

1. Given the information from the question.

(a)

$$\theta_x = \frac{V}{Q} = 10d$$

For effluent concentration:

$$S = K \frac{1 + b\theta_x}{Y\hat{q}\theta_x - (1 + b\theta_x)} = 0.26 \frac{\text{mg BOD}}{L}$$

For biomass produced per day:

$$X_v = X_i^0 + \frac{Y(S^0 - S)(1 + (1 - f_d)b\theta_x)}{1 + b\theta_x} = 129.9 \frac{\text{mg BOD}}{L}$$

$$M_v = X_v \times Q = 1299 \frac{g}{day}$$

(b)

A dual CSTR then $\theta_{dual} = \frac{\theta_x}{2}$

For effluent concentration:

$$S = K \frac{1 + b\frac{\theta_x}{2}}{Y\hat{q}\frac{\theta_x}{2} - (1 + b\frac{\theta_x}{2})} = 0.36 \frac{\text{mg BOD}}{L}$$

For this dual CSTR treatment, retention time in one reactor decline then it should be considered if it reach the sufficient SRT. On the other hand, the substrate flowing into the second reactor, should be sufficient, allowing the higher removal efficiency.

For biomass produced per day:

$$X_v = X_i^0 + \frac{Y(S^0 - S)(1 + (1 - f_d)b\frac{\theta_x}{2})}{1 + b\frac{\theta_x}{2}} = 164.2 \frac{\text{mg BOD}}{L}$$

$$M_v = X_v \times Q = 1642 \frac{g}{day}$$

(c)

$$\theta_x = \frac{1}{b_{bet}} = 20d$$

For effluent concentration:

$$S = K \frac{1 + b\theta_x}{Y\hat{q}\theta_x - (1 + b\theta_x)} = 0.20 \frac{mg \text{ BOD}}{L}$$

As the result we can see, this system is capable of achieving the same level of treatment as the CSTR in section a.

The removal is 99% removal efficiency and the numerical values are substituted into Equation 4.42

$$S = S^0 - \frac{J_{ss}aV}{Q} = S^0 - \frac{J_{ss}A}{Q}$$

Where A is in m^2 and J_{ss} is in $\frac{mg}{cm^2 \cdot d}$

$$K^* = \frac{D}{L} \left[\frac{K_s}{\hat{q}X_f D_f} \right]^{0.5} = \frac{\frac{1cm^2}{d}}{0.008cm} \times \left(\frac{10mg \text{ BOD}/L}{20 \frac{mg}{mg VSS \cdot d} \times 25 \frac{mg VSS}{cm^3} \times 1 \frac{cm^2}{d}} \right)^{0.5} = 2.5$$

$$S_{min}^* = \frac{b'}{Y\hat{q} - b'} = \frac{0.15 \text{ } d^{-1}}{0.5 \times 20 - 0.15} = 0.015$$

Using the Figure 4.11

$$J^*/S_{min}^* = 2.8 \text{ when } S_{min}^* = 0.015 \text{ Then } J^* = 0.042$$

$$J_R = J^*(K\hat{q}X_f D_f)^{0.5} = 0.042 \times \left(\frac{10mg}{L} \times \frac{20 mg}{mg VSS \cdot d} \frac{25 mg VSS}{cm^3} \times \frac{1 cm^2}{d} \right)^{0.5} = 2.97$$

$$\text{Then } S_{min} = K \frac{b'}{Y\hat{q} - b'} = 0.15 \text{ mg/L and } \frac{S}{S_{min}} = \frac{5}{0.15} = 33.3$$

According to the Appendix B.1 $S_{min}^* = 0.015$ and $K^* = 2.5$

$$J_{ss}/J_R = 200 \text{ where } \frac{S}{S_{min}} = 33.3 \text{ then } J_{ss} = 594 \text{ mg/cm}^2 \cdot d$$

$$A = \frac{Q(S^0 - S)}{J_{ss}} = \frac{10000 \frac{L}{d} \times 595 \frac{mg}{L}}{33.3 \frac{mg}{cm^2 \cdot d} \times \frac{10000 cm^2}{1m^2}} = 17.87 \text{ m}^2$$

2. Given the information from the question.

$$Q = 1000 \text{ m}^3/d \text{ and } S^0 = 300 \text{ mg BOD/L}$$

$$X_v^0 = 0, Y = 0.5 \text{ g VSSa/g BOD}, \hat{q} = 20 \text{ g BOD/g VSSa} \cdot d \text{ and } b = 0.1d$$

$$\text{a. } [\theta_x^{min}]_{lim} = \frac{1}{Y\hat{q} - b} = 0.10 \text{ day}$$

$$\text{b. } \theta_x^d = [\theta_x^{min}]_{lim} \times SF = 4 \text{ days}$$

$$\text{c. } S = K \frac{1 + b\theta_x}{Y\hat{q}\theta_x - (1 + b\theta_x)} = 0.36 \text{ mg/L}$$

$$\text{d. } U = \frac{\hat{q}S}{K + S} = 0.69 \text{ kg/kg VSSa} \cdot d$$

$$\text{e. } \frac{\Delta S}{\Delta t} = -Q(S^0 - S) = 299.6 \text{ kg/day}$$

$$\text{f. } \frac{\Delta X_a}{\Delta t} = V \frac{dX_a}{dt} = -\frac{\Delta S}{\Delta t} \frac{Y}{1 + b\theta_x} = -Q(S^0 - S) \frac{Y}{1 + b\theta_x} = 107.0 \text{ kg VSSa/day}$$

$$\frac{\Delta X_i}{\Delta t} = V \frac{dX_i}{dt} = -\frac{\Delta X_a}{\Delta t} (1 - f_d) b\theta_x = 8.6 \text{ kg VSS}_i/\text{day}$$

$$\text{g. } X_v V = \theta_x \left(\frac{\Delta X_a}{\Delta t} + \frac{\Delta X_i}{\Delta t} \right) = 462.8 \text{ kg}$$

$$\text{h. } \frac{X_a}{X_v} = \frac{X_a V}{X_a V + X_i V} = \frac{X_a V}{X_v V} = 0.93$$

$$\text{i. } Q^w = \frac{\frac{\Delta X_v}{\Delta t} - QX_v^e}{X_v^R - X_v^e} = 10.6 \text{ m}^3/\text{day} , \text{ where } X_v^R = X_a^R + X_i^R \text{ and } X_v^e = X_a^e + X_i^e$$

$$\text{j. } R = \frac{Q^R}{Q} = \frac{X_v(1 - \theta/\theta_x)}{X_v^R - X_v} = 0.081$$

$$\text{k. } Q^R = Q \times R = 81.0 \text{ m}^3/\text{day}$$

$$\text{l. } V.L. = \frac{QS^0}{V} = 0.6 \text{ kg/m}^3 \cdot \text{day}$$

$$\text{m. } X_{a(BOD_L)}^e = X_a^e \cdot 1.42 \frac{mg \text{ BOD}_L}{mg \text{ VSS}} \cdot f_d = 10.6 \text{ mg BOD/L}$$

$$\text{n. } r_{ut} = -\frac{S^0 - S}{\theta_x} = 74.9 \text{ mg/L/day}$$

$$UAP = \frac{-(\hat{q}_{UAP}X_a\theta + K_{UAP} + k_1r_{ut}\theta) + \sqrt{(q_{UAP}X_a\theta + K_{UAP} + k_1r_{ut}\theta)^2 - 4K_{UAP}k_1r_{ut}\theta}}{2}$$

$$BAP = \frac{-(K_{BAP} + (\hat{q}_{BAP} - k_2)X_a\theta) + \sqrt{(K_{BAP} + (\hat{q}_{BAP} - k_2)X_a\theta)^2 + 4K_{BAP}k_2X_a\theta}}{2}$$

Where

$$k_1 = 0.12g \text{ COD}_p/g \text{ COD}_s \quad \hat{q}_{UAP} = 1.8g \text{ COD}_p/g \text{ VSS}_a$$

$$k_2 = 0.09g \text{ COD}_p/g \text{ COD}_s \quad K_{UAP} = 100mg \text{ COD}_p/g \text{ VSS}_a$$

$$\hat{q}_{BAP} = 0.1g \text{ COD}_p/g \text{ VSS}_a \quad K_{BAP} = 85mg \text{ COD}_p/g \text{ VSS}_a$$

$$\text{Soluble COD} = S + UAP + BAP = 353.2 \text{ mg/L}$$

$$r_{O_2} = Q \times COD = 163.8 \text{ kg/d}$$

$$\text{o. Efficiency per day} = 1 \text{ kg O}_2/\text{khW}\cdot\text{d}$$

$$E = \frac{\text{Efficiency per day}}{r_{O_2}} = 4.2 \text{ kWh}$$

$$p. \frac{\Delta N}{\Delta t} = \frac{\Delta X_v}{\Delta t} \cdot 0.124 \frac{kg \ N}{kg \ VSS} = 14.3 \frac{kg \ N}{day}$$

$$\frac{\Delta P}{\Delta t} = \frac{\Delta X_v}{\Delta t} \cdot 0.025 \frac{kg \ P}{kg \ VSS} = 2.9 \frac{kg \ P}{kg \ VSS}$$

q.

3.

Variable	Operating Characteristic			
	$\theta_x(d)$ (1)	Sludge Production (kg/d) (2)	Effluent Substrate Concentration S^e (3)	Oxygen Consumption (kg/d)
Y	-	+	-	+
\hat{q}	-	+	-	+
Q^0	-	?	?	+
S^0	No	+	No	+
b	+	-	+	?
K	+	-	+	-
X^e	No	+	No	?
X_i^0	No	+	No	?

$$(1) \theta_x = \frac{1}{Y \frac{\hat{q}S}{K+S} - b} \text{ or } \theta_x = \frac{V}{Q^0}$$

$$(2) X_v = X_i^0 + Y(S^0 - S)\theta_x + (1 - f_d)b\theta_x$$

$$(3) S = K \frac{1 + b\theta_x}{Y\hat{q}\theta_x - (1 + b\theta_x)}$$