HOMEWORK 2

Problem 2.1. The data set accompanying this homework gives daily returns for three stocks: TSLA, AAPL, and IBM.

(a) Calculate the historical (regressed, no intercept) beta, for each of these assets as of Dec 31, 2014. In each case, calculate the appropriate t-statistic on the coefficient to test the null hypothesis $\beta = 0$ and state whether you reject the null hypothesis.

(b) Compute the holdings vector $h \in \mathbb{R}^3$ for the unique portfolio which is dollar-neutral (ie. self-financing) and which has unit exposure to AAPL and zero exposure to beta as of Dec 31, 2014.

(c) Compute the daily returns of the portfolio from (b) over the period Jan 1, 2015 to Dec 31, 2015. Assume that each day, the portfolio is rebalanced back to the initial holdings vector $h \in \mathbb{R}^3$. Plot the cumulative sum of the log returns.

(d) Compute the realized correlation of the returns in part (c) to the market's return. Construct a statistical test of the null hypothesis that the correlation is zero. Is the realized correlation significantly different from zero at the 95% level?

Problem 2.2. Use the Sherman-Morrison-Woodbury matrix inversion lemma to derive a simple expression for the inverse of the covariance matrix in an APT model. In other words, derive an expression for Σ^{-1} where

$$\Sigma = \mathbb{V}[R] = X'FX + D$$

where D is diagonal and X is $n \times p$ and as usual we assume $p \ll n$. In your answer, any matrices being inverted should be either diagonal or $p \times p$.

Problem 2.3. Show that, for any $n \times p$ real matrix X (not necessarily full rank) and n-vector Y, the following are equal:

(1) $\lim_{\delta \to 0+} (X'X + \delta I)^{-1} X'Y$
(2) The smallest-norm element of $\operatorname{argmin}_b \|Y - Xb\|$.
(3) $VS^+U'Y$ where $X = USV'$ is the SVD of X.