Planning Sales Territories - A Facility Location Approach

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Abstract. Planning sales territories may be viewed as the problem of grouping small geographic sales coverage units (e.g. counties, zip code areas, company trading areas) into larger geographic clusters called sales territories in such a way that the latter are acceptable according to managerially relevant criteria (e.g. minimal/maximal/average sales potentials or travel times). In contrast to the currently available set–partitioning oriented models, we propose a facility location approach for planning sales territories. The integer programming formulation we developed for this approach is a discrete capacitated m–median problem with the additional requirement that sales territories have to be connected. As this problem is NP–hard, we devised an efficient, interchange based heuristic procedure for solving large scale problems. The algorithms are embedded in the so–called *BusinessManager*, a software tool developed by *Geomer GmbH*, Germany.

1 Introduction

Due to fast changing markets or the introduction of new products, sales territory planning decisions have frequently to be re–evaluated. Especially for large sales territories this is a lengthy process and therefore an algorithmic approach for expediting the process is often desired. Well–planned decisions enable an efficient market penetration and lead to decreased costs and improved customer service.

Another advantage of an algorithmic approach is the possibility of performing sensitivity analysis to find out how robust the obtained solutions are. One aspect concerns the possibly inaccurate forecasts of relevant data. Questions which commonly arise in this context are for example: How much do sales forecasts influence the optimal solution? Do we obtain the same solution if sales figures were 10% higher/lower (worst-case, best-case)?

In spite of the fact that planning sales territories is an area where management science techniques are likely to be accepted and implemented, it is also an area in which marketing science literature does not propose a generalized optimization approach to the problem. Several heuristic methods exist (see e.g. [4]) but all have the major shortcoming of being trial and error procedures which rely on an adjust–and–evaluate mechanism to arrive at reasonable territory configurations. Two types of mathematical programming models have been employed. In [5] the problem is formulated as a set–partitioning model. Alternatively, in [2] so-called geographic sales coverage unit (SCU) assignment models are introduced, and subsequently refined in [7]. The latter model takes multiple criteria into account and eliminates the problems of disconnected sales regions encountered in the first model by introducing a hierarchical SCU–adjacency tree. Both models have the major drawback that they require the centers of the sales territories, i.e. the location of the sales person(s), to be fixed in advance.

In this paper we report on the development of a tool to support sales territory planning, where the centers of the sales territories can but need not be fixed in advance. In the latter case, the optimal locations of sales persons within the territories are also determined. The developed algorithms are embedded in the so–called *BusinessManager*, a software extension developed by *Geomer GmbH, Germany* for the Geographic Information System ArcView GIS[1].

In the next section we will formally introduce the problem. Section 3 is dedicated to solution techniques that assist managers in planning sales territories. In Section 4 we present the numerical results obtained for randomly generated test problems. Some concluding remarks are given in Section 5.

2 Problem Formulation

We are given a set of n sales coverage units. These SCUs may be just points (locations), but may also represent geographical areas (see Fig. 1). In case of 2-dimensional objects these are represented by the coordinates of the center of the area. Counties, zip code areas, company trading areas and other predetermined prospect clusters are some examples of SCUs. Furthermore, a sales figure is assigned to each SCU allowing to formulate additional planning criteria. These figures may be combinations of different values. Typical examples range from demographical data, e.g. number of inhabitants, purchasing power, to company specific values like turnover figures.

Each sales territory (ST) consists of a subset of SCUs. In addition, a sales person (SP) is assigned to each ST and sited in one of the SCUs belonging to the ST. The generation of sales territories must fulfill certain planning criteria. Typical examples are the following:

- The sales figure of each ST has to be within certain limits, i.e. a minimal or maximal ST size is required. The sales figure of the ST is simply the sum of the figures of its SCUs. For example, we would like to plan STs in such a way that the sales person assigned to each neither has to serve too many or too less SCUs.

[1] ArcView® GIS is a registered trademark of Environmental Systems Research Institute, Inc. (ESRI), USA.

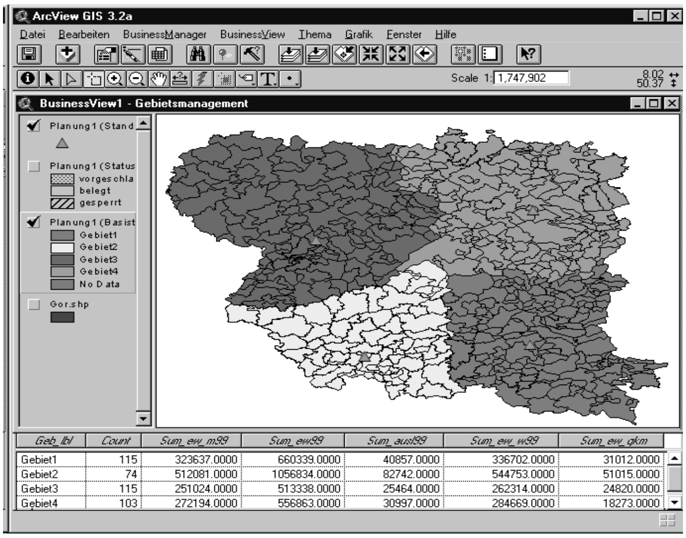

Fig. 1. Graphical display of sales territories consisting of zip–code areas in the *Geomer – BusinessManager*.

- A maximal distance/travel time from the SCU where a sales person is located to all SCUs served by him/her. For example, the distance a sales person can drive in one day starting from his/her home location may be used as a planning criterium.

Furthermore, STs have to be disjoint, i.e. an SCU belongs to exactly one ST, and connected (see the four shaded areas in Fig. 1). Finally, we are given a matrix indicating the distance between the centers of any two SCUs. The entries of the matrix may be measured in kilometers based on Euclidean distances or correspond to distances on an underlying road–network. Alternatively, travel times or costs can be considered. Where required, the distances can be weighted by additional factors which may differ from the SCU sales figures.

The objective is to find a given number m of new sales territories minimizing the sum of distances from the SCUs to the locations of the sales persons within the corresponding ST to which the SCUs belong. The planning can either be performed from scratch or in the presence of already existing STs or even locations of sales persons. In the former case, it is allowed to add additional SCUs to an existing ST. In the latter case, a new sales territory will be built around the location of the SP. Moreover, the number m of STs needs not be fixed in advance. Instead, the algorithm will choose it in such a way that the planning criteria are fulfilled. A last option is that some SCUs may remain unassigned if the maximal allowed ST size is too restricting.

3 A Facility Location Approach

In contrast to the previously known set–partitioning oriented models, we propose a facility location approach for planning sales territories. The problem is modeled as a discrete capacitated m–facility location problem. SCUs correspond to existing sites, while locations of sales persons correspond to new facilities, whose optimal locations are sought. Candidate sites for these facilities are (a subset of) the SCUs.

The assignment of SCUs to a sales person is based on the minimum distance criterium. The set of SCUs assigned to the same candidate site now defines a sales territory. This means that the catchment area of an SP contains all sales coverage units that are closer to this sales person than to all other SPs. The main advantages of this approach are:

- Locations of sales persons can but need not be fixed in advance.
- Connected STs are obtained as a result of assigning each SCU to the closest selected candidate site, in the case that the distance matrix fulfills the triangular inequality.
- No neighborhood information (of SCUs) is needed for the planning process.
- A broad range of techniques for solving discrete facility location problems is available.

In addition, the above facility location approach always yields the optimal solution, if no size constraints are specified for the STs, i.e. we have an unconstrained planning problem. The drawback of this approach is that for very size restrictive problems a feasible solution may not be found although one exists.

3.1 Integer Programming Formulation

The integer programming (IP) formulation we developed for this approach is a discrete capacitated m–median model with the additional requirement that SCUs are always allocated to the closest selected candidate location.

We denote by $SCU_1, \ldots, SCU_n$ the sales coverage units, and by SF_i the sales figure of SCU_i, $i \in \mathcal{I} := \{1, \ldots, n\}$. Let d_{ij} be the (weighted) distance from SCU_i to SCU_j, $i, j \in \mathcal{I}$. Furthermore, let $MinSF$ and $MaxSF$ denote the minimal and maximal sales limits of a sales territory, respectively. Finally, the following decision variables are defined:

$$y_j = \begin{cases} 1 \text{ if a sales person is located at } SCU_j \\ 0 \text{ otherwise} \end{cases}$$

$$x_{ij} = \begin{cases} 1 \text{ if } SCU_i \text{ is assigned to a sales person located at } SCU_j \\ 0 \text{ otherwise} \end{cases}$$

The following formulation includes constraints on the upper and lower sales figures of the sales territories. Moreover, it is assumed that planning is carried out from scratch.

$$
\begin{array}{llcllr}
\textbf{MIN} & \displaystyle\sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}x_{ij} & & & & \\
\textbf{s.t.} & \displaystyle\sum_{j=1}^{n} x_{ij} & = & 1 & \forall i = 1,\ldots,n & (1) \\
 & \displaystyle\sum_{j=1}^{n} y_j & = & m & & (2) \\
 & x_{ij} & \leq & y_j & \forall i,j = 1,\ldots,n & (3) \\
 & \displaystyle\sum_{i=1}^{n} SF_i\, x_{ij} & \geq & MinSF\, y_j & \forall j = 1,\ldots,n & (4) \\
 & \displaystyle\sum_{i=1}^{n} SF_i\, x_{ij} & \leq & MaxSF\, y_j & \forall j = 1,\ldots,n & (5) \\
 & y_k(d_{ij} - d_{ik}) & \leq & (1 - x_{ij})(d_{ij} + d_{ik}) & \forall i,j,k = 1,\ldots,n & (6) \\
 & x_{ij},\, y_j & \in & \{0,1\} & \forall i,j = 1,\ldots,n & (7)
\end{array}
$$

The objective is to minimize the sum of (weighted) distances from the SCUs to the new sites. Constraints (1) insure that an SCU is assigned to exactly one new site, i.e. sales person. By (2) we select exactly m new sites and by (3) SCUs can only be allocated to SCU sites where an SP is located. Furthermore, constraints (4) and (5) insure that the sales figures of STs are within the desired limits. By (6), if SCU_i is assigned to a new facility located at SCU_j, i.e. $x_{ij} = 1$, then for every other SP located at some SCU_k, $k \neq j$, the distance from i to k is not smaller than the distance from i to j, i.e. SCU_i is assigned to the closest SP.

Note that we have $O(n^2)$ binary decision variables and $O(n^3)$ constraints.

3.2 Heuristic Procedure

Observe that our problem is NP–hard since it contains as a special case the discrete $m-$median problem. Hence, to solve large scale problems, we developed an efficient heuristic procedure which is based on a variable neighborhood search (VNS) technique [1] and utilizes as local search method the interchange algorithm of [6].

An initial site selection of sales persons is obtained by solving a continuous m-facility location problem. This is done using the heuristic method described in [3], which can also take already existing SPs into account. For each site generated for a sales person by this procedure, the closest SCU site is taken as the actual SP location.

Based on the initial site selection, a combined interchange and variable neighborhood search heuristic is applied in order to determine a better solution. In our case, we slightly modified the local exchange step of the interchange procedure [6]. Namely, the current SP site is only exchanged by another SCU site, if the latter belongs to the catchment area, i.e. the sales territory, of this SP. This simplification of the basic interchange procedure is applied, since from our computational experience we could observe that the initial site selection already provides very good solutions (see Table 1).

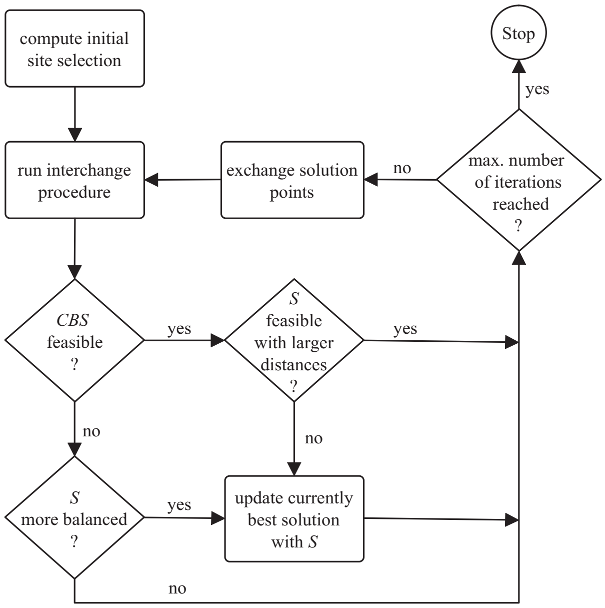

Fig. 2. Outline of the heuristic.

Fig. 2 gives an outline of the heuristic. Using the initial solution, we run the modified interchange procedure and store the new site selection as currently best solution, CBS. Afterwards, we exchange a certain number of SP sites of the CBS by SCU sites which belong to the sales territories of the corresponding SPs. The exchange number hereby depends on the number of new SPs, m. Using this modified site selection, we run again the interchange procedure and obtain a modified solution S. We accept this new site selection S as currently best solution either if CBS is infeasible and S is more balanced, i.e. the min. and max. sales figure limits for the STs are less violated, or both site selections are feasible but S has a lower total distance. Next, we iterate this process of exchanging SP sites and evaluating the new solution. The algorithm stops after a certain, predetermined, number of iterations.

4 Computational Results

In this section we evaluate the quality of our heuristic procedure on a set of test problems.

The coordinates of SCUs were drawn randomly from a uniform distribution between 0 and 100, and the sales figures between 4 and 8. For a fixed number of SCUs, we generated 20 instances and solved every instance 5 times, each with a different number of new sales persons. For unconstrained planning problems, input data was generated for six problem types ranging from 100 up to 600 SCUs. The number of new sales persons ranged from 4 to 25 depending on the size of the problem.

Table 1. Computational results for unconstrained problems

		Initial			VNS		Exact
SCUs	A–Gap	W–Gap	A–CPU	A–Gap	W–Gap	A–CPU	A–CPU
100	2.77	8.82	0.04	0.39	4.85	0.32	4.26
200	3.39	10.19	0.06	0.50	4.08	0.90	35.03
300	2.90	10.23	0.13	0.31	3.06	2.53	128.01
400	3.66	11.86	0.17	0.39	2.00	4.37	351.90
500	3.61	7.80	0.33	0.43	2.67	10.14	779.73
600	3.92	7.80	0.38	0.36	1.92	15.35	1649.92

Table 1 summarizes the results obtained for the 600 test problems. We compare the initial solution given by the procedure in [3] ('Initial') and our heuristic approach ('VNS') with the optimal solution ('Exact'). The first column of the table indicates the total number of SCUs in each problem category. The 'A–Gap' and 'W–Gap' columns present the average and worst, respectively, relative percentage gap with respect to the optimal objective function values. All tests were performed on a Pentium III, 600 MHz with 1 GB RAM, and the CPU times (in seconds) are given in the columns 'A–CPU'. Finally, all problems were solved optimally with CPLEX 7.0.

By comparing the gaps of the initial solution with the VNS approach, we observe that the latter are considerably better and are obtained at the expense of an acceptable increase in the computational time. As expected, solving a problem to optimality involves a considerable computational burden, even for relatively small instances. In contrast, the VNS approach yields very good solutions for large scale problems in reasonable time.

For the constrained planning case, input data was generated for problems with 50 and 75 SCUs with the above settings. The minimal and maximal sales figure limits, $MinSF$ and $MaxSF$, for the STs are 90% and 110%, respectively, of the sum of the sales figures of the SCUs divided by the number of new SPs. Table 2 shows the results obtained.

Table 2. Computational results for constrained planning problems

		VNS		Exact
SCUs	A–Gap	W–Gap	A–CPU	A–CPU
50	2.46	12.06	0.54	11819
75	1.54	4.07	0.76	60617

Constrained problems are much more difficult to solve optimally as the computational times show. Therefore, the use of heuristic procedures for solving real life problems is necessary. Although the average gap of the VNS approach worsens compared to the unconstrained case, it is still acceptable.

5 Conclusions and Further Research

In this paper we presented an efficient, interchange based heuristic procedure for planning sales territories. Some issues require further investigation. One topic is the problem that in constrained planning it is often desired by managers that the STs not only fulfill the sales figure restrictions but should also be of equal size, i.e. somehow balanced. Another yet un–modeled issue is a multi–criteria approach which allows the user to specify more than one sales figure for each SCU and therefore, also more than one planning restriction on the STs. Concerning the IP formulation, polyhedral properties will be investigated.

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