Emerging Markets and Inflation

Fall 2017
Yury Blyakhman

Desk Quant Intro

1. **Personal intro**

2. **What do Desk Quant supporting Market Making business do?**

   - **Salespeople help clients to determine the best strategies for hedging or investment**
   - **Traders work on price quotes for the products, and manage portfolio risks they take through trading with our customers**
   - **Quants work with trading on pricing models, risk analysis, hedging strategy as well as marketing on new products**

Emerging Markets and Inflation

Lecture 1. Introduction to Emerging Markets

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Agenda for Today

1. **Class introduction: syllabus and structure**

2. **Lecture 1. Introduction to Emerging Markets**

Agenda for Today. Class Introduction

1. **Class introduction: syllabus and structure**
   a) **Summary**
   b) **Detailed Course Outline**
   c) **Class Structure**

2. **Lecture 1. Introduction to Emerging Markets**

Class Introduction

Summary:

- Seven three-hour classes
- Two main topics:
  - Emerging Markets (EM)
  - Inflation
- Particulars:
  - Always start from Economics
  - Turn to derivatives, discuss practical pricing and risk management
  - Follow the details of Linear (Flow) products in Rates and FX
  - Use a lot of EM examples, spend the whole day in Brazil
  - Inflation and special Latin America (LatAm) cases. Real Rates
  - Classic Inflation modelling: 3-Factor, Libor Models

Detailed Course Outline:

1. **Introduction to Emerging Markets**
   a. **Definitions and market' drivers**
   b. **Local Markets overview: economics and derivatives' trading**
   c. **Emerging Markets as an Asset Class: compare to classic Rates, FX or Credit markets**
   d. **EM tradable Indices and their derivatives. Pricing and hedging practices**
   e. **Practical overview of Debt Exchange Warrants**

2. **Linear Rates and FX Introduction. Part 1: FX**
   a. **Introduction to Interest Rates and FX**
   b. **Basic Interest Rates Concepts**
   c. **Linear FX Instruments**
      i. **FX Spot and Forward**
      ii. **Non-Deliverable or Cash Settled FX Forward**
      iii. **Forward Starting FX Forward and Convexity Adjustment(s)**
      iv. **FX Future and Convexity Adjustment(s)**

Detailed Course Outline (continued):

3. **Linear Rates and FX Introduction. Part 2: Rates**
   - a. **Fixed Income Instruments and Curves**
     - i. Bonds and Asset Swaps
     - ii. Evolution of a Bond market into a Swap market in Emerging Markets
     - iii. Single Currency and Cross Currency Swaps
   - b. **Interest Rate (Yield) Curve Bootstrapping**
     - i. Calibration
     - ii. Markets and Patterns across Developed and Emerging Markets
     - iii. New Instruments evolution in Emerging Markets
   - c. **Differential Discounting**
     - i. Multiple CSA Discounting Introduction
     - ii. Special cases of Local Collateral in Emerging Markets

Detailed Course Outline (continued):

4. **Brazil**
   a. **Brazil benchmarks and Day Count Conventions**
   b. **FX Products. Onshore and Offshore market. FX Convertibility**
   c. **Linear Interest Rate products**
      i. **IR Futures. IR/FX Futures**
      ii. **Onshore USD rates and extension to other Currencies**
      iii. **CDI Swap and Percentage CDI Swap**
   d. **Interest Rate options**
      i. **CDI Swaption and DI Future Option**
      ii. **CDI Cap and IDI Options**

Detailed Course Outline (continued):

5. Introduction to Inflation

a. Economics of Inflation. Inflation measures
b. Inflation-Linked products and markets
    i. Securities
    ii. Breakeven Inflation
    iii. Futures and Inflation swaps
    iv. Inflation-linked derivatives market
c. Bootstrapping Inflation curve
    i. Breakeven and Zero Coupon Swap Inflation curve
    ii. Mean-reverting process in building the inflation curve and forward inflation pricing
d. Inflation seasonality

Detailed Course Outline (continued):

6 – 7. Inflation in Latin America and Classic Inflation Modelling

a. The Real Rate Economy: Inflation in Latin America
   i. Real Rate Currency
   ii. Latin America Inflation-Indexed bonds overview
   iii. A Little Bit of History
b. Inflation in Chile: Forwards
c. Inflation in Brazil: Inflation-linked Bonds and Swaps
d. Inflation in Colombia: Term-on-Term Inflation in Real Rates Economy
e. The 3-Factor Jarrow - Yildirim model of Inflation
   i. HJM and Short Rate refresher
   ii. Detailed JY derivation
f. Application of the 3-F inflation model in Colombia
g. Pricing of Zero Coupon Inflation Indexed Swap (ZCIIS)
h. Pricing of Year-on-Year Inflation Indexed Swap (YYIIS)
   i. YYIIS pricing with JY model
   ii. YYIIS pricing with First Market Model
   iii. YYIIS pricing with Better Market Model

Course Structure

- **Lectures**
  - PowerPoint slides available on the class' forum site before the class
  - Slides will stay available throughout the semester
  - There will be no hard copies
- **Home works**
  - All 100% practical covering structures virtually identical to real trades
  - Strictly due before the next class submitted to your TA
  - Submission format is free
  - Each lecture will start with the homework review
  - E-copy of solutions will not be available
  - Cumulative Home works grade will be worth 20% of the final grade
- **In-class final exam**

Agenda for Today. Introduction to Emerging Markets

1. Class introduction: syllabus and structure

2. **Lecture 1. Introduction to Emerging Markets**

   a) **Emerging Markets definition, drivers, map**

   b) EM Local Markets. Latest Overview

   c) Emerging Markets as an Asset Class. Regional differences

   d) Emerging Markets Indices and Derivatives on them
      - EMBI+ and the EM Contagion effect
      - GBI-EM and Derivatives pricing

   e) Debt Exchange Warrants
      - Pricing model and risk management
      - Parameters estimation

Emerging Markets Definitions

Definitions

Collection of countries with economic similarities

- **Collection of BRICs:**
  - **BRIC = Brazil, Russia, India, China as in [O'N 2001]**
  - **BRICS = BRIC + South Africa**
  - **BRICET = BRIC + Eastern Europe, Turkey**
  - **BRICM = BRIC + Mexico**
  - **BRICK = BRIC + South Korea**
- **And mortars:**
  - **MIST: Mexico, Indonesia, South Korea, Turkey**
  - **MINT: Mexico, Indonesia, Nigeria, Turkey**
  - **CIVETS: Colombia, Indonesia, Vietnam, Egypt, Turkey, South Africa**
- **Come up with your acronym?**

Definitions (continued)

- **Development over last 20 years, forming G20 with 11 EM countries**
- ***Less Economically Developed Countries* term used in 1970s**
- ***"Emerging Markets"* coined in early 1980s by A. van Agtmael from the World Bank's International Financial Corporation**
- **No agreed upon definition, but common characteristics are →**
- **[JPMorgan 2006]:**
  - **Markets and economies in transition, hence not stable. Countries have begun to open up their markets and "emerge". They have stock and capital markets that foreign investors can participate in;**
  - **Lack of historical economic and political stability, regulatory / legal framework and transparency;**
  - **High growth and liberalization / privatizations, but heavily regulated with barriers and capital controls;**
  - **Markets susceptible to financial and currency crisis, often a reflection of the domestic banking system strength**

Emerging Markets Definitions

Drivers

1. **Systemic:**
   - **General risk appetite and liquidity constraints**
   - **State of economy, monetary policy, levels of inflation and interest rates**
   - **Scare of contagion effect (will be shown later)**
2. **Idiosyncratic:**
   - **Political process, fiscal discipline**
   - **History of payments, defaults (at least 7 for Argentina so far)**
   - **Economic growth, reforms, transparency, etc.**

Emerging Markets Definitions

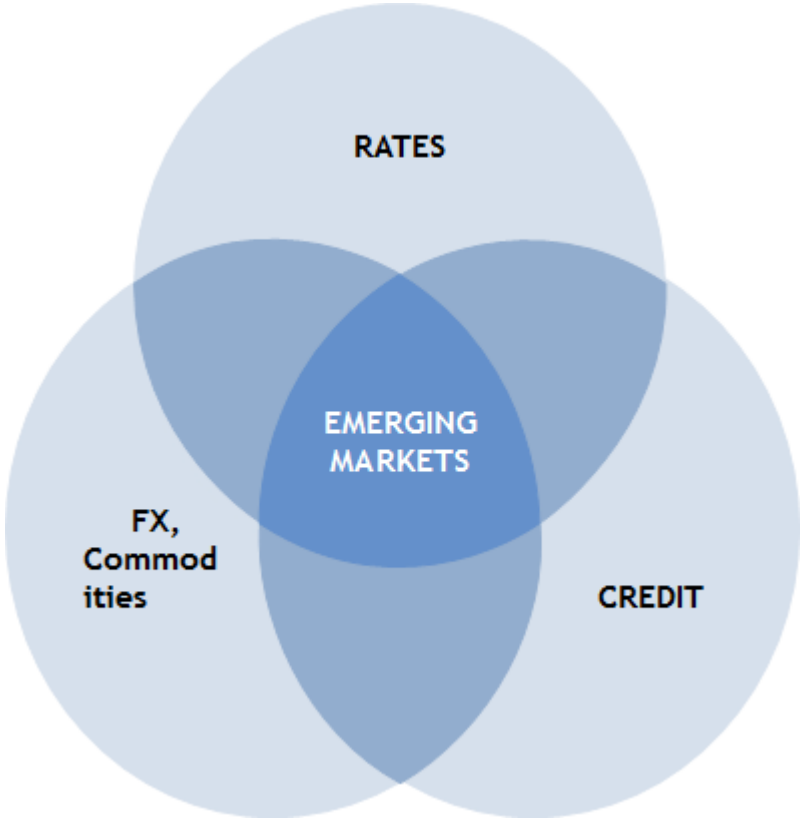
Map [Mirae 2013]

Local Markets Overview [JPMorgan 2014]

- **Continue to depend on US monetary policy and even more so now than before**
- **Local yields recovery was very moderate connected to low money inflow into dedicated EM strategies and capital flights from them**
- **Regionally Emerging Eastern Europe Middle East and Africa (EMEA EM) sees less of the inflow compared to Asia and Latin America (LatAm) due to the ever increased geopolitical risks**
- **EM FX liquidity continues to improve since 2012, but is still low**

What makes it special?

- **[Fabozzi 2002] suggests Brady debt restructuring in 1990's as first step towards Asset Class recognition**
- **Look at XCcy IR Swap in EM [JPMorgan 2006]:**
  1. **IR Fixed - Float Single Ccy Swap**
  2. **Deliverable XCcy Swap**
  3. **Non-Deliverable XCcy Swap**
  4. **Local Ccy Collateral in off-market mode**
  5. **Rates and FX correlated to Credit**
  6. **Commodities dependency**



Regional differences. Latin America

- **Mexico:**
  - Deep capital markets, high products sophistication
  - Economy highly correlated to US
  - Only free floating and convertible currency in LatAm
  - Heavy regulatory environment limits development in the derivatives space
- **Chile:**
  - Strong and stable capital markets with high products sophistication
  - Freely convertible, but non-deliverable FX (will be defined later)
  - Heavy attention to the inflation market due to historical reasons
- **Brazil** (to be covered in more details later):
  - Large scope of exchange traded products and deep capital control
  - Non Convertible FX
  - Regulatory framework is becoming more fluid
  - Largest local bonds market in EM
  - High inflation and wide range of inflation linked products
  - Special conventions historical going back to high interest rates

Regional differences. Europe, Africa. Middle East

- **Poland, Czech, Hungary:**
  - Free floating and convertible FX
  - Not in Eurozone, but market is following EUR both in depth and products sophistication
- **Turkey:**
  - Low product sophistication with visible growing and liberalization
  - Free floating and convertible FX
  - Liquid Fixed Income market
- **South Africa:**
  - One of the deepest FX and Rates markets in EM
  - Fully convertible and free floating FX
  - Fixed coupon bonds are among the most liquid in EM, but FX Options are less liquid
  - Heavy regulatory requirement from SA Reserve Bank (SARB)
- **Israel:**
  - Deep capital markets
  - FX is managed float and convertible
  - Highly liquid local bonds, FX Options, Rates derivatives

Regional differences. Asia

- **South Korea:**
  - Most developed, dynamic and liquid markets in EM Asia
  - Credit Default Swap (CDS) spread is close to Japan
  - Large and active IR Options market

- **China:**
  - Complex variety of onshore (CNY) and Hong Kong traded deliverable CNB
  - Gradual evolution of regulations
  - Active and liquid government bonds market

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   c) Emerging Markets as an Asset Class. Regional differences

   d) **Emerging Markets Indices and Derivatives on them**
      - **EMBI+ and the EM Contagion effect**
      - **GBI-EM and Derivatives pricing**

   e) Debt Exchange Warrants
      - Pricing model and risk management
      - Parameters estimation

Emerging Markets Indices

Overview [J.P.Morgan Markets]

- **Powerful tool providing investors with an access to multi-component markets without trading components directly**
- **Scalar collection of assets (benchmarks), or investment strategy replicating underlyings in a transparent manner**
- **In EM justified by lack of access to Local Markets directly and thus are among the most popular ones**
- **Think of your own Index as it will be needed for the Home work later**

Emerging Markets Indices

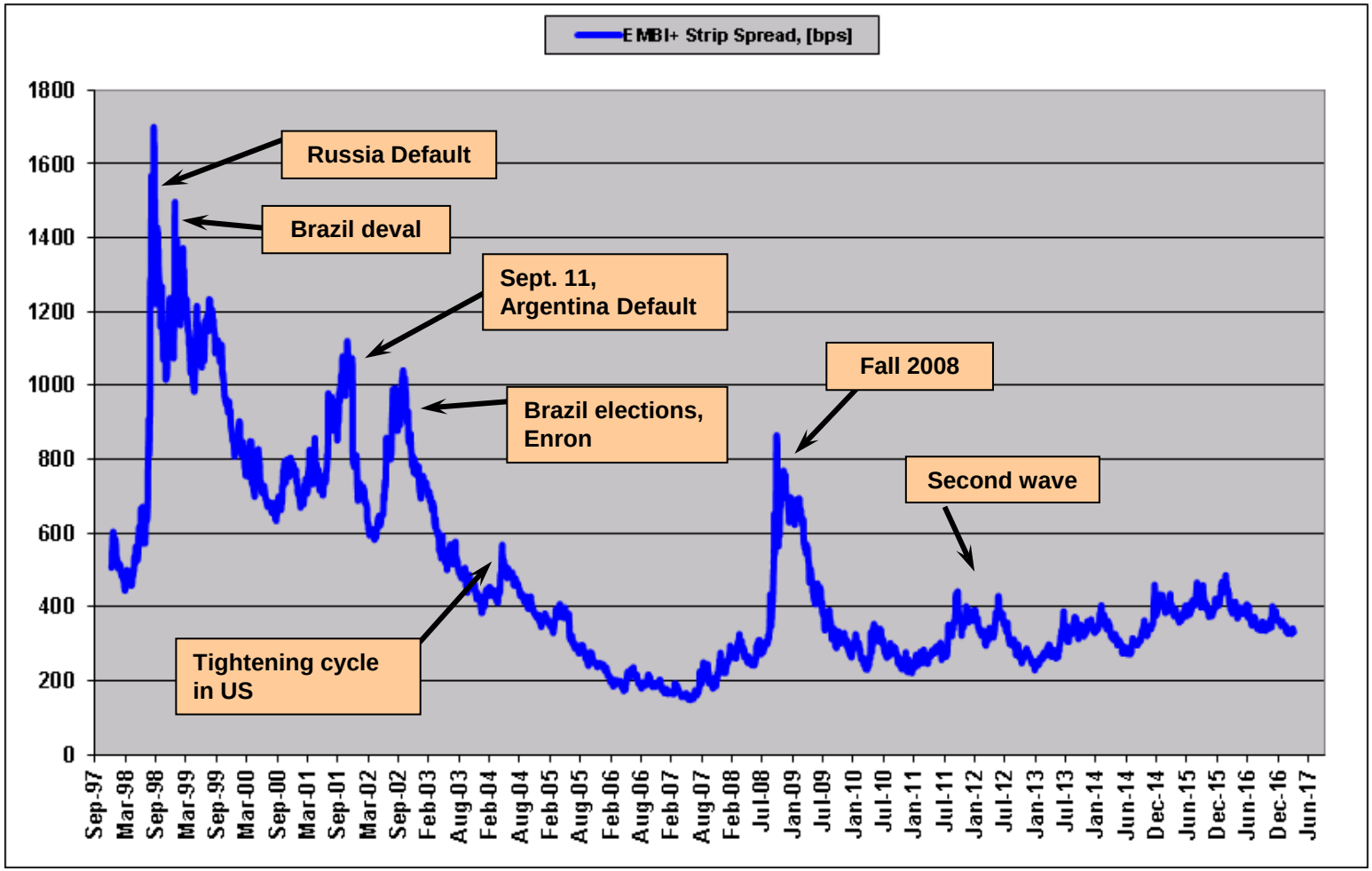
EM families

- **External Sovereigns**
  - EMBI (Emerging Markets Bond Index) family: USD EM debt benchmark covering securities issued by sovereign and quasi-sovereign entities
  - NEXGEM (Next Generation Market Index) for less liquid population of EM Economies with frequent and large debt issuance
- **External Credit**
  - CEMBI (Corporate Emerging Markets Bond Index)
  - JACI (JPMorgan Credit Asia Index)
  - CACI (Central America and Caribbean Index)
- **FX**
  - ELMI (EM Local Market Index) good at tracking short term returns
  - LACI (Latin America Currencies Index)
- **Local Government Bonds**
  - GBI-EM provides exposure to local currency denominated sovereign debt and FX rates

EMBI+ and EM Contagion effect

- **Among the first offered and proved to be a great measure for EM investors**
- **Weighting of its components uses complex liquidity criteria with rebalance**
- **Stripped Spread measures the credit risk premium over US Treasury bonds:**
  - **A difference between Yield to Maturity of a bond to the Yield to Maturity of the corresponding point on the US Treasury curve**
  - **Measures constituents' sovereign risk since it also accounts for collateral**

EMBI+ and EM Contagion effect (continued)



GBI-EM Index [JPMorgan GBI-EM]

- Tracks local currency bonds issued by EM governments
- Includes the following 16 countries split in 4 regions:
  - Asia: Indonesia, Malaysia, Philippines and Thailand
  - Europe: Hungary, Poland, Romania, Russia, Turkey
  - Latin America: Brazil, Chile, Colombia, Mexico, Peru
  - Africa: Nigeria, South Africa
- Excludes countries with explicit capital control
- Positioned as the investable benchmark

GBI-EM Index (continued)

- **Monthly rebalanced with inclusion / removal of countries and issuances**

- **Includes fixed coupon bonds with very strict liquidity requirements:**
  - **Pricing: trading with enough frequency to prevent stale pricing**
  - **Availability: regularly traded at size at acceptable bid-offer**
  - **Replication cost: no additional cost to replicate index via weighted components**

*** Liquidity as ability to trade in sizable amounts without affecting the market. We will need this later for hedging**

GBI-EM Index. Definition

Define some notations:

- ***CP(t)*** **:** **Clean Price of a bond at time *t***
- ***IA(t)*** **:** **Accrued Interest of a bond at time *t***
- ***DP(t)*** **:** **Dirty Price of a bond at time t. *DP(t) = CP(t) + IA(t)***
- ***MV(t)*** **:** **Market Value (outstanding amount) of a bond at time *t***
- ***FX(t)*** **:** **FX rate in terms of foreign currency per US dollars at time *t***
- **r(t) :** **One-day return at time *t***
- **w(t) :** **Weight at time *t***
- **I(t) :** **Index value at time *t***

GBI-EM Index. Definition (contd)

- Global Index consists of collection weighted Regional Indices
- Regional Index is a weighted collection of USD denominated Country Indices
- Country USD denominated index is FX converted from Local Ccy denominated country Index
- Ccy denominated country index is collection of weighted returns on individual bond prices

GBI-EM Index. One bond return

- **One-day return of a bond in local currency:**

$$r_B^{(L)}(t) = \frac{CP(t) + IA(t)}{CP(t-1) + IA(t-1)} - 1 = \frac{DP(t)}{DP(t-1)} - 1$$

- **One-day return of a bond in USD:**

$$r_B^{(U)}(t) = [1 + r_B^{(L)}(t)][1 + r_{FX}(t)] - 1$$

where

$$r_{FX}(t) = \frac{FX(t-1)}{FX(t)} - 1$$

GBI-EM Index. Country Index

- **Country Index in Local currency for N_{CB} bonds:**

$$I_C^{(L)}(t) = I_C^{(L)}(t-1)\cdot\left[1+r_C^{(L)}(t)\right]$$
$$= I_C^{(L)}(t-1)\cdot\left[1+\sum_{i=1}^{N_{CB}} w_{B_i}(t)\cdot r_{B_i}^{(L)}(t)\right]$$

- **Here $w_{B_i}(t)$ is weight of i-th bond for country C:**

$$w_i(t) = \frac{MV_i^{(L)}(t-1)}{\sum_{k=1}^{NC} MV_k^{(L)}(t-1)}$$

- **Country index in USD then:**

$$I_C^{(U)}(t) = I_C^{(U)}(t-1)\cdot\left[\left(1+r_C^{(L)}(t)\right)\cdot\left(1+r_{FX}(t)\right)\right]$$

GBI-EM Index. Regional Index

- **Weighted combo of Country Indices**
- **Defined as month-to-date return since last rebalance at *s*:**

$$\hat{r}_C^{(L)}(t) = \frac{I_C^{(L)}(t)}{I_C^{(L)}(s)} - 1$$

$$\hat{r}_C^{(U)}(t) = [1 + \hat{r}_C^{(L)}(t)][1 + \hat{r}_{FX}(t)] - 1$$

- **So for N_{RC} countries in a Region:**

$$I_R{}^{(U)}(t) = I_R{}^{(U)}(s)\left[1 + \sum_{j=1}^{N_{RC}} w_{C_j}(s) \cdot \hat{r}_{C_j}^{(U)}(t)\right]$$

- **With country weight $w_C(s)$ as of last rebalance defined as**

$$w_C(s) = \frac{MV_C^{(U)}(s)}{\sum_{j=1}^{N_{RC}} MV_{C_j}^{(U)}(s)}$$

GBI-EM Index. Global Index

- **Expressed similarly to Regional via weighted collection:**

$$I(t) = I(s)\left[1 + \sum_{k=1}^{N_R} w_{R_k}(s) \cdot \hat{r}_{R_k}^{(U)}(t)\right]$$

$$w_R(s) = \frac{MV_R^{(U)}(s)}{\sum_{j=1}^{N_R} MV_{R_j}^{(U)}(s)}$$

where $w_R(s)$ is the weight of the Region in the global index at the last rebalancing date in terms of its total market value in USD

- **So, why all the details?..**

GBI-EM Index. Total Return Swap

- **Total Return Swap (TRS): financial contract that fully transfers risk of an underlying asset from one party to another**
- **No asset ownership is required**
- **Risk transferred includes Market and Credit risk**
- **For position *Q* on an Index *I* till maturity *T* Future Value *FV(T)* is**

$$FV_T = Q \cdot \left\{ \frac{I_T}{I_o} - \Phi \right\},$$

$$\Phi = \begin{cases} 0, \text{ with final exchange} \\ 1, \text{ without final exchange} \end{cases}$$

(1)

- **Now we need some dynamics assumptions!**

GBI-EM Index. Total Return Swap. Pricing

Funding rate or general rate of growth:

- **Forward price of an index via Forward price of a bond:**

$$P_{t,T} = \frac{P_{t,t}}{Z^{Fund}{}_{t,T}}$$

- **here Z^{Fund} is standard price of a Discount Zero Coupon bond paying \$1 at time T. Comes from Repo market for a bond**
- **Repo: a combination of a security's sale and an agreement of later purchase for a pre-agreed price on a pre-agreed day**
- **So let us start building Forward price of an Index via assumptions on Repo rates for 200+ bonds in it and market information on FX growth:**

GBI-EM Index. Total Return Swap. Pricing

Use Country index as a starting point:

$$I_T^U = I_o^U \left\{1+\left(\frac{I_T^L}{I_o^L}\frac{FX_o}{FX_T}-1\right)\right\} = I_o^U \left\{1+\left(\frac{I_{T-1}^L\left(1+R/365\right)}{I_o^L}\frac{FX_o}{FX_T}-1\right)\right\},$$

$$R = \sum_i w_i\left(DP_T^i / DP_{T-1}^i - 1\right) = \sum_i w_i\left(DP_{T-1}^i\left(1+\rho/365\right) / DP_{T-1}^i - 1\right) = \sum_i w_i \rho^i/365$$

$$FX_T = FX_{T-1}\frac{Z_{T-1,T}^U}{Z_{T-1,T}^L}$$

here ρ is an overnight repo rate for individual bond.

- **Next is to extend overnight repo to a term one…**

GBI-EM Index. Total Return Swap. Pricing

- **Stop here as it already is getting too complicated…**
- **Instead use generic arbitrage free assumption in Eq. (1):**

$$FV_T = Q \cdot \left\{ \frac{I_T}{I_o} - 1 \right\} = Q \cdot \left\{ \frac{I_t}{I_o} \cdot \frac{I_T}{I_t} - 1 \right\} = Q \cdot \left\{ \frac{I_o(1+R_t)}{I_o} \cdot \frac{I_t(1+\rho_T)}{I_t} - 1 \right\}$$
$$= Q\{(1+R_t)\cdot(1+\rho_T) - 1\} = Q(R_t + \rho_T + R_t \cdot \rho_T) \qquad (2)$$

- **Now TRS depends only on realized Index return and generic Funding rate ρ from time *t* to maturity *T***
- **Could even dissect into regional (hedge-able) funding (using *fv* for Regional)**

$$fv_T = q_i \cdot \left\{ \frac{I^R{}_T}{I^R{}_o} - 1 \right\} = q_i \cdot \left(R^R{}_t + \rho^R{}_T + R^R{}_t \cdot \rho^R{}_T \right)$$

Debt Exchange Warrant

Mexico Debt Exchange Warrants of 2005

- **Switch foreign currency (debt) holding into locally denominated**
- **Enticing as allows to rely on domestic based financing**
- **And allows to exclude FX risk due to convertible nature of MXN Peso**
- **More details:**
  - **Compare local market MBono securities to offshore UMS**
  - **Option (warrant) involves cash-neutral switch basket ATMF option**
  - **Option to exchange any UMS bond from deliverable basket into pre-defined MBono with price quanto'd into USD**

Quanto is a derivative where payoff measured in currency X is made in currency Y. Good introduction to quanto math could be found in Ch. 19.9 of (Hull, 2000)

Debt Exchange Warrant

General description

- **Actual pay-off at maturity:**

$$\pi_T = MAX\left[f_B \hat{P}_B(T) - f_{U_1} P_{U_1}(T),\ f_B \hat{P}_B(T) - f_{U_2} P_{U_2}(T),\ldots, 0\right]$$

where:

f_B – face value of MBono;

$\hat{P}_B(T)$ - MXN spot price of MBono at option maturity, quanto'd in USD

f_{U_i} - face value of the *i*-th UMS bond from the deliverable basket

$P_{U_i}(T)$ - spot price of the *i*-th UMS bond at option maturity

Option valuation

- **Start with 1 UMS bond in a basket**
- **Two assets exchange option valuation as in [Margrabe'78]:**

$$\begin{aligned}\pi_o &= \mathrm{E}\left[Z_{0,T}\cdot\max\left(f_B\hat{P}_B(T)-f_U P_U(T),0\right)\right]\\ &= \mathrm{E}\left[Z_{0,T}\cdot\max\left(f_B P_B(T)e^{\rho_{B,FX}\cdot\sigma_B\cdot\sigma_{FX}\cdot\tau}-f_U P_U(T),0\right)\right]\\ &\overset{def}{=} \mathrm{E}\left[Z_{0,T}\cdot\max(B-U,0)\right]\\ &= Z_{0,T}\cdot B\cdot\left\{B\cdot\mathrm{N}(d_1)-U\cdot\mathrm{N}(d_2)\right\}\end{aligned} \qquad (3)$$

$Z_{0,T}$ - USD discount factor from today to expiry T

$P_U(T)$ - forward price of UMS bond at expiry

$P_B(T)$ - forward price of MBono bond at expiry in MXN

$\hat{P}_B(T)$ - MXN forward price of MBono bond at expiry quanto'd into USD

Option valuation

- **Continue with two assets case as in [Margrabe'78]:**

$e^{\rho_{B,FX}\cdot\sigma_B\cdot\sigma_{FX}\cdot\tau}$ - standard quanto adjustment from numeraire change

$\rho_{B,FX}$ - MBono price and FX rate correlation

σ_B - MBono price volatility

σ_{FX} - MXN forward price of MBono bond at expiry quanto'd into USD

τ - option's tenor

And standard notations for cumulative normal density *N(*), d_1 and d_2*

$$d_1 = \frac{\ln\left(B/U\right) + \frac{\hat{\sigma}^2\tau}{2}}{\hat{\sigma}\sqrt{\tau}}$$

$$d_2 = d_1 - \hat{\sigma}\sqrt{\tau}$$

Debt Exchange Warrant

Option valuation

- **Two assets variance $\hat{\sigma}^2$ could be easily derived as**

$$\hat{\sigma}^2 = \sigma_B{}^2 - 2\rho_{B,U}\sigma_B\sigma_U + \sigma_U{}^2$$

σ_U - UMS price volatility

$\rho_{B,U}$ - MBono price and UMS price correlation

HW1: Derive variance of two assets exchange warrant

Option valuation for full deliverable basket

- **Price of a basket at time 0:**

$$\pi_o = \mathrm{E}\left[Z_{0,T} \cdot MAX\left(f_B \hat{P}_B(T) - f_{U_1} P_{U_1}(T),\ f_B \hat{P}_B(T) - f_{U_2} P_{U_2}(T),\ldots,0\right)\right]$$
$$\overset{def}{=} \mathrm{E}\left[Z_{0,T} \cdot MAX\left(B - U_1, B - U_2,\ldots,0\right)\right] \qquad (4)$$

- **Margrabe doesn't help us, so what can we do?→**

- **Multi-dimensional Monte-Carlo**

Parameters estimation

- What parameters do we need? Let's look at Eq. [4] again →
- Individual Volatilities and Correlations matrix for all assets
- Where and how can we get them? →
- Of course in the market →
- But no Warrants market to calibrate parameters directly. Next? →
- Bond Derivatives with vols. Bond Options, Bond Future Options →
- Bond Derivatives with correlations →
- So we have nothing…

Parameters estimation. Historical estimates

1. Assets distribution:

- Goodness of Fit (GoF) or Chi Square
- What measure will let us say “lognormal enough”? →
- Compare to other assets with traded options:
  - US treasury Options
  - Mexico IR (TIIE) Swaptions + Correlation of Swap rates to Bond yields

Parameters estimation. Historical estimates

2. Volatilities. Suggest two approaches:

A. Standard Deviation of bond prices returns in log space:

$$\sigma = n\sqrt{\frac{n\left(\sum_i^n x_i^{\,2}\right) - \left(\sum_i^n x_i\right)^2}{n(n-1)}}$$

$$x_i = \ln\left(\frac{P_{i+1}}{P_i}\right)$$

Questions to consider:

- **How big rolling window to take?**
- **What do we do with vol as a function of time or asset prices?**

Parameters estimation. Historical estimates

2. Volatilities. Suggest two approaches:

B. Correlation to IR Options:

- Yield vs. IR Swap ➔ Yield vol. How do we get Price vol? →
- Simple assumption for price P_t and yield to maturity Y_t:
- Assuming

$$\begin{cases} dP_t = \sigma_P P_t dW_t \\ dY_t = \mu dt + \sigma_Y Y_t dW_t \end{cases}$$

$$\sigma_P \equiv \mathrm{E}\left[\frac{dP_t}{P_t}\right] \quad \text{and} \quad \sigma_Y \equiv \mathrm{E}\left[\frac{dY_t}{Y_t}\right]$$

- Results in a simple

$$\begin{cases} dP_t = \dfrac{dP_t}{dY_t} dY_t \\ \sigma_P P_t \propto \dfrac{dP_t}{dY_t} \sigma_Y Y_t \end{cases} \Rightarrow \sigma_P = \sigma_Y \cdot BondDuration \cdot Y_t$$

Parameters estimation. Historical estimates

3. Correlations:

- Same as before, go for simple historical correlations:

$$\begin{cases} A_i = \ln\left(\dfrac{(P_1)_{i+1}}{(P_1)_i}\right) \\ B_i = \ln\left(\dfrac{(P_2)_{i+1}}{(P_2)_i}\right) \end{cases} \Rightarrow \quad \rho = \mathrm{Corr}(\mathrm{A}, \mathrm{B})$$

- **Parameters are ready, but how stable is our Monte-Carlo?**

Emerging Markets Indices

HW2 discuss pricing vanilla CALL Option on a generic Index

Given:

- Index is newly created: not much of historical index prices is available
- Index components have been traded for a while and do have historical prices
- Feel free to use earlier discussed GBI-EM index as example to make question more detailed
- There is no option market for this index

Questions:

- Discuss what model we could use to price this option. Can we start with Black-Sholes?
- Discuss how could we extract or derive parameters for this model: Index' volatility?
- If we are to deviate from Black-Sholes a bit and to introduce some simple Local Vol as vol for strike, how could we mark this smile and where from?
- How would we hedge this option if ever traded?

Summary

- Justified and agreed on pricing model
- Estimated parameters via some simple assumptions
- Implemented stand-alone pricing engine
- Discussed stability and potential ways of improvement
- What is next? →
  - Hedging: what can we hedge and how? →
  - Credit considerations? →
  - Stochastic behavior and jumps of EM bond yields [Matovu'07]
  - In times of economic distress when needed the most

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