

The Mathematician

π Day

The (Not So) Simple Concept of π

Every year, on the 14th day of the 3rd month, we celebrate π day, a day in celebration of the intriguing concept of π . This concept has intrigued many mathematicians for centuries, and will further intrigue them for more to come. But what is this concept? Well, π was found and used by almost every ancient civilization. The Greeks defined π as the ratio of the circumference of a circle with its diameter, in which the circumference is the perimeter of the circle, and the diameter is any line that can split the circle directly in half. The Chinese and Indians approximated π to 7 and 5 digits, approximately. The Egyptians and Babylonians calculated fairly accurate values of π as well. This ratio is an irrational decimal, or a decimal that cannot be expressed as a fraction, and is used in many theorems and proofs. π is usually used as 3.14, though modern technology can calculate trillions of digits of π . The most prominent use of π could be NASA and other space observatories, to calculate the orbit of exoplanets and asteroids. They use π , along with

Kepler's third law, to calculate how long it takes the exoplanet to make one full orbit of its star, which reveals the planet's location and whether it's in the habitable zone. But π does not limit to geometry. It can appear in many interesting places. Take this equation: $1 + 1/4 + 1/9 + 1/16 + \dots + 1/n^2 + \dots$ Try to find what this is equal to? Well, believe it or not, the answer to the question is $\pi^2/6$. This is a famous series, called the Basel's series, solved by Leonard Euler. Try to solve this yourself first, and if you're interested, you can search up Basel's series, and you will get a plethora of answers and other interesting questions. Aside from this, there are many more of these problems, and I will leave you with one last problem:

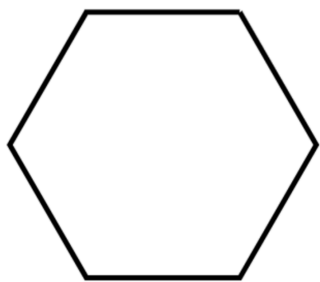
The Leibniz formula for π , named after Gottfried Leibniz, states that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots$$

Try to prove it!

Calculating π

π is known to be estimated as 3.14 or $22/7$ (the last one is not π , beware of the misconception), but can you calculate further? Well, yes. Let's see how Archimedes calculated π to around 8 digits. You might have noticed that as we add more sides to a regular polygon, the figure gets closer and closer to a circle. Hence, the perimeters of those polygons get closer and closer to the circumference. The distance from the center of the polygon and one of its vertices are also getting closer and closer to the ratio. The more sides you add, the more accurate it is. These values are a lot easier to calculate than if they were on the circle. For example, try calculating π using this:

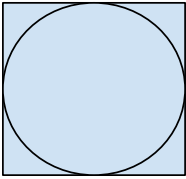


Assuming each side had the length of one, we can easily see that the circumference is $6 \times 1 = 6$. The ratio is a

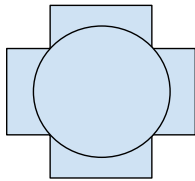
bit harder. We can split the hexagon into six triangles, all of them sharing a vertex in the center. The center of the hexagon is a 360 degree angle, and when split into 6 angles, each angle is 60 degrees. Each outer angle of the hexagon is $(180 \times 6 - 360) \div 6 = 120$. This is because the hexagon can be split into 6 triangles, each with 180 degrees total. For 6 triangles, this is 180×6 . However, we must subtract the 360 degrees in the middle. Split between 6 angles, each angle is $720 \div 6 = 120$. Split between 2 different angles, each angle is 60 degrees as well. Since each angle is 60 degrees, the triangles are all equilateral. Then, the ratio of the hexagon would be equal to one side of the hexagon. So, the ratio is 1. As we said before, π is the ratio of the circumference and the diameter, and the diameter is twice the ratio, so the diameter is 2. Then, our estimation of π would be $6 \div 2 = 3$. This is pretty good! As the sides increase, the ratio is more precise, and so is π . If you're interested, try calculating π using a dodecagon, or a twelve sided shape. Have fun!

$$\pi = 4?$$

After all this, I think it's safe to say that π is 3.1415... There shouldn't be any doubt about that. However, what if I told you that this is all a joke and that π is 4? Let's prove this. You can prove this by inscribing a circle in a square with the sides touching like shown below. The radius of the circle is one, and then you can figure out the perimeter of the square. And since the radius of the circle is one, the area of the square is eight. With that we can move on.



We can then turn the image into the shape shown below.



And the perimeter of this new shape is still eight. So, we can continue this on and on forever until the new shape touches the circle. This is called **Limit**, and this is super important for very very very hard questions from Calculus. So, π is the perimeter of the circle or circumference of the circle divided by the diameter. And since the circumference of the circle is eight and the diameter is two. Then π is eight divided by two which is four. So π is four.

Well, actually this proof is incorrect. Do you know why? Try to disprove this proof, and prove that π is actually and truly 3.1415...

$$\pi \neq 4$$

Ten Cool Facts about π and π day

1. Stephan Curry, a well known basketball player, and Albert Einstein, were all born on π day!
2. The world record for calculating π with a computer is 62,831,853,071,796 digits, which is actually MORE than the positions a Rubik's Cube can be in (beats it by 19×10^{18}) (you can see π up to 1 million digits on <https://www.piday.org/million/>)
3. π was used as something to help the ancient Egyptians build pyramids, somehow.
4. The symbol of π has been used for 250 years since today.
5. There is an entire LANGUAGE made by the digits of π . The first word has 3 letters, the second has 1 letter, and the 3rd has 4 letters and so on for the sentence. This has actually been used in a book!
6. A mathematician calculated 707 digits of π , but the 527th digit was incorrect, so the rest after were as well, so he only got 526 by hand which is still very good.
7. One way that will take forever to calculate π is to add sides to a polygon. This will approach the shape of a circle, but never exactly.
8. Some mathematicians say there are infinitely many corners in a circle because π has infinitely many digits.
9. If you tried to use π for calculating the circumference of the earth, for every 25,000 inches when you have π rounded to the 9th decimal, so 10 digits, you would be off by 1/4th of an inch. How accurate!
10. τ (tau) might take over π , so shouldn't a τ day become official? I mean it makes sense to use it in more places as we know....

About the Authors

A plethora of authors, editors, and revisors went through this article to refine it again and again. Firstly, we would like to thank Brooks Wang for writing these articles on Calculating π and The (Not So) Simple Concept of π . Thanks to Eddie Yang for writing $\pi = 4?$, and lastly for Rohan Hallur for writing the 10 Fun Facts About π And π Day. Thanks to Junxiao Wu, Abhinav Shah, Daniel Pei, Sooriyan Thiruchilvem, and lastly, Hady Jalloul. If you would like to join the group, ask any of us, and we will get you in the squad immediately! Help is greatly appreciated, and thank you for reading this article.