MTH 9821 Numerical Methods for Finance I

Fall 2015

Midterm Exam

October 22, 6-8pm

Work is to be done individually. This rule is going to be strictly enforced under severe penalties. No discussions or email exchanges between students are permitted.
Write your results in the file firstname_lastname_midterm_9821_fall2015.xls provided with the test. Write the last four digits of your social security number at the top of the file, and change the name of the file to include your name. DO NOT MODIFY THE FORMAT/STYLE OF THE FILE!

Email the file to
Dan.Stefanica@baruch.cuny.edu

You will receive full credit for a correct answer, meaning 9 decimal digits correct (unless tolerance requires otherwise), partial credit if some of the last decimals do not match, and no credit if you are off at the fourth decimal or worse. Please note that the 9 decimals rule cannot apply to residual errors. Report decimal errors as follows: $2.51 \cdot 10^{-12}$, i.e., the first nonzero decimal which may be close to epsilon machine and should be reported as accurately as possible.

Good luck!

1. Let A_1 be the following 9×9 matrix:

$$\begin{aligned} A_1(i,i) &= 4, \quad \forall\, i = 0:8; \\ A_1(i,i-1) &= -1, \quad \forall\, i = 1:8; \\ A_1(i,i-2) &= 5, \quad \forall\, i = 2:8; \\ A_1(i,i+2) &= -2, \quad \forall\, i = 0:6. \end{aligned}$$

(10 points) Find the matrices P_1, L_1, and U_1 for the LU decomposition with row pivoting of A_1.

Let b_1 be the following column vector:

$$b_1(i) = \sqrt{\frac{i^2}{2} - i + 5}, \qquad i = 0:8.$$

Solve

$$A_1 v_1 = b_1.$$

Compute the residual error

$$||b_1 - A_1 v_1||,$$

where the norm is the 2-norm.

(10 points) Compute L_1^{-1}, U_1^{-1} and A_1^{-1}.

Use matrix–vector multiplication to compute the vector

$$v_2 \;=\; A_1^{-1} b_1.$$

Compute the residual error

$$||b_1 - A_1 v_2||.$$

2. Let

$$A_2 = A_1^t \ A_1.$$

(10 points) Find the upper triangular matrix U_2 from the Cholesky decomposition on A_2.

(5 points) Let b_2 be the following column vector:

$$b_2(i) = \frac{i^2 - 9}{2i + 5}, \qquad i = 0 : 8.$$

Solve

$$A_2 x_2 = b_2.$$

Compute the residual error

$$||b_2 - A_2 x_2||.$$

(5 points) Let $A_3 = A_1^t + A_1$. Solve

$$A_3 x_3 = b_2.$$

Compute the residual error

$$||b_2 - A_3 x_3||.$$

3. Let A_4 be an 8×8 matrix given by

$$\begin{aligned} A_4(i,i) &= 10, \quad \forall\, i = 0:7 \\ A_4(i,i+2) &= -2, \quad \forall\, i = 0:5 \\ A_4(i,i-2) &= 4, \quad \forall\, i = 2:7 \\ A_4(i,i+3) &= -2, \quad \forall\, i = 0:4 \\ A_4(i,i-3) &= -1, \quad \forall\, i = 3:7 \end{aligned}$$

Let b_2 be a column vector given by

$$b_4(i) = \frac{4i-3}{2i^2+1}, \qquad i = 0:7.$$

Solve

$$A_4 x_4 = b_4,$$

using Jacobi **(10 points)**, Gauss–Siedel **(10 points)**, and SOR **(10 points)** with two different values for ω:

$$\omega \in \{0.95, 1.21\}.$$

For each method, the initial guess is a vector x_0 with all entries equal to 0. Use a tolerance of 10^{-6} and a residual–based stopping criterion. Report the first three approximations and the final result, as well as the number of iterations to convergence for each method.

4. The file `financials2012-8.xlsx` contains the weekly prices adjusted for dividends from January 11, 2012, through October 15, 2012, (i.e., 41 prices) for the following financial stocks: JPM, GS, MS, BAC, RBS, CS, UBS, BCS (Barclays).
Compute the 40×8 matrix X of weekly log returns of these stocks.

(5 points) (i) Compute the 8×8 covariance matrix Σ_X of the weekly log returns of the stocks.

(5 points) (ii) Compute the Cholesky factor U_X of the matrix Σ_X.

(5 points) (iii) Find the linear regression of the MS weekly log returns with respect to the weekly log returns of the other financial stocks. Report the residual error of the linear regression.

Repeat the linear regression of the MS weekly log returns with respect to the weekly log returns of the other financial stocks and a constant vector (with entries 1). Report the residual error of the linear regression; the coefficient of the constant vector should be the last one reported.

(5 points) (iv) Given the independent standard normal variables Z_i, $i = 1 : 8$, find normal random variables X_i, $i = 1 : 8$, with covariance matrix Σ_X and given by $\mathbf{X} = M\mathbf{Z}$, where M is an 8×8 matrix and $\mathbf{X} = (X_i)_{i=1:8}$, $\mathbf{Z} = (Z_i)_{i=1:8}$. Report the matrix M

5. A snapshot taken on March 9, 2012, of the mid prices of the S&P 500 options (which are European options), maturing on December 22, 2012, corresponding to a spot price of the index of $1,370$ can be found in Table 1:

Table 1: SPX option prices 3/9/2012

Call Strike	Price	Put Strike	Price
C1175	225.40	P1175	46.60
C1200	205.55	P1200	51.55
C1225	186.20	P1225	57.15
C1250	167.50	P1250	63.30
C1275	149.15	P1275	70.15
C1300	131.70	P1300	77.70
C1350	99.55	P1350	95.30
C1375	84.90	P1375	105.30
C1400	71.10	P1400	116.55
C1450	47.25	P1450	143.20
C1500	29.25	P1500	173.95
C1550	15.80	P1550	210.80
C1575	11.10	P1575	230.90
C1600	7.90	P1600	252.40

Recall from the Put–Call parity that

$$Fe^{-rT} - Ke^{-rT} \;=\; C - P, \tag{1}$$

where $F = Se^{(r-q)T}$ is the forward price of the asset at time T.

Let $disc = e^{-rT}$ be the discount factor and let $PVF = Fe^{-rT}$ be the present value of the forward price. Then, (1) is the same as

$$PVF - K \cdot disc \;=\; C - P. \tag{2}$$

(3 points) (i) From (2), it follows that the values of PVF and $disc$ can be obtained by solving a least square problem

$$y \;\approx\; Ax,$$

with $x = \begin{pmatrix} PVF \\ disc \end{pmatrix}$, where A is a 14×2 matrix A, and y is a 14×1 column vector.

Identify A and y.

(7 points) (ii) Use least squares to compute PVF and $disc$.