MATH405: HW12, Q3

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Part a.

```
X <- matrix(c(1,5,1,2,1,0,1,-3), 4, 2, byrow=TRUE)
X
```

```
##      [,1] [,2]
## [1,]    1    5
## [2,]    1    2
## [3,]    1    0
## [4,]    1   -3
```

Part b.

```
solve(t(X) %*% X)
```

```
##             [,1]        [,2]
## [1,]  0.27941176 -0.02941176
## [2,] -0.02941176  0.02941176
```

Part c.

```
#install.packages("Matrix")
library(Matrix)
```

Part d.

```
rankMatrix(X) == rankMatrix(t(X) %*% X)
```

```
## [1] TRUE
```

```
rankMatrix(t(X) %*% X) == rankMatrix(solve(t(X) %*% X))
```

```
## [1] TRUE
```

```
rankMatrix(X) == rankMatrix(solve(t(X) %*% X))
```

```
## [1] TRUE
```

Part e.

i.

```
eigen(t(X) %*% X)$values
```

```
## [1] 38.464249  3.535751
```

ii.

```
det(t(X) %*% X)
```

```
## [1] 136
```

iii.

```
prod(eigen(t(X) %*% X)$values)
```

```
## [1] 136
```

iv.

```
t(X) %*% X
```

```
##      [,1] [,2]
## [1,]    4    4
## [2,]    4   38
```

We see that the diagonal elements of $X^T X$ are positive. Thus, $X^T X$ is positive definite.

Part f.

i.

```
H <- X %*% solve(t(X) %*% X) %*% t(X)
H
```

```
##            [,1]      [,2]      [,3]       [,4]
## [1,]  0.7205882 0.3676471 0.1323529 -0.2205882
## [2,]  0.3676471 0.2794118 0.2205882  0.1323529
## [3,]  0.1323529 0.2205882 0.2794118  0.3676471
## [4,] -0.2205882 0.1323529 0.3676471  0.7205882
```

ii.

```
H %*% H
```

```
##            [,1]      [,2]      [,3]       [,4]
## [1,]  0.7205882 0.3676471 0.1323529 -0.2205882
## [2,]  0.3676471 0.2794118 0.2205882  0.1323529
## [3,]  0.1323529 0.2205882 0.2794118  0.3676471
## [4,] -0.2205882 0.1323529 0.3676471  0.7205882
```

It appears that H is indeed idempotent since $HH = H$.