

UNIT 7: Infectious disease

Outline

Introduction

Rate of spread

Single-epidemic model

Epidemic size

Recurrent epidemic models

Reproductive numbers and risk

Infectious disease

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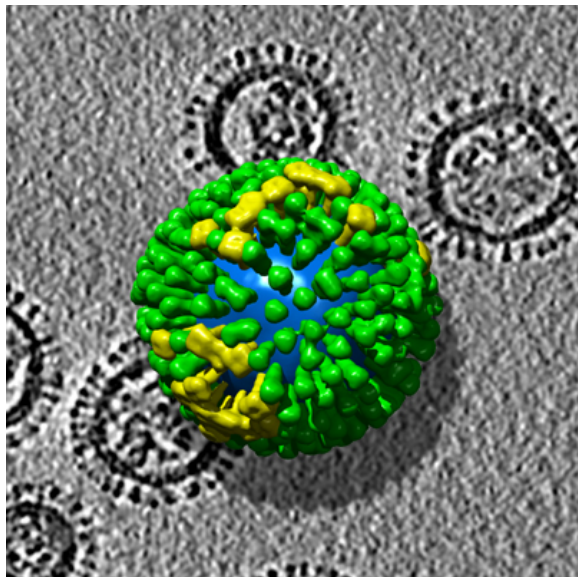
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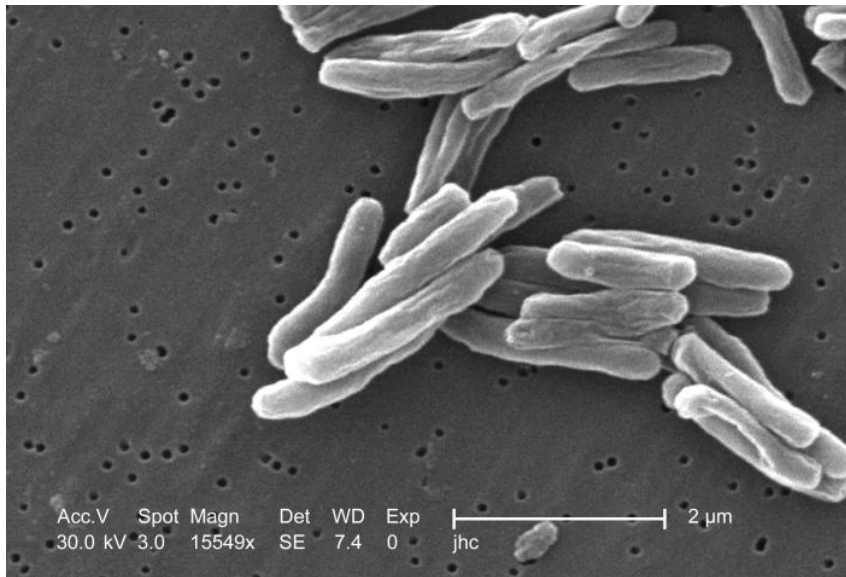
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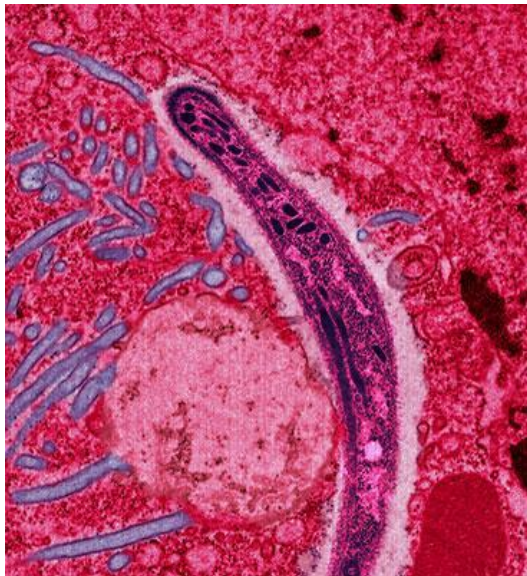
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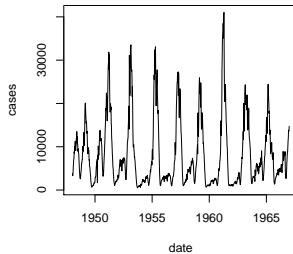
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Measles reports from England and Wales

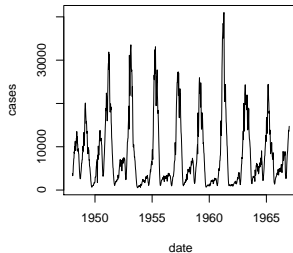


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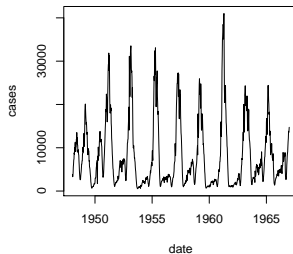


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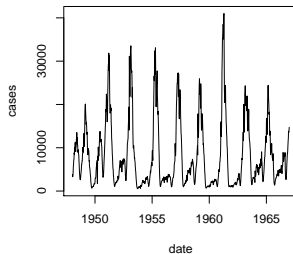


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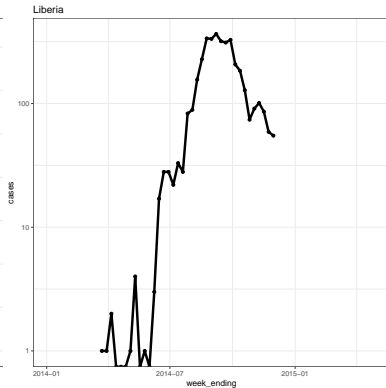
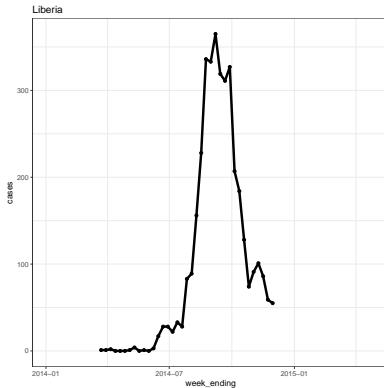
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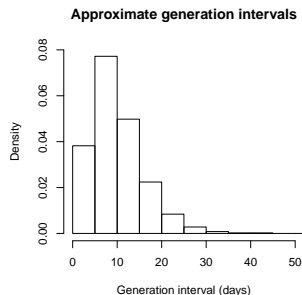
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Example: the West African Ebola epidemic



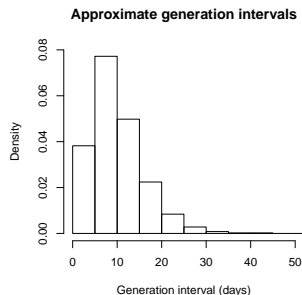
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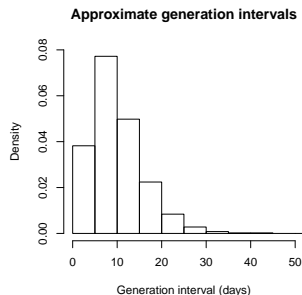
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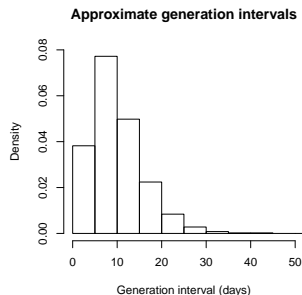
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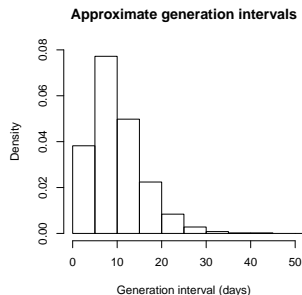
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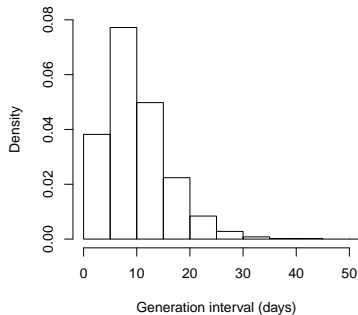
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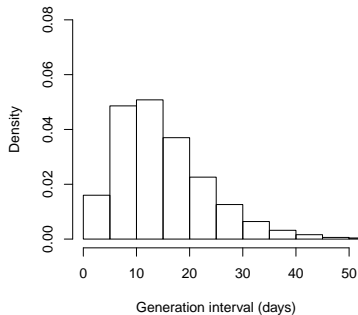


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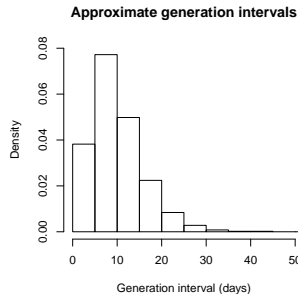


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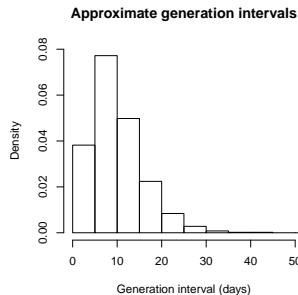
Speed and risk

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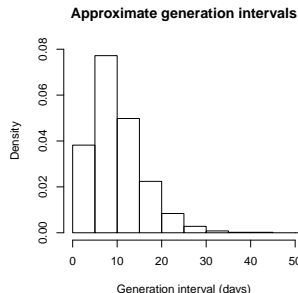
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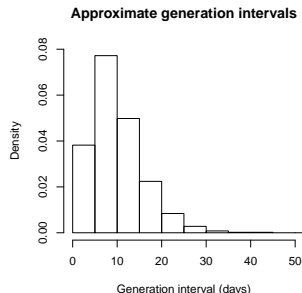
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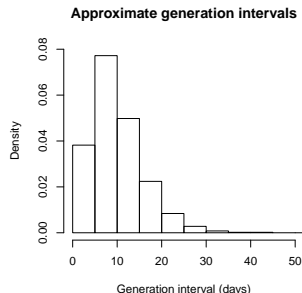
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Fighting Ebola



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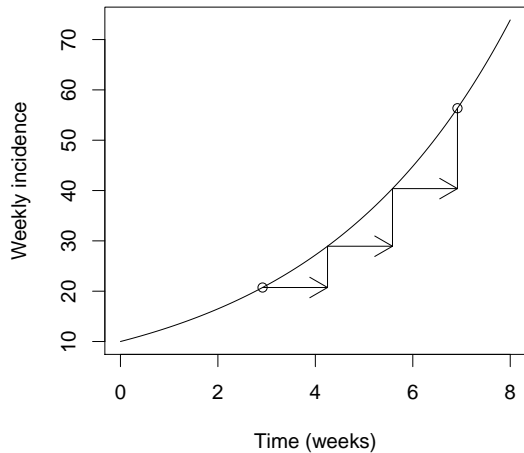
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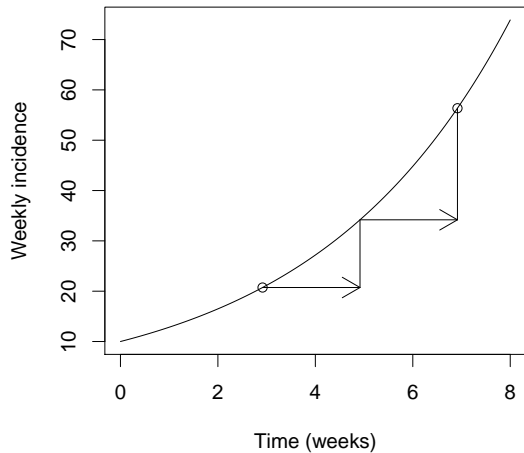
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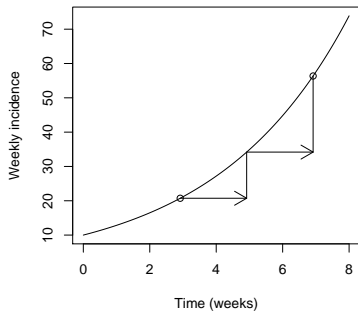
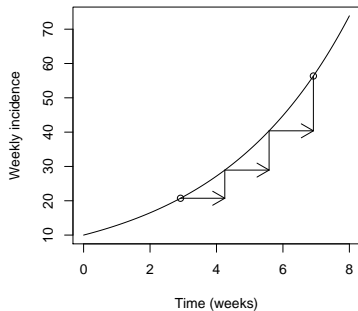
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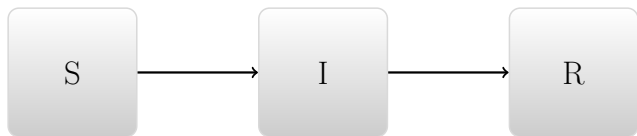
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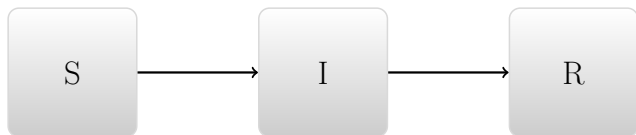
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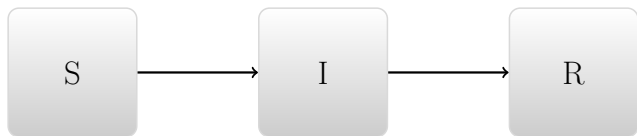
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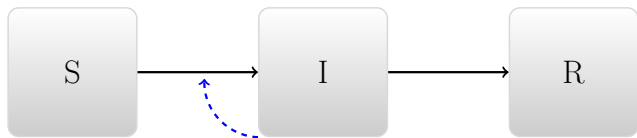
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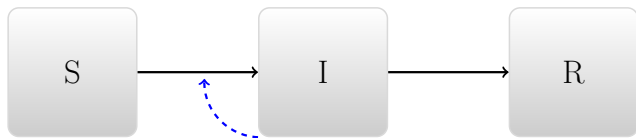
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Transition rates



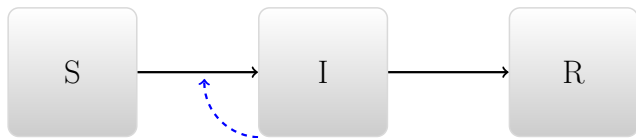
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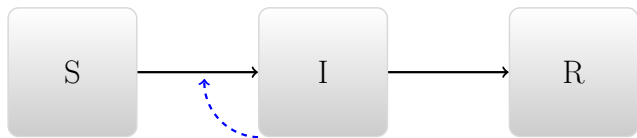
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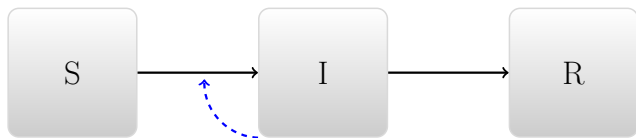
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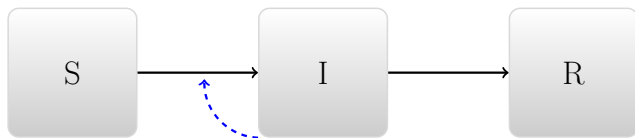
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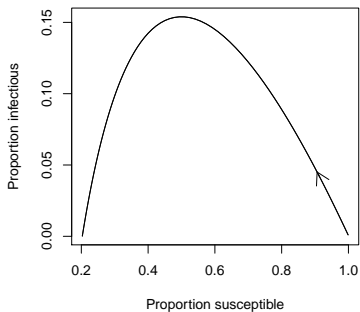
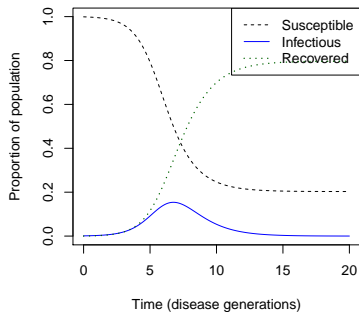
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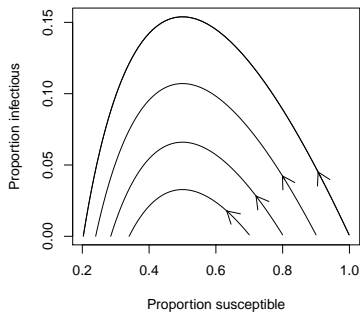
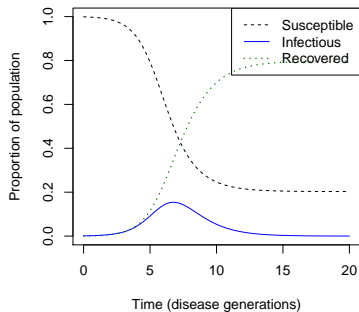
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Subsection 1

Epidemic size

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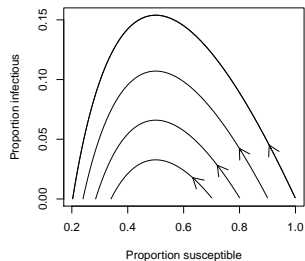
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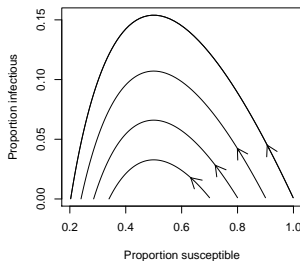
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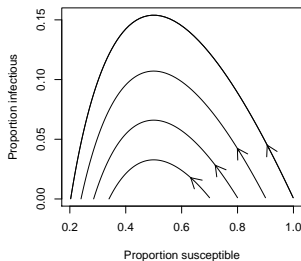
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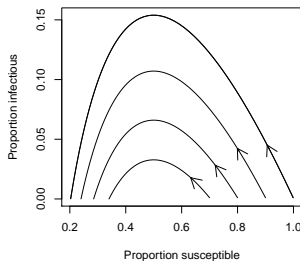


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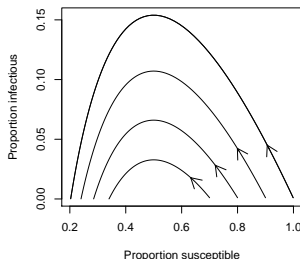
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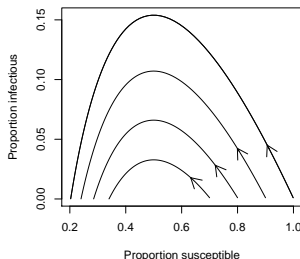
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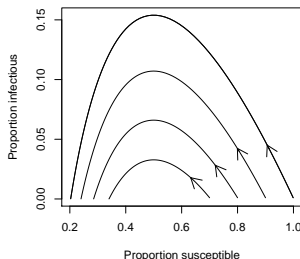
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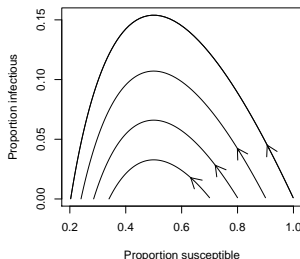
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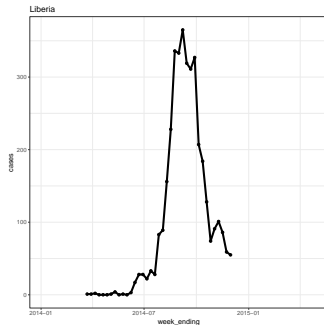
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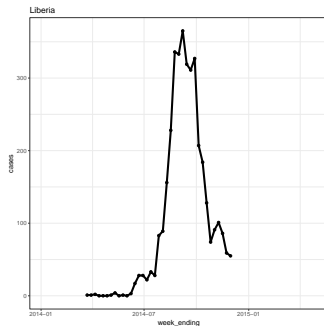
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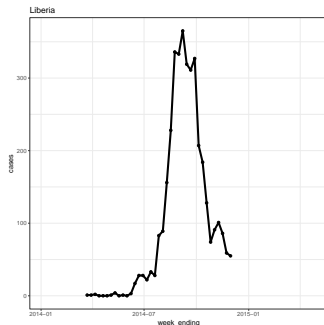
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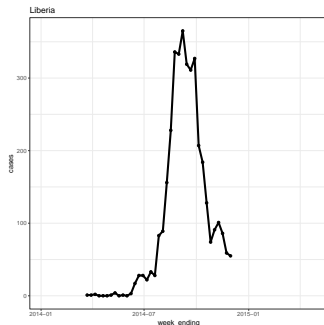
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Outline

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Rate of spread

Single-epidemic model

Epidemic size

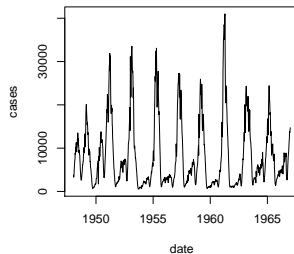
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Measles reports from England and Wales

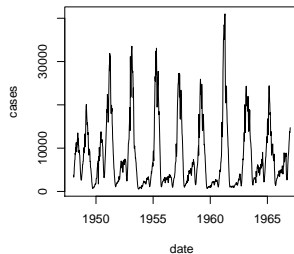


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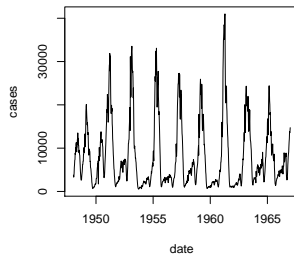
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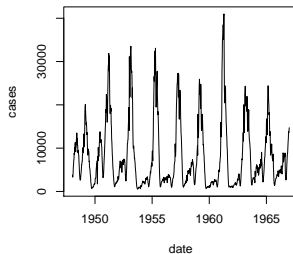
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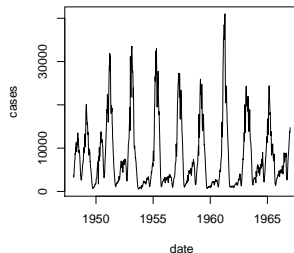
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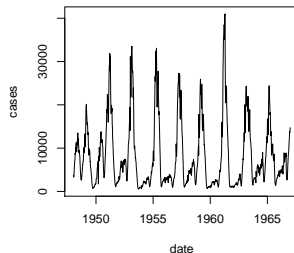
Measles reports from England and Wales



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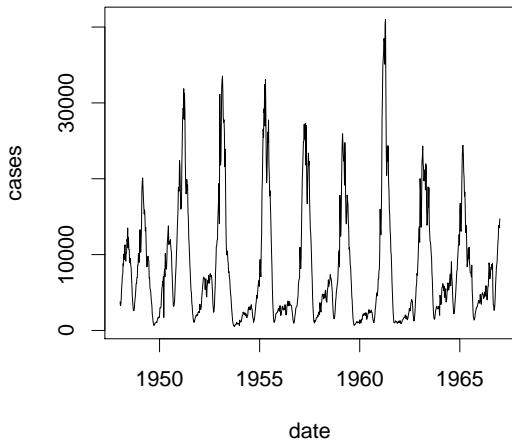
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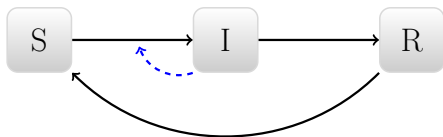


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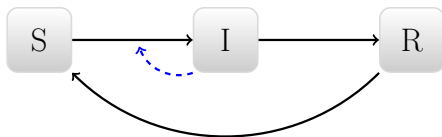
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Closing the circle

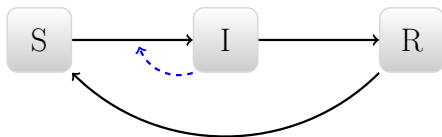


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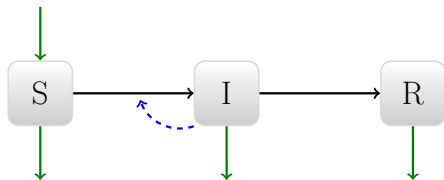
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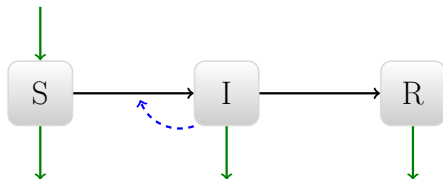


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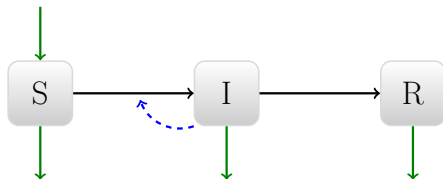


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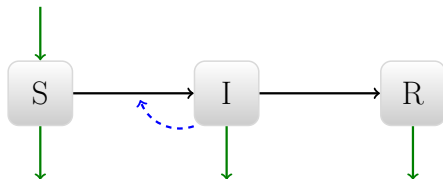
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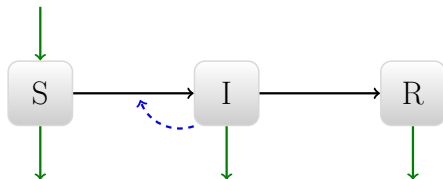


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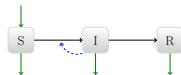


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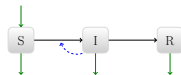
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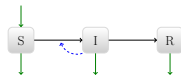
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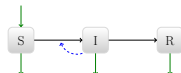
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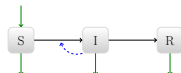
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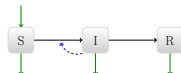
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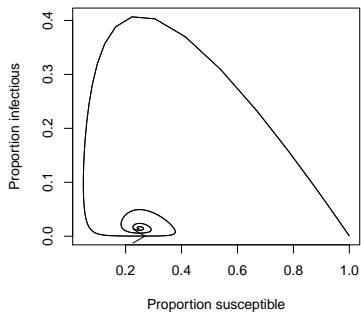
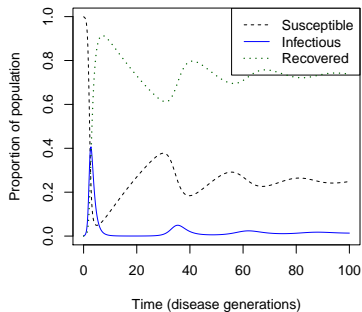
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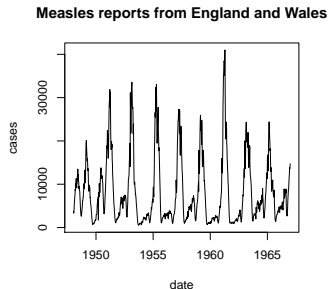
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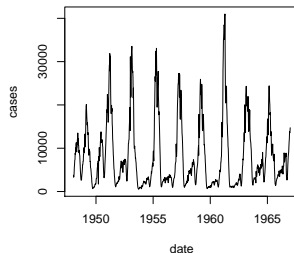


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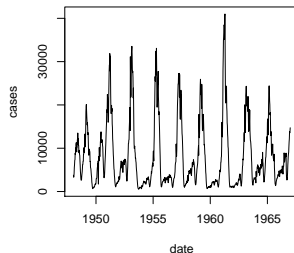
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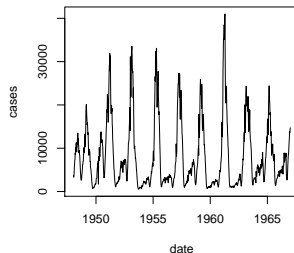
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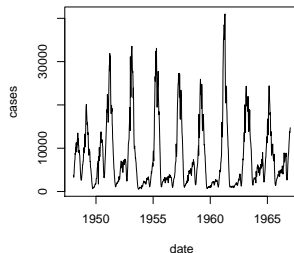
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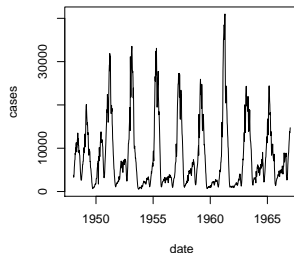
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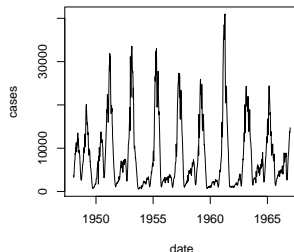
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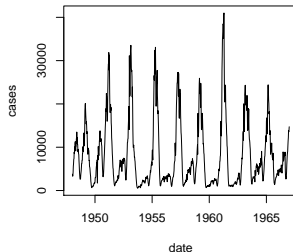
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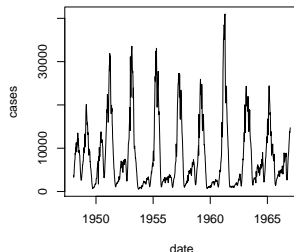
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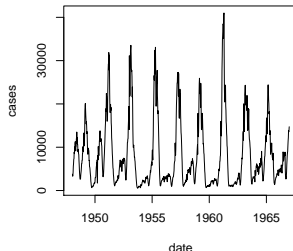


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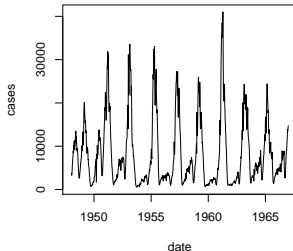
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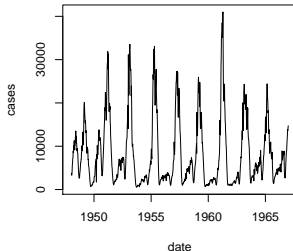
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Outline

Introduction

Rate of spread

Single-epidemic model

Epidemic size

Recurrent epidemic models

Reproductive numbers and risk

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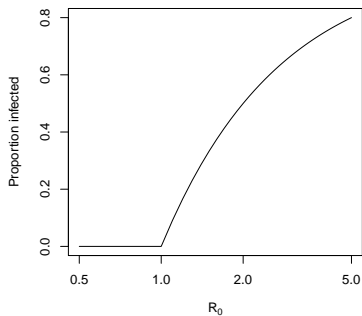
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- ▶ Proportion “affected” (infectious or immune) should be approximately $V/N = 1 - 1/\mathcal{R}_0$
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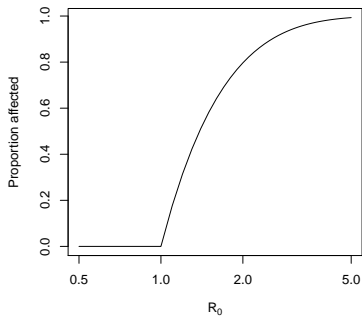
- ▶ At equilibrium, the proportion of people who are susceptible to disease should be approximately $S/N = 1/\mathcal{R}_0$
- ▶ Proportion “affected” (infectious or immune) should be approximately $V/N = 1 - 1/\mathcal{R}_0$
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endemic equilibrium



epidemic size



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