Bio 3SS Math review

Notation (e is cool)

- In this course log or $\log_e$ means the *natural* logarithm (written *ln* in the textbooks)
  - If I mean $\log_{10}$, I will say so
- exp() is a synonym for "e to the power of"

Logarithms

- Multiplying on the original scale is equivalent to adding on the log scale
  - $\log(ab) = \log(a) + \log(b)$
- Division is the opposite of multiplication and subtraction is the opposite of addition:
  - $\log(a/b) \equiv \log\left(\frac{a}{b}\right) = \log(a) - \log(b)$
- Exponentiation is to multiplication like multiplication is to addition:
  - $\log(\lambda^k) = k\log(\lambda)$

Complementarity

- $\log_e$ and exp are complementary functions
  - $\exp(\log_e(x)) \equiv e^{\log_e(x)} = x$
  - $\log_e(\exp(x)) \equiv \log_e(e^x) = x$

Exponentiation

- The rules for exp are exactly complementary to those for log
  - $\exp(a+b) = \exp(a) \cdot \exp(b)$
  - $\exp(a-b) = \exp(a)/\exp(b)$
  - $\exp(k\log_e(\lambda)) = \lambda^k$
- Remember: you can't take the log or exponent of something with units

Problems

- You should be able to use these rules to solve simple problems, algebraically, or on your calculator
  - If $\lambda^{20} = 0.1$, what is λ?
- Also, word problems
  - If a population declines by 90% in 20 generations, what is its reproductive number per generation?

Algebra

- If $\lambda^{20} = 0.1$, what is λ?
  - **Answer:** $20 \log_e(\lambda) = \log_e(0.1)$
  - **Answer:** $\log_e(\lambda) = \log_e(0.1)/20$
  - **Answer:** $\lambda = \exp(\log_e(0.1)/20)$
  - **Answer:** $\lambda = 0.89$
- If $\exp(rt) = 5$, and $r = 0.1/\,\mathrm{yr}$, what is t?
  - **Answer:** $rt = \log_e(5)$
  - **Answer:** $t = \frac{\log_e(5)}{0.1/\,\mathrm{yr}}$
  - **Answer:** $t = 16\,\mathrm{yr}$

Units are our friends

- Keep track of units at all times
- Use units to confirm that your answers make sense
  - Or to find quick ways of getting the answer
- Get used to manipulating and cancelling units
- 36 mpg = ?? L/100km
  - **Answer:** $\dfrac{\mathrm{gal}}{36\,\mathrm{mi}} \cdot \dfrac{\mathrm{mi}}{1.6\,\mathrm{km}} \cdot \dfrac{3.8\,\mathrm{L}}{\mathrm{gal}} \cdot \dfrac{100\,\mathrm{km}}{100\,\mathrm{km}}$
  - **Answer:** 6.6 L/100km