4. Pricing Instruments with Credit Risk

Exercises

1. The term structure of zero rates is as follows:

t	rate (continuous)
1	0.03
2	0.035

(a) A bond paying annual coupons of 5% per year matures 1 year from today and currently trades at 101.25. What is the z-spread for this bond?
(b) A bond from a different issuer paying annual coupons of 5% per year matures 2 years from today and currently trades at 102. What is the constant z-spread for this bond?
(c) Suppose a second bond from the same issuer as in part (b) is found. This bond pays a 2% annual coupon and matures in 1 year; its price is 98.75. Use this bond and the one from part (b) to find the 2-year term structure of z-spreads for this issuer.

2. The z-spreads for a particular issuer at years 1 and 2 are:

t	z-spread
1	0.005
2	0.0068

For the purposes of these questions, assume no recovery in the event of default.

(a) What is the unconditional probability of default in the second year?
(b) What is the probability of default in the second year conditional on survival for the first year?
(c) What is the forward hazard rate for the second year?

3. A particular issuer has a constant hazard rate of 0.01. For the purposes of this question, assume that the risk-free interest rate is 0.05 to all maturities.

(a) A semiannual bond with maturity 10 years paying a coupon rate of 4% trades at a price of 89. What is the present value of the recovery on the bond?
(b) What is the recovery rate implied by this value?
(c) Suppose that the bond in question is backed by collateral posted by the issuer, and due to recent changes, that collateral has appreciated to such a degree that you now consider the recovery rate on the bond to be 100%. What is the price of the bond under this assumption?
(d) Compare your price above in (c) to the price this bond would have if it were free of credit risk. The two prices differ. Explain why one is greater. What change to the bond's coupon, if any, would cause the other to be greater?

4. For the following, assume that the risk-free interest rate is 2.5% (expressed with continuous compounding) to all maturities, and the z-spread is 50bp (also expressed continuously) to all maturities.

(a) What is the NPV per 100 face of a fixed-coupon bond from this issuer paying a 2.515% coupon semiannually for five years? Estimate its duration and spread duration via numerical approximation.
(b) What is the NPV per 100 face of an FRN from this issuer paying LIBOR flat quarterly for five years? Estimate its duration and spread duration via numerical approximation.
(c) Consider a portfolio consisting of a long position in the bond from (a) and a short position in the bond from (b), with both positions having 100 face. Calculate the NPV and estimate the duration and spread duration of the portfolio. (Note: Express your durations by dividing the central finite difference estimate of the derivative by $-N$.)
(d) Calculate the NPV and estimate the duration as in (c) of a swap with semiannual payments on the fixed leg and quarterly payments on the float leg paying 2.515% fixed for 5 years. The portfolio above in (c) can be thought of as this same swap contract entered into with a risky counterparty. Does credit risk make an appreciable difference to the value or interest rate sensitivity of the contract? How does this result change if we consider a 6.5% fixed coupon on the risky and risk-free swaps instead of the original coupon?

5. For the following questions, assume the risk-free rate is 2% (expressed with continuous compounding) and that the recovery rate on the issuer is 40%.

(a) What is the par CDS spread on a 3Y contract with the premium paid quarterly if the issuer has a hazard rate of 0.01? To simplify your calculation, assume that default in any premium period occurs at the *end* of the period—that is, if default occurs in a particular period, the protection buyer owes the full period's premium and is compensated for credit loss at that time.
(b) Using your result in (a), determine the arbitrage-free minimum par spread for a 5Y contract on the same issuer.

Applications

Corporate Bond Sensitivity Profiles

The sensitivity of bonds to interest rate and credit movements under various market circumstances are crucial aspects of risk management for fixed income portfolios. In this problem, we will examine two simple bonds under a range of rate regimes. In the process, we will extract spread information from the traded price of the bond.

For this problem, consider the term structure of risk-free zero rates:

T	zero (continuous)
0.25	0.005

0.5	0.007
1	0.0085
1.5	0.01
2	0.011
3	0.0125
5	0.013
7	0.0125
10	0.0185
20	0.0225
30	0.025

In all cases, when duration or convexity is asked for, the value you should compute is via the Fisher-Weil method—that is, sensitivities with respect to a parallel shift in the term structure of risk-free rates.

(a) Consider a bond paying a fixed coupon with maturity 10Y, paying a coupon rate of 4% per annum with semiannual coupon payments. Assuming no credit risk and the interest rates above, what are the value, duration, and convexity of the bond? (Note: As mentioned previously, it is conventional to express bond prices per 100 face.)

(b) Suppose the bond in part (a) has a market price of 102.5. What is the constant credit spread (same z-spread to all maturities) implied by this price?

(c) Using the credit spread you determined in part (b), graph the value and duration of the bond (same graph, separate axes) under a range of parallel shifts to interest rates ranging from -200bp to 800bp, in increments of 25bp. Apply the shock to the zero rates defining the risk-free curve.

(d) Using the credit spread you determined in part (b) as the base spread and the risk-free interest rates, graph the value and duration of the bond (same graph, separate axes) under a range of parallel shifts to the credit spread ranging from -200bp to 800bp, in increments of 25bp.

(e) It is common for managers of portfolios of corporate bonds to hedge away some or part of their interest rate exposure, so that the portfolio's primary exposure is to the credit of the bonds in the portfolio. Using intuition gained from the above, suppose you have such a portfolio hedged against interest rate moves. Considering the fact that spread movements and interest rates are negatively correlated, then if spreads widen, is the portfolio under the new market circumstances more likely to be overhedged or underhedged with respect to interest rate movements?

(f) Consider an FRN with maturity 5Y, paying a margin of 100bp over the floating rate index with quarterly payments. The floating rate index is the same one from which the risk-free interest rate curve above was inferred. Assuming no credit risk, under that set of risk-free rates, what are the value, duration, and convexity of the bond?

(g) Suppose the FRN in part (f) has a market price of 99.5. What is the credit spread (same z-spread to all maturities) implied by this price.

(h) Using the credit spread you determined in part (g), graph the value and duration of the FRN (same graph, separate axes) under a range of parallel shifts to interest rates ranging from -200bp to 800bp, in increments of 25bp. Apply the shock to the zero rates defining the risk-free curve.

(i) Using the credit spread you determined in part (g) as the base spread and the risk-free interest rates, graph the value and duration of the FRN (same graph, separate axes) under a range of parallel shifts to the credit spread ranging from -200bp to 800bp, in increments of 25bp.