Lecture 4: Option Markets

Modeling and Marketing Making in Foreign Exchange

Volatility Interpolation: Time

- Consider what happens to implied volatility as time crosses an event
- Event: Friday morning at 8:30am, Non-Farm Payrolls announcement
  - Assume that counts as 0.5 days worth of variance
- Pricing options for Monday 10am exercise
  - 10am is the “New York cut” – the standard expiration time for most G7 options
  - We’ll assume weekend days have zero variance for simplicity
- Implied volatility for Monday exercise is quoted at 10% on Friday at 8:29:59am: what should it be at 8:30:01am?

Volatility Interpolation: Time

- What is the trading time?
  - We'll choose to count hours here, but units aren't that important because we go back and forth
  - 25.5 normal "trading hours" from Fri 8:30am to Mon 10:00am
    - We're counting weekend hours as worth zero, and weekend goes from Fri 5pm->Sun 5pm
  - Plus 12 extra trading hours from the NFP event
  - Totals 37.5 trading hours

- What is the calendar time?
  - Actual days/365 = 3/365

Volatility Interpolation: Time

- Get the total variance from implied vol^2 * calendar time
  - $0.1^2 * 3/365 = 8.219\text{x}10^{-5}$ (unitless)

- Convert that into a trading time vol using trading time vol^2 * trading time = total variance
  - TT vol = $\text{sqrt}(8.219\text{x}10^{-5}/37.5) = 0.001480$
  - Units are per sqrt(hours), so it doesn't look like a market-convention volatility – but that's fine

Volatility Interpolation: Time

- Now we roll time ahead across the event
  - Zero move in calendar time since that's still just days/365
  - Trading time drops by the event weight to 25.5 hours
- Use new trading time to get new total variance
  - Variance = $(\text{TT vol})^2$ * (trading time in hours)
  - Keep the same TT vol as before – we assume this is smooth in calendar time
  - Variance = $0.001480^2 * 25.5 = 5.589\text{x}10^{-5}$
- Convert total variance to market-convention implied vol
  - Implied vol = sqrt(total variance/calendar time)
  - Results in a vol of 8.25%, so crossing the event made vol drop

Source of RR and BF

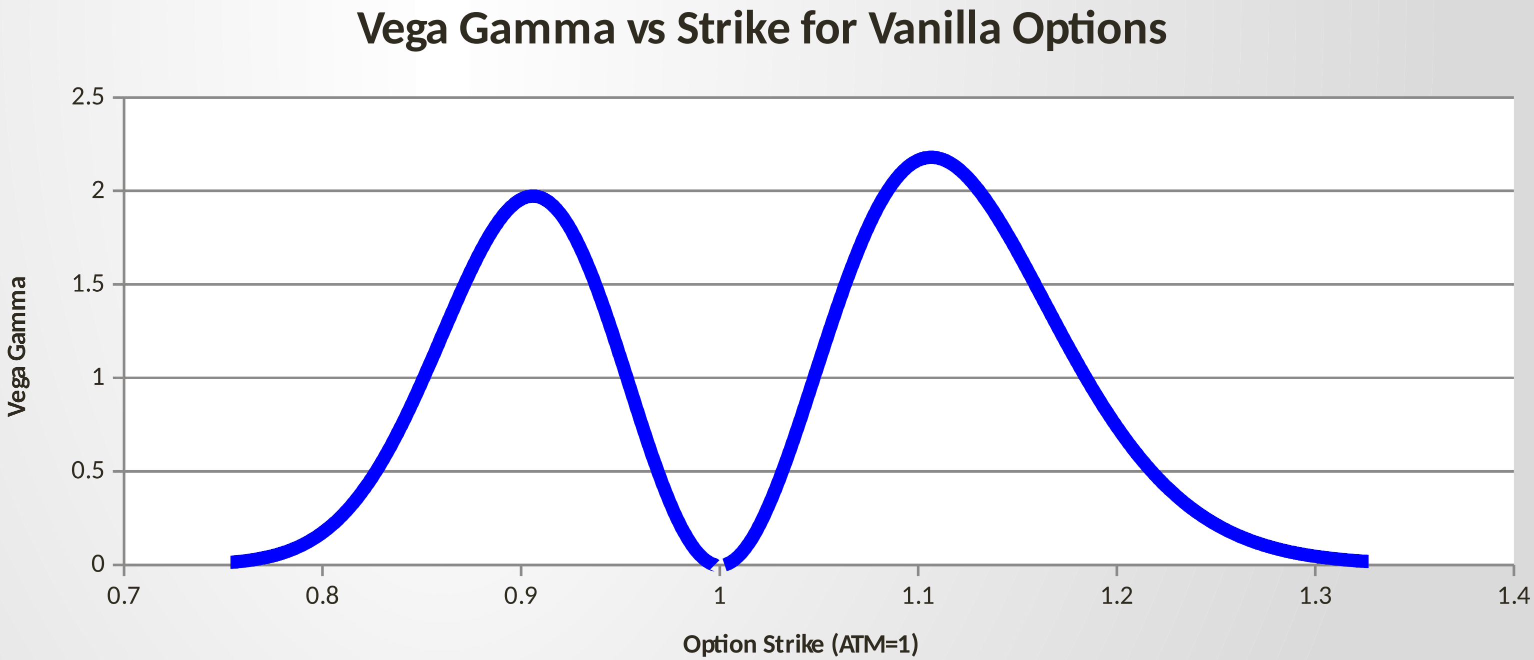
- All stochastic volatility models have a qualitatively similar behavior in driving risk reversal and butterfly
  - Spot/vol correlation drives RR
  - Volatility of volatility drives BF

- Specific models give different quantitative impacts, but one can develop useful intuition about stochastic volatility models without getting to that level of detail

- We’ll think about RR and BF value from the perspective of how traders really work
  - Won’t talk about skewness and kurtosis of spot distribution

Source of BF

- Start with the butterfly (smile)
- The butterfly comes from “vega gamma”, the derivative of vega with respect to volatility
  - Convexity of option price with respect to volatility (“vomma”)



Source of BF

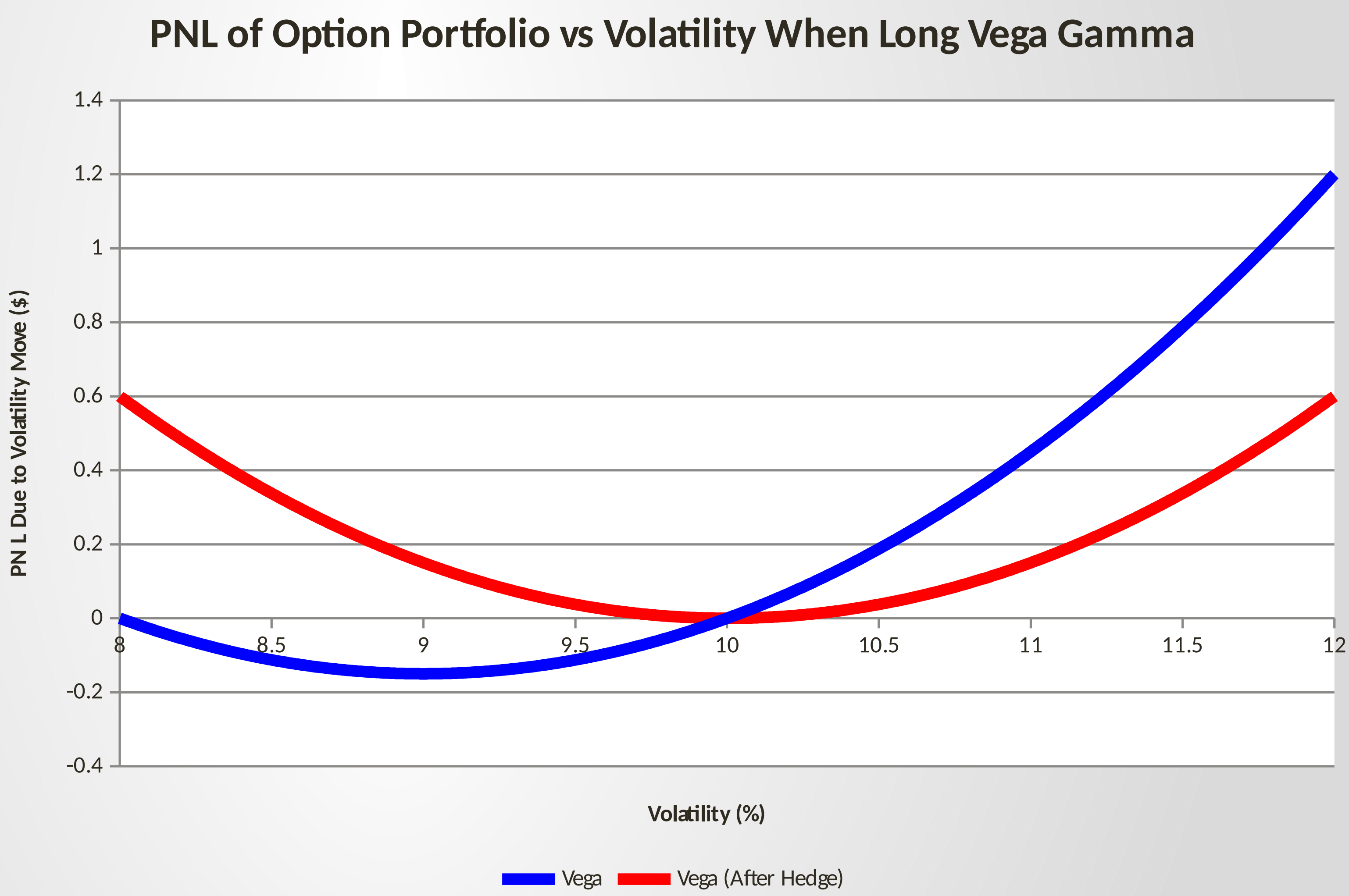
- Buy an option with strike such that vega gamma is large
  - Now long vega (long an option) and long vega gamma

- Vega hedge with an appropriate amount of ATM option
  - ATM option has zero vega gamma, so still long vega gamma

- Now whichever way vol moves you make money!
  - Happy to buy high strike or low strike options at the flat-vol price
  - Bids them up and raises their prices, and therefore their implied volatilities: that's a smile because the effective is the same for high and low strike options

- More volatility of volatility, bigger the smile

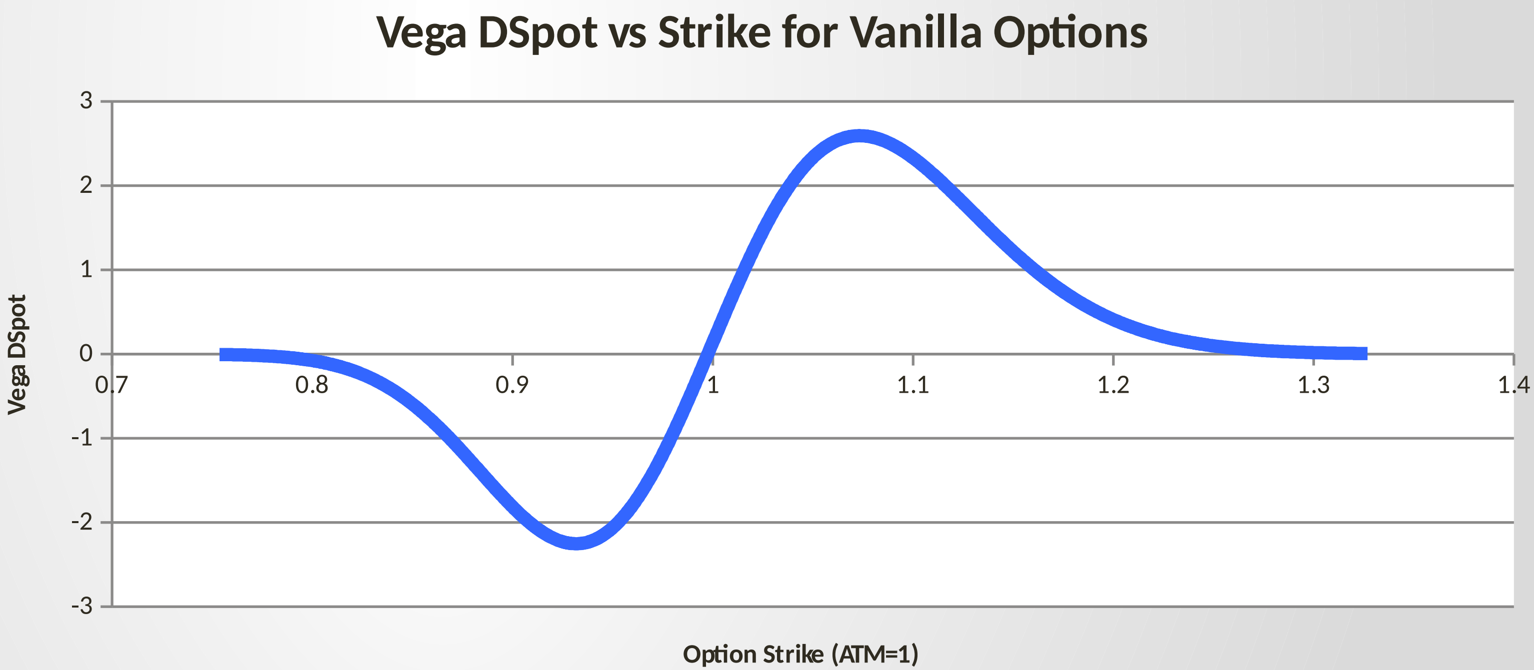
Source of BF



PNL of Option Portfolio vs Volatility When Long Vega Gamma
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
8
8.5
9
9.5
10
10.5
11
11.5
12
PN L Due to Volatility Move ($)
Volatility (%)
Vega
Vega (After Hedge)

Source of RR

- The convexity that drives risk reversal (skew) value is vega dspot: d(vega)/d(spot), or the cross gamma between spot and volatility ("vanna")



Source of RR

- Buy a high strike option
  - Positive vega dspot, positive vega
- Sell enough ATM to vega hedge
  - ATM option has zero vega dspot, still long vega dspot
- Assume positive spot/vol correlation
  - Spot moves up, vol moves up, vega turns positive
    - Make money!
  - Spot moves down, vol moves down, vega turns negative
    - Make money!

Source of RR

- Reverse is true if you buy a low strike option and hedge its vega with an ATM option
  - Negative vega dspot

- Assume positive spot/vol correlation
  - Spot moves up, vol moves up, vega turns negative
    - Lose money.
  - Spot moves down, vol moves down, vega turns positive
    - Lose money.

- With positive spot/vol correlation, traders bid up high strike options and offer on low strike options
  - Drives the skew

Source of RR

- Positive spot/vol correlation leads to positive skew
  - Magnitude of the skew is proportional to spot/vol correlation and volatility of volatility
- Negative spot/vol correlation leads to negative skew
  - All the signs of PNLs in previous examples flip

Source of RR and BF

- This analysis holds for any stochastic volatility model
  - Including local volatility models
  - And just qualitatively – details depend on specific model
- Butterfly comes from volatility of volatility
  - Via symmetric vega gamma profile of vanilla options
- Risk reversal comes from spot/vol correlation
  - Via asymmetric vega dspot profile of vanilla options
  - Also proportional to volatility of volatility

Delta

- What does delta mean?
- Market convention for quoting options/volatility
  - Keeps vol-by-strike fixed
- However, implied volatility as a function of strike does not stay fixed as spot moves
- Need some “market model” that defines how implied volatility moves when spot moves

Delta and Market Models

Some example volatility market models

- “Sticky delta”: vol-by-delta stays fixed as spot moves
  - Most common in FX markets since vol-by-delta is quoting convention
- “Sticky strike”: vol-by-strike stays fixed as spot moves
  - Rare in FX markets, as market doesn’t move that way
  - Common in equity markets as vol quoted by strike
- Models
  - eg local volatility or Heston model predict how vol moves
- Ad hoc
  - eg incorporate historical covariance btw vols and spot

Delta and Market Models

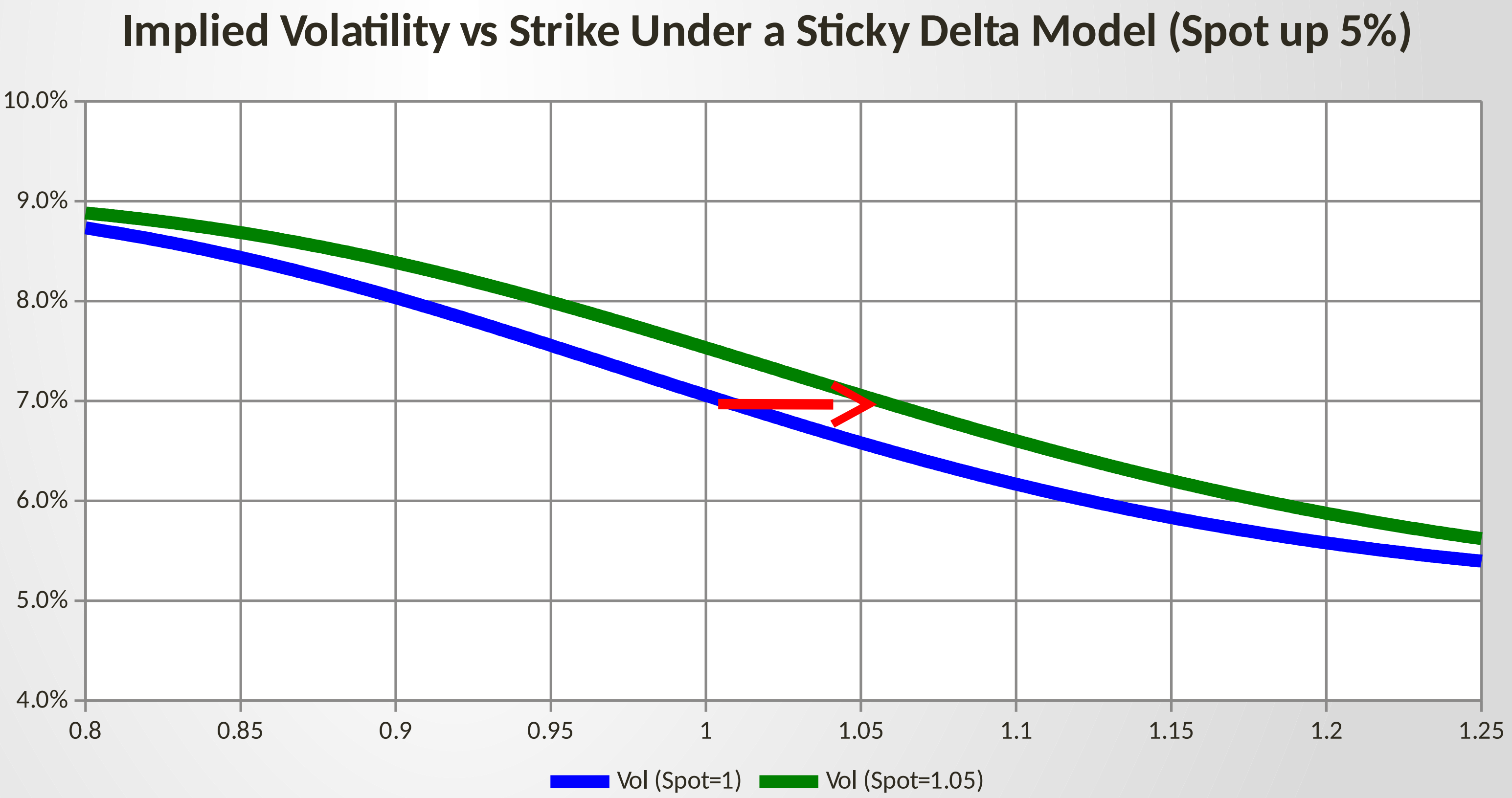
- For a vanilla option: the delta differs from BS delta by a correction based on vega

$$\frac{dv}{dS} = \frac{\partial v_{BS}}{\partial S} + \frac{\partial v_{BS}}{\partial \sigma} \frac{d\sigma}{dS}$$

- v is the price of an option; v_{BS} is the Black-Scholes price of the option using the implied volatility for the strike σ
- First term is the Black-Scholes delta, assuming strike vol is unchanged
- Second term is Black-Scholes vega * derivative of strike vol with respect to spot (which comes from market model)

Delta and Market Models

- What does the slope of strike vol vs spot look like?
  - Sticky delta: a function of the slope of implied vol vs strike



Delta and Market Models

- What is the right market model to use?
- Most desks use sticky delta
  - Mostly because they mark vol by delta and want to think about PNL that way
- Not the most efficient way to delta hedge!
  - There really is some correlation between moves in spot and moves in volatility
- If volatility (as an asset) is relatively liquid then it doesn’t matter much

Delta and Market Models

- The delta adjustment we saw before works for portfolios of options in addition to regular options

- If a portfolio's vega is small, the vega correction to delta is small, and the choice of market model doesn't impact delta

- In general: if an option market is liquid, people don't spend a lot of time thinking about how to hedge vol moves most efficiently with spot
  - True in G7 FX markets
  - Less true in some EM FX markets, or in other markets like interest rates

Volatility Market Models

- What is the best market model to use to have the most effective delta?
  - One that includes spot/vol correlation and that includes spot/RR correlation
  - We already saw that spot/RR correlation is significant (the risk reversal beta)

- Spot/vol correlation is not that stable
  - But the risk reversal is proportional to spot/vol correlation, as we saw earlier
  - Can use that in a regression

Volatility Market Models

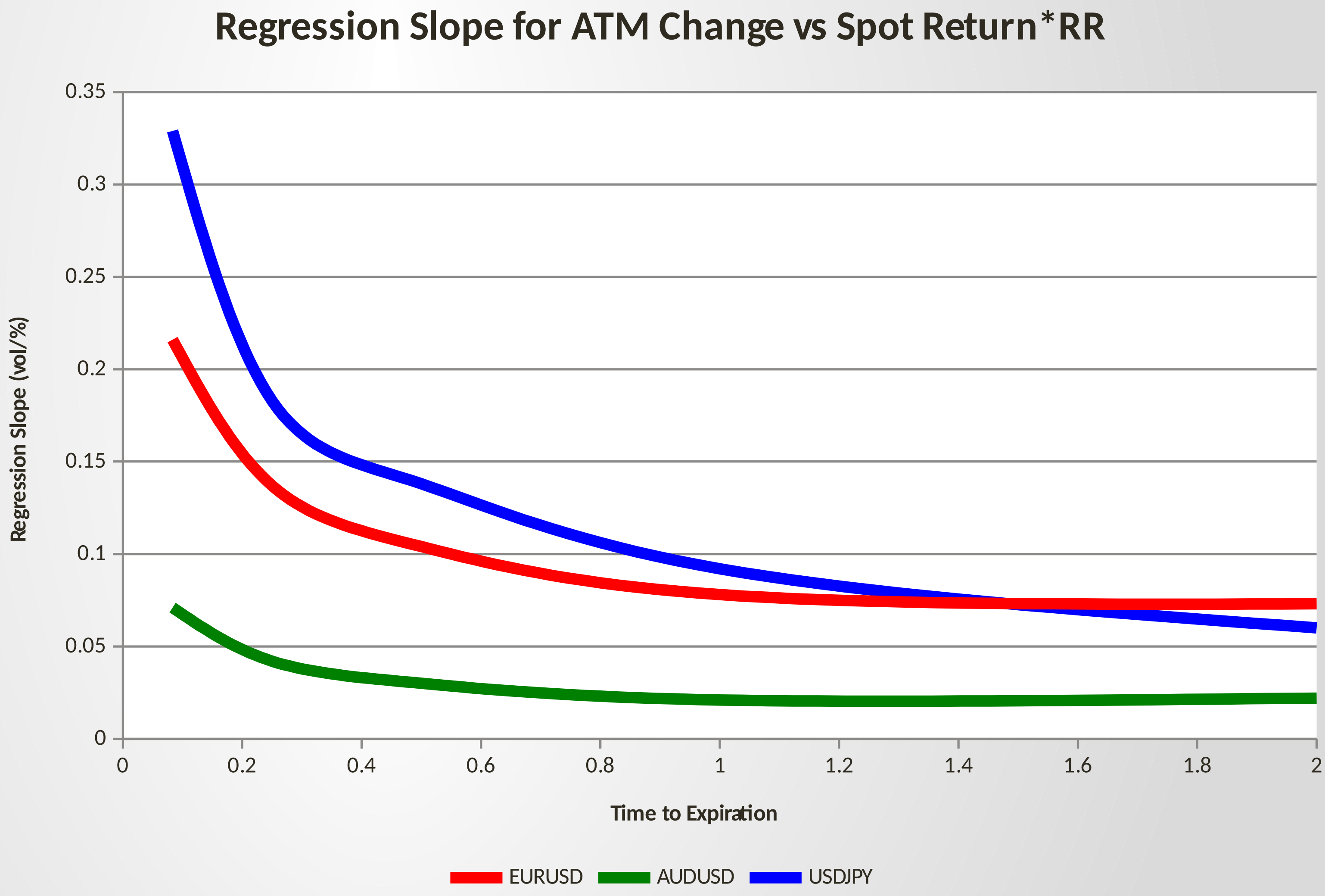
- Sensible way to regression vol move against spot return:

$$\Delta\sigma(T) = A\, RR(T)\, \Delta \ln(S)$$

- $\sigma(T)$ is implied volatility for tenor T
- RR(T) is 25-delta risk reversal for tenor T
- S is the underlying spot
- A is the regression slope (to be determined)

- Get decent R^2 with this regression
  - Significantly better than if you leave out the RR(T) factor if RR moves a lot

Volatility Market Models



Vega

- Vega is risk to moves in implied volatility
- More specifically in FX it means risk to moves in ATM implied volatility
  - Separate risks to moves in RR and moves in BF
  - In vega calculation, keep RR and BF fixed
    - Does not result move vol-by-strike in parallel; instead vol-by-delta moves in parallel
  - Risk orthogonalized along the ATM/RR/BF axes because that's how options are quoted in the inter-dealer market
- Separate vegas for each benchmark tenor
  - Dimensionality reduction is key here to get manageable risks

RR/BF Risk

- Vega is risk to ATM vol moves; RR risk is risk to RR moves; BF risk is risk to BF moves

- RR risk: 25-delta or 10-delta?
  - In general consider risks to both, separately
  - In practice, look at risks to 25d RR assuming that the 10d:25d RR ratio stays fixed

- BF risk: same thing

- Separate RR and BF risks for each benchmark tenor

Correlation Risk

- How is vega defined for a cross pair?
- For liquid pairs, just look at vega to implied volatilities
  - They trade in the market so are valid hedge instruments
- Less liquid pairs, however, have less liquid option markets
  - Maybe try to hedge with USD pairs?

Correlation Risk

- Consider a Black-Scholes world where there are two USD pairs, and a cross pair defined as the ratio of the two USD pair spots

$$dS_1 = \sigma_1 S_1 dz_1$$
$$dS_2 = \sigma_2 S_2 dz_2$$
$$E\left[dz_1 dz_2\right] = \rho\, dt$$
$$dS_x = d\left(\frac{S_1}{S_2}\right) = \left(\sigma_1 dz_1 - \sigma_2 dz_2\right) S_x = \sigma_x S_s dz_x$$
$$\sigma_x = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}$$

- So the cross volatility σ_x can be expressed in terms of the two USD-pair volatilities plus a correlation

Correlation Risk

- Can think of cross-pair vega then as vega to the two USD-pair volatilities plus risk to a correlation
- That correlation is observable in the market (via the cross-pair options market implied volatilities)
- If it is sufficiently stable, hedging cross-pair options with USD-pair options can be efficient
  - No need to do this if the cross-pair options market is liquid
  - But hedging costs this way can be lower if the cross-pair market is illiquid

Volatility Relative Value

- Options risk management is complex and has a lot of moving parts
- That complexity also leads to opportunities for market inefficiencies
  - Usually small, but if you are efficient about picking up nickels and dimes every day you can make a nice profit over the long run
- In fact it's hard to make money market-making if you do not look for relative value opportunities
  - Bid/ask spreads too tight currently to make much money with naïve market-making
  - Increasingly this is what quants spend time doing on the sell side

Volatility Relative Value

- One relative value signal: ATM curve relative value

- If one point on the ATM volatility curve seems out of whack with the rest, buy/sell that point and sell/buy the rest
  - In practice you do not run a separate strategy for this and pay spreads on all the legs
  - Instead you use relative value to decide which is the best hedge to do
    - eg imagine long 6m vega
    - Could hedge by selling 6m vega: low residual risk
    - Or could hedge by selling 3m vega if 3m vol is too high relative to 6m
      - Higher residual risk but perhaps good return

Volatility Relative Value

- Need to start with a model to help define relative value

- Consider a Black-Scholes model with time-dependent but deterministic “mean reverting” volatility
  - Like Heston but with zero vol of vol

$$dv(t) = \beta\left(\bar{v} - v(t)\right) dt$$

$$v(0) = v_0$$

- v(t) is instantaneous volatility^2 of spot=$\sigma^2(t)$
- Three parameters: initial v, mean v, and mean reversion speed

Volatility Relative Value

- Best-fit those three parameters to ATM vols at benchmark tenors
  - Really should fit against trading time vols, especially if you include short-dated tenors

$$\sigma_I(T) = \sqrt{\bar{v} + \frac{(v_0 - \bar{v})}{\beta T}(1 - e^{-\beta T})}$$

- $\sigma_I(T)$ is the implied volatility for expiration time T in this simple model
  - Function of the three parameters

Volatility Relative Value

- Define a trading signal:
  - Do a best fit of that form against benchmark ATM vols from 1w to 1y tenor
  - If any residual is above a given threshold (in absolute value), signal to do a trade
  - Trading signal is buy(/sell) ATM option at the tenor with the residual, sell(/buy) ATM options at spanning tenors
  - Notional of hedge options set to hedge triangle shocks
  - Delta hedge package
  - Hold for one day then take off and record PNL
  - All trades done at mid, trade 1 unit of vega of the target option
    - Then average PNL represents average vol spread capture
  - Can have more than one trade/day if multiple residuals appear, and may have days with no trades at all

Volatility Relative Value