle with an area of 16π.

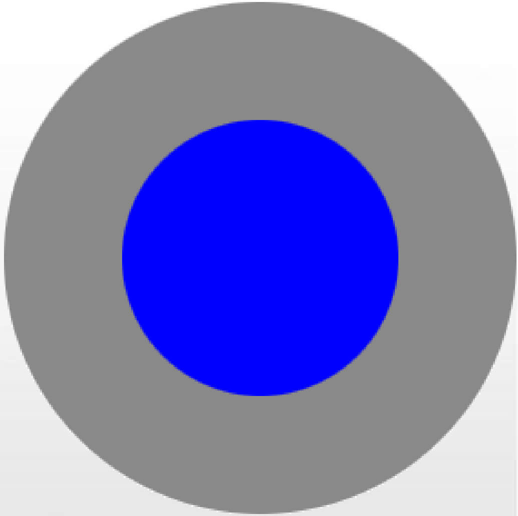
A point is selected at random fro inside the larger circle. What is  probability that the point also in

the same circle.

Area of smaller circle = 16 π

Area of larger circle = π * (36 π / 2 π )^2

= 324 π



Prob of point in same circle = 16 / 324

# Monty Hall Problem

Probability of Switch ?

Probability of not switch ?

# Probability with counting outcomes

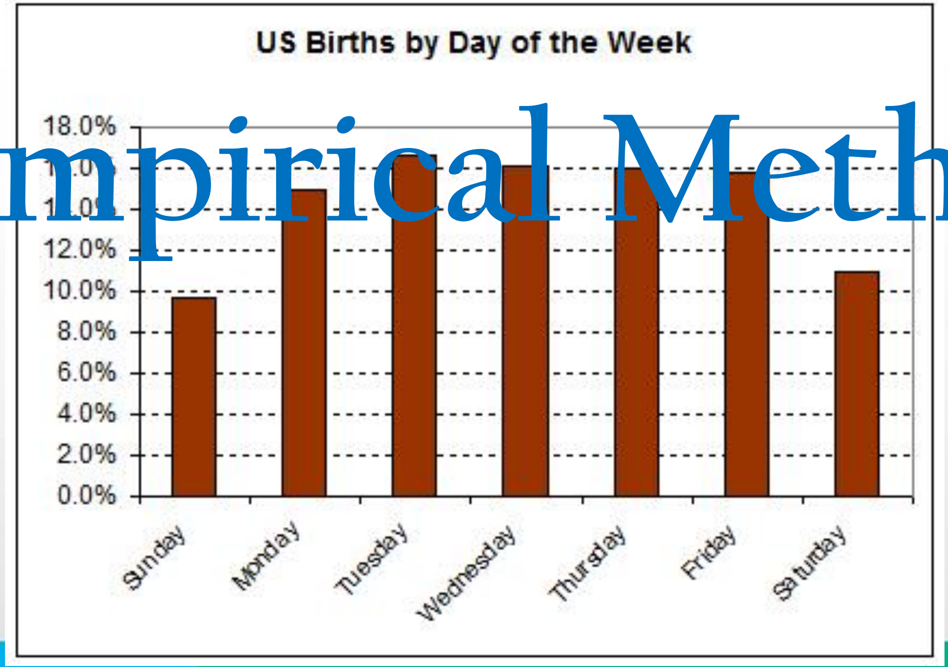
Find the probability of flipping exactly two heads on three coins ?

$P = 3 / 8$

# Computing Probability

What is the probability of a baby begin born on Wednesday ?

A-priori probability = 1/ 7 = 14.3 %




Empirical Method

# Computing Probability

## Empirical Method – A Posteriori or Frequentist

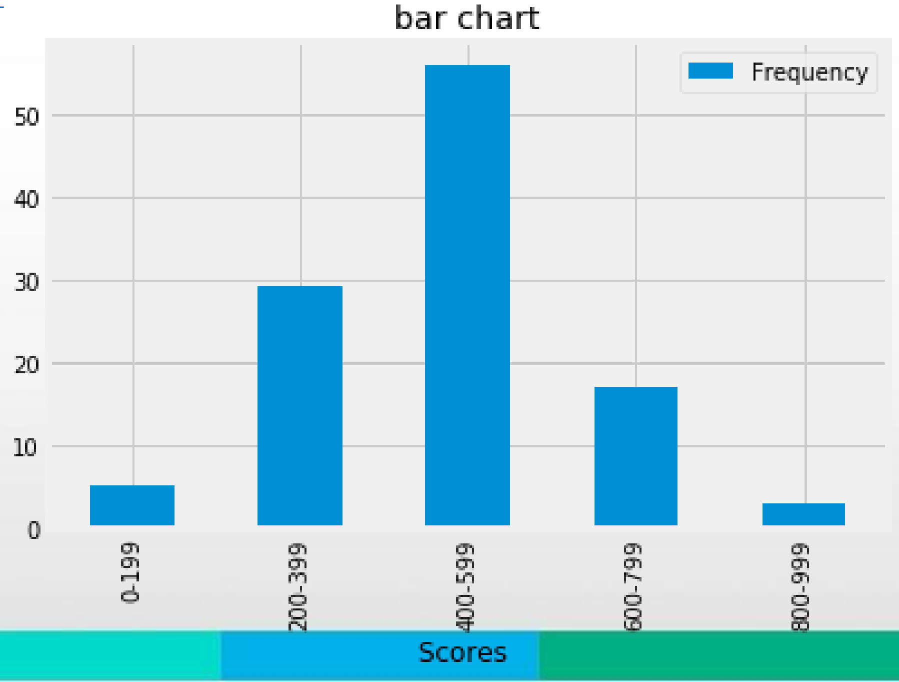
Probability can be determined post conducting a thought experiment

- This is most used method in statistical inference

# Computing Probability

Experimental probability:

“ Is to get estimate of something based on data and experience the we had in the past. “




What is probability of that for next game you will have a score greater than 600 ?

Ans: total games = 110

p (score >= 600) = (16 + 2) / 110

.

# Making Prediction with Probability

Predict number of times we will get "FREE GIFT" if I rotate roulette for 1200 times.

P (free gift) = 2 /12

No. of time we might get free gift = 1200 * 2 /12

= 200

# Computing Probability

## Subjective Method

Probability based on feelings, insights, knowledge, etc. of a person.

What is the probability of India wining upcoming World Cup- 2019

# Random Variable

# Random Variables

- Random variables is the ways to measure Random process numbers.
  - Like flipping coin
  - Rolling a die
- Mapping outcomes to numbers

# Random Variables

Discrete Random Variable

- Distinct / separate value

Continuous Random Variable

- Any value in interval

# Discrete vs Continuous Random Variable

1. Tossing as Coin

$$X = \begin{cases} 1 & Head \\ 0 & Tail \end{cases}$$

2. Year that a random student was born

1992 , 1990, 2000, 2005

1. Exact Mass of a random animal selected at zoo

75.124818181861.....

0 2 Ton

2. Exact Winning time men's 100 m race

9.568198498191895l2546 .......

# Discrete Random Variable

# Discrete Random Variable Probability

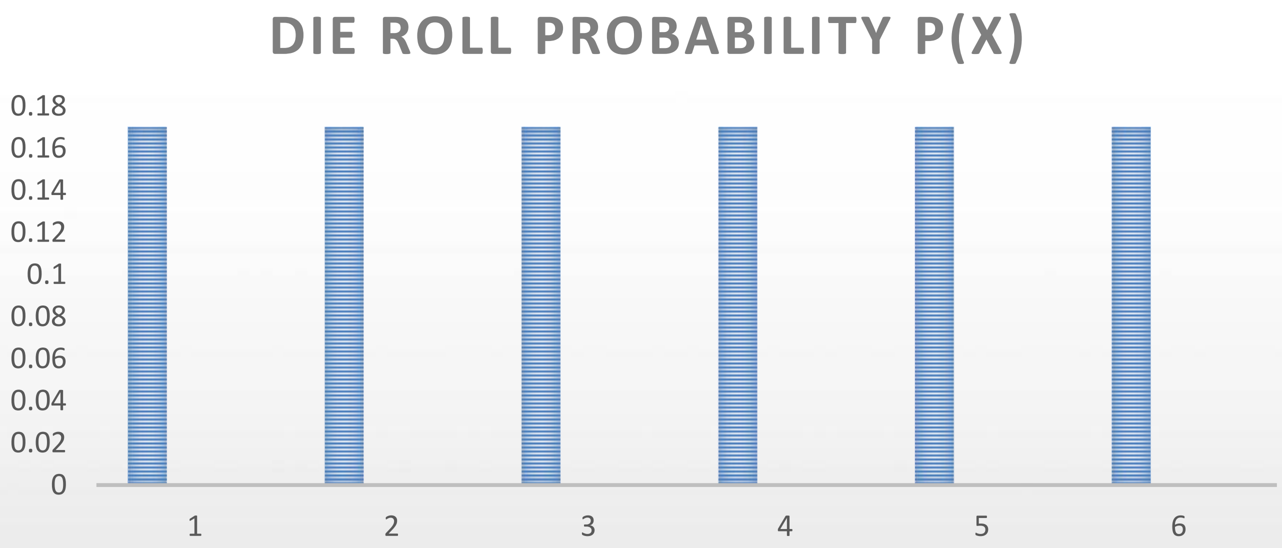
Let 1 through 6 represent the outcomes for a die roll. So our discrete random variable X is described as:

$$x = 1,2,3,4,5,6$$

What will be our probability distribution look like? What will our probability function P(x) values look like?

| X | P(X) |
|---|---|
| 1 | 1/6 |
| 2 | 1/6 |
| 3 | 1/6 |
| 4 | 1/6 |
| 5 | 1/6 |
| 6 | 1/6 |

# Uniform Probability Distribution



# Conditions for Discrete Probabilities

$$0 \leq \ P(x) < 1$$

$$\Sigma \ \mathrm{P(x)} \ = 1$$

# Compound Probabilities

What is the probability of rolling a 2 or a 5 during a die roll ?

$X = 2, 5$

$P(x) = 1/6 + 1/6 = 1/3$

# Expected Value

# Problem

On your Psychology 101 syllabus it states that your final grade will be determined in the following manner:

| ASSIGNMENT | FINALE GRADE |
|---|---|
| Home work | 30 % |
| Quizzes | 20 % |
| Midterm | 25 % |
| Final | 25 % |

# Problem

In your Psychology 101 class you received the following grades:

| ASSIGNMENT | GRADE % | WEIGHT | SUBSCORE |
|---|---|---|---|
| Home work | 89 % | 0.30 | 26.7 |
| Quizzes | 79 % | 0.20 | 15.8 |
| Midterm | 84 % | 0.25 | 21.0 |
| Final | 92 % | 0.25 | 23.0 |
| | | | 86.5 % |

# Expected Value ?

The expected value is simply the mean of a random variable; the average expected outcome. It does not have to be a value the discrete random variable can assume

- $E(x) = \mu = \sum x \; P(x)$
  - $\mu$ is the mean

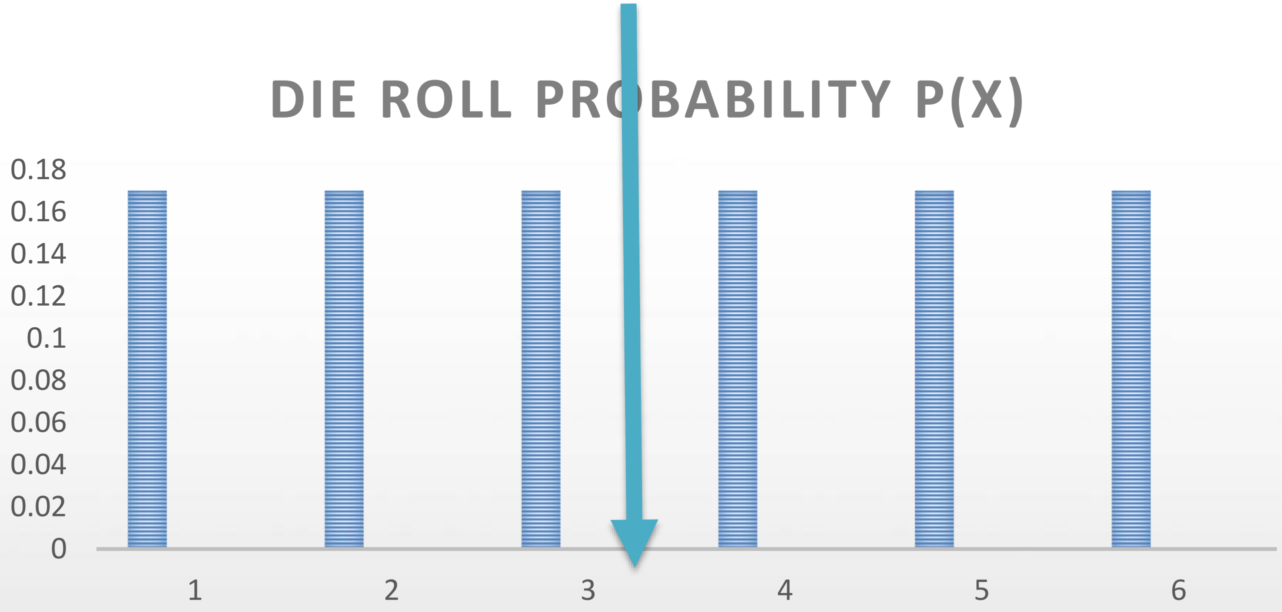
# Question 1

If I roll a die many times and then average my rolls, what should I expect for $\mu$. ?

- $E(x) \;= \mu = \sum x \;\; P(x)$
  - $\mu$ is the mean

| X | P(x) | xP(x) |
| --- | --- | --- |
| 1 | 1/6 | 1/6 |
| 2 | 1/6 | 2/6 |
| 3 | 1/6 | 3/6 |
| 4 | 1/6 | 4/6 |
| 5 | 1/6 | 5/6 |
| 6 | 1/6 | 6/6 |
| | | 21/6 = 3.5 |

$$\mu=(b+a)/2$$


DIE ROLL PROBABILITY P(X)
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6

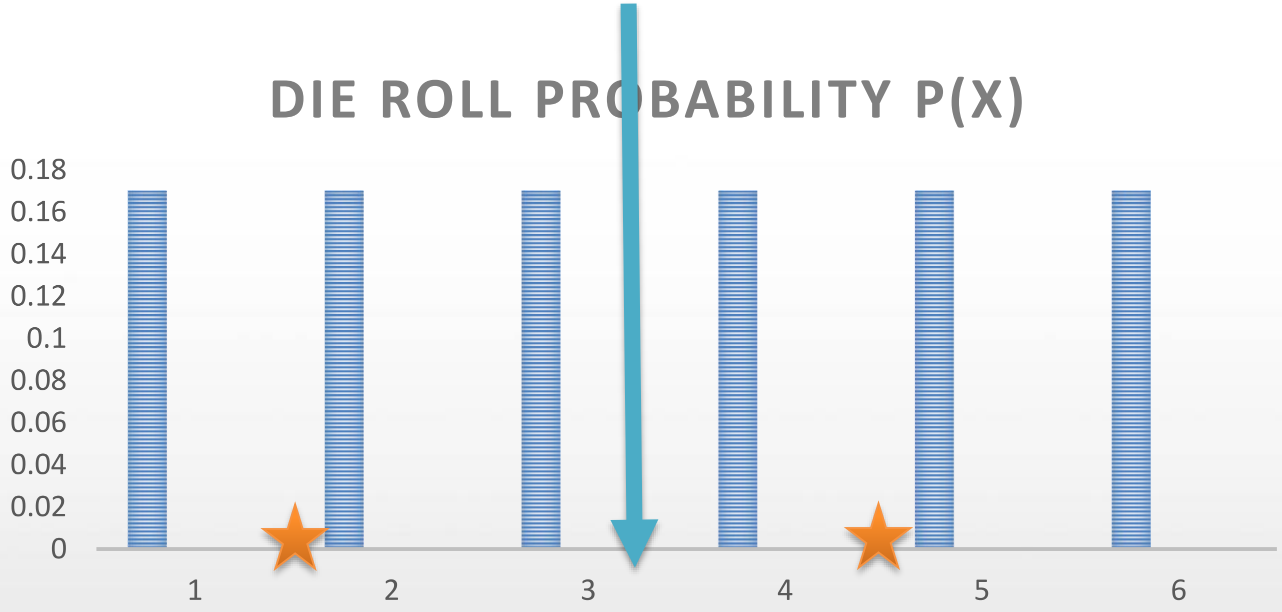
# Random Variable Variance

# Random Variable Variance

Though the expected value tells us the mean of a random variable oftentimes we need to know that variability, or how spread out, the random variable is from its mean.

- We can use the variance and standard deviation of a random variable to learn about how **dispersed** it is relative to its mean.

$$\sigma^2 = \sum (x - \mu)^2 \, P(x)$$



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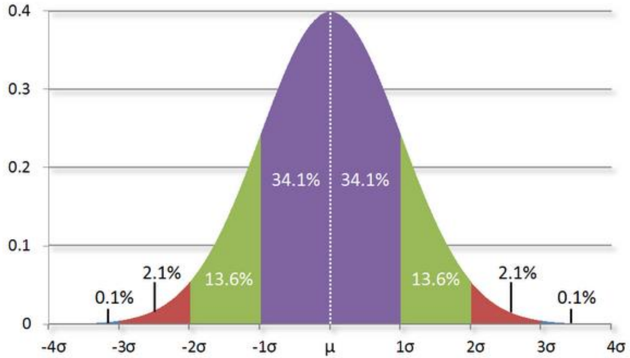


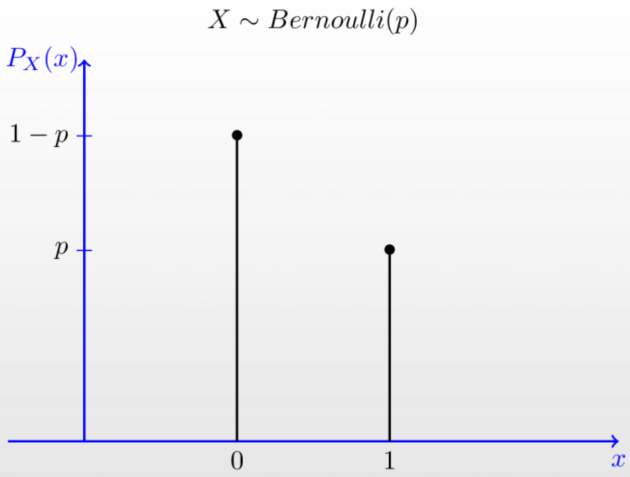

# Distribution

# Bernouli Distribution

# Bernouli

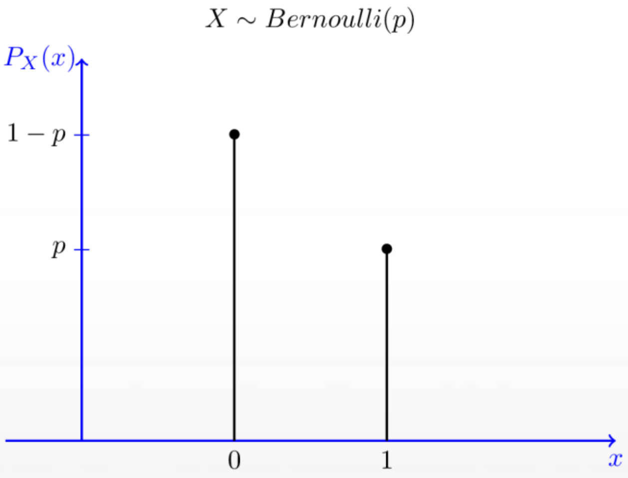
- There are two possibilities (pass or fail) with probability *p* of success and

  $q = 1\text{-}p$ of failure..




Expectation : p
Variance : pq

# Bernouli




$$Expectation, E(x) = \sum x_i \, P(x_i)$$

$= 1 * p + 0 * q$

$= p$

$$Variance, Var = \sum (x_i - \mu)^2 \; P(x_i)$$

$= (1 - p)^2 * p + (0 - p)^2 * (1 - p)$

$= p\,(1 - p)$

$= pq$

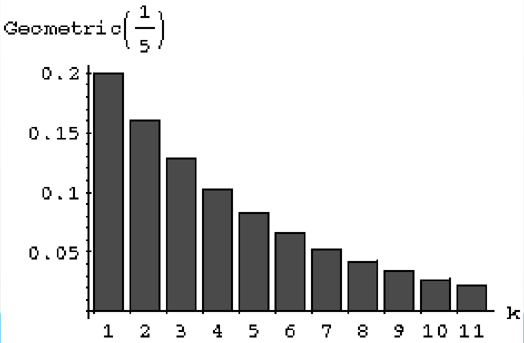
# Geometric Distribution

Number of independent and identical Bernoulli trails needed to get ONE success.

Eg. Number of attempts before I pass the exam

# Geometric Distribution

- You run a series of independent trail.
- There can be either a success or failure for each trail, and the probability of success is the same for each trail.
- How many trails are needed in order to get the first successful outcome.



# Geometric Distribution

- PMF* = P(X = r) $= q^{r-1}p$
- $P(X > r) = q^r$
- CDF**, $P(X \leq r) = 1 - q^r$
- $E(X) = \frac{1}{p}$ $var(x) = \frac{q}{p^r}$

PMF : Probaility Mass Funciton
CDF : Cumulative Distribution Function

# Binomial Distribution

# Binomial Experiment

- The process consists of a sequence of n trials.
- Only two exclusive outcomes are possible in each trail. One outcome is called “Success” and other a “failure”.
- The probability of a success denotes *p* , does not change from trail to trail. The probability of failure is 1-*p* and is also fixed from trail to trail.
- The trails are independent; the outcome of previous trail not influence future trail.

# Binomial Variable

- Let's consider a coin P(H) = 0.6
- P(T) = 0.4

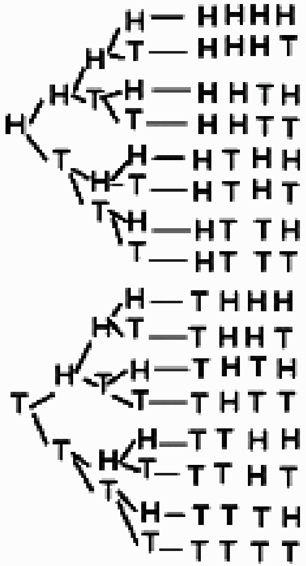
- X = # of heads after 10 flip of my coin
- Made up of independent trails
- Each trail can be classified as either success or failure
- Probability of success on each trail is constant

# Binomial distribution

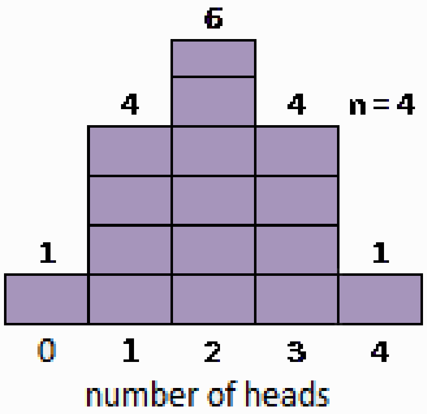
- X = # of heads after flipping coin 4 times
- Possible outcomes = 2 * 2 * 2 * 2 = 16
- P(X = 0) = $\frac{C_0^4}{32} = \frac{1}{32}$
- P(X = 1) = $\frac{C_1^4}{32} = \frac{5}{32}$
- P(X = 2) = $\frac{C_2^4}{32} = \frac{6}{32}$

# Binomial distribution

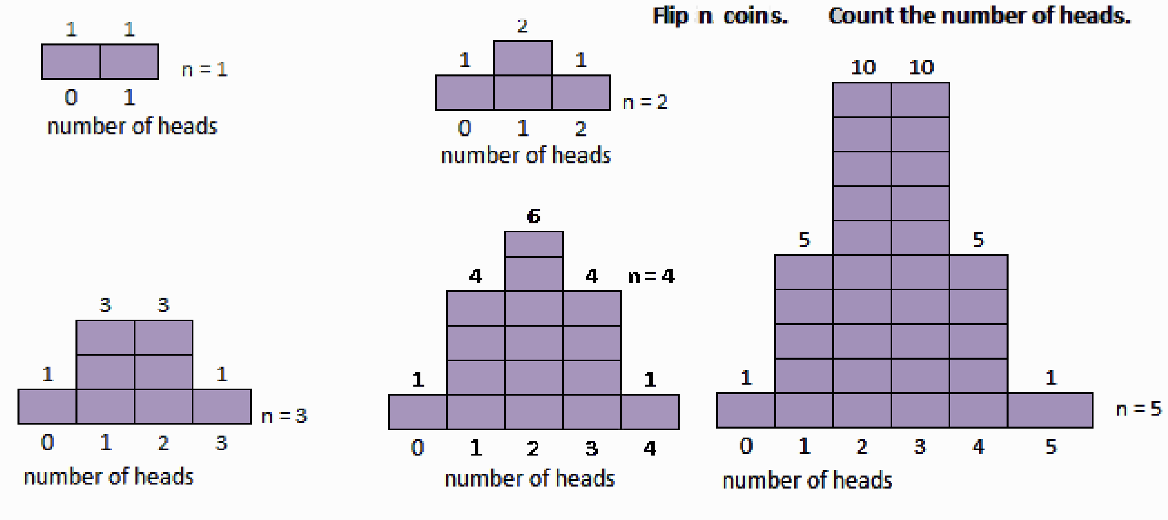
**Flip a coin four times. Count the number of heads.**



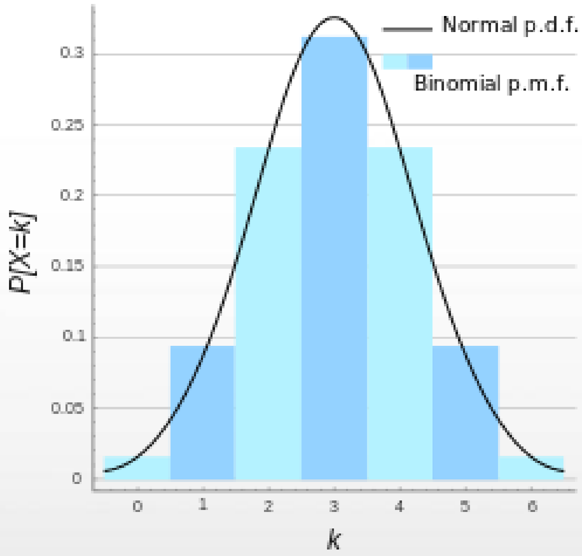

| x | P(x) = p | |
|---|---|---|
| 0 | 0.0625 | 1 /16 |
| 1 | 0.25 | 4 /16 |
| 2 | 0.375 | 6 / 16 |
| 3 | 0.25 | 4 /16 |
| 4 | 0.0625 | 1/16 |



# Binomial distribution



# Binomial distribution



# Binomial Probability example

P(Score) = 70 % or 0.7
P(miss) = 30 % or 0.3

P(Exactly 2 scores in 6 attempts) =

$$= C_2^6 \, (0.7)^2 \, (0.3)^4$$

P(Exactly k scores in n attempts) =

$$= C_k^n \, (p)^k \, (1 - p)^{n-k}$$

# Question

In this area it is expected that 9 cars are passing in a hours. What is the probability that there are exactly 6 cars are passing in this hour

$E(X) = \lambda = n * p = 9$ cars/hr

Since , 1 hours = 60 minutes

So, n = 60

K = 6 (number of expected cars in hour)

$$P(x = k) = C_k^n \, (p)^k \, (1 - p)^{n-k}$$

$$p = \lambda / n = 9 / 60$$

# Question

In this area it is expected that 9 cars are passing in a hours. What is the probability that there are exactly 6 cars are passing in this hour

$p = \lambda / n = 9 / 60$

$1 - p = 51 / 60$

Therefore, probability of exactly 6 cars are passing in an hours is

$$P(x = 6) = C_6^{60} \left(\frac{9}{60}\right)^6 \left(\frac{51}{60}\right)^{54}$$

$n = 3600$ (no. of seconds in an hour)

# Poisson Distribution

# Poisson

E(X) = $\lambda \quad = n * p$

P(X = k ) $\quad = \lim_{n\to\infty} C_k^n \, (p)^k \; (1 - p)^{n-k}$

$$= \frac{\boldsymbol{\lambda^k}}{\boldsymbol{k\,!}} \boldsymbol{e}^{-\lambda}$$

$\lambda$ = 9 cars pass

P(X = 2)

# Question

Q. Probability that a cars will not pass in n min

$$= \frac{\lambda^{k}}{k\,!} e^{-\lambda}$$

For 1st second:

P (x = 0) = $\frac{\lambda^{0}}{0!} e^{-\lambda} = e^{-\lambda}$ ( for 1 sec)

For *n* second:

Probability that a cars will pass for n sec.

$$= e^{-n\lambda}$$

# Exponential Distribution

Q. Probability that a cars will pass in n sec

$$1 - e^{-n\lambda}$$

CDF = $1 - e^{-n\lambda}$ , $n \geq 0$

PDF= $\lambda e^{-n\lambda}$ , $n \geq 0$

# Exponential Distribution

- Poisson process
- Continuous analog of Geometric distribution

$$E(X) = \frac{1}{\lambda} \qquad var(X) = \frac{1}{\lambda^2}$$

# IOTGYAN