Applications

1. *A dispersion trade*

Dispersion trading seeks to capture gains relating to the correlation among individual constituents of an equity basket, such as an index. Because of diversification effects, the variance of the basket return should be less than the sum of the constituents' variances, with the degree of difference attributable to the correlations of the constituents to one another. The purpose of this problem is to construct a simplified dispersion trade and investigate its correlation behaviors.

Consider the two assets, which pay no dividends, and a basket of the two constructed of fixed quantities of each:

asset	S0	basket quantity (shares)
A	125	0.4
B	40	1.25
basket	100	1

For 6-month European options on these underlyings, the Black-Scholes implied volatilites for at-the-money forward (ATMf) options are 0.325 for asset A, 0.45 for asset B, and 0.31 for the basket. Note that the implied volatility for the basket option is calculated as if the basket price followed a geometric Brownian motion in its own right.

The risk-free interest rate is 1% to all maturities, expressed with continuous compounding.

(a) Determine the price of ATMf straddles on 0.4 shares of asset A, 1.25 shares of asset B, and a single share of the basket.
(b) Using your preferred method (quasi-analytic or Monte Carlo), build a pricer for an option on the basket using the correlation as an input. With your pricer, determine the values of ATMf call and put options on one share of the basket under the following correlation scenarios: -1, 0, 1. (Note that a quasi-analytic pricer will likely fail with inputs at either extreme; use correlations very, very near the extreme values to answer the question if you have chosen this implementation. If you use a Monte Carlo pricer, in addition to your final prices provide 95% confidence intervals for the prices, and report how many trials were used to obtain these results.)
(c) Determine the correlation implied by the market price of the ATMf straddle.
(d) You purchase, in the same quantities as the basket, AMTf straddles on asset A and asset B individually. What quantity of ATMf straddles on the index is needed to finance your purchase? Is the resulting portfolio long or short the correlation between the assets?
(e) Use your pricer to determine the PL of the portfolio in part d under an increase of 0.01 in the market-implied asset correlation; do the same for a decrease of 0.01 in the market-implied correlation.
(f) Use your pricer to estimate the deltas of the portfolio in part d with respect to each of the underlying assets.

2. *Miscellaneous FX Options…..*

The spot EURUSD exchange rate is 1.29 dollars per Euro. The volatility of the USDEUR rate is 8%. The 3M risk-free rate in USD is 0.3095%, and the 3M risk-free rate in EUR is 0.183%. Calculate the following, making it clear where appropriate in which currency your result is denominated:

(a) Long an option to pay \$1.3 million and receive €1 million in 3M.

(b) The delta hedge that would be used by a party short the option in part a. (This will consist of a certain quantity of the USD money market asset and a certain quantity of the EUR money market asset.)

(c) The value of a one-touch option that pays \$1 million if EUR touches the level 1.275 USD within the next 3M. The cash amount is paid at the time of the touch.

The general form of the pricing formula for one-touch options is shown below. Define the following quantities:

Q = the quantity of pricing currency paid in the event of touch

η = upper / lower barrier indicator: $\eta = 1$ for an upper barrier, $\eta = -1$ for a lower barrier

ω = payment time indicator: $\omega = 0$ for pay-at-hit; $\omega = 1$ for pay-at-expiry

X_0 = the spot value of one unit of foreign in pricing currency

H = the barrier FX level, expressed as the price of one unit of foreign in pricing currency

r = the pricing currency risk-free rate

r_f = the foreign currency risk-free rate

T = time to expiry

σ = FX rate volatility

Then the value of the one-touch is given by:

$$V_{one-touch} = Qe^{-\omega rT}\left[\left(\frac{H}{X_0}\right)^{\frac{\theta+\vartheta}{\sigma}} N\left(\eta e_+\right) + \left(\frac{H}{X_0}\right)^{\frac{\theta-\vartheta}{\sigma}} N\left(-\eta e_-\right)\right]$$

$$\theta = \frac{r - r_f}{\sigma} - \frac{\sigma}{2}$$

$$\vartheta = \sqrt{\theta^2 + 2\left(1-\omega\right)r}$$

$$e_{\pm} = \frac{1}{\sigma\sqrt{T}}\left(\pm\ln\frac{X_0}{H} - \sigma\vartheta T\right)$$

(d) The value of a no-touch option that pays €1 million if EUR does not touch the level 1.3 USD within the next 3M.