Credit Risk Models

9. XVA I

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Outline

Objectives

- *Risk management (PFE)*
- *Valuation (CVA)*

Approaches to calculating credit exposure

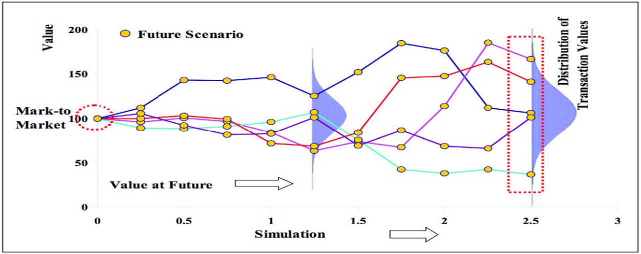
- *Add-ons*. This approach is simplistic and assumes that the future exposure is approximately equal to the current exposure. It forms a basis for the Basel I capital rules.
- *Semi-analytical methods*. This approach is more sophisticated than the add-on approaches, and are similar to the calculations discussed in Lecture Notes #8.
- *Monte Carlo simulations*. This is a universal approach to quantifying credit exposure, and is considered state of the art. Its complexity can be high.

Monte Carlo methodology

- The first task is to identify the relevant risk factors and specify their dynamics.
- A compromise has to be struck between the model comprehensiveness and parsimony. For example, one can settle on using a one-factor interest rate model rather than three- or four-factor.
- Adding a new risk factor also requires understanding its dependence on the other risk factors. This leads to additional complexity of the model.
- Calibration decisions have to be made: historical or cross-sectional data (or a mix of both).
- Generally, risk models tend to be less detailed and complex than front office models.

Monte Carlo methodology

The graph below illustrates the Monte Carlo approach to credit exposure.



Monte Carlo methodology

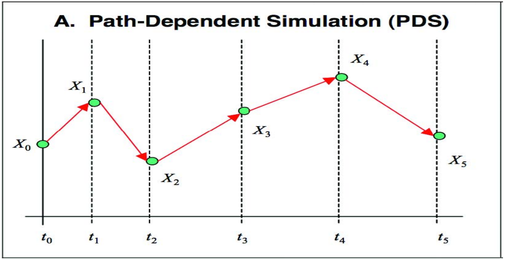
- *Scenario generation*:
  - (i) Time grid selection.
  - (ii) Path dependent simulation vs long jump.
- *Revaluation*. This step requires using efficient valuation model. If need be, approximations have to be made as in the Longstaff-Schwartz approach to valuation of American option.
- *Aggregation*. Next, let $V(k, s, t)$ denote the value of instrument k under scenario s at future time t. We now compute the aggregates over each netting set (NS):

$$V_{NS}(s, t) = \sum_{k \in NS} V(k, s, t).$$

- *Postprocessing*. This step requires applying the logic corresponding to risk mitigants such as collateral, additional termination events, etc.
- *Extraction*. We can now collapse the scenarios to metrics such as EE, other metrics can be computed out of scenario level data.

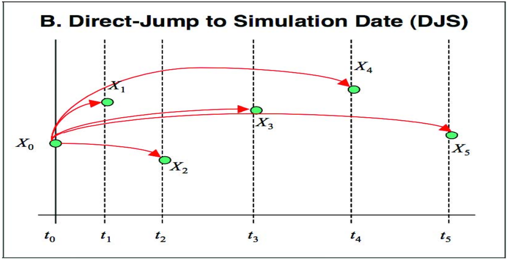
Monte Carlo methodology

The graph below illustrates the path dependent simulation method.



Monte Carlo methodology

The graph below illustrates the long jump simulation method.



Interest rates

- A simple (but still used) model of interest rates is the Hull-White model. It is a one-factor model based on the short rate r with the dynamics:

$$dr(t) = \lambda\Big(\mu(t) - \frac{d\mu(t)}{dt} - r(t)\Big)dt + \sigma(t)\, dW(t),$$

  where $\mu(t)$ is the mean reversion level, λ is the mean reversion speed, and $\sigma(t)$ is the instantaneous volatility.
- This model can be calibrated exactly to the current snapshot of the swap curve, with $\mu(t)$ directly related to current instantaneous forward curve.
- The mean reversion speed λ has the moderating effect on the volatility of zero coupon bonds $P(t, T)$:

$$\sigma_P = \sigma \frac{1 - e^{-\lambda(T-t)}}{\lambda(T-t)}$$

  (assuming constant σ).

Equities

- The standard geometric Brownian motion (lognormal) of a stock price $S(t)$ is given by

$$\frac{dS(t)}{S(t)} = \mu(t)\,dt + \sigma(t)\,dW(t)\,,$$

where $\mu(t)$ is the instantaneous rate of return and $\sigma(t)$ is the instantaneous volatility.

FX

- Short dated FX instruments are modeled using the lognormal model.
- Longer dated FX instruments require using a lognormal with mean reversion:

$$\frac{dX(t)}{X(t)} = \lambda(\theta - \log X(t))dt + \sigma(t)\, dW(t)\,,$$

where θ is the mean reversion level, and λ is the mean reversion speed.

Commodities

- A popular model for commodities is given by the following specification:

$$\begin{aligned} \log S(t) &= f(t) + Z(t), \\ dZ(t) &= \lambda(\theta - Z(t))dt + \sigma(t)\, dW(t). \end{aligned}$$

  where $f(t)$ is a deterministic periodic function, and $S(t)$ is the spot price.
- Note that this model attempts to capture the seasonal character of commodity prices.

Credit

- For credit, we assume the CIR model for the intensity $\lambda(t)$:

$$d\lambda(t) = \kappa(\theta - \lambda(t))dt + \sigma(t)\sqrt{\lambda(t)}dW(t),$$

discussed in Lecture Notes #5.

Risk neutral versus physical valuation

- The choice of measure is dictated by the practicalities of available financial data.
- As a rule:
  (i) Risk management metrics, such as PFE, are calculated under the P-measure.
  (ii) Valuation (and thus CVA) is done under the Q-measure.

CVA formula

- We will now combine the concepts introduced earlier in the course (default probability and recovery rate) with the concept of credit exposure - this is important for accurate pricing of counterparty risk.
- We will first make three simplifying assumptions:
  (i) The institution (bank, denoted B in the following) is default free, while the counterparty (denoted C) is credit risky. We are ignoring here the debt valuation adjustment (DVA).
  (ii) It is possible to perform risk free valuation. We are thus assuming the existence of a discount rate.
  (iii) The credit exposure and default probability are independent. This ignores the wrong way risk.
- Later we will relax these assumptions.
- We first derive the formula for CVA and then discuss its use within an institution.

CVA formula

- When valuing a financial transaction, it is possible to separate the counterparty risk as follows:
$$\text{Risky Value} = \text{Riskless Value} - \text{CVA}.$$
To see this, we proceed as follows.
- Let $\widehat{V}(t, T)$ denote the (counterparty credit) risky value of a netted set of derivatives positions with a maximum maturity T, and let $V(t, T)$ denote its risk-free value of the positions. We assume that the mark to market of the positions is given by $V(t, T)$
- Then $V(s, T)$, where $t \leq s \leq T$, denotes the future (uncertain) MtM. It is related to $V(t, T)$ by $V(t, T) = P(t, s)V(s, T)$, where $P(t, u)$ is the riskless discount factor (zero coupon bond).
- As usual, the default time of the counterparty is denoted by τ.
- We consider two cases:

CVA formula

- Case 1. *Counterparty does not default before T*. In this case, the risky position is equivalent to the risk-free position and we write the corresponding payoff as

$$1_{\tau > T} V(t, T).$$

- Case 2. *Counterparty defaults before T*. In this case, the payoff consists of two terms: the value of the position that would be paid before the default time (all cash flows before default will still be paid by the counterparty)

$$1_{\tau \leq T} V(t, \tau),$$

  and the payoff at default calculated as follows.

- Here, if the MtM of the trade at the default time, $V(\tau, T)$, is positive, then the institution will receive a recovery rate R of the risk-free value of the derivatives positions. If it is negative then they will still have to settle this amount. Hence, the default payoff at time τ is:

$$1_{\tau \leq T} \big(RV(\tau, T)^{+} + V(\tau, T)^{-} \big).$$

CVA formula

- Putting these payoffs together, we have the following expression for the value of the risky position under the risk-neutral measure:

$$\widehat{V}(t,T) = \mathsf{E}^{\mathsf{Q}}\big[1_{\tau>T}V(t,T) + 1_{\tau\leq T}\big(V(t,\tau) + RV(\tau,T)^{+} + V(\tau,T)^{-}\big)\big].$$

- Using the relation $x^{-} = x - x^{+}$, we rearrange the terms as follows:

$$\begin{aligned}\widehat{V}(t,T) &= \mathsf{E}^{\mathsf{Q}}\big[1_{\tau>T}V(t,T) + 1_{\tau\leq T}\big(V(t,\tau) + RV(\tau,T)^{+} + V(\tau,T) - V(\tau,T)^{+}\big)\big]\\ &= \mathsf{E}^{\mathsf{Q}}\big[1_{\tau>T}V(t,T) + 1_{\tau\leq T}\big(V(t,\tau) + (R-1)V(\tau,T)^{+} + V(\tau,T)\big)\big].\end{aligned}$$

- Since $V(t,\tau) + V(\tau,T) = V(t,T)$, we can rewrite this as

$$\widehat{V}(t,T) = \mathsf{E}^{\mathsf{Q}}\big[1_{\tau>T}V(t,T) + 1_{\tau\leq T}\big(V(t,T) + (R-1)V(\tau,T)^{+}\big)\big].$$

CVA formula

- Finally, using $1_{\tau > T} V(t, T) + 1_{\tau \leq T} V(t, T) = V(t, T)$, we can write:

$$\widehat{V}(t, T) = V(t, T) - \mathsf{E}^{\mathsf{Q}}\big[1_{\tau \leq T}(1 - R)\, V(\tau, T)^{+}\big].$$

  The above equation defines the risky value of a netting set of derivatives positions with respect to the risk-free value.

- We write it as

$$\begin{aligned}\widehat{V}(t, T) &= V(t, T) - CVA(t, T),\\ CVA(t, T) &= \mathsf{E}^{\mathsf{Q}}\big[1_{\tau \leq T}(1 - R)\, V(\tau, T)^{+}\big],\end{aligned}$$

  where the term $CVA(t, T)$ is called the credit value adjustment (CVA).

CVA formula

- The CVA formula is very useful because it allows us to value a transaction and computing counterparty risk separately. In principle, the CVA components can be handled centrally.
- Within a bank, one desk is responsible for risk free valuation (front office) and one is responsible for counterparty risk valuation (CVA desk).
- In reality, things are a bit more complicated. Due to risk mitigants such as netting and collateral, CVA is not truly linear.
- This means that the risky value of a transaction is defined withing the netting set of other transactions, and cannot be calculated individually.

Standard CVA formula

- The standard CVA formula is an approximation to the CVA formula above, and is used throughout the industry. To derive it, we write:

$$CVA(t, T) = (1 - \overline{R})\mathrm{E}^{\mathrm{Q}}\left[1_{u \leq T} V^*(u, T)^+\right],$$

where $\overline{R}$ is the expected recovery rate, and

$$V^*(u, T) \triangleq V(u, T) \,|\, \tau = u.$$

- This is a key point in the analysis, as the above statement requires the exposure at a future date $V(u, T)$, conditioned on the counterparty default at $\tau = u$. Ignoring wrong-way risk means that $V^*(u, T) = V(u, T)$.
- Since the expected value above is taken over all times before the final maturity, we represent it as an integral over all possible default times u:

$$CVA(t, T) = (1 - \overline{R})\mathrm{E}^{\mathrm{Q}}\Big[\int_t^T P(t, u) V(u, T)^+ dQ(t, u)\Big],$$

where $Q(t, u)$ is the cumulative counterparty default probability.

Standard CVA formula

- We denote by
$$EE_t(u,T) \triangleq \mathsf{E}^{\mathsf{Q}}\left[P(t,u)V(u,T)^+\right]$$
the discounted expected exposure calculated under the risk neutral measure.
- Assuming for simplicity that the default probabilities are deterministic, we thus have:
$$CVA(t,T) = (1-\overline{R})\int_t^T EE_t(u,T)dQ(t,u).$$
- Finally, we compute approximately the above expression using:
$$CVA(t,T) \approx (1-\overline{R})\sum_{i=1}^{m} EE_t(t,T_i)(Q(t,T_i)-Q(t,T_{i-1})),$$
where $t = T_0 < T_1 < \ldots < T_m = T$ (using the trapezoid rule would, of course, lead to a better accuracy).

Standard CVA formula

- The last formula is the *standard CVA formula*. We will write it schematically as

$$\mathrm{CVA} \approx (1 - \mathrm{Rec}) \sum_{i=1}^{m} \mathrm{DiscFact}(T_i)\mathrm{EE}(T_i)\mathrm{DefProb}(T_{i-1}, T_i).$$

- It has four components:
  (i) LGD represented by $1 - \mathrm{Rec}$, where Rec stands for recovery rate.
  (ii) Discount factors to the event dates.
  (iii) Expected exposure to the event dates.
  (iv) Default probabilities between to the event dates.

CVA as a spread

- With further simplifying assumptions, we can obtain a simple expression for CVA linking it to the credit spread of the counterparty. To do this, we work with the undiscounted expected exposure and write the CVA formula as:

$$CVA(t,T) = (1-\overline{R}) \int_t^T P(t,u) EE(u,T) dQ(t,u).$$

- Suppose that we can approximate the undiscounted expected exposure term, $EE(u,T)$, as a fixed known amount, namely the EPE. The fixed EPE could be approximately computed as the EE averaged over time:

$$\begin{aligned} EPE &= \frac{1}{T-t} \int_t^T EE(t,u) du \\ &\approx \frac{1}{m} \sum_{i=1}^m EE(t,T_i). \end{aligned}$$

- Clearly, this approximation can be made if the relationship between EPE, default probability, and discount factors can be assumed homogeneous through time.

CVA as a spread

- Using this approach, the CVA can be expressed as

$$CVA(t,T) = (1-\overline{R})\mathrm{E}^{\mathrm{Q}}\Big[\int_t^T P(t,u)dQ(t,u)\Big]\, EPE.$$

- We see that this is simply the value of CDS protection on a notional equal to the EPE. Hence we have the following approximation giving a running CVA (i.e. expressed as a spread):

$$\mathrm{CVA} \approx \mathrm{EPE} \times \mathrm{Spread}.$$

- The approximate formula, which is a variant of the credit triangle studied earlier in the course, is often useful for intuitive understanding of the drivers of CVA.
- We can use the corresponding risky annuity to convert the spread into an upfront value.

CVA calibration issues

- For CVA calibration, we should be calculating the exposure under the risk neutral measure rather than the physical measure. The drift terms associated with the physical measure may lead to price distortions.
- This requires calibration to the cross-sectional market data rather than historical market data.
- For example, in the case of interest rate derivatives, we should be using currently observed swap and money market rates, cap / floor and swaption volatilities, smile data, etc, rather then historical time series of swap rates.

Semi-analytical methods for CVA

- In case of specific instruments it is possible to evaluate CVA in closed form.
- Such explicit formulas are of limited use, as they apply only to stand-alone positions without accounting for netting or collateral.
- These formulas are, however, of interest as they can be used for quick calculations and intuitive understanding of CVA.
- Below we discuss two examples of such formulas, namely an option and an interest rate swap.

CVA for an option

- In this case we have a simplification since the exposure of the long option position can never be negative:

$$\begin{aligned} CVA_{\text{opt}}(t,T) &= (1-\overline{R})\mathsf{E}^{\mathsf{Q}}\left[1_{\tau<T}\right]\mathsf{E}^{\mathsf{Q}}\left[P(t,\tau)V_{\text{opt}}(\tau,T)\right] \\ &= (1-\overline{R})Q(t,T)V_{\text{opt}}(t,T), \end{aligned}$$

  where $V(t,T)_{\text{opt}}$ is the option premium.
- This means that the value of the risky option can be calculated as:

$$\begin{aligned} \widehat{V}_{\text{opt}}(t,T) &= V_{\text{opt}}(t,T) - CVA_{\text{opt}}(t,T) \\ &= V_{\text{opt}}(t,T) - (1-\overline{R})Q(t,T)V_{\text{opt}}(t,T) \\ &= V_{\text{opt}}(t,T)S(t,T) + \overline{R}Q(t,T)V_{\text{opt}}(t,T). \end{aligned}$$

- We see that, with zero recovery, the risky premium is the risk-free value multiplied by the survival probability over the life of the option.

CVA for a swap

- In the case of a swap, the exposure can be positive or negative.
- Consider a receiver swap (the calculation is analogous for a payer); its CVA is

$$
\begin{aligned}
CVA_{\text{rec swap}}(t,T) &= (1-\overline{R})\mathrm{E}^{\mathrm{Q}}\Big[\int_t^T P(t,u)V_{\text{rec swap}}(u,T)^+ dQ(t,u)\Big] \\
&\approx (1-\overline{R})\sum_{i=1}^{m}\mathrm{E}^{\mathrm{Q}}\big[P(t,T_i)V_{\text{rec swap}}(T_i,T)^+\big(Q(t,T_i)-Q(t,T_{i-1})\big)\big],
\end{aligned}
$$

  where T_i, $i=1,\ldots,m$ are the fixed coupon payment days.
- Assuming that rates are independent of the credit of C, we write it as

$$
CVA_{\text{rec swap}}(t,T) \approx (1-\overline{R})\sum_{i=1}^{m}\mathrm{E}^{\mathrm{Q}}\big[P(t,T_i)V_{\text{rec swap}}(T_i,T)^+\big)\big]\big(Q(t,T_i)-Q(t,T_{i-1})\big).
$$

CVA for a swap

- We note that
$$\left[P(t,T_i)V_{\text{rec swap}}(T_i,T)^{+})\right]$$
represents the time t premium on a payer swaption expiring on T_i.
- Therefore, the CVA of a receiver swap can be written as:

$$CVA_{\text{rec swap}}(t,T)=(1-\overline{R})\sum_{i=1}^{m}(Q(t,T_i)-Q(t,T_{i-1}))V_{\text{pay swaption}}(t,T_i,T).$$

- The intuition is that the counterparty has the “option” to default at any point in the future and therefore cancel the trade (execute the reverse position).
- The values of these swaptions are weighted by the relevant default probabilities and recovery is taken into account.
- This formula was obtained by Sorensen and Bollier in 1994.

CVA for a swap

- The Sorensen and Bollier gives us a useful insight on CVA calculations, namely that a CVA calculation is at least as complex as pricing the underlying instrument.
- To price the swap CVA we need to know swaption volatility across expirations and strikes. The price of the swap does not depend on volatility and yet its CVA does.
- The asymmetry between payer and receiver swaps is captured naturally. The receiver (payer) swaptions corresponding to the payer (receiver) swaps are ITM (OTM).
- In the latter case, the strike of the swaptions moves significantly out of the money when the bank receives a quarterly cash flow while not having yet made a semiannual payment.
- The above analysis can be extended to other products where any transaction can be represented as a basket of options.

Incremental CVA formula

- The presence of a netting agreement is likely to reduce the CVA and it cannot increase it:

$$CVA^{NS} \leq \sum_{i=1}^{N} CVA^{i},$$

where CVA^{NS} denotes the CVA of a netting set, and CVA^{i} is the stand-alone CVA for instrument i in the netting set.

- This reduction can be substantial, and the question arises how to allocate it to the individual instruments.
- This is done with the help of *incremental CVA* defined as:

$$CVA^{i}_{incr} = CVA^{NS\cup\{i\}} - CVA^{NS}.$$

- This guarantees that the CVA of a new trade is given by its contribution to the overall CVA at the time it is executed.

Incremental CVA formula

- The following formula holds for the incremental CVA

$$\mathrm{CVA}^{i}_{incr} \approx (1 - \mathrm{Rec}) \sum_{i=1}^{m} \mathrm{DiscFact}(T_i)\mathrm{EE}^{i}_{incr}(T_{j-1}, T_j)\mathrm{DefProb}(T_{i-1}, T_i),$$

where EE^{i}_{incr} denotes incremental expected exposure.

Incremental CVA formula

- To calculate incremental CVA, we need to quantify the change before and after added a new trade i:

$$
\begin{aligned}
& CVA^{NS\cup\{i\}}(t,T) - CVA^{NS}(t,T) \\
&\qquad = (1-\overline{R})\sum_{i=1}^{m} EE^{NS\cup\{i\}}(t,T_i)\big(Q(t,T_i)-Q(t,T_{i-1})\big) \\
&\qquad - (1-\overline{R})\sum_{i=1}^{m} EE^{NS}(t,T_i)\big(Q(t,T_i)-Q(t,T_{i-1})\big) \\
&\qquad = (1-\overline{R})\sum_{i=1}^{m} \big(EE^{NS\cup\{i\}}(t,T_i)-EE^{NS}(t,T_i)\big)\big(Q(t,T_i)-Q(t,T_{i-1})\big).
\end{aligned}
$$

- We therefore simply need to use the incremental EE,

$$
EE^{i}_{incr} \triangleq EE^{NS\cup\{i\}}(t,T_i) - EE^{NS}(t,T_i)
$$

in the standard CVA formula.

Bilateral CVA formula

- One of the assumptions used in the derivation of the CVA formula was that the bank could not default. Historically, banks have charged their corporate counterparties CVA linked to their credit quality and bank's exposure.
- In the post-Lehman world it became painfully clear that banks themselves are credit risky entities.
- Since credit exposure has a liability component (the negative exposure defined earlier), this could be include in the counterparty credit valuation model.
- This leads us to the concept of the *debt valuation adjustment* (DVA).
- DVA is a somewhat controversial concept. While it resolves some theoretical issues with CVA, it leads to some unintuitive consequences that we will discuss later.

Bilateral CVA formula

- We shall now find an expression for the risky value $\widehat{V}(t, T)$ of a netted set of derivatives positions with a maximum maturity date T, where, unlike above, we assume that both institution B may and its counterparty C may default.
- Denote the default times of the institution and its counterparty by τ_B, and τ_C, respectively, and their recovery rates by as R_B and R_C, respectively. Let $\tau^1 = \min(\tau_B, \tau_C)$ denote the first-to-default time of the institution and counterparty.
- Following the notation and logic of the previous section, we consider the following cases:

Bilateral CVA formula

- 1. *Neither counterparty nor bank defaults before T*. In this case, the risky position is equivalent to the risk-free position and we write the corresponding payoff as:

$$1_{\tau^1 > T} V(t, T).$$

- 2. *Counterparty defaults first and also before time T*. This is the default payoff as in the previous section

$$1_{\tau^1 \leq T} 1_{\tau^1 = \tau_C} \big(R_C V(\tau^1, T)^+ + V(\tau^1, T)^- \big).$$

- 3. *Bank defaults first and also before time T*. This is an additional term compared with the unilateral CVA case and corresponds to the institution itself defaulting. If B owes money to C (negative MtM) then it will pay only a recovery fraction of this. If the C owes B money (positive MtM) then it will still receive this. Hence, the payoff is the opposite of case 2 above:

$$1_{\tau^1 \leq T} 1_{\tau^1 = \tau_B} \big(R_B V(\tau^1, T)^- + V(\tau^1, T)^+ \big).$$

Bilateral CVA formula

- 4. *If either the bank or counterparty does default then all cash flows prior to the first-to-default date will be paid*. The payoff is

$$1_{\tau^1 \leq T} V(t, \tau).$$

- Putting the above payoffs together, we obtain the following expression for the value of the risky position:

$$\begin{aligned}\widehat{V}(t, T) = \mathsf{E}^{\mathsf{Q}}\big[&1_{\tau^1 > T} V(t, T) + 1_{\tau^1 \leq T} 1_{\tau^1 = \tau_C} \big(R_C V(\tau^1, T)^+ + V(\tau^1, T)^-\big) \\ &+ 1_{\tau^1 \leq T} 1_{\tau^1 = \tau_B} \big(R_B V(\tau^1, T)^- + V(\tau^1, T)^+\big) + 1_{\tau^1 \leq T} V(t, \tau)\big].\end{aligned}$$

Bilateral CVA formula

- We can simplify the above expression as:

$$\begin{aligned}\widehat{V}(t,T) = \mathsf{E}^{\mathsf{Q}}\big[&1_{\tau^1>T}V(t,T) + 1_{\tau^1\leq T}V(t,\tau^1) + 1_{\tau^1\leq T}V(\tau^1,T)\\&+1_{\tau^1\leq T}1_{\tau^1=\tau_C}\big(R_C V(\tau^1,T)^+ - V(\tau^1,T)^+\big)\\&+1_{\tau^1\leq T}1_{\tau^1=\tau_B}\big(R_B V(\tau^1,T)^- - V(\tau^1,T)^-\big)\big].\end{aligned}$$

- This can finally be written as

$$\begin{aligned}\widehat{V}(t,T) = V(t,T) + \mathsf{E}^{\mathsf{Q}}\big[&1_{\tau^1\leq T}1_{\tau^1=\tau_C}\big(R_C V(\tau^1,T)^+ - V(\tau^1,T)^+\big)\\&+1_{\tau^1\leq T}1_{\tau^1=\tau_B}\big(R_B V(\tau^1,T)^- - V(\tau^1,T)^-\big)\big].\end{aligned}$$

- After rearranging the terms, we write this as

$$\begin{aligned}\widehat{V}(t,T) = V(t,T) - \mathsf{E}^{\mathsf{Q}}\big[&1_{\tau^1\leq T}1_{\tau^1=\tau_C}(1-R_C)V(\tau^1,T)^+\\&+1_{\tau^1\leq T}1_{\tau^1=\tau_B}(1-R_B)V(\tau^1,T)^-\big].\end{aligned}$$

Bilateral CVA formula

- We can thus identify the *bilateral CVA* (BCVA) as

$$\begin{aligned} BCVA(t,T) = \mathrm{E}^{\mathrm{Q}}\big[&1_{\tau^1\leq T}1_{\tau^1=\tau_C}(1-R_C)V(\tau^1,T)^+ \\ &+ 1_{\tau^1\leq T}1_{\tau^1=\tau_B}(1-R_B)V(\tau^1,T)^-\big]. \end{aligned}$$

- Finally, under the assumptions of no wrong-way risk and no simultaneous default between the default of the institution and its counterparty, we obtain a formula analogous to that derived in the case of unilateral CVA:

$$\begin{aligned} BCVA(t,T) = (1-\overline{R}_C)\mathrm{E}^{\mathrm{Q}}\Big[&\int_t^T P(t,u)V(u,T)^+ S_B(t,u)dQ_C(t,u)\Big] \\ &+ (1-\overline{R}_B)\mathrm{E}^{\mathrm{Q}}\Big[\int_t^T P(t,u)V(u,T)^- S_C(t,u)dQ_B(t,u)\Big], \end{aligned}$$

where $S(t,u)$ denotes survival probability.

Bilateral CVA formula

- Proceeding as in the calculation above for CVA, we can approximate this formula by

$$
\begin{aligned}
BCVA(t,T) \approx (1-\overline{R}_C)\sum_{i=1}^{m} EE_t(t,T_i)S_B(t,T_{i-1})\big(Q_C(t,T_i)-Q_C(t,T_{i-1})\big)\\
+(1-\overline{R}_B)\sum_{i=1}^{m} NEE_t(t,T_i)S_C(t,T_{i-1})\big(Q_B(t,T_i)-Q_B(t,T_{i-1})\big)
\end{aligned}
$$

  where $t = T_0 < T_1 < \ldots < T_m = T$ is a suitable time grid.
- This approximation is a basis for the standard BCVA formula.

Standard BCVA formula

- We will state the standard BCVA formula as

$$
\begin{aligned}
\mathrm{BCVA} = {} & (1-\mathrm{Rec}_C)\sum_{i=1}^{m}\mathrm{DiscFact}(T_i)\mathrm{EE}(T_i)\mathrm{SurvProb}_B(0,T_{i-1})\mathrm{DefProb}_C(T_{i-1},T_i) \\
& + (1-\mathrm{Rec}_B)\sum_{i=1}^{m}\mathrm{DiscFact}(T_i)\mathrm{NEE}(T_i)\mathrm{SurvProb}_C(0,T_{i-1})\mathrm{DefProb}_B(T_{i-1},T_i).
\end{aligned}
$$

- The first term is essentially the CVA formula (adjusted for the bank's own survival).
- The second term represents a negative contribution (since NEE is negative) and is called the *debt valuation adjustment* (DVA).

Standard BCVA formula

- In practice, the survival probability of B is not included in the CVA fromula, and the survival probability of C is not included in the DVA formula.
- Also, the correlation of defaults between B and C is ignored. Such a correlation, if positive, would impact CVA and DVA.
- Finally, the definitions of EE and ENE rely on standard valuation methodologies, rather than the actual close-out values that may be realized in case of a default.

BCVA as a spread

- To gain understanding the meaning of BCVA we can carry out the same approximations as in the case of CVA to derive:

$$\mathrm{BCVA} \approx \mathrm{EE} \times \mathrm{Spread}_C + \mathrm{ENE} \times \mathrm{Spread}_B .$$

- This can be interpreted as the fact that the institution should subtract from CVA the component accounting for its own credit.
- In particular, if we can assume that $ENE \approx -EE$, then

$$\mathrm{BCVA} \approx \mathrm{EE} \times (\mathrm{Spread}_C - \mathrm{Spread}_B),$$

and the credit charge is proportional to the difference in the spreads.

Properties of DVA and the DVA controversy

- A bank using DVA attaches value to its own potential default. This may have consequences that may (or may not) be reasonable.
- A credit risky derivative can be worth more than a risk-free derivative: the BCVA can be negative (unlike CVA).
- If all counterparties agree on using BCVA, pricing counterparty is a zero sum game.
- Risk mitigants can increase BCVA. For example, netting (which increases CVA) may increase DVA.
- A bank can show accounting profits as a result of widening (own) credit spreads.
- DVA is largely disregarded in pricing and replaced as a funding benefit adjustment (FBA). We shall discuss this relationship later.
- Basel III requirements ignore DVA.

References

- Brigo, D., Morini, M., and Pallavicini, A.: *Counterparty Credit Risk, Collateral and Funding*, Wiley Finance (2014).
- Gregory, J.: *The xVA Challenge: Counterparty Credit Risk, Funding, Collateral, and Capital*, Wiley Finance (2015).
- Pykhtin, M., and Zhu, S.: A Guide to Modelling Counterparty Credit Risk, *GARP*, July / August, 16 - 22 (2007).
- Ruiz, I.: *XVA Desks - A New Era for Risk Management*, Palgrave MacMillan (2015).