MTH 9821 Numerical Methods for Finance I

Fall 2016

Midterm Exam

September 22, 6-8:30pm

Work is to be done individually. This rule is going to be strictly enforced under severe penalties. No discussions or email exchanges between students are permitted.
Write your results in the file firstname_lastname_midterm_9821_fall2016.xlsx provided with the test. Write the last four digits of your social security number at the top of the file, and change the name of the file to include your name. DO NOT MODIFY THE FORMAT/STYLE OF THE FILE!

Email the file to
kristoforos.joanidis@baruch.cuny.edu

You will receive full credit for a correct answer, meaning 9 decimal digits correct (unless tolerance requires otherwise), partial credit if some of the last decimals do not match, and no credit if you are off at the fourth decimal or worse. Please note that the 9 decimals rule cannot apply to residual errors. Report decimal errors as follows: $2.51 \cdot 10^{-12}$, i.e., the first nonzero decimal which may be close to epsilon machine and should be reported as accurately as possible.

Good luck!

1. Let A_1 be the following 9×9 matrix:

$$\begin{aligned} A_1(i,i) &= 4, \quad \forall\, i = 0:8; \\ A_1(i,i-1) &= -1, \quad \forall\, i = 1:8; \\ A_1(i,i-2) &= 3, \quad \forall\, i = 2:8; \\ A_1(i,i+2) &= -2, \quad \forall\, i = 0:6. \end{aligned}$$

(10 points) Find the matrices P_1, L_1, and U_1 for the LU decomposition with row pivoting of A_1.

Let b_1 be the following column vector:

$$b_1(i) = \sqrt{\frac{i^2}{2} - i + 6}, \qquad i = 0:8.$$

Solve

$$A_1 v_1 = b_1.$$

Compute the residual error

$$||b_1 - A_1 v_1||,$$

where the norm is the 2-norm.

(5 points) Compute L_1^{-1}, U_1^{-1} and A_1^{-1} without using Eigen's invert method.

Use matrix–vector multiplication to compute the vector

$$v_2 \;=\; A_1^{-1} b_1.$$

Compute the residual error

$$||b_1 - A_1 v_2||.$$

2. Let

$$A_2 = A_1^t \; A_1.$$

(10 points) Find the upper triangular matrix U_2 from the Cholesky decomposition on A_2.

(5 points) Let b_2 be the following column vector:

$$b_2(i) = \frac{i^2 - 9}{i + 5}, \qquad i = 0 : 8.$$

Solve

$$A_2 x_2 = b_2.$$

Compute the residual error

$$||b_2 - A_2 x_2||.$$

(5 points) Let $A_3 = A_1^t + A_1$. Solve

$$A_3 x_3 = b_2.$$

Compute the residual error

$$||b_2 - A_3 x_3||.$$

3. Let A_4 be an 8×8 matrix given by

$$\begin{aligned} A_4(i,i) &= 9, \quad \forall\, i = 0:7 \\ A_4(i,i+2) &= -2, \quad \forall\, i = 0:5 \\ A_4(i,i-2) &= 4, \quad \forall\, i = 2:7 \\ A_4(i,i+3) &= -2, \quad \forall\, i = 0:4 \\ A_4(i,i-3) &= -1, \quad \forall\, i = 3:7 \end{aligned}$$

Let b_2 be a column vector given by

$$b_4(i) = \frac{4i-3}{2i^2+1}, \qquad i = 0:7.$$

Solve

$$A_4 x_4 = b_4,$$

using Jacobi **(10 points)**, Gauss–Siedel **(10 points)**, and SOR **(10 points)** with two different values for ω:

$$\omega \in \{0.95, 1.21\}.$$

For each method, the initial guess is a vector x_0 with all entries equal to 0. Use a tolerance of 10^{-6} and a residual–based stopping criterion. Report the first three approximations and the final result, as well as the number of iterations to convergence for each method.

4. The file `financials2016-8.xlsx` contains the weekly prices adjusted for dividends from November 6, 2015, through September 16, 2016, (i.e., 46 prices) for the following financial stocks: JPM, GS, MS, BAC, RBS, CS, UBS, BCS (Barclays).
Compute the 45×8 matrix X of weekly log returns of these stocks.

(5 points) (i) Compute the 8×8 covariance matrix Σ_X of the weekly log returns of the stocks along with the Cholesky factor U_X of the matrix Σ_X.

(5 points) (ii) Find the linear regression of the MS weekly log returns with respect to the weekly log returns of the other financial stocks. Report the residual error of the linear regression.

Repeat the linear regression of the MS weekly log returns with respect to the weekly log returns of the other financial stocks and a constant vector (with entries 1). Report the residual error of the linear regression; the coefficient of the constant vector should be the last one reported.

(5 points) (iii) Given the independent standard normal variables Z_i, $i = 1 : 8$, find normal random variables X_i, $i = 1 : 8$, with covariance matrix Σ_X and given by $\mathbf{X} = M\mathbf{Z}$, where M is an 8×8 matrix and $\mathbf{X} = (X_i)_{i=1:8}$, $\mathbf{Z} = (Z_i)_{i=1:8}$. Report the matrix M

5. The file `sp-options-2016.xlsx` contains a snapshot taken on September 21, 2016, of bid and ask prices of the S&P 500 options (which are European options), maturing on Jan 20, 2017 corresponding to a spot price of the index of $2,139$.

Recall from the Put–Call parity that

$$Fe^{-rT} - Ke^{-rT} \;=\; C - P, \tag{1}$$

where $F = Se^{(r-q)T}$ is the forward price of the asset at time T.

Let $disc = e^{-rT}$ be the discount factor and let $PVF = Fe^{-rT}$ be the present value of the forward price. Then, (1) is the same as

$$PVF - K \cdot disc \;=\; C - P. \tag{2}$$

(3 points) (i) From (2), it follows that the values of PVF and $disc$ can be obtained by solving a least square problem

$$y \;\approx\; Ax,$$

with $x = \begin{pmatrix} PVF \\ disc \end{pmatrix}$, where A is a 14×2 matrix A, and y is a 14×1 column vector.

Identify A and y.

(7 points) (ii) Use least squares to compute PVF and $disc$.

6. The following discount factors were obtained from market data:

Date	Discount Factor
1 months	0.9980
4 months	0.9935
10 months	0.9820
15 months	0.9775
21 months	0.9620

The overnight rate is 0.7%.

(2 points) (i) What are the corresponding 1 month, 4 months, 10 months, 15 months, and 21 months zero rates?

(5 points) (ii) What is the 4×4 tridiagonal system that must be solved in the efficient implementation of the natural cubic spline interpolation for finding the zero rate curve for all times less than 21 months?

(3 points) Use the efficient implementation of the natural cubic spline interpolation to find a zero rate curve for all times less than 21 months matching the discount factors above, and find the value of a 19 months semi-annual coupon bond with 3.5% coupon rate.