INTEREST RATES MODELS

Homework assignment #5

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March 27, 2017

Problems

1. Using the market data sets provided for the previous homework assignments, calculate approximately the following CMS rates:

   – 10 year CMS settling in 1 year and paying 3 months later, and
   – 10 year CMS settling in 5 years and paying 3 months later.

   For simplicity, use the Black model approach (method (i) on page 24 of Lecture Notes #5). It produces less accurate results but is much simpler than the replication method. What fraction of the CMS convexity correction comes from the payment delay?

2. Derive the formula for the valuation of a call option on a zero coupon bond in the one factor Hull-White model (formulas (27) - (30) on pages 21 - 22 of Lecture Notes #6). Hint: in a your computation, use expression (22) for the zero coupon bond.

3. In this problem, all formula numbers refer to Lecture Notes #6. We consider an affine term structure model, i.e. a short rate model, with Q_0-dynamics given by equation (1), in which the zero coupon price is given by equation (42).

(i) Let, as usual, $P(t, T)$ denote the time t price of a zero coupon bond for maturity T. Argue that $P(t, T)$ satisfies the following partial differential equation:

$$\begin{aligned} \frac{\partial P}{\partial t} + \frac{1}{2}\,\sigma(t,r)^2\,\frac{\partial^2 P}{\partial r^2} + \mu(t,r)\frac{\partial P}{\partial r} &= rP, \\ P(T,T) &= 1. \end{aligned}$$

This fact does not, of course, require that the model be affine.

(ii) Conclude that in an affine term structure model with coefficients $A(t, T)$ and $B(t, T)$,

$$\frac{\partial \log A}{\partial t} - r\,\frac{\partial B}{\partial t} + \frac{1}{2}\,\sigma(t,r)^2 B^2 - \mu(t,r)B = r,$$

with

$$\begin{aligned} A(T,T) &= 1, \\ B(T,T) &= 0. \end{aligned}$$

(iii) Show that if the coefficients in the SDE (1) are of the form (43), then the partial differential equation above reduces to the system (44) of ordinary differential equations.

This assignment is due on April 3.