

# 1 Supplemental 1

## 1.1 Derivation of Eq. 1 in the main text

We assume that there are  $n$  species for which per capita growth rates ( $\mu$ ) are measured at different temperatures ( $T$  in Kelvin) in the pooled dataset. For species  $j$ , there are  $m_j$  paired measurements of temperature and growth rate. Because we only use the data with the temperatures below optimal growth temperature ( $T_{opt}$ ) to compute  $E_{app}$ , we can assume a linear equation for the relationship between  $\ln$  growth rate ( $y = \ln\mu$ ) and Boltzmann temperature ( $x = \frac{1}{k_b}(\frac{1}{T_r} - \frac{1}{T})$ ) of each species:

$$y_{ij} = E_{intra,j}x_{ij} + b_j + \epsilon_{ij} \quad (1)$$

in which  $E_{intra,j}$  is the intraspecific activation energy of species  $j$ .  $y_{ij}$  and  $x_{ij}$  is the  $i^{th}$  measurement of  $\ln$  growth rate and Boltzmann temperature of species  $j$ .  $b_j$  is the growth rate normalized to the reference temperature  $T_r$ .  $\epsilon_{ij}$  is the residual of the  $i^{th}$  measurement of species  $j$  and has the mean of 0 and variance of  $\sigma_j^2$  ( $\epsilon_{ij} \sim N(0, \sigma_j^2)$ ).

The total number of paired observations in the pooled dataset is  $M = \sum_{j=1}^n m_j$ . The mean Boltzmann temperature of species  $j$  is  $\bar{x}_j = \frac{\sum_{i=1}^{m_j} x_{ij}}{m_j}$ . The grand mean Boltzmann temperature of the pooled dataset is defined as  $\bar{\bar{X}} = \frac{1}{M} \sum_{j=1}^n m_j \bar{x}_j$ .

We assume that  $b_j$  is a linear function of  $\bar{x}_j$ , which can be fitted via a weighed ordinary least-squares (OLS) regression shown below:

$$b_j = E_{inter}\bar{x}_j + b_0 + \beta_j \quad (2)$$

in which  $E_{inter}$  and  $b_0$  are the slope and intercept, respectively, that minimize the weighed sum of residual squares ( $\sum_{j=1}^n \frac{m_j}{M} \beta_j^2$ ).  $E_{inter}$  can be considered as a form of interspecific activation energy as explained in the main text. We assume that the residual  $\beta_j$  follows a normal distribution with a weighed mean 0 and variance  $\sigma_\beta^2$  (i.e.,  $\frac{m_j}{M} \beta_j \sim N(0, \sigma_\beta^2)$ ).

The mean of  $b_j$  ( $\bar{b}$ ) can be calculated as:

$$\begin{aligned}
\bar{b} &= \frac{1}{M} \sum_{j=1}^n m_j b_j \\
&= \frac{1}{M} \sum_{j=1}^n m_j (E_{inter} \bar{x}_j + b_0 + \beta_j) \\
&= E_{inter} \left( \sum_{j=1}^n \frac{m_j}{M} \bar{x}_j \right) + b_0 + \sum_{j=1}^n \frac{m_j}{M} \beta_j \\
&= E_{inter} \bar{\bar{X}} + b_0
\end{aligned} \tag{3}$$

The grand mean of  $y$  is  $\bar{\bar{Y}} = \frac{1}{M} \sum_{j=1}^n m_j \bar{y}_j = \frac{1}{M} \sum_{j=1}^n m_j (E_{intra,j} \bar{x}_j + b_j) = \frac{1}{M} \sum_{j=1}^n m_j E_{intra,j} \bar{x}_j + \bar{b}$ .

$E_{app}$  is calculated as the slope of the (OLS) regression of  $y$  against  $x$ :

$$E_{app} = \frac{\sum_{j=1}^n \sum_{i=1}^{m_j} (x_{ij} - \bar{\bar{X}}) y_{ij}}{\sum_{j=1}^n \sum_{i=1}^{m_j} (x_{ij} - \bar{\bar{X}})^2} \tag{4}$$

The numerator of Eq.(4) can be manipulated as:

$$\begin{aligned}
\sum_{j=1}^n \sum_{i=1}^{m_j} (x_{ij} - \bar{\bar{X}}) y_{ij} &= \sum_{j=1}^n \sum_{i=1}^{m_j} (x_{ij} - \bar{\bar{X}}) (E_{intra,j} x_{ij} + b_j + \epsilon_{ij}) \\
&= \sum_{j=1}^n \sum_{i=1}^{m_j} (x_{ij} - \bar{\bar{X}}) (E_{intra,j} x_{ij} - E_{intra,j} \bar{\bar{X}} + E_{intra,j} \bar{\bar{X}} + b_j + \epsilon_{ij}) \\
&= \sum_{j=1}^n E_{intra,j} \sum_{i=1}^{m_j} (x_{ij} - \bar{\bar{X}})^2 + \sum_{j=1}^n \sum_{i=1}^{m_j} (x_{ij} - \bar{\bar{X}}) (E_{intra,j} \bar{\bar{X}} + b_j - \bar{b}) + \sum_{j=1}^n \sum_{i=1}^{m_j} (x_{ij} - \bar{\bar{X}}) \epsilon_{ij}
\end{aligned} \tag{5}$$

Because  $b_j - \bar{b} = E_{inter}(\bar{x}_j - \bar{\bar{X}}) + \beta_j$ , we have:

$$\begin{aligned}
\sum_{j=1}^n \sum_{i=1}^{m_j} (x_{ij} - \bar{X})(E_{intra,j} \bar{X} + b_j - \bar{b}) &= \sum_{j=1}^n (E_{intra,j} \bar{X} + b_j - \bar{b} + \beta_j) \sum_{i=1}^{m_j} (x_{ij} - \bar{X}) \\
&= \sum_{j=1}^n (E_{intra,j} \bar{X} + b_j - \bar{b} + \beta_j) m_j (\bar{x}_j - \bar{X}) \\
&= \sum_{j=1}^n [E_{intra,j} \bar{X} + E_{inter}(\bar{x}_j - \bar{X}) + \beta_j] m_j (\bar{x}_j - \bar{X}) \\
&= \bar{X} \sum_{j=1}^n m_j E_{intra,j} (\bar{x}_j - \bar{X}) + E_{inter} \sum_{j=1}^n m_j (\bar{x}_j - \bar{X})^2 + \sum_{j=1}^n m_j \beta_j (\bar{x}_j - \bar{X})
\end{aligned} \tag{6}$$

We can show that:

$$\begin{aligned}
\sum_{j=1}^n \frac{m_j}{M} E_{intra,j} (\bar{x}_j - \bar{X}) &= \sum_{j=1}^n \frac{m_j}{M} (E_{intra,j} - \overline{E_{intra}}) (\bar{x}_j - \bar{X}) \\
&= Cov(E_{intra}, \bar{x})
\end{aligned} \tag{7}$$

in which  $\overline{E_{intra}} = \frac{\sum_{j=1}^n m_j E_{intra,j}}{M}$ .  $Cov(E_{intra}, \bar{x})$  is the covariance between  $E_{intra,j}$  and  $\bar{x}_j$  with unequal probability  $\frac{m_j}{M}$ . Similarly,  $\sum_{j=1}^n \frac{m_j}{M} (\bar{x}_j - \bar{X})^2$  is the variance of  $\bar{x}_j$  and  $\sum_{j=1}^n \frac{m_j}{M} \beta_j (\bar{x}_j - \bar{X})$  is the covariance between  $\beta_j$  and  $\bar{x}_j$  ( $Cov(\beta, \bar{x})$ ), both with unequal probabilities  $\frac{m_j}{M}$ .

Similarly, the last term at the right-hand side of Eq. 5 becomes:

$$\sum_{j=1}^n \sum_{i=1}^{m_j} (x_{ij} - \bar{X}) \epsilon_{ij} = MCov(\epsilon, x) \tag{8}$$

Therefore,  $E_{app}$  can be decomposed as:

$$E_{app} = \frac{\sum_{j=1}^n E_{intra,j} \sum_{i=1}^{m_j} (x_{ij} - \bar{X})^2}{MVar(x)} + E_{inter} \frac{Var(\bar{x})}{Var(x)} + \bar{X} \frac{Cov(E_{intra}, \bar{x})}{Var(x)} + \frac{Cov(\beta, \bar{x})}{Var(x)} + \frac{Cov(\epsilon, x)}{Var(x)} \tag{9}$$

Eq. 9 can be interpreted as follows.  $\langle E_{intra} \rangle = \frac{\sum_{j=1}^n E_{intra,j} \sum_{i=1}^{m_j} (x_{ij} - \bar{X})^2}{MVar(x)}$  is the variance-weighted mean intraspecific activation energy.  $E_{inter} \frac{Var(\bar{x})}{Var(x)}$  represents the contribution of interspecific thermal

adaptation to  $E_{app}$ . The covariance term of  $Cov(E_{intra}, \bar{x})$  can be interpreted as the contribution of the covariance of  $E_{intra}$  and  $\bar{x}$  to  $E_{app}$ . If warm adapted species tends to have a greater  $E_{intra}$ ,  $E_{app}$  will be greater, and vice versa. The second covariance term ( $Cov(\beta, \bar{x})$ ) can be interpreted as the contribution of covariance between  $\beta$ , the residuals of the linear regression of log growth rate normalized to a reference temperature ( $b_j$ ) against mean temperature ( $\bar{x}_j$ ) of each taxon, and  $\bar{x}$ . In other words, if the relationship between  $b_j$  and  $\bar{x}_j$  is curvilinearly convex,  $E_{app}$  will be greater, and vice versa. Likewise,  $Cov(\epsilon, x)$  is the covariance between  $\epsilon$ , the residuals of each individual growth rate in each OLSR that estimated  $E_{intra}$  for each taxon, and temperature ( $x$ ). If in general the relationship between log growth rate and temperature within each taxon is curvilinearly convex, the final  $E_{app}$  will be greater, and vice versa.

Note that because  $Cov(E_{intra}, \bar{x})$ ,  $Cov(\beta, \bar{x})$  and  $Cov(\epsilon, x)$  are negligible compared to the first two terms at the right side in our datasets, Eq. (9) can be approximated as:

$$E_{app} \approx \langle E_{intra} \rangle + E_{inter} \frac{Var(\bar{x})}{Var(x)} \quad (10)$$

## 1.2 Relationship between $E_{app}$ , $E_L$ , and $E_{inter}$

The interspecific (long-term) activation energy can also be expressed by the slope of  $\ln$  maximal growth rate ( $y_m = \ln \mu_m$ ) and Boltzmann optimal temperature ( $x_m$ ):

$$y_{m,j} = E_L x_{m,j} + B_0 + \nu_j \quad (11)$$

in which  $y_{m,j}$  is the maximal  $\ln$  growth rate of species  $j$ .  $x_{m,j}$  is the Boltzmann optimal temperature of species  $j$ .  $E_L$  is the regression slope of the OLS regression line between  $y_m$  and  $x_m$  weighed by the number of measurements  $m_j$ .  $E_L$  is often used as the interspecific (long-term) activation energy in the literature (1; 6).  $E_L$  differs from  $E_{inter}$  in that  $E_L$  is zero in the case of perfect adaptation ( $E_{inter} < 0$ ) and equals to  $E_{intra}$  if there is no adaptation at all ( $E_{inter} = 0$ ).  $B_0$  is the regression intercept which is a constant.  $\nu_j$  is the residual of species  $j$  which follows a normal distribution with a weighed mean of zero and variance of  $\sigma_\nu^2$  (i.e.,  $\frac{m_j}{M} \nu_j \sim N(0, \sigma_\nu^2)$ ).

To examine the relationship between  $E_{app}$  and  $E_L$ , we express the  $\ln$  growth rate at the  $i^{th}$

temperature of species  $j$  as a function of  $x_{m,j}$  and  $y_{m,j}$  instead of Eq. 1:

$$y_{i,j} = E_{intra,j}(x_{ij} - x_{m,j}) + y_{m,j} + \xi_{ij} \quad (12)$$

in which  $x_{ij}$ ,  $y_{ij}$ ,  $E_{intra,j}$ ,  $x_{m,j}$  and  $y_{m,j}$  are the same as defined in Eq. 1 and 11.  $\xi_{ij}$  is the residual of the  $i^{th}$  measurement of species  $j$  and follows a normal distribution ( $\xi_{ij} \sim N(0, \sigma_{\xi,j}^2)$ ).

To prepare for the following derivation, we need to define the average of  $x_{m,j}$  ( $\overline{X_m}$ ) and the average of  $y_{m,j}$  ( $\overline{Y_m}$ ) as  $\overline{X_m} = \frac{1}{M} \sum_{j=1}^n m_j x_{m,j}$  and  $\overline{Y_m} = \frac{1}{M} \sum_{j=1}^n m_j y_{m,j}$ , respectively.

The numerator of Eq.(4) can be rewritten as:

$$\begin{aligned} \sum_{j=1}^n \sum_{i=1}^{m_j} (x_{ij} - \overline{X}) y_{ij} &= \sum_{j=1}^n \sum_{i=1}^{m_j} (x_{ij} - \overline{X}) [E_{intra,j}(x_{ij} - x_{m,j}) + y_{m,j} + \xi_{ij}] \\ &= \sum_{j=1}^n \sum_{i=1}^{m_j} (x_{ij} - \overline{X}) [E_{intra,j}(x_{ij} - \overline{X}) + E_{intra,j}(\overline{X} - x_{m,j}) + y_{m,j} + \xi_{ij}] \\ &= \sum_{j=1}^n E_{intra,j} \sum_{i=1}^{m_j} (x_{ij} - \overline{X})^2 - \sum_{j=1}^n E_{intra,j} (x_{m,j} - \overline{X}) \sum_{i=1}^{m_j} (x_{ij} - \overline{X}) \\ &\quad + \sum_{j=1}^n y_{m,j} \sum_{i=1}^{m_j} (x_{ij} - \overline{X}) + \sum_{j=1}^n \sum_{i=1}^{m_j} \xi_{ij} (x_{ij} - \overline{X}) \\ &= \sum_{j=1}^n E_{intra,j} \sum_{i=1}^{m_j} (x_{ij} - \overline{X})^2 - \sum_{j=1}^n m_j E_{intra,j} (x_{m,j} - \overline{X}) (\overline{x_j} - \overline{X}) \\ &\quad + \sum_{j=1}^n m_j y_{m,j} (\overline{x_j} - \overline{X}) + \sum_{j=1}^n \sum_{i=1}^{m_j} \xi_{ij} (x_{ij} - \overline{X}) \\ &= \sum_{j=1}^n E_{intra,j} \sum_{i=1}^{m_j} (x_{ij} - \overline{X})^2 - \sum_{j=1}^n m_j E_{intra,j} (x_{m,j} - \overline{X}) (\overline{x_j} - \overline{X}) \\ &\quad + \sum_{j=1}^n m_j (E_L x_{m,j} + B_0 + \nu_j) (\overline{x_j} - \overline{X}) + \sum_{j=1}^n \sum_{i=1}^{m_j} \xi_{ij} (x_{ij} - \overline{X}) \\ &= \sum_{j=1}^n E_{intra,j} \sum_{i=1}^{m_j} (x_{ij} - \overline{X})^2 - \sum_{j=1}^n m_j E_{intra,j} x_{m,j} (\overline{x_j} - \overline{X}) + \overline{X} \sum_{j=1}^n m_j E_{intra,j} (\overline{x_j} - \overline{X}) \\ &\quad + E_L \sum_{j=1}^n m_j x_{m,j} (\overline{x_j} - \overline{X}) + \sum_{j=1}^n m_j \nu_j (\overline{x_j} - \overline{X}) + \sum_{j=1}^n \sum_{i=1}^{m_j} \xi_{ij} (x_{ij} - \overline{X}) \\ &= \sum_{j=1}^n E_{intra,j} \sum_{i=1}^{m_j} (x_{ij} - \overline{X})^2 \\ &\quad + M [E_L Cov(x_m, \overline{x}) - Cov(E_{intra} x_m, \overline{x}) + \overline{X} Cov(E_{intra}, \overline{x}) + Cov(\nu, \overline{x}) + Cov(\xi, \overline{x})] \end{aligned} \quad (13)$$

Thus,  $E_{app}$  can be expressed as:

$$E_{app} = \langle E_{intra} \rangle + E_L \frac{Cov(x_m, \bar{x})}{Var(x)} - \frac{Cov(E_{intra}x_m, \bar{x})}{Var(x)} + \bar{X} \frac{Cov(E_{intra}, \bar{x})}{Var(x)} + \frac{Cov(\nu, \bar{x})}{Var(x)} + \frac{Cov(\xi, x)}{Var(x)} \quad (14)$$

Realizing that the three terms,  $Cov(E_{intra}, \bar{x})$ ,  $Cov(\nu, \bar{x})$ , and  $Cov(\xi, x)$  are negligible compared to  $Var(x)$  (Table S1), Eq. 14 can be approximated as:

$$E_{app} \approx \langle E_{intra} \rangle + E_L \frac{Cov(x_m, \bar{x})}{Var(x)} - \frac{Cov(E_{intra}x_m, \bar{x})}{Var(x)} \quad (15)$$

By comparing Eq. 15 with Eq. 10, we can obtain an approximate relationship between  $E_L$  and  $E_{inter}$ :

$$E_L \approx \frac{E_{inter}Var(\bar{x}) + Cov(E_{intra}x_m, \bar{x})}{Cov(x_m, \bar{x})} \quad (16)$$

### 1.3 Estimates of covariance terms in Eq. 9 and Eq. 14 in autotrophic and heterotrophic prokaryotes and protists as well as insects

As described in the main text, we also applied the above framework onto autotrophic and heterotrophic prokaryotes as well as insects besides the dataset of protists. The dataset of autotrophic prokaryotes (i.e., cyanobacteria) was compiled at the same time as that of the autotrophic protists (2; 4). The dataset of heterotrophic prokaryotes was obtained from Smith et al.(6). The dataset of insects was obtained from Rezende et al.(5).

Table S1 shows that the covariance terms in Eq. 9 and Eq. 14 are usually negligible for all five groups of taxa, although the terms of  $\frac{Cov(\nu, \bar{x})}{Var(x)}$  tend to be greater than other terms which is another reason that Eq. 9 is preferred over Eq. 14.

Table S1. Estimates of covariance terms in Eq. 9 and Eq. 14 for autotrophic and heterotrophic prokaryotes and protists as well as insects.

Term	Autotrophic protists	Heterotrophic protists	Autotrophic prokaryotes	Heterotrophic prokaryotes	Insects
$\overline{X} \frac{Cov(E_{intra}, \bar{x})}{Var(x)}$	-0.005	-0.01	0.01	0.002	-0.025
$\frac{Cov(\beta, \bar{x})}{Var(x)}$	$6.0 \times 10^{-18}$	$1.2 \times 10^{-17}$	$-2.5 \times 10^{-17}$	$1.8 \times 10^{-17}$	$-2.1 \times 10^{-17}$
$\frac{Cov(\nu, \bar{x})}{Var(x)}$	-0.033	-0.07	0.01	-0.086	-0.11
$\frac{Cov(\xi, x)}{Var(x)}$	$3.2 \times 10^{-18}$	$7.7 \times 10^{-19}$	$-3.3 \times 10^{-18}$	$-3.2 \times 10^{-18}$	$-5.1 \times 10^{-18}$
$\frac{Cov(\epsilon, x)}{Var(x)}$	$2.2 \times 10^{-18}$	$-2.9 \times 10^{-18}$	$-7.9 \times 10^{-18}$	$-1.9 \times 10^{-18}$	$-7.2 \times 10^{-19}$

#### 1.4 Estimates of $E_{app}$ , $E_{intra}$ , $E_{inter}$ and $E_L$ and other relevant terms in autotrophic and heterotrophic prokaryotes and insects

The following Table S2 shows the estimated terms in Eq. 10 and the simplified Eq. 14 of autotrophic and heterotrophic prokaryotes and insects. The results show that  $E_{app}$  appears similar between autotrophic and heterotrophic prokaryotes, which results from an even greater Intraspecific activation energy ( $\langle E_{intra} \rangle$ ) in autotrophic prokaryotes than in heterotrophic prokaryotes but a stronger thermal adaptation (i.e., a more negative  $E_{inter}$ ) in autotrophic prokaryotes. Note that Smith et al.(6) did not observe a thermal adaptation in mesophilic bacteria either.

While the estimates of  $E_{app}$ ,  $E_{inter}$ , and  $E_L$  of insects appear to be biased by other confounding factors such as body size, the estimate of  $E_{intra}$  is consistent with previous studies (3).

Table S2. Estimates of various terms in Eq. 9 and Eq. 14 for autotrophic and heterotrophic prokaryotes and insects.

Term	Definition	Autotrophic prokaryotes	Heterotrophic prokaryotes	Insects
$n$	Number of taxa	145	81	45
$M$	Total number of paired observations	703	1300	185
$E_{app}$ (OLSR; Mean $\pm$ SE)	Apparent activation energy calculated via OLSR	$0.63 \pm 0.03$	$0.69 \pm 0.21$	$0.30 \pm 0.08$
$E_{inter}$ (Mean $\pm$ SE)	OLSR slope of $\ln$ normalized growth rate against $\bar{x}$	$-0.32 \pm 0.15$	$0.04 \pm 0.44$	$-1.38 \pm 0.22$
$E_L$ (Mean $\pm$ SE)	OLSR slope of $y_m$ against $x_m$	$0.53 \pm 0.09$	$1.02 \pm 0.36$	$-0.65 \pm 0.17$
$\langle E_{intra} \rangle$	Variance weighted mean $E_{intra}$	$0.76 \pm 0.04$	$0.67 \pm 0.06$	$0.89 \pm 0.03$
$E_{inter} \frac{Var(\bar{x})}{Var(x)}$	Interspecific term in Eq. 9	$-0.14 \pm 0.07$	$0.02 \pm 0.20$	$-0.57 \pm 0.08$
$E_L \frac{Cov(x_m, \bar{x})}{Var(x)}$	Interspecific (2 <sup>nd</sup> ) term in Eq. 14	$0.22 \pm 0.04$	$0.33 \pm 0.14$	$-0.22 \pm 0.06$
$Var(x)$	Variance of $x$	0.88	1.73	0.58
$Var(\bar{x})$	Variance of $\bar{x}$	0.40	0.81	0.24
$Var(x_m)$	Variance of $x_m$	0.47	0.93	0.19
$\frac{Cov(E_{intra} x_m, \bar{x})}{Var(x)}$	3 <sup>rd</sup> term in Eq. 14	$0.37 \pm 0.07$	$0.43 \pm 0.07$	$0.24 \pm 0.06$
Calculated $E_{app}$	$E_{app}$ calculated based on Eq. 9	$0.62 \pm 0.03$	$0.69 \pm 0.06$	$0.32 \pm 0.03$



## References

- [1] BARTON, S., AND YVON-DUROCHER, G. Quantifying the temperature dependence of growth rate in marine phytoplankton within and across species. *Limnology and Oceanography* 64, 5 (2019), 2081–2091.
- [2] CHEN, B., AND LAWS, E. A. Is there a difference of temperature sensitivity between marine phytoplankton and heterotrophs? *Limnology and Oceanography* 62, 2 (2017), 806–817.
- [3] FRAZIER, M., HUEY, R. B., AND BERRIGAN, D. Thermodynamics constrains the evolution of insect population growth rates: “warmer is better”. *The American Naturalist* 168, 4 (2006), 512–520.
- [4] KREMER, C. T., THOMAS, M. K., AND LITCHMAN, E. Temperature-and size-scaling of phytoplankton population growth rates: Reconciling the eppley curve and the metabolic theory of ecology. *Limnology and Oceanography* 62, 4 (2017), 1658–1670.
- [5] REZENDE, E. L., AND BOZINOVIC, F. Thermal performance across levels of biological organization. *Philosophical Transactions of the Royal Society B* 374, 1778 (2019), 20180549.
- [6] SMITH, T. P., THOMAS, T. J., GARCÍA-CARRERAS, B., SAL, S., YVON-DUROCHER, G., BELL, T., AND PAWAR, S. Community-level respiration of prokaryotic microbes may rise with global warming. *Nature communications* 10, 1 (2019), 1–11.