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A Rational Approach to Wave Friction Coefficients for Rough, Smooth and Transitional Turbulent Flow

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ABSTRACT

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The wave friction coefficient (f_w) and the phase lead of the bottom shear stress over the free-stream velocity (ϕ) for rough, smooth and transitional smooth-to-rough turbulent flow are presented. The analogy between wave boundary layer flow and planetary boundary layer flow is utilized using similarity theory. Results are obtained for f_w and ϕ by determining the constants, as required by similarity theory*. An approximation for f_w by disregarding the phase ϕ is also presented. Comparisons are also made with experimental data.

INTRODUCTION

In shallow and intermediate water depths of wave-dominated areas, kinematics and dynamics of the fluid motion in the wave boundary layer near the seabed govern sediment transport and scour. Furthermore, boundary layer related friction between the fluid and the bottom contributes to the dissipation of surface gravity waves. Gravity waves represent the driving mechanism of the oscillatory motion in the boundary layer.

Analytical descriptions of the wave-boundary layer are provided by semi-empirical theories developed in close connection with experiments and theoretical modelling. The theoretical models are mainly based on simple eddy viscosity models or on the closely related models based on Prandtl's mixing length hypothesis. Reviews of wave boundary layers and related works are given in Jonsson (1980) and Myrhaug (1986). Results of theoretical work using more refined turbulence modelling techniques have also been reported recently

*Data from Sumer et al. (1987).

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by Hagatun and Eidsvik (1986) and Justesen (1988). Laboratory experiments on the turbulent wave boundary layer structure have also recently been reported by Sumer et al. (1987), Jensen et al. (1987) and Sleath (1987).

This paper presents the wave friction coefficient (f_w) and the phase lead of the bottom shear stress over the free-stream velocity (ϕ) for rough, smooth and transitional smooth-to-rough turbulent flow. The analogy between wave boundary layer flow and planetary boundary layer flow is utilized following Gill (1982) using similarity theory. Results are obtained for f_w and ϕ by determining the constants, as required by similarity theory, from rough turbulent flow data taken from Sumer et al. (1987). An approximation for f_w by disregarding the phase lead ϕ is also presented. Comparisons are also made with experimental data, and the present semi-empirical formulas for f_w and ϕ agree well with some of the experimental values.

THEORY

The analogy between wave boundary layer flow and planetary boundary layer flow is well known (Soulsby, 1983; Grant and Madsen, 1986). By utilizing this analogy and following Gill (1982), the free-stream velocity amplitude associated with the oscillatory wave motion can be written as:

$$u_0=\frac{u_*}{\kappa}\left(\ln\frac{u_*}{\omega z_0}-A-iB\right) \tag{1}$$

according to the similarity theory. Here $u_0=u_{0r}+iu_{0i}$ is the free-stream velocity in complex form with u_{0r} and u_{0i} as real and imaginary parts, respectively. $u_*=(\tau_0/\rho)^{\frac{1}{2}}$ is the friction velocity, ρ is the density of the fluid and τ_0 is the maximum shear stress at the seabed, which has a phase lead ϕ over the maximum value of the free-stream velocity. κ is Von Karman's constant ($=0.4$), z_0 is the roughness parameter of the seabed, ω is the cyclic frequency of wave motion, A and B are constants as required by the similarity theory and $i=(-1)^{\frac{1}{2}}$. The wave friction coefficient associated with the maximum shear stress at the seabed is defined as:

$$f_w=\frac{\tau_0}{\frac{1}{2}\rho|u_0|^2}=2\left(\frac{u_*}{|u_0|}\right)^2 \tag{2}$$

where $|u_0|$ is the magnitude of u_0 or the maximum free-stream velocity. By introducing the free-stream particle amplitude $a_0=|u_0|/\omega$ and by using Eq. 2, the magnitude of Eq. 1 can be written as:

$$\frac{2\kappa^2}{f_w}=\left[\ln\left(c\frac{a_0}{z_0}\sqrt{f_w/2}\right)\right]^2+B^2 \tag{3}$$

where $c=\mathrm{e}^{-A}$ is a constant. Since $u_{0r}=|u_0|\cos\phi$ and $u_{0i}=-|u_0|\sin\phi$, which expresses the phase lag of the maximum value of the free-stream velocity after the maximum shear stress at the seabed, and by using Eq. 2, A and B are given from Eq. 1 as:

$$A = \ln\left(\frac{a_0}{z_0}\sqrt{f_w/2}\right) - \frac{\kappa}{\sqrt{f_w/2}}\cos\phi \tag{4}$$

and:

$$B = \frac{\kappa}{\sqrt{f_w/2}}\sin\phi \tag{5}$$

Thus Eq. 4 and 5 determine A and B for a given data set. Then f_w can be determined from Eq. 3, and ϕ is determined from Eq. 5, i.e.:

$$\sin\phi = \frac{B}{\kappa}\sqrt{f_w/2} \tag{6}$$

An approximation to Eq. 3 is given by:

$$\frac{\kappa^2}{\sqrt{f_w/2}} = \left[\ln\left(c'\frac{a_0}{z_0}\sqrt{f_w/2}\right)\right]^2 \tag{7}$$

which equivalently can be written as:

$$|u_0| = \frac{u_*}{\kappa}\ln\frac{c'u_*}{\omega z_0} \tag{8}$$

In this approximation the phase of the velocity in the boundary layer is disregarded, and the logarithmic boundary layer flow model (Eq. 8) is extended beyond its range of validity. The logarithmic boundary layer flow model is only valid in a region where the shear stress is constant. Here it is extended to a height $c'u_*/\omega$ where the velocity is equal to the maximum free-stream velocity. Thus, for a given data set the constant c' can be determined from Eq. 8.

For rough surfaces the roughness parameter is given by:

$$z_0 = \frac{k}{30} \tag{9}$$

where k is the Nikuradse's equivalent sand roughness of the surface, i.e. the characteristic dimension of the physical roughness of the surface (Schlichting, 1979). However, k may be very different from what the physical roughness of the surface would suggest. For smooth surfaces the roughness parameter is given by:

$$z_0 = \frac{\nu}{9u_*} \tag{10}$$

where ν is the kinematic viscosity of the fluid (Schlichting, 1979). This means that the velocity is taken to be zero at a fixed level above the surface rather than at $z=0$.

For transitional smooth-to-rough turbulent flow the roughness parameter is given by:

$$z_0=\frac{k}{30}\left[1-\exp\left(-\frac{1}{27}\frac{ku_*}{\nu}\right)\right]+\frac{\nu}{9u_*} \tag{11}$$

according to Christoffersen and Jonsson (1985). Equation 11 is obtained from a fit to the data points in Schlichting (1979; his figs. 20. 21, on p. 620). For $\nu = 0$ and $k = 0$, Eq. 11 reduces to Eq. 9 and 10, respectively.

According to Schlichting (1979) the following turbulent flow regimes are given by:

$$\text{smooth} \qquad 0<\frac{ku_*}{\nu}<5 \tag{12}$$

$$\text{transition} \qquad 5<\frac{ku_*}{\nu}<70 \tag{13}$$

$$\text{rough} \qquad \frac{ku_*}{\nu}>70 \tag{14}$$

The given quantities in these flow regimes are:

rough $\quad |u_0|, \quad \omega, \quad k$

smooth $\quad |u_0|, \quad \omega, \quad \nu$

transition $\quad |u_0|, \quad \omega, \quad k, \quad \nu$

From this the following dimensionless quantities are obtained:

$$\left.\begin{array}{lll} \text{rough} & \dfrac{a_0}{k} & ; \quad f_w = f_w\left(\dfrac{a_0}{k}\right) \\ \text{smooth} & Re = \dfrac{|u_0|a_0}{\nu} & ; \quad f_w = f_w(Re) \\ \text{transition} & \dfrac{a_0}{k}, Re & ; \quad f_w = f_w\left(\dfrac{a_0}{k}, Re\right) \end{array}\right| \tag{15}$$

Here Re is the Reynolds number associated with the oscillatory wave motion.

By substituting Eq. 9–11 in Eq. 3 and by using Eq. 15, the wave friction coefficients for rough, smooth and transitional turbulent flow are given by:

rough:

$$\frac{2\kappa^2}{f_w} = \left[\ln\left(30c\frac{a_0}{k}\sqrt{f_w/2} \right) \right]^2 + B^2 \tag{16}$$

smooth:

$$\frac{2\kappa^2}{f_w} = [\ln(4.5c\, Re\, f_w)]^2 + B^2 \tag{17}$$

transition:

$$\frac{2\kappa^2}{f_w} = \left[\ln\left(30c\frac{a_0}{k}\sqrt{f_w/2} \right) - \ln \right.$$
$$\left. \left(1 - \exp\left(-\frac{1}{27}\frac{Re}{a_0/k}\sqrt{f_w/2} \right) + \frac{30a_0/k}{9Re\sqrt{f_w/2}} \right) \right]^2 + B^2 \tag{18}$$

RESULTS AND DISCUSSION

The constants A and B in Eq. 1 will now be determined according to Eq. 4 and 5 by using a data set from measurements of the turbulent flow structure in rough turbulent oscillatory boundary layers reported by Sumer et al. (1987). The main parameters and results from the experiments are summarized in Table 1. The experiments were carried out in an oscillating water tunnel and a one-component Laser Doppler Anemometer (LDA) was used to measure the streamwise and transverse velocity components, respectively. For the rough bottom experiments the bottom was covered with sand of fairly uniform size.

The determination of u_* and k was performed by fitting straight lines to the logarithmic-layer portion of the velocity data. u_* was also determined from the momentum integral equation by using the velocity distribution of the experiments. It appeared that the u_* values determined in both ways were in good agreement. More details are given in Sumer et al. (1987).

Thus the values of A and B and the corresponding value of c in Eq. 3 are determined as:

$$A = 1.21; \quad B = 1.28; \quad c = 0.30 \tag{19}$$

$c' = 0.34$ is determined from Eq. 7. Thus, the approximation in Eq. 7 corresponds to the use of Eq. 3 with:

TABLE 1

Main parameters and results from Sumer et al. (1987) for rough turbulent flow conditions

$\lvert u_0 \rvert$ (m/s)	a_0 (m)	ω (rad s^{-1})	k (mm)	u_* (cm s^{-1})	f_w	Re[1]	a_0/k
2.1	2.71	0.774	3.75	13.6	0.0084	5×10^6	720

[1] $\nu = 0.0114$ cm^2 s^{-1} at $T = 15$°C.

$$B=0; \quad c=0.34 \tag{20}$$

It appears that Eq. 3 using Eq. 20 is a good approximation to Eq. 3 using Eq. 19, both for rough and smooth turbulent flow (see Fig. 1 and Table 2). It should be noted that Eq. 3 and 7 can be applied with other values of B, c and c' if in the future other very precise experiments should give other values.

Figures 1 and 2 show the wave friction coefficient f_w and the phase ϕ, respectively, vs. the amplitude to roughness ratio for rough turbulent flow. Figure

TABLE 2

Semi-empirical calculations for f_w for smooth turbulent flow

Re	f_w (Eq. 17 using Eq. 19)	f_w (Eq. 17 using Eq. 20)
10^5	0.00668	0.00667
10^6	0.00419	0.00417
10^7	0.00283	0.00281
10^8	0.00202	0.00201

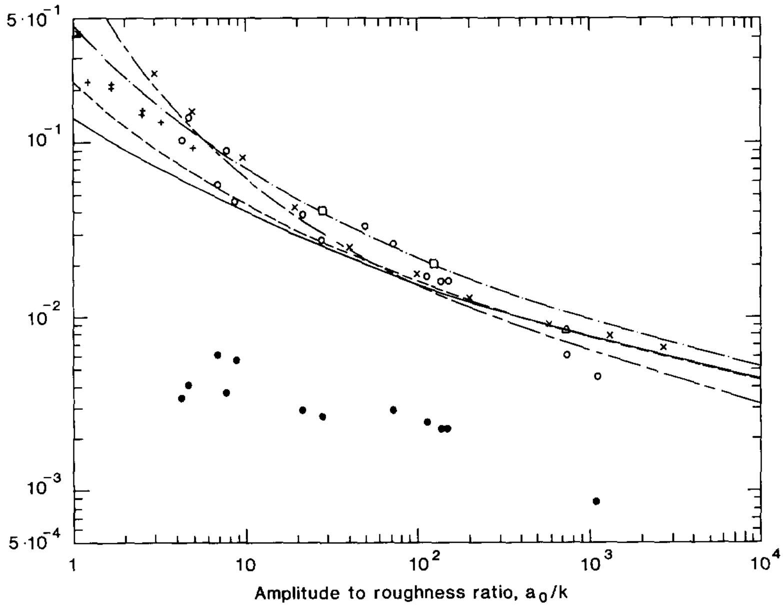

Fig. 1. Wave friction coefficient vs. amplitude to roughness ratio for rough-turbulent flow: —— Eq. 16, using Eq. 19; - - - Eq. 16 using Eq. 20; —·—· Eq. 25 (Jonsson and Carlsen, 1976); — — — - Eq. 26 (Kamphuis, 1975); + Bagnold (1946); × Kamphuis (1975); □ Jonsson and Carlsen (1976); △ Sumer et al. (1987); ○ (from momentum integral equation); ● (from Reynolds shear stress) Sleath (1987).

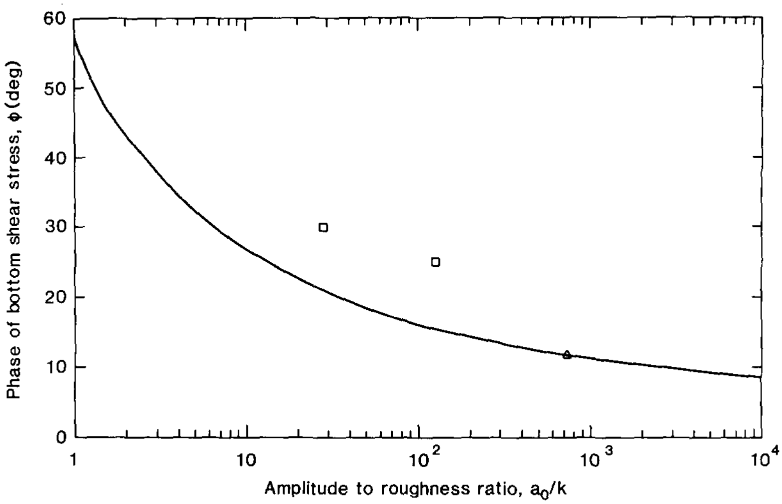

Fig. 2. Phase of bottom shear stress vs. amplitude to roughness ratio for rough turbulent flow: — Eq. 6 and 16 using Eq. 19. Other symbols as in Fig. 1.

1 includes the data from Sumer et al. (1987), Bagnold (1946), Kamphuis (1975), Jonsson and Carlsen (1976), Sleath (1987), and the semi-empirical predictions according to Eq. 16 using Eq. 19 and Eq. 16 using Eq. 20. The figure also includes the semi-empirical relationships by Jonsson and Carlsen (1976) and Kamphuis (1975), which will be given subsequently. Figure 2 includes the data by Sumer et al. (1987) and Jonsson and Carlsen (1976), and the semi-empirical predictions according to Eq. 6 and 16 using Eq. 19. Brief descriptions of the experiments referred to above will be given subsequently.

Figure 3 shows the wave friction coefficient vs. Reynolds number for smooth turbulent flow according to Eq. 17 using Eq. 19. In the same figure f_w vs. a_0/k for rough turbulent flow according to Eq. 16 using Eq. 19 and f_w vs. a_0/k and Re for transitional smooth-to-rough turbulent flow according to Eq. 18 using Eq. 19 are shown, together with the data from Sumer et al. (1987), Kamphuis (1975), Jonsson and Carlsen (1976), Hino et al. (1983) and Jensen et al. (1987). These data will be described briefly subsequently. The lower limit of rough turbulent flow according to Eq. 14 is also shown, which can be expressed as:

$$\frac{ku_*}{\nu} = \frac{Re}{a_0/k}\sqrt{f_w/2} = 70 \tag{21}$$

This is in good agreement with Jonsson (1980), which gives a review of transition to turbulence for both smooth and rough walls in oscillatory flow.

For comparison it should be noted that the laminar wave friction coefficient according to the solution of Stokes' second problem (Schlichting, 1979) is given by $f_w = 2Re^{-\frac{1}{2}}$ with $\phi = 45°$.

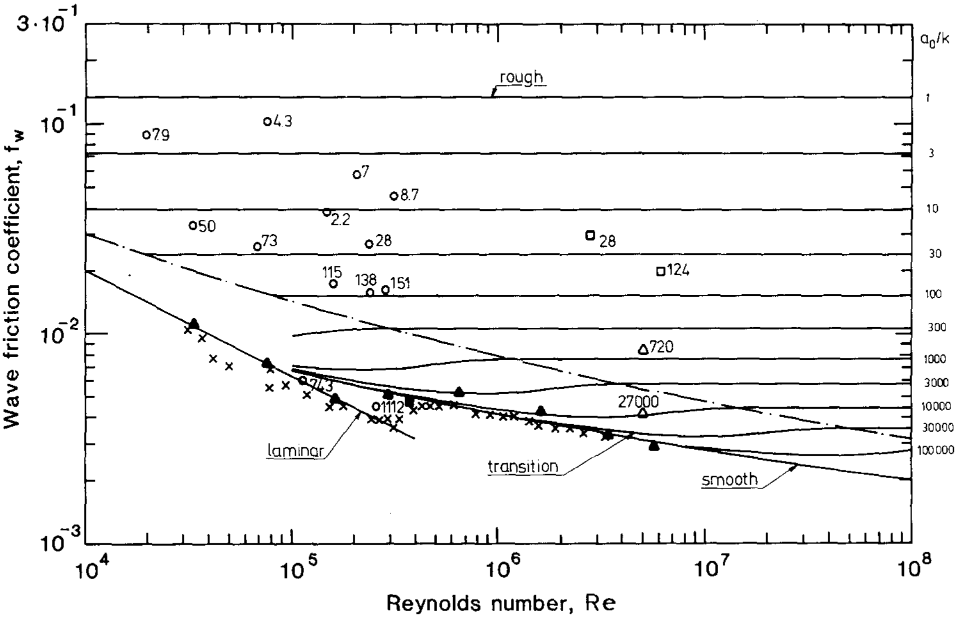

Fig. 3. Wave friction coefficient vs. Reynolds number. Rough turbulent flow: Eq. 16 using Eq. 19; Smooth turbulent flow: Eq. 17 using Eq. 19; Transitional smooth-to-rough turbulent flow: Eq. 18 using Eq. 19. —·—· lower limit for rough turbulent flow (Eq. 21); □ Jonsson and Carlsen (1976), rough; △ Sumer et al. (1987), rough/transition rough to smooth; ○ (from momentum integral equation) Sleath (1987), rough (numbers refer to a_0/k values); × Kamphuis (1975), laminar/transition laminar to smooth turbulent/smooth; ■ Hino et al. (1983), transition laminar to smooth turbulent; ▲ Jensen et al. (1987), laminar/transition laminar to smooth turbulent/smooth.

Figure 4 shows the phase ϕ vs. the Reynolds number for smooth turbulent flow according to Eq. 6 and 17 using Eq. 19. In the same figure ϕ vs. a_0/k for rough turbulent flow and ϕ vs. a_0/k and Re for transitional smooth-to-rough turbulent flow are shown, together with the lower limit of rough turbulent flow. The data from Sumer et al. (1987), Jonsson and Carlsen (1976) and Jensen et al. (1987) are also shown.

Bagnold (1946) did oscillating wall tests with a plate which was artificially roughened with ripples. By measuring the drag on the plate, friction coefficients for small a_0/k values in the rough turbulent flow regime were obtained. Kamphuis (1975) did oscillating water tunnel tests and measured the shear stress on both smooth and sand-roughened plates. The experiments covered a wide range of a_0/k and Re values in the laminar, transitional laminar to turbulent and turbulent flow regimes. Jonsson and Carlsen (1976) were supplementary to Jonsson (1963) and included oscillating water tunnel tests (in the same tunnel as used by Sumer et al. (1987)). Here the velocities were measured by a micro-propeller under rough turbulent flow conditions. Hino et al. (1983) did oscillating wind tunnel tests measuring velocities with an LDA. These tests

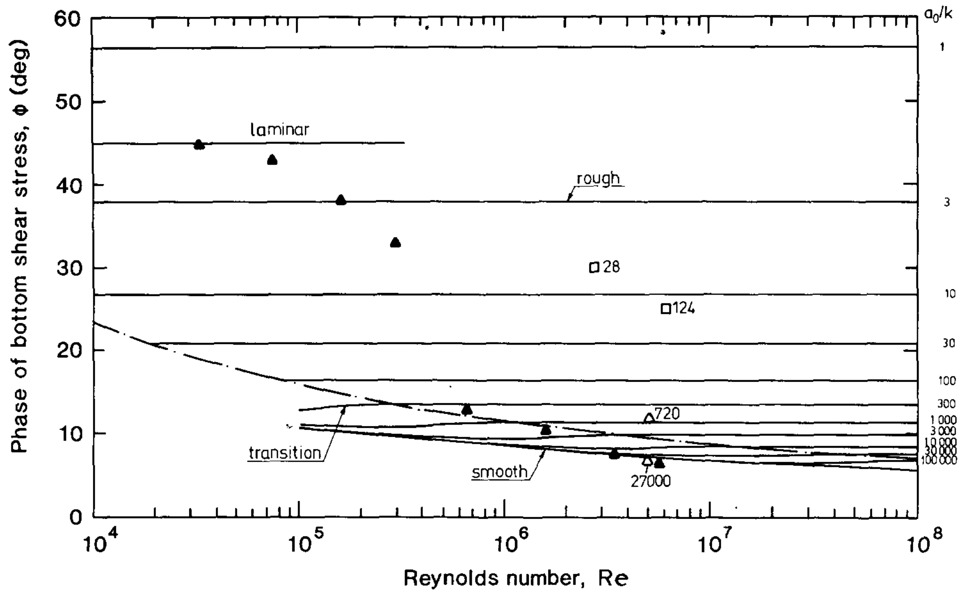

Fig. 4. Phase of bottom shear stress vs. Reynolds number. Rough-turbulent flow: Eq. 6 and 16 using Eq. 19. Smooth-turbulent flow: Eq. 6 and 17 using Eq. 19. Transitional smooth-to-rough turbulent flow: Eq. 6 and 18 using Eq. 19. Other symbols as in Fig. 3.

were in the transitional laminar to smooth turbulent flow regime. Jensen et al. (1987) did oscillating water tunnel tests (in the same tunnel as used by Sumer et al. (1987)). Wall shear stress measurements were made with a smooth wall covering laminar, transitional laminar to smooth turbulent and smooth turbulent flow regimes. They concluded, among other things, that the transition occurs over the Reynolds number range $1\times10^5 < Re < 3\times10^6$, and that the flow becomes fully turbulent after the Reynolds number reaches the value of approximately 3×10^6. Sleath (1987) did oscillating water tunnel tests measuring velocities with a two-component LDA over four different rough beds, covering a wide range of a_0/k values. The friction coefficient values were determined both from the Reynolds shear stress and from the momentum integral equation. More details are given in Sleath (1987). As shown in Fig. 1 there is a significant difference between the values determined in these two ways. It appears that the values of f_w calculated from the momentum integral equation are between five to ten times those given by the Reynolds shear stress. However, there is fairly good agreement between the values of f_w calculated from the momentum integral equation and those values given by the other investigators in Fig. 1. The reason for the large differences in Fig. 1 is unclear, although some suggestions for it are given in Sleath (1987). The agreement between the present semi-empirical predictions and some of the data from Sleath (1987) and Kamphuis (1975) is very good. The agreement is also reasonably good between Bagnold's (1946) data and the semi-empirical predictions according to Eq. 16 using Eq. 20 in the lower a_0/k range.

By using Eq. 20, Eq. 16–18 can be expressed as:

rough:

$$\frac{1}{4.07\sqrt{f_w}}+\log_{10}\frac{1}{4\sqrt{f_w}} = 0.256+\log_{10}\frac{a_0}{k} \tag{22}$$

smooth:

$$\frac{1}{8.14\sqrt{f_w}}+\log_{10}\frac{1}{4\sqrt{f_w}} = -0.51+\log_{10}\sqrt{Re} \tag{23}$$

transition:

$$\frac{1}{4.07\sqrt{f_w}}+\log_{10}\frac{1}{4\sqrt{f_w}} = 0.256-\log_{10}\left[1-\exp\left(-0.0262\frac{Re}{a_0/k}\sqrt{f_w}\right)+4.71\frac{a_0/k}{Re\sqrt{f_w}}\right] \tag{24}$$

Jonsson and Carlsen (1976) and Kamphuis (1975) have also given semi-empirical expressions for f_w based on their data. Jonsson and Carlsen (1976) determined their relationship by using the test data for $a_0/k=124$ and obtained:

$$\frac{1}{4\sqrt{f_w}}+\log_{10}\frac{1}{4\sqrt{f_w}} = -0.08+\log_{10}\frac{a_0}{k} \tag{25}$$

Kamphuis (1975) obtained the following relationship for rough turbulent flow as the best fit to his data points:

$$\frac{1}{4\sqrt{f_w}}+\log_{10}\frac{1}{4\sqrt{f_w}} = -0.35+\frac{4}{3}\log_{10}\frac{a_0}{k} \tag{26}$$

Equations 25 and 26 are also shown in Fig. 1. It should be noted that Eqs. 22–24 are valid by disregarding the phase ϕ.

Figure 2 shows that the present semi-empirical predictions give lower values than the data of Jonsson and Carlsen (1976).

Figure 3 shows very good agreement between the present semi-empirical predictions for smooth turbulent flow and the corresponding data from Kamphuis (1975) and Jensen et al. (1987). However, the f_w values from Sleath (1987) for $a_0/k=743$ and 1112 calculated from the momentum integral equation and representing rough-turbulent flow conditions, do not agree with the other data. The reason for this is unclear.

Figure 4 shows very good agreement between the present semi-empirical predictions for smooth turbulent mlowand the corresponding data from Jensen et al. (1987). The f_w and ϕ values for $a_0/k=27{,}000$ of Sumer et al. (1987) representing transitional smooth-to-rough turbulent flow conditions show reasonably good agreement with the present predictions.

SUMMARY AND CONCLUSIONS

The wave friction coefficient (f_w) and the phase lead of the bottom shear stress over the free stream velocity (ϕ) for rough, smooth and transitional smooth-to-rough turbulent flow are presented. The analogy between wave-boundary layer flow and planetary boundary layer flow is utilized following Gill (1982) using similarity theory. The constants as required by similarity theory are determined from rough-turbulent flow data taken from Sumer et al. (1987). An approximation for f_w by disregarding the phase lead ϕ of the wall shear stress over the free stream velocity is also presented. Comparisons are also made with experimental data and the present semi-empirical formulas for f_w and ϕ agree well with some of the experimental values.

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