

MTH 9831, Fall 2015. Syllabus (tentative).

- Recommended materials:
 - Lecture notes, handouts, forum discussions, homeworks (see MFE Forum and BlackBoard).
 - S. Shreve. Stochastic Calculus for Finance. Vol. II.
 - J. Jacod, P. Protter. Probability Essentials.
- Basic skills:
 - (1) Computation of quadratic variation, cross variation for Itô and jump processes.
 - (2) Computation of the distribution of a process that can be expressed as an integral of a deterministic integrand with respect to a BM.
 - (3) Finding out whether a given process is a martingale, super- or sub-martingale (using definitions, Itô's formula, MGF or characteristic function, whichever is appropriate).
 - (4) Finding out whether a given process is a standard BM (dimensions 1 and 2).
 - (5) Computations involving a change of measure for BM with a drift and jump processes.
 - (6) Finding a closed form solution of simple SDEs (for example: generalized GBM, Ornstein-Uhlenbeck process, various short rate models, geometric Poisson process, Doléans-Dade exponential).
 - (7) Computation of the expected value of a function of an Ito or jump process (i.e. finding the expectation, variance, MGF, etc.).
 - (8) Evaluation of simple stochastic integrals.
 - (9) Writing equations for the dynamics of a portfolio value process and a discounted portfolio value process. Self-financing condition.
 - (10) Using Feynman-Kac formula (dimensions 1 and 2).
 - (11) Computing generators and their adjoints for diffusion processes. Writing Kolmogorov forward and backward equations.
 - (12) Identifying Markov processes and using Markov property.
 - (13) Understanding and using optional stopping theorems.
- Students have to know rigorous definitions and basic properties of:
 - (1) Brownian Motion (BM), Geometric Brownian Motion (GBM), generalized GBM, Poisson process, compound Poisson process.
 - (2) Martingale.
 - (3) Stopping time.

- (4) Quadratic and cross variation (for Itô and jump processes).
 - (5) Stochastic integral with respect to BM, Ito process, and jump process (in particular, Itô's isometry).
 - (6) Risk-neutral measures.
 - (7) Arbitrage.
 - (8) Complete market.
 - (9) Markov process.
 - (10) Convex/concave functions.
 - (11) Stochastic differential equation, its components (drift and diffusion); diffusion process.
 - (12) Jump process.
 - (13) Black-Scholes-Merton (BSM) model and Black-Scholes-Merton PDE.
- Students have to be able to state and apply the following theorems:
 - (1) Ito-Doeblin formula (for BM, Itô, and jump processes, all types and dimensions). Particular case: integration by parts (i.e. Ito's product rule).
 - (2) Levy's characterization of BM (dimensions 1 and 2)
 - (3) Girsanov theorem (all dimensions).
 - (4) Martingale representation theorem (all dimensions).
 - (5) Two fundamental theorems of asset pricing.
 - (6) Feynman-Kac representation (dimensions 1 and 2).
 - (7) Optional stopping (=sampling) theorems (Shreve II, p. 342, 373; refresher notes).
 - (8) Analytic and probabilistic characterization of the price of American put options (perpetual and finite expiration).
 - (9) Jensen's inequality and conditional Jensen's inequality.