UNIT 1: Linear population models

Outline

Subsection 1

Dandelions

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Subsection 2

Gypsy moths

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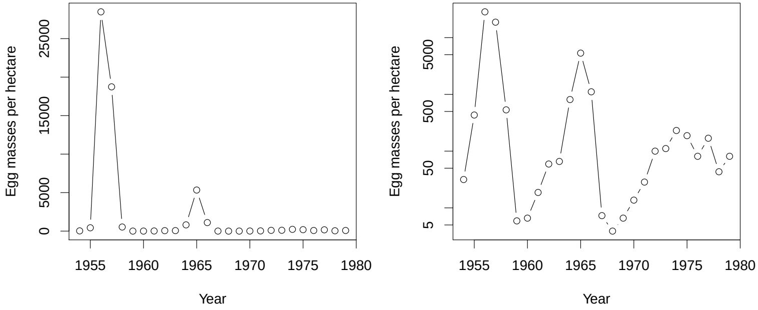
Gypsy moth populations



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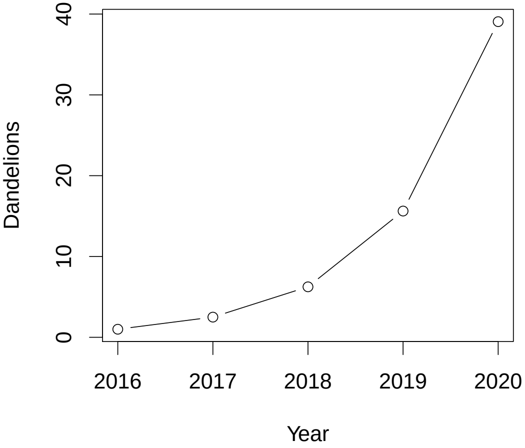

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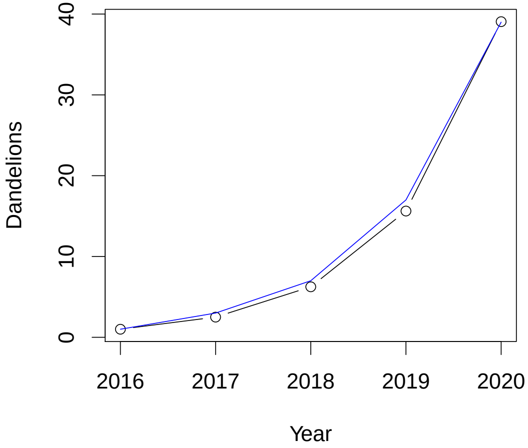

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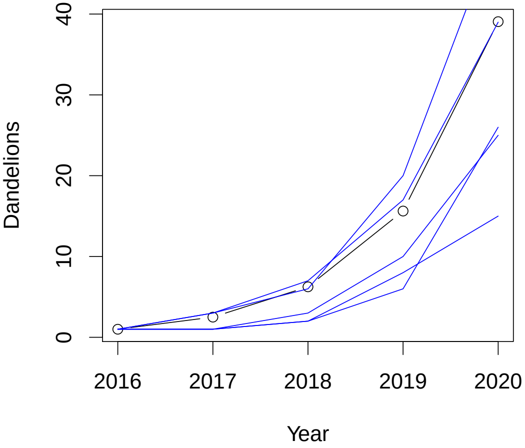

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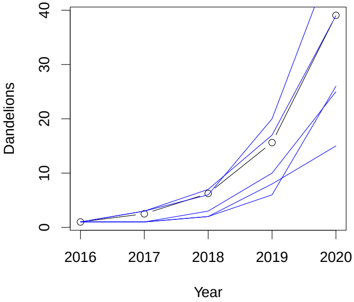

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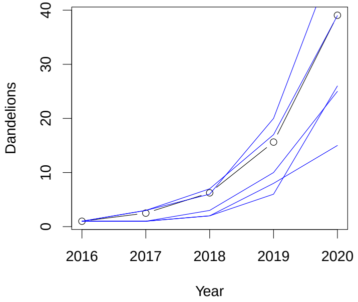

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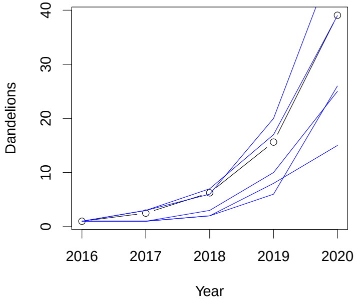

Subsection 3

Bacteria

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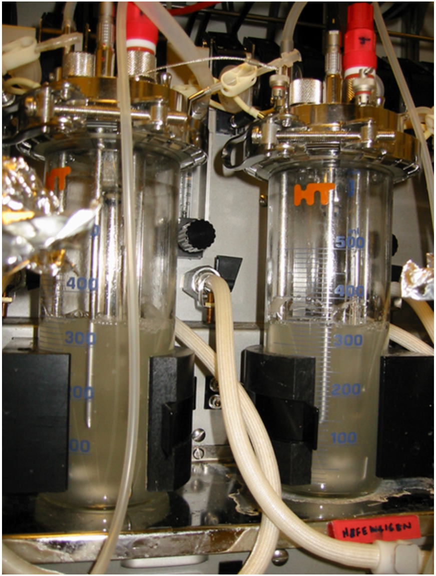

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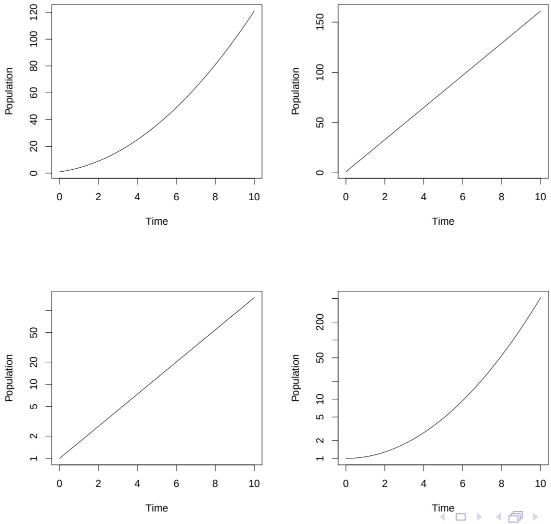

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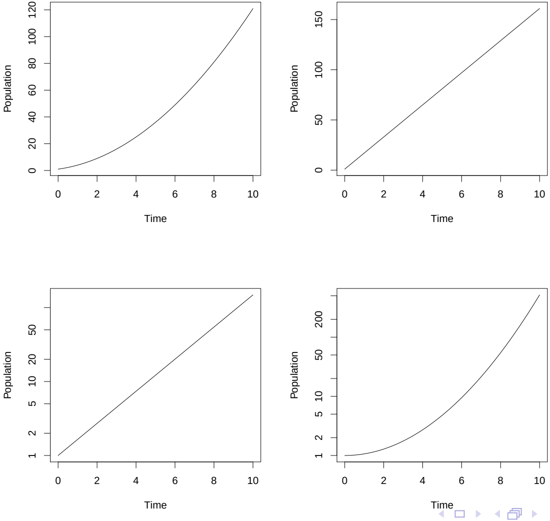

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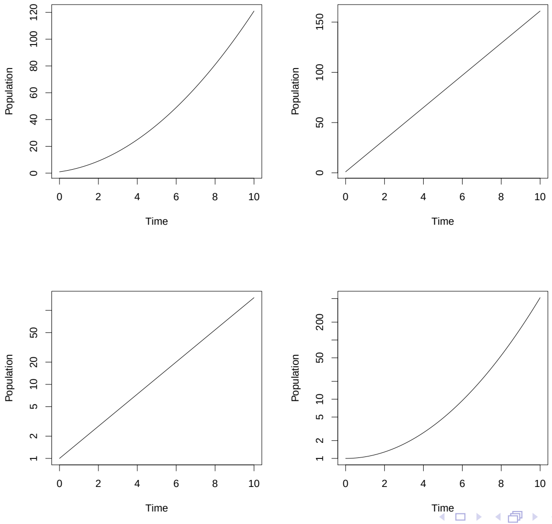

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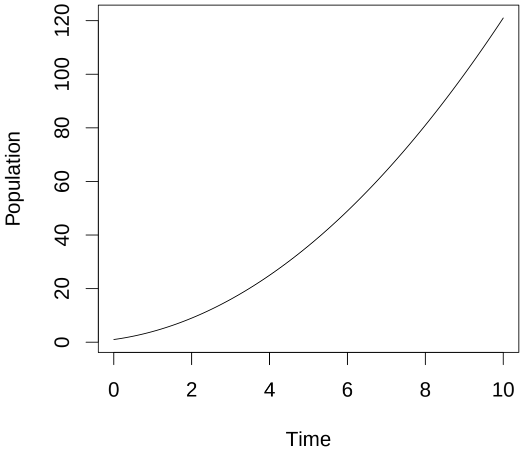

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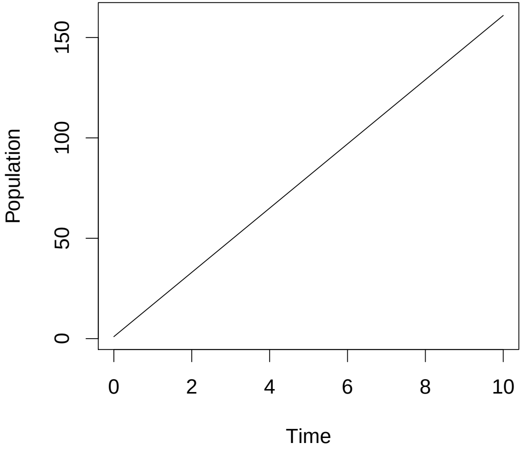
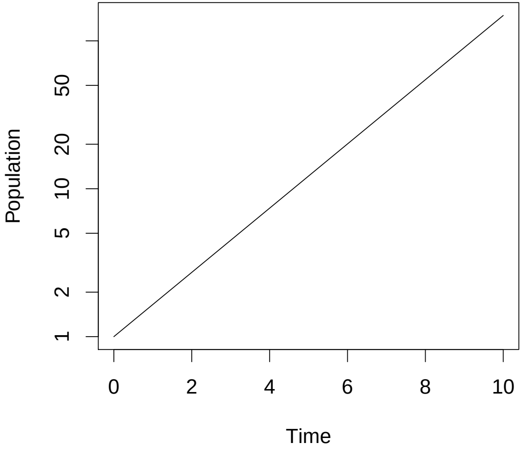


C



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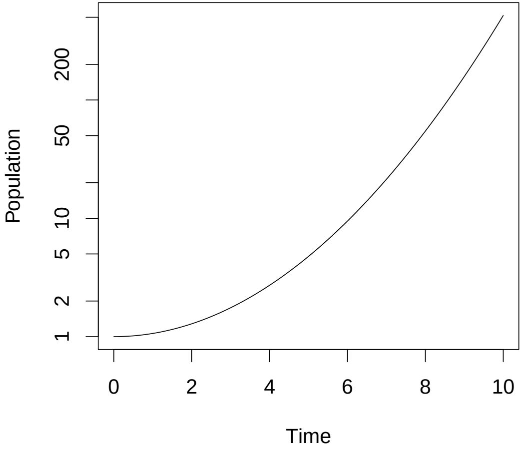

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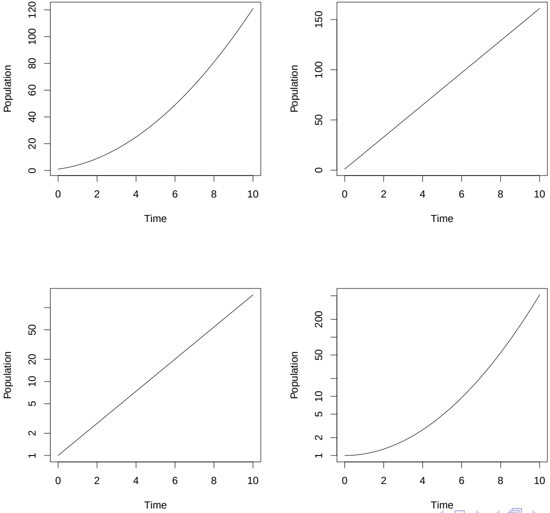

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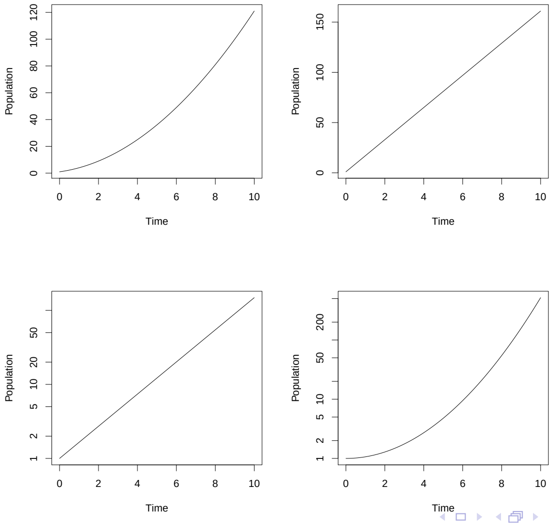

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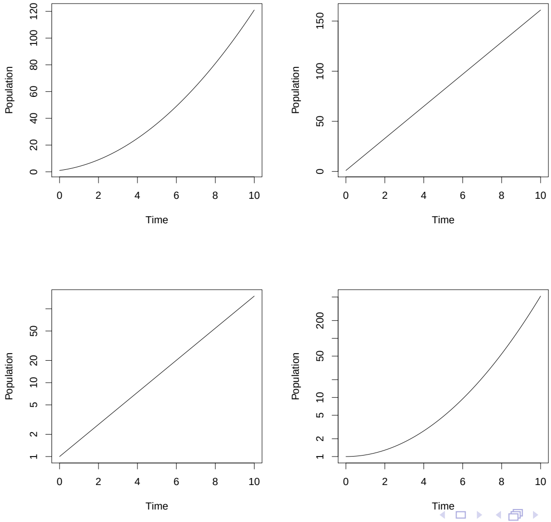

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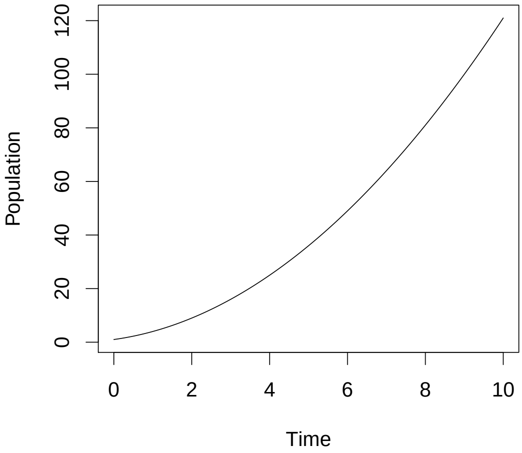

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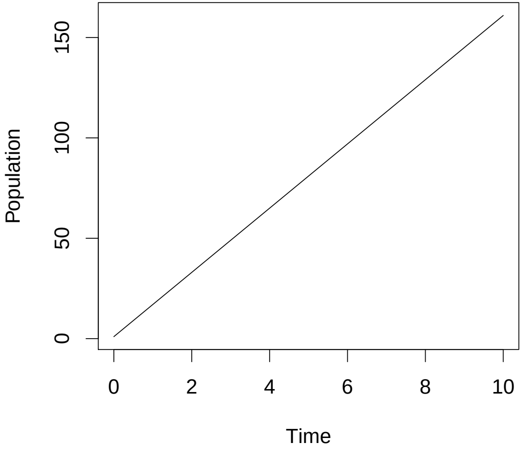

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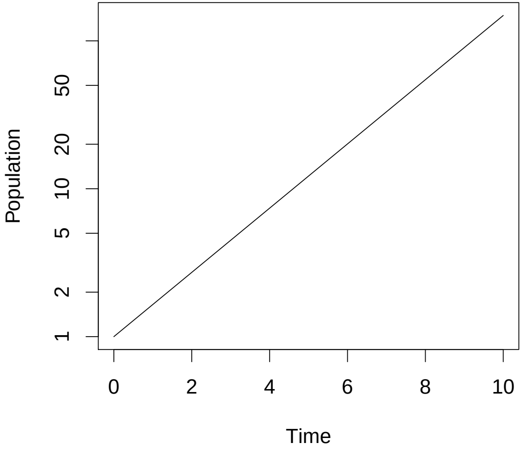

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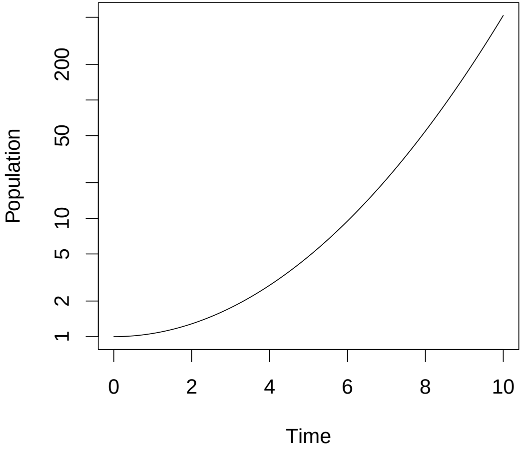

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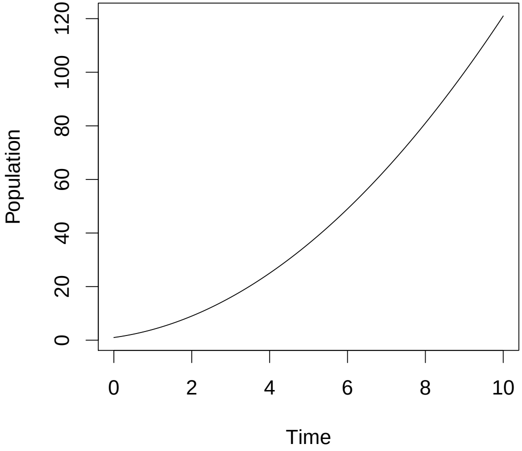
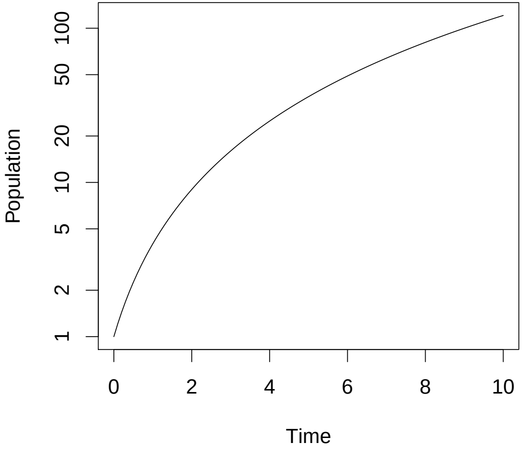


A on the log scale



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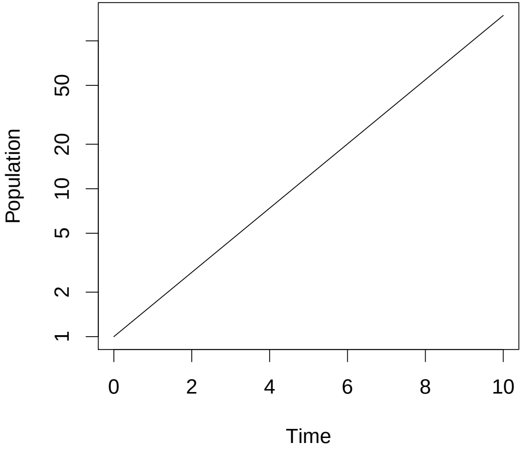
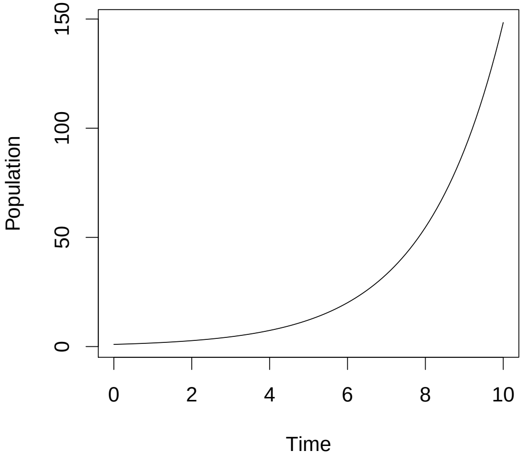


C on the linear scale



Types of growth

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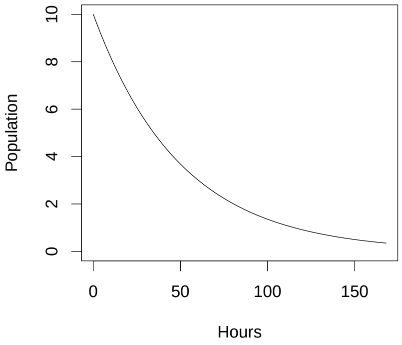

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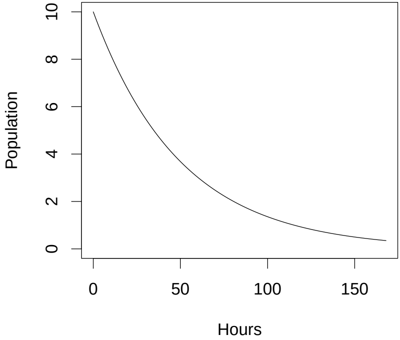

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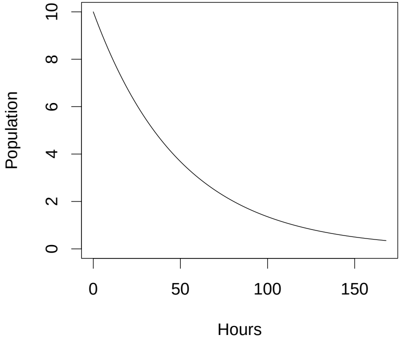

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Subsection 1

Log and linear scales

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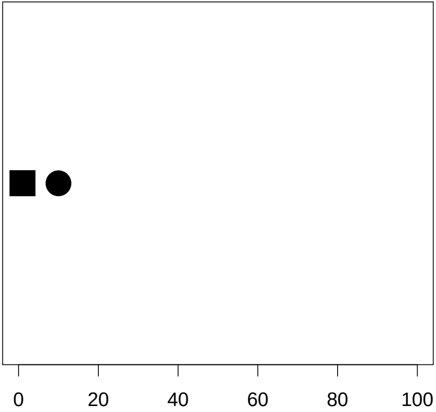

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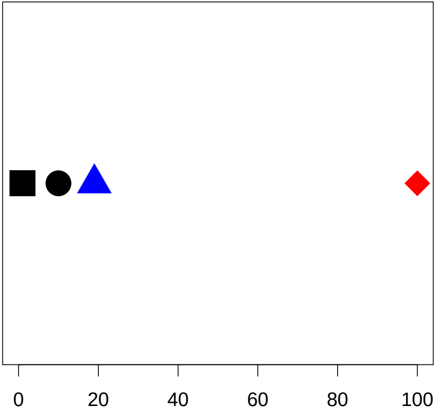

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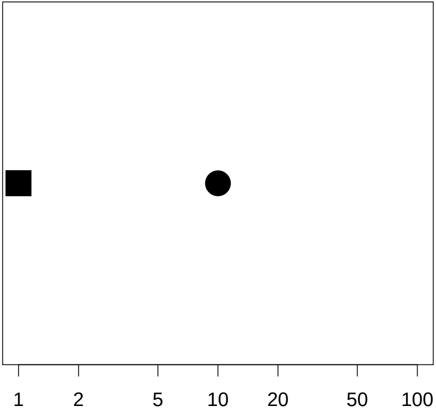

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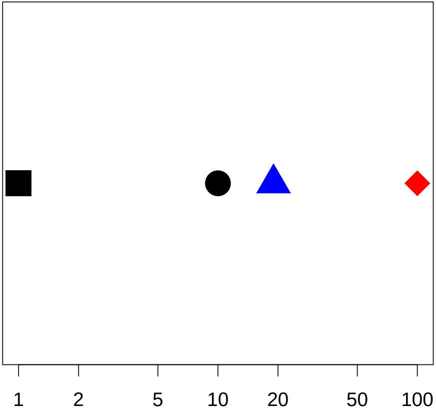

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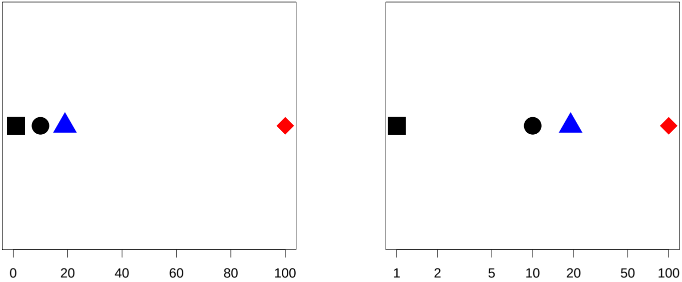
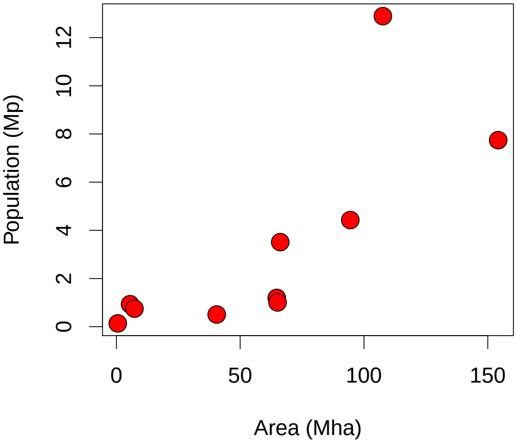


There is only one log scale; it doesn't matter which base you use!

Canadian provinces



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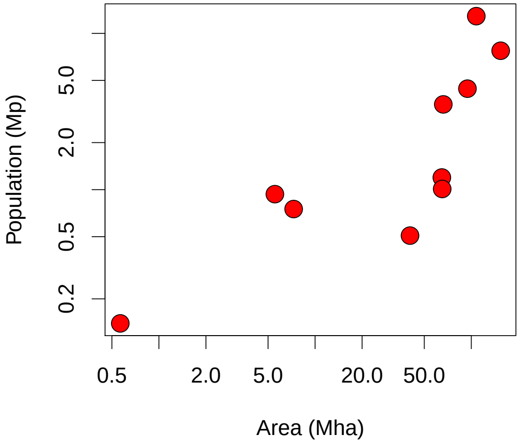

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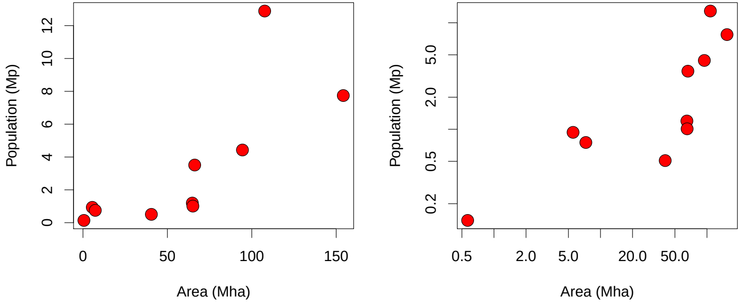

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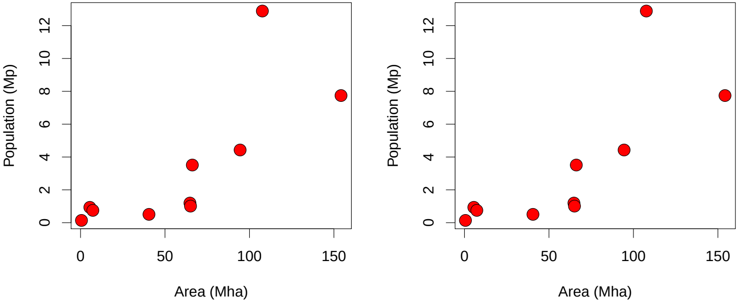

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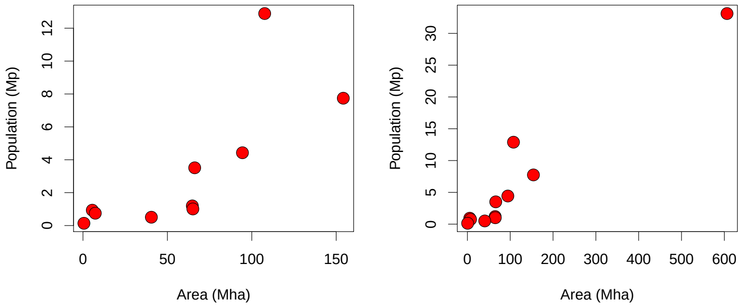

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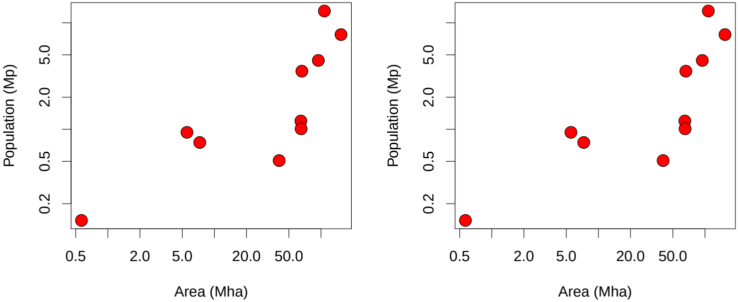

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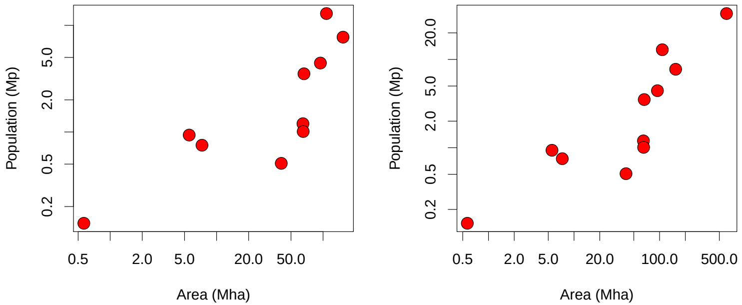

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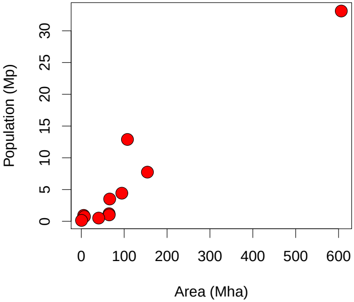

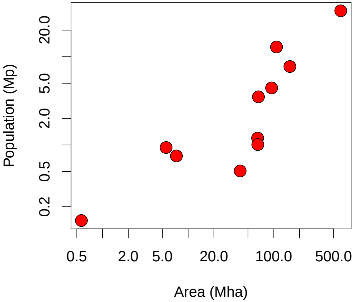

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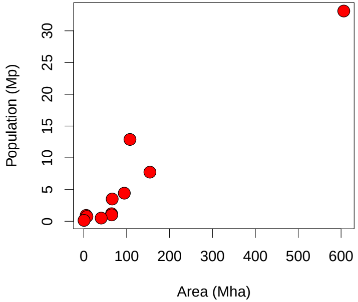

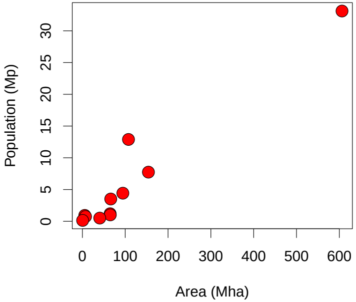

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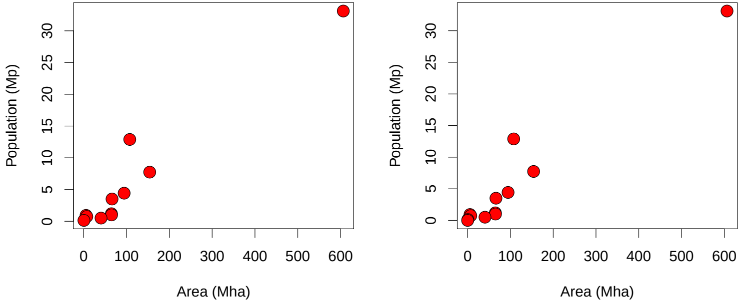

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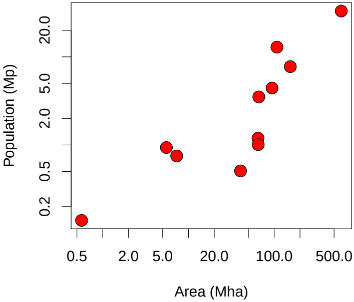

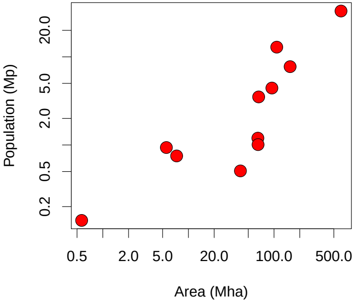

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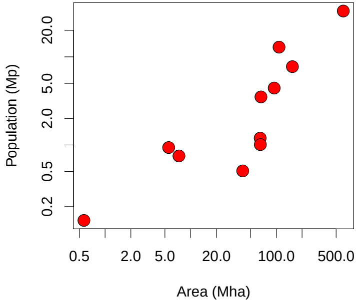

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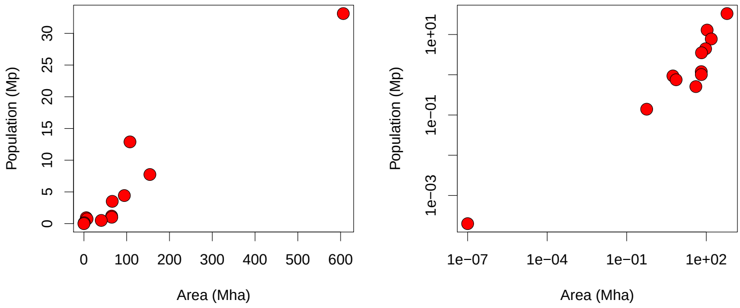

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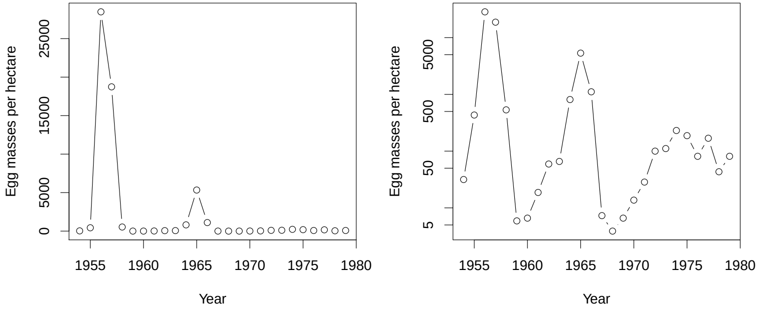

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Subsection 2

Time scales

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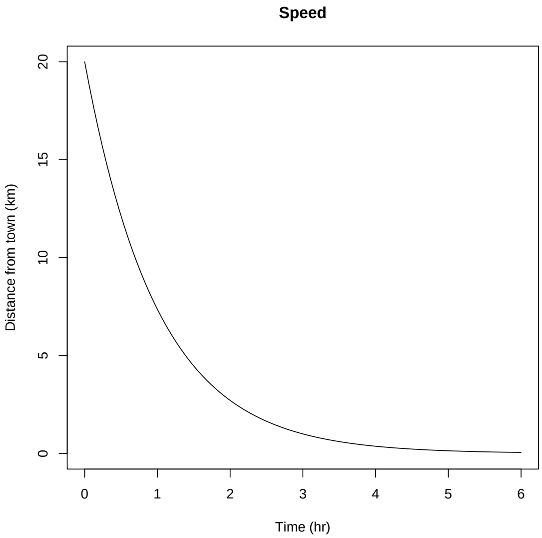

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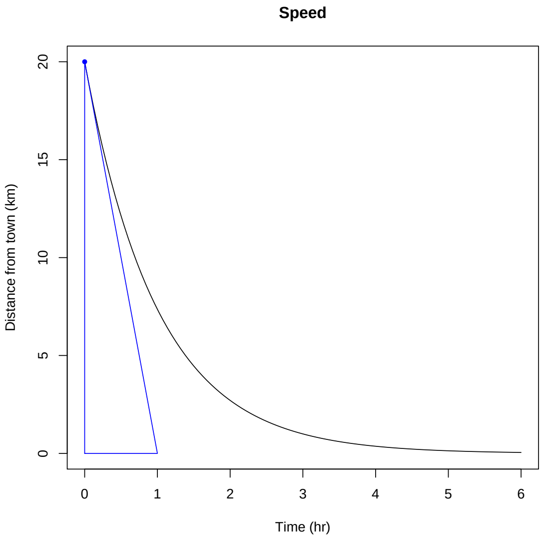

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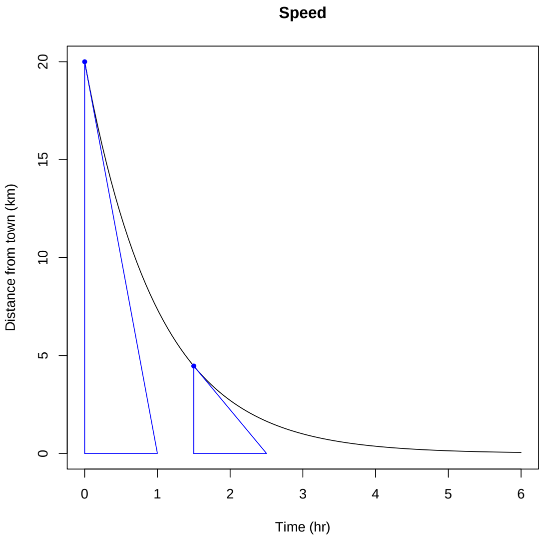

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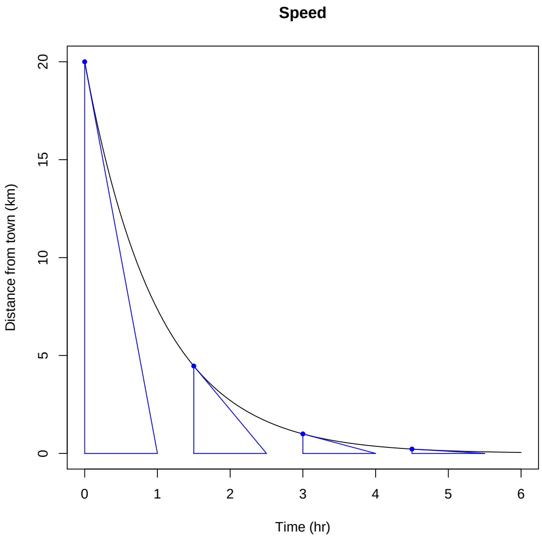

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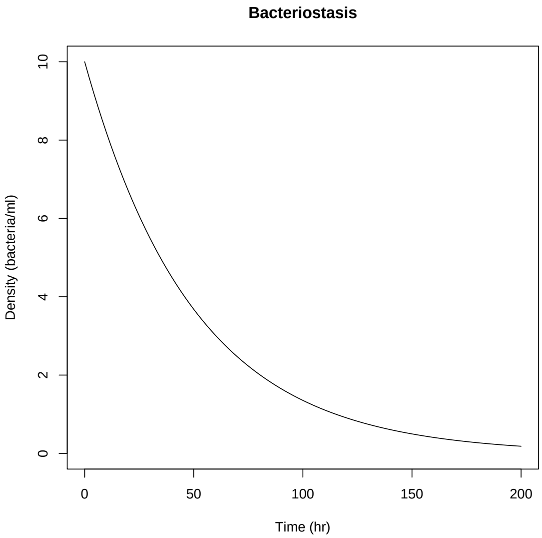

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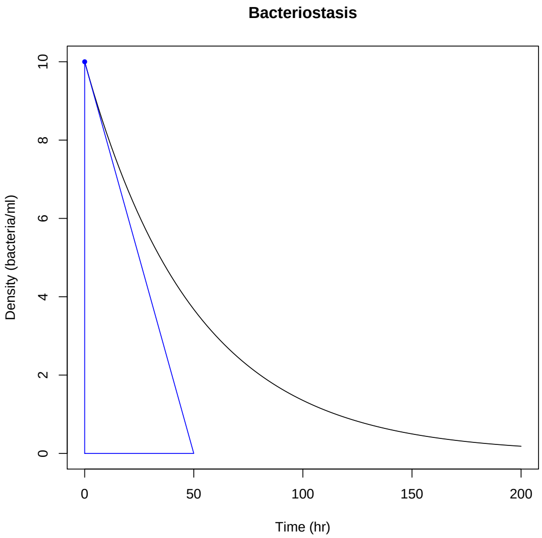

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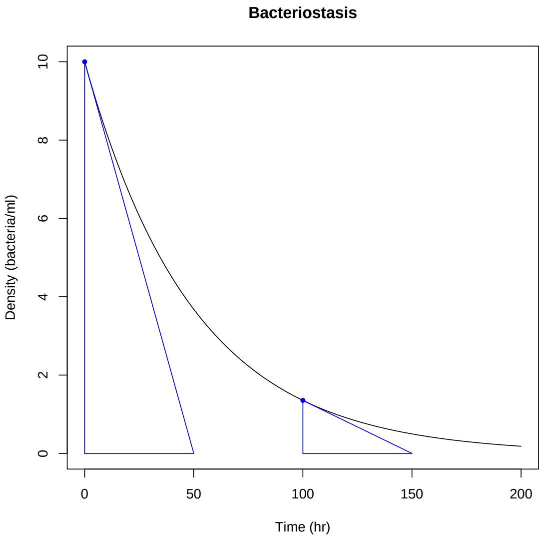

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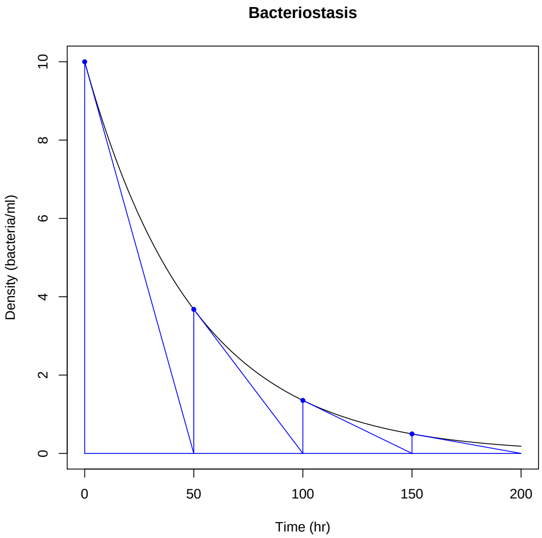

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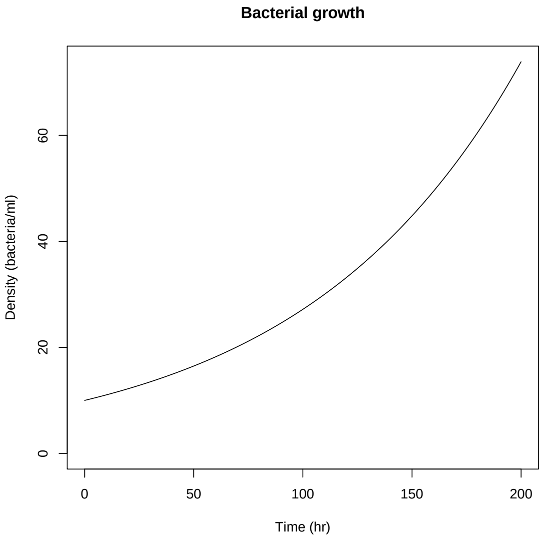

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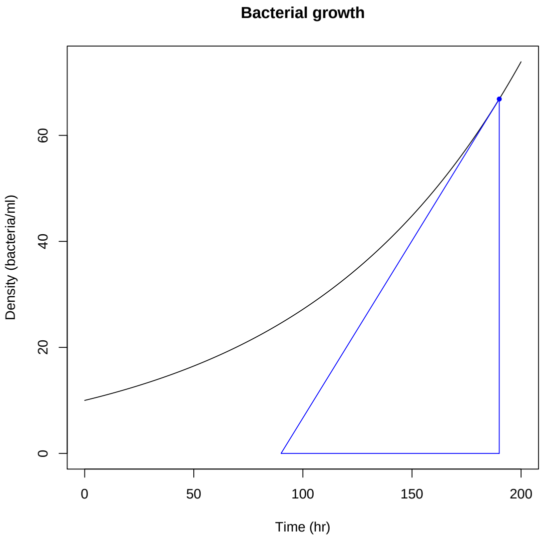

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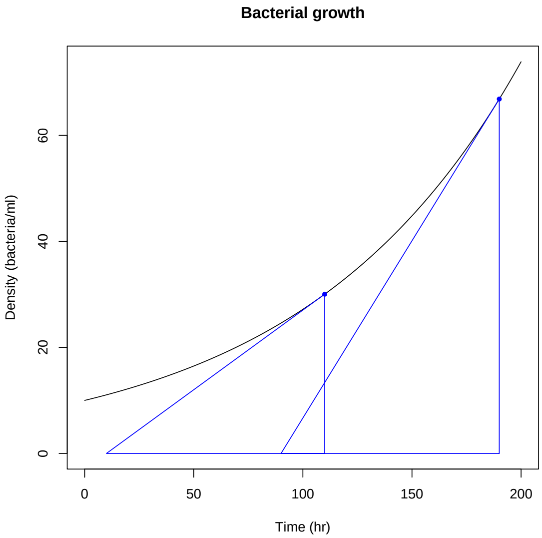

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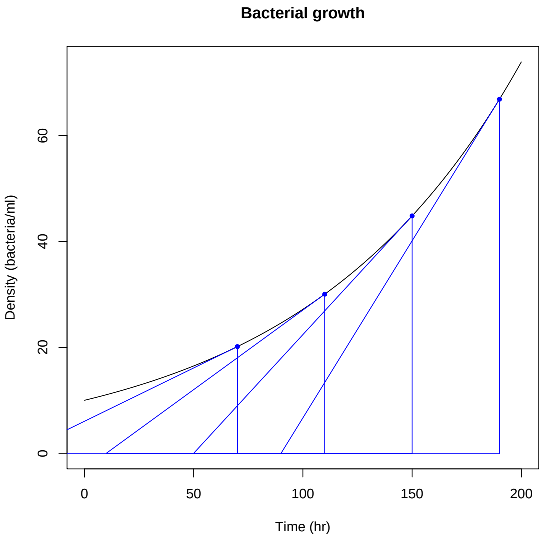

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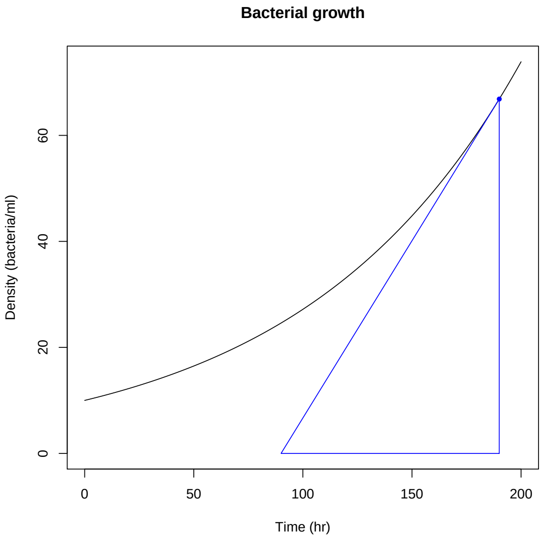
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Density (bacteria/ml)
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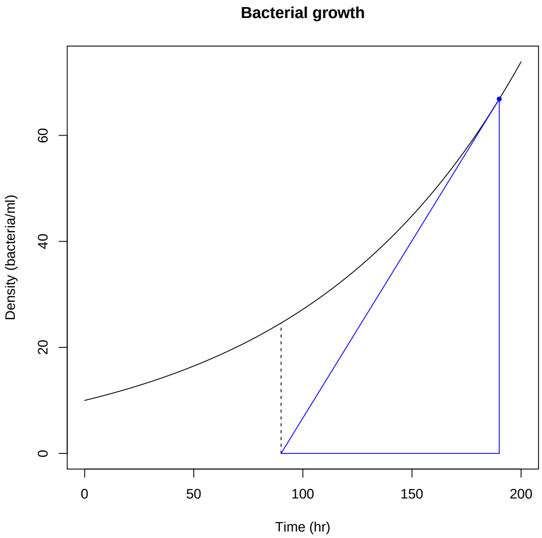
Time (hr)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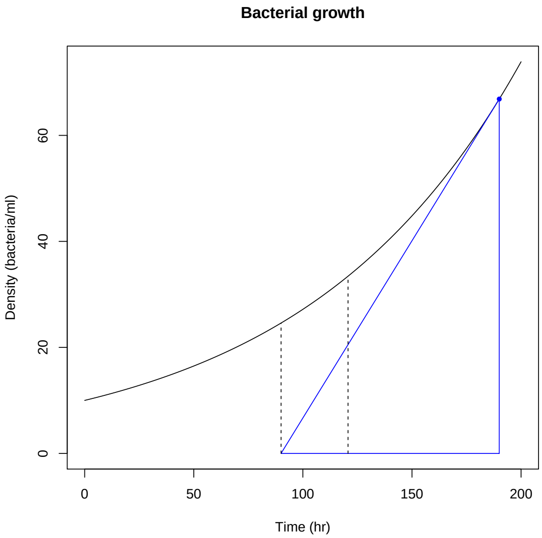
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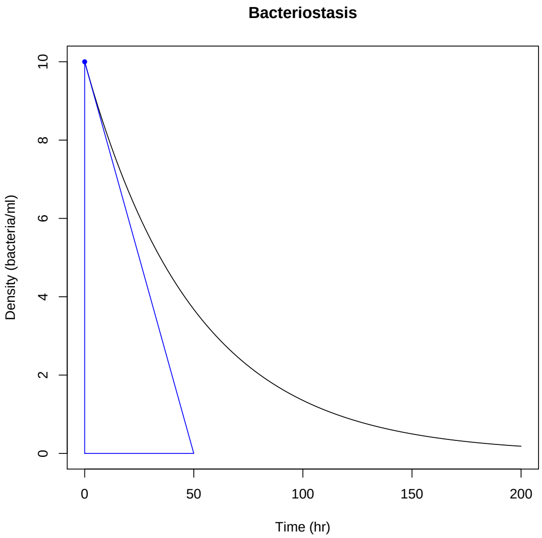
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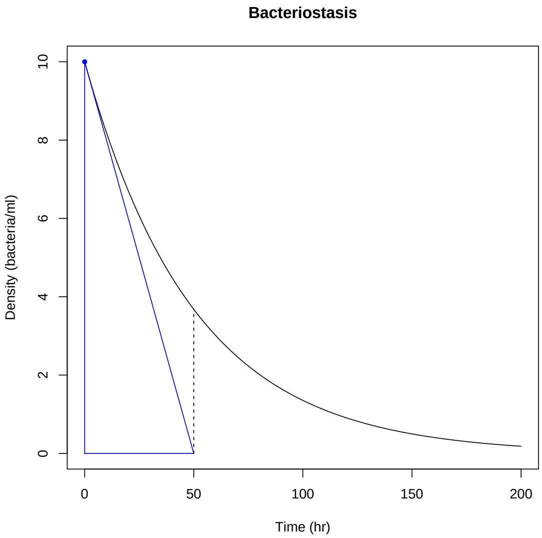
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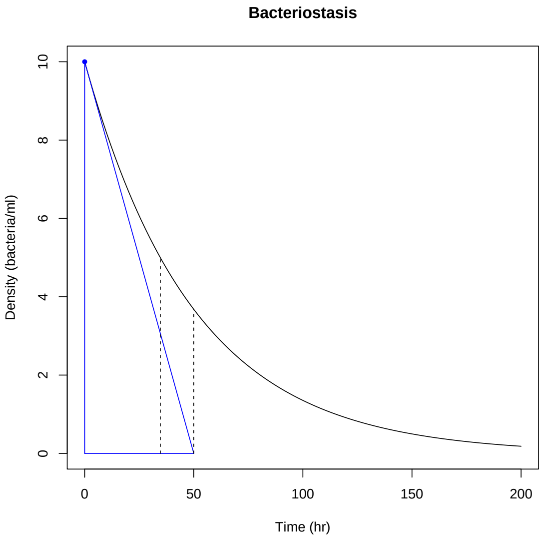

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Outline

Subsection 1

Dynamical models

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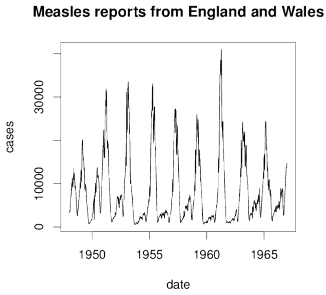

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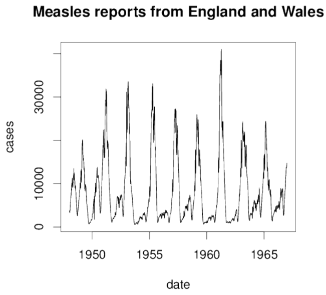

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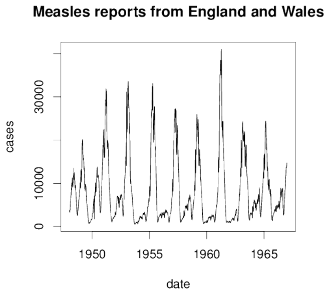

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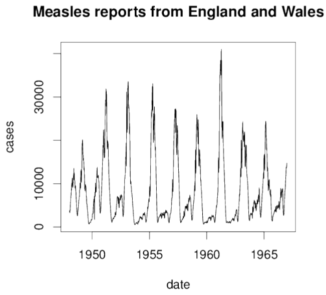

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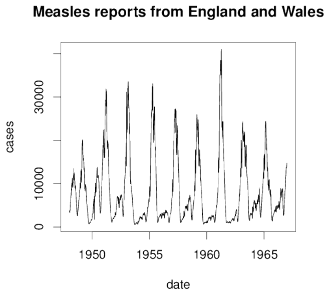

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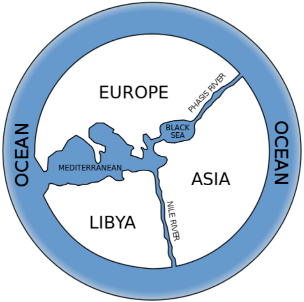

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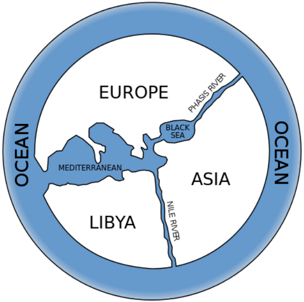

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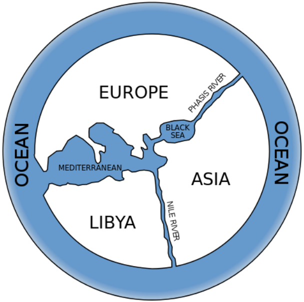

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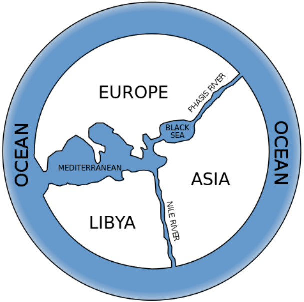

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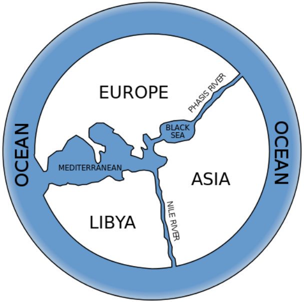

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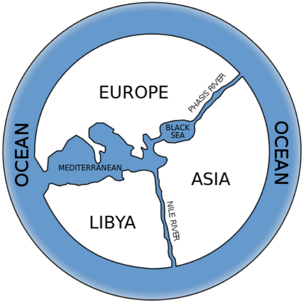

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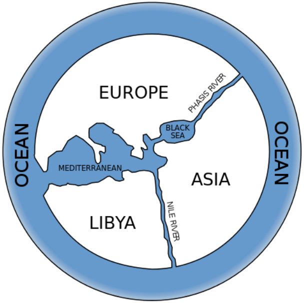

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Subsection 2

Examples

Gypsy moths

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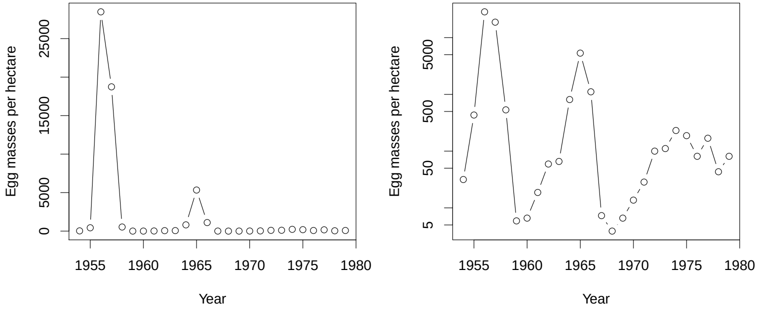

Moth example

- State variables

Moth example

- State variables
    - *

Moth example

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Moth example

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Bacteria

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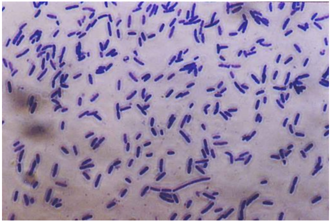

Bacteria

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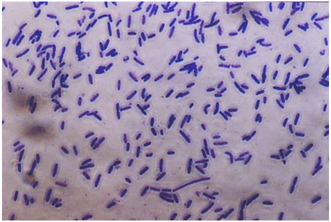

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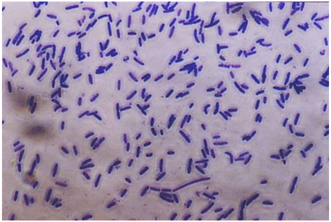

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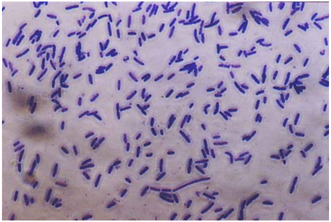

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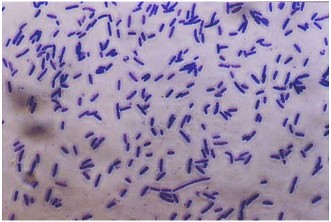

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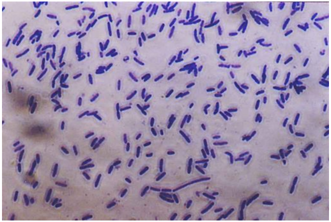

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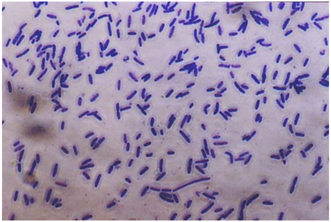

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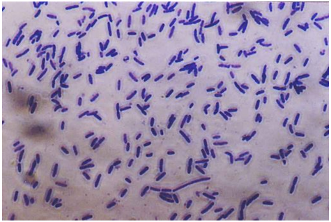

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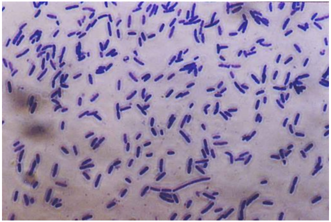

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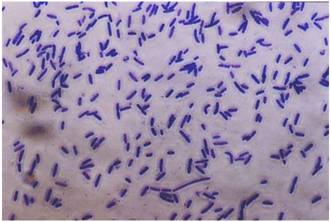

Dandelions

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Subsection 3

A simple discrete-time model

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5	2005-12-20	232 €	132 €	100 €
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9	2005-12-24	256 €	121 €	135 €
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- Spreadsheet (see resource page)

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A simple continuous-time model

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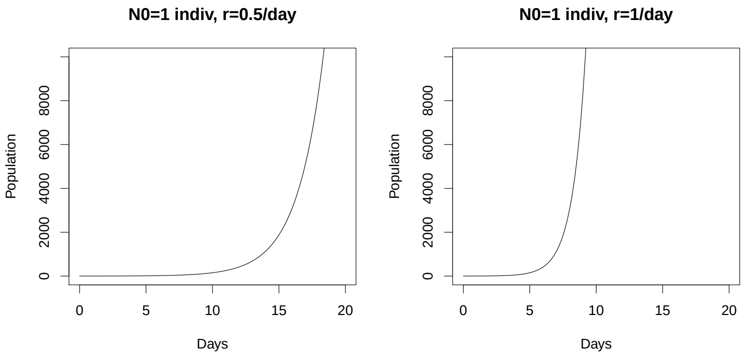
N0=1 indiv, r=0.5/day
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Population
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2000
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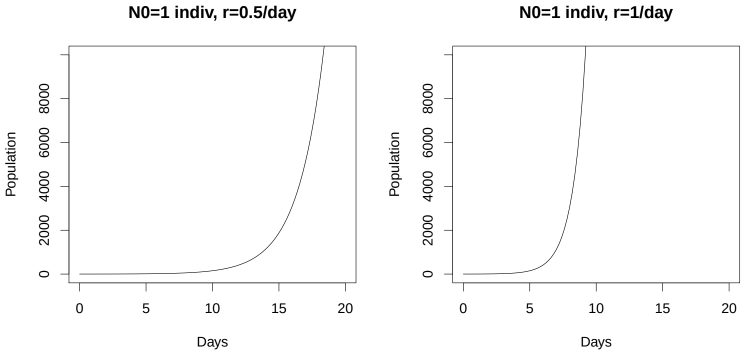

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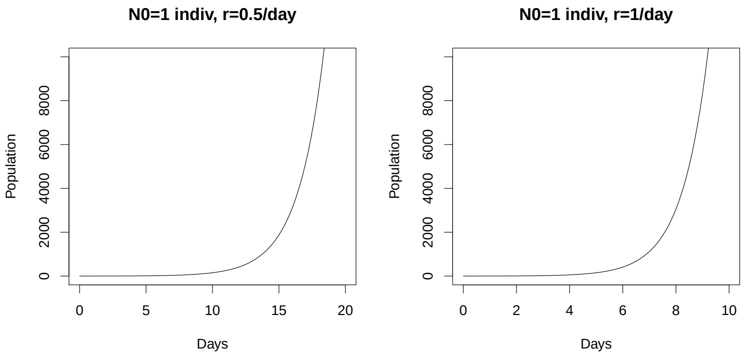

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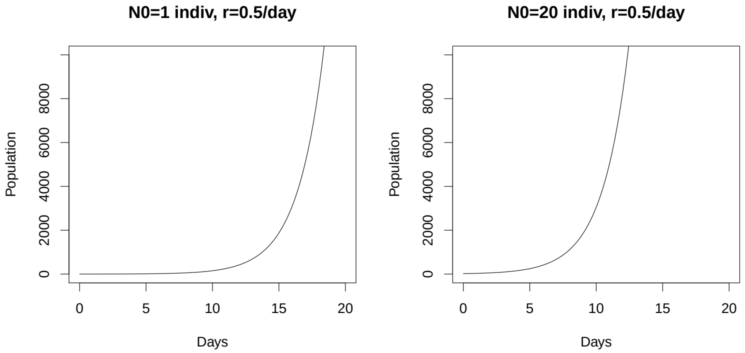

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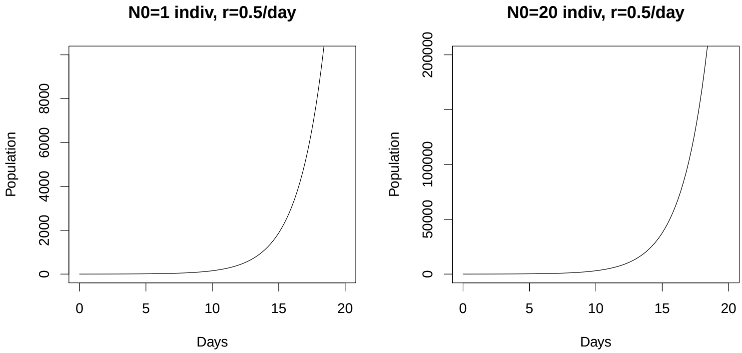

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Subsection 1

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- The reproductive number $\mathcal{R}$measures the average number of offspring produced by a single individual over the course of its lifetime
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Subsection 2

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