1 (a)

Listing 1: **sol_tri_matrix.m**

```
function sol = sol_tri_matrix(d,l,u,f)
    % l1 = 0;   un = 0;
    n = length(d);
    sol = zeros(1,n);
    for i = 2:n
        d(i) = d(i) - u(i-1)*l(i)/d(i-1);
        f(i) = f(i) - f(i-1)*l(i)/d(i-1);
    end

    sol(n) = f(n)/d(n);
    for i = n-1:-1:1
        sol(i) = (f(i) - u(i)*sol(i+1))/d(i);
    end

end
```

2 (b)

Listing 2: **get_coefficients.m**

```
function c = get_coefficients(f)
    n = length(f)-1;
    d = ones(1,n-1)*4;
    l = [0,ones(1,n-2)];
    u = [ones(1,n-2),0];
    b = f(2:n);
    b(1) = b(1) - 1/6*f(1);
    b(end) = b(end) - 1/6*f(end);

    c_mid = sol_tri_matrix(d,l,u,b);

    c_0 = f(1)/6;
    c_n = f(end)/6;
    c_left = 2*c_0-c_mid(1);
    c_right = 2*c_n-c_mid(end);
    c = [c_left,c_0,c_mid,c_n,c_right];
end
```

3 (c)

Listing 3: **get_yy.m**

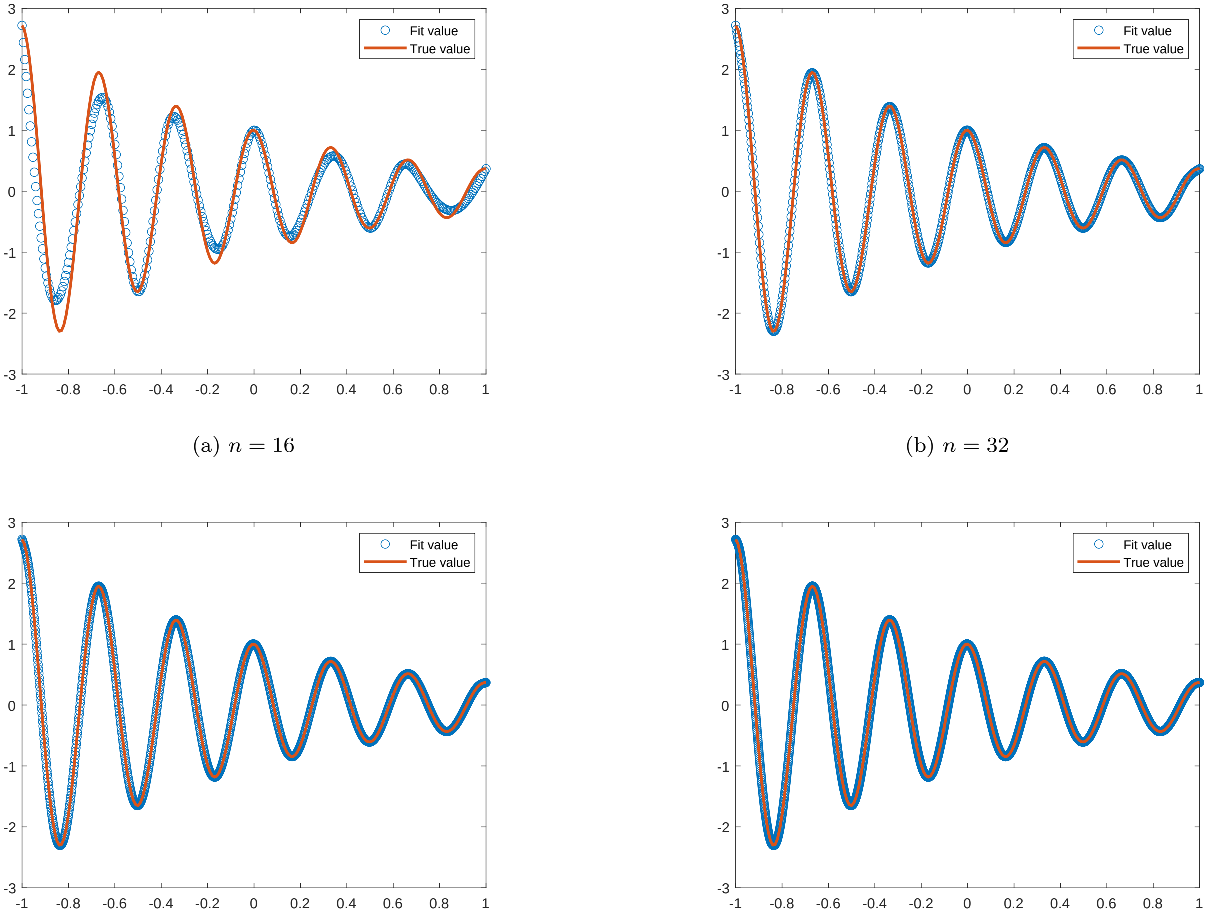
```
function yy = get_yy(x,c)

n = length(x);
a = x(1);
b = x(end);
```

```
xx = linspace(a,b,20*n+1);

h = x(2)-x(1);
syms t

B = @(t) 0.*(t<=-2) + ((t+2).^3) .* (t<=-1 & t>-2) + (1 + 3*(t+1) + 3*(t+1).^2 - 3*(t+1).^3) .* (t<=0 & t>-1) + (1 + 3*(1-t) + 3*(1-t).^2 - 3*(1-t).^3) .* (t<=1 & t>0) + ((2-t).^3) .* (t<=2 & t>1) + 0 .* (t>2);

x_extend = [x(1)-h,x,x(end)+h];


n = length(x_extend);
yy = 0;

for i = 1:n
    yy = yy + c(i)*B((xx-x_extend(i))/h);
end

end
```

4 (d)

Listing 4: **main.m**

```
clc,clear,close all
nn = [16,32,64,128];
my_error = zeros(1,length(nn));
for i = 1:4
    n = nn(i);
    x = linspace(-1,1,n+1);
    f = exp(-x).*cos(6*pi*x);
    c = get_coefficients(f);
    yy = get_yy(x,c);
    a = x(1);
    b = x(end);
    xx = linspace(a,b,20*(n+1)+1);
    figure
    plot(xx,yy)
    hold on
    plot(x,f,'*')
    hold off
    legend('Fit value','True value')
    my_error(i) = max(abs(yy-exp(-xx).*cos(6*pi*xx)));
end
loglog(nn,my_error)
xlabel('n')
ylabel('$||f-q_3^n||_{\infty}$','Interpreter','latex')
```



(a) $n = 16$

(b) $n = 32$

(c) $n = 64$

(d) $n = 128$

Figure 1: The fitting effect varies with n

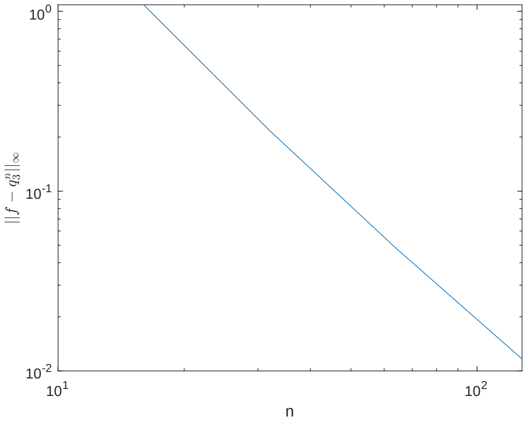


Figure 2: Error analysis

The magnitude of the error E of the solution for the maximum error for different n values is shown. In the log-log plot, the error is a function of n, which is essentially a straight line with a slope of -2, meaning that $lgEapproxalpha + blgn$, where $b = -2$; in other words, the error is $EapproxKn^{-2}$