1. Given the information from the question.

(a)

$$\theta_x = \frac{V}{Q} = 10d$$

For effluent concentration:

$$S = K\frac{1 + b\theta_x}{Y\hat{q}\theta_x - (1 + b\theta_x)} = 0.26\frac{mg\ BOD}{L}$$

For biomass produced per day:

$$X_v = X_i^0 + \frac{Y(S^0 - S)(1 + (1 - f_d)b\theta_x)}{1 + b\theta_x} = 129.9\ \frac{mg\ BOD}{L}$$

$$M_v = X_v \times Q = 1299\ \frac{g}{day}$$

(b)

A dual CSTR then $\theta_{dual} = \dfrac{\theta_x}{2}$

For effluent concentration:

$$S = K\frac{1 + b\frac{\theta_x}{2}}{Y\hat{q}\frac{\theta_x}{2} - (1 + b\frac{\theta_x}{2})} = 0.36\frac{mg\ BOD}{L}$$

For this dual CSTR treatment, retention time in one reactor decline then it should be considered if it reach the sufficient SRT. On the other hand, the substrate flowing into the second reactor, should be sufficient, allowing the higher removal efficiency.

For biomass produced per day:

$$X_v = X_i^0 + \frac{Y(S^0 - S)(1 + (1 - f_d)b\frac{\theta_x}{2})}{1 + b\frac{\theta_x}{2}} = 164.2\frac{mg\ BOD}{L}$$

$$M_v = X_v \times Q = 1642\ \frac{g}{day}$$

(c)

$$\theta_x = \frac{1}{b_{bet}} = 20d$$

For effluent concentration:

$$S = K\frac{1 + b\theta_x}{Y\hat{q}\theta_x - (1 + b\theta_x)} = 0.20\frac{mg\ BOD}{L}$$

As the result we can seen, this system is capable of achieving the same level of treatment as the CSTR in section a.

The removal is 99% removal efficiency and the numerical values are substituted into Equation 4.42

$$S = S^0 - \frac{J_{ss}aV}{Q} = S^0 - \frac{J_{ss}A}{Q}$$

Where A is in m^2 and J_{ss} is in $\frac{mg}{cm^2{\cdot}d}$

$$K* = \frac{D}{L}[\frac{K_s}{\hat{q}X_fD_f}]^{0.5} = \frac{\frac{1cm^2}{d}}{0.008cm} \times (\frac{10mg\ BOD/L}{20\frac{mg}{mg\ VSS\ d} \times 25\frac{mg\ VSS}{cm^3} \times 1\frac{cm^2}{d}})^{0.5} = 2.5$$

$$S^*_{min} = \frac{b'}{Y\hat{q} - b'} = \frac{0.15\ d^{-1}}{0.5 \times 20 - 0.15} = 0.015$$

Using the Figure 4.11

$J*/S^*_{min} = 2.8$ when $S^*_{min} = 0.015$ Then $J* = 0.042$

$$J_R = J*(K\hat{q}X_fD_f)^{0.5} = 0.042 \times (\frac{10mg}{L} \times \frac{20\ mg}{mg\ VSS{\cdot}d}\frac{25\ mg\ VSS}{cm^3} \times \frac{1\ cm^2}{d})^{0.5} = 2.97$$

Then $S_{min} = K\frac{b'}{Y\hat{q} - b'} = 0.15\ mg/L$ and $\frac{S}{S_{min}} = \frac{5}{0.15} = 33.3$

According to the Appendix B.1 S^*_{min} = 0.015 and K* = 2.5

$$J_{ss}/J_R = 200 \text{ where } \frac{S}{S_{min}} = 33.3 \text{ then } J_{ss} = 594\ mg/cm^2{\cdot}d$$

$$A = \frac{Q(S^0 - S)}{J_{ss}} = \frac{10000\frac{L}{d} \times 595\frac{mg}{L}}{33.3\frac{mg}{cm^2{\cdot}d} \times \frac{10000cm^2}{1m^2}} = 17.87\ m^2$$

2. Given the information from the question.

$Q = 1000m^3/d$ and $S^0 = 300\ mg\ BOD/L$

$X_v^0 = 0$, $Y = 0.5\ g\ VSSa/g\ BOD$, $\hat{q} = 20\ g\ BOD/g\ VSSa{\cdot}d$ and $b = 0.1d$

a. $[\theta_x^{min}]_{lim} = \dfrac{1}{Y\hat{q} - b} = 0.10\ day$

b. $\theta_x^d = [\theta_x^{min}]_{lim} \times SF = 4\ days$

c. $S = K\dfrac{1 + b\theta_x}{Y\hat{q}\theta_x - (1 + b\theta_x)} = 36.3\ mg/L$

d. $U = \dfrac{\hat{q}S}{K + S} = 15.68kg/kg\ VSSa{\cdot}d$

e. $\dfrac{\triangle S}{\triangle t} = -Q(S^0 - S) = 263.7\ kg/day$

f. $\dfrac{\triangle X_a}{\triangle t} = V\dfrac{dX_a}{dt} = -\dfrac{\triangle S}{\triangle t}\dfrac{Y}{1 + b\theta_x} = -Q(S^0 - S)\dfrac{Y}{1 + b\theta_x} = 94.2\ kg\ VSSa/day$

$\dfrac{\triangle X_i}{\triangle t} = V\dfrac{dX_i}{dt} = -\dfrac{\triangle X_a}{\triangle t}(1 - f_d)b\theta_x = 7.5\ kg\ VSS_i/day$

g. $X_vV = \theta_x(\dfrac{\triangle X_a}{\triangle t} + \dfrac{\triangle X_i}{\triangle t}) = 406.8\ kg$

h. $\dfrac{X_a}{X_v} = \dfrac{X_aV}{X_aV + X_iV} = \dfrac{X_aV}{X_vV} = 0.93$

i. $Q^w = \dfrac{\frac{\triangle X_v}{\triangle t} - QX_v^e}{Xv^R - X_v^e} = 9.18\ m^3/day$, where $X_v^R = X_a^R + X_i^R$ and $X_v^e = X_a^e + X_i^e$

j. $R = \dfrac{Q^R}{Q} = \dfrac{X_v(1-\theta/\theta_x)}{X_v^R - X_v} = 0.078$

k. $Q^R = Q \times R = 78.7 m^3/day$

l. $V.L. = \dfrac{QS^0}{V} = 0.6\ kg/m^3{\cdot}day$

m. $X^e_{a(BOD_L)} = X_a^e{\cdot}1.42\dfrac{mg\ BOD_L}{mg\ VSS}{\cdot}f_d = 212.6\ mg\ BOD/L$

n. $r_{ut} = -\dfrac{S^0 - S}{\theta_x} = 65.9\ mg/L{\cdot}day$

$$UAP = \frac{-(\hat{q}_{UAP}X_a\theta + K_{UAP} + k_1 r_{ut}\theta) + \sqrt{(\hat{q}_{UAP}X_a\theta + K_{UAP} + k_1 r_{ut}\theta)^2 - 4K_{UAPk}k_1 r_{ut}\theta}}{2}$$

$$BAP = \frac{-(K_{BAP} + (\hat{q}_{BAP} - k_2)X_a\theta) + \sqrt{(K_{BAP} + (\hat{q}_{BAP} - k_2)X_a\theta)^2 + 4K_{BAP}k_2X_a\theta}}{2}$$

Where

$k_1 = 0.12 g\ COD_p/g\ COD_s$ $\qquad$ $\hat{q}_{UAP} = 1.8 g\ COD_p/g\ VSS_a$

$k_2 = 0.09 g\ COD_p/g\ COD_s$ $\qquad$ $K_{UAP} = 100 mg\ COD_p/g\ VSS_a$

$\hat{q}_{BAP} = 0.1 g\ COD_p/g\ VSS_a$ $\qquad$ $K_{BAP} = 85 mg\ COD_p/g\ VSS_a$

$Soluble\ COD = S + UAP + BAP =$

$r_{O_2} = Q \times COD =$

o. $Efficiency\ per\ day = 1\ kg\ O_2/khW{\cdot}d$

$E = \dfrac{Efficiency\ per\ day}{r_{O_2}} =$

p. $\frac{\triangle N}{\triangle t} = \frac{\triangle X_v}{\triangle t} \cdot 0.124 \frac{kg\ N}{kg\ VSS} = 12.6\ \frac{kg\ N}{day}$

$$\frac{\triangle P}{\triangle t} = \frac{\triangle X_v}{\triangle t} \cdot 0.025 \frac{kg\ P}{kg\ VSS} = 2.54 \frac{kg\ P}{kg\ VSS}$$

q.

3.

	Operating Characteristic			
Variable	$\theta_x(d)$	Sludge Production (kg/d)	Effluent Substrate Concentration S^e	Oxygen Consumption (kg/d)
Y	-		-	
$\hat{q}$	-		-	
Q^0	-	?		
S^0	No	+	No	
b	+	-	+	
K		-	+	
X^e		+	No	
X_i^0		+	No	

(1) $\theta_x = \frac{1}{Y \frac{\hat{q}S}{K+S} - b}$ or $\theta_x = \frac{V}{Q^0}$

(2) $X_v = X_i^0 + Y(S^0 - S)\theta_x + (1 - f_d)b\theta_x$

(3) $S = K \frac{1 + b\theta_x}{Y\hat{q}\theta_x - (1 + b\theta_x)}$

(4)