

UNIT 8: Infectious disease

1 Introduction

Infectious disease

- Extremely common
- Huge impacts on ecological interactions
- A form of exploitation, but doesn't fit well into our previous modeling framework
 - How many people are there?
 - How many influenza viruses are there?
 - How do they find each other?

Disease agents

- Poll: Name an infectious agent that causes disease in humans.
- Disease agents vary tremendously:
 - Most **viruses** have just a handful of genes that allow them to hijack a cell and get it to make virus copies
 - * **Answer:** influenza virus, Ebola virus, HIV, SARS-CoV-2
 - **Bacteria** are independent, free-living cells with hundreds or thousands of chemical pathways
 - * **Answer:** Tuberculosis, anthrax, pertussis
 - **Eukaryotic** pathogens are nucleated cells who are more closely related to you than they are to bacteria
 - * **Answer:** Malaria, various worms

Microparasites

- For infections with small pathogens (viruses and bacteria), we don't attempt to count pathogens, but instead divide disease into stages
 - Latently infected (infected but not yet infectious)
 - Infectious
 - Recovered

Microparasite models

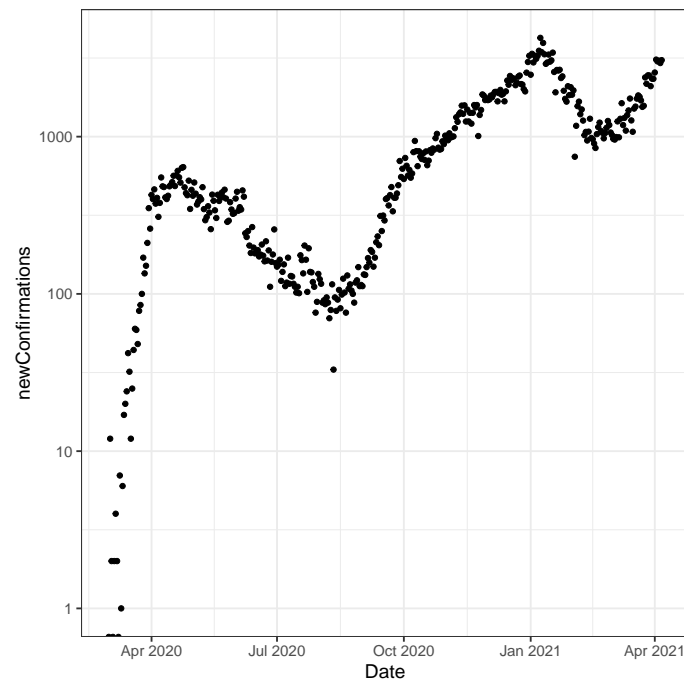
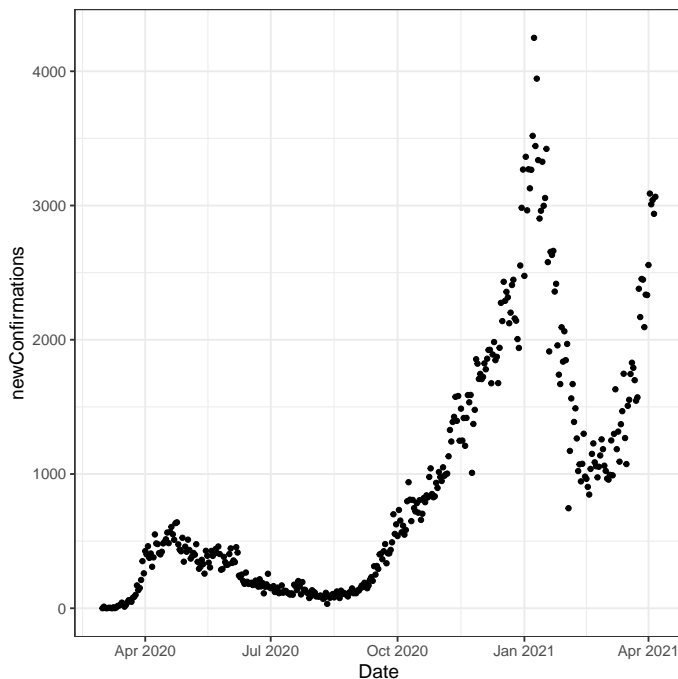
- We model microparasites by counting the number of hosts in various **states**:
 - **Susceptible** individuals can become infected
 - **Infectious** individuals are infected and can infect others
 - **Resistant** individuals are not infected and cannot become infected
- More complicated models include other states:
 - **Answer:** Not yet infectious
 - **Answer:** Severe infections
 - **Answer:** Asymptomatic but infectious

Models as tools

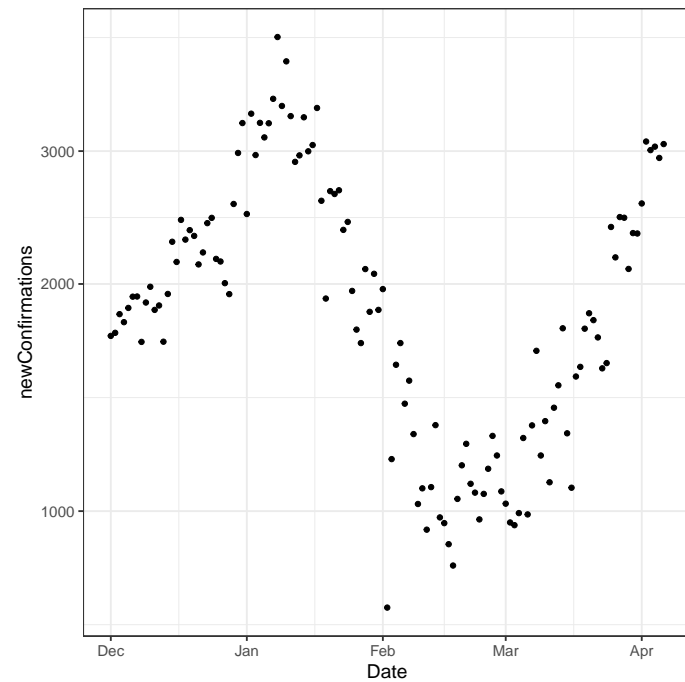
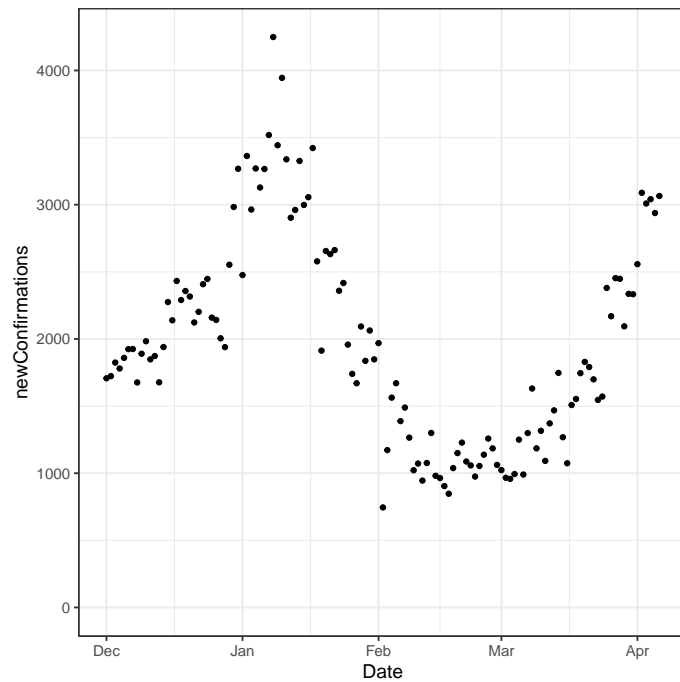
- Models are the tools that we use to connect scales:
 - individuals to populations
 - single actions to trends through time

2 Rate of spread

COVID in Ontario



COVID in Ontario



Scales

- Which scale should we look at?
 - Answer: Log scale is better for looking at trends
 - Answer: Linear scale is better for looking at impacts

Population biology

- What quantities do we want to look at?
 - Answer: Speed of exponential growth r
 - Answer: Finite rate of increase λ
 - * Answer: Skipped this year
 - Answer: Lifetime reproduction

Instantaneous rate of growth r

- What are the components?
 - Answer: Birth rate
 - * Answer: Instantaneous rate of a case producing new cases
 - * Answer: $[\text{case}/(\text{case} \cdot \text{time})]$
 - Answer: Death rate
 - * Answer: Virus-centered!
 - * Answer: Rate of death, recovery, or effective quarantine

- How do you think we estimate?
 - Answer: We estimate r from the population-level increase in disease
 - * Answer: Then using that to estimate b
 - Answer: Models go both directions!
 - * Individuals \leftrightarrow Populations

Reproductive number \mathcal{R}

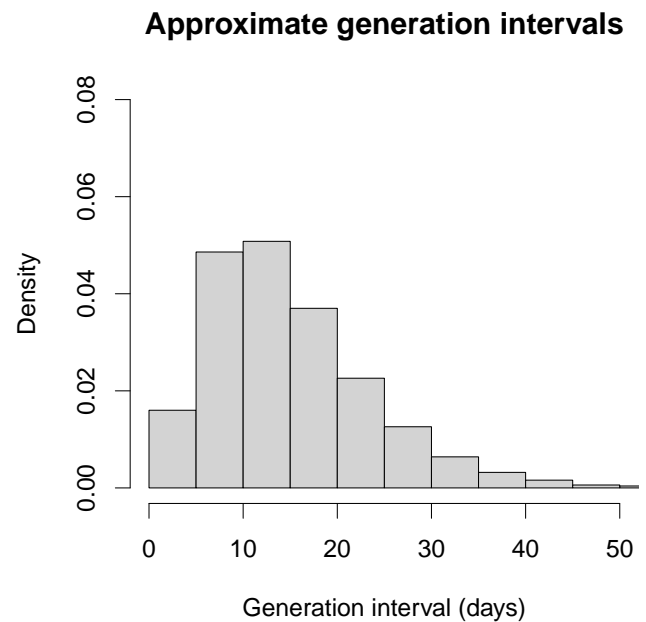
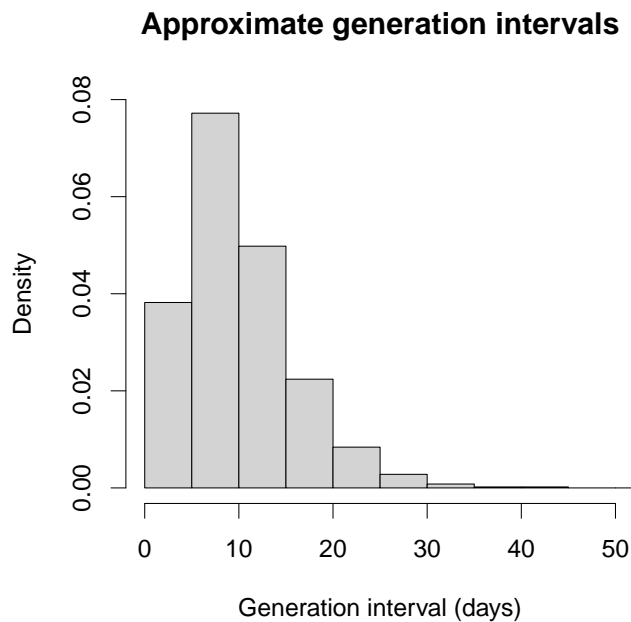
- What is it?
 - Answer: Expected number of new cases per case over the lifetime of a case
- Why do we want this?
 - Answer: An important measure of how hard the epidemic will be to stop
- How do we calculate it?
 - Answer: $\mathcal{R} = b/d$; if we can estimate those

Example

- $r \approx 0.14/\text{day}$
- What is our estimate of \mathcal{R} ?
 - When average length of infection $L = 5 \text{ day}$?
 - When average length of infection $L = 10 \text{ day}$?

Generation intervals

- Researchers try to estimate the *proportion* of transmission that happens for different **ages of infection**
- How long from the time you are *infected* to the time you *infect someone else*?
- Analogous to a life table
- The effective generation time \hat{G} has units of time
 - And is hard to calculate, like λ in a structured model

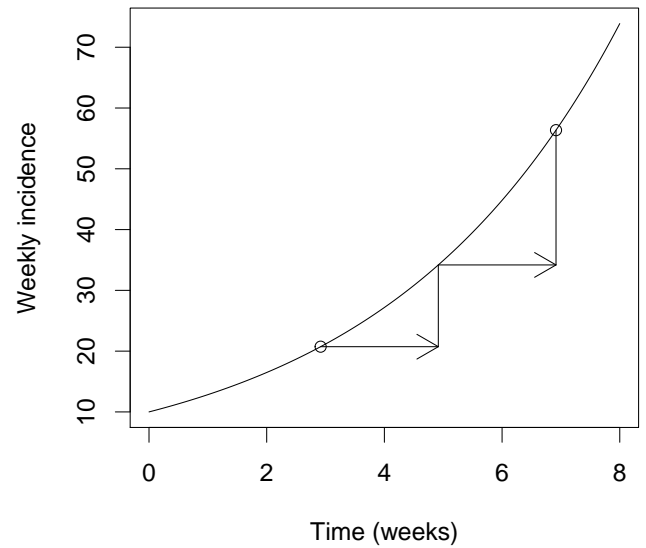
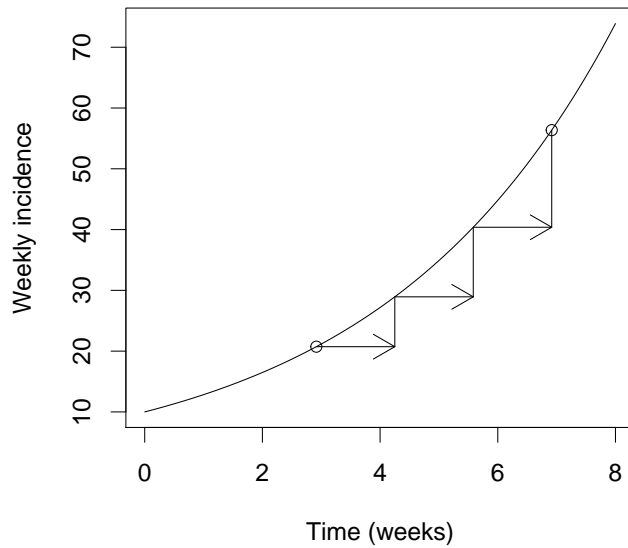


Speed and risk

- Which is more dangerous, a fast disease, or a slow disease?
 - How are we measuring speed?
 - How are we measuring danger?
 - *What do we already know?*

Generation time and risk

- If we know \mathcal{R} , what does the generation time tell us about r ?
 - **Answer:** The faster the generations (small \hat{G}), the faster the exponential growth (large r)
- If we know r , what does the generation time tell us about \mathcal{R} ?
 - **Answer:** The faster the generations (small \hat{G}), the the *smaller* the strength of the epidemic (small reproductive number \mathcal{R})
- $\mathcal{R} = \exp(r\hat{G})$

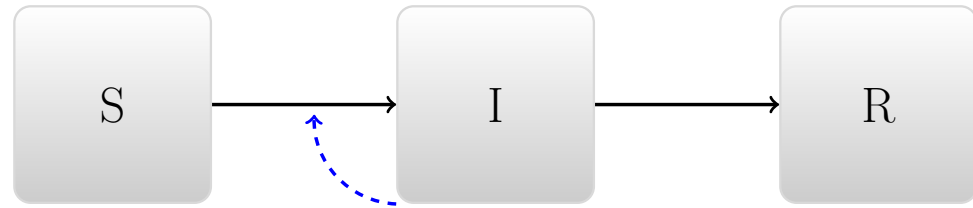


- $\mathcal{R} = \exp(r\hat{G})$
- An intuitive view:
 - Epidemic speed = Generation strength \times Generation speed
 - Comment: Mathematically: $r = \log(\mathcal{R}) * (1/\hat{G})$
- If we know generation speed, then a faster epidemic speed means:
 - Answer: More strength required (greater \mathcal{R})
- If we know epidemic speed, a faster generation speed means
 - Answer: Less strength required (smaller \mathcal{R})

3 Single-epidemic model

- Susceptible \rightarrow Infectious \rightarrow Recovered
- We also use N to mean the total population

Transition rates



- What factors govern movement through the boxes?
 - People get better independently
 - People get infected by infectious people

Conceptual modeling

- Poll: What happens in the long term if we introduce an infectious individual?
 - **Answer:** The *may be* an **epidemic** – an outbreak of disease
 - **Answer:** Disease burns out
 - **Answer:** Everyone winds up either recovered ...
 - * **Answer:** or susceptible!

Interpreting

- Why might there not be an epidemic?
 - **Answer:** If the disease can't spread well enough in the population
 - * **Answer:** Could depend on season, or immunity ...
 - **Answer:** Demographic stochasticity: if we only start with one individual, we expect an element of chance
- Why doesn't everyone get infected?
 - **Answer postponed:**

Quantities

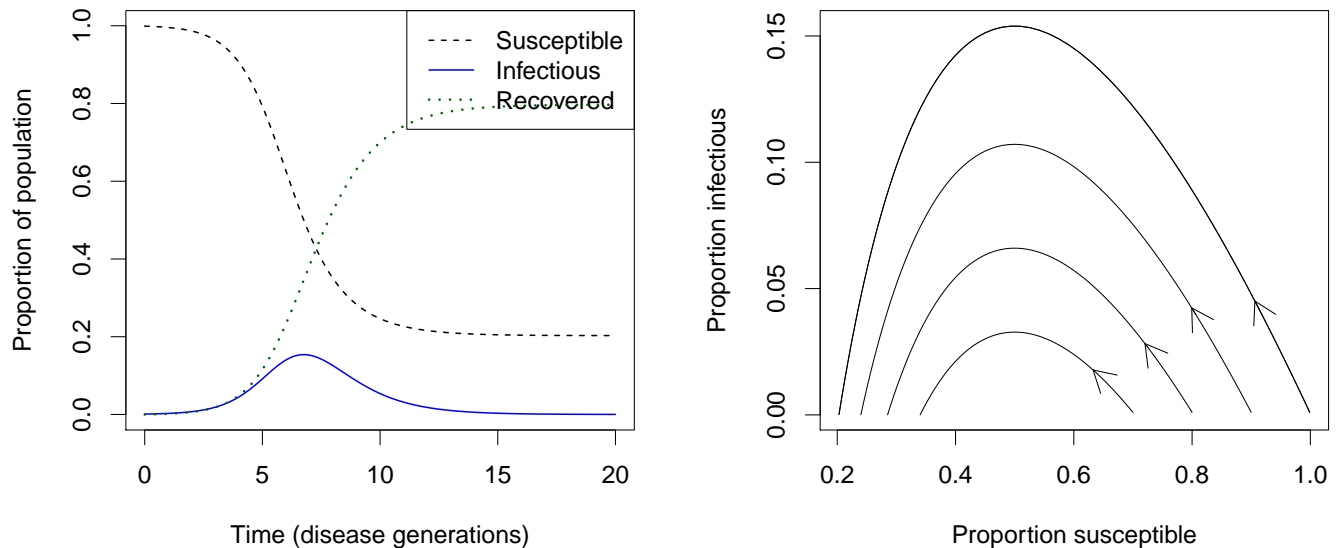
State variables

- S, I, R, N : [people] or [people/ha]

Parameters

- Susceptible people have **potentially effective** contacts at rate β (units [1/time])
 - These are contacts that would lead to infection if the person contacted is infectious
 - Total infection rate is $\beta I/N$, because I/N is the proportion of the population infectious
- Infectious people recover at *per capita* rate γ (units [1/time])
 - Total recovery rate is γI
 - Mean time infectious is $D = 1/\gamma$ (units [time])

Simulating the model



Basic reproductive number

- Poll: What *unitless* parameter can you make from the model above?
 - **Answer:** $\mathcal{R}_0 = \beta D = \beta/\gamma$ is the **basic reproductive number**
 - **Answer:** The *potential* number of infections caused by an average infectious individual
 - * **Answer:** That is: the number they would cause on average if everyone else were susceptible
 - **Answer:** The product of the rate β (units [1/t]) and the duration D ([t])

- Poll: What happens early in the epidemic if $\mathcal{R}_0 > 1$?
 - **Answer:** Number of infected individuals grows exponentially
- What happens early in the epidemic if $\mathcal{R}_0 < 1$?
 - **Answer:** Number of infected individuals does not grow (disease cannot invade)

Effective reproductive number

- The effective reproductive number gives the number of new infections per infectious individual in a partially susceptible population:
 - **Answer:** $\mathcal{R}_e = \mathcal{R}_0 S/N$
- Is the disease increasing or decreasing?
 - **Answer:** It will increase when $\mathcal{R}_e > 1$ (more than one case per case)
 - **Answer:** This happens when $S/N > 1/\mathcal{R}_0$
- Why doesn't everyone get infected?
 - **Answer:** When susceptibles are low enough $\mathcal{R}_e < 1$
 - **Answer:** When $\mathcal{R}_e < 1$, the disease dies out on its own (less than one case per case)