The Mathematician

February Edition

Unary, Base 5/4, and more?

In many other Mathematicians, we talked about bases. Now today, we're going to have some more complex bases. Many of you may be familiar with other bases, like base 2, binary, base 3, ternary, base 6, heximal, or also called seximal, base 8, base 16, and so on and so forth. Now, let's talk about some weird bases.

First, how do bases work? Let's take Base 4. If we want to represent 0, we use 0, 1 for 1, 2 for 2, 3 for 3, and 4 for 4. When we get to 5, though, we can't use the digit 5. Instead, we use 10. 11, 12, 13, 14, then 100, and so on. How do we convert this back to decimal, though? Well, I'll leave that for you. If you want an explanation, find any of us(names listed in credits, and we'll be more than happy to explain.

Mystery 1: Base 1?

But what about base 1? With the rule, it means that the only digit you could use is 0, and because 0 multiplied by anything is 0, the only number you can make is zero. Right? Well it turns out, in this base, you have to use a tally, like this |. But to represent big numbers, like one million, you have to use many tallies. This base is called unary, but it isn't very efficient in representing big numbers.

Mystery 2: Base -10?

Base -10? Well, following our rule, we find that the one's place will be positive, the "tens" negative, the hundreds positive, and so on. But what digits will we use? Well it turns out, we use the digits from 0 to the **absolute value** of the number - 1. The absolute value of a number is the distance the number is from zero. For example, the absolute value of -483 is 483.

Mystery 3: Base ⅓?

How do we represent the number 9 in Base ⅓? Well, it turns out, we have to use the decimal places. When you go to the decimal places, the numbers are raised to the negative power. Base ⅓ to the power of -1 is 3, Base ⅓ to the power of -2 is 9, and so on. This means that 9 in Base ⅓ is 0.01! But wait! This is the exact same thing as Base 3, but reversed! In Base 3, the one's digit represents 3, second for 9, third for 27, and so on and so forth. In Base ⅓, it is the exact same thing, except it is to the right of the decimal point. This means that they share the same exact digits being used. Base ⅓ uses the digits 0,1,2,3, just like Base 3.

Mystery 4: Base 5/4?

When you have a fraction like this, it's a little bit complex. But there is a very cool way to calculate this. But first, we have to start with Base 5. How do you turn a 364 to Base 5? Well there is a trick to find this. Start by dividing 364 by 5. You get a remainder of 4 and a quotient of 72. Now, divide the quotient, 72, by 5 again. Now you get a remainder of 2 and a quotient of 14. Now divide 14 by 5, getting us a remainder of 4 and a quotient of 2. Divide 2 by 5 and get a remainder of 2 and a quotient of 0. Now take all the remainders starting from the last one to the first and line them up. What do you get? 2424. This is 364 in Base 5. Now, apply this to 5/4, but after you divide by 5, multiply by 4. Take that number and divide it by 5, and take the remainder from the bottom to the top, and now, you have just made a number in Base 5/4.

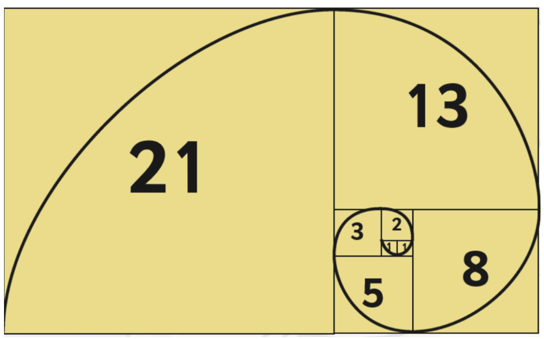
Mystery 5: Base ⅘?

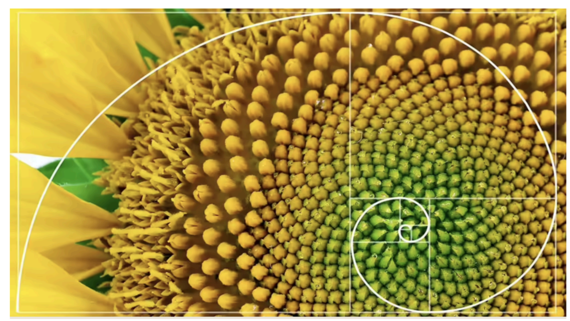
When you use the trick we used to find numbers converted in Base 5/4 for this type of base, you'll find that the numbers you are dividing by are getting bigger, and you will just be in an infinite loop. So for numbers with numerators that are less than the denominators, you could use the same trick we used for Base ⅓. For Base ⅓, we found a connection with its reciprocal, 3. Because ⅘ is the reciprocal of 5/4, we can flip the digits from base 5/4. For example, if we have a decimal, say 45 hundredths in Base 5/4, we could flip it to make 540. This might be a little confusing, but bear with me. Well for both fractions, how do we find how many numbers are used? Well, if A< B, use digits from 0 to B-1. If A>B, use digits from 0 to A-1.

Phi

Do you think numbers are actually in this world? In other words, do you think humans invented numbers, or have numbers just been waiting, hidden in nature? Well, Phi might change your answer.

But to understand Phi first, we must talk about a simpler topic: the Fibonacci Sequence. The Fibonacci Sequence is a sequence of numbers that start with 1, 1, and each number after that is the sum of the two numbers before it. So, the third number would be 1 + 1 = 2. The sequence starts 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89... Found by Italian mathematician Fibonacci, this sequence is most commonly used to map a spiral pattern, like in figure 1.1 below. Squares of side length of each term of the sequence are placed in a pattern like shown below, and a curve is drawn through each square.





The Fibonacci Sequence in a Sunflower's seeds

Figure 1.1

However, there is much more to this than to the Fibonacci than just some numbers. The Golden Ratio, also known as Phi, is also hidden in this sequence. Calculate the ratio between one term and the term before it. Start from the second term and keep on calculating:

1:1 = 1, 2:1 = 2, 3:2 = 1.5, 5:3 = 1.6..., 8:5 = 1.6, 13:8 = 1.625, Keep on calculating.

You might not see the pattern, but if you calculate to enough terms, you can see the ratios gradually come to 1.618033... This irrational(going forever) ratio of phi might seem quite irrelevant and useless, but just you wait. Phi, also called the Golden Ratio, is seen everywhere!

We can start with Fibonacci in nature. If you look carefully, almost all plants have petals that come in fibonacci terms. Each layer can start with 3 and keep on going. This may not seem like much—Yet. The angle that each petal made by two consecutive petals is often around 0.1618033... out of a 360 degree full circle. This allows each leaf to have the most access to sunlight and photosynthesis. In addition, trees also follow such a pattern. Branches are observed to split from tree trunks only at a measurement of a fibonacci term. The same can be said about roots and algae. Lastly, measurements of animal's body parts and human faces follow closely with fibonacci and phi. Even though every creature's body is different, they all seem to be around the fibonacci numbers.

Aside from the ones mentioned here, there are even more significant appearances of phi and the fibonacci in nature and especially in our art, infrastructure and even music! For example, many famous art pieces like the Mona Lisa, the Last Dinner, and so on, can create many examples of phi. But why is there so many, too many, appearances of phi? Compared to any other random ratio, is Phi even significant? If so, why is Phi significant? I will leave that for you to research and discover. If you find anything cool, please tell us!

e

Many kids think that calculus is the hardest subject. And that is maybe true. e stands for Euler's Number. The value of e is 2.718281828459045... so on. Just like pi, e is an irrational number. If you haven't read Sooriyan's newspaper, make sure to read that. It talks about pi.

There are many ways of calculating the value of *e*, but none of them ever give a totally exact answer, because *e* is irrational and its digits go on forever without repeating.

But it is known to have over 1 trillion digits of accuracy!

For example, the value of $(1 + 1/n)^n$ approaches *e* as *n* gets bigger and bigger:

Try it! Put "(1 + 1/100000)^100000" into the calculator:

$$(1 + 1/100000)^{100000}$$

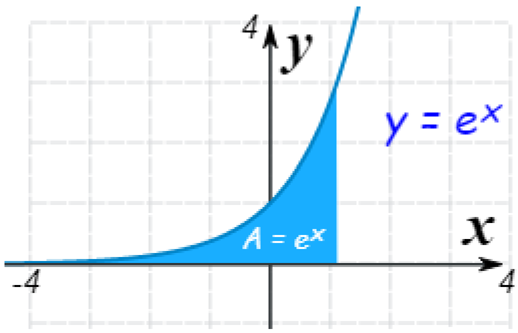
What do you get?

= 2.71666...

In fact Euler himself used this method to calculate *e* to 18 decimal places.

A cool fact is that if you multiply the square root of -1 (a type of imaginary number) by π, and call this value n, e^n will equal -1! So e^iπ=-1. As you see with this graph, e^x is never negative. And if you are confused, i=√-1 which means something times the same thing equals -1, yet a square can't be real if it is a negative number, so, we use i for imaginary.

The area up to any x-value is also equal to e^x :



100 Decimal Digits

Here is *e* to 100 decimal digits:

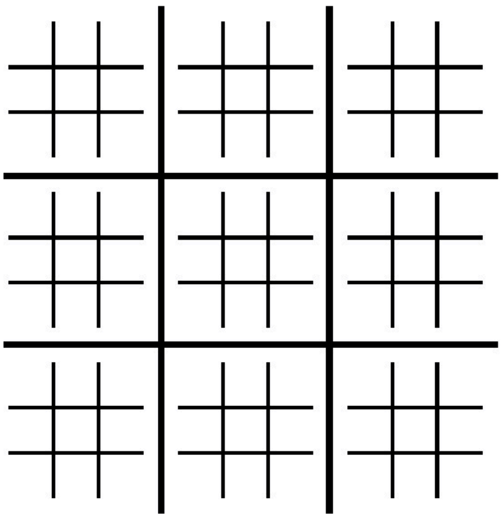
2.71828182845904523536028747135266249775724709369995957
49669676277240766303535475945713821785251664274...

To start, 1 has an end, so does ½, but does ⅙, and remember, after ⅙, every denominator will be a multiple of 3, and just to prove it, ⅙= 0.16666666666666666666666666666666666666666.........
which also has infinite digits, so if we multiply this in a calculator (just the first 2 6s) it gets 0.996, so it will go to 0.999.......6 if you multiply, which is the same as 0.999... and we know how *that* is 1.

That's all about e. I hope you enjoyed it. Next, I will teach you how to play ultimate tic tac toe.

How to play ultimate tic tac toe

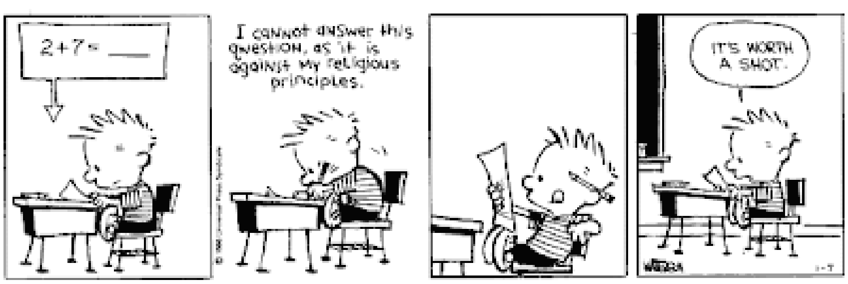
Ultimate tic-tac-toe is like normal tic tac toe but
more varieties of ways to place your mark on yo

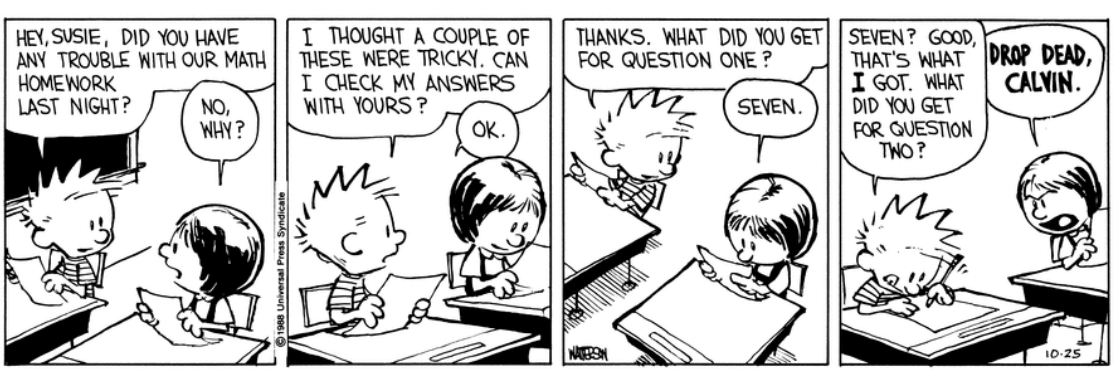


The board looks like this). The first person can pl
wherever she or he wants. The next player need:
place their turn on the corresponding place on th

. So, if player 1 places their turn on the very middle
layer 2 has to place their turn in the middle small
. If player 2 then plays in the top left corner
e of the middle board, then player 1 must play in
p left board in their next turn. If a person gets 3
in a row on the small board, they win that board.
a person wins 3 boards in a row, they win the
There are many choices to make in the ultimate
toe game. Have fun!

Math comics!!!





About the authors

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