MTH9886 Emerging Markets and Inflation, Fall 2017

ShengQuan Zhou

1 Present value correction for various settlements

Estimate price and risk differences when evaluating non-standard settling NDFs as standard using market data provided. Use ARS-USD FX pair with market data supplied. Estimate present value correction for 5, 10, and 30 days of Settlement after the Fixing compared to the standard 2 days offset rule coinciding with Spot Date rule.

Solution: Present value of NDF

$$\begin{aligned}
\mathrm{pv}^{\$} &= \frac{F(T_{\mathrm{fix}})-K}{K}\cdot Z^{\mathrm{CCY}}(T_{\mathrm{spot}}, T_{\mathrm{fix}}+\text{Spot Date Rule})\cdot Z^{\$}(T_{\mathrm{fix}}+\text{Spot Date Rule}, T_{\mathrm{settle}})\\
&= \frac{F(T_{\mathrm{fix}})-K}{K}\cdot Z^{\mathrm{CCY}}(T_{\mathrm{spot}}, T_{\mathrm{fix}}+\text{Spot Date Rule})\cdot Z^{\$}(T_{\mathrm{fix}}+\text{Spot Date Rule}, T_{\mathrm{fix}}+\text{Settle Date Offset}).
\end{aligned}$$

then

$$\frac{\mathrm{pv}^{\$}}{\mathrm{pv}^{\mathrm{CCY}}} = \frac{Z^{\$}(T_{\mathrm{fix}}+\text{Spot Date Rule}, T_{\mathrm{settle}})}{Z^{\mathrm{CCY}}(T_{\mathrm{fix}}+\text{Spot Date Rule}, T_{\mathrm{settle}})}$$

Today is assumed to be 2016/09/09, and let $\tau = T_{\mathrm{settle}} - T_{\mathrm{fix}} - \text{Spot Date Rule}$,

$$Z = \exp\left(-\frac{r\tau}{365}\right),$$

for either currency.

Days from today	End date	$r^{\$}$	r^{ARS}	τ	$Z^{\$}$	Z^{ARS}	$\frac{Z^{\$}}{Z^{\mathrm{ARS}}}$
5	2016/09/14	0.58%	19.67%	3	0.9999523299	0.9983845938	1.0015702727
10	2016/09/19	0.62%	19.67%	8	0.9998641188	0.9956980471	1.0041840714
30	2016/10/09	0.75%	19.59%	28	0.9994248230	0.9850844108	1.0145575466

In other words, for 5 days, non-standard settling NDF price is 100.16% of standard NDF price. For 10 days, non-standard price is 100.42% of standard price. For 30 days, non-standard price is 101.46% of standard price.

2 Convexity Adjustment in Forward Starting NDF

Make some numerical estimates of the Convexity Adjustment in Forward Starting NDF using market data provided. Argue the validity limits.

Solution: According to lecture notes, $\mathbb{E}\left[\frac{1}{1+\beta\cdot\mathbf{X}}\right] \propto \exp(\beta^2)$,

$$\mathbb{I} \triangleq \mathbb{E}\left[\frac{S(T_{\text{Strike Set}})}{S(T_{\text{Strike Set}})+\Delta S}\right] = 1-\alpha\cdot\mathbb{E}\left[\frac{1}{1+\beta\cdot\mathbf{X}}\right]$$

$$= 1-\frac{\Delta S}{F(T_{\text{Strike Set}})+\Delta S}\exp\left(\left[\frac{F(T_{\text{Strike Set}})}{F(T_{\text{Strike Set}})+\Delta S}\right]^2\sigma^2 T_{\text{Strike Set}}\right),$$

$$\mathbb{III} \triangleq \mathbb{E}\left[\frac{1}{S(T_{\text{Strike Set}})+\Delta S}\right] \propto \frac{1}{F(T_{\text{Strike Set}})}\mathbb{E}\left[\frac{S(T_{\text{Strike Set}})}{S(T_{\text{Strike Set}})+\Delta S}\right].$$

Choose $\Delta S = 0.23$, $T_{\text{Strike Set}} = 1$, $F(T_{\text{Strike Set}}) = 4.2$,

σ	$\mathbb{I}$	$\mathbb{III}$	Percentage change of $\mathbb{III}$
0.0%	0.94808	0.22573	
2.00%	0.94806	0.22573	-0.002%
4.00%	0.94801	0.22572	-0.008%
6.00%	0.94791	0.22569	-0.018%
8.00%	0.94778	0.22566	-0.032%
10.00%	0.94761	0.22562	-0.049%
12.00%	0.94740	0.22557	-0.071%
14.00%	0.94716	0.22551	-0.097%
16.00%	0.94687	0.22545	-0.127%
18.00%	0.94655	0.22537	-0.162%
20.00%	0.94618	0.22528	-0.200%
22.00%	0.94577	0.22518	-0.243%
24.00%	0.94532	0.22508	-0.291%
26.00%	0.94483	0.22496	-0.343%
28.00%	0.94429	0.22483	-0.400%
30.00%	0.94371	0.22469	-0.461%

The percentage change in $\mathbb{III}$ is relatively small in magnitude for σ up to 30%, indicating that convexity adjustment for short-dated forward starting forward price is negligible.