Formulas
discrete time growth:

- $N_T = N_0 \lambda^T$

- $\lambda = f + p$

- $\mathcal{R} = f/(1 - p)$

continuous time growth:

- $N(t) = N(0) \exp(rt)$

- $r = b - d$

- $\mathcal{R} = b/d$

Use this information for the next two questions. A microbial population grows in a flask with discrete, non-overlapping generations (i.e., survival to next generation $p = 0$), and finite rate of increase $\lambda = 2$. Its generation time is 1 day. The population takes 20 days to fill the whole flask.

1. How much of the flask is filled after 19 days?

    A. 5%
    B. 50%
    C. 67%
    D. 95%
    E. There is not enough information to tell

2. Which of the following *most* accurately describes the reproductive number $\mathcal{R}$ for this population?

    A. $\mathcal{R} > 1$
    B. $1 < \mathcal{R} < 2$
    C. $\mathcal{R}=2$
    D. $\mathcal{R} > 2$
    E. There is not enough information to tell

3. A certain large island does not have any native snakes, despite the fact that snakes are occasionally washed there by storms. Which of the following is *not* a likely explanation for their failure to thrive?

    A. Snakes experience Allee effects on the island
    B. Snakes experience density dependence on the island
    C. Snakes have very high death rates on the island
    D. Snakes have very low birth rates on the island

Use this information for the next two questions. A researcher estimates that a moth population has a density of 10 pupae/ha in 2012, and finite rate of growth $\lambda = 1.4$ (associated with a time step of one year). The sex ratio of the population is 2:1 (twice as many females as males at each stage).

4. If λ remains constant, what is the approximate density of pupae she will expect to see in 2020?

    A. 14 pupae/ha
    B. 27 pupae/ha
    C. 54 pupae/ha
    D. 74 pupae/ha
    E. 148 pupae/ha

5. What value of the instantaneous growth rate r corresponds to the finite growth model described in the question above?

    A. 0.34/yr
    B. 0.34 yr
    C. 1.4/yr
    D. 1.4 yr
    E. There is not enough information to tell

6. You invest \$100 at 1% monthly interest (so the total at the end of each month is 1% more than it was at the beginning of that month). How much money do you have after 8 years?

    A. \$108
    B. \$109
    C. \$196
    D. \$260
    E. \$272

7. Choose the most precise correct answer. For a population to be regulated, in the long term, its per-capita reproductive number *must*:

    A. respond directly to the population size
    B. respond either directly or indirectly to the population size
    C. respond directly either to the population size or to external factors like climate
    D. respond either directly or indirectly either to the population size or to external factors like climate

8. Compared to the instantaneous rate 0.05/hr, the instantaneous rate 1.2/day:

    A. Means exactly the same thing
    B. Is not directly comparable, because they refer to different time steps
    C. Is comparable, and refers to a larger (faster) rate
    D. Is comparable, and refers to a smaller (slower) rate

9. A simple population model has *structure*, but not *regulation* (individuals are assumed to be independent). If the model has $\mathcal{R}_0 > 1$, then: The modeled population _____ grow exponentially at first, and _____ grow exponentially as it approaches a stable age distribution (SAD)

    A. will; will
    B. may not; will
   **C.** will; may not
    D. may not; may not

Use the picture below for the next two questions. It shows a time series for a continuous-time birth-death model.
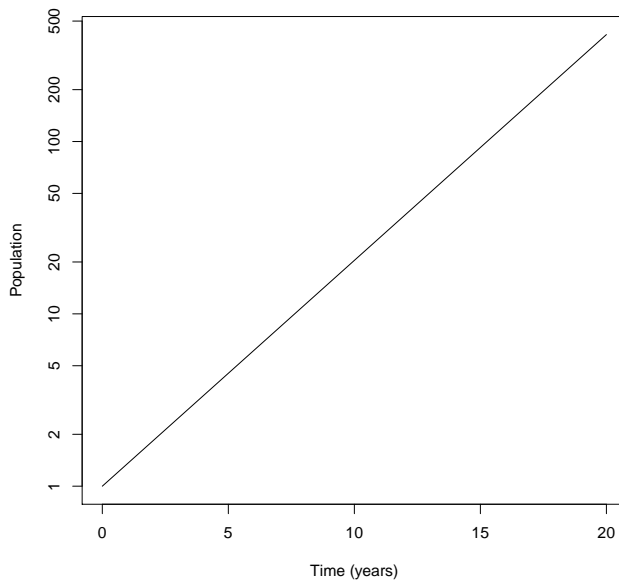

10. This picture shows a population that is:

    A. Increasing arithmetically
    B. Increasing geometrically
    C. Increasing arithmetically on the log scale, but geometrically on a linear scale
   **D.** Increasing geometrically on the log scale, but arithmetically on a linear scale

11. Which of the four pictures below shows the assumptions that generated this time plot?
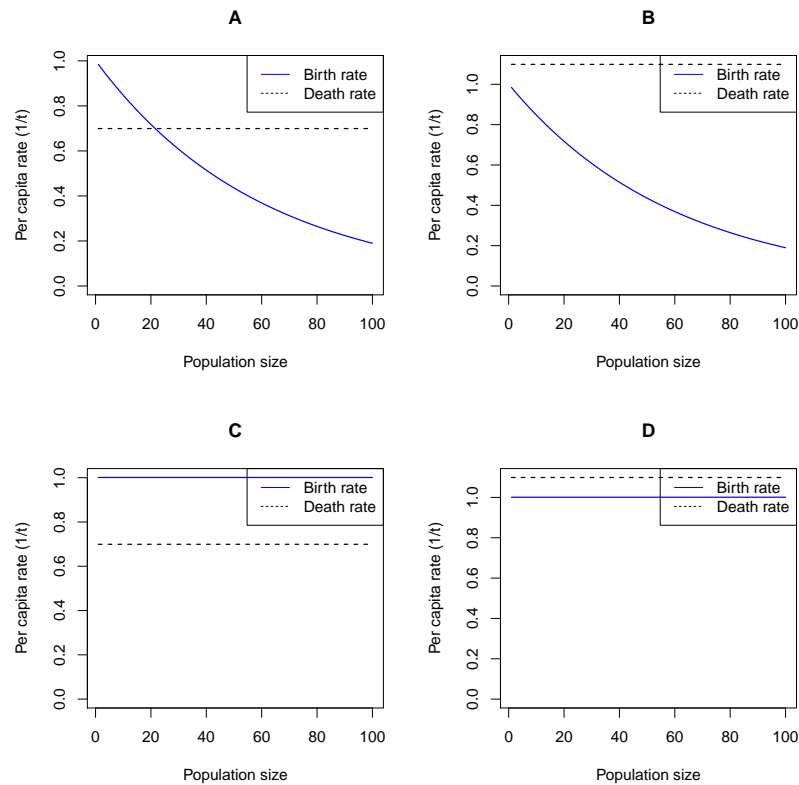
Use the picture below for the next three questions. It shows the assumptions made for a continuous-time birth-death model.
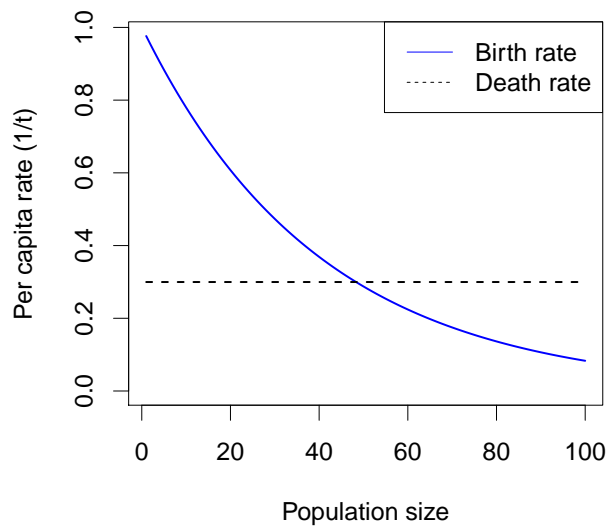
12. Which of the four pictures below could be generated by the same model as the picture above?
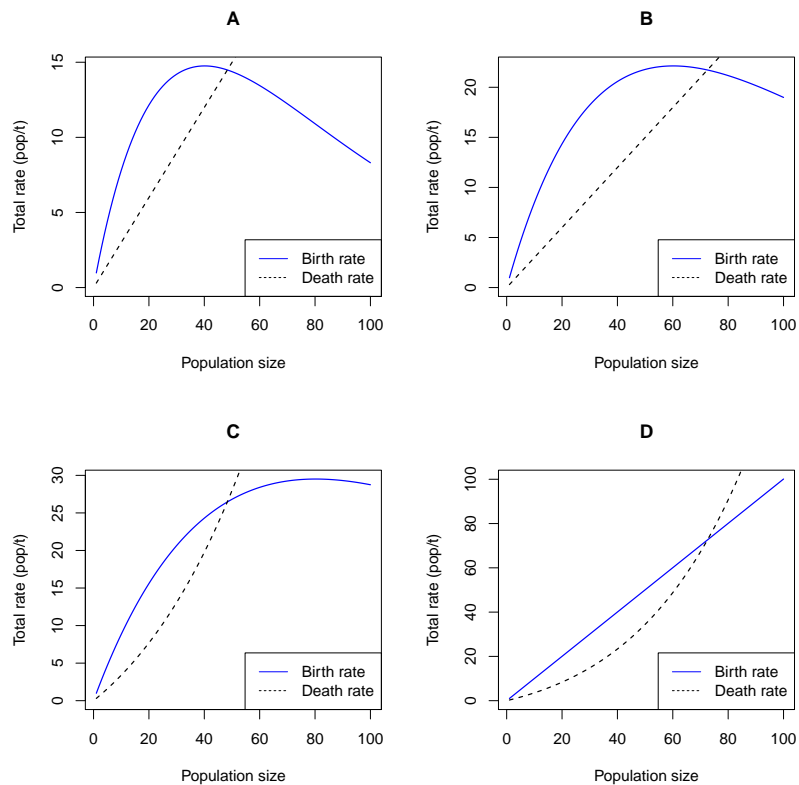
13. The model illustrated above predicts that the population will increase:

    A. When the population is very small (only)
    B. When the population is very large (only)
    C. When the population is very small or very large
    D. When the population is between the two equilibria
    E. When the population is at the nonzero equilibrium

14. The highest *per capita* net growth rate (r) in this model is seen:

    A. When the population is very small
    B. When the population is between the two equilibria
    C. When the population is at the nonzero equilibrium
    D. When the population is very large

Use the following figure for the next two questions.
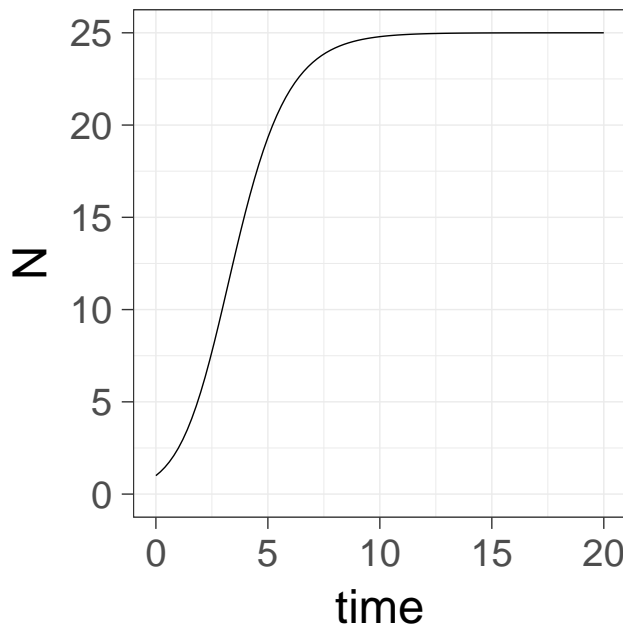
15. What can we say about the equilibrium at $N = 0$ for this population?

    A. It must be stable
    B. It must be unstable
    C. The *per capita* death rate must equal the *per capita* birth rate at $N = 0$
    D. There must be an Allee effect
    E. There must not be an Allee effect

16. If these dynamics are the outcome of an unstructured, continuous-time model, what would you expect to happen if we started the population at $N = 50$?

    A. The population would crash down to zero
    B. The population would increase exponentially
    C. The population would decrease to ≈ 25
    D. The *per capita* growth rate would be larger than when $N \approx 25$
    E. The population would decrease exponentially to zero

17. Which of the following is necessary for a population to reach a stable equilibrium?

    A. $R(0)$ must be < 1
    B. The death rate must be independent of the population size
    C. The population growth rate must be positive just above zero
    D. The population growth rate must be negative for very large population size
    E. The population growth rate must be negative just above zero

Answer questions on this page. *Briefly* show necessary work and equations.

18. (4 points) A population of sea turtles was observed to decline from 1500 breeding females in the year 2005 to 1000 in 2015. The instantaneous death rate d was estimated at 0.06/year. The sea turtle population has a 1:1 sex ratio. For the purposes of this question, assume the population is changing exponentially, on average.

a) Why does d have units of [1/year] only (no turtles)?

b) What is the instantaneous rate of change r for this population?

c) What is the instantaneous birth rate b?

d) What is the lifetime reproductive number $\mathcal{R}$?

19. (4 points) A population of voles with a carrying capacity of 200 indiv/ha experiences simple density dependence in its birth rate, and a *strong* Allee effect (so that $\mathcal{R}_0 < 1$, $\mathcal{R}_{\max} > 1$) in its death rate.

a) Draw a plot of the *per capita* birth and death rates for a simple continuous-time model of this population. Draw both lines on the same graph, indicating clearly which line represents births and which represents deaths. Label your axes clearly, with units and tick marks showing values.
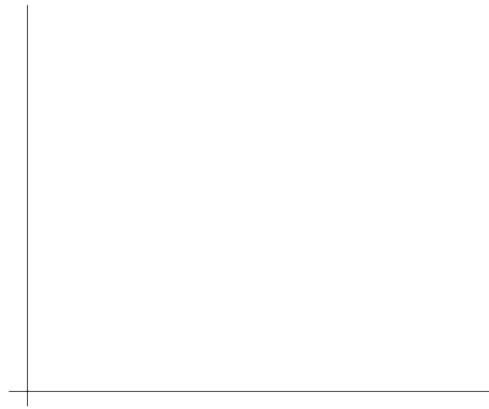
b) How many equilibria does this population have? Indicate them on your graph, remembering that the graph may not call attention to all of the equilibria.

c) Say whether each equilibrium is stable or unstable, and why. Add arrows to your graph to illustrate.

d) Give one plausible ecological reason that the Allee effect might occur.