UNIT 1: Linear population models

Outline

Subsection 1

Dandelions

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Subsection 2

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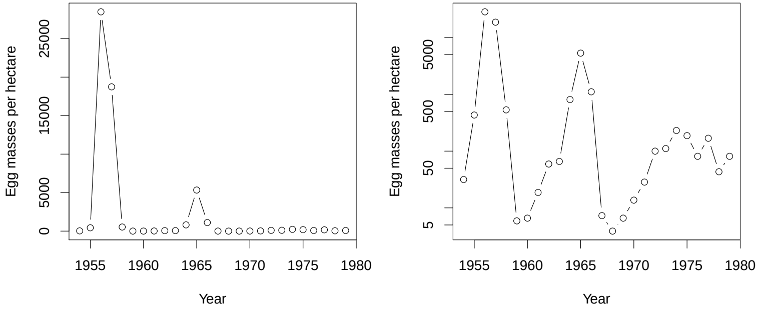
Gypsy moth populations



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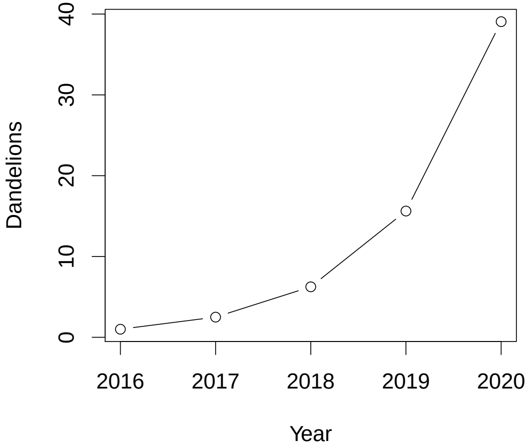

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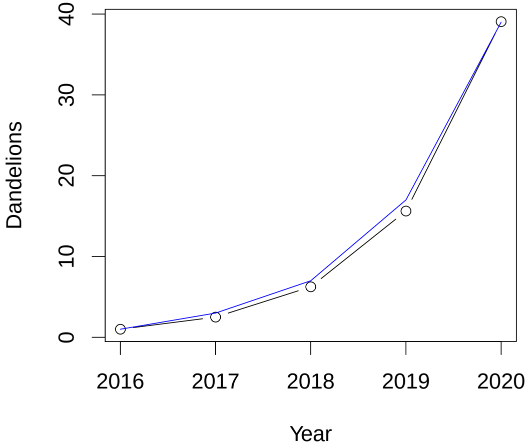

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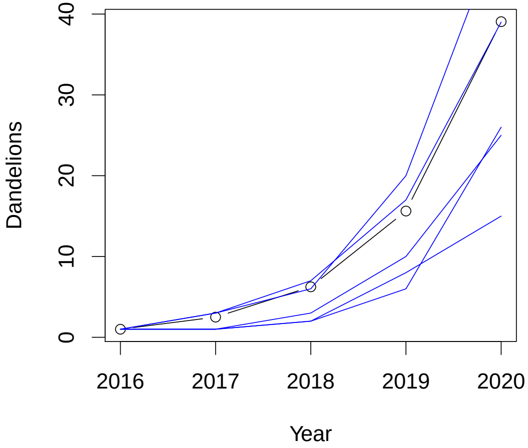

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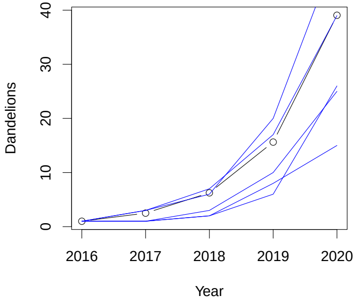

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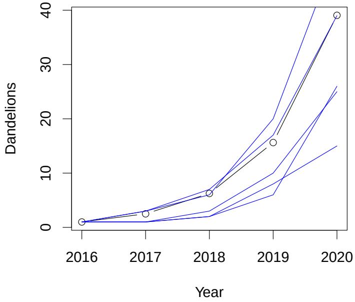

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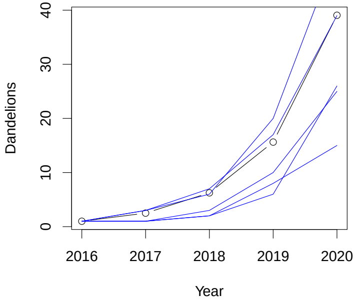

Subsection 3

Bacteria

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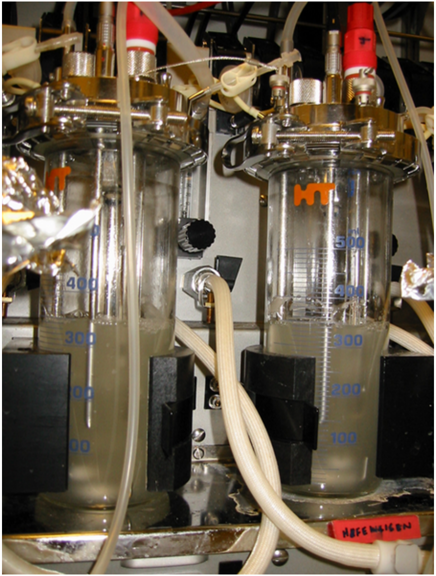

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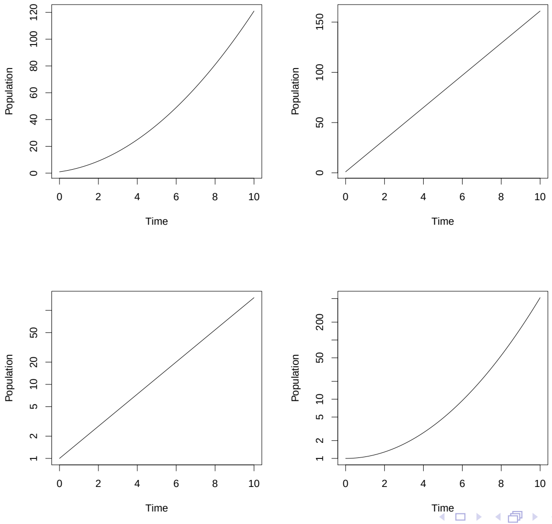

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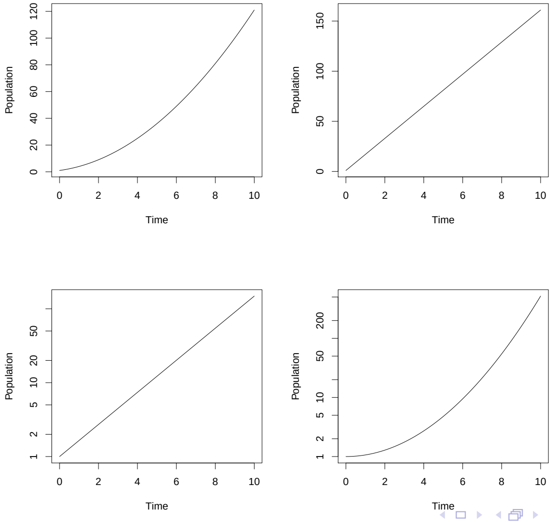

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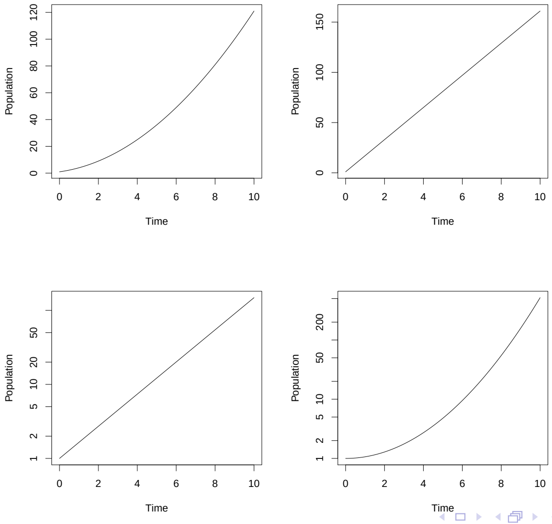

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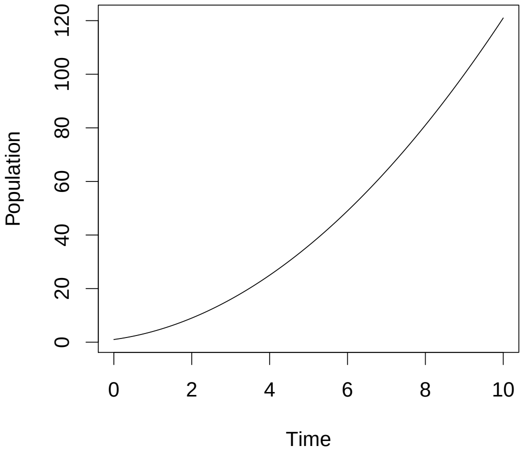
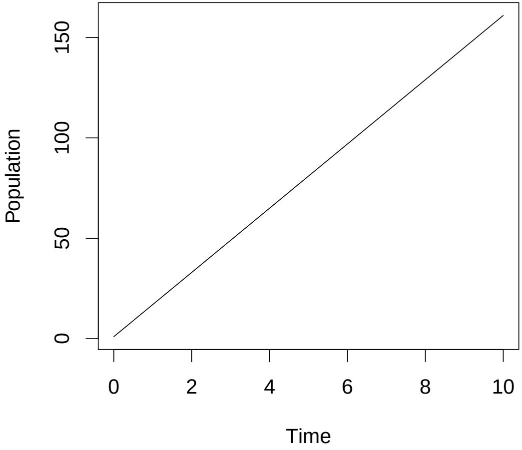


B



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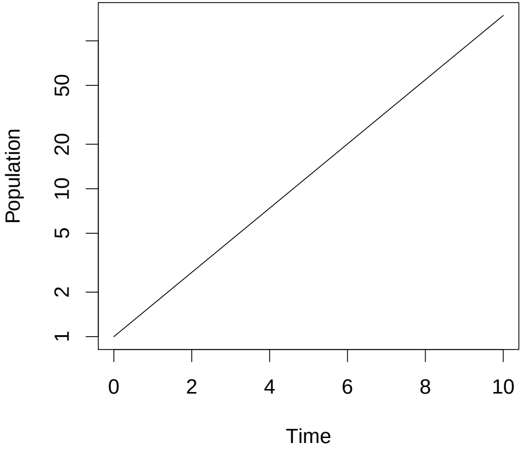

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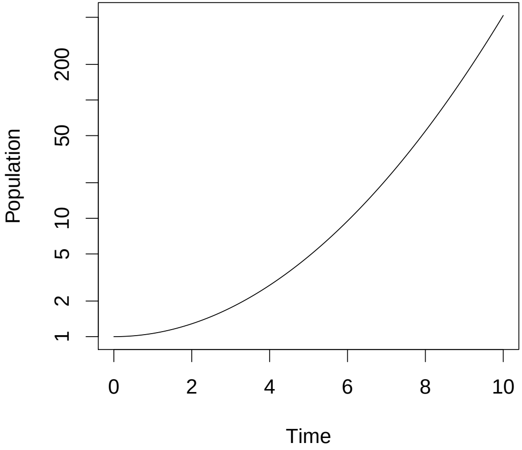

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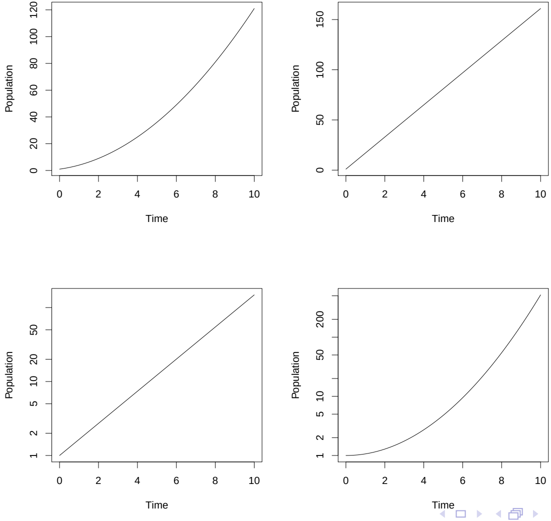

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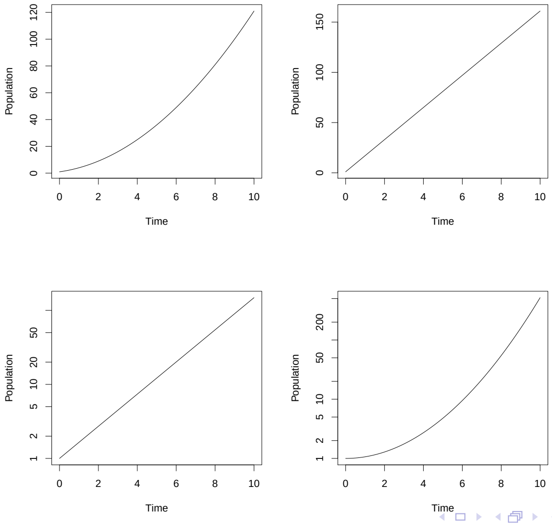

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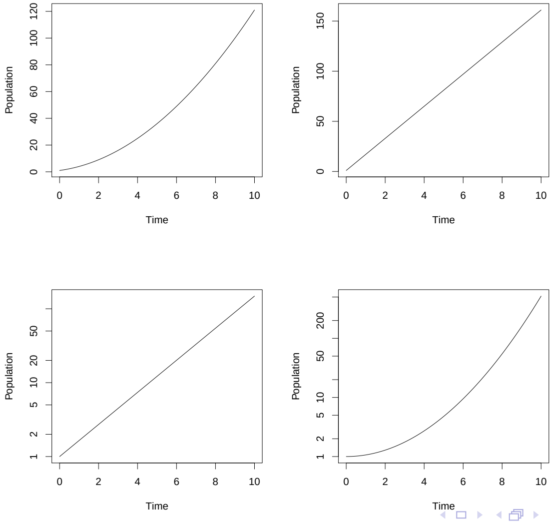

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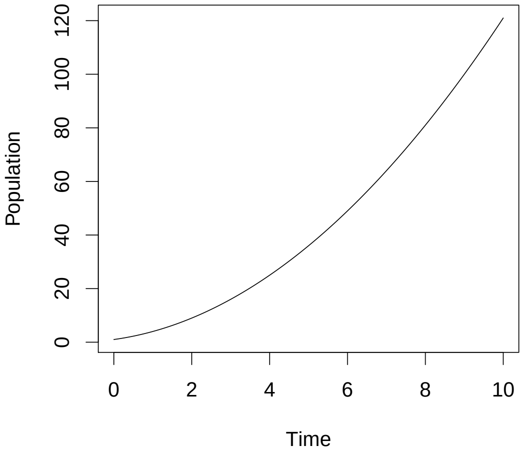

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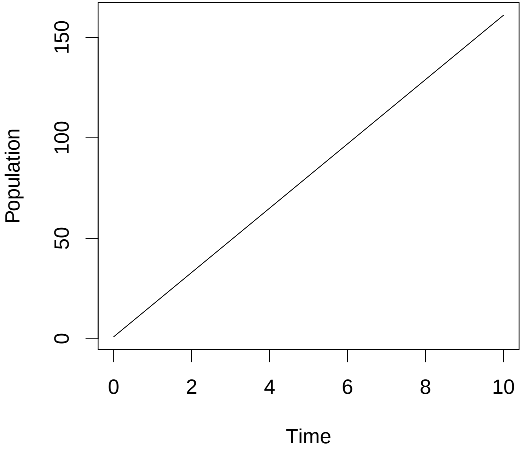

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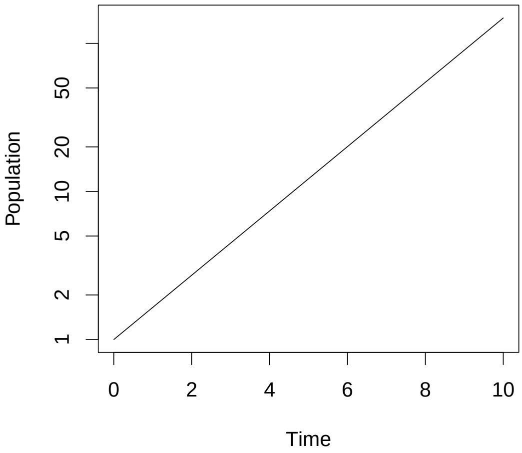

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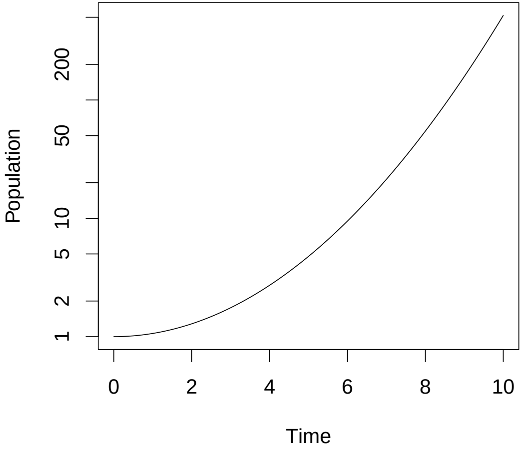

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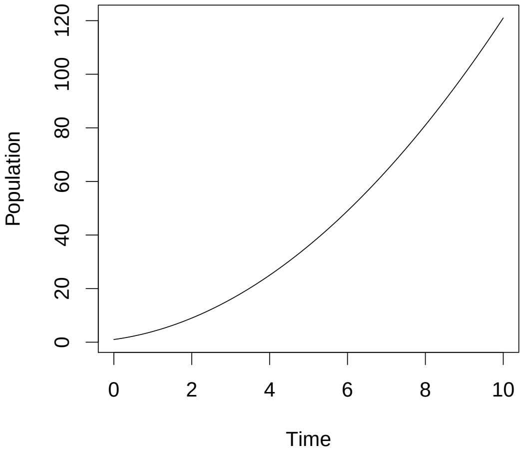
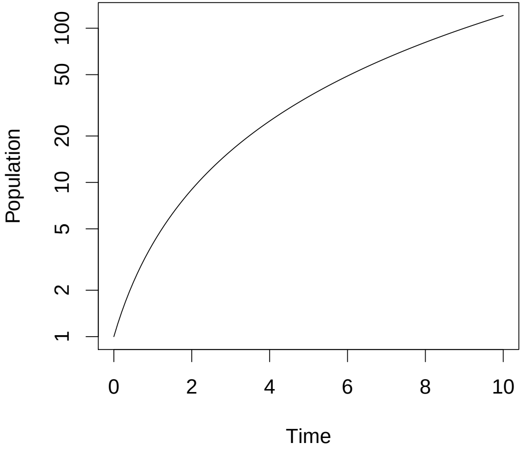


A on the log scale



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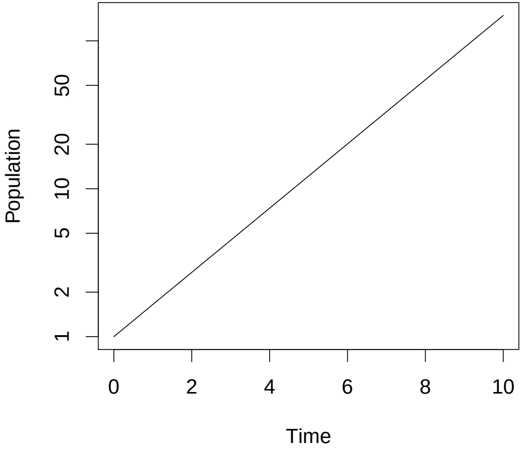
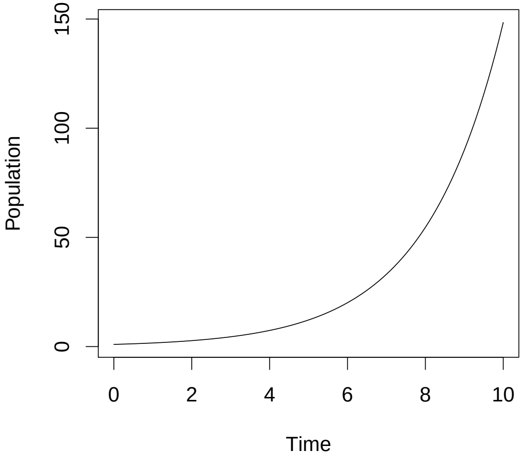


C on the linear scale



Types of growth

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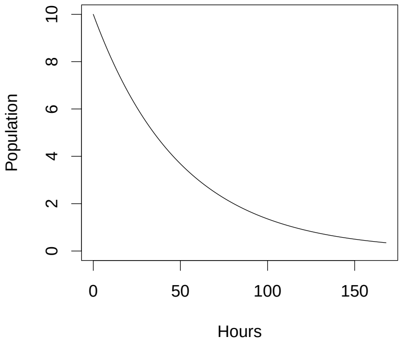

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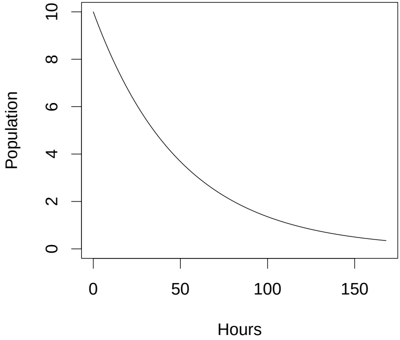

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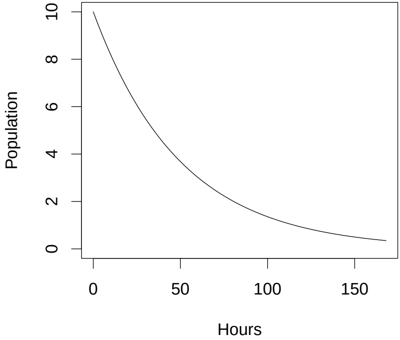

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Subsection 1

Log and linear scales

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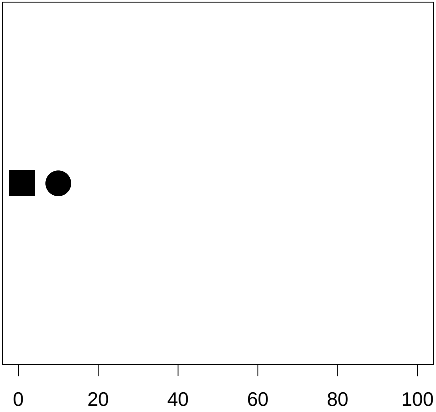

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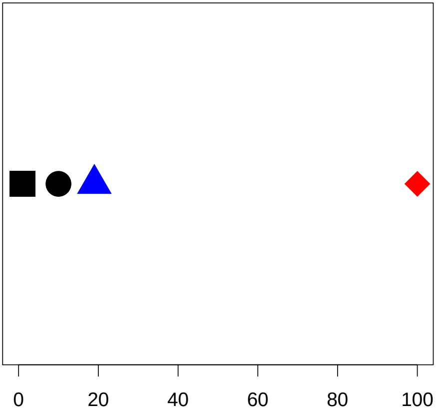

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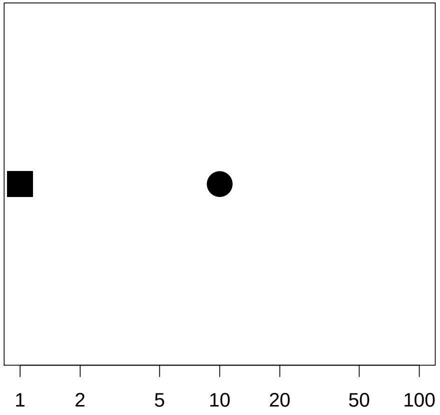

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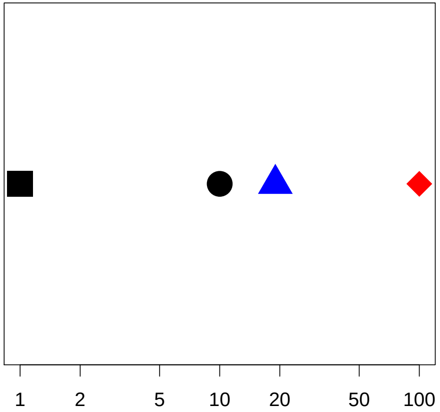

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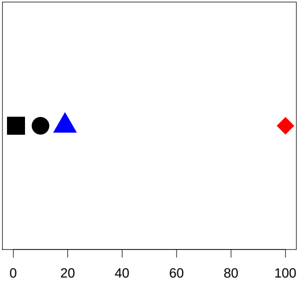
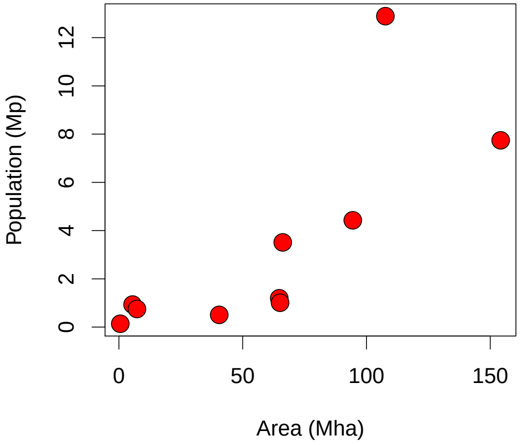


There is only one log scale; it doesn't matter which base you use!

Canadian provinces



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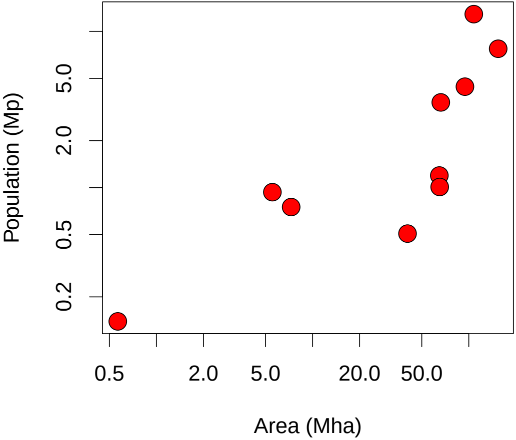

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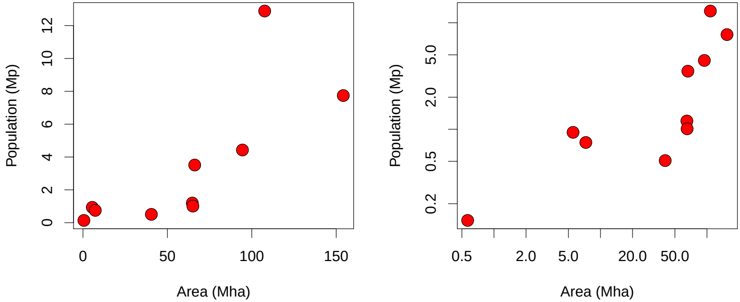
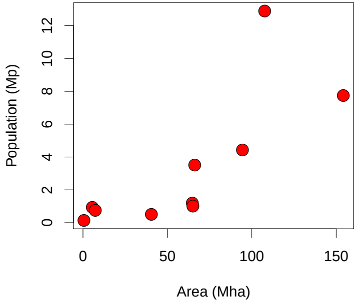
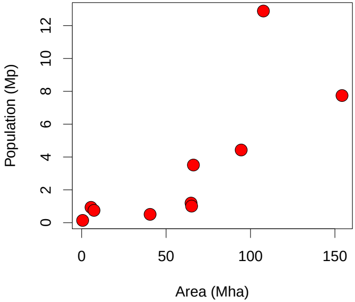


Canadian provinces plus Canada?





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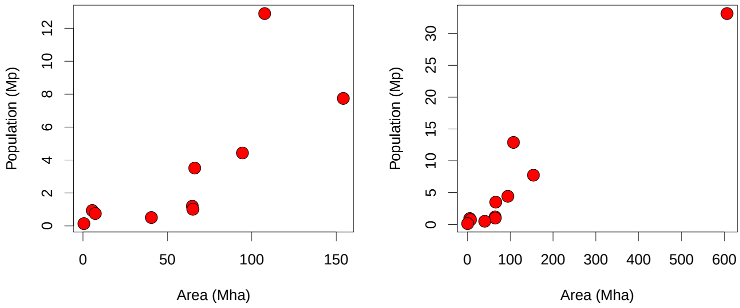

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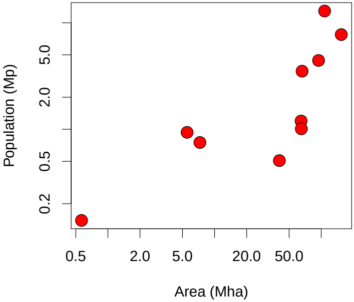

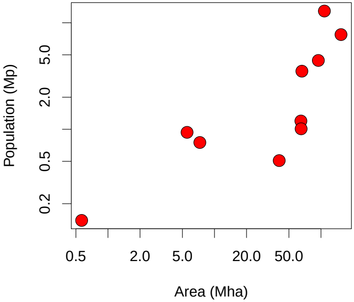

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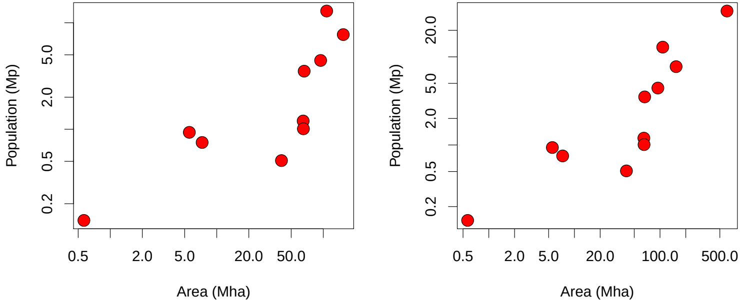

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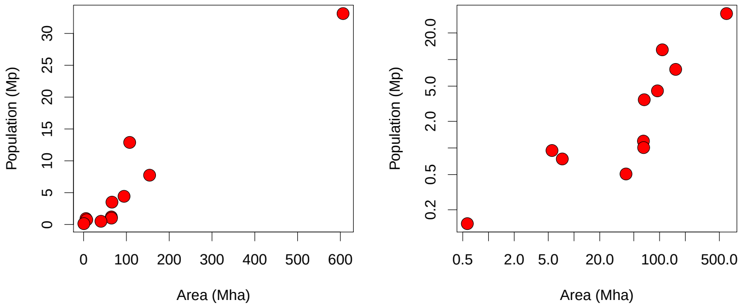

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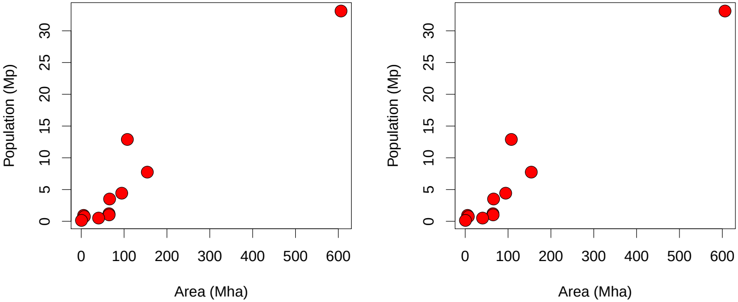

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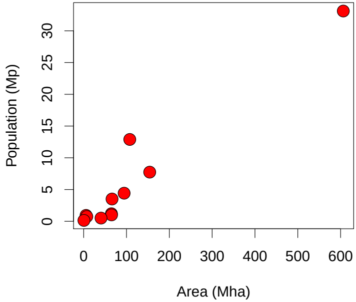

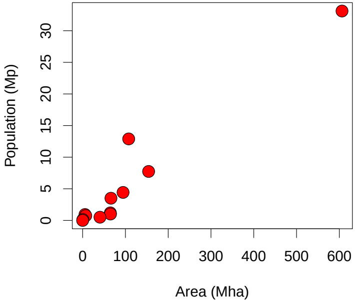

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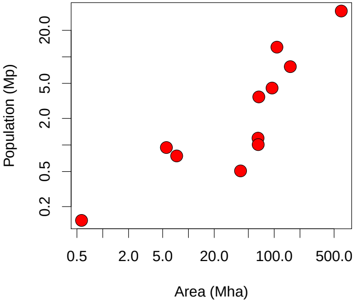

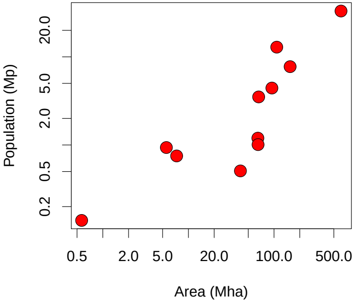

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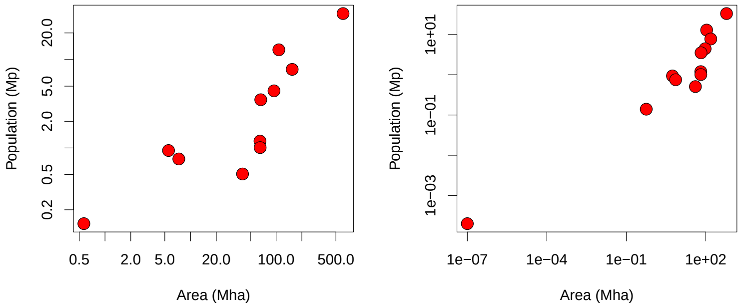

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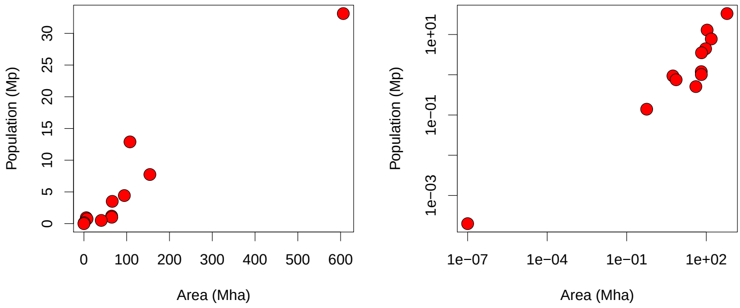

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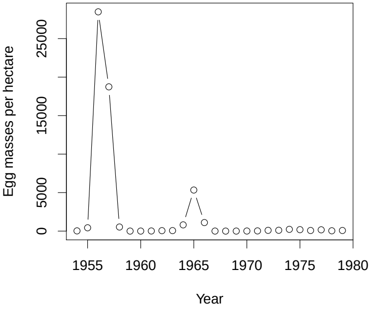
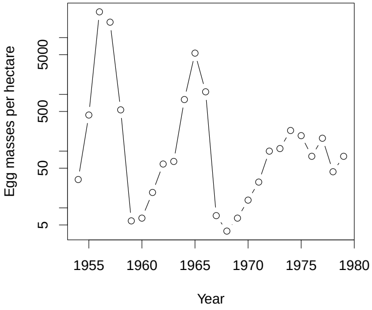
Gypsy-moth example





Scales in population biology

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Subsection 2

Time scales

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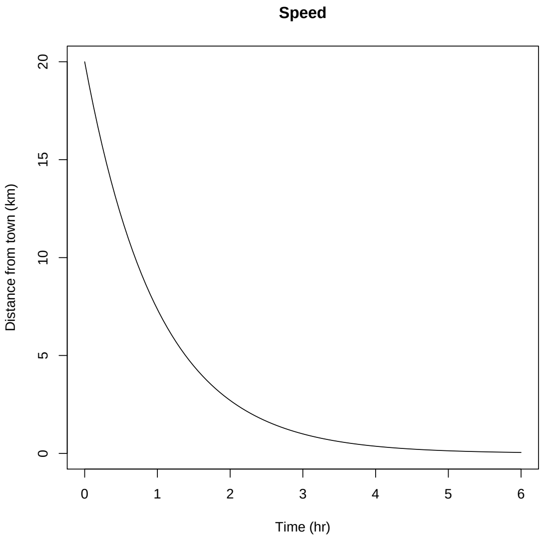

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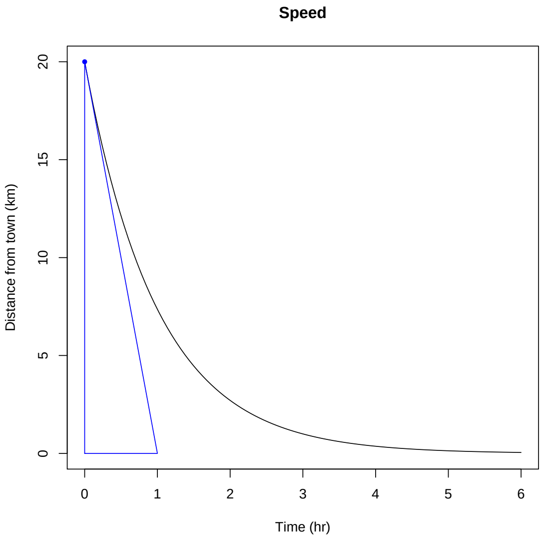

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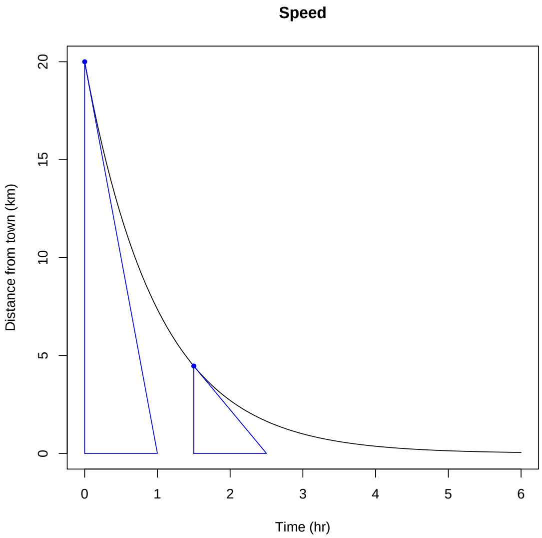

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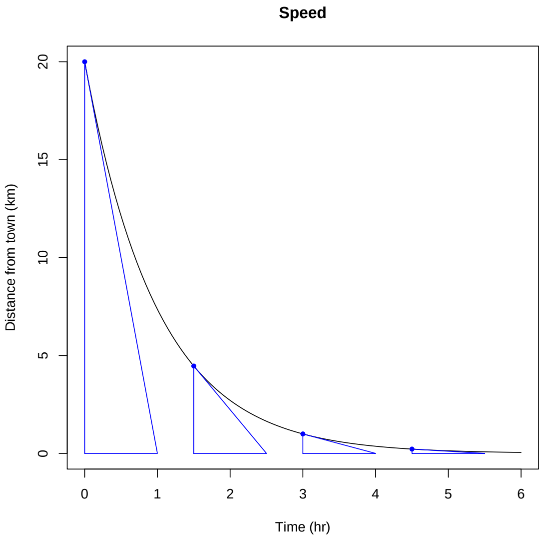

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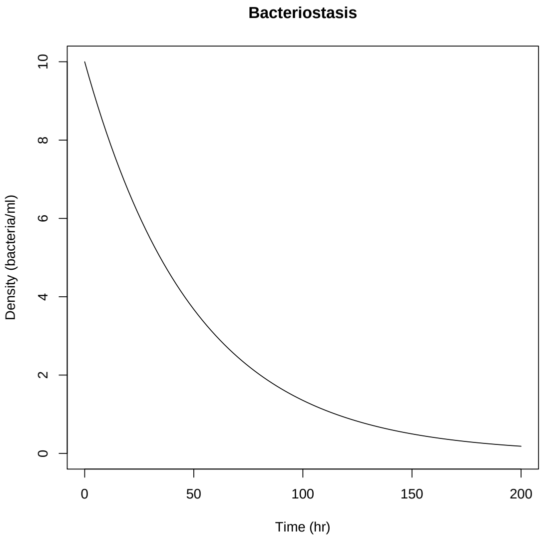

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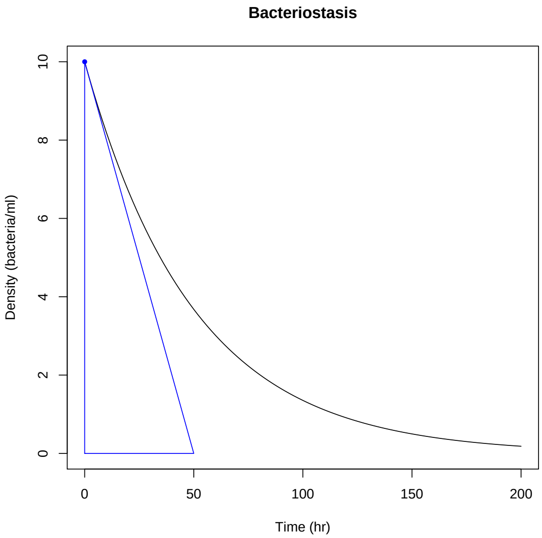

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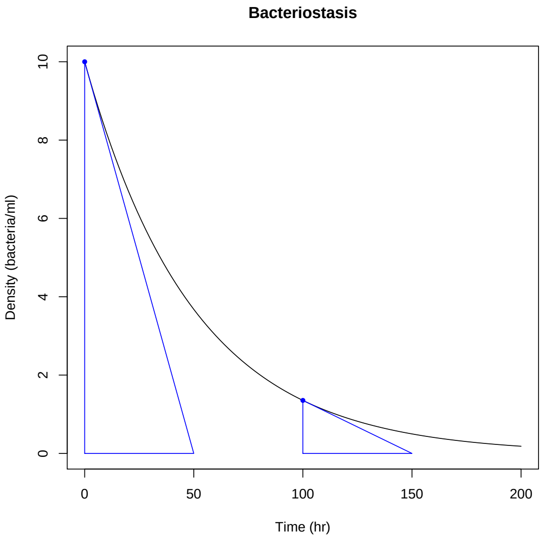

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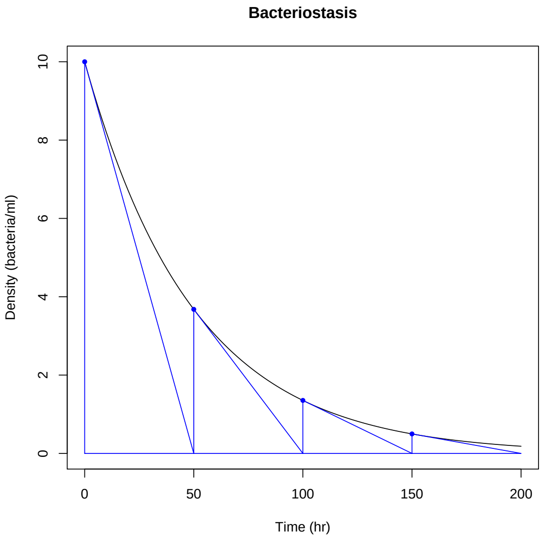

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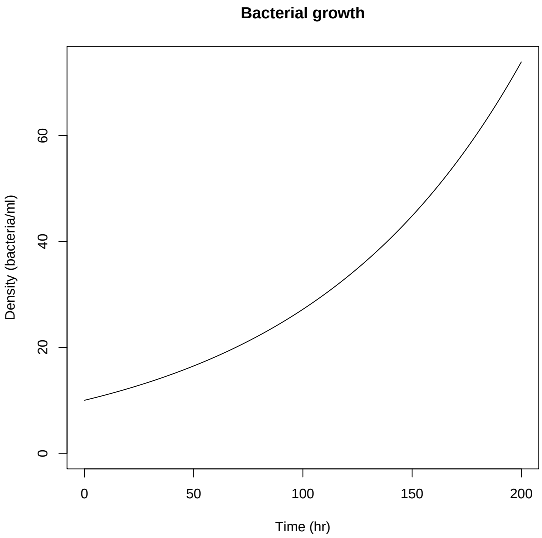

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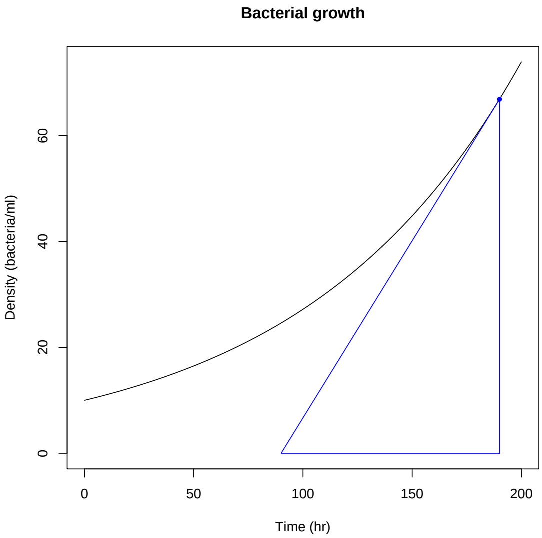

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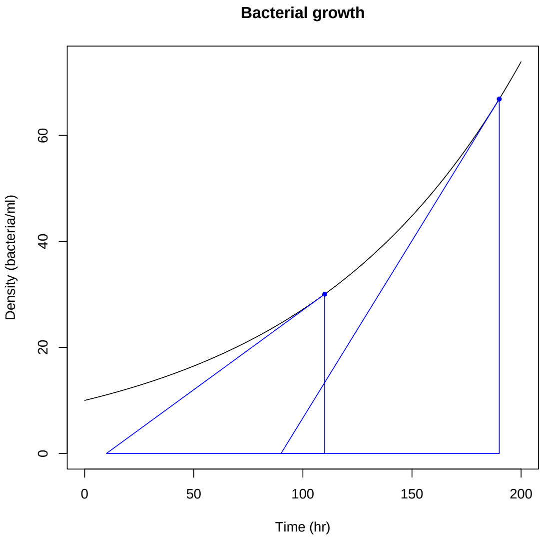

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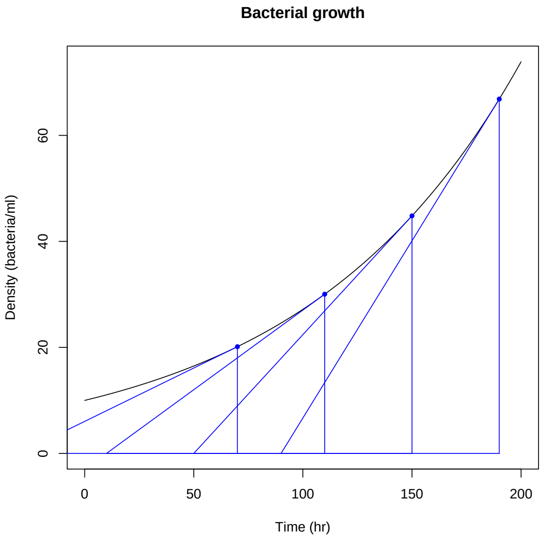

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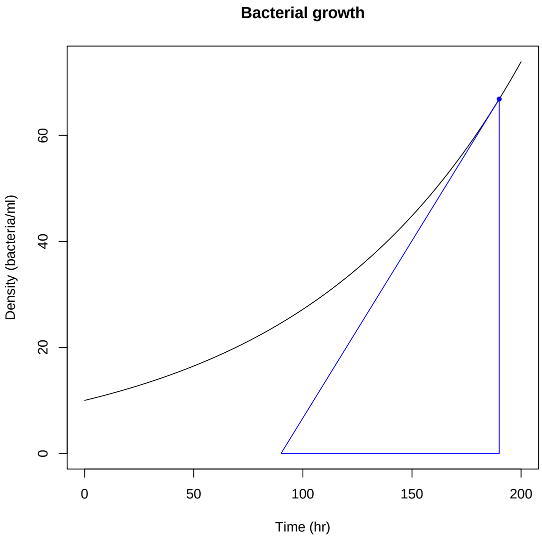

Bacterial growth
Density (bacteria/ml)
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20
40
60
0
50
100
150
200
Time (hr)

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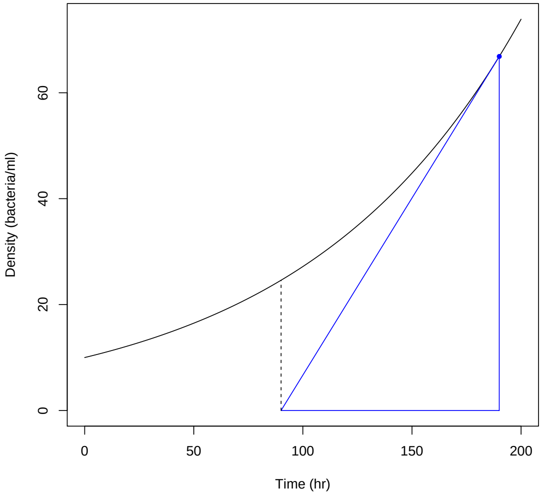

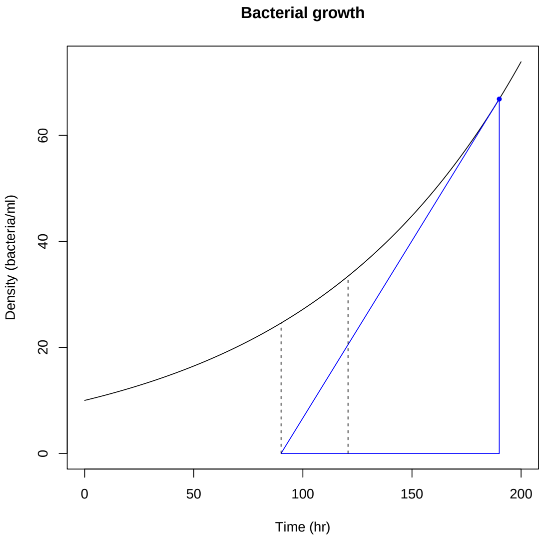
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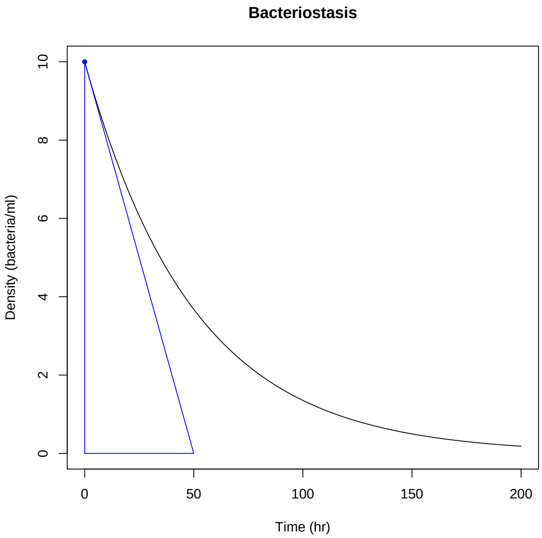

Bacteriostasis
Density (bacteria/ml)
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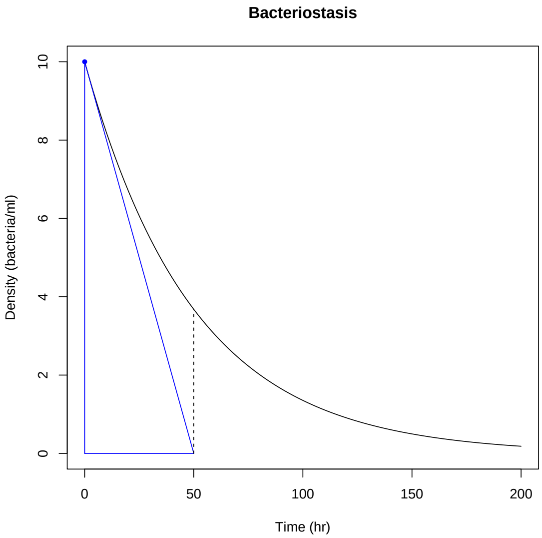
2
4
6
8
10
0
50
100
150
200
Time (hr)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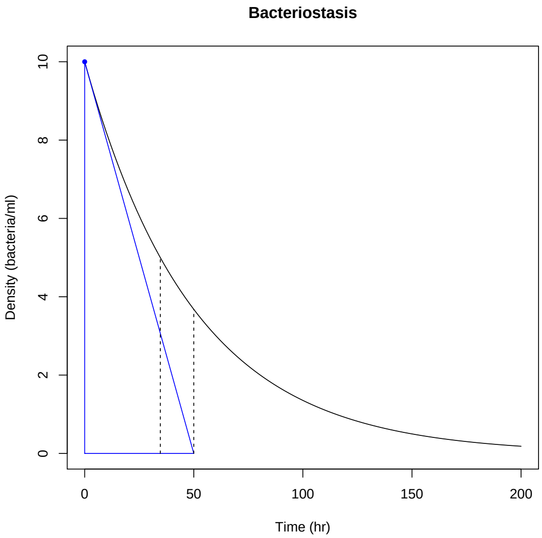
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Outline

Subsection 1

Dynamical models

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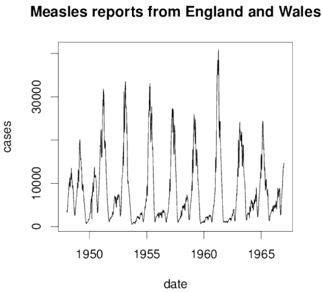

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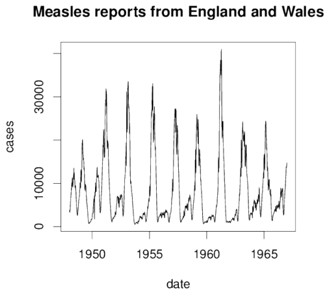

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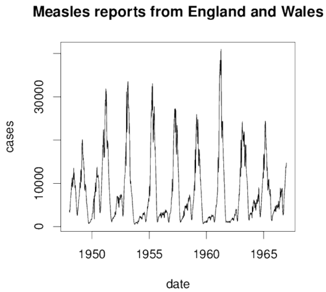

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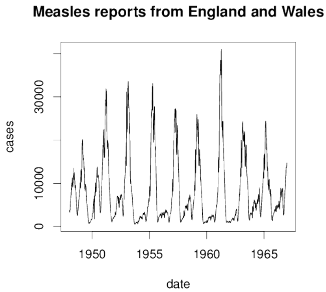

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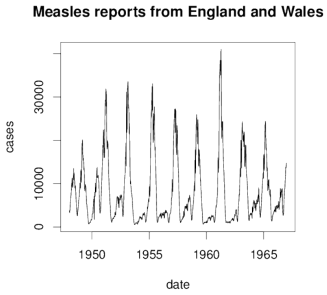

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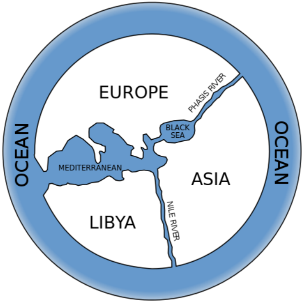

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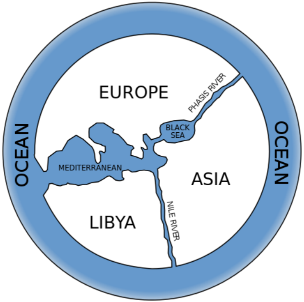

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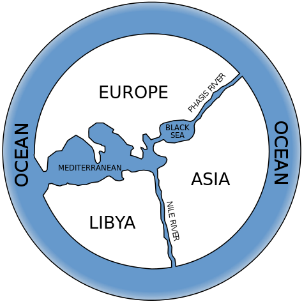

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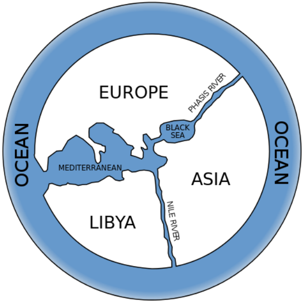

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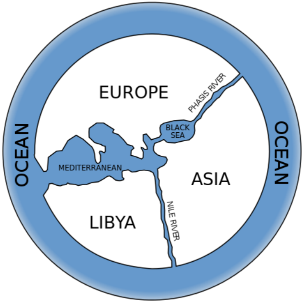

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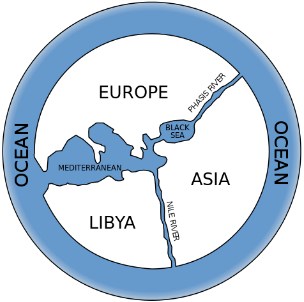

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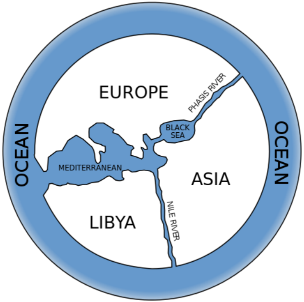

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  - * Birth
  - * Death
  - * Immigration and emigration
  - * Sampling (ie., my counts are not perfectly correct)

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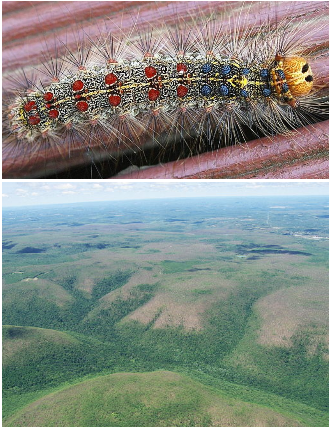
Subsection 2

Examples

Gypsy moths

- A pest species that feeds on deciduous trees



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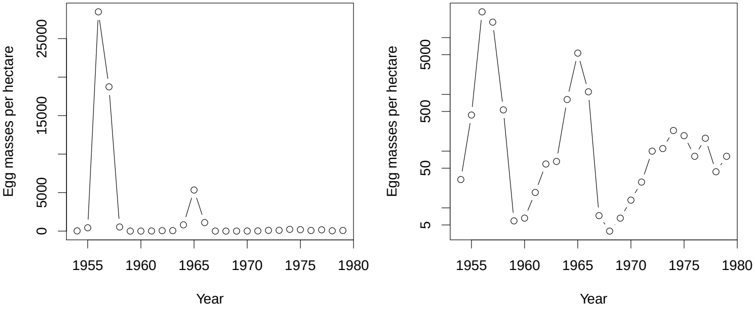

Moth example

- State variables

Moth example

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  - *

Moth example

- State variables
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Moth example

- State variables
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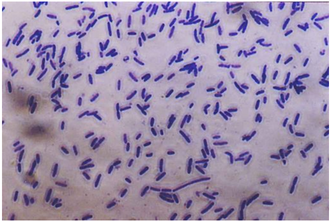
- State variables



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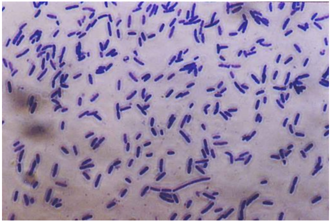

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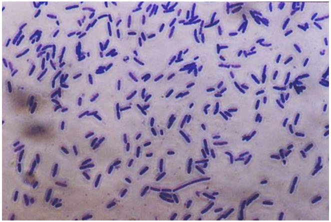

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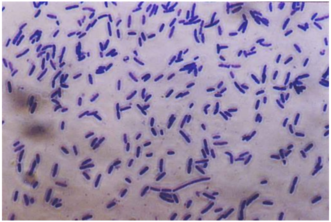

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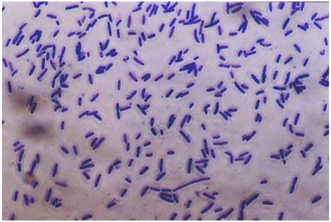

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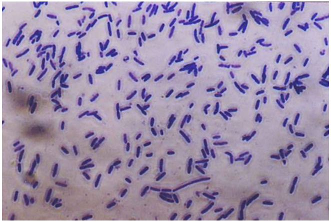

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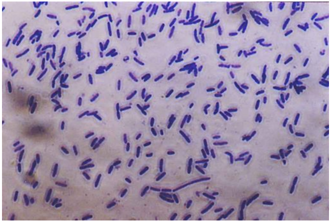

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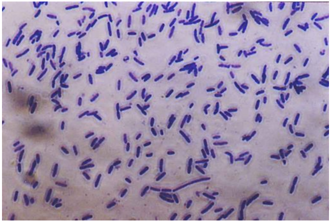

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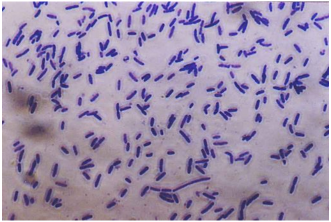

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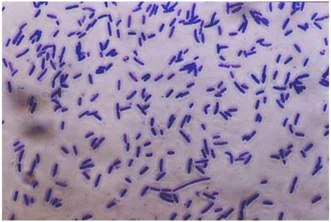

Dandelions

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Subsection 3

A simple discrete-time model

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5	2005-12-20	232 €	132 €	100 €
6	2005-12-21	122 €	134 €	-12 €
7	2005-12-22	128 €	223 €	-95 €
8	2005-12-23	432 €	218 €	214 €
9	2005-12-24	256 €	121 €	135 €
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- Spreadsheet (see resource page)

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Subsection 4

A simple continuous-time model

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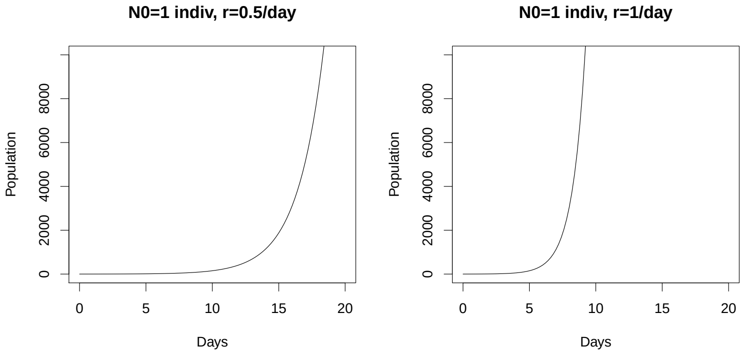
N0=1 indiv, r=0.5/day
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Population
Days
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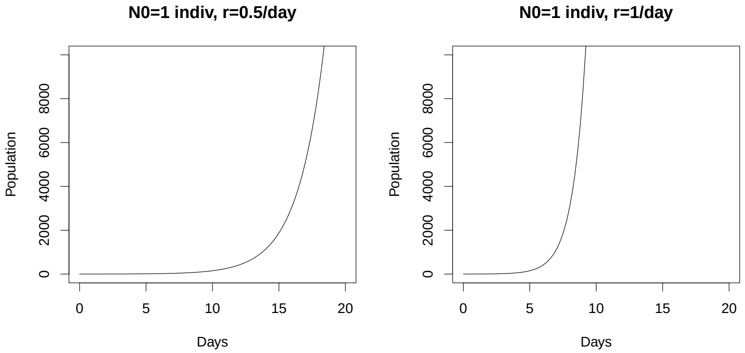

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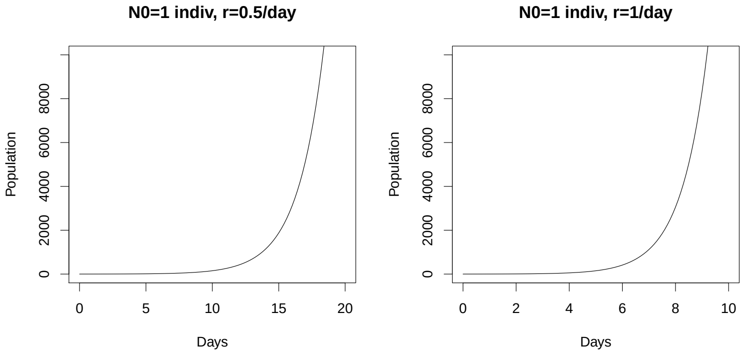

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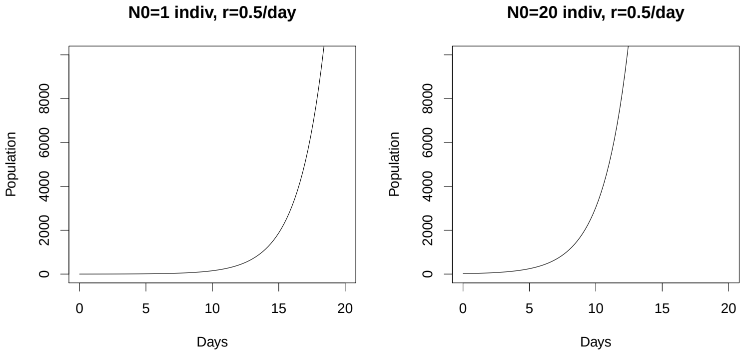

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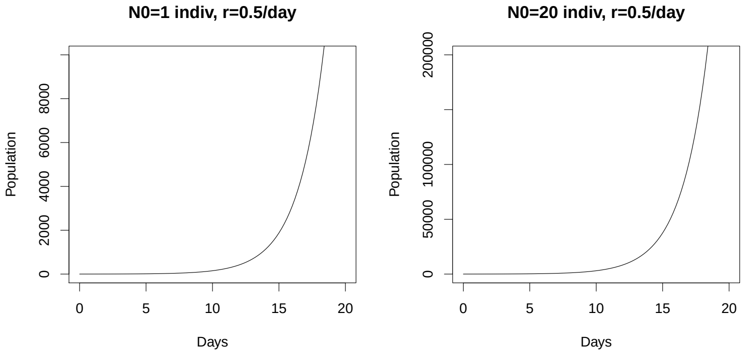

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Subsection 2

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