UNIT 2 Non-linear population models

Outline

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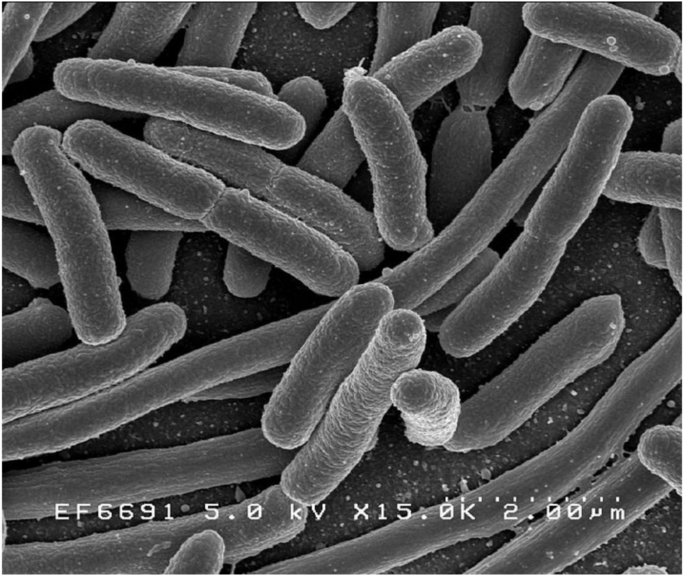

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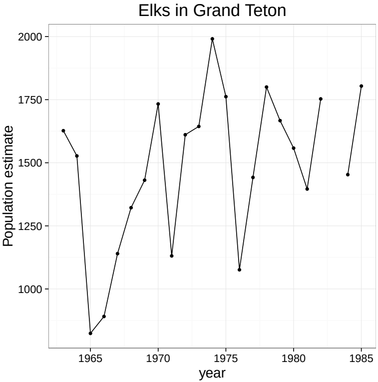
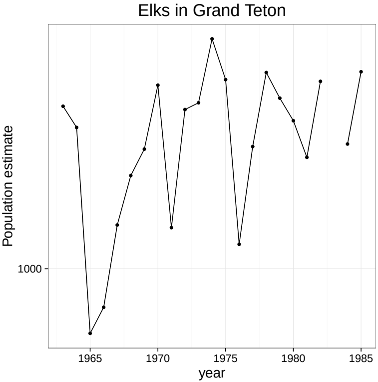
Subsection 1

Population Examples

Elk


Elks in Grand Teton
2000
1750
1500
1250
1000
Population estimate
1965
1970
1975
1980
1985
year


Elks in Grand Teton
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Population estimate
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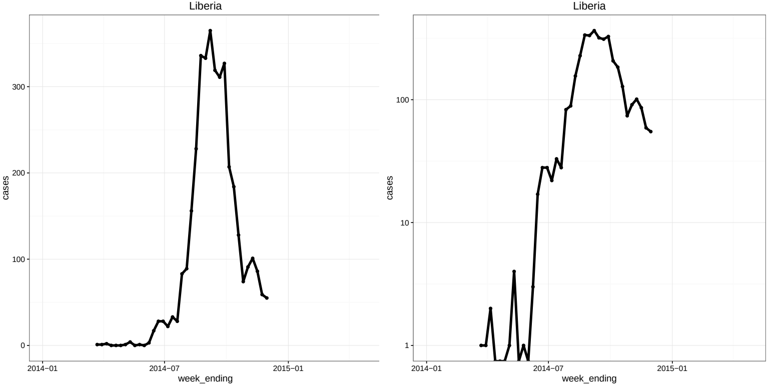
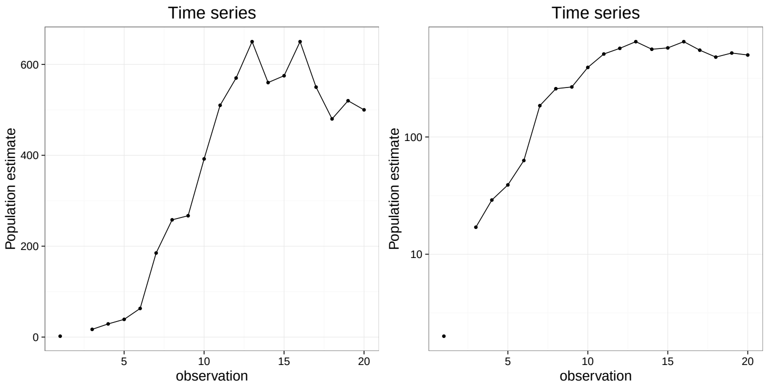
Ebola

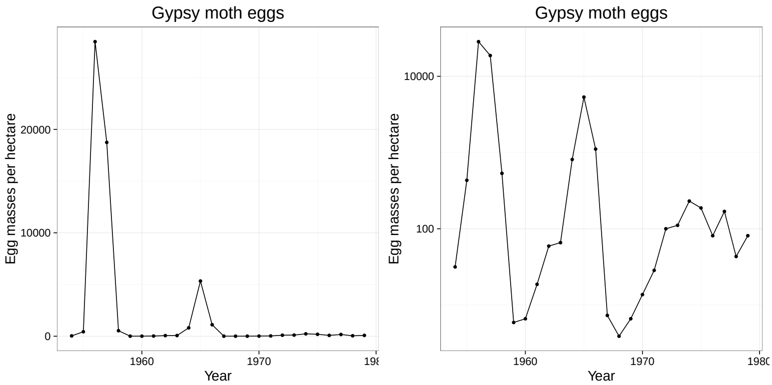

Liberia
cases
300
200
100
0
2014−01
2014−07
2015−01
week_ending
Liberia
cases
100
10
1
2014−01
2014−07
2015−01
week_ending

Paramecia



Time series
Population estimate
600
400
200
0
5
10
15
20
observation
Time series
Population estimate
100
10
5
10
15
20
observation

Gypsy moths


Gypsy moth eggs
Egg masses per hectare
20000
10000
0
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Year
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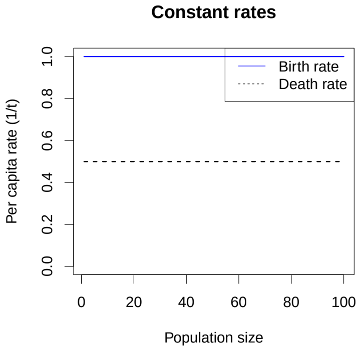

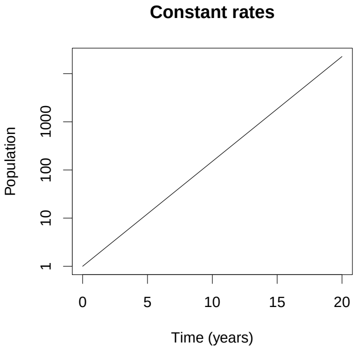

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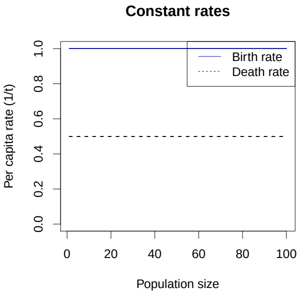

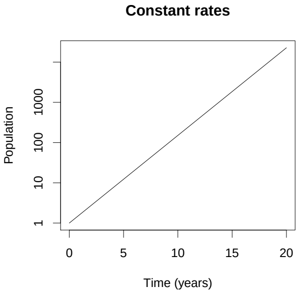

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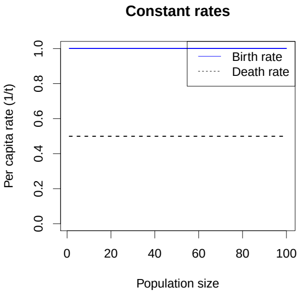

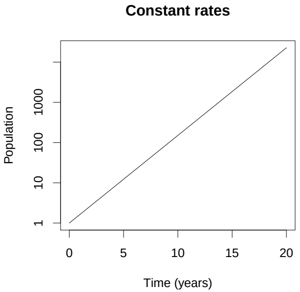

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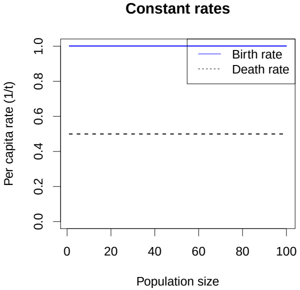

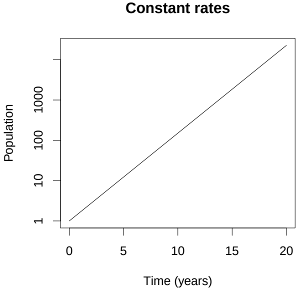

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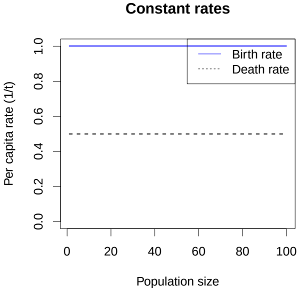

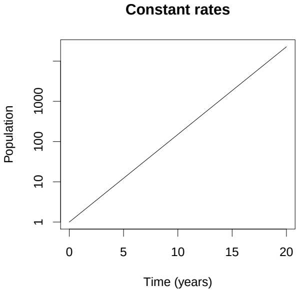

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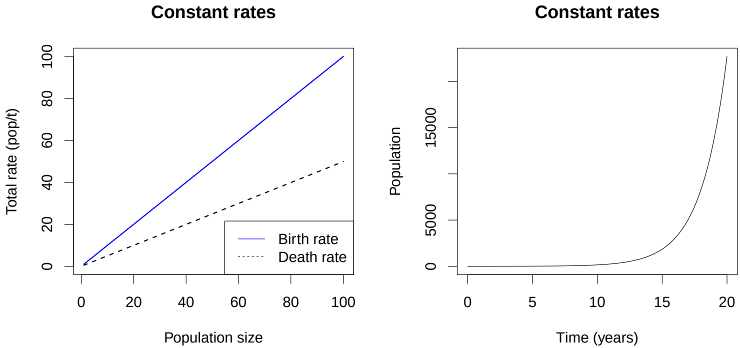

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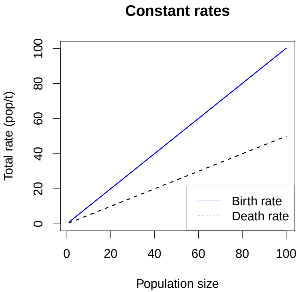

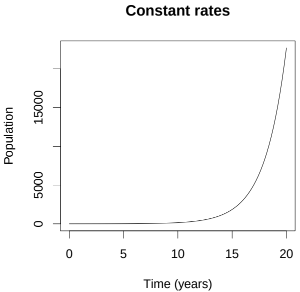

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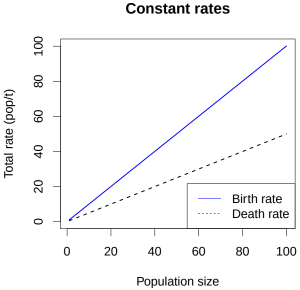

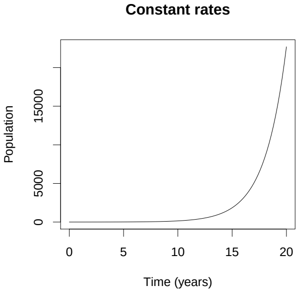

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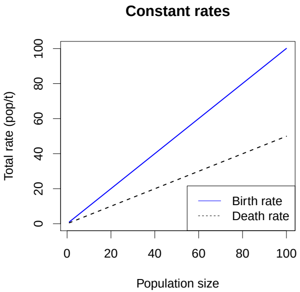

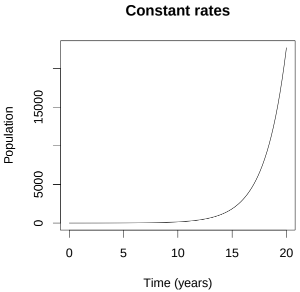

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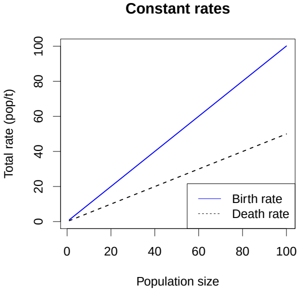

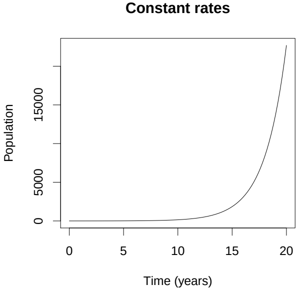

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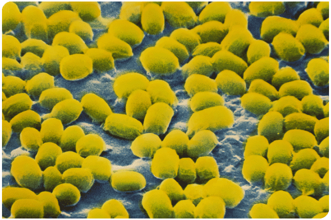

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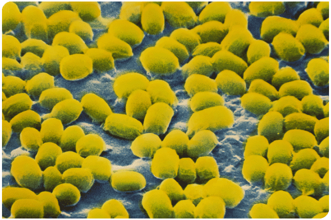

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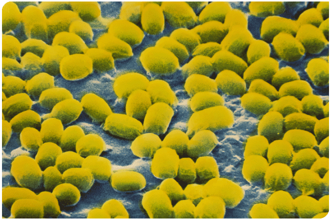

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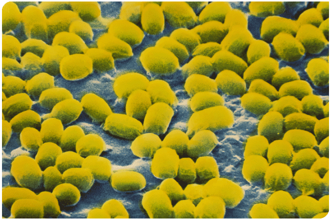

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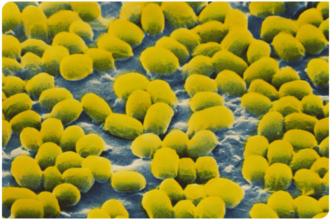

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Subsection 1

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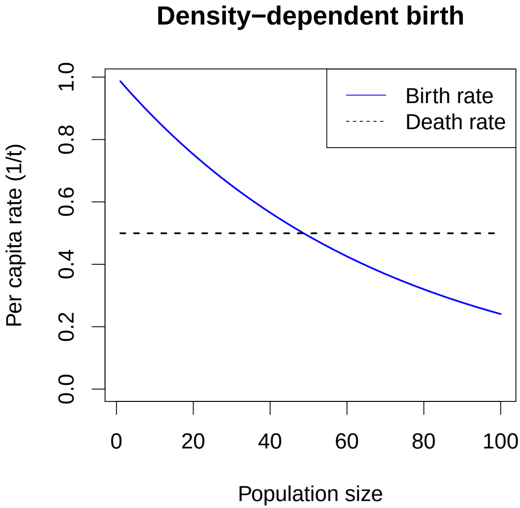

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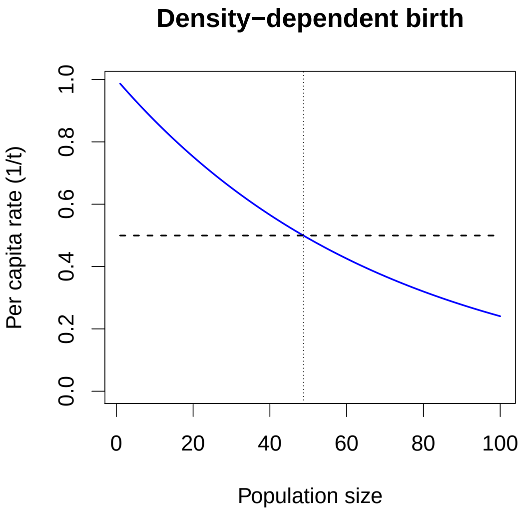

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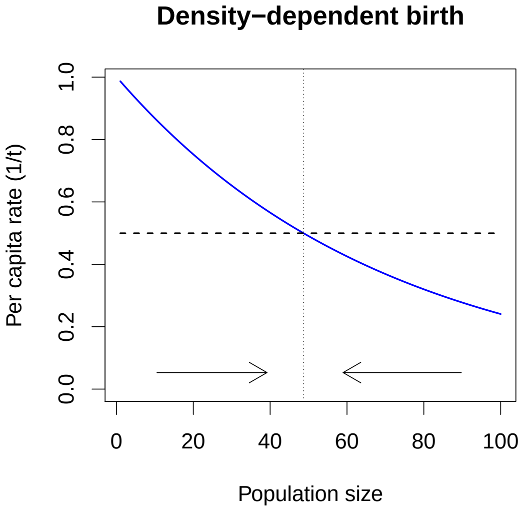

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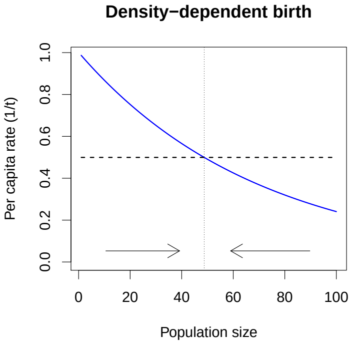

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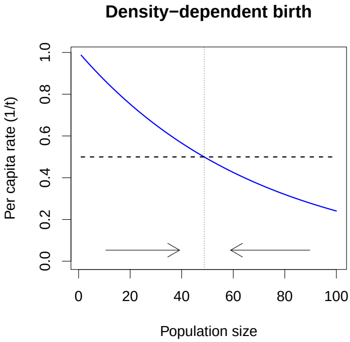

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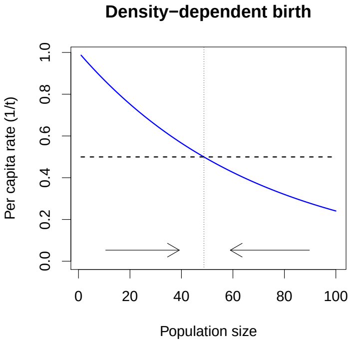

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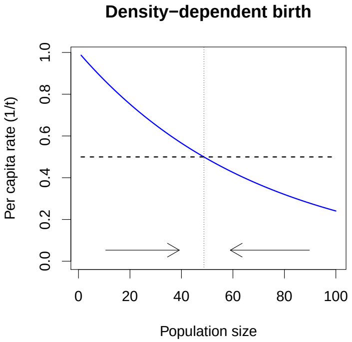

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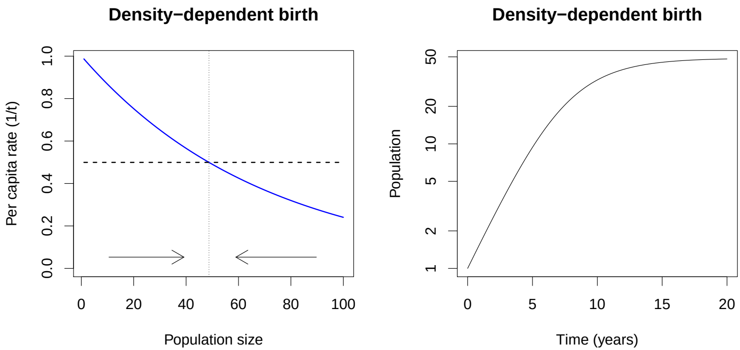

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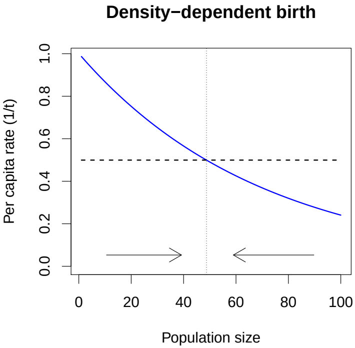

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Subsection 2

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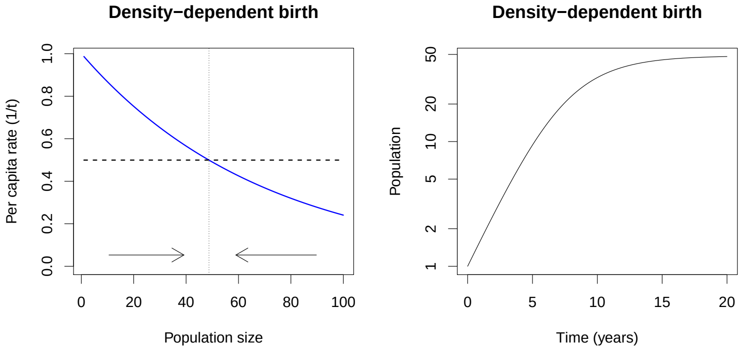

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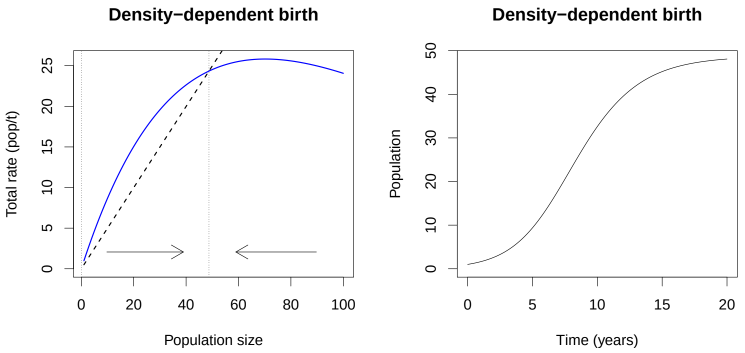
    - or [density/time] = [(indiv/area)/time]
  - what is changing at the population level?
  - corresponds to the dynamics we visualize on a linear-scale graph of the population

Population perspective picture



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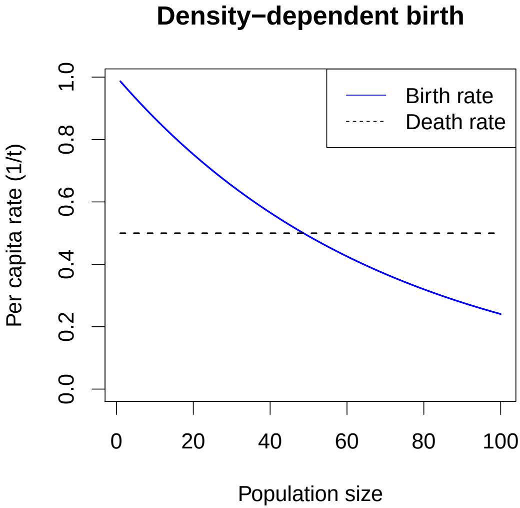

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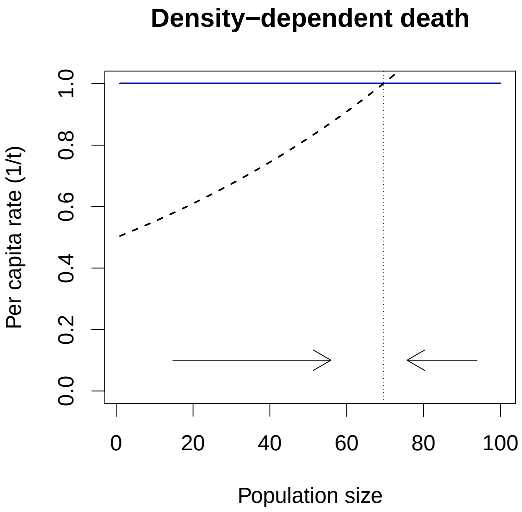

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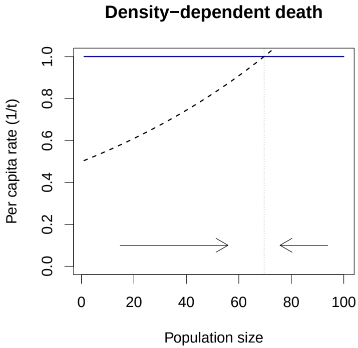

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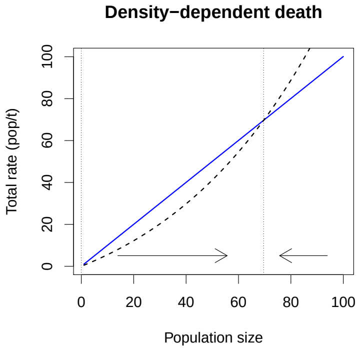

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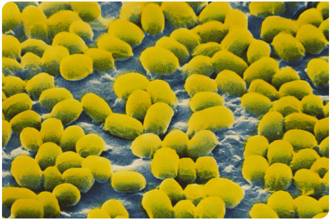

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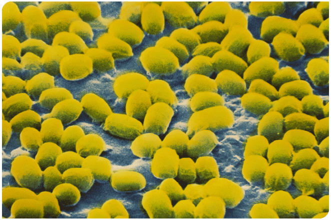

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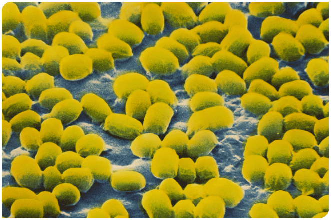

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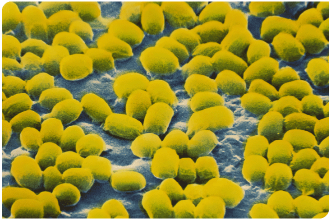

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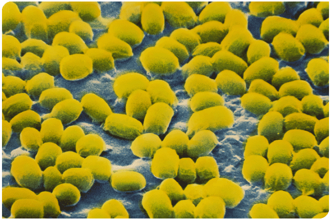

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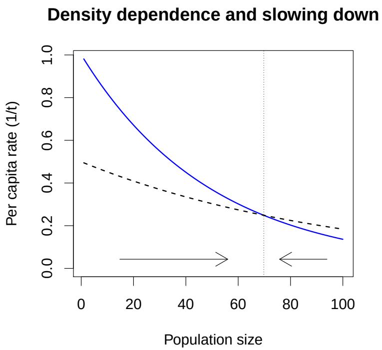

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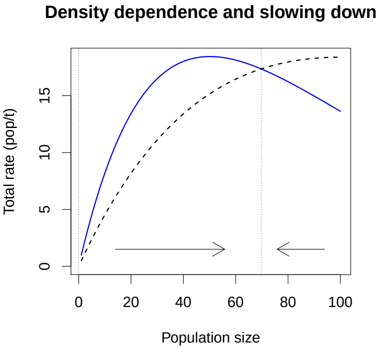

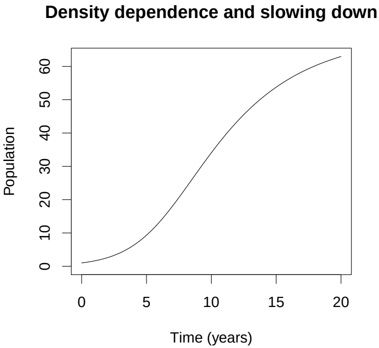

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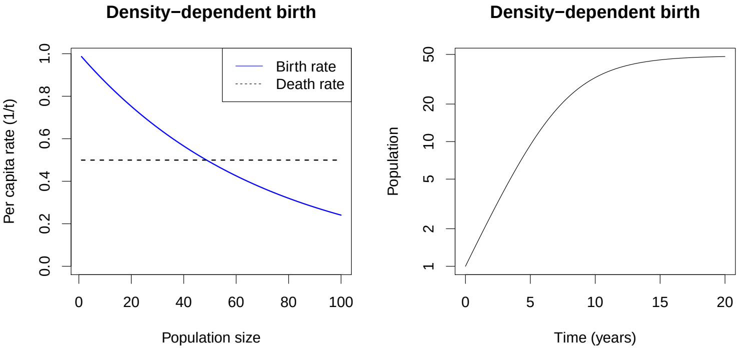

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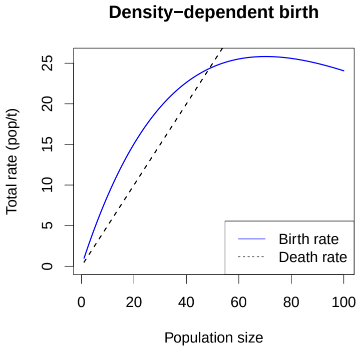

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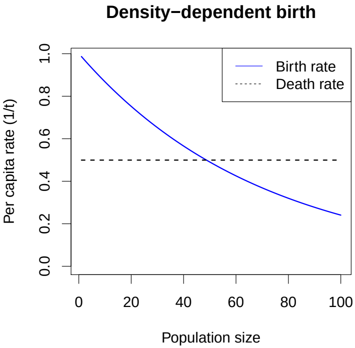

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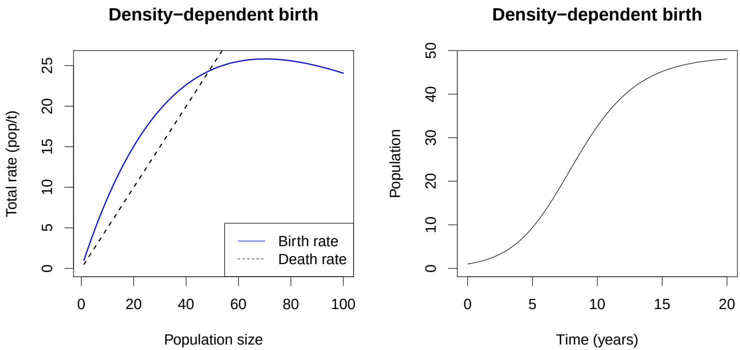

Subsection 3

Equilibria and time scales

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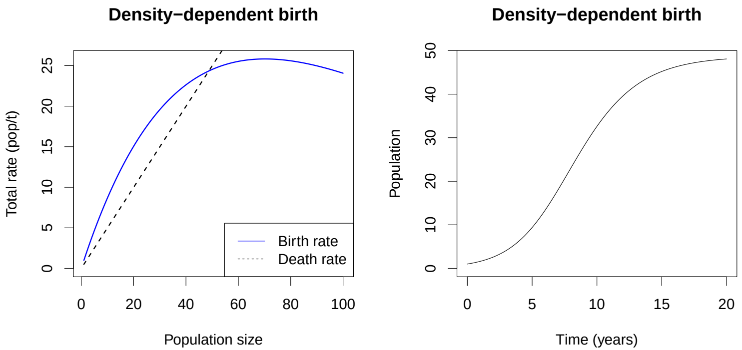

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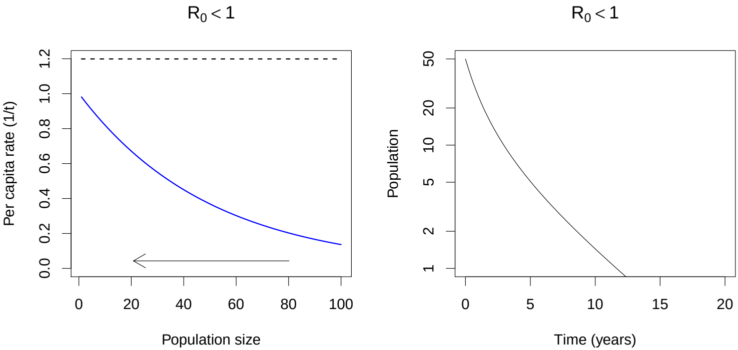

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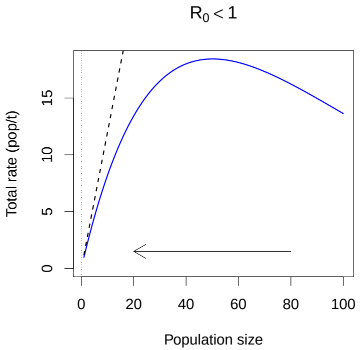

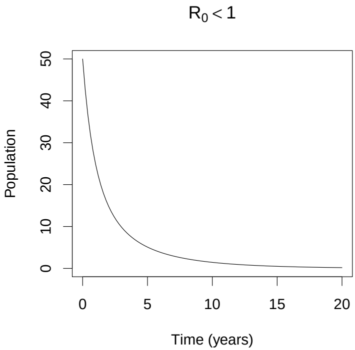

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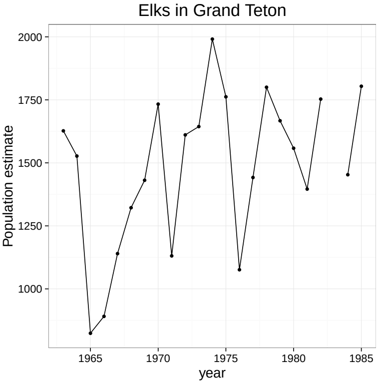
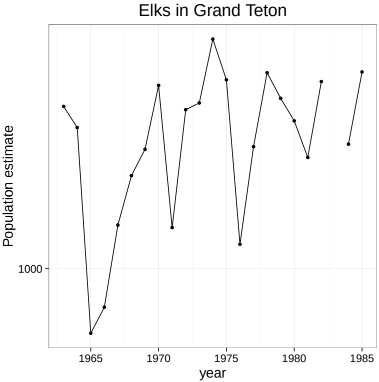


Elks in Grand Teton
2000
1750
1500
1250
1000
Population estimate
1965
1970
1975
1980
1985
year



Elks in Grand Teton
1000
Population estimate
1965
1970
1975
1980
1985
year

Dynamics of density-dependent populations

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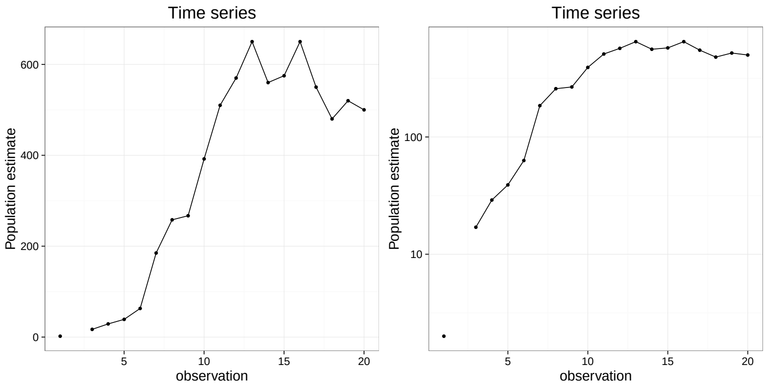
Paramecia



Time series
Population estimate
600
400
200
0
5
10
15
20
observation
Time series
Population estimate
100
10
5
10
15
20
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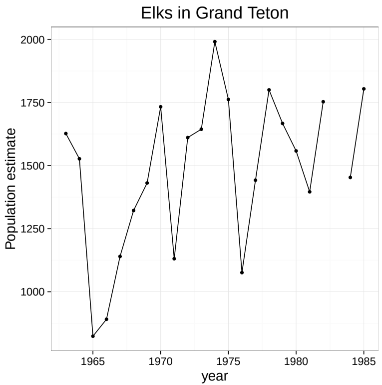
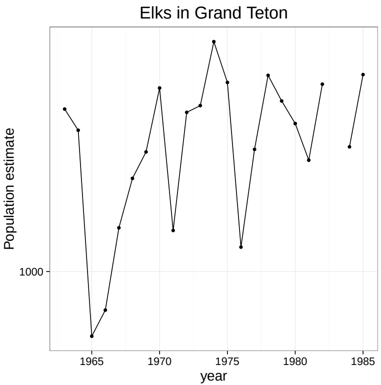
Elk



Elks in Grand Teton
Population estimate
2000
1750
1500
1250
1000
1965
1970
1975
1980
1985
year



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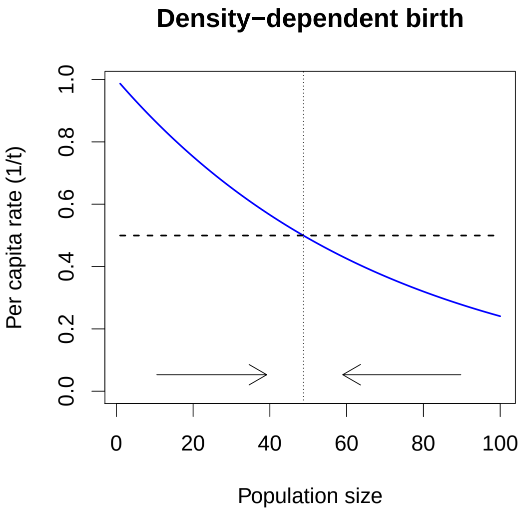
Arrows with time delay



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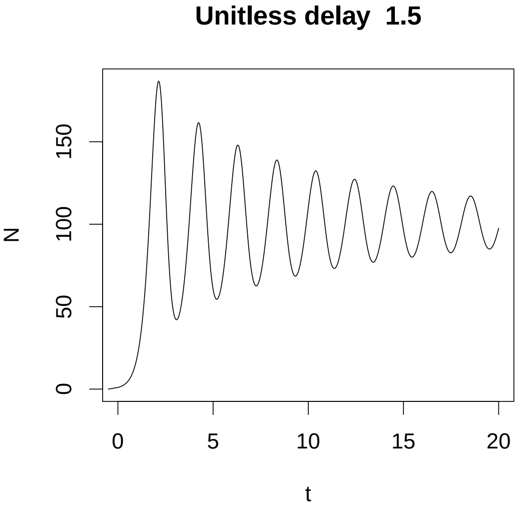

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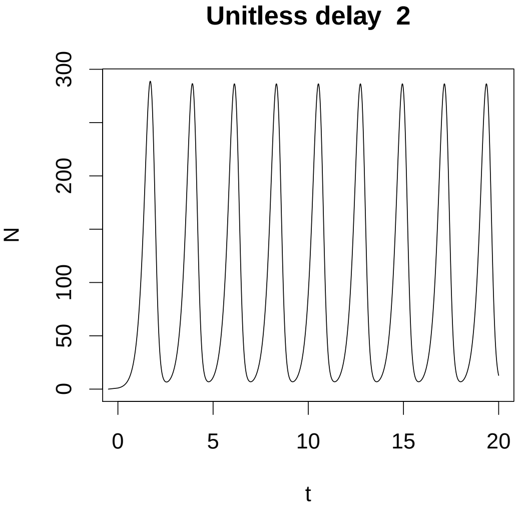

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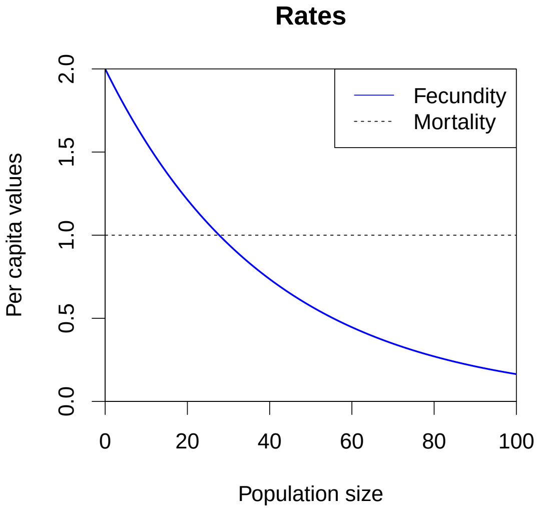

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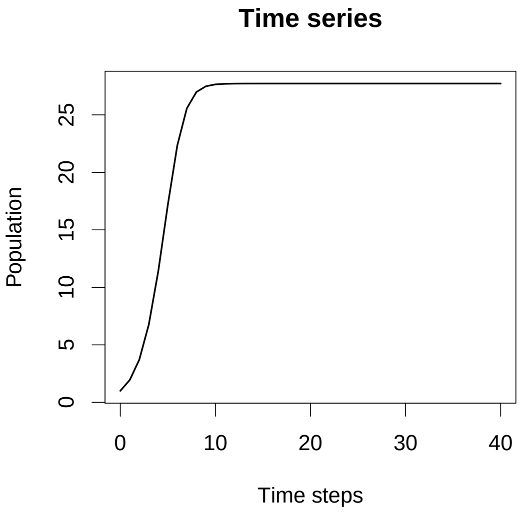
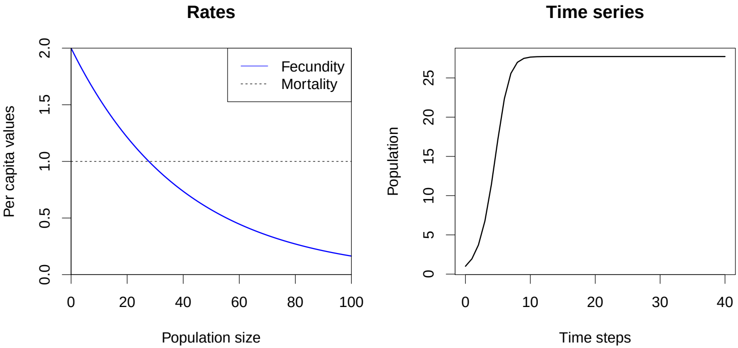


We expect simple dynamics



What dynamics do we get?

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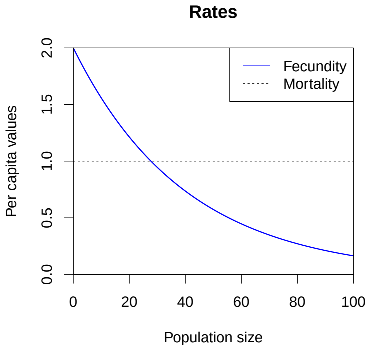

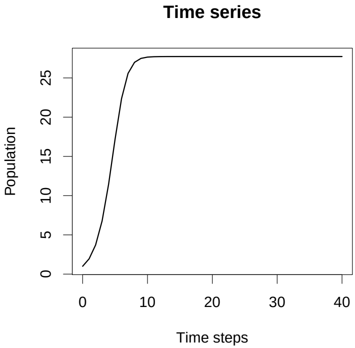

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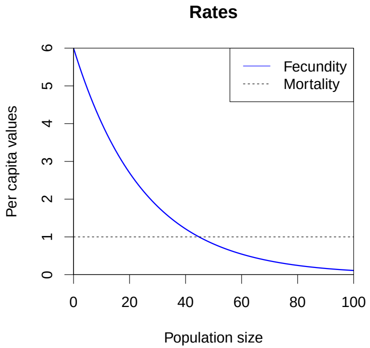

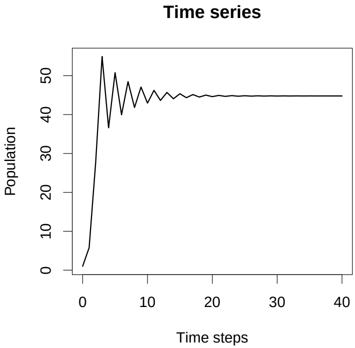

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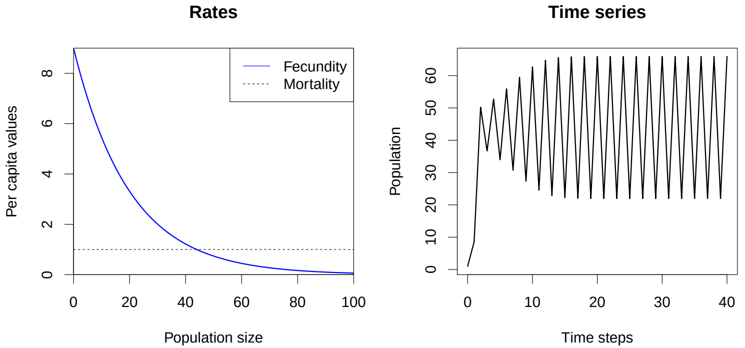
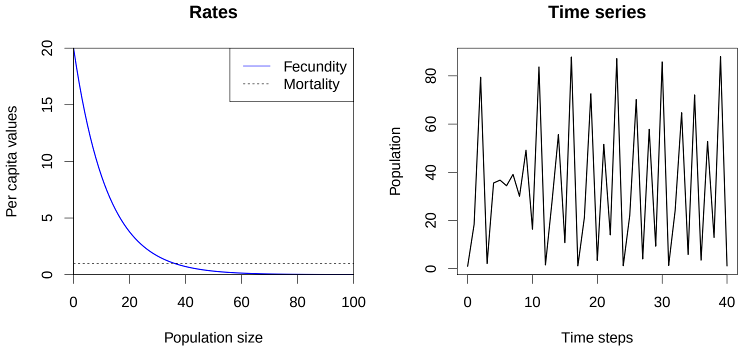


Lots of other behaviours



Subsection 3

Interpreting complex behaviour

Interpreting complex behaviour

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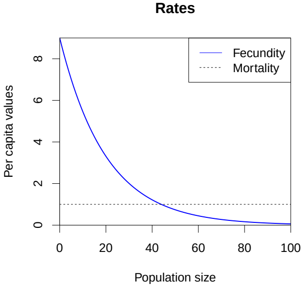

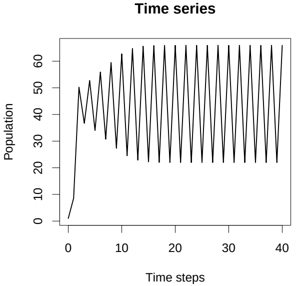

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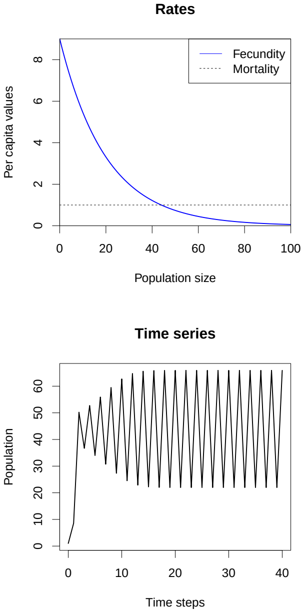

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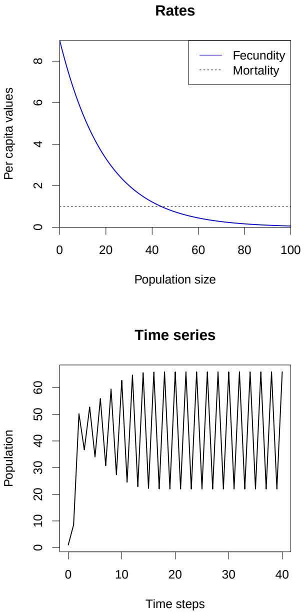

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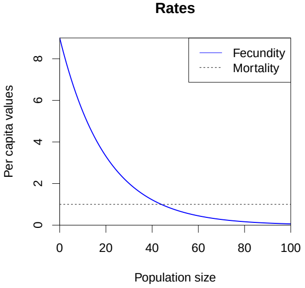

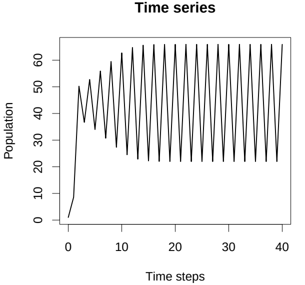

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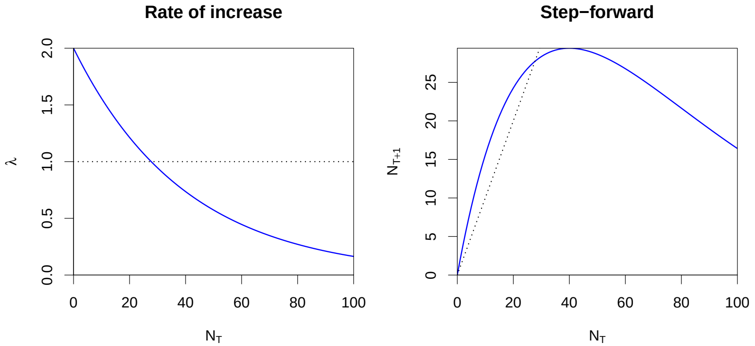
Response to population increase



Turnover

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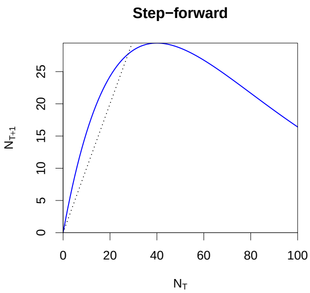

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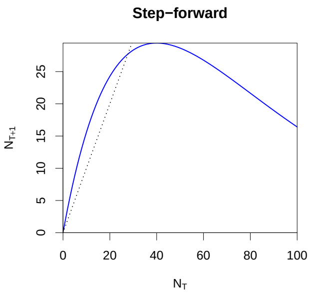

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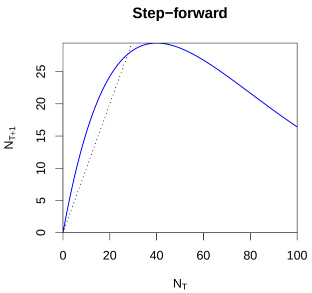

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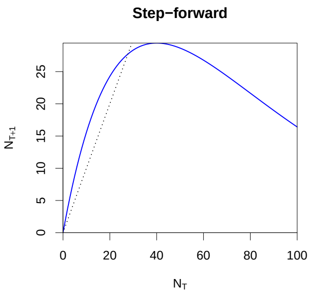

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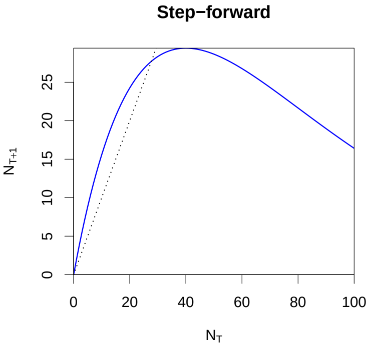

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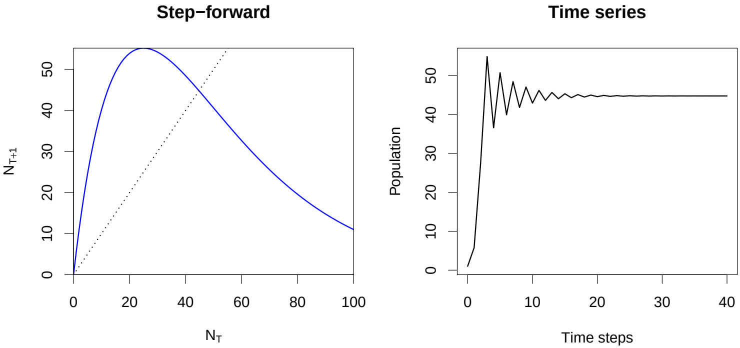

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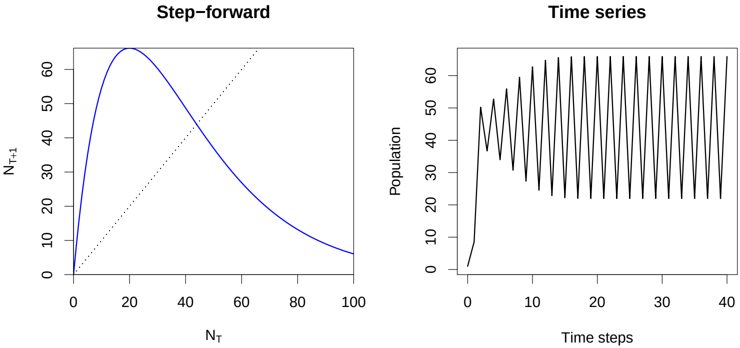

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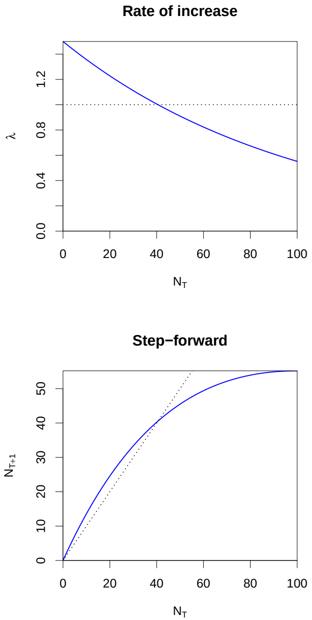

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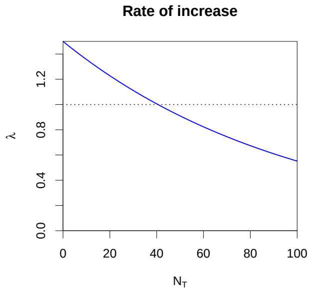

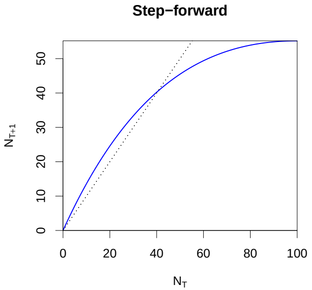

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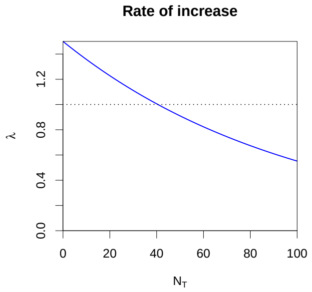

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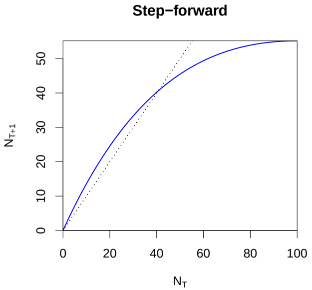

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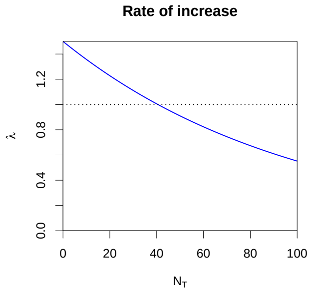

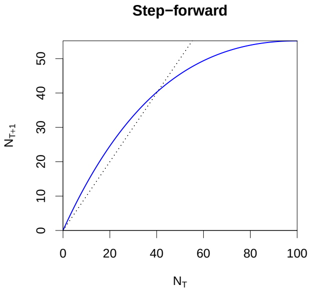

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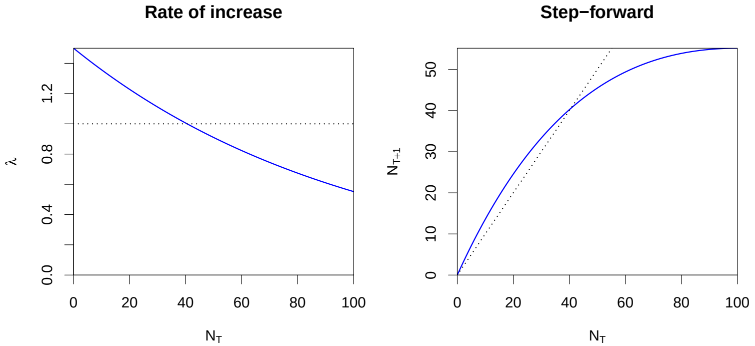

Songbirds

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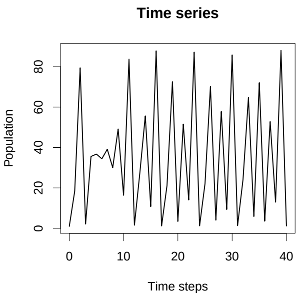

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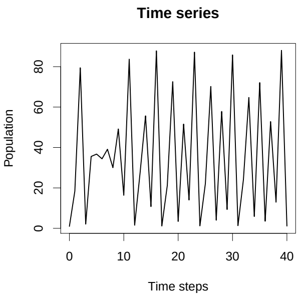

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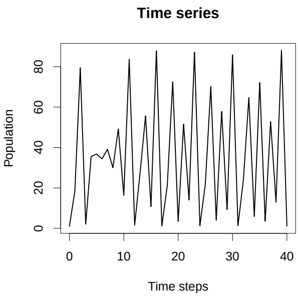

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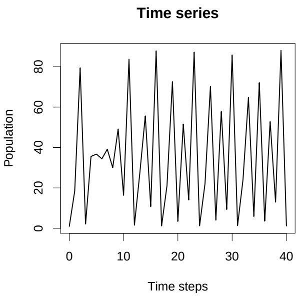

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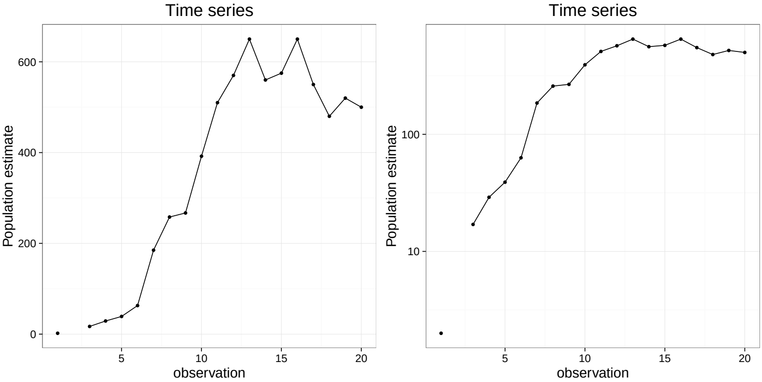
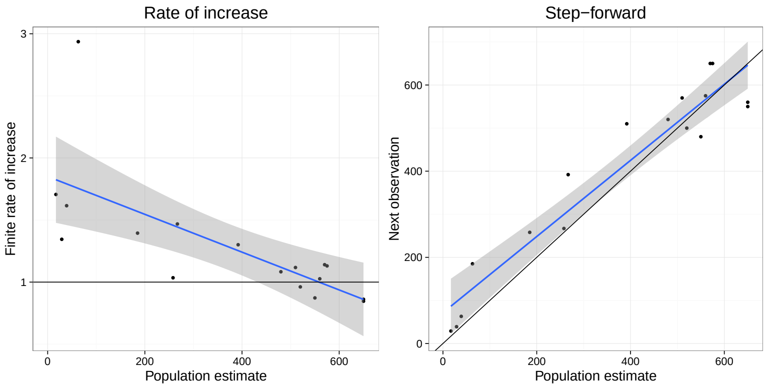
Paramecia



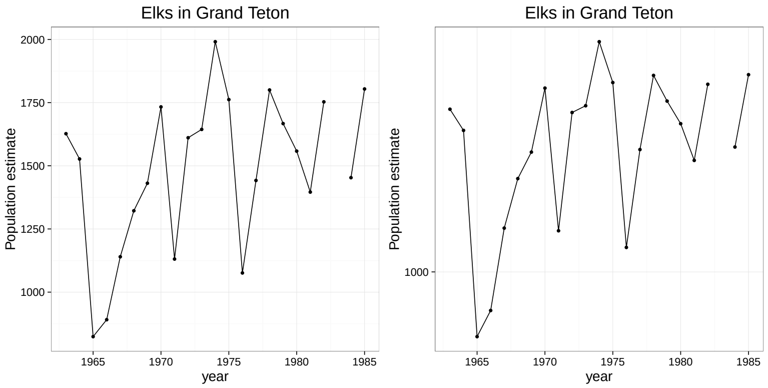
Time series
Population estimate
0
200
400
600
5
10
15
20
observation
Time series
Population estimate
10
100
5
10
15
20
observation

Paramecia



Rate of increase
Finite rate of increase
3
2
1
0
200
400
600
Population estimate
Step−forward
Next observation
600
400
200
0
0
200
400
600
Population estimate

Elk


Elks in Grand Teton
Population estimate
2000
1750
1500
1250
1000
1965
1970
1975
1980
1985
year
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1000
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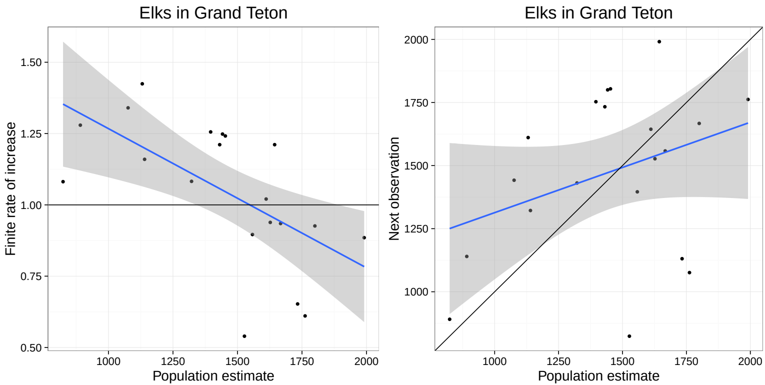

Elks in Grand Teton
Finite rate of increase
1.50
1.25
1.00
0.75
0.50
1000
1250
1500
1750
2000
Population estimate
Elks in Grand Teton
Next observation
2000
1750
1500
1250
1000
1000
1250
1500
1750
2000
Population estimate

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Subsection 1

Allee effects

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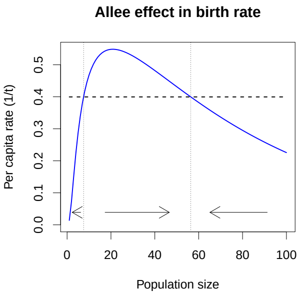

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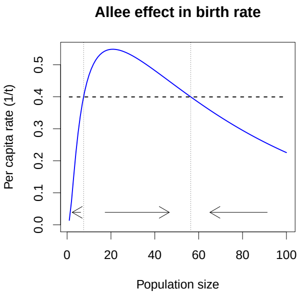

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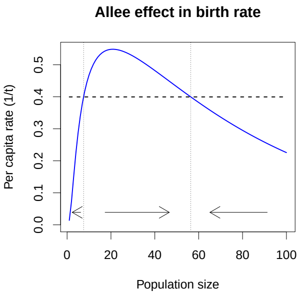
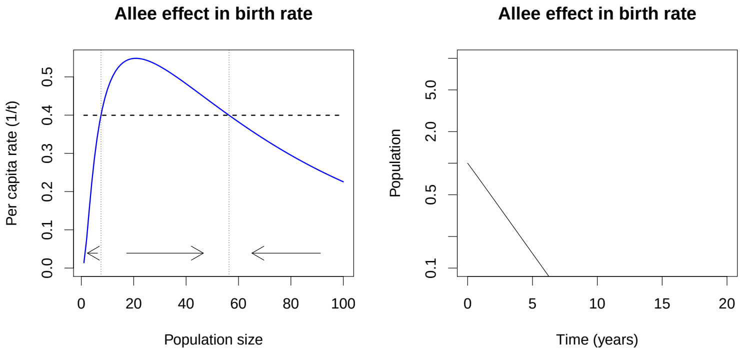


Individual perspective



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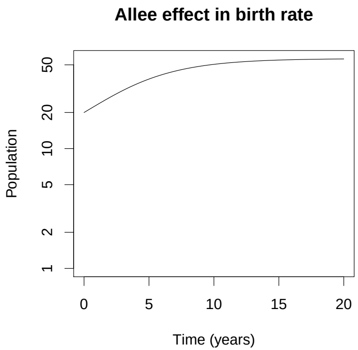

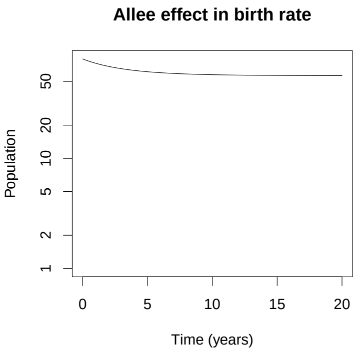

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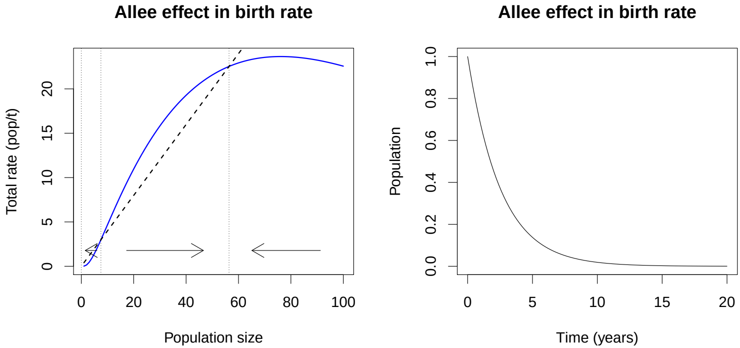

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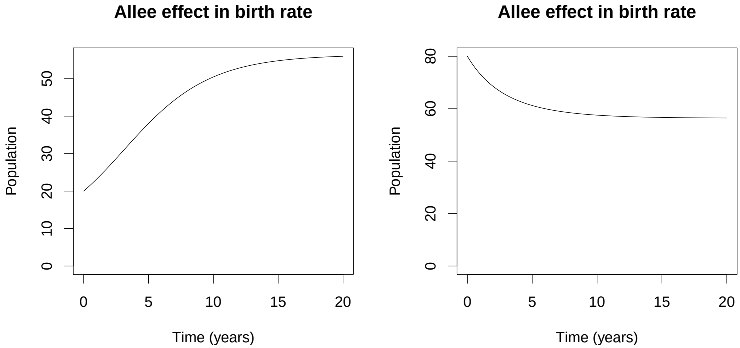

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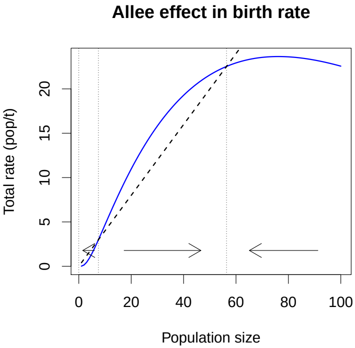

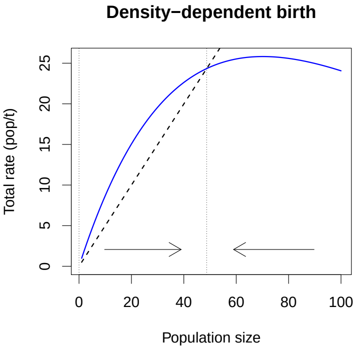

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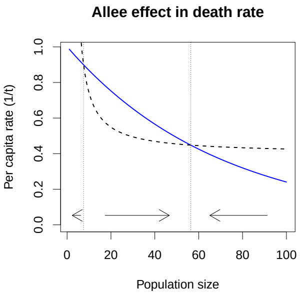

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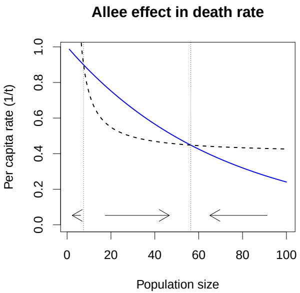

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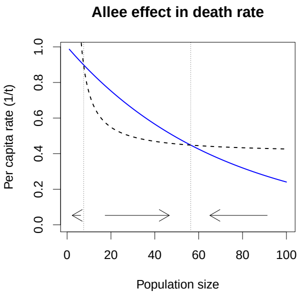

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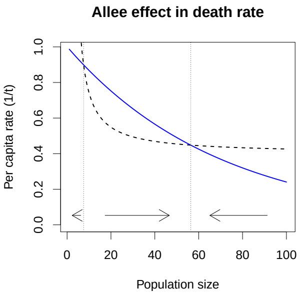

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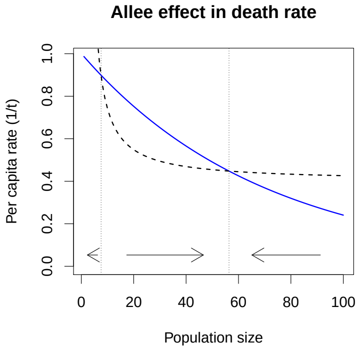

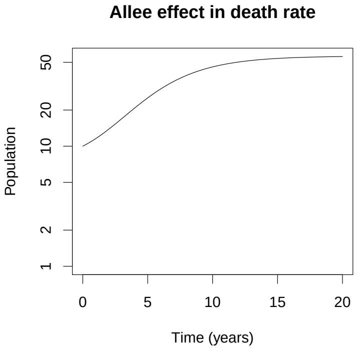

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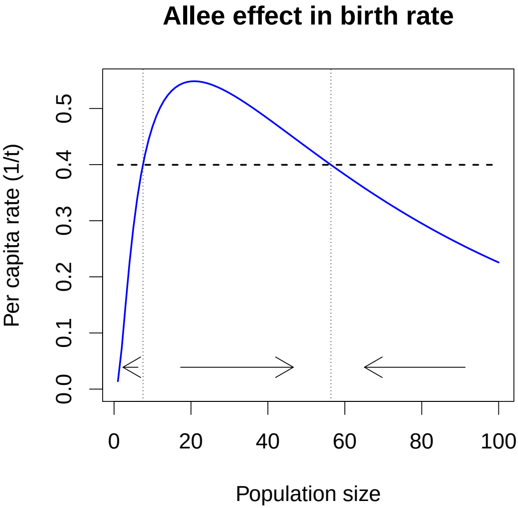

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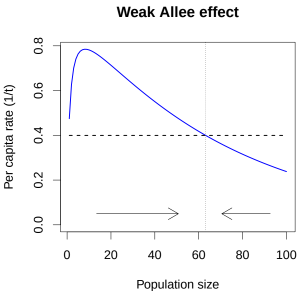

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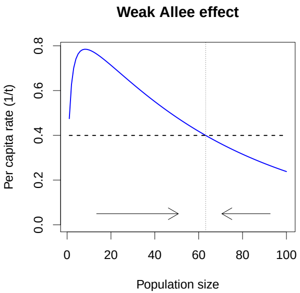

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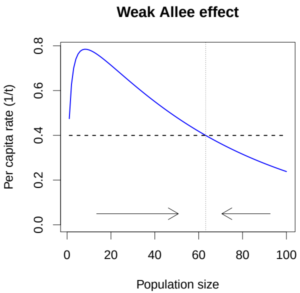

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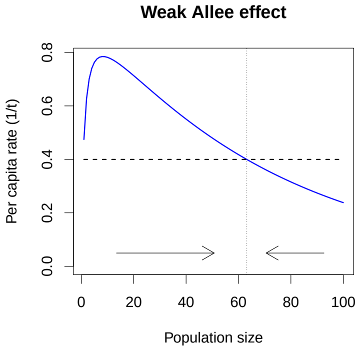

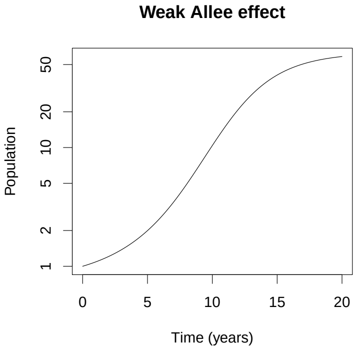

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Subsection 2

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Simulations

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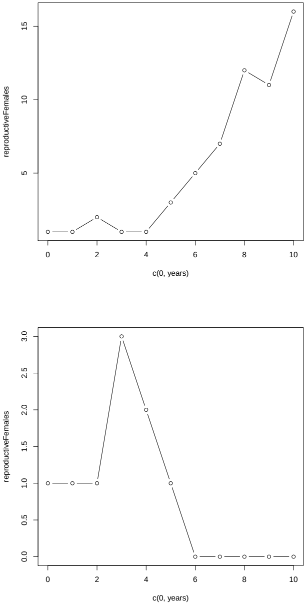

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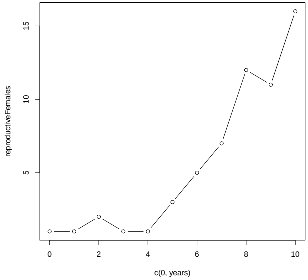

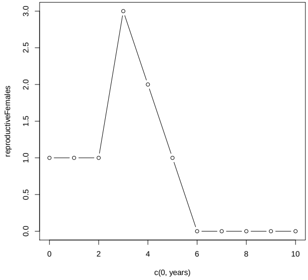

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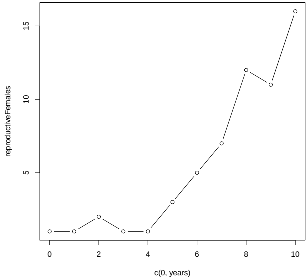

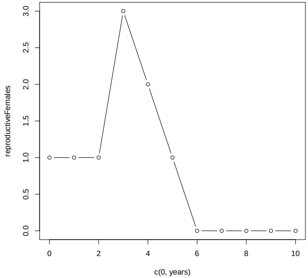

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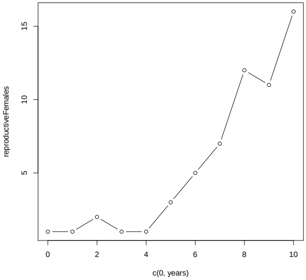

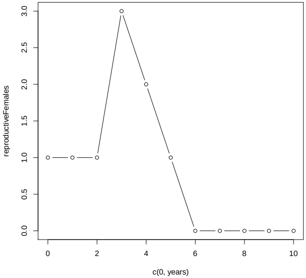

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