

Dear readers,

Eq.(10) can have a different version of the Kronecker product.

According to Theorem 2.2 in [1], the following should hold:

$$\frac{\partial \text{vec}(aA(x))}{\partial x} = (I \otimes a) \frac{\partial \text{vec}(A(x))}{\partial x}$$

where  $a$  is a vector,  $A$  is a matrix depending on a vector  $x$ . In this case, the order of terms in the Kronecker product of Eq.(10) should be reversed.

However, my simulation results show that strictly following this definition can cause numerical issues, like complex eigenvalues. Therefore, we opted to use the definition in Eq.(4) of [2] instead.

This is an interesting phenomenon worth further investigation.

[1]. Magnus, Jan R., and Heinz Neudecker. *Matrix differential calculus with applications in statistics and econometrics*. John Wiley & Sons, 2019.

[2]. Dyro, Robert, Edward Schmerling, Nikos Arechiga, and Marco Pavone. "Second-Order Sensitivity Analysis for Bilevel Optimization." In *International Conference on Artificial Intelligence and Statistics*, pp. 9166-9181. PMLR, 2022.

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