

1 MODULE *AdditionalSequenceOperators*

3 Copyright: <https://github.com/bringhurst/tlaplus/blob/master/org.lamport.tla.toolbox.uitest/farsite/AdditionalSequenceOperators>

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6 EXTENDS *Naturals, Sequences, FiniteSets*

The TLA+ *Sequences* module defines the operators *Head* and *Tail* for retrieving the first element of a sequence and all-but-the-first elements of a sequence, respectively. This module provides four operators that slightly generalize the notions of *Head* and *Tail*:

First returns the first element of a sequence, equivalently to *Head*. *Last* returns the last element of a sequence. *AllButFirst* returns all-but-the-first elements of a sequence, equivalently to *Tail*.

AllButLast returns all-but-the-last elements of a sequence.

This module also provides several additional operators on sequences: *IsElementInSeq* is a predicate that is true when the specified value is an element of the specified sequence. *IsSequenceOfSetElements* is a predicate that is true when the specified sequence contains all and only elements of the specified set. *IsSortedSequenceOfSetElements* is a predicate that is true when the *IsSequenceOfSetElements* is true and the sequence is also sorted in increasing order. *DeleteElement* produces a sequence by deleting an indicated element from another sequence.

29 Defn $Prepend(s, e) \triangleq \langle e \rangle \circ s$

31 Defn $First(seq) \triangleq seq[1]$

33 Defn $Last(seq) \triangleq seq[Len(seq)]$

35 Defn $AllButFirst(seq) \triangleq [i \in 1 \dots (Len(seq) - 1) \mapsto seq[(i + 1)]]$

37 Defn $AllButLast(seq) \triangleq [i \in 1 \dots (Len(seq) - 1) \mapsto seq[i]]$

39 Defn $DoesSeqPrefixSeq(seq1, seq2) \triangleq$

40 $\wedge Len(seq1) \leq Len(seq2)$

41 $\wedge (\forall i \in 1 \dots Len(seq1) : seq1[i] = seq2[i])$

43 Defn $DoesSeqProperlyPrefixSeq(seq1, seq2) \triangleq$

44 $\wedge Len(seq1) < Len(seq2)$

45 $\wedge (\forall i \in 1 \dots Len(seq1) : seq1[i] = seq2[i])$

47 Defn $IsElementInSeq(el, seq) \triangleq \exists i \in \text{DOMAIN } seq : seq[i] = el$

49 Defn $IsSequenceOfSetElements(seq, set) \triangleq$

50 $\wedge Len(seq) = Cardinality(set)$

51 $\wedge (\forall el \in set : IsElementInSeq(el, seq))$

53 Defn $IsSortedSequenceOfSetElements(seq, set) \triangleq$

54 $\wedge IsSequenceOfSetElements(seq, set)$

55 $\wedge (\forall i \in \text{DOMAIN } seq, j \in \text{DOMAIN } seq : i < j \Rightarrow seq[i] < seq[j])$

57 Defn $DeleteElement(seq, index) \triangleq$

58 $[i \in 1 \dots (Len(seq) - 1) \mapsto \text{IF } i < index \text{ THEN } seq[i] \text{ ELSE } seq[(i + 1)]]$

It requires that $index \geq 1$.

If $index > Len(seq) + 1$, then it appends the element to seq .

(ADDED by hengxin; July 04, 2018)

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67 Defn  InsertElement(seq, elem, index)  $\triangleq$ 
68   [i  $\in$  1 .. (Len(seq) + 1)  $\mapsto$  IF i < index
69     THEN IF i = (Len(seq) + 1)
70       THEN elem
71       ELSE seq[i]
72     ELSE IF i = index
73       THEN elem
74       ELSE seq[(i - 1)]] i > index

76 Defn  IsSorted2Partition(n, seq1, seq2)  $\triangleq$ 
77    $\wedge seq1 \in Seq(1 .. n)$ 
78    $\wedge seq2 \in Seq(1 .. n)$ 
79    $\wedge n = Len(seq1) + Len(seq2)$ 
80    $\wedge (\forall i \in \text{DOMAIN } seq1, j \in \text{DOMAIN } seq1 : i < j \Rightarrow seq1[i] < seq1[j])$ 
81    $\wedge (\forall i \in \text{DOMAIN } seq2, j \in \text{DOMAIN } seq2 : i < j \Rightarrow seq2[i] < seq2[j])$ 
82    $\wedge (\forall i \in \text{DOMAIN } seq1, j \in \text{DOMAIN } seq2 : seq1[i] \neq seq2[j])$ 

84 Defn  IsSequenceInterleaving(seq, subSeq1, subSeq2, indSeq1, indSeq2)  $\triangleq$ 
85    $\wedge indSeq1 \in Seq(Nat)$ 
86    $\wedge indSeq2 \in Seq(Nat)$ 
87    $\wedge IsSorted2Partition(Len(seq), indSeq1, indSeq2)$ 
88    $\wedge Len(indSeq1) = Len(subSeq1)$ 
89    $\wedge Len(indSeq2) = Len(subSeq2)$ 
90    $\wedge (\forall i \in \text{DOMAIN } indSeq1 : seq[indSeq1[i]] = subSeq1[i])$ 
91    $\wedge (\forall i \in \text{DOMAIN } indSeq2 : seq[indSeq2[i]] = subSeq2[i])$ 
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\ * Last modified Wed Jul 04 11:21:54 CST 2018 by hengxin
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