

# Neighbour-Joining Unit Tests

Hint: Many test values are taken from project Algorithms for Bioinformatics of Alexander Mattheis or the lectures.

Example from: [https://en.wikipedia.org/wiki/Neighbor\\_joining](https://en.wikipedia.org/wiki/Neighbor_joining) (but other minima chosen)  
Terms are taken from several internet sources (lecture slides).

## Test 1

### Input

<i>D</i>	a	b	c	d	e
a	0	5	9	9	8
b		0	10	10	9
c			0	8	7
d				0	3
e					0

### Iteration 1

**Step 1:** Calculate neighbor-joining matrix  $D^*$  from  $N \times N$  distance matrix  $D$

$$D_{i,j}^* = (N - 2) \cdot D_{i,j} - D_{i,J} - D_{J,j}$$

Hint: we know a similar formula from lecture Machine Learning in Life Sciences (without  $N - 2$ ), which allows to find out if a matrix is additive

where  $D_{i,J} = \sum_{k=1}^N D_{i,k}$  is the total-distance (sum of distances from cluster  $i$  to all other clusters)

**Step 1.1:** Compute total distances

<i>D</i>	a	b	c	d	e	$\Sigma$
a	0	5	9	9	8	31
b	5	0	10	10	9	34
c	9	10	0	8	7	34
d	9	10	8	0	3	30
e	8	9	7	3	0	27
$\Sigma$	31	34	34	30	27	

**Step 1.2:** Calculate neighbor-joining matrix

$D^*$	a	b	c	d	e
a		-50	-38	-34	-34
b			-38	-34	-34
c				-40	-40
d					-48
e					

first row:

$$D_{a,b}^* = (5 - 2) \cdot 5 - D_{a,B} - D_{A,b} = 3 \cdot 5 - 34 - 31 = -50$$

$$D_{a,c}^* = (5 - 2) \cdot 9 - D_{a,C} - D_{A,c} = 3 \cdot 9 - 34 - 31 = -38$$

$$D_{a,d}^* = (5 - 2) \cdot 9 - D_{a,D} - D_{A,d} = 3 \cdot 9 - 30 - 31 = -34$$

$$D_{a,e}^* = (5 - 2) \cdot 8 - D_{a,E} - D_{A,e} = 3 \cdot 8 - 27 - 31 = -34$$

second row:

$$D_{b,c}^* = (5 - 2) \cdot 10 - D_{b,c} - D_{B,c} = 30 - 34 - 34 = -38$$

$$D_{b,d}^* = (5 - 2) \cdot 10 - D_{b,d} - D_{B,d} = 30 - 30 - 34 = -34$$

$$D_{b,e}^* = (5 - 2) \cdot 9 - D_{b,e} - D_{B,e} = 27 - 27 - 34 = -34$$

third row:

$$D_{c,d}^* = (5 - 2) \cdot 8 - D_{c,d} - D_{C,d} = 24 - 30 - 34 = -40$$

$$D_{c,e}^* = (5 - 2) \cdot 7 - D_{c,e} - D_{C,e} = 21 - 27 - 34 = -40$$

fourth row:

$$D_{d,e}^* = (5 - 2) \cdot 3 - D_{d,e} - D_{D,e} = 9 - 27 - 30 = -48$$

**Step 2:** Find minimum element in  $D^*$  and create new cluster  $ij$

$D^*$	a	b	c	d	e
a		-50	-38	-34	-34
b			-38	-34	-34
c				-40	-40
d					-48
e					

$$D_{min} = D_{a,b} = -50 \quad \text{and} \quad ab = a \cup b$$

$D$	c	d	e	ab
c				
d				
e				
ab				

**Step 3:** Recompute distances

**Step 3.1:** Pair-members and new cluster (distance in tree)

$$d(i, ij) = \frac{1}{2}(D_{i,j} - \Delta_{i,j}) = \frac{1}{2}\left(D_{i,j} - \frac{D_{i,j} - D_{I,j}}{N - 2}\right)$$

$$d(j, ij) = \frac{1}{2}(D_{i,j} + \Delta_{i,j}) = D_{i,j} - d(i, ij)$$

where  $\Delta_{i,j} = \frac{D_{i,j} - D_{I,j}}{N - 2}$  is the ratio between the total difference and the remaining number of iterations

Hint: with these formulae you get better results than in UPGMA, because neighbor-joining does not assume the same evolution rate for both i.e. the branch lengths of merged taxa are different

$$d(a, ab) = \frac{1}{2}\left(D_{a,b} - \frac{D_{a,b} - D_{A,b}}{5 - 2}\right) = \frac{1}{2}\left(5 - \frac{34 - 31}{3}\right) = 2$$

$$d(b, ab) = \frac{1}{2}(D_{a,b} + \Delta_{a,b}) = \frac{1}{2}(5 + 1) = 3 = D_{a,b} - d(a, ab) = 5 - 2$$

**Step 3.2:** Remaining clusters and new node

$$D_{ij,k} = \frac{(D_{i,k} + D_{j,k} - D_{i,j})}{2}$$

<i>D</i>	c	d	e	ab
c	0	8	7	7
d		0	3	7
e			0	6
ab				0

$$D_{ab,c} = \frac{1}{2}(D_{a,c} + D_{b,c} - D_{a,b}) = \frac{1}{2}(9 + 10 - 5) = 7$$

$$D_{ab,d} = \frac{1}{2}(D_{a,d} + D_{b,d} - D_{a,b}) = \frac{1}{2}(9 + 10 - 5) = 7$$

$$D_{ab,e} = \frac{1}{2}(D_{a,e} + D_{b,e} - D_{a,b}) = \frac{1}{2}(8 + 9 - 5) = 6$$

**Iteration 2**

<i>D</i>	c	d	e	ab
c	0	8	7	7
d		0	3	7
e			0	6
ab				0

**Step 1:** Calculate neighbor-joining matrix  $D^*$  from  $N \times N$  distance matrix  $D$

**Step 1.1:** Compute total distances

<i>D</i>	c	d	e	ab	$\Sigma$
c	0	8	7	7	22
d	8	0	3	7	18
e	7	3	0	6	16
ab	7	7	6	0	20
$\Sigma$	22	18	16	20	

**Step 1.2:** Calculate neighbor-joining matrix

$D^*$	c	d	e	ab
c		-24	-24	-28
d			-28	-24
e				-24
ab				

first row:

$$D_{ab,c}^* = (4 - 2) \cdot 7 - D_{ab,c} - D_{AB,c} = 14 - 20 - 22 = -28$$

$$D_{ab,d}^* = (4 - 2) \cdot 7 - D_{ab,d} - D_{AB,d} = 14 - 20 - 18 = -24$$

$$D_{ab,e}^* = (4 - 2) \cdot 6 - D_{ab,e} - D_{AB,e} = 12 - 20 - 16 = -24$$

second row:

$$D_{c,d}^* = (4 - 2) \cdot 8 - D_{c,D} - D_{C,d} = 16 - 18 - 22 = -24$$

$$D_{c,e}^* = (4 - 2) \cdot 7 - D_{c,E} - D_{C,e} = 14 - 16 - 22 = -24$$

third row:

$$D_{d,e}^* = (4 - 2) \cdot 3 - D_{d,E} - D_{D,e} = 6 - 16 - 18 = -28$$

**Step 2:** Find minimum element in  $D^*$  and create new cluster  $ij$

$D^*$	c	d	e	ab
c		-24	-24	-28
d			-28	-24
e				-24
ab				

Hint: Implementation chooses pair (d, e).

$$D_{min} = D_{d,e} = -28 \quad \text{and} \quad de = d \cup e$$

$D$	c	ab	de
c			
ab			
de			

**Step 3:** Recompute distances

**Step 3.1:** Pair-members and new cluster (distance in tree)

$$d(d, de) = \frac{1}{2} \left( D_{d,e} - \frac{D_{d,E} - D_{D,e}}{4 - 2} \right) = \frac{1}{2} \left( 3 - \frac{16 - 18}{2} \right) = 2$$

$$d(e, de) = \frac{1}{2} (D_{d,e} + \Delta_{d,e}) = \frac{1}{2} (3 - 1) = 1 = D_{d,e} - d(e, de) = 3 - 2$$

**Step 3.2:** Remaining clusters and new node

$D$	c	ab	de
c	0	7	6
ab		0	5
de			0

$$D_{de,c} = \frac{1}{2} (D_{d,c} + D_{e,c} - D_{d,e}) = \frac{1}{2} (8 + 7 - 3) = 6$$

$$D_{de,ab} = \frac{1}{2} (D_{d,ab} + D_{e,ab} - D_{d,e}) = \frac{1}{2} (7 + 6 - 3) = 5$$

### Iteration 3

$D$	c	ab	de
c	0	7	6
ab		0	5
de			0

**Step 1:** Calculate neighbor-joining matrix  $D^*$  from  $N \times N$  distance matrix  $D$

**Step 1.1:** Compute total distances

$D$	c	ab	de	$\Sigma$
c	0	7	6	13
ab	7	0	5	12
de	6	5	0	11
$\Sigma$	13	12	11	

**Step 1.2:** Calculate neighbor-joining matrix

$D^*$	c	ab	de
c			
ab			
de			

first row:

$$D_{c,ab}^* = (3 - 2) \cdot 7 - D_{c,AB} - D_{C,ab} = 7 - 12 - 13 = -18$$

$$D_{c,de}^* = (3 - 2) \cdot 6 - D_{c,DE} - D_{C,de} = 6 - 11 - 13 = -18$$

second row:

$$D_{ab,de}^* = (3 - 2) \cdot 5 - D_{ab,DE} - D_{AB,de} = 5 - 11 - 12 = -18$$

**Step 2:** Find minimum element in  $D^*$  and create new cluster  $ij$

$D^*$	c	ab	de
c		-18	-18
ab			-18
de			

$$D_{min} = D_{c,ab} = -18 \quad \text{and} \quad cab = c \cup ab$$

$D$	de	cab
de		
cab		

**Step 3:** Recompute distances

**Step 3.1:** Pair-members and new cluster (distance in tree)

$$d(c, cab) = \frac{1}{2} \left( D_{c,ab} - \frac{D_{c,AB} - D_{C,ab}}{3 - 2} \right) = \frac{1}{2} \left( 7 - \frac{12 - 13}{1} \right) = 4$$

$$d(ab, cab) = \frac{1}{2} (D_{c,ab} + \Delta_{c,ab}) = \frac{1}{2} (7 + (-1)) = 3 = D_{c,ab} - d(c, cab) = 7 - 4$$

**Step 3.2:** Remaining clusters and new node

$D^*$	de	cab
de	0	2
cab		0

$$D_{cab,de} = \frac{1}{2}(D_{c,de} + D_{ab,de} - D_{c,ab}) = \frac{1}{2}(6 + 5 - 7) = 2$$

**Final matrix**

$D^*$	decab
decab	0

$$d(i, ij) = \frac{1}{2}(D_{i,j} - \Delta_{i,j}) = \frac{1}{2}\left(D_{i,j} - \frac{D_{i,J} - D_{I,j}}{N - 2}\right)$$

theoretical stuff: (only to see correctness)

$$d(de, decab) = \frac{1}{2}\left(D_{de,cab} - \frac{D_{de,CAB} - D_{DE,cab}}{2 - 2}\right) = \frac{1}{2}\left(2 - \frac{2 - 2}{2 - 2}\right) = \frac{1}{2}(2 - 0) = 1$$

$$d(cab, decab) = \frac{1}{2}(D_{c,ab} + 0) = \frac{1}{2}(2 + 0) = 1 = D_{de,cab} - d(de, decab) = 2 - 1$$

Hint:  $0/0 := 0$  in this case

computation:

store last computed distance and divide by 2

but:

we want same result as on Wikipedia and no floating points and so we set

$$d(de, decab) = 2 \quad \text{and} \quad d(cab, decab) = 0$$

(set on last computed distance)

**Final-Output**

```
((a:2,b:3):3,c:4):0,(e:1,d:2):2);
```