1 Redis List OT 函数设计

1.1 Redis List API 分类：根据“Effects”分为三类

单个元素的删除、修改、插入：$Insert(pos, ele), delete(pos), set(pos, ele),$ $(Lpush(ele), Lpushx), (Rpush(ele), Rpushx), Lpop, Rpop$

单个区间的删除、插入：$Ins(pos, str), Del(pos, len)$

多个区间的删除：$Rem(pos1, len1; pos2, len2; ...; posk, lenk), Trim(pos1, pos2)$
而$Trim$操作可以转换为$Rem(0, pos1-1; pos2+1, len-pos2-1)$

1.2 第一类OT 函数的设计

$$Lpush\begin{cases} OT(Lpush(x), Lpush(y)) = Lpush(x) \\ OT(Lpush(x), Rpush(y)) = Lpush(x) \\ OT(Lpush(x), Lpop) = Lpush(x) \\ OT(Lpush(x), Rpop) = Lpush(x) \\ OT(Lpush(x), Set(i,y)) = Lpush(x) \\ OT(Lpush(x), Ins(i,y)) = Lpush(x) \\ OT(Lpush(x), Del(i)) = Lpush(x) \end{cases} \tag{1}$$

$$Rpush\begin{cases} OT(Rpush(x), Lpush(y)) = Rpush(x) \\ OT(Rpush(x), Rpush(y)) = Rpush(x) \\ OT(Rpush(x), Lpop) = Rpush(x) \\ OT(Rpush(x), Rpop) = Rpush(x) \\ OT(Rpush(x), Set(i,y)) = Rpush(x) \\ OT(Rpush(x), Ins(i,y)) = Rpush(x) \\ OT(Rpush(x), Del(i)) = Rpush(x) \end{cases} \tag{2}$$

$$Lpop\begin{cases} OT(Lpop, Lpush(x)) = Del(1) \\ OT(Lpop, Rpush(x)) = Lpop \\ OT(Lpop, Lpop) = no-op \\ OT(Lpop, Rpop) = Lpop \\ OT(Lpop, Set(i,x)) = Lpop \\ OT(Lpop, Ins(i,x)) = \begin{cases} Del(1) & i = 0 \\ Lpop & i \neq 0 \end{cases} \\ OT(Lpop, Del(i)) = n\begin{cases} no-op & i = 0 \\ Lpop & i \neq 0 \end{cases} \end{cases} \tag{3}$$

$$
Rpop \begin{cases}
OT(Rpop, Lpush(x)) = Rpop \\
OT(Rpop, Rpush(x)) = Del(-2) \\
OT(Rpop, Lpop) = Rpop \\
OT(Rpop, Rpop) = no-op \\
OT(Rpop, Set(i,x)) = Rpop \\
OT(Rpop, Ins(i,x)) = \begin{cases} Del(-2) \quad i = len-1 \\ Rpop \quad i \neq len-1 \end{cases} \\
OT(Rpop, Del(i)) = \begin{cases} no-op \quad i = len-1 \\ Rpop \quad i \neq len-1 \end{cases}
\end{cases} \tag{4}
$$

$$
Set \begin{cases}
OT(Set(i,x), Lpush(y)) = Set(i+1,x) \\
OT(Set(i,x), Rpush(y)) = Set(i,x) \\
OT(Set(i,x), Lpop) = \begin{cases} no-op \quad i = 0 \\ Set(i-1,x) \quad i \neq 0 \end{cases} \\
OT(Set(i,x), Rpop) = \begin{cases} no-op \quad i = -1 \\ Set(i,x) \quad i \neq -1 \end{cases} \\
OT(set(i,x), set(j,y)) = set(i,x) \\
OT(Set(i,x), Ins(j,y)) = \begin{cases} Set(i,x) \quad i < j \\ Set(i+1,x) \quad i = j \\ Set(i+1,x) \quad i > j \end{cases} \\
OT(Set(i,x), Del(j)) = \begin{cases} Set(i,x) \quad i < j \\ no-op \quad i = j \\ Set(i-1,x) \quad i > j \end{cases}
\end{cases} \tag{5}
$$

$$Ins\begin{cases} OT(Ins(i,x),Lpush(y)) = Ins(i+1,x) \\ OT(Ins(i,x),Rpush(y)) = Ins(i,x) \\ OT(Ins(i,x),Lpop) = \begin{cases} Ins(i,x) & i=0 \\ Ins(i-1,x) & i \neq 0 \end{cases} \\ OT(Ins(i,x),Rpop) = \begin{cases} Ins(i-1,x) & i=-1 \\ Ins(i,x) & i \neq -1 \end{cases} \\ OT(Ins(i,x),set(j,y)) = Ins(i,x) \\ OT(ins(i,x),ins(j,y)) = \begin{cases} ins(i+1,x) & i>j \\ ins(i,x) & i<j \\ ins(i,x) & i=j \end{cases} \\ OT(Ins(i,x),Del(j)) = \begin{cases} Ins(i,x) & i<j \\ Ins(i,x) & i=j \\ Ins(i-1,x) & i>j \end{cases} \end{cases} \tag{6}$$

$$Del\begin{cases} OT(Del(i),Lpush(y)) = Del(i+1) \\ OT(Del(i),Rpush(y)) = Del(i) \\ OT(Del(i),Lpop) = \begin{cases} no-op & i=0 \\ Del(i-1) & i!=0 \end{cases} \\ OT(Del(i),Rpop) = \begin{cases} no-op & i=-1 \\ Del(i) & i \neq -1 \end{cases} \\ OT(Del(i),Set(j,x)) = Del(i) \\ OT(Del(i),Ins(j,x)) = \begin{cases} Del(i+1) & i>j \\ Deli & i<j \\ Del(i+1) & i=j \end{cases} \\ OT(del(i),del(j)) = \begin{cases} Del(i-1) & i>j \\ Del(i) & i<j \\ no-op & i=j \end{cases} \end{cases} \tag{7}$$

1.3 第二类OT 函数设计

$$OT(Ins(p1,s1),Ins(p1,s2)) = \begin{cases} Ins(p1,s1) & p1<p2 \\ Ins(p1+|s2|,s1) & p1=p2 \\ Ins(p1+|s2|,s1) & p1>p2 \end{cases} \tag{8}$$

$$OT(Ins(p1,s1),Del(p2,l1)) = \begin{cases} Ins(p1,s1) & p1 \leq p2 \\ no-op & p2<p1<p2+l1 \\ Ins(p1-l1,s1) & p1 \geq p2+l1 \end{cases} \tag{9}$$

$$OT(Del(p1,l1),Ins(p2,s1)) = \begin{cases} Del(p1,l1) & p1+l1 \le p2 \\ Del(p1,l1+|s1|) & p1 < p2 < p1+l1 \\ Ins(p1+|s1|,l1) & p1 \ge p2 \end{cases} \tag{10}$$

$$OT(Del(p1,l1),Del(p2,l2)) = \begin{cases} Del(p1,l1) & p1+l1 \le p2 \\ Del(p1-l2,s1) & p1 \ge p2+l2 \\ Del(p1,p2-p1) & p1 < p2 < p1+l1 \le p2+l2 \\ Del(p2,p1+l1-p2-l2) & p2 \le p1 < p2+l2 < p1+l1 \\ Del(p1,l1-l2) & p1 \le p2 < p2+l2 < p1+l1 \\ no-op & else \end{cases} \tag{11}$$

1.4 第三类OT 函数设计

$$OT(Ins(p_{k+1},s_{k+1}),Del(p_1,l_1;p_2,l_2;...;p_k,l_k))$$

$$= \begin{cases} Ins(p_{k+1},s_{k+1}) & p_{k+1} \le p_1 \\ no-op & p_i < p_{k+1} < p_i+l_i \\ Ins(p_{k+1}-l_1-l_2-...-l_i,s_{k+1}) & p_i+l_i \le p_{k+1} \le p_{i+1} \\ Ins(p_{k+1}-l_1-l_2-...-l_i,s_{k+1}) & p_{k+1} \ge pk+|sk| \end{cases} \tag{12}$$

$$OT(Del(p_1,l_1;p_2,l_2;...;p_k,l_k),Ins(p_{k+1},s_{k+1}))$$

$$= \begin{cases} Del(p_1,l_1;p_2,l_2;...;p_k,l_k) & P_k+l_k \le p_{k+1} \\ Del(p_1,l_1;p_2,l_2;...;p_{i-1},l_{i-1};p_i,l_i+|s_{k+1}|;p_{i+1}+|s_{k+1}|,l_{i+1};...;p_k+|s_{k+1}|,l_k) & p_i < p_{k+1} \le pi+li \\ Del(p_1,l_1;p_2,l_2;...;p_i,l_i;p_{i+1}+|s_{k+1}|,l_{i+1};...;p_k+|s_{k+1}|,l_k) & p_i+l_i < p_{k+1} \le p_{i+1} \end{cases} \tag{13}$$

$$OT(Del(p_{k+1},l_{k+1}),Del(p_1,l_1;p_2,l_2;...;p_k,l_k))$$

$$
= \begin{cases}
Del(p_{k+1}, l_{k+1}) & p_{k+1} < p_1 \quad p_{k+1} + l_{k+1} \le p_1 \\
Del(p_{k+1}, p_j - l_1 - l_2 - ... - l_{j-1} - p_{k+1}) & p_{k+1} < p_1 \quad p_j < p_{k+1} + l_{k+1} \le p_j + l_j \\
Del(p_{k+1}, l_{k+1} - l_1 - l_2 - ... - l_j) & p_{k+1} < p_1 \quad p_j + l_j < p_{k+1} + l_{k+1} \le p_{j+1} \\
Del(p_{k+1}, l_{k+1} - l_1 - l_2 - ... - l_k) & p_{k+1} < p_1 \quad p_{k+1} + l_{k+1} > P_k + l_k \\
Del(p_i - l_1 - l_2 - ... - l_{i-1}, p_j - p_i - l_i - l_{i+1} ... - l_{j-1}) & p_i \le p_{k+1} < p_i + l_i \\
 & p_j < p_{k+1} + l_{k+1} \le p_j + l_j \\
Del(p_i - l_1 - l_2 - ... - l_{i-1}, p_{k+1} + l_{k+1} - p_i - l_i - l_{i+1} - ... - l_j) & p_i \le p_{k+1} < p_i + l_i \\
 & p_j + l_j < p_{k+1} + l_{k+1} \le p_{j+1} \\
Del(p_i - l_1 - l_2 - ... - l_{i-1}, p_{k+1} + l_{k+1} - p_i - l_i - l_{i+1} - ... - l_k) & p_i \le p_{k+1} < p_i + l_i \\
 & p_{k+1} + l_{k+1} > P_k + l_k \\
Del(p_{k+1} - l_1 - l_2 - ... - l_{i-1}, p_j - p_{k+1} - l_{i+1} - l_{i+2} - ... - l_{j-1}) & p_i + l_i \le p_{k+1} < p_{i+1} \\
 & p_j < p_{k+1} + l_{k+1} \le p_j + l_j \\
Del(p_{k+1} - l_1 - l_2 - ... - l_{i-1}, l_{k+1} - l_{i+1} - l_{i+2} - ... - l_j) & p_i + l_i \le p_{k+1} < p_{i+1} \\
 & p_j + l_j < p_{k+1} + l_{k+1} \le p_{j+1} \\
Del(p_{k+1} - l_1 - l_2 - ... - l_{i-1}, l_{k+1} - l_{i+1} - l_{i+2} - ... - l_k) & p_i + l_i \le p_{k+1} < p_{i+1} \\
 & p_{k+1} + l_{k+1} > P_k + l_k \\
Del(p_{k+1} - p_1 - p_2 ... - p_k, l_{k+1}) & p_{k+1} \ge p_k + l_k \\
 & (i \ge j)
\end{cases} \tag{14}
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