

**Formulas**

*discrete time growth:*

- $N_T = N_0 \lambda^T$
- $\lambda = f + p$
- $\mathcal{R} = f/(1 - p)$

*continuous time growth:*

- $N(t) = N(0) \exp(rt)$
- $r = b - d$
- $\mathcal{R} = b/d$

Use this information for the next two questions. A population of small plants has discrete, overlapping generations. Adults survive each year with a probability of  $1/2$  (and thus they have an average lifespan of two years). Each reproducing adult produces an average of 20 seeds *each* year, of which an average of 10% survive to reproduce in the next year. We model this population using a discrete-time model with time step of 1 year, and we count individuals just before reproduction.

1. What are the values for survival  $p$  and fecundity  $f$  for this model?

- A.  $p = 1/4$  and  $f = 1$
- B.  $p = 1/2$  and  $f = 1$
- C.  $p = 1/4$  and  $f = 2$
- D.  $p = 1/2$  and  $f = 2$

2. What can you say about the finite rate of increase  $\lambda$  and the reproductive number  $\mathcal{R}$  for this population?

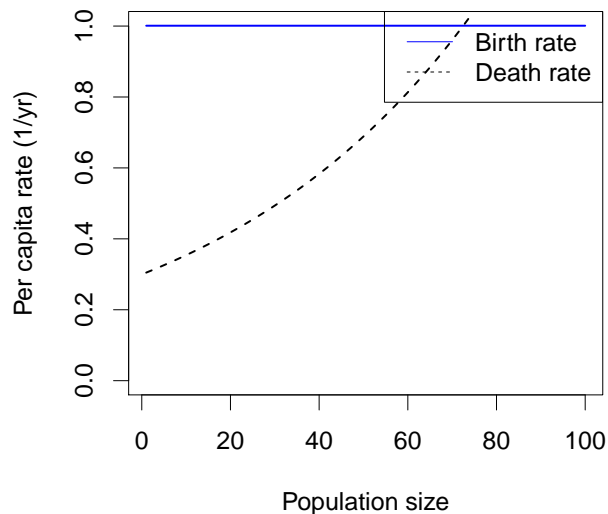
- A.  $\mathcal{R}$  and  $\lambda$  both  $= p + f$
- B.  $\mathcal{R}$  and  $\lambda$  are both  $> p + f$
- C.  $\mathcal{R} = p + f$ , but  $\lambda$  is larger
- D.  $\lambda = p + f$ , but  $\mathcal{R}$  is larger

3. A population of shrubs is growing exponentially with a characteristic time of 4 yr. Its doubling time will be approximately

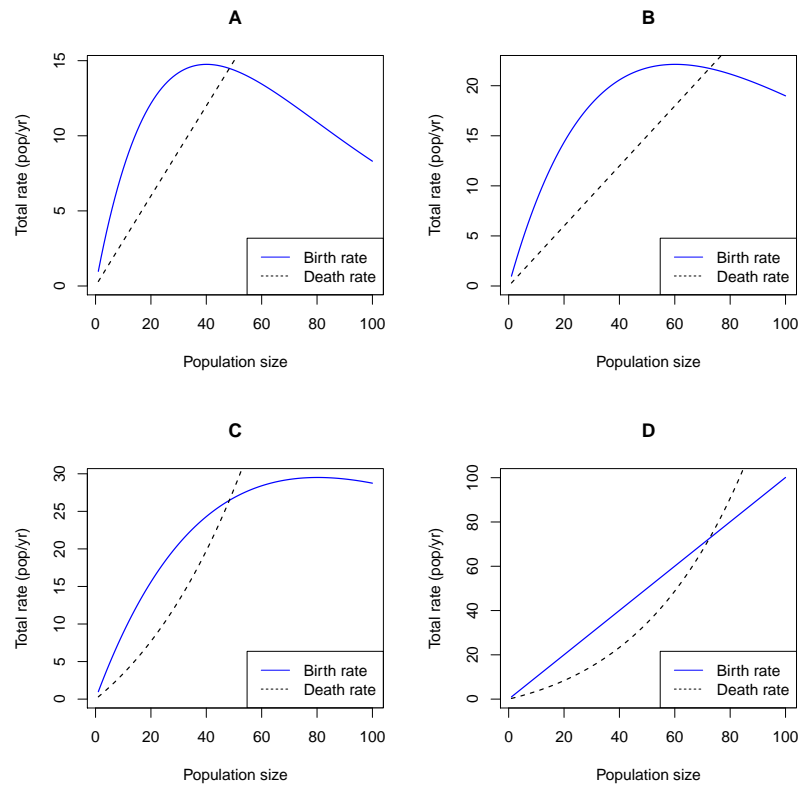
- A. 0.17 yr
- B. 0.36 yr
- C. 1 yr
- D. 2.8 yr
- E. 5.8 yr

4. Simple models of continuous-time regulation can be useful, but *cannot* explain:
- A. Why exponential growth often occurs at low density
  - B. Why total population growth is usually highest at intermediate density
  - C. Why some populations fluctuate periodically
  - D. Why populations may not persist in an area even if they can complete each step of their reproductive cycle
5. If a simple model assumes individuals are independent of each other, then \_\_\_\_\_ death rates should \_\_\_\_\_ with the size of the population.
- A. per capita; increase
  - B. per capita; decrease
  - C. total; increase
  - D. total; decrease

Use the picture below for the next three questions. It shows the assumptions made for a continuous-time birth-death model.



6. Which of the four pictures below could be generated by the same model as the picture above?



7. The highest *total population* net growth rate ( $dN/dt$ ) in this model is seen:

- A. When the population is very small
- B. When the population is between the two equilibria
- C. When the population is at the nonzero equilibrium
- D. When the population is very large

8. The model illustrated above predicts that the population will decrease:

- A. When the population is very small (only)
- B. When the population is very large (only)
- C. When the population is very small or very large
- D. When the population is between the two equilibria
- E. When the population is at the nonzero equilibrium

9. Which of the following is the *least* likely scenario for a density-dependent per capita response? As population density goes up:

- A. Birth rate goes down and death rate goes up
- B. Birth rate goes down and death rate goes down faster
- C. Birth rate goes up and death rate goes up faster

10. Modern humans have been very successful over the last 100 kiloyears. Considering the lifespan of a single human, it would be most accurate to say that the instantaneous rate of increase  $r$  has been:

- A. A little greater than zero
- B. Much greater than zero
- C. A little greater than one
- D. Much greater than one

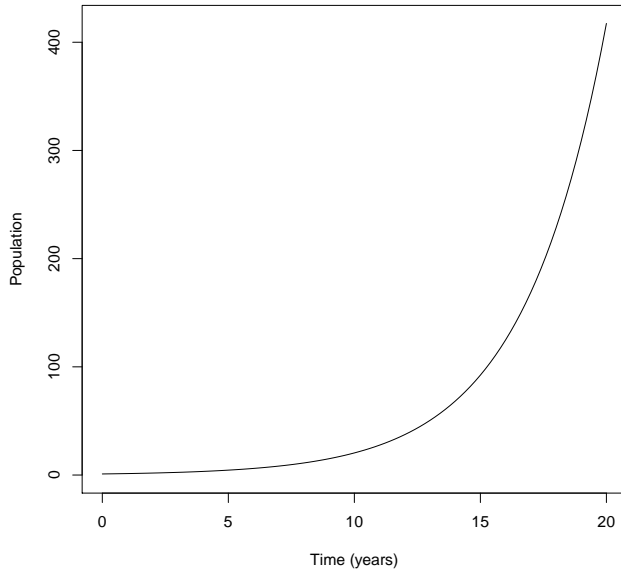
11. If we are modeling the spread of coronavirus with one of our population models, then a “death” would correspond to a person:

- A. Catching the disease
- B. Either catching the disease or recovering
- C. Dying from the disease
- D. Either dying from the disease or recovering

12. Which of these is *not* a likely mechanism for the population of coronavirus to regulate itself?

- A. People recovering from the disease and becoming immune
- B. People changing behaviour in response to the disease
- C. Viral evolution
- D. A vaccination campaign

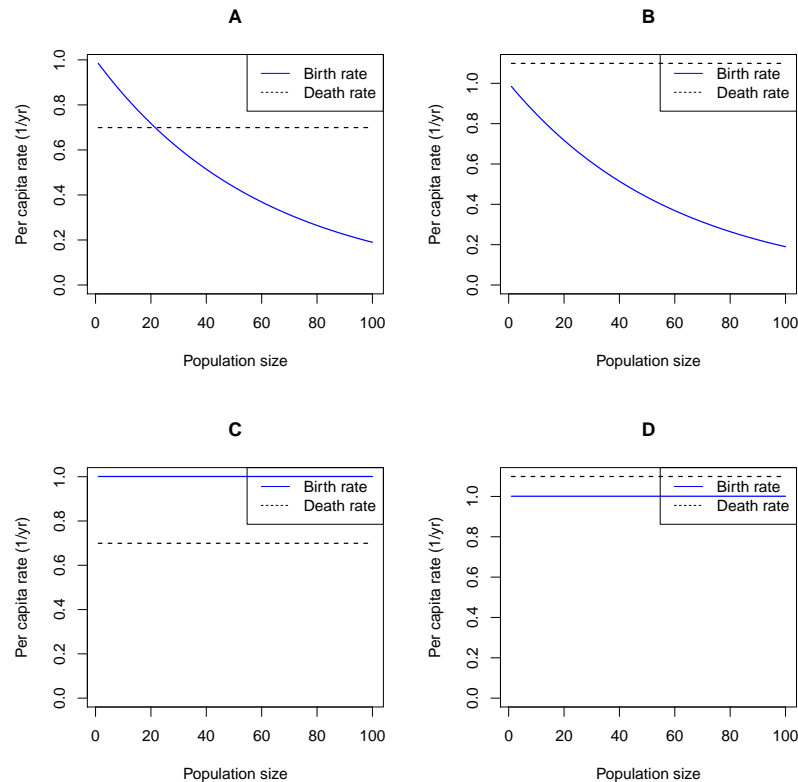
Use the picture below for the next two questions. It shows a time series for a continuous-time birth-death model.



13. This picture shows a population that is:

- A. Increasing arithmetically
- B. Increasing geometrically
- C. Increasing arithmetically on the log scale, but geometrically on a linear scale
- D. Increasing geometrically on the log scale, but arithmetically on a linear scale

14. Which of the four pictures below shows the assumptions that generated this time plot?



15. This class argues that every population regulates itself:

- A. Directly and immediately
- B. Directly, but not necessarily immediately
- C. Immediately, but not necessarily directly
- D. Either directly or indirectly and either immediately or with a delay

16. Choose the most precise correct answer. A gypsy moth population grew from 100 pupae/hectare to 2000 pupae/hectare in 2008, and then to 5000 pupae/hectare in 2009. The 2009 change was larger than the 2008 change:

- A. On the linear scale
- B. On the log scale
- C. On both scales
- D. On neither scale

17. All of the following mechanisms can change gypsy moth population growth rates. Which of the following is *least* like to *regulate* growth rate in the sense discussed in class?

- A. Conflict between gypsy-moth caterpillars
- B. Viral diseases
- C. Gypsy-moth damage to the trees
- D. Drought damage to the trees

### Short-answer questions

Answer questions *in pen*. *Briefly* show necessary work and equations. Points may be *deducted* for wrong information, even when the correct information is also there.

**18.** (4 points) Imagine some bacteria in a favorable environment. They are continuously reproducing at a constant per-capita rate of 0.2 new indiv per indiv per day and continuously dying at a constant per-capita rate of 0.1 per day. We start with a density of 5 indiv/ml.

a) Write an equation describing our *assumptions* about how this population changes through time (not the result).

b) What are the birth rate  $b$ , death rate  $d$  and growth rate  $r$ ?

c) How many bacteria do we expect to see after a day?

d) How many bacteria do we expect to see after a week?



**19.** A car uses 8L of gasoline per 100km.

a) (2 points) How far can the car go on 10L of gasoline? Show work with units.

b) (2 points) If  $1\text{L} = 1\text{m}^3$  and  $1\text{km} = 1000\text{m}$ , write the fuel consumption of the car in the simplest form (consumption should be higher if the car uses more gas for a given distance).

c) (1 point, extra credit). Can you find a clear explanation for the simplest units of fuel consumption?