MTH 9821 Numerical Methods for Finance I

Fall 2014

Midterm Exam
October 2, 6-8pm

Work is to be done individually. This rule is going to be strictly enforced under severe penalties. No discussions or email exchanges between students are permitted.
Write your results in the file firstname_lastname_midterm_9821_fall2013.xls provided with the test. Write the last four digits of your social security number at the top of the file, and change the name of the file to include your name. DO NOT MODIFY THE FORMAT/STYLE OF THE FILE!

Email the file to
Dan.Stefanica@baruch.cuny.edu

You will receive full credit for a correct answer, meaning 9 decimal digits correct (unless tolerance requires otherwise), partial credit if some of the last decimals do not match, and no credit if you are off at the fourth decimal or worse. Please note that the 9 decimals rule cannot apply to residual errors. Report decimal errors as follows: $2.51 \cdot 10^{-12}$, i.e., the first nonzero decimal which may be close to epsilon machine and should be reported as accurately as possible.

Good luck!

1. Let A_1 be the following 9×9 matrix:

$$\begin{aligned} A_1(i,i) &= 3, \quad \forall\, i = 0:8; \\ A_1(i,i-1) &= -2, \quad \forall\, i = 1:8; \\ A_1(i,i-2) &= 4, \quad \forall\, i = 2:8; \\ A_1(i,i+2) &= -1, \quad \forall\, i = 0:6. \end{aligned}$$

(10 points) Find the matrices P_1, L_1, and U_1 for the LU decomposition with row pivoting of A_1.

(5 points) Let b_1 be the following column vector:

$$b_1(i) = \sqrt{i^2 - 2i + 5}, \qquad i = 0:8.$$

Solve

$$A_1 v_1 = b_1.$$

Compute the residual error

$$||b_1 - A_1 v_1||,$$

where the norm is the 2-norm.

(10 points) Compute L_1^{-1}, U_1^{-1} and A_1^{-1}.

Use matrix–vector multiplication to compute the vector

$$v_2 \;=\; A_1^{-1} b_1.$$

Compute the residual error

$$||b_1 - A_1 v_2||.$$

2. Let

$$A_2 = A_1^t \ A_1.$$

(10 points) Find the upper triangular matrix U_2 from the Cholesky decomposition on A_2.

(5 points) Let b_2 be the following column vector:

$$b_2(i) = \frac{2i^2 - 5}{2i + 3}, \qquad i = 0 : 8.$$

Solve

$$A_2 x_2 = b_2.$$

Compute the residual error

$$||b_2 - A_2 x_2||.$$

(5 points) Let $A_3 = A_1^t + A_1$. Solve

$$A_3 x_3 = b_2.$$

Compute the residual error

$$||b_2 - A_3 x_3||.$$

3. Let A_4 be an 8×8 matrix given by

$$\begin{aligned}
A_4(i,i) &= 9, \quad \forall\, i = 0:7 \\
A_4(i,i+2) &= -2, \quad \forall\, i = 0:5 \\
A_4(i,i-2) &= 3, \quad \forall\, i = 2:7 \\
A_4(i,i+3) &= -1, \quad \forall\, i = 0:4 \\
A_4(i,i-3) &= 1, \quad \forall\, i = 3:7
\end{aligned}$$

Let b_2 be a column vector given by

$$b_4(i) = \frac{3i-4}{i^2+1}, \qquad i = 0:7.$$

Solve

$$A_4 x_4 = b_4,$$

using Jacobi **(10 points)**, Gauss–Siedel **(10 points)**, and SOR **(10 points)** with two different values for ω:

$$\omega \in \{0.90, 1.15\}.$$

For each method, the initial guess is a vector x_0 with all entries equal to 0. Use a tolerance of 10^{-6} and a residual–based stopping criterion. Report the first three approximations and the final result, as well as the number of iterations to convergence for each method.

4. The file `financials2012-short.xlsx` contains the weekly prices adjusted for dividends from January 11, 2012, through September 4, 2012, (i.e., 35 prices) for the following financial stocks: JPM, GS, MS, BAC, RBS, CS, UBS, RY (RBC), BCS (Barclays).
Compute the 34×9 matrix X of weekly log returns of these stocks.

(5 points) (i) Compute the 9×9 covariance matrix Σ_X of the weekly log returns of the stocks.

(5 points) (ii) Compute the Cholesky factor U_X of the matrix Σ_X.

(5 points) (iii) Find the linear regression of the JPM weekly log returns with respect to the weekly log returns of the other eight financial stocks. Report the residual error of the linear regression.

5. The following discount factors were obtained from market data:

Date	Discount Factor
1 months	0.9980
4 months	0.9935
10 months	0.9820
14 months	0.9775
20 months	0.9620

The overnight rate is 0.75%.

(2 points) (i) What are the corresponding 1 month, 4 months, 10 months, 14 months, and 20 months zero rates?

(5 points) (ii) What is the 4×4 tridiagonal system that must be solved in the efficient implementation of the natural cubic spline interpolation for finding the zero rate curve for all times less than 20 months?

(3 points) Use the efficient implementation of the natural cubic spline interpolation to find a zero rate curve for all times less than 20 months matching the discount factors above, and find the value of a 19 months semi-annual coupon bond with 2.5% coupon rate.