Discrete-Time Signals and Systems

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Discrete-Time Signals

- Graphical
- Functional
- Tabular
- Sequence

PRESENTATION

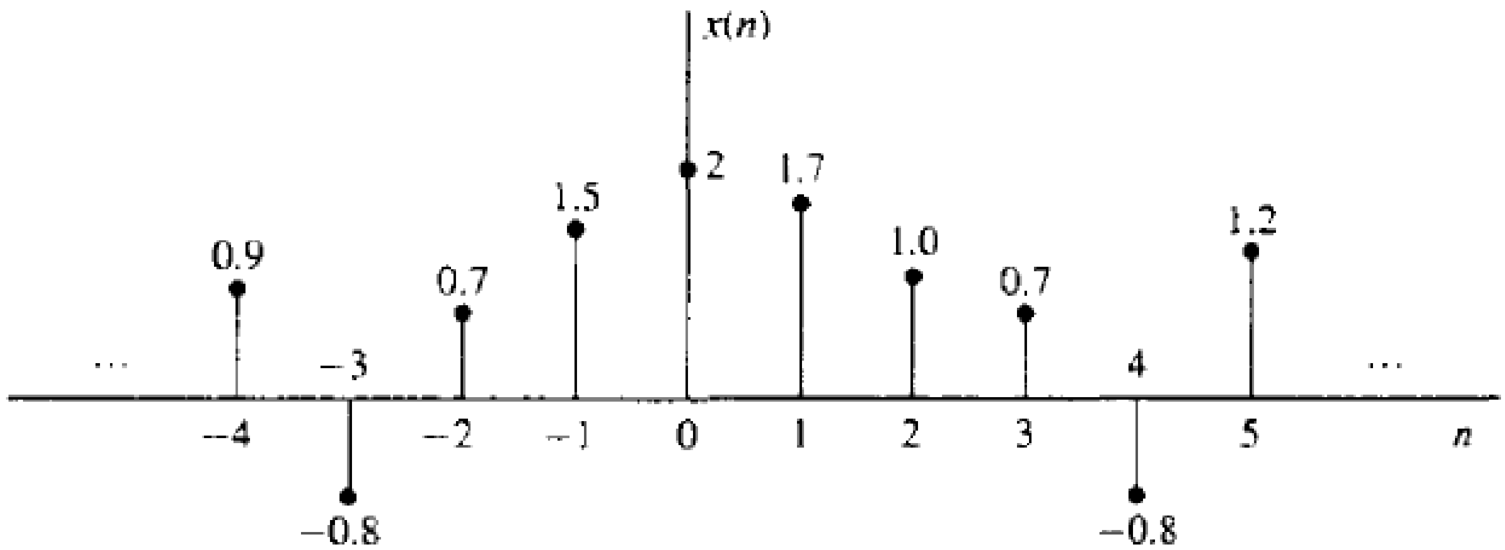


Figure 2.1.1 Graphical representation of a discrete-time signal

Some Elementary Discrete-Time Signals

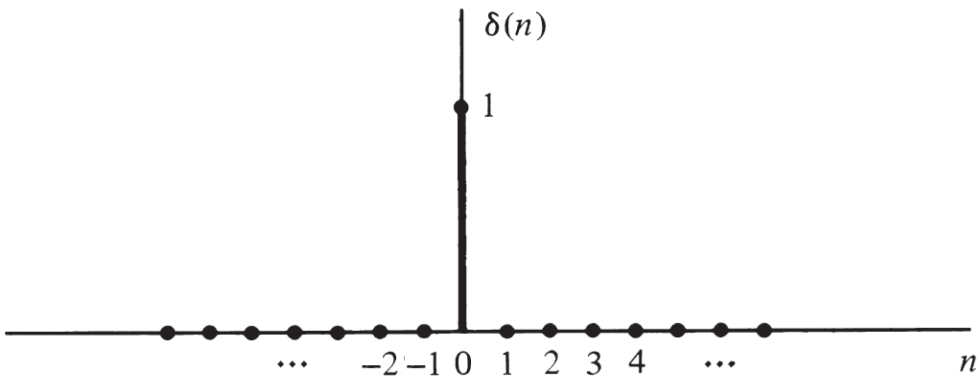


Figure 2.1.2 Graphical representation of the unit sample signal.

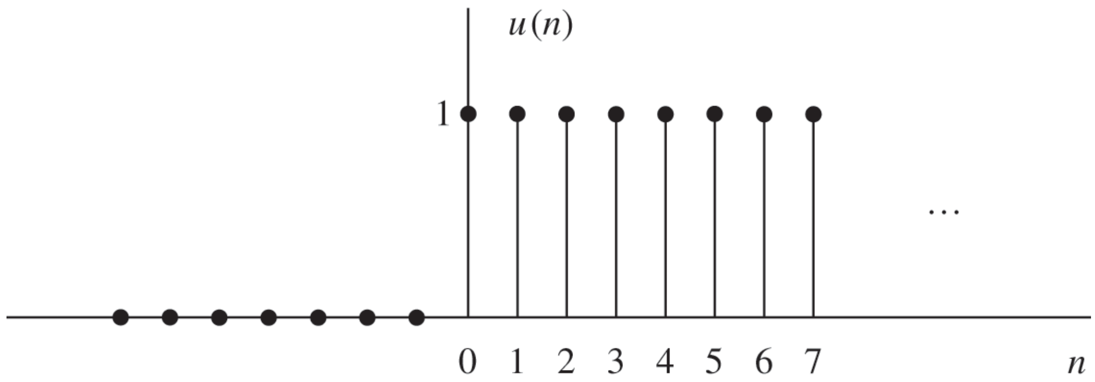


Figure 2.1.3 Graphical representation of the unit step signal.

Some Elementary Discrete-Time Signals

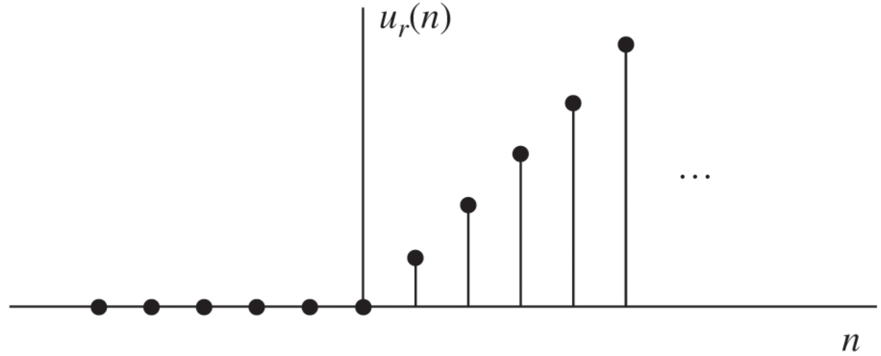


Figure 2.1.4 Graphical representation of the unit ramp signal.

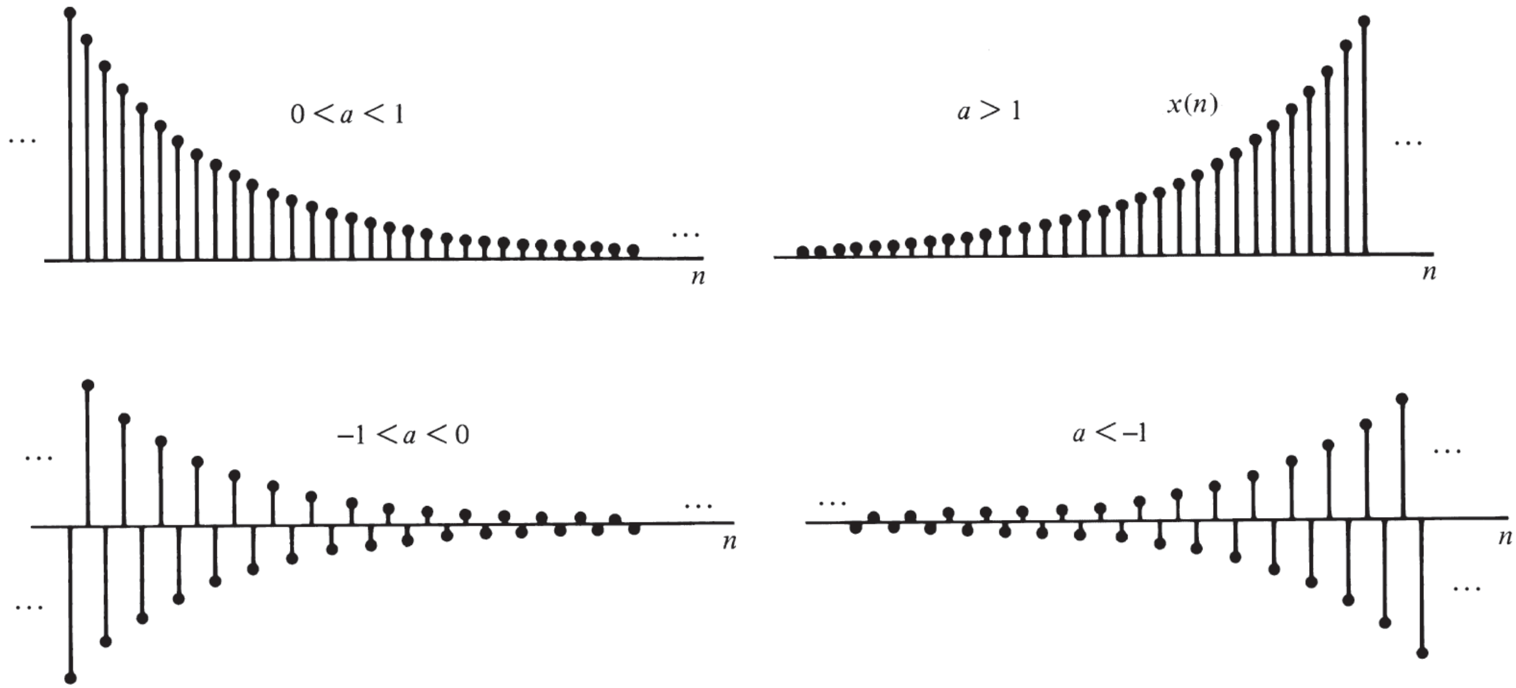


Figure 2.1.5 Graphical representation of exponential signals.

Classification of Discrete-Time Signals

- Energy signals and power signals
- Periodic signals and aperiodic signals
- Symetric (even) and antisymmetric (odd) signals

Simple Manipulations of Discrete-Time Signals

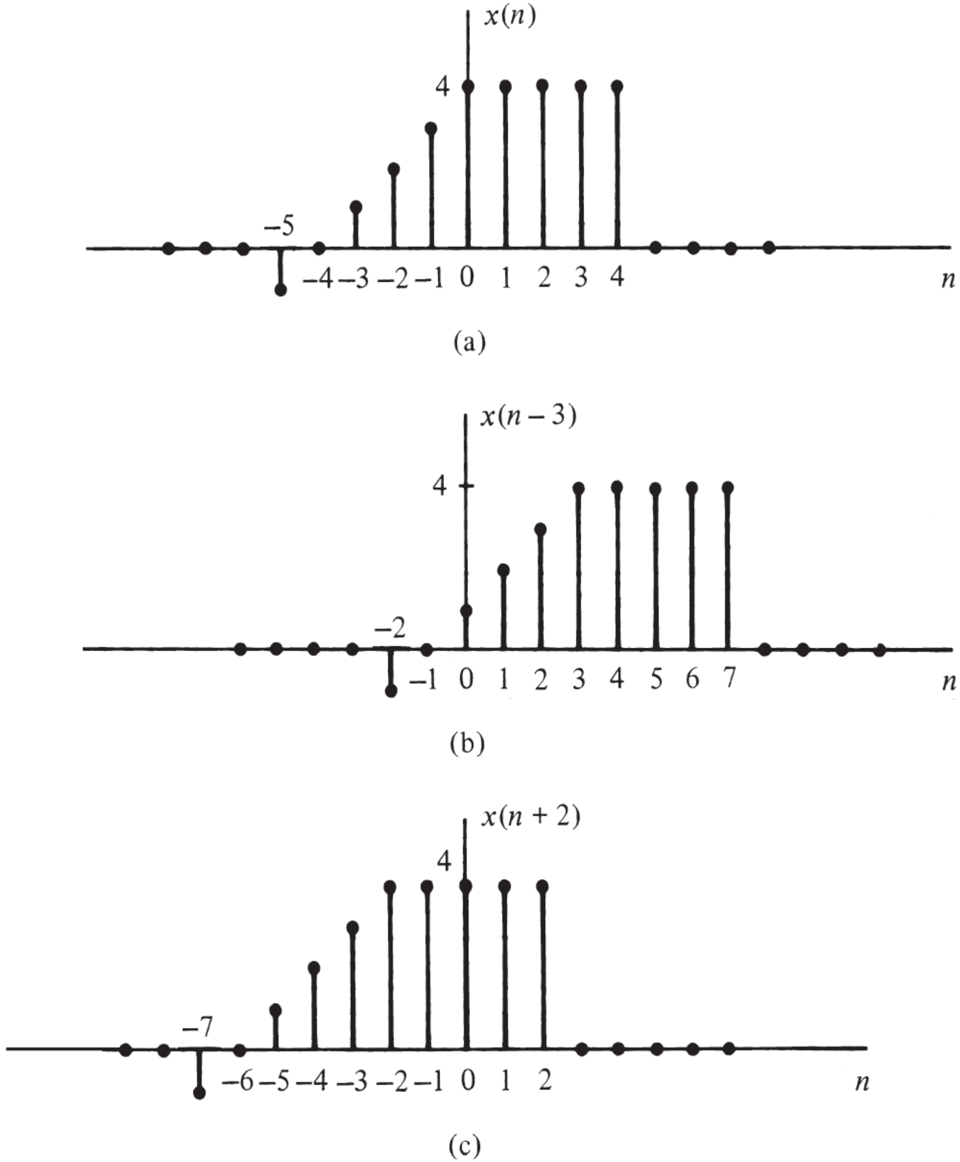


Figure 2.1.9 Graphical representation of a signal, and its delayed and advanced versions.

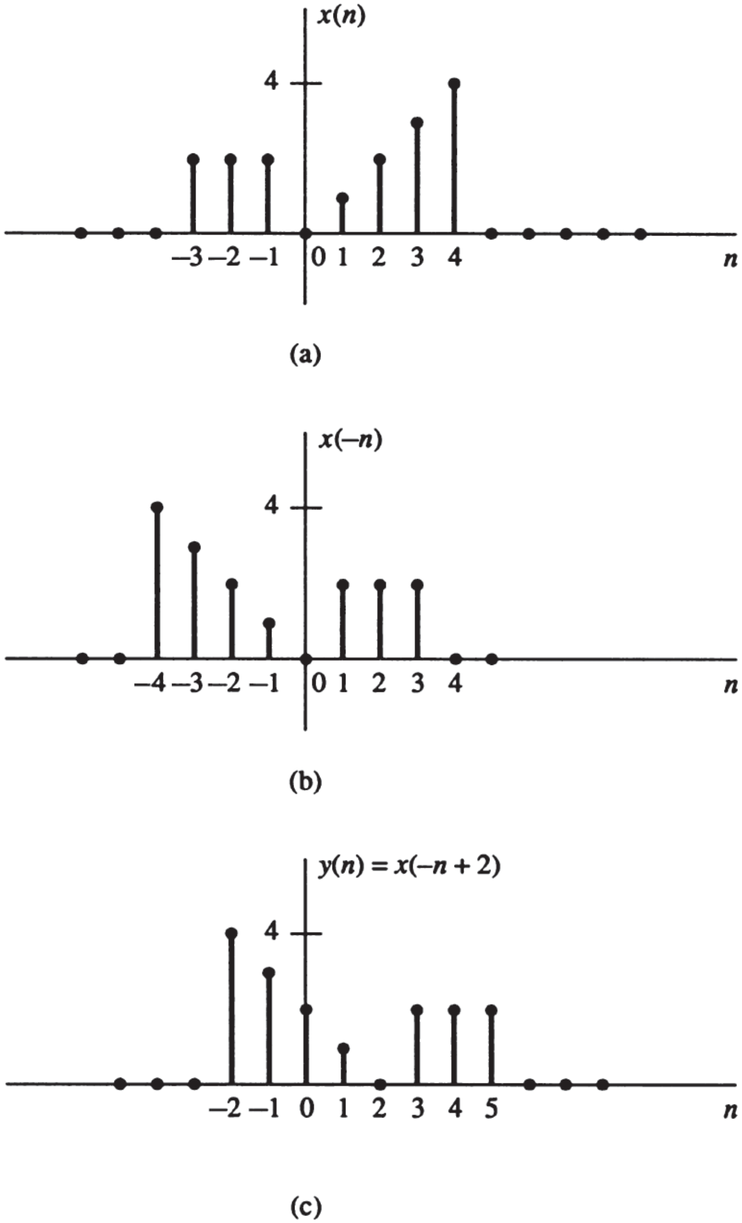


Figure 2.1.10 Graphical illustration of the folding and shifting operations.

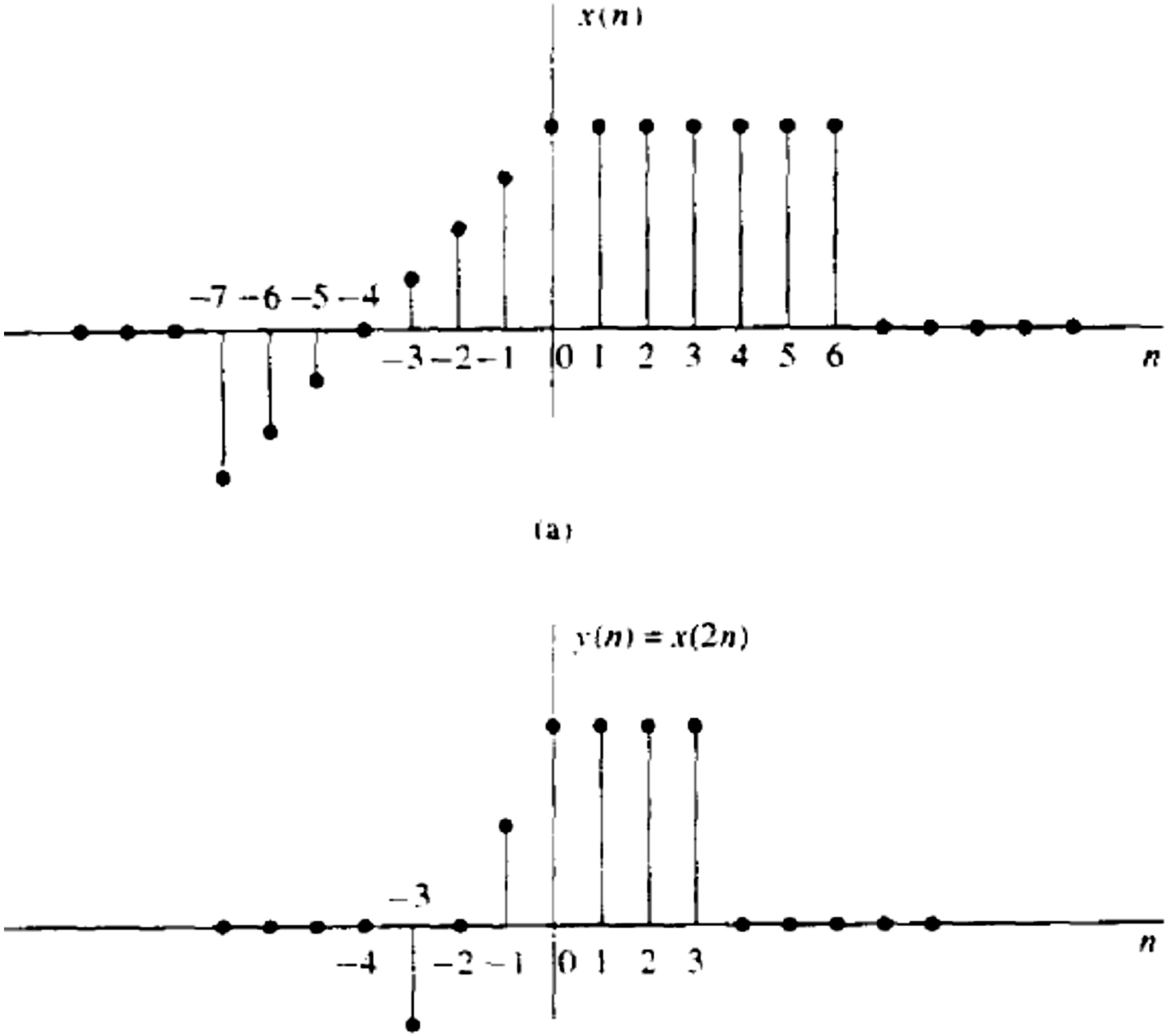


Figure 2.1.11 Graphical representation of down-sampling operation

Discrete-Time Systems

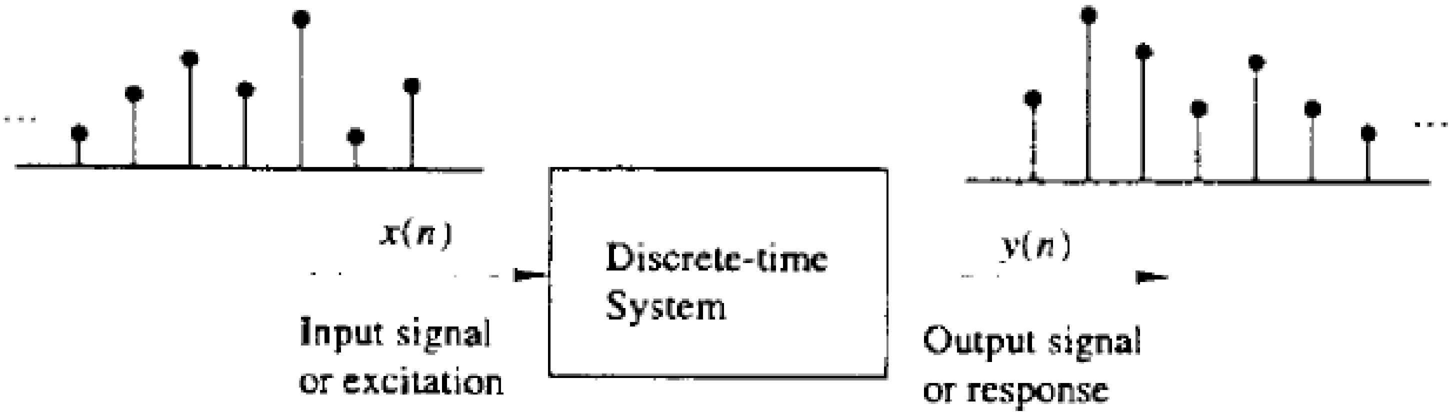
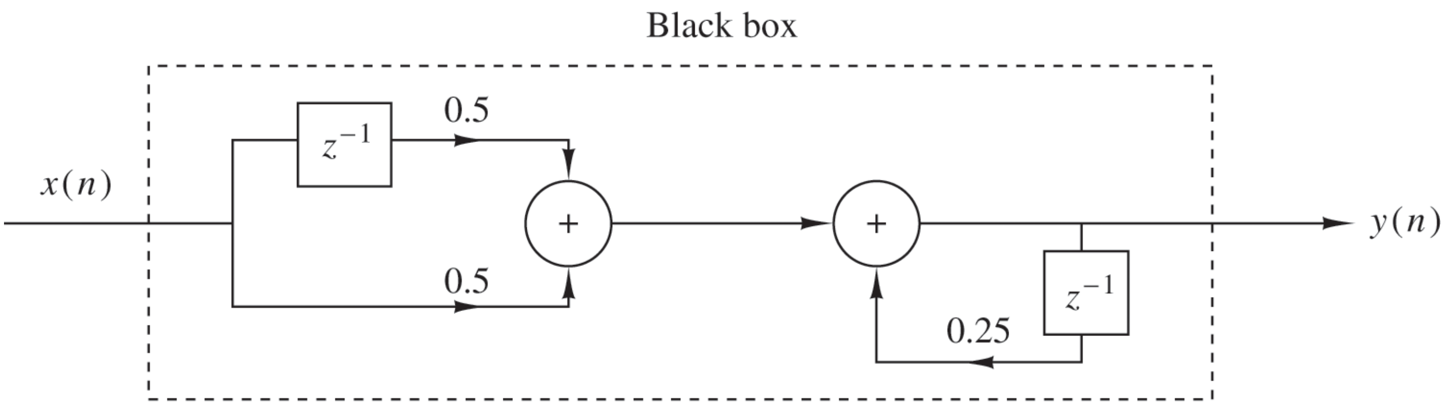


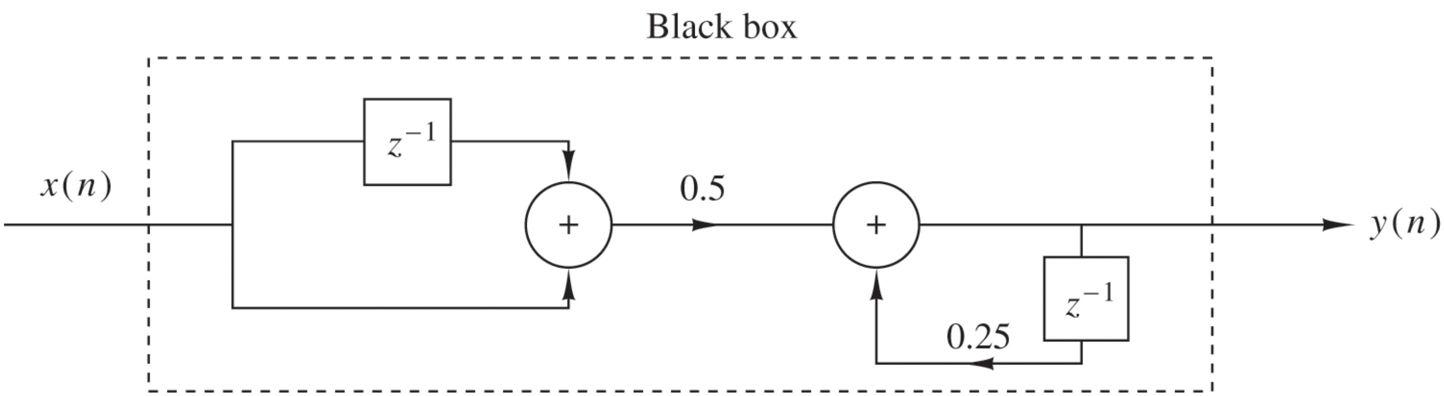
Figure 2.1.11 Block diagram representation of a discrete-time system

- Input-Output Description of Systems

- Block Diagram Representation of Discrete-Time Systems



(a)



(b)

Figure 2.2.7 Block diagram realizations of the system $y(n) = 0.25y(n-1) + 0.5x(n) + 0.5x(n-1)$.

Classification of Discrete-Time Systems

- Static vs dynamic systems
- Time-invariant vs time-variant systems
- Linear vs nonlinear systems
- Causal vs noncausal systems
- Stable vs unstable systems

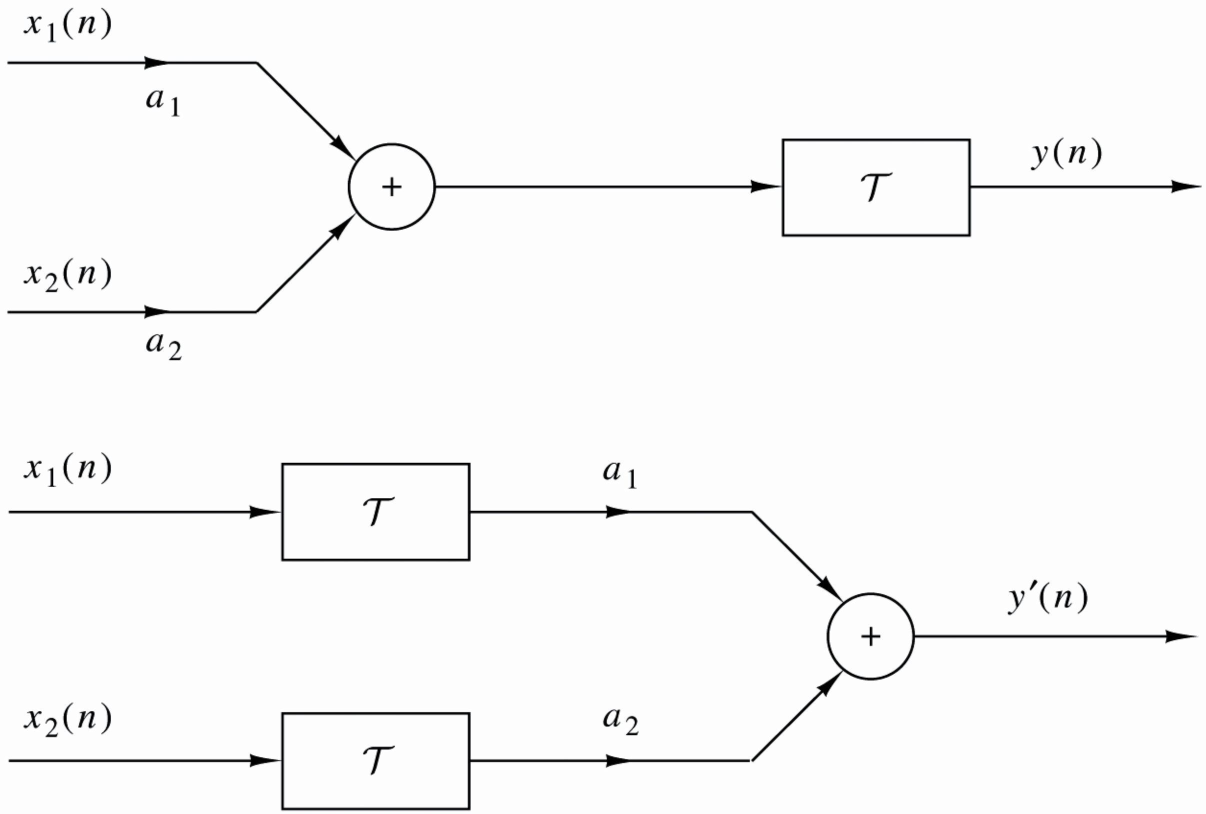


Figure 2.2.9 Graphical representation of the superposition principle. $\mathcal{T}$ is linear if and only if $y(n) = y'(n)$.

Interconnection of Discrete-Time Systems

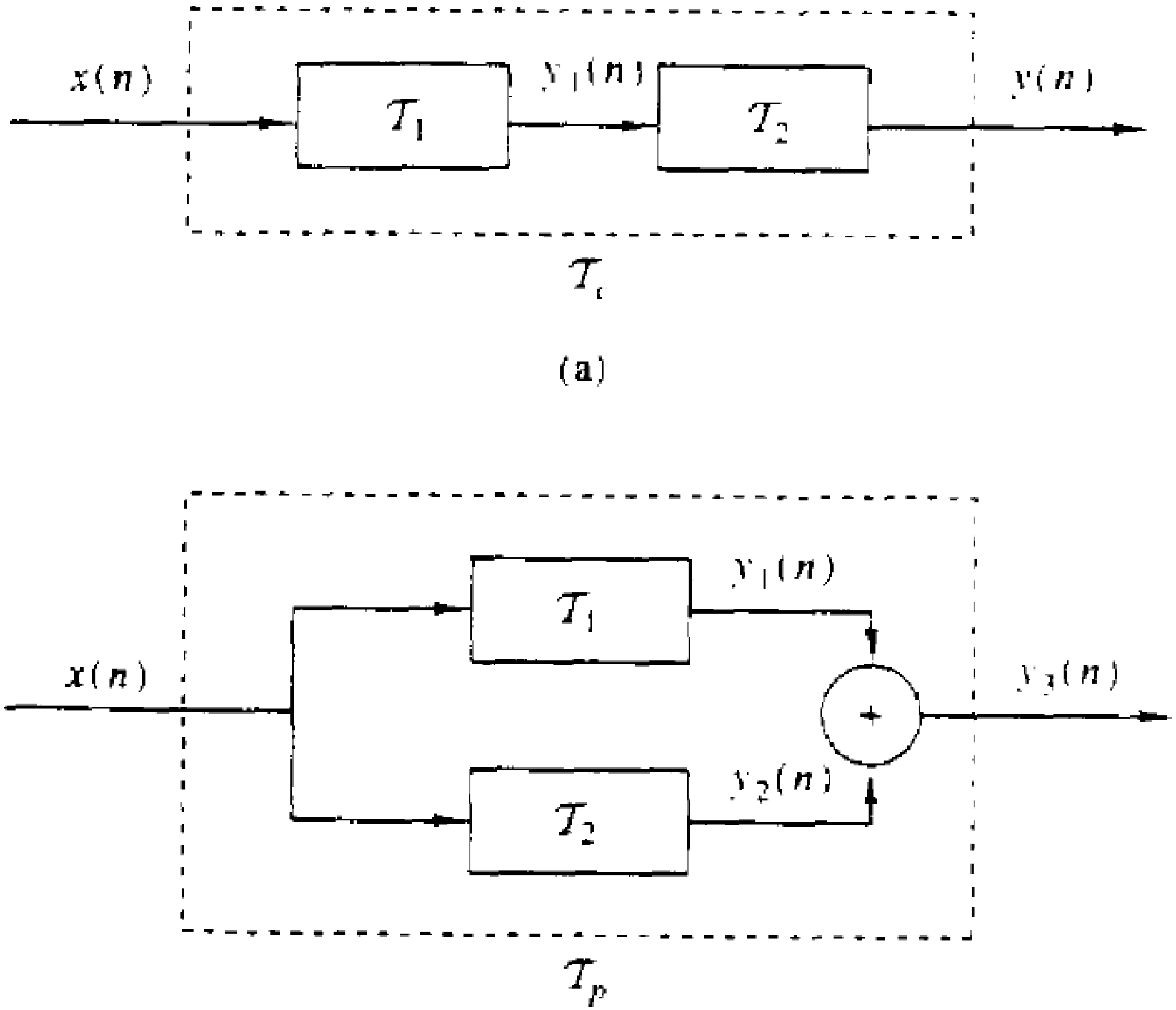


Figure 2.2.10 Cascade (a) and parallel (b) interconnections of systems

Analysis of Discrete-Time Linear Time-Invariant Systems

- Techniques for the Analysis of Linear Systems

    + Based on the direct solution of the in-out equation

    + Decompose input signal into a sum the of elementary signals, which are selected so that the response of the system to each component is easily determined.

Figure 2.2.10 Cascade (a) and parallel (b) interconnections of systems

Resolution of a Descrete-Time Signal into Impulses

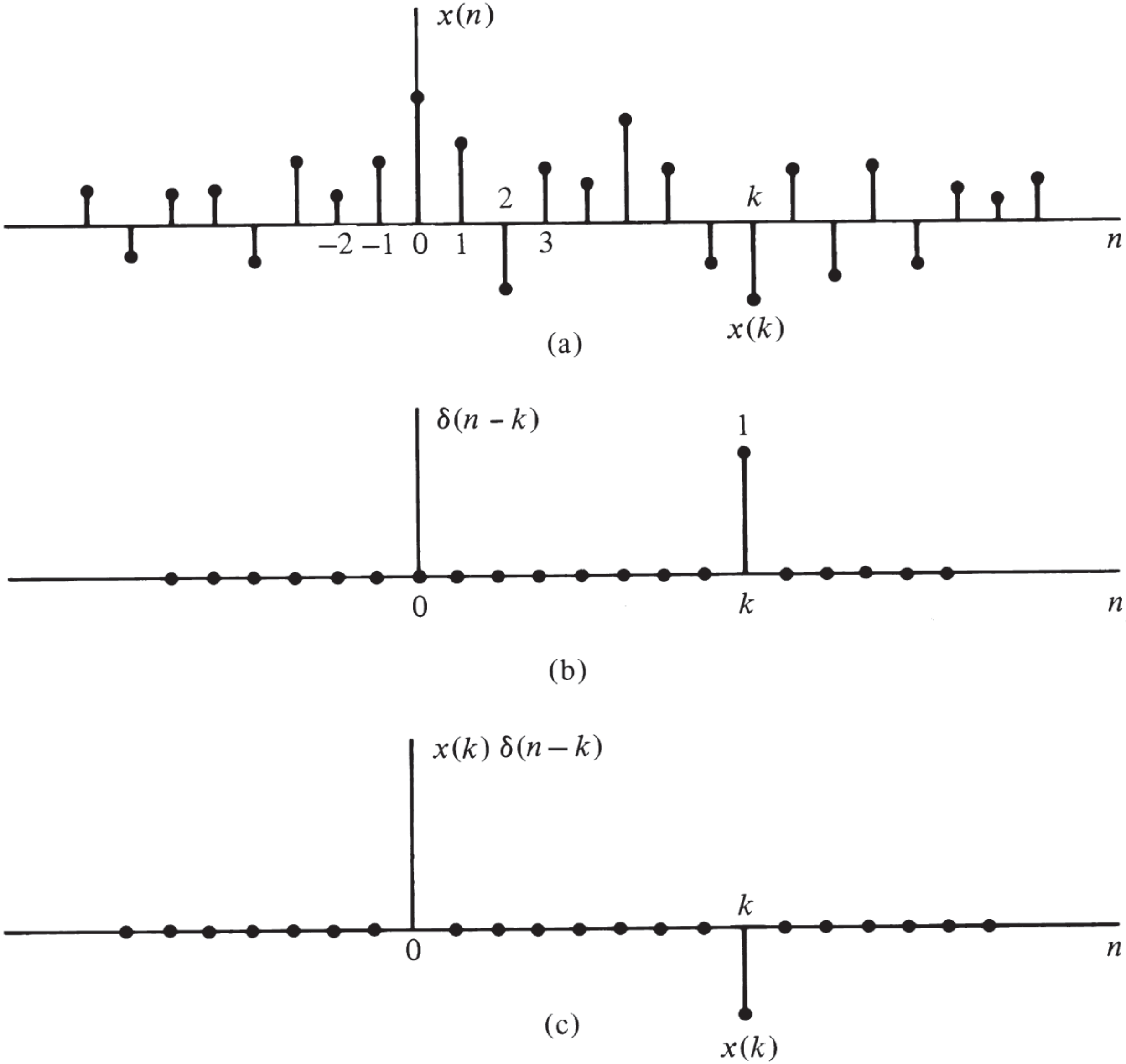


Figure 2.3.1 Multiplication of a signal $x(n)$ with a shifted unit sample sequence.

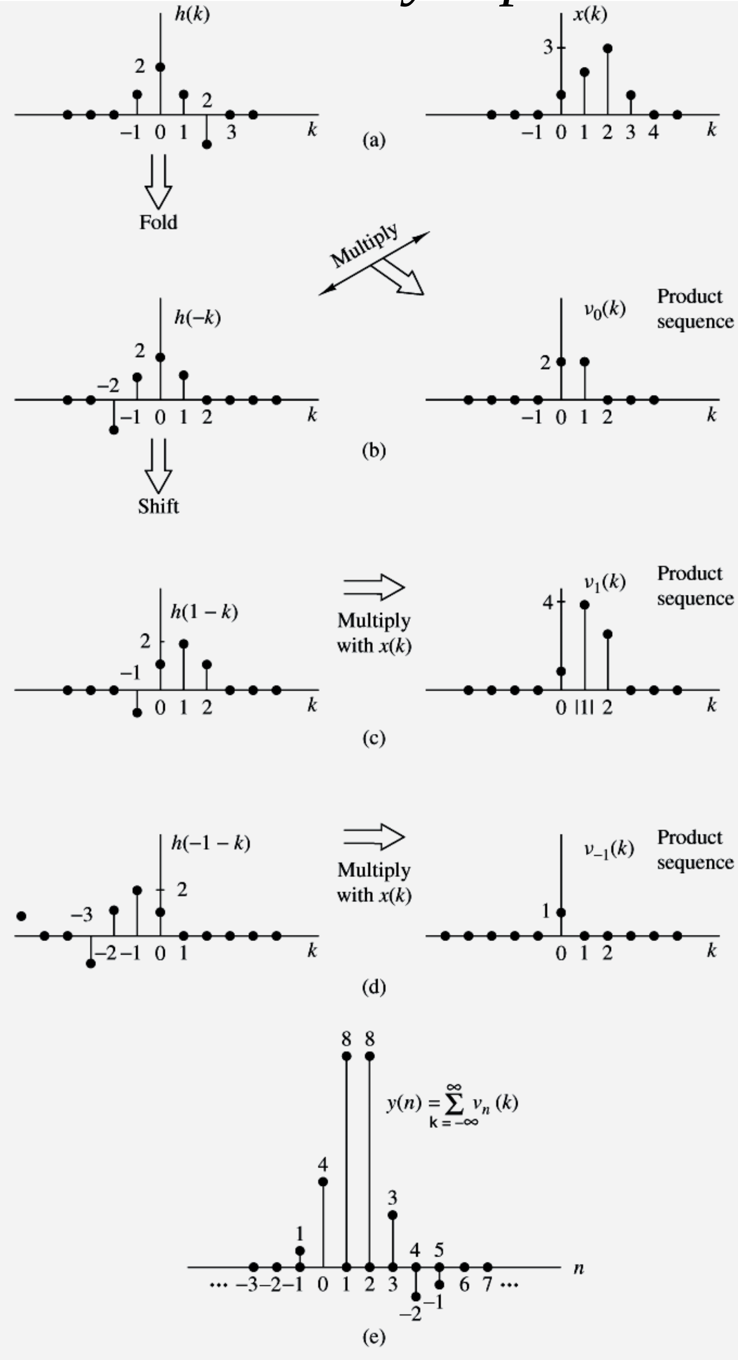


Figure 2.3.2 Graphical computation of convolution.

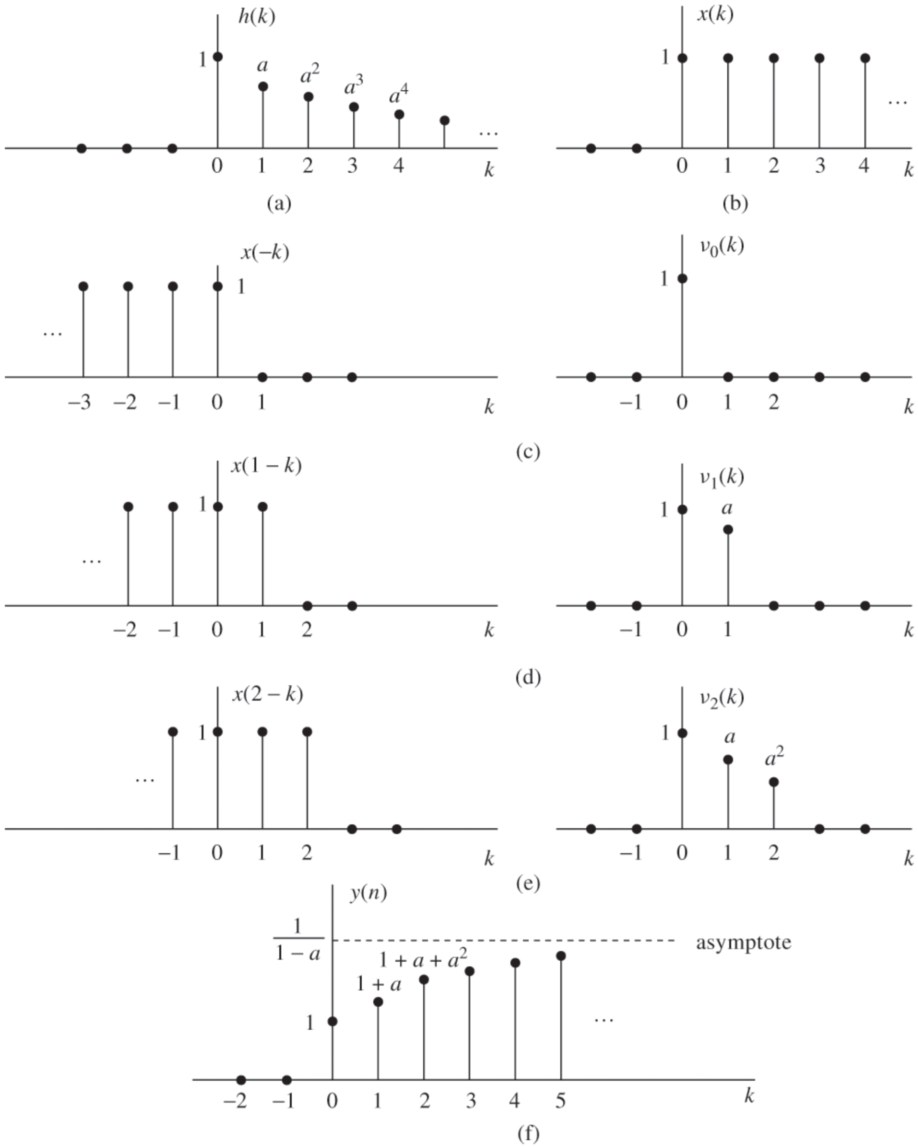


Figure 2.3.3 Graphical computation of convolution in Example 2.3.3.

Properties of Convolution and the Interconnection of LTI Systems

- Identify and Shifting Properties

- Commutative law

- Associative law

- Distributive law

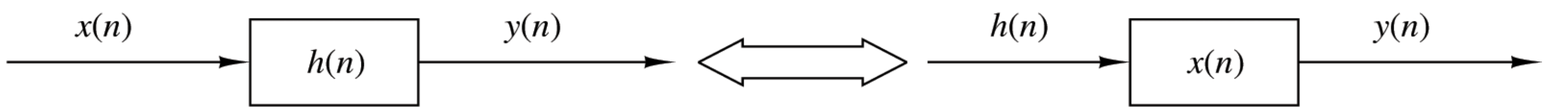


Figure 2.3.4 Interpretation of the commutative property of convolution.

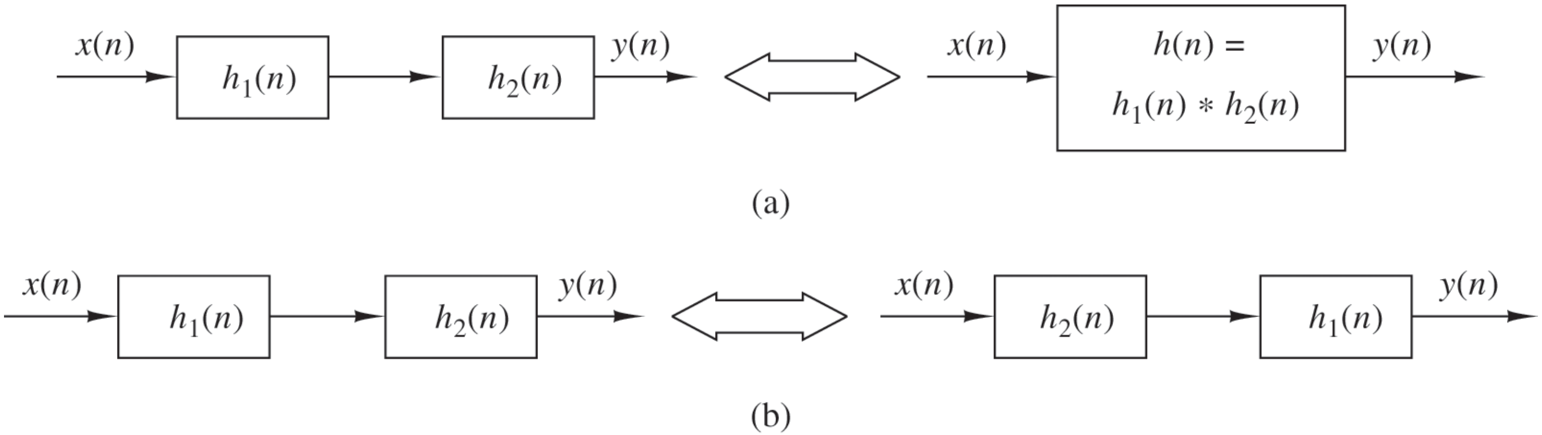


Figure 2.3.5 Implications of the associative (a) and the associative and commutative (b) properties of convolution.

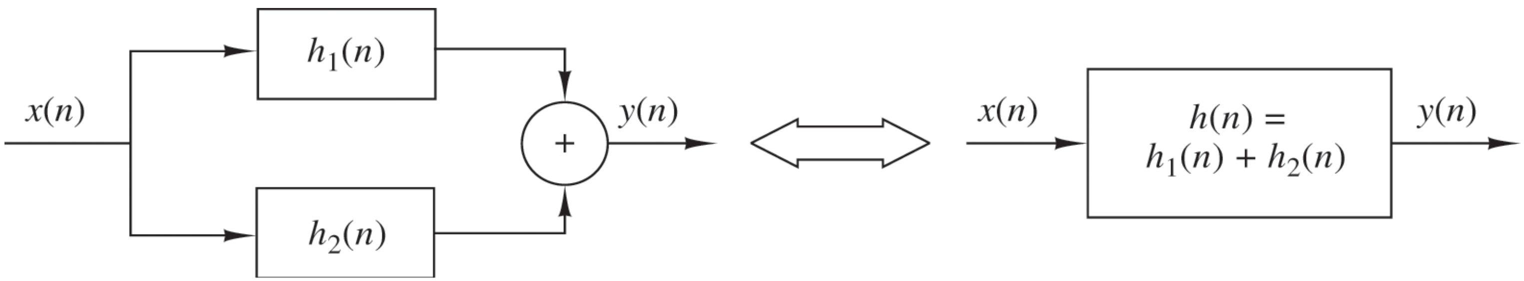


Figure 2.3.6 Interpretation of the distributive property of convolution: two LTI systems connected in parallel can be replaced by a single system with $h(n) = h_1(n) + h_2(n)$.

Causal Linear Time-Invariant Systems

$$\Leftrightarrow h(n) = 0, \; n < 0$$

Stability of Linear Time-Invariant Systems

$$\Leftrightarrow \quad S_h = \sum_{k=-\infty}^{\infty} \left| h(k) \right| < \infty$$

Systems with Finite-Duration and Infinite-Duration Impulse Response (FIR and IIR)

FIR : $h(n) = 0,\ n < 0$ and $n \geq M$

$$y(n) = \sum_{k=0}^{M-1} h(k)\,x(n-k)$$

IIR: $$y(n) = \sum_{k=0}^{\infty} h(k)\,x(n-k)$$

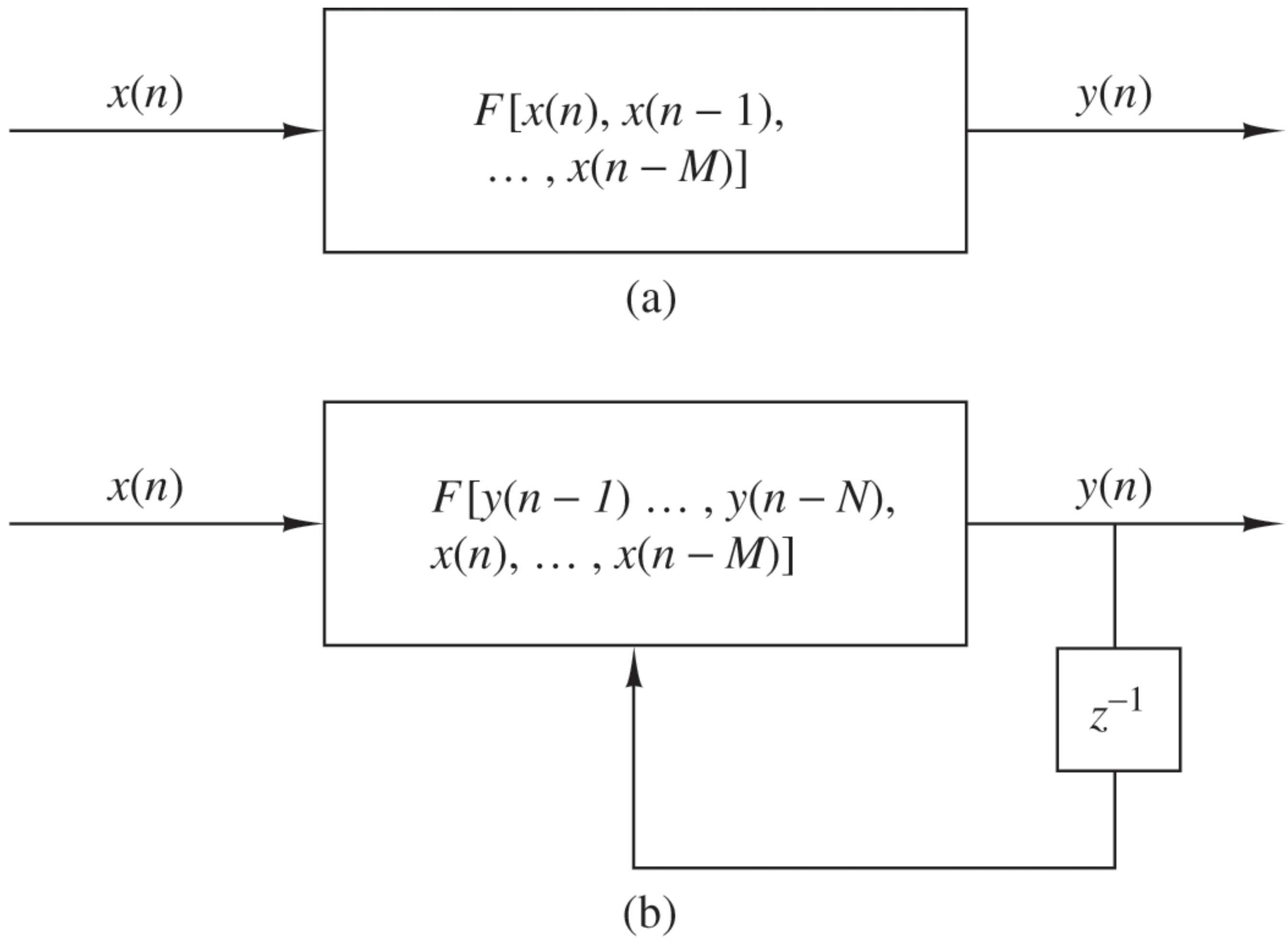


Figure 2.4.3 Basic form for a causal and realizable (a) nonrecursive and (b) recursive system.

Implementation of Discrete-Time Systems

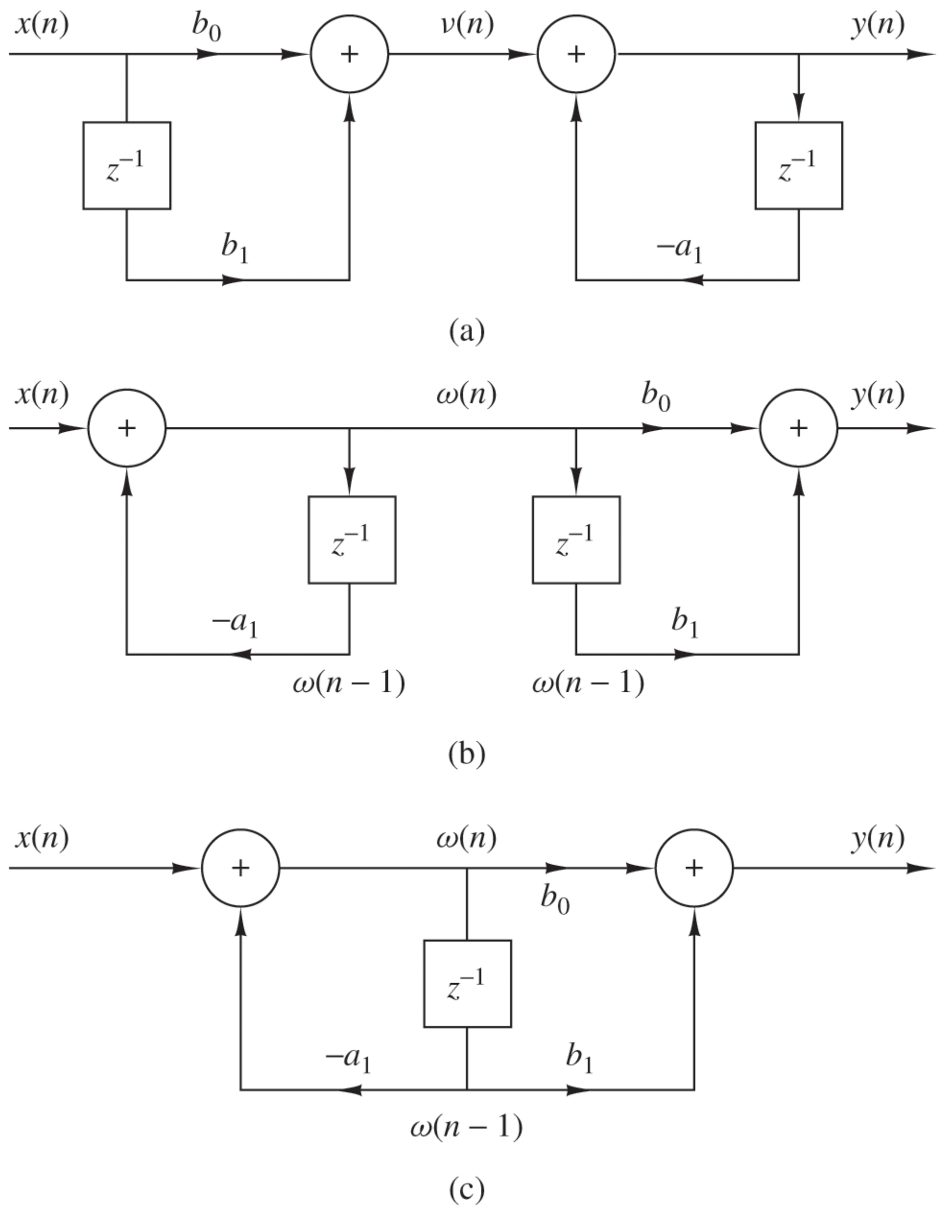


Figure 2.5.1 Steps in converting from the direct form I realization in (a) to the direct form II realization in (c).

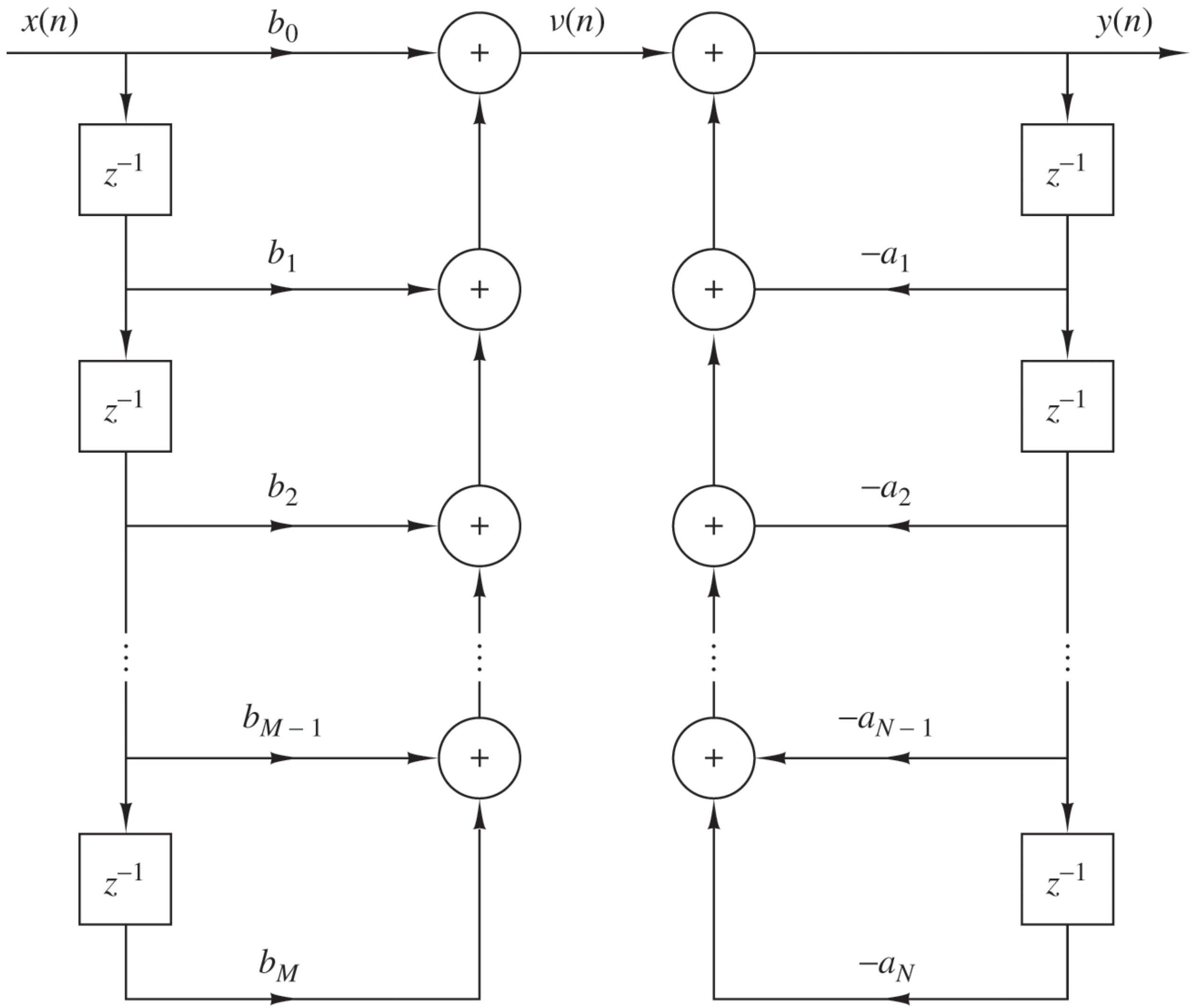


Figure 2.5.2 Direct form I structure of the system described by (2.5.6).

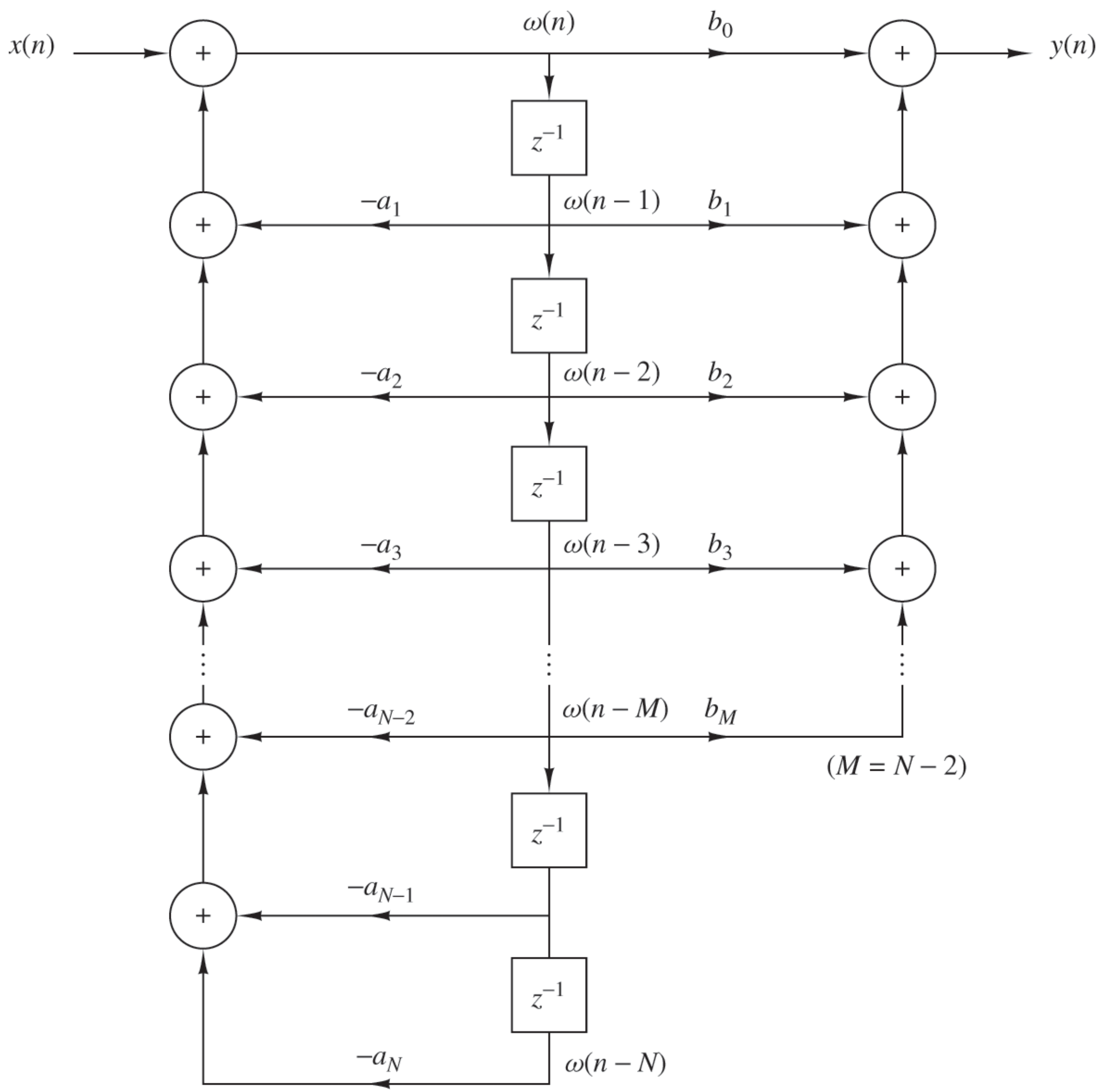


Figure 2.5.3 Direct form II structure for the system described by (2.5.6).

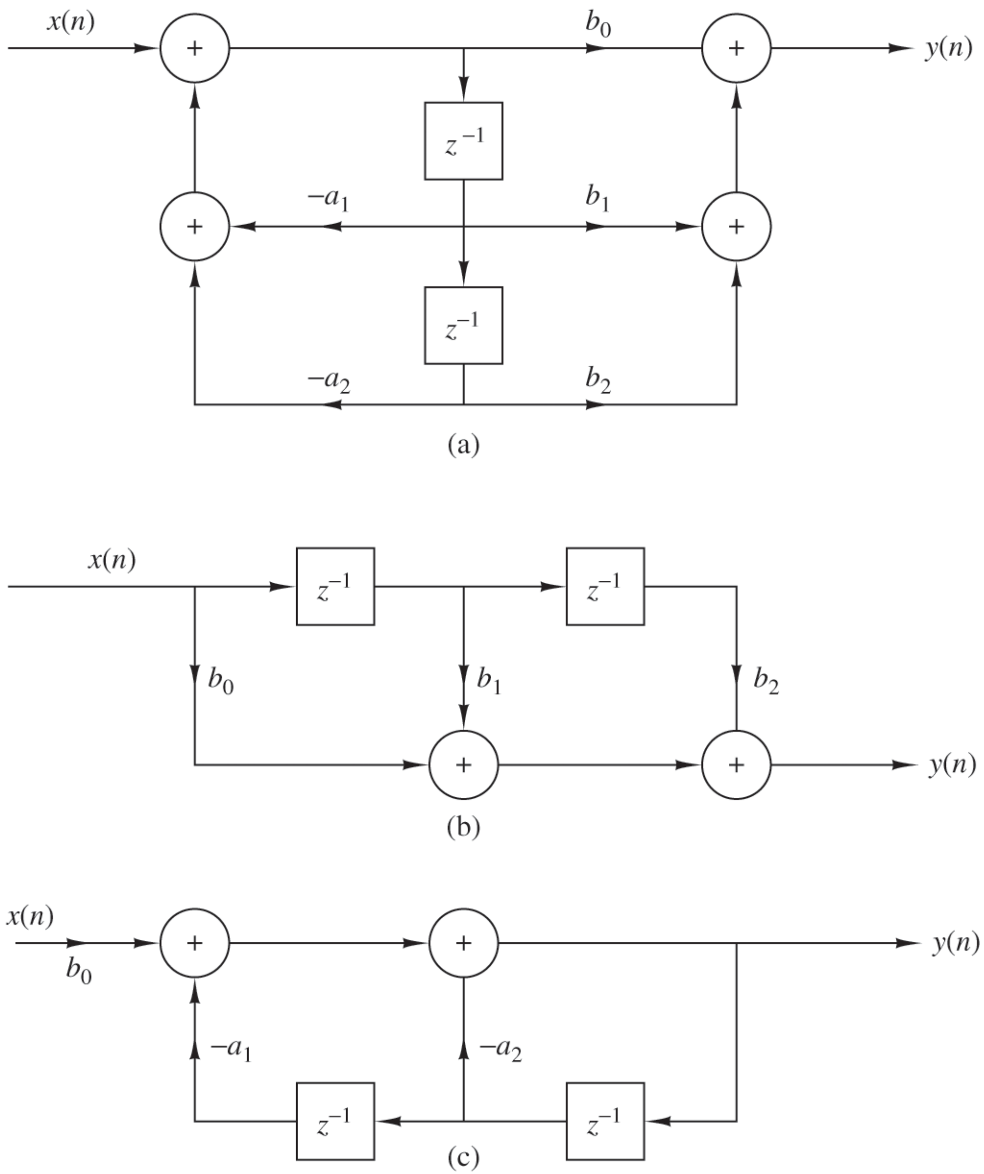


Figure 2.5.4 Structures for the realization of second-order systems: (a) general second-order system; (b) FIR system; (c) "purely recursive system."

Correlation of Discrete-Time Signals

- Crosscorelation

- Autocorrelation

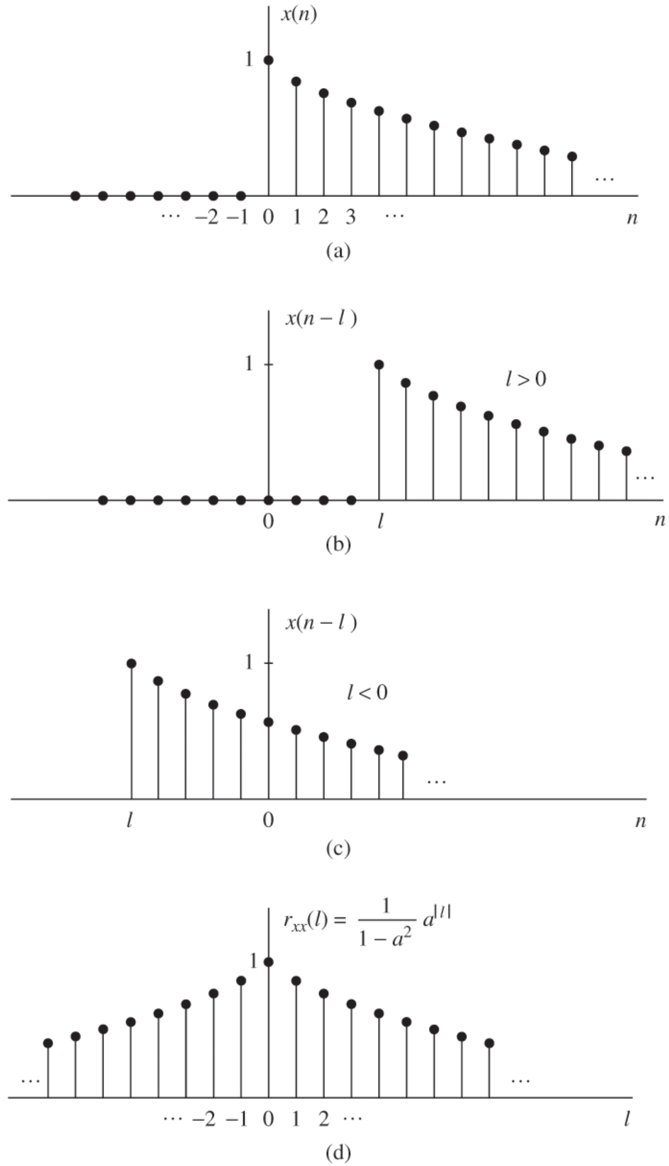


Figure 2.6.2 Computation of the autocorrelation of the signal $x(n) = a^n$, $0 < a < 1$.

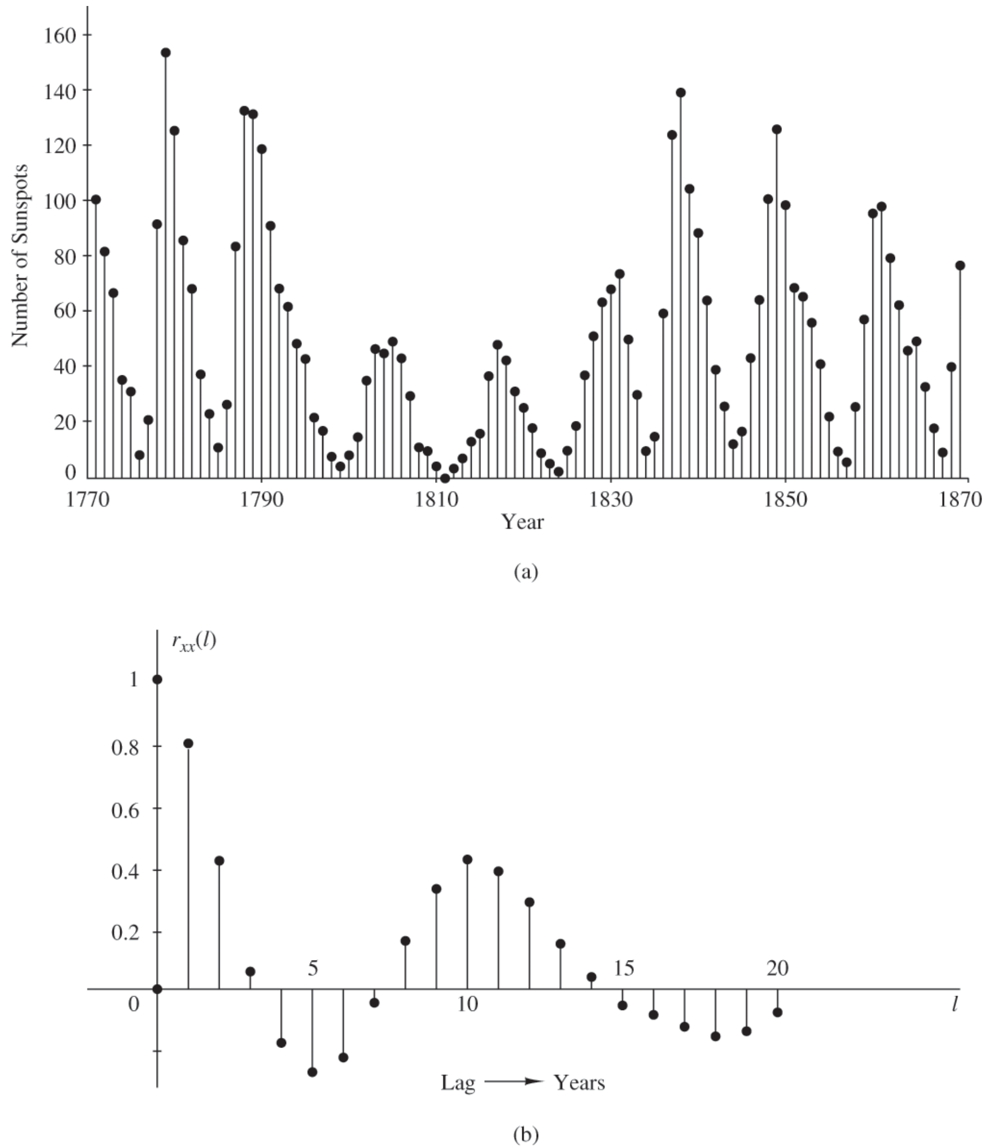


Figure 2.6.3 Identification of periodicity in the Wölfer sunspot numbers: (a) annual Wölfer sunspot numbers; (b) normalized autocorrelation sequence.

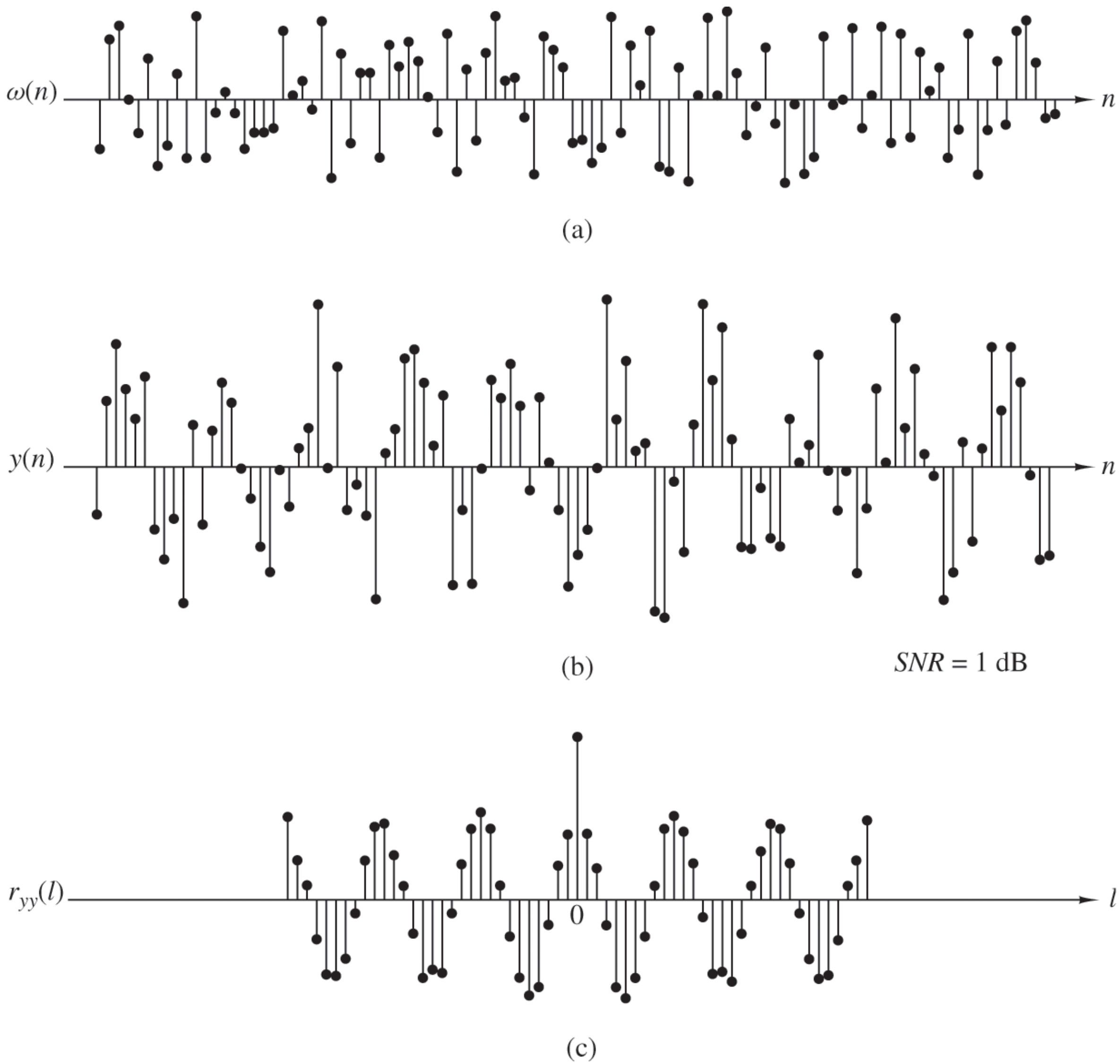


Figure 2.6.4 Use of autocorrelation to detect the presence of a periodic signal corrupted by noise.