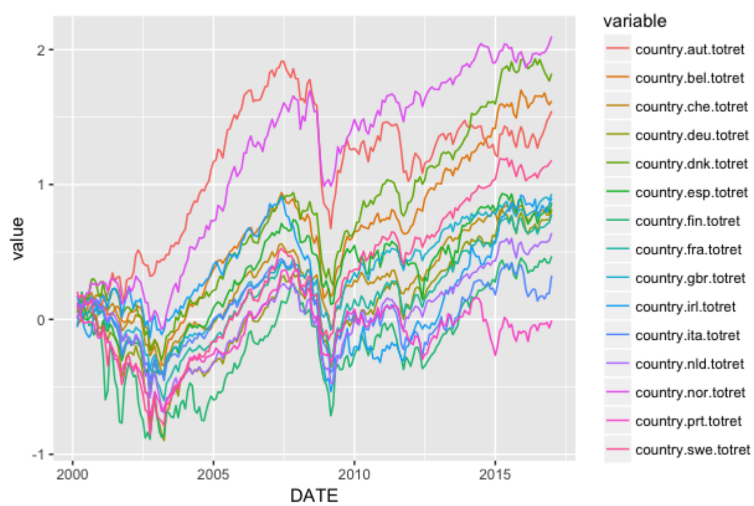
Homework 4

This assignment will guide you through the implementation and backtesting of a simple Black–Litterman optimization model.

Your input data consist of monthly total returns and end-of-month market capitalizations of the 15 most liquid countries in Europe over the sample period 2000–2016. Each row in the provided file contains the country total return in month t and the USD market cap on the last trading day of the previous month $t-1$, where t is the month represented in the DATE column. For the notational conventions in the following problems, refer to the lecture notes on the Black–Litterman model.

Problem 4.1. Load the data and plot the cumulative sum of the raw returns for all countries together, as a way of verifying that you have the data correctly loaded. Your picture should look something like the following:



Problem 4.2. Write a function which takes an input data frame, and estimates the volatilities and correlations that you will need. Specifically, let t be the date of the last row in the input. As discussed in Lecture, estimate volatilities and correlations separately. Let S be a diagonal matrix with the vols on the diagonal, and let R be the correlation matrix, so that the covariance matrix is $\Sigma = SRS$. Use a shrinkage estimator, specifically,

```
R <- shrinkage * R + (1 - shrinkage) * Ident
```

where `shrinkage` is a parameter, and `Ident` is an identity matrix of the appropriate dimension. Taking the full sample, create a plot of the condition number of the covariance matrix versus the shrinkage parameter. How much shrinkage is necessary to achieve a well-conditioned covariance matrix? For the rest of the exercises below, set shrinkage to 0.5.

Problem 4.3. Let X be a portfolio with weight $1/4$ in each of the four countries `nor, swe, dnk, fin`. Let Y be a portfolio with weight zero in those four countries, but weights proportional to market caps in all other countries in the data set we have. Renormalize Y so that $\sum_i Y_i = 1$. Suppose you have one single view, which is that $X - Y$ will have a return of $q = 0.01$ with uncertainty $\omega = 0.015$. Write code to compute the $1 \times k$ matrix P, the 1×1 matrix Ω and the one-dimensional vector q. Here, k is the number of countries, 15 in this case.

Problem 4.4. Write a function which takes an input data frame and computes the Black-Litterman portfolio appropriate for use in the last month of the input. Use the views computed in the previous part, and use parameter values $\tau = 0.01, \kappa = 1.0$. Compute the Black–Litterman portfolio h^*, print out its weights, and also create a scatterplot with h_{eq} weights on the x-axis and h^* weights on the y-axis.
Hints: Let t be the date of the last row in the input data. Your program should first compute the covariance matrix Σ using the function you wrote earlier, but be careful about what you pass as input to that function. Specifically, be sure to use only returns up to and including $t - 1$ in order to avoid lookahead bias. Then compute the CAPM equilibrium h_{eq} using the provided market capitalizations, and normalize so that $\sum_i h_{\text{eq},i} = 1$.

Problem 4.5. Using the function you wrote in the previous part, write a simple backtest which, for each t, computes the BL optimal portfolio h^* and equilibrium portfolio h_{eq} that were knowable as of the last trading day of month $t - 1$, and compute the dot product of these portfolios with the returns in month t. Plot the cumulative sum of the two time-series of returns $h^*_{t-1} \cdot r_t$ and $h_{\text{eq},t-1} \cdot r_t$ on the same axes. Start from $t = \lfloor N/4 \rfloor$ where N is the total number of dates (after all, you need some history to compute the covariance matrix.)