3. Floating-Rate Instruments

Exercises

1. Suppose the LIBOR discount factor curve for the first year is as follows:

t	discount factor
0	1
0.25	0.99782972
0.5	0.994845185
1	0.987439389

(a) What is the fair rate for an FRA on 3M LIBOR where the floating rate is observed in 3 months?

(b) What is the present value of an FRA on 3M LIBOR where the floating rate is observed in 6 months if you receive the fixed rate of 1% on a notional of $10 million? (Interpolate by assuming that the instantaneous forward rate is constant between time points.)

(c) You encounter an FRA from 6M to 1Y where the fixed leg is quoted, as usual, as a simple rate of interest, but the floating rate is 3M LIBOR compounded over the term of the FRA. That is, in 6M the 3M LIBOR rate r_1 is observed, then in 9M the 3M LIBOR rate r_2 is observed, and the floating amount paid on the terminating date is:

$$N\left[(1+r_1\Delta t)(1+r_2\Delta t)-1\right]$$

Determine the fair fixed rate for this contract. Support your answer by showing how such a contract may be hedged or replicated.

2. When the term structure of zero rates is…

t	r (continuous)
0.25	0.012
0.5	0.0155
1	0.018
2	0.02

…you are asked to report certain characteristics of an FRN with the following terms and conditions:

principal: 100
reset and coupon frequency: quarterly
time to maturity: 1.2 years
floating rate margin: 2%
most recent reset rate: 0.0145

For the purposes of this question, disregard the possibility that the bond may default. Interpolate in the term structure of interest rates by assuming that the continuously compounded zero rates are linear, and that the rate is constant between 0 and 0.25 years.

(a) What are the projected cash flows of the bond?
(b) What is the present value of the bond?
(c) What is the bond's duration? Its convexity? (Use the more general form for these analytics based on dollar duration.)

3. For the following term structure of discount factors….

t	df
0.5	0.975309912
1	0.946485148
1.5	0.917364861
2	0.884263663

(a) Determine the par swap rate for a 2-year swap whose fixed leg pays annually.
(b) Determine the par swap rate for a 2-year swap whose fixed leg pays semiannually.
(c) Determine the forward par swap rate for a 1-year swap commencing in 1 year whose fixed leg pays semiannually.

Applications
OIS Discounting
Although the most commonly traded interest rate swaps reference a tenor of IBOR, the entities that most often deal in these contracts fund at a lower rate referenced by Overnight Indexed Swaps. This funding curve—the "OIS" curve—is the appropriate one for banks to use in determining the fair rate for the swaps they enter into.

The method for determining OIS and IBOR curves depends upon the instruments available in the currency of interest, and the FE's choices of interpolation method in the various curves. This problem is modeled on the EUR swaps market (although the rates are invented), where OIS trade as fixed for EONIA compounded, with semiannual payments on both legs, and vanilla swaps trade as fixed for 6M EURIBOR, again with semiannual payments on both legs.

(i) Suppose the term structure of OIS swaps is:

T	par swap (s/a)
0.5	-0.0025
1	-0.00125
1.5	-0.0005
2	0
3	0.0012
5	0.0025
7	0.0075
10	0.01
30	0.0205

The rates above are all decimals. Use the sandBox utilities to develop a bootstrapper that produces the zero rates for the OIS discounting curve, and show its results using these par swap rates. Assume linear interpolation in the continuously compounded zero rates.

(ii) Suppose the term structure of vanilla swaps is:

T	par swap (s/a)
0.5	0.0055
1	0.007
2	0.0125
3	0.0165
5	0.01725
7	0.0225
10	0.0325
20	0.04
30	0.045

Use the sandBox utilities to develop a bootstrapper that produces the EURIBOR rates implied by the above, given the term structure of OIS rates you determined above. Once again, assume linear interpolation in the continuously compounded zero rates.

(iii) A basis swap is a float-float interest rate swap where the two legs reference different interest rates. The leg referencing the rate that is generally lower pays that rate plus a spread, in exchange for the rate that is generally larger. For example, in EUR a basis swap between OIS and IBOR would pay EONIA compounded + spread in exchange for 6M EURIBOR, with payments exchanged semiannually.

Determine the term structure of basis swap spreads for maturities from 1 year to 30 years, spaced one year apart. Again assume linear interpolation in the continuously compounded zero rates.

(iv) Consider three fixed-for-EURIBOR swaps in which you receive fixed: one with negative present value, one at par, and one with positive present value. We wish to characterize the risk of each of these positions; you may illustrate your answer with examples constructed using the sandBox utilities:

(a) What is the exposure of each to a simultaneous parallel shift in both the EURIBOR and OIS rates?

(b) What is the exposure of each to a compression in the spread between the EURIBOR and OIS rates? (Is there a notable difference if this is thought of as OIS rising while EURIBOR remains the same, or EURIBOR dropping while OIS remains the same?)

(c) Similar to (b), what is the exposure of each to a widening in the spread between the EURIBOR and OIS rates?