# Credit Risk Models

## 8. Counterparty Credit Risk

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# Outline

# Financial risk

- Tradionally, financial risk has been broken up into several (partly overlapping) categories:
  (i) Market risk
  (ii) Credit risk
  (iii) Liquidity risk
  (iv) Operational risk
  (v) Model risk
- The financial crisis of 2007-2008 heightened the importance of two other categories of risk:
  (i) Counterparty risk
  (ii) Systemic risk

# Market risk

- Market risk has been the most studied and best understood of the financial risks.
- It arises from movements of asset prices.
- The impact of price movement can be:
  (i) linear (for underlying instruments such as stocks, commodities, FX,...) or
  (ii) nonlinear (for options and instruments with embedded options).
- Market risk can be magnified by leverage.
- Market risk can, in principle, be eliminated by entering into an offsetting trade ("hedge").
- Unless the hedge is done with the same counterparty, counterparty credit risk is created.

# Credit risk

- We have studied credit risk outside of the context of counterparty risk in the previous Lecture Notes.
- Credit risk gets complicated once counterparty credit risk is included.

# Liquidity risk

- There are two forms of liquidity risk:
    (i) *Asset liquidity* risk arises when it is impossible to execute a transaction at a reasonable price.
    (ii) *Funding liquidity* refers to the inability to make contractual payments or meet collateral requirements.
- Somewhat disappointingly, reducing counterparty credit risk in the financial markets may lead to increased liquidity risk.

# Operational risk

- Operational risk arises from people, systems, internal and external events.
- Legal risk.
- Political risk.
- Model risk:
  (i) From the quantitative perspective, model risk takes a particular place.
  (ii) It arises as a result of using misspecified or miscalibrated models.
  (iii) Correlation and dependency.

# Risks of derivatives

- Leverage.
- Concentration risk.
- Credit derivatives pose significant counterparty risk.

# Value at Risk (VaR)

- VaR is a popular and very useful quantitative metric used summarizing for summarizing portfolio risk.
- Let $\Phi$ denote the probability distribution of portfolio losses $L$. Given a significance level $\alpha$ (say, $\alpha = 1\%$) it is defined as

$$\begin{aligned} VaR_\alpha(L) &= \sup_l \{ l : \ \mathrm{P}(L \geq l) \geq \alpha \} \\ &= \sup_l \{ l : \ \Phi(l) \leq 1 - \alpha \}. \end{aligned}$$

  The number $1 - \alpha$ is referred to as the *confidence level*.
- If the cumulative distribution is continuous and monotone increasing, then $VaR_\alpha(L) = \Phi^{-1}(1-\alpha)$. For the standard normal distribution, the 99% VaR is $N^{-1}(0.99) = 2.326347874$.
- VaR's reliability is often put into question, as it says nothing about what lies above the assumed confidence level. Overreliance on VaR leads to false sense of complacency.

# Coherent risk measures

- A serious problem with VaR is that it has some counter-intuitive features, namely lack of *coherence*.
- Ideally, a conceptually satisfactory risk measure should be a *coherent risk measure*.
- Let $L$ denote a random variable representing potential portfolio losses (drawn from a distribution $\Phi$). A real valued function $\rho(L)$ is called a coherent risk measure if it satisfies the following conditions:
  (i) *Normalization*: $\rho(0) = 0$,
  (ii) *Monotonicity*: If $L_1 \leq L_2$, then $\rho(L_1) \leq \rho(L_2)$,
  (iii) *Subadditivity*: $\rho(L_1 + L_2) \leq \rho(L_1) + \rho(L_2)$,
  (iv) *Positive homogeneity*: For $\lambda > 0$, $\rho(\lambda L) = \lambda\rho(L)$,
  (v) *Translation invariance*: If $A$ has deterministic loss $a$, then $\rho(L + A) = \rho(L) + a$.

# Coherent risk measures

- In particular, the subadditivity is a natural requirement stating that the risk of a combined portfolio cannot exceed the sum of the risks of its parts.
- It reflects the fact that diversification may offset parts of the risks inherent in the portfolio.
- In general, VaR *does not* have the subadditivity property. Exception is the normal distribution (and more generally, elliptical distributions), for which VaR is subadditive.

# Conditional Value at Risk

- Conditional Value at Risk (CoVaR), a.k.a. Expected Shortfall (ES) is defined as

$$\begin{aligned} CoVaR_\alpha(L) &= \mathrm{E}[L \,|\, L \geq VaR_\alpha] \\ &= \frac{1}{\alpha}\int_{VaR_\alpha}^{\infty} l d\Phi(l) \end{aligned}$$

- If the CDF $\Phi$ is continuous, then CoVaR can alternatively be expressed as

$$CoVaR_\alpha(L) = \frac{1}{\alpha}\int_0^{\alpha} VaR_\gamma(L) d\gamma,$$

# Conditional Value at Risk

- To see this, we use the fact that $VaR_\gamma(L) = \Phi^{-1}(1-\gamma)$, and so

$$
\begin{aligned}
\int_0^\alpha VaR_\gamma(L)d\gamma &= \int_0^\alpha \Phi^{-1}(1-\gamma)d\gamma \\
&= -\int_\infty^{VaR_\alpha} l\Phi'(l)dl \\
&= \int_{VaR_\alpha}^\infty l d\Phi(l),
\end{aligned}
$$

  where we have made the substitution $l = \Phi^{-1}(1-\gamma)$, i.e. $\gamma = 1 - \Phi(l)$.
- CoVaR is a coherent risk measure. It is, however, harder to measure than VaR.
- For the standard normal distribution, the 99% CoVaR is 2.66521422.

# Definition of counterparty credit risk

- Counterparty credit risk is the risk that the counterparty on a financial transaction will fail to fulfill their contractual obligation.
- Counterparty risk arises from two categories of asset classes:
  (i) OTC derivatives such as interest rate swaps (IRS) and swaptions, CDS and default swaptions, FX forwards.
  (ii) Securities financing transactions such as repos and reverse repos, and securities borrowing and lending.

# Counterparty risk versus lending risk

- There are significant differences between lending risk and counterparty credit risk.
- Lending risk:
  (i) The notional amount on the loan is known.
  (ii) Only one party (the lender) takes on lending risk.
- Counterparty risk:
  (i) The value of the contract in the future is uncertain.
  (ii) Since the value of the contract can be positive or negative (e.g. an interest rate swap), counterparty risk tends to be bilateral.

# Settlement and pre-settlement risk

- *Pre-settlement risk* is the risk that a counterparty will default prior to the final settlement of the transaction (at expiration).
- *Settlement risk* arises if there are timing differences between when the counterparties perform on their obligations.
- Exchange traded derivatives.
- OTC derivatives.
- ISDA Master Agreement.
- Repos and reverse repos.

# Netting

- *Payment netting* offers counterparties the ability to net cash flows occurring on the same day. For example, through floating leg compounding, payments on IRS are netted.
- *Closeout netting* allows the termination of all transactions between the solvent and insolvent counterparties by offsetting the values of the transactions. This may occur in a variety of situations:
  (i) Trades may be parts of a hedged transaction.
  (ii) A trade may an unwind of another trade.
  (iii) Trades are independent.
- Closeout netting allows the solvent institution to immediately recognize the gains vs losses in case of a counterparty default.

# Collateral

- Collateral is an asset supporting a risk in a legally enforceable way.
- The collateral receiver becomes the owner of the collateral only if the party posting the collateral defaults.
- The use of collateral in derivatives transactions is regulated by ISDA's credit support annex (CSA).

# Credit support annex (CSA)

- The ISDA Master Agreement is usually amended with a CSA.
- CSA specifies issues such as:
  (i) Method and timing of the underlying valuation.
  (ii) Amount of collateral to be posted.
  (iii) The mechanics and timing of collateral transfer.
  (iv) Eligible collateral.
  (v) Dispute resolution.
  (vi) Haircuts.
  (vii) Possible rehypothecation of collateral.
  (viii) Interest payments on collateral.

# Collateral terms

- Valuation agent.
- Types of collateral: cash, government securities, other (GSA securities, AAA MBS securities, corporate bonds, equities).
- Margin calls.
- Haircuts.
- As long as the counterparty posting collateral is not in default, it receives all the coupons, dividends and other cash flows. Interest is paid on cash collateral at the overnight indexed swap (OIS) rate.

# Collateral terms

- No CSA. Rarely applied.
- Two-way CSA. This is typical among similar counterparties.
- One-way CSA. This is a typical arrangement in bank / hedge fund relations.
- Collateral terms may be linked to the counterparties credit quality.

# Default remote entities

- A *default remote entity* (DRE) is a legal entity whose probability of default is supposed to be negligible.
- Prior to the 2007 - 2008 crisis, common types of DREs included entities within corporate groups insulated from their credit:
    - (i) Special purpose entities (SPE) are legal wrappers giving preferential treatment to their counterparties in the event of default.
    - (ii) Derivative product companies (DPC).
- Monolines: These are insurance companies guaranteeing scheduled payments of interest and principal on a bond or other security in the event of a payment default by the issuer of the bond or security (AMBAC, MBIA, ...).
- Central counterparties (CCP): DTCC, CME, ICE, LCH, OCC, ... .
- *Systemically Important Financial Institutions* (SIFI).

# Central clearing

- In 2010 both the US and EU published legislative proposals, according to which all standard OTC derivatives should be cleared through central counterparties (CCP).
- The key function of a CCP is contract *novation*.
- Novation is the replacement of a bilateral contract with several contracts, as a result of which the CCP interposes itself between the counterparties and acts as insurer of the counterparty risk.
- This is achieved through rigorous counterparty risk management such as variation margin and risk mutualization.

# Functions of a CCP

- As a result of contract novation, a CCP bears no net market risk, but it takes on counterparty risk.
- A CCP mitigates the counterparty risk by collecting financial resources from its members that cover potential losses in the event of a default (initial margin, variation margin, liquidity fund, default fund).
- With the collected funds, a CCP converts counterparty risk into other from of risks.

# Advantages of central clearing

- *Multilateral netting*: Increases the flexibility to enter new transactions and terminate existing ones.
- *Increased position size transparency*: The CCP is able to impose punitive measures on members with outsized positions.
- *Loss mutualization*: Part of the losses are distributed among clearing members.
- *Legal and operational efficiency*: A CCP working directly with regulators is likely to be more efficient than individual market participants.
- *Liquidity*: Frequent (at least daily) marking to market of the member positions will lead to more transparent valuation of cleared instruments.
- *Default management*: A CCP managing the liquidation of the positions of a defaulted member will be less disruptive to the market than uncoordinated actions by individual market participants.

# Multilateral netting

- Policymakers and regulators make the point that CCPs facilitate multilateral netting, which can alleviate systemic risk.
- The effect may be diminished by the presence of multiple CCPs (e.g. CME, ICE, LCH, ...), even though this fragmentation may be beneficial from the systemic risk point of view.
- This effect has been studied in a simple toy models setting by Duffie and Zhu.

  Among their findings are:

  (i) The required number of members of a CCP to achieve netting reduction is a function of the correlation among the assets and the number of assets.

  (ii) Increased netting benefit can only be achieved by a small number of CCPs clearing a large number of transactions. It may thus be inefficient to have multiple CCPs.

# Initial margin

- Initial margin is calculated using a VaR model. Occasionally, older systems (such as CME's SPAN) are used.
- Most CCPs use historical simulations to calculate VaR applied to 5-day portfolio returns.
- The 5-day period is referred to as the market period of risk (MPOR).
- For VaR calculation, a long look-back period (typically, between 1 and 5 years) is used.
- In some cases, for the sake of performance efficiency, parametric VaR models (such as Gaussian or Student t distribution based) are used.
- VaR level is calculated as the appropriate $k$-th worst return.
- VaR model's performance is subject to regulatory control based on daily back tests.

# Initial margin

- Portfolio returns tend to form a non-stationary time series.
- In order to bring this time series closer to stationarity, volatility scaling is applied to the time series of returns.
- Specifically, we define the exponentially weighted moving average (EWMA) model by
$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)r_t^2,$$
where $\lambda$ is a scaling parameter (typically 0.97), and $r_t$ is the return on day $t$.
- Portfolio return on day $t$ is scaled by $\sigma_t$
- In order to account for fatter tails, *stressed VaR* is used. In additional to the scenarios from the look-back period, additional scenarios are used.

# Variation margin

- Variation margin is an adjustment to the initial margin calculated frequently (at least daily).
- A CCP can make an intraday margin call if large market moves threaten to deplete a clearing member's margin funds.
- Such practices are increasingly common, owing to the advancing technology.

# Default Fund

- Default Fund (DF) is a facility whose purpose is to mitigate potential losses in excess of the initial margin.
- As such, DF is often associated with tail risk events.
- The size of the DF is determined through the selection of stress scenarios which are applied to the clearing members' portfolios.
- The scenarios used for sizing of the DF consist of a number of historical and hypothetical scenarios.

# Additional charges

- A CCP may also impose additional charges due to the specific nature of the cleared instruments.
- For example, a liquidity charge may be added to portfolios containing illiquid positions or outsized positions in liquid assets.

# Default management

- In the case of a clearing member default, the CCP takes the following actions.
- Auctions the positions of the defaulted member. Depending on liquidity conditions, some assets will require warehousing.
- May transfer some positions to surviving clearing members.
- Allocate the losses associated with portfolio liquidation according to the following loss waterfall (this may vary depending on the CCP):
  - (i) Initial margin
  - (ii) Member Default Fund
  - (iii) CCP equity ("skin in the game")
  - (iv) Default Fund (of surviving members)
  - (v) Right of reassessment
  - (vi) Remaining CCP capital
  - (vii) Liquidity support or failure

# Have CCPs ever failed?

- CCP failures have been infrequent. Notable examples include:
  (i) In 1974 French *Caisse de Liquidation* failed as a result of a sharp drop in sugar prices and the inability of a large member to post additional margin.
  (ii) In 1983 *Commodity Clearinghouse* of Kuala Lumpur failed as a result of a crash in oil futures and the inability of several brokers to follow up on margin calls.
  (iii) In 1987 *Hong Kong Futures Exchange Clearing Corporation* had to be bailed out as a result of margin calls following the global stock market crash.
- Note that the recent failures of Lehman Brothers, MF Global, and Knight Capital did not cause losses for CCPs (let alone failures...).

# Counterparty exposure

- A defining feature of counterparty risk is the asymmetry of potential losses.
- In the event of a counterparty default, the institution determines the value of the transactions:
  - *Negative value*. The institution is in debt to the defaulted counterparty, and is still obliged to settle the amount, but from the valuation perspective the position is essentially unchanged.
  - *Positive value*. The defaulted counterparty is incapable to fulfill its commitments, and the institution will have a claim on the assets of the defaulted party. It can expect to recover a certain fraction of its claim.
- We can thus define the exposure of the institution to the counterparty as

$$\text{Exposure} = \max(\text{Value}, 0).$$

- This asymmetry is similar to being short an option.

# Bilateral exposure

- Counterparty risk is bilateral since both counterparties can default.
- This can be captured by defining

$$\text{NegativeExposure} = \min(\text{Value}, 0).$$

# Future exposure

- While the current exposure is known with certainty, the future exposure can be defined only probabilistically.
- Quantifying future exposure is difficult as it may involve long periods.
- This bears some similarities to the VaR methodology. However, unlike VaR, future exposure has to be understood in multi-period terms.
- Consequently, future exposure depends not only on volatility but also on the drift.

# Metrics of future exposure

- *Expected future value* (EFV) is the expected value (forward) of the netted positions at some point in the future. Note that EFV may significantly differ from the current value of the positions.
- *Potential future exposure* (PFE) is, roughly, the worst exposure at a certain time point in the future, given a confidence level $\alpha$ (say 99%). Its definition is analogous to the definition of $VaR_\alpha$.
- *Expected exposure* (EE) is defined as the expected value of exposure values.

# Metrics of future exposure

- In the case of the normal distribution $N(\mu, \sigma^2)$, the future value of the portfolio is

$$E = \mu + \sigma Z,$$

where $Z \sim N(0, 1)$.

- PFE is given by the same formula as VaR:

$$PFE_\alpha = \mu + \sigma N^{-1}(\alpha).$$

- Exposure is given by

$$\begin{aligned} E &= \max(V, 0) \\ &= \max(\mu + \sigma Z, 0), \end{aligned}$$

and thus

$$\begin{aligned} EE &= \int_{-\mu/\sigma}^{\infty} (\mu + \sigma x)\varphi(x)dx \\ &= \mu N(\mu/\sigma) + \sigma\varphi(\mu/\sigma), \end{aligned}$$

where $\varphi(x)$ is the density of the normal distribution.

# Metrics of future exposure

- Maximum PFE is the highest value of PFE over a given time horizon.
- *Expected positive exposure* (EPE) is the average exposure over a given time horizon. For example, in the normal model with $V(t) = \sigma\sqrt{t}Z$ where $\sigma$ is the annualized volatility, the EPE is given by:

$$
\begin{aligned}
EPE &= \frac{1}{T}\int_0^T \sigma\sqrt{t}\varphi(0)dt \\
&= \frac{2}{3\sqrt{2\pi}}\,\sigma T^{1/2} \\
&= 0.27\sigma T^{1/2}.
\end{aligned}
$$

- *Negative expected exposure* (NEE) is the average negative exposure over a given time horizon.
- *Effective expected positive exposure* (EEPE) takes into account rollover risk of short dated transactions.

# Factors driving credit exposure

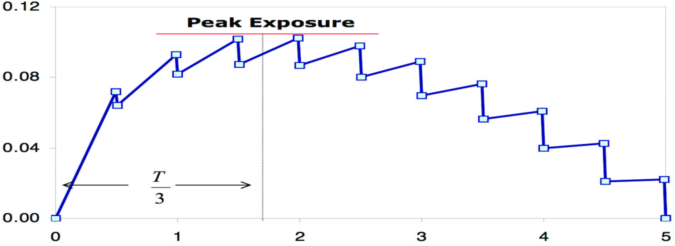
- Loans and bonds.
- Future uncertainty associated with FRAs and FX forwards. It is characterized by the square root law:
$$\text{Exposure} \propto t^{1/2}.$$
- In order to see it, suppose we wish to calculate the exposure on a forward contract and assume the normal model for its future value:
$$dV(t) = \mu dt + \sigma dW(t).$$
- Then $V(t) \sim N(\mu t, \sigma^2 t)$, and thus, by the previous calculations,
$$\begin{aligned} PFE_\alpha(t) &= \mu t + \sigma\sqrt{t}N^{-1}(\alpha), \\ EE_\alpha(t) &= \mu t N(\mu/\sigma\sqrt{t}) + \sigma\sqrt{t}\varphi(\mu/\sigma\sqrt{t}). \end{aligned}$$

# Factors driving credit exposure

- Future uncertainty associated with periodic cash flows such as swaps is approximately given by
$$\text{Exposure} \propto (T - t)t^{1/2},$$
where $T$ is the tenor of the swap. This is a consequence of the balance between future uncertainties over payments, and the roll-off of coupon payments over time.
- In order to see it, we model the future value of a swap as $V(t) \sim N(0, \sigma t(T - t)^2)$, where $T - t$ is the approximate duration of the swap at time $t$ (this is valid for a flat forward curve). The maximum exposure is at $t = T/3$.

# Factors driving credit exposure

- The graph below shows the exposure profile of an interest rate swap.



# Factors driving credit exposure

- The exposure of a payer swap is higher than the receiver swap due to the expectation to net pay initially, and net receive in later stages of the swap.
- This effect is even more pronounced in cross-currency swaps. The overall high interest rates paid are expected to be offset by the gain on the notional exchange at the maturity of the contract. This expected gain on exchange of notional leads to a significant exposure for the payer of the high interest rate.
- Options.
- CDSs.

# Impact of netting and correlation

- If netting agreements are absent, exposures are considered additive.
- With netting allowable, one can offset at the netting set levels, and the exposures are zero.
- A high positive correlation between two instruments.

# Impact of netting and correlation

- Netting factor is defined as

$$\text{NettingFactor} = \frac{\sqrt{n + n(n-1)\overline{\rho}}}{n},$$

  where $n$ is the number of exposures, and $\overline{\rho}$ is the average correlation.
- The netting factor is the ratio of net exposure to gross exposure, and is 1 if there is no netting benefit ($\overline{\rho} = 1$) and 0 if there is maximum netting benefit $\overline{\rho} = -\frac{1}{n-1}$.
- The netting benefit improves (lower netting value) for large number of exposures and low correlations.
- For $\overline{\rho} = 0$, the netting factor is $\frac{1}{\sqrt{n}}$, a significant reduction of the exposure.

# Impact of netting and correlation

- In order to understand the meaning of the netting factor, we assume that each asset $X_i$, $i = 1, \ldots, n$ within a netting set is modeled as $X_i \sim N(\mu_i, \sigma_i^2)$. Then the total mean and standard deviation are given by

$$\mu = \sum_{i=1}^{n} \mu_i,$$
$$\sigma^2 = \sum_{i,j=1}^{n} \rho_{ij}\sigma_i\sigma_j,$$

  where $\rho_{ij}$ is the correlation between assets $i$ and $j$.
- Assuming average $\mu_i$ to be zero, and average $\sigma_i$ to be $\overline{\sigma}$, this gives:

$$\mu = 0,$$
$$\sigma^2 = (n + n(n-1)\overline{\rho})\overline{\sigma}^2,$$

  where $\overline{\rho}$ is the average correlation.

# Impact of netting and correlation

- Therefore, the EE of the netting set is

$$EE_{NS} = \frac{\overline{\sigma}}{\sqrt{2\pi}} \sqrt{n + n(n-1)\overline{\rho}}\,.$$

- The gross (no netting) EE is

$$EE_{NN} = \frac{\overline{\sigma}}{\sqrt{2\pi}}\, n.$$

- The netting benefit is thus

$$\frac{EE_{NS}}{EE_{NN}} = \frac{\sqrt{n + n(n-1)\overline{\rho}}}{n}\,.$$

# References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