Defining Fairness and Fairly Correct Systems

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Outline

- We define fairness,
  - i.e., the set of all fairness properties wrt a given system,
  - in line with definitions of *safety* and *liveness* by Lamport, Alpern and Schneider,
  - fairness properties are similar to — but different from — properties of measure 1
- We motivate and define fairly correct systems
  = a system that is correct under *some* fairness assumption
- We study fair model checking:
  - has usually the same complexity as standard model checking, but can be less expensive
  - can be seen as a useful approximation of standard model checking, where no specification of a fairness assumption is needed

Road Map

What is fairness? – A: Examples

- Traditional fairness notions
- Stronger fairness notions

What is fairness? – B: Characterisation

- A first, language-theoretical characterisation
- A game-theoretical characterisation
- A topological characterisation

Fairness and probability

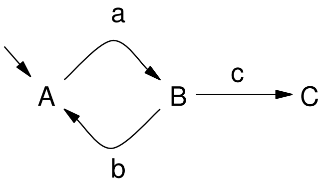
Fairly correct systems

- Motivation and definition
- Fair model checking
- Complete fairness

Road Map

Model of a system

Safety as a transition system



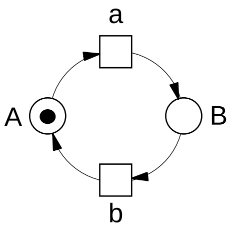
What *may* happen;
generate set of all runs

Liveness as a *fairness assumption*

- *Maximality* ∩
- *Strong fairness* wrt. *c*

What *must* happen;
selects a subset of runs

Sequential maximality

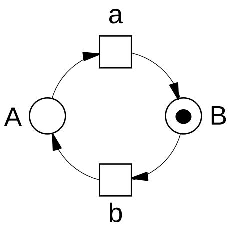


$A, a, B, b, A, \ldots$

Linear-time semantics

- All runs: $a, ab, aba, \ldots, (ab)^\omega$
- Undesired: e.g.. aba
- Assume: Maximality
- Unique maximal run: $(ab)^\omega$

Sequential maximality



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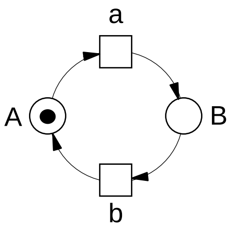

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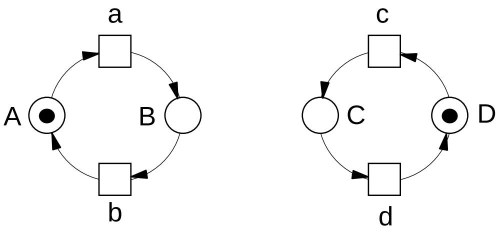
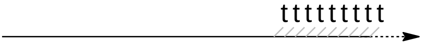
Weak fairness



- Undesired: e.g. $(ab)^\omega, (cd)^\omega$
- Weak fairness wrt. t: $\Diamond \Box\, \mathit{enabled}(t) \Longrightarrow \Box \Diamond\, \mathit{taken}(t)$



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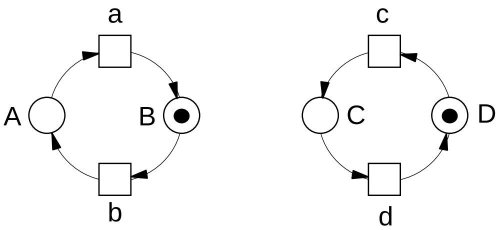

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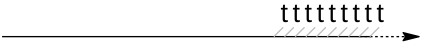

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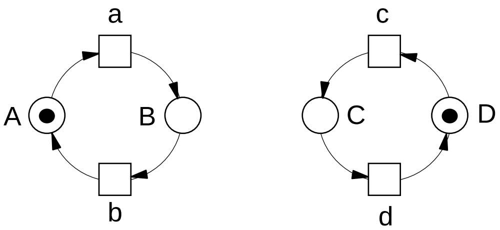

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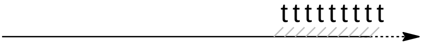

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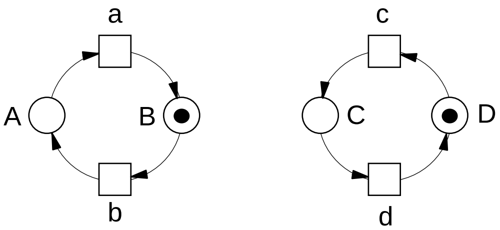

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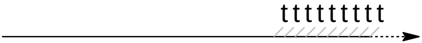

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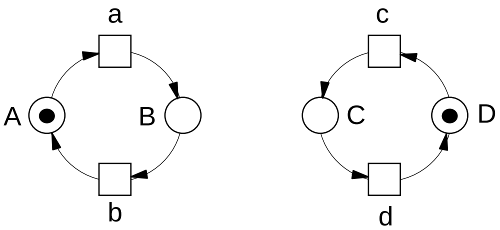

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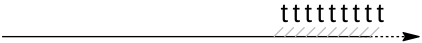

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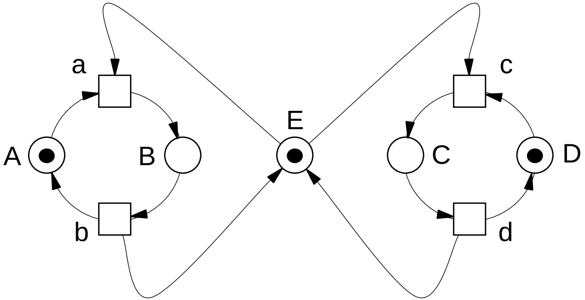

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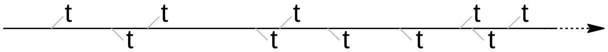

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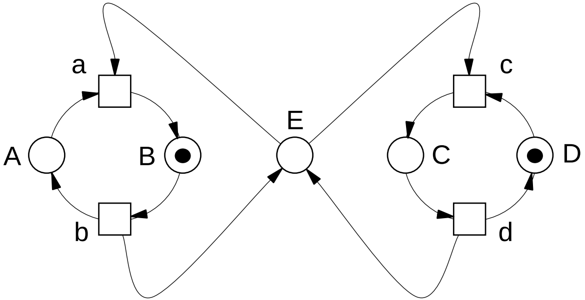

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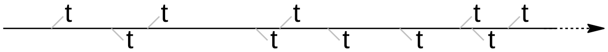

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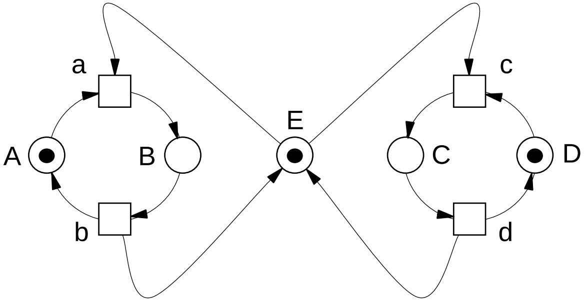

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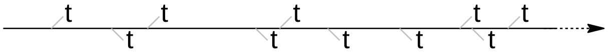

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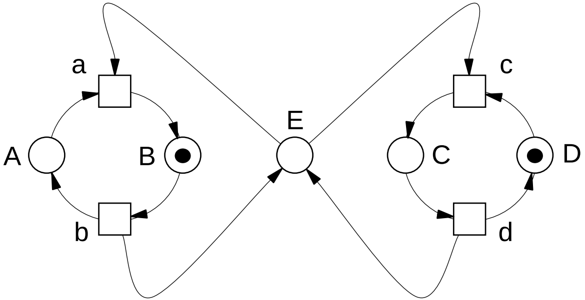

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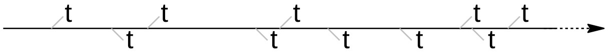

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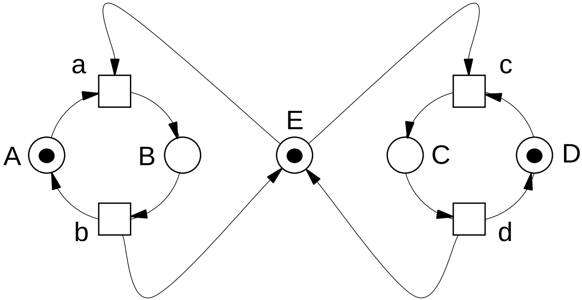

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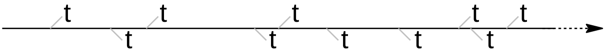
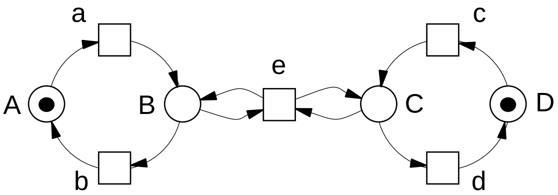


Road Map

∞-Fairness (E. Best 84)



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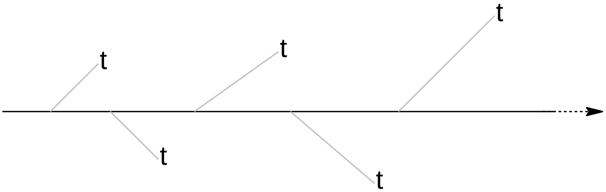

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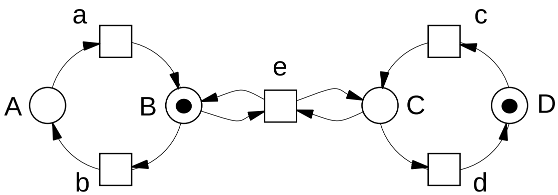

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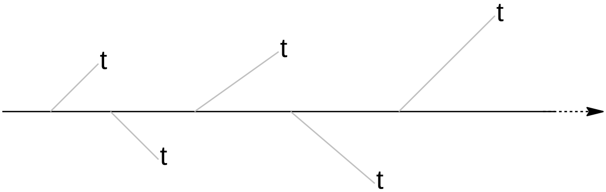

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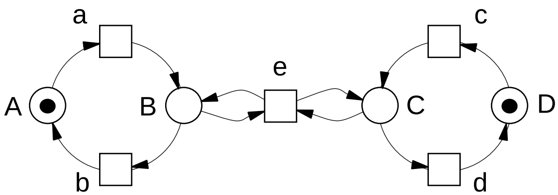

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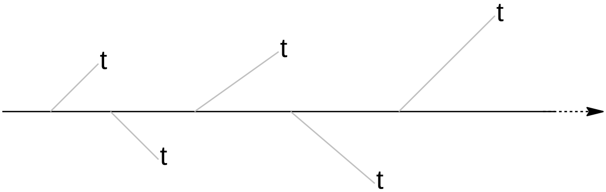

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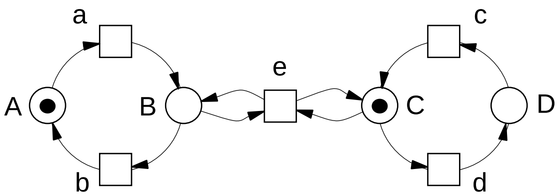

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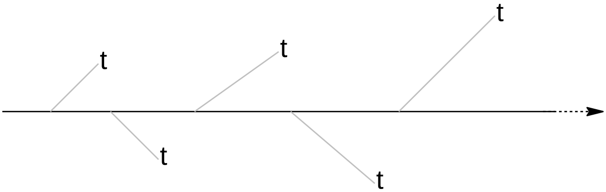

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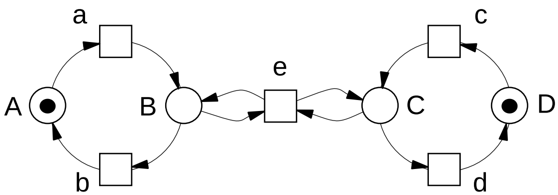

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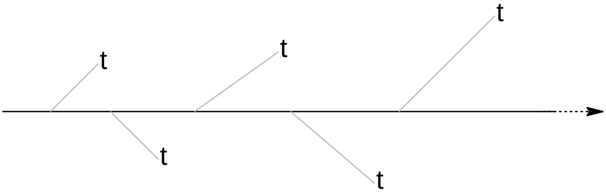

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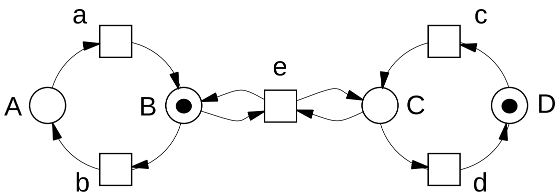

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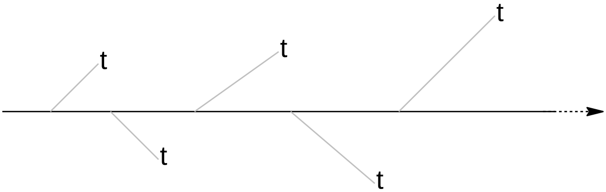

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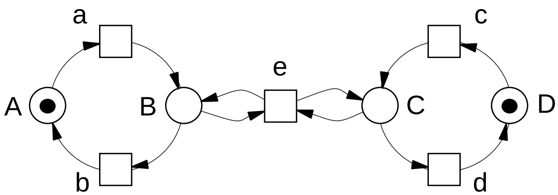

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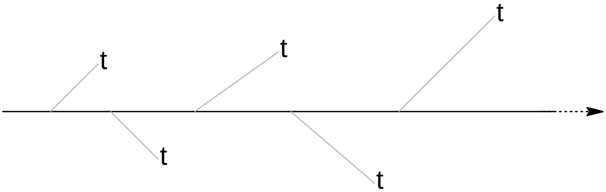
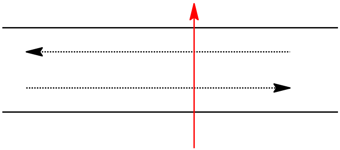


What does fairness mean?

A two-lane road



- Always prefer the weaker assumption
- Stronger (than traditional) assumptions can still be ok

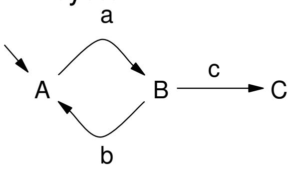
The stronger the fairness assumption, the stronger the potential performance problems

Road Map

Setting

$$x = s_0, s_1, \ldots$$

- Run: finite or infinite sequence of states: $x \in \Sigma^\infty = \Sigma^+ \cup \Sigma^\omega$
- Temporal property: $E \subseteq \Sigma^\infty$
- System $S \subseteq \Sigma^\infty$ = all runs generated by a given transition system



$$\{A, AB, ABA, ABC, ABAB, \ldots\}$$

Question

When is a temporal property F a fairness property for a given system S?

Common pattern:

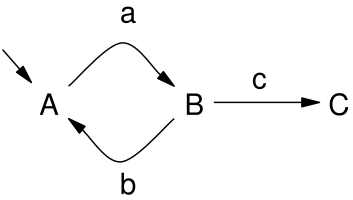
Sufficiently often enabled $\Longrightarrow$ sufficiently often taken.

Defining fairness – History

- Lamport 85, Alpern and Schneider 85: Defined *safety* and *liveness*
- No characterisation of fairness
- Apt, Francez, Katz 88: Necessary criteria for fairness
  - machine closure,
  - *equivalence robustness*, and *liveness enhancement*
- Lamport 2000: Machine closure is the only relevant criterion

Requirement: Machine closure of (S, F)

= each finite run of S can be extended into $S \cap F$



- $\Diamond$ *taken*(b) is not m.c.
- $\Diamond$ *taken*(c) is m.c.

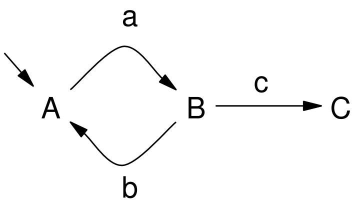
= Fairness does not rule out finite runs of the transition system

  (Transition system cannot 'paint itself into a corner')

- If (S, F) is an implementation (S, F) should be m.c.

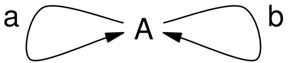
Requirement: Closure under intersection

= intersection of two (countably many) fairness is fairness



- x-Fairness wrt transition 1 $\cap$
- y-Fairness wrt process 2 $\cap$
- z-Fairness wrt ...

Machine closure is not enough



- $E_1 = \Box\Diamond\, taken(a)$
- $E_2 = \Diamond\Box\, taken(b)$
- $E_1 \cap E_2 = \emptyset$

- E_2 prescribes that some choice is not taken sufficiently often
- E_1, E_2 are both machine closed
- $E_1 \cap E_2$ is not machine closed

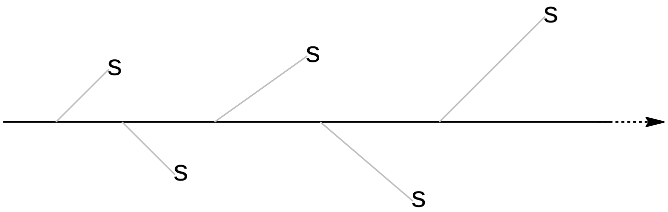
$\Longrightarrow$ machine-closed properties are not closed under intersection (bad for composition)

What do we want?

1. Machine-closed : Minimal requirement for implementability
2. Modular : Intersection of two fairness assumptions is a fairness assumption
3. Popular existing fairness notions fit
4. Otherwise as liberal as possible

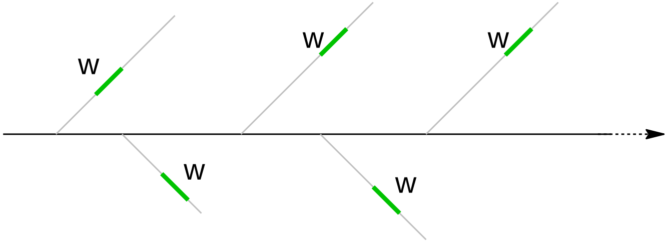
∞-Fairness wrt a state $s \in \Sigma$

Strongest fairness wrt a state



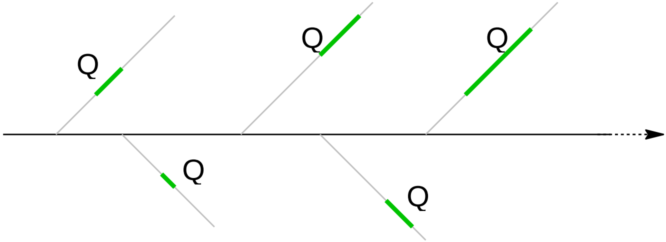
$\Box\, reachable_S(s) \implies \Box \Diamond\, taken(s)$

∞-Fairness wrt a word $w \in \Sigma^+$



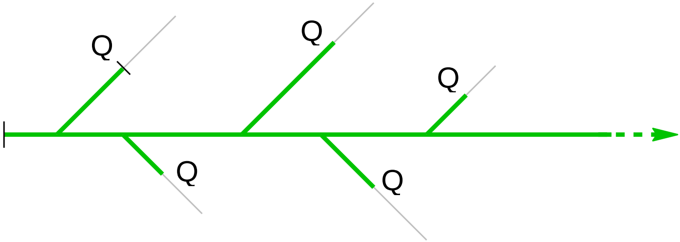
$$\Box\, reachable_S(w) \Longrightarrow \Box \Diamond\, taken(w)$$

∞-Fairness wrt $Q \subseteq \Sigma^+$



$\Box\, reachable_S(Q) \Longrightarrow \Box\Diamond\, taken(Q)$ (informal notation)

Memoryful ∞-fairness wrt $Q \subseteq \Sigma^+$

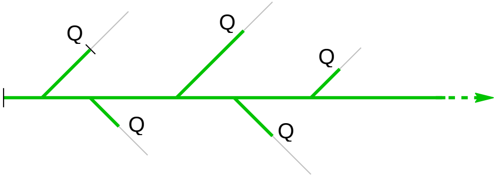


$\Box\, live_S(Q) \Longrightarrow \Box \Diamond Q$ (informal notation)

Examples:

- $Q = \Sigma^+ w$ (∞-Fairness wrt w)
- $Q =$ "$\#a = \#b$" (truly memoryful) "∞-Equifairness"

Defining fairness



Definition

$E \subseteq \Sigma^\infty$ is a fairness property for S iff it contains a property of the form $\Box\, live_S(Q) \Longrightarrow \Box \Diamond Q$ for some $Q \subseteq \Sigma^+$.

Example:

- $\Box \Diamond(\Phi \wedge enabled(t)) \Longrightarrow \Box \Diamond(\Phi \wedge taken(t))$ where Φ is a past formula (α-Fairness (Lichtenstein, Pnueli, Zuck 85))

What do we want? — Revisited

1. Machine-closed : Minimal requirement for implementability ✓
2. Modular : Intersection of two fairness assumption is a fairness assumption ?
3. Popular existing fairness notions fit ✓
4. Otherwise as liberal as possible ?
5. Moreover:
   - Fairness is closed under superset (and arbitrary union) ✓
   - Is $\Diamond \Box \mathit{taken}(b)$ a fairness property ?

Road Map

Game-theoretical characterisation

- Helps to prove or disprove that a given property is a fairness property
- Helps to construct model checking algorithms

Banach-Mazur game

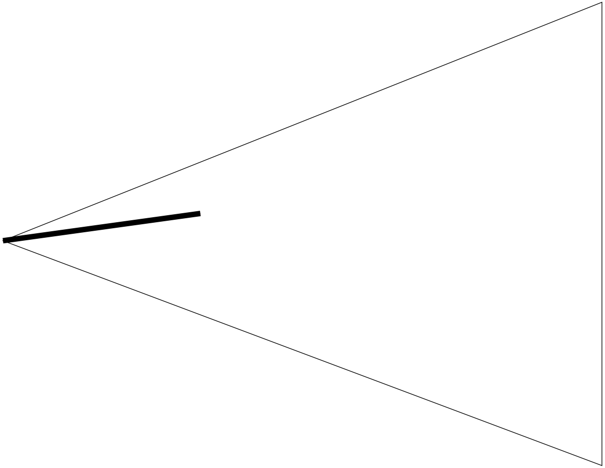
Two players: Scheduler and Opponent

- *Target:* (fairness) property E
- Opponent tries to produce an unfair run $x \notin E$
- Scheduler tries to produce a fair run $x \in E$

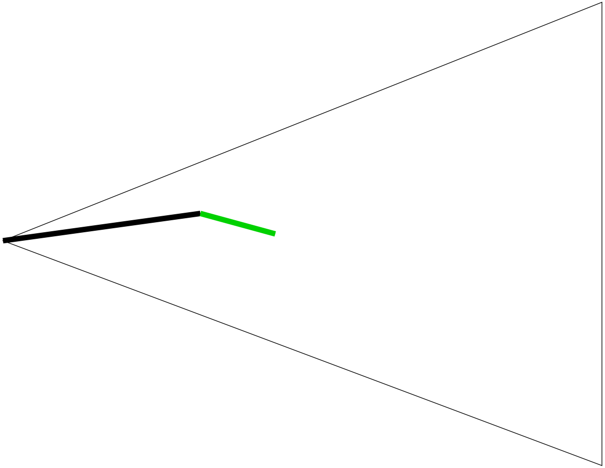
Banach-Mazur game

Run x:

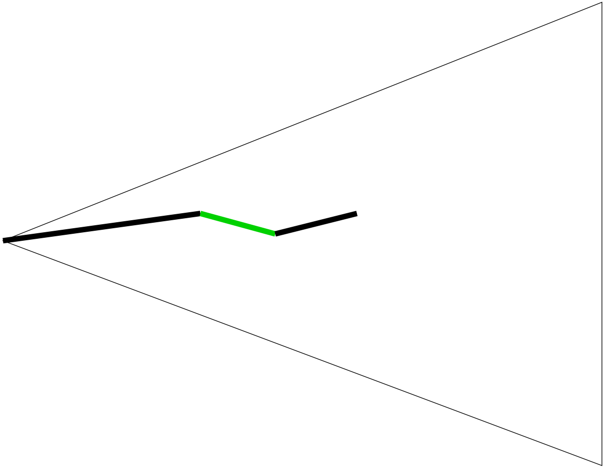
Banach-Mazur game



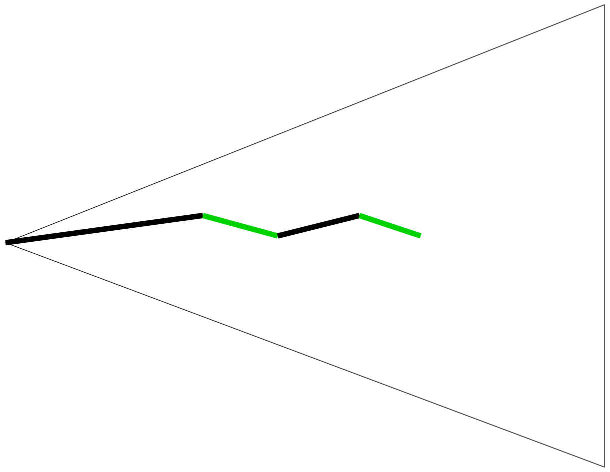
Banach-Mazur game



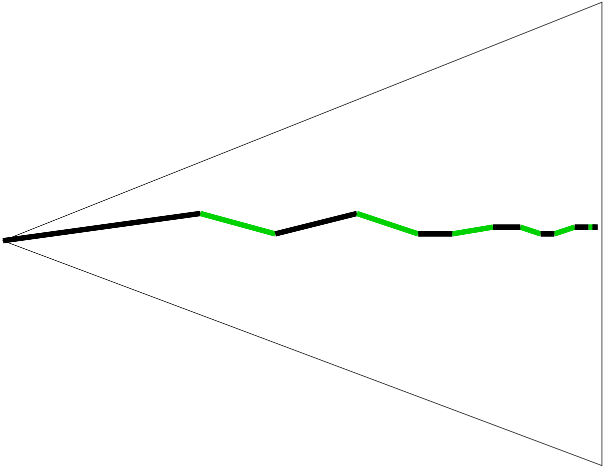
Banach-Mazur game



Banach-Mazur game

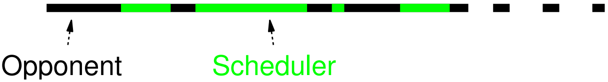


Banach-Mazur game



Banach-Mazur game

Run x:



- Target: $E \subseteq S$
  - scheduler wins if $x \in E$
  - otherwise, opponent wins
- Scheduler can enforce a finite behaviour to be taken infinitely often
- It cannot prevent another finite behaviour from being taken infinitely often

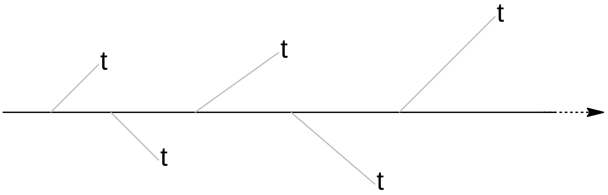
Theorem

F is a fairness property for S iff the Scheduler has a winning strategy for F.

Scheduler has winning strategy for F

Examples

- ∞-Fairness wrt. transition t



- Any weaker property

Counterexamples

- Σ^+
- $\Diamond \Box \, taken(b)$
- $\{\alpha x \mid \alpha \in \Sigma^+\}$ for $x \in \Sigma^\omega$

Closure under Intersection

Strategy: $f : \Sigma^+ \to \Sigma^+$ s.t. α is prefix of $f(\alpha)$.

Theorem

Fairness is closed under countable intersection.

Proof: Let f_i be a winning strategy for $E_i, i = 0, \ldots$. Define

$$f(\alpha) = f_k(f_{k-1}(\ldots f_0(\alpha)\ldots)) \quad \text{where } k = |\alpha|$$

f is a winning strategy for $\bigcap_i E_i$

What do we want? — Revisited

1. Machine-closed : Minimal requirement for implementability ✓
2. Modular : Intersection of two fairness assumption is a fairness assumption ✓
3. Popular existing fairness notions fit ✓
4. Otherwise as liberal as possible ?
5. Moreover:
   - Fairness is closed under superset (and arbitrary union) ✓
   - Is $\Diamond\Box\,\mathit{taken}(b)$ a fairness property ✓

Determinacy

$E \subseteq \Sigma^{\infty}$ is **determinate** if either Scheduler or Opponent has a winning strategy for it.

Determinate

$\uparrow$

Borel

$\uparrow$

Definable by
LTL, Buchi-automata, Linear time μ-calculus

Existence of indeterminate property can be shown using the axiom of choice.

NB. Determinacy yields complete proof strategy for fairness.

Maximality theorem

Theorem

Fairness is a maximal class of determinate properties such that fairness is machine-closed wrt the system and fairness is closed under finite intersection.

Maximality theorem

Theorem

Fairness is a maximal class of determinate properties such that fairness is machine-closed wrt the system and fairness is closed under finite intersection.

Suppose: Scheduler has no strategy for E.

$\Longrightarrow$ Opponent has winning strategy for E, let α be its first move.

$\Longrightarrow$ Scheduler has strategy for $F = \overline{E} \cup \overline{\alpha\uparrow}$

$\Longrightarrow$ α has no extension into $E \cap F$.

$\Longrightarrow$ $E \cap F$ is not machine-closed wrt the system

What do we want? — Revisited

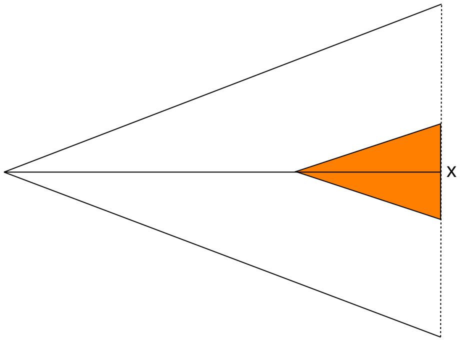
1. Machine-closed : Minimal requirement for implementability $\checkmark$
2. Modular : Intersection of two fairness assumption is a fairness assumption $\checkmark$
3. Popular existing fairness notions fit $\checkmark$
4. Otherwise as liberal as possible $\checkmark$
5. Moreover:
   - Fairness is closed under superset (and arbitrary union) $\checkmark$
   - Is $\Diamond \Box \mathit{taken}(b)$ a fairness property $\checkmark$

Road Map

Topological characterisation

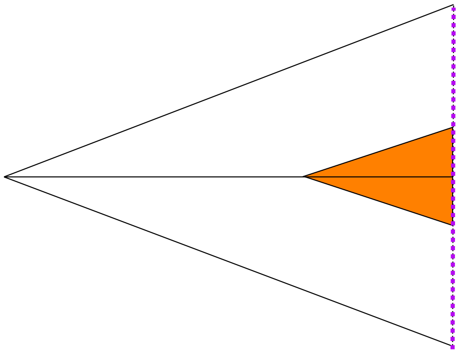
- Fairness properties are the large sets from a topological point of view
- Formalises that most runs are fair
- Leads to an important link to probability theory

Neighbourhood of a run *x*



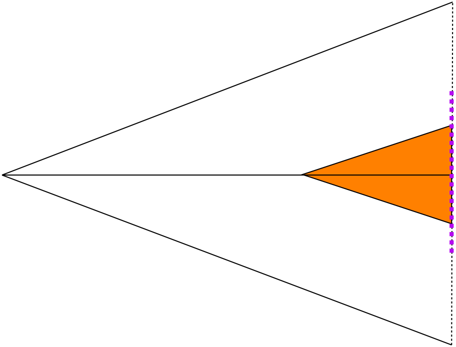
- = includes the set of all runs that share a specific prefix with *x* (*a basic open set*)
- the longer the prefix the smaller the neighbourhood

Dense set E



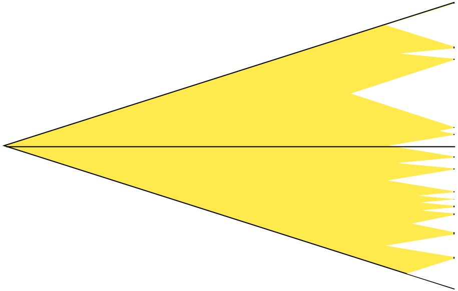
= intersects every neighbourhood
= machine closure
= pollutes the whole space (every neighbourhood is polluted)

Dense set E



- Somewhere dense = dense inside some basic open set (pollutes part of the space)

Nowhere dense set



= not somewhere dense

- Clean runs can stay clear of dirty runs
- Nowhere dense $\Longrightarrow$ small
- Full of holes (holes reachable from everywhere)

Topological characterisation of fairness

Meager set = small:

- = union of countably many nowhere dense sets
- No neighbourhood is meager

Co-meager set = large:

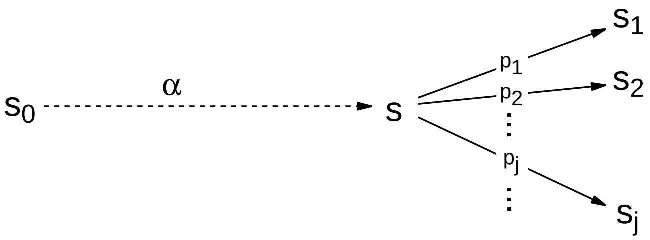
- = complement of a meager set
- $\Leftrightarrow$ fairness

Intermediate set:

- = neither large nor small

Road Map

Probability measure μ on a system S

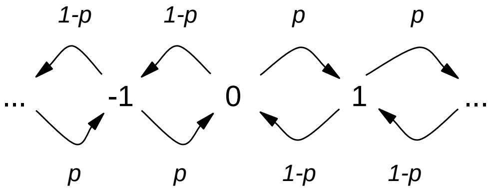


- $\sum_j p_j = 1$
- Assume $p_j \neq 0$
- Bounded: $\exists c > 0$: for all α and p_j : $p_j > c$
- Markov: p_j depends on last state only

Shared properties of topological and probabilistic largeness

- If a set is large its complement is not.
- Any superset of a large set is large.
- The intersection of countably many large sets is large.
- Intersection with a large set preserves size.
- Every large set is dense.
- ...

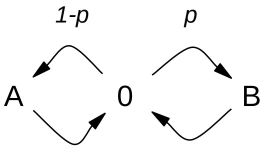
Notions do not coincide! (1/2)



$p \neq \frac{1}{2}$

- $E = \Box \Diamond 0$
- $\mu(E) = 0$ but E is co-meager
- $\mu(\overline{E}) = 1$ but $\overline{E}$ is meager
- System is infinite!

Notions do not coincide! (2/2)



$p \neq \frac{1}{2}$

- $E = \Box \Diamond (\#A = \#B)$
- $\mu(E) = 0$ but E is co-meager
- Property is not ω-regular, hence not expressible in LTL!

Coincidence — Main theorem

Theorem

If S is finite-state, E is ω-regular, μ a bounded probability measure on S then

$$E \text{ is co-meager in } S \Leftrightarrow \mu(E) = 1$$

Coincidence — Main theorem

Theorem

If S is finite-state, E is ω-regular, μ a bounded probability measure on S then

$$E \text{ is co-meager in } S \Leftrightarrow \mu(E) = 1$$

In particular true when μ is a Markov measure and E is LTL expressible.

Proof sketch (E is co-meager in $S \Leftrightarrow \mu(E) = 1$)

Suppose: Scheduler has strategy for E.

$\Longrightarrow$ Scheduler has memoryless strategy for E. (Berwanger, Grädel, and Kreutzer 2003)

$\Longrightarrow$ Scheduler has bounded strategy for E.

$\Longrightarrow$ $\mu(E) = 1$

Proof sketch (E is co-meager in $S \Leftrightarrow \mu(E) = 1$)

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Suppose: Scheduler has no strategy for E.

$\Longrightarrow$ Opponent has winning strategy for E, let α be its first move.

$\Longrightarrow$ Scheduler has strategy for $F = \overline{E} \cup \overline{\alpha\uparrow}$

$\Longrightarrow$ $\mu(\overline{E} \cup \overline{\alpha\uparrow}) = 1$ (furthermore: $\mu(\overline{\alpha\uparrow}) < 1$)

$\Longrightarrow$ $\mu(\overline{E}) > 0$ hence $\mu(E) < 1$

Some consequences

- Any ω-regular fairness property has probability 1 under randomised scheduling
- Obtain alternative characterisations for probability 1 (language-theoretic, game-theoretic, topological) in the considered case
- Obtain nice proof for the known fact that concrete values of probabilities do not matter for probability 1

Some consequences

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- Obtain alternative characterisations for probability 1 (language-theoretic, game-theoretic, topological) in the considered case
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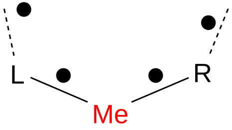
- Obtain algorithms for model checking fairly correct systems ...

Road Map

Introduction

Five Philosophers



System is not correct!

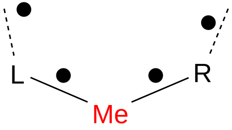
- L and R may ‘conspire’ against Me

SPEC

- mutual exclusion and
- starvation-freedom

Introduction

Five Philosophers



SPEC

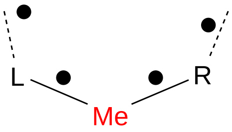
- mutual exclusion and
- starvation-freedom

Get a better system!

- May not be possible (e.g. fault-tolerant consensus, fault-tolerant dining philosophers)
- May not be desirable (price to pay)
- May not be necessary (SPEC is satisfied in practice)

Introduction

Five Philosophers



SPEC

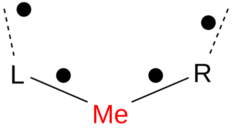
- mutual exclusion and
- starvation-freedom

Live with the system at hand!

- System is almost correct
- ‘Most’ runs satisfy SPEC
- Occurs e.g. in fault-tolerant distributed algorithms

Introduction

Five Philosophers



SPEC

- mutual exclusion and
- starvation-freedom

How to verify the system?

- System is not correct wrt SPEC
  - Pragmatic approach: weaken SPEC
- System is almost correct (‘most’ runs satisfy SPEC)
  - How to formalize this?
  - How to verify this?

Relaxations of correctness

Let S be the set of all runs of the system.

Almost Correct

- SPEC is probabilistically large (i.e. $\mu(SPEC) = 1$)
- needs probability measure μ on S

Fairly Correct (New!)

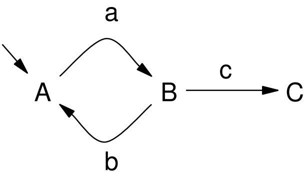
- SPEC is topologically large (i.e. SPEC is co-meager wrt S)

$\Leftrightarrow$ there is a fairness assumption F for S such that $S \cap F \subseteq SPEC$

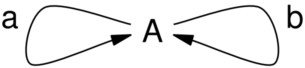
$\Leftrightarrow$ $SPEC$ is a fairness property for S !

Coincide for finite-state systems and ω-regular specifications.

Technical examples



- SPEC: No *a* after a *c*, correct and fairly correct
- SPEC: Termination, not correct but fairly correct



- SPEC: $\Diamond\Box\, taken(a)$, not correct and not fairly correct
- SPEC: $\Box\Diamond\, taken(a)$, not correct but fairly correct

Road Map

What is fairness? – A: Examples

Traditional fairness notions

Stronger fairness notions

What is fairness? – B: Characterisation

A first, language-theoretical characterisation

A game-theoretical characterisation

A topological characterisation

Fairness and probability

Fairly correct systems

Motivation and definition

Fair model checking

Complete fairness

Fair linear-time model checking

$$M \models_{\approx} \phi$$

- Fair model checking for LTL is PSPACE-complete; time linear in $|M|$ and exponential in $|\phi|$ (Courcoubetis and Yannakakis 95, Vardi 85)
- The same is true for specifications given by Büchi automata (Vardi 85)
- Standard model checking has the same complexity in both cases
- No implementation available

Alternative: Use of existing technology

Assume ϕ is in reactivity normal form:

$$\phi = \bigwedge_{i=1}^{n} (\Box \Diamond h_i \vee \Diamond \Box g_i)$$

where h_i and g_i are *past formulas*.

- We have linear translation of ϕ into CTL+past formula Φ s.t. M is fairly correct wrt ϕ if and only if M is correct wrt Φ
- There is a model checker for CTL+past: TLV
- Model checking CTL+past is PSPACE-complete

Branching time: CTL, CTL*

- We obtained optimal algorithms for “relaxed” versions of CTL and CTL*
  (Replace ‘for all paths’ by ‘for almost all paths’)
- Complexity is the same as for standard model checking for these languages
- Algorithm for CTL uses translation into standard CTL

Fair model checking can be less expensive than standard model checking

- Usually they have the same complexity (see above)
- Remains true for many subclasses, e.g. $L(\Diamond)$ is co-NP-complete (New: Schmalz 2007)
- But not always:

Theorem

Fair model checking of $L(\Box\Diamond)$ can be done in linear time (while standard model checking of $L(\Box\Diamond)$ is co-NP-complete)

More results from Schmalz 2007

- Algorithm for $L(\Box\Diamond)$ can be integrated with algorithm of Courcoubetis and Yannakakis (sometimes exponentially better, never worse)
- Algorithm can be extended to handle past operators
- Algorithm can be extended to return diagnostic information
- Crucial tool: Game-theoretic characterisation

Road Map

What is fairness? – A: Examples

- Traditional fairness notions
- Stronger fairness notions

What is fairness? – B: Characterisation

- A first, language-theoretical characterisation
- A game-theoretical characterisation
- A topological characterisation

Fairness and probability

Fairly correct systems

- Motivation and definition
- Fair model checking
- Complete fairness

Question

Is there a strongest fairness property F for S, i.e.,

S is fairly (almost) correct wrt $SPEC$ iff $F \cap S \subseteq SPEC$?

Benefit: Reduces proving fair correctness to proving satisfaction conditioned on F.

Answer (1/2)

- No, not in general.
  (Fairness is not closed under arbitrary intersection.)

Answer (1/2)

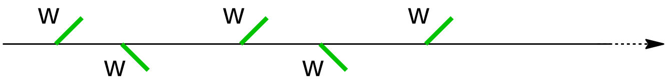
- No, not in general.
  (Fairness is not closed under arbitrary intersection.)
- Yes, if we are interested in a countable class $\mathcal{F}$ of properties only (e.g. LTL, ω-regular)

$$F_{\mathcal{F}} = \bigcap \{F \in \mathcal{F} \mid F \text{ is a fairness property for } S\}$$

is complete for $\mathcal{F}$.

Answer (2/2)

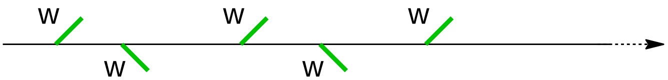
- Word fairness is complete for LTL and ω-regular



- α-Fairness is also known to be complete

Answer (2/2)

- Word fairness is complete for LTL and ω-regular



- α-Fairness is also known to be complete
- Word fairness is not ω-regular
- No ω regular-property is complete in general
- State fairness is complete for — as well as expressible in $L(\diamond)$

Conclusion (1/3)

Performance

$\uparrow$

Full functional correctness (under fairness)

$\uparrow$

Fair correctness

$\uparrow$

Machine closure

$\uparrow$

Safety

Higher assurance

Lower assurance

There are more generic relaxations of correctness (Berwanger et al. 2003, Pistore and Vardi 2003)

Conclusion (2/3)

Fair model checking

- allows to verify systems that are only fairly correct
- can be used as an approximation to standard model checking
  - has the same complexity for LTL, can even be better for subclasses
  - no need to specify any fairness assumption
  - difference in bugs covered is small

Conclusion (3/3)

- Definition of fairness carries over to other domains (Mazurkiewicz traces, pomsets, ...)
- Game-theoretic characterisation could help to simplify other algorithms in probabilistic model checking

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