FPT Education
FPT UNIVERSITY

Motion Models

Objectives

- What is image stitching ?
- Understand about motion models.
- Study some motion model such as Planar perspective motion, Rotational panoramas, Gap closing, Cylindrical and spherical coordinates
- Applications of motion models

What is image stitching?

1 or 2 images ?

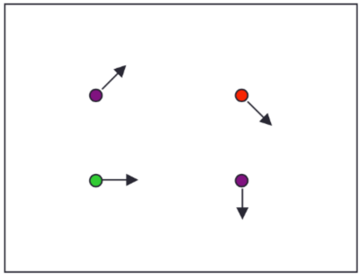
Any stitching application do you know ?

Image stitching

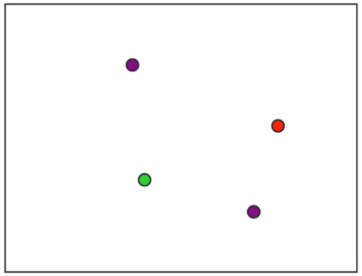
- Image stitching is the process of combining multiple photographic images with overlapping fields of view to produce a segmented panorama or high-resolution image.
- Image stitching is a widely used technique for recovering original data from ripped data.
- Image stitching is used in forensic and investigative science for the reconstruction of torn paper which is a big problem.
- In image mapping, stitching of images is done to do the complete mapping of a particular place.

Motion

- How to estimate pixel motion from image I(x,y,t) to I(x,y,t+1) ?•
  - Solve pixel correspondence problem– given a pixel in I(x,y,t), look for nearby pixels of the same color in I(x,y,t+1)
- This is called the optical flow problem



$I(x,y,t)$



$I(x,y,t+1)$

Optical flow problem

- Intensity equation:

$$\begin{aligned} 0 &= I(x+u, y+v, t+1) - I(x, y, t) \\ &\approx I(x, y, t+1) + I_x u + I_y v - I(x, y, t) \\ &\approx [I(x, y, t+1) - I(x, y)] + I_x u + I_y v \\ &\approx I_t + I_x u + I_y v \\ &\approx I_t + \nabla I \cdot < u, v > \end{aligned}$$

- In the limit as u and v go to zero, this becomes exact

- Brightness constancy constraint equation

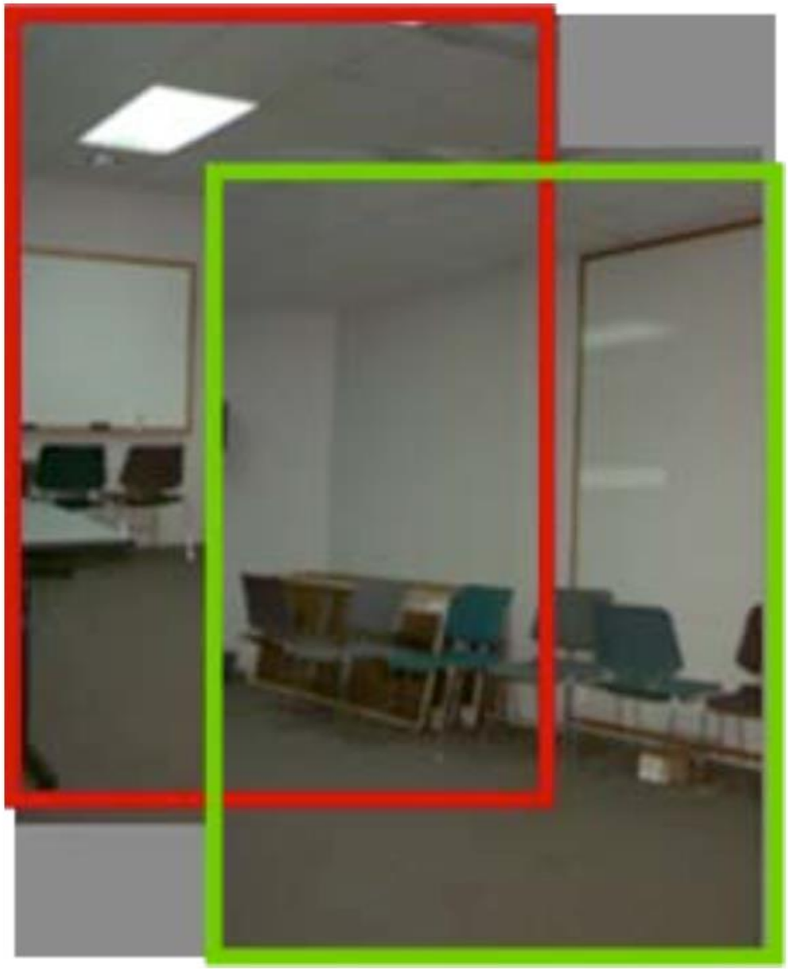
$$0 = I_t + \nabla I \cdot < u, v >$$

$$I_x\, u + I_y\, v + I_t = 0$$

Planar perspective motion

- Aligning images is to simply translate and rotate them in 2D
- This is exactly the same kind of motion that you would use if you had overlapping photographic prints.
- The perspective transformation deals with the conversion of 3d world into 2d image.

Planar perspective motion

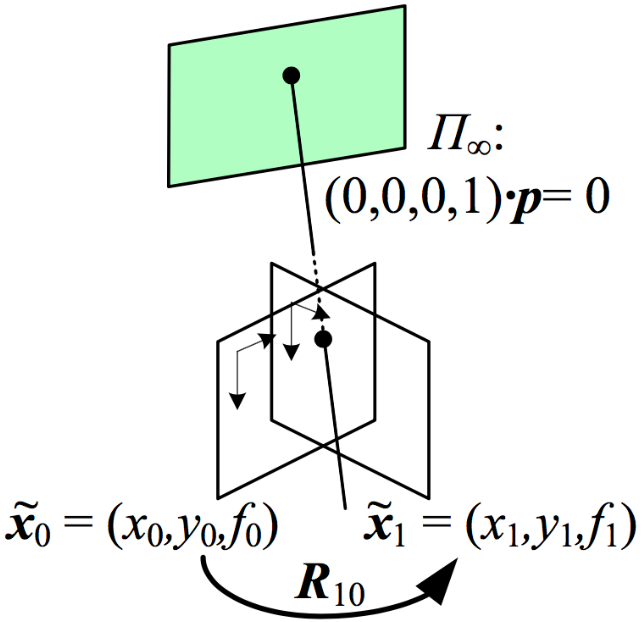


Application: Whiteboard and document scanning

- The most obvious application is used in scanning large documents - beyond the size of the scanner--> so what is the solution?
- We simply use multiple scanners, scan parts of the document, and then stitching them together.
- Note that the component images must overlap with amount enough so as not to miss any feature of the other large document.

Rotational panoramas

- Assume the camera is doing pure 3D rotation
- The most common panoramic image stitching, e.g., when taking images of the Grand Canyon
- Assumes that all points are very



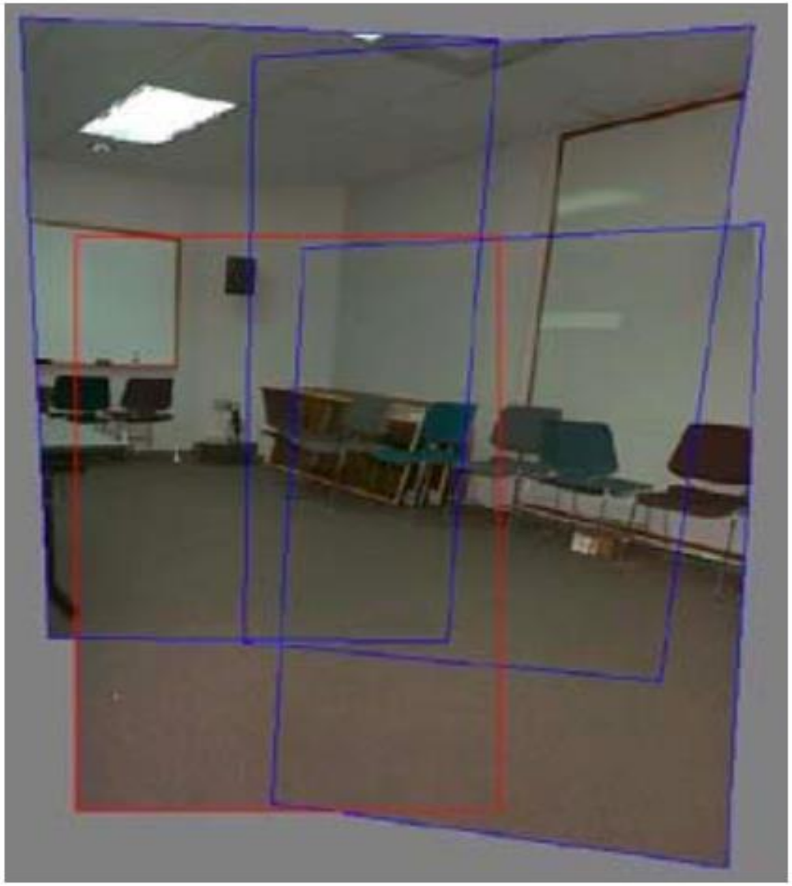
Rotational panoramas

- In this case simplified homography
  - Where K the camera intrinsic matrix assuming cx = cy = 0 $$\tilde{\boldsymbol{H}}_{10} = \boldsymbol{K}_1 \boldsymbol{R}_1 \boldsymbol{R}_0^{-1} \boldsymbol{K}_0^{-1} = \boldsymbol{K}_1 \boldsymbol{R}_{10} \boldsymbol{K}_0^{-1}$$
- This can be written as
- Or more $$\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} \sim \begin{bmatrix} f_1 & & \\ & f_1 & \\ & & 1 \end{bmatrix} \boldsymbol{R}_{10} \begin{bmatrix} f_0^{-1} & & \\ & f_0^{-1} & \\ & & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ y_1 \\ f_1 \end{bmatrix} \sim \boldsymbol{R}_{10} \begin{bmatrix} x_0 \\ y_0 \\ f_0 \end{bmatrix}$$

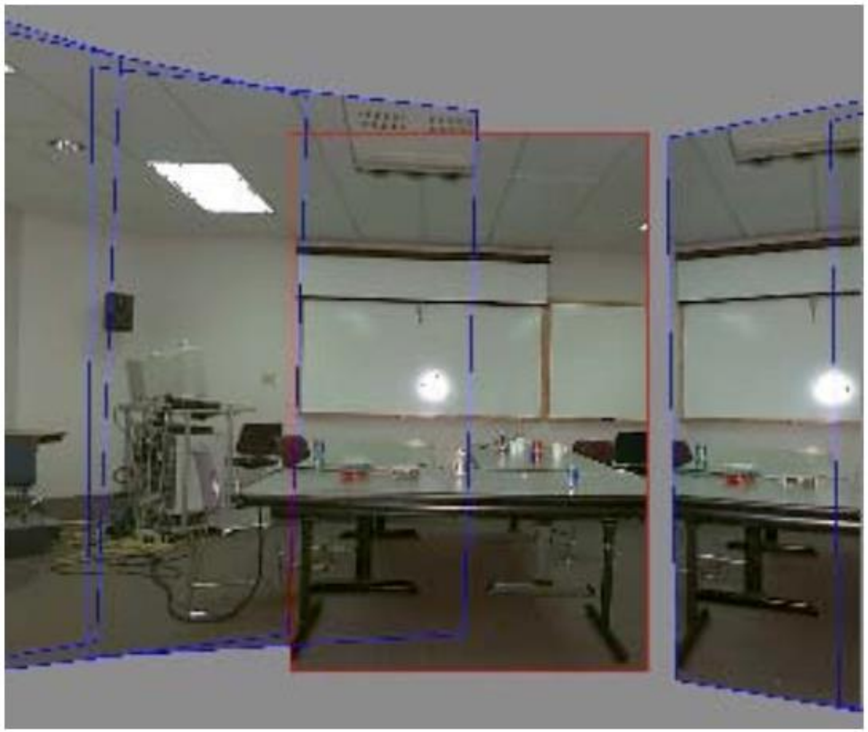
Rotational panoramas

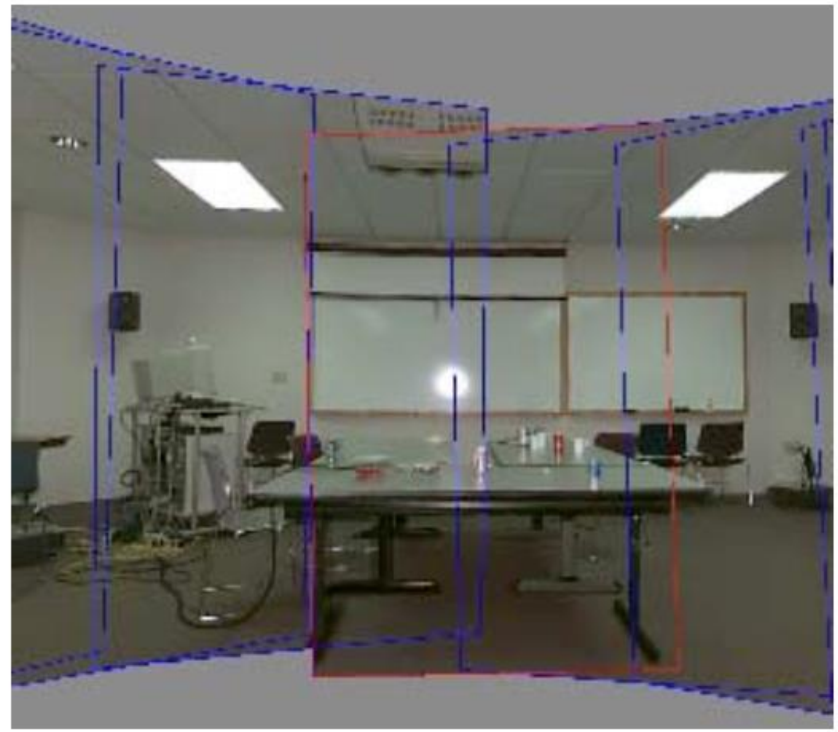
- Four images taken with a hand-held camera registered using a 3D rotation motion model



Gap closing

- A gap is visible when the focal length is wrong (f = 510)





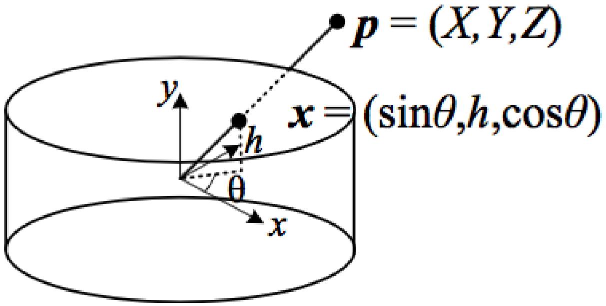
- No gap is visible for the correct focal length (f = 468).

Cylindrical and Spherical Coordinates

- An alternative to using homographies or 3D motions to align images is to first warp the images into cylindrical coordinates and then use a pure translational model to align them
- This only works if the images are all taken with a level camera or with a known tilt angle.
- Assume for now that the camera is in its canonical position, i.e., R = I and the optical axis is aligned with the z axis and the y axis is aligned vertically
- We wish to project this image onto a cylindrical surface of unit radius
- Points on this surface are parameterized by an angle θ and a height h with the 3D cylindrical given by $(\sin\theta, h, \cos\theta) \propto (x, y, f)$

Cylindrical and Spherical Coordinates

- We can compute the correspondence between warped and mapped coordinates



$$
\begin{aligned}
x' &= s\theta = s \tan^{-1}\frac{x}{f}, \\
y' &= sh = s\frac{y}{\sqrt{x^2+f^2}}
\end{aligned}
$$

$$
\begin{aligned}
x &= f\tan\theta = f\tan\frac{x'}{s}, \\
y &= h\sqrt{x^2+f^2} = \frac{y'}{s}f\sqrt{1+\tan^2 x'/s} = f\frac{y'}{s}\sec\frac{x'}{s}
\end{aligned}
$$

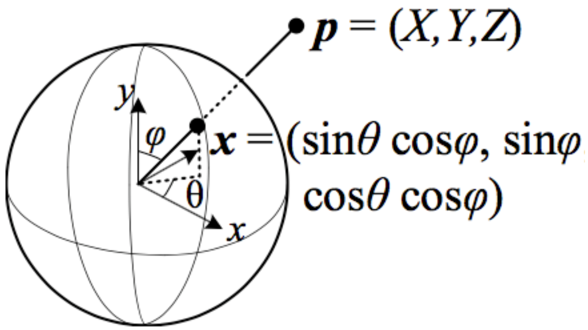
Cylindrical Panorama

- Cylindrical is used if the camera is level and we have only rotation around its vertical axis
- Then we only need to estimate a translation

Spherical Projection

- while the inverse is given by

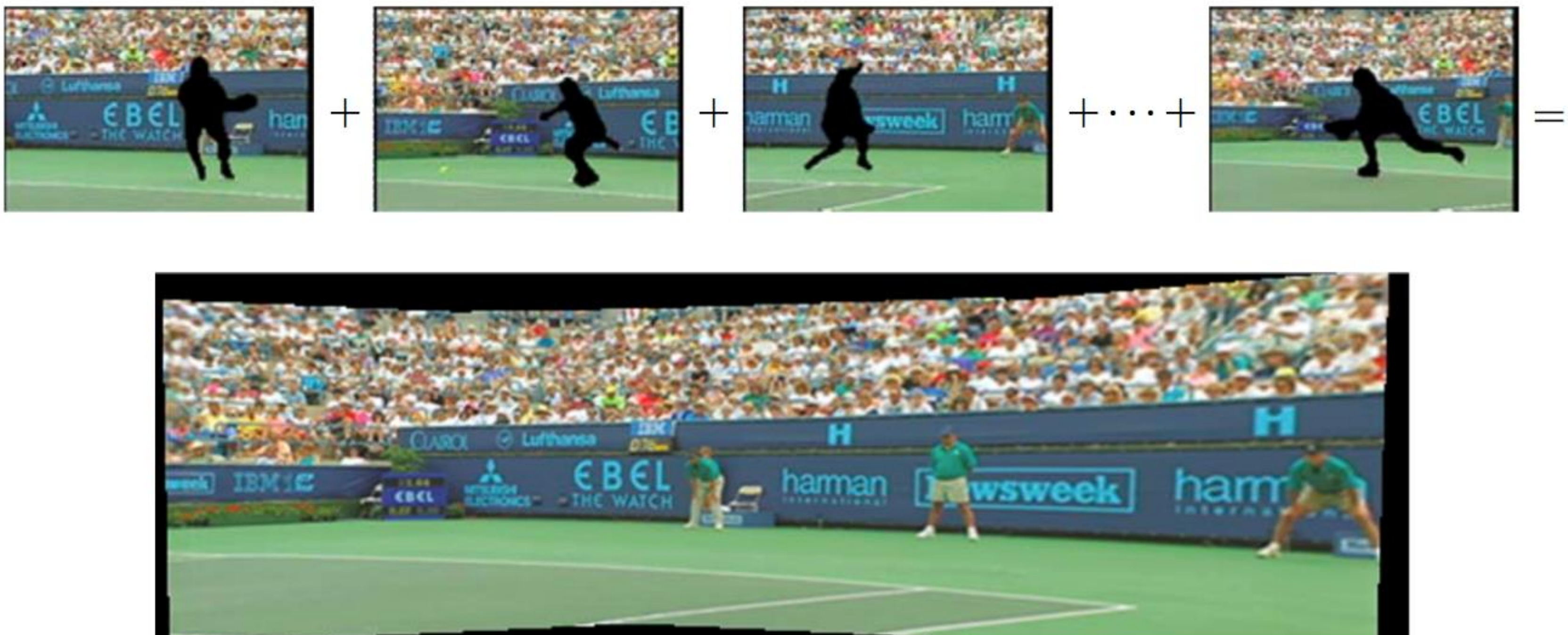
$$
\begin{aligned}
x' &= s\theta = s \tan^{-1} \frac{x}{f}, \\
y' &= s\phi = s \tan^{-1} \frac{y}{\sqrt{x^2 + f^2}}
\end{aligned}
$$



$$
\begin{aligned}
x &= f \tan\theta = f \tan \frac{x'}{s}, \\
y &= \sqrt{x^2 + f^2} \tan\phi = \tan \frac{y'}{s} f \sqrt{1 + \tan^2 x'/s} = f \tan \frac{y'}{s} \sec \frac{x'}{s}
\end{aligned}
$$

Application: Video summarization and compression

- Video stitching the background scene to create a single sprite image that can be transmitted and used to re-create the background in each frame



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