Complex Numbers

Lecture 01

Complex Numbers

DEFINITION OF COMPLEX NUMBERS

A **complex number** is an expression of the form

$$a + bi$$

where a and b are real numbers and $i^2 = -1$. The **real part** of this complex number is a and the **imaginary part** is b. Two complex numbers are **equal** if and only if their real parts are equal and their imaginary parts are equal.

Arithmetic Operations

Definition

Addition

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

Subtraction

$$(a + bi) - (c + di) = (a - c) + (b - d)i$$

Multiplication

$$(a + bi) \cdot (c + di) = (ac - bd) + (ad + bc)i$$

Express the following in the form $a + bi$.

(a) $(3 + 5i) + (4 - 2i)$ **(b)** $(3 + 5i) - (4 - 2i)$

(c) $(3 + 5i)(4 - 2i)$ **(d)** i^{23}

Dividing Complex Numbers

DIVIDING COMPLEX NUMBERS

To simplify the quotient $\dfrac{a + bi}{c + di}$, multiply the numerator and the denominator by the complex conjugate of the denominator:

$$\frac{a + bi}{c + di} = \left(\frac{a + bi}{c + di}\right)\left(\frac{c - di}{c - di}\right) = \frac{(ac + bd) + (bc - ad)i}{c^2 + d^2}$$

Express the following in the form $a + bi$.

(a) $\dfrac{3 + 5i}{1 - 2i}$ **(b)** $\dfrac{7 + 3i}{4i}$

Square Root of a Negative Number

SQUARE ROOTS OF NEGATIVE NUMBERS

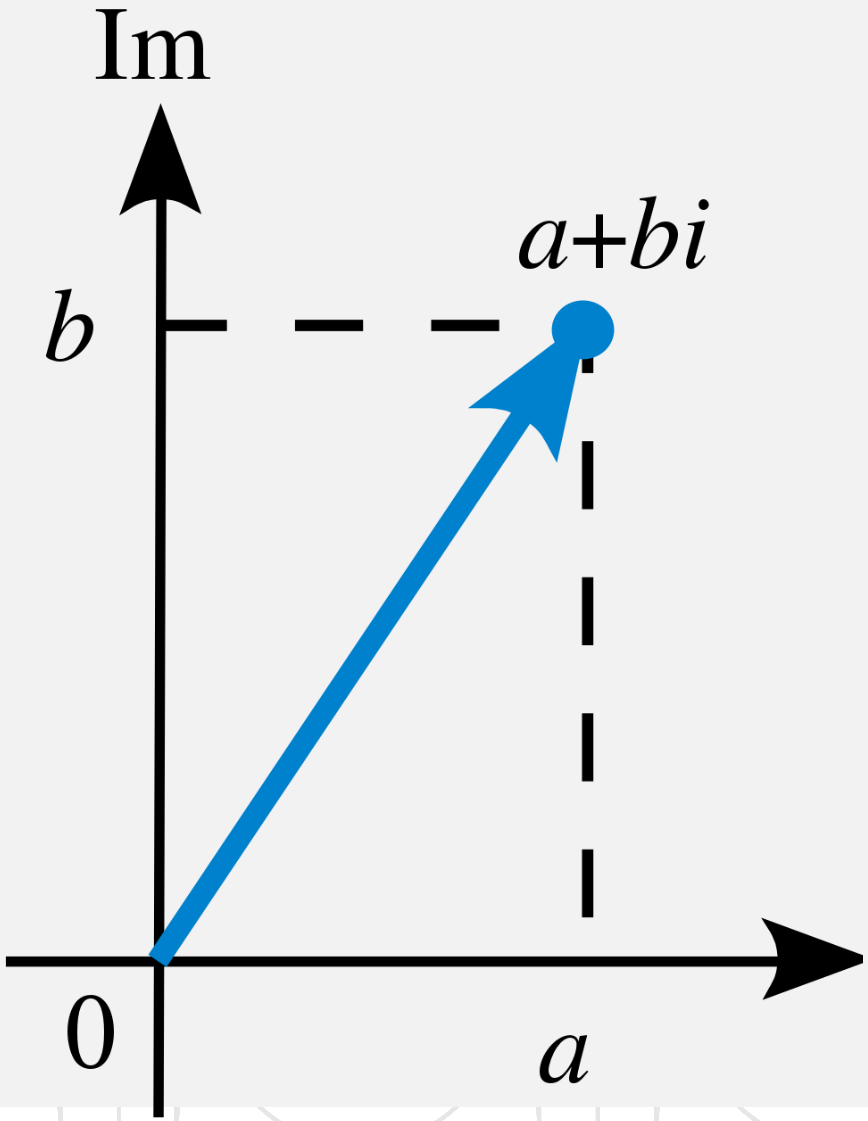
If $-r$ is negative, then the **principal square root** of $-r$ is

$$\sqrt{-r} = i\sqrt{r}$$

The two square roots of $-r$ are $i\sqrt{r}$ and $-i\sqrt{r}$.

(a) $\sqrt{-1} = i\sqrt{1} = i$ **(b)** $\sqrt{-16} = i\sqrt{16} = 4i$ **(c)** $\sqrt{-3} = i\sqrt{3}$

Graphing Complex Numbers



Modules of a Complex Number

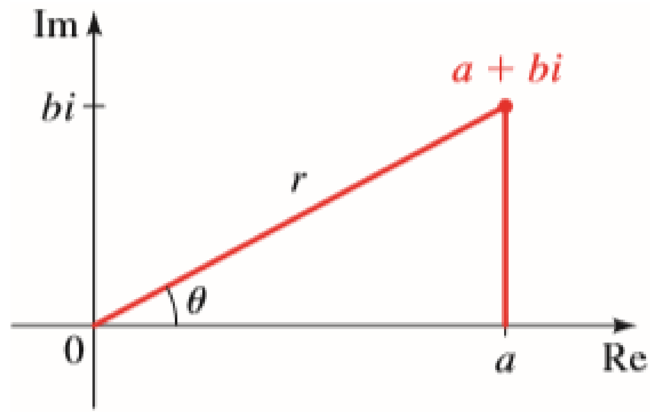
MODULUS OF A COMPLEX NUMBER

The modulus (or absolute value) of the complex number $z = a + bi$ is

$$|z| = \sqrt{a^2 + b^2}$$

Find the moduli of the complex numbers $3 + 4i$ and $8 - 5i$.

POLAR FORM OF A COMPLEX NUMBER



POLAR FORM OF COMPLEX NUMBERS

A complex number $z = a + bi$ has the **polar form** (or **trigonometric form**)

$$z = r(\cos\theta + i\sin\theta)$$

where $r = |z| = \sqrt{a^2 + b^2}$ and $\tan\theta = b/a$. The number r is the **modulus** of z, and θ is an **argument** of z.

Regions in Argand Plane

$$\{z = a + bi \,|\, a \le 0, b \ge 0$$

$$\{z = a + bi \,|\, a > 1, b > 1$$

$$\{z \,\|\, z \,| = 3\}$$

$$\{z \,\|\, z \,| < 2\}$$

$$\{z = a + bi \,|\, a + b < 2\}$$

$$\{z = a + bi \,|\, a \ge b\}$$

THE END