The Pattern Recognition 's homework

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1 Ex 1

The log-likelihood function is:

$$L(\mu) = \ln p(\mathscr{K}|\mu) = \sum_{i=1}^{N} lnp(x_i|\mu) = \frac{1}{2}\sum_{i=1}^{N}(x_i - \mu)^2 + C \tag{1}$$

The derivation of Ł(μ) ,we could get that :

$$\frac{\partial L(\mu)}{\partial \mu} = \sum_{i=1}^{N} x_i - N\mu = 0$$

So the maximum likelihood estimation of $\hat{\mu} = \frac{1}{N}\sum_{i=1}^{N} x_i$

The bayesian estimation :

First we get the posterior:

$$\begin{aligned}
p(\mu|\mathscr{K}) &= \frac{p(\mathscr{K}|\mu)p(\mu)}{\int p(\mathscr{K}|\mu)p(\mu)du} \\
&= \alpha \prod_{i=1}^{N} p(x_i|u)p(u) \\
&= \alpha \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi}\sigma} exp[-\frac{(x_i - u)^2}{2\sigma^2}] \cdot \sqrt{2\pi}\sigma_0 exp[-\frac{(u - u_0)^2}{2\sigma_0^2}] \\
&= \alpha^{'}[-\frac{1}{2}[(\frac{N}{\sigma^2} + \frac{1}{\sigma_0^2})\mu^2 - 2(\frac{1}{\sigma^2}\sum_{i=1}^{N} x_i + \frac{\sigma_0}{\sigma_0^2})\mu]]
\end{aligned}$$

According to the formula ,apply the method of undetermined coefficients ,we can get that :

$$\hat{u} = \int \mu p(\mu|\mathscr{K})du = \int \mu \frac{1}{\sqrt{2\pi}\sigma_N} exp[-\frac{1}{2}(\frac{u - u_N}{\sigma_N})^2]du = u_N$$

Combine with $\sigma = 1, u_0 = 0, \sigma_0 = 1$,we can get the bayesian estimation :

$$\hat{\mu} = \frac{1}{N+1}\sum_{i=1}^{N} x_i$$

2 Ex 2

According to the theory of maximum likelihood estimation and derivation of EX1,

$$l(\theta) = P(x|\theta) = \prod_{k=1}^{n} p(x_k|\theta)$$

$$H(\theta) = \ln l(\theta) = \sum_{k=1}^{n} \ln p(x_k|\theta) = \sum_{k=1}^{n} \sum_{i=1}^{d} [x_{ki} ln(\theta_i) + (1 - x_{ki} \ln(1 - \theta_i))]$$

In terms of $0 = \dfrac{\partial H(\theta)}{\partial \theta_i} = \sum_{k=1}^{n} [\dfrac{x_{ki}}{\theta_i} - \dfrac{1 - x_{ki}}{1 - \theta_i}]$

We get that:

$$\hat{\theta} = \frac{1}{n} \sum_{k=1}^{n} x_{ki}, i = 1, \cdots, d;$$

So,$\hat{\theta} = \dfrac{1}{n} \sum_{k=1}^{n} x_k$

3 Ex3

According to the theory of bayes estimation,