MODULE *Euclid*

EXTENDS *Integers*, *GCD*, *TLC*, *TLAPS* *TLAPS* for *PTL*

CONSTANTS M, N

ASSUME $\land M \in Nat \setminus \{0\}$
$\land N \in Nat \setminus \{0\}$

ASSUME $MNPosInt \triangleq \land M \in Nat \setminus \{0\}$
$\land N \in Nat \setminus \{0\}$

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```
--algorithm Euclid{
 variables x = M, y = N ;
 { while ( x ≠ y ) { if ( x < y ) { y := y − x }
                           else       { x   := x − y }
                         } ;
  }
 }
```

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BEGIN TRANSLATION

VARIABLES x, y, pc

$vars \triangleq \langle x, y, pc \rangle$

$Init \triangleq$ Global variables
$\land x = M$
$\land y = N$
$\land pc =$ "Lbl_1"

$Lbl\_1 \triangleq \land pc =$ "Lbl_1"
$\land$ IF $x \neq y$
THEN $\land$ IF $x < y$
THEN $\land y' = y - x$
$\land x' = x$
ELSE $\land x' = x - y$
$\land y' = y$
$\land pc' =$ "Lbl_1"
ELSE $\land pc' =$ "Done"
$\land$ UNCHANGED $\langle x, y \rangle$

$Next \triangleq Lbl\_1$
$\lor$ Disjunct to prevent deadlock on termination
$(pc =$ "Done" $\land$ UNCHANGED $vars)$

$Spec \triangleq Init \land \Box[Next]_{vars}$

$Termination \triangleq \Diamond(pc =$ "Done")

END TRANSLATION

$PartialCorrectness \triangleq (pc = \text{“Done”}) \Rightarrow (x = y) \land (x = GCD(M,\ N))$

$$TypeOK \triangleq \begin{array}{l} \land\ x \in Nat \setminus \{0\} \\ \land\ y \in Nat \setminus \{0\} \end{array}$$

$$Inv \triangleq \begin{array}{l} \land\ TypeOK \\ \land\ GCD(x,\ y) = GCD(M,\ N) \\ \land\ (pc = \text{“Done”}) \Rightarrow (x = y) \end{array}$$

THEOREM $Spec \Rightarrow \Box PartialCorrectness$
$\langle 1 \rangle 1.\ Init \Rightarrow Inv$
BY DEF $Init$, Inv, $TypeOK$
BY $MNPosInt$ DEF $Init$, Inv, $TypeOK$
$\langle 1 \rangle 2.\ Inv \land [Next]_{vars} \Rightarrow Inv'$
BY $MNPosInt$, $GCD2$, $GCD3$ DEF Inv, $TypeOK$, $Next$, Lbl_1, $vars$
$\langle 1 \rangle 3.\ Inv \Rightarrow PartialCorrectness$
BY $MNPosInt$, $GCD1$ DEF Inv, $TypeOK$, $PartialCorrectness$
$\langle 1 \rangle 4.$ QED
OBVIOUS
BY $\langle 1 \rangle 1$, $\langle 1 \rangle 2$, $\langle 1 \rangle 3$
BY $\langle 1 \rangle 1$, $\langle 1 \rangle 2$, $\langle 1 \rangle 3$ DEF $Spec$
BY $\langle 1 \rangle 1$, $\langle 1 \rangle 2$, $\langle 1 \rangle 3$, PTL DEF $Spec$

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