UNIT 1: Linear population models

Outline

Subsection 1

Dandelions

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Subsection 2

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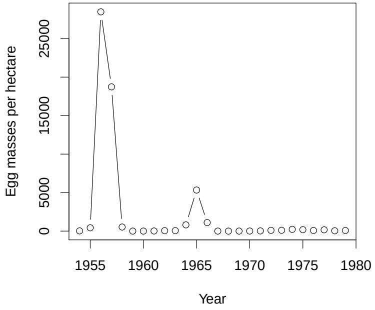
Gypsy moth populations



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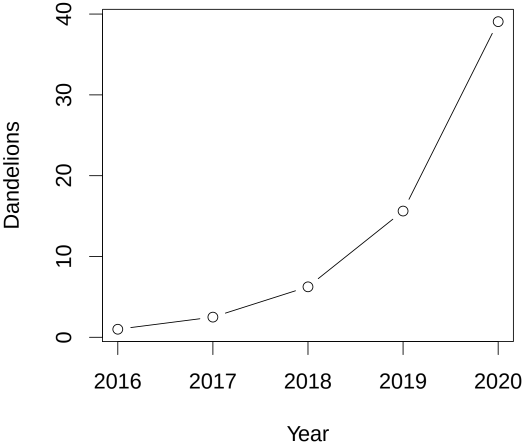

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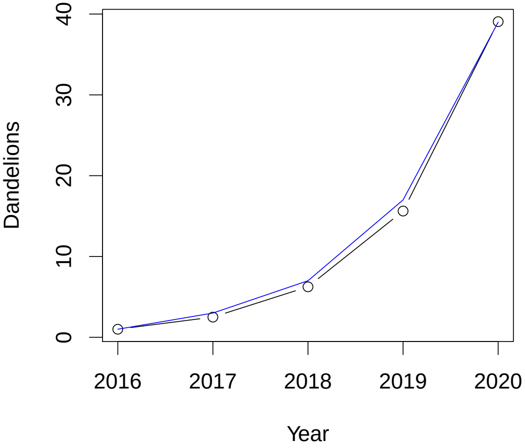

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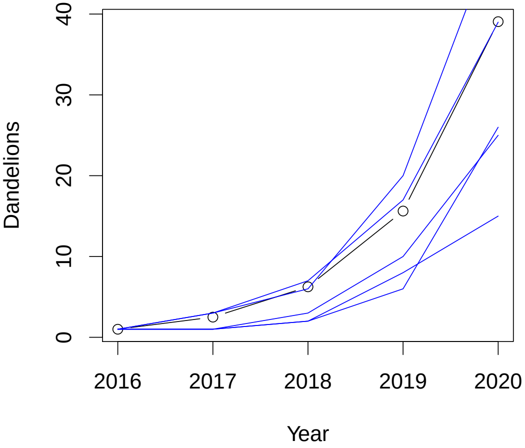

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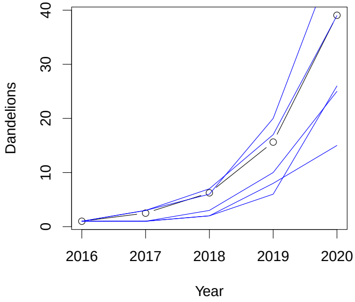

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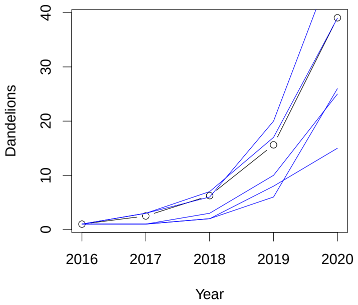

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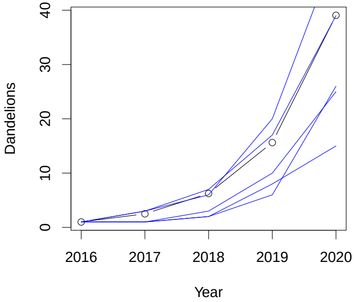

Subsection 3

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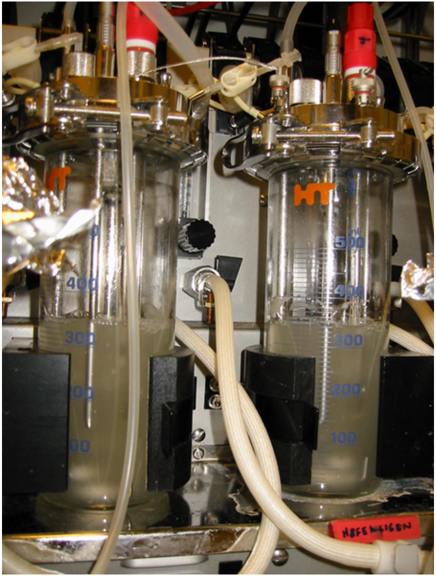

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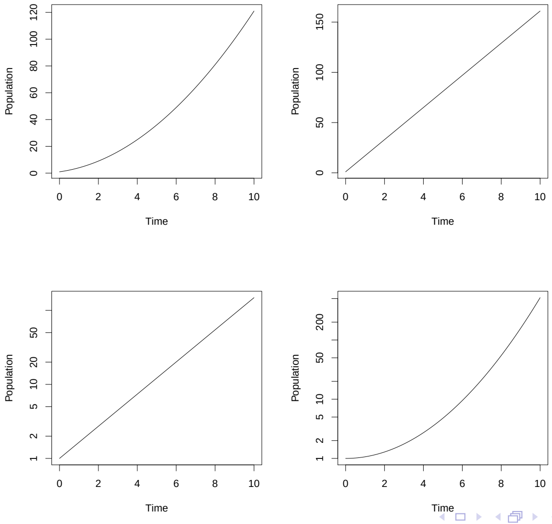

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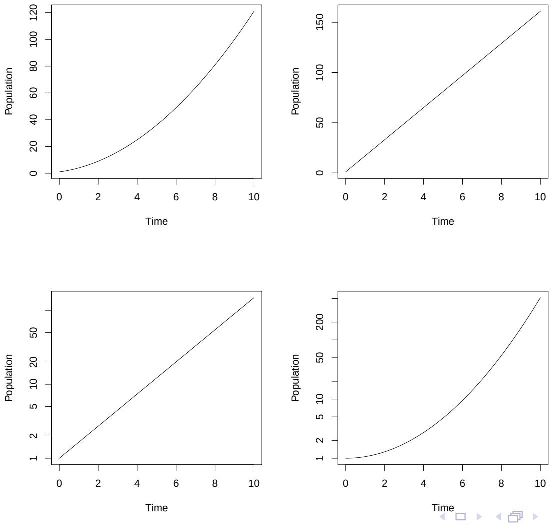

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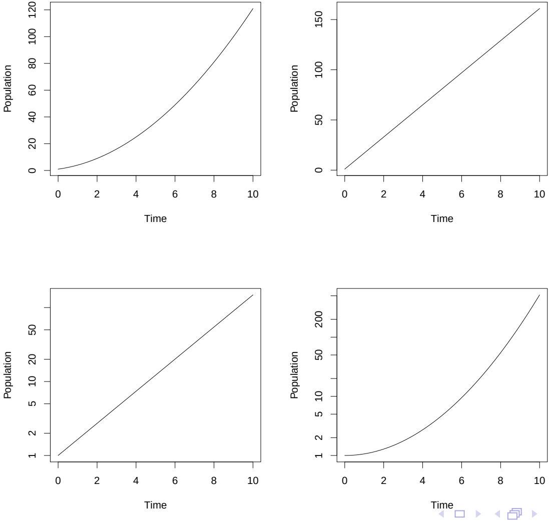

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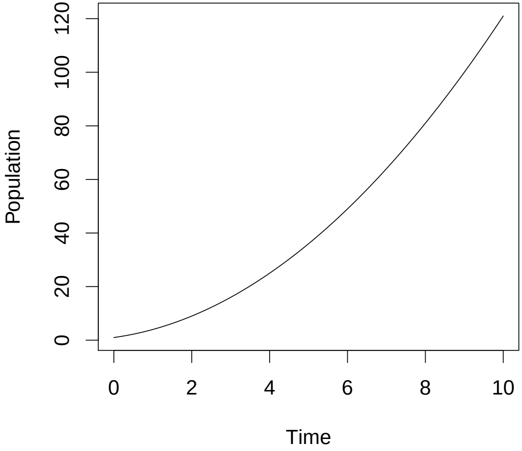

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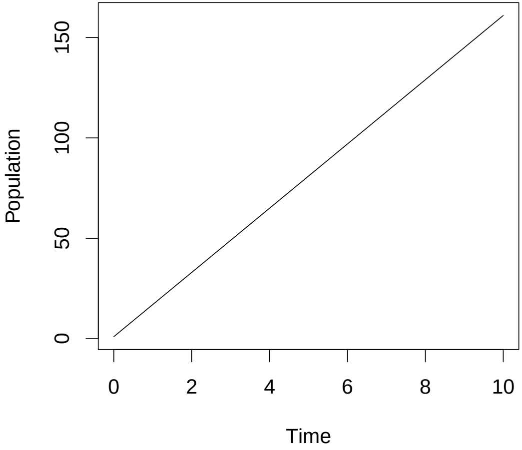
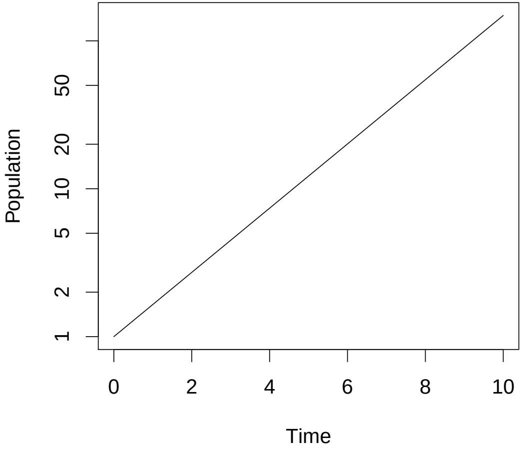
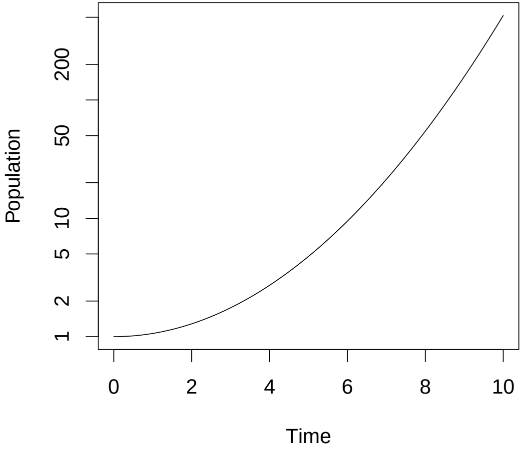


C



D



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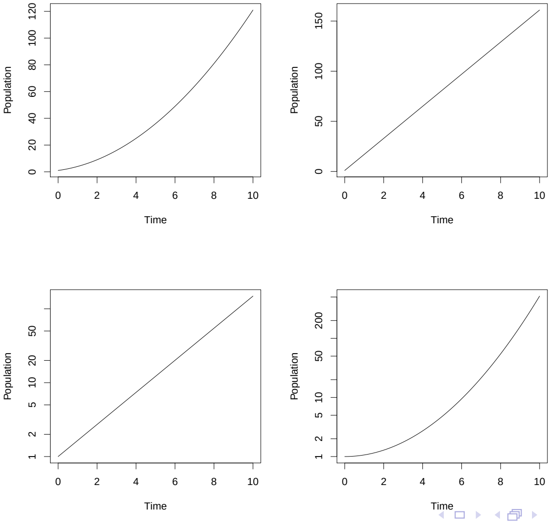

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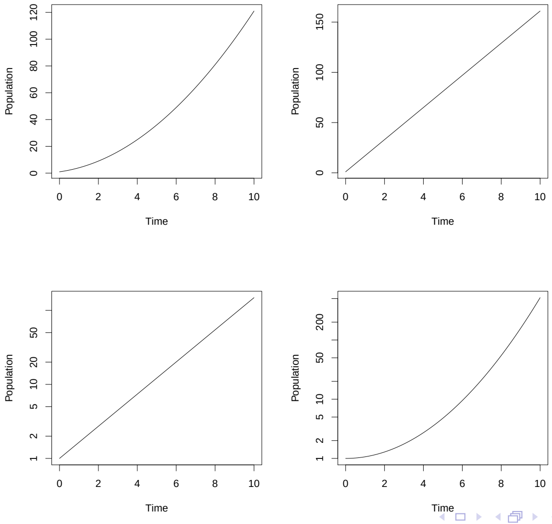

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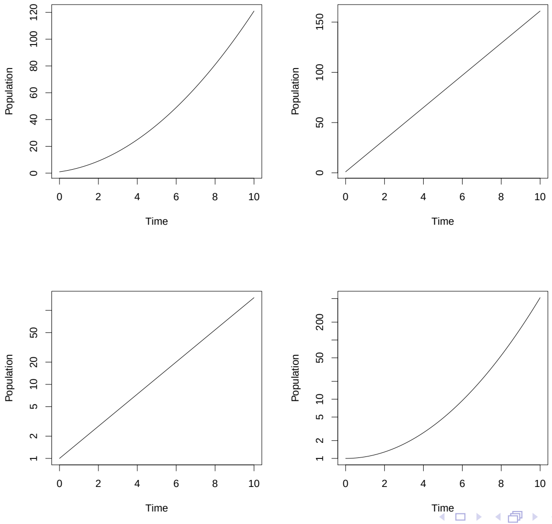

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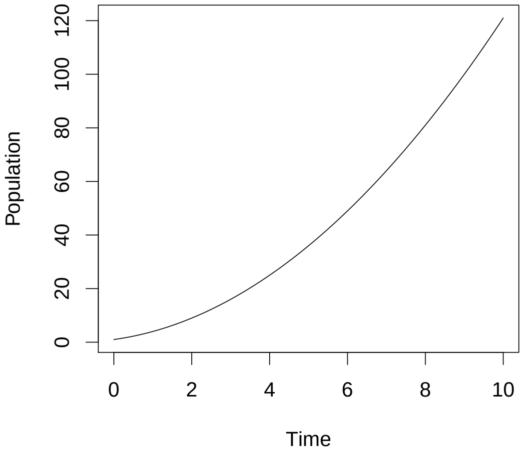

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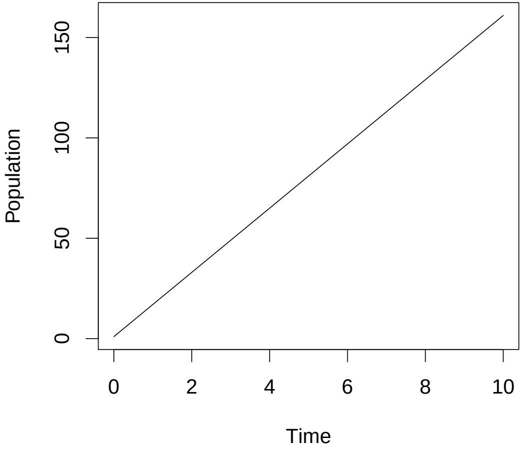

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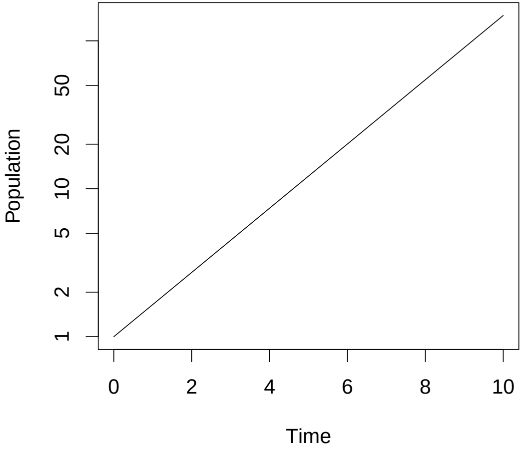

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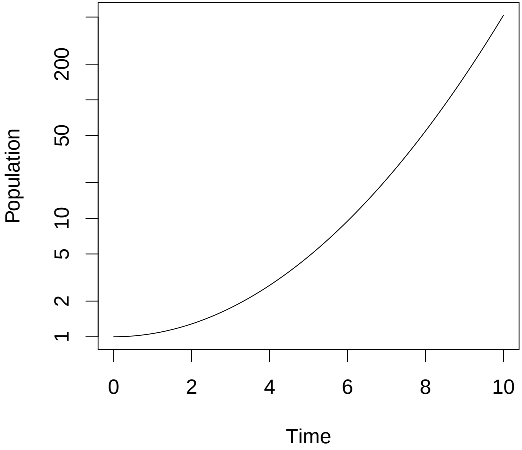

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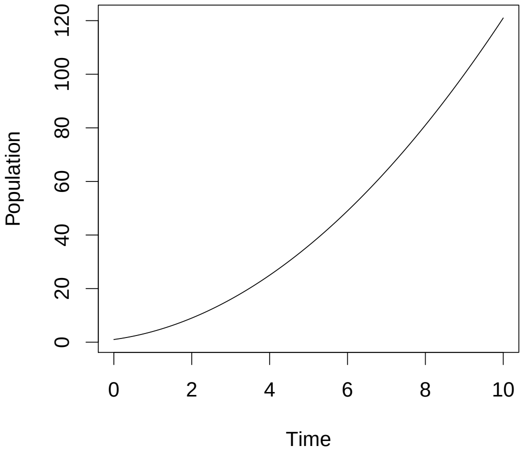
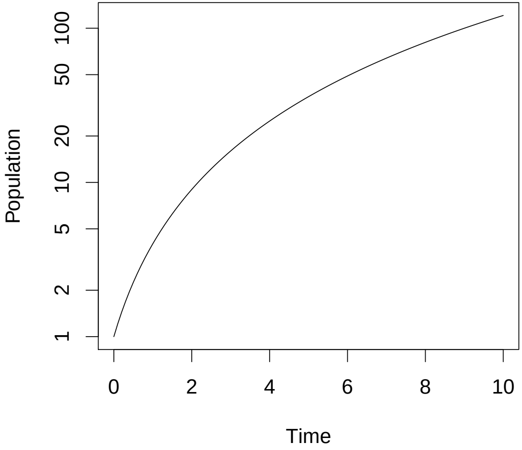


A on the log scale



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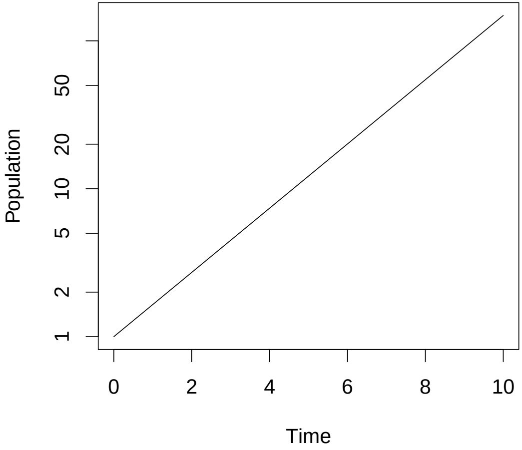
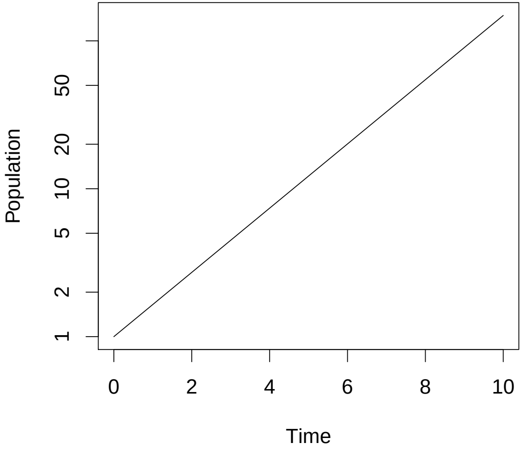


C on the linear scale



Types of growth

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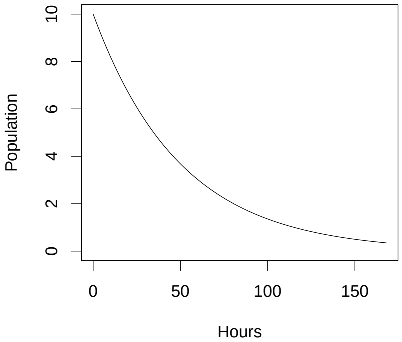

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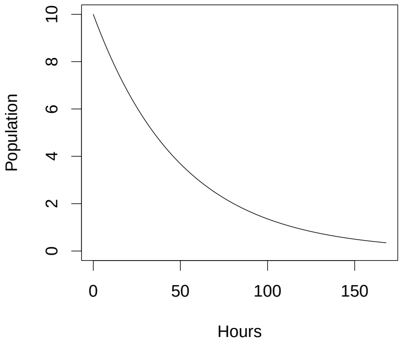

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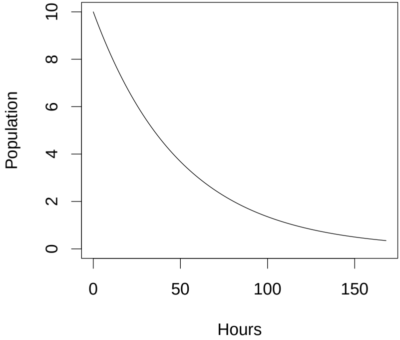

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Subsection 1

Log and linear scales

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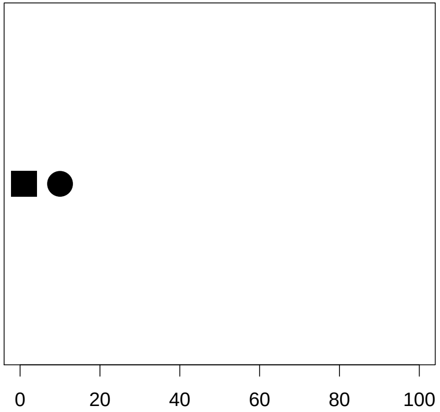

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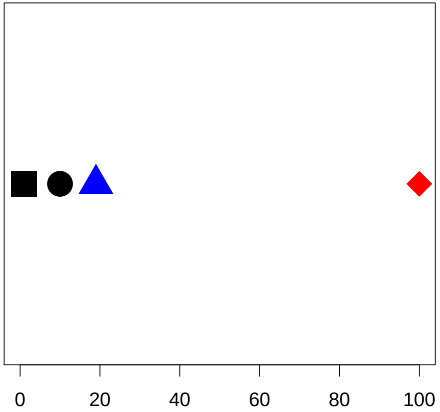

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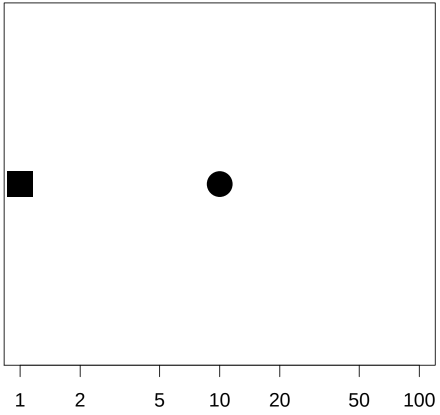

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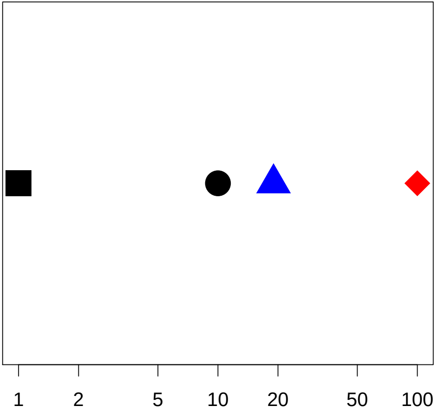

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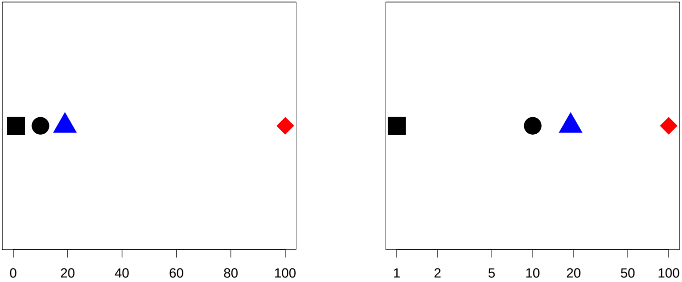
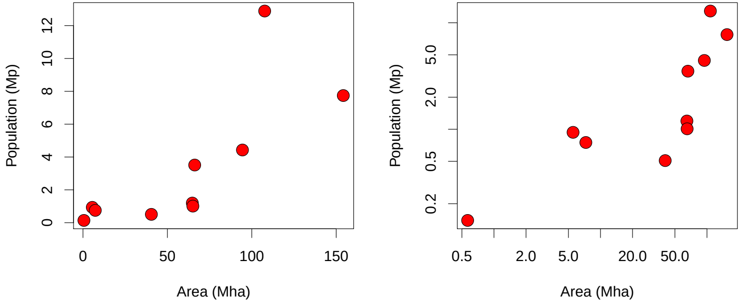
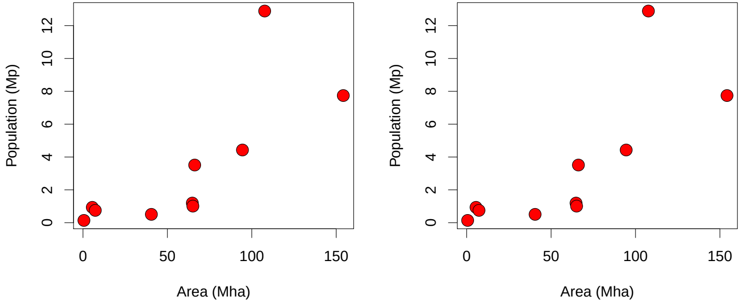


There is only one log scale; it doesn't matter which base you use!

Canadian provinces



Canadian provinces plus Canada?



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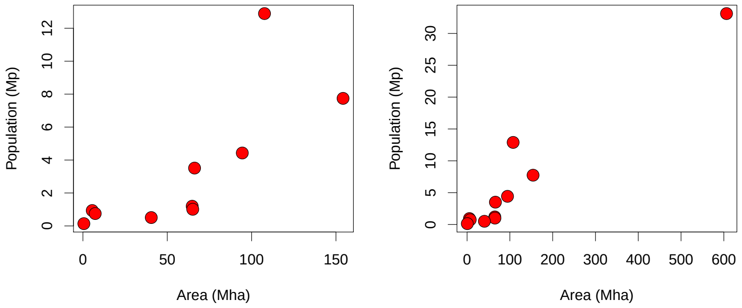

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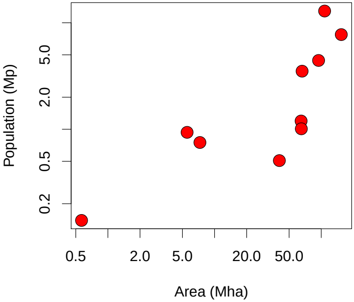

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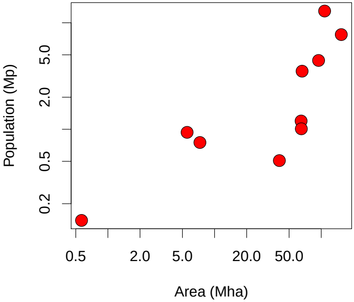

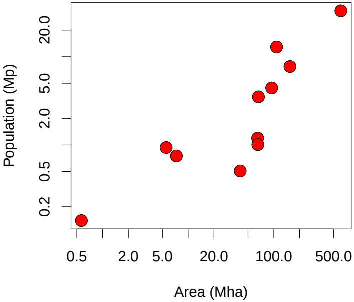

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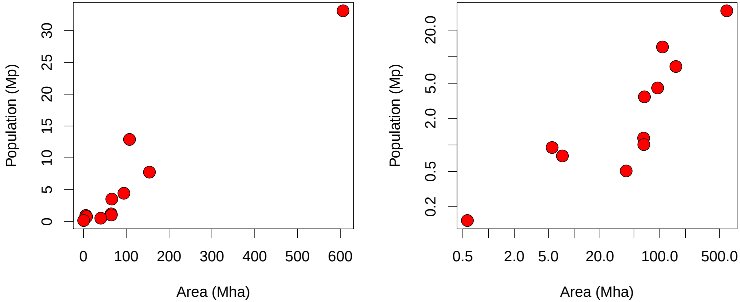
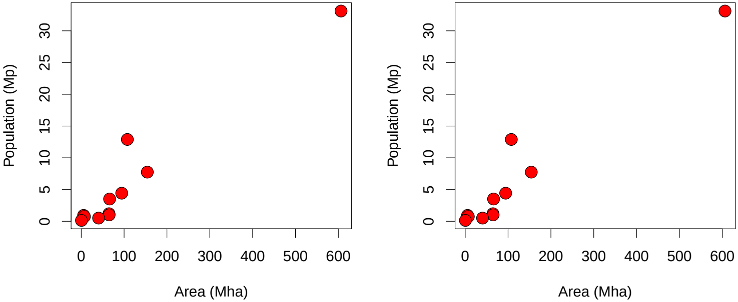


Canada plus room 1105?



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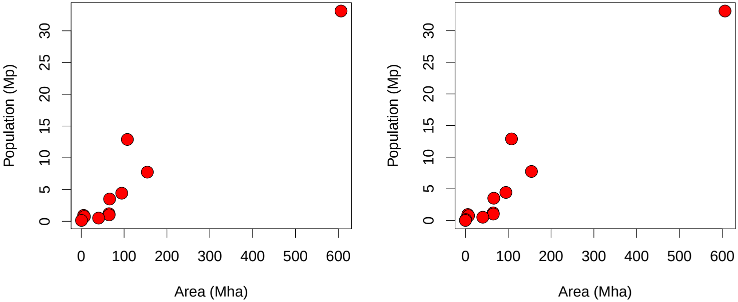

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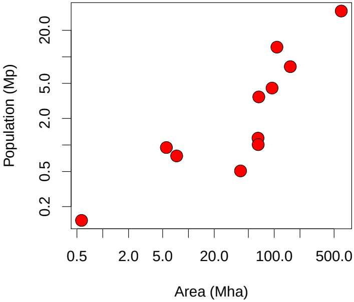

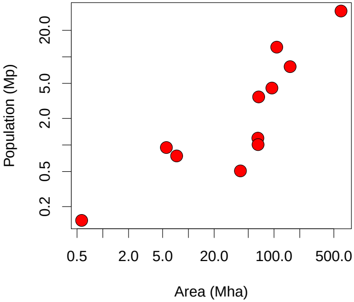

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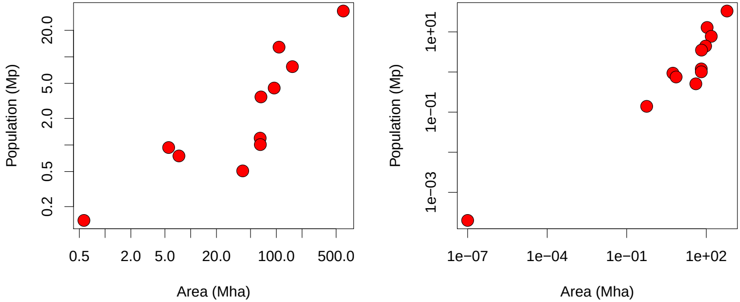

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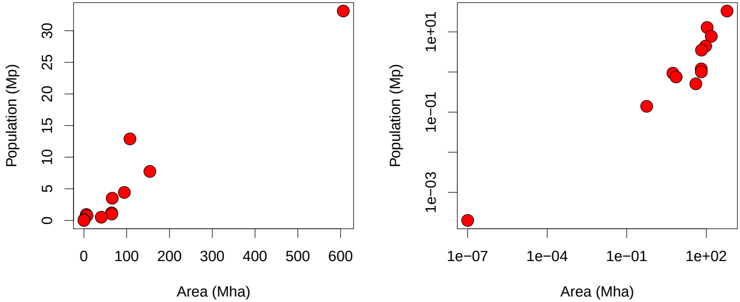

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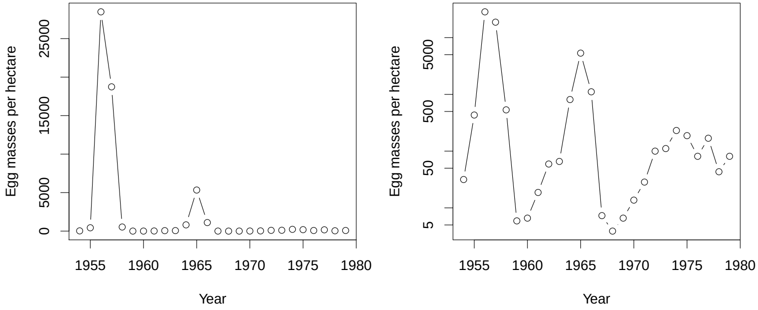

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Subsection 2

Time scales

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Outline

Subsection 1

Dynamical models

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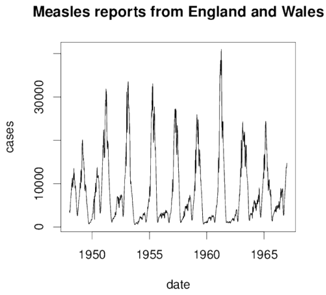

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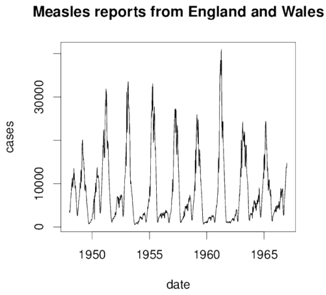

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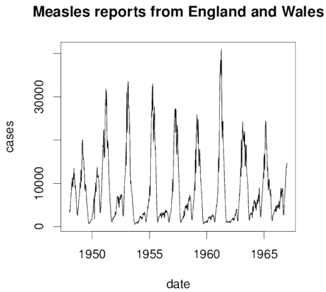

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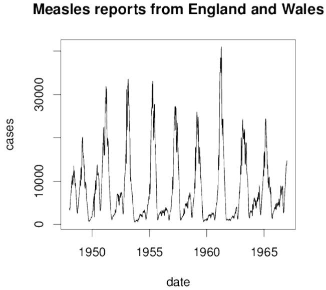

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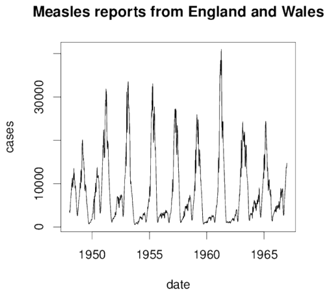

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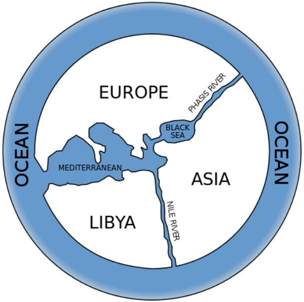

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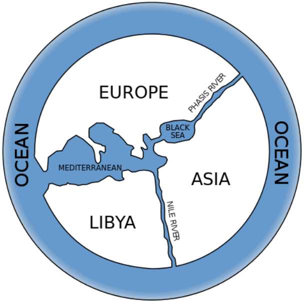

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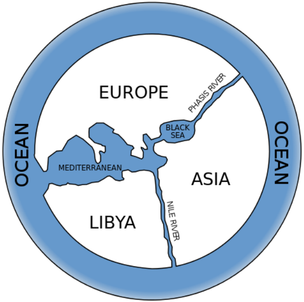

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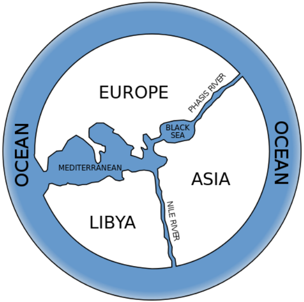

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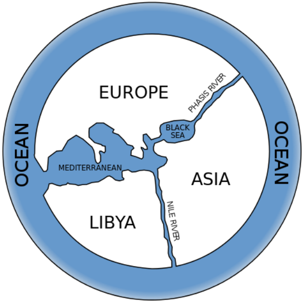

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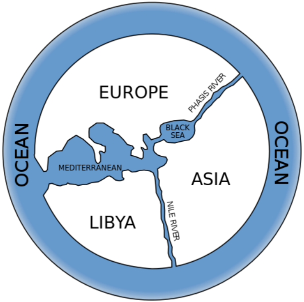

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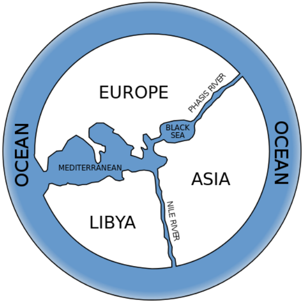

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Subsection 2

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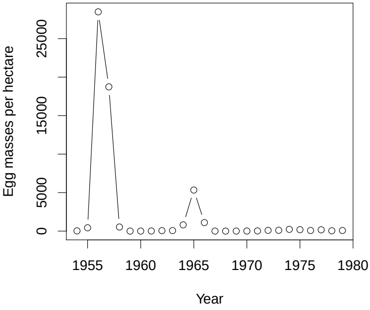

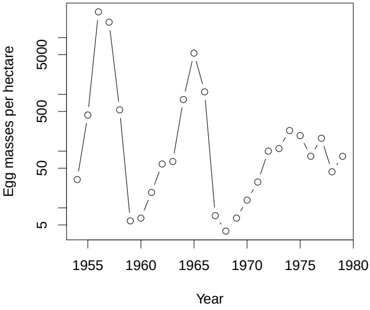

Moth example

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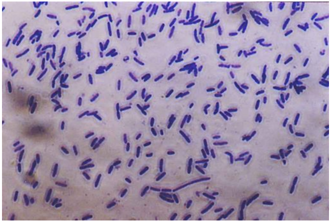

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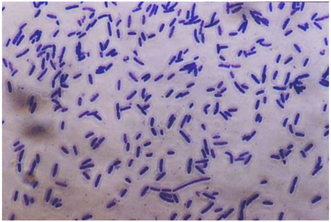

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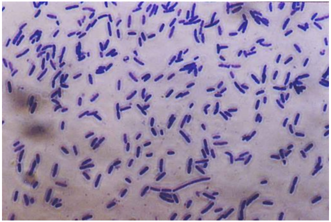

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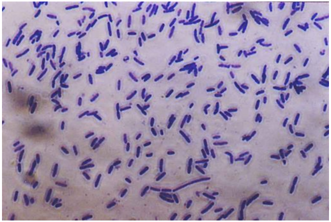

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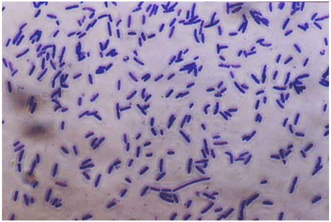

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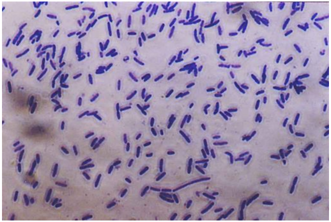

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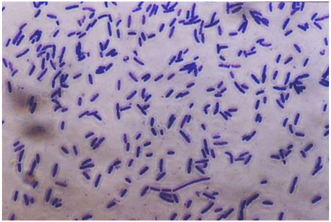

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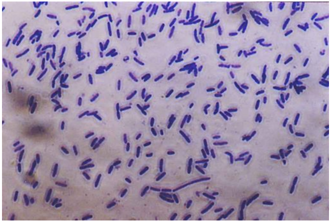

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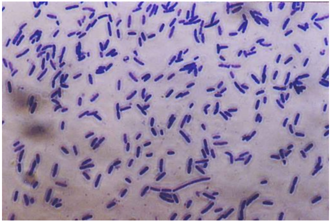

Bacteria

- State variables
  - * Number of bacteria/ml
- Parameters
  - * Division rate, death rate, washout rate
- Census time
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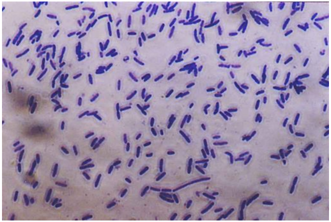

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Subsection 3

A simple discrete-time model

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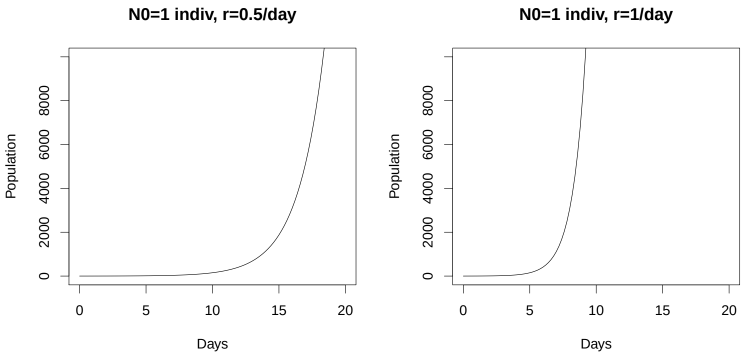

N0=1 indiv, r=0.5/day
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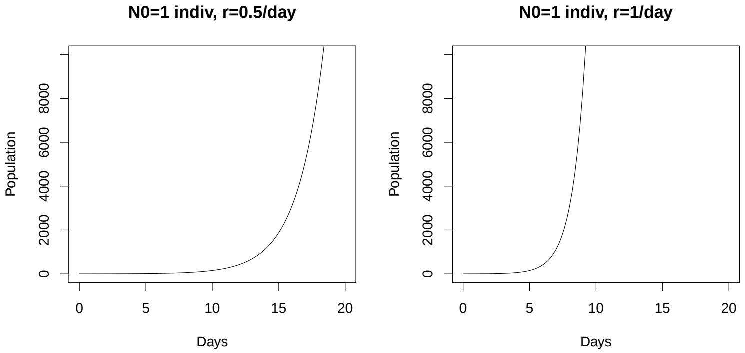

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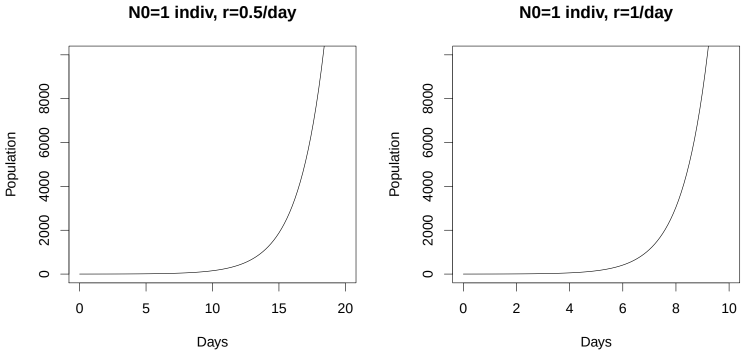

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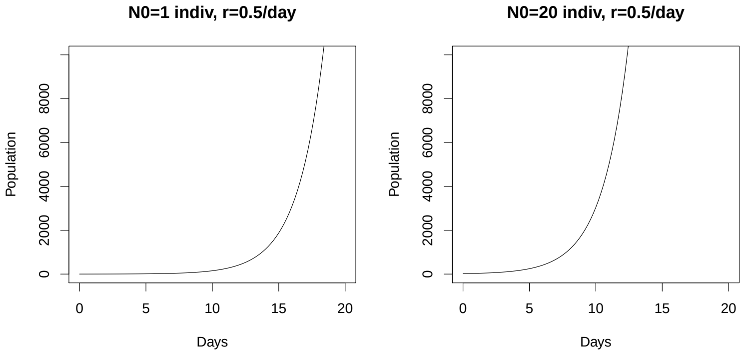

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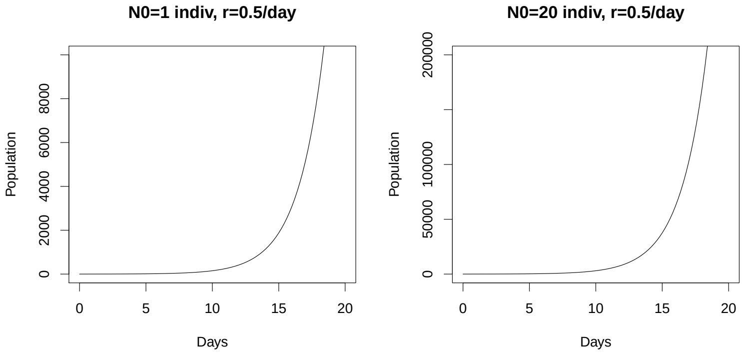

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Outline

Subsection 1

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Changing growth rates

- Poll: What sort of factors can make species growth rates change?
  - * Seasonality
  - * Environmental changes
  - * Competition within species
  - * Competition between species
  - * Predators and diseases
  - * Resources (food and space)
  - * Natural disasters

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