The Mathematician News

Halloween Edition!

Exponents and Beyond

Have you ever seen these little numbers at the top right corner of a number?: 2^3 ? They are called exponents. The big number(2) is called the base. The smaller number(^{3}) is called the exponent. The whole thing(2^3) is called a power. How do we calculate powers? Well, the exponent is how many times the base is multiplied by itself. For example, 2^3 means 2 is multiplied by itself 3 times, or 2×2×2=8. Another way of writing exponentiation is using ^. For example, 2^3 is the same thing as 2×2×2. Now look at the problem $2^3 \times 2^4$. Well, this can be expanded into 2×2×2×2×2×2×2. That can be shortened to 2^7. We can look at a couple more. $3^4 \times 3^2 = 3^6$; $4^8 \times 4^7 = 4^{15}$; $6^2 \times 6^9 = 6^{11}$; You can write it down in your own time, or come to the math club meetings. With all these patterns, we can see that $x^y \times x^z = x^{y+z}$.

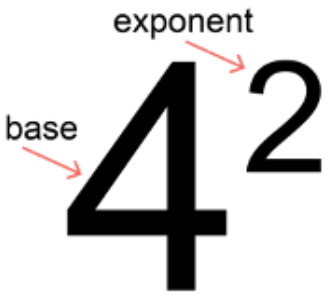


Figure 1

One of the most famous exponential numbers is googol. This number is represented by 10^100. It has so many digits that it even has more digits than the number of atoms in the world. That's cool! Are there any other patterns with exponents? I'll let you figure it out. Hint: what other signs have we not used. Addition? Subtraction? Division? You might think of another thing. Can we repeat exponentiation? Yes! This term is called tetration. Tetration doesn't have a very specific way to write, but many people write the 'exponent' on the top left corner, like this: $^3 2$. It is equal to $(2)^{2^2} = 16$. The exponent is the amount of 2's the base is powered to. So $^4 2 = (2)^{2^{(2^2)}} = 65536$. As you can see, the value increases tremendously. That is why tetration is used rarely in mathematics. But obviously we have an operation that makes these numbers look small. This operation is pentation. This is basically repeated tetration. For example, 2 pentated to 4 is 2 tetrated to 2 tetrated to 2. We start from the end and we go down. 2 tetrated to 2 is 4. 2 tetrated to 4 is 65536. 2 tetrated 65536, well this number just makes the others look like ants. This number is so big you cannot compare this to anything in existence. But pentation is not the most powerful operation. Hexation is basically... you guessed it! Repeated pentation. 3 hexated to 3 is 3 pentated to 3 pentated to 3. This number is huge. But with this number, we can start explaining how to make Graham's number. First we must imagine hexation as 4 up arrows. That means pentation is 3 up arrows, tetration as two, and exponentiation is 1 up arrow. Now imaging 3 hexated to 3 as x. 3 followed by x up arrows 3 is our second number, y. 3 followed by y up arrows and then a 3 is our new number v, and so on. Graham's number is the result when you repeat this 63 more times. This number is Graham's number. This number is the biggest number of that time, but now there are bigger numbers, like tree(3). But we can explain that another time.

FACTORIALS

Have you ever heard of factorials? Well, today is your lucky day to learn about them. Let's start with just plain Factorials. Factorial means you multiply every number below the number you see before the !. So,4! Means 4x3x2x1 which is 24. Try 5!. What about 6!? 7!? How far can you get? In addition, there are more complicated factorials. For example, double factorials. Double factorials look like this: !!. If the number being double factorial is even, you multiply all the even numbers below it. If it's odd, multiply all the odd numbers below it. For example, 4!! means 4×2 which is 8. 6!! Is 6×4×2 which is 48. Again, can you find 5!!? Or higher? How much can you calculate? However, there is an even more complicated factorial. And you guessed it, triple factorials, or this: !!!. Basically, triple factorials will multiply all the third numbers below the number being triple factorial. Or, you can think of it like this: Any other number that is smaller and has the same remainder when divided by 3 is multiplied with n. For example, 5!!! Is 10 because when you do 5/3, you get a remainder of 2. Or, because that 5-3=2, and 2-3 is negative, so 5×2 is 5!!!, which is calculated as 10. Again, can you calculate it larger? But, why are we writing about this? What makes this useful? Well, We are going to talk about it now. First of all, factorials are a short cut to writing long multiplication equations. For example, we can rewrite 6×5×4×2×3 as 6! And we can rewrite 10×7×4 as 10!!!. In addition, in arrangement problems, we can use factorials. First try this question out: If you want to line up 4 differently colored balls in a row, how many possible lines are there. In this question, we must find the number of arrangements possible by lining up these balls. Perhaps before, you've drawn a tree diagram to solve this problem, but today, we are using factorials. Let's assume that the balls are labeled A, B, C, and D. A, B, C, and D all can be first in line, so there are 4 possibilities for the first spot. Lets assume the ball A is first. Then, B, C, and D can be second. That makes 3 more possibilities. Let's assume again that B is in the second spot. This leaves balls c and d having no spots. The third spot is either c, or, so there are 2 choices. The Last spot has only one more choice. So, for the first spot we have 4, then 3, then 2, then 1. We multiply these together to get 4×3×2×1. We can rewrite this as 4!, which calculates as 24. We can learn from this to see that is x balls need to be arranged, then there are x! Possible arrangements If you would like to know why we must multiply, or want a more concise explanation, come to the math club, and we will explain it there. However, there are still more types of factorials. Now, it is your turn. Brief the internet to find more, If you would like, you can talk to the math club all about these new factorials.These are just the simple ones. If you want to use harder ones , there's a game for that on the next page.

GAME TIME!

Are you familiar with four four's? This very popular game is played by any age, old or young. This game gives you four 4's, AND FOUR FOURS ONLY. You must use these four fours to get as far as you can. For example, 1 can be made with four 4's by writing 4/4 times

4/4. But make sure to come on wednesdays and fridays to get your own four four's notebook! BUt anyway, you can add anything to the four four's that is not a number. Examples can be addition, subtraction, multiplication, division, factorials, square root, and other very rare ones you've probably never heard of, like double factorial or triple factorial.. We've gone very far with four four's and so far, we are on 171. Let's see if you can beat us!About what i said earlier, you can also use something like 4!!(!!!). Try your best. Once you get to bigger numbers, make it simple.Don't try to show off. It doesn't work that way. It will be very hard then.Have fun. I don't care if you can't beat us. I want you to just try hard on it.

1.

2.

3.

4.

5.

6.

7.

8.

9.

10.

11.

12.

If you would like, keep writing these numbers in a private notebook. See if you can beat us!

Glossary

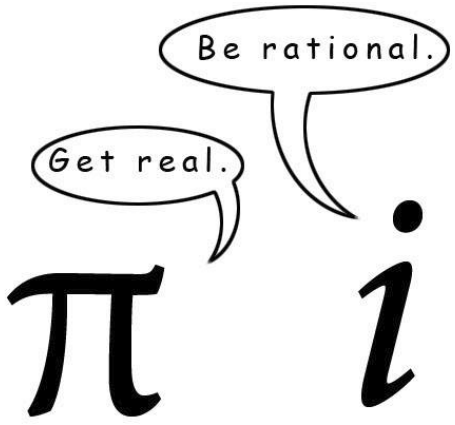
Graham's number: a tremendously large infinite number that is a proven upper bound to the solution of a certain problem in Ramsey theory.

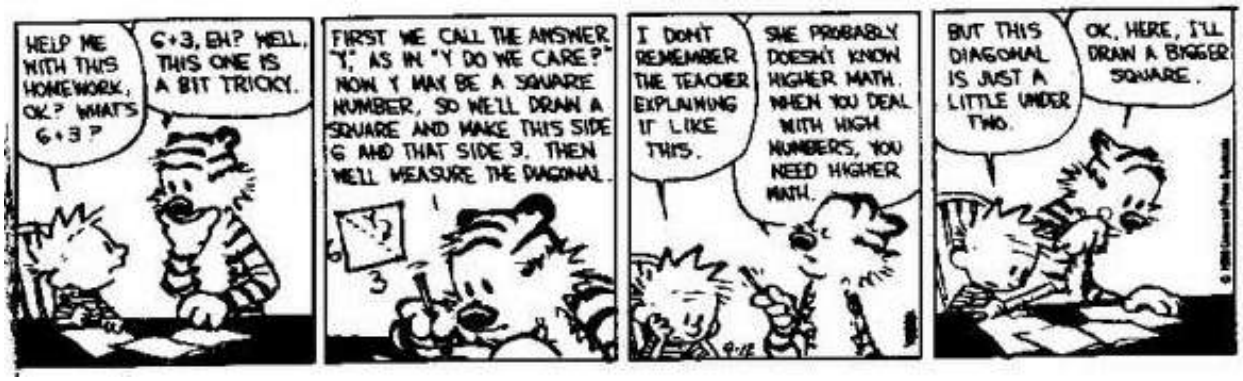
Hexation: The arithmetic operation of repeated pentation

Pentation: The arithmetic operation of repeated tetration.

Tetration: The arithmetic operator consisting of repeated exponentiation.

MATH JOKES!





About the authors

Hello! I see you have finished the halloween addition of the mathematician newspaper that soooo many people wrote in. Although some of them weren't as good as the others, we are still going to list everyone's names. So, our authors of this newspaper are Brooks Wang, Daniel Pei, Junxiao Wu, Dhun Patel, and Hady Jalloul.. Again, feel free to contact any of them. Thank you for reading.