MODULE *AdditionalSequenceOperators*

Copyright: https://*github.com*/bringhurst/tlaplus/blob/master/*org.lamport.tla.toolbox.uitest*/farsite/*AdditionalSequenceOperat*

EXTENDS *Naturals*, *Sequences*, *FiniteSets*

The TLA+ *Sequences* module defines the operators *Head* and *Tail* for retrieving the first element of a sequence and all-but-the-first elements of a sequence, respectively. This module provides four operators that slightly generalize the notions of *Head* and *Tail*:

First returns the first element of a sequence, equivalently to *Head*. Last returns the last element of a sequence. *AllButFirst* returns all-but-the-first elements of a sequence, equivalently to *Tail*.

*AllButLast* returns all-but-the-last elements of a sequence.

This module also provides several additional operators on sequences: *IsElementInSeq* is a predicate that is true when the specified value is an element of the specified sequence. *IsSequenceOfSetElements* is a predicate that is true when the specified sequence contains all and only elements of the specified set. *IsSortedSequenceOfSetElements* is a predicate that is true when the *IsSequenceOfSetElements* is true and the sequence is also sorted in increasing order. *DeleteElement* produces a sequence by deleting an indicated element from another sequence.

$$Prepend(s,\ e) \triangleq \langle e \rangle \circ s$$

$$First(seq) \triangleq seq[1]$$

$$Last(seq) \triangleq seq[Len(seq)]$$

$$AllButFirst(seq) \triangleq [i \in 1\ ..\ (Len(seq) - 1) \mapsto seq[(i + 1)]]$$

$$AllButLast(seq) \triangleq [i \in 1\ ..\ (Len(seq) - 1) \mapsto seq[i]]$$

$$DoesSeqPrefixSeq(seq1,\ seq2) \triangleq$$
$$\land\ Len(seq1) \leq Len(seq2)$$
$$\land\ (\forall\, i \in 1\ ..\ Len(seq1) : seq1[i] = seq2[i])$$

$$DoesSeqProperlyPrefixSeq(seq1,\ seq2) \triangleq$$
$$\land\ Len(seq1) < Len(seq2)$$
$$\land\ (\forall\, i \in 1\ ..\ Len(seq1) : seq1[i] = seq2[i])$$

$$IsElementInSeq(el,\ seq) \triangleq \exists\, i \in \text{DOMAIN } seq : seq[i] = el$$

$$IsSequenceOfSetElements(seq,\ set) \triangleq$$
$$\land\ Len(seq) = Cardinality(set)$$
$$\land\ (\forall\, el \in set : IsElementInSeq(el,\ seq))$$

$$IsSortedSequenceOfSetElements(seq,\ set) \triangleq$$
$$\land\ IsSequenceOfSetElements(seq,\ set)$$
$$\land\ (\forall\, i \in \text{DOMAIN } seq,\ j \in \text{DOMAIN } seq : i < j \Rightarrow seq[i] < seq[j])$$

$$DeleteElement(seq,\ index) \triangleq$$
$$[i \in 1\ ..\ (Len(seq) - 1) \mapsto \text{IF } i < index \text{ THEN } seq[i] \text{ ELSE } seq[(i + 1)]]$$

It requires that $index \geq 1$.

If $index > Len(seq) + 1$, then it appends the element to $seq$.

(ADDED by hengxin; *July* 04, 2018)

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InsertElement(seq, elem, index) ≜
  [i ∈ 1 .. (Len(seq) + 1) ↦ IF i < index
                             THEN IF i = (Len(seq) + 1)
                                  THEN elem
                                  ELSE seq[i]
                             ELSE IF i = index
                                  THEN elem
                                  ELSE seq[(i − 1)]]
```

```
IsSorted2Partition(n, seq1, seq2) ≜
  ∧ seq1 ∈ Seq(1 .. n)
  ∧ seq2 ∈ Seq(1 .. n)
  ∧ n = Len(seq1) + Len(seq2)
  ∧ (∀ i ∈ DOMAIN seq1, j ∈ DOMAIN seq1 : i < j ⇒ seq1[i] < seq1[j])
  ∧ (∀ i ∈ DOMAIN seq2, j ∈ DOMAIN seq2 : i < j ⇒ seq2[i] < seq2[j])
  ∧ (∀ i ∈ DOMAIN seq1, j ∈ DOMAIN seq2 : seq1[i] ≠ seq2[j])
```

```
IsSequenceInterleaving(seq, subSeq1, subSeq2, indSeq1, indSeq2) ≜
  ∧ indSeq1 ∈ Seq(Nat)
  ∧ indSeq2 ∈ Seq(Nat)
  ∧ IsSorted2Partition(Len(seq), indSeq1, indSeq2)
  ∧ Len(indSeq1) = Len(subSeq1)
  ∧ Len(indSeq2) = Len(subSeq2)
  ∧ (∀ i ∈ DOMAIN indSeq1 : seq[(indSeq1[i])] = subSeq1[i])
  ∧ (∀ i ∈ DOMAIN indSeq2 : seq[(indSeq2[i])] = subSeq2[i])
```

Sequences up to length $n$, including the empty sequence $\langle\rangle$.

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```
SeqMaxLen(S, n) ≜  UNION {[1 .. m → S] : m ∈ 0 .. n}
```

Map on a sequence.

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SeqMap(Op(_), seq) ≜ [x ∈ DOMAIN seq ↦ Op(seq[x])]
```

* Modification History
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