Easy

E1

$y_i \sim Normal(\mu, \sigma)$ is the likelihood.

E2

There are two parameters: μ and σ.

E3

$$
\begin{aligned}
[\mu, \sigma \mid y] \propto \prod_i^n & Normal(y_i \mid \mu, \sigma) \times \\
& Normal(\mu \mid 0, 10) \times \\
& Uniform(\sigma \mid 0, 10)
\end{aligned}
$$

E4

$\mu_i = \alpha + \beta x_i$ is the linear model.

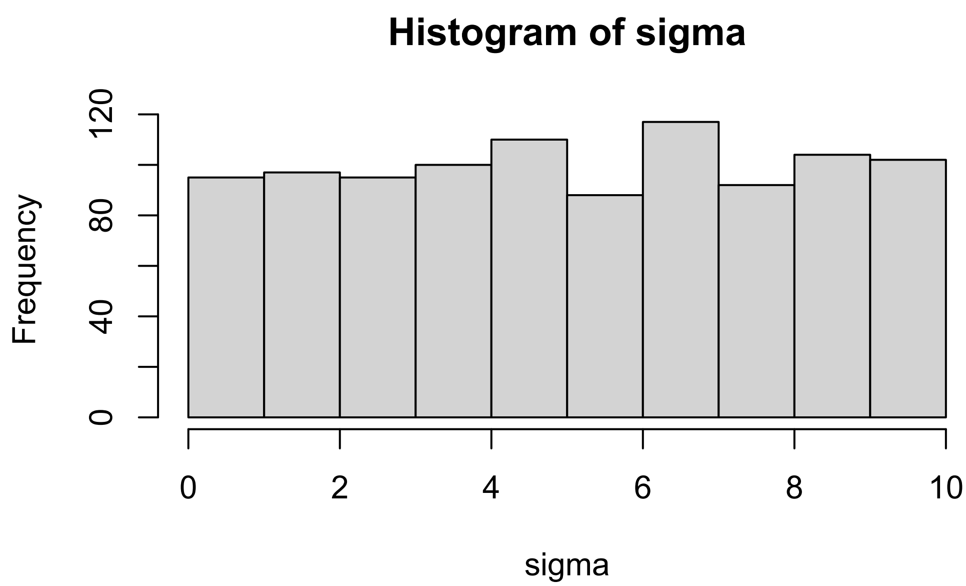
E5

There are 3 parameters: α, β, and σ.

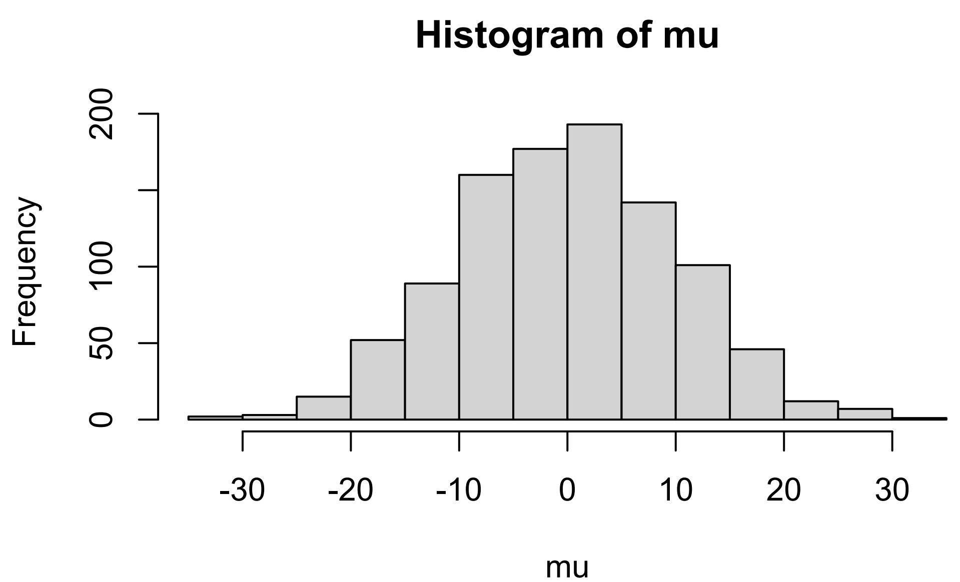
Medium

M1

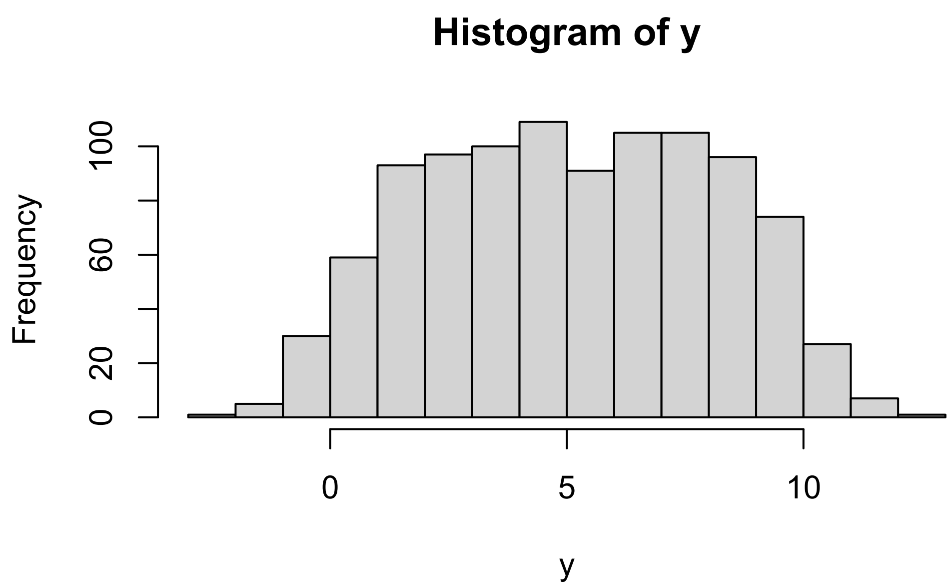
```r
sigma <- runif(1000, 0, 10)
mu <- rnorm(1000, 0, 10)
y <- rnorm(mu, sigma)
hist(sigma)
```




```r
hist(mu)
```




```r
hist(y)
```



M2

```r
alist(
  y ~ dnorm(mu, sigma),
  mu ~ dnorm(0, 10),
  sigma ~ dunif(0,10)
)
```

```
[[1]]
y ~ dnorm(mu, sigma)

[[2]]
mu ~ dnorm(0, 10)

[[3]]
sigma ~ dunif(0, 10)
```

M3

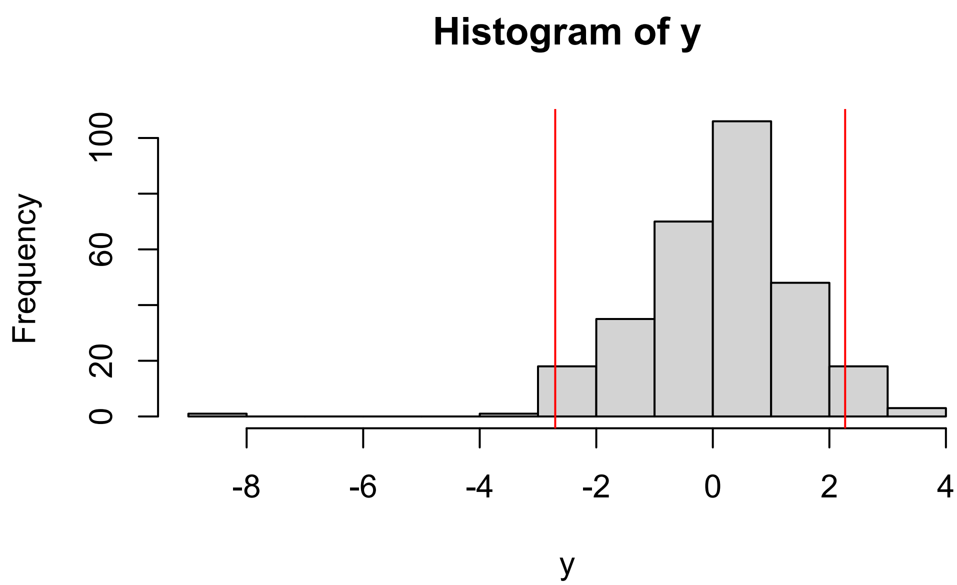
$$y_i \sim Normal(\mu_i, \sigma)$$
$$\mu_i = \alpha + \beta x_i$$
$$\alpha \sim Normal(0, 50)$$
$$\beta \sim Uniform(0, 10)$$
$$\sigma \sim Uniform(0, 50)$$

M4

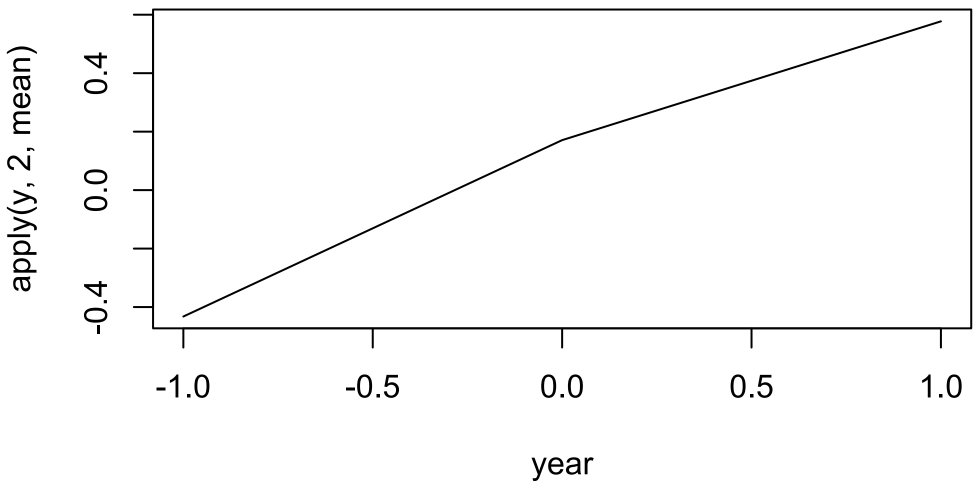
I will standardize height across all three years to make choosing priors easier. I will also code the first year as -1, the second as 0 and the third year as 1. This will make α very close to zero. Setting β to likely be positive (kids tend to grow). And $\log(\sigma)$ has a distribution around 1. These give a prior predictive distribution that seems reasonable

```
# center year so that it is (-1, 0, 1)
# standardize height measurements so that they have a mean of 0 and a stdv of 1
n <- 100
a <- rnorm(n,0, .5)
b <- rnorm(n,.5, .5)
y <- matrix(nrow = n, ncol = 3)
sigma <- rnorm(n, 0, .25)
for(i in 1:3){
  y[,i] <- rnorm(n, a + b*(i-2), exp(sigma))
}

hist(y)
abline(v = quantile(y,c(.025, .975)),
       col = "red")
```




```
year <- c(-1,0,1)
plot(apply(y,2,mean) ~ year, type = "l")
```



$$
\begin{aligned}
height_i &\sim Normal(\mu_i, \sigma) \\
\mu_i &= \alpha + \beta \times year_i \\
\alpha &\sim Normal(0, .5) \\
\beta &\sim Normal(.5, .5) \\
\log(\sigma) &\sim Normal(0, .25)
\end{aligned}
$$

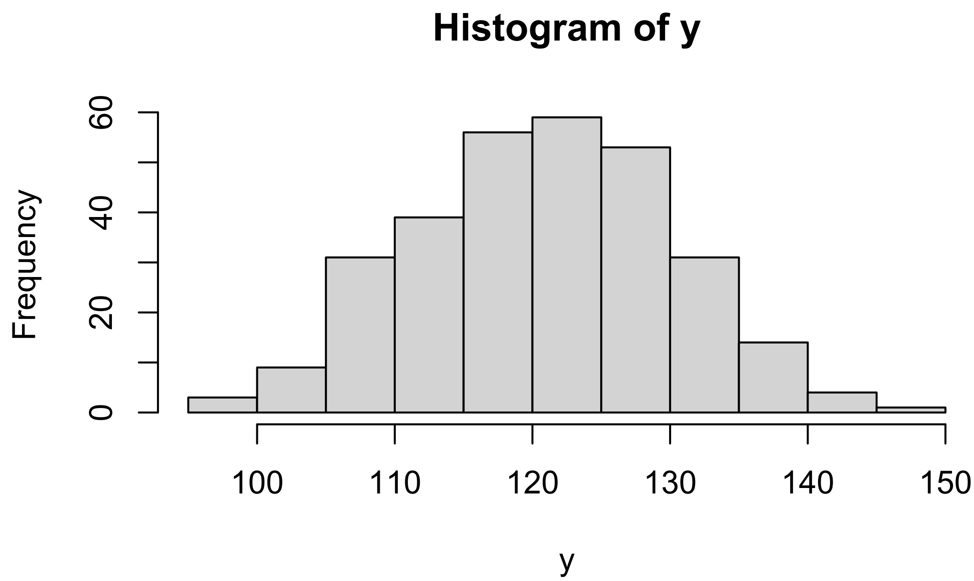
M5

It wouldn't with how I decided to standardize the variables, but in the spirit of the question, I would amend α to have a mean of 120 with an sd of 10. And let's say they grow an average of 2 ish centimeters per year. Giving $\log(\sigma)$ a mean 0f 0 and sd of 1 seems to give reasonable results from the prior predictive.

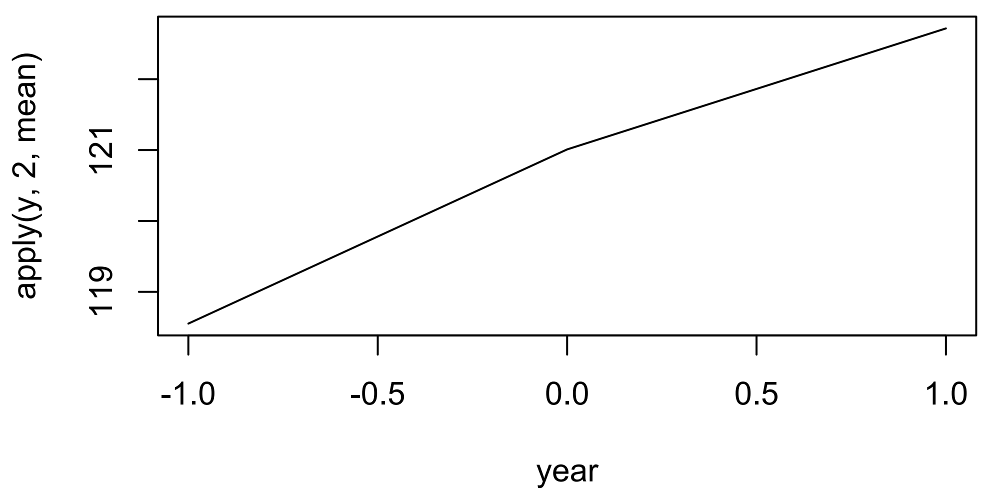
```
a <- rnorm(n,120, 10)
b <- rnorm(n,2,.5)
sigma <- rnorm(n,0, 1)

y <- matrix(nrow = n, ncol = 3)
for(i in 1:3){
  y[,i] <- rnorm(n, a + b*(i-2), exp(sigma))
}

hist(y)
```




```r
plot(apply(y, 2, mean) ~ year, type = "l")
```

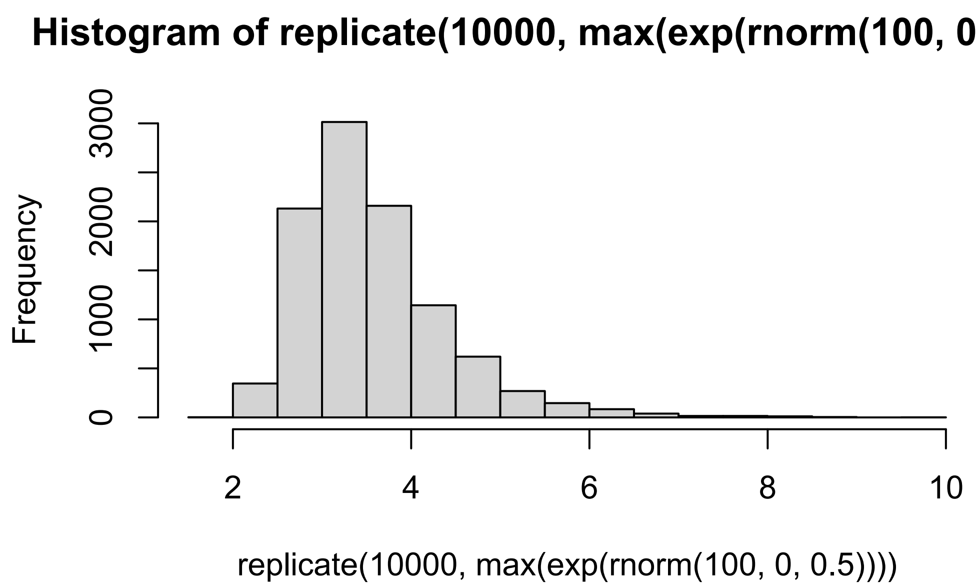


$$
\begin{aligned}
height_i &\sim Normal(\mu_i, \sigma) \\
\mu_i &= \alpha + \beta \times year_i \\
\alpha &\sim Normal(120, 10) \\
\beta &\sim Normal(2, .5) \\
\log(\sigma) &\sim Normal(0, 1)
\end{aligned}
$$

M6

A variance of 64 is a standard deviation of 8, and my prior is predicting more variation than that, so I can probably tighten it down. Going to $\log(\sigma) \sim Normal(0, .5)$ seems to work well.

```
hist(replicate(1e4, max(exp(rnorm(100, 0, .5)))))
```



$$
\begin{aligned}
height_i &\sim Normal(\mu_i, \sigma) \\
\mu_i &= \alpha + \beta \times year_i \\
\alpha &\sim Normal(120, 10) \\
\beta &\sim Normal(2, .5) \\
\log(\sigma) &\sim Normal(0, .5)
\end{aligned}
$$