# Design of Digital Filters IIR

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#### Design of IIR Filters From Analog Filters

$$H(s) = s$$

$$U(a)$$

$$H(z) = \frac{1 - z^{-1}}{T}$$

$$U(b)$$

$$H(s) = s$$

$$U(a)$$

$$U(b)$$

$$U(b)$$

$$U(b)$$

$$U(b)$$

$$U(c) = \frac{dy(t)}{dt}$$

$$U(c) = \frac{dy(t)}{dt}$$

$$U(c) = \frac{1 - z^{-1}}{T}$$

Figure 10.3.1 Substitution of the backward difference for the derivative implies the mapping  $s = (1 - z^{-1})/T$ .

#### Design of IIR Filters by Approximation of Derivatives

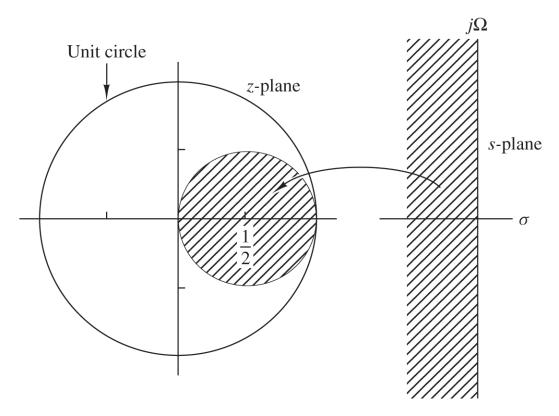


Figure 10.3.2 The mapping  $s=(1-z^{-1})/T$  takes LHP in the s-plane into points inside the circle of radius  $\frac{1}{2}$  and center  $z=\frac{1}{2}$  in the z-plane.

#### Design of IIR Filters by Impulse Invariance

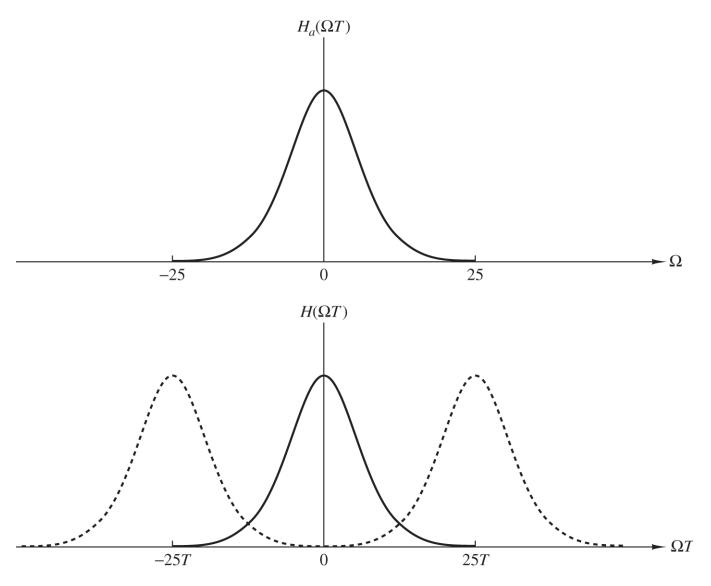


Figure 10.3.3 Frequency response  $H_a(\Omega)$  of the analog filter and frequency response of the corresponding digital filter with aliasing.

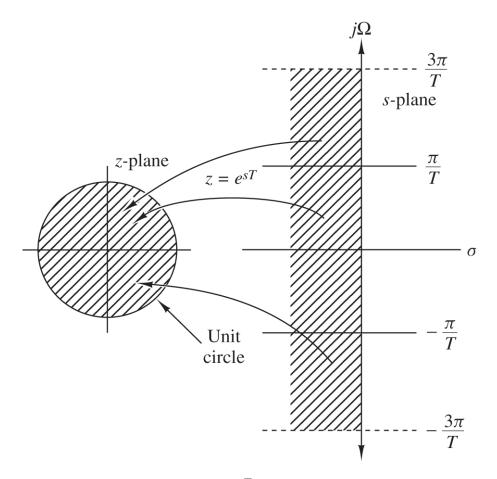


Figure 10.3.4 The mapping of  $z=e^{sT}$  maps strips of width  $2\pi/T$  (for  $\sigma<0$ ) in the s-plane into points in the unit circle in the z-plane.

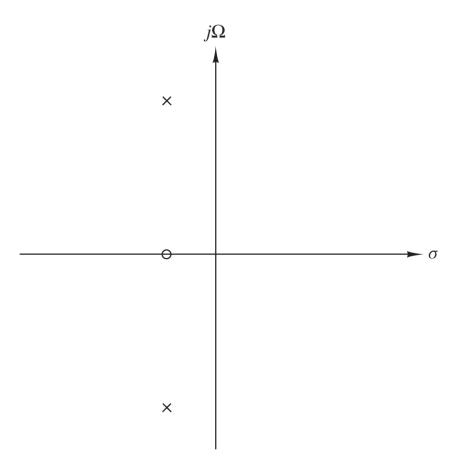


Figure 10.3.5 Pole–zero locations for analog filter in Example 10.3.3.

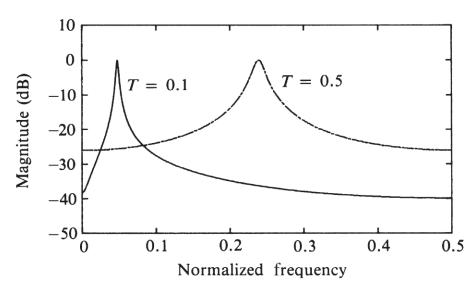


Figure 10.3.6 Frequency response of digital filter in Example 10.3.3.

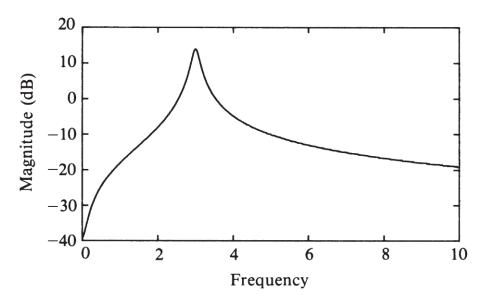


Figure 10.3.7 Frequency response of analog filter in Example 10.3.3.

#### IIR Filter Design by the Bilinear Transformation

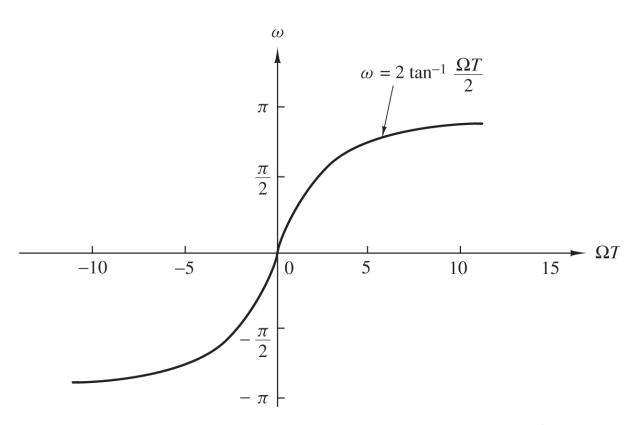


Figure 10.3.8 Mapping between the frequency variables  $\omega$  and  $\Omega$  resulting from the bilinear transformation.

### Characteristic of Commonly Used Analog Filters

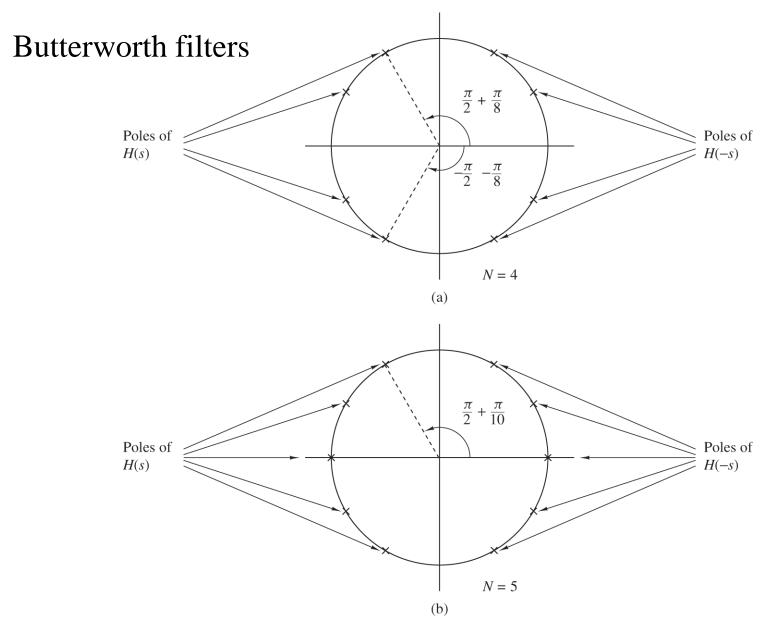


Figure 10.3.9 Pole positions for Butterworth filters.

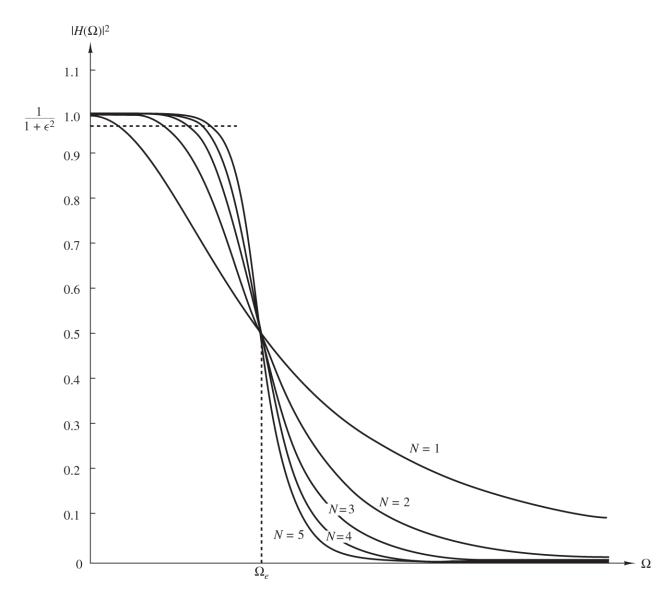


Figure 10.3.10 Frequency response of Butterworth filters.

## Chebyshev filters

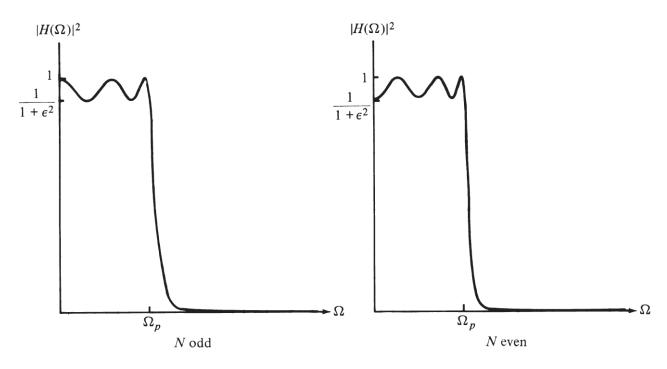


Figure 10.3.11 Type I Chebyshev filter characteristic.

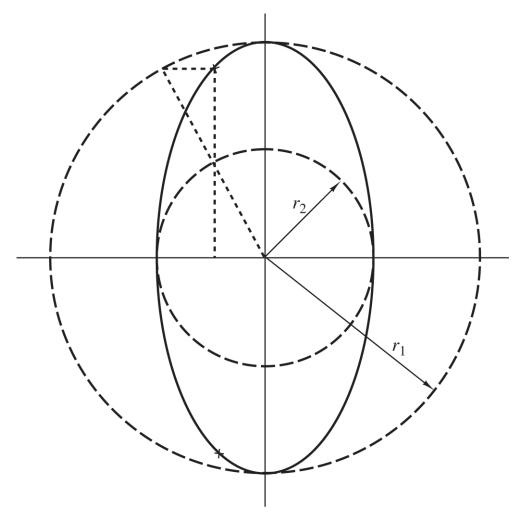


Figure 10.3.12 Determination of the pole locations for a Chebyshev filter.

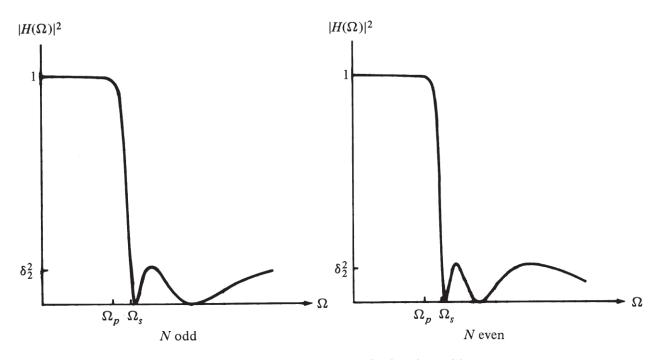


Figure 10.3.13 Type II Chebyshev filters.

## Elliptic filters

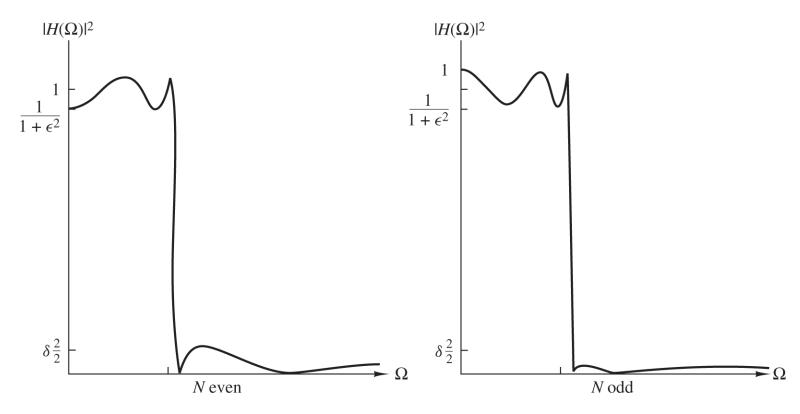


Figure 10.3.14 Magnitude-squared frequency characteristics of elliptic filters.

#### Bessel filters

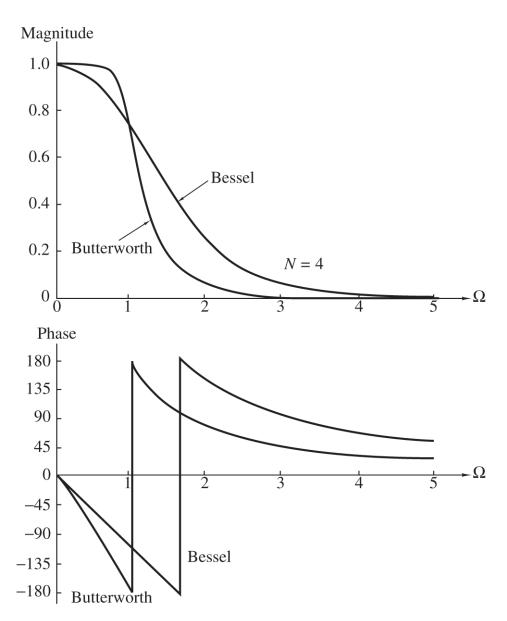


Figure 10.3.15 Magnitude and phase responses of Bessel and Butterworth filters of order N=4.