

The general linear model:

$$y_i \sim \text{Normal}(\mu_i, \sigma)$$
$$\mu_i = \beta_0 + x_i\beta_1$$

Simulate from the model and draw  $x$  from a standard normal distribution:

$$y_i \sim \text{Normal}(\mu_i, \sigma)$$
$$\mu_i = \beta_0 + x_i\beta_1$$
$$\beta_0 = 0.5$$
$$\beta_1 = 0.5$$
$$\sigma = 0.75$$

Generalized linear models have:

1. A response distribution
2. A linear function trying to predict the response from the predictors
3. A link function that maps the linear prediction to a parameter in the response distribution.

If we call the link function  $g()$ , the general linear model looks like:

$$y_i \sim \text{Normal}(\mu_i, \sigma)$$
$$g(\mu_i) = \beta_0 + x_i\beta_1$$

In this case we use the identity link function (think just multiplying by 1). So the general linear model is a *generalized* linear model with a normal response distribution and an identity link.

How do we bring Allie's lizard example into the linear model framework since it doesn't have normal residuals? We could use a binomial response distribution and the logit link function:

$$y_i \sim \text{Binomial}(1, p_i)$$
$$\text{logit}(p) = \beta_0$$

What is a logit? First turn probability into odds:

$$\text{odds} = \frac{p}{1-p}$$

Next we take the log of the odds and get the logit:

$$\text{logit}(p) = \log\left(\frac{p}{1-p}\right)$$

Allie's lizard example with size.

$$\text{dive}_i \sim \text{Binomial}(1, p_i)$$
$$\text{logit}(p_i) = \beta_0 + \text{size}_i\beta_1$$

The model for germination as a function of area:

$$\begin{aligned}
\textit{germ}_i &\sim \textit{Binomial}(1, p_i) \\
\textit{logit}(p_i) &= \beta_0 + \textit{area}_i \beta_1
\end{aligned}$$

Let's add in the crosses now. This works by indexing the intercept so that there is a unique intercept for each cross.

$$\begin{aligned}
\textit{germ}_i &\sim \textit{Binomial}(1, p_i) \\
\textit{logit}(p_i) &= \beta_{0[\textit{cross}_i]} + \textit{area}_i \beta_1
\end{aligned}$$