

Problem 1: Comparing Growth Rates (16 pts)

Arrange the following functions by growth rate (slowest growth to fastest growth). Indicate which functions grow at the same rate (i.e. $f(N) = \Theta(g(N))$)
 $2/N < 128 < \log N < \sqrt{N} < 23N < N \log N < N^2 < 42N^3 < N! < 2^n = 3^n = 2^{n+1}$

Problem 2: Running Time Analysis (16 pts)

Give an analysis of the Big-O running time for each of the following program fragments (provide a tight bound for the running time).

$O(N^2)$

```
int sum = 0;
for (int i = 0; i < n; i++)
    for(int k=i; k < n; k++)
        sum++;
```

$O(N)$

```
int sum = 0;
for (int i = 0; i < 23; i++)
    for (int j = 0; j < n; j++)
        sum++;
```

$O(\log N)$

```
public int foo(int x,int k) {
    if (x <= k)
        return 1;
    else
        return foo(x / k, k) + 1;
}
```

Problem 3: Re-arranging Train Cars (16 pts)

Answer the following questions:

- **(a) (8 pts)** Provide a solution for this specific input train and 3 holding tracks as a sequence of steps.
- 3 - S3
- 6 - S2
- 9 - S1
- 2 - S3
- 4 - S2

- 7 - S1
- 1 - OUTPUT
- 2 - OUTPUT
- 3 - OUTPUT
- 8 - S3
- 5 - S1
- 4 - OUTPUT
- 5 - OUTPUT
- 6 - OUTPUT
- 7 - OUTPUT
- 8 - OUTPUT
- 9 - OUTPUT
- **(b) (8 pts)** Is there a solution for any train of length 9 with 3 holding tracks? Explain why or why not.
- No, there's no solution for some situation. For example, 198765432. 1 should be in the first one in output. So 98765432 should be hold in the holding tracks before 1 be moved in output. And when holing the trucks, the smaller ones should always on top of the larger ones. Since this is the only way they'll come out in the right order. 432 would be in S1 S2 S3 seperately, and we'll have no place to put 5. So there's no solution in this situation.
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