Complex Numbers (Part ii) Tutorial

81–90 ■ Find the indicated roots, and graph the roots in the complex plane.

81. The square roots of $4\sqrt{3} + 4i$

82. The cube roots of $4\sqrt{3} + 4i$

83. The fourth roots of $-81i$

84. The fifth roots of 32

85. The eighth roots of 1

86. The cube roots of $1 + i$

87. The cube roots of i

88. The fifth roots of i

89. The fourth roots of -1

90. The fifth roots of $-16 - 16\sqrt{3}i$

91–96 ■ Solve the equation.

91. $z^4 + 1 = 0$

92. $z^8 - i = 0$

93. $z^3 - 4\sqrt{3} - 4i = 0$

94. $z^6 - 1 = 0$

95. $z^3 + 1 = -i$

96. $z^3 - 1 = 0$

97. **(a)** Let $w = \cos \dfrac{2\pi}{n} + i \sin \dfrac{2\pi}{n}$ where n is a positive integer. Show that $1, w, w^2, w^3, \ldots, w^{n-1}$ are the n distinct nth roots of 1.

(b) If $z \neq 0$ is any complex number and $s^n = z$, show that the n distinct nth roots of z are

$$s, sw, sw^2, sw^3, \ldots, sw^{n-1}$$

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Transform the complex numbers into Cartesian form:

$a)\ z = 2e^{i\frac{\pi}{6}}$

$b)\ z = 2\sqrt{3}\,e^{i\frac{\pi}{3}}$

$c)\ z = 4e^{3\pi i}$

$d)\ z = 4e^{i\frac{\pi}{2}}$

$e)\ z = \sqrt{2}\,e^{i\frac{3\pi}{4}}$

$f)\ z = 2\sqrt{3}\,e^{i\frac{2\pi}{3}}$

Sketch the following set of inequalities

a) $|z + 3| \leq 1$

b) $|z + 2| \leq |z - 2|$

c) $|z - i| \leq |z + i|$

d) $|z| \leq |2z + 1|$