

UNIT 7: Infectious disease

Outline

Introduction

Rate of spread

Single-epidemic model

Epidemic size

Recurrent epidemic models

Reproductive numbers and risk

Infectious disease

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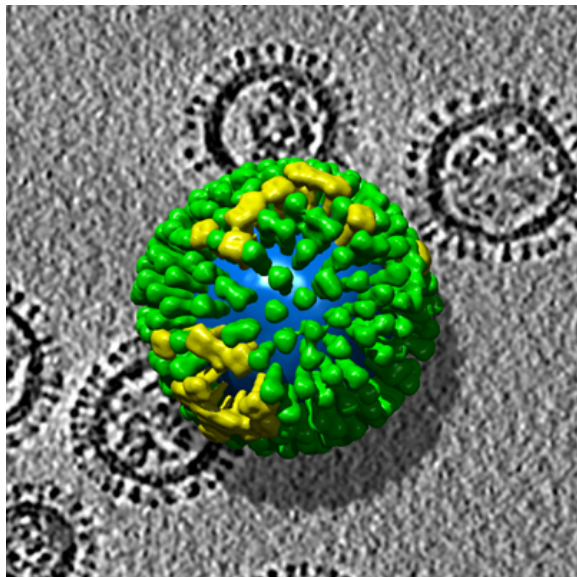
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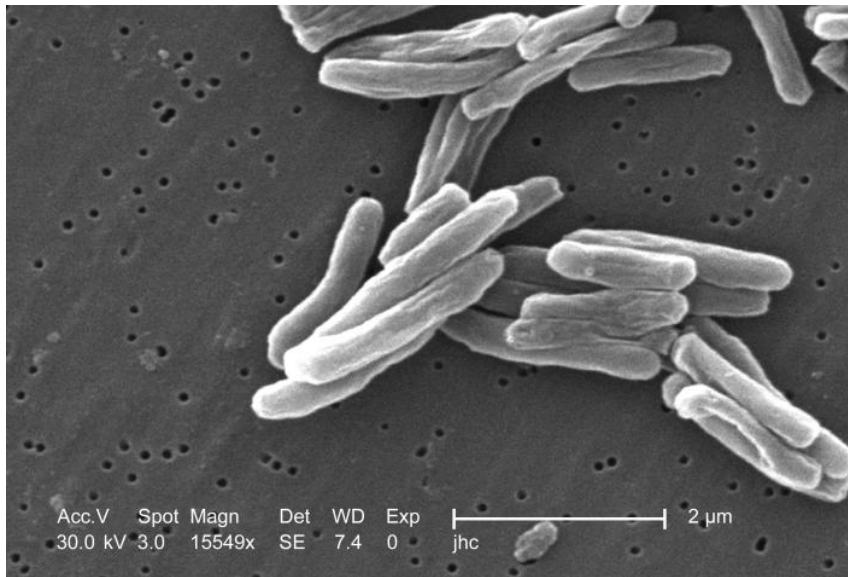
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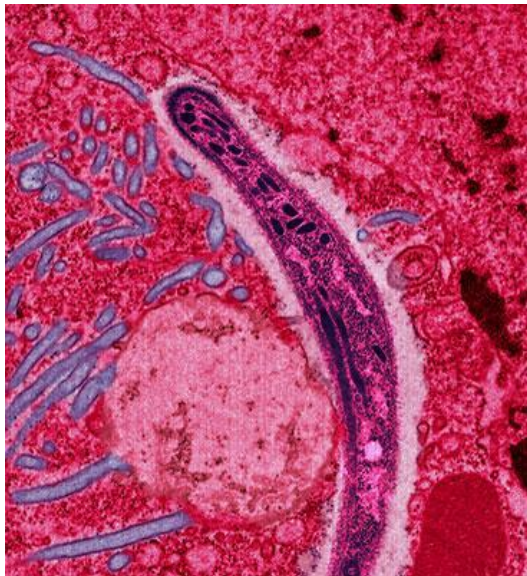
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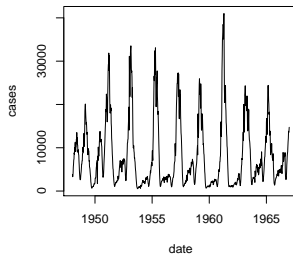
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Measles reports from England and Wales

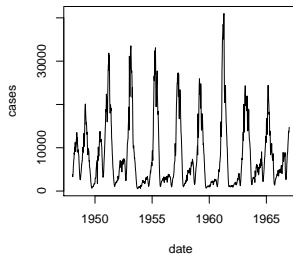


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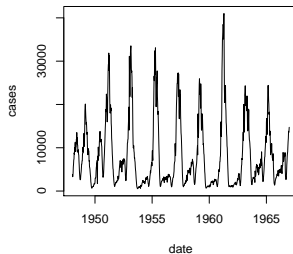


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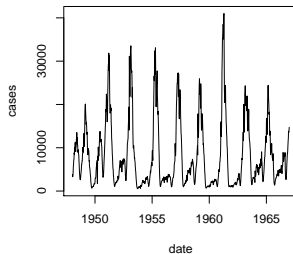


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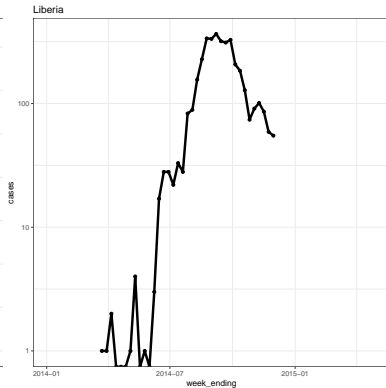
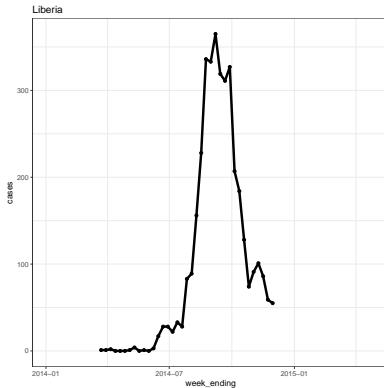
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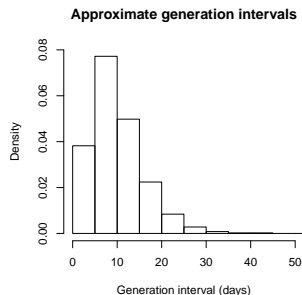
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Example: the West African Ebola epidemic



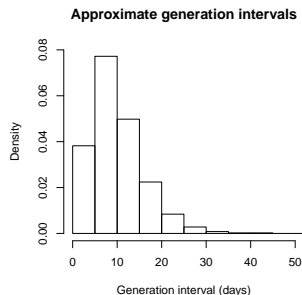
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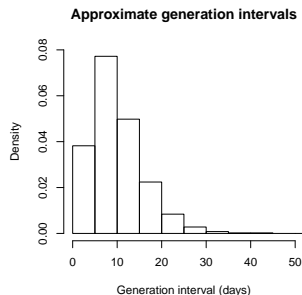
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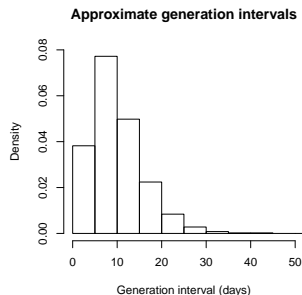
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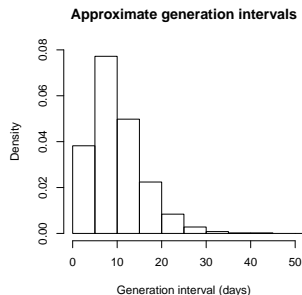
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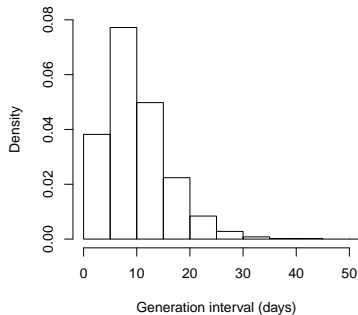
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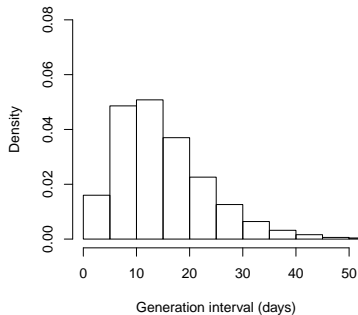


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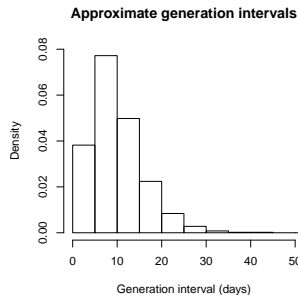


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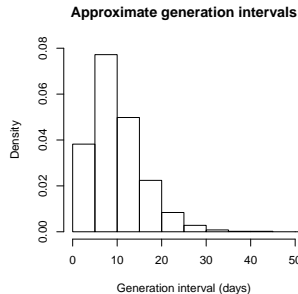
Speed and risk

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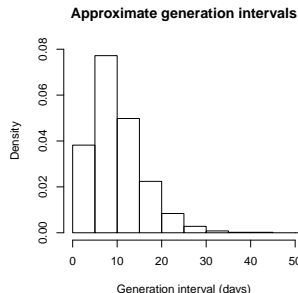
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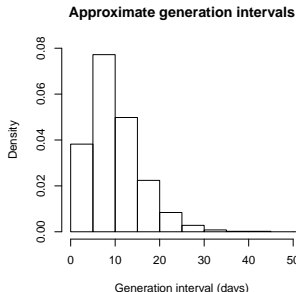
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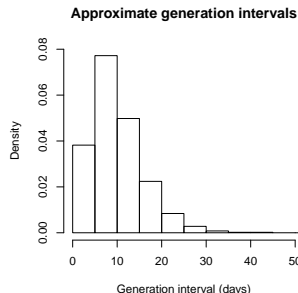
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Fighting Ebola



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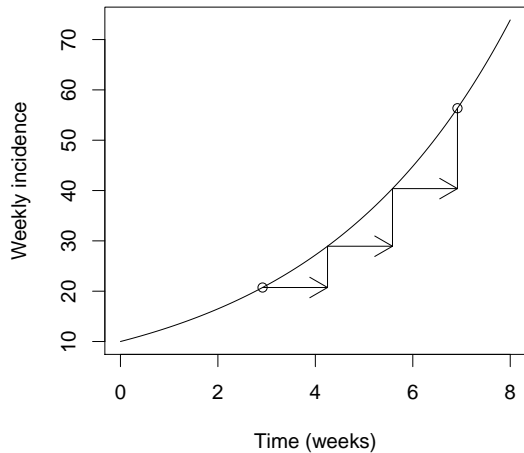
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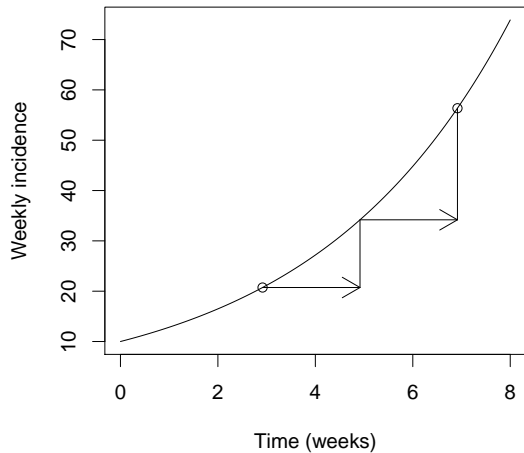
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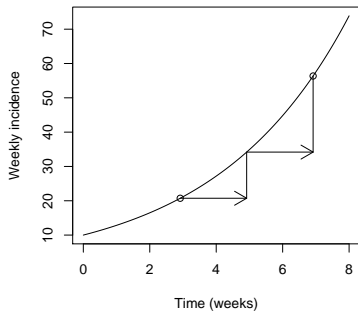
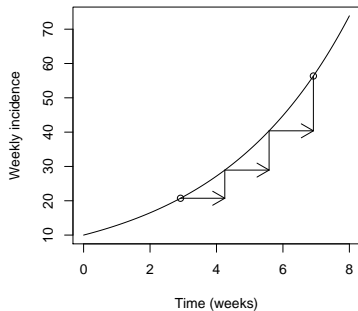
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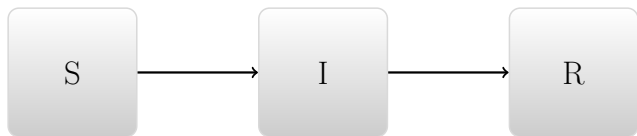
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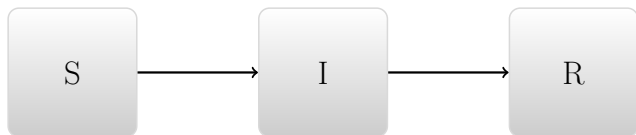
Reproductive numbers and risk

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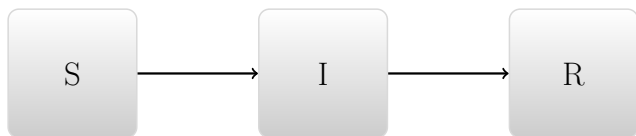
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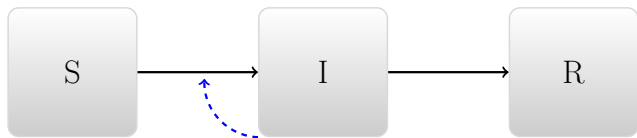
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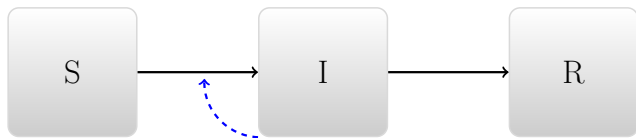
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Transition rates



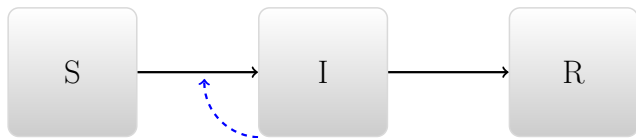
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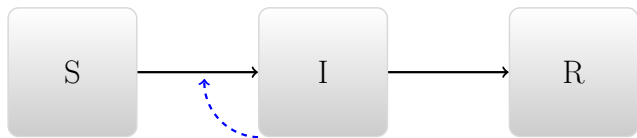
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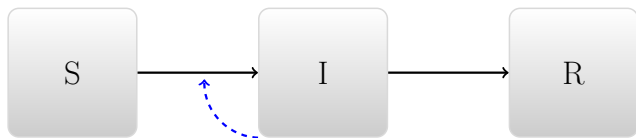
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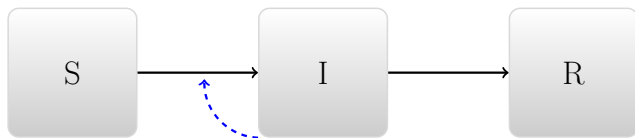
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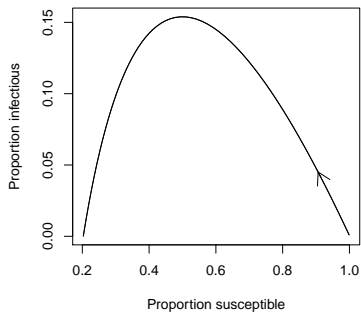
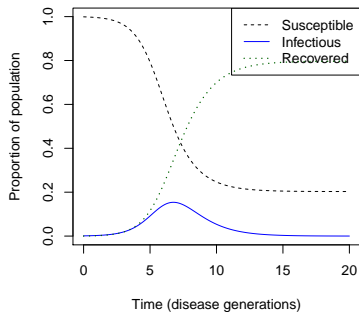
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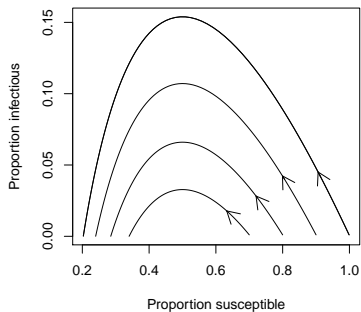
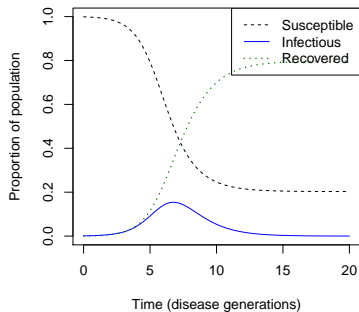
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Subsection 1

Epidemic size

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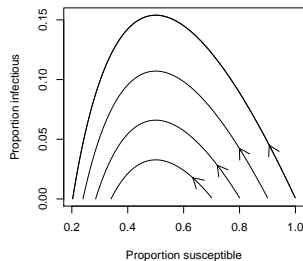
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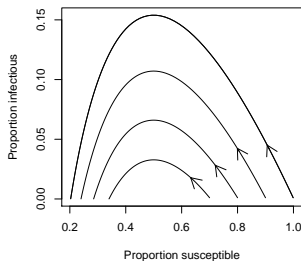
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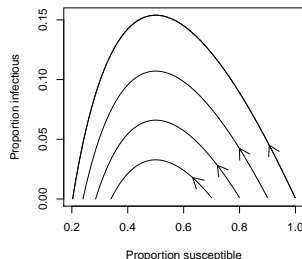
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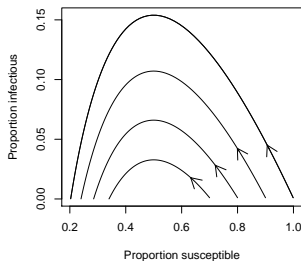


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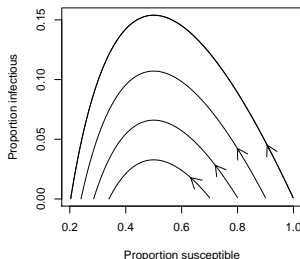
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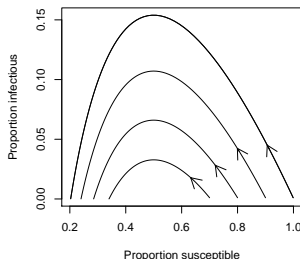
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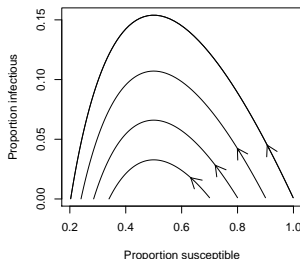
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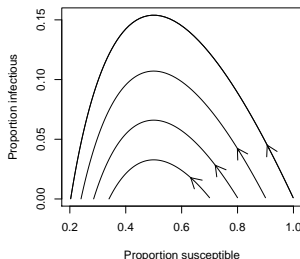
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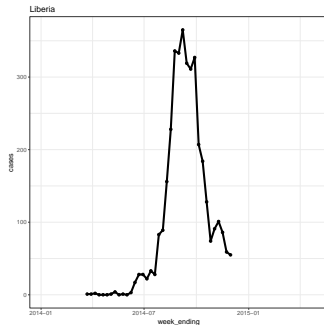
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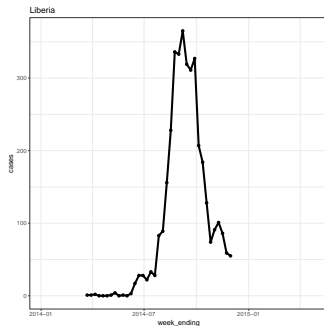
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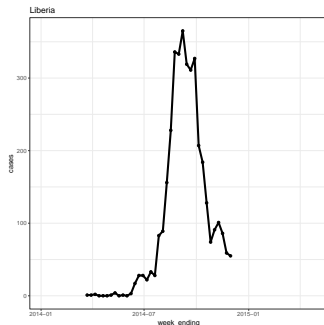
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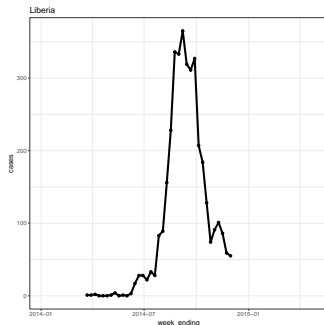
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Outline

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Single-epidemic model

Epidemic size

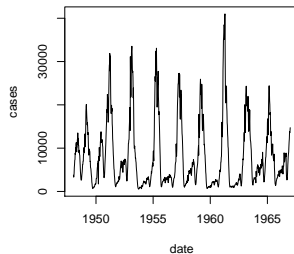
Recurrent epidemic models

Reproductive numbers and risk

Recurrent epidemic models

- Poll: If epidemics tend to burn out, why do we often see repeated epidemics?

Measles reports from England and Wales

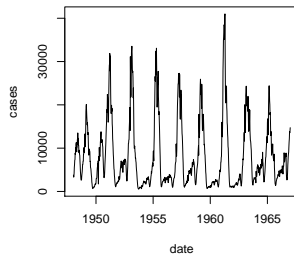


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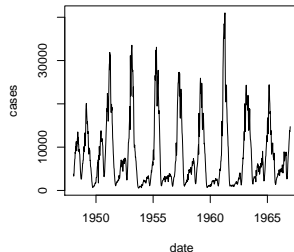
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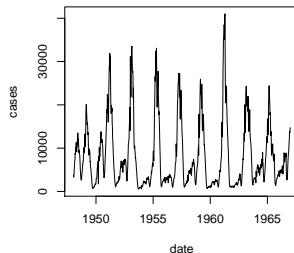
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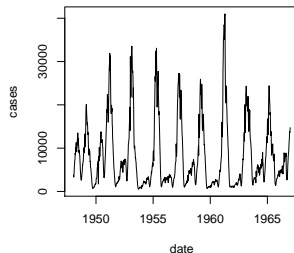
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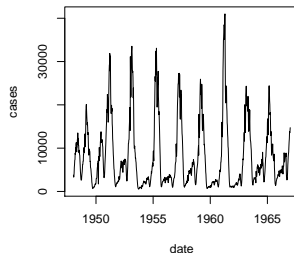
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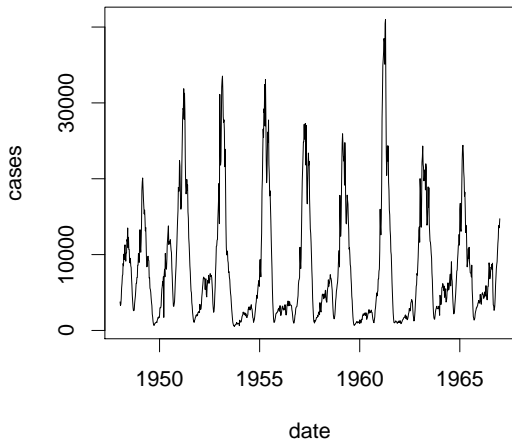
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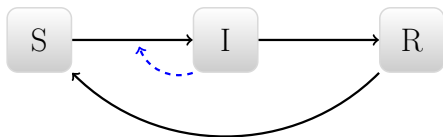


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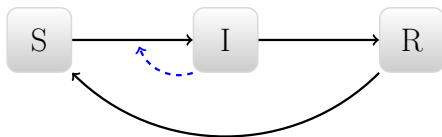
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Closing the circle

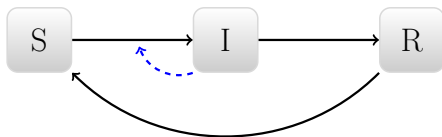


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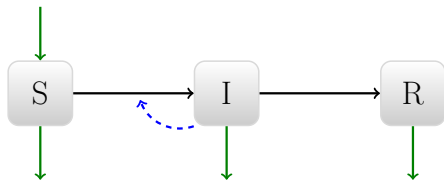
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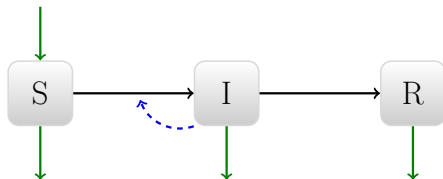


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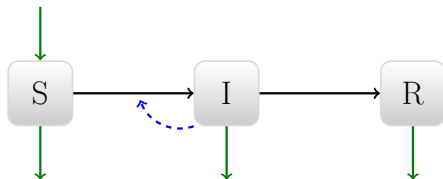


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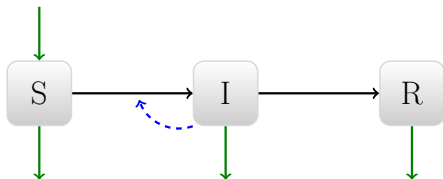
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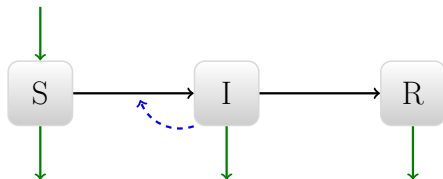


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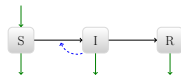


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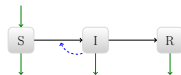
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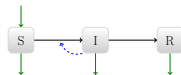
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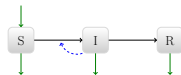
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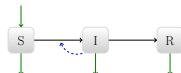
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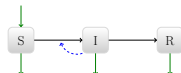
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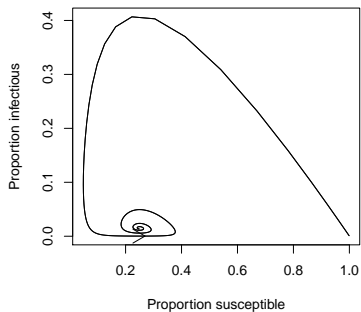
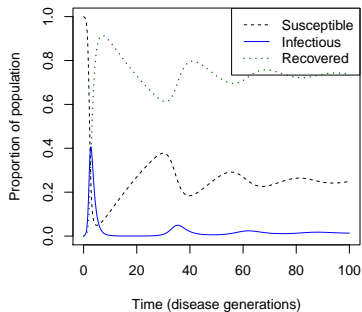
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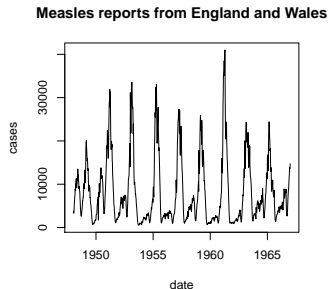
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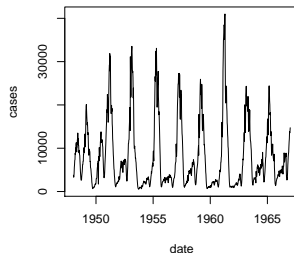


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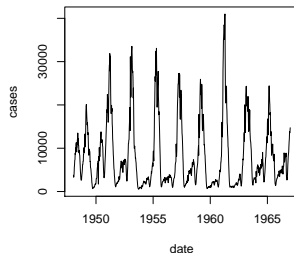
Measles reports from England and Wales



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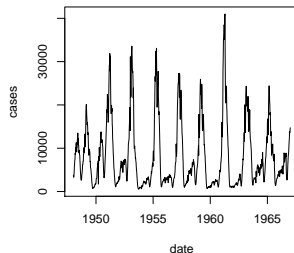
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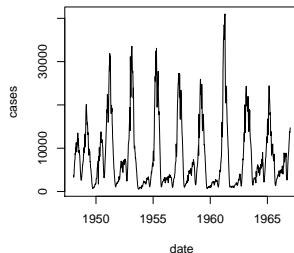
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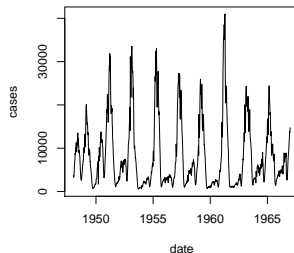
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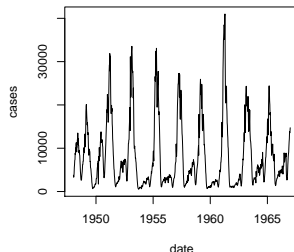
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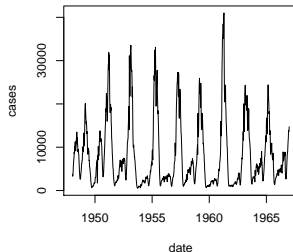
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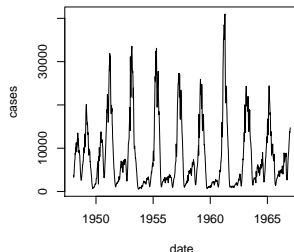
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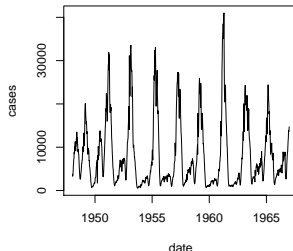


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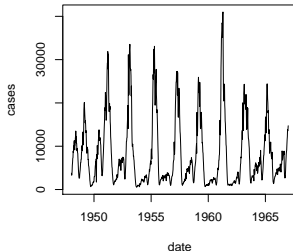
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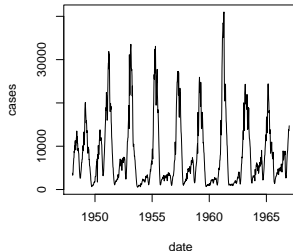
Measles reports from England and Wales



Persistent oscillations

- ▶ Poll: If oscillations tend to be damped in simple models, why do they persist in real life?
 - ▶ * Weather
 - ▶ * School terms
 - ▶ * Demographic stochasticity
 - ▶ * Changes in Behaviour
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Outline

Introduction

Rate of spread

Single-epidemic model

Epidemic size

Recurrent epidemic models

Reproductive numbers and risk

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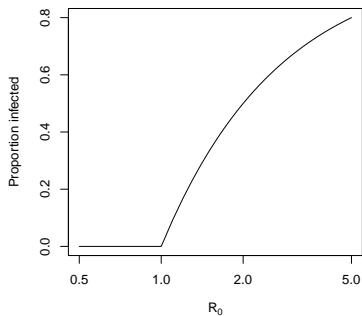
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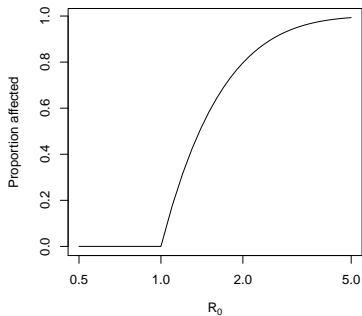
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endemic equilibrium



epidemic size



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