Lecture 7: Algorithmic Indexes

Modeling and Marketing Making in Foreign Exchange

Regular Indexes

- An index is meant to be a representation of a particular market
  - eg S&P 500 Index: representative average of the US large-cap equity market
  - eg GSCI: representative average of the commodity markets

- Indexes are used as benchmarks for real trading strategies

- Most indexes are not themselves tradable
  - eg S&P 500: there is no set of trades you can put on such that the portfolio value tracks the index level exactly

Tradable Indexes

- Tradable indexes are meant to track the value of a real trading strategy
  - Bid/ask spreads
  - Dividends/interest
  - Actual securities

- A market making desk can offer derivative products on tradable indexes
  - Swaps
    - Can hedge the risk exactly by trading the underlying strategy
  - Options
    - Trickier if the underlying portfolio isn't very simple

Tradable Indexes

- Trading strategies are defined by a set of rules
  - Rules rarely, if ever, change for a given index
  - No discretion by index publisher in choice of investments at all

- The index defines a "synthetic portfolio"
  - The result of all the synthetic trades done via the index rules
  - The synthetic portfolio ages over time and rules must account for its evolution
    - Paying/receiving interest on currencies
    - Paying/receiving dividend payments on equities
    - Exercising/expiring options

Tradable Index

- eg: EUR tracker index

- Non-tradable index
  - Equal to EURUSD spot: the price of a EUR in USD
  - Tries to represent the value of a strategy where you hold 1 EUR
  - Does not represent the interest you get paid on your EUR

- Tradable index
  - Equals the price of a portfolio that holds 1 EUR and receives interest on it
  - Accounts for bid/ask on interest payments

EUR Tracker Index

- Rules:
  - Day 1: buy $100 of EUR at spot at the ECB fix
    - Portfolio is a EUR zero coupon bond
    - Notional 100/(fix on day 1), value date = spot date of initial date
  - Day i>1: roll the EUR (now T+1, not T+2) back to spot using tom/next forward points
    - Notional increases every day at the appropriate EUR rate
    - Value date of EUR keeps stepping ahead
  - Day i value: EUR notional * (fix on day i)

- Use the FX forward curve to roll because that is how, in practice, one access the short-term interest rate market in EUR

EUR Tracker Index

- Let's consider the roll process in practice

- Start with N EUR value date T+1
  - Was T+2 (spot) yesterday, but now it's only T+1

- Short a forward contract value date T+1 for N EUR
  - Sell N EUR T+1, buy N F_1 USD T+1
  - F_1 is the forward to T+1

- Long a forward contract value date T+2 (spot) for N EUR
 - Buy N' EUR T+2, sell N' S USD T+2
  - S is spot: the forward to T+2

EUR Tracker Index

- Now we've got a portfolio which is a little complex
  - +N F_1 USD T+1
  - +N' EUR T+2
  - -N' S USD T+2

- We need to roll that USD T+1 to USD T+2
  - Also need the USD rate in here! Use Fed funds effective rate as the fix (where a bank can roll overnight)
    - Not quite right since that's a T+0 to T+1 rate, but pretty close
  - Sell N F_1 USD T+1, buy N F_1 $(1+r \, \Delta t)$ USD T+2
  - Δt equals [number of calendar days from T+1 to T+2] / 360
    - Need to know holiday calendars now <SIGH>
    - These details matter if you don't want to bleed away your fee

EUR Tracker Index

- Now we have a portfolio all on T+2
  - N' EUR T+2
  - $[N F_1 (1+r \Delta t) - N' S]$ USD T+2

- We'll choose N' such that the USD notional is zero
  - $N' = N F_1 (1+r \Delta t)/S$
  - Remember that F_1 is the forward to T+1
  - $F_1 = S (1+q \Delta t)/(1+r \Delta t)$, so $N' = N (1+q \Delta t)$
    - q = EUR interest rate; so accruing up at EUR interest rate implied from forwards and the Fed funds effective rate

- Lot of work just to figure out how to roll!
  - Typical of index construction – loads of finicky little details

EUR Tracker Index
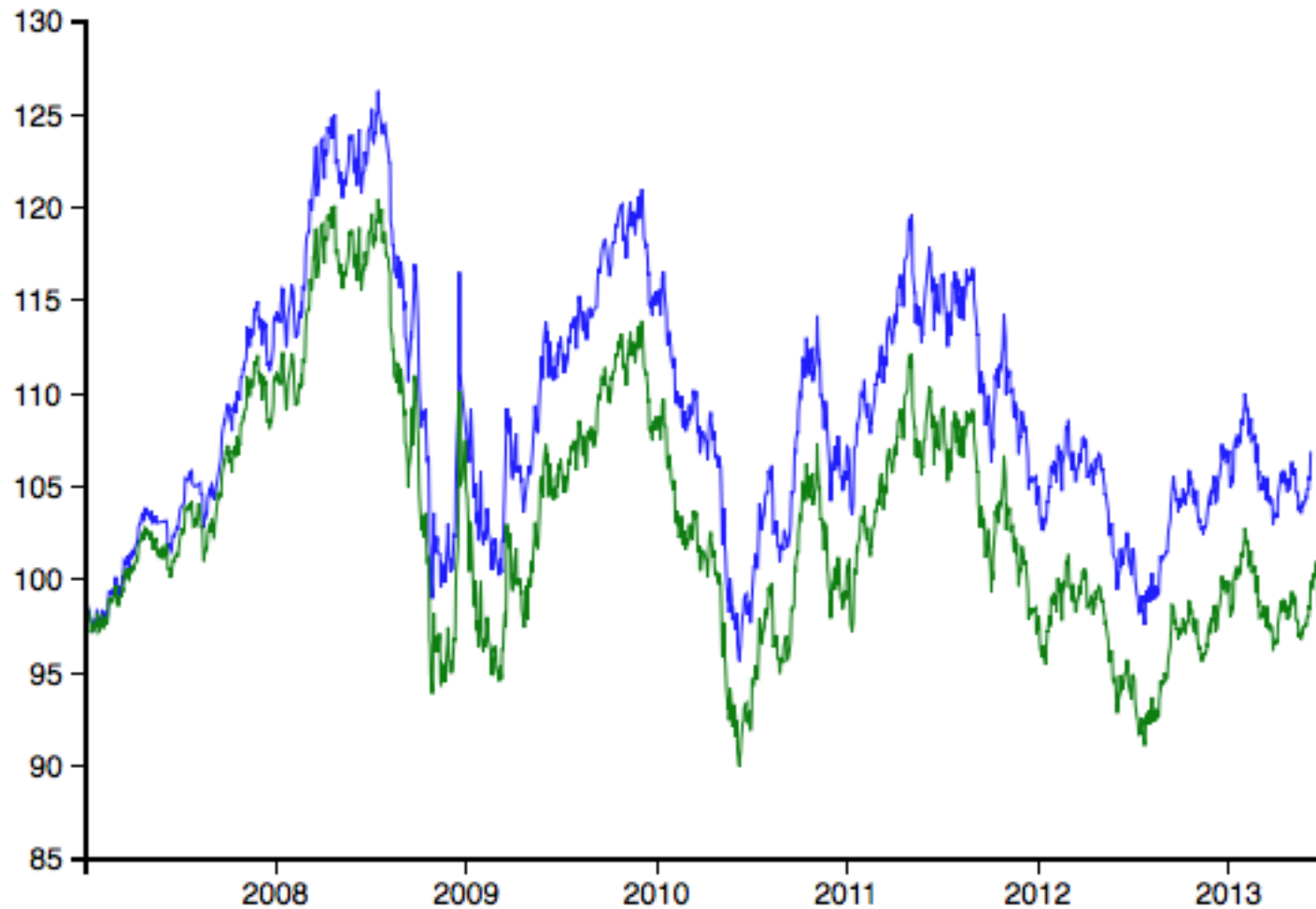
EUR Tracker Index

- This EUR tracker index wouldn't be marketable, though!
  - We used Bloomberg LN close forward points
  - Where do those come from?
    - Bloomberg's not saying! Some opaque aggregation.
  - The fixes in the index need to be transparent
    - Published by a trusted 3rd party (government, exchange)
    - Or based on tradable levels (eg WMR fixes)
  - We could replace the tom/next forward point fixes with WMR forward point fixes
    - Index publisher must pay license fees to be able to use & view those

- Also no bid/ask
  - Need to include that in tom/next forward points, not USD rates

EUR Tracker Index

- Other constructions are possible

- Trade FX forwards with value dates further out on the curve
  - Initially trade a 3m forward
  - Hold it until it's spot, then roll it to 3m again
  - Results in lower bid/ask but doesn't track spot as closely

- Trade FX futures contracts
  - Hold the prompt contract
  - Roll into the next contract when the futures settles

Index Trading

- Swaps
  - Payoff is [(Index at End)/(Index at Start) – 1] * Notional

- Notes
  - Bond that pays Notional * (Index at End)/(Index at Start)
  - Normally zero coupon

- Notes with principal protection
  - Bond that pays Notional * Max(1,(Index at End)/(Index at Start))
  - Equivalent to a zero coupon bond paying Notional plus a call option on the index level struck ATM

Hedging a Tradable Index

- A market maker always sells the tradable index; client always buys
  - Needs to be direction-specific since the bid/ask is built into the index value
  - Can construct two indexes, one for a long position and one for a short position, but client always buys the index

- Client receives the synthetic trades, which means dealer delivers them

- Dealer then executes real trades to offset the synthetic trade risk

Hedging a Tradable Index

- Really important to think about hedging process when constructing an index

- eg EUR tracker index example
  - What bid/ask will we really pay on tom/next rolls when hedging with the forwards desk?
  - What rate do we receive from the internal treasury when rolling USD cash balances? Is it really Fed funds effective rate?

Tradable Indexes & OTC

- Tradable indexes are hard to sell if they include fixes that are not transparent and based off dealable prices
  - Clients are concerned that dealers will manipulate fixes against them
    - LIBOR scandal
    - WMR FX scandal even

- Big deal for mostly-OTC markets like FX

- Often index products will use the futures markets to avoid this
  - Exchange settles are canonical example of a good fix
  - … but futures markets often only cover a corner of the real market

Fees in Tradable Indexes

- Some fees are explicitly defined in the index rules
    - eg: index decays at 25bp/year all else being equal
    - That decay for the client is a pickup for the dealer

- Some fees are wrapped up in internal bid/ask logic in the index
    - Generally rules-defined bid/ask is bigger than market bid/ask
    - Way of getting paid, but also a way of protecting against widening bid/ask in crisis events etc

- Fees are almost always very transparent in tradable indexes
    - Clients demand it

Beta Indexes

- A "beta" index is one where the index is meant to track some particular market factor
  - "Beta" from the CAPM beta parameter that regresses an asset price against an index representing a broad market

- Not meant to have any particular edge
  - No smart analytics inside

- Just a way to access a particular market
  - Lets someone trade a simply financially-settled swap and get access to a market they might not know the operational details of
  - Pay a fee in return for access – usually quite small for simple beta indexes

Beta Indexes

- Beta indexes are usually not big money-makers for dealers

- Fees are low

- Lots of competition
  - Other dealers offering similar indexes
  - Other products (eg ETFs) that give similar exposure

- Big investors can trade the markets directly
  - Don't need an access vehicle

- Exceptions for harder-to-access markets like EM

Carry Indexes

- A "carry" trade in general is one where you get paid a risk premium in return for taking tail risk
  - Like an insurance premium
  - Not risk-neutral!

- In FX it means something more specific
  - Borrow money in a low-interest rate currency
  - Invest it in a high-interest rate currency
  - Make the interest rate differential…
  - … so long as spot doesn't move against you!

Carry Indexes

- Idea is that a country in trouble will need to raise interest rates significantly to attract investment, and will try to defend its currency against devaluation
  - While that's true, the carry trade works well
  - Not much spot PNL, so just make the interest rate differential

- But eventually the country's central bank runs out of bullets
  - Currency devalues
  - Carry trade gets creamed

- Very common trade in the FX world
  - Especially with EM currencies vs G10 currencies

Short Vol Indexes

- Another risk premium: implied volatility vs realized volatility
  - Implied volatility tends to be higher than realized volatility
  - Options price in some "insurance premium" above the theoretical value from Black-Scholes-style risk neutral pricing

- Most of the time shorting options makes money, picking up the premium

- In a big move, the short option position gets creamed
  - Few people are able to weather the drawdowns even if on average the returns are positive

Short Vol Indexes

- Simple strategy: sell ATM EURUSD 3m options every day
  - Delta hedge the aging portfolio

- Need to be able to calculate the price of the options to sell on each new day
  - What fixing sources do you use for spot, forward points, and vol?
  - Note that the new options are benchmark options (ATM, 3m)

- Need to be able to value the aging portfolio
  - How do you value an arbitrary portfolio?
  - Aging options do not have benchmark expiration dates or strikes

- Need to define some volatility interpolation scheme

Short Vol Indexes

Short Vol Indexes

- Most short vol indexes add some intelligence about which asset to sell options in
  - Normally some measure of the spread of current implied to recent realized vol

- Many tradable indexes include signals like this
  - Want some intelligence to improve performance
  - Not too complex or no investor will understand/believe it

- Have to be very careful about overfitting!
  - Investors always want to see real, live performance
  - Too easy to get good backtested performance due to overfitting

Short Vol Indexes

- Really need to be careful about bid/ask when trading vol index products

- Think about hedging
  - Every time a dealer sells/buys the index to/from a client, they need to rebalance the hedges
  - Hedges correspond to the exact synthetic portfolio underlying the index
  - With vol index products, that synthetic portfolio contains a lot of discrete aging options

- This affects the ability of a client to eg unwind an index swap before the settlement date

Vol Control

- One way of trading index products is with a note with principal protection
  - Note pays Max(1,(Index at End)/(Index at Start))
  - Equivalent to a zero coupon bond paying 1 plus a call option Max((Index at End/Index at Start)-1,0)

- If a dealer sells such a note, they are selling options on the index
  - What volatility should one use?
  - Simple tracker indexes, fairly easy to hedge
  - Complex strategy indexes (eg short vol index), no obvious option market to use

Vol Control

- To make this easier, dealers invented "vol control"

- Idea: construct a new index that allocates between the real index and cash
  - When index vol is higher, allocate more to cash

- Define a volatility target and set the allocation based on that
  - Assume that future index volatility equals eg recent realized vol of index
  - Choose cash vs index allocation so that mixed index volatility will match the target

Vol Control

- Imagine a portfolio with some fraction α allocated to the underlying index and remaining value allocated to cash
  - Cash is valued at 1 always, notional of cash = C

$$M_t = \alpha_t S_t + C_t, \quad \alpha_t = Min(1, \sigma_{\text{Target}} / \sigma_t)$$

$$dS_t = \sigma_t S_t dz_t$$

$$dC_t = -d\alpha_t S_t, \quad C_0 = (1 - \alpha_0) S_0$$

- Here the volatility σ is a time-dependent estimate of volatility, usually realized volatility over some rolling window

- σ_{Target} is the target level of volatility
 - Should be lower than the index volatility will ever go

Vol Control

- That's just one vol control scheme – lots of different (but similar) ways to define how the allocation works between the index and cash
  - Instead of cash might be a treasury bond portfolio or similar

- Basic idea is to create the new (meta-)index M that has a fixed volatility equal to the target volatility

- Then can write options on M with a lot more confidence

Vol Control

EUR tracker index and vol-controlled index
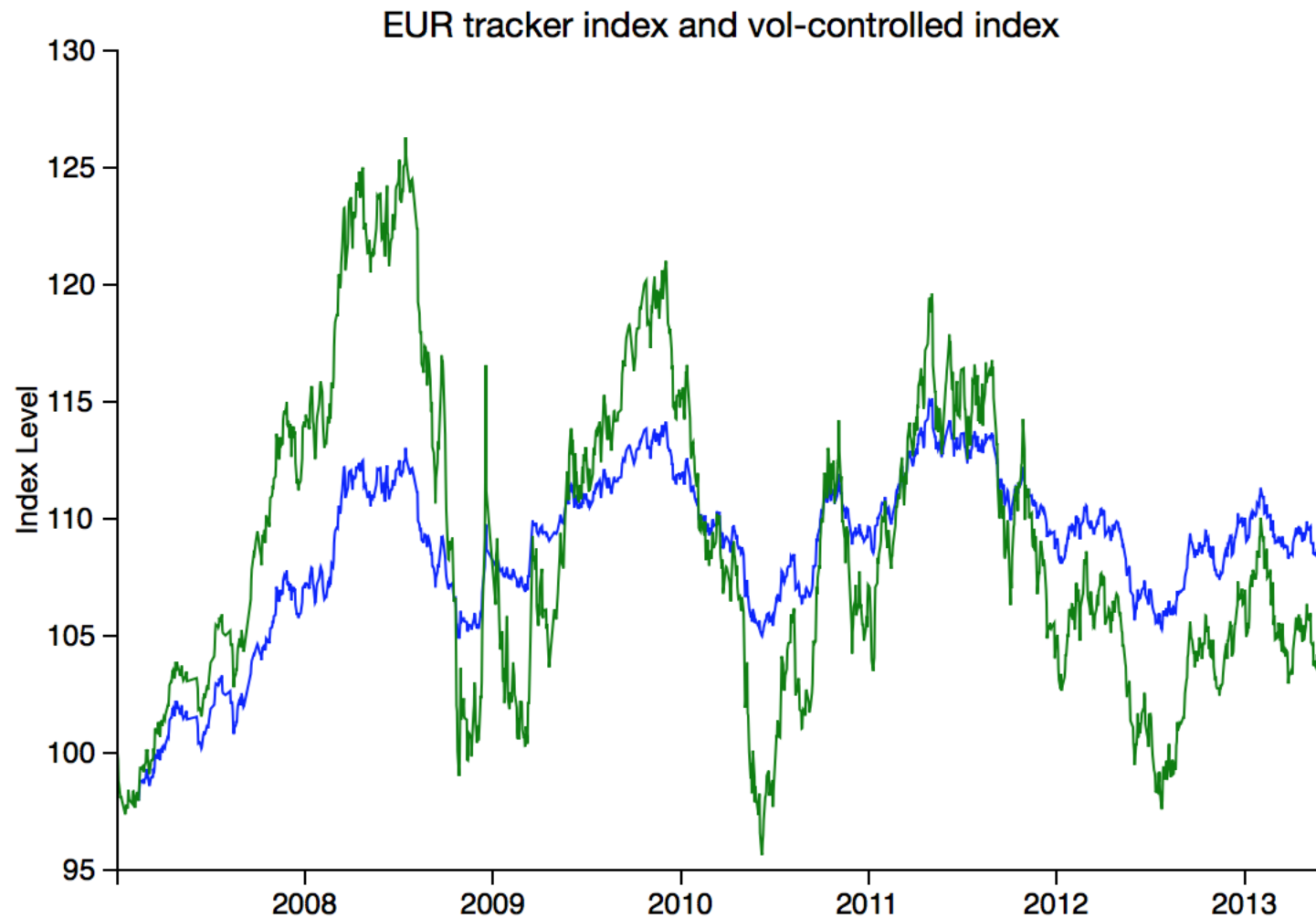
Vol Control
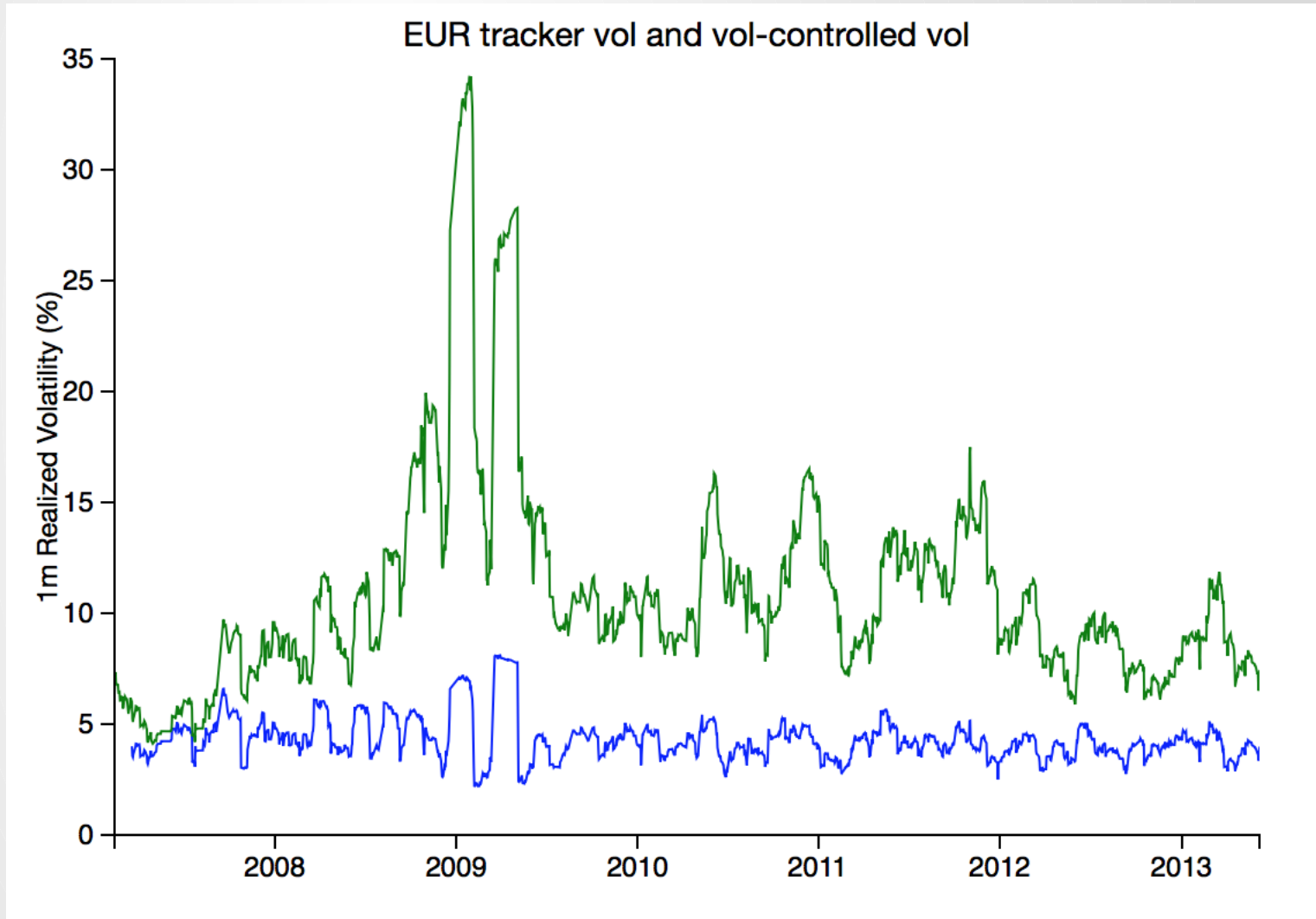
Course Summary

Stuff to focus on

- Spot
  - OTC voice business model & roles
  - OTC electronic business model & roles
  - Basics of electronic price-making algos & hedging algos

- Forwards
  - Spot vs forward arb and how it works in practice
  - Why FX forwards businesses are really short-term rates businesses
  - Ways of reducing dimensionality of risk

Course Summary

- Options
  - Conventions (delta, risk reversal, butterfly)
  - Risk reversal beta
  - Vol interpolation in the strike direction
  - Vol interpolation in the time direction & trading time/theta
  - Why vega gamma and vega dspot lead to skew & smile
  - Definition of delta/volatility market models
  - Regarding cross pair volatility in terms of correlation
  - Volatility relative value

Course Summary

- Exotics
  - Pricing European payoffs through replication
  - Why RR beta drives barrier option pricing
  - LV/SV mixture model definitions
  - The local vol 3-state model
  - Pricing vanillas using characteristic functions
  - Gaussian copulas
  - Why realized vol of implied volatility drives vol swap pricing

- Index
  - What makes an index tradable
  - Types of tradable index strategy
  - How vol control works

Final Exam

- Final exam is TBD – will post on forum when it's announced

- Written exam only
  - No calculators, computers, notes, etc

- Questions will of course not be numerical
  - Qualitative questions about market structure, market dynamics involved in pricing, etc
  - Math questions like the ones in the assignments