Describing univariate distributions

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Overview

- Terminology for describing univariate distributions
- Measures of location (centrality)
- Measures of dispersion (spread)

Example data set: butterfat data

We'll use a data set that records the butter fat percentage in milk from 120 Canadian dairy cows (Sokal and Rohlf, Biometry, 4th ed)

- See the link on the course wiki for `butterfat.csv`
- Load `butterfat.csv` using the `read.csv` function

Generate a histogram

Using the `ggplot2` library, generate a histogram for the butterfat data set.

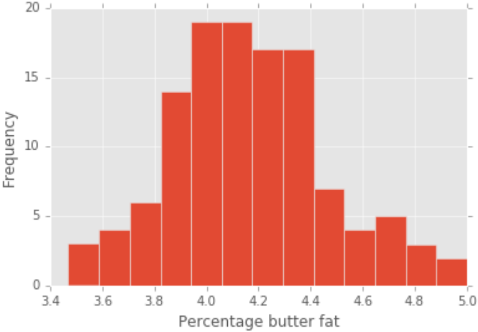


Figure: Histogram of butter fat percentage from 120 Canadian cows.

Mean

- Most common measure of location
- Measure of location that minimizes the sum of the squared deviations around it
- Statistical measure of location that has the smallest standard error (to be defined later)
- Physical analogy: If we think of observations as points of mass on a line, the mean is the center of mass (balance point)

Let $X = \{x_1, x_2, \ldots, x_n\}$. The mean of x is:

$$\overline{X} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

Median

- The middle point of a frequency distribution
- The value of the variable that has an equal number of items on either side of items

The median is a robust estimator of location. Robust statistics are those that are not strongly affected by outliers our violations of model assumptions.

Robustness of median: Example

Changes in estimates of location when three outlier values (8, 10, 15) are added to butterfat data.

Mode

- The most common value (or interval) in a distribution
- Unimodal, bimodal, multi-modal

Some other "means"

Weighted mean – useful when there is some a priori notion of weight or importance for different observations

$$\overline{X}_w = \frac{1}{\left(\sum^n w_i\right)} \sum^n w_i x_i$$

where the w_i represent the weights attached to each observation.

Geometric mean – most often used to study proportional growth (populations, tissues, organs, etc)

$$GM_X = \sqrt[n]{\prod^n x_i}$$

Harmonic mean – rarely used in biology.

$$HM_X = \frac{1}{n} \sum^n \frac{1}{x_i}$$

Range

- The difference between the largest and smallest items in a sample

$$\max(x) - \min(x)$$

Deviates

Deviate – the difference between an observation and the mean; can be negative or positive. Units same as the x_i.

$$x_i - \overline{X}$$

Squared deviate – the square of a deviate; always ≥ 0 (units^2).

$$(x_i - \overline{X})^2$$

Sum of squared deviations – the sum of all the squared deviations in a sample (units^2).

$$\sum_{i=1}^{n} (x_i - \overline{X})^2$$

Variance and standard deviation

Variance – the mean squared deviation (units^2).

$$\sigma_X^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{X})^2$$

Standard deviation – the square root of the variance (units same as the x_i).

$$\sigma_X = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{X})^2}$$

The above are the <u>population</u> variance and standard deviation.

Sample estimators of variance and standard deviation

The *unbiased* sample estimators of the variance and standard deviation are given by:

Variance: $$s_X^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{X})^2$$

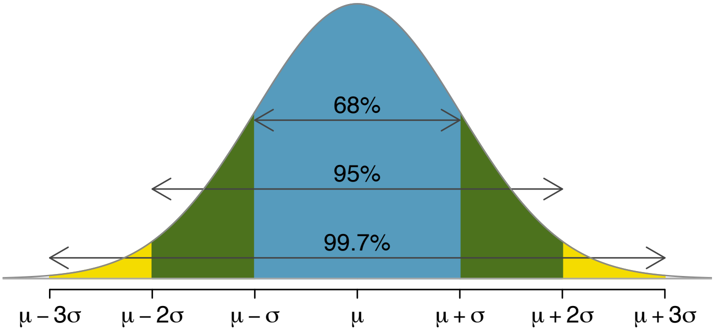
Standard deviation: $$s_X = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{X})^2}$$

You almost always want to use the sample estimators of variance and standard deviation.

Standard deviation rules of thumb

If data are normally distributed:

- Approximately 68% of observations fall within 1 standard deviation about the mean
- Approximately 95% of observations fall within 2 standard deviations about the mean
- Approximately 99.7% of observations fall within 3 standard deviations about the mean



Coefficient of variation

- Standard deviation expressed as percentage of mean
- Unitless measure

$$V = \frac{s_X \times 100}{\overline{X}}$$

Quantiles, quartiles, interquartile range

- Quantiles – points that will divide a frequency distribution into equal sized groups
  - quartiles – points dividing a distribution into 4 equal groups
  - deciles – points dividing a distribution into 10 equal groups
  - percentiles – points dividing a distribution into 100 equal groups
- Interquartile range (IQR)– range of values that captures the central 50% of the distribution
  - Q1 = lower quartile, Q3 = upper quartile

Boxplots typically depict information about quartiles

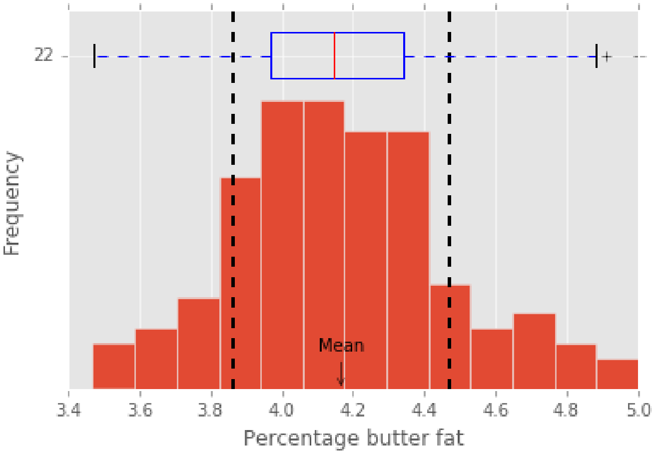


Figure: Histogram of butterfat data set, with superimposed boxplot.

Median absolute deviation (MAD)

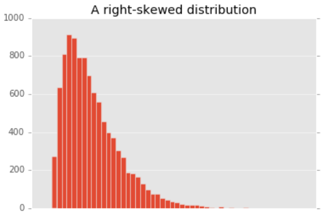
- A robust estimator of dispersion

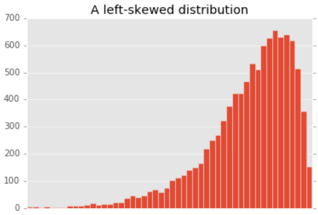
$$\mathrm{MAD}(X) = \mathrm{median}(|x_i - \mathrm{median}(X)|)$$

For normal distribution, $\sigma_X \approx 1.486 \times \mathrm{MAD}(X)$.

Skewness

- Skewness describes asymmetry of distributions





Common measure of skewness:

$$\text{skewness} = E\left[\left(\frac{(x-\mu)}{\sigma}\right)^3\right]$$