

## UNIT 8: Infectious disease

### 1 Introduction

#### Infectious disease

- Extremely common
- Huge impacts on ecological interactions
- A form of exploitation, but doesn't fit well into our previous modeling framework
  - How many people are there?
  - How many influenza viruses are there?
  - How do they find each other?

#### Disease agents

- Poll: Name an infectious agent that causes disease in humans.
- Disease agents vary tremendously:
  - Most **viruses** have just a handful of genes that allow them to hijack a cell and get it to make virus copies
  - \*
  - **Bacteria** are independent, free-living cells with hundreds or thousands of chemical pathways
  - \*
  - **Eukaryotic** pathogens are nucleated cells who are more closely related to you than they are to bacteria
  - \*

#### Microparasites

- For infections with small pathogens (viruses and bacteria), we don't attempt to count pathogens, but instead divide disease into stages
  - Latently infected (infected but not yet infectious)
  - Infectious
  - Recovered

## Microparasite models

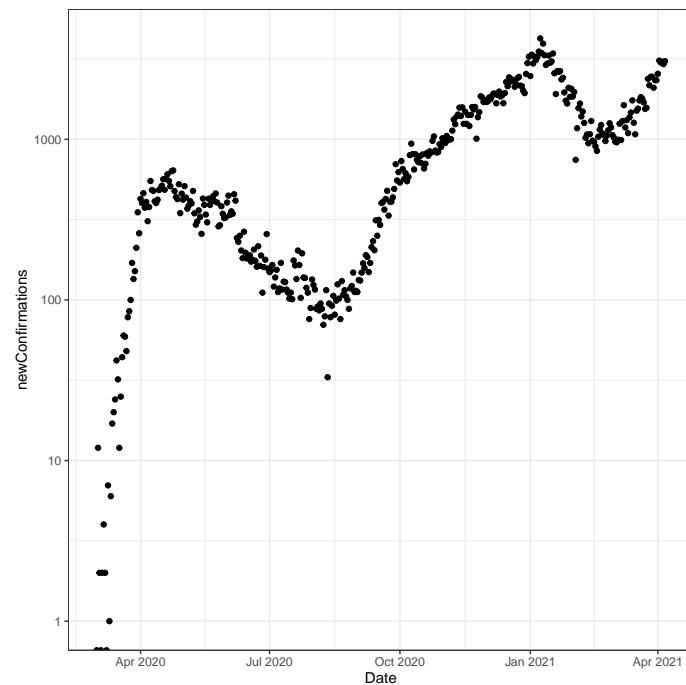
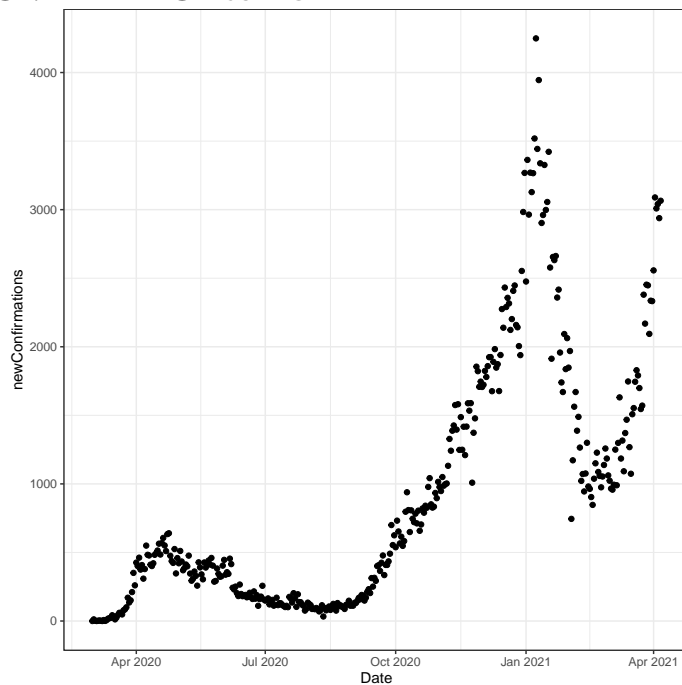
- We model microparasites by counting the number of hosts in various **states**:
  - **Susceptible** individuals can become infected
  - **Infectious** individuals are infected and can infect others
  - **Resistant** individuals are not infected and cannot become infected
- More complicated models include other states:
  - 
  - 
  -

## Models as tools

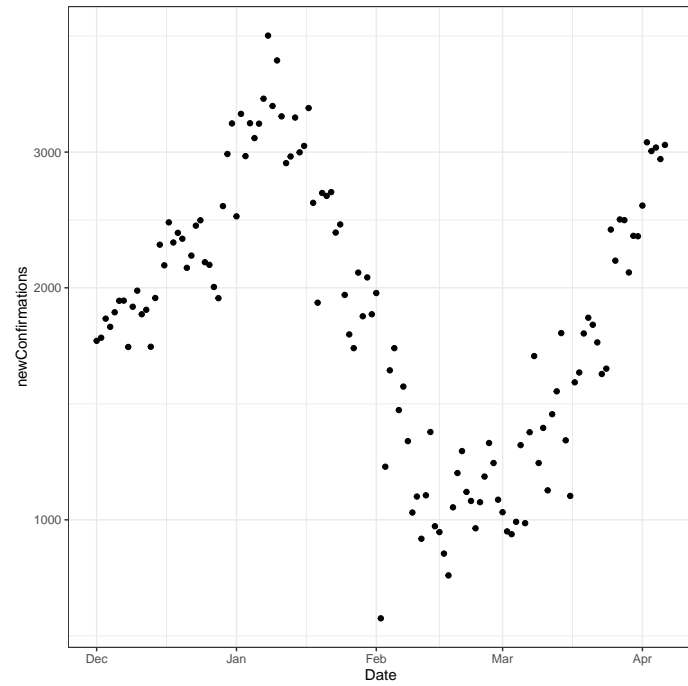
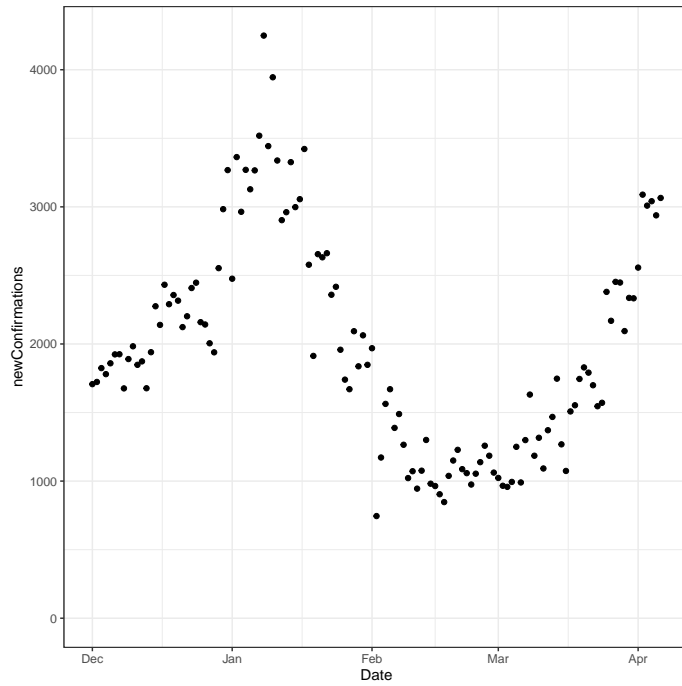
- Models are the tools that we use to connect scales:
  - individuals to populations
  - single actions to trends through time

## 2 Rate of spread

### COVID in Ontario



## COVID in Ontario



## Scales

- Which scale should we look at?

—  
—

## Population biology

- What quantities do we want to look at?

—  
—  
\*  
—

## Instantaneous rate of growth $r$

- What are the components?

—  
\*  
\*  
—  
\*  
\*

- How do you think we estimate?

—

\*

—

\* Individuals  $\leftrightarrow$  Populations

## Reproductive number $\mathcal{R}$

- What is it?

—

- Why do we want this?

—

- How do we calculate it?

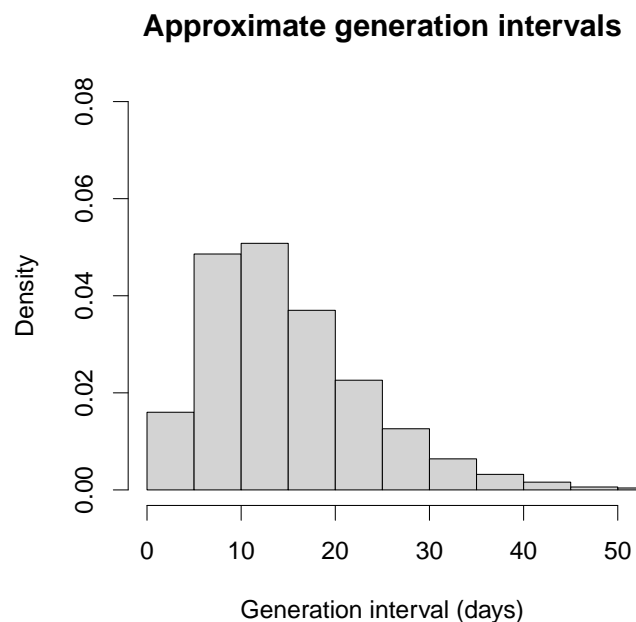
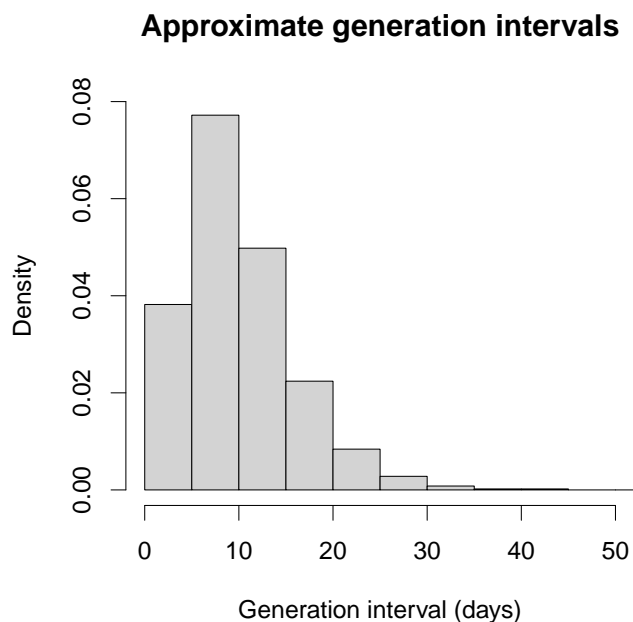
—

## Example

- $r \approx 0.14/\text{day}$
- What is our estimate of  $\mathcal{R}$ ?
  - When average length of infection  $L = 5$  day?
  - When average length of infection  $L = 10$  day?

## Generation intervals

- Researchers try to estimate the *proportion* of transmission that happens for different **ages of infection**
- How long from the time you are *infected* to the time you *infect someone else*?
- Analogous to a life table
- The effective generation time  $\hat{G}$  has units of time
  - And is hard to calculate, like  $\lambda$  in a structured model

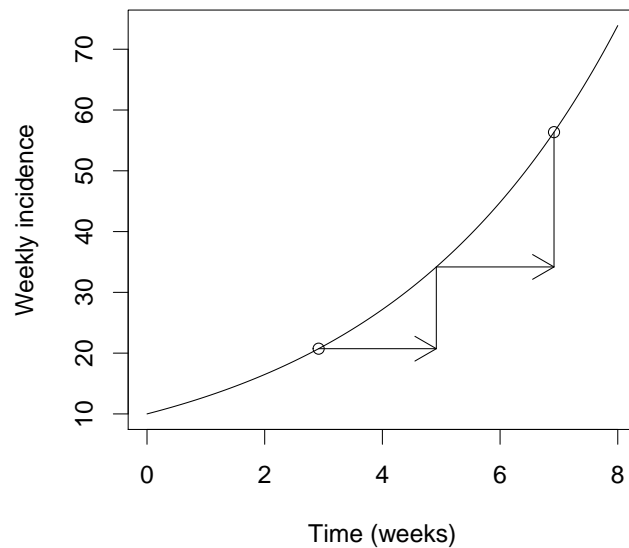
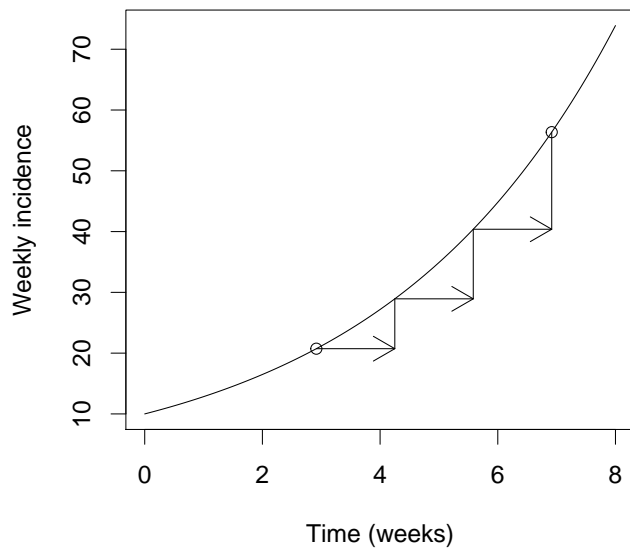


## Speed and risk

- Which is more dangerous, a fast disease, or a slow disease?
  - How are we measuring speed?
  - How are we measuring danger?
  - *What do we already know?*

## Generation time and risk

- If we know  $\mathcal{R}$ , what does the generation time tell us about  $r$ ?
  -
- If we know  $r$ , what does the generation time tell us about  $\mathcal{R}$ ?
  -
- $\mathcal{R} = \exp(r\hat{G})$

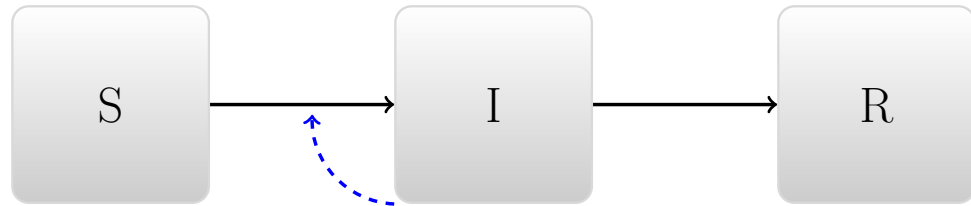


- $\mathcal{R} = \exp(r\hat{G})$
- An intuitive view:
  - Epidemic speed = Generation strength  $\times$  Generation speed
- If we know generation speed, then a faster epidemic speed means:
  -
- If we know epidemic speed, a faster generation speed means
  -

### 3 Single-epidemic model

- Susceptible  $\rightarrow$  Infectious  $\rightarrow$  Recovered
- We also use  $N$  to mean the total population

## Transition rates



- What factors govern movement through the boxes?
  - People get better independently
  - People get infected by infectious people

## Conceptual modeling

- Poll: What happens in the long term if we introduce an infectious individual?
  - 
  - 
  -
- \*

## Interpreting

- Why might there not be an epidemic?
  -
- \*
- 
- Why doesn't everyone get infected?
  -

## Quantities

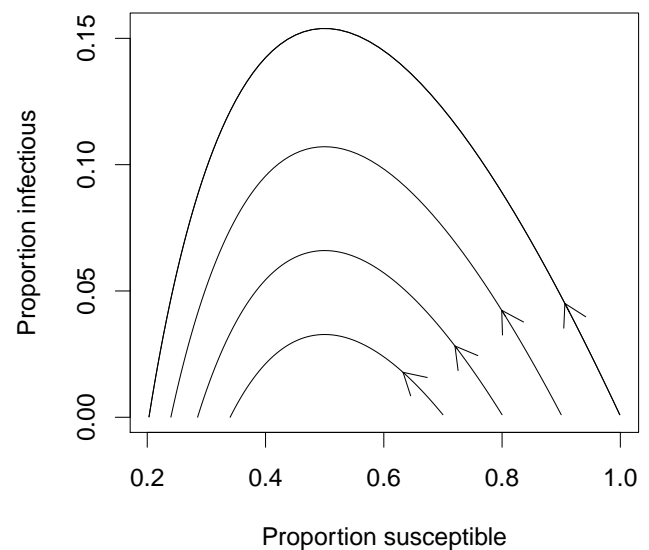
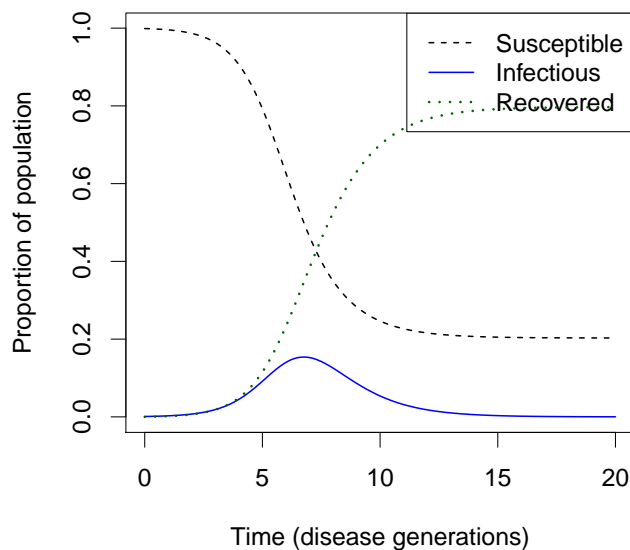
State variables

- $S, I, R, N$ : [people] or [people/ha]

## Parameters

- Susceptible people have **potentially effective** contacts at rate  $\beta$  (units [1/time])
  - These are contacts that would lead to infection if the person contacted is infectious
  - Total infection rate is  $\beta I/N$ , because  $I/N$  is the proportion of the population infectious
- Infectious people recover at *per capita* rate  $\gamma$  (units [1/time])
  - Total recovery rate is  $\gamma I$
  - Mean time infectious is  $D = 1/\gamma$  (units [time])

## Simulating the model



## Basic reproductive number

- Poll: What *unitless* parameter can you make from the model above?

—  
—  
—  
\*  
—

- Poll: What happens early in the epidemic if  $\mathcal{R}_0 > 1$ ?

—

- What happens early in the epidemic if  $\mathcal{R}_0 < 1$ ?

—



## Effective reproductive number

- The effective reproductive number gives the number of new infections per infectious individual in a partially susceptible population:

—

- Is the disease increasing or decreasing?

—

—

- Why doesn't everyone get infected?

—

—

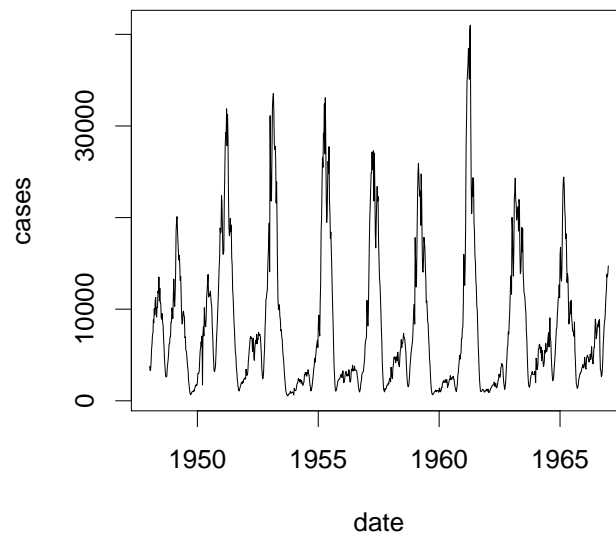
## 4 Recurrent epidemic models

- Poll: If epidemics tend to burn out, why do we often see repeated epidemics?

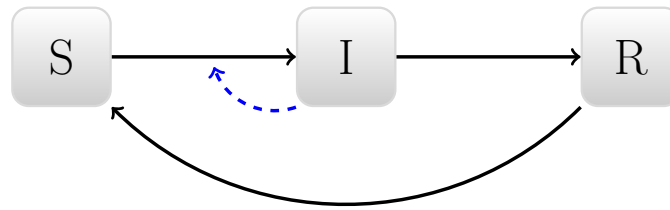
—

—

**Measles reports from England and Wales**

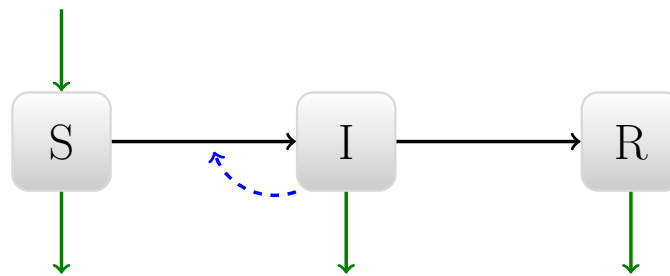


Closing the circle



•

Closing the circle



•

—

**Equilibrium**

- At equilibrium, we know that  $\mathcal{R}_e = 1$ 
  - One case per case
  - Number of susceptibles at equilibrium determined by the number required to keep infection in balance
    - \*  $S/N = 1/\mathcal{R}_0$

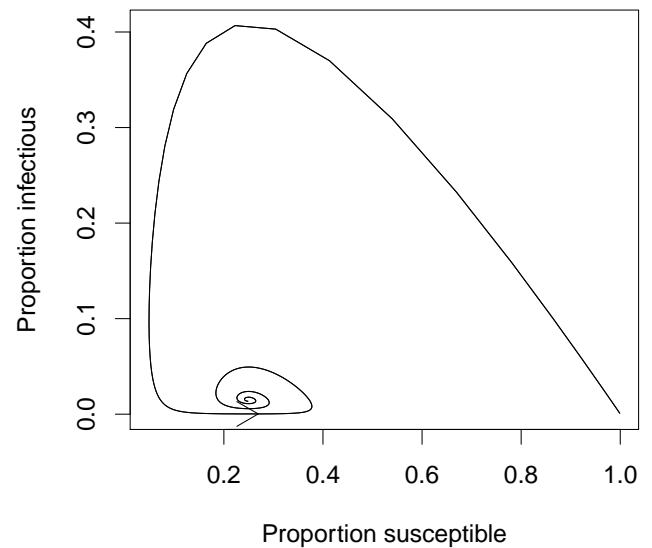
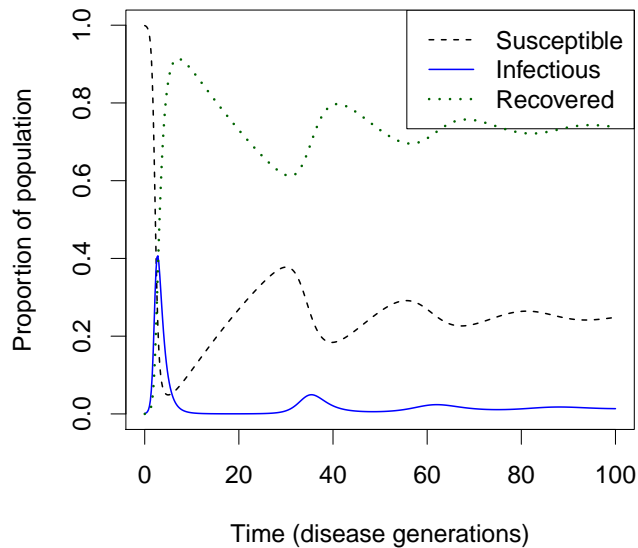
- What does this remind you of?
  -
- Number of infectious individuals determined by number required to keep susceptibles in balance.
- As susceptibles go up, what happens?
  - Rate of susceptible increase goes down
    - \* per capita, at least
  - Infections required goes down

## Reciprocal control

- What happens to *equilibrium* if we protect susceptibles (move them to  $R$  class)?
  - 
  - 
  - 
  -
- What else could happen?
  - 
  - 
  - \*
    -

## Reciprocal control

- Poll: What happens if we remove infectious individuals at a constant rate (find them and cure them or isolate them)?
  - 
  - 
  - \*
    -



- “Closed-loop” SIR models (ie., with births or loss of immunity):
  - Tend to show damped oscillations
    - \* System reaches an **endemic** equilibrium – disease persists

## Source of oscillations

- Similar to predator-prey systems
- What happens if we start with too many susceptibles?

—  
—  
—  
—  
—

## Persistent oscillations

- Poll: If oscillations tend to be damped in simple models, why do they persist in real life?

—  
\*  
—  
—  
—  
\*