

# The Mathematician News

2024, New Year's Copy

*Special New Year  
2024 copy!*

$$1 - 1 + 1 - 1 + 1 - 1 \dots = ?$$

Infinity is a mystical concept that has baffled mathematicians for centuries. However, we are going to unpack some of the mysteries. For example, take this problem:  $1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 \dots$  going on into infinity. In other terms, this pattern continues forever. What do you think this equals to? Well, what if we group it like this:

$(1 - 1) + (1 - 1) + (1 - 1) + (1 - 1) + \dots$  off into infinity. We get infinite zeroes, as  $1 - 1 = 0$ . Hence, I could say that the answer is 0. However, what if I do this:  $1 + (-1 + 1) + (-1 + 1) + (-1 + 1) + \dots$  off into infinity. We start with one, and we continuously add  $-1 + 1 = 0$ . So, we get  $1 +$  infinite zeroes, which equals 1.

So which answer is it? 1 or 0? Or perhaps you think it's another answer? Well, I'm going to prove that it's something else. I am going to subtract this expression from one. In other words, I will do this:

$1 - (\text{the equation}) = 1 - (1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 \dots)$  Now expand the parentheses. In doing this, we must reverse every single sign, like this:  $1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 \dots$  Hey! That's our expression at the beginning! So,  $1 - \text{the expression} = \text{the expression}$ . If we solve for the expression, we see that this is just algebra!

We can see the  $2 \times \text{the expression} = 1$ . Dividing 2 from each side, we see that the expression  $= \frac{1}{2}$ , or half. Huh. So this could equal to 1, 0, or  $\frac{1}{2}$ . Each answer has their own flawless logic. What is the correct answer? Well, Although most people believe it is  $\frac{1}{2}$ , this answer is controversial, and you can believe whatever you believe. However, for the rest of this copy, we will assume it is  $\frac{1}{2}$ .

# What is $1+2+3+4+5+6+7+8+\dots$ *INFINITY*

When we talk about Grandi's series, we can't ignore this:  
 $1 + 2 + 3 + 4 + \dots$  into infinity. But before we get to that step, we first have to learn about the series  $1 - 2 + 3 - 4 + 5 - 6 + \dots$  and forever. We will take a different approach to this problem. We will call this series  $x$ . Now let's multiply  $x$  by 2. We could just double every number in  $x$ , but we could also do it like in diagram 1:

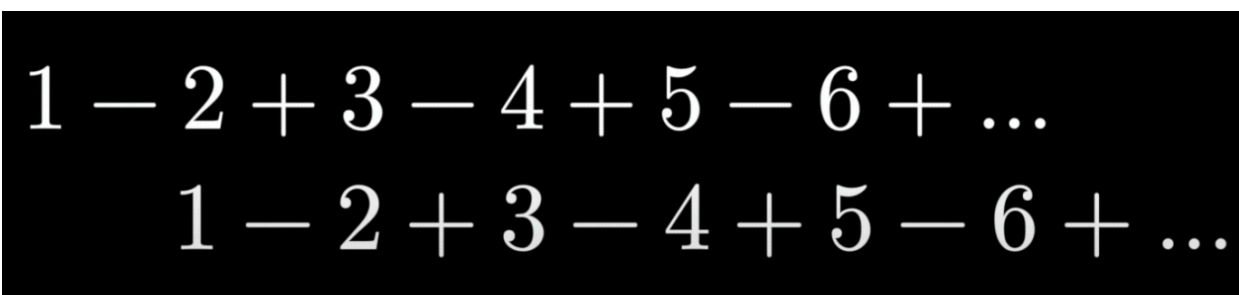

$$\begin{array}{l} 1 - 2 + 3 - 4 + 5 - 6 + \dots \\ 1 - 2 + 3 - 4 + 5 - 6 + \dots \end{array}$$

Diagram 1

We add -2 to 1, 3 to -2, -4 to 3, and so on. If you calculate this, you see that  $2x$  is equal to  $1-1+1-1+1-\dots$ . Wait. That's Grandi's series, which we proved is  $\frac{1}{2}$ . So this means  $2x = \frac{1}{2}$ ! So  $x = \frac{1}{4}$ ! Surprising, isn't it? But if you remember our title, how is this going to help with the sum of all the counting numbers? You'll see. Again, we will set this new series as a variable. This time, it is  $Y$ . Now we can ask, what is  $Y-X$ ? Challenge: Try to do this difference on your own.

Did you get it? Well, if you did it correctly, you would see that all the odd numbers would get canceled out, and the even ones would get doubled, or this:  $4+8+12+16+\dots$ . But wait! Isn't this just  $4Y$ ? Multiplying every single number in  $Y$  will give us  $Y-X$ . And because  $X$  is  $\frac{1}{4}$ , as we recently calculated, we know that  $Y - \frac{1}{4} = 4Y$ . After solving this equation, we find that the sum of all counting numbers is **NEGATIVE ONE TWELFTH!!!** Not only is this answer negative, it's a fraction! The sum of infinite whole, positive numbers is a **NEGATIVE FRACTION!!!** Surprising, right? Well I think many of these answers in these two passages surprised you. That's ok. Honestly, that's the point.

0.999999999999... is equal to what?

You might say that the answer to this question is obvious.  $0.999999999\dots$  is equal to  $0.999999999\dots$ , and that's that. But let me tell you something more surprising.  $0.999999999\dots$  is equal to 1. That's right. Now you know that, try to prove it yourself. Here are our solutions:

1. We know that  $\frac{1}{3} = 0.33333\dots$   $\frac{1}{3} * 3 = 1$ . So,  $0.33333\dots * 3 = 1$ . We know, using basic multiplication,  $0.33333 * 3 = 0.99999\dots$ , so  $0.99999 = 1$ .

2. First, let's set a variable for  $0.99999\dots$ , like  $X$ . So,  $X = 0.99999$ . Now, we know that  $10X = 10 \cdot 0.99999\dots = 9.99999\dots$ . Let's subtract these equations:  $10X - X = 9.99999\dots - 0.99999$ ,  $9X = 9$ . So, by dividing each side by 9, we see that  $X = 1$ .

Proof 1:

$$\frac{1}{3} = 0.3333333333...$$

$$\frac{1}{3} * 3 = 1$$

$$0.3333333333... * 3 = 0.9999999999...$$

$$1 = 0.999999999999...$$

Proof 2:

$$X = 0.99999\dots$$

$$10X = 9.99999\ldots$$

$$10X - X = 9.99999 - 0.99999$$

$$9X = 9$$

$$X = 1$$

$$0.999999 = 1$$

[illegible]

Proof 2 above is the textbook way for calculating numbers with repeating numbers. Try to solve some other repeating numbers!

## 4D

Now this newspaper probably already blew your minds. Well, this might bend it more. In Jean le Rond d'Alembert's *Dimensions*, published in 1754, he stated the idea of a fourth dimension. However, you probably know that you don't live in a cartoon world and that in the real world, there are three dimensions. Height, Width, and Length, and there isn't a fourth. However, ever since, the idea that there is a secret fourth dimension has sparked the interest of plenty aspiring mathematicians and thinkers, and we will understand the concept today. To understand 4D, we must start with 0D. 0D has zero dimensions. Imagine living on a dot. That's it. You can't move left or right, up or down. Now, imagine that that dot is extended both ways forever. You would only be able to move left or right, and you are not able to walk in any other direction. Still kind of boring. Now imagine that this infinitely long line is raised up and down forever, making a flat plane. Yay! We can move right, left, forward, backward. But up or down? Well, not really. However, let's move our plane up and down forever. Now, we get to jump and dig underground all we want. You probably know what 3D feels like. It's part of our everyday life! But then what is 4D? Well 4D adds a new dimension: spacetime. Because time and space can both be measured, they could be a combined dimension theoretically, right? Well, yes. But, since we haven't worked with this dimension in real life, we need the help of patterns. In one D, your shadow is a dot. In 2D, your shadow is a line. In 3D, well obviously, your shadow is 2D. Then, using this pattern, we can infer that your shadow in 4D is 3D. I know that sounds confusing, but bear with me. We then think of a circle in a square. In 2D, you would only see a line, and when you look down from it, well, you can't look down, but it would just be a line. And then in 3D, it wouldn't be a circle in a square, it would be a sphere in a box, and the outline would be 2D. So in 4D, when you look from above, it would all be 3D, and you are able to see ALL the faces of the box. Remember the shadow? Well when I said the shadow was 3D, it was 3D. We have a name for it: **tesseract**. Anyway, A tesseract is a cube in a cube, connected by the vertices. But when you rotate this monster, this creates a cursed shape. The inside cube is switching with the outside cube! Why? Well in 2D, when you turn the flat shape, the side facing you changes. In 3D, the same thing happens. Take a cube. When you turn the cube, the face that is facing you changes. So in 4D, when you turn the tesseract, the cube facing you changes. Huh? I know this may seem impossible in our lives, but in a 4D world, you wouldn't even be able to think about it! Here's another way you can think about it. In 2D, when someone in 3D tries to put a sphere in their world, all they will see is a circle, growing bigger as it gets to the center, and smaller as it leaves the center. So in 3D, when someone in 4D tries to place a 4D sphere, or a hypersphere into the 3D world, all you will see is a sphere, gradually getting bigger, and then smaller until it disappears into thin air. So, this is 4D in a nutshell. Obviously, we have only brushed the 4D iceberg. If this interests you, we encourage you to read and study more of this.

# Prove It!

Ok. I will give you an easy task. How do you prove that  $13 - 8 = 5$

## Problem 1: The number flaw

Seems easy, isn't it? Well before we get to that, let's try to prove  $1 + 1 = 2$ . What is 1? One is just the first number. What does that mean? How do we prove that  $1+1$  is 2? Well the thing is, this is a trick question. Why can't we think that  $1+1$  is 3? Or 4? It seems impossible, because that is the "definition" of 1.  $1+1$  has always been taught to you as 2. This is because it is the "definition" of 2. Us humans have defined 1 and 2 so that  $1+1$  is defined as 2. This could mean that  $13-8$  is also just defined as 5, and cannot be proved. But, before we give up, why don't we try?

## Problem 2: The hilarious subtraction flaw

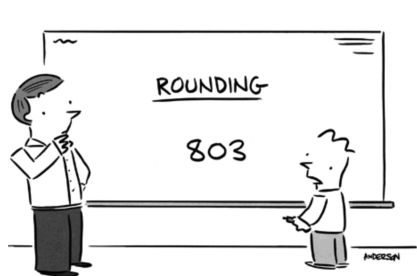
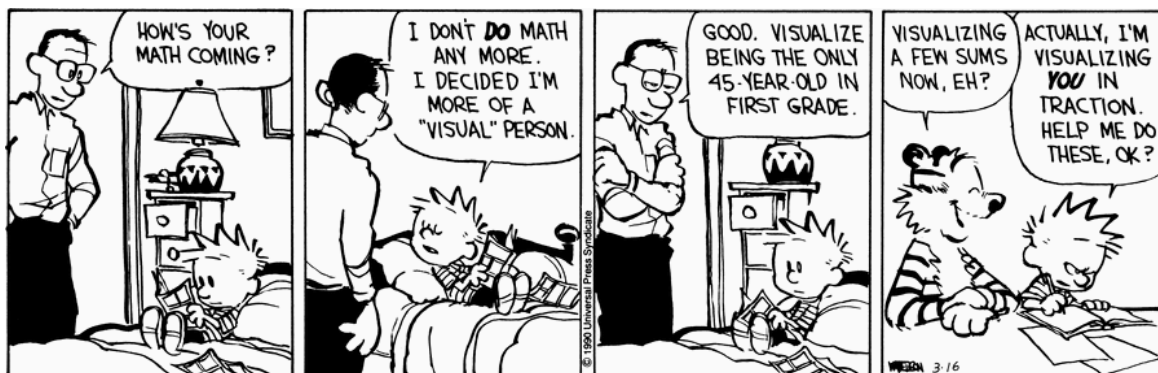
Let's tackle the real problem.  $13-8$ . Let's do this. First, let's try to do  $13-8$  normally as you would do any other subtraction problem. We try to subtract the ones. This gives a negative number, so we must regroup. We regroup, giving us the problem  $13-8$ .

Okay, this is interesting, so let's regroup again, giving us  $13-8$ . Ugh! This is taking us nowhere! Instead, we could try using another way. Hopefully we don't fail this time.

## The Answer?

Ok. We could use our fingers, I guess. Let's pretend we have 13 fingers. Then subtract 8 of those fingers. That sounds disgusting, but who cares? We have 5 fingers left!

# Comics!



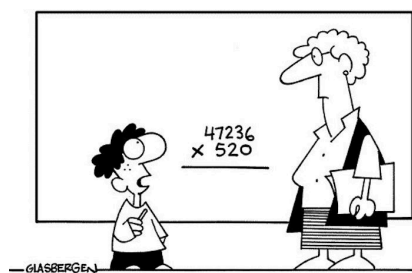
"I suppose I could try, but I don't think I can make them any rounder."



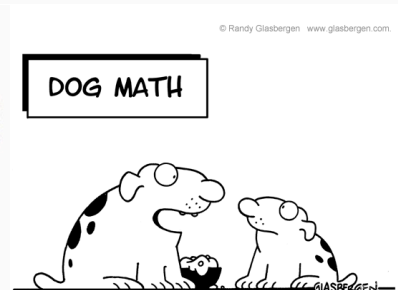
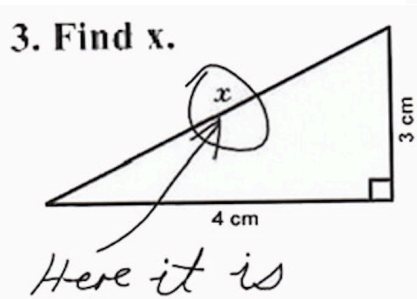
"To show you how well I understand fractions, I only did half of my homework."



"What if someone hits the mute button?"



"Aren't there enough problems in the world already?"



"If I have 3 bones and Mr. Jones takes away 2, how many fingers will he have left?"

## About the Authors

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Lastly, thank you, dear reader, for reading our newspaper, and if you would like to help, just tell any of us! We'll be delighted to add you to our staff.