

① 强行展开

② 下标法：求和约定 —— 同一单项式中具有相同下标的字母进行求和。

$$\text{eg. } \vec{A} = A_1 \vec{x}_1 + A_2 \vec{x}_2 + A_3 \vec{x}_3 = A_i \vec{x}_i \quad (i=1,2,3)$$

$$A_i \cdot B_{i,k} = \sum_{i=1}^3 A_i B_{i,k}$$

• 对于单线积的坐标系基矢 \vec{x}_i , $\vec{x}_i \cdot \vec{x}_j = \delta_{ij} = \begin{cases} 1, & i=j \\ 0, & i \neq j \end{cases}$

• Levi-Civita 符号 $\epsilon_{ijk} = \begin{cases} 1, & ijk \text{ 为偶排列} \\ -1, & ijk \text{ 为奇排列} \\ 0, & \text{其他, 例如 } i=j, j=k \end{cases} \quad (i,j,k=1,2,3)$

$\Rightarrow \epsilon_{ijk}$ 是反对称的, 即 $\epsilon_{ijk} = -\epsilon_{ikj} = \epsilon_{kij}$

$$\begin{aligned} \text{点乘 } \vec{A} \cdot \vec{B} &= (\underbrace{A_i \vec{x}_i}) \cdot (\underbrace{B_j \vec{x}_j}) \\ &= A_i B_j (\vec{x}_i \cdot \vec{x}_j) = A_i B_j \delta_{ij} = \underline{A_i B_i} \end{aligned}$$

$$\text{叉乘 } \vec{A} \times \vec{B} = \epsilon_{ijk} A_i B_j \vec{x}_k$$

$$\begin{aligned} \text{③ 符号法: } \nabla \cdot (\vec{A} + \vec{B}) &= \nabla_A \cdot (\vec{A} + \vec{B}) + \nabla_B \cdot (\vec{A} + \vec{B}) \\ &\quad \downarrow \text{仅作用于 } \vec{A} \\ &= \underline{\nabla_A \cdot \vec{A}} + \nabla_B \cdot \vec{B} = \underline{\nabla \cdot \vec{A}} + \nabla \cdot \vec{B} \end{aligned}$$

$$\text{证明: } d\varphi = d\vec{r} \cdot \nabla \varphi$$

$$\begin{aligned} \text{左: } d\varphi &= \partial_i \varphi dx_i \\ &= \partial_i \varphi dx_j \delta_{ij} \\ &= (\partial_i \varphi \vec{x}_i) \cdot (\partial_j dx_j \vec{x}_j) \end{aligned}$$

$$\cdot \nabla f = f' u \nabla u$$

$$\text{右: } \nabla f = \partial_i f \vec{x}_i$$

$$= \frac{\partial f}{\partial x_i} \vec{x}_i = \frac{\partial f}{\partial u} \frac{\partial u}{\partial x_i} \vec{x}_i = f' u \nabla u$$

$$\cdot \nabla \cdot (\varphi \vec{A}) = \varphi \nabla \cdot \vec{A} + \vec{A} \cdot \nabla \varphi$$

$$\text{右下标: } \nabla \cdot (\varphi \vec{A}) = \partial_i \vec{x}_i \cdot (\varphi A_{ij} \vec{x}_j)$$

$$\begin{aligned} &= \partial_i (\varphi A_{ij}) \vec{x}_j \\ &= \partial_i (\varphi A_i) \\ &= A_i \partial_i \varphi + \varphi \partial_i A_i \\ &= \vec{A} \cdot \nabla \varphi + \varphi \nabla \cdot \vec{A} \end{aligned}$$

$$\star \cdot \nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B})$$

下标: $\nabla \cdot (\vec{A} \times \vec{B}) = \partial_i x_i \cdot (\epsilon_{jkl} A_j B_k x_l)$

$$= \epsilon_{jkl} (\partial_i (A_j B_k)) (x_i \cdot x_l)$$

$$= \epsilon_{jkl} (\partial_i (A_j B_k)) \delta_{il}$$

$$= \epsilon_{jki} (\partial_i (A_j B_k))$$

$$= \underline{\epsilon_{jik} B_k \partial_i A_j} + \underline{\epsilon_{ijk} A_j \partial_i B_k}$$

$$= \underline{\partial_k \epsilon_{jik} A_j} \underline{\partial_i A_j} \underline{\partial_i B_k}$$

符号: $\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B})$

$$= \nabla_A \cdot (\vec{A} \times \vec{B}) + \nabla_B \cdot (\vec{A} \times \vec{B})$$

$$= \vec{B} \cdot (\nabla_A \times \vec{A}) - \vec{B} \cdot (\nabla_B \times \vec{A})$$

$$= \vec{B} \cdot (\nabla_A \times \vec{A}) - \vec{A} \cdot (\nabla_B \times \vec{B}) \quad (\text{混和积: } \vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{C} \cdot (\vec{A} \times \vec{B}))$$

$$= \vec{B} \cdot (\nabla_A \times \vec{A}) - \vec{A} \cdot (\nabla_B \times \vec{B}) = \vec{B} \cdot (\vec{C} \times \vec{A})$$

$$\begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix} = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$$

$$\star \nabla (\vec{A} \cdot \vec{B})$$

下标: $\nabla (\vec{A} \cdot \vec{B}) = \partial_i (A_i B_j) x_i$

$$= A_i \partial_i B_j x_i + B_j \partial_i A_i x_i$$

$$= (\vec{A} \cdot \nabla) \vec{B} + (\vec{B} \cdot \nabla) \vec{A} + \vec{A} \times (\nabla \times \vec{B}) + \vec{B} \times (\nabla \times \vec{A})$$

$$= A_i \partial_i B_j x_i + B_j \partial_i A_i x_i + \epsilon_{ijk} A_i (\epsilon_{mnj} \partial_m B_n) x_k$$

$$= A_i \partial_i B_j x_i + B_j \partial_i A_i x_i + (A_i \partial_j B_i - A_i \partial_j B_i) x_i + (B_i \partial_j A_i - B_i \partial_j A_i) x_j$$

注: $\epsilon_{ijk} A_i (\epsilon_{mnj} \partial_m B_n) x_k$

$$= -\epsilon_{ijk} \epsilon_{mnj} A_i \partial_m B_n x_k$$

$$= -(\sum_{l \neq m} f_{kn} - \sum_{l \neq m} f_{km}) A_i \partial_m B_n x_k$$

$$= -A_i \partial_j B_k x_k + A_i \partial_k B_j x_k$$

符号: $\nabla (\vec{A} \cdot \vec{B}) = \nabla_A \vec{A} \cdot \vec{B} + \nabla_B \vec{B} \cdot \vec{A} \quad (\text{将 } k \text{ 换为 } j, \text{ 不影响求和})$

$$= (\nabla_A \vec{A} - \vec{A} \nabla_A) \cdot \vec{B} + \vec{A} \cdot \nabla_A \cdot \vec{B} + (\nabla_B \vec{B} - \vec{B} \nabla_B) \cdot \vec{A} + \vec{B} \cdot \nabla_B \cdot \vec{A}$$

注: $\vec{A} \times (\vec{C} \times \vec{B}) = \vec{B} \cdot (\vec{A} \vec{C} - \vec{C} \vec{A}) = (\vec{C} \vec{A} - \vec{A} \vec{C}) \cdot \vec{B}$

∇_A 因只作用于 A 上, 此时可完全当成一个普通矢量

其他公式方法类似, 不再详解, 有问题可向任一助教 (不要求掌握, 仅是即可)

Remark: 强制微分中 $\nabla \cdot (\nabla \times \vec{A}) = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = 0$ (这题最好加一句二阶偏导连续)

$$\vec{A} \times (\vec{A} \times \vec{A}) = \frac{1}{2} \nabla A^2 - (\vec{A} \cdot \nabla) \vec{A}$$

简单证法: 由 $\nabla(\vec{A} \cdot \vec{B}) = \vec{B} \nabla \cdot \vec{A} - \vec{A} \nabla \cdot \vec{B}$
 $\vec{r} = (x, y, z) \Rightarrow d\vec{r} = (dx, dy, dz)$, 不是 $\vec{r}(x, y, z)$ 的数
 及其未知

“要书写规范, 表点出来! $(\vec{A} \cdot \nabla) \vec{B} \neq \nabla \cdot \vec{A}$
 X 权量与矢量不能点乘

又作用于后面!!! 错误示例 $(\vec{A} \cdot \nabla) \vec{B} = \frac{\partial \vec{A}}{\partial x} \dots$

$$\vec{B}(\vec{A} \cdot \nabla) \neq (\vec{A} \cdot \nabla) \vec{B}$$

$\vec{A} \times (\vec{B} \times \vec{C}) = \dots$ 不可直接令 $\vec{A} = \nabla$, 公式无法保证什么地方角了交换律,
 只有另如 ∇A , 使其 ∇ 作用于 A 时可直接把 ∇ 当矢量, 见得书

* 1.2 $F = k q_1 q_2 / r^2$

$$\text{令 } q_1 = q_2 = C, r = 1m$$

$$\text{则 } F \approx 8.9876 \times 10^9 \text{ kg} \cdot \text{m/s}^2$$

$$= 8.9876 \times 10^{14} \text{ g} \cdot \text{cm/s}^2$$

$$1C \Leftrightarrow x \text{ esu}$$

$$\text{则 } F = q_1 q_2 / r^2$$

$$\Rightarrow x = \sqrt{8.9876 \times 10^{14} \times 100}$$

$$\approx 2.9979 \times 10^9.$$

$$1C \Leftrightarrow 2.9979 \times 10^9 \text{ esu}$$

$$1 \text{ esu} \Leftrightarrow 3.3356 \times 10^{-10} C$$

$$1e \approx 4.774 \times 10^{-10} \text{ esu}$$

$$\approx 1.5924 \times 10^{-9} C$$

注意: esu 与 C 只是对应关系, 并非 km 与 m 之间有直接的转换关系 (类比于温度单位 K, °C)

$$\text{正确的转换关系应为 } \frac{1C}{\sqrt{4\pi\epsilon_0}} \approx 2.9979 \times 10^9 \text{ esu}$$

1.3 (1) $\frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2} = \frac{G m_1 m_2}{r^2}$

$$\Rightarrow Q_1 Q_2 = 4\pi\epsilon_0 G m_1 m_2$$

$$Q = Q_1 + Q_2 \geq 2\sqrt{Q_1 Q_2} \approx 1.14 \times 10^{14} C$$

(2) $\left\{ \begin{array}{l} Q_1 Q_2 = 4\pi\epsilon_0 G m_1 m_2 \\ \frac{Q_1}{m_1} = \frac{Q_2}{m_2} \end{array} \right.$

$$\Rightarrow \left\{ \begin{array}{l} Q_1 = \sqrt{4\pi\epsilon_0 G} m_1 \\ Q_2 = \sqrt{4\pi\epsilon_0 G} m_2 \end{array} \right. \therefore Q = Q_1 + Q_2 \approx 5.21 \times 10^{14} C$$

$$1.6 \text{ ① 距圆心 } z \text{ 处 } \vec{E} = \frac{Qz}{4\pi\epsilon_0(R^2+z^2)^{\frac{3}{2}}} \hat{z}$$

②  对称等效

$$d\vec{F} = \vec{E} dq, dq = \lambda dz$$

$$\vec{F} = \int d\vec{F} = \int_0^{+R} \frac{Q\lambda}{4\pi\epsilon_0} \frac{z}{(R^2+z^2)^{\frac{3}{2}}} dz = \frac{Q\lambda}{4\pi\epsilon_0} \left(-\frac{1}{\sqrt{R^2+z^2}}\right) \Big|_0^{+R} = \frac{Qx}{4\pi\epsilon_0 R^3} \hat{x}$$

* 1.7 距圆心 x_0 处 $\vec{F}_z = -\frac{Qq x_0}{4\pi\epsilon_0 (R^2+x_0^2)^{\frac{3}{2}}} \hat{z}$

$$= -\frac{Qq x_0 R^{-3}}{4\pi\epsilon_0 R} \left(1 + \frac{x_0^2}{R^2}\right)^{-\frac{3}{2}} \hat{z} \quad \text{真正的力量}$$

$$\approx -\frac{Qq x_0}{4\pi\epsilon_0 R^3} \hat{z} \quad \left(\frac{d}{dx_0} \left[\frac{x_0}{R} \left(1 + \frac{x_0^2}{R^2}\right)^{-\frac{3}{2}}\right]\right) \Big|_{x_0=0} = \frac{1}{R} \cdot \frac{x_0}{R} \left(1 + \frac{x_0^2}{R^2}\right)^{-\frac{3}{2}} \approx \frac{x_0}{R}$$

$$\therefore m\omega^2 = \frac{Qq}{4\pi\epsilon_0 R^3}, \quad \omega = \sqrt{\frac{Qq}{4\pi\epsilon_0 m R^3}}, \quad x = x_0 \cos \sqrt{\frac{Qq}{4\pi\epsilon_0 m R^3}} t$$

Remark: 近似计算时将分子同时作为一个整体展开后保留带有小量的第一项

此题可以仅近似分母是因分子仅为 x_0 的影响!!!

若 1.14, 1.15 题分别近似分子母易得到错误答案!

② 不要抄答案, 自己算, 不少同学抄答案写成了 $x = x_0 \cos \sqrt{\frac{Qq_0}{4\pi\epsilon_0 m R^3}} t$

1.8 (忽略条件: 质心系中径向初始速度为 0!)

$$\text{① } F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}, \quad \mu = \frac{m_1 m_2}{m_1 + m_2}, \quad a = \frac{F}{\mu} \quad \text{② } t =$$

$$a = \frac{dv}{dt} = \frac{dv}{dr} \frac{dr}{dt} = \frac{dv}{dr} v$$

$$\Rightarrow v = \sqrt{\frac{q_1 q_2 (r_0 - r)}{2\pi\epsilon_0 \mu r_0 r}} = \frac{dr}{dt}$$

$$\Rightarrow t = \sqrt{\frac{\pi^3 \epsilon_0 \mu r_0^3}{-2 q_1 q_2}}$$

Remark: *

* 1.10 $\epsilon_0 \cdot E \cdot 2\pi rl = l \int_0^r (ar - br^3) \cdot 2\pi r dr, \quad 0 < r < R$

$$\Rightarrow \vec{E} = \left(\frac{ar^2}{3\epsilon_0} - \frac{br^4}{5\epsilon_0} \right) \hat{r} \quad *$$

$$\epsilon_0 \cdot E \cdot 2\pi rl = l \int_0^K (ar - br^3) \cdot 2\pi r dr, \quad r > R$$

$$\Rightarrow \vec{E} = \left(\frac{ar^3}{3\epsilon_0 r} - \frac{br^5}{5\epsilon_0 r} \right) \hat{r}, \quad r > R$$

Remark: ① 柱坐标 $dr \, r d\phi \, dz$

球坐标 $dr \, r \sin\theta d\phi \, r d\theta$

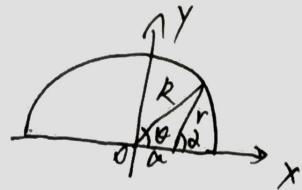
② 场强为矢量, 有方向 **

*1.11

① 确算 $E_{||} = 0 + E_L \neq 0$

$$dE_L = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} \cos\alpha$$

$$dq = \lambda_0 \sin\theta \frac{R}{r} d\phi \quad \cos\alpha = \frac{R^2 - a^2 - r^2}{2ra}$$



② 计算电势 $\psi = \frac{\lambda_0}{2\pi\epsilon_0}$

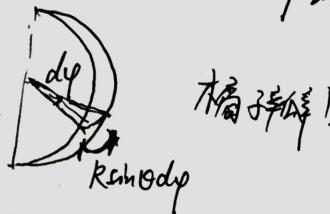
③ 等效性: $E_{||} = \int_0^\pi \frac{\lambda_0 \sin\theta R d\phi}{4\pi\epsilon_0 r^2} \cos\alpha = ?$

考虑均匀分布的带电球壳: 已知 $\int_0^{2\pi} \int_0^\pi \frac{\sigma \cdot R \sin\theta d\phi \cdot R d\psi}{4\pi\epsilon_0 r^2} \cos\alpha$

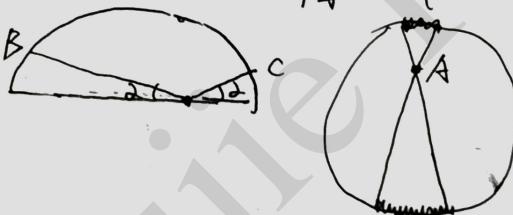
$$= 2\pi \int_0^\pi \frac{\sigma R^2 \sin\theta d\phi}{4\pi\epsilon_0 r^2} \cos\alpha = 0 \quad (\text{Gauss 定理对称性})$$

∴ $E_{||} = 0$

从物理上理解, 相当于将球壳沿轴向无限长线切开, 相同 $d\psi$ 下, 两壳切成的环的电荷线密度与弧长成正比, 即与 $\theta \sin\theta$ 的关系



④ 由球壳对称面元在A点, 场强抵消, 联想到圆弧上B, C两点, 面元在直线上场强抵消, 可自己尝试, 但要注意, A极点不在圆心, 并非简单的“距离=2R/2倍*距离”



*1.12 ① 确算

② 等效性



$$\frac{1}{4\pi\epsilon_0} \frac{d(2R \tan\theta)}{(R \cos\theta)^2} = \frac{1}{4\pi\epsilon_0} \frac{\lambda d\phi}{R}$$

仅在 O 点有上述等式!
且对等效面元积分时积分范围相同!

$$1.13 \text{ 无限长直细导线 } \vec{E} = \frac{\lambda}{2\pi R \epsilon_0} \hat{z}, \quad \lambda = \sigma R d\theta$$

$$\begin{aligned}\therefore \vec{E} &= \int_0^\pi \frac{\lambda}{2\pi R \epsilon_0} \sin\theta \hat{z} \\ &= \int_0^\pi \frac{\sigma R \sin\theta}{2\pi R \epsilon_0} d\theta = \frac{\sigma}{\pi \epsilon_0} \hat{z}\end{aligned}$$

注意: ① 正负未知, 方向不可说向上或向下
 ② 不可认为半球面, 因为面电荷密度是均匀分布的, 未解麻烦

$$1.14 \quad \vec{E} = \frac{1}{4\pi \epsilon_0} \left(\frac{-q}{r^2} + \frac{q}{(r-l)^2} + \frac{q}{(r+l)^2} \right) \hat{r}$$

$$\therefore (1+x)^{-2} \approx 1 - 2x + 3x^2 + \dots$$

$$\text{而 } (r-l)^{-2} = r^{-2} \left(1 - \frac{l}{r} \right)^{-2} \approx$$

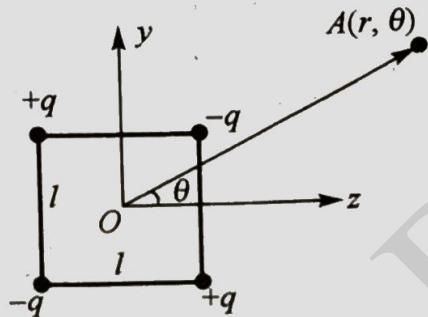
$$= r^{-2} \left(1 + 2 \frac{l}{r} + \frac{2l^2}{r^2} \right)$$

$$\therefore \vec{E} \approx \frac{1}{4\pi \epsilon_0} \cdot 6q \frac{l^2}{r^4} \hat{r} \quad \text{保留到带有} \frac{l}{r} \text{的第一项}$$

② 先求电势再求 \vec{E}

~~待续~~

*1.18 面电四极子如习题 1.18 图所示, 点 $A(r, \theta)$ 与四极子共面, 极轴 ($\theta = 0$) 通过正方形中心并与两边平行. 设 $r \gg l$, 求面电四极子在点 A 处产生的电势和电场强度.



习题 1.18 图

解 取无限远处为电势零点, 电荷到 O 点的距离为 $a = l/\sqrt{2}$, 则

$$\begin{aligned}
 U &= \frac{q}{4\pi\epsilon_0} \left[\frac{-1}{\sqrt{r^2 + a^2 - 2ra \cos(0.25\pi - \theta)}} + \frac{1}{\sqrt{r^2 + a^2 - 2ra \cos(0.25\pi + \theta)}} \right. \\
 &\quad \left. + \frac{1}{\sqrt{r^2 + a^2 - 2ra \cos(0.75\pi - \theta)}} + \frac{-1}{\sqrt{r^2 + a^2 - 2ra \cos(0.75\pi + \theta)}} \right] \\
 &= \frac{q}{4\pi\epsilon_0 r} \left\{ - \left[1 + \left(\frac{a}{r} \right)^2 - \frac{2a}{r} \cos\left(\frac{\pi}{4} - \theta\right) \right]^{-1/2} + \left[1 + \left(\frac{a}{r} \right)^2 - \frac{2a}{r} \cos\left(\frac{\pi}{4} + \theta\right) \right]^{-1/2} \right. \\
 &\quad \left. + \left[1 + \left(\frac{a}{r} \right)^2 + \frac{2a}{r} \cos\left(\frac{\pi}{4} + \theta\right) \right]^{-1/2} - \left[1 + \left(\frac{a}{r} \right)^2 + \frac{2a}{r} \cos\left(\frac{\pi}{4} - \theta\right) \right]^{-1/2} \right\}
 \end{aligned}$$

推导中用到如下恒等式：

$$\cos\left(\frac{3\pi}{4} \pm \theta\right) = -\cos\left(\frac{\pi}{4} \mp \theta\right)$$

对远处 ($r \gg l$), 利用泰勒展开式

$$\frac{1}{\sqrt{1+x}} = 1 - \frac{1}{2}x + \frac{1 \cdot 3}{2 \cdot 4}x^2 + \dots = 1 - \frac{1}{2}x + \frac{3}{8}x^2 + \dots$$

仅保留至二级小量, 则上述电势表达式化为

$$\begin{aligned} U &= \frac{q}{4\pi\epsilon_0 r} \left[-1 + \frac{a^2}{2r^2} - \frac{a}{r} \cos\left(\frac{\pi}{4} - \theta\right) - \frac{3a^2}{2r^2} \cos^2\left(\frac{\pi}{4} - \theta\right) \right. \\ &\quad + 1 - \frac{a^2}{2r^2} + \frac{a}{r} \cos\left(\frac{\pi}{4} + \theta\right) + \frac{3a^2}{2r^2} \cos^2\left(\frac{\pi}{4} + \theta\right) \\ &\quad + 1 - \frac{a^2}{2r^2} - \frac{a}{r} \cos\left(\frac{\pi}{4} + \theta\right) + \frac{3a^2}{2r^2} \cos^2\left(\frac{\pi}{4} + \theta\right) \\ &\quad \left. - 1 + \frac{a^2}{2r^2} + \frac{a}{r} \cos\left(\frac{\pi}{4} - \theta\right) - \frac{3a^2}{2r^2} \cos^2\left(\frac{\pi}{4} - \theta\right) \right] \\ &= \frac{3qa^2}{4\pi\epsilon_0 r^3} \left[\cos^2\left(\frac{\pi}{4} + \theta\right) - \cos^2\left(\frac{\pi}{4} - \theta\right) \right] = \frac{3ql^2}{4\pi\epsilon_0 r^3} \sin\theta \cos\theta \end{aligned}$$

下面从所得电势表达式出发计算电场 (限于面电四极子所在平面)

$$\begin{aligned} \mathbf{E} &= -\nabla U = -\frac{\partial U}{\partial r} \mathbf{e}_r - \frac{1}{r} \frac{\partial U}{\partial \theta} \mathbf{e}_\theta \\ &= -\frac{9ql^2 \sin\theta \cos\theta}{4\pi\epsilon_0 r^4} \mathbf{e}_r + \frac{3ql^2 \cos 2\theta}{4\pi\epsilon_0 r^4} \mathbf{e}_\theta \end{aligned}$$

1.2 | 取高斯球面: $\epsilon_0 E(r) \cdot 4\pi r^2 = \int_0^r \frac{p_0 e^{-kr}}{r} 4\pi r^2 dr$

$$= p_0 \int_0^r r e^{-kr} dr = -\frac{p_0}{k} (r e^{-kr} \Big|_0^r - \int_0^r e^{-kr} dr)$$

$$= -\frac{p_0}{k} r e^{-kr} - \frac{p_0}{k} (e^{-kr} - 1)$$

故 $E(r) = \frac{p_0}{\epsilon_0 r^2} \left(\frac{1}{k^2} - \frac{1}{k} e^{-kr} - \frac{1}{k} r e^{-kr} \right)$

$$= \frac{p_0}{\epsilon_0 r^2 k^2} [1 - e^{-kr} (1 + kr)]$$

p_0 为密度

注意:

1) $\vec{E} = -\nabla \varphi$

2) $\varphi = \int \vec{E} \cdot d\vec{l}$ 注意
积分上下限

若无穷远处 $\varphi = 0$

1.29 . $\begin{cases} E_x = -\frac{\partial \varphi}{\partial x} = +\frac{A}{r} (x^2 + y^2 + a^2)^{-\frac{3}{2}} \cdot 2x \\ E_y = Ay (x^2 + y^2 + a^2)^{-\frac{3}{2}} \\ E_z = 0 \end{cases} \Rightarrow |\vec{E}| = A \sqrt{\frac{x^2 + y^2}{(x^2 + y^2 + a^2)^3}}$

1.30 $\hookrightarrow \varphi(r < ra) = \int_r^{ra} E dr = \frac{ze}{4\pi\epsilon_0} \left(-\frac{1}{r} - \frac{r^2}{2ra^3} \right) \Big|_{r=\frac{ra}{2}}^{\frac{ra}{2}} = \frac{ze}{4\pi\epsilon_0} \left(\frac{1}{ra} - \frac{r}{ra^2} \right)$

$\hookrightarrow \varphi(r > ra) = \frac{ze}{4\pi\epsilon_0 r} + \int_r^{ra} \frac{p_0 r^2 ds}{4\pi\epsilon_0 s^3} + \int_r^{ra} \frac{p_0 r^2 ds}{4\pi\epsilon_0 s^3} = \frac{ze}{4\pi\epsilon_0 r} - \frac{ze}{4\pi\epsilon_0 r^2} \frac{1}{3} - \frac{ze}{4\pi\epsilon_0} \left(\frac{1}{2} r_a^2 - \frac{1}{2} r^2 \right)$

1.31 $- \frac{p_0 \frac{4\pi r^2 a}{3\epsilon_0 r^2} q}{4\pi\epsilon_0 r^2} = m\ddot{r} \Rightarrow \ddot{r} = -\frac{pq}{3\epsilon_0 m} r$ 简谐. $\hookrightarrow E(r < ra) = -\frac{\partial \varphi}{\partial r} \hat{r} = \dots$

$\vec{E} = -\frac{\partial \varphi}{\partial r} \hat{r} = -\frac{ze}{4\pi\epsilon_0} \left(\frac{1}{ra} + \frac{r}{ra^2} \right) \hat{r}$

1.32 $\sigma = p d\lambda \Rightarrow \int_{-\infty}^x -ds \times \delta dy \hookrightarrow \vec{E} = \int_{-\infty}^x \frac{p dx}{2\epsilon_0} - \int_x^{\infty} \frac{p dx}{2\epsilon_0} = \frac{p}{2\epsilon_0} [(x + \frac{d}{2}) - (\frac{d}{2} - x)] = \frac{p}{\epsilon_0} x$

$x > \frac{d}{2} \Rightarrow E = \frac{p}{2\epsilon_0} \hat{d}$

$x < -\frac{d}{2} \Rightarrow E = -\frac{p}{2\epsilon_0} \hat{d}$

补偿原理

$$\Psi_p' = \int_0^r \frac{\sigma \cdot 2\pi r dr}{4\pi\epsilon_0 \sqrt{r^2 + x^2}} = \frac{\sigma}{4\epsilon_0} \int_0^r \frac{dr}{\sqrt{r^2 + x^2}}$$

$$= \frac{\sigma}{2\epsilon_0} (\sqrt{r^2 + x^2} - x)$$

$$\hookrightarrow E_p' = -\frac{\partial \Psi_p'}{\partial x} = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{x}{\sqrt{r^2 + x^2}} \right)$$

$$\hookrightarrow E_p = E_p - E_p' = \frac{\sigma}{2\epsilon_0} \frac{x}{\sqrt{r^2 + x^2}}$$

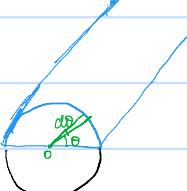
以 0 为电势零点 $\Rightarrow \Psi_p = - \int_0^x E_p d\lambda = \frac{-\sigma}{2\epsilon_0} \int_0^x \frac{x}{\sqrt{r^2 + x^2}} d\lambda$

$$= -\frac{\sigma}{4\epsilon_0} 2\sqrt{r^2 + x^2} \Big|_0^x = \frac{\sigma}{2\epsilon_0} (r - \sqrt{r^2 + x^2})$$

对称性

为什么 $\vec{F} = \int \langle \vec{E} \rangle dq$

* 1.35



内部: $E(r < R) = 0$

$$\text{外: } \sigma \cdot 2\pi R^2 h = \epsilon_0 E(r) \cdot 2\pi R^2 h$$

$$\hookrightarrow E(r > R) = \frac{\sigma}{\epsilon_0} \frac{R}{r}$$

及 单位长之力 $f_z = \int_0^{\pi/2} \frac{\pi}{2\epsilon_0} \cdot \sigma R d\theta \cdot \sin\theta = \frac{\sigma^2 R}{\epsilon_0}$

$\vec{E}_{\text{总}} = \vec{E}_{\text{外}} + \vec{E}_{\text{内}}$

$\vec{E}_{\text{总}} = \vec{E}_{\text{外}} - \vec{E}_{\text{内}}$



$$\therefore \vec{E}_{\text{外}} = \frac{1}{2} (\vec{E}_{\text{总}} + \vec{E}_{\text{总}})$$

$$= \frac{1}{2} \langle \vec{E} \rangle$$

1.36 (1) $\vec{E}(r < R_1) = 0$; (2) $\vec{E}(r < R_1) = \vec{E}(r > R_2) = 0$.

(2) $\vec{E}(R_1 < r < R_2) = \frac{\lambda_1}{2\pi\epsilon_0 r}$; (3) $\vec{E}(R_1 < r < R_2) = \frac{\lambda_1}{2\pi\epsilon_0 r}$

(3) $\vec{E}(r > R_2) = \frac{\lambda_1 + \lambda_2}{2\pi\epsilon_0 r}$)

有通量的为例面

关于唯一性定理：

真真

$$\left. \begin{array}{l} \nabla \cdot \vec{E} = \rho / \epsilon_0 \\ \nabla \cdot \vec{B} = 0 \end{array} \right\} \nabla \times \vec{E} = -\partial_t \vec{B}$$

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \partial_t \vec{E}$$

+足解条件

从数学上理解：给足了一组足解条件，即有唯一的解。eg. ~~通过~~ 边界的电势
 \Rightarrow 保证了解的可行性，设置像电荷，其电场一定满足 Maxwell 方程，而若能满足足解条件，即满足了方程组的所有数学式，即为唯一的正确的解。

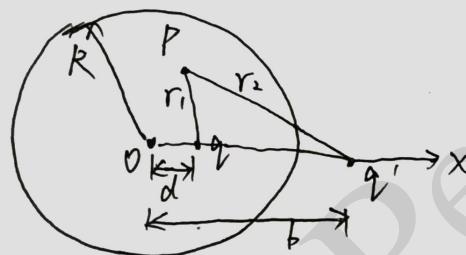
电像性：唯一性原理的应用

eg. 半径为 R 的导体球壳，内部距球心 a 处有点电荷 q ，求

(1) 球壳接地时球内 E 与 ψ

(2) --- 不 --- 带电量为 Q 时球内 E 与 ψ

解(1)



$$\left. \begin{array}{l} \partial b = R^2 \\ q/d = -Rq \end{array} \right\}$$

根据题意有时

直接使用

* 别忘了！
但最好别
直接搬出来

$$\Rightarrow E = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r^2 + d^2 - 2rd\cos\theta} \hat{e}_r \right) = \frac{R/d}{r^2 + R^2/d^2 - 2r(R^2/d)\cos\theta} \hat{e}_r$$

$$\psi = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{\sqrt{r^2 + d^2 - 2rd\cos\theta}} - \frac{R}{\sqrt{r^2 + R^2/d^2 - 2rd(R^2/d)\cos\theta}} \right)$$

(1) 中 计算球壳带电量：求出后根据边值关系算出 ψ 后积分可得 Q
 ψ 与 q 或 q' 并无任何直接联系

(此题可用 Gauss 定理)

\Rightarrow 带电量为 $-q$ ，但若将电荷在球外移为 q' ，则带电量则为 $-\frac{R}{d'} q'$ (见老师 PPT)

2. 设球壳外表面上均匀分布虚拟面电荷 $(Q' = Q + q)$ (满足总电量为 Q 和等势条件)
 $(猜!!!)$

$$\therefore \psi_1 = \psi + \frac{Q+q}{4\pi\epsilon_0 R}$$

注意：像电荷不可以设在求解区域内，否则不满足电荷解，即 $\nabla \cdot \vec{E} = \rho / \epsilon_0$ 中 ρ 改变，即改变了方程