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KKT conditions for Auxiliary linear MHE system.

$$\nabla_{\hat{X}_{0|N}} \mathcal{L}_2 = (P + \bar{L}_0^{xx})\hat{X}_{0|N} - P\bar{X}_0 + L_0^{x\theta} + L_0^{xw}\hat{W}_{0|N} - F_0^T \Lambda_0^* = 0. \quad (1)$$

$$\nabla_{\hat{X}_{k|N}}\mathcal{L}_2 \Rightarrow \Lambda_{k-1}^* = F_k^T\Lambda_k^* - \bar{L}_k^{xx}\hat{X}_{k|N} - L_k^{xw}\hat{W}_{k|N} - L_k^{x\theta} \qquad \forall k = 1, \cdots, N \quad (2)$$

$$\nabla_{\hat{W}_k}\mathcal{L}_2 \Rightarrow \hat{W}_{k|N} = (L_k^{ww})^{-1}(G_k^T\Lambda_k^* - L_k^{wx}\hat{X}_{k|N} - L_k^{w\theta}) \qquad \forall k = 0, \cdots, N-1 \quad (3)$$

$$\nabla_{\Lambda_k^*}\mathcal{L}_2 \Rightarrow \hat{X}_{k+1|N} = F_k\cdot\hat{X}_{k|N} + G_k\hat{W}_{k|N} \qquad \forall k = 0, \cdots, N-1. \quad (4)$$

$$\underbrace{\Lambda_{k-1}^*}_{\text{ranging from } 0\sim N-1} = F_k^T \underbrace{\Lambda_k^*}_{\text{ranging from } 1\sim N.} - \bar{L}_k^{xx}\hat{X}_{k|N} - L_k^{xw}\hat{W}_{k|N} - L_k^{x\theta} \qquad \forall k = 1, \cdots, N$$

$$\hat{W}_{k|N} = (L_k^{ww})^{-1}(G_k^T \underbrace{\Lambda_k^*}_{\text{ranging from } 0\sim N-1} - L_k^{wx}\hat{X}_{k|N} - L_k^{w\theta}) \quad (5)$$

They are the same

Substituted into $\nabla_{\Lambda_k^*}\mathcal{L}_2$

$$\underbrace{\hat{X}_{k+1|N}}_{\text{ranging from } 1\sim N.} = F_k\cdot\hat{X}_{k|N} - G_k(L_k^{ww})^{-1}L_k^{wx}\hat{X}_{k|N} + G_k(L_k^{ww})^{-1}G_k^T\Lambda_k^* - G_k(L_k^{ww})^{-1}L_k^{w\theta} \quad (6).$$

$\forall k = 0, \cdots, N-1$

※ $\underbrace{\Lambda_k^*}_{0\sim N-1}$ in the R.H.S of (6) is exactly the same as $\underbrace{\Lambda_{k-1}^*}_{1\sim N}$ in (2) [L.H.S of]

Note that the ranges of k in these two are different!

So, it's not allowed to plug the R.H.S of (2) directly into (6)!

We need to express 'k' in (2) using the 'k' in (6):

$$\underbrace{k_{(in\,(2))}}_{1\sim N} = \underbrace{k_{(in\,(6))}}_{0\sim N-1.} + 1. \quad (7)$$

(as detailed in Appendix-C ["Substituting P_{k+1} from (18b) and Λ_k^* from (37) for $\underline{k+1}$])

The resulting (2) in terms of $k_{(in\,(6))}$ can be written as:

$$\Lambda_k^* = F_{k+1}^T\underbrace{\Lambda_{k+1}^*}_{\text{ranging from } 1\sim N.} - \bar{L}_{k+1}^{xx}\hat{X}_{k+1|N} - L_{k+1}^{xw}\hat{W}_{k+1|N} - L_{k+1}^{x\theta} \qquad \forall k = 0, \cdots, N-1 \quad (8)$$

* Note that: $\hat{W}_{k+1|N} = (L_{k+1}^{ww})^{-1}(G_{k+1}^T\Lambda_{k+1}^* - L_{k+1}^{wx}\hat{X}_{k+1|N} - L_{k+1}^{w\theta})$ still holds for $k+1$.

When (8) is plugged into (6), we can establish a relationship between $\hat{X}_{k+1|N}$ and Λ_{k+1}^*. $\forall k = 0, \cdots, N-1$

ranging from $1\sim N$, the same.

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