Population ecology worksheet 1: Introductory concepts

Some of these are intentionally tricky, so it's probably OK if you're confused. Use the opportunity to try to get the ideas straight in your head.

1. A population is growing geometrically, according to the equation $N_T = N_0\lambda^T$. Describe the behaviour of the population in the following cases:

a. $\lambda > 1$

The population is growing

b. $0 < \lambda < 1$

The population is declining

c. $\lambda < 0$

It makes no sense for λ to be negative; this would imply a negative population in the year following a positive population.

d. Choose *two* of these three cases. For each, pick parameters for the dandelion spreadsheet to give you λ in the right range, and make a graph on the spreadsheet that shows the population doing what you say.

2. A population is growing exponentially, according to the equation $N(t) = N(0)\exp(rt)$. Describe the behaviour of the population in the following cases:

a. $r < 0$

The population is declining. $r < 0$ makes perfect sense, and corresponds to a negative *rate of change*.

b. $r > 0$

The population is growing

c. $r > 1$

Sorry, this is a trick question! Don't worry if you didn't get it; the main point is to make you think (sort of like the $\lambda < 0$, but trickier). We won't be this mean when we're marking for credit.

The desired answer is that the *question* makes no sense. r has units of 1/[time], so it makes no sense to compare it to the number 1. In general, there are no "special" values (except 0) for quantities with units. It is worth thinking about why 0 is an exception. One way to think of it is that 0 when multiplied by *anything* gives zero, so that the units disappear. Another way is to think about a concrete example: are 1 foot, 1 meter and 1 light year the same thing? How about 0 feet, 0 meters and 0 light years?

3. If Niagara Falls moves backward by 6cm/month, how many years will it take to move 1km? Use this question to practice cancelling units.

A good trick is to put the fraction in the form we want for the answer. Since we want an answer in yr/km (not km/yr), we start with month/cm instead of cm/month:

$$\frac{\text{month}}{6\,\text{cm}} \cdot \frac{100\,\text{cm}}{1\,\text{m}} \cdot \frac{1000\,\text{m}}{1\,\text{km}} \cdot \frac{\text{yr}}{12\,\text{month}} = \frac{1400\ \text{yr}}{\text{km}}$$

It will take about 1400 years.

4. A population of bacteria is growing exponentially at the rate 0.7/day. How long will it take its density to increase from 20/ml to 100/ml?

$N(t) = N(0)\exp(rt)$. Divide by N_0 to make both sides unitless, and take the log of both sides.

$\log(N(t)/N(0)) = rt$. Divide both sides by r and plug in all the numbers: $t = \frac{\log(5)}{0.7/\text{day}}$. Remember that log means natural log, and think carefully about the 1/day in the denominator, to get 2.3 days.

5. The state of Wisconsin specifies that gypsy moth control efforts will be implemented in an area when the estimated density of pupae reaches 100/ha. If the estimated density in 2010 in a particular county is 10/ha, and each year each adult female lays 600 eggs, of which (on average) 1/400 survive to become an adult female, in what year will the state begin its control efforts?

If 1/400 eggs survive, then the average number that survives for each female is 600/400 = 1.5. So the population will grow by a factor of 1.5 per year. This value of λ should be the same no matter what life stage we look at, so the number of pupae should be 15/ha in 2011; 22.5/ha in 2012; 33.7/ha in 2013; 50.6/ha in 2014; 75.9/ha in 2015; 114.0/ha in 2015. So control efforts would start in 2016. The part about applying the λ calculated from egg to egg to a calculation involving pupae is worth thinking about a little bit.

To solve directly we would say $N_T = N_0\lambda^T > N_f$, where $N_f = 100(\text{pupae}/\text{ha})$, $N_0 = 10(\text{pupae}/\text{ha})$, and $\lambda = 1.5$. $10(\text{pupae}/\text{ha})1.5^T > 100(\text{pupae}/\text{ha})$.

Cancel the units and divide by 10 to get $1.5^T > 10$. This works because λ and T are both unitless. Take the log of both sides, and divide: $T > \log(10)/\log(1.5) = 5.7$. So we want $T = 6$; if $T = 0$ is 2010, then $T = 6$ is 2016.