

UNIT 4: Structured populations

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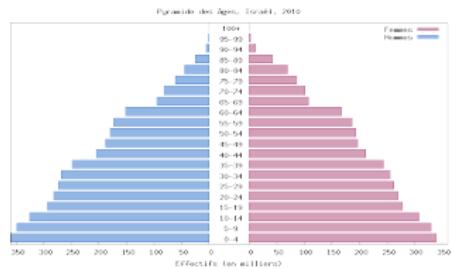
Other structured models

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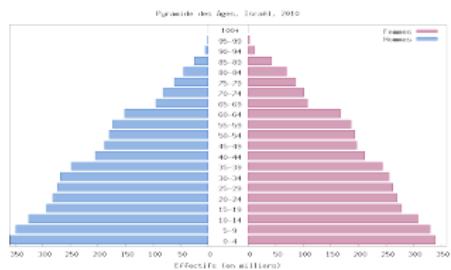
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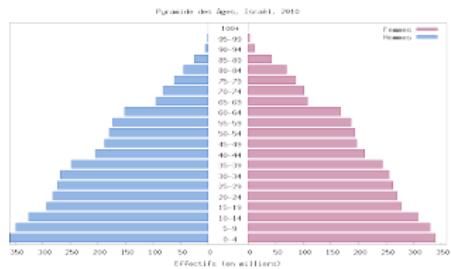
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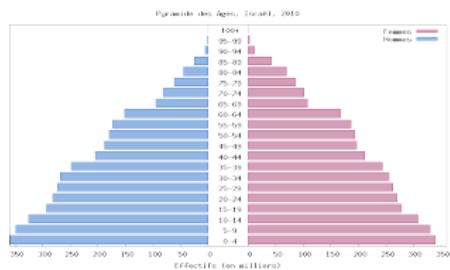
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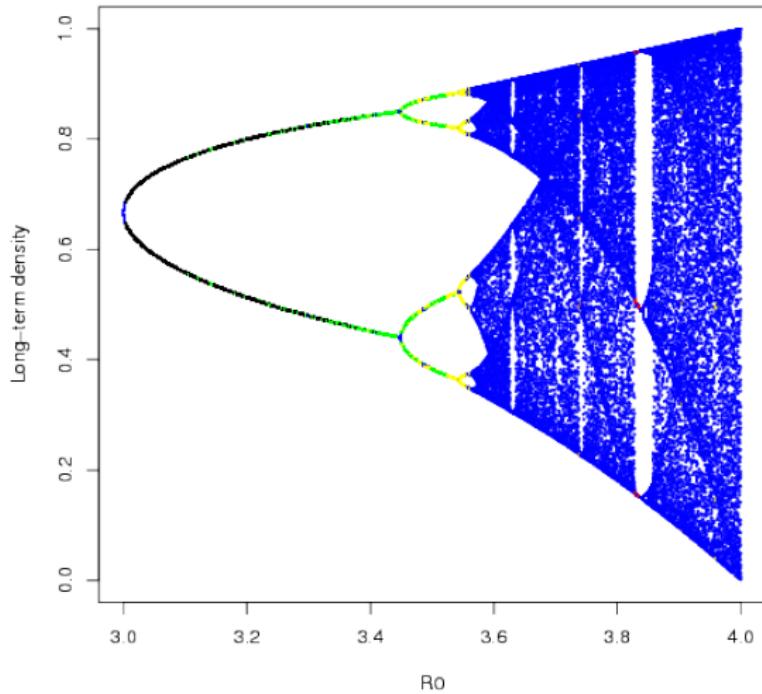
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Complexity



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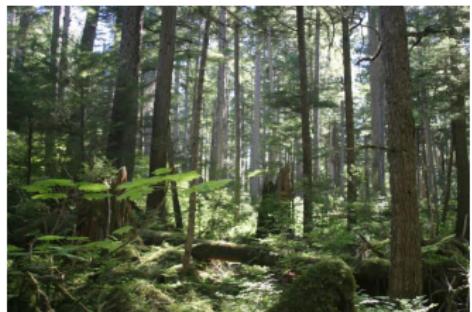
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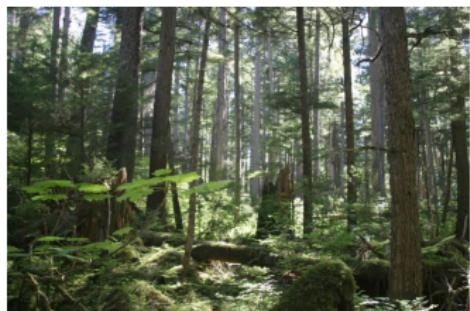
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Discrete vs. continuous time in unstructured models

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Discrete vs. continuous time in unstructured models

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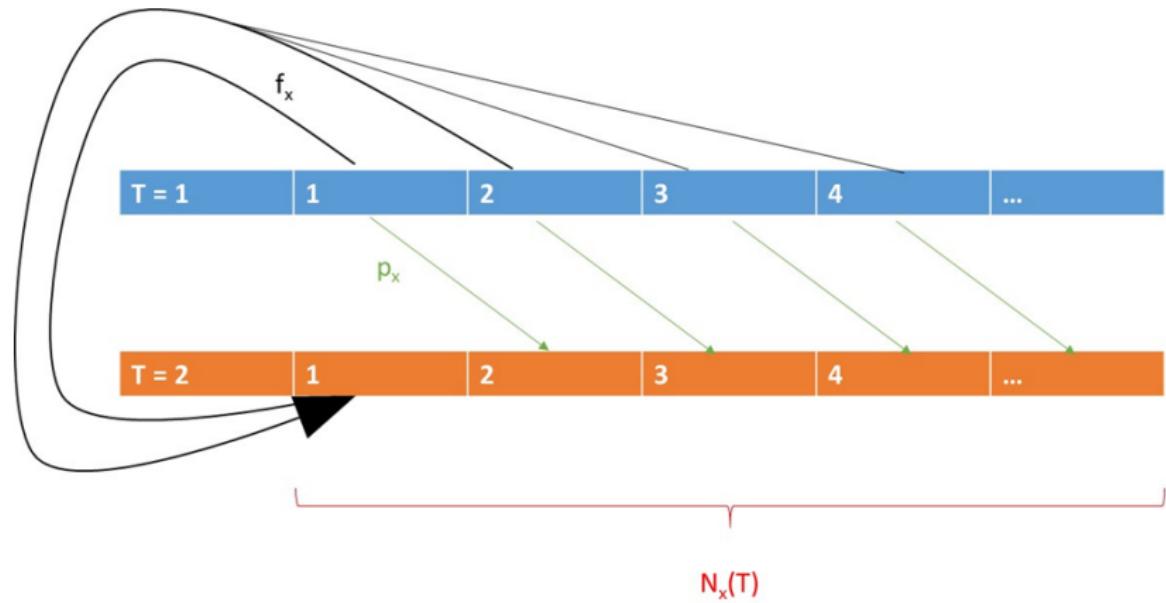
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x	f_x	p_x	ℓ_x	$\ell_x f_x$
1				
2				
R				

Dandelion life table (repeat)

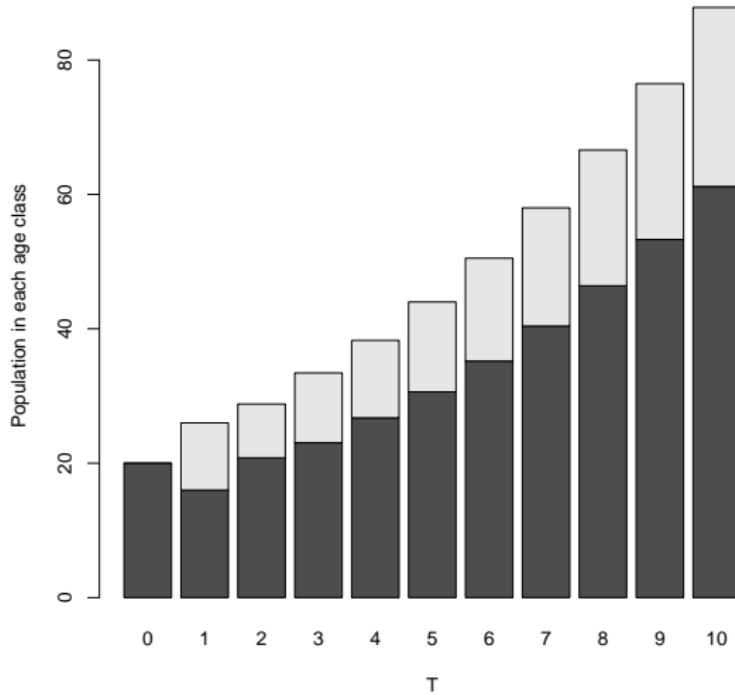
x	f_x	p_x	ℓ_x	$\ell_x f_x$
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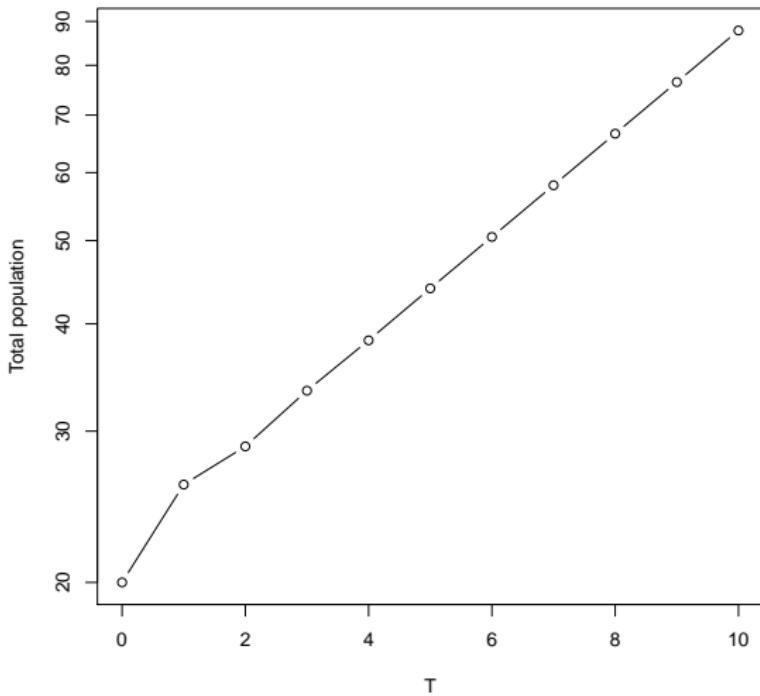
Dandelion dynamics

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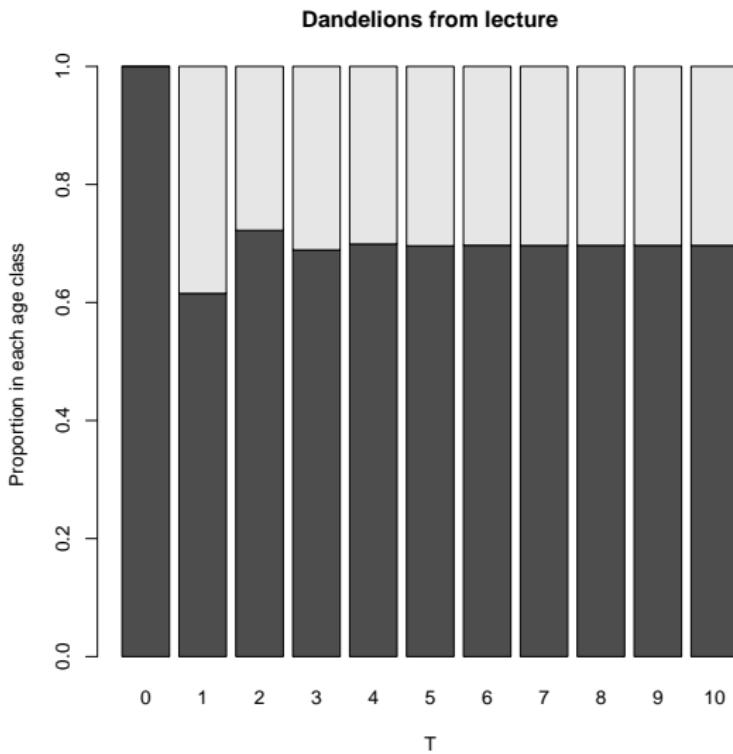


Dandelion population dynamics

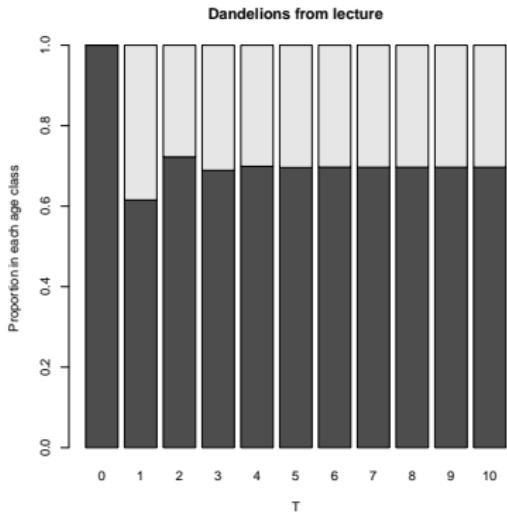
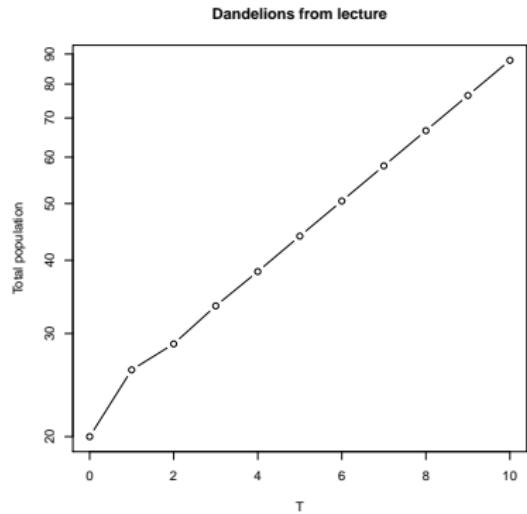
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Dandelion age dynamics



Dandelion dynamics



Squirrel example



Gray squirrel population example

x	f_x	p_x	ℓ_x	$\ell_x f_x$
1	0	0.25		
2	1.28	0.46		
3	2.28	0.77		
4	2.28	0.65		
5	2.28	0.67		
6	2.28	0.64		
7	2.28	0.88		
8	2.28	0.0		
R				

Squirrel observations

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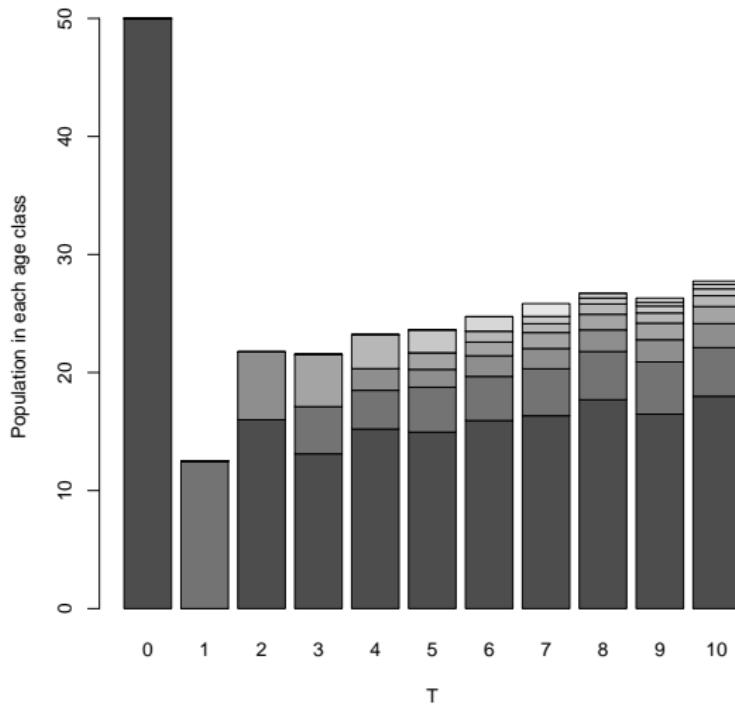
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Gray squirrel population example

x	f_x	p_x	ℓ_x	$\ell_x f_x$
1	0	0.25	1.000	0.000
2	1.28	0.46	0.250	0.320
3	2.28	0.77	0.115	0.262
4	2.28	0.65	0.089	0.202
5	2.28	0.67	0.058	0.131
6	2.28	0.64	0.039	0.088
7	2.28	0.88	0.025	0.056
8	2.28	0.0	0.022	0.050
R				1.109

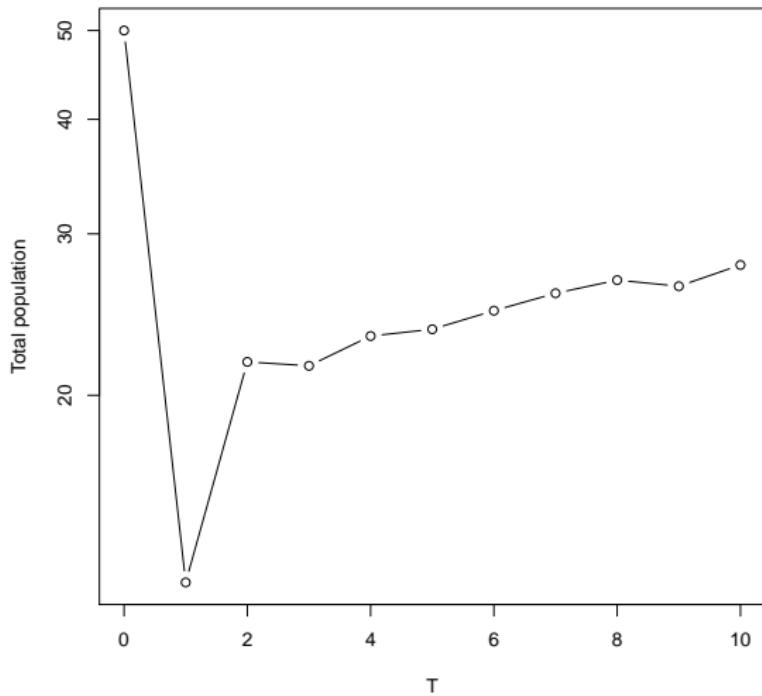
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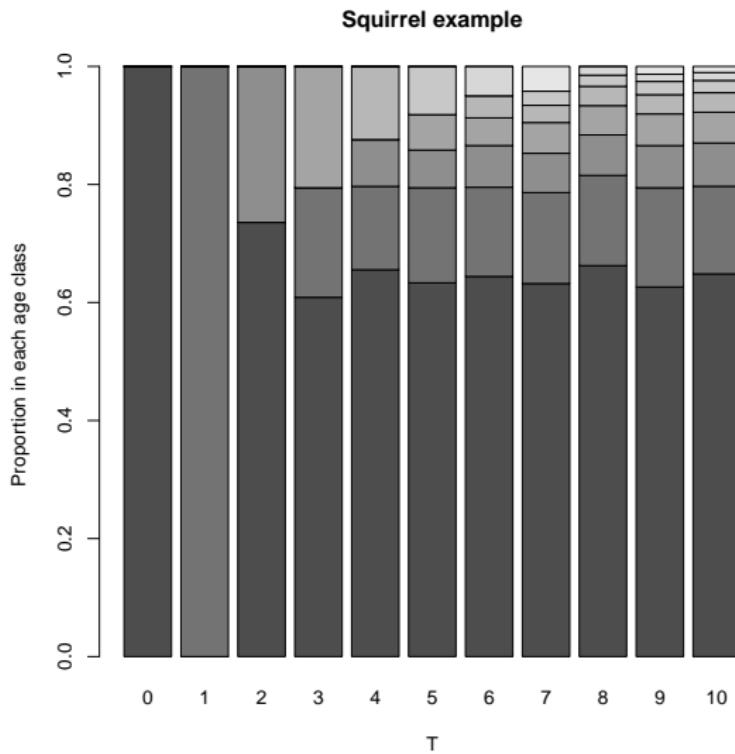


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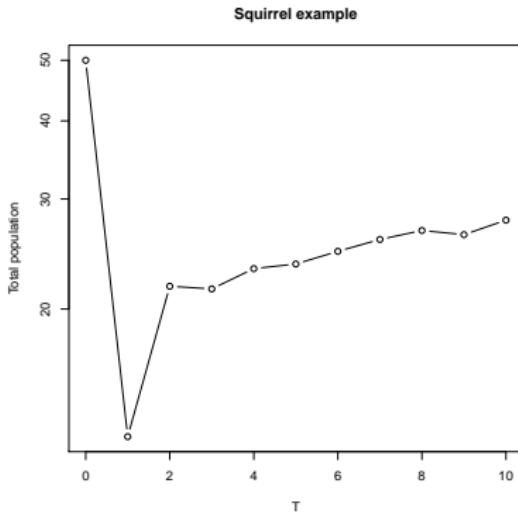
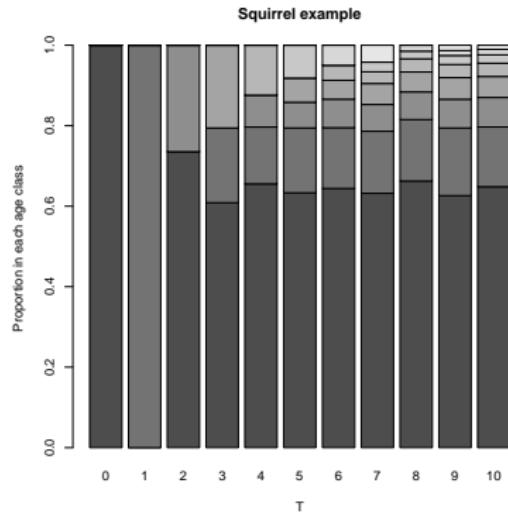
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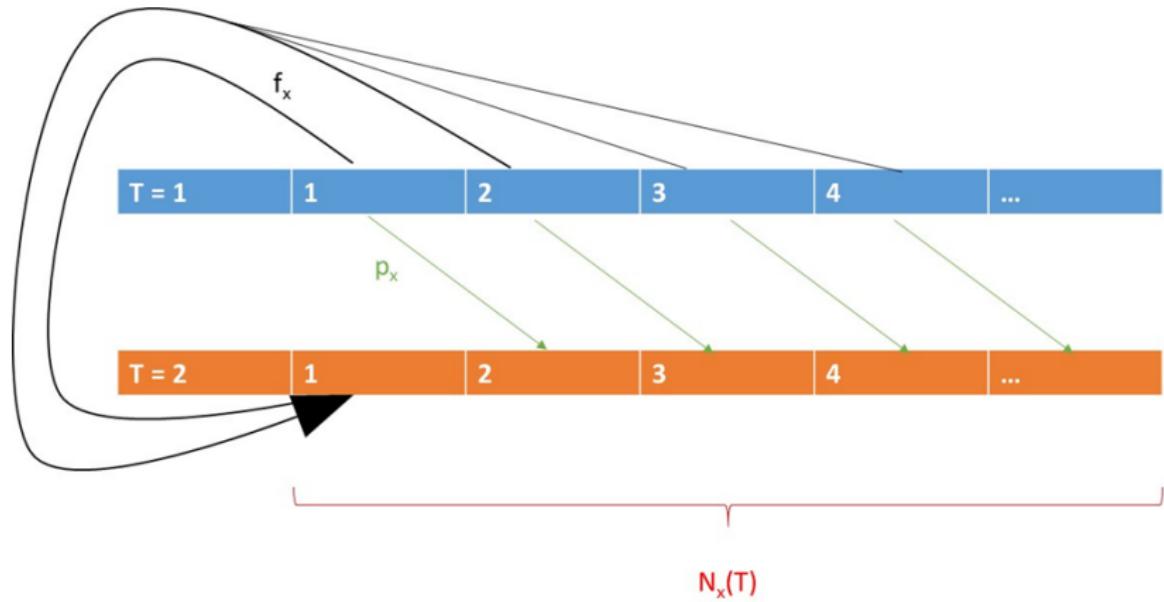
Gray squirrel age dynamics



Gray squirrel dynamics



The structured model (repeat)



Salmon example

- What happens when a population has independent cohorts?

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x	f_x	p_x	ℓ_x	$\ell_x f_x$
1	0	0.1		
2	0	0.5		
3	0	0.6		
4	50	0		
R				

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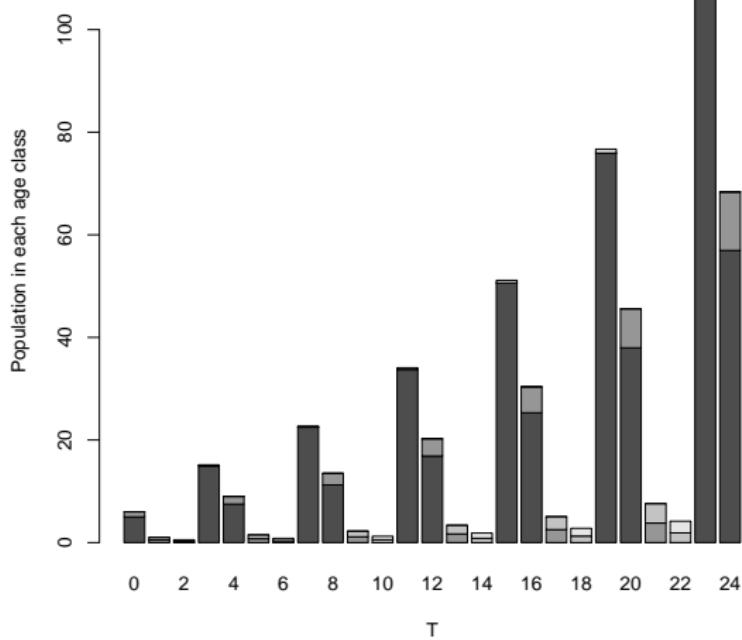
x	f_x	p_x	ℓ_x	$\ell_x f_x$
1	0	0.1	1.000	0.000
2	0	0.5	0.100	0.000
3	0	0.6	0.050	0.000
4	50	0	0.030	1.500
R				1.500

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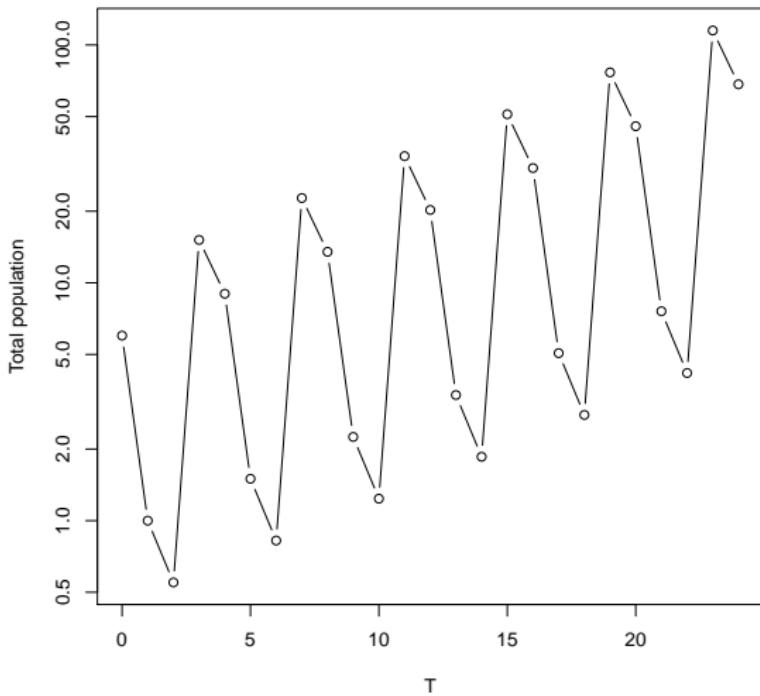
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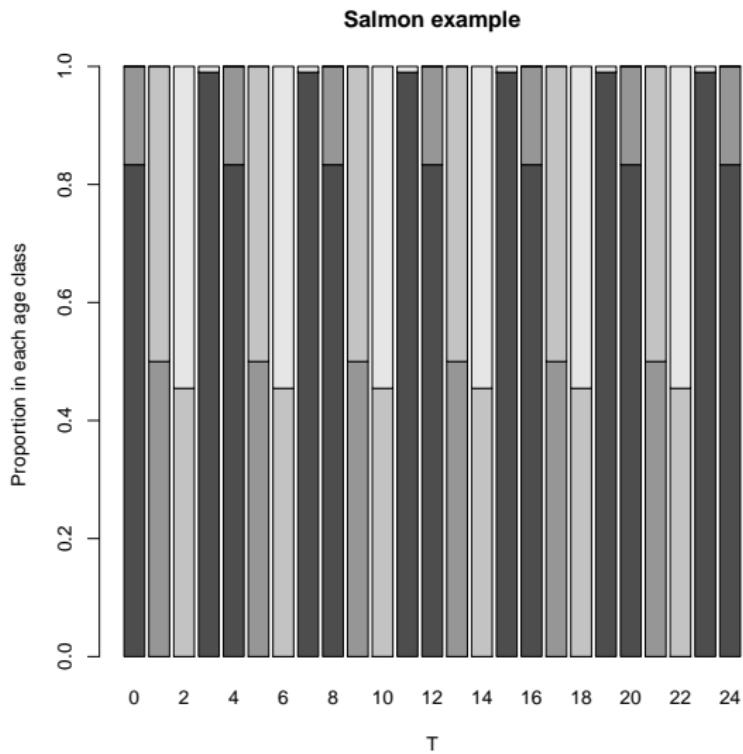


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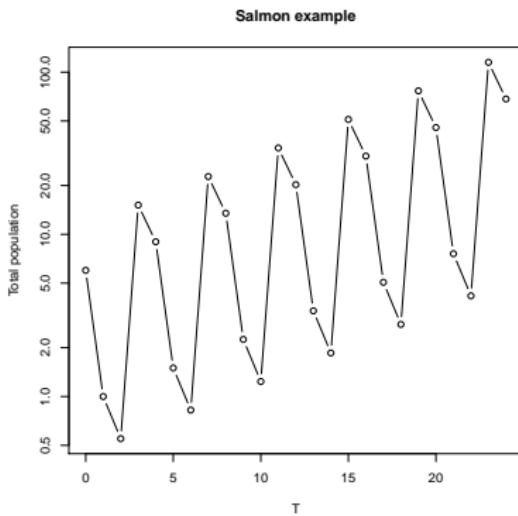
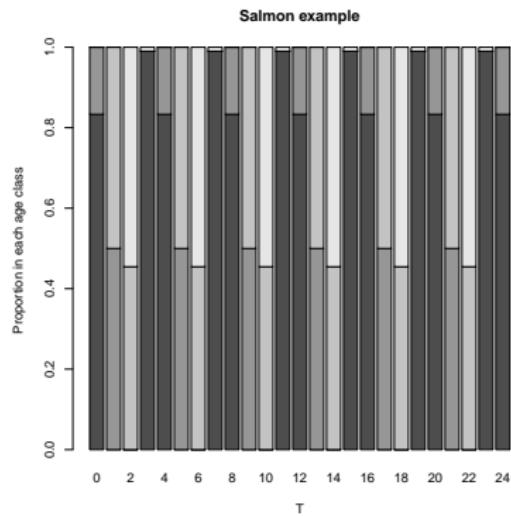
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Counting after reproduction

x	f_x	p_x	ℓ_x	$\ell_x f_x$
1	0.8	0.01	1.000	0.800
2	40	0	0.010	0.400
R				1.200

There are two different approaches

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- ▶ In an exponentially changing population, each year's **cohort** is a factor of λ bigger (or smaller) than the previous one
 - ▶ λ is the finite rate of increase, like before
- ▶ Looking back in time, the cohort x years ago is λ^{-x} as large as the current one
- ▶ The existing cohorts need to make the next one:
 - ▶ $\sum_x \ell_x f_x \lambda^{-x} = 1$

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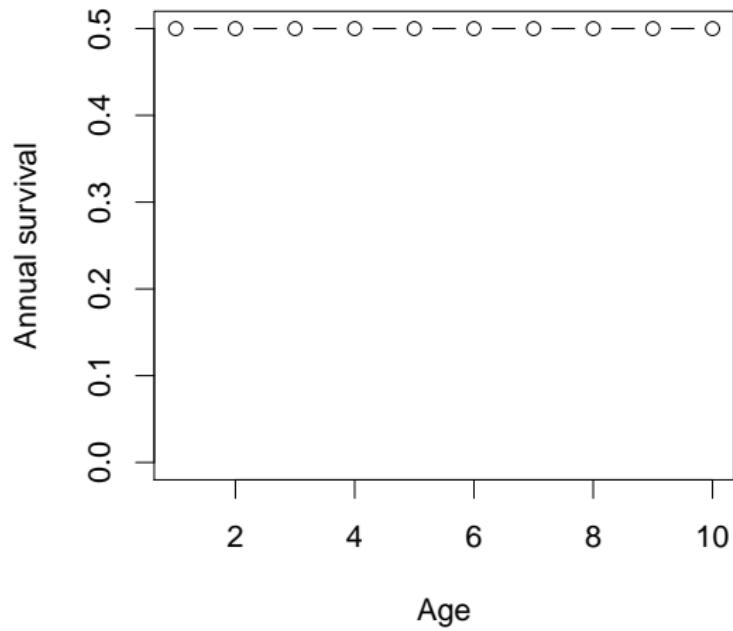
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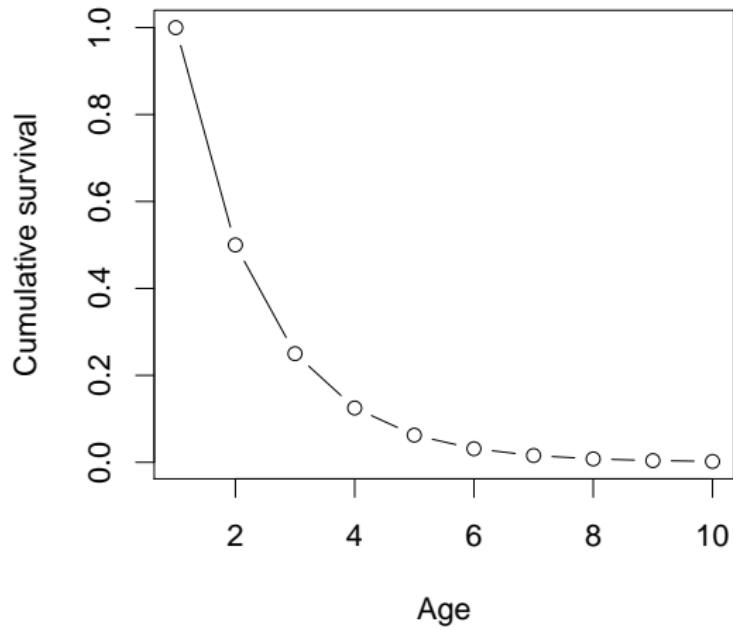
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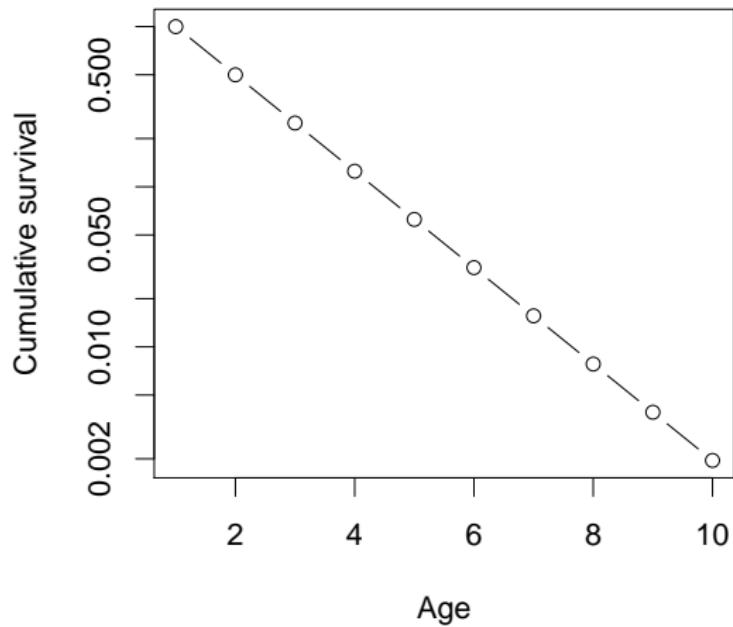
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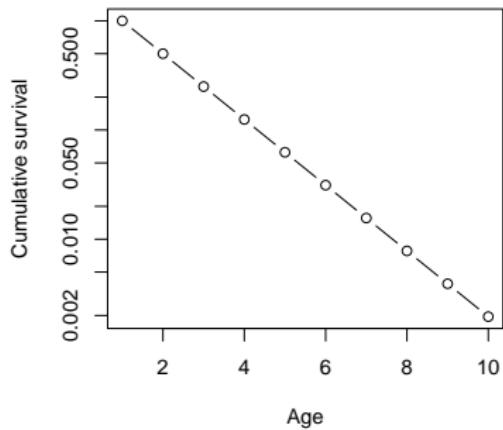
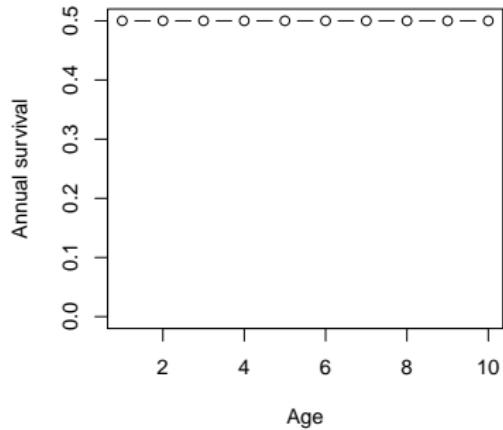
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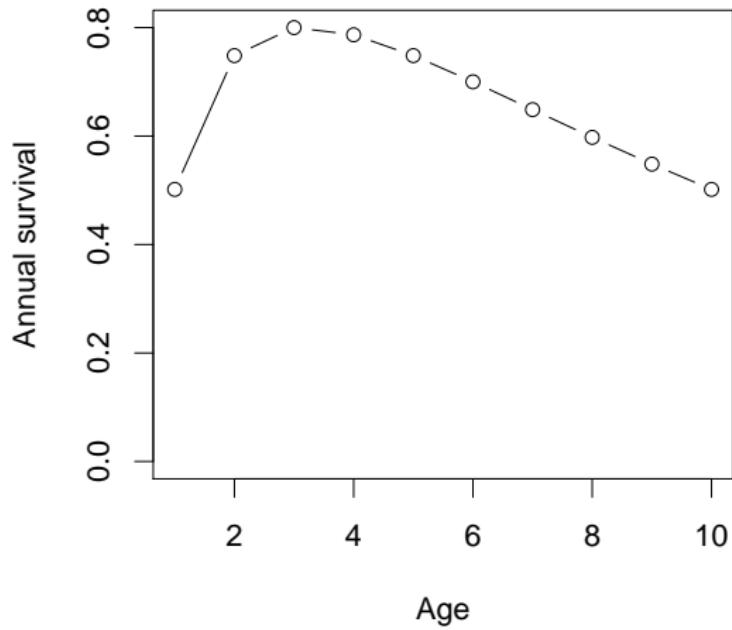
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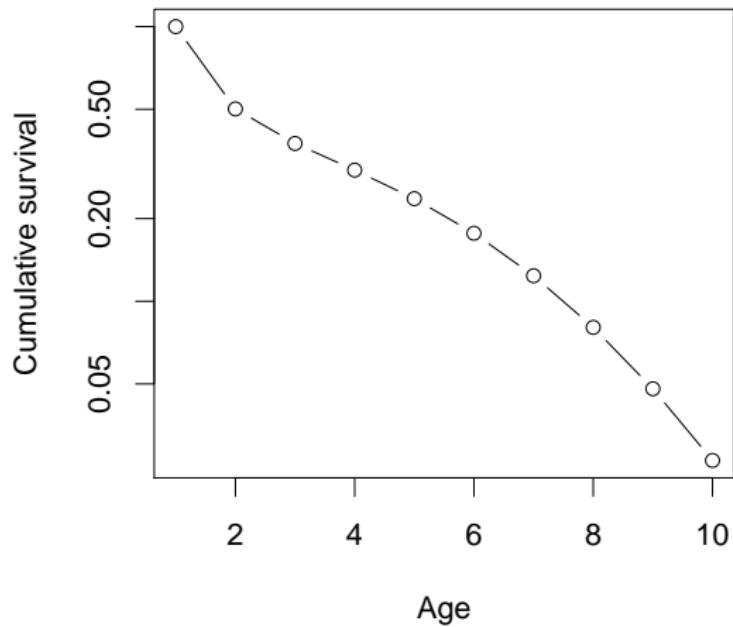
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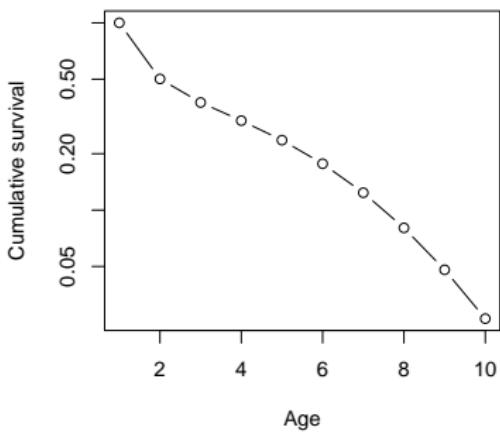
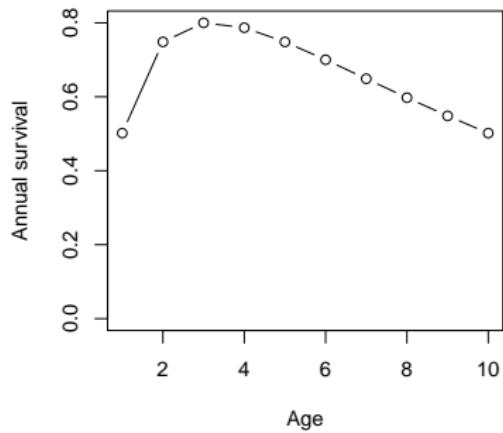
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- ▶ Just as in our simple population growth models, we define fecundity as the expected number of offspring at the next census produced by an individual observed at this census
 - ▶ Parent must survive from counting to reproduction
 - ▶ Parent must give birth
 - ▶ Offspring must survive from birth to counting
- ▶ Remember to think clearly about gender when necessary
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► <http://www.gapminder.org/population/tool/>

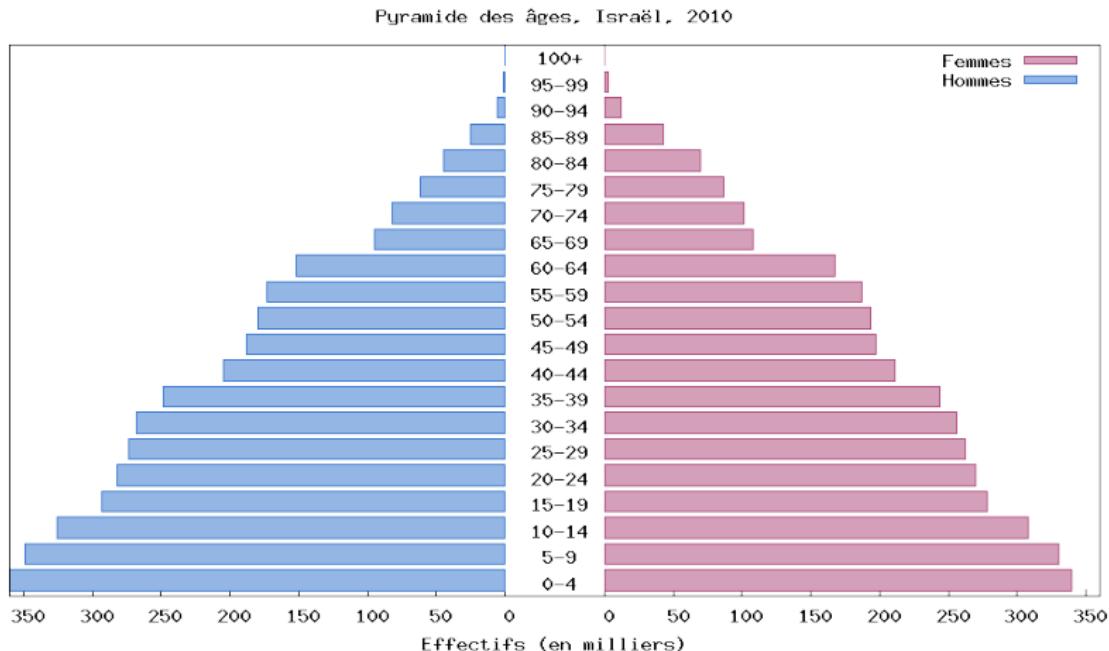
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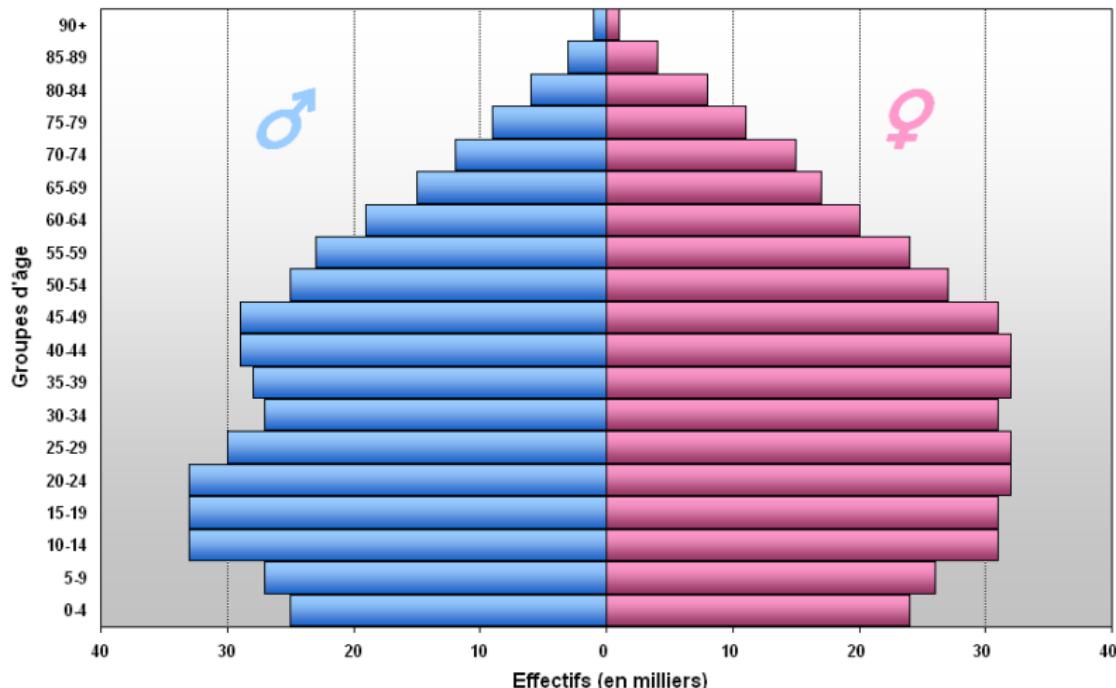
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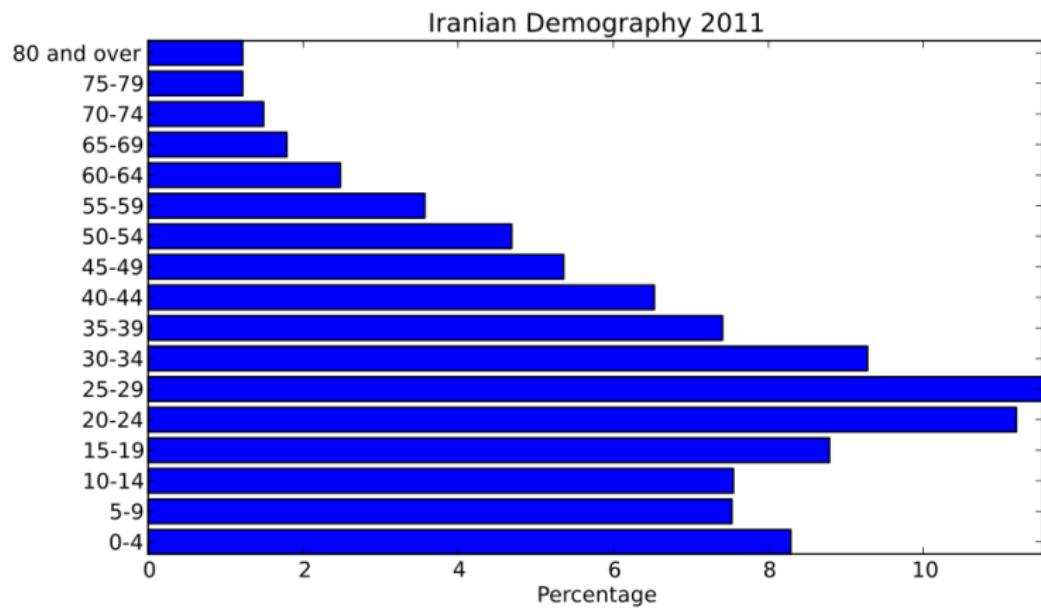
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Pyramide des âges, Chypre, 2005

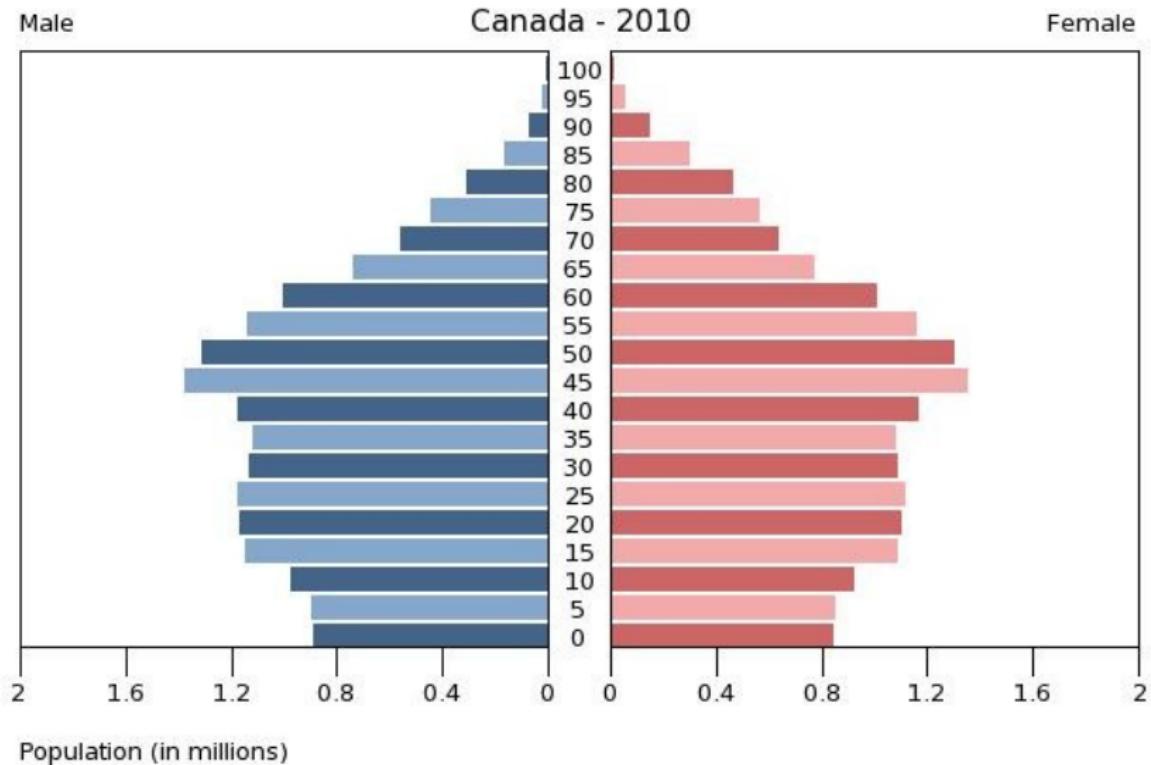


Source: Organisation des Nations Unies (World Population Prospects: The 2004 Revision)

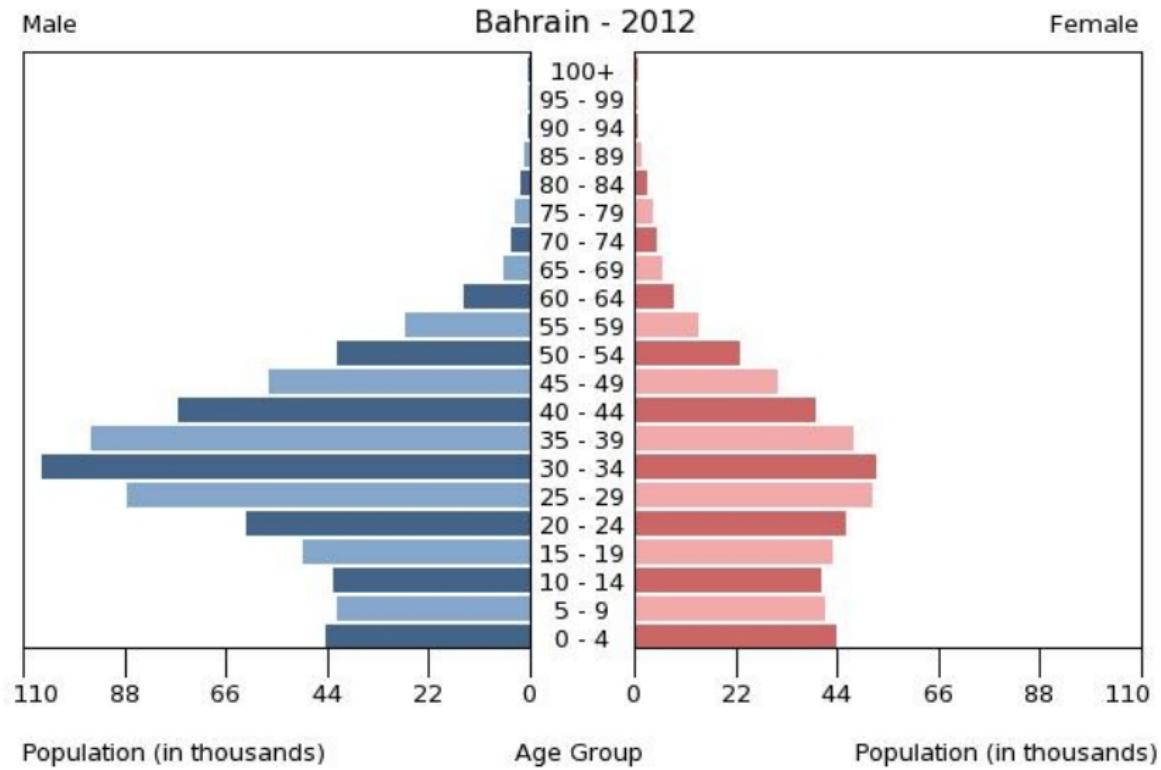
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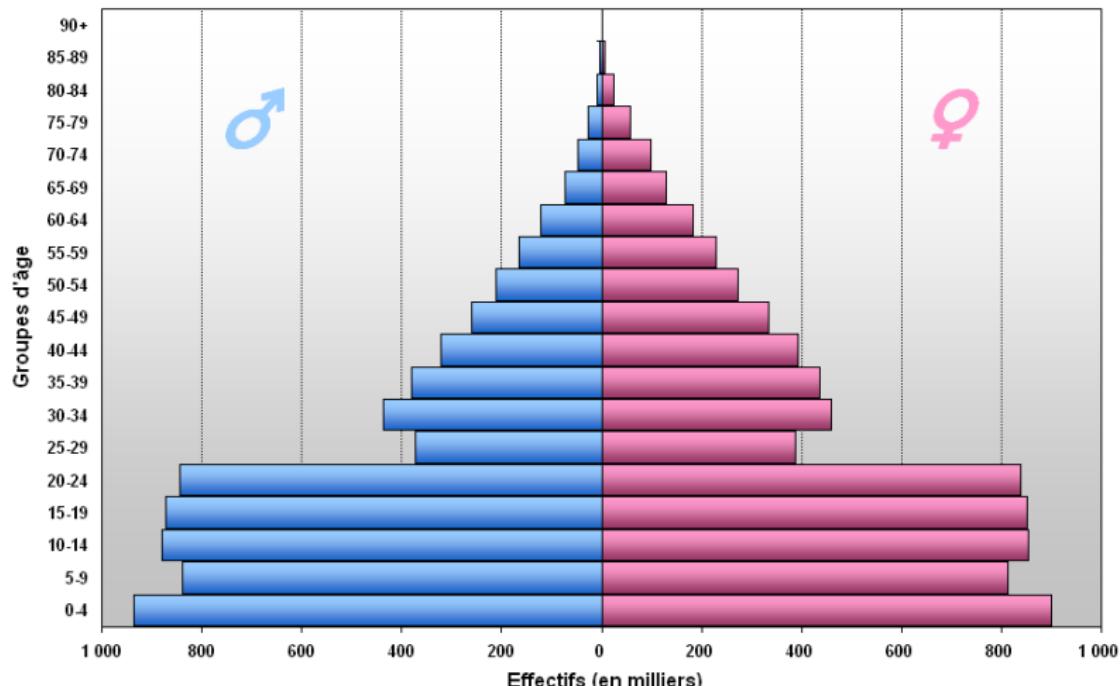


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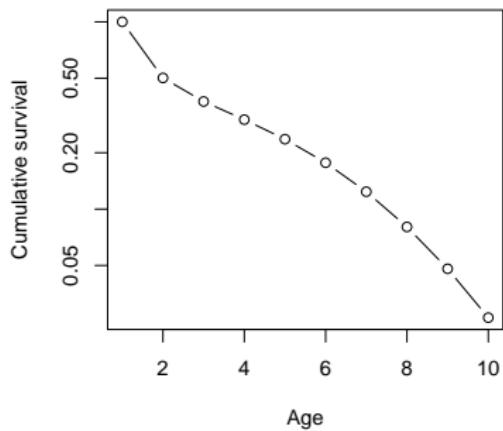
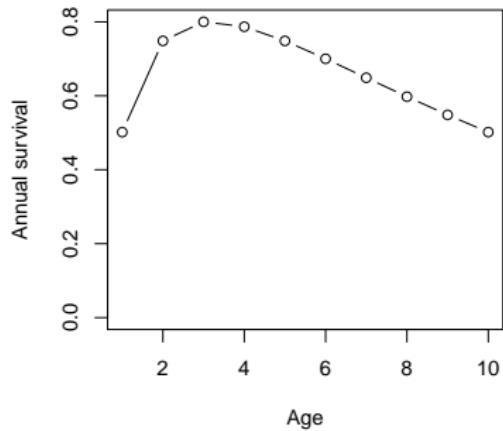
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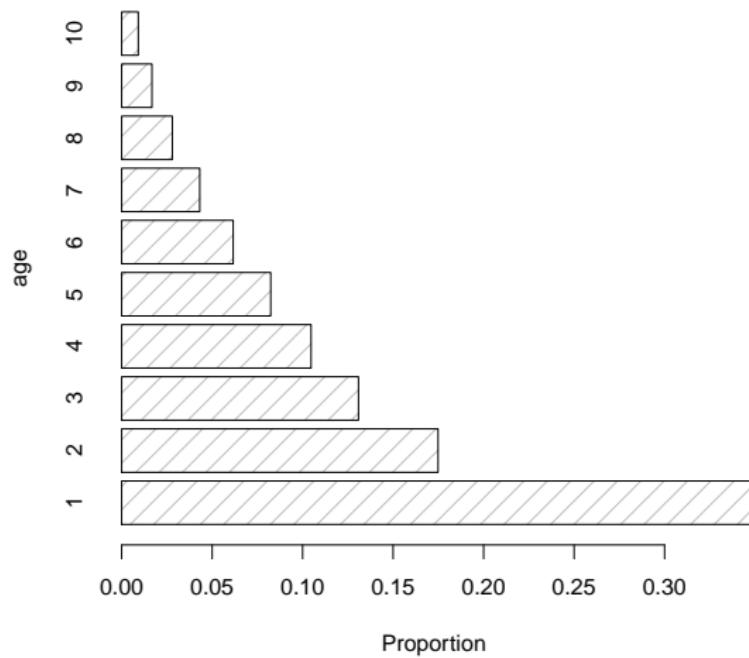
Changing survivorship (repeat)



Age distributions

Stable age distribution

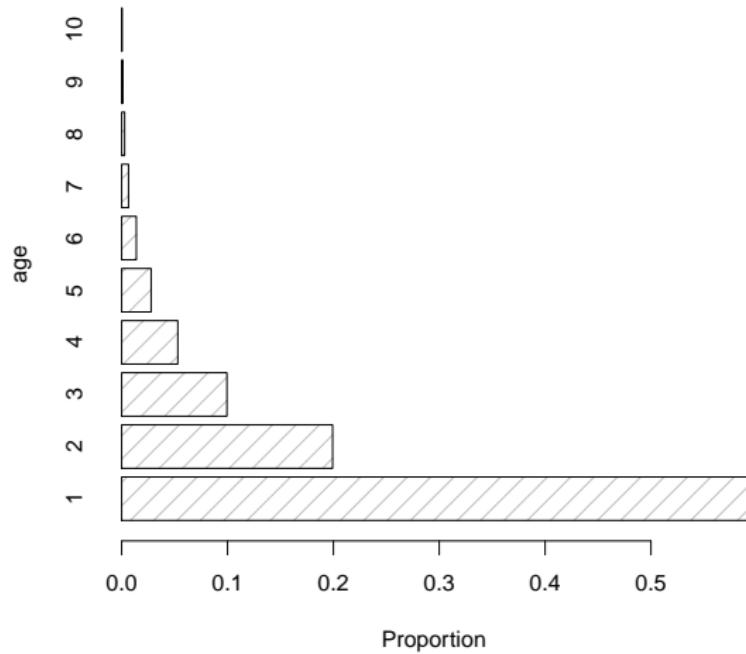
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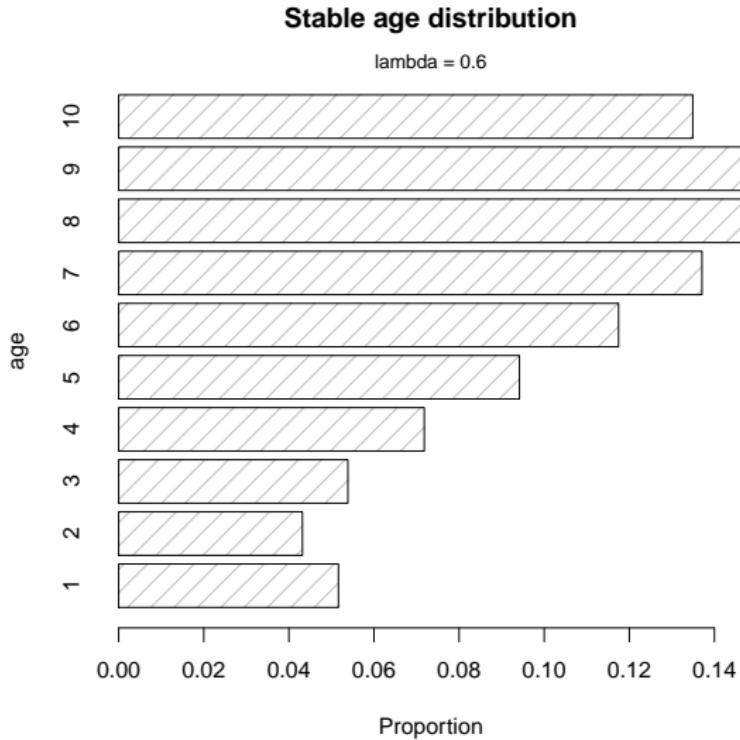
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Outline

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Model dynamics

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Measuring growth rates

Life-table patterns

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Fecundity

Age distributions

Other structured models

Stage structure

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