MTH 9831 Assignment 8 (11/11/2015 - 11/18/2015).

(1) (Bond price for CIR model) Read Example 6.5.2 and do Exercise 6.4.

(2) (Derivation of Kolmogorov backward equation) Review Section 4 of Lecture 2, which covers a special case when the SDE is $dX(t) = dB(t)$. Do Exercise 6.8.

(3) (Pricing down-and-out call options)

(a) Use a probabilistic technique to find the time zero price of a down-and-out call option with strike K, barrier $B < K$, and expiration T. Assume the Black-Scholes framework with a constant interest rate r. You may leave the answer as an integral, but make sure that limits of integration are spelled out.

(b) Let $v(t, x)$ be the price of the above option at time t if $S(t) = x$ assuming that the call has not been knocked out prior to t. Show that $v(t, x)$ solves the BSM PDE in $[0, T) \times [B, \infty)$ and argue that it has to satisfy the following boundary conditions:

$$v(T, x) = (x - K)_+, \quad x \geq B; \tag{1}$$

$$v(t, B) = 0, \quad t \in [0, T]; \tag{2}$$

$$\frac{v(t, x)}{x} \to 1, \quad \text{as } x \to \infty. \tag{3}$$

(c) What is the relationship between the price of a down-and-out call option and the price of a down-and-in call option with the same parameters? Using this relationship write down the price of a down-and-in call option with the same parameters.

(4) (Zero strike Asian call option) Exercise 7.7.