Frequency Analysis of Signals

Phan Duy Hùng

Why we need the Fourier transform ? (frequency domain)

Frequency Analysis of Continuous-Time Signals

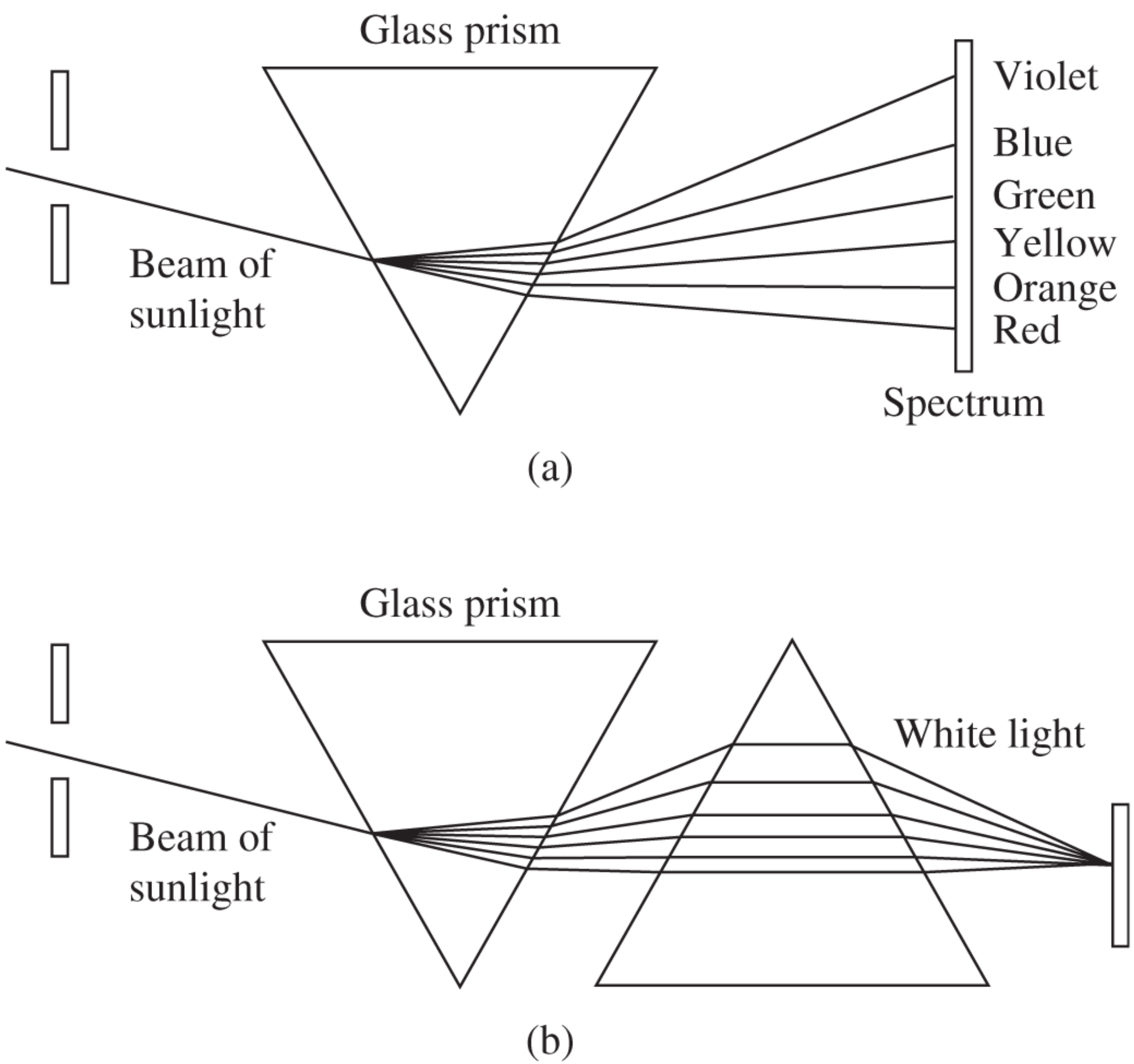

Figure 4.1.1 (a) Analysis and (b) synthesis of the white light (sunlight) using glass prisms.

The Fourier Seriers for Continuous-Time Periodic Signals

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k F_0 t}$$

$$c_k = \frac{1}{T_p} \int_{T_p} x(t) e^{-j2\pi k F_0 t} dt$$

Power Density Spectrum of Periodic Signals

$$P_x = \frac{1}{T_p} \int_{T_p} |x(t)|^2 \, dt = \sum_{k=-\infty}^{\infty} |c_k|^2$$

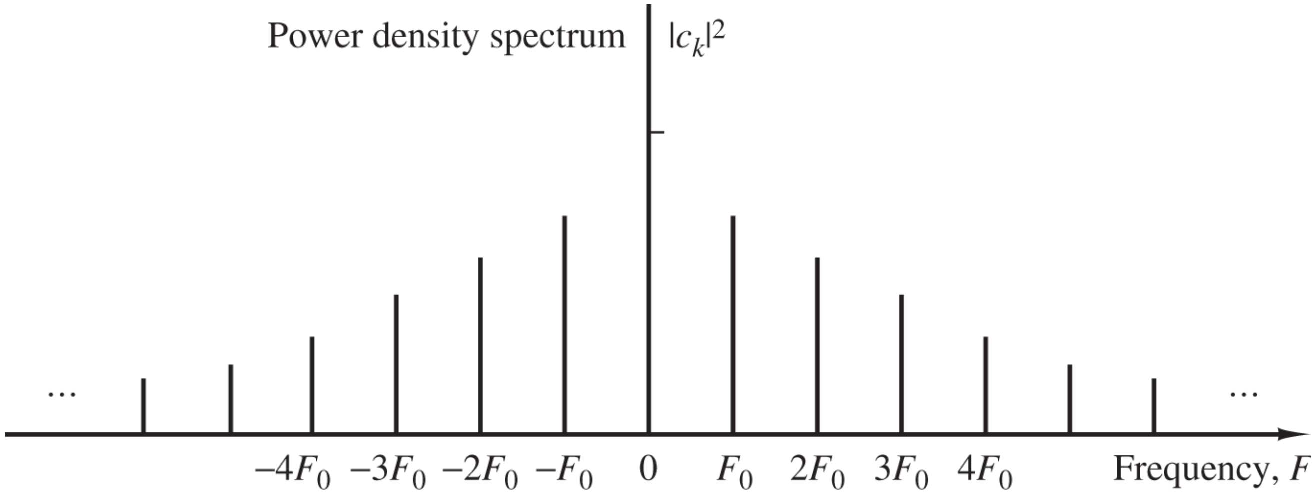

Figure 4.1.2 Power density spectrum of a continuous-time periodic signal.

Example:

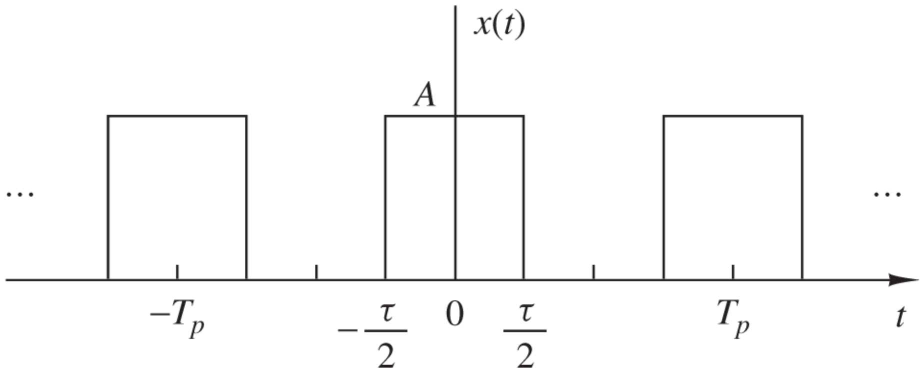

Figure 4.1.3 Continuous-time periodic train of rectangular pulses.

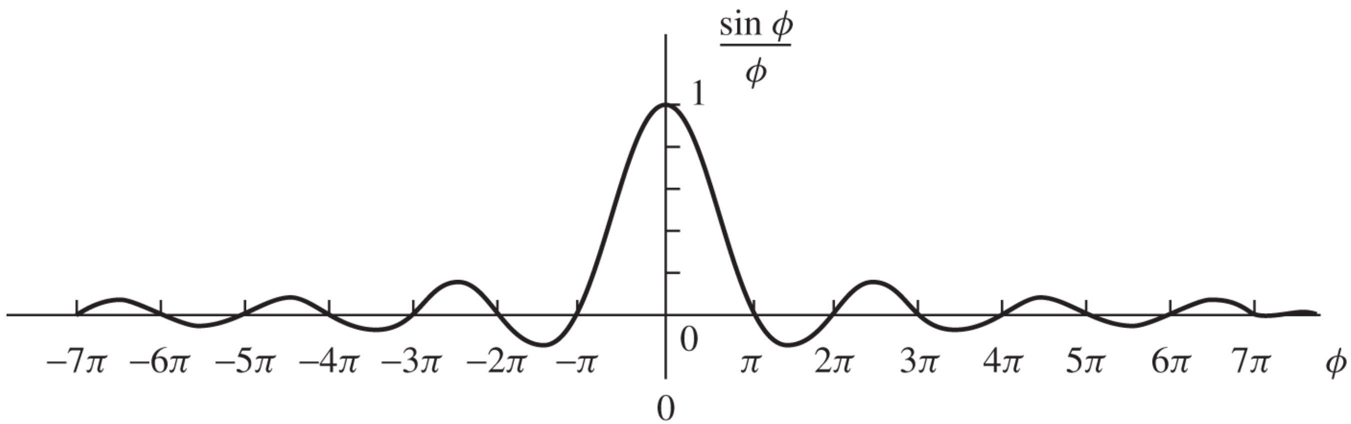

Figure 4.1.4 The function $(\sin\phi)/\phi$.

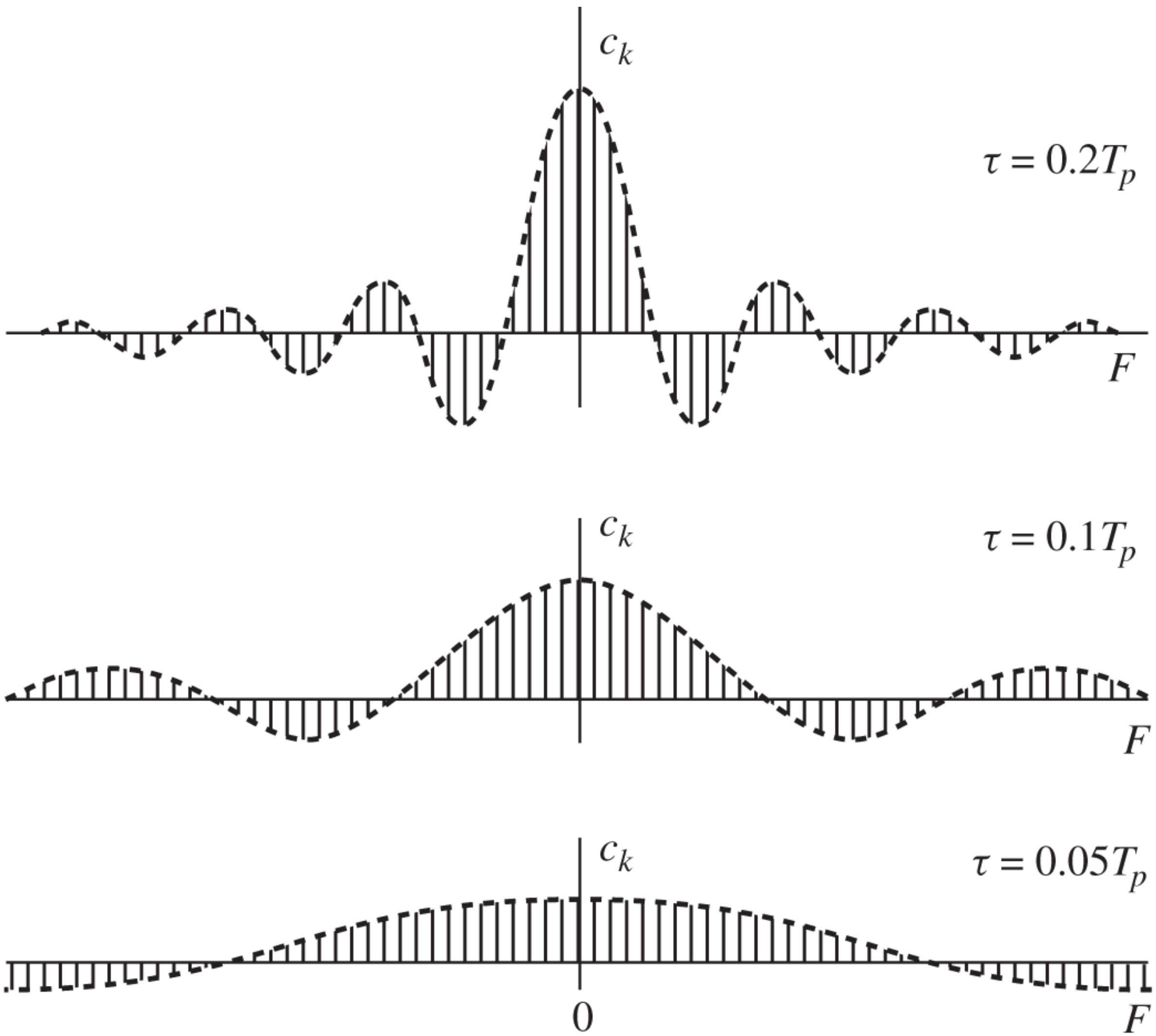

Figure 4.1.5 Fourier coefficients of the rectangular pulse train when T_p is fixed and the pulse width τ varies.

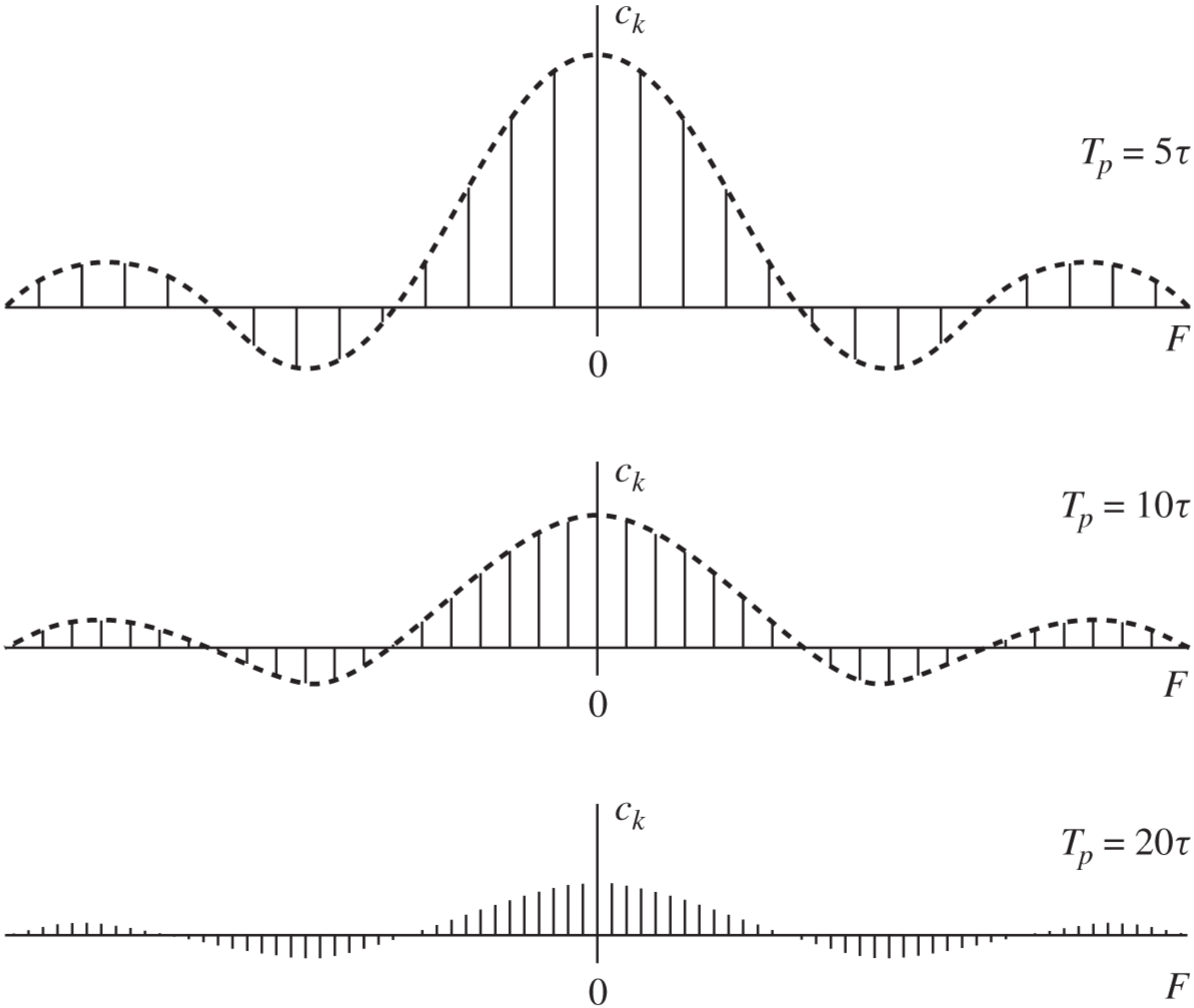

Figure 4.1.6 Fourier coefficient of a rectangular pulse train with fixed pulse width τ and varying period T_p.

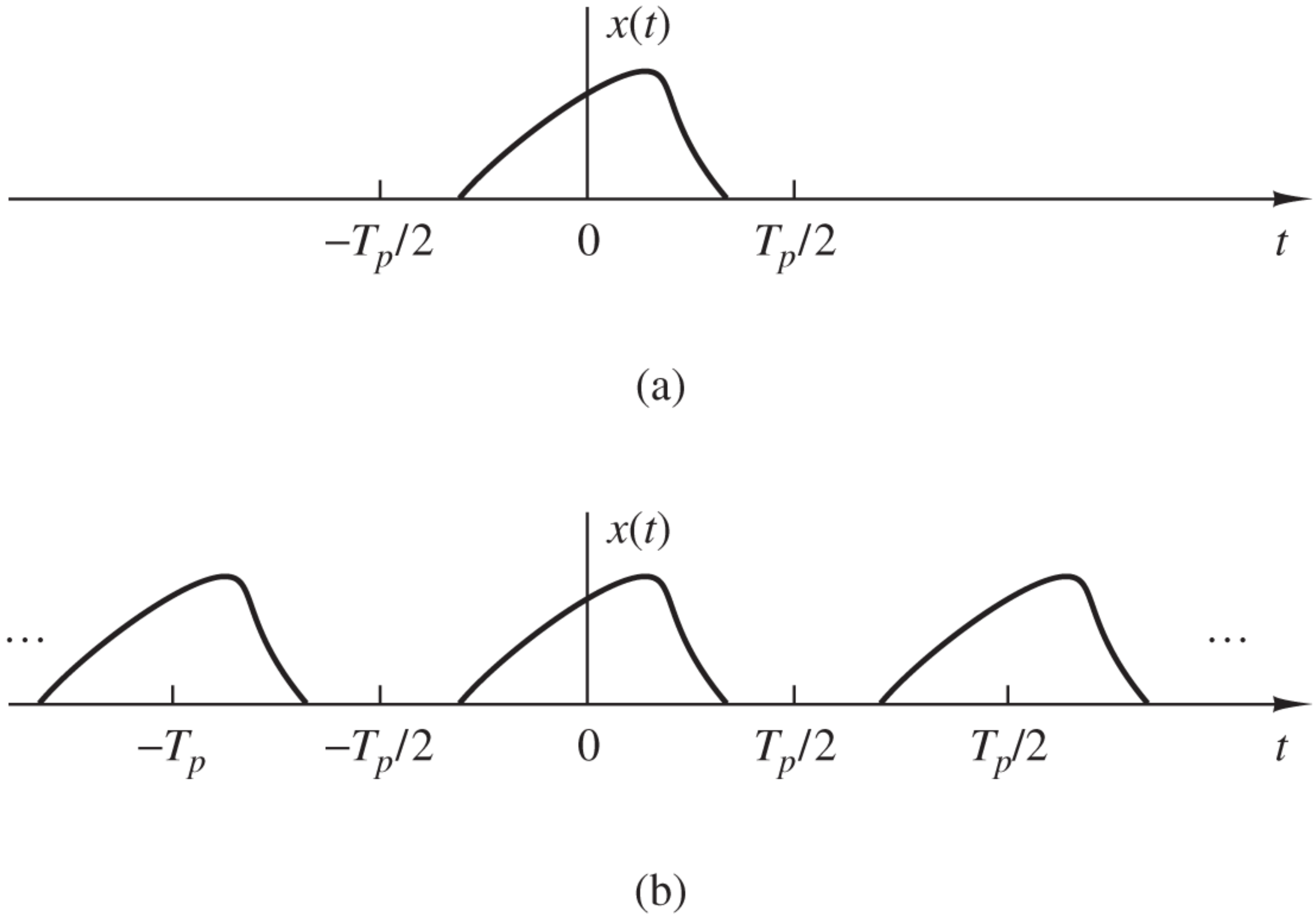

Figure 4.1.7 (a) Aperiodic signal $x(t)$ and (b) periodic signal $x_p(t)$ constructed by repeating $x(t)$ with a period T_p.

The Fourier Transform for Continuous-Time Aperiodic Signals

$$x(t) = \int_{-\infty}^{\infty} X(F)\, e^{j2\pi Ft}\, dF$$

$$X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi Ft}\, dt$$

Energy Density Spectrum of Aperiodic Signals

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2\, dt = \int_{-\infty}^{\infty} |X(F)|^2\, dF$$

$$S_{xx}(F) = |X(F)|^2$$

Example

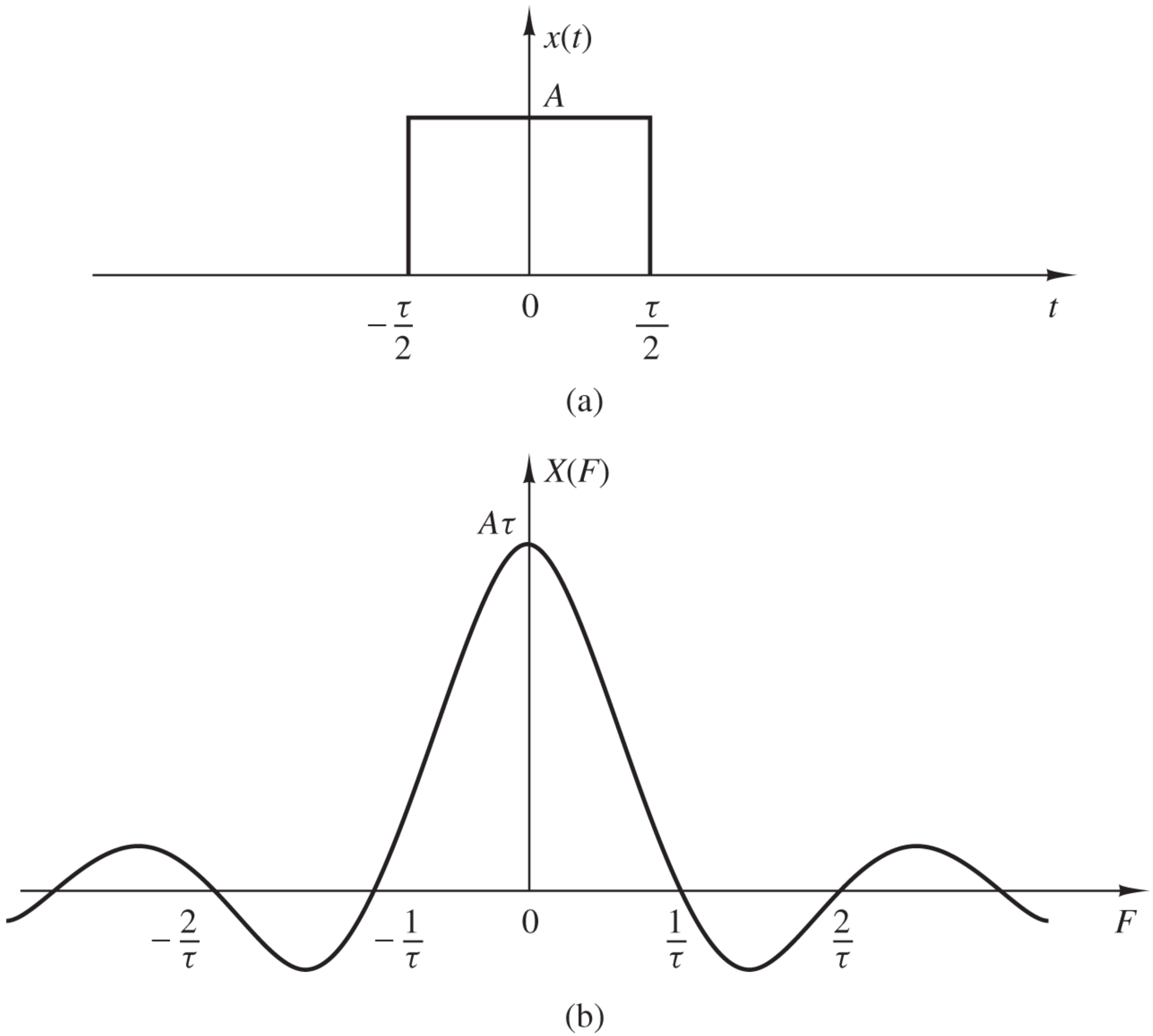

Figure 4.1.8 (a) Rectangular pulse and (b) its Fourier transform.

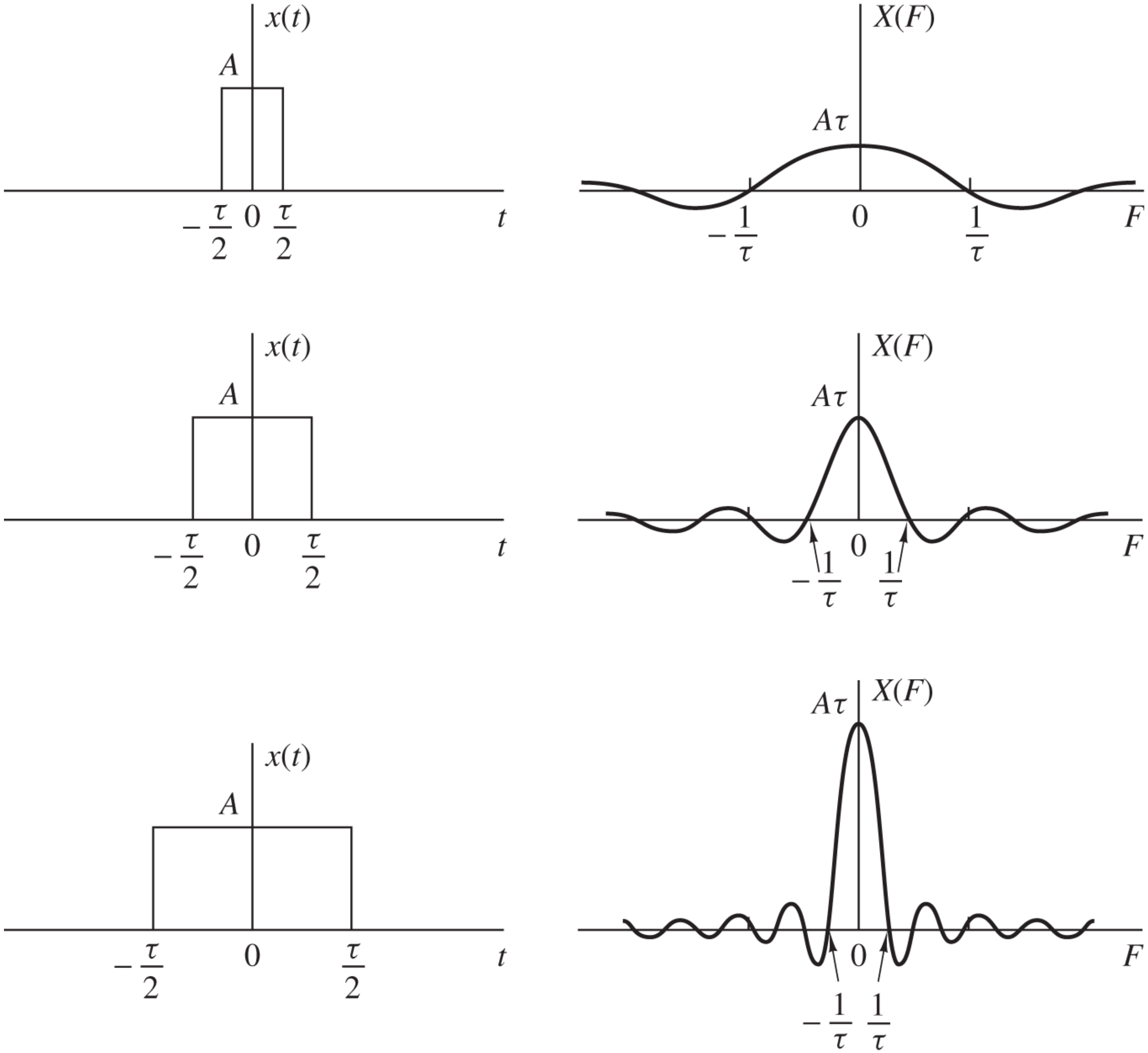

Figure 4.1.9 Fourier transform of a rectangular pulse for various width values.

The Fourier Series for Discrete-Time Periodic Signals

$$x(n) = \sum_{k=0}^{N-1} c_k e^{j2\pi kn/N}$$

$$c_k = \frac{1}{N} \sum_{k=0}^{N-1} x(n) e^{-j2\pi kn/N}$$

Example

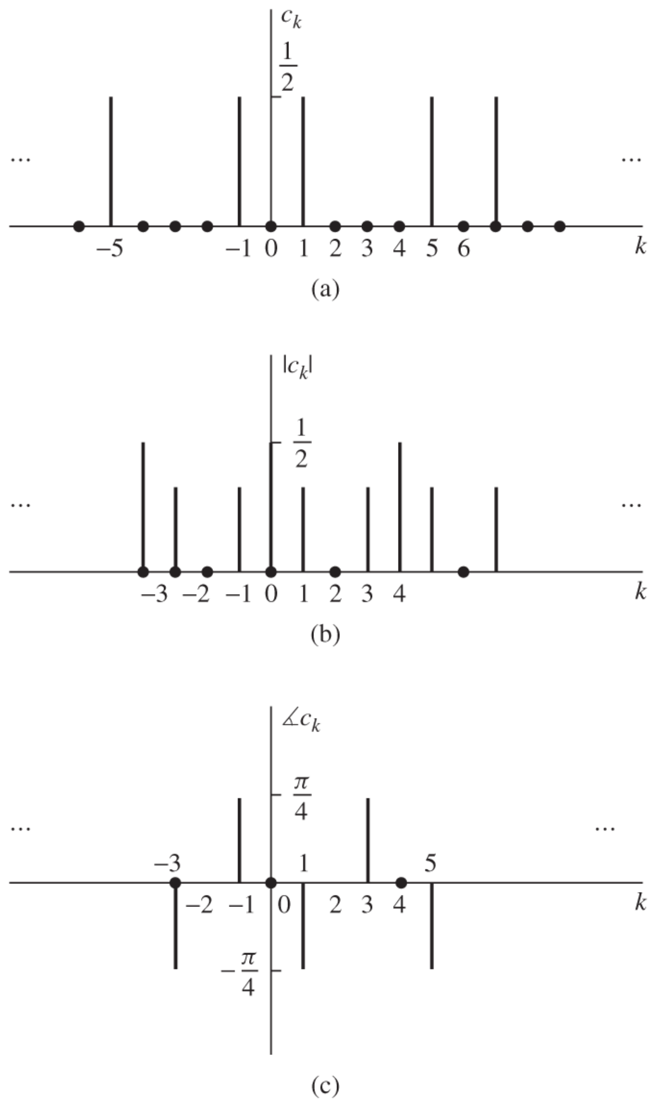

Figure 4.2.1 Spectra of the periodic signals discussed in Example 4.2.1 (b) and (c).

Power Density Spectrum of Periodic Signals

$$P_x = \frac{1}{N}\sum_{n=0}^{N-1}\left|x(n)\right|^2 = \sum_{n=0}^{N-1}\left|c_k\right|^2$$

Example

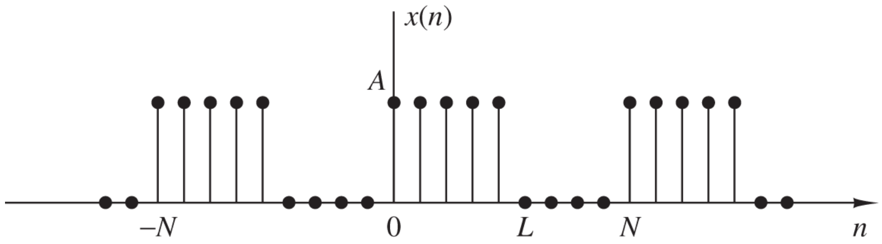

Figure 4.2.2 Discrete-time periodic square-wave signal.

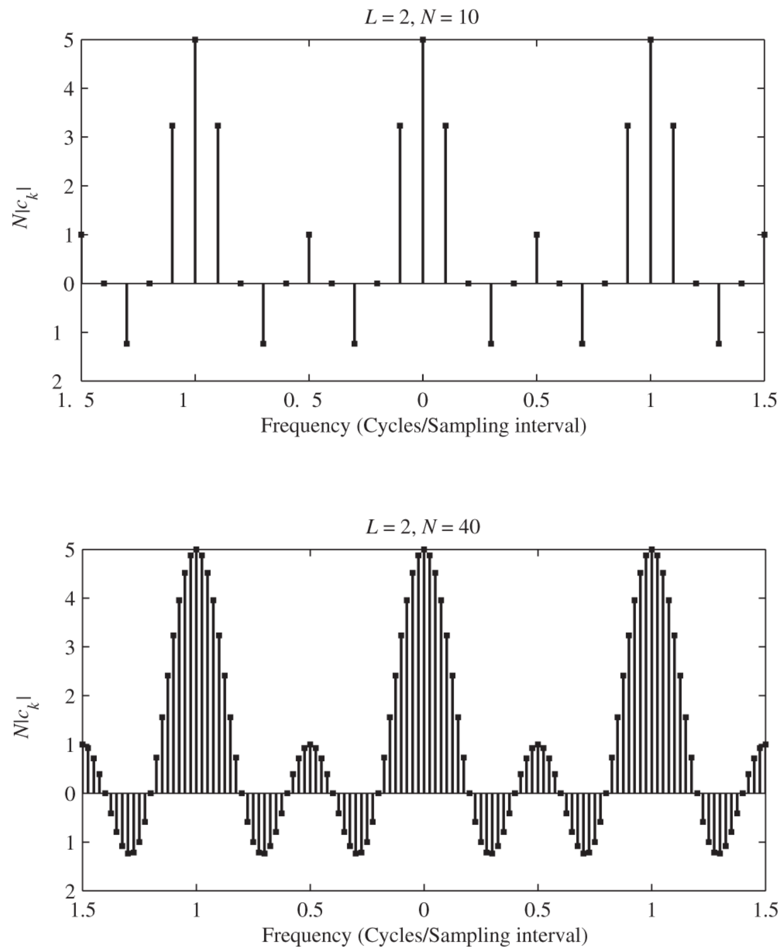

Figure 4.2.3 Plot of the power density spectrum given by (4.2.22).

The Fourier Series for Discrete-Time Aperiodic Signals

$$x(n) = \frac{1}{2\pi} \int_{2\pi} X(\omega) e^{j\omega n} d\omega$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

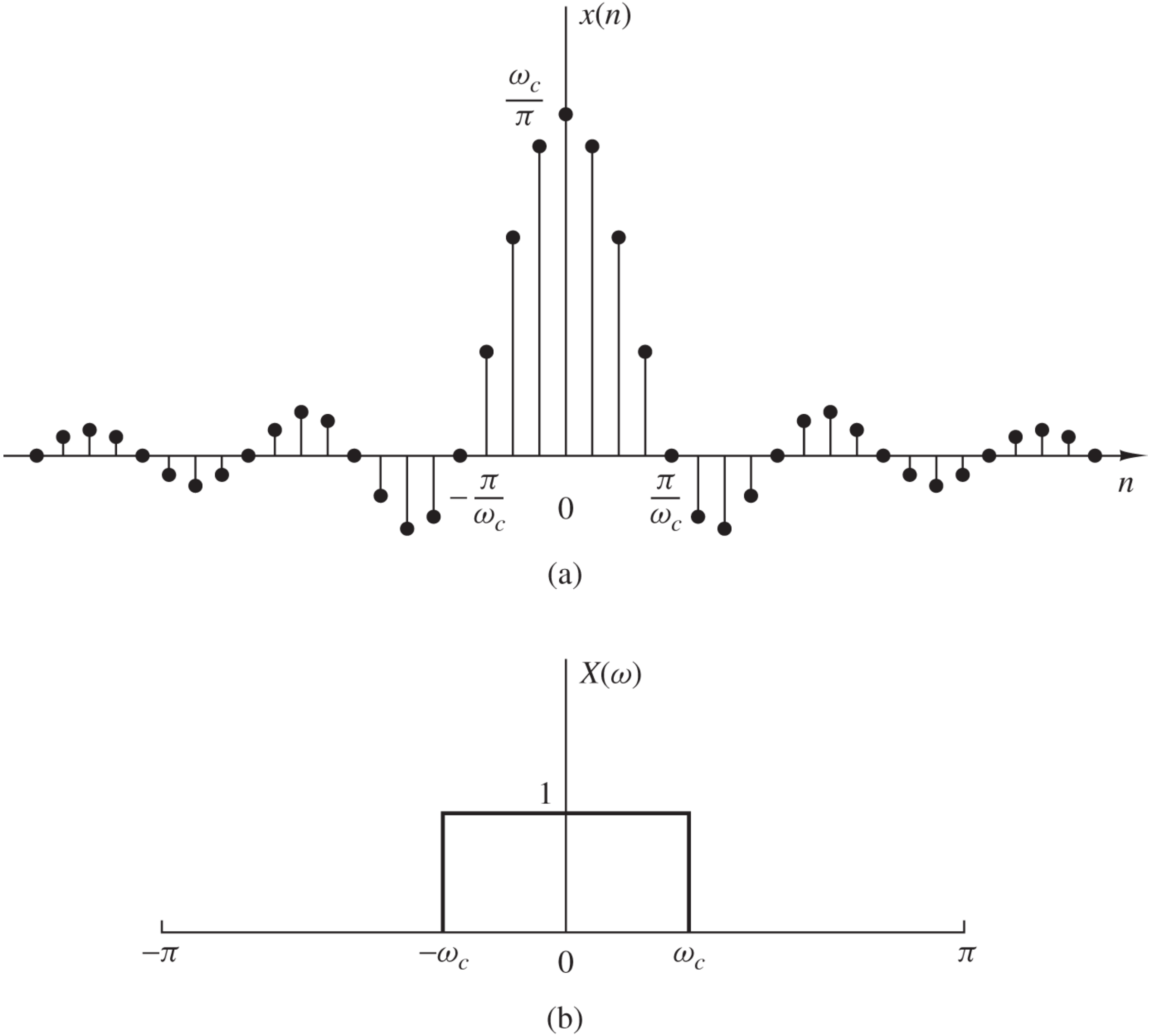

Figure 4.2.4 Fourier transform pair in (4.2.35) and (4.2.36).

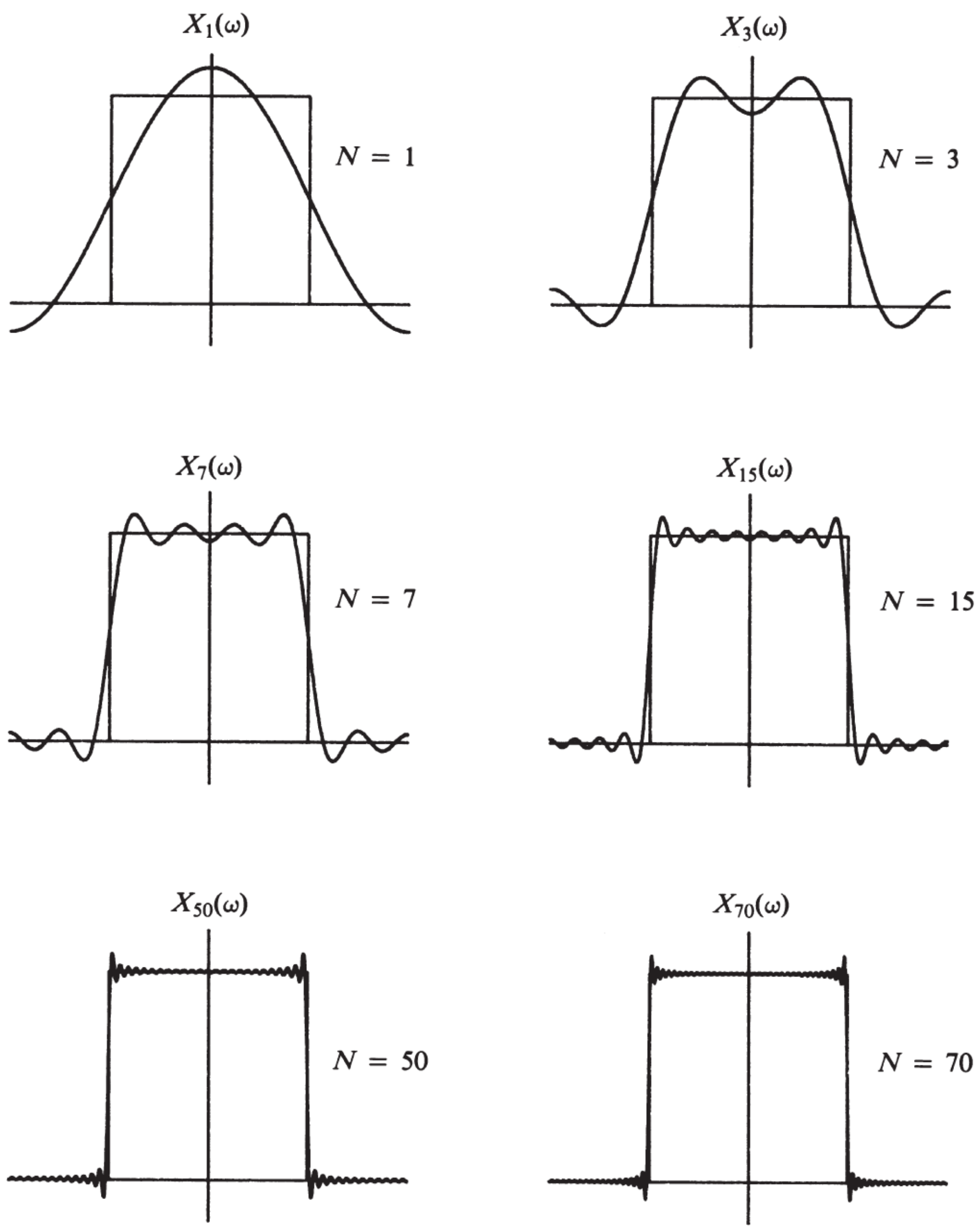

Figure 4.2.5 Illustration of convergence of the Fourier transform and the Gibbs phenomenon at the point of discontinuity.

Energy Density Spectrum of Aperiodic Signals

$$E_x = \sum_{-\infty}^{\infty} \left| x(n) \right|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| X(\omega) \right|^2 d\omega$$

$$S_{xx}(\omega) = \left| X(\omega) \right|^2$$

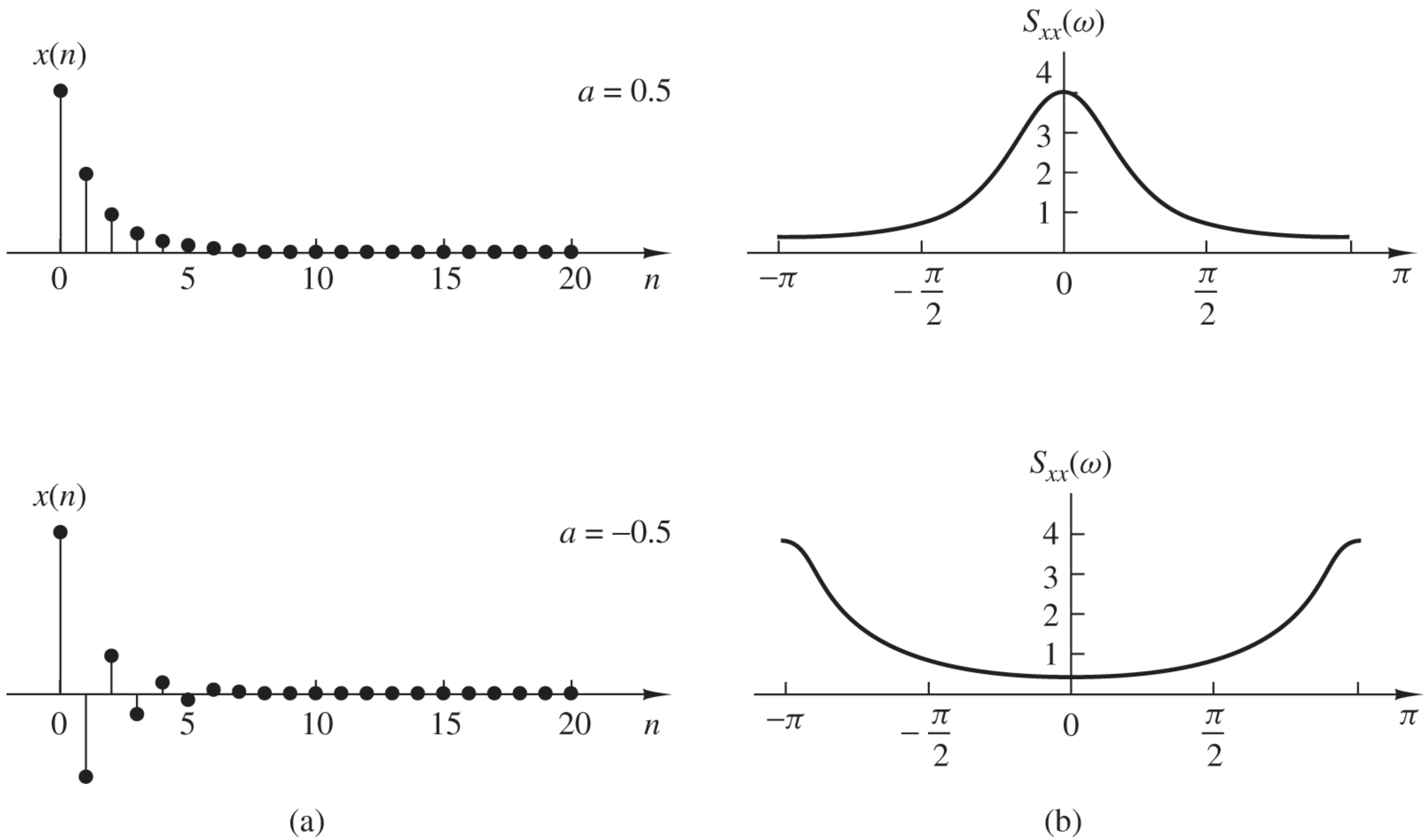

Figure 4.2.6 (a) Sequence $x(n) = (\frac{1}{2})^n u(n)$ and $x(n) = (-\frac{1}{2})^n u(n)$; (b) their energy density spectra.

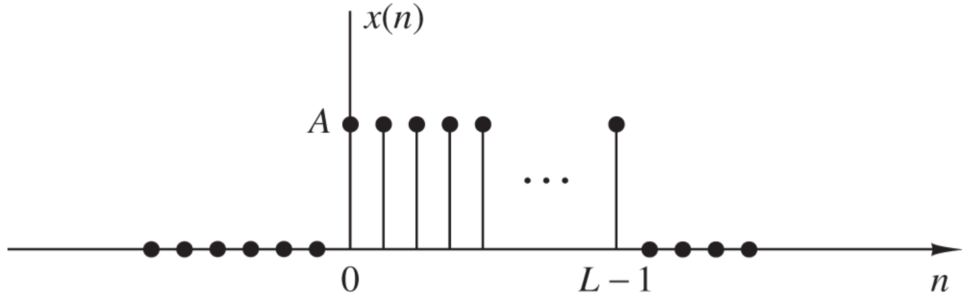

Figure 4.2.7 Discrete-time rectangular pulse.

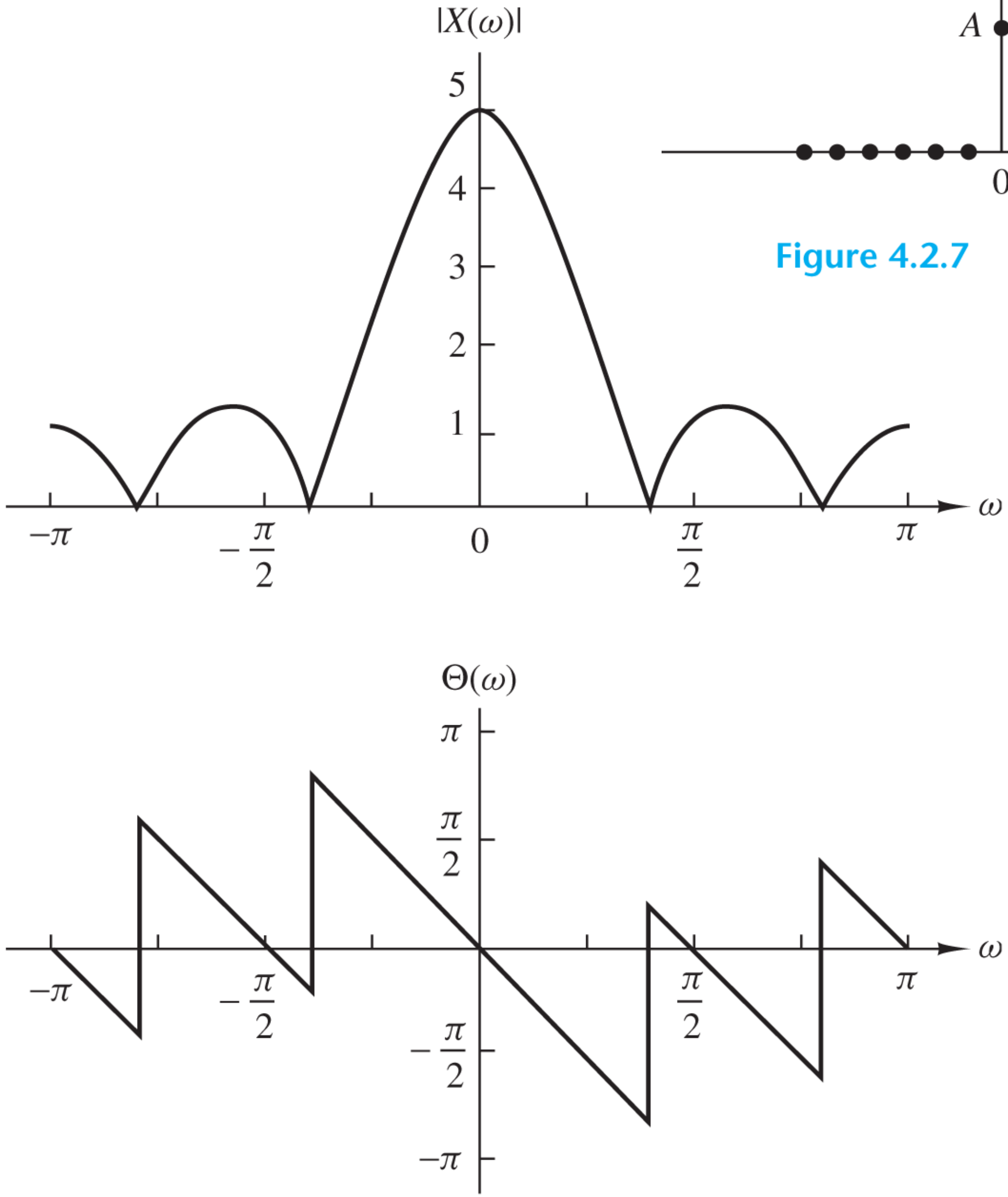

Figure 4.2.8 Magnitude and phase of Fourier transform of the discrete-time rectangular pulse in Fig 4.2.7.

Relationship of the Fourier Transform to the z-Transform

$$X(z)\big|_{z=e^{j\omega}} = X(\omega)$$

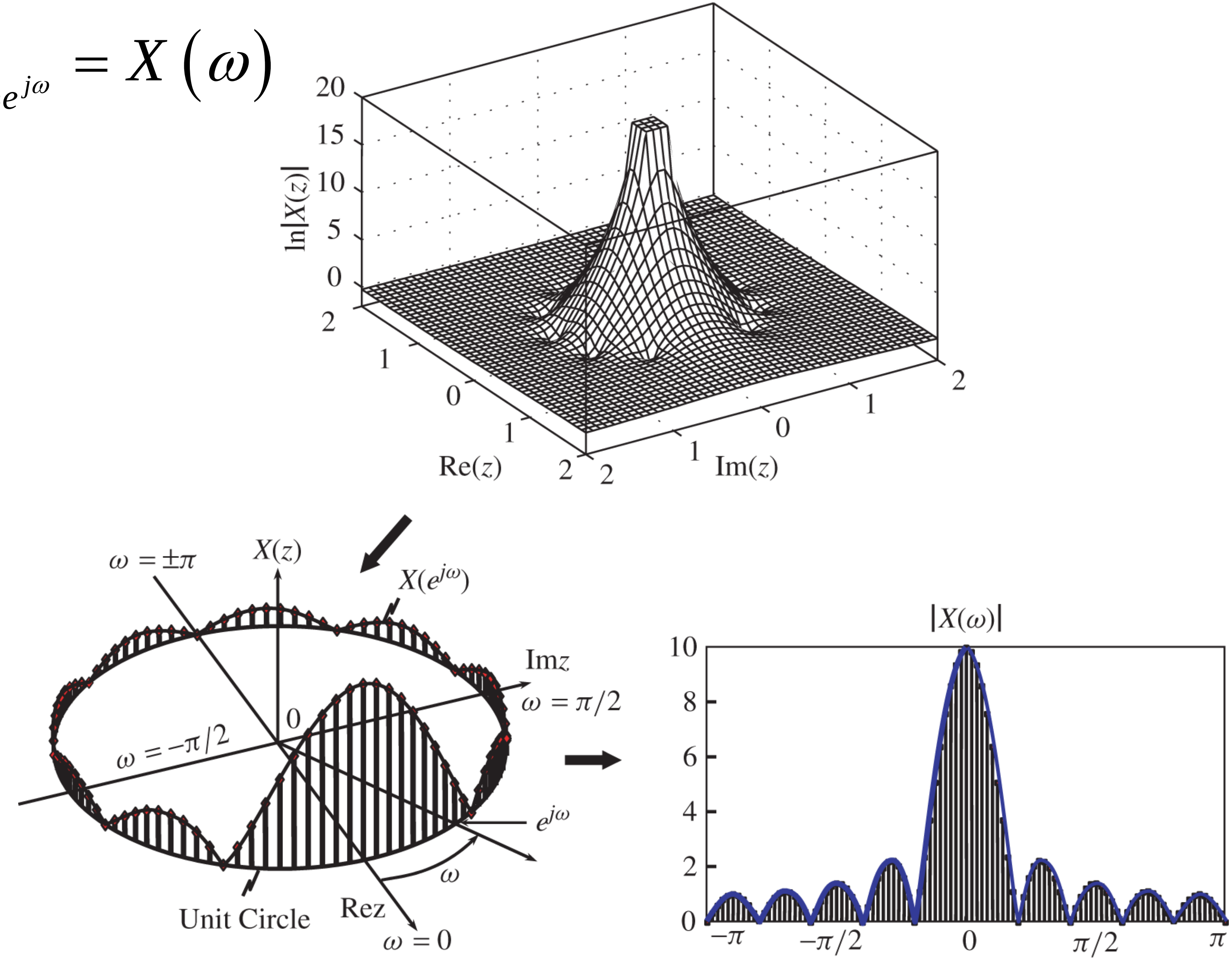

Figure 4.2.9 relationship between $X(z)$ and $X(\omega)$ for the sequence in Example 4.2.4, with $A = 1$ and $L = 10$

Frequency –Domain Classification of Signals: The Concept of Bandwidth

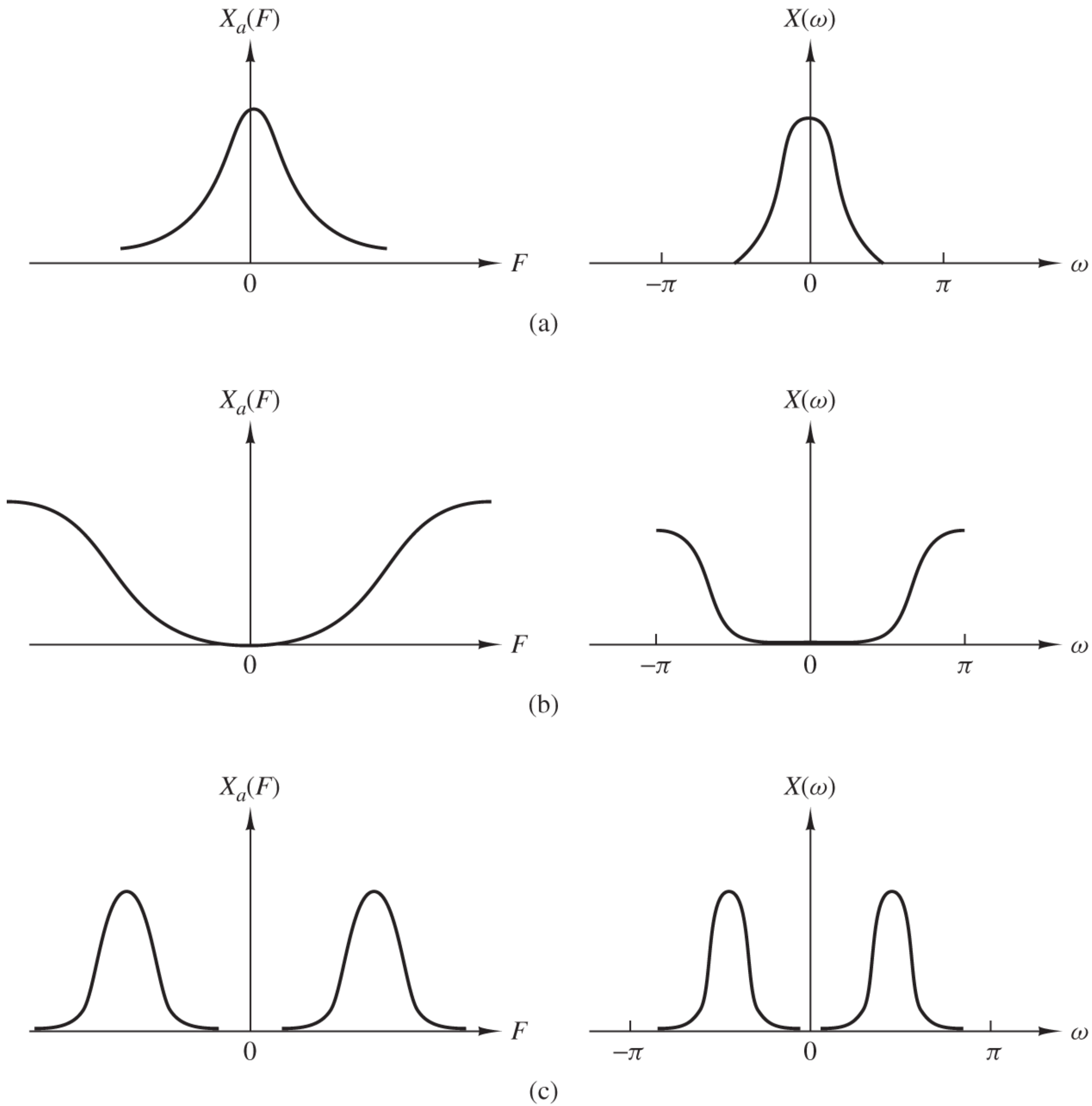

Figure 4.2.10 (a) Low-frequency, (b) high-frequency, and (c) medium-frequency signals.

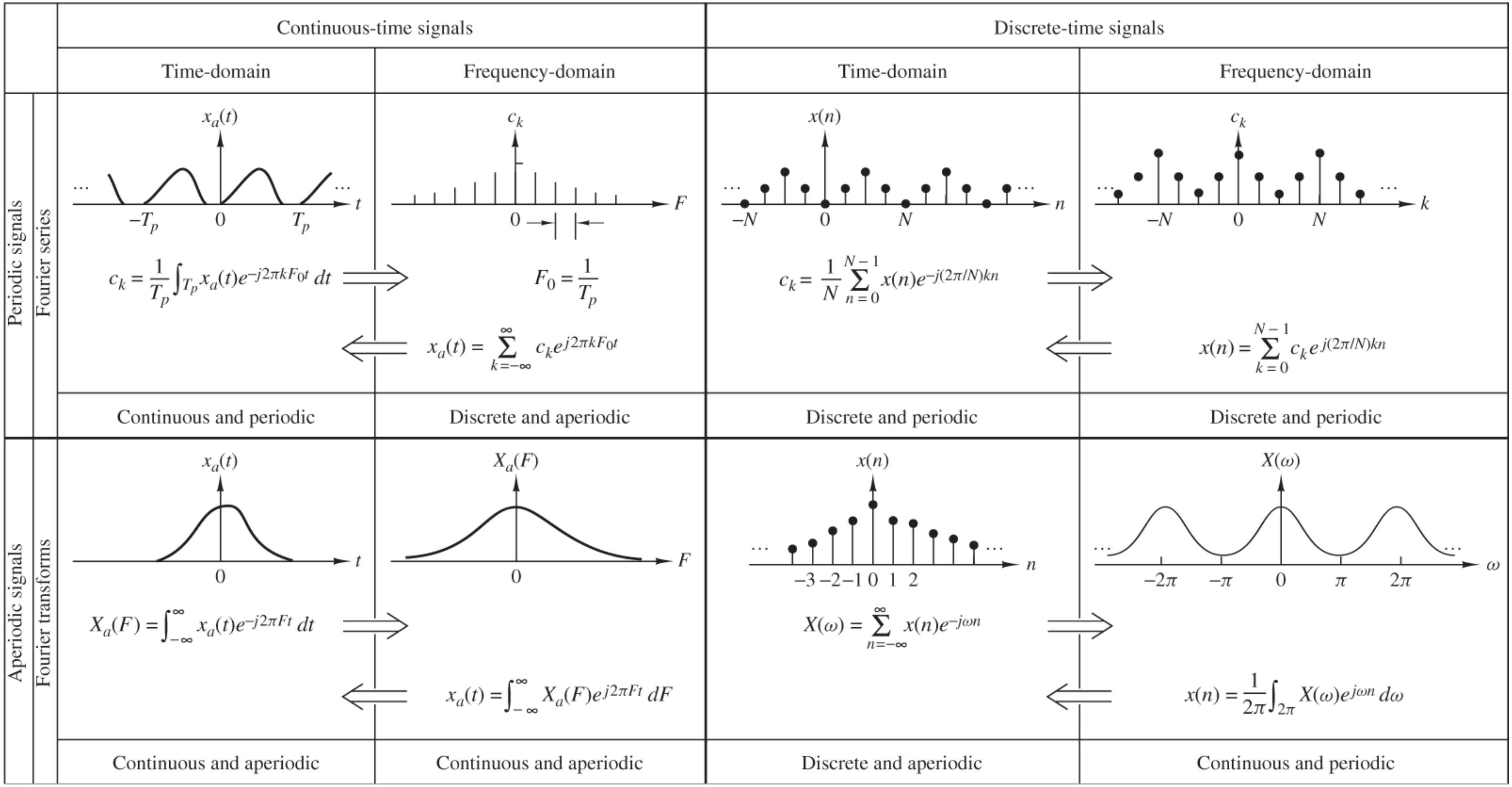

Figure 4.3.1 Summary of analysis and synthesis formulas.

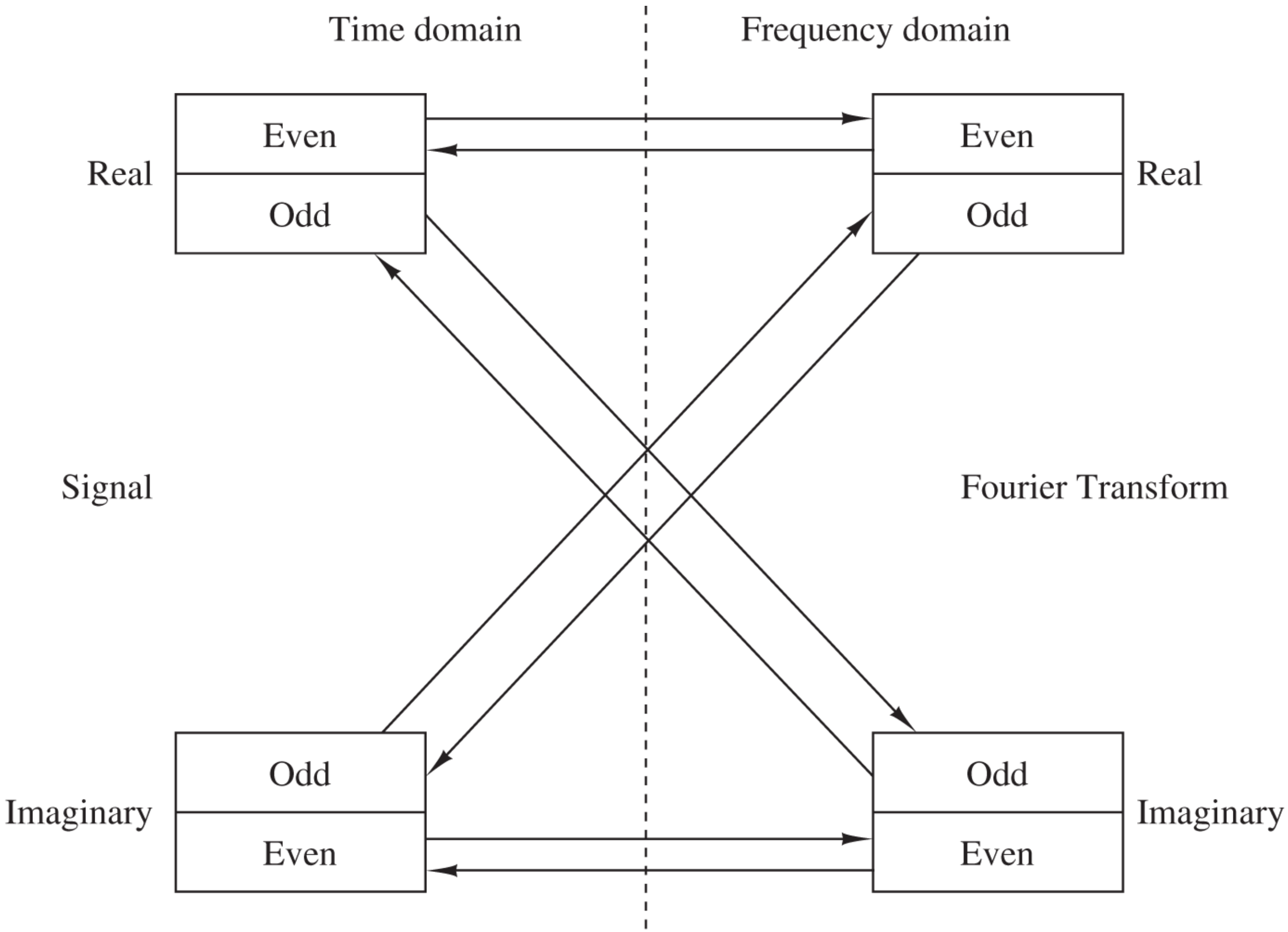

Figure 4.4.2 Summary of symmetry properties for the Fourier transform.

Fourier Transofrm Theorems and Properties

- Linearity
- Time shifting
- Time reversal
- Convolution theorem
- Correlation theorem
- Wiener-Khintchine theorem
- Frequency shifting
- Modulation theorem
- Parseval’s theorem
- Multiplication of two sequences (Windowing theorem)
- Differentiation in the frequency domain

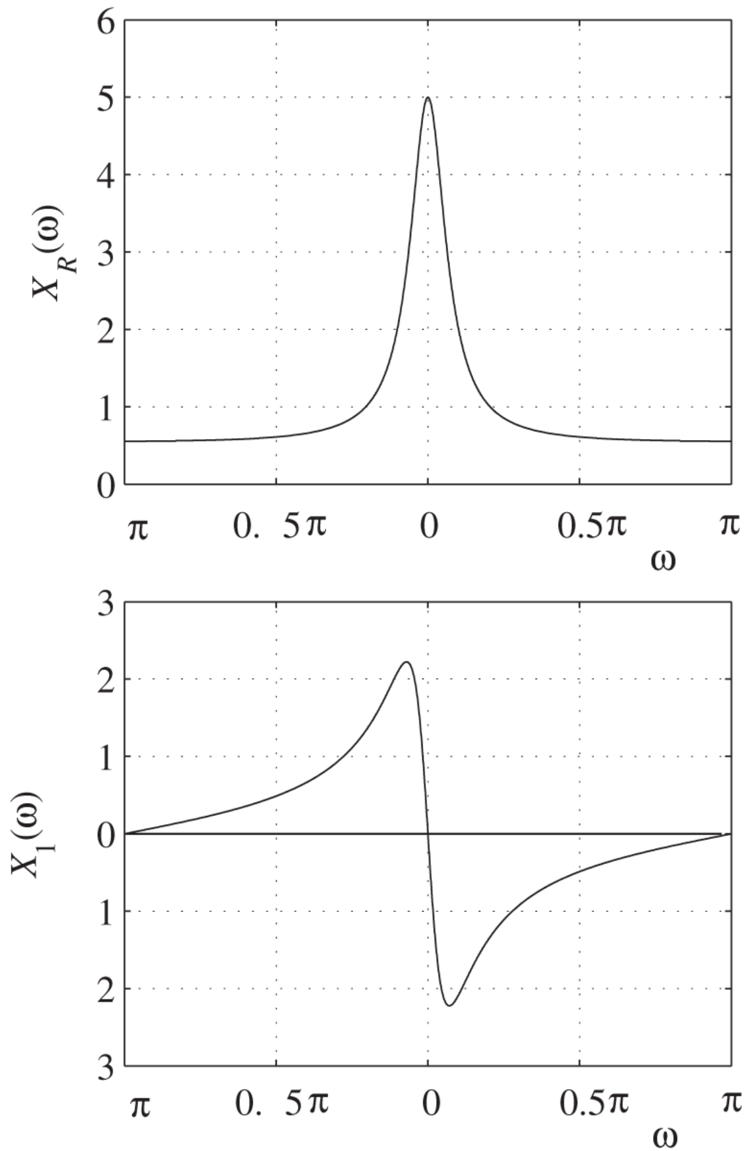

Figure 4.4.3 Graph of $X_R(\omega)$ and $X_I(\omega)$ for the transform in Example 4.4.1.

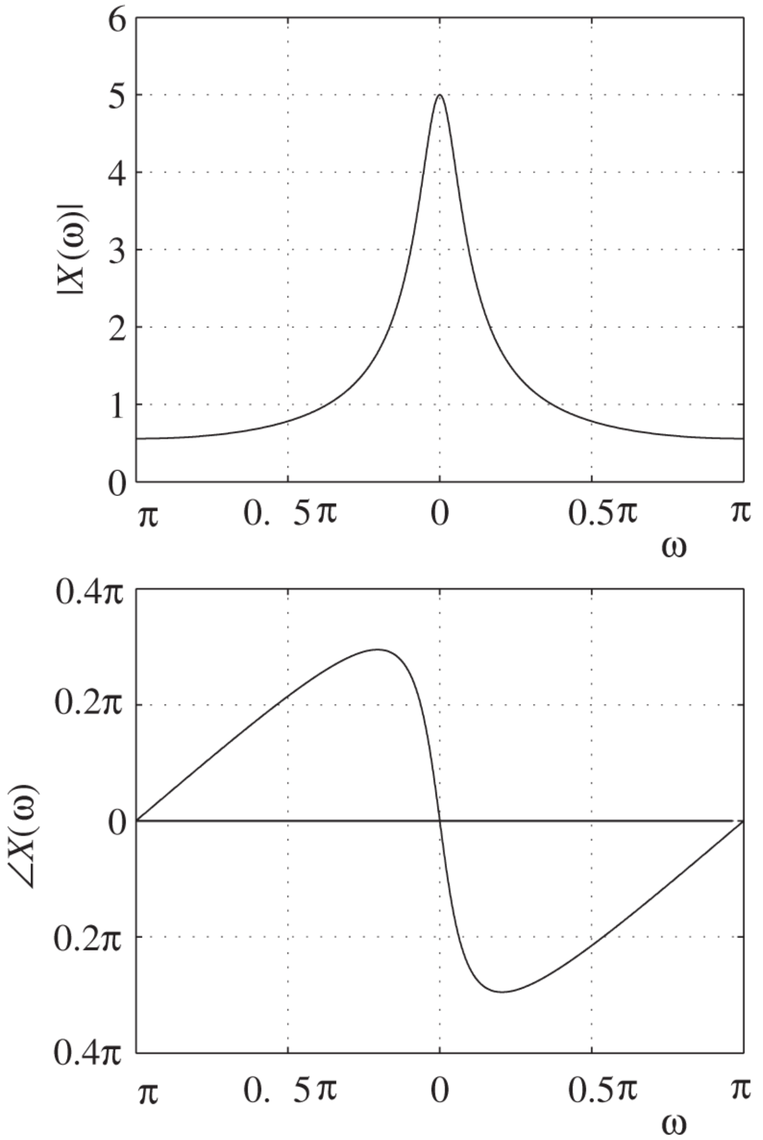

Figure 4.4.4 Magnitude and phase spectra of the transform in Example 4.4.1.

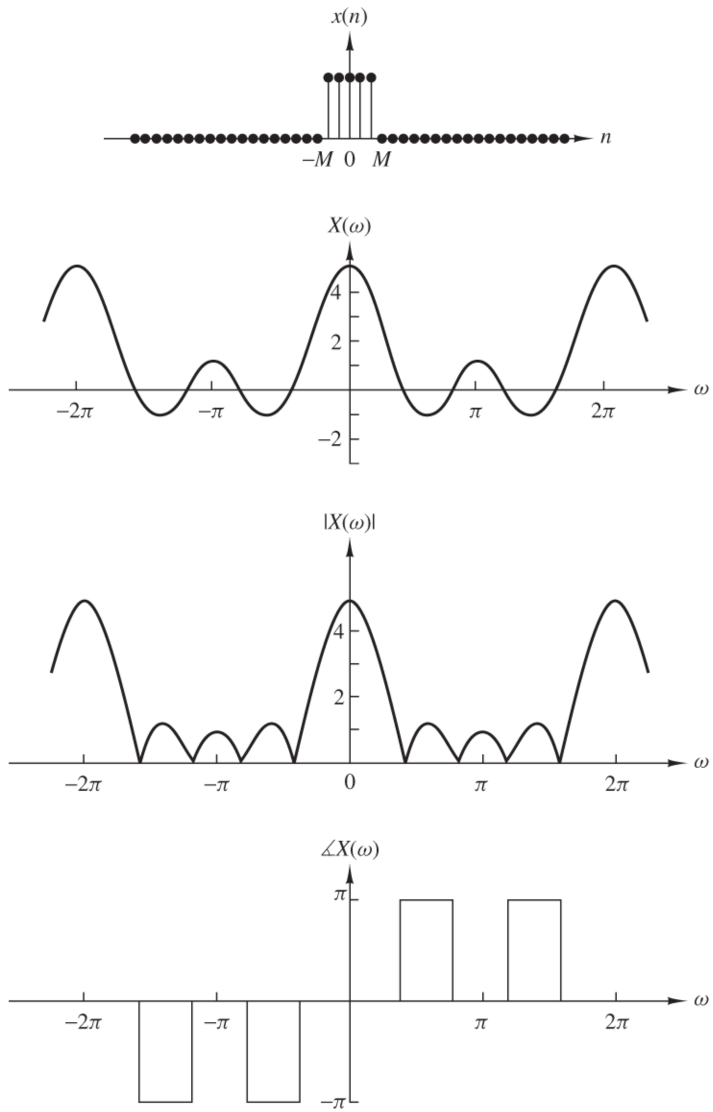

Figure 4.4.5 Spectral characteristics of rectangular pulse in Example 4.4.2.

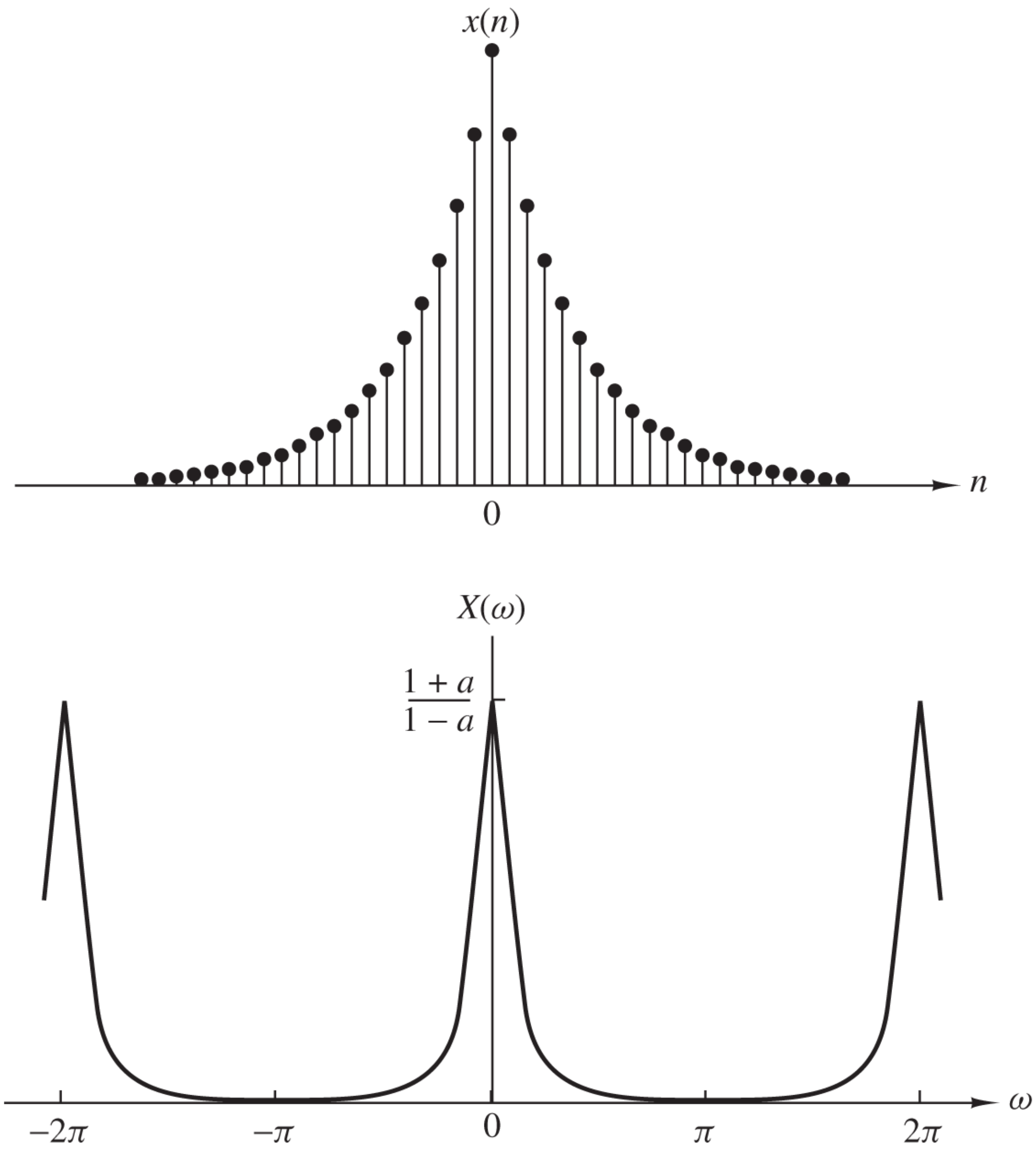

Figure 4.4.6 Sequence $x(n)$ and its Fourier transform in Example 4.4.3 with $a = 0.8$.

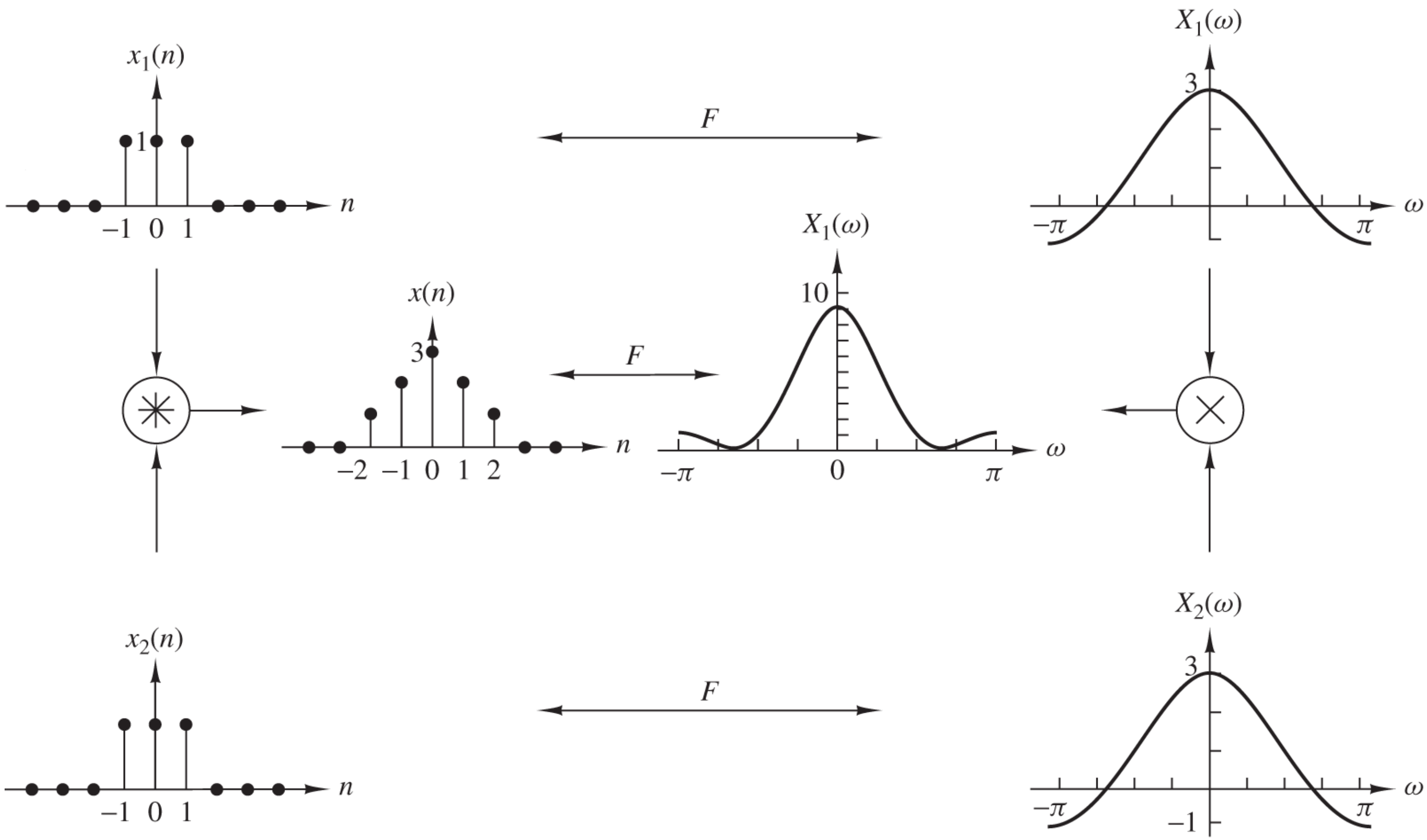

Figure 4.4.7 Graphical representation of the convolution property.

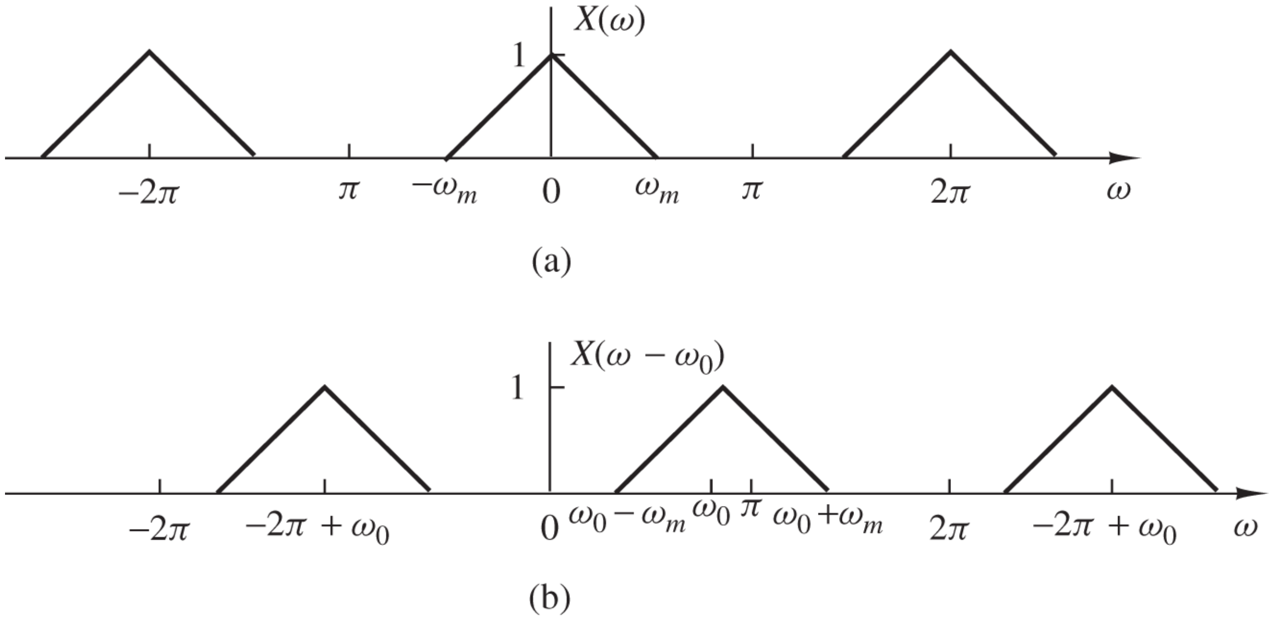

Figure 4.4.8 Illustration of the frequency-shifting property of the Fourier transform ($\omega_0 \leq 2\pi - \omega_m$).

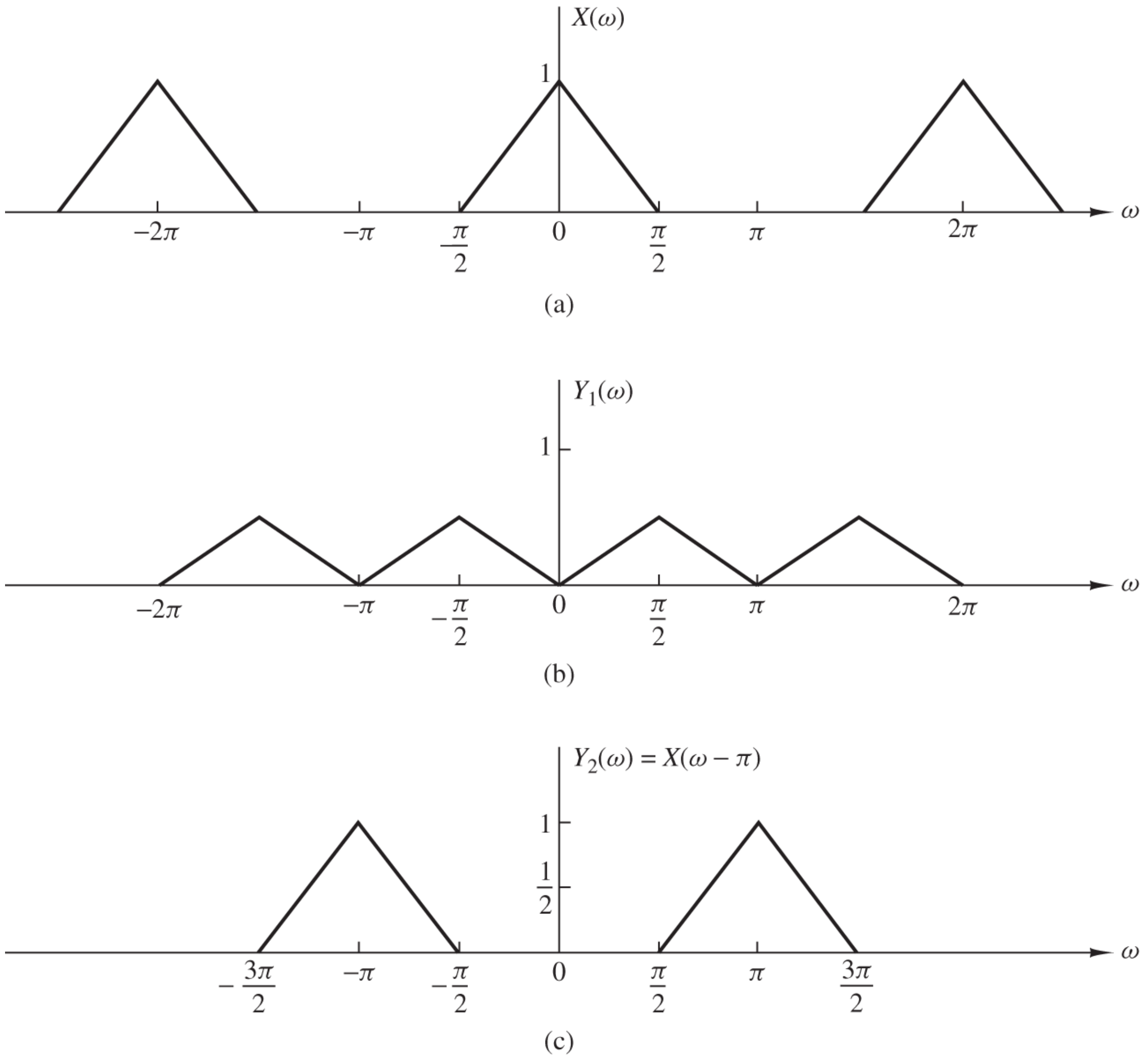

Figure 4.4.9 Graphical representation of the modulation theorem.