UNIT 3: Structured populations

Outline

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Subsection 1

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Subsection 2

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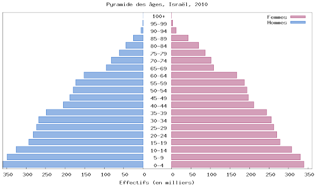

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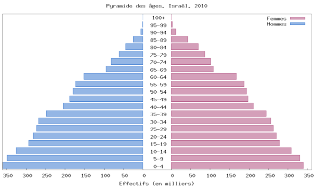

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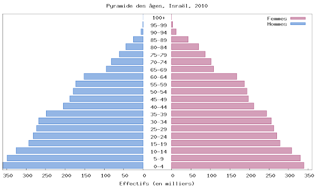

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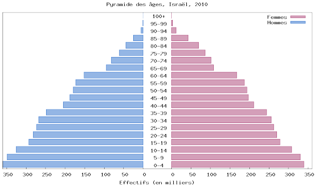

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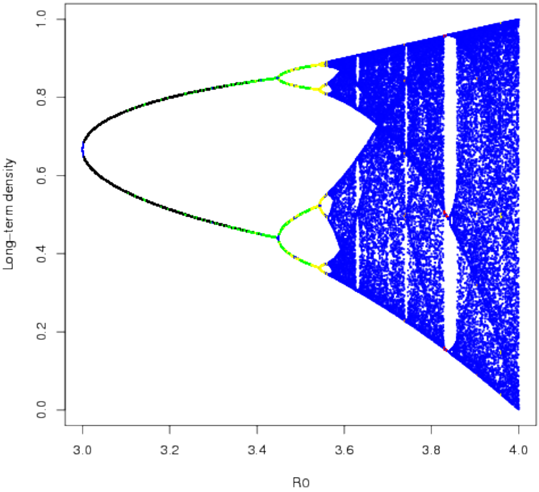

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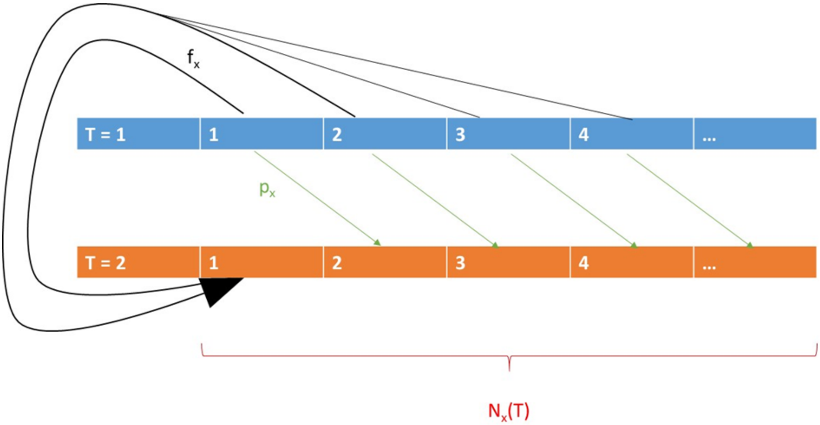

Subsection 1

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Subsection 1

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1				
2				
R				

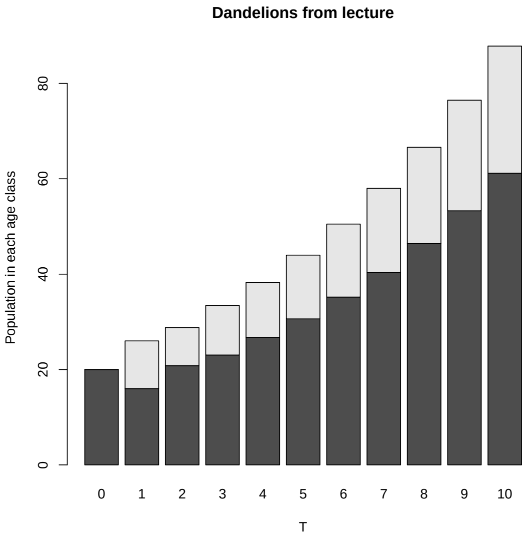
Dandelion life table

x	f_x	p_x	ℓ_x	$\ell_x f_x$
1	0.8	0.5		
2	0.8	0		
R				

Dandelion life table

x	f_x	p_x	ℓ_x	$\ell_x f_x$
1	0.8	0.5	1.000	0.800
2	0.8	0	0.500	0.400
R				1.200

Dandelion dynamics



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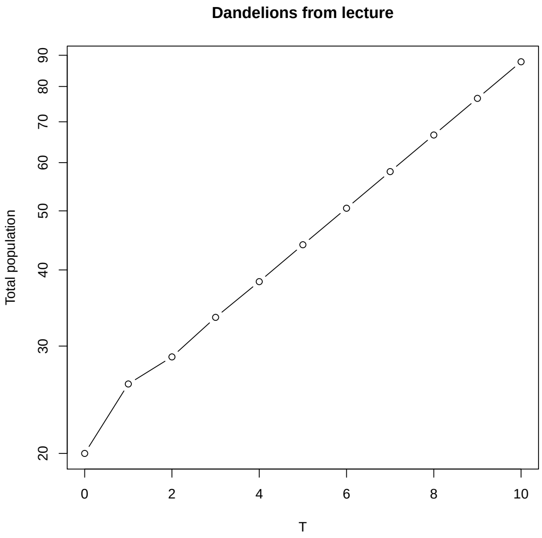

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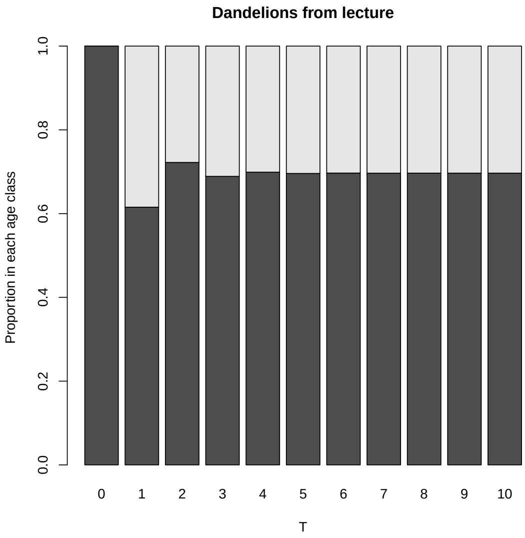

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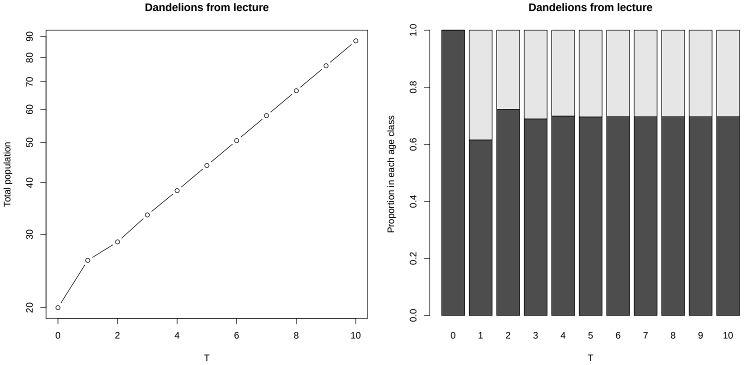

Squirrel example

Gray squirrel population example

x	f_x	p_x	ℓ_x	$\ell_x f_x$
1	0	0.25		
2	1.28	0.46		
3	2.28	0.77		
4	2.28	0.65		
5	2.28	0.67		
6	2.28	0.64		
7	2.28	0.88		
8	2.28	0.0		
R				

Squirrel observations

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3	2.28	0.77	0.115	0.262
4	2.28	0.65	0.089	0.202
5	2.28	0.67	0.058	0.131
6	2.28	0.64	0.039	0.088
7	2.28	0.88	0.025	0.056
8	2.28	0.0	0.022	0.050
R				1.109

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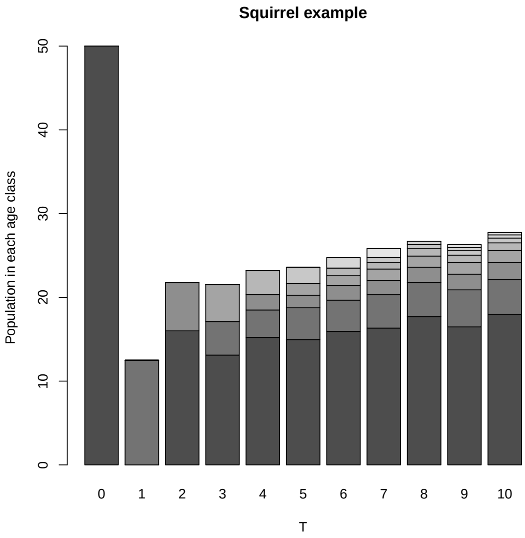

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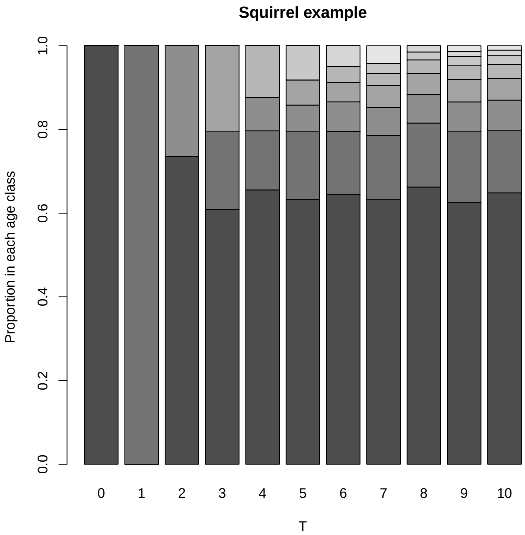

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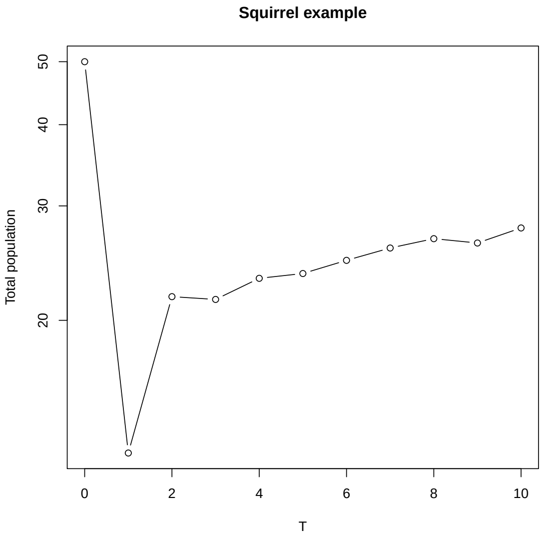

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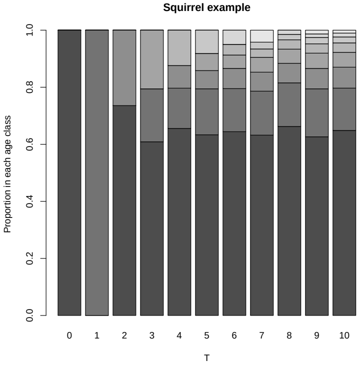
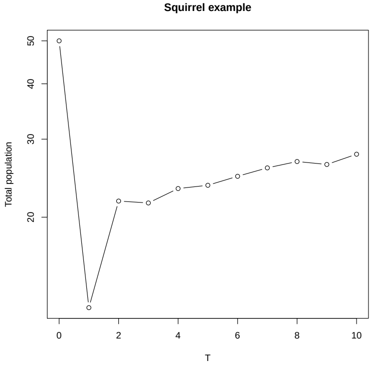
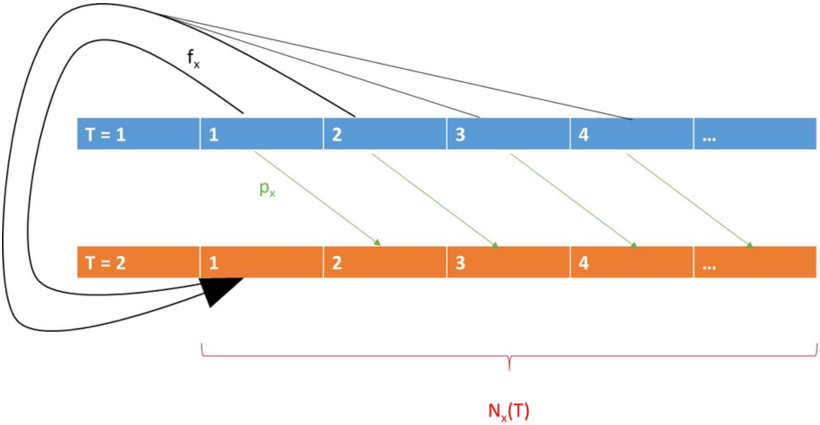




The structured model



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3	0	0.8		
4	10	0		
R				

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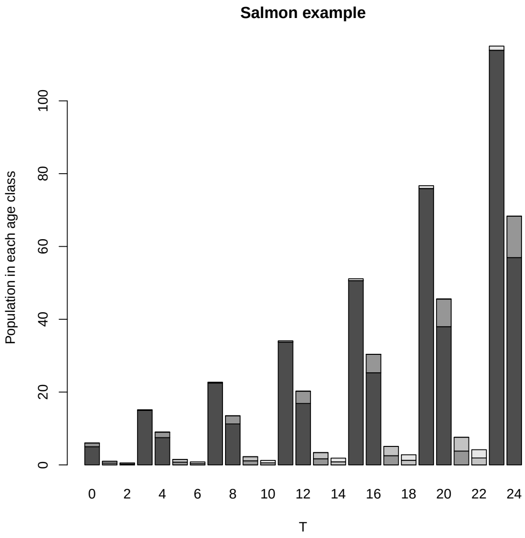

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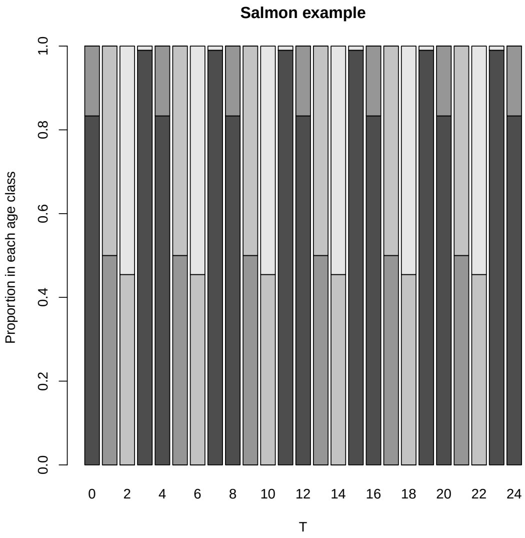

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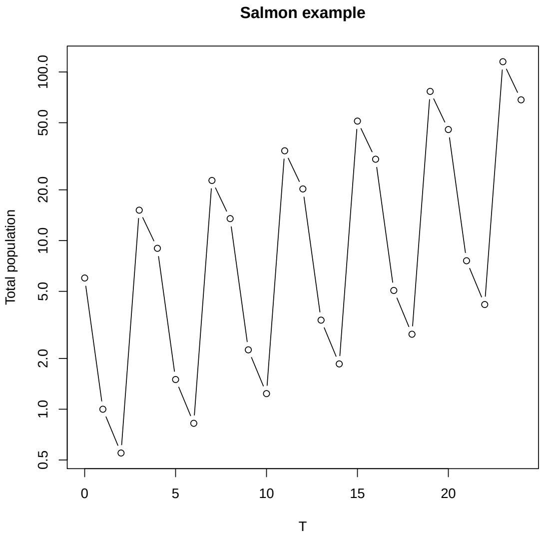

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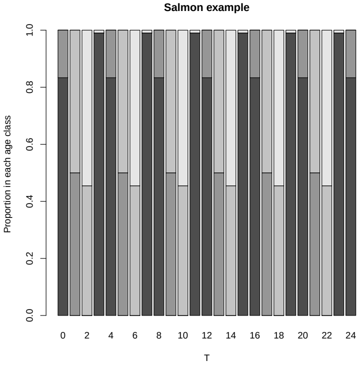

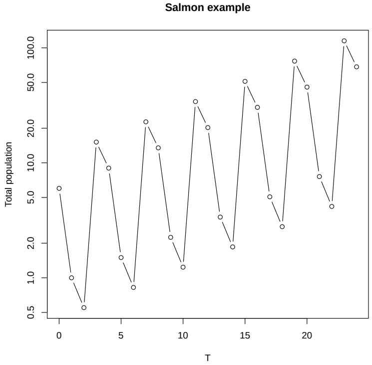

Subsection 2

Calculation details

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Dandelion life table

x	f_x	p_x	ℓ_x	$\ell_x f_x$
1	0.8	0.5	1.000	0.800
2	0.8	0	0.500	0.400
R				1.200

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x	f_x	p_x	ℓ_x	$\ell_x f_x$
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Subsection 1

Survivorship

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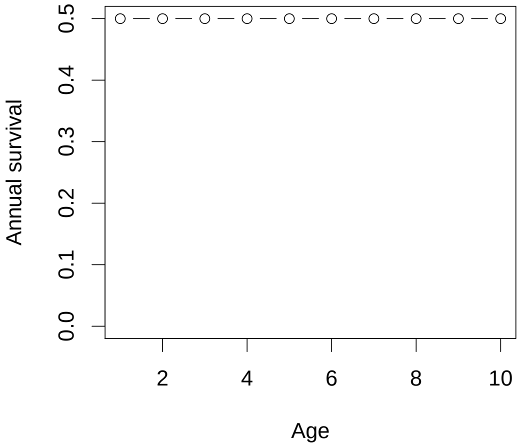

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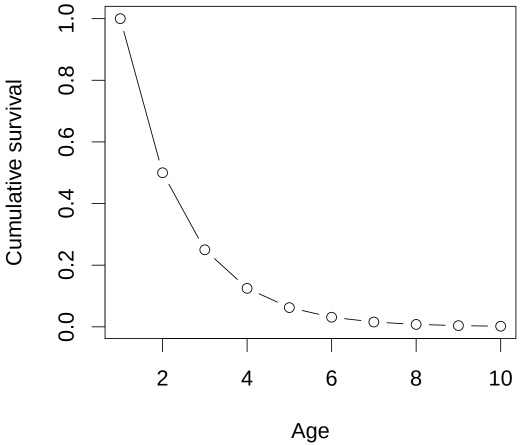

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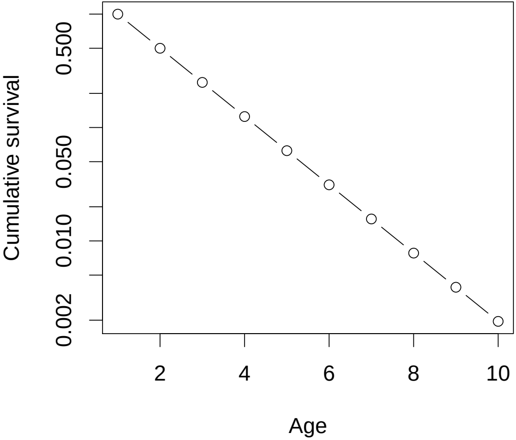

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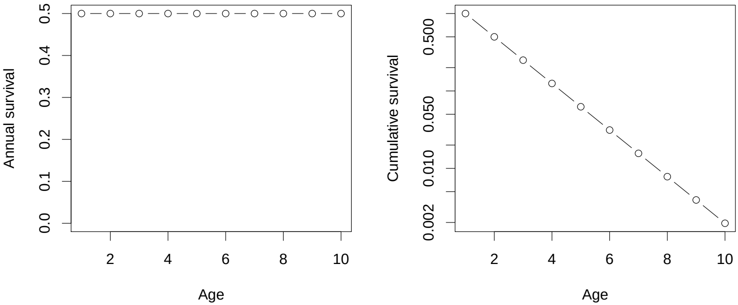

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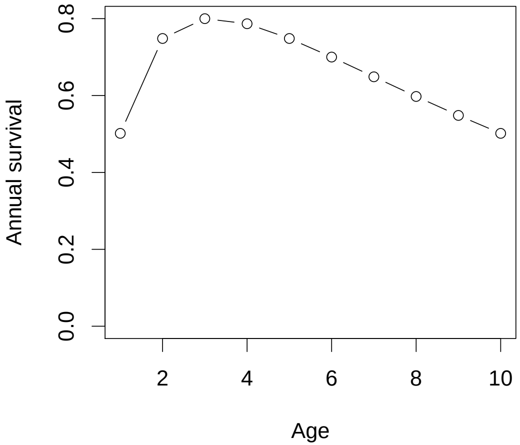

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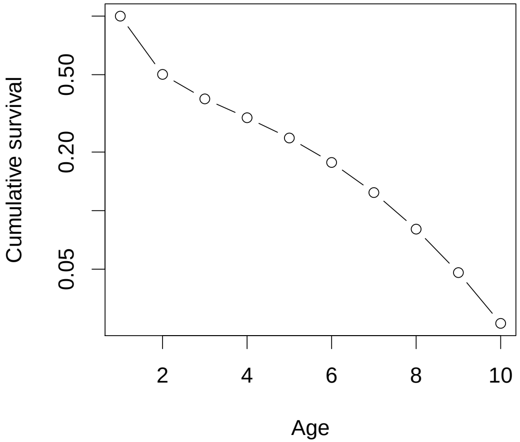

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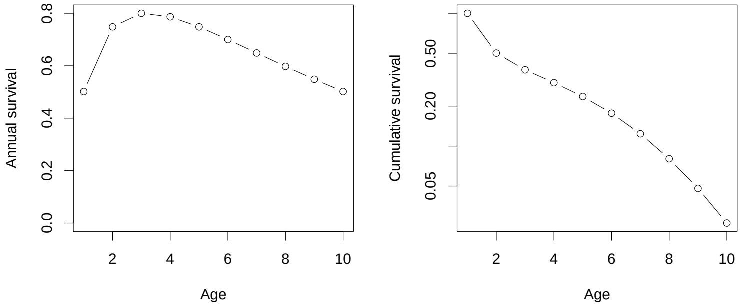

Subsection 2

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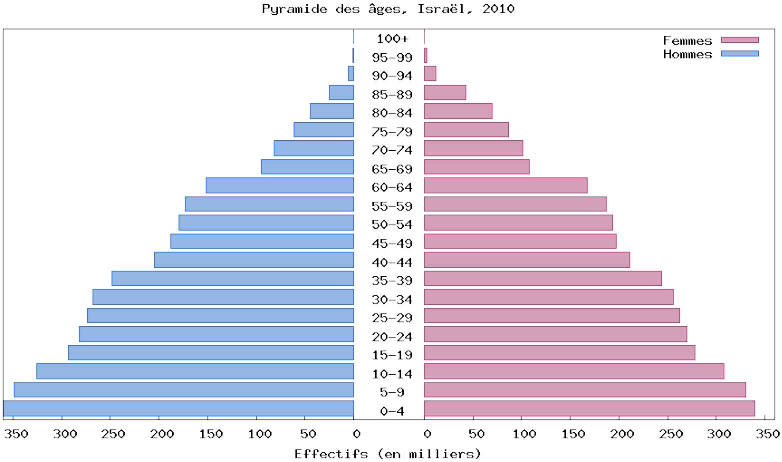

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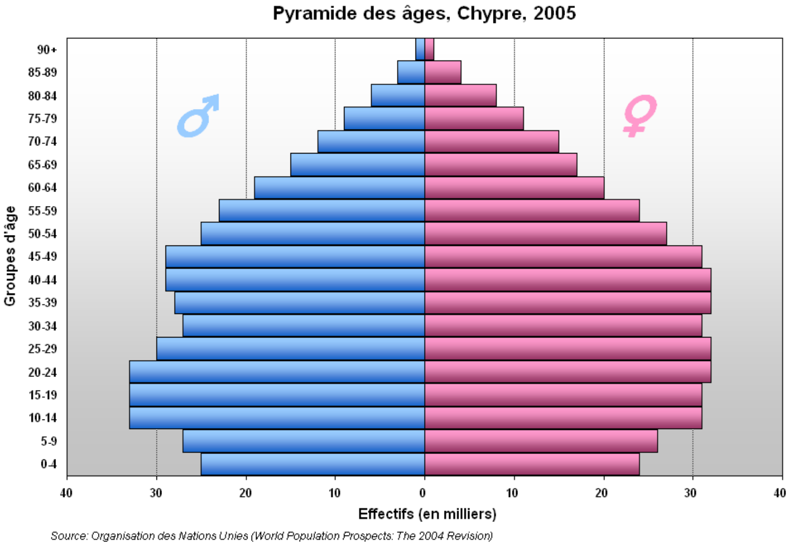

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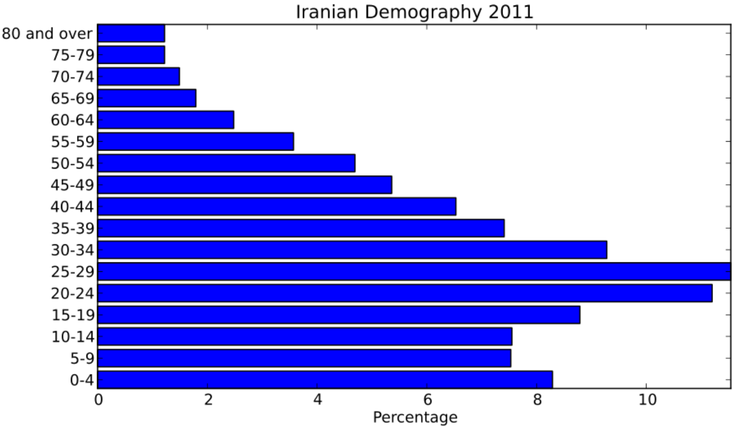

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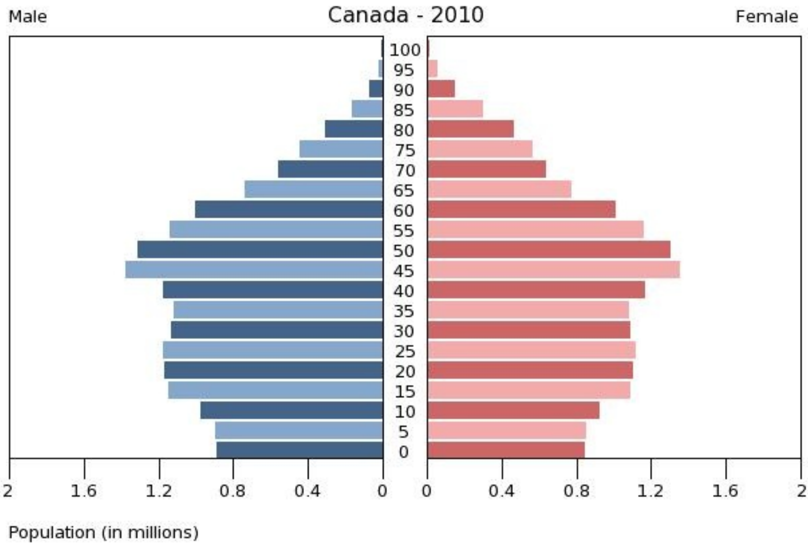

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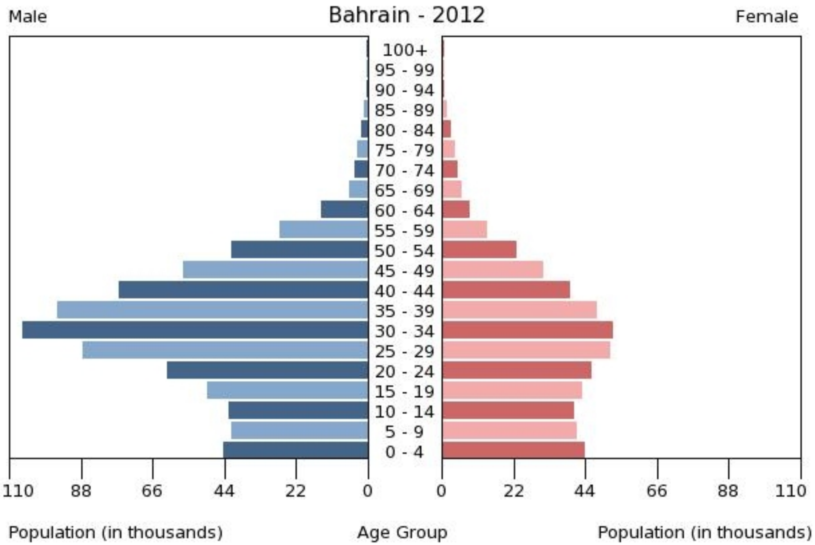

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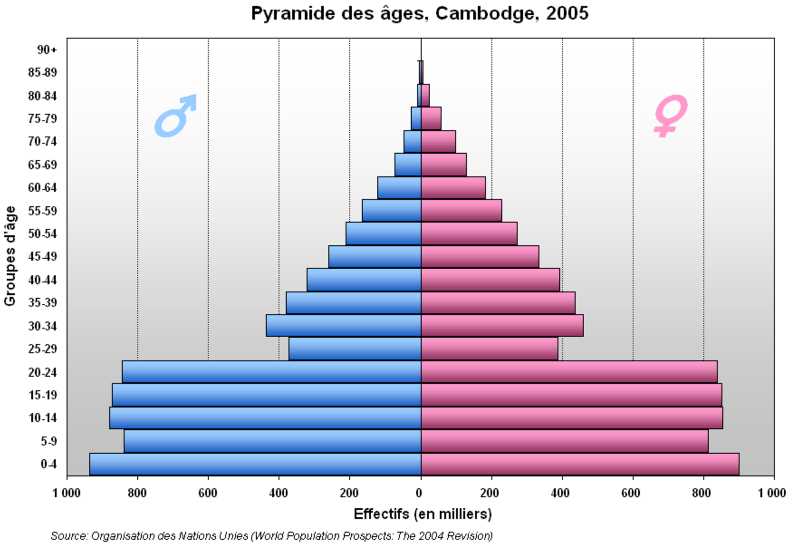

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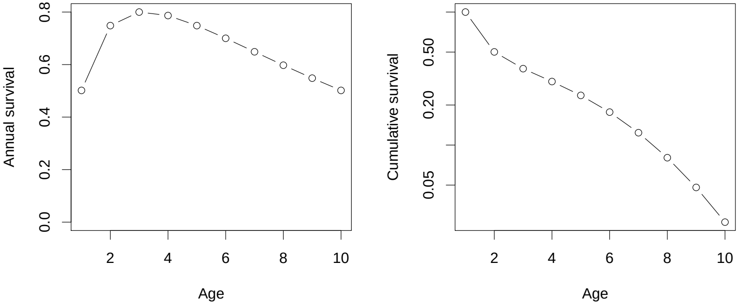
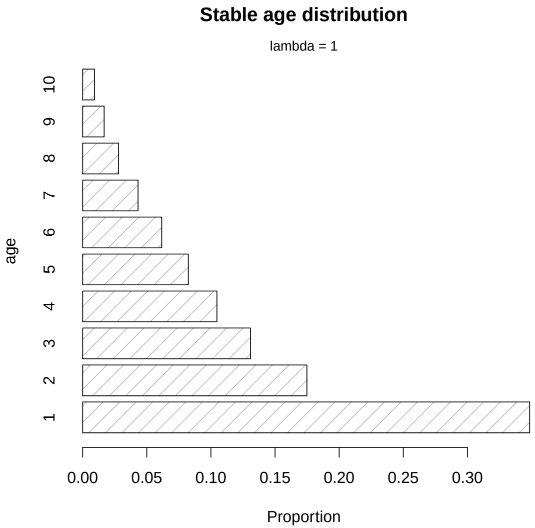
Changing survivorship



Age distributions



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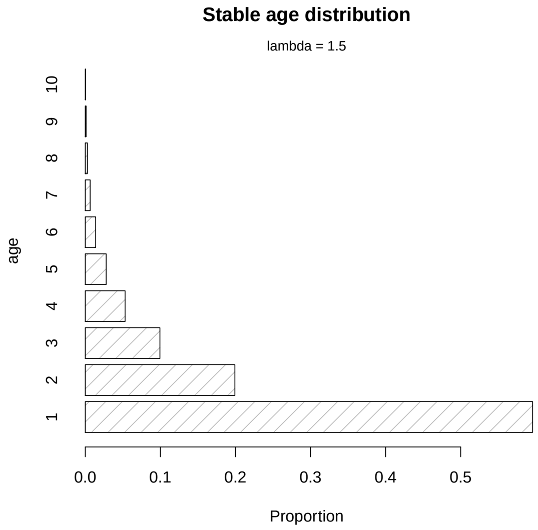

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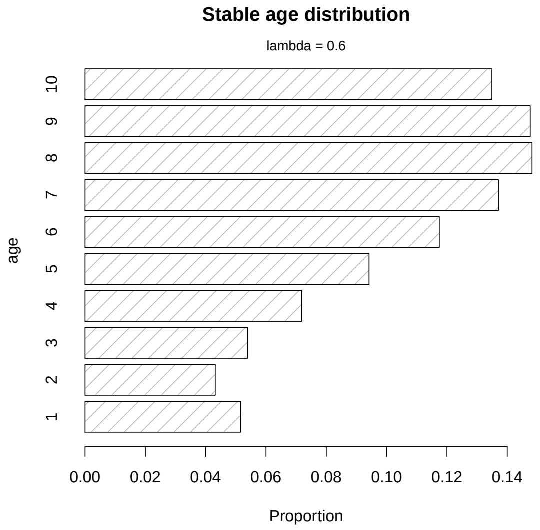

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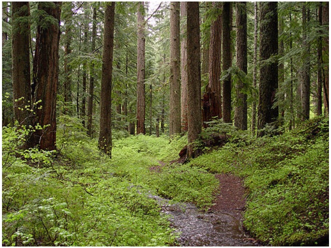
Outline

Forest example

- Forests have obvious population structure



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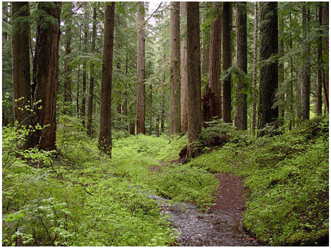

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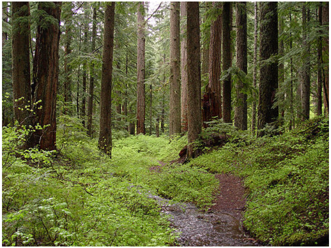

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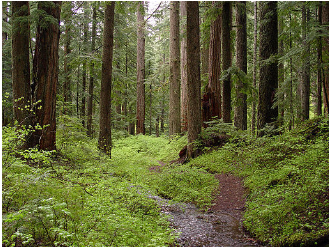

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Subsection 1

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