Pattern Recognition homework
Chapter 4

Wu Bingzhe
1200010666
The School of mathmatical science

October 23, 2015

1 Ex1

According to $x_j \in R_i, j = 1, 2$,we have :

$$g_i(x_j) \leq (\geq)0 \tag{1}$$

In terms of $g(\lambda x_1 + (1-\lambda)x_2) = \lambda g(x_1) + (1-\lambda)g(x_2)$, combine (1) and $0 \leq \lambda \leq 1$,we get:

$$g(\lambda x_1 + (1-\lambda)x_2) \leq (\geq)0 \tag{2}$$

So the decision area is convex

2 Ex2

2.1 (a)

The projection of x_a can be represented as :

$$x_a = x_p + r\frac{\omega}{\|\omega\|} \tag{3}$$

Where x_p is the projection point on the hyperplane ,r is the distance between the x_a and the hyperplane $g(x) = 0$.

Because the point x_p is on the hyperplane ,we have $g(x_p) = 0$.Further more:

$$\begin{aligned} g(x_a) &= \omega^T x_a + \omega_0 \\ &= \omega^T (x_p + r\frac{\omega}{\|\omega\|}) + \omega_0 \\ &= g(x_p) + r\|\omega\| \\ &= r\frac{\omega}{\|\omega\|} \end{aligned}$$

So we get $r = \dfrac{|g(x_a)|}{\|\omega\|}$

On the other hand,for an arbitrarily point x_q on the hyperplane ,

$$\begin{aligned}\|x_q - x_a\|^2 &= \|x_q - x_p + x_p - x_a\|^2 \\ &= \|x_q - x_a\|^2 + r^2 + (x_q - x_p)(x_p - x_a) \\ &= \|x_q - x_a\|^2 + r^2 \\ &\geq r^2\end{aligned}$$

The conclusion established.

2.2 (b)

In terms of $r = \frac{g(x_a)}{\|\omega\|}$, combine (3) ,we can get the projection of x_a is :

$$x_p = x_a - \frac{g(x_a)}{\|\omega\|^2}\omega$$

3 Ex3

3.1 (a)

Assumption y_i satisfy $a^T y_i \geq b$,hence,we have $a^T y_i \geq b \geq 0$. So after bringing the b,the solution area is located in the original problem area.

3.2 (b)

The boundary of the solution after bringing the b is a hyperplane $a^T y_i = b$,and the original boundary is another hyperplane $a^T y_i = 0$. So the distance between the two hyperplane is $d = \frac{b}{\|y_i\|}$

4 Ex4

The criterion function of perceptron has the form :

$$J(a) = \sum_{y \in Y} (-a^t y)$$

5 Ex5

6 Ex6