Multiple Objective Districting: A General Heuristic Approach Using Multiple Criteria
Author(s): Richard F. Deckro
Source: *Operational Research Quarterly (1970-1977)*, Vol. 28, No. 4, Part 2 (1977), pp. 953-961
Published by: Palgrave Macmillan Journals on behalf of the Operational Research Society
Stable URL: http://www.jstor.org/stable/3008868
Accessed: 04-11-2015 07:52 UTC

Opl Res. Q., Pergamon Press 1977. Vol. 28, 4, ii, pp. 953 to 961. Printed in Great Britain

Multiple Objective Districting: A General Heuristic Approach Using Multiple Criteria

RICHARD F. DECKRO

Department of Business Administration, Virginia Polytechnic Institute and State University

This paper presents a heuristic approach for the creation of similar districts which are required to satisfy multiple and potentially divergent goals. The heuristic creates districts whose members fall within acceptable ranges of variation on a lexiographic ordering of prescribed criteria. Through the variation of the ordering of the criteria and the allowable ranges of variation, alternative formulations are obtained from which the final structure can be selected.

INTRODUCTION

DISTRICTS, zones, or territories are often used to distinguish geographical or political areas of responsibility, service and/or representation in both the public and private sectors. Increasing numbers of these districts are required to satisfy multiple (and potentially conflicting) objectives. Where a clear measure of these objectives exist, the task of creating districts is substantially simplified. However, many districts are required to satisfy such ill-defined goals as equality of service, balance of work load, and maintenance of goodwill or penetration. If acceptable measures of performance for such criteria are not available in comparable units, it may be difficult to obtain an appropriate optimizing model.

In addition to the difficulty of incompatible criteria, the decision-maker often has indistinct objectives. There may be no "optimal" district configuration which is actually discernible. An additional complication arises from the imprecision of the measures used. Such factors as sales potential, anticipated contact hours, or predicted travel times are often only estimates. "Optimal" solutions to specific districting problems may be highly dependent upon the accuracy of subjective criteria.

The model presented below attempts to create similar districts which consider multiple and potentially divergent goals. Rather than attempt to optimize a series of objectives, the heuristic creates districts whose members fall within acceptable ranges of variation on a lexiographic ordering of prescribed criteria. The selection of the criteria, their ordering and the range of variation allowed are inputs to the model. The efficiency of the procedure precludes the necessity of creating weights and allows the decision-maker to utilize his intuition in selecting the final district structure.

The heuristic

The heuristic approach constructs districts from already existing sub-divisions or units, the selection of which is, in general, independent of the solution procedure. The basic units should be distinct and non-overlapping and the appropriate data for each criteria, which should be additive, must be available for each basic unit. The particular size of the basic unit is immaterial to the districting process. Unit size may be entered as a criterion if desired. Information on the contiguity of the units must be known.

The heuristic uses a decision rule similar to that proposed by Garfinkel and Nemhauser.[1] Districts are created such that a particular criterion is satisfied within a range of acceptable variation. Easingwood[2] also used the concept of satisfaction of a range of variation in his model to create sales districts with balanced work loads.

The actual decision rule varied from the Garfinkel–Nemhauser model in two ways. The first is the specification of the target value, rather than strict reliance on the calculated average per district. (The author is indebted to an anonymous reviewer in pointing out the weakness of the Garfinkel–Nemhauser rule in a generalized model. In cases where the mode value of a criteria is not sufficiently smaller than the mean value, districting with regard to that criterion can only occur for units below the mean per district value. This would only by desirable if the average per district value were the target value.) Second, the upper and lower bounds, establishing the acceptable range of variation about the target value, need not be equidistant from the target value.

p_{ij} = the value of the jth criteria of the ith unit where $j = 1, 2, \ldots m$

$\overline{p_j}$ = the target value per unit for the jth criteria

$$p_j(k) = \sum_{i=1}^{n} b_{ik} p_{ij} \quad \text{where} \quad \begin{array}{l} b_{ik} = 1 \text{ when unit } i \in \text{district } k \\ b_{ik} = 0, \text{ otherwise} \end{array}$$

α_{1j} = portion of the target value used to calculate the lower bound on the target value of j; $0 < \alpha_{1j} < 1$

α_{2j} = portion of the target value used to calculate the upper bound on target value of j; $\alpha_{2j} > 0$

C_k = the set of units or districts contiguous to district k, which are potential candidates for addition to k, $s \in C_k$.

District k will be considered acceptable with regard to the jth criteria if

$$\overline{p_j} - (\alpha_{1j})\overline{p_j} < p_j(k) < \overline{p_j} + (\alpha_{2j})\overline{p_j}.$$

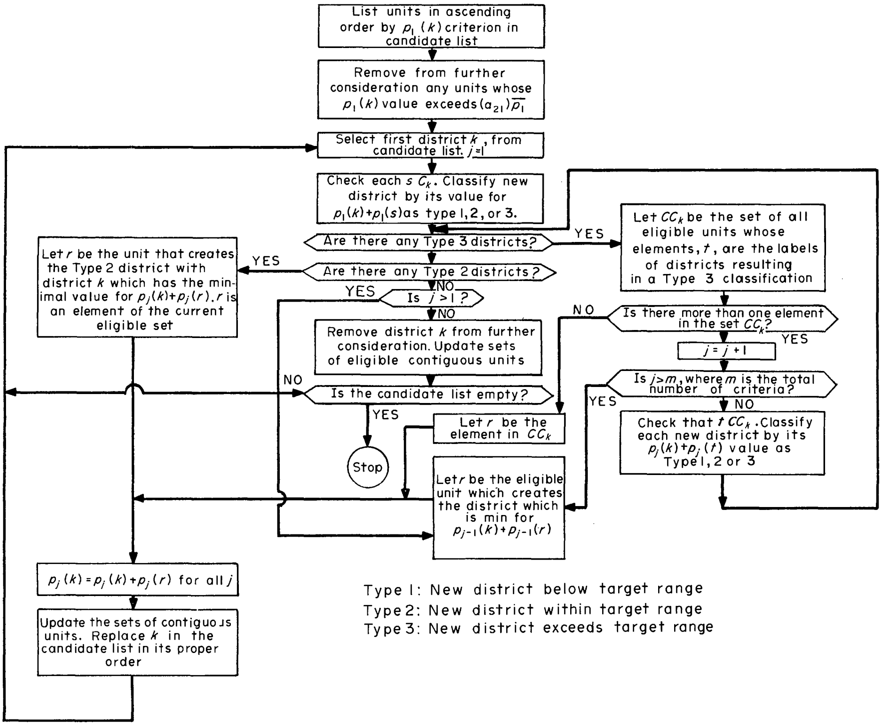

FIG. 1. *Flow of the heuristic.*

That is, a district, k, is acceptable with regard to a particular criterion, j, if the sum of the jth criterion for each unit in district k is greater than or equal to the lower bound $[\overline{p_j} - (\alpha_{1j})\overline{p_j}]$ and less than or equal to the upper bound $[\overline{p_j} + (\alpha_{2j})\overline{p_j}]$ of the range of allowable variation for the target value, $\overline{p_j}$.

Figure 1 outlines the flow of logic for the heuristic procedure. Each run is initialized by the establishment of a lexiographic ordering of the priority of the criteria with the highest priority goal or criteria labelled $j = 1$.

Each unit is initially a district composed of one unit. $p_j(k)$ will equal p_{ij} for each district where $i = k$. The units are then sorted in ascending order according to their district value of the initial criterion, $p_1(k)$. If any unit or units have initial $p_1(k)$ values which are greater than the upper bound on criterion 1, these units remain districts composed of one unit. They are removed from the sets of contiguous units and are not considered further.

The remainder of the units, still sorted in ascending order by their $p_1(k)$ values, are placed in the candidate list which thus includes all those districts or units which are eligible for districting. When two units or districts are added to one another to create a new district, the old districts are replaced by the new district in its proper order. The first district in the candidate list will always be the district with the lowest $p_1(k)$ value. A district will be removed from the candidate list if no contiguous districts remain to which the object district may be added without causing the new district to exceed the upper bound on the primary criterion.

Once the heuristic is initialized, the process continues in an interative fashion. The first district in the candidate list is selected as the object district. A check is made of each of the eligible districts which are contiguous to the object district to determine which one, if any, should be added to the object district.

If the addition of a contiguous district, s, would cause the sum of the primary criterion for the object district, k, to exceed the upper limit on that criterion, the addition of district s to district k is considered infeasible. This condition will be referred to as Type 1. If all the contiguous districts give new districts in the Type 1 classification, it indicates that the object district, k, should be removed from further consideration.

A Type 2 district results where the addition of a contiguous district creates a new district whose sum for the primary criteria is below the lower limit of the acceptable range of variation for the primary criteria. Should Type 2 districts result from all of the members of C_k, the contiguous district, s, with the minimum $p_1(s)$ value would be added to the object district, k. $p_j(k)$ would then total $p_j(k) + p_j(s)$ for all values of j. District s would be removed from the candidate list and would be replaced by district k in the sets of contiguous districts. Finally, the enlarged district k would be replaced in the ordered candidate list according to its newly calculated value for $p_1(k)$.

Type 3 districts result when the check of contiguous districts indicates that one or more of the possible new district structures would result in a district within the range of acceptable variation for the primary criterion. If only one district structure is within the acceptable range of variation, that new structure is adopted and the various lists are updated.

If more than one structure satisfies the test, CC_k a subset of C_k, is formed. This subset will contain the elements of C_k which have satisfied the initial test on the primary criterion. Each of the elements of CC_k is then tested against the second priority goal, $j = 2$, giving one of the three cases.

If all Type 1 districts result from a test of a lower priority goal, j, the districts with the lowest value for $p_{j-1}(s)$ is added to the object district. If all Type 2 districts occur, the decision rule for all Type 2 districts for a primary goal is used. Finally, if Type 3 districts occur, CC_k is updated to contain only those elements whose member satisfy the goals already tested. The $j + 1$ goal is then tested.

The heuristic constructs districts by adding one unit or district at a time. In the check of potential new districts created by the addition of elements of the contiguous set, it is likely that all three types may be obtained. In the event of multiple type classification, preference is given to those potential district structures which are classified Type 3, then Type 2 districts and finally Type 1. In this manner, feasible districts are created as rapidly as possible when districts approach the acceptable range of variation. Until that time, however, the districts are built-up from as many units as possible by the second order preference of districts classified as Type 2.

After each new district is created, it is returned to the candidate list and a new object district is selected. Potential district structures which satisfy the primary objective are checked for their conformity to the secondary goals. Through the variation of the ordering of the priorities and the widths of the acceptable ranges on these priorities, one may create sets of feasible districts. The process is completed when the candidate list is emptied.

The decision-maker has the opportunity to accept a particular district pattern resulting from an initial specification. If he chooses to generate a set of district structures, the user may make a subjective evaluation to select the desired formulation from the set of alternative formulations.

Illustration of the heuristic

Easingwood[2] develops a measure of work load for sales district measures, which combines hierarchial classifications of salesmen's calling policies as a means for comparing sales districts. Lodish[3] points out that Easingwood's measure assumes that travel times are equal in each district and the criterion of inequal times may have to be included to balance work load. Lucas and Weinberg[4] suggest that the sales potential of a particular area should be included in any sales district formulation.

Although these three measures are not all-inclusive, they illustrate the heuristic technique.

A sales manager wishes to create sales districts from a group of 10 basic units. He has the appropriate data for these units and feels that the following criteria are important to his decision: sales potential, work load as given by Easingwood's measure, and travel time in each district. Each criteria is expressed in additive terms for each basic unit.

The sales manager feels that the sales potential of the districts is the most important criterion. He wishes each district to have a minimum sales potential of £100,000 and an upper limit of £125,000 per year ($\alpha_{11} = 0$, $\alpha_{21} = 0.25$). The manager would like to balance the work loads per district at a target value of 70 per month and a range of variation of 10% of the target value in either direction from the target ($\alpha_{12} = 0.1$, $\alpha_{22} = 0.1$). Travel times per month in each district are the third priority goal, with a target value of 80 hr per month and a range of acceptable variation from 72 to 88 hr ($\alpha_{13} = 0.1$, $\alpha_{23} = 0.1$). Table 1 summarizes the initial data for the 10 basic units.

TABLE 1. INITIAL UNIT CHARACTERISTICS

j k unit	1 Sales potential (in thousands)	2 Workload per unit	3 Travel time per unit	Contiguous set, C_k
1	128	74	85	2,6
2	47	23	32	1,3,7
3	61	49	39	2,4,8
4	57	41	23	3,5,9
5	35	24	21	4,10
6	39	28	19	1,7
7	36	25	15	2,6,8
8	49	28	37	3,7,9
9	107	62	82	4,8,10
10	26	19	10	5,9
lower limit $\alpha_{1j}\,\bar{p}_j$	100	63	72	
upper limit $\alpha_{2j}\,\bar{p}_j$	125	77	88	

Creating the initial candidate list, unit 1 is removed from further consideration as its sales potential exceeds the upper limit of 125. Unit 1 becomes a district composed of one unit. The initial candidate list, sorted in ascending order by sales potential, is: 10, 5, 7, 6, 2, 8, 4, 3, 9.

Unit 10 is selected as the object district. Its contiguous set, C_{10}, contains the labels 5 and 9. If unit 5 is added to unit 10, $p_1(5) + p_1(10) = 61$. If unit 9 is added to unit 10, $p_1(9) + p_1(10) = 133$. $p_1(9) + p_1(10)$ is greater than the upper limit on priority 1, 125. Unit 5 is added to unit 10 to create a new district.

After the first iteration, the candidate list and associated criteria are given by Table 2.

Unit 7 is selected as the next objective district. It is contiguous with units 2, 6 and 8. The addition of any one of these units to unit 7 will result

TABLE 2. CANDIDATE LIST AND CHARACTERISTICS AFTER FIRST ITERATION

Unit	Sales potential (in thousands)	Workload per unit	Travel time per unit	Contiguous set, C_k
7	36	25	15	2,6,8
6	39	28	19	7
2	47	23	32	3,7
8	49	28	37	3,7,9
4	57	41	23	3,9,10
3	61	49	39	2,4,8
10	61	43	31	4,9
9	107	62	82	4,8,10
District	Units in district			
1	1			
2	5,10			

TABLE 3. CANDIDATE LIST AND CHARACTERISTICS AFTER SECOND ITERATION

Unit	Sales potential (in thousands)	Workload per unit	Travel time per unit	Contiguous set, C_k
2	47	23	32	3,7
8	49	28	37	3,7,9
4	57	41	23	3,9,10
3	61	49	39	2,4,8
10	61	43	31	4,9
7	75	53	34	2,8
9	107	62	82	4,8,10
District	Units in district			
1	1			
2	5,10			
3	6,7			

in a new district which is below the minimum limit on priority 1. Unit 6, the unit with the smallest $p_1(s)$ value, is therefore selected for addition to unit 7.

Unit 2 is selected as the object district for the third iteration. It is contiguous to unit 3 and the new district 3, which has been given the label of 7. If unit 3 is added to unit 2, the new district would have a sales potential of £108,000. If unit 7 (District 3) is added to unit 2, the new district would have a sales potential of £122,000. The addition of either unit 3 or unit 7 will result in a new district within the acceptable range of variation. The subset CC_2 is defined to be $\{3,7\}$ and the second priority goal, work load, is checked.

The test of the second priority goal, also results in districts within the range of acceptable variation. The addition of unit 3 would result in a new district with a work load value of 72. The addition of unit 7 to unit 2 would result in a new district with a work load value of 76. The elements of CC_2 remain unchanged, and a lower goal is evaluated.

A check of the third priority indicates that neither district configuration would reach the lower limit on travel time per district. Since unit 7 has the lower value on this criteria, it is selected for addition to unit 2.

The heuristic process continues in this fashion. Ultimately no new districts can be formed without exceeding the upper limit on the variation of the target value. The final district structure resulting from the run of the heuristic is given below:

Primary objective—sales potential

District	Units in district	Sales potential	Workload	Travel time
1	1	128	74	85
2	4,5,10	118	84	54
3	2,6,7	122	76	66
4	3,8	110	77	76
5	9	107	62	82

If work load had been used as the primary objective, an altered structure would have been obtained:

Primary objective—workload

District	Units in district	Workload	Sales potential	Travel time
1	9,10	81	133	92
2	2,3	72	108	71
3	4,5	65	92	44
4	6,7	53	75	34
5	8	28	49	37
6	1	74	128	85

A review of the structure created with work load as the primary criteria indicates that one may wish to arbitrarily add districts 4 and 5 together. They are contiguous, but the new district would slightly exceed the call rate desired. This new district, however, would have an improved sales potential, as compared to district 4 and 5 individually. Such a decision would be left to the discretion of the sales manager.

Alternative formulations could be created by varying the target values, the width of the acceptable ranges and the priorities. This flexibility allows the decision-maker some latitude in selecting the final district structure. It may be desirable to utilize this heuristic approach in an iterative fashion. Specification would be stated and a particular district structure created by the heuristic. After a review of the structure, the specifications could be restated to account for any undesirable variation observed in the initial structure. Such an interfacing procedure may be particularly useful when districts are to be created to satisfy some public sector need requiring legislative or executive approval.

CONCLUSION

The general heuristic presented here has a wide range of applications to the problem of creating similar districts based on multiple criteria. The model requires only that a basic unit can be designated and the required additive data for each criterion can be obtained.

The relative efficiency of the procedure allows the user some latitude in experimenting with formulations. By altering the range widths and ordering of the parameters, the effects of various decision criteria on the district structure can be tested. It is expected that the user would make a series of runs, altering his or her criteria in an iterative fashion. Final selection of desired structure from the formulations generated is left to the ultimate decision-maker. In this way, the benefits of experience, intuition and the other intangibles of the decision process can be incorporated.

ACKNOWLEDGEMENT

The author is indebted to the anonymous reviewers for their helpful comments.

REFERENCES

[1] R. S. GARFINKEL and G. L. NEMHAUSER (1970) Optimal political districting by implicit enumeration techniques. *Mgmt Sci.* **16**, B495–B508.
[2] C. EASINGWOOD (1973) A heuristic approach to selecting sales regions and territories. *Opl Res. Q.* **24**, 527–534.
[3] L. M. LODISH (1975) Sales territory alignment to maximize profit. *JMR* **11**, 30–36.
[4] H. C. LUCAS, C. D. WEINBERG and K. W. CLOVES (1975) Sales response as a function of territorial potential and sales representative workload. *JMR* **12**, 298–305.