Credit Risk Models

10. XVA II

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Outline

OIS rate

- The classic option valuation theory assumes the existence of a riskless rate (used in risk neutral valuation).
- The reality is that there is no risk neutral rate, and a surrogate has to be used.
- Traditionally, until the 2007 - 2008 crisis, LIBOR was used for risk free discounting of cash flows.
- This was justified by the fact that banks could lend and borrow from each at LIBOR.
- However, the unsecured lending has lost its significance in the aftermath of the crisis.
- The discounting rate typically used now is the overnight indexed swap (OIS) rate.

OIS rate

- OIS usage as discounting rate is justified by the way transactions (such as derivatives) are collateralized through the applications of ISDA's CSA.
- The typical frequency of posting cash collateral is daily, and the holder of collateral pays Federal Fund Effective Rate.
- Using OIS discounting leads to the multicurve in pricing interest rate derivatives.

Funding value adjustment

- Despite the increased use of collateral, a significant portion of OTC derivatives remain uncollateralized or undercollateralized.
- Funding costs (or benefits) arise in the following situations:
  (i) Undercollateralized transactions give rise to both costs and benefits. For example, a non-CSA counterparty creates a funding requirement for a bank trading with it (this relates to the need to hedge the transaction with a CSA counterparty).
  (ii) Even if collateral is posted, it may not be usable. Collateral that cannot be rehypothecated is useless from a funding perspective.

Funding value adjustment

- The funding cost / benefits can be illustrated by means of Figure 1. A bank enters into uncollateralized trades with a counterparty which are hedged through collateralized trades with another bank.
- If the trades have a positive value, than the hedges have a negative (off-setting) value and the bank needs to post a collateral.
- The return on the collateral is (typically) OIS, and so there is an associated *cost*, if the bank cannot fund at OIS.

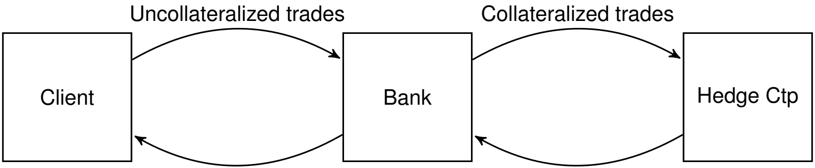

Figure: 1. Uncollateralized trade hedged with a collateralized counterparty

Funding value adjustment

- On the other hand, if the trades have a negative value, the counterparty on the hedges posts collateral, which results in a funding *benefit* for the bank.
- Note that the benefit can only arise if the collateral can be reused, i.e. rehypothecation is allowed.
- The above analogy does not apply if:
  - (i) The bank does not hedge its trades. There should still be an FVA without exchange of collateral.
  - (ii) The profit (spread) on the trades is not part of its value, but the FVA may be charged.
  - (iii) ...
- In general, funding costs arise from the uncollateralized positive value of a portfolio.

FVA formula

- The FVA formula below has a structure similar to the BCVA formula discussed earlier. We shall state this formula without justification and will derive it later.
- Let $\mathrm{Spr_b}(T_{i-1}, T_i)$ and $\mathrm{Spr_l}(T_{i-1}, T_i)$ denote the funding spread for borrowing and lending, respectively. Then

$$
\begin{aligned}
\mathrm{FVA} = &\sum_{i=1}^{m} \mathrm{DiscFact}(T_i)\mathrm{EE}(T_i)\mathrm{SurvProb}_B(0, T_{i-1})\mathrm{SurvProb}_C(0, T_{i-1})\mathrm{Spr_b}(T_{i-1}, T_i)\delta_i \\
&+ \sum_{i=1}^{m} \mathrm{DiscFact}(T_i)\mathrm{NEE}(T_i)\mathrm{SurvProb}_C(0, T_{i-1})\mathrm{SurvProb}_B(0, T_{i-1})\mathrm{Spr_l}(T_{i-1}, T_i)\delta_i \\
&= \mathrm{FCA} + \mathrm{FBA}.
\end{aligned}
$$

- The first term is called the *funding cost adjustment* (FCA), and the second term represents the *funding benefit adjustment* (FBA).

FVA formula

- Note that we have the following (approximate) credit traingle formula for FVA:

$$\mathrm{FVA} \approx \mathrm{Spr_b} \times \mathrm{EE} + \mathrm{Spr_l} \times \mathrm{NEE}.$$

  Here, as usual, we assume that the funding spreads and exposures are constant.
- In case if $\mathrm{Spr_b}(T_{i-1}, T_i) \approx \mathrm{Spr_l}(T_{i-1}, T_i)$, we can use the identity

$$\mathrm{EE} + \mathrm{NEE} = \mathrm{EFV}$$

  to conclude that

$$\mathrm{FVA} \approx \sum_{i=1}^{m} \mathrm{DiscFact}(T_i)\mathrm{EE}(T_i)\mathrm{SurvProb}_B(0, T_{i-1})\mathrm{SurvProb}_C(0, T_{i-1})\mathrm{Spr}(T_{i-1}, T_i)\delta_i.$$

FVA formula

- This is analogous to BCVA which is a sum of CVA and DVA: CVA and FCA are related to the exposure, while DVA and FBA are related to the negative exposure.
- There are, however, important differences between FVA and BCVA.
- The FVA references a bank's own spread (both for borrowing and lending), while BCVA references counterparty's spread in the CVA term and the bank's spread in the DVA term.
- Funding spread should not be confused with the CDS spread (used in the BCVA formula). This spread reflects funding cost in excess of OIS.

FVA in practice

- We mentioned above that DVA is generally viewed as a funding benefit.
- Although DVA and FBA are mathematically similar, there are some differences between them:
  (i) DVA requires risk neutral default probability calibrated to the CDS market, while FBA requires a banks own funding spread.
  (ii) DVA and FBA are applied at the level of a netting set. In fact, FBA is relevant to the entire portfolio of all counterparties.

FVA in practice

From *Risk Magazine*, October 2014:

New angles

Citi takes $474 million FVA charge

Citi has become the latest bank to recognise the funding effects associated with uncollateralised derivatives trades, booking a $474 million loss in its third-quarter results, published on October 14.

Funding valuation adjustment (FVA) reflects the costs and benefits incurred when uncollateralised trades are hedged with collateralised ones, or where received collateral is not reusable. At least ten banks now recognise FVA in their earnings.

In Citi's investor call, chief financial officer John Gerspach said the bank had been looking at taking the charge during 2014, but first wanted to monitor the new adjustment over a full quarter.

"So we had got everything in place and we felt comfortable then pulling the trigger on the formal adoption in the third quarter instead of waiting until the fourth quarter. We think it's the way the industry is going and therefore we wanted to adopt it as soon as we felt comfortable we had the systems in place to track it," said Gerspach.

Citi: the bank is the tenth known to have applied an FVA charge

FVA arises in cases where, for example, a dealer is in-the-money on a trade with a corporate client and is not receiving collateral but would have to post it to its hedge counterparty (*www.risk.net/2257428*). The same situation arises when the client trade is collateralised but the assets cannot be rehypothecated or are not easy to fund (*www.risk.net/2341905*).

On the other hand, if the dealer is out-of-the-money on the client trade, it will receive collateral from its hedge counterparty, and, if it is rehypothecable, should be able to lend the collateral to its treasury, which should be recognised as a funding benefit. Many dealers have been incorporating FVA when pricing trades, but recognising it in earnings can require a significant one-off adjustment as the portfolio is revalued.

Citi is the tenth bank to take an FVA charge. BNP Paribas and Crédit Agricole applied an FVA charge in their second-quarter results, resulting in losses of €166 million and €167 million, respectively. JP Morgan disclosed a $1.5 billion FVA loss in its fourth quarter 2013 results, while Barclays, Deutsche Bank, Goldman Sachs, Lloyds Banking Group, Nomura and Royal Bank of Scotland have all applied FVA charges (*www.risk/net/ 2322843*).

As *Risk* went to press on October 28, UBS also implemented FVA in its third-quarter earnings, resulting in a Str 267 ($282) million loss.

Citi's Gerspach said the industry has made up its mind that FVA needs to be recognised. "I think the move is to go that way and we just felt it was better to be at least in the middle of the pack," he said.

Lukas Becker and Joe Rennison

Margin value adjustment (MVA)

- Margin valuation adjustment arises from the initial margin (IM) requirements imposed by CCPs on their clearing members.
- It is appropriate to consider it separately with MVA as it quantifies the cost of posting IM as well as other funds such as default fund or liquidity fund discussed in Lecture Notes #8.
- It is significant to separate it from FVA, as the methodologies to determine IM are conceptually quite complicated.

Margin value adjustment (MVA)

- Similarly to the xVA's discussed thus far, the MVA is an integral of the expected initial margin profile.
- Specifically, the MVA is given by the following formula:

$$\mathrm{MVA} = \sum_{i=1}^{m} \mathrm{DiscFact}(T_i)\mathrm{EIM}(T_i)\mathrm{SurvProb}(0, T_{i-1})\mathrm{Spr}_{IM}(T_{i-1}, T_i)\delta_i .$$

  Here EIM is the expected initial margin, and Spr_{IM} is the spread between the funding cost of posting the initial margin and the remuneration rate.
- The remuneration rate on initial margin may be lower than OIS.

Margin value adjustment (MVA)

- As discussed in LN #8, IM is calculated via Monte Carlo simulations.
- Since this has to be done for each time slice going forward inside a Monte Carlo simulation, the problem of calculating MVA is computationally expensive.
- One way around it is to use a Longstaff-Schwarz style algorithm.

CME / LCH basis

- The impact of MVA has recently manifested itself as the *CME-LCH basis* for interest rate swaps.
- Until April 2015 the cost of clearing a swap had been steady and was about 0.15 bp.
- In May 2015, this basis started widening dramatically, affecting primarily longer dated swaps.
- At the peak, the cost of clearing of 10Y swap at CME was higher by about 2.5 bp than at LCH.
- For a swap with a notional value of $1 bn, that represents a difference of around $2.5 mm.

CME / LCH basis

- Figure 2 plots the historical time series of the CME-LCH basis, and is taken from the Clarus Technology blog.

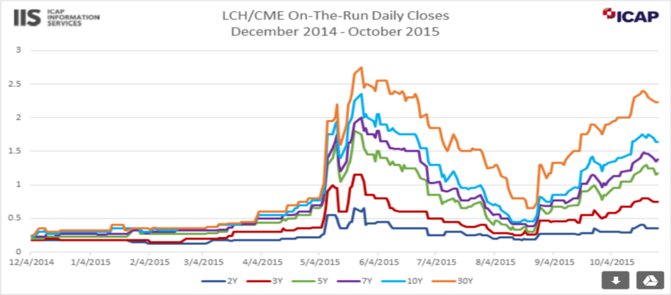

Figure: 2. CME-LCH spread: historical time series

CME / LCH basis

- Figure 3 shows a market snapshot on October 28, 2015, and is also taken from the Clarus Technology blog.

19981

10/28 17:00 GMT [REUTERS CAPITAL MARKETS SOURCE:ICAP] 19981

USD INDICATIVE IRS SEMI BOND LCH / CME BASIS

SB Term	LCH MID	CME(bps)	CME MID
1Y	0.49685	+0.1500	0.49835
2Y	0.74477	+0.3500	0.74827
3Y	0.98391	+0.7500	0.99141
4Y	1.19722	+1.0000	1.20722
5Y	1.38652	+1.1700	1.39822
6Y	1.55191	+1.3000	1.56491
7Y	1.69400	+1.3800	1.70780
8Y	1.81150	+1.4600	1.82610
9Y	1.90895	+1.5500	1.92445
10Y	1.99148	+1.6400	2.00788
12Y	2.12866	+1.7000	2.14566
15Y	2.26651	+1.7800	2.28431
20Y	2.40151	+1.9300	2.42081
25Y	2.46493	+2.0800	2.48573
30Y	2.50155	+2.2300	2.52385

Figure: 3. Term structure of the CME-LCH spread

Mechanics of the CME / LCH basis

- CME cleared volume is mostly driven by fixed income asset managers and other end users of swaps. They generally look to swap the fixed coupons on their bond holdings into floating coupons, and thus they tend to pay fixed on swaps.
- As a consequence, on the client-dealer swap trade, the dealer is receiving fixed. To hedge their risk, the dealers pay fixed in the interdealer swap market or sell ED futures.
- Traditionally, the dealers have been clearing their swap trades at LCH.
- The result is that dealer books at CME are largely directional, with little offsetting exposure, and the resulting margin and funding costs are thought to be the source of the price difference.

Capital value adjustment (KVA)

- Banks have to hold considerable resources, *regulatory capital*, against their OTC derivatives positions.
- Capital requirements against OTC derivatives consist of three components:
  (i) *CVA charge*. It reflects the MTM volatility of the counterparty risk due to credit spread volatility.
  (ii) *Default risk capital charge*. It is a liquidity reserve for the case of a counterparty default.
  (iii) *Market risk capital charge*. Banks are expected to hedge their market risk, this charge is usually not assessed.

Capital value adjustment (KVA)

- The KVA is given by the following formula:

$$\mathrm{KVA} = \sum_{i=1}^{m} \mathrm{DiscFact}(T_i)\mathrm{EC}(T_i)\mathrm{SurvProb}(0, T_{i-1})\mathrm{CC}(T_{i-1}, T_i)\delta_i .$$

  Here EC is the expected capital profile, and CC is the cost of capital.
- In order to be compute KVA, one needs a methodology for generating future capital scenarios.

Wrong way risk

- We showed in Lecture Notes #9 that, under the assumption of independence of counterparty credit and exposure, the CVA can be expressed as a product of the counterparty credit spread and the exposure.
- If the independence assumption does not hold, the CVA can
  (i) increase relative to the standard value, in which case we talk about the *wrong way risk* (WWR),
  (ii) or decrease, which is termed the *right way risk* (RWR).
- In the following, WWR means both WWR and RWR.

Examples of wrong way risk

- Buying a put option on an underlying positively correlated with the counterparty.
- Trading FX forwards or cross currency interest rate swaps with a sovereign.
- Interest rate swaps: high interest rates may trigger defaults.
- Buying protection on a CDS on a name correlated with the counterparty.
- Commodity forwards / swaps with counterparties exposed to the underlying.

Quantifying WWR

- A natural way of incorporating WWR into the standard CVA formula is through the conditional EE.
- We can rewrite the general expression for CVA in the form

$$\begin{aligned} CVA(t, T) &= (1 - \overline{R})\mathrm{E}^{\mathrm{Q}}\left[1_{\tau \leq T} V(\tau, T)^{+}\right] \\ &= (1 - \overline{R}) \int_t^T \mathrm{E}^{\mathrm{Q}}\left[V(\tau, T)^{+} \,\middle|\, \tau = u\right] dQ(t, u). \end{aligned}$$

- Replacing, as usual, the integral with its discrete approximation, we can write this as

$$CVA(t, T) \approx (1 - \overline{R}) \sum_{i=1}^{m} EE_t(t, T_i | \tau = T_i)(Q(t, T_i) - Q(t, T_{i-1})),$$

where $EE_t(t, T_i | \tau = T_i)$ denotes the expected exposure conditioned on the time T_i being the default time of the counterparty.

Quantifying WWR

- In Lecture Notes #8 we derived an expression for the expected exposure in a simple normal model where the asset value follows the process, namely

$$V(t) = \mu t + \sigma\sqrt{t}Y,$$

where Y is a standard normal variable.

- Then

$$EE(s) = s\mu N(\mu\sqrt{s}/\sigma) + \sqrt{s}\sigma\varphi(\mu\sqrt{s}/\sigma),$$

where s denotes the time horizon.

- We can derive an analogous expression for EE in the presence of WWR assuming a constant default intensity:

$$Q(t) = 1 - e^{-\lambda t}.$$

- Assume that the time to default is given by:

$$\tau = Q^{-1}(N(Z)),$$

where Z is standard normal.

Quantifying WWR

- Finally (we are mimicking here the Gaussian copula model), we set

$$Y = \rho Z + \sqrt{1-\rho^2}\varepsilon,$$

with standard normal ε.

- Then,

$$\begin{aligned} EE[s|\tau = s] &= \mathrm{E}\big[V(s)^+ \,\big|\, \tau = s\big] \\ &= \mathrm{E}\big[V(s)^+ \,\big|\, Z = N^{-1}(Q(\tau))\big] \\ &= \int_{-\mu(s)/\sigma(s)}^{\infty} \Big(\tilde{\mu}(u) + \tilde{\sigma}(u)\, x\Big)\varphi(x)\, dx, \end{aligned}$$

where

$$\begin{aligned} \tilde{\mu}(s) &= \mu(s) + \rho\sigma(s)\, N^{-1}(Q(\tau)) \\ \tilde{\sigma}(s) &= \sigma(s)\sqrt{1-\rho^2}\,. \end{aligned}$$

Quantifying WWR

- Carrying out the integral we find that the conditional EE is given by

$$EE[s|\tau = s] = \tilde{\mu}(s) N(\tilde{\mu}(s) / \tilde{\sigma}(s)) + \tilde{\sigma}(s) \varphi(\tilde{\mu}(s) / \tilde{\sigma}(s)).$$

- Note that this expression reduces to the previous formula if $\rho = 0$.

CVA for a CDS

- CDSs, which are rather simple instrument, have significant counterparty risks as a direct consequence of their structure.
- In 2007 - 2008 market participants realized the severe nature of counterparty risks in single name CDS products and portfolio credit derivatives.
- A successful future for the credit derivative market is very much linked on the ability to control the inherent counterparty risks.

CVA for a CDS

- Assume that bank B buys protection from counterparty C on a credit name U. The following cases have to be considered:
  (i) *Counterparty defaults after the reference name does*. Here, there is no loss since the reference entity default has been settled.
  (ii) *Counterparty defaults before the reference name does*. Here, there is a significant loss since the default payment will not be made.
  (iii) *Reference entity defaults but the counterparty does not*. Here, there is no counterparty risk since the reference name's default will be settled as required.
  (iv) *Counterparty defaults but reference name does not*. This is the most complex case. The counterparty defaults and, although the reference entity does not default, any potential positive MtM of the contract will be lost, less some recovery value. In case of a correlated reference name the counterparty default implies a significantly positive MtM on the CDS protection, since the entity it is expected to widen the reference entity's spreads. This loss MtM could be thus significant (a manifestation of wrong way risk).

CVA for a CDS

- When calculating the CVA for a CDS, we should account for the order in which the defaults of the reference name and the counterparty occur.
- The pricing requires valuing the two legs of a CDS contingent on the counterparty surviving (once the counterparty has defaulted the bank would neither make premium payments nor receive default payments) and adding a final term depending on the MtM of the CDS at the default time.
- Let $S(t, T)$ denote the joint risk neutral survival probability of both C and U. The time t premium payments made on the CDS contract of final maturity T represent an annuity stream with cash flows contingent on joint survival:

$$\widehat{V}_{prem}(t, T) = C \sum_{j=1}^{N} P(t, T_j) S(t, T_j) \delta_j .$$

CVA for a CDS

- The value of the protection leg is

$$\widehat{V}_{prot}(t,T) = \mathrm{E}^{\mathrm{Q}}\big[(1-R_U)P(t,\tau_U)1_{\tau_U<T}1_{\tau_C>\tau_U}\big],$$

where $\tau^1 \triangleq \min(\tau_U, \tau_C)$.

- Discretizing, we approximate this expression by

$$\widehat{V}_{prot}(t,T) = (1-R_U)\sum_{j=1}^{N} P(t,T_j)\mathrm{P}\big(\tau_U \in [T_{j-1},T_j] \,|\, \tau_C > \tau_U\big),$$

where $\mathrm{P}\big(\tau_U \in [T_{j-1},T_j] \,|\, \tau_C > \tau_U\big)$ is the probability of default of U conditional on the survival of C.

CVA for a CDS

- We also have to add the payment made at the counterparty default time (case 4 above).
- Let $V(\tau, T)$ denote the discounted counterparty risk free MtM of the CDS at the default time τ.
- If this value is positive then B will receive only a fraction $R_C V(\tau, T)$ of the MtM; if it is negative then the MtM must be paid to the defaulted counterparty. Hence the payoff in default is $R_C V(\tau, T)^+ + V(\tau, T)^-$.
- Finally, the total value of the CDS with counterparty risk is given by

$$\widehat{V}(t, T) = \widehat{V}_{prot}(t, T) - \widehat{V}_{prem}(t, T) + \mathrm{E}^{\mathrm{Q}}\big[R_C V(\tau, T)^+ + V(\tau, T)^-\big].$$

CVA for a CDS

- In order to evaluate the above quantity we define the random default time of the reference name via a Gaussian copula. Specifically, we set $\tau_U = S_U^{-1}(N(Z))$, where $S_U(t, T)$ is the survival probability of the reference name and $N(x)$ denotes the cumulative Gaussian distribution function of a standard Gaussian random variable Z.
- Similarly, we set $\tau_C = S_C^{-1}(N(Y))$, where $S_C(t, T)$ is the survival probability of the counterparty.
- Then we relate C and U via $Y = \rho Z + \sqrt{1-\rho^2}\varepsilon$, with $\varepsilon \backsim N(0, 1)$, where ρ is the correlation parameter.
- The joint survival probability is then given by

$$S(t, T) = C_\rho\big(N^{-1}(S_U(t, T)), N^{-1}(S_C(t, T))\big),$$

where $C_\rho(x, y)$ denotes the bivariate normal cumulative probability distribution with correlation ρ.

CVA for a CDS

- The conditional default probability factor can be approximated by:

$$\begin{aligned}\mathrm{P}(\tau_U \in [T_{j-1}, T_j] \mid \tau_C > \tau_U) &\approx \mathrm{P}(\tau_U > T_{j-1} \mid \tau_C > T_j) - \mathrm{P}(\tau_U > T_j \mid \tau_C > T_j)\\ &= C_\rho(N^{-1}(S_U(t, T_{j-1})), N^{-1}(S_C(t, T_j))) - C_\rho(N^{-1}(S_U(t, T_j)), N^{-1}(S_C(t, T_j))),\end{aligned}$$

  which is accurate for short time intervals where the probability of both U and C defaulting simultaneously is negligible.
- The contingent premium and protection terms, $\widehat{V}_{prem}(t, T)$ and $\widehat{V}_{prot}(t, T)$, can then be computed analytically using the above approximations.
- The number of points used for discretization should be large (at least 20 per year), especially when correlation is high.

CVA for a CDS

- The computation of the last term in the expression for $\widehat{V}(t, T)$ is more complicated since it involves the risk-free value of the CDS at some future date τ_C.
- The option-like payoff of this term means that not only the expected value of the CDS is required but also the distribution of future CDS values conditional on the counterparty default time.
- While the expected value of the CDS at the default time can be calculated using the copula approach, the optionality inherent in the counterparty risk calculation requires the use of a dynamic credit model.
- Also, this computation involves an American option Monte Carlo valuation problem.

CVA for a CDS

- A simple approach consists in replacing the last term by its upper and lower bounds:

$$\begin{aligned}\text{Upper bound} &= R_C \mathrm{E}^{\mathrm{Q}}\big[V(\tau, T)\big]^{+} + \mathrm{E}^{\mathrm{Q}}\big[V(\tau, T)\big]^{-}\\ \text{Lower bound} &= \mathrm{E}^{\mathrm{Q}}\big[R_C Csh(\tau, T)^{+} + Csh(\tau, T)^{-}\big],\end{aligned}$$

  where $Csh(\tau, T)$ is the of the cash flows on the CDS at time τ_C in a given scenario.
- The upper and lower bounds defined above can be computed by straightforward Monte Carlo simulation.

CVA for a CDS

- Using Monte Carlo simulation and analytical expressions one can now calculate the break-even CDS spread in the presence of counterparty risk. We note a final complexity, which is that, since the term $V(\tau_C, T)$ depends on the spread itself, we need to solve recursively for this spread. In practice, due to the relatively linearity in the region of the solution, the convergence is almost immediate.
- We note that the above expressions describe the risky MtM of a CDS without explicit reference to a CVA term. Given the wrong way risk inherent in the product, this is more appropriate. The CVA could be computed by subtracting the risk-free MtM of the CDS.
- We have also ignored the impact of any collateral on the valuation. This is conservative since the use of collateral significantly reduces CDS counterparty risk. However, due to the highly contagious and systemic nature of CDS risks, the impact of collateral may be hard to assess and indeed may be quite limited, especially in cases of high correlation.
- Note also that many protection sellers in the CDS market (such as monolines) have not traditionally entered into collateral agreements.

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