

HOMEWORK 3

Problem 3.1. Consider optimally trading a single stock over $T = 30$ days. Each period is one day, and you can trade once per day. The stock's daily return volatility is σ . Suppose your forecast is 50 basis points for the first period, and decays exponentially with half-life 5 days. This means that

$$(3.1) \quad \alpha_t := \mathbb{E}[r_{t,t+1}] = 50 \times 10^{-4} \times 2^{-t/5}.$$

Let $c(\delta)$ be the cost, in dollars, of trading δ dollars of this stock. For selling, $\delta < 0$. Following Almgren, we assume that

$$c(\delta) = PX \left(\frac{\gamma\sigma}{2} \frac{X}{V} \left(\frac{\Theta}{V} \right)^{1/4} + \text{sign}(X) \eta\sigma \left| \frac{X}{V} \right|^\beta \right), \quad X = \delta/P$$

where P is the current price in dollars, X is the signed trade size in shares, V is the daily volume in shares, Θ is the total number of shares outstanding, and finally $\gamma = 0.314$ and $\eta = 0.142$ and $\beta = 0.6$ are constants fit to market data. For concreteness, suppose the asset we are trading has

$$P = \$40, \quad V = 2 \times 10^6, \quad \Theta = 2 \times 10^8, \quad \sigma = 0.02.$$

For a trading path $\mathbf{x} = (x_0, x_1, \dots, x_T)$ where x_t denotes dollar holdings of the stock at time t , define the profit (also in dollars) as

$$\pi(\mathbf{x}) = \sum_{t=1}^t [x_t r_{t,t+1} - c(x_t - x_{t-1})]$$

This is a random variable due to the presence of $r_{t,t+1}$ which you can assume is Gaussian with mean $\alpha_t := \mathbb{E}[r_{t,t+1}]$ and variance $\mathbb{V}[r_{t,t+1}] = \sigma^2$. In this problem, always assume $x_0 = 0$ is fixed.

(a) Find the sequence of positions x_1, x_2, \dots, x_T that maximizes

$$(3.2) \quad u(x_1, \dots, x_T) = \sum_{t=1}^T \left[x_t \alpha_t - \frac{\kappa}{2} \sigma^2 x_t^2 - c(x_t - x_{t-1}) \right]$$

with risk-aversion $\kappa = 10^{-7}$. Set tolerance so that your algorithm does not terminate unless each $x_t \in \mathbb{R}$ is within a distance of one dollar to the true optimal path. Plot the optimal path $\mathbf{x}^* := (x_0 = 0, x_1^*, \dots, x_T^*)$ and also report its values in a table. Also report the computation time.

Submit your code and a clear explanation of the algorithm you used, why you chose it over other possible algorithms, and how you

know that it converges. For example, if you used a method that requires convexity, explain why the function you are optimizing is convex.

- (b) Use the program you wrote in part (a) to plot expected profit of the optimal path, $\mathbb{E}[\pi(\mathbf{x}^*)]$ and *ex ante* Sharpe ratio of the optimal path, defined as

$$\text{Sharpe}(\mathbf{x}^*) = \sqrt{252} \frac{\mathbb{E}[\pi(\mathbf{x}^*)]}{\sqrt{\mathbb{V}[\pi(\mathbf{x}^*)]}}$$

as a function of κ , as a function of the half-life (which was taken to be 5 in equation (3.1) above), as a function of the initial strength (taken to be 50 in equation (3.1)), and as a function of σ . So you need to do eight plots in all: profit and Sharpe ratio, each as a function of one of four parameters (holding the others fixed). Choose appropriate intervals around the parameter values in part (a). Note that κ cannot be negative in reasonable models.