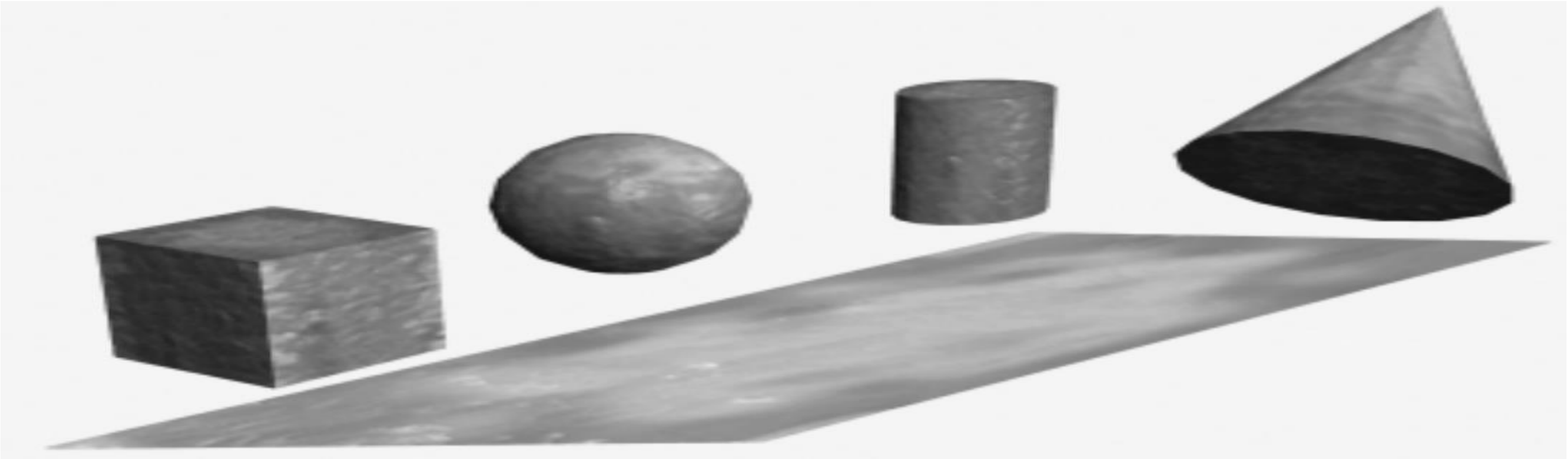
Geometric primitives and transformations

Objectives

- Study the basic geometric primitive: points, lines, and planes
- 2D, 3D geometric
- The geometric transformations
- Lens distortions

What is a geometric primitive?

- Geometric primitives are the basic shapes we all know and recognize
  - Cubes, spheres, cylinders, and cones,
  - They work just like the blocks in a typical preschool building set



Geometric primitive

- Homogeneous coordinates→ way to approach the projective plane analytically
  - Represent coordinates in 2 dimensions with 3 –vector
  - Add a 3rd coordinate to every 2D point
    - (x, y, w) ➔ $(x/w, y/w)$
    - w = 0→ point at infinity
    - (0,0,0) is undefined
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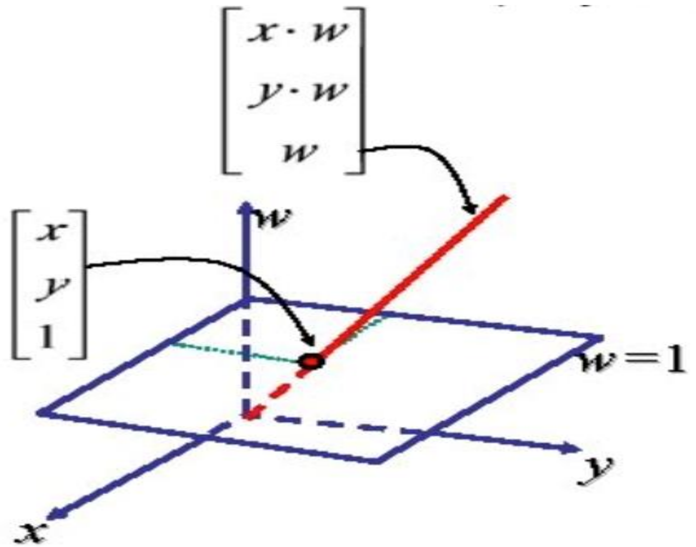
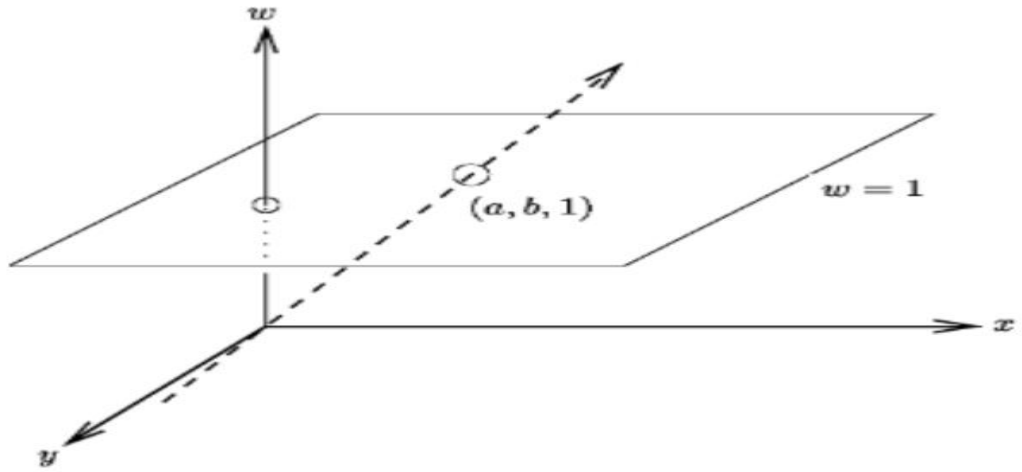


Geometric primitive

- 2D Point: two-dimensional primitive we can implement. It is infinitely small; it has x and y coordinates: $x = (x, y) \in \mathcal{R}^2$
  - Homogeneous coordinates: $\tilde{x} = (\tilde{x}, \tilde{y}, \widetilde{w}) \in \mathcal{P}^2$
  - The 2D projective space: $\mathcal{P}^2 = \mathcal{R}^3 - (0,0,0)$



Geometric primitive

- 2D Lines: A line is a straight one-dimensional figure having no thickness and extending infinitely in both directions
  - Equation: $ax + by + c = 0$
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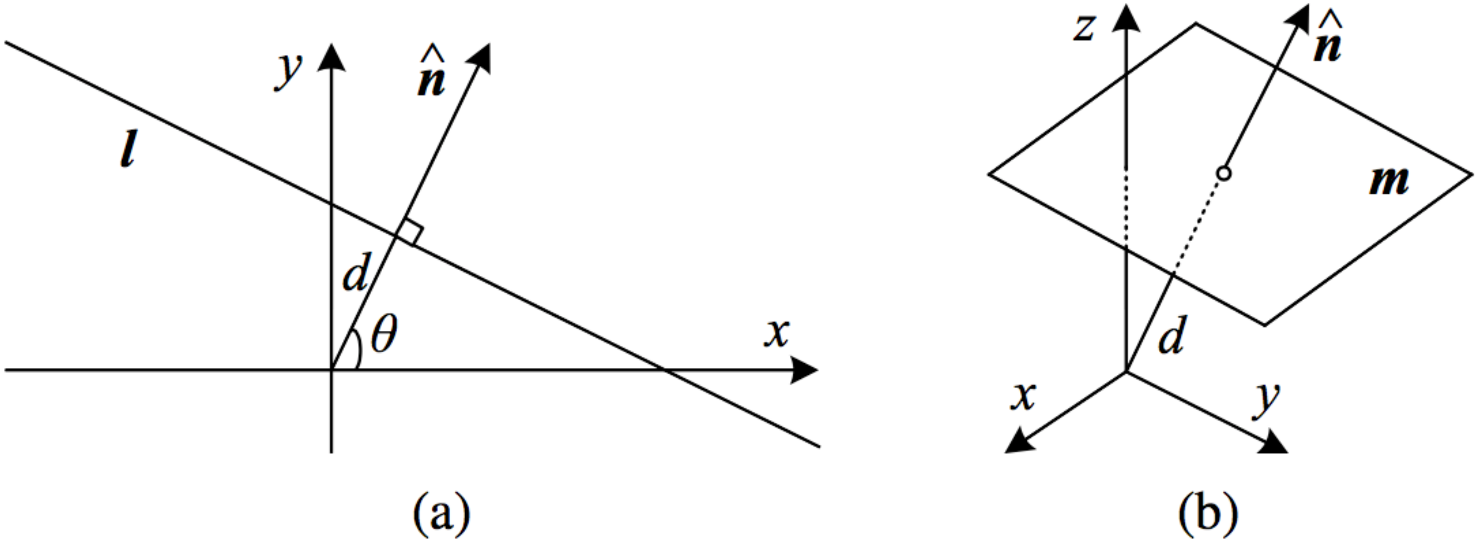
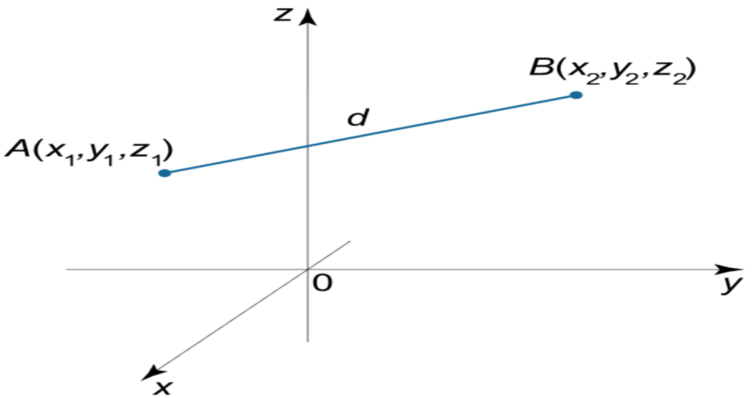


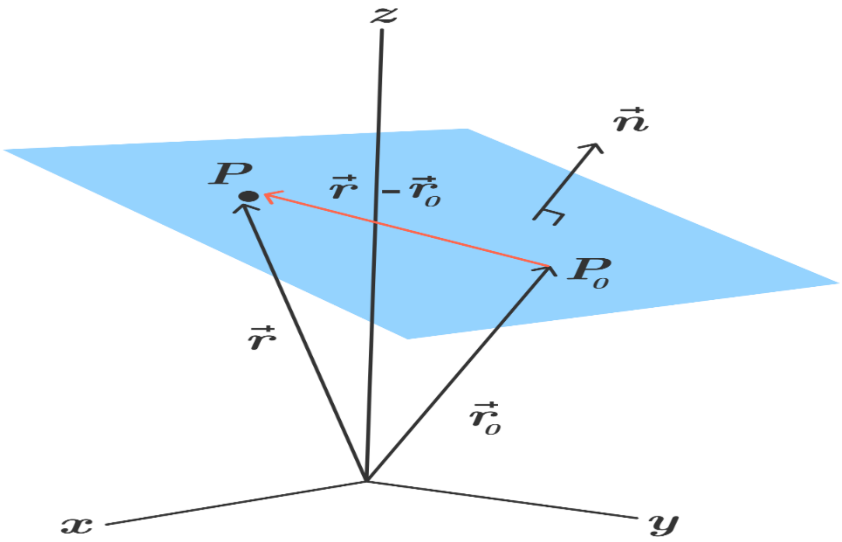
Geometric primitive

- 3D point: basis consisting of three mutually perpendicular vectors. These vectors define the three coordinate axes: the x–, y–, and z–axis
  - Denoted: $\mathrm{A(x, y, z)}$
  - Distance between two points $\mathrm{A(x1, y1, z1)}$ and $\mathrm{B(x2, y2, z2)}$:
  - $d = |\, AB \,| = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$



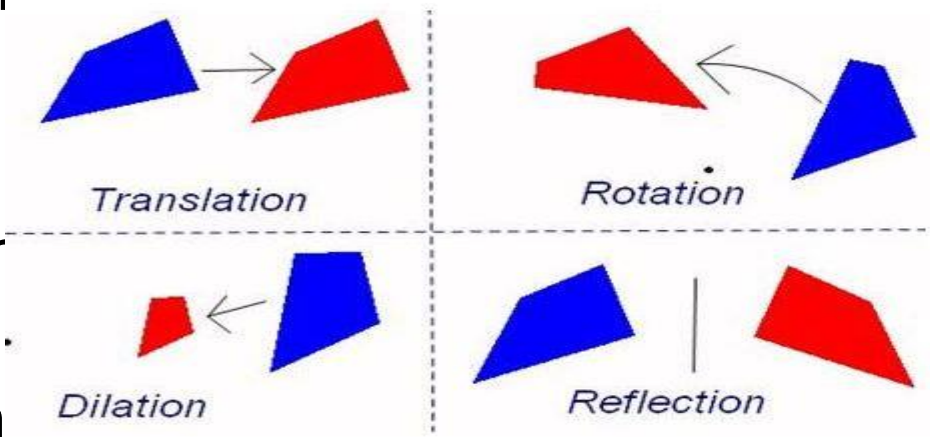
Geometric primitive

- 3D planes:
  - A plane is a flat, two-dimensional surface that extends infinitely far
  - A plane in 3D coordinate space is determined by a point and a vector that is perpendicular to the plane
  - Equation: $ax + by + cz + d = 0$



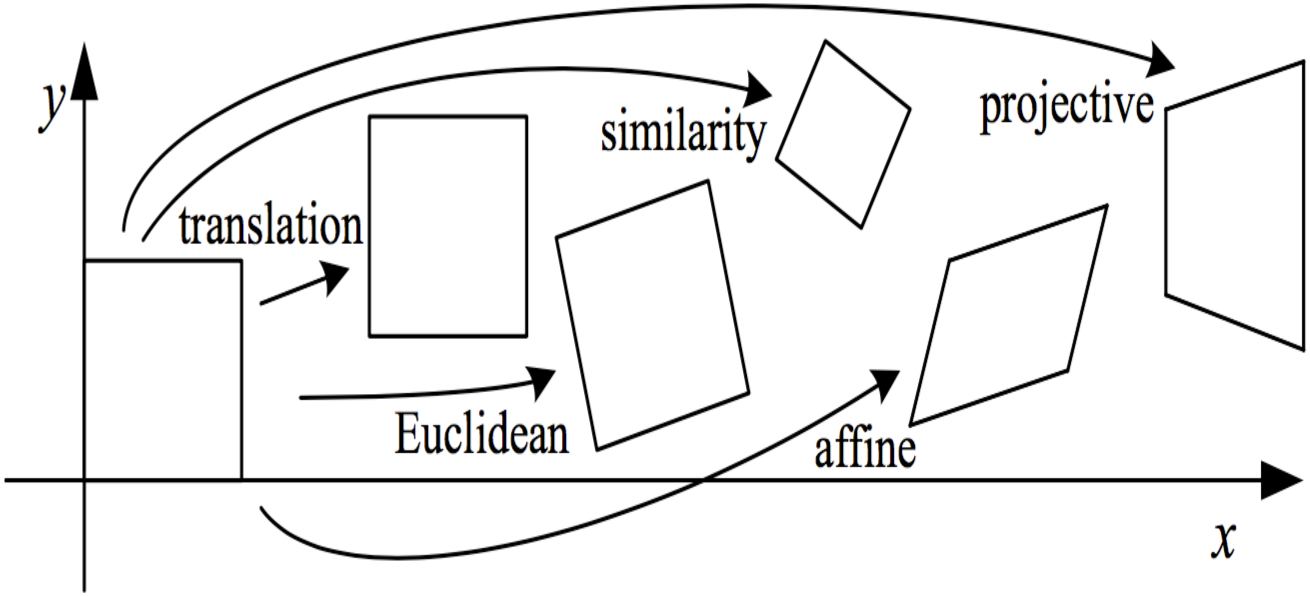
Transformations

- Graph transformation is the process by which an existing graph, or graphed equation, is modified to produce a variation of the proceeding graph.
- Image transformation is a function or operator that takes an image as its in
as its output.
- Geometric transformati
geometry of an im
transformations where
changed without alterin
commonly referred to as "Geometric" transformation.



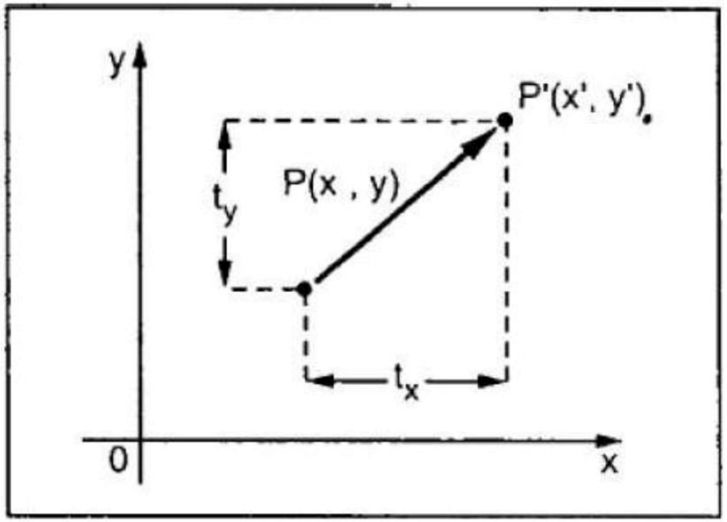
2D Transformations

- Basic set of 2D planar transformations.
  - Translation
  - Euclidean
  - Similarity
  - Affine
  - Projective



2D Transformations

- Translation:
  - A translation moves an object to a different position on the screen.
  - You can translate a point in 2D by adding translation coordinate (tx, ty) to the original coordinate (X, Y) to get the new coordinate (X’, Y’).
  - Genaral equation $\boldsymbol{p}' = \boldsymbol{p} + \boldsymbol{t}$



- ▻ $\mathrm{X}' = \mathrm{X} + \mathrm{tx}$
- ▻ $\mathrm{Y}' = \mathrm{Y} + \mathrm{t}y$
- ▻ (tx, ty): translation vector or shift vector

2D Transformations

- Rotation:
  - We rotate the object at particular angle θ (theta) from its origin..
  - You can translate a point in 2D by adding translation coordinate (tx, ty) to the original coordinate (X, Y) to get the new coordinate (X', Y').
  - Genaral equation: $\boldsymbol{p}' = \boldsymbol{\theta}\boldsymbol{p} + \boldsymbol{t}$

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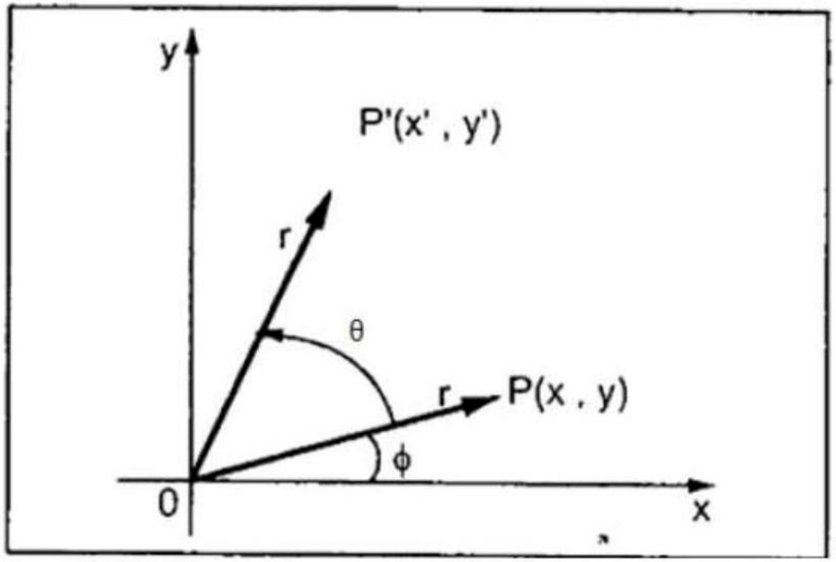
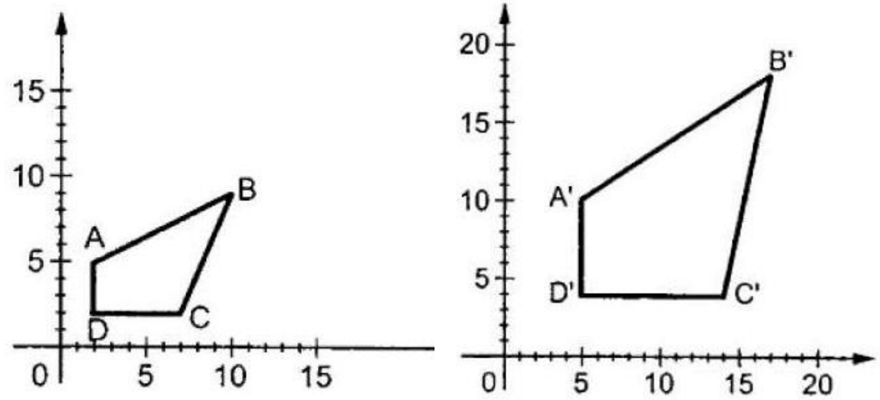


2D Transformations

- Scaling:
  - To change the size of an object.
  - You either expand or compress the dimensions of the object.
  - Scaling can be achieved by multiplying the original coordinates of the object with the scaling factor to get the desired result.
  - The scaling factor SX, SY scales the object in X and Y direction respectively
  - Genaral equation $\boldsymbol{p}' = \boldsymbol{p}.S$

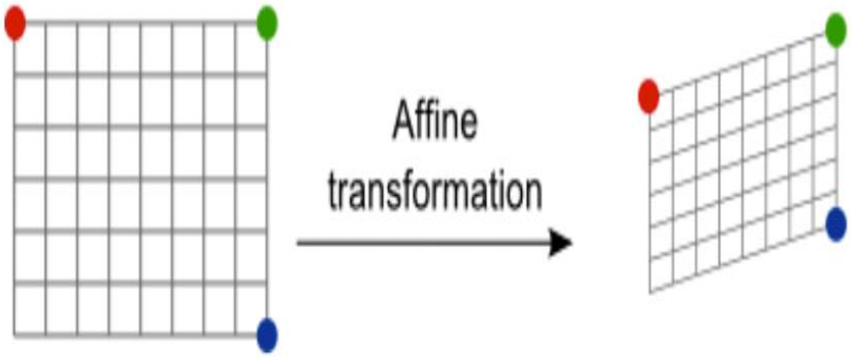


▻ $X' = X . S X$
▻ $Y' = Y . S Y$
▻ (Sx, Sy): the scaling factors

2D Transformations

- Affine:
  - Affine transformations are used for scaling, skewing and rotation.
  - Lines map to line.
  - Parallel line remain parallel under affine transformations
  - Genaral equation $x' = A\bar{x}$ where A is an arbitrary 2 x 3 matrix

$$x' = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \end{bmatrix} \bar{x}$$



2D Transformations

- Projective:
  - A projective transformation shows how the perceived objects change as the observer's viewpoint changes.
  - These transformations allow the creating of perspective distortion.
  - Perspective transformations preserve straight lines (i.e., they remain straight after the trans- formation).
  - Genaral equation $\tilde{x}' = \tilde{H}\tilde{x}$ where H is an arbitrary 3 x 3 m

$$x' = \frac{h_{00}x + h_{01}y + h_{02}}{h_{20}x + h_{21}y + h_{22}}$$

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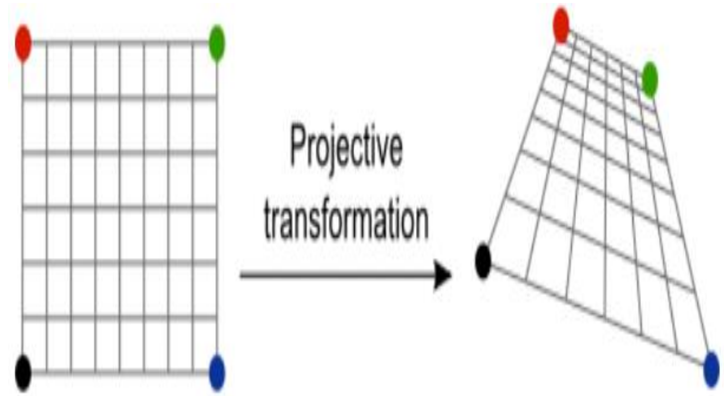
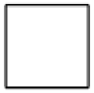
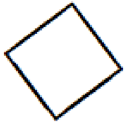
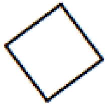
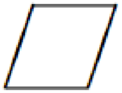
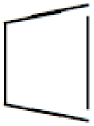
$$y' = \frac{h_{10}x + h_{11}y + h_{12}}{h_{20}x + h_{21}y + h_{22}}$$

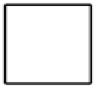
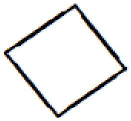
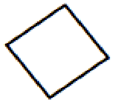
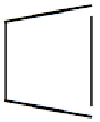


2D Transformations

Transformation	Matrix	# DoF	Preserves	Icon
translation	$\left[\begin{array}{c\|c} \boldsymbol{I} & \boldsymbol{t} \end{array}\right]_{2\times 3}$	2	orientation	
rigid (Euclidean)	$\left[\begin{array}{c\|c} \boldsymbol{R} & \boldsymbol{t} \end{array}\right]_{2\times 3}$	3	lengths	
similarity	$\left[\begin{array}{c\|c} s\boldsymbol{R} & \boldsymbol{t} \end{array}\right]_{2\times 3}$	4	angles	
affine	$\left[\begin{array}{c} \boldsymbol{A} \end{array}\right]_{2\times 3}$	6	parallelism	
projective	$\left[\begin{array}{c} \tilde{\boldsymbol{H}} \end{array}\right]_{3\times 3}$	8	straight lines	

3D Transformations

- What are different to 2D transformations?→ write your answer !

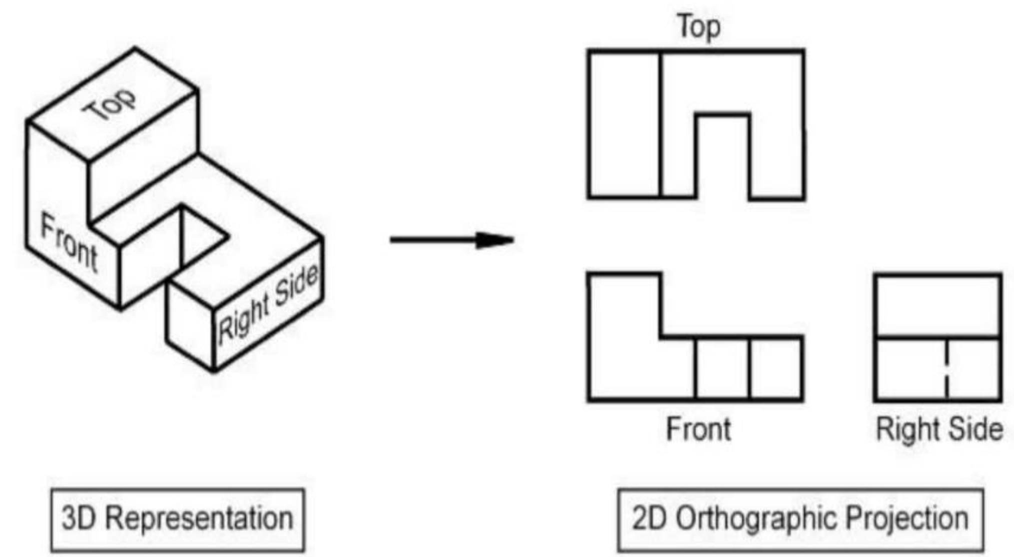
Transformation	Matrix	# DoF	Preserves	Icon
translation	$\begin{bmatrix} \boldsymbol{I} \mid \boldsymbol{t} \end{bmatrix}_{3\times 4}$	3	orientation	
rigid (Euclidean)	$\begin{bmatrix} \boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{3\times 4}$	6	lengths	
similarity	$\begin{bmatrix} s\boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{3\times 4}$	7	angles	
affine	$\begin{bmatrix} \boldsymbol{A} \end{bmatrix}_{3\times 4}$	12	parallelism	
projective	$\begin{bmatrix} \tilde{\boldsymbol{H}} \end{bmatrix}_{4\times 4}$	15	straight lines	

3D to 2D projections

- What is projections ?
- How 3D primitives are projected onto the image 3D plane?
  - Linear 3D to 2D projection matrix.
  - Orthography, which requires no division to get the final (inhomogeneous) result.
  - Perspective since this more accurately models the behavior of real cameras
- Commonly used projection models
  - Orthography
  - Scaled orthography
  - Para-perspective
  - Perspective / Object-centered

3D to 2D projections

- Orthography
  - Drops the z component of the three-dimensional coordinate p to obtain the 2D point x
  - Equation : $x = [I_{2x2}|0]p$ where p denote 3D point and x denote 2D point
  - In homogeneous coordinates



$$\tilde{\boldsymbol{x}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tilde{\boldsymbol{p}}$$

3D to 2D projections

- Perspective
  - When we view scenes in everyday life far away items appear small relative to nearer items.
  - A side effect of perspective projection is that parallel lines appear to converge on a vanishing point.

Inhomogeneous coordinates

$$\bar{\boldsymbol{x}} = \mathcal{P}_z(\boldsymbol{p}) = \begin{bmatrix} x/z \\ y/z \\ 1 \end{bmatrix}$$

Homogeneous coordinates

$$\tilde{\boldsymbol{x}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tilde{\boldsymbol{p}},$$

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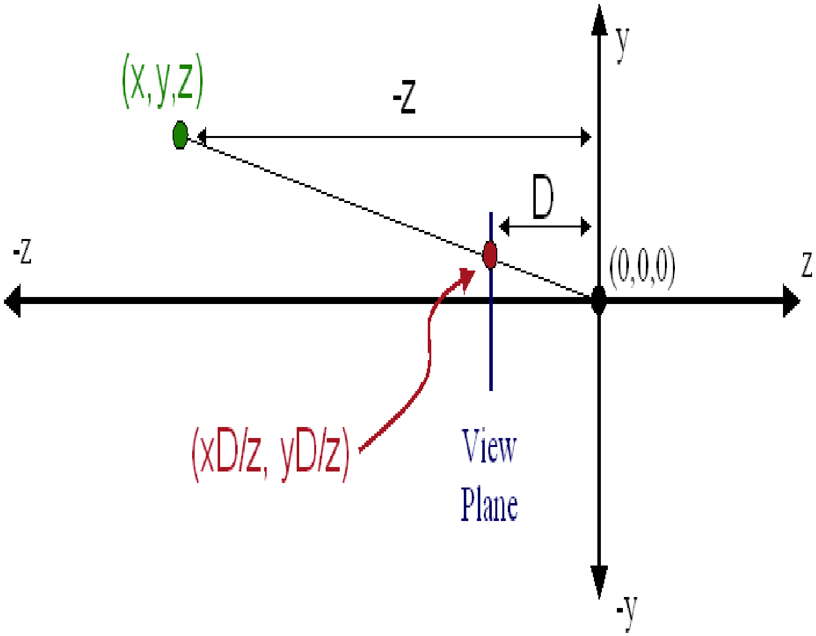
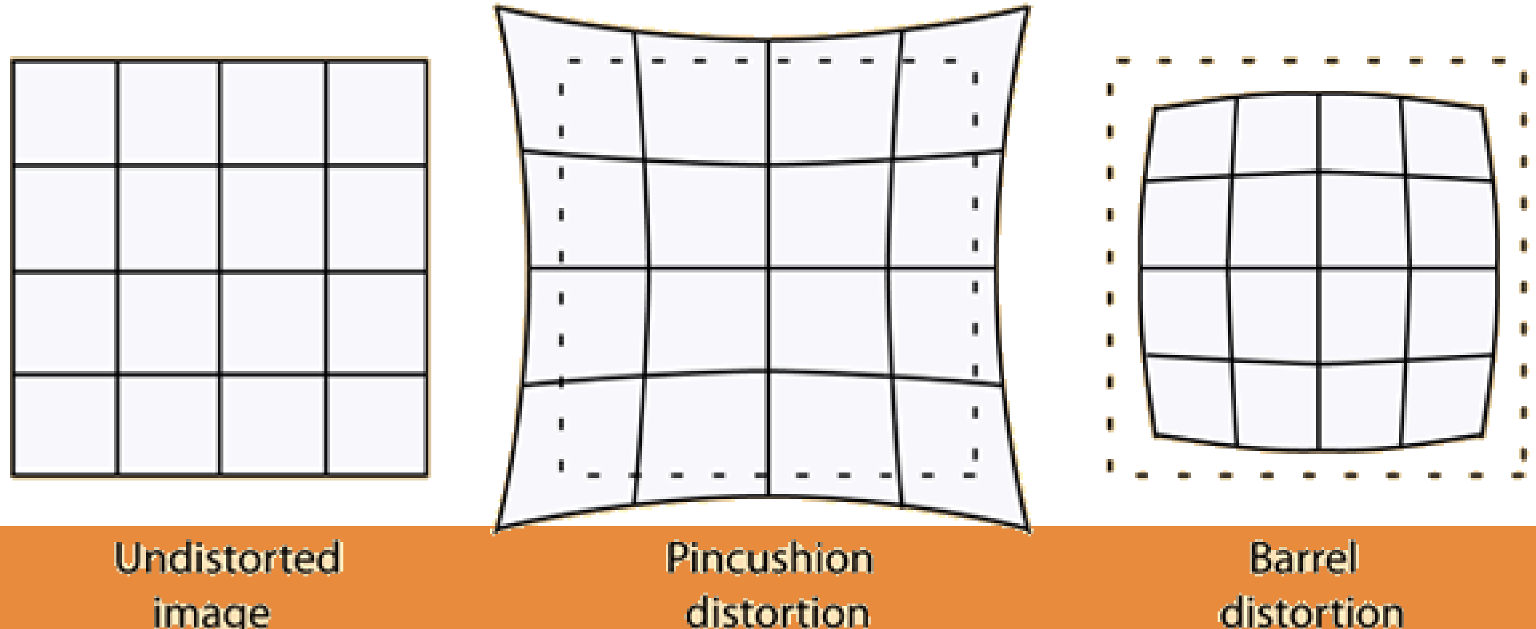


Lens distortions

- Cameras obey a *linear* projection model where straight lines in the world result in straight lines in the image. Is it true in all cases?→ **No**
- Many wide-angle lenses have distortion --> visible curvature in the projection of straight lines.
- Distortion occurs when the linear magnification is a function of the off-axis distance.



Summary

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- Study the basic geometric primitive: points, lines, and planes
- 2D, 3D geometric
- The geometric transformations
- Lens distortions.