```
- Module XJupiterExtended
1 [
    XJupiter extended with Cop with the sctx field.
   EXTENDS Integers, OT, TLCUtils, AdditionalFunctionOperators, AdditionalSequenceOperators
    CONSTANTS
         Client.
                        the set of client replicas
         Server.
                         the (unique) server replica
 9
         Char,
                        set of characters allowed
10
         InitState
                        the initial state of each replica
11
    Replica \triangleq Client \cup \{Server\}
    List \stackrel{\triangle}{=} Seq(Char \cup Range(InitState)) all possible lists/strings
    MaxLen \stackrel{\triangle}{=} Cardinality(Char) + Len(InitState) the max length of lists in any states;
          We assume that all inserted elements are unique.
17
    ClientNum \triangleq Cardinality(Client)
19
    Priority \triangleq CHOOSE f \in [Client \rightarrow 1 ... ClientNum] : Injective(f)
     direction flags
    Local \triangleq 0
    Remote \stackrel{\triangle}{=} 1
    ASSUME
26
         \land Range(InitState) \cap Char = \{\} due to the uniqueness requirement
27
         \land Priority \in [Client \rightarrow 1 .. ClientNum]
28
    The set of all operations. Note: The positions are indexed from 1.
   Rd \stackrel{\Delta}{=} [type : \{ \text{``Rd''} \}]
    Del \stackrel{\triangle}{=} [type : \{ "Del" \}, pos : 1 ... MaxLen]
    Ins \stackrel{\triangle}{=} [type: \{ \text{"Ins"} \}, pos: 1... (MaxLen + 1), ch: Char, pr: 1... ClientNum] pr: priority
    Op \triangleq Ins \cup Del
39 ⊢
    Cop: operation of type Op with context
   Oid \stackrel{\Delta}{=} [c:Client, seq:Nat] operation identifier
    Cop with the sctx field (the extended part)
   Cop \stackrel{\Delta}{=} [op: Op \cup \{Nop\}, oid: Oid, ctx: SUBSET Oid, sctx: SUBSET Oid]
    OT of two operations of type Cop.
    COT(lcop, rcop) \triangleq [lcop \ EXCEPT \ !.op = Xform(lcop.op, rcop.op), !.ctx = @ \cup \{rcop.oid\}]
53 F
   VARIABLES
        For the client replicas:
                   cseq[c]: local sequence number at client c \in Client
58
         cseq.
```

```
For the server replica (the extended part):
          soids.
                      the set of operations the Server has executed
 62
         The 2D state spaces (ss, for short). Each client maintains one 2D state space. The server
         maintains n 2D state spaces, one for each client.
                      c2ss[c]: the 2D state space at client c \in Client
           c2ss,
 68
                      cur[c]: the current node of c2ss[c]
 69
           ccur,
           s2ss,
                       s2ss[c]: the 2D state space maintained by the Server for client c \in Client
 70
 71
           scur,
                      scur[c]: the current node of s2ss[c]
        For all replicas
          state,
                      state[r]: state (the list content) of replica r \in Replica
 75
         For communication between the Server and the Clients:
                             cincoming[c]: incoming channel at the client c \in Client
 79
           cincoming,
           sincoming,
                             incoming channel at the Server
 80
         For model checking:
           chins
                     a set of chars to insert
 84
 85 F
      comm \stackrel{\triangle}{=} INSTANCE \ CSComm \ WITH \ Msg \leftarrow Cop
 86
 87
      eVars \triangleq
                   \langle chins \rangle
                               variables for the environment
      c Vars \triangleq
                   \langle cseq \rangle
                               variables for the clients
 89
     sVars \triangleq
                   \langle soids \rangle
                               variables for the Server
                     \triangleq \langle c2ss, ccur \rangle
                                              variables for 2D state spaces at clients
      c2ssVars
                     \stackrel{\scriptscriptstyle \Delta}{=}\ \langle s2ss,\,scur\rangle
      s2ssVars
                                              variables for 2D state spaces at the Server
      commVars \stackrel{\triangle}{=} \langle cincoming, sincoming \rangle
                                                             variables for communication
     vars \triangleq \langle eVars, eVars, eVars, ecmmVars, e2ssVars, s2ssVars, state \rangle all variables
 95 F
      A 2D state space is a directed graph with labeled edges. It is represented by a record with node
     field and edge field. Each node is characterized by its context, a set of operations. Each edge is
     labeled with an operation and a direction flag indicating whether this edge is LOCAL or REMOTE.
     For clarity, we denote edges by records instead of tuples.
     IsSS(G) \triangleq
104
105
            \land G = [node \mapsto G.node, edge \mapsto G.edge]
            \land G.node \subseteq (SUBSET \ Oid)
106
            \land G.edge \subseteq [from: G.node, to: G.node, cop: Cop, lr: \{Local, Remote\}]
107
      TypeOK \triangleq
109
          For the client replicas:
           \land cseq \in [Client \rightarrow Nat]
113
          For the 2D state spaces:
           \land \forall c \in Client : IsSS(c2ss[c]) \land IsSS(s2ss[c])
117
           \land ccur \in [Client \rightarrow SUBSET \ Oid]
118
           \land scur \in [Client \rightarrow SUBSET \ Oid]
119
           \land state \in [Replica \rightarrow List]
120
          For communication between the server and the clients:
```

```
\land comm! TypeOK
124
          For model checking:
           \land chins \subseteq Char
128
129 |
     The Init predicate.
133 Init \stackrel{\triangle}{=}
          For the client replicas:
           \land cseq = [c \in Client \mapsto 0]
137
          For the Server replica (the extended part):
141
           \land soids = \{\}
          For the 2D state spaces:
           \land c2ss = [c \in Client \mapsto [node \mapsto \{\{\}\}, edge \mapsto \{\}]]
145
           \land ccur = [c \in Client \mapsto \{\}]
146
           \land s2ss = [c \in Client \mapsto [node \mapsto \{\{\}\}, edge \mapsto \{\}]]
147
           \land scur = [c \in Client \mapsto \{\}]
148
          For all replicas:
           \land \mathit{state} = [r \in \mathit{Replica} \mapsto \mathit{InitState}]
152
          For communication between the server and the clients:
           \land comm!Init
156
          For model checking:
           \wedge chins = Char
160
161 ⊢
     Locate the node in the 2D state space ss which matches the context ctx of cop.
     Locate(cop, ss) \stackrel{\triangle}{=} CHOOSE \ n \in (ss.node) : n = cop.ctx
165
     xForm: iteratively transform cop with a path through the 2D state space ss at some client,
     following the edges with the direction flag d.
     xForm(cop, ss, cur, d) \stackrel{\Delta}{=}
172
           Let u \triangleq Locate(cop, ss)
173
                 v \triangleq u \cup \{cop.oid\}
174
                 RECURSIVE xFormHelper(\_, \_, \_, \_)
175
                  'h' stands for "helper"; xss: eXtra ss created during transformation
176
                 xFormHelper(uh, vh, coph, xss) \triangleq
177
                      If uh = cur
178
                       THEN xss
179
                       ELSE LET e \stackrel{\Delta}{=} \text{CHOOSE } e \in ss.edge : e.from = uh \land e.lr = d
180
                                      uprime \stackrel{\triangle}{=} e.to
181
                                      copprime \triangleq e.cop
182
                                      coph2copprime \stackrel{\triangle}{=} COT(coph, copprime)
183
                                      copprime2coph \triangleq COT(copprime, coph)
184
                                       vprime \stackrel{\Delta}{=} vh \cup \{copprime.oid\}
185
                                      xFormHelper(uprime, vprime, coph2copprime,
186
                                           [xss except !.node = @ \circ \langle vprime \rangle,
187
```

```
the order of recording edges here is important
188
                                                                so that the last one is labeled with the final transformed operation
189
                                                              !.edge = @ \circ \langle [from \mapsto vh, to \mapsto vprime, cop \mapsto copprime2coph, lr \mapsto vprime]
190
                                                                                 [from \mapsto uprime, to \mapsto vprime, cop \mapsto coph2copprime,
191
                 xFormHelper(u, v, cop, [node \mapsto \langle v \rangle,
192
                                                    edge \mapsto \langle [from \mapsto u, to \mapsto v, cop \mapsto cop, lr \mapsto 1 - d] \rangle])
193
194
      Client c \in Client perform operation cop guided by the direction flag d.
      ClientPerform(cop, c, d) \triangleq
198
           LET xss \stackrel{\triangle}{=} xForm(cop, c2ss[c], ccur[c], d)
199
                 xn \stackrel{\triangle}{=} xss.node
200
                 xe \stackrel{\triangle}{=} xss.edge
201
                  xcur \triangleq Last(xn)
202
                 xcop \stackrel{\triangle}{=} Last(xe).cop
203
                 \wedge c2ss' = [c2ss \ EXCEPT \ ![c].node = @ \cup Range(xn),
204
                                             ![c].edge = @ \cup Range(xe)]
205
                  \wedge ccur' = [ccur \ EXCEPT \ ![c] = xcur]
206
                  \wedge state' = [state \ EXCEPT \ ![c] = Apply(xcop.op, @)]
207
      Client c \in Client issues an operation op.
      DoOp(c, op) \triangleq
                              op: the raw operation generated by the client c \in Client
211
               \land cseq' = [cseq \ EXCEPT \ ![c] = @ + 1]
212
213
                    op with the sctx field (the extended part)
               \land LET cop \stackrel{\triangle}{=} [op \mapsto op, oid \mapsto [c \mapsto c, seq \mapsto cseq'[c]], ctx \mapsto ccur[c], sctx \mapsto \{\}]
214
                         \land ClientPerform(cop, c, Remote)
215
                          \land comm! CSend(cop)
216
      DoIns(c) \triangleq
218
219
           \exists ins \in Ins:
               \land ins.pos \in 1 \dots (Len(state[c]) + 1)
220
               \land ins.ch \in chins
221
               \wedge ins.pr = Priority[c]
222
223
               \wedge chins' = chins \setminus \{ins.ch\} We assume that all inserted elements are unique.
               \wedge DoOp(c, ins)
224
225
               \land Unchanged \langle sVars, s2ssVars \rangle
      DoDel(c) \triangleq
227
           \exists del \in Del:
228
               \land del.pos \in 1 \dots Len(state[c])
229
               \wedge DoOp(c, del)
230
               \land UNCHANGED \langle sVars, s2ssVars, eVars \rangle
231
      Do(c) \triangleq
233
              \vee DoIns(c)
234
              \vee DoDel(c)
235
      Client c \in Client receives a message from the Server.
```

```
Rev(c) \triangleq
239
             \land comm! CRev(c)
240
             \wedge LET cop \stackrel{\triangle}{=} Head(cincoming[c]) the received (transformed) operation
241
                      ClientPerform(cop, c, Local)
242
             \land Unchanged \langle eVars, cVars, sVars, s2ssVars \rangle
243
244
      The Server performs operation cop.
     ServerPerform(cop) \triangleq
248
           LET c \stackrel{\triangle}{=} cop.oid.c
249
                    \stackrel{\Delta}{=} xForm(cop, s2ss[c], scur[c], Remote)
250
                   \stackrel{\Delta}{=} xss.node
251
                   \stackrel{\triangle}{=} xss.edge
              xe
252
            xcur \triangleq Last(xn)
253
            xcop \stackrel{\triangle}{=} Last(xe).cop
254
                 \wedge s2ss' = [cl \in Client \mapsto
255
                                 If cl = c
256
                                  THEN [s2ss[cl]] EXCEPT !.node = @ \cup Range(xn),
257
                                                            ! .edge = @ \cup Range(xe)]
258
                                  ELSE LET scurcl \stackrel{\triangle}{=} scur[cl]
259
                                                 scurclprime \stackrel{\Delta}{=} scurcl \cup \{cop.oid\}
260
                                                [s2ss[cl]] EXCEPT !.node = @ \cup \{scurclprime\},
261
                                                                   ! .edge = @ \cup \{[from \mapsto scurcl, to \mapsto scurclprime, \}
262
                                                                                            cop \mapsto xcop, \ lr \mapsto Remote]\}]
263
264
                 \land scur' = [cl \in Client \mapsto
265
                                 IF cl = c THEN xcur ELSE scur[cl] \cup \{cop.oid\}
266
                 \land state' = [state \ EXCEPT \ ! [Server] = Apply(xcop.op, @)]
267
                 \land comm! SSendSame(c, xcop) broadcast the transformed operation
268
      The Server receives a message.
      SRev \triangleq
272
           \land comm!SRev
273
           \land LET cop \triangleq [Head(sincoming) \ EXCEPT !.sctx = soids]
274
275
                 cop with the sctx field (the extended part)
               IN
                      \land soids' = soids \cup \{cop.oid\}
276
                      \land ServerPerform(cop)
277
           \land UNCHANGED \langle eVars, cVars, c2ssVars \rangle
278
279
      The next-state relation.
      Next \triangleq
283
            \vee \exists c \in Client : Do(c) \vee Rev(c)
284
285
      The Spec. (TODO: Check the fairness condition.)
     Spec \stackrel{\Delta}{=} Init \wedge \Box [Next]_{vars} \wedge WF_{vars}(Next)
289
290 L
```