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MODULE *BasicPaxos*

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EXTENDS *Integers, FiniteSets*

$Maximum(S) \triangleq$  IF  $S = \{\}$  THEN  $-1$   
ELSE CHOOSE  $n \in S : \forall m \in S : n \geq m$

CONSTANTS *Acceptors, Ballot, Value*

$NoneValue \triangleq$  CHOOSE  $v : v \notin Value$

$Messages \triangleq$   $[type : \{\text{"prepare"}\}, bal : Ballot]$   
 $\cup$   
 $[type : \{\text{"promise"}\}, bal : Ballot, maxVBal : Ballot \cup \{-1\}, maxVVal : Value \cup \{NoneValue\}, acc : Acceptors]$   
 $\cup$   
 $[type : \{\text{"accept"}\}, bal : Ballot, val : Value]$   
 $\cup$   
 $[type : \{\text{"accepted"}\}, maxVBal : Ballot, maxVVal : Value, acc : Acceptors]$

$Quorums \triangleq \{Q \in \text{SUBSET } Acceptors : Cardinality(Q) * 2 > Cardinality(Acceptors)\}$

ASSUME  $\wedge Ballot \subseteq Nat$   
 $\wedge 0 \in Ballot$   
 $\wedge \forall Q1, Q2 \in Quorums : Q1 \cap Q2 \neq \{\}$

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VARIABLES *state, msgs*

$vars \triangleq \langle state, msgs \rangle$

$TypeOK \triangleq \wedge state \in [Acceptors \rightarrow [maxBal : Ballot \cup \{-1\},$   
 $maxVBal : Ballot \cup \{-1\},$   
 $maxVVal : Value \cup \{NoneValue\}]]$   
 $\wedge msgs \subseteq Messages$

$Send(m) \triangleq msgs' = msgs \cup \{m\}$

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$Init \triangleq \wedge state = [a \in Acceptors \mapsto [maxBal \mapsto -1, maxVBal \mapsto -1, maxVVal \mapsto NoneValue]]$   
 $\wedge msgs = \{\}$

$Prepare(b) \triangleq \wedge \neg \exists m \in msgs : m.type = \text{"prepare"} \wedge m.bal = b$   
 $\wedge Send([type \mapsto \text{"prepare"}, bal \mapsto b])$   
 $\wedge \text{UNCHANGED } state$

$Promise(acc) \triangleq \exists msg \in msgs : \wedge msg.type = \text{"prepare"}$   
 $\wedge state[acc].maxBal < msg.bal$   
 $\wedge state' = [state \text{ EXCEPT } ![acc].maxBal = msg.bal]$   
 $\wedge Send([type \mapsto \text{"promise"},$

[illegible]

There exists some value being chosen eventually

And it should be false, because *Paxos* does not satisfy liveness

$Liveness \triangleq \Diamond(ChosenSet \neq \{\})$

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