# Lecture 2: Forward Markets

Modeling and Marketing Making in Foreign Exchange

# The FX Forward Markets

- A forward trade is an agreement to exchange some amount of one currency for another amount of another currency at some point in the future
  - Spot is one point on the forward curve

- Bilateral, over-the-counter transactions
  - Not exchange traded, or even traded on SEFs (because of a Treasury exemption from DFA)

- Traded as “outright forwards” or “swaps”
  - Outright forward: just a forward contract
  - Swap: long a forward, short spot (or the reverse), in same notional

# The FX Futures Markets

- Currency futures trade on the CME
  - Other exchanges too but most futures liquidity on the CME
- Quarterly settlements going out six contracts
  - Most liquidity is in “prompt” (closest-to-settle) contract
- Relatively small compared to OTC market
  - $5.3T/day in OTC market (spot+forward)
  - $0.16T/day in exchange markets
- A lot of trading in the prompt contract is electronic hedgers using it as a close-to-spot product for extra spot liquidity

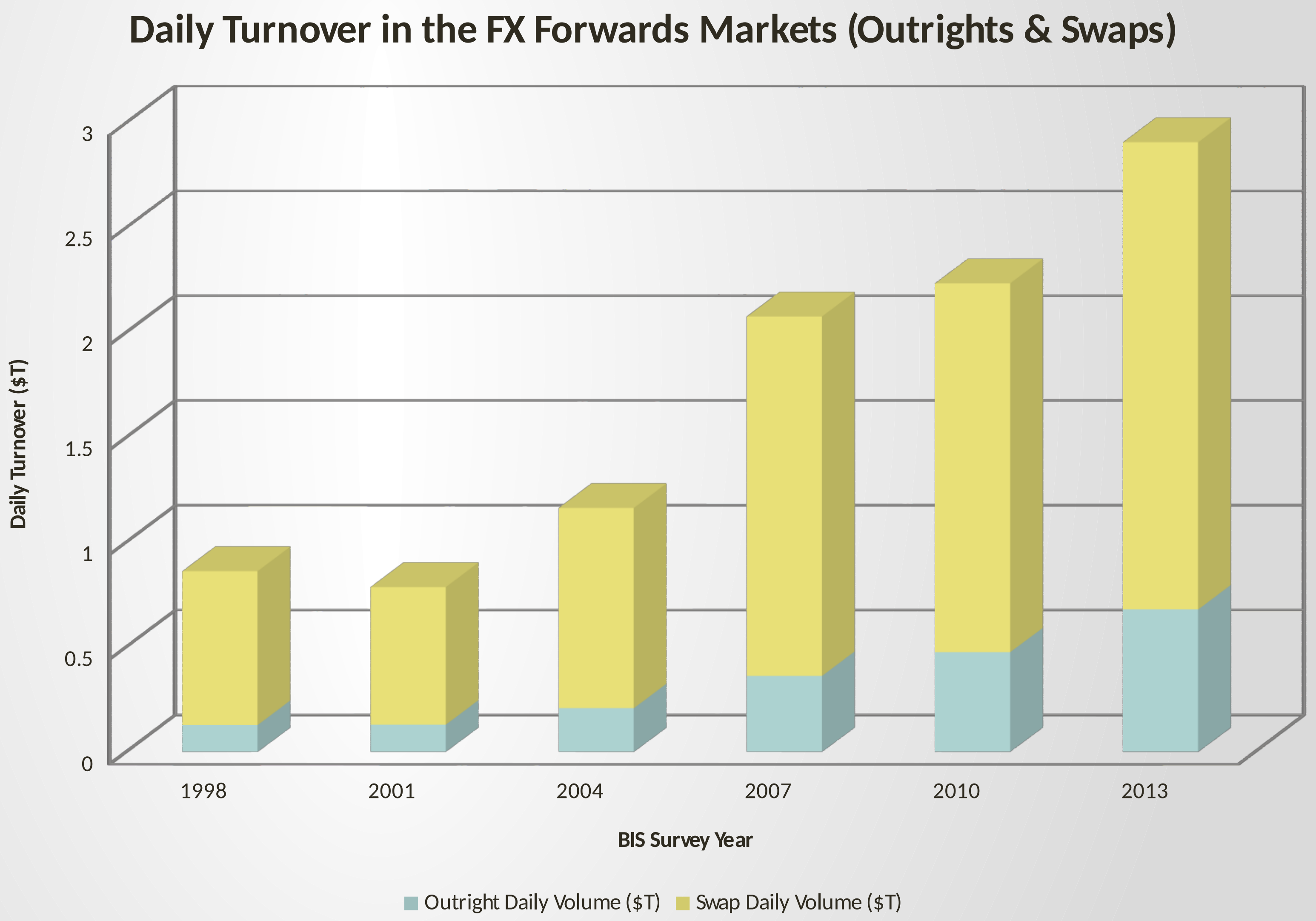
# Forward Market Statistics

- The OTC forward market is a bit bigger than the spot market
  - Spot: $2.05T/day
  - Outright forwards: $0.68T/day
  - FX swaps: $2.23T/day

- Most trading is inside 1y

- Smaller fraction of trades executed electronically than spot
  - For swaps, 53% by volume, vs 65% for spot
  - Electronic market significantly less developed than spot

# Forward Market Statistics

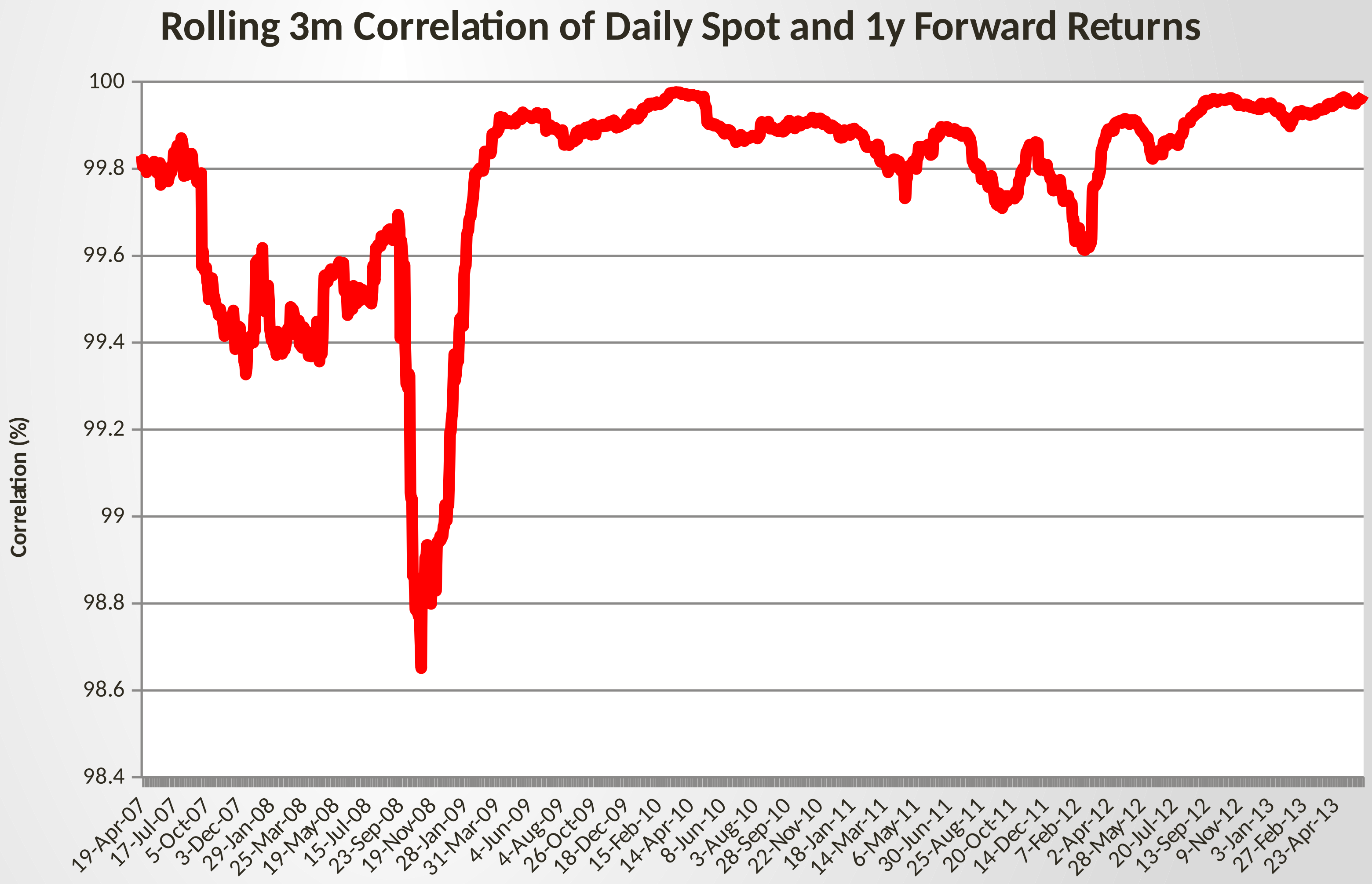


# Forward Market Conventions

- “Outright”, or all-in, forward: regular forward contract
- Forward point: difference between all-in forward and spot prices
  - Mostly a function of interest rates
  - Much more stable than spot

- Tenor of the forward contract adds another dimension
  - Not all trades are fungible with each other, like with spot
  - “Benchmark” tenors trade in the broker market for inter-dealer trades
  - Clients can trade any settlement date they like
    - Spreads are not tighter for benchmark settlement dates
  - Liquidity runs out to 2-3y for most currency pairs

# Spot/Forward Correlation



# The Spot/Forward Arbitrage

- FX is a financial market where the two requirements for spot/forward arbitrage hold:
  - You can store currencies (and receive an interest rate for them)
  - You can borrow & short currencies (and pay an interest rate)
- Interest rate markets for G7 currencies are all well-developed so you can really execute the arbitrage
- In fact, you can view an FX forward trade as just two zero coupon bonds, one in each currency, one long, one short

# The Spot/Forward Arbitrage

- eg: forward settling in 1y, receiving 1M EUR, paying 1.13M USD
  - Equivalent to being long 1 unit of a zero coupon bond in EUR with a 1y settlement date and being short 1.13 units of a zero coupon bond in USD

$$v(t) = S(t)e^{-Q(t,T)(T-t)} - Ke^{-R(t,T)(T-t)}$$

v(t): price in denominated currency of a forward contract settling at T with strike K

S(t): current spot price

Q(t,T): zero coupon bond rate for asset currency to settlement T

R(t,T): zero coupon bond rate for denominated currency to T

# The Spot/Forward Arbitrage

- Traders execute the arbitrage all day, every day
  - Mostly it's FX forwards market makers executing the arb by deciding whether it's more efficient to hedge with a forward directly, or hedge with spot and interest rate contracts

- That arbitrage fixes the market forward price to the fair forward price

$$F(t,T) = S(t)e^{(R(t,T)-Q(t,T))(T-t)}$$

# Voice Trading

- Same basic structure as we saw with the spot markets:
  - Clients: market takers who call dealers to get a price
  - Salespeople: take orders from clients and relay prices from traders
  - Traders: market makers who make prices to clients and manage the market risk they accrue by taking the other side of client trades
  - Inter-dealer market: where traders (at dealers) trade with each other to hedge

- Pricing and risk management more complex than for spot because of the tenor dimension of a forward contract
  - Not every forward contract is fungible with every other

# Voice Trading

- The “forwards” traders are really FX swap traders
  - They trade outright forward vs spot as their product
  - Not much spot risk; really they are trading interest rates
- Outright forward trade involves two different traders
  - Spot trader quotes the spot bid/ask
  - Forwards trader quotes the bid/ask on “forward points”
  - Client is shown an aggregated bid/ask by sales, and trade is broken into two pieces for risk management
- FX swap trade involves just the forwards trader

# Voice Trading Example

- Hedge fund ABC wants to buy 10M EUR vs USD for 1y settlement

- ABC trader calls Dealer A
  - Talks on the phone to salesperson, and asks him for A's outright 2-way market on 10M EUR in 1y settlement
  - Salesperson stands up and yells at the EURUSD spot trader and asks for the 2-way market on 10M EUR for hedge fund ABC
  - Salespeople yells at the EURUSD forwards trader for their two-say on the 1y swap on 10M EUR for hedge fund ABC
  - Traders make their prices and sales aggregates for the client

- ABC trader calls Dealers B and C and checks the pricing and deals on the lowest offer price across the dealers

# Voice Market Making

- Same basic inputs for price-making as with spot trading

- Inter-dealer market
  - Market where a dealer can hedge (if desired)
    - Dealing here is by voice as well, via brokers or “direct”
- Current risk position
  - Already long, bias down prices; already short, bias up
- Market views
  - Dealer wants to take risk one way or the other?
- Client behavior
  - Does the client typically buy vs sell? Is the client typically right about market direction?

# Forwards Portfolio Risk

- An FX forwards portfolio might contain forwards settling on many different dates
- Usually represent risk to benchmark dates only, since those are the easiest points to trade in the inter-dealer market on the hedge

- What is the risk to?
  - Forward points: the difference between all-in forward prices and the spot price
  - Or equivalently, non-USD interest rates, but risk displayed in notional of FX swap equivalents
  - Forwards traders also always monitor delta (risk to spot) but hedge that with their friends on the spot desk

# Forwards Portfolio Risk

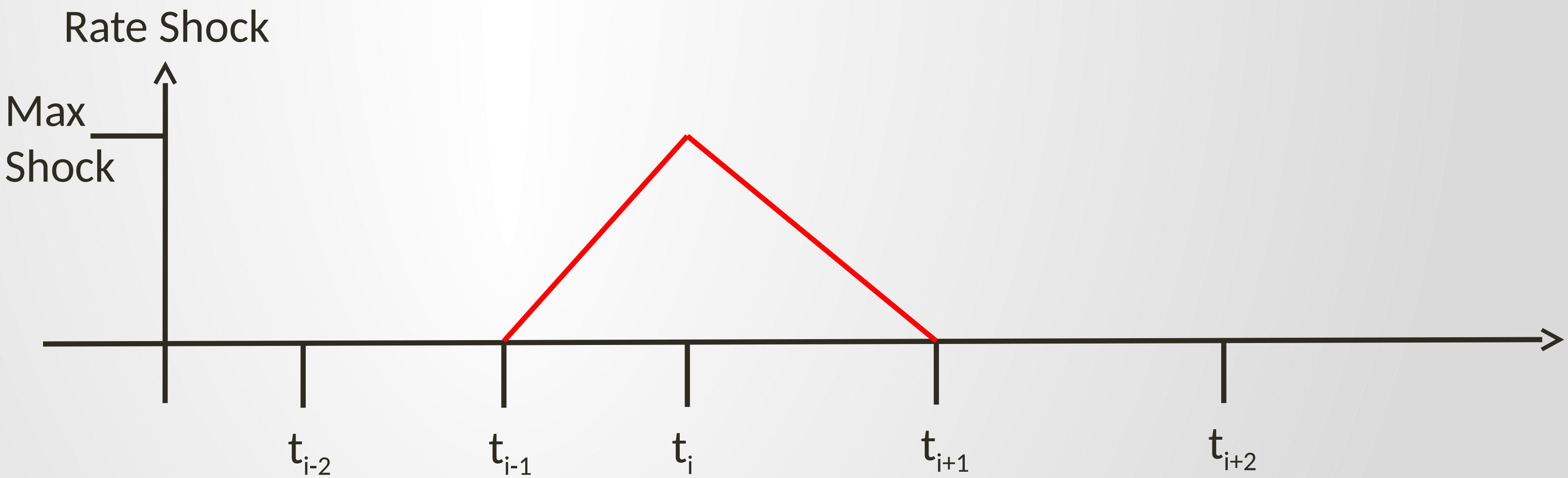
- What are the benchmark settlement dates?
  - 1w, 2w, 1m, 2m, 3m, 6m, 9m, 1y, 2y; maybe 3y, 4y, 5y

- Also some short-dated forward tenors with unusual conventions
  - t/n: "tom/next", the points from 1d before the spot date (ie, tomorrow) to spot (ie, the next date)
    - t/n points = spot price – forward price for 1d settlement
  - o/n: "overnight", the points from today to tomorrow
    - o/n points = forward price for 1d settlement – forward point for 0d settlement
  - A lot of trading happens here for "rolls"
    - Trades to move ahead the settlement date of a forward about to settle

# Forwards Portfolio Risk

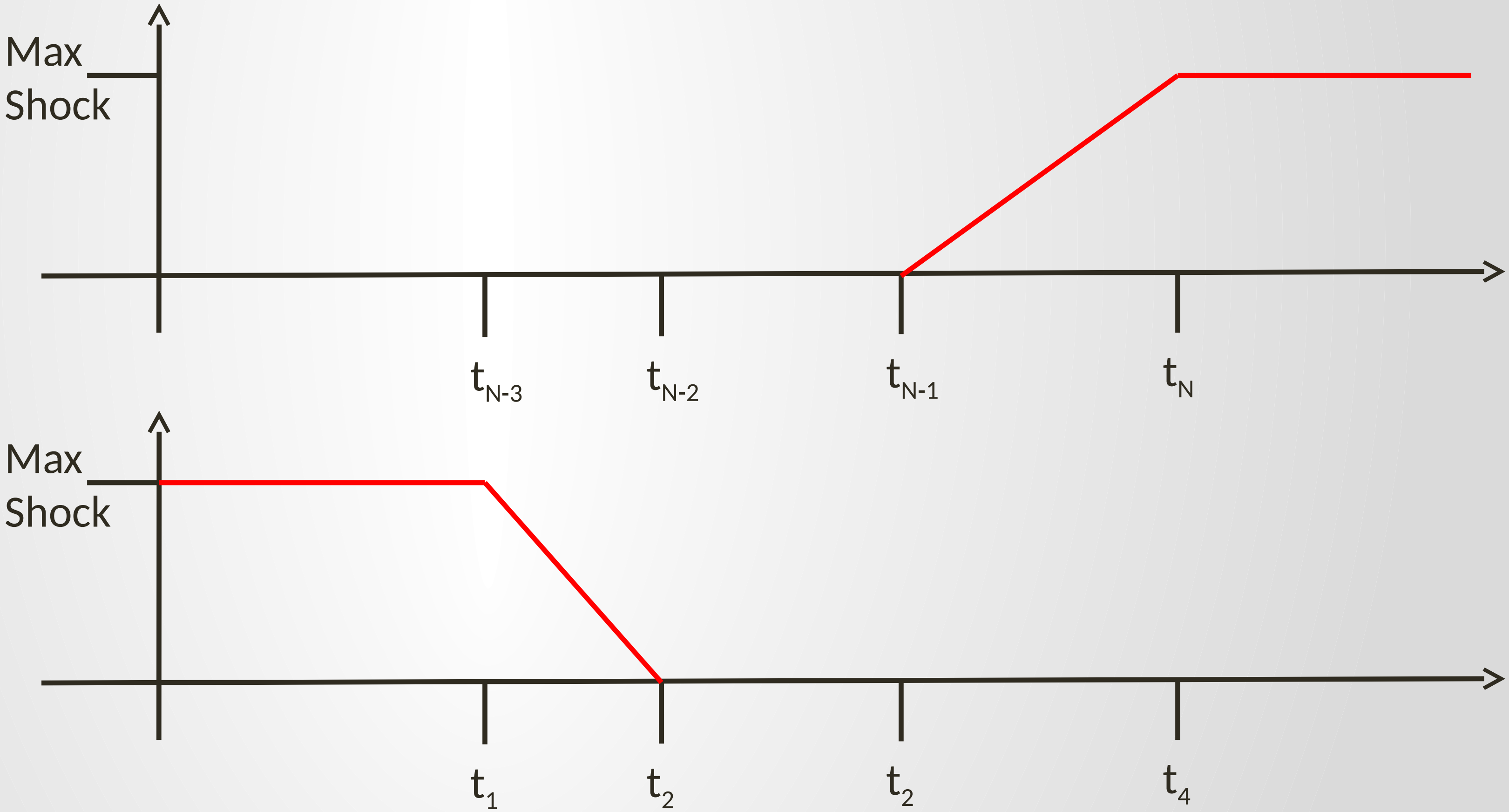
- Need a way to define "bucketed" interest rate risk that sensibly aggregates risk for every possible settlement date into the (much smaller) set of benchmark dates

- Lots of ways to define this but the most common is the "triangle shock", shown below for time $t_i$



# Forwards Portfolio Risk

- At the edges the shock is not triangular; it goes flat



# Forwards Portfolio Risk

- The sum of all those triangle shocks (with the adjusted shocks at the edge points) is a parallel shift to the interest rate curve
  - Nothing is left out, at least to first order

- If a portfolio contains a forward contract with a settlement in between two benchmark settlement dates, its risk will be split (roughly) linearly between the two benchmark buckets
  - Tells the trader which inter-dealer benchmark hedges to do
  - After those hedges the book is still not perfectly hedged, but its remnant risk should be “small”

# Forwards Portfolio Risk

- What would the suggested hedges be for that example?
  - Assume asset interest rate curve is flat at 2% for all times for simplicity, and unit notional for the forward in the portfolio

- Start with expression for the price of a forward contract with settlement time T and strike K, then take partial derivative with respect to Q:

$$v = Se^{-QT} - Ke^{-RT}$$

$$\frac{\partial v}{\partial Q} = -STe^{-QT}$$

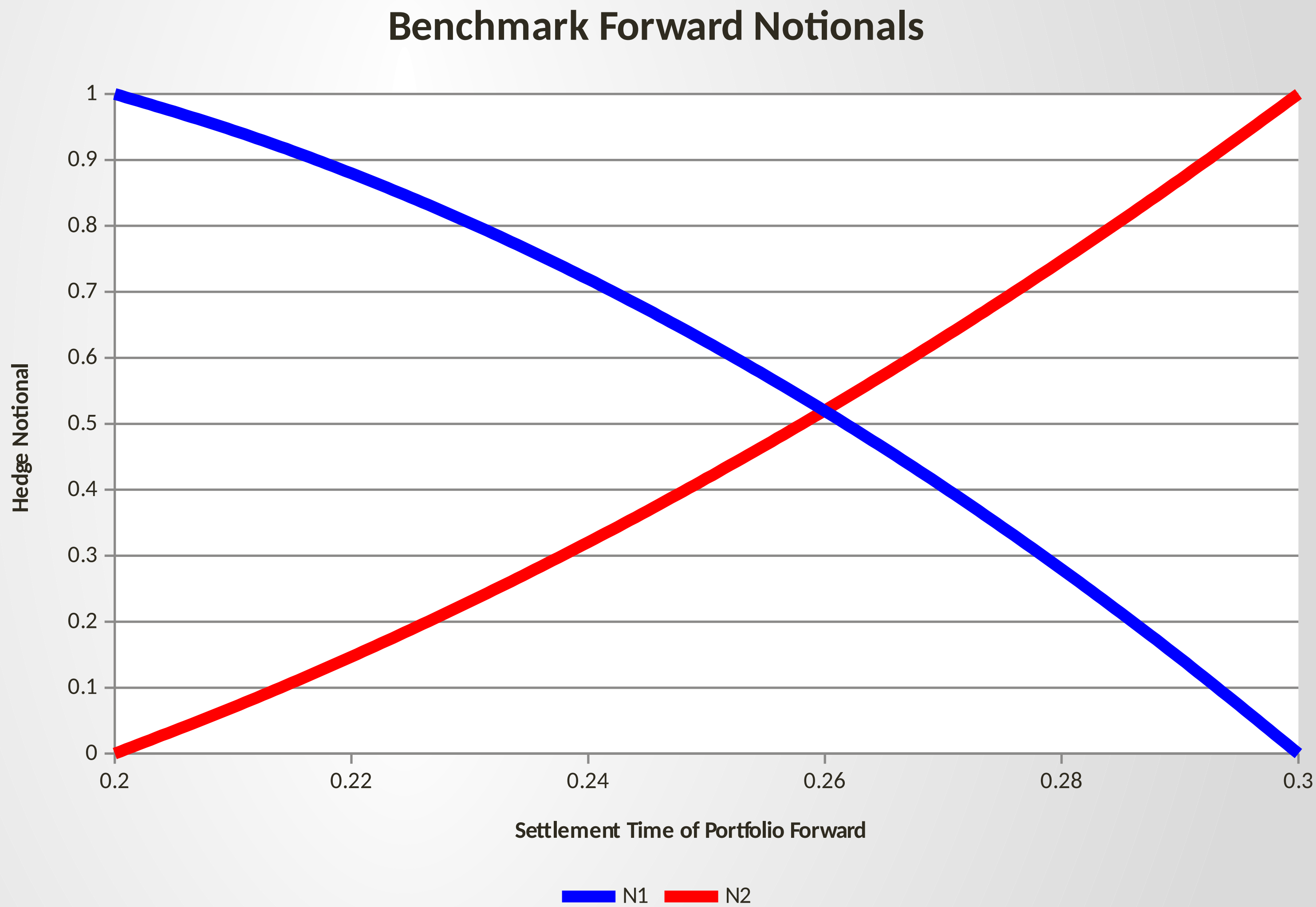
# Forwards Portfolio Risk

- Assume the actual forward trade in the portfolio has settlement time T (=0.22 years) and the benchmark times are T1 (=0.2 years) and T2 (=0.3 years).
- We want to find the notionals of the two benchmark forwards that hedge the risk of the portfolio:

$$N_1 = \left(\frac{T_2 - T}{T_2 - T_1}\right)\frac{T}{T_1} e^{-Q(T - T_1)}$$

$$N_2 = \left(\frac{T - T_1}{T_2 - T_1}\right)\frac{T}{T_2} e^{Q(T_2 - T)}$$

# Forwards Portfolio Risk



# Forwards Portfolio Risk

- Example of a forward risk report

| Forward Risk Report | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | |
| **Currency** | **o/n** | **t/n** | **1w** | **2w** | **1m** | **2m** | **3m** | **6m** | **9m** | **1y** | **2y** | **5y** | **Total** |
| EUR | 100 | 31 | -96 | -94 | -67 | 46 | 90 | 81 | -16 | 12 | 46 | 95 | **228** |
| JPY | -93 | -90 | 1 | -51 | -11 | 87 | 20 | 10 | 12 | 86 | -89 | -85 | **-203** |
| CHF | -57 | 83 | 77 | 89 | -17 | -38 | 95 | -19 | -20 | 31 | 61 | -39 | **246** |
| AUD | 77 | -78 | -50 | 11 | 29 | -93 | 74 | 3 | -90 | 92 | 7 | -22 | **-40** |
| | | | | | | | | | | | | | |

# Reducing Risk Dimensionality

- Lots of numbers even for a single currency on that report
  - Not unmanageable, but maybe we can do better
- Want a way to efficiently reduce the dimensionality of the market risk
  - Market risk to interest rate spreads, that is (currency rate minus USD rate); which is how people tend to think about risk to FX forwards
- Two common approaches:
  - Principal component analysis
  - Parametric factor models

# Principal Component Analysis

- Look for most important (non-parametric) shocks that tend to drive moves in the whole curve
  - eg parallel shift, tilt move, bend move, etc

- Use historical daily interest rate spread moves, particularly covariances between them, to figure out the factors
  - Assumes all daily returns of constant-tenor rate spreads are drawn from an identical distribution
  - Pick off the component shocks (eigenfunctions of the covariance matrix) corresponding to the largest eigenvalues of the covariance matrix

# Principal Component Analysis

- In practice people don’t use this very often
- Non-parametric shocks are hard to understand properly
  - Can have unusual shapes due to specific data points in the history you’re using
- Non-parametric shock shapes change over time
  - Do you fix the shock shape based on a particular historical run?
  - Do you recalculate the shock shapes every day based on rolling historical runs?
- Together they make traders unsure what their risk numbers mean

# Factor Models

- More common are factor models for the interest rate spread curve

- We'll focus on forward curve models; one variation is

$$dS(t,T) = \sum_{i=1}^{N} \sigma_i(t,T)\, dz_i(t)$$

- $S(t,T)$: term interest rate spread for tenor T as seen at time t
- $\sigma_i(t,T)$: volatility of $i^{th}$ factor
- $dz_i(t)$: Brownian motion driving $i^{th}$ factor (all correlated)

# Two Factor Model

- Common form for a 2-factor forward curve model:

$$dS(t,T) = \sigma_1 e^{-\beta_1 T} dz_1(t) + \sigma_2 e^{-\beta_2 T} dz_2(t)$$

$$E\left[ dz_1(t)\, dz_2(t) \right] = \rho\, dt$$

- $\sigma_1$, $\sigma_2$, $\beta_1$, $\beta_2$, and $\rho$ are factor model parameters
- Calibrate them to historical dynamics

# Two Factor Model

- Covariance of log returns in the two-factor model

$$Cov\left[dS(T_1), dS(T_2)\right] = \left(\sigma_1^2 e^{-\beta_1(T_1+T_2)} + \sigma_2^2 e^{-\beta_2(T_1+T_2)} + \rho\sigma_1\sigma_2\left(e^{-\beta_1 T_1-\beta_2 T_2} + e^{-\beta_1 T_2-\beta_2 T_1}\right)\right) dt$$

- Can compare the model covariances with historical covariances
- Then do a 5-dimensional non-linear rootfinding to find the model parameters that best match the historical dynamics
- Need to choose two elements of historical data:
  - The set of tenors to include in the historical data
  - The length of the historical data set

# Two Factor Model

- The two factor model can give shocks that look a lot like the first two principal component shocks
  - Roughly parallel: one shock with relatively low mean reversion strength
  - Tilt: another shock with a high mean reversion that's correlated with the first shock
    - Then the orthogonal shocks look like parallel plus a shock that looks like a difference between exponentials, which shapes a tilt

$$dS(t,T) = \underbrace{\left(\sigma_1 e^{-\beta_1 T} + \rho\sigma_2 e^{-\beta_2 T}\right) dz_1(t)}_{\text{Tilt Shock } (\rho<0)} + \underbrace{\sigma_2 \sqrt{1-\rho^2}\, e^{-\beta_2 T} dz_3(t)}_{\text{Roughly parallel shock}}$$

$$E\left[dz_1(t)dz_3(t)\right] = 0$$