Forwards and Futures
Options in Discrete Time

Exercises
1. The term structure of risk-free interest rates is as follows:

t	r
3m	0.025
6m	0.027
9m	0.03
1y	0.0285
15m	0.0295
18m	0.03
21m	0.0305
2y	0.031
3y	0.04
4y	0.0465
5y	0.05

All rates are expressed with continuous compounding. Determine the forward price in 6 months of the following assets:

(a) A zero-coupon bond with maturity nine months, per 100 face.
(b) A bond paying a 5% coupon semiannually with maturity 1.25 years, per 100 face.
(c) An equity with spot price 100 paying a 1.2% dividend continuously.
(d) An equity with spot price 300 that is expected to split 3 for 1 in 2 months. (Express the forward as a price per share.)
(e) A foreign currency with spot exchange rate 1.2—expressed as number of units foreign currency per unit domestic currency—in which the 6-month risk-free rate is 2.2%, expressed with continuous compounding. (Express your answer as number of units foreign currency per unit domestic currency.)

2. An equity's spot price S_0 is 100 at the moment. Using other traded market instruments, you have determined that the term structure of zero rates and equity forward prices is as follows:

t	r (continuous)	Ft
1m	0.02	100.0625195
2m	0.022	100.2628448
3m	0.0225	100.3882518
4m	0.0245	100.5934205
5m	0.025	100.8200105
6m	0.026	101.0732181

(a) What is the term structure of dividend rates in each month implied by these values? (Express your answer as the continuously compounded dividend rate from now to 1m, from 1m to 2m, 2m to 3m, and so on.)
(b) Suppose that the equity has an established record of paying its dividend, if any, at the end of each month, relative to the date for which you extracted this data. Under this assumption, what the is amount of the lump-sum cash dividend at the end of each month implied by this data? (Assume that the forward price as expressed here is *after* the payment of the fixed dividend.)

3. Use a two period binomial model and the CRR up and down steps with the following parameters…
$S_0 = 30$
$r = 0.05$
$q = 0$
$T = 0.5$
$\sigma = 0.4$
…to price each of the following options:

(a) A European call option with strike $K = 25$
(b) A European put option with strike $K = 25$

At each node in your lattice, give:
the spot price
the value of the option
the delta of the option

4. Use a two period binomial model and the CRR up and down steps with the following parameters…
$S_0 = 50$
$r = 0.03$
$q = 0.025$
$T = 0.25$
$\sigma = 0.6$
…to price each of the following options:

(a) An American call option with strike $K = 41$
(b) An American put option with strike $K = 52$

At each node in your lattice, give:
the spot price
the value of the option if its holder chooses not to exercise (the continuation value)
the value of the option if its holder chooses to exercise (the intrinsic value)
the value of the option
the delta of the option

Applications

1. *Binomial Tree Pricing*

For this exercise, you will implement a binomial tree pricer in VBA for European and American options. We will construct the asset's tree using the initial values:

$$S_0 = 100$$
$$r = 0.01$$
$$q = 0.0025$$
$$T = 0.5$$
$$\sigma = 0.3$$

(a) Unit test 1: Verify that your tree construction is correct by pricing long forward contracts at three strikes: ATM, 10% ITM, 10% OTM and demonstrating agreement with the theoretical prices of these contracts.

(b) Unit test 2: Verify your tree's option prices by comparing the prices of vanilla European calls and puts to the Black-Scholes value using the averaging binomial method with the following series of time steps: 125, 250, 500, 1000, 2000, 4000, demonstrating convergence to the B-S values.

(c) Use your tree pricer to estimate the early exercise region for an ATM American put as a function of time. Price at 500 time steps and show the upper bound price for early exercise at each time step.

(d) Do the same for an ATM American call, pricing at 500 time steps and showing the lower bound price for early exercise at each step.

(e) Conventionally, early exercise of an American call is never optimal when the dividend rate is zero. However, in the presence of negative interest rates, the reasoning behind this conclusion no longer holds, and early exercise may be optimal. Demonstrate this by setting $q = 0$ for the asset and estimating the early exercise region when $r = -0.01$, again using 500 time steps.

(f) Do you think there is some configuration of interest rate and dividend rate that could cause an American option with a vanilla payoff to have *two* early exercise regions? If not, explain your reasoning. If so, then give an example and show your estimate of the exercise regions using your binomial tree pricer.