MODULE OT

EXTENDS *Naturals*, *Sequences*

CONSTANTS
CH,
POS,
PR,
LOP, for test only
$LOPS$, for test only
ROP for test only

$OP \triangleq [type : \{\text{“ins”}, \text{“del”}\}, pos : POS, ch : CH, pr : PR]$
$NOP \triangleq$ CHOOSE $v : v \notin OP$

$XformII(lins, rins) \triangleq$ the left insertion transformed against the right insertion
IF $lins.pos < rins.pos$
THEN $lins$
ELSE IF $lins.pos > rins.pos$
THEN $[lins$ EXCEPT $!.pos = @ + 1]$
ELSE IF $lins.ch = rins.ch$
THEN NOP
ELSE IF $lins.pr > rins.pr$
THEN $[lins$ EXCEPT $!.pos = @ + 1]$
ELSE $lins$

$XformID(ins, del) \triangleq$ the left insertion transformed against the right deletion
IF $ins.pos < del.pos$
THEN ins
ELSE $[ins$ EXCEPT $!.pos = @ - 1]$

$XformDI(del, ins) \triangleq$ the first deletion transformed against the right insertion
IF $del.pos < ins.pos$
THEN del
ELSE $[del$ EXCEPT $!.pos = @ + 1]$

$XformDD(ldel, rdel) \triangleq$ the first deletion transformed against the right deletion
IF $ldel.pos < rdel.pos$
THEN $ldel$
ELSE IF $ldel.pos > rdel.pos$
THEN $[ldel$ EXCEPT $!.pos = @ - 1]$
ELSE NOP

$Xform(lop, rop) \triangleq$ the left operation is transformed against the right operation
CASE $lop.type = \text{“ins”} \land rop.type = \text{“ins”} \to XformII(lop, rop)$
$\square\ lop.type = \text{“ins”} \land rop.type = \text{“del”} \to XformID(lop, rop)$
$\square\ lop.type = \text{“del”} \land rop.type = \text{“ins”} \to XformDI(lop, rop)$
$\square\ lop.type = \text{“del”} \land rop.type = \text{“del”} \to XformDD(lop, rop)$

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XformOpOps(op, ops) ≜  the left operation is transformed against the right operation sequence
    LET X[j ∈ 1 .. Len(ops) + 1] ≜
        IF j = 1
        THEN ⟨op⟩
        ELSE Append(X[j − 1], Xform(X[j − 1][j − 1], ops[j − 1]))
    IN   X[Len(ops) + 1]

XformOpsOp(ops, op) ≜  the left operation sequence is transformed against the right single operation
    LET T[i ∈ 0 .. Len(ops)] ≜
        IF i = 0
        THEN ⟨⟩
        ELSE LET X ≜ XformOpOps(op, ops)
             IN   Append(T[i − 1], Xform(ops[i], X[i]))
    IN   T[Len(ops)]
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