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26. May. 2022.

Discussions of singularity problem of $\bar{L}_t^{x\eta}$

at $k=t$, there is no such a term $\bar{L}_t^{x\eta}$! Nevertheless, our proposed algorithm still works. This is to prove $S_t = -\bar{L}_t^{xx}$, and $T_t = -L_t^{x\theta}$.

(*. p.s. for $k<t$, $S_k = \bar{L}_k^{x\eta}(\bar{L}_k^{\eta\eta})^{-1}\bar{L}_k^{\eta x} - \bar{L}_k^{xx}$, $T_k = \bar{L}_k^{x\eta}(\bar{L}_k^{\eta\eta})^{-1}L_k^{\eta\theta} - L_k^{x\theta}$)

At $k=t$.

Original KKT conditions become:

$$\nabla_{x_t} L = \lambda_{t-1} - H_t^T \cdot R_t \left(y_t - h(\hat{x}_{t|t})\right) = 0 \quad \text{as } \underline{\lambda_t = 0}$$

Then,

$$\frac{d\nabla_{x_t} L}{d\theta} = \underbrace{\frac{\partial \lambda_{t-1}}{\partial \theta}}_{\Lambda_{t-1}} + \underbrace{\frac{\partial \nabla_{x_t} L}{\partial \hat{x}_{t|t}}}_{\bar{L}_t^{xx}} \cdot \underbrace{\frac{\partial \hat{x}_{t|t}}{\partial \theta}}_{\hat{X}_{t|t}} + \underbrace{\frac{\partial \nabla_{x_t} L}{\partial \theta}}_{L_t^{x\theta}} = 0.$$

$$\Rightarrow \quad \Lambda_{t-1} = -\bar{L}_t^{xx} \hat{X}_{t|t} - L_t^{x\theta}$$

Comparing with the second step of Lemma 2,

$$\Lambda_{t-1} = (I + S_t \cdot C_t) \cdot \bar{F}_t^T \cdot \underset{0}{\underset{\shortparallel}{\Lambda_t}} + S_t \cdot \hat{X}_{t|t}^{KF} + T_t$$

$$= S_t \cdot \hat{X}_{t|t}^{KF} + T_t$$

where $\hat{X}_{t|t}^{KF} = \hat{X}_{t|t}$,

we can have: $S_t = -\bar{L}_t^{xx}$, $T_t = -L_t^{x\theta}$. # This completes the proof!

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