Credit Risk Models

3. Credit Default Swaps

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Outline

Mechanics of a CDS

A credit default swap (CDS) is the most commonly traded type of credit derivatives. It is an unfunded contract written on the credit worthiness of a reference entity ("name").

- CDSs used to be traded over the counter (OTC). However, over the past few years, there has been a move toward central clearing of CDS contracts (ICE and CME).
- A CDS has a stated amount of notional, i.e. the face value of the reference name.
- A CDS settles on the next business day following the trade date. It has a stated maturity (most liquid terms are 3, 5, 7, and 10 years).
- A CDS has two *legs*:
    - *Protection leg:* if a credit event (default) occurs prior to the contracts maturity, protection seller has to compensate the protection buyer.
    - *Premium leg:* in return for assuming the credit risk of the underlying name, the protection buyer pays a premium (upfront fee + periodic coupon). These payments end at contract's maturity or following a default, whichever happens first

Mechanics of a CDS

- The counterparty paying the premium is *long* the CDS (short the credit of the refernce name). This counterparty benefits from deteriorating credit of the name.
- The counterparty receiving the premium is *short* the CDS (long the credit of the reference name). This counterparty benefits from improving credit of the name.

Mechanics of a CDS

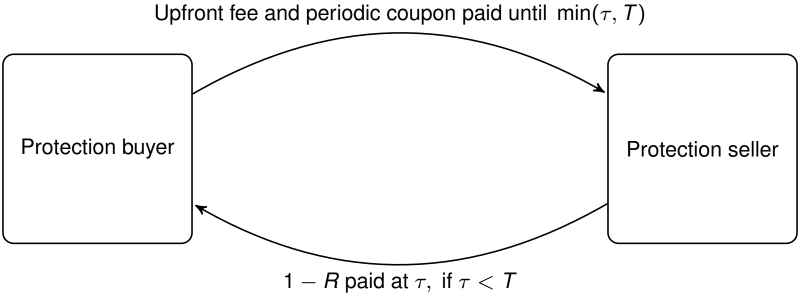

Figure: Cash flows on a credit default swap

Mechanics of a CDS

- Credit default swaps are reminiscent of insurance contracts.
- Some important differences between CDSs and insurance polices are:
  - An insurance policy provides protection against losses suffered by the policy holder on the insured asset (property, health, life,...) that they hold. In contrast, the holder of a CDS does not have to hold the reference asset.
  - CDSs are subject to mark to market, while insurance contracts are not.
  - CDSs are tradeable, while policies are not.
  - CDSs are terminated by unwinding at the mark to market. In order to terminate an insurance policy, it is enough for the holder to stop paying the premium.

Quarterly roll

- Until 2003, the standard convention was that the maturity date of a CDS was T years from the settlement date (adjusted by the *following business days convention*).
- Since 2003, the market adopted standard *quarterly roll dates* on March 20, June 20, September 20, and December 20 (similar to the IMM dates[1] from interest rates markets). A T-year CDS matures on the first IMM date (adjusted for business days) following T years from the settlement date.

[1] We will refer to these dates as the IMM dates, even though, stricly speaking, they are not identical with the IMM dates.

Mechanics of the premium leg

- The maturity of the CDS is the first IMM date (adjusted for business dates) following T years from the settlement date.
- All the IMM dates between the settlement dates and the maturity date are called the roll dates.
- Coupon payments take place on the roll dates (adjusted for business dates).
- The premium amounts are calculated based on the “act/360” day count fraction convention. The dollar amount of a premium payment equals

$$cpn \times \frac{numDays}{360} \times Notional,$$

where *cpn* is the annualized periodic coupon, and *numDays* is the number of days in the accrual periods.

Mechanics of the premium leg

- The first accrual period is usually a *stub* period:
  - *short stub*: if the trade date is before 30 days prior to the first IMM date, the premium is paid on the first accrual date for the number of days of effective protection during that period.
  - *long stub*: if the trade date is within 30 days before the first coupon date, no premium payment is made on the first accrual date.
- In the event of a default, accrued premium at default has to be paid.
- In the USD market the coupon is usually 100 bp (for investment grade reference name) or 500 bp (high-yield reference names). The reminder of the premium is paid as an upfront fee.

Mechanics of the protection leg

In the case of a default of the reference name, the protection leg is responsible for making up to the par value of the defaulted instrument.

- *Physical settlement.* Protection buyer delivers a defaulted instrument to the protection seller in order to receive the face value of the defaulted debt.
- *Cash settlement.* The protection seller pays the protection buyer the face value of the defaulted instrument less its recovery price in cash. The recovery price is determined by a dealer poll or an auction.

Pricing the protection leg

We continue assuming that there is no counterparty credit risk.

- In Lecture Notes 1 we priced a risky discount bond, which had zero recovery value in case of a default. In practice, upon default the assets of the defaulted company are split among the various claim holders according to their priority.
- The present value of receiving the cash amount C at time τ is given the expectation:
$$\mathcal{C}(0,T) = \mathrm{E}\Big[C(\tau)e^{-\int_0^\tau r(t)dt}1_{\tau\leq T}\Big].$$
We wish to express $\mathcal{C}(0,T)$ in terms of the credit intensity process.
- Consider first a contract that pays $C(s)$ if the default takes place in time small interval $[s, s+ds]$. The value of this cash flow is
$$\mathrm{E}\Big[C(s)\,e^{-\int_0^s r(t)dt}1_{\tau\in[s,s+ds]}\Big].$$

Pricing the protection leg

- We can rewrite it as

$$\mathrm{E}\Big[C(s)\, e^{-\int_0^s r(t)dt} \lambda(s)\, e^{-\int_0^s \lambda(t)dt}\Big] = \mathrm{E}\Big[C(s)\, \lambda(s)\, e^{-\int_0^s (r(t)+\lambda(t))dt}\Big]$$

- Integrating over s from 0 to T we find that

$$\mathcal{C}(0,T) = \mathrm{E}\Big[\int_0^T C(s)\, \lambda(s)\, e^{-\int_0^s (r(t)+\lambda(t))dt} ds\Big].$$

- If interest rates and the credit intensity are independent, we can write this expression as:

$$\mathcal{C}(0,T) = -\int_0^T \mathrm{E}[C(s)] P(0,s) dS(0,s).$$

- In particular, if we write $C(s) = 1 - R(s)$, i.e. $\mathrm{E}[C(s)] = 1 - R$, where R is the recovery rate, then the value of the protection leg is

$$\begin{aligned} V_{prot}(T) &= \mathcal{C}(0,T) \\ &= -(1-R)\int_0^T P(0,s) dS(0,s). \end{aligned}$$

Pricing the protection leg

- This is a *Stieltjes integral* of the form $\int_0^T f(s)\,dg(s)$ and it can effectively be calculated by means of a numerical approximation.
- For example, we can use the “trapezoid rule”: For $0 = s_0 < s_1 < \ldots < s_n =$, where $s_j = \frac{jT}{n}$, for $j = 0, \ldots, n$,

$$\int_0^T f(s)\,dg(s) \approx \sum_{j=1}^{n} \frac{f(s_{j-1}) + f(s_j)}{2}\,(g(s_j) - g(s_{j-1})).$$

- Using this approximation, we find that

$$V_{prot} \approx (1 - R) \sum_{j=1}^{n} \frac{P(0, s_{j-1}) + P(0, s_j)}{2}\,(S(0, s_{j-1}) - S(0, s_j)).$$

- In practice, it is sufficient to chose the time steps monthly for sufficient accuracy. It is also possible to develop more accurate numerical approximations.

Pricing the premium leg

Consider a basic CDS maturing on T, with the premium consisting of the periodic coupon payments only. Let C denote the annualized coupon. Assume first that we are on a coupon payment date.

- The PV of a \$1 paid on T_j is given by the risky discount factor

$$\begin{aligned}\mathcal{P}(0, T_j) &= \mathsf{E}\Big[e^{-\int_0^{T_j} r(s)ds}1_{\tau > T_n}\Big] \\ &= \mathsf{E}\Big[e^{-\int_0^{T_j}(r(s)+\lambda(s))ds}\Big].\end{aligned}$$

- Let δ_j denote the act/360 based day count fraction factor between T_{j-1} and T_j. Hence, the PV of the coupon streams is

$$C\sum_{j=1}^{N}\delta_j\mathcal{P}(0, T_j)$$

If rates and credit are independent, then it is equal to

$$C\sum_{j=1}^{N}\delta_j P(0, T_j)S(0, T_j).$$

Pricing the premium leg

- This formula is not quite the value of the premium leg, as it does not include the premium accrued at default
- It is equal to the premium accrued from the previous payment date to the default time:

$$-C\int_{T_{j-1}}^{T_j}(s-T_{j-1})P(0,s)dS(0,s)$$

- Using the trapezoid rule for the Stieltjes integral, we can approximate

$$-\int_{T_{j-1}}^{T_j}(s-T_{j-1})P(0,s)dS(0,s)\approx\frac{1}{2}\,\delta_j P(0,T_j)(S(0,T_{j-1})-S(0,T_j)).$$

- As a result, the total value of the accrued is given by

$$-C\sum_{j=1}^{N}\int_{T_{j-1}}^{T_j}(s-T_{j-1})P(0,s)dS(0,s)\approx\frac{C}{2}\sum_{j=1}^{N}\delta_j P(0,T_j)(S(0,T_{j-1})-S(0,T_j))$$

Pricing the premium leg

- As a consequence, the PV of the premium leg is equal to

$$V_{prem}(T) = C\mathcal{A}(T),$$

where $\mathcal{A}(T)$ is the *risky annuity*:

$$\begin{aligned}\mathcal{A}(T) &= \sum_{j=1}^{N} \delta_j P(0, T_j) S(0, T_j) + \frac{1}{2} \sum_{j=1}^{N} \delta_j P(0, T_j)(S(0, T_{j-1}) - S(0, T_j)) \\ &= \frac{1}{2} \sum_{j=1}^{N} \delta_j P(0, T_j)(S(0, T_{j-1}) + S(0, T_j))\end{aligned}$$

Pricing the premium leg

Let us now relax the assumption that we are on a premium payment date. We assume that we have a seasoned CDS, and that the valuation date ("today") falls between two payment dates.

- Let T_k be the first payment date after today. Then,

$$V_{prem}(T) = -C\int_0^{T_k}(s - T_{k-1})P(0,s)dS(0,s) + C\sum_{j=k}^{N}\delta_j P(0,T_j)S(0,T_j)$$
$$- C\sum_{j=k+1}^{N}\int_{T_{j-1}}^{T_j}(s - T_{j-1})P(0,s)dS(0,s)$$

- Approximating as above, we find

$$V_{prem}(T) = \frac{C}{2}\sum_{j=k+1}^{N}\delta_j P(0,T_j)(S(0,T_{j-1}) + S(0,T_j)) + \frac{C}{2}\delta_k^{+}P(0,T_k)(1 - S(0,T_k))$$
$$+ C\delta_k^{-}P(0,T_k)(1 - S(0,T_k)) + C\delta_k P(0,T_k)S(0,T_k),$$

where δ_k^{+} is the day count fraction from today to T_k, and $\delta_k^{-} = \delta_k - \delta_k^{+}$.

Par credit spread

- Mark to market (MTM) of a CDS is given by

$$\begin{aligned} V_{CDS}(T) &= V_{prot}(T) - V_{prem}(T) \\ &= V_{prot}(T) - C\mathcal{A}(T) . \end{aligned}$$

- At inception, the MTM of the swap is zero, $V_{CDS} = 0$, which implies that

$$C_0 = \frac{V_{prot}(T)}{\mathcal{A}(T)} .$$

- This is the *par credit spread* on the swap.

Conventional spread CDS

- The calculations above can easily be extended to a CDS with a combination of an upfront fee and a running spread.
- Consider a new CDS with a conventional coupon C_{run} (usually 100 or 500 bp), and let U denote the upfront fee. Then

$$V_{prem}\left(T\right) = U + C_{run}\mathcal{A}\left(T\right).$$

- Therefore,

$$\begin{aligned} U &= V_{prem}\left(T\right) - C_{run}\mathcal{A}\left(T\right) \\ &= (C_0 - C_{run})\mathcal{A}\left(T\right). \end{aligned}$$

  where C_0 is the par spread.
- It is straightforward to generalize these calculations to the case of seasoned swaps.

The credit triangle

- Assume that the credit intensity λ is constant, and the protection leg is calculated continuously.
- Then

$$
\begin{aligned}
C_0 &= \frac{V_{prot}(T)}{\mathcal{A}(T)} \\
&\approx -\frac{(1-R)\int_0^T P(0,s)dS(0,s)}{\int_0^T \mathcal{P}(0,s)ds} \\
&= \frac{(1-R)\lambda \int_0^T \mathcal{P}(0,s)ds}{\int_0^T \mathcal{P}(0,s)ds}\,.
\end{aligned}
$$

- This implies the *credit triangle*:

$$
C_0 \approx (1-R)\lambda.
$$

The credit triangle

- The credit triangle is a very useful relation allowing us to quickly estimate the default probabilities implied from the credit spreads.
- Consider, for example, a credit name with $R = 40\%$, and credit spread $C = 200$ bp. Then $\lambda \approx 0.0333$. Hence,

$$
\begin{aligned}
Q(1) &\approx 3.3\% \\
Q(5) &\approx 15.4\%
\end{aligned}
$$

Recovery rate assumptions

- At inception, the standard recovery rate assumption on a CDS is:
  - 40% for senior debt
  - 20% for subordinated debt
  - 25% is used in for types of debt in emerging markets

Bootstrapping par CDS spreads

- Market participants use the term structures of available par CDS spreads from the market, and build continuous intensity curves by bootstrapping (or "stripping"). Typically, hundreds or thousands of such curves are built every day on a trading desk.
- Rather than constructing the survival curves $S(0, t)$, we can focus on calibrating a forward intensity curve $\lambda(t)$, so that

$$S(0, t) = e^{-\int_0^t \lambda(s)ds},$$

or

$$\lambda(t) = -\frac{d \log S(0, t)}{dt}.$$

Bootstrapping par CDS spreads

- For practical reasons, we focus on a term intensity $\overline{\lambda}(t)$ defined as

$$S(0,t) = e^{-t\overline{\lambda}(t)}$$

or

$$\overline{\lambda}(t) = -\frac{\log S(0,t)}{t}$$

- Assume that we have K CDSs with maturities $T_1 < T_2 < \ldots < T_K$. The market observable quantities are the par spread for these securities
- In the simplest (and frequently used by the industry) approach, we assume that $\overline{\lambda}(t)$ is piecewise constant between the times T_i, $i = 1, 2, \ldots, K$. Let $\overline{\lambda}_i$ be the constant forward intensity on the interval $[T_{i-1}, T_i]$, where $T_0 = 0$.
- We discretize the expression for the protection leg using a discrete set of times t_j that contains the maturity dates T_i, $i = 1, 2, \ldots, K$.. Let $V_{prot}(T_i)$ denote the value of the protection leg of the CDS that matures at T_i. Then,

$$V_{prot}(T_i) = (1-R)\sum_{j=1}^{n_i} \frac{P(0,s_{j-1}) + P(0,s_j)}{2}\,(S(0,s_{j-1}) - S(0,s_j))$$

Bootstrapping par CDS spreads

We now proceed in following steps:

1. Make an initial guess for $\overline{\lambda}_1$ (for example, we can use the credit triangle for the shortest maturity spread), and compute $V_{prot}(T_1)$ as well as the risky annuity $\mathcal{A}(T_1)$. The corresponding par spread C_1 for the CDS maturing at T_1 is the ratio of these two numbers
2. Use a search algorithm (such as the secant method) to find the value of $\overline{\lambda}_1$ until C_1 matches the market quote
3. Now that $\overline{\lambda}_1$ is known, make a guess for $\overline{\lambda}_2$. We can now compute survival probabilities out to T_2, and find the corresponding par spread on the CDS maturing at T_2. Using a search algorithm, find the value of $\overline{\lambda}_2$ until C_2 matches the market spread
4. Continue the process until all the $\overline{\lambda}_i$, $i = 1, 2, \ldots, K$ have been found

Bootstrapping and no-arbitrage

- Other bootstrap techniques rely on:
  - linear interpolation of $-\log S(0, t)$
  - linear interpolation of $-\frac{d}{dt} \log S(0, t)$
- All these techniques lead frequently to zig zagged curves or curves violating the no-arbitrage condition:
$$\frac{dS(0, t)}{dt} \leq 0$$
(survival probability cannot increase in time!)
- Better results are obtained by spline fitting combined with global optimization

Forward starting CDS

- So far we have discussed a pricing model for credit default swaps (CDSs). The tacit assumption behind this model is that the CDS is either a brand new spot starting CDS or a seasoned instrument.
- A *forward starting* CDS is a contract which allows us to transfer the credit risk of a reference name in the future:
   (i) The swap settles on a date T_0 (say, in 5 months) and it bears the contractually defined *forward credit spread*.
   (ii) If a default occurs prior to T_0, the contract if torn up at no cost to the counterparties.
   (iii) If no default occurs prior to T_0, the counterparties enter into a usual CDS.

Forward starting CDS

- Having built a survival curve out of the liquid benchmark CDSs, it is possible to consider forward starting credit default swaps
- Let us first establish some notation:
  - By $C(t, T_0, T)$, for $t \leq T_0$, we denote the break-even par spread at time t on a CDS starting on T_0 and maturing on T.
  - By $C_0(T_0, T)$ we denote today's value of the forward spread. In particular, $C_0(T) \triangleq C_0(0, T)$ is the spot spread introduced before.
  - We will also write $C(t, T) \triangleq C(t, t, T)$.

Valuation of a forward starting CDS

- Valuation of a forward starting CDS is based on the same principles as a spot starting CDS
- The time t value of the protection leg is

$$V_{prot}(t, T_0, T) = -(1 - R) \int_{T_0}^{T} P(t, s) dS(t, s).$$

- The time t value of the premium leg is

$$V_{prem}(t, T_0, T) = C\mathcal{A}(t, T_0, T),$$

where $\mathcal{A}(t, T_0, T)$ is the *forward risky annuity*:

$$\mathcal{A}(t, T_0, T) = \frac{1}{2} \sum_{j=1}^{N} \delta_j P(t, T_j)(S(t, T_{j-1}) + S(t, T_j)).$$

Valuation of a forward starting CDS

- At inception, a forward starting CDS has PV of zero:

$$V_{prot}(t, T_0, T) - V_{prem}(t, T_0, T) = 0.$$

- Hence, the break-even forward spread is given by

$$C(t, T_0, T) = \frac{V_{prot}(t, T_0, T)}{\mathcal{A}(t, T_0, T)}\,.$$

- Assuming no default through T_0, $C(t, T_0, T)$ is independent of t and is equal to $C(T_0, T)$.

Corporate bonds revisited

Consider again a corporate bond that pays a coupon C on payment dates T_j, $j = 1, \ldots, n$, and returns the principal (equal \$1 for simplicity) at maturity to the bond holder.

- If default occurs prior to T_n, remaining coupon payments cease, and a fraction R of the bond principal is returned on the default date τ.
- The value of the coupon payment at T_j is given by

$$C\delta_j \mathcal{P}(0, T_j) = C\delta_j \mathsf{E}\Big[e^{-\int_0^{T_j} r(s)ds} 1_{\tau > T_j}\Big].$$

- Likewise, the PV of the final return of the principal at T_n is simply $\mathcal{P}(0, T_n)$.
- The value of receiving recovery R at the time of default is $\mathcal{C}(0, T_n)$ with $C(s) = R$.
- Adding up all these elements leads to the valuation formula:

$$Price = C \sum_{j=1}^{n} \delta_j \mathcal{P}(0, T_j) + \mathcal{P}(0, T_n) + \mathcal{C}(0, T_n).$$

CDS-cash basis trade

- It is possible (in principle) to synthesize a corporate bond of maturity T through the following trades:
  - long US Treasury of maturity T of the same face value.
  - short CDS on the corporate bond of maturity T and notional equal to the face value.
- This observation underlies the concept of the *CDS-cash basis*, defined as the difference of the CDS spread and the spread of a bond of similar (or same) maturity.
- This basis can be positive or negative (a negative basis means that the CDS spread is lower than the spread implied by the bond). A view on the spread is implemented by *basis trading*.

CDX and iTraxx

- In 2001 JPMorgan originated the first synthetic CDS indices. There are currently two main families of CDS indices: CDX and iTraxx, both of them issued and administered by Markit (since 2007). CDX indices contain North American and emerging market corporate names, while iTraxx contain companies from Europe, Asia, and Australia.
- Currently, credit indices are the most liquid form of credit default swaps (traded volumes of single name CDS have significantly declined since 2008). They serve as an important gauge of credit conditions, and allow market participants to take a broad view on credit spreads.
- CDX contains three main series: IG (125 investment grade names), HY (100 high yield names) and EM (14 names).
- Indices are quoted in standardized maturities (3,5,7,10 years), and are rolled semiannually. The new issue may have a new composition of names (for example, the names whose credits were downgraded cannot be part of the basket). For example, the current IG series is CDX.NA.IG.26 (to be rolled into series 27 on September 20, 2016).
- Quote conventions are: 100 bp running spread (for IG) and 500 bp (for HY and EM) plus upfront fee.

Mechanics of a credit index

- A credit index is a bilateral OTC transaction. According to the market convention, a *buyer* of the index is the party receiving the spread (and thus assuming the credit risk). This is to be contrasted with the single name CDS convention, where the buyer pays the spread.
- The upfront fee is paid 3 business days following the trade.
- The coupon is paid quarterly on the IMM dates using the act/360 basis.

Mechanics of a credit index

- The prices of the indices are calculated as follows. We start out with N firms (say, $N = 125$) in the basket. On each firm, we set up a standard CDS with a fixed (conventional) coupon C. Each of the N component CDSs pays the same coupon, on the same payment dates.
- The index is quoted as the total upfront required to buy this basket of CDSs. If C is larger than the average par spread of the firms in the basket, we expect the upfront amount to be negative.
- Often the market quotes the price following the bond market convention: “bond price” = 100 + upfront fee in %.

Mechanics of a credit index

- It is convenient to convert the upfront price into an intrinsic “par spread”, according to the following convention:
    - (i) Assume that all firms in the basket are identical and have a flat par spreads of $\overline{C}$.
    - (ii) Assuming 40% recovery, find the value of $\overline{C}$ that yields the upfront value of the index.
- The intrinsic index spread $\overline{C}$ is not a straight arithmetic average of the par spreads of the component spreads. It is close to it in case if the basket consists of similar credits.

Default handling

- If there are no defaults in the basket, then the coupon is paid until contract's maturity.
- If name j defaults:
  - The buyer pays $1/N$ of the notional to the seller against the delivery of the defaulted name or the payment of $1 - R_j$ applied to the appropriate fraction of the notional.
  - The buyer receives the accrued coupon from the defaulted name from the last payment date.
  - The notional gets reduced by $1/N$.

Scaling

- The credit indices have a somewhat different definition of what constitutes default from that of single name CDSs. In the case of CDX, the payment on the protection leg is triggered in case of bankruptcy or failure to pay but not in case of restructuring.
- As a consequence, if one replicates the index by buying all the individual CDSs, the replicated portfolio price does not match the index price.
- Sometimes this is expressed in terms of an intensity basis scale ε: take all regular single-name survival curves $S_i(0,t)$, $i = 1, \ldots, N$, and scale them according to

$$\widetilde{S}_i(0,t) = S_i(0,t)^{\varepsilon}.$$

- This is equivalent to scaling all forward intensities by a factor of ε. We determine a value of ε (typically in the range 90-95%) such that when we use $\widetilde{S}_i(0,t)$ as the survival curves, we match the index price.

References

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