

MTH 9814  
Fall 2014  
Course Outline

I. Discount Factors and Interest Rates

Instruments:

Zero-coupon bond

Concepts:

Arbitrage-free value, "Law of One Price"

Valuation via replication / hedging arguments

Discount factor, forward discount factor

Interest rates

Interest rate quotation methods:

Simple interest

Discrete compounding

Continuous compounding

Conversions between / among interest rates, discount factors

Interest rate interpolation methods:

Linear spot rate interpolation

Constant forward rate interpolation

Cubic spline

Instantaneous forward rate

II. Yield and Duration

Instruments:

Fixed-coupon bond

Fixed-rate annuity, forward-starting annuity

Concepts:

Bond yield, forward yield

Par yield, forward par yield

Bootstrapping term structure of interest rates from bond prices

Solution methods for nonlinear equations:

Bisection method

Newton's method

Secant method

Discretely compounded yield

Interest rate sensitivities:

Macaulay duration / convexity

Modified duration

Approximation of bond PL using duration / convexity

### III. Floating-Rate Instruments

#### Instruments:

- FRA
- FRN
- Fixed / float IR swap

#### Concepts:

- Present value of a floating-rate coupon
- Interest rate sensitivities:
  - IR DV01
  - Dollar duration / convexity
  - Effective duration / convexity
  - Aggregate duration / convexity of a portfolio of bonds
- Par swap rate, forward par swap rate
- Methods for valuation of fixed / float IR swaps:
  - Long / short fixed bond + short / long FRN
  - Strip of FRA's
  - Multiple of annuity
- Bootstrapping term structure of interest rates from FRA's and IR swaps

### IV. Valuation of Credit-Risky Instruments

#### Concepts:

- Survival probability, unconditional and conditional
- Default as a Poisson process (memorylessness)
- Hazard rate, forward hazard rate
- Z-spread
- No-recovery default model:
  - Yield as decomposition of IR and hazard rate
  - Correspondence of z-spread and hazard rate
  - Term structure of hazard rate and relationship to z-spread
- Recovery and loss given default (LGD)

### V. Hedging Credit-Risky Instruments

#### Instruments:

- CDS

#### Concepts:

- Duration of credit-risky cash flows
- Duration of credit-risky FRN
- Fair spread of a credit-risky FRN
- Par CDS spread
- CDS valuation concepts:
  - Premium leg, including accrued premium at default time
  - Protection leg (recovery and LGD)
- CDS upfront payment

## VI. Forwards and Futures

### Instruments:

#### Forwards on:

- Zero-coupon bonds
- Fixed-coupon bonds
- Equities
- FX

#### Futures

### Concepts:

- Static hedge of a forward contract
- Using zero-coupon bonds in the hedge of an instrument paying fixed cash flows
- Using the underlying asset in the hedge of an instrument paying proportional cash flows
- Adjusted spot price
- Differences between forward contracts and futures contracts
- Relationship of forward price and futures price

## VII. Options in Discrete Time

### Instruments:

- European / American Call / Put Options

### Concepts:

- Single- and multiple-period binomial model:
  - Construction of tree (risk-neutral transition probabilities)
  - CRR parameterization
  - Calibration to forward price
  - Valuation of European / American claims on the binomial lattice
  - Delta hedge of European / American claims on the binomial lattice
  - Convergence properties
  - Averaging binomial method
- Self-financing replication in the binomial model

## VIII. Options in Continuous Time

### Concepts:

- Lognormal distribution of terminal asset price
- Divisibility and the lognormal distribution
- Valuation of European claim using lognormal distribution
- Closed-form valuation under lognormal assumption:
  - European call option
  - European put option
- Wiener process
- Geometric Brownian motion:
  - SDE of asset with GBM dynamics
  - SDE of that asset's log-return
- Put-call parity
- Intrinsic value and time value of an option
- Optimal exercise of American call options
- Optimal exercise of American put options

## IX. BS PDE and Greeks of European Vanillas

### Concepts:

- Ito's Lemma
- Self-financing replication in continuous time
- Black-Scholes PDE
- Greeks definitions:
  - Delta
  - Gamma
  - Vega
  - Theta
  - Rho
- Closed-form formulas for Greeks of European vanilla options

## X. Option Combinations

### Instruments:

- European Binary Call / Put Options

### Concepts:

- Option combinations:
  - Vertical spreads:
    - Bull spread
    - Bear spread
    - Butterfly
    - Risk reversal
    - Strangle
    - Straddle
  - Calendar spread
  - Diagonal spread
- Binary put as limit of bear spread
- Relationship between / among:
  - Forward prices of binary puts
  - Risk-neutral CDF of terminal spot price
  - Derivative of put prices with respect to strike
- Breeden-Litzenberger Result
- Dirac delta function as limit of butterfly spread
- Relationship between / among:
  - Forward prices of (imaginary) "Dirac delta" options
  - Risk-neutral PDF of terminal spot price
  - Second derivative of put prices with respect to strike
  - Second derivative of call prices with respect to strike
- No-arbitrage requirements of implied vol surface:
  - Per maturity: All butterfly spreads have positive value
  - Across maturities: Options with same discounted strike have nondecreasing value

## XI. Chocolate Options

### Instruments:

- Barrier Options (Up / Down and In / Out Call / Put)

- Spread Options

### Concepts:

- Trinomial tree:

  - Construction

  - Valuation on the lattice

- Monte Carlo:

  - Valuation estimate:

    - Updating

    - Convergence

  - Standard deviation of estimate

  - Variance-based stopping criterion

  - Variance reduction via control variate:

    - Requirements of control variate

    - Best choice of control variate

    - Optimal coefficient of control variate in value estimate

    - Successive estimation / updating of coefficient

- Quasi-analytic valuation of spread options (tower property)

## XII. FX Derivatives

### Concepts:

- Symmetry of FX forward valuation in underlying currencies

- Garman-Kohlhagen valuation formula

- Put-call duality of FX options

- Siegel's paradox:

  - Change of measure for currency pairs

- Risk-neutral probabilities of FX option exercise

- Reflection principle of Brownian motion

## XIII. IR Options

### Instruments:

- Caps / Floors

- Swaptions

### Concepts:

- Arbitrage-free GBM dynamics in forward measure

- Black 76 valuation formula

- Zero-coupon bond as numeraire for FRA's and caps / floors

- Annuity as numeraire for swaps and swaptions