1/

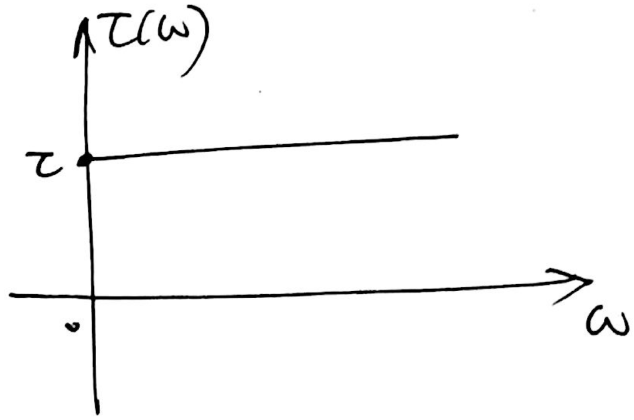
$$h_1(t) = \delta(t-\tau)$$

$$\tilde{h}_1(\omega) = \int_{-\infty}^{+\infty} h_1(t)\, e^{-i\omega t}\, dt = \int_{-\infty}^{+\infty} \delta(t-\tau)\, e^{-i\omega t}\, dt$$

$$= e^{-i\omega\tau}$$

$$\therefore \angle \tilde{h}_1(\omega) = -\omega\tau$$

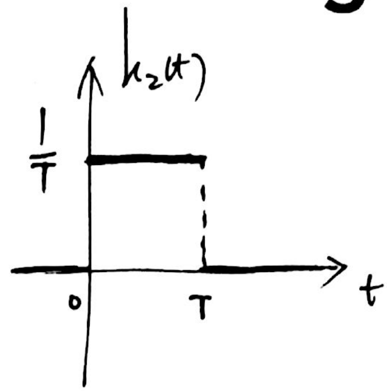
$$\tau(\omega) = -\frac{\partial}{\partial\omega}\angle\tilde{h}_1(\omega) = \tau.$$



<u>D. C. Value.</u>

$$\tau(\omega=0) = \tau.$$

2/ $$h_2(t) = \frac{1}{T}\left[u(t) - u(t-T)\right]$$



$$\tilde{h}_2(\omega) = \int_{-\infty}^{+\infty} h_2(t) e^{-i\omega t} dt$$

$$= \frac{1}{T}\int_0^T e^{-i\omega t} dt = \frac{1}{-i\omega T} e^{-i\omega t}\Big|_0^T$$

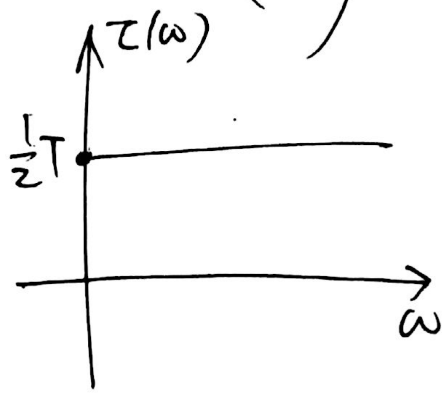
$$= \frac{1 - e^{-i\omega T}}{i\omega T}$$

$$= \frac{1}{i\omega T}\left(1 - \cos(\omega T) + i\sin(\omega T)\right)$$

$$= \frac{1}{\omega T}\left[\sin(\omega T) - i\left(1 - \cos(\omega T)\right)\right]$$

$$\tan\left(\angle\tilde{h}_2(\omega)\right) = -\frac{1-\cos(\omega T)}{\sin(\omega T)} = -\tan\left(\frac{\omega T}{2}\right)$$

$$= \tan\left(-\frac{\omega T}{2}\right)$$



$$\therefore \angle\tilde{h}_2(\omega) = -\frac{1}{2}\omega T$$

$$\tau(\omega) = -\frac{\partial}{\partial\omega}\angle\tilde{h}_2(\omega) = \frac{1}{2}T.$$

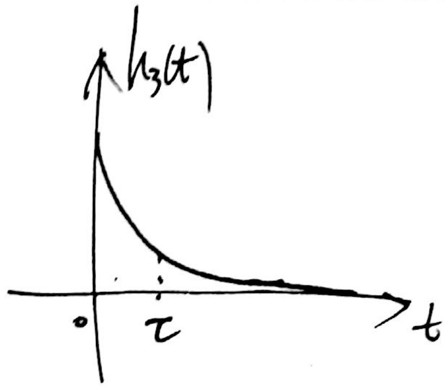
<u>D. C. value:</u>

$$\tau(\omega = 0) = \frac{1}{2}T$$

Bloomberg

3/

$$h_3(t) = \frac{1}{\tau} e^{-\frac{t}{\tau}} u(t)$$



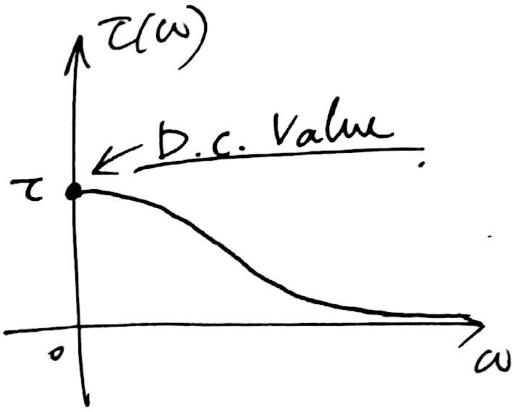
$$\hat{h}_3(\omega) = \int_{-\infty}^{+\infty} h_3(t) e^{-i\omega t} dt$$

$$= \frac{1}{\tau} \int_0^{\infty} e^{-\left(\frac{1}{\tau} + i\omega\right)t} dt$$

$$= -\frac{1}{\tau\left(\frac{1}{\tau} + i\omega\right)} e^{-\left(\frac{1}{\tau} + i\omega\right)t} \Big|_0^{\infty}$$

$$= \frac{1}{1 + i\omega\tau}$$

$$= \frac{1 - i\omega\tau}{1 + \omega^2\tau^2}$$



$$\therefore \tan\left(\angle \hat{h}_3(\omega)\right) = -\omega\tau$$

$$\therefore \angle \hat{h}_3(\omega) = \arctan\left(-\omega\tau\right) = -\arctan\left(\omega\tau\right)$$

Differentiate with respect to ω,

$$\frac{1}{\cos^2\left(\angle \hat{h}_3(\omega)\right)} \cdot \frac{\partial}{\partial\omega} \angle \hat{h}_3(\omega) = -\tau$$

$$\therefore \tau(\omega) = -\frac{\partial}{\partial\omega} \angle \hat{h}_3(\omega) = \tau \cos^2\left(\angle \hat{h}_3(\omega)\right)$$

$$= \frac{\tau}{1 + \tan^2\left(\angle \hat{h}_3(\omega)\right)} = \frac{\tau}{1 + \omega^2\tau^2}.$$

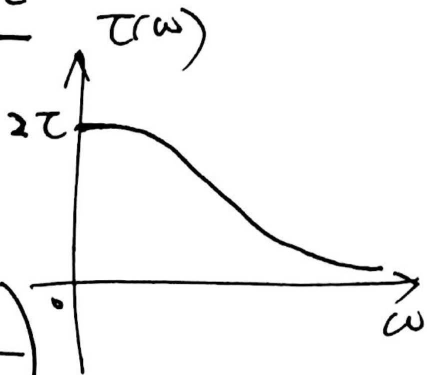
4./ $h_4(t) = \frac{1}{\tau^2} t e^{-\frac{t}{\tau}} u(t)$

$$\tilde{h}_4(\omega) = \int_{-\infty}^{+\infty} h_4(t) e^{-i\omega t} dt$$

$$= \int_0^{\infty} \frac{1}{\tau^2} t e^{-\frac{t}{\tau} - i\omega t} dt$$

$$= \frac{1}{\tau^2} \times \frac{1}{\left(\frac{1}{\tau} + i\omega\right)^2} = \frac{1}{(1 + i\omega\tau)^2}$$

$$= \frac{(1 - i\omega\tau)^2}{(1 + \omega^2\tau^2)^2} = \frac{(1 - \omega^2\tau^2) - 2i\omega\tau}{(1 + \omega^2\tau^2)^2}$$



$$\therefore \tan\left(\angle \tilde{h}_4(\omega)\right) = -\frac{2\omega\tau}{1 - \omega^2\tau^2}$$

$$\therefore \angle \tilde{h}_4(\omega) = -\arctan\left(\frac{2\omega\tau}{1 - \omega^2\tau^2}\right)$$

Differentiate with respect to ω,

$$\frac{1}{\cos^2\left(\angle \tilde{h}_4(\omega)\right)} \cdot \frac{\partial}{\partial\omega} \angle \tilde{h}_4(\omega) = -\frac{2\tau(1 + \omega^2\tau^2)}{(1 - \omega^2\tau^2)^2}$$

$$\therefore \tau(\omega) = \frac{2\tau(1 + \omega^2\tau^2)}{(1 - \omega^2\tau^2)^2} \times \frac{1}{1 + \frac{4\omega^2\tau^2}{(1 - \omega^2\tau^2)^2}}$$

$$= \frac{2\tau(1 + \omega^2\tau^2)}{(1 - \omega^2\tau^2)^2 + 4\omega^2\tau^2} = \frac{2\tau}{1 + \omega^2\tau^2}$$

D.C. value:

$$\tau(\omega = 0) = 2\tau$$

5/ $$h_5(t) = \frac{e^{-\frac{t}{\tau_+}} - e^{-\frac{t}{\tau_-}}}{\tau_+ - \tau_-} u(t)$$

$$\tilde{h}_5(\omega) = \int_{-\infty}^{+\infty} h_5(t) e^{-i\omega t} dt$$

$$= \frac{1}{\tau_+ - \tau_-} \int_0^{\infty} \left[e^{-\frac{t}{\tau_+}} - e^{-\frac{t}{\tau_-}} \right] e^{-i\omega t} dt$$

$$= \frac{1}{\tau_+ - \tau_-} \times \left(\frac{1}{\frac{1}{\tau_+} + i\omega} - \frac{1}{\frac{1}{\tau_-} + i\omega} \right)$$

$$= \frac{1}{(1 + i\omega\tau_+)(1 + i\omega\tau_-)} = \frac{(1 - i\omega\tau_+)(1 - i\omega\tau_-)}{(1 + \omega^2\tau_+^2)(1 + \omega^2\tau_-^2)}$$

$$= \frac{(1 - \omega^2\tau_+\tau_-) - i\omega(\tau_+ + \tau_-)}{(1 + \omega^2\tau_+^2)(1 + \omega^2\tau_-^2)}$$

$$\therefore \tan\left(\angle \tilde{h}_5(\omega)\right) = -\frac{\omega(\tau_+ + \tau_-)}{1 - \omega^2\tau_+\tau_-}$$

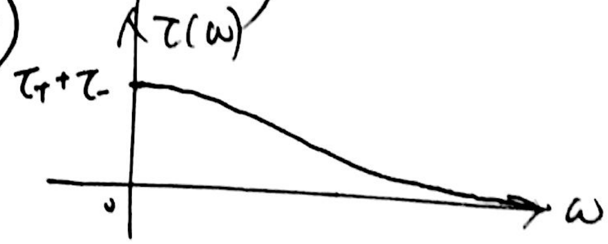
$$\therefore \angle \tilde{h}_5(\omega) = -\arctan\left(\frac{\omega(\tau_+ + \tau_-)}{1 - \omega^2\tau_+\tau_-} \right)$$

Differentiate with respect to ω

$$\frac{1}{\cos^2\left(\angle \tilde{h}_5(\omega)\right)} \times \frac{\partial}{\partial\omega} \angle \tilde{h}_5(\omega) = -\frac{(\tau_+ + \tau_-)(1 + \omega^2\tau_+\tau_-)}{(1 - \omega^2\tau_+\tau_-)^2}$$

$$\therefore \tau(\omega) = \frac{(\tau_+ + \tau_-)(1 + \omega^2\tau_+\tau_-)}{(1 - \omega^2\tau_+\tau_-)^2} \times \frac{1}{1 + \frac{\omega^2(\tau_+ + \tau_-)^2}{(1 - \omega^2\tau_+\tau_-)^2}}$$

$$= \frac{1 + \omega^2\tau_+\tau_-}{(1 + \omega^2\tau_+^2)(1 + \omega^2\tau_-^2)} \times (\tau_+ + \tau_-)$$



$\therefore$ D.C. Value

$$\tau(\omega = 0) = \tau_+ + \tau_-.$$