Project 1: Martingale (Report): OMSCS ML4T

Bing Sun
bsun302@gatech.edu

Abstract—This paper explores a American Roulette wheel[1] Simulator to evaluate Professor Balch's initial and an improved betting strategy within a casino setting. Through multiple simulated bets, the study analyzes risk, probability, and the comparative efficacy of both strategies in the casino context. Results highlight key statistical metrics, providing insights into the performance of the strategies in a real-world gambling scenario. The research not only assesses the existing strategy but also introduces and evaluates an improved, more realistic approach on enhancing probabilistic approaches in real life.This research contributes to understanding probabilistic approaches in gambling, drawing parallels with financial risk assessment. The paper showcases the integration of mathematical tools, research, programming, and academic writing, emphasizing their role in machine learning education.

1 INTRODUCTION

The initial strategy is to accumulate winnings up to $80 within successive betting episodes. Each series of 1000 successive bets is called an "episode." The strategy conduct an iterative process where bets are placed on black with an initial amount of $1. If a bet is successful, the winnings add the bet amount, and if failed, the bet amount doubles for the next attempt and winnings minus the bet amount. This process continues until the cumulative winnings reach or exceed the target of $80. The improved realistic strategy sets the gambler's bankroll to maximum $256. The study aims to analyze the performance of these two strategies, its risk management aspects, and overall effectiveness in a casino environment.

2 QUESTION 1

The probability of achieving a $80 win within 1000 consecutive bets is 100% based on experiment. All episodes equating to $80 within 1000 sequential bets for 1000

1 Here is wiki for American Roulette Wheel.

episodes have been recorded(see Figure 7, the result of Q1 experiment output). In each episode, the winnings successfully reach \$80. Based on the outcomes of Experiment 1, the calculated probability is 100%, denoted as 1000/1000 * 100%. As illustrated in Figure 2, it shows that after approximately 180 bets, the winnings of all ten episodes converge to \$80. This observation aligns seamlessly with our earlier analysis.

3 QUESTION 2

The estimated expected value of winnings after 1000 sequential bets is \$80. In Experiment1, an expectation[2] can be thought of as an arithmetic mean and write as:

$$\mathbb{E}[X] = \sum_{i=1}^{n} x_i * p_i,$$

where x1, x2, ... are the possible outcomes of the random variable X and p1, p2, ... are their corresponding probabilities. Considering the results(output as Figure 1) from Question 1, it's evident that all winning values converge to \$80 after 1000 episodes of bets, denoted as x1, x2, and so forth. Their corresponding probabilities are uniformly 100%. Based on the given equation, we can calculate the result as below,

$$\mathbb{E}[X] = 80 * 100\% = 80,$$

The mean of winnings, as depicted in Figure 3, stabilizes and converges to \$80 after 210 bets. This consistency with the deduced conclusion reinforces our earlier analysis.

4 QUESTION 3

Yes. In Figure 3, it is obvious that the values of "mean + stdev" and "mean - stdev" stabilize and achieve a consistent state at around 210 sequential bets. With the progressive increase in the number of sequential bets, the three lines representing the mean, mean + stdev, and mean - stdev become parallel and establish a stable equilibrium around the 210th bet. Additionally, the standard deviation (std) approaches and eventually becomes 0.

2 Here is wiki for Expected Value.

5 QUESTION 4

In Experiment 2, the probability of achieving a $80 winning within 10000 consecutive bets is 62.08% based on experiment. All episodes equating to $80 within 1000 sequential bets for 10000 episodes have been recorded(see Figure 7, the result of Q4 experiment output). In all 10000 episodes, numbers of episodes whose the winnings successfully reach $80 is 6208. Based on the outcomes of Experiment 2, the calculated probability is 6208/10000 * 100% = 62.08%.

6 QUESTION 5

In Question 4, the probability of winning $80 is 62.08% then winning $-256 is (1 - 62.08%) = 37.92%. As the expectation equation we mentioned before in Question 2, the estimated expected value of winnings after 1000 sequential bets is:

$$\mathbb{E}[X] = 62.08\% * 80 + 37.92\% * (-256) = -47.4$$

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7 QUESTION 6

Yes. The upper std reaches about $116 and lower std reaches -$198 then stabilize. In the arrays which represent the mean, mean + std, and mean - std across 1000 episodes, the last ten elements in each array are nearly identical (see Figure 6 output). This consistency is indicative of the stabilization of each line, reaching a maximum (or minimum) value. Additionally, with the progression of sequential bets, approximately 250 bets into the simulation, the standard deviation lines converge.

8 QUESTION 7

Using expected values has advantages over relying on the result of a single random episode. Expected values provide a more accurate estimate by averaging outcomes over many repetitions, reducing the impact of randomness,. Additionally, expected values offer repeatability and verification, making it easier to assess results through multiple trials. The consistency and predictability of expected values make them a reliable measure, especially in situations with uncertainties and fluctuations. Overall, expected values offer a more comprehensive and stable basis for decision-making and analysis compared to individual random occurrences.

9 CHARTS

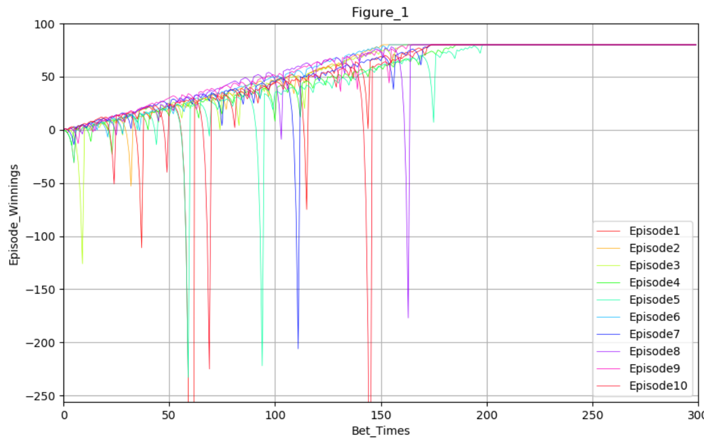

Figure 1—Track winnings for running 10 episodes.

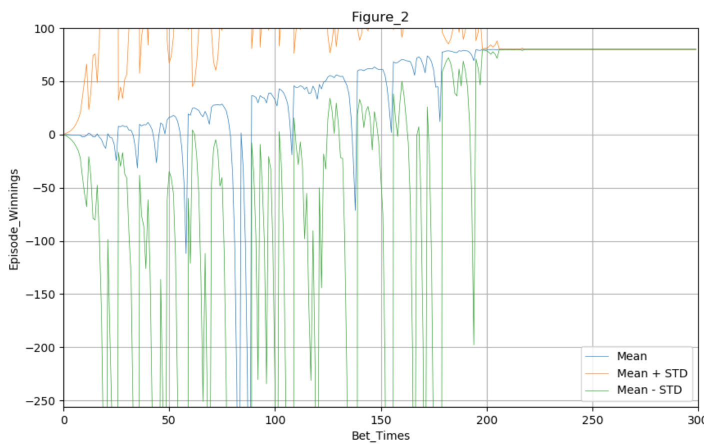

Figure 2—mean, mean + std, mean - std of the winnings at each point. (original strategy)

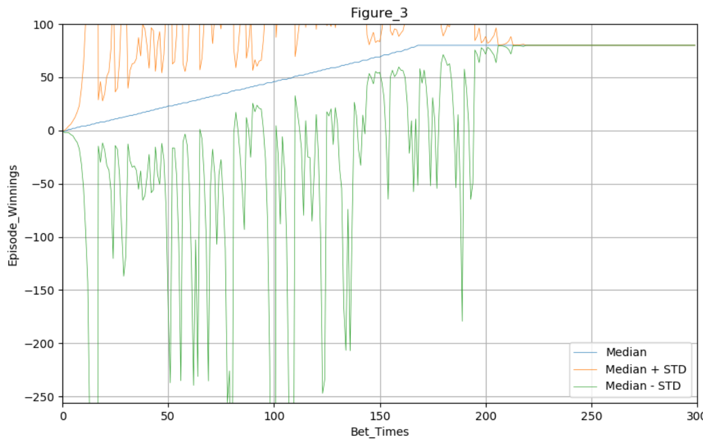

Figure 3—median, median + std, median - std of the winnings at each point.(original strategy)

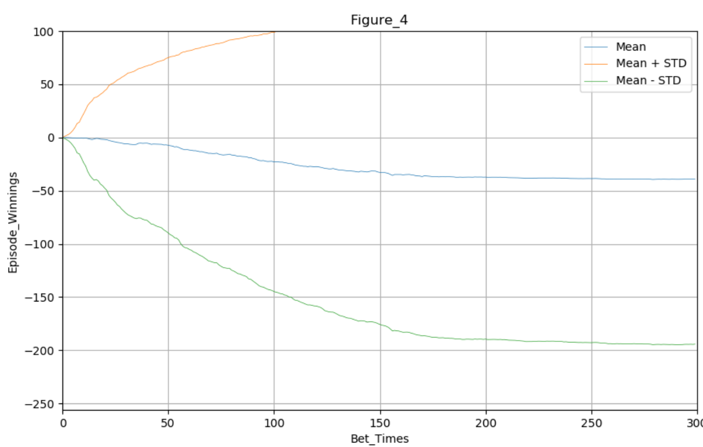

Figure 4—mean, mean + std, mean - std of the winnings at each point.(improved strategy)

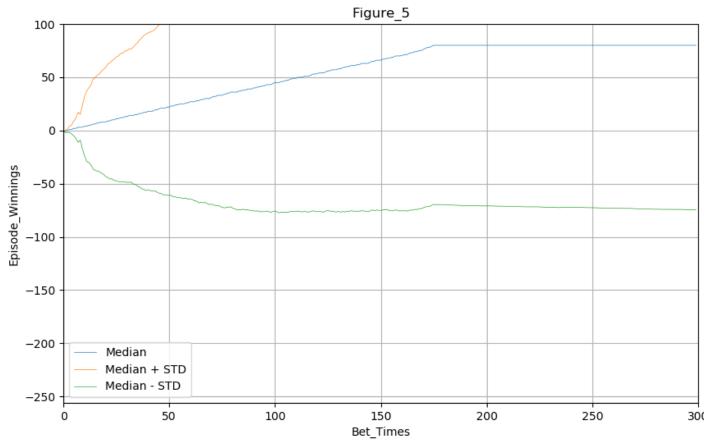

Figure 5—median, median + std, median - std of the winnings at each point.(improved strategy)

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Question 6:
    The simulation is executed across 1000 episodes
    of improved stratege (realistic_strategy) to
    output the last ten elememnts in the array of
    mean, mean + std, and mean - std.
The Q6 output is:
(array([-40.806, -40.825, -40.881, -40.938, -40.896, -40.866, -40.912,
       -40.883, -40.938, -40.974]), array([116.38871481, 116.3650136 , 116.36964973, 116.37268036,
       116.42775276, 116.45430398, 116.46164537, 116.5011012 ,
       116.47428083, 116.44800394]), array([-198.00071481, -198.0150136 , -198.13164973, -198.24868036,
       -198.21975276, -198.18630398, -198.28564537, -198.2671012 ,
       -198.35028083, -198.39600394]))
```

Figure 6—screenshot of Question 6 output.

```
martingale.py    p1_results.txt    figure_2.png    figure_4.png

Question 1:
    The simulation is executed across 1000 episodes
    of initial strategy (strategy) to determine the
    numbers of episodes required for the winnings to
    reach $80.
The Q1 output is:
1000
Question 2:
    The simulation is executed across 10000 episodes
    of improved strategy (realistic_strategy) to
    determine the numbers of episodes required for
    the winnings to reach $80.
The Q4 output is:
6208
```

Figure 7—screenshot of Question 1 & Question 4 output.

10 REFERENCES

[1] *6.4 EXPECTED VALUE* (2024). URL: https://louis.pressbooks.pub/finitemathematics/chapter/6-4-expected-value/ (visited on 01/21/2024).