UNIT 1: Linear population models

Outline

Subsection 1

Dandelions

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Subsection 2

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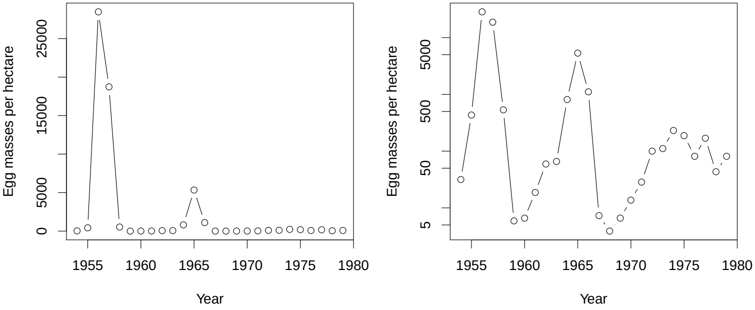

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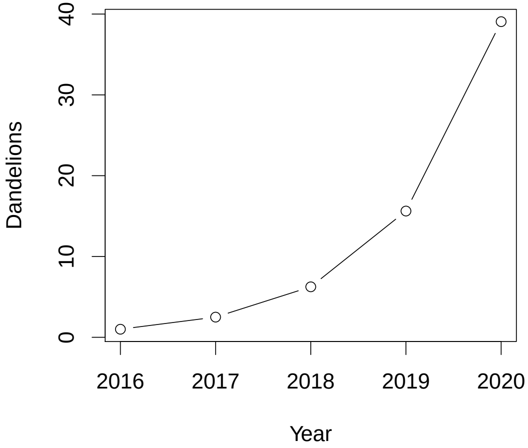

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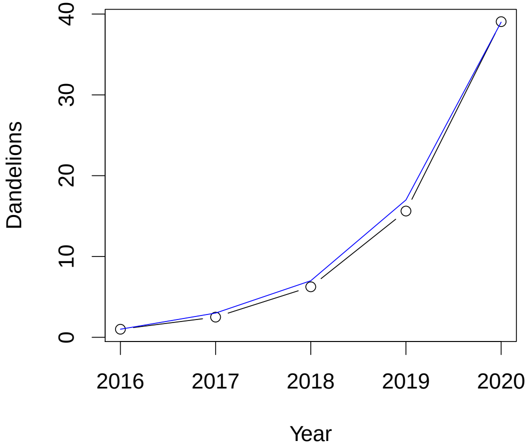

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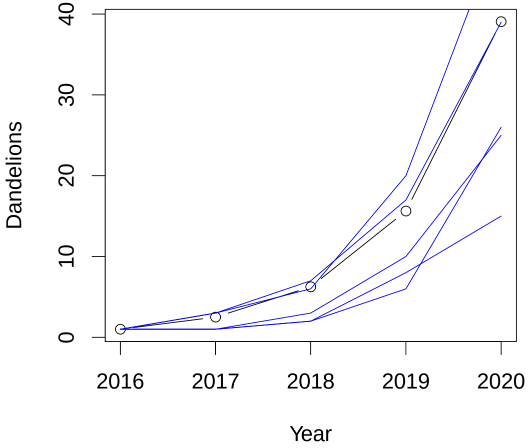

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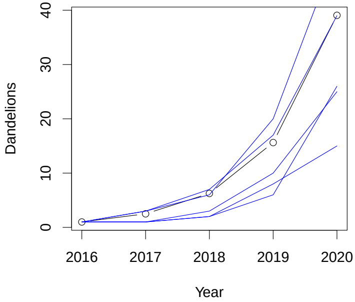

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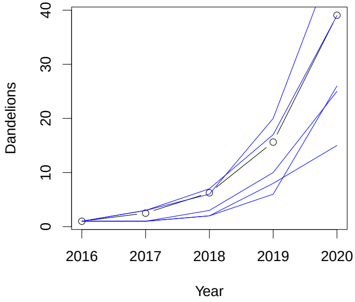

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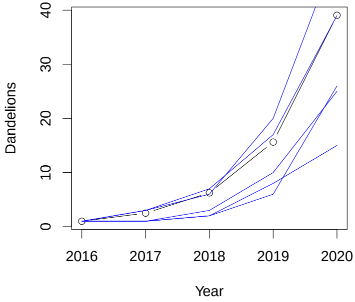

Subsection 3

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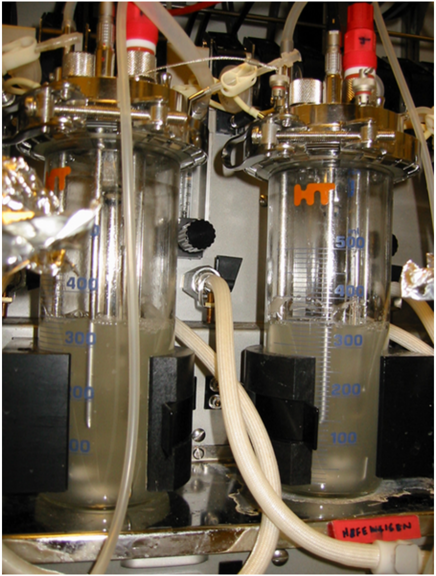

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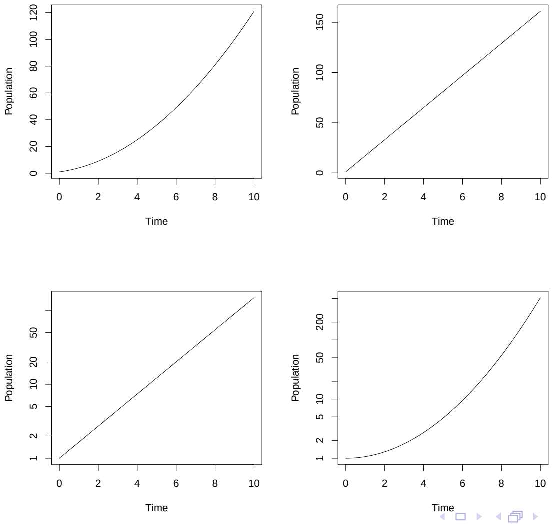

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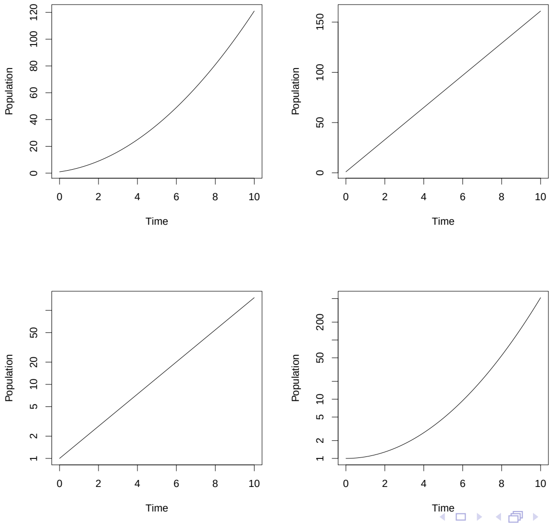

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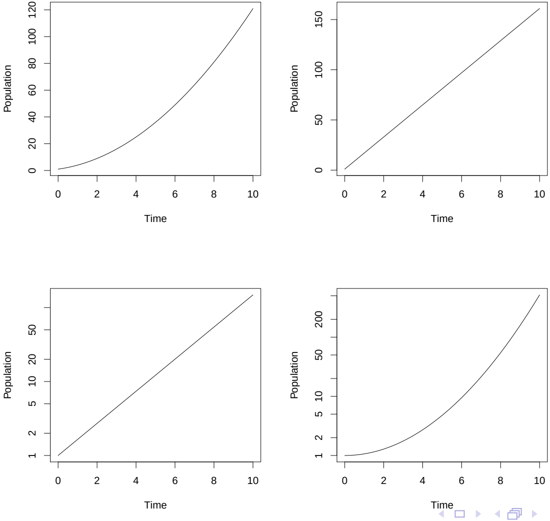

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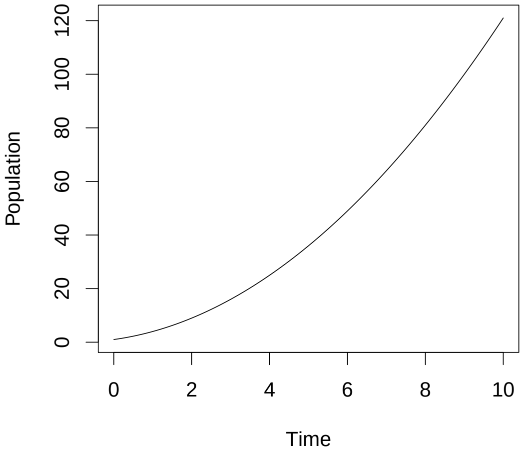
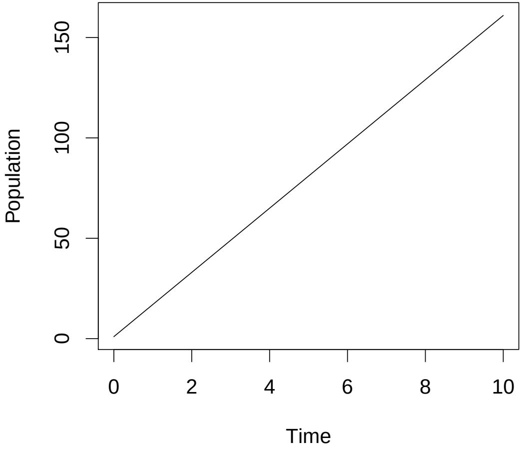


B



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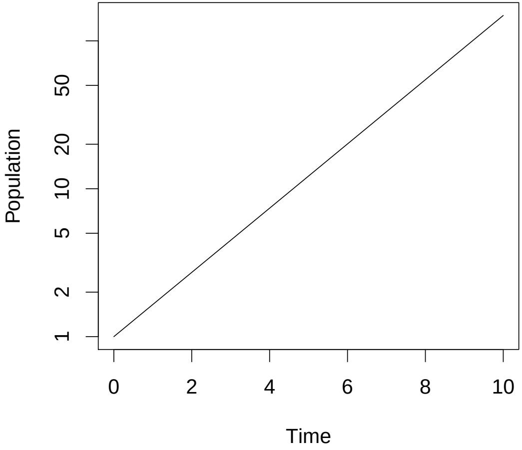

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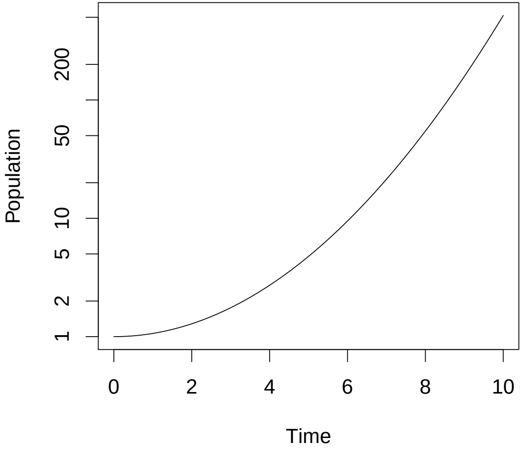

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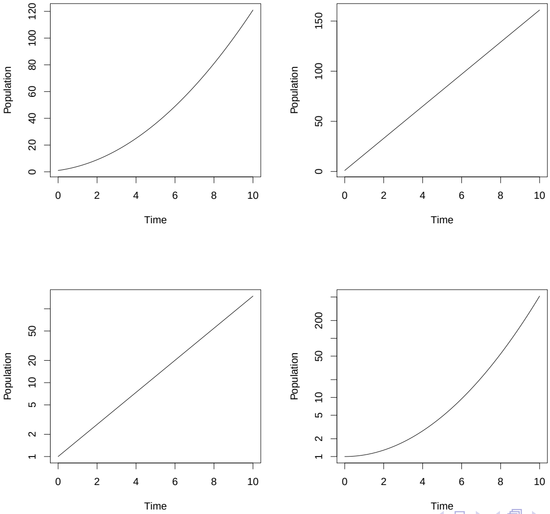

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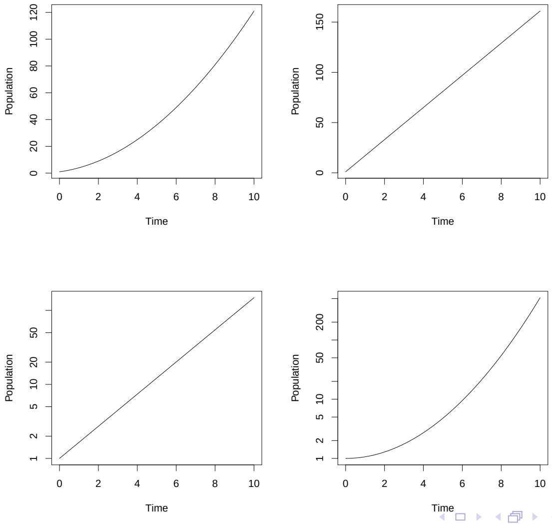

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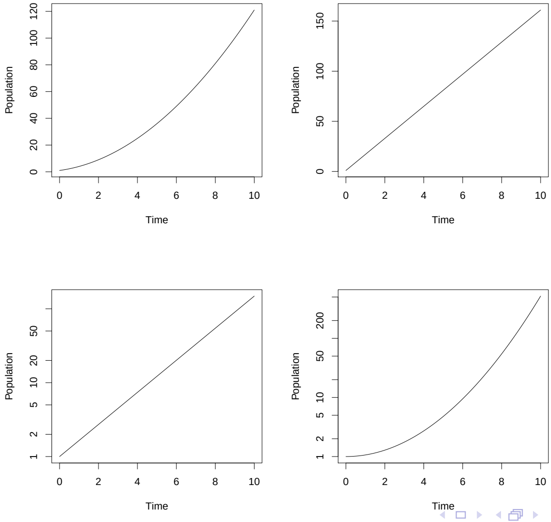

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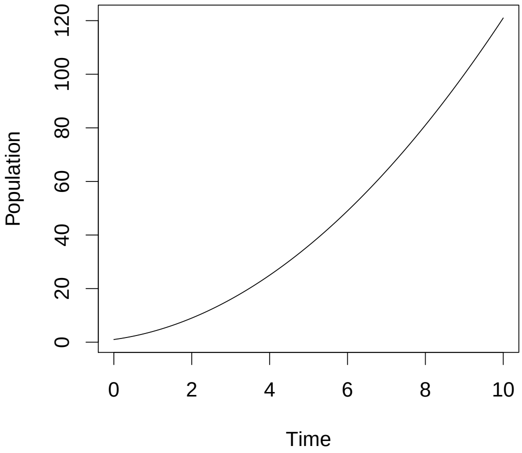

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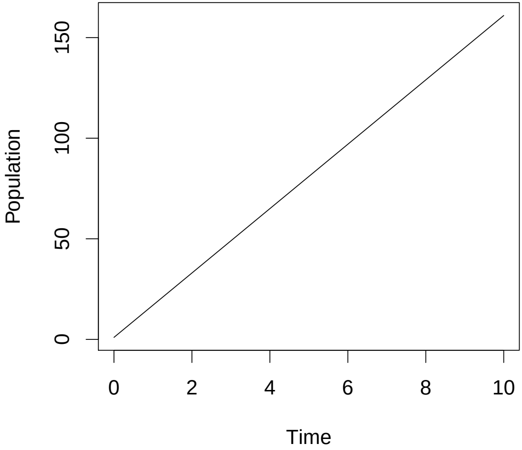

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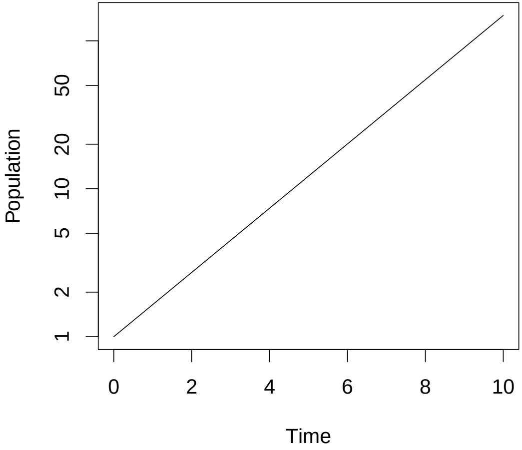

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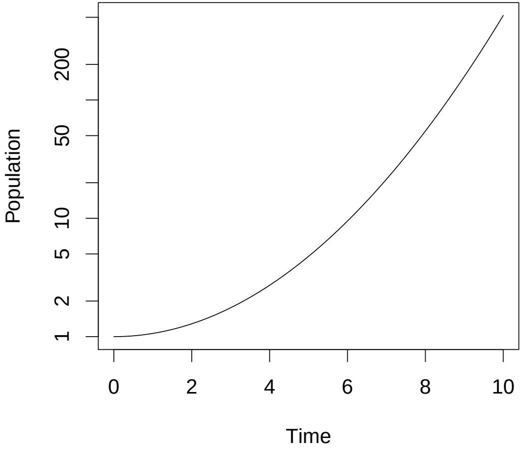

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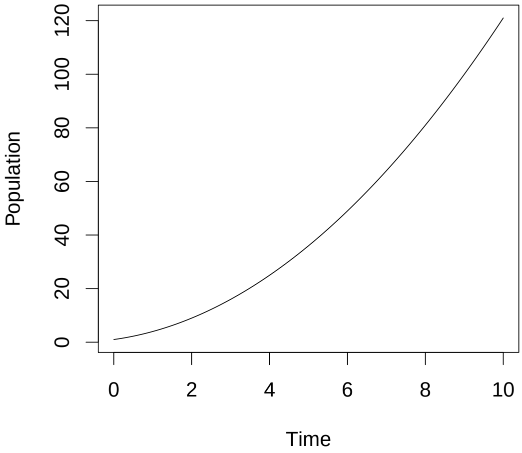
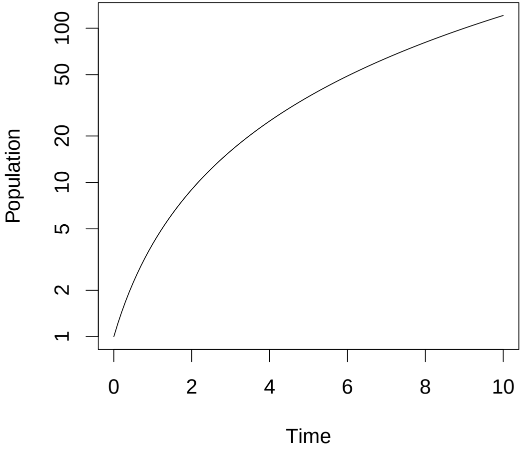


A on the log scale



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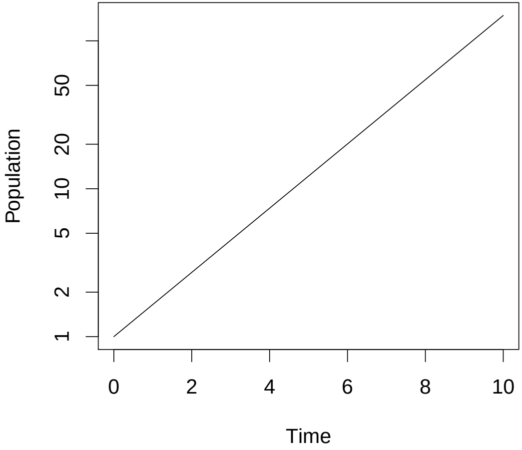
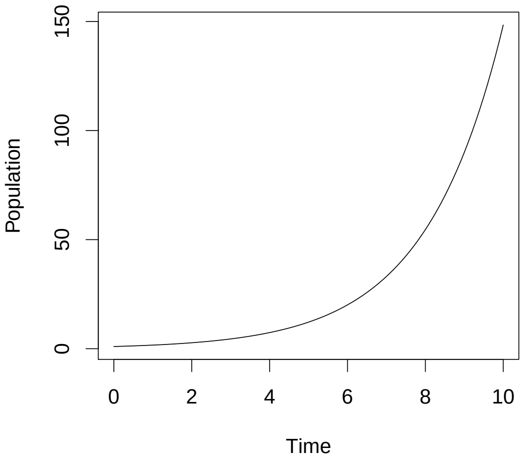


C on the linear scale



Types of growth

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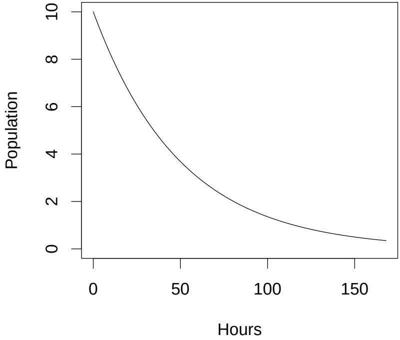

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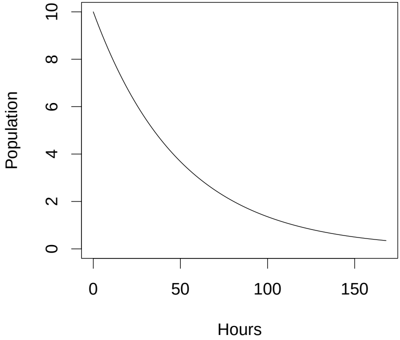

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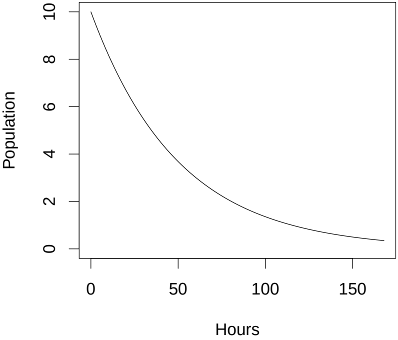

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Subsection 1

Log and linear scales

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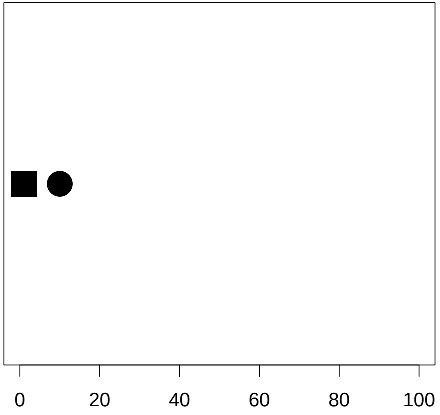

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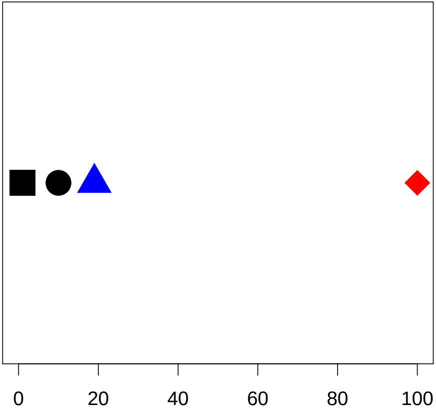

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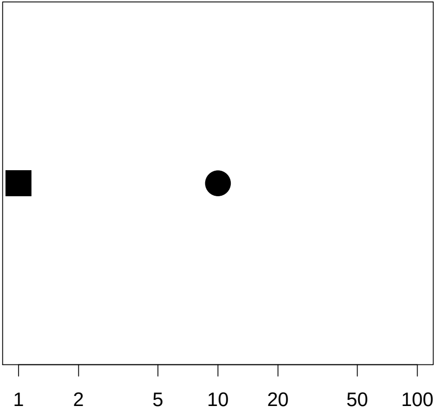

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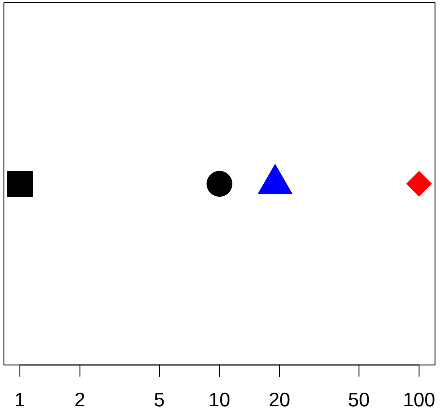

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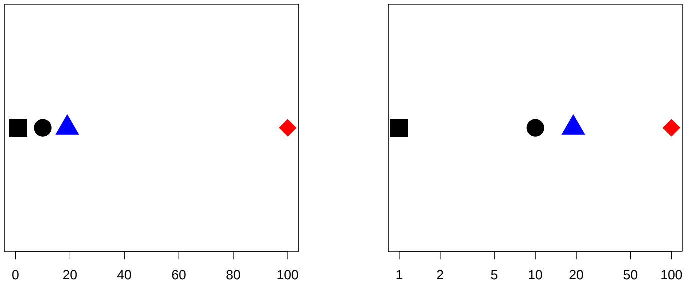
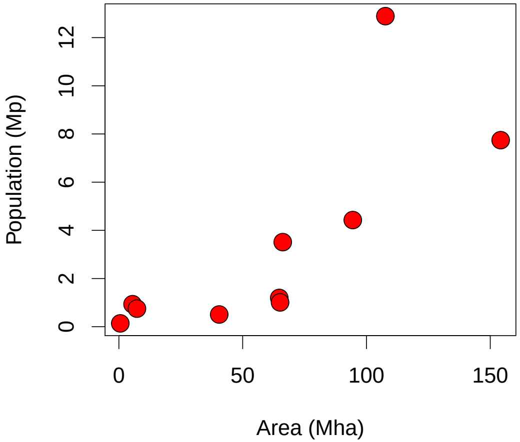


There is only one log scale; it doesn't matter which base you use!

Canadian provinces



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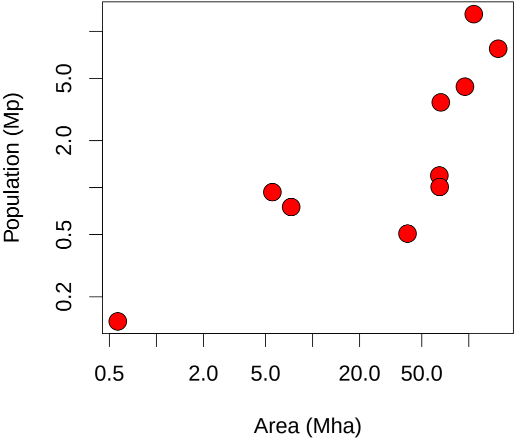

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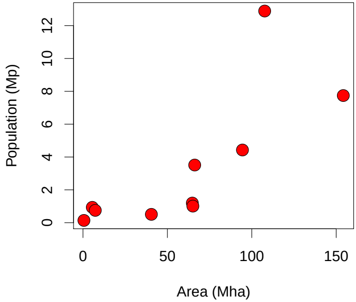
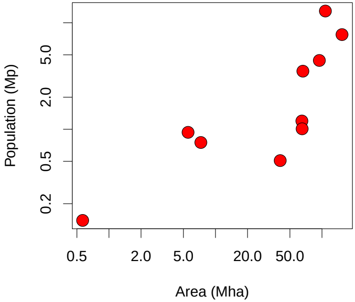
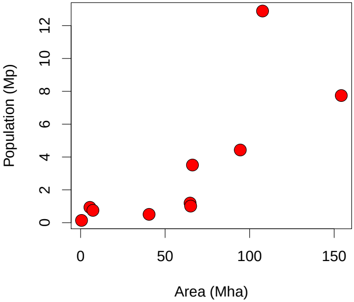
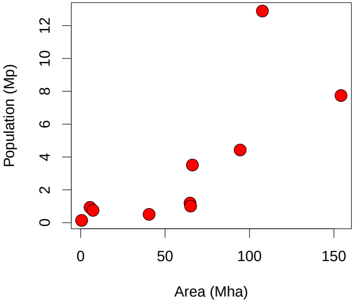




Canadian provinces plus Canada?





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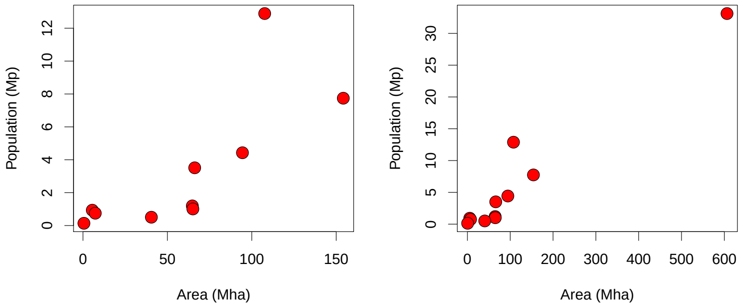

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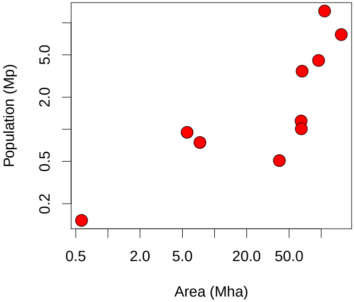

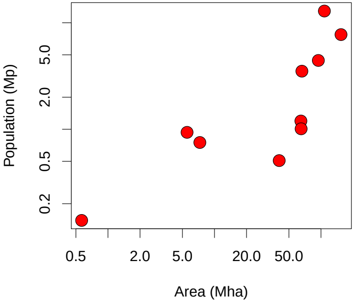

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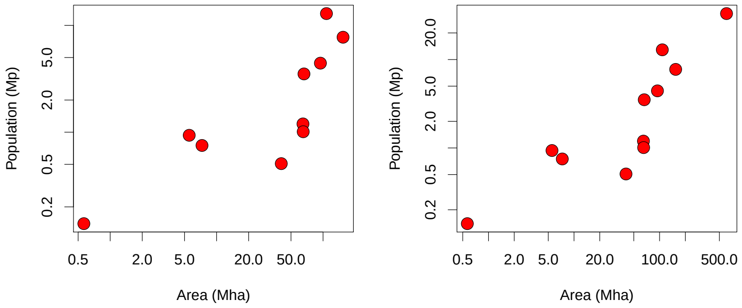

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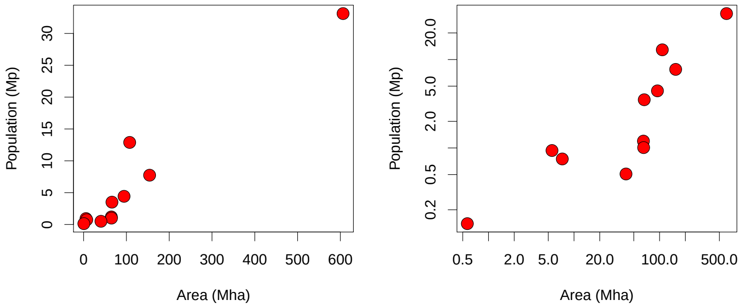

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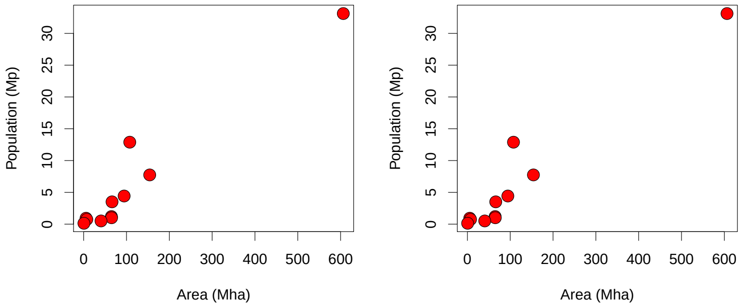

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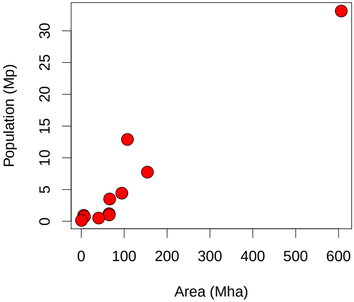

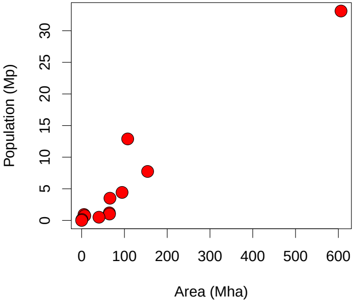

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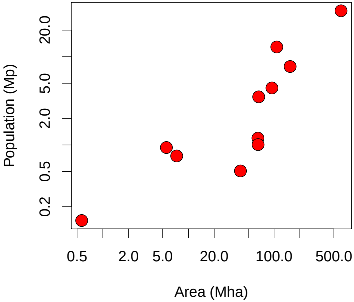

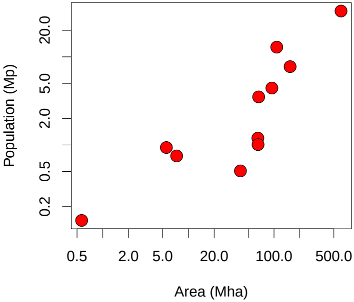

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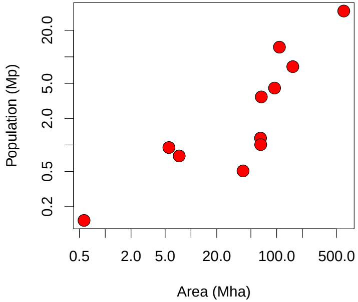

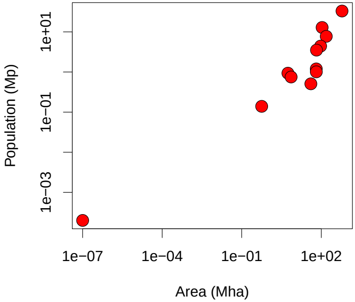

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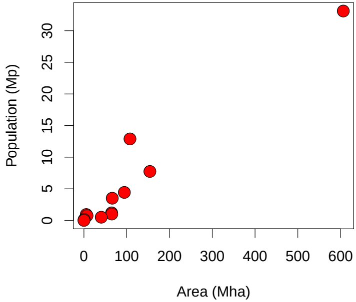

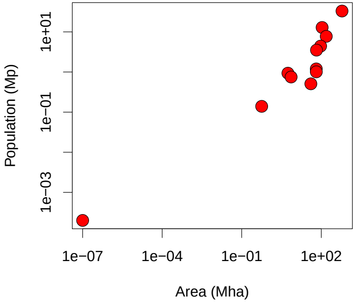

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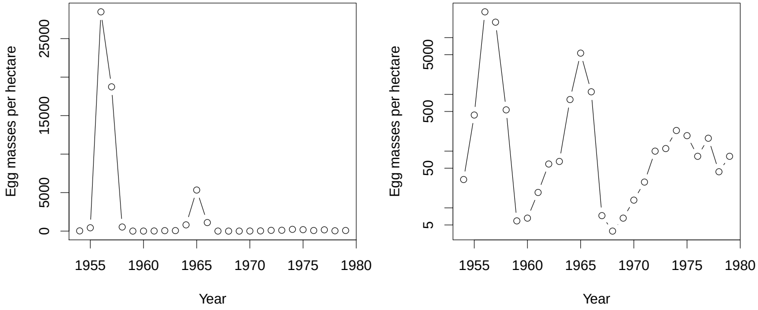
Gypsy-moth example



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Subsection 2

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Outline

Subsection 1

Dynamical models

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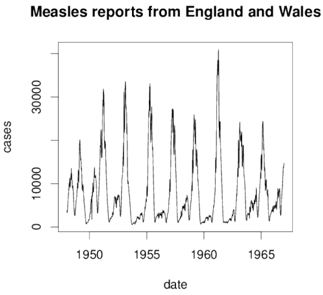

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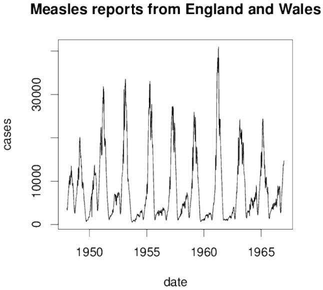

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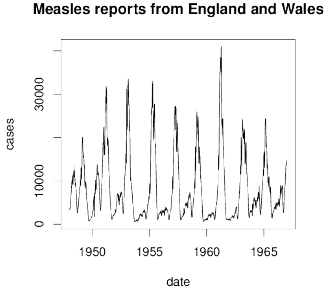

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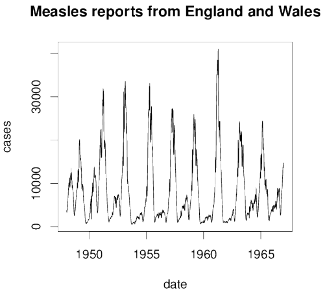

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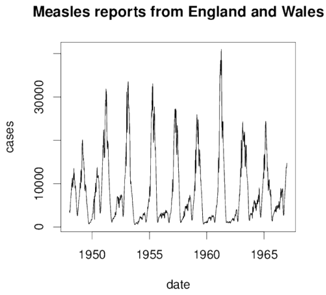

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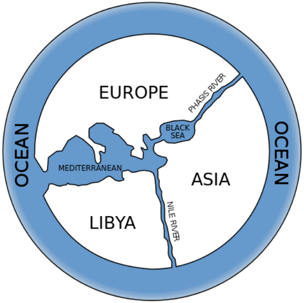

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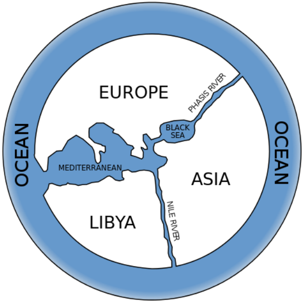

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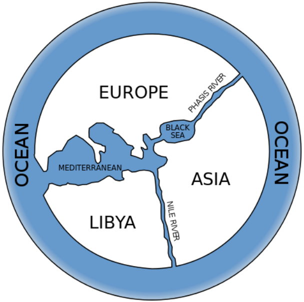

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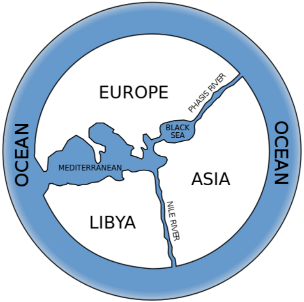

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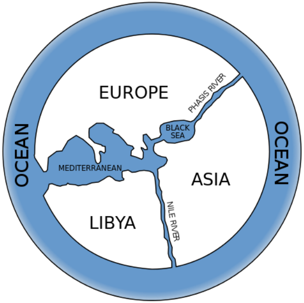

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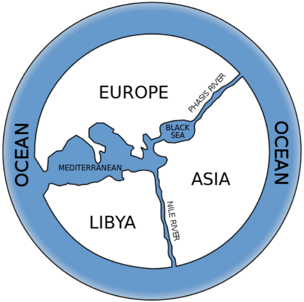

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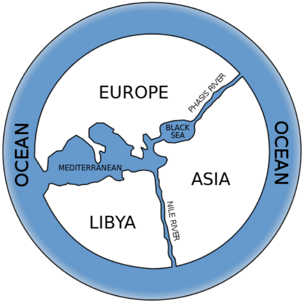

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Subsection 2

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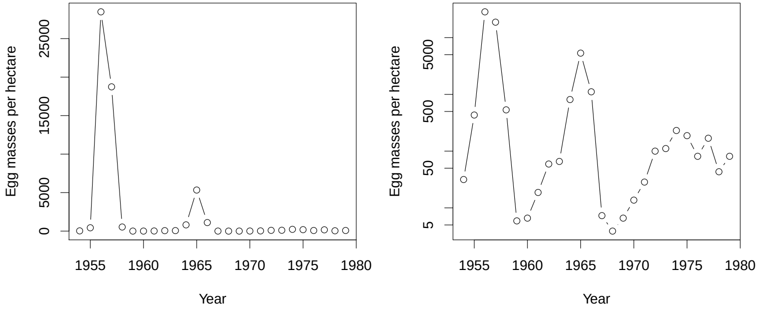

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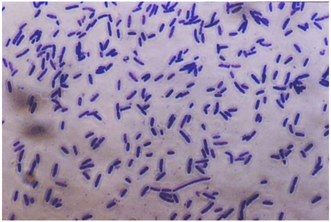

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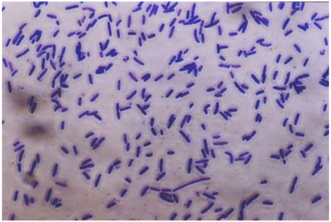

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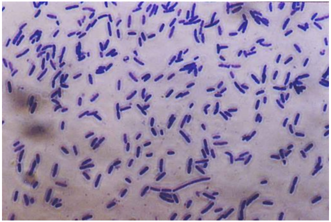

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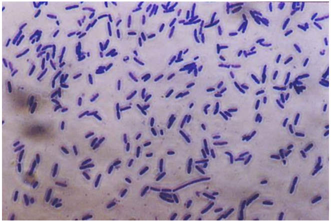

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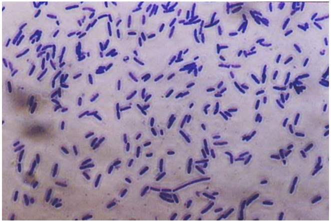

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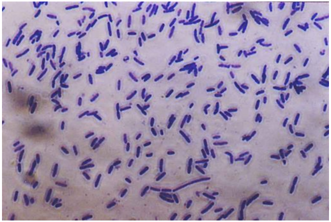

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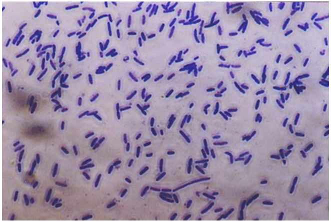

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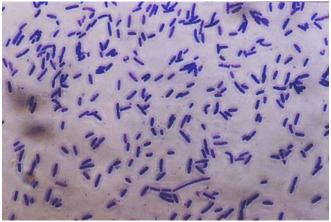

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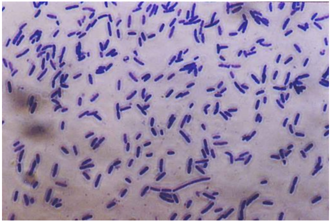

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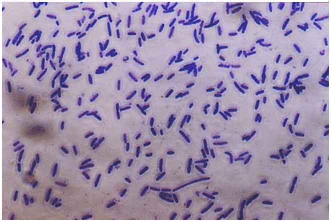

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Subsection 3

A simple discrete-time model

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Subsection 4

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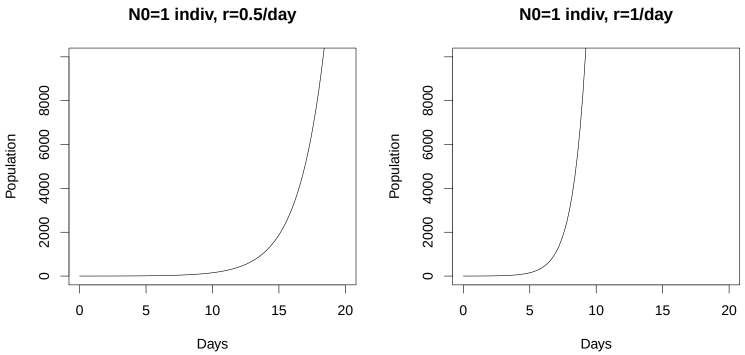

N0=1 indiv, r=0.5/day
Population
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4000
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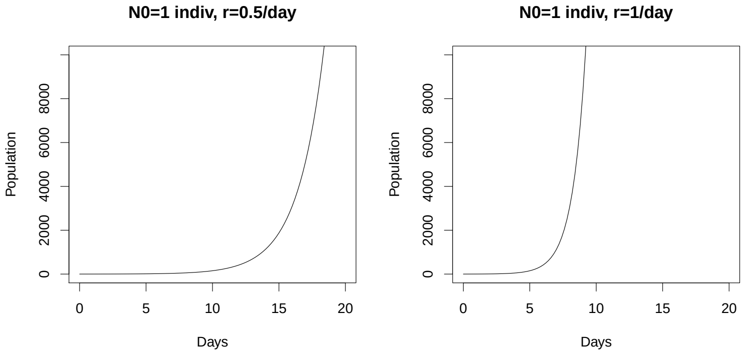

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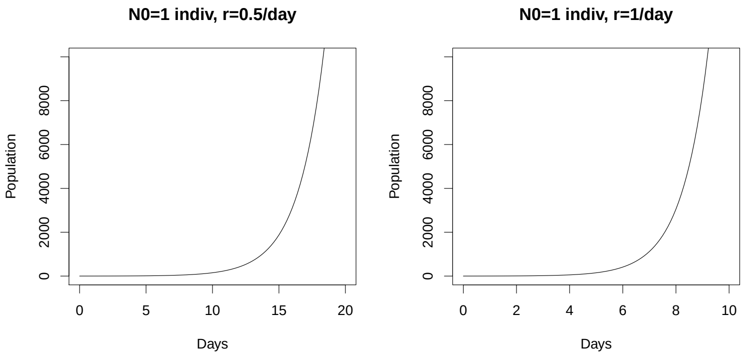

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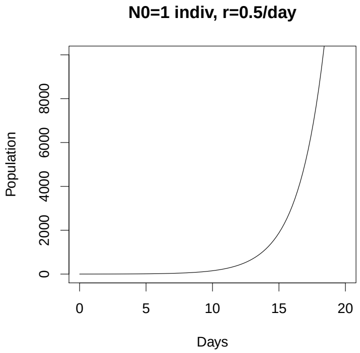

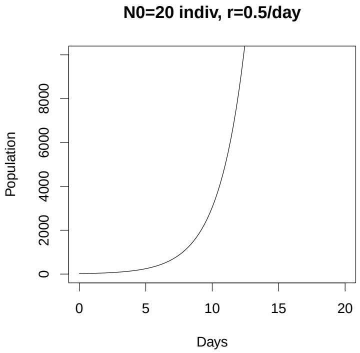

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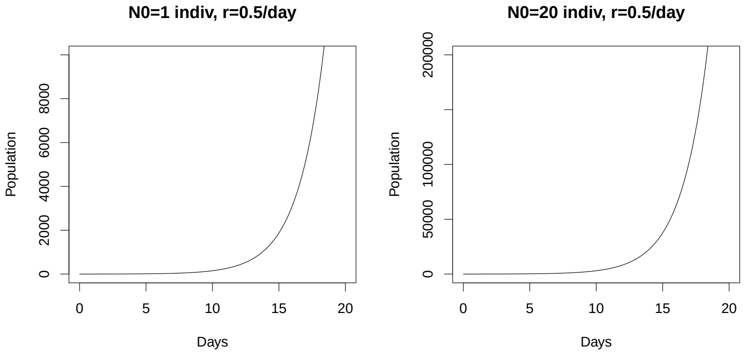

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Subsection 2

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