UNIT 2 Non-linear population models

Outline

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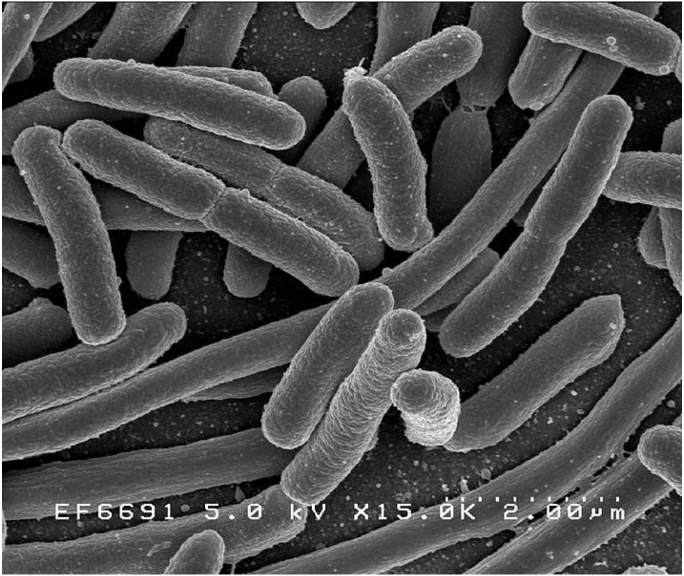

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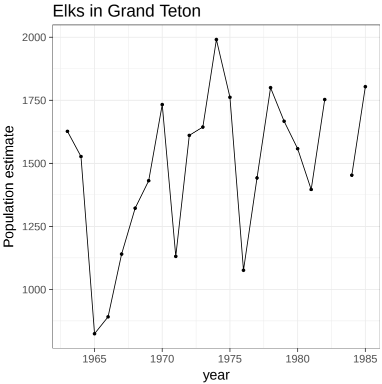
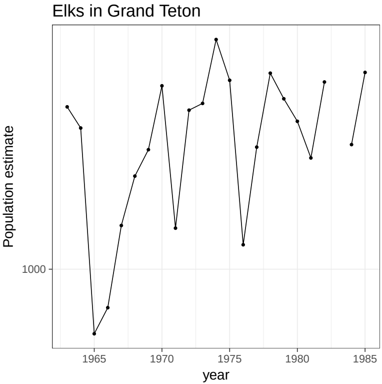
Subsection 1

Population Examples

Elk


Elks in Grand Teton
2000
1750
1500
1250
1000
Population estimate
1965
1970
1975
1980
1985
year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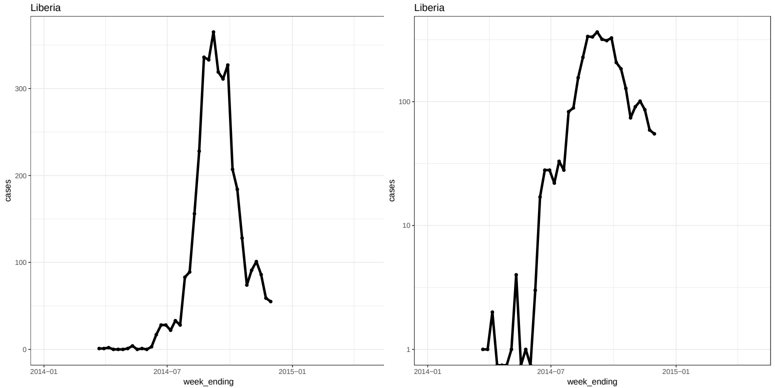
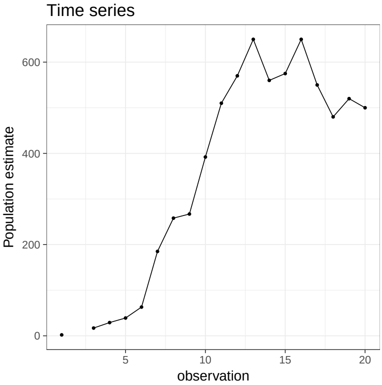
Ebola

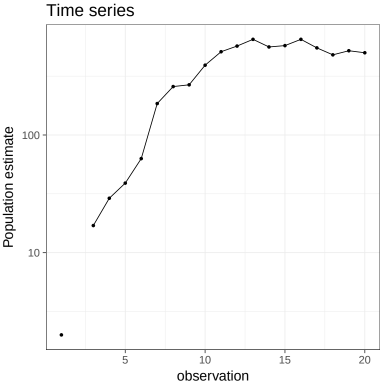


Liberia
cases
300
200
100
0
2014−01
2014−07
2015−01
week_ending
Liberia
cases
100
10
1
2014−01
2014−07
2015−01
week_ending

Paramecia



Time series
Population estimate
600
400
200
0
5
10
15
20
observation



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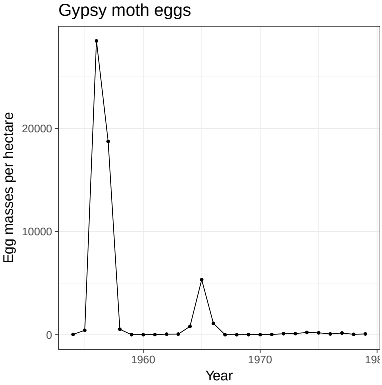
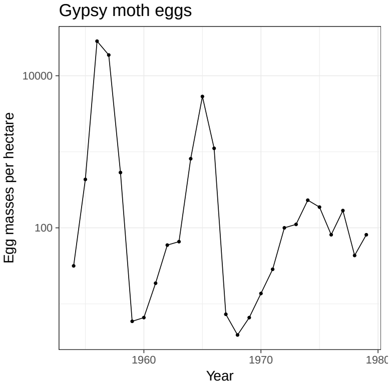
Gypsy moths



Gypsy moth eggs
Egg masses per hectare
20000
10000
0
1960
1970
198
Year



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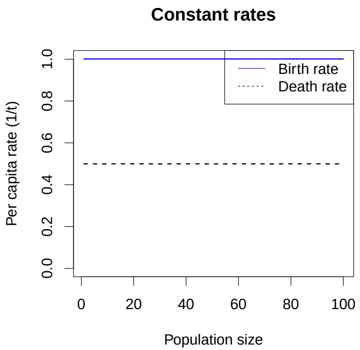

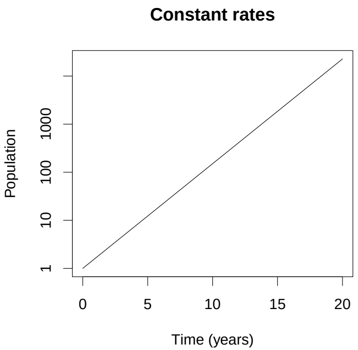

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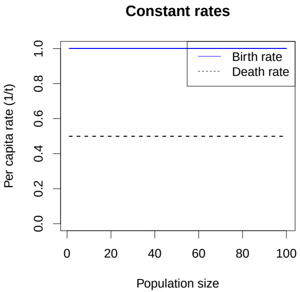

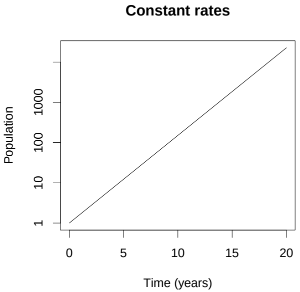

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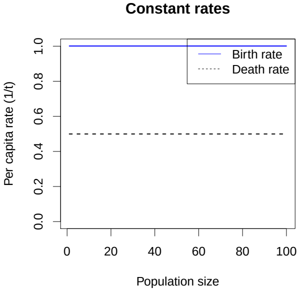

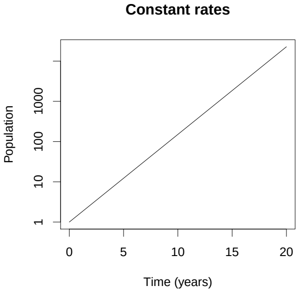

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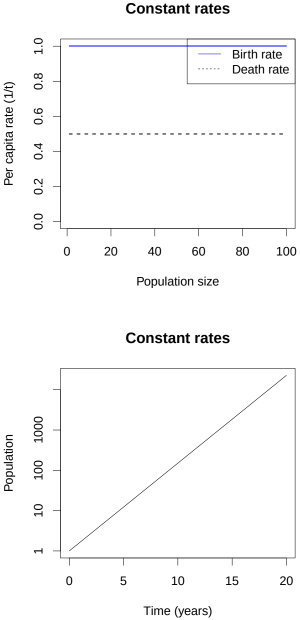

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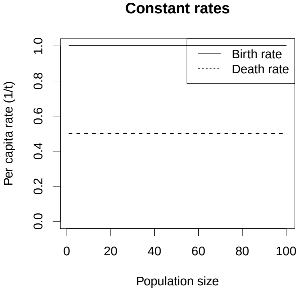

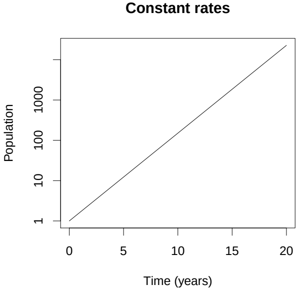

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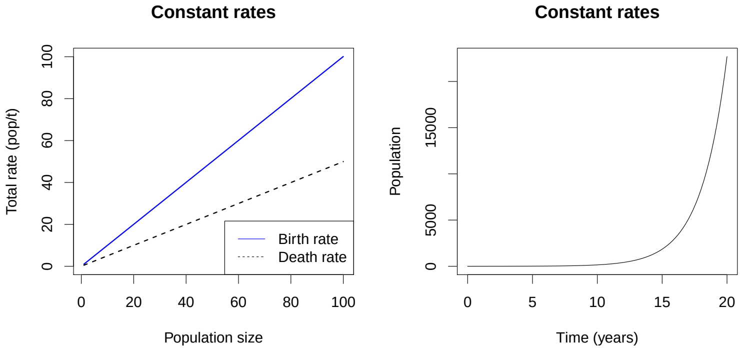

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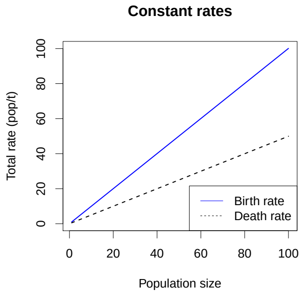

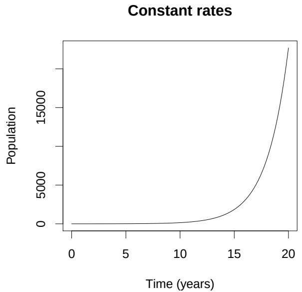

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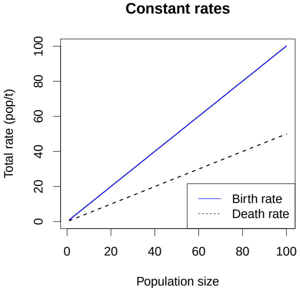

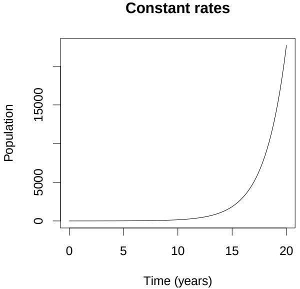

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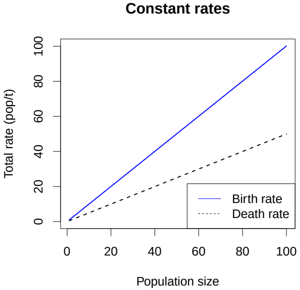

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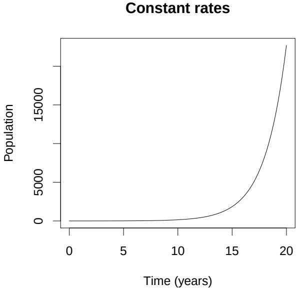

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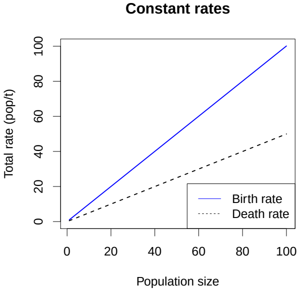

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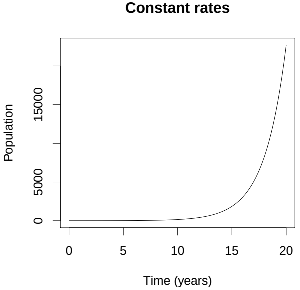

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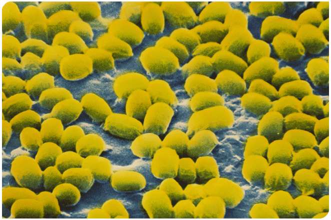

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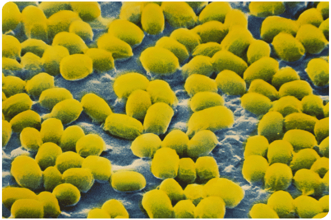

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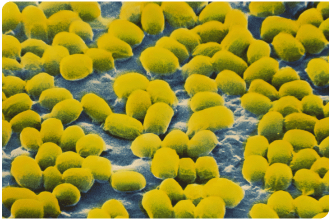

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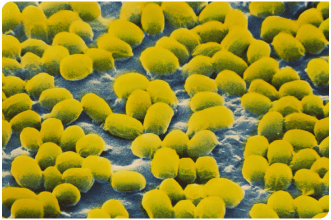

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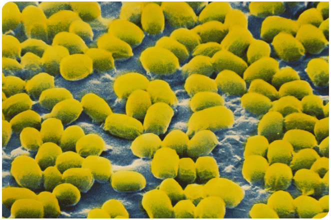

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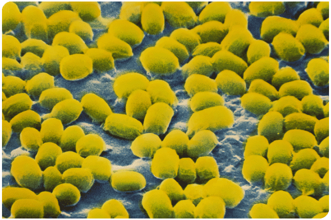

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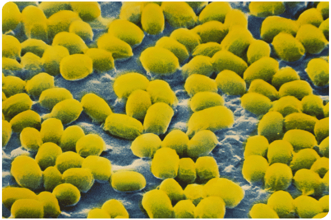

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Subsection 1

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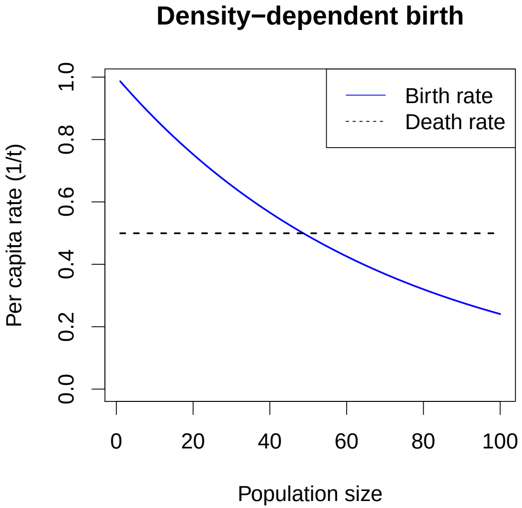

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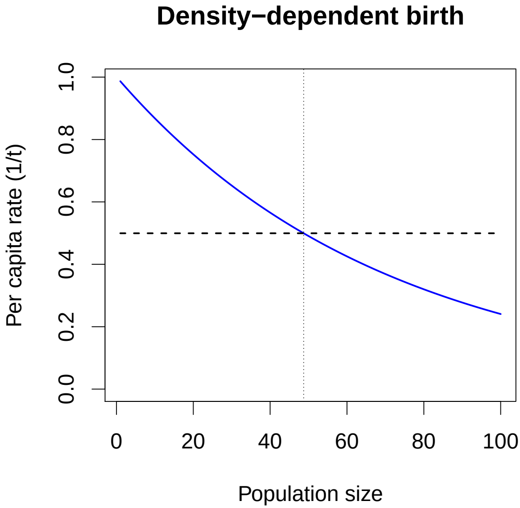

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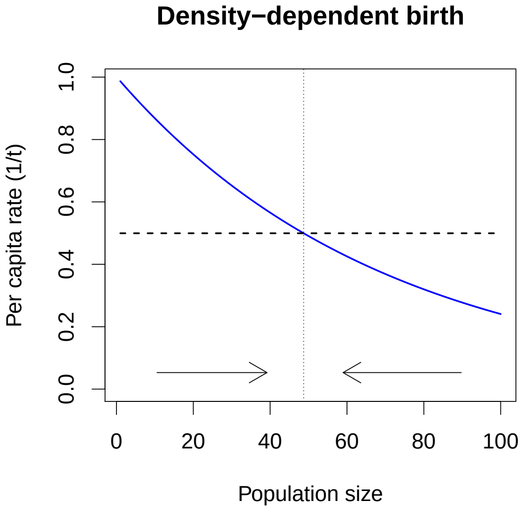

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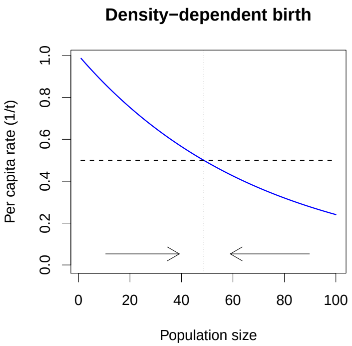

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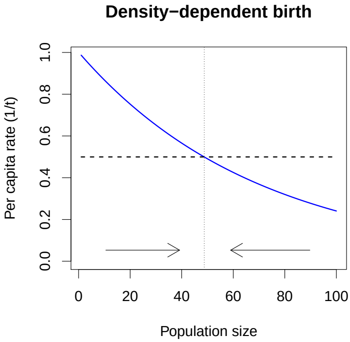

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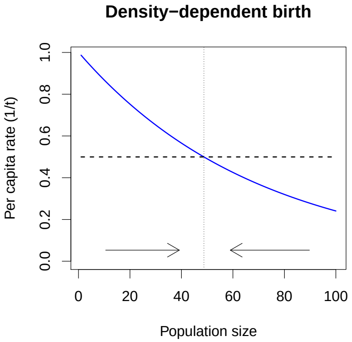

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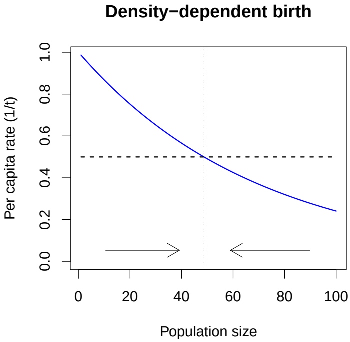

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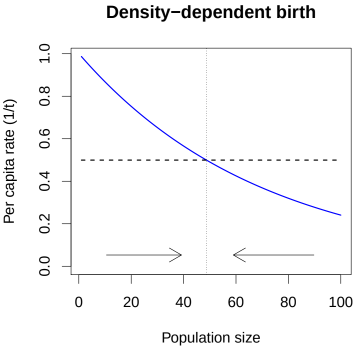

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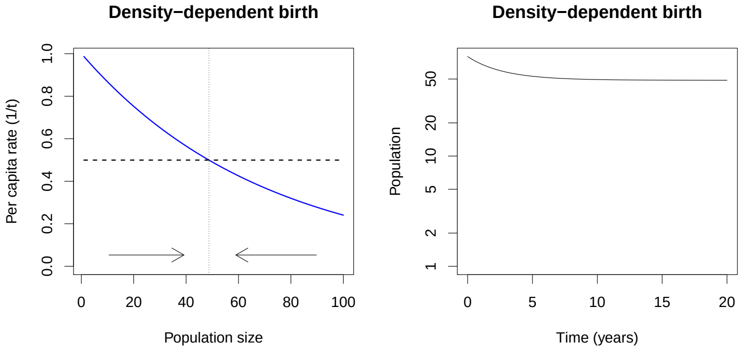

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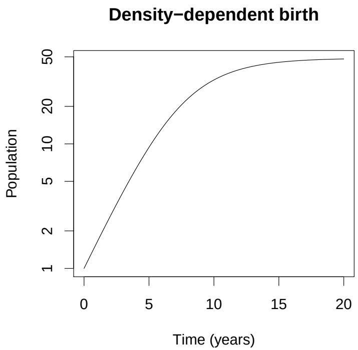

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Subsection 2

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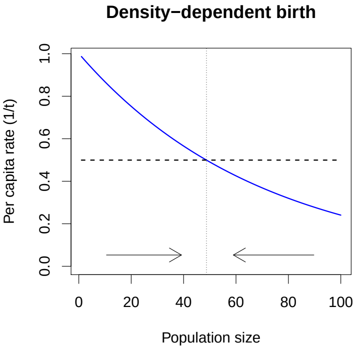

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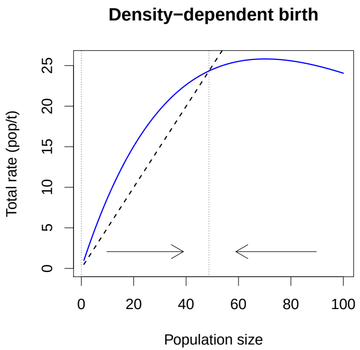

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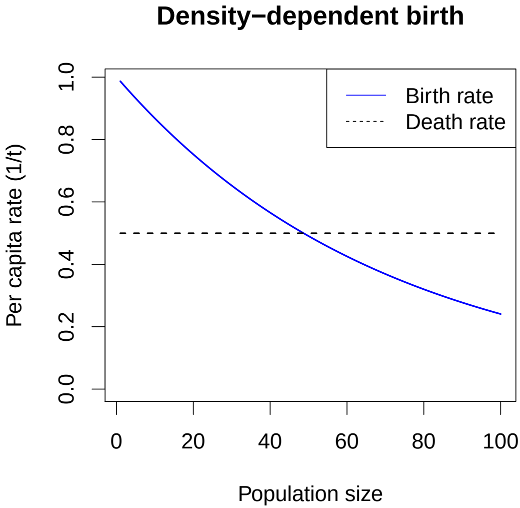

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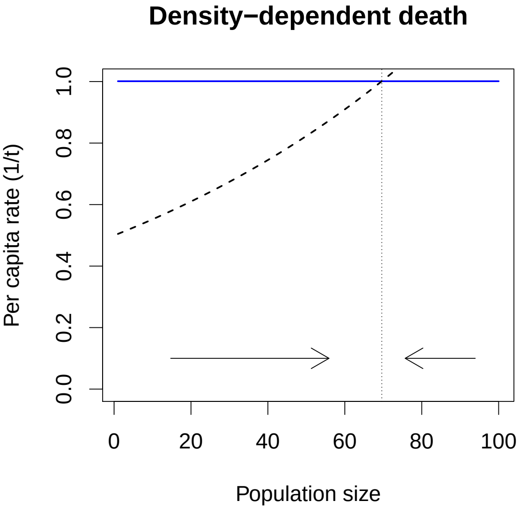

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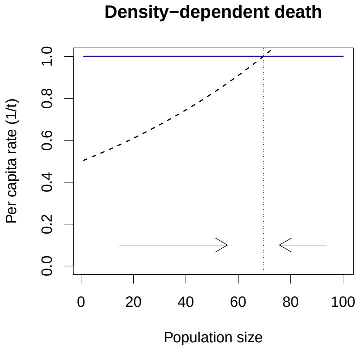

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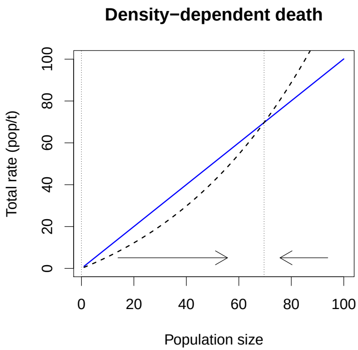

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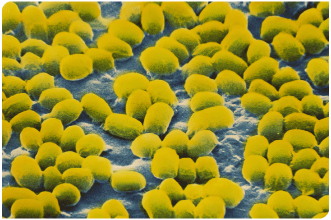

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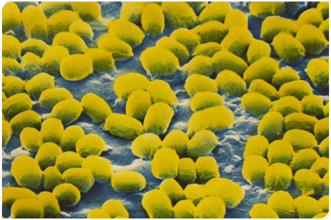

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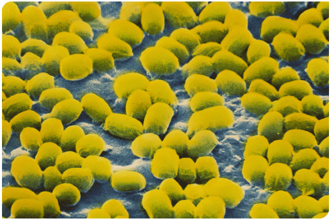

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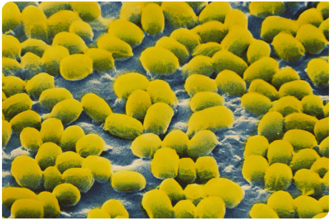

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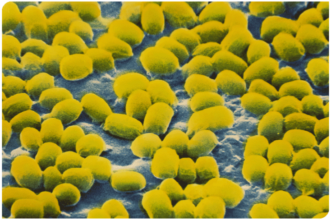

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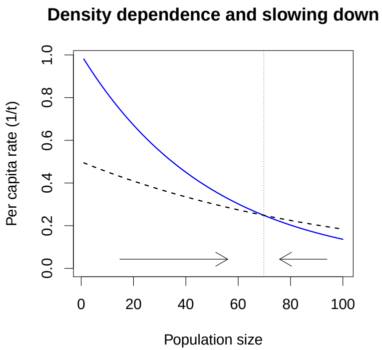

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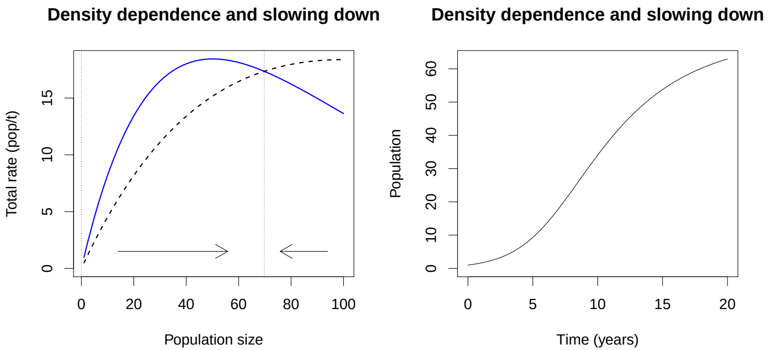

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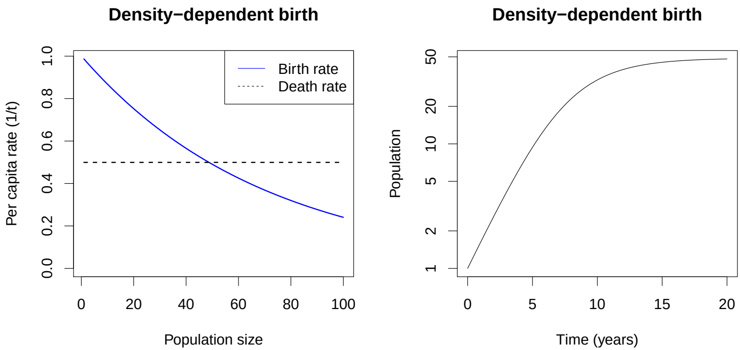

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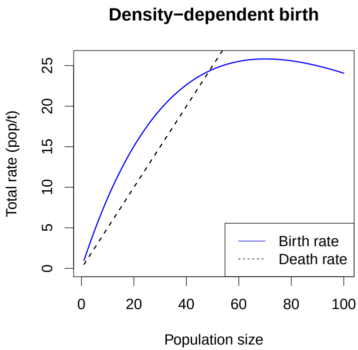

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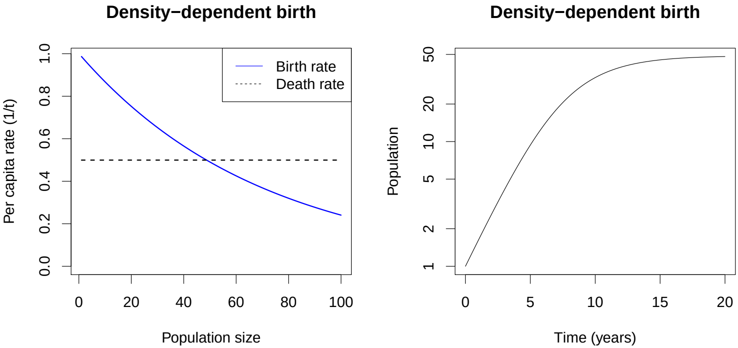

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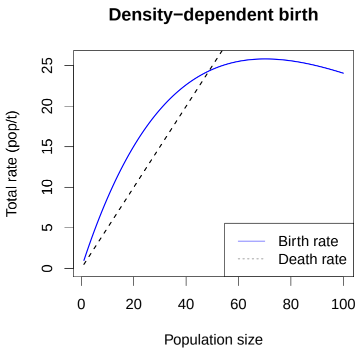

Subsection 3

Equilibria and time scales

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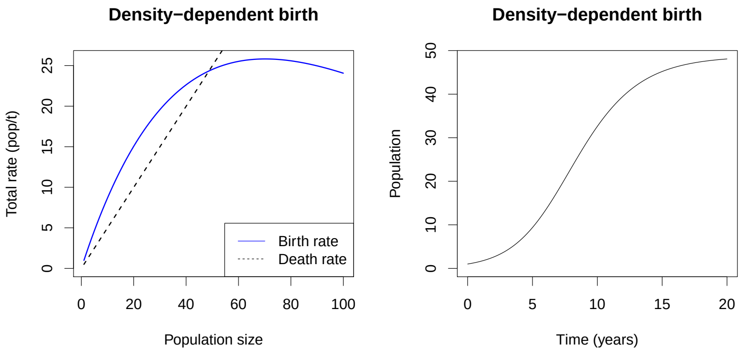

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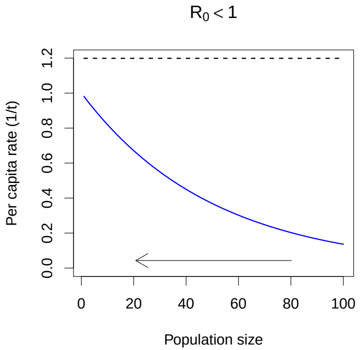

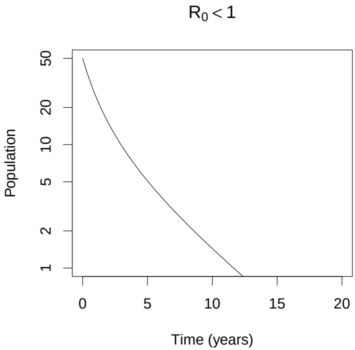

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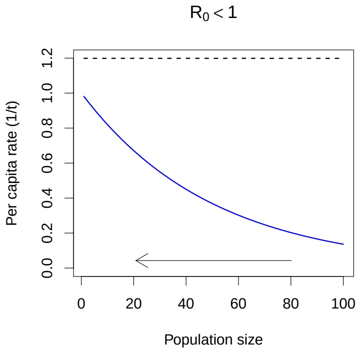

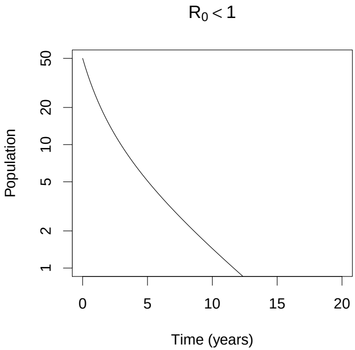

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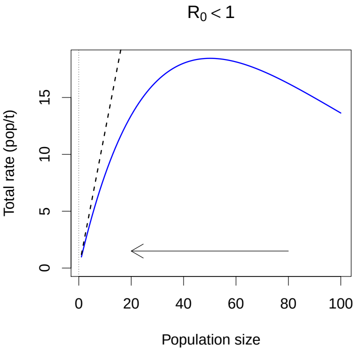

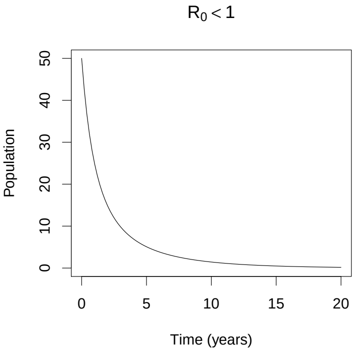

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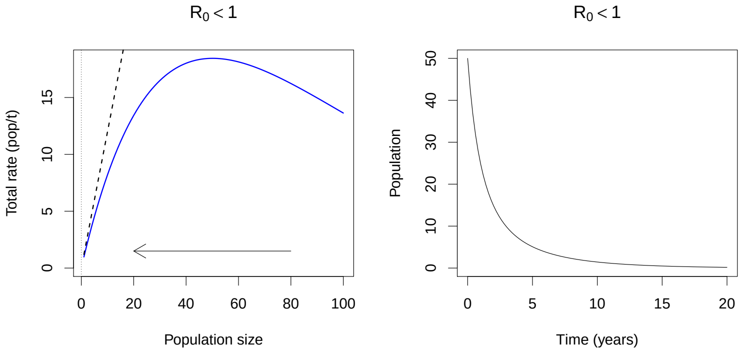

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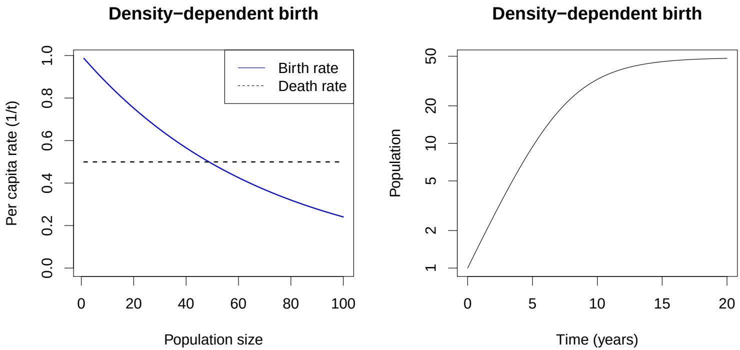

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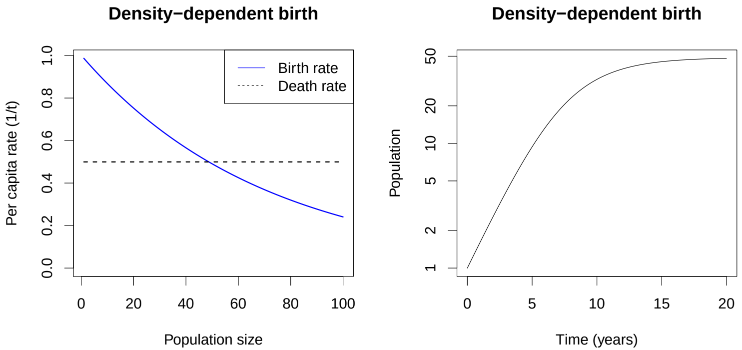

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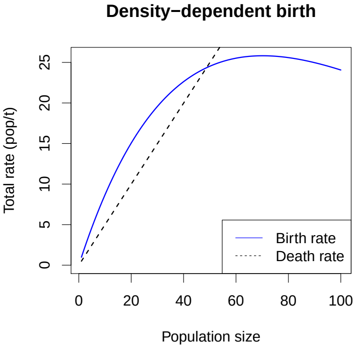

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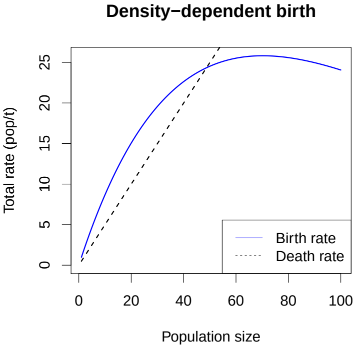

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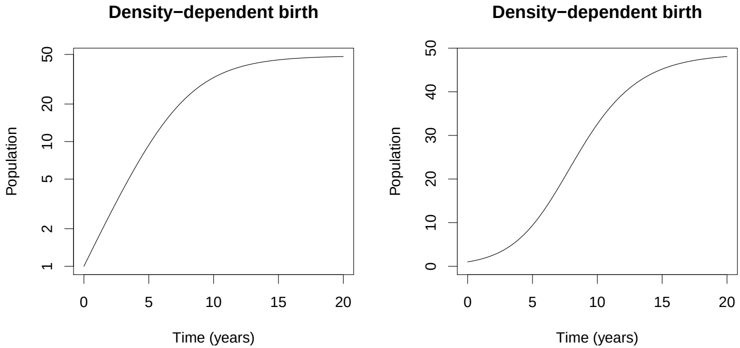

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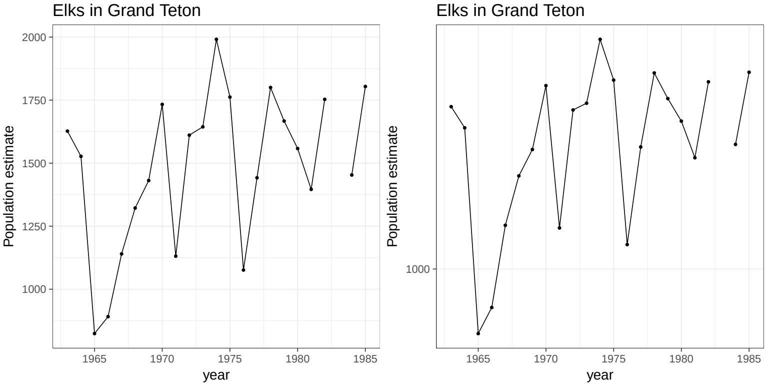

Elks in Grand Teton
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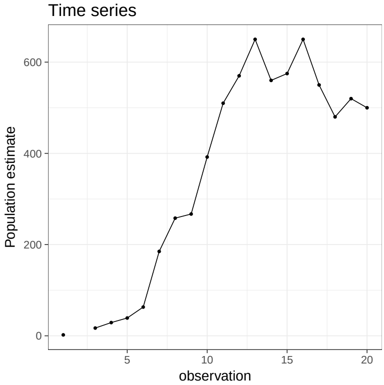
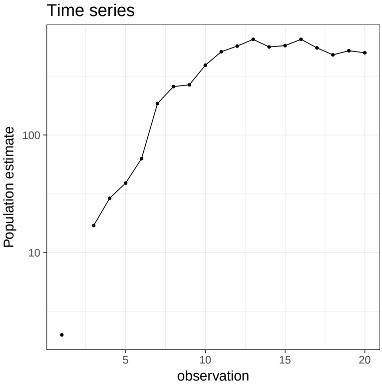


Time series
Population estimate
600
400
200
0
5
10
15
20
observation



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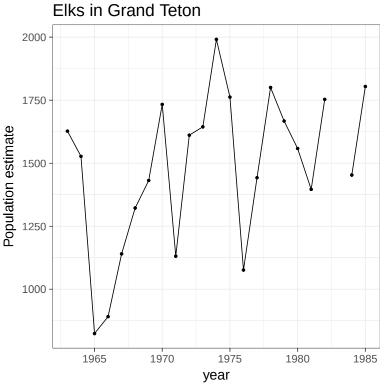
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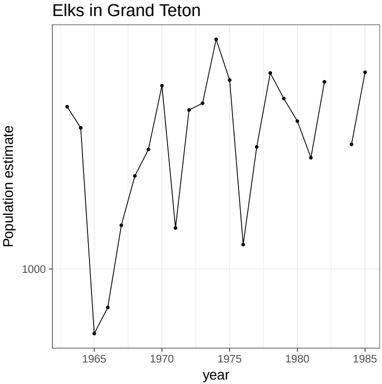
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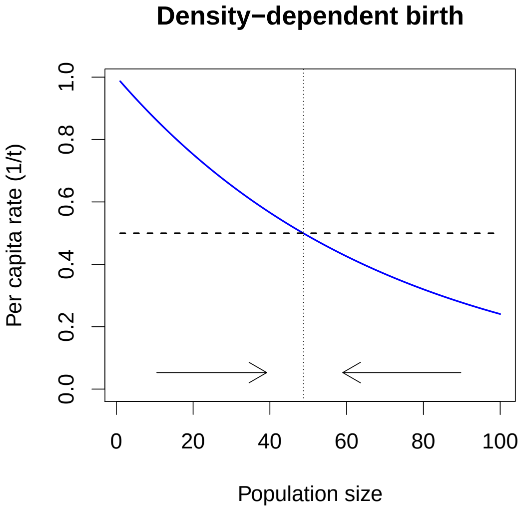

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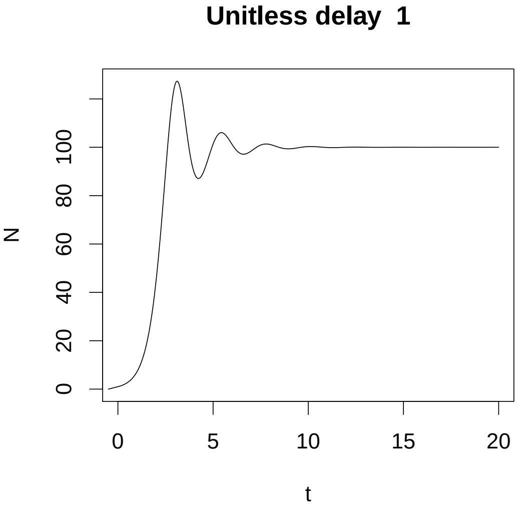

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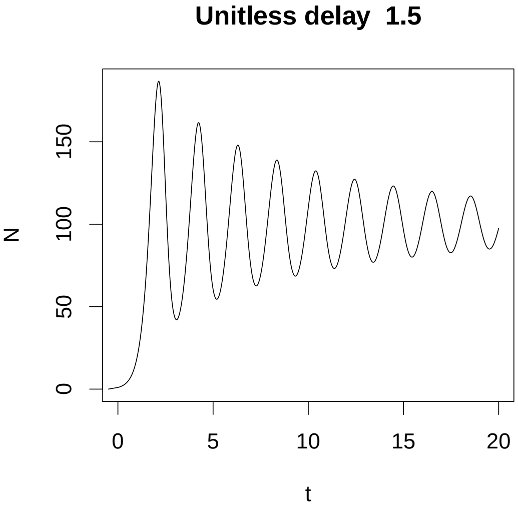

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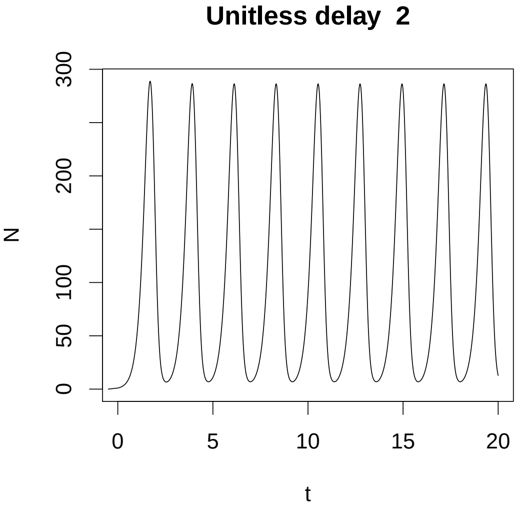

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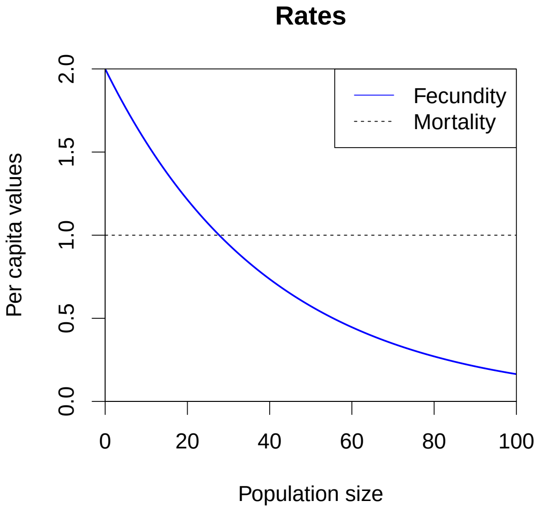

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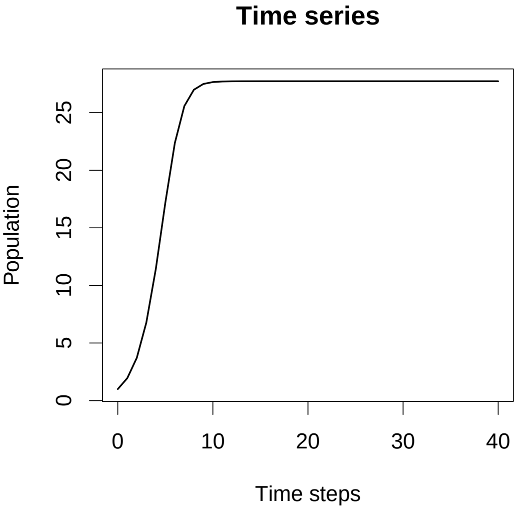
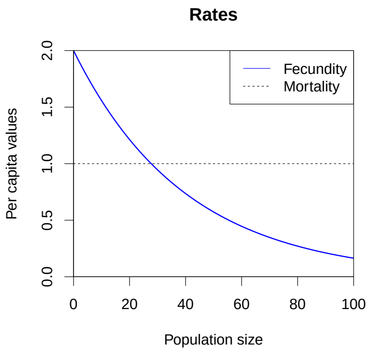
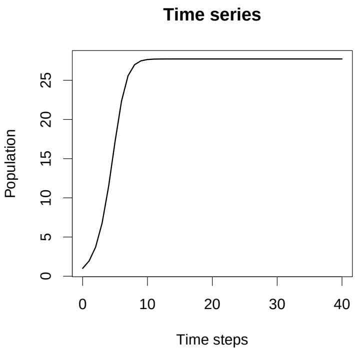


We expect simple dynamics





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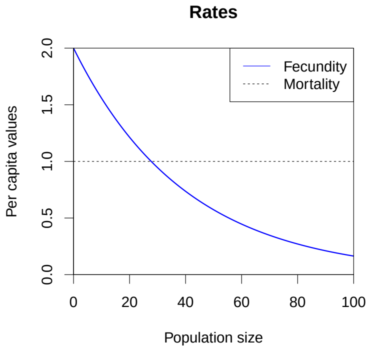

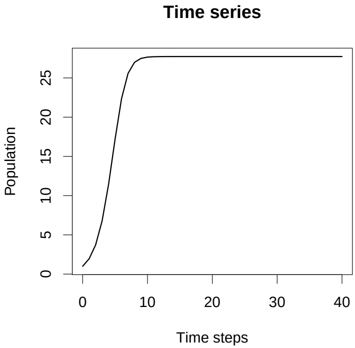

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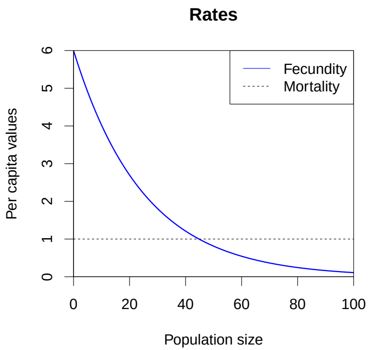

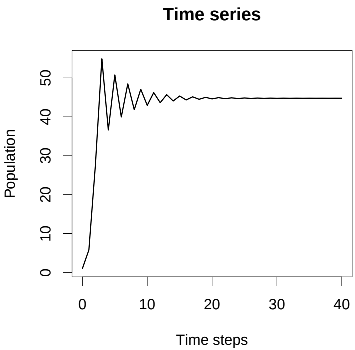

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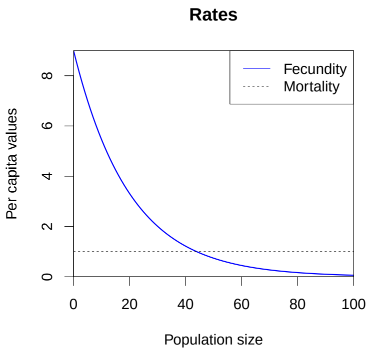
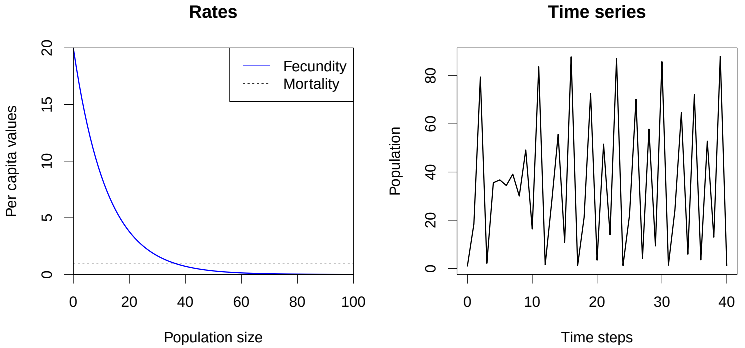


Lots of other behaviours



Subsection 3

Interpreting complex behaviour

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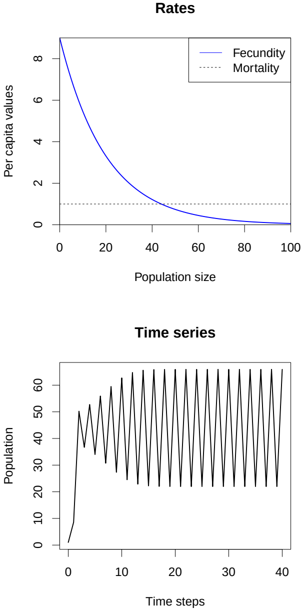

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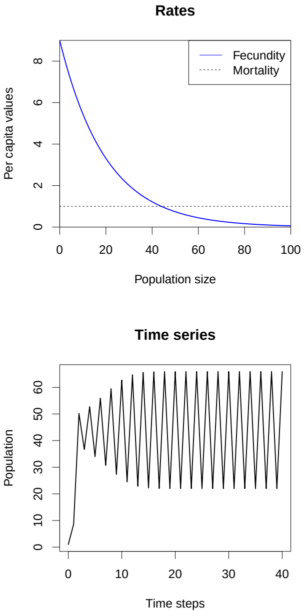

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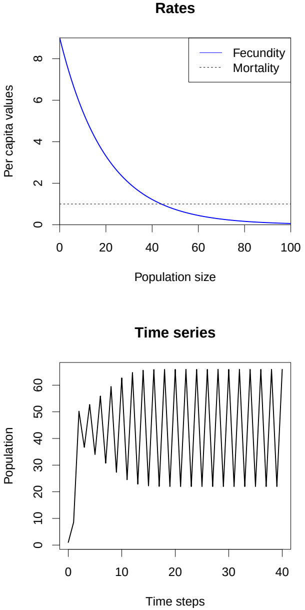

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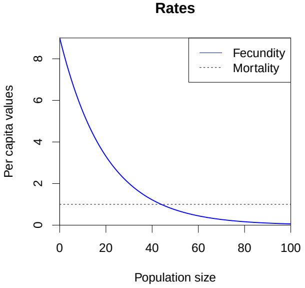

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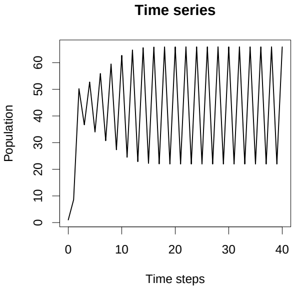

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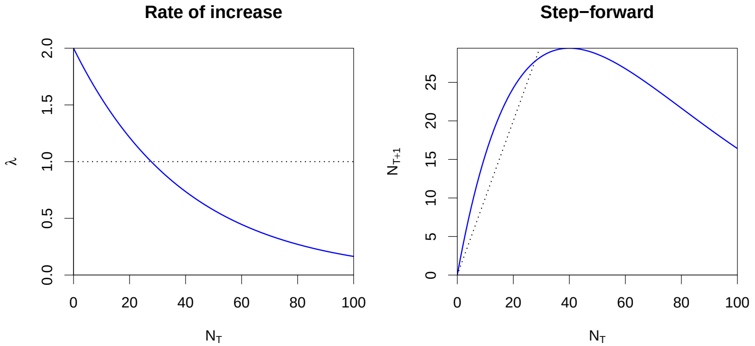

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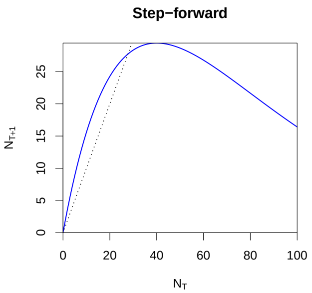

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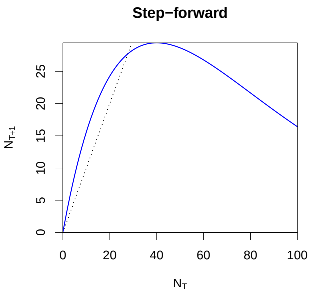

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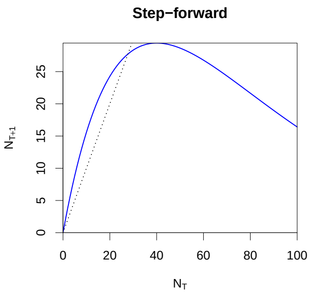

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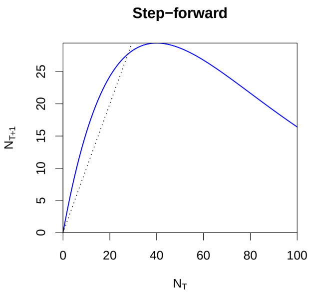

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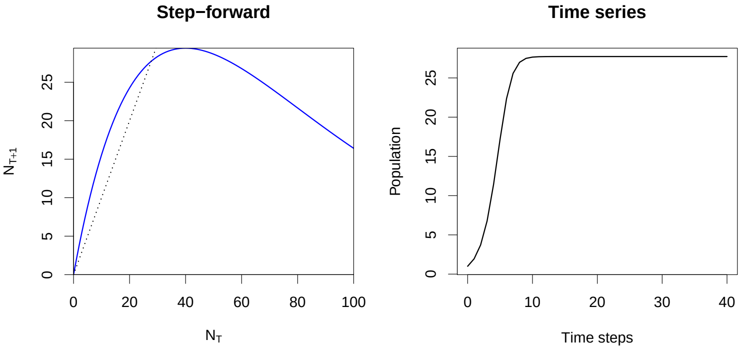

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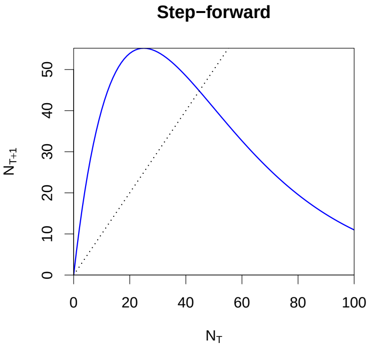

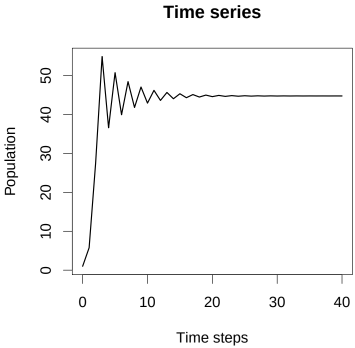

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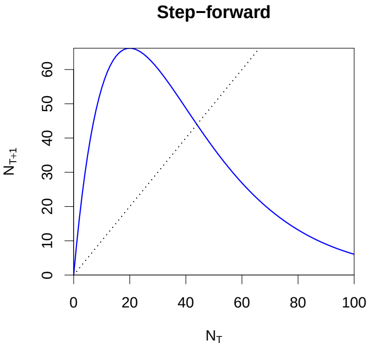

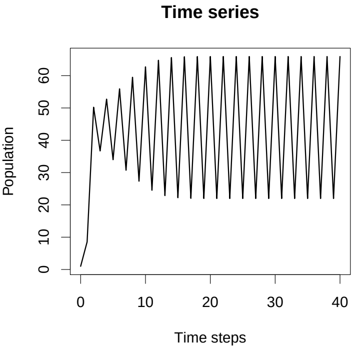

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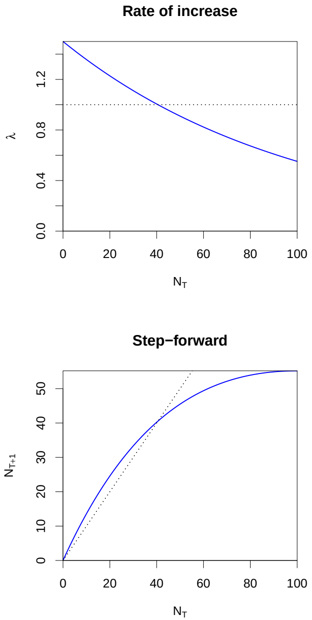

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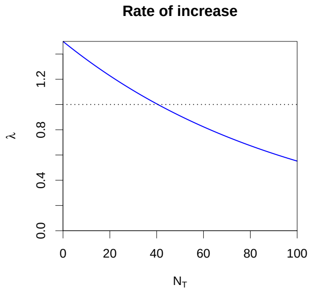

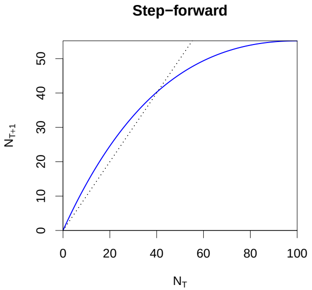

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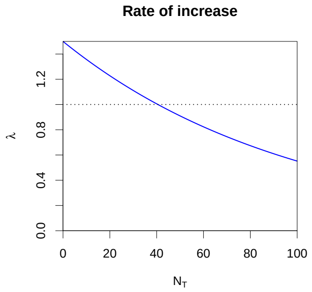

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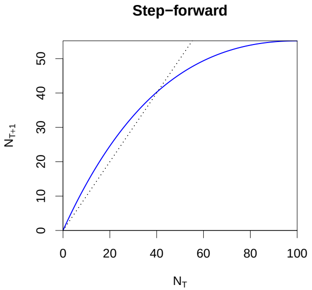

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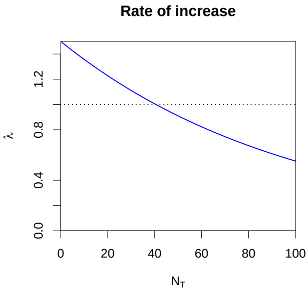

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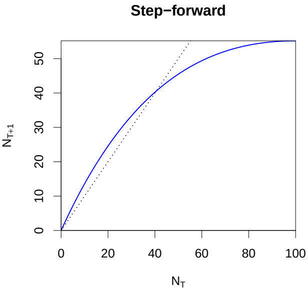
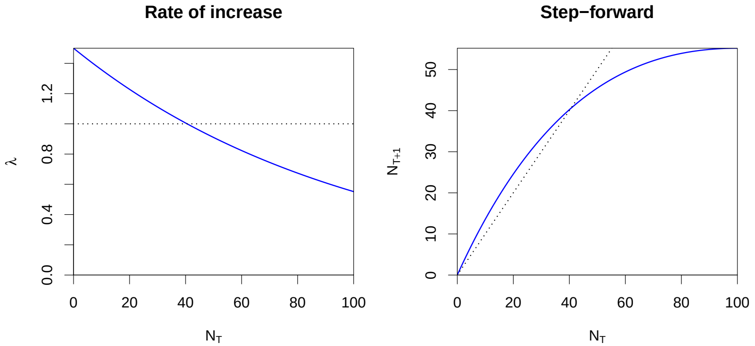


Contest regulation



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  - * The ones limited by food are more likely to have scramble competition and turnover

Plants

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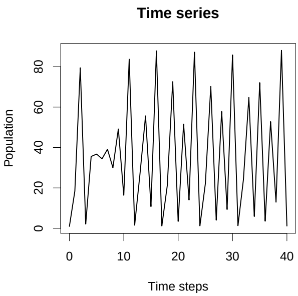

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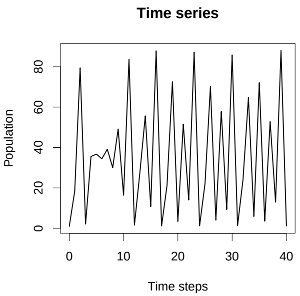

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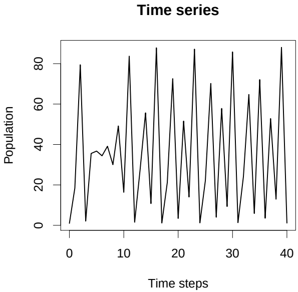

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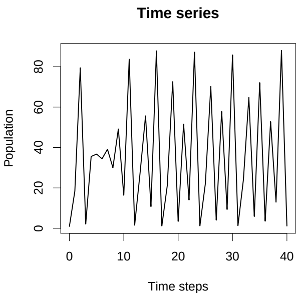

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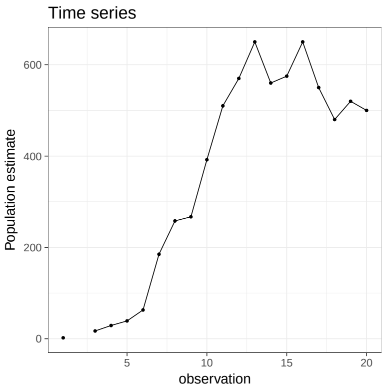
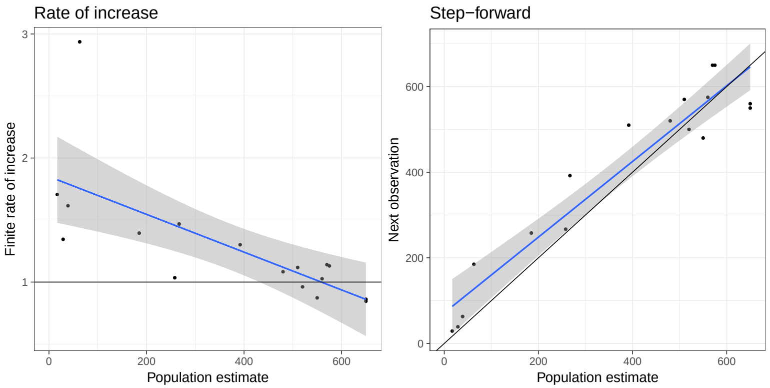
Paramecia

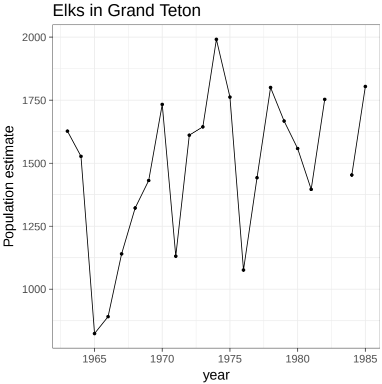


Time series
Population estimate
observation
0
200
400
600
5
10
15
20
Time series
Population estimate
observation
10
100
5
10
15
20

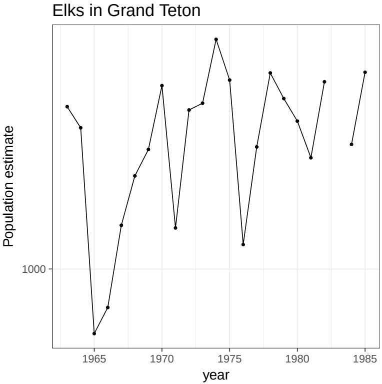
Paramecia


Rate of increase
Finite rate of increase
3
2
1
0
200
400
600
Population estimate
Step−forward
Next observation
600
400
200
0
0
200
400
600
Population estimate

Elk



Elks in Grand Teton
Population estimate
2000
1750
1500
1250
1000
1965
1970
1975
1980
1985
year



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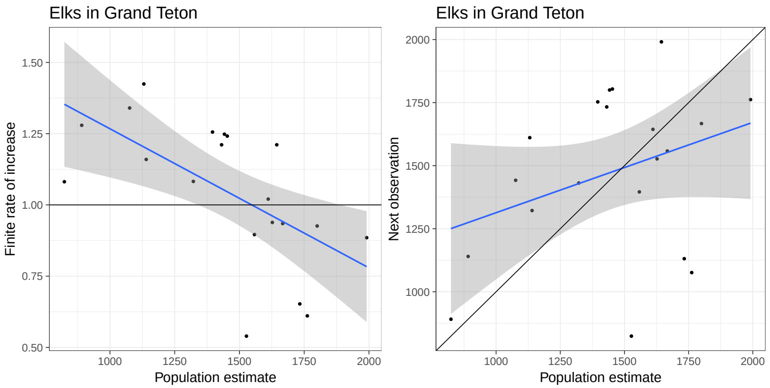

Elks in Grand Teton
Finite rate of increase
1.50
1.25
1.00
0.75
0.50
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1250
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Population estimate
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Subsection 1

Allee effects

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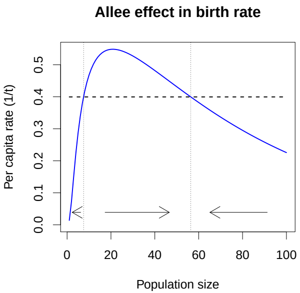

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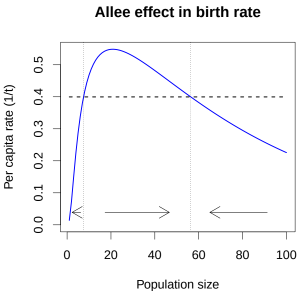

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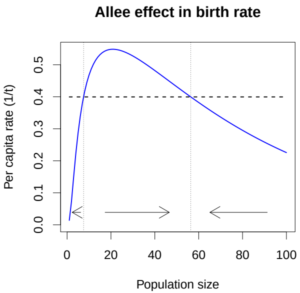

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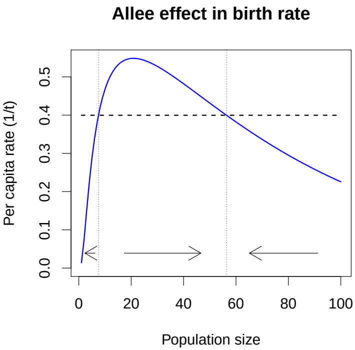

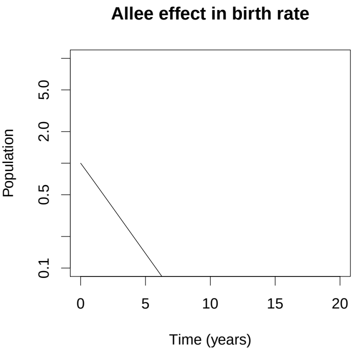

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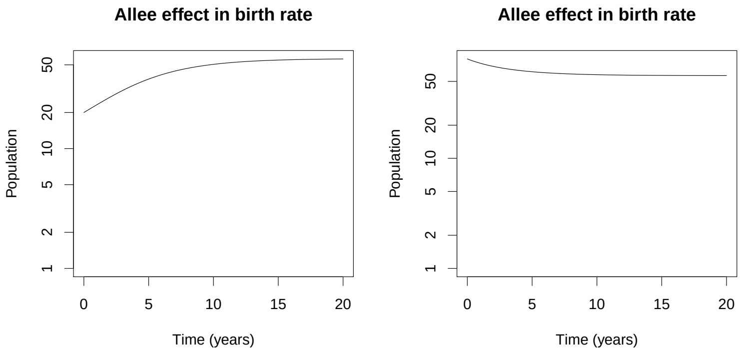

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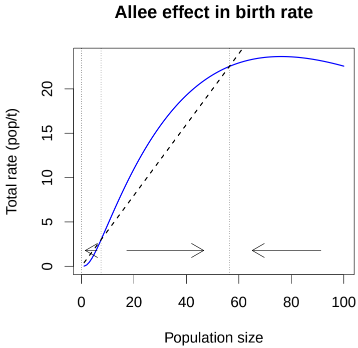

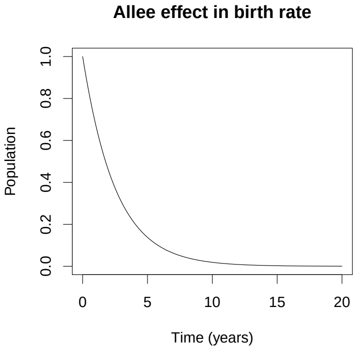

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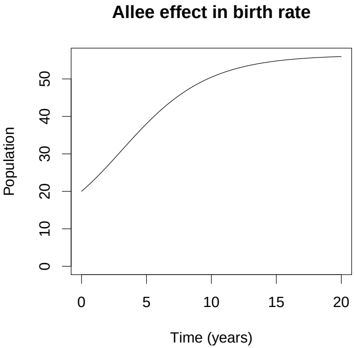
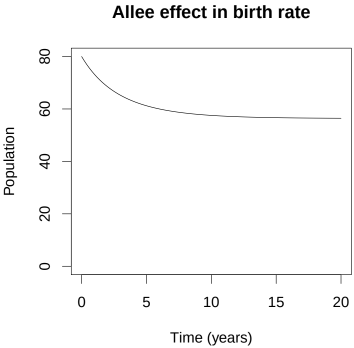
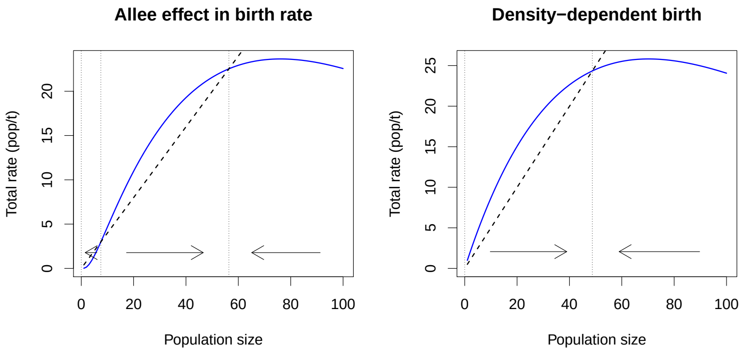




Population comparison



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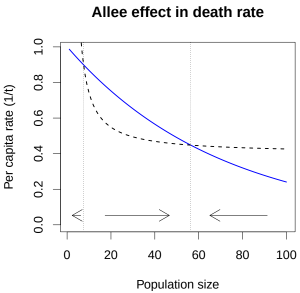

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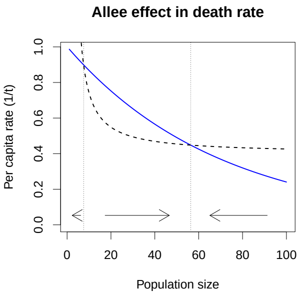

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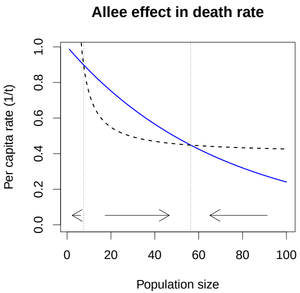

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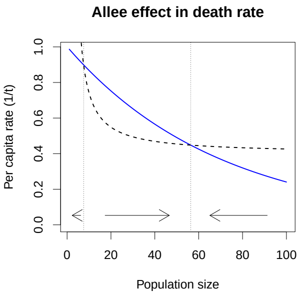

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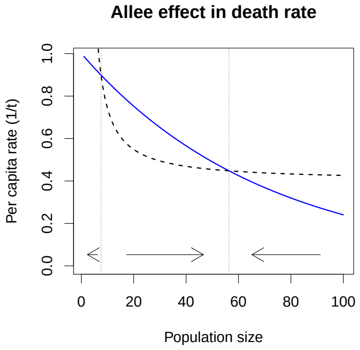

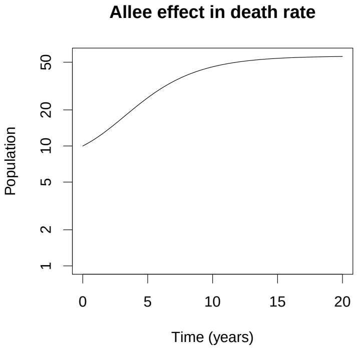

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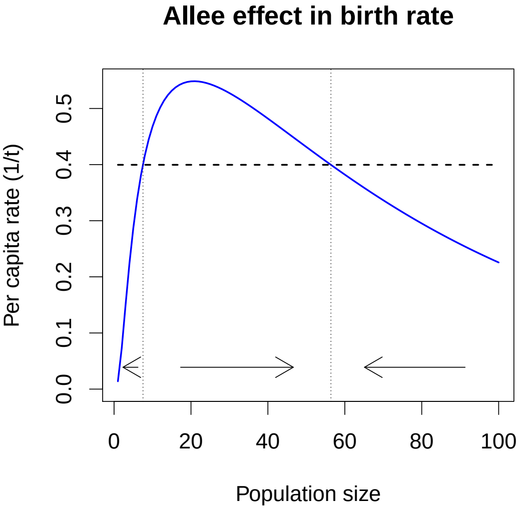
$\mathcal{R}_0$ and $\mathcal{R}_{max}$



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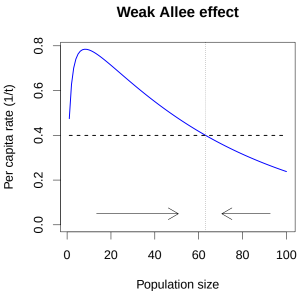

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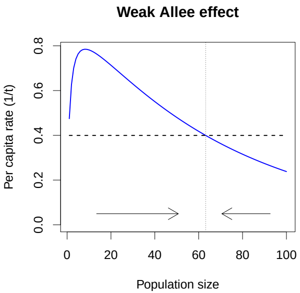

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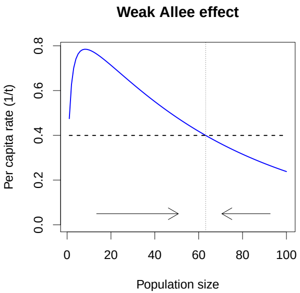

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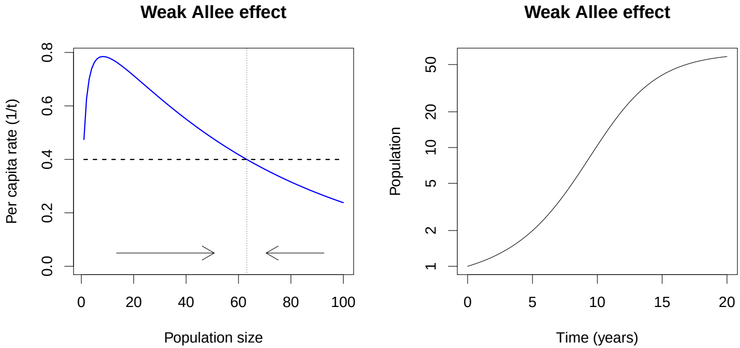

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Allee effect summary

- Population may go extinct if it drops below a certain threshold
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Subsection 2

Stochastic effects

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Simulations

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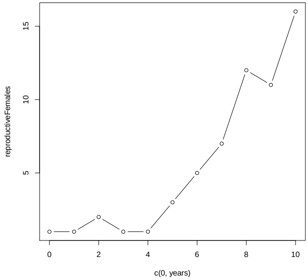

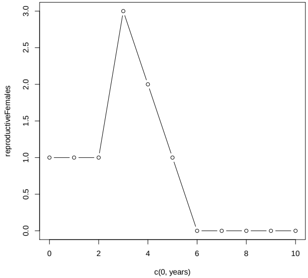

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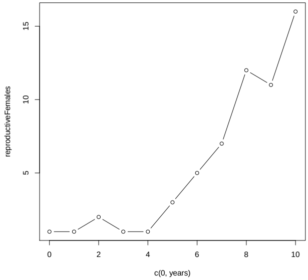

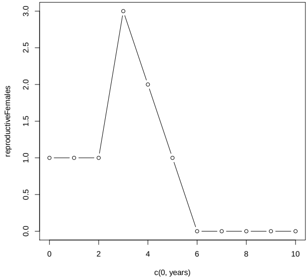

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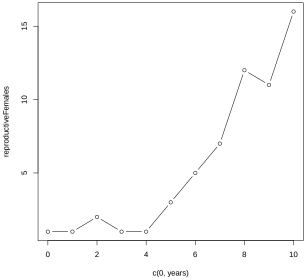

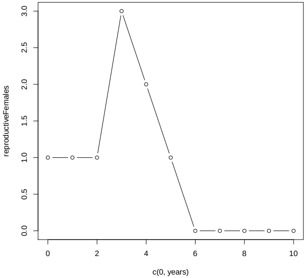

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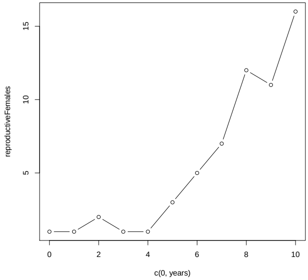

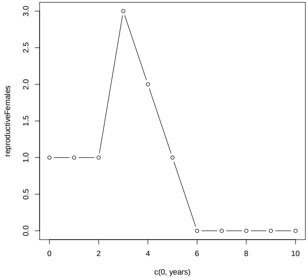

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