MTH 9831 Assignment 11 (12/02/2015 - 12/09/2015).

Let $\{N(t)\}_{t\geq 0}$ be a Poisson process with intensity λ, $M(t) = N(t) - \lambda t$, $t \geq 0$, be a compensated Poisson process, $\{Q(t)\}_{t\geq 0}$ be a compound Poisson process with jump distribution $\mathbb{P}(Y_1 = y_m) = p_m$, $m \in \{1, 2, \ldots, M\}$, and $M_Q(t) = Q(t) - \beta\lambda t$ ($\beta = \mathbb{E}Y_1$), $t \geq 0$, be a compensated Poisson process.

(1) Find the process $[Q, Q](t)$, $t \geq 0$, and compute its expectation (not just quote, compute!). What is $[M_Q, M_Q](t)$, $t \geq 0$?

(2) Let $\varphi(t)$, $t \geq 0$, be a left-continuous square-integrable process adapted to the filtration of $M(t)$, $t \geq 0$. Show that

$$\mathbb{E}\left(\int_0^t \varphi(s)dM(s)\right)^2 = \lambda\mathbb{E}\int_0^t \varphi^2(s)\,ds.$$

Find

$$\mathrm{Var}\left(\int_0^t 2^{M(s-)}dM(s)\right).$$

Hint: adapt the calculation between equations (5) and (6) of Lecture 12.

(3) Let $\varphi(t)$, $t \geq 0$, be a left-continuous square-integrable process adapted to the filtration of $M_Q(t)$, $t \geq 0$. Show that

$$\mathbb{E}\left(\int_0^t \varphi(s)dM_Q(s)\right)^2 = \lambda\mathbb{E}(Y_1^2)\mathbb{E}\int_0^t \varphi^2(s)\,ds.$$

Hint: the method of the previous problem works here too.

(4) Exercise 11.6 from the textbook.

(5) Let under $\mathbb{P}$ the process $S(t)$, $t \geq 0$ be a solution to

$$dS(t) = \mu S(t)dt + \sigma S(t)dB(t) + S(t-)dQ(t).$$

Find a change of measure, such that under the new measure the process $e^{-rt}S(t)$, $t \geq 0$, is a martingale.

(6) Suppose that under a risk-neutral measure the stock price can be represented as follows:

$$S(t) = S^*(t)e^{Q(t)},$$

where $S^*(t) = e^{\sigma B(t)+\mu t}$, $B(t)$ is a standard Brownian motion, $Q(t) = \sum_{i=1}^{N(t)} Y_i$ is a compound Poisson process with intensity λ, random variables Y_i, $i \geq 1$, are normal with mean μ_0 and variance σ_0^2, and $\mu = r - \sigma^2/2 - \lambda(Ee^{Y_1} - 1)$, where r is the annual nominal interest rate. Find the time 0 cost of a European call option with strike price K and expiration t. Hint: condition on $N(t) = n$ and recognize each term of the obtained series as a version of a standard Black-Scholes price.