2022.4.2

1.1 电荷守恒

$\frac{\partial\rho}{\partial t}+\nabla\cdot\vec{j}=0$

1.2 Coulomb 定律

$\vec{E}=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\vec{e}_r$, 电场强度平方反比.

电磁场的 7 个层面: 1.物理规律; 2.原像; 3.介质; 4.位形; 5.边界; 6.频率; 7.参考系.

1.3 叠加原理

$\vec{E}=\sum_i\vec{E}_i$.

1.4 Gauss 定理与环路定理

数学上, $\oiint_\Sigma\vec{E}\cdot\vec{e}_n\,\mathrm{d}S=\iiint\nabla\cdot\vec{E}\,\mathrm{d}V$, 结合 Maxwell 方程组的第一项$\nabla\cdot\vec{E}=\frac{\rho_e}{\varepsilon_0}$, 得到

$$\oiint_\Sigma\vec{E}\cdot\vec{e}_n\,\mathrm{d}S=\frac{1}{\varepsilon_0}\iiint\rho_e\,\mathrm{d}V.$$

静电场环路定理$\oint_L\vec{E}\cdot d\vec{l}=0$.

1.5 电势

静电场环路定理可以引入相差常数的电势φ, 对点电荷有$\varphi(\vec{r})=\frac{1}{4\pi\varepsilon_0}\frac{q}{r}$. $\varphi(\vec{r})$满足叠加原理.

有$\vec{E}(\vec{r})=-\nabla\varphi(\vec{r})$.

1.5.1 矢量分析

Lamé 系数$h_i=\left|\frac{\partial\vec{r}}{\partial q_i}\right|$,

$\nabla\psi=\frac{1}{h_1}\frac{\partial\psi}{\partial q_1}\vec{e}_1+\frac{1}{h_2}\frac{\partial\psi}{\partial q_2}\vec{e}_2+\frac{1}{h_1}\frac{\partial\psi}{\partial q_1}\vec{e}_3$,

$\nabla\cdot\vec{f}=\frac{1}{h_1h_2h_3}\left(\frac{\partial}{\partial q_1}(h_2h_3f_1)+\frac{\partial}{\partial q_2}(h_3h_1f_2)+\frac{\partial}{\partial q_3}(h_1h_2f_3)\right)$,

$\nabla\times\vec{f}=\frac{1}{h_1h_2h_3}\begin{vmatrix}h_1\vec{e}_1 & \frac{\partial}{\partial q_1} & h_1f_1\\ h_2\vec{e}_2 & \frac{\partial}{\partial q_2} & h_2f_2\\ h_3\vec{e}_3 & \frac{\partial}{\partial q_3} & h_3f_3\end{vmatrix}$,

$$\nabla^2\psi=\frac{1}{h_1h_2h_3}\left(\frac{\partial}{\partial q_1}\left(\frac{h_2h_3}{h_1}\frac{\partial\psi}{\partial q_1}\right)+\frac{\partial}{\partial q_2}\left(\frac{h_3h_1}{h_2}\frac{\partial\psi}{\partial q_2}\right)+\frac{\partial}{\partial q_3}\left(\frac{h_1h_2}{h_3}\frac{\partial\psi}{\partial q_1}\right)\right).$$

(i)柱坐标系

$$q_1=r,q_2=\phi,q_3=z,$$

$$h_1=1,h_2=r,h_3=1,$$

$$\nabla\psi=\frac{\partial\psi}{\partial r}\vec{e}_r+\frac{1}{r}\frac{\partial\psi}{\partial\phi}\vec{e}_\phi+\frac{\partial\psi}{\partial z}\vec{e}_z,$$

$$\nabla\cdot\vec{f}=\frac{1}{r}\frac{\partial}{\partial r}(rf_r)+\frac{1}{r}\frac{\partial f_\phi}{\partial\phi}+\frac{\partial f_z}{\partial z},$$

$$\nabla\times\vec{f}=\frac{1}{r}\begin{vmatrix}\vec{e}_r & \frac{\partial}{\partial r} & f_r\\ r\vec{e}_\phi & \frac{\partial}{\partial\phi} & rf_\phi\\ \vec{e}_z & \frac{\partial}{\partial z} & f_z\end{vmatrix},$$

$$\nabla^2\psi=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right)+\frac{1}{r}\frac{\partial^2\psi}{\partial\phi^2}+\frac{\partial^2\psi}{\partial z^2}.$$

(ii)球坐标系

$$q_1=r,q_2=\theta,q_3=\phi,$$

$$h_1=1,h_2=r,h_3=r\sin\theta,$$

$$\nabla\psi=\frac{\partial\psi}{\partial r}\vec{e}_r+\frac{1}{r}\frac{\partial\psi}{\partial\theta}\vec{e}_\theta+\frac{1}{r\sin\theta}\frac{\partial\psi}{\partial\phi}\vec{e}_\phi,$$

$$\nabla\cdot\vec{f}=\frac{1}{r^2}\frac{\partial}{\partial r}(r^2f_r)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(f_\theta\sin\theta)+\frac{1}{r\sin\theta}\frac{\partial f_\phi}{\partial\phi},$$

$$\nabla\times\vec{f}=\frac{1}{r^2\sin\theta}\begin{vmatrix}\vec{e}_r & \frac{\partial}{\partial r} & f_r\\ r\vec{e}_\theta & \frac{\partial}{\partial\theta} & rf_\theta\\ r\sin\theta\,\vec{e}_\phi & \frac{\partial}{\partial\phi} & r\sin\theta\, f_\phi\end{vmatrix},$$

$$\nabla^2\psi=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}.$$

Poisson 方程$\nabla^2\varphi=-\frac{\rho_e}{\varepsilon_0}$, Laplace 方程$\nabla^2\varphi=0$.

Earnshaw 定理: 点粒子集不能被稳定维持在仅由电荷的静电相互作用构成的一个稳定静止的力学平衡结构.

1.6 电偶极子

真空中$\varphi(\vec{r})=\frac{1}{4\pi\varepsilon_0}\iiint_V\frac{\rho\left(\overrightarrow{r'}\right)}{r'}\mathrm{d}V'$.

多级展开$\varphi(\vec{r})=\frac{1}{4\pi\varepsilon_0}\left(\frac{q_0}{r}-(\overrightarrow{p_0}\cdot\nabla)\frac{1}{r}+\frac{1}{6}\left(\overleftrightarrow{D}:\nabla\nabla\right)\frac{1}{r}+\cdots\right)$, 其中$q_0=\iiint_V\rho\left(\overrightarrow{r'}\right)\mathrm{d}V'$, $\overrightarrow{p_0}=\iiint_V\rho\left(\overrightarrow{r'}\right)\overrightarrow{r'}\,\mathrm{d}V'$, $\overleftrightarrow{D}=\iiint_V 3\rho\left(\overrightarrow{r'}\right)\overrightarrow{r'}\overrightarrow{r'}\,\mathrm{d}V'$.

$\varphi^{(2)}(\vec{r})=-\frac{1}{4\pi\varepsilon_0}(\overrightarrow{p_0}\cdot\nabla)\frac{1}{r}=\frac{1}{4\pi\varepsilon_0}\frac{\overrightarrow{p_0}\cdot\vec{r}}{r^3}$.

1.7 电场对带电体的作用

电偶极子$\vec{F}=(\vec{p}\cdot\nabla)\vec{E}$，$\vec{M}=\vec{r}\times\vec{F}+\vec{p}\times\vec{E}_e$.

面电荷受力.

2.1 静电场中的导体

2.1.1 静电屏蔽

2.1.2 电容器

$C=\frac{q}{U}$

(i)球形电容器

$C=4\pi\varepsilon_0\frac{r_1r_2}{r_1-r_2}$;

(ii)平行板电容器

$C=\frac{\varepsilon_0 S}{d}$;

(iii)圆柱形电容器

$C=\frac{2\pi\varepsilon_0 l}{\ln{}^{r_1}/_{r_2}}$.

电容器的串联($\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}$) 与并联（$C=C_1+C_2$）.

2.2 唯一性定理与静电镜像法

2.2.1 唯一性定理

成立条件: 介质不为铁介质; 边界为闭合边界

(i) 设计电场的依据; (ii) 求解电场的基础; (iii) 理解电场的重要方法.

静电问题的唯一性定理

设区域V内给定自由电荷分布$\rho\left(\overrightarrow{r'}\right)$, 在$\partial V$上给定(i)电势$\varphi|_{\partial V}$或(ii)电势的法线方向偏导数$\left.\frac{\partial\varphi}{\partial n}\right|_{\partial V}$, 则$V$内电场唯一地确定.

有导体存在时的唯一性定理

设区域V内给定自由电荷分布$\rho(\overrightarrow{r'})$, 以及一些导体. 给定各导体上的电荷$q_i$或导体上的电势$\varphi_i$, 以及$\partial V$上的电势$\varphi|_{\partial V}$或电势的法线方向偏导数$\left.\frac{\partial\varphi}{\partial n}\right|_{\partial V}$, 则$V$内电场唯一地确定.

2.2.2 静电镜像法

原则: (i) 镜像电荷必须位于求解区域以外的空间; (ii) 镜像电荷的最终引入不能改变原问题的边界条件; (iii) 不能将此方法用于虚拟空间;(iv) 求移动电荷做功一定不能用电势能之差, 只能用经典能之差.

(i)无限大导体平板

(ii)导体球