UNIT 2 Non-linear population models

Outline

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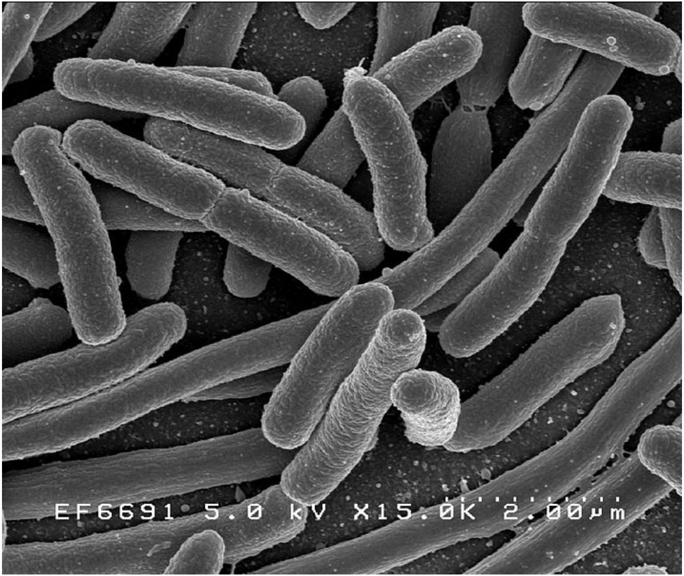

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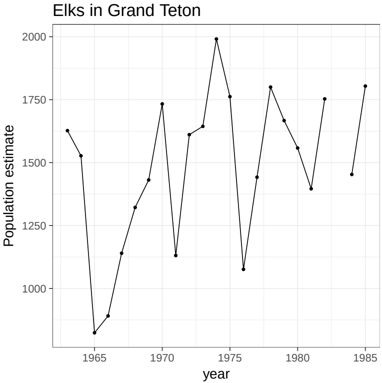
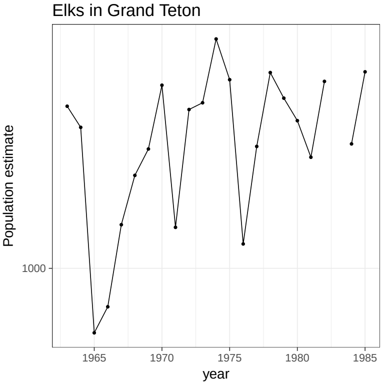
Subsection 1

Population Examples

Elk


Elks in Grand Teton
2000
1750
1500
1250
1000
Population estimate
1965
1970
1975
1980
1985
year


Elks in Grand Teton
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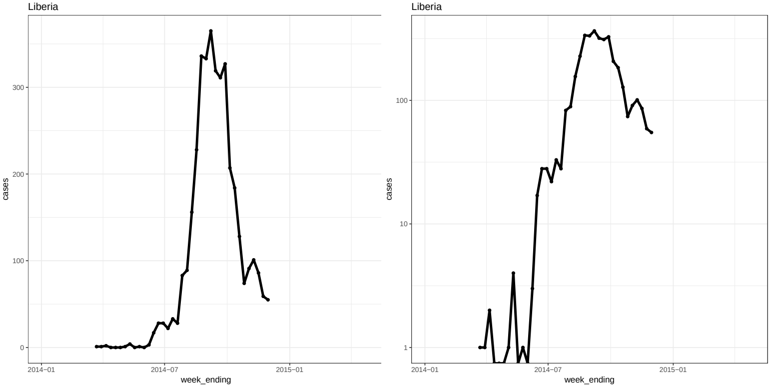
Ebola


Liberia
300
200
100
0
cases
2014−01
2014−07
2015−01
week_ending
Liberia
100
10
1
cases
2014−01
2014−07
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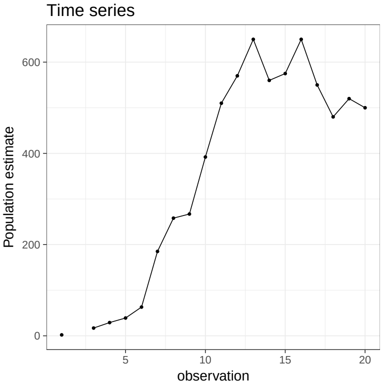
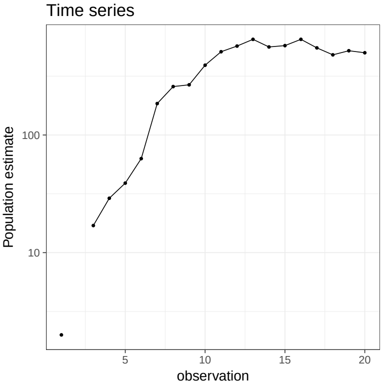
week_ending

Paramecia


Time series
Population estimate
600
400
200
0
5
10
15
20
observation


Time series
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100
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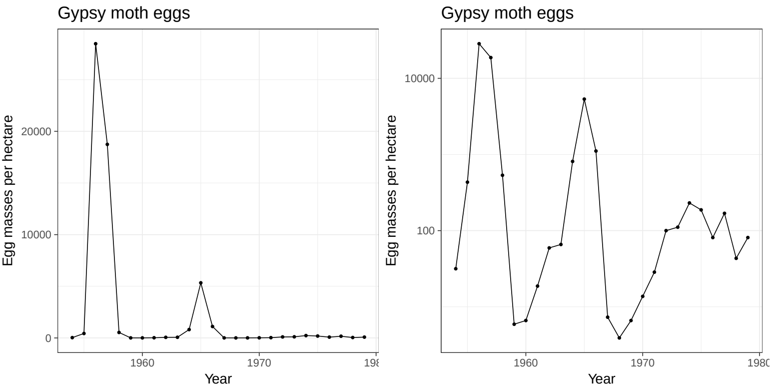
Gypsy moths



Gypsy moth eggs
Egg masses per hectare
20000
10000
0
1960
1970
198
Year
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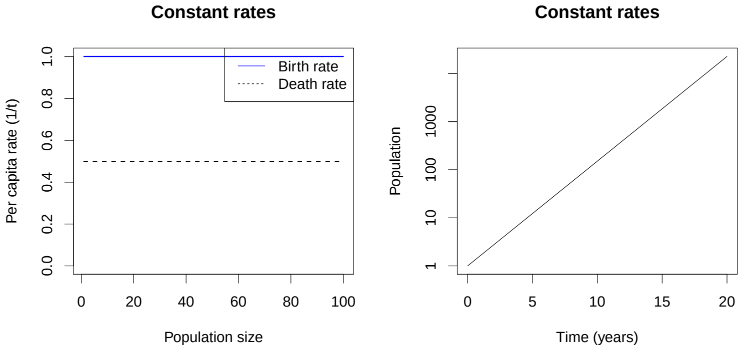

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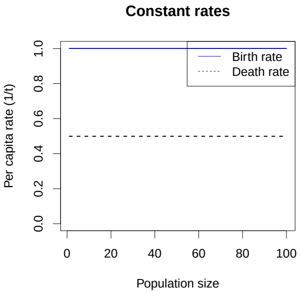

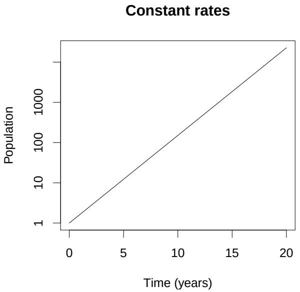

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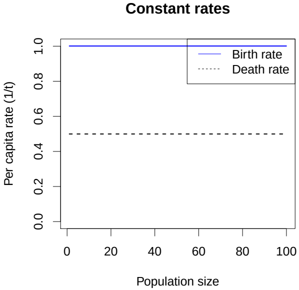

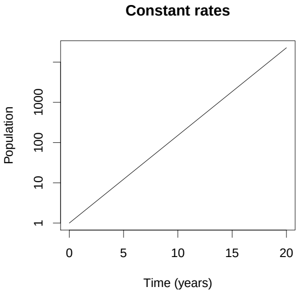

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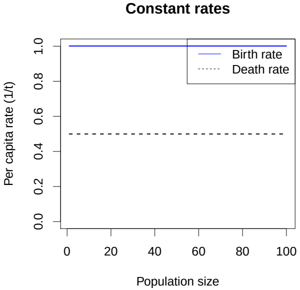

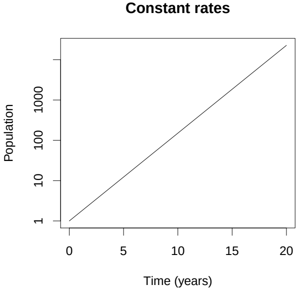

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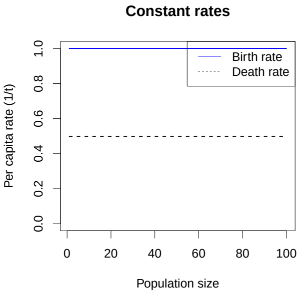

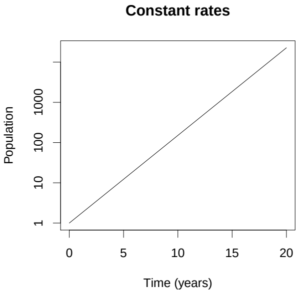

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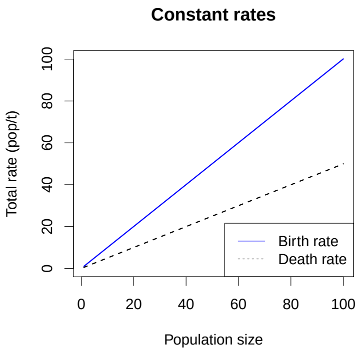

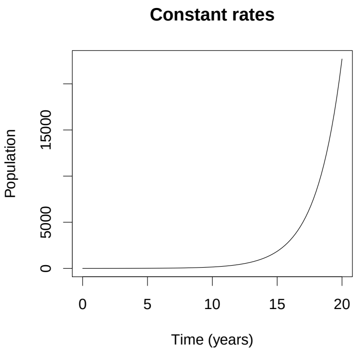

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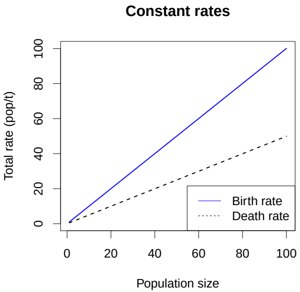

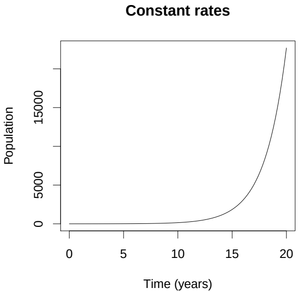

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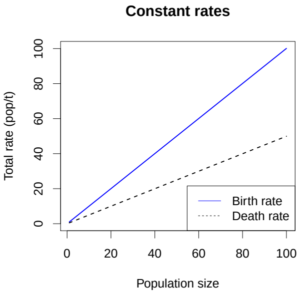

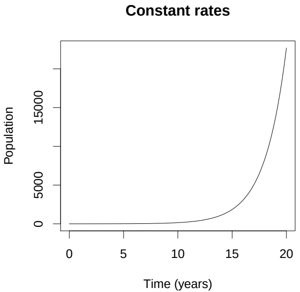

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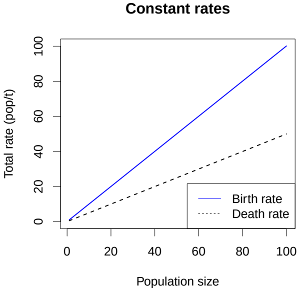

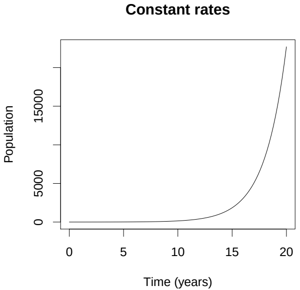

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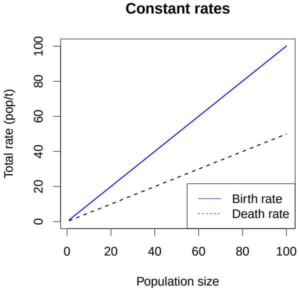

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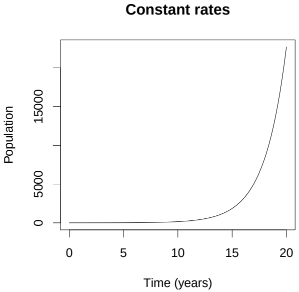

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Birth rates

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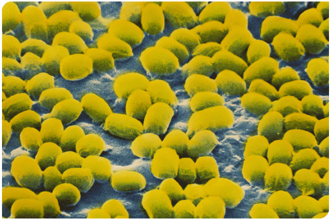

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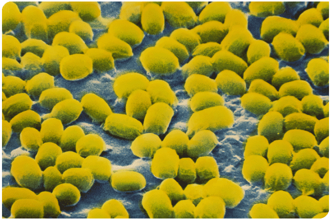

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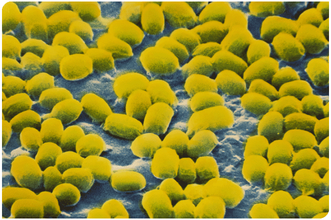

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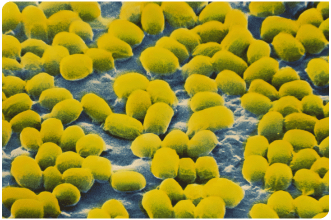

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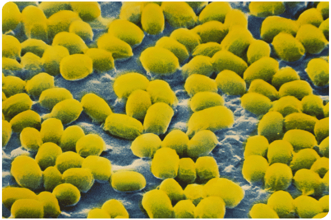

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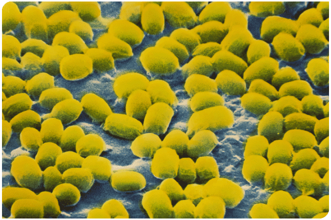

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Subsection 1

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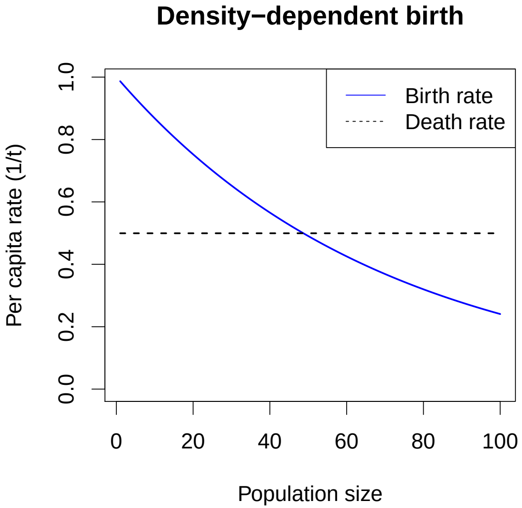

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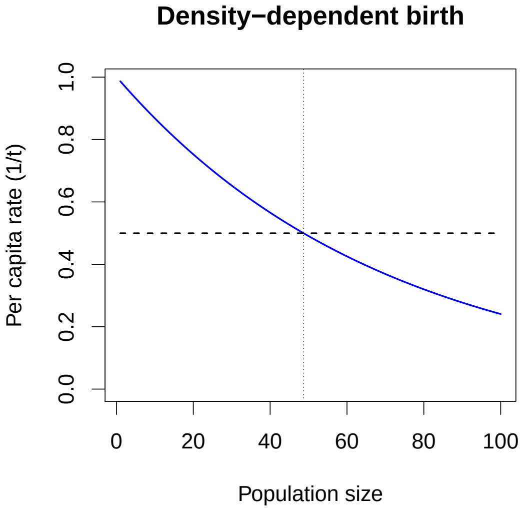

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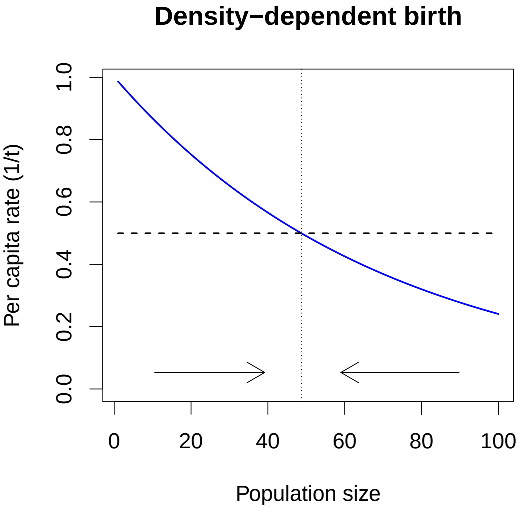

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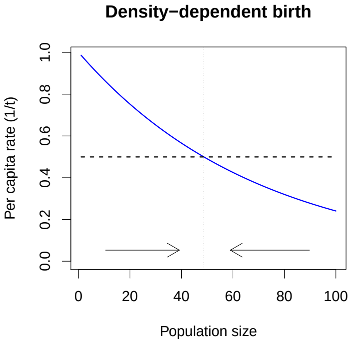

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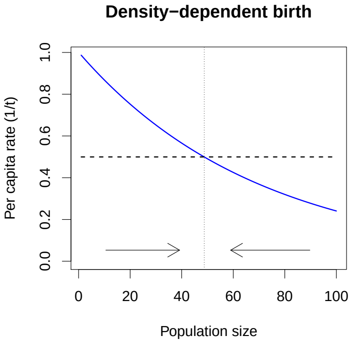

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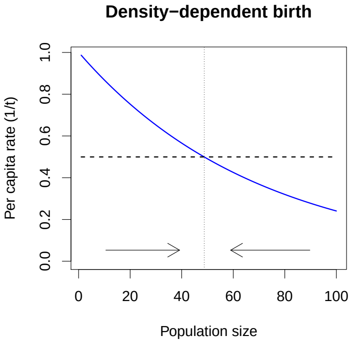

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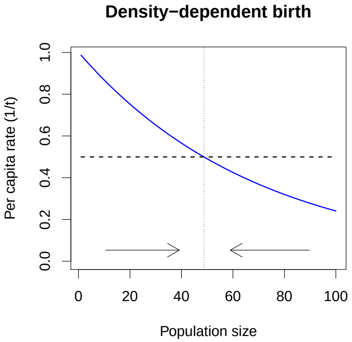

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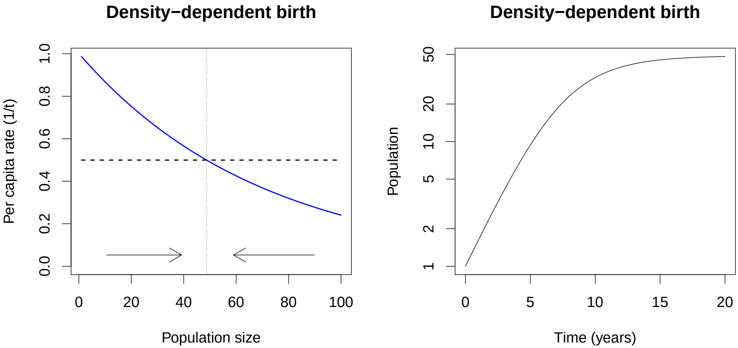

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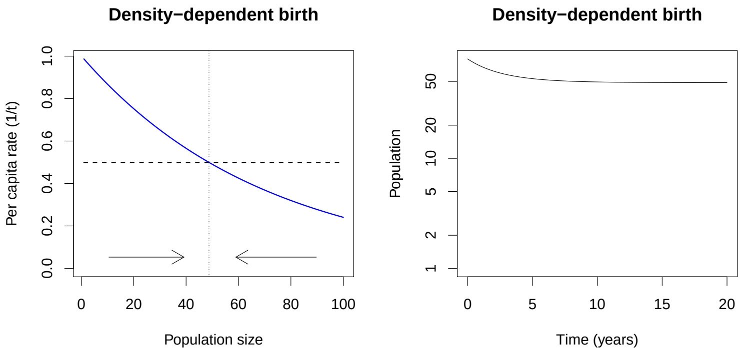

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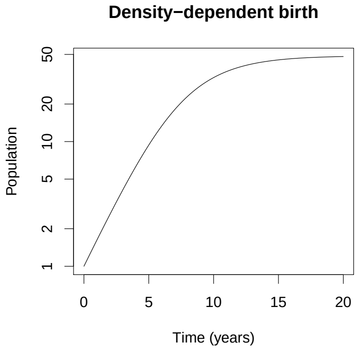
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Subsection 2

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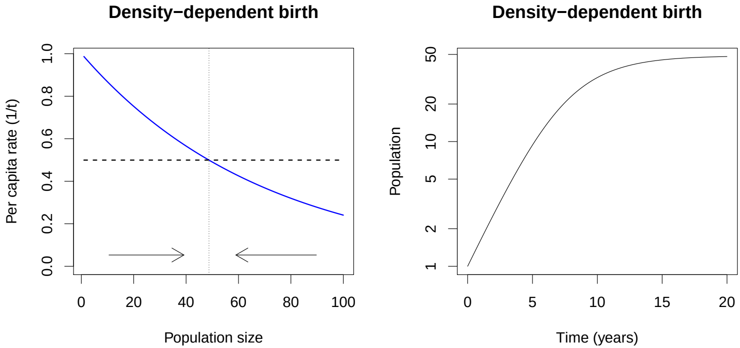

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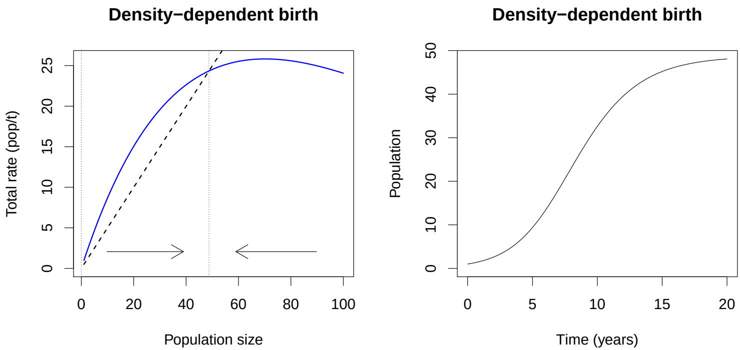

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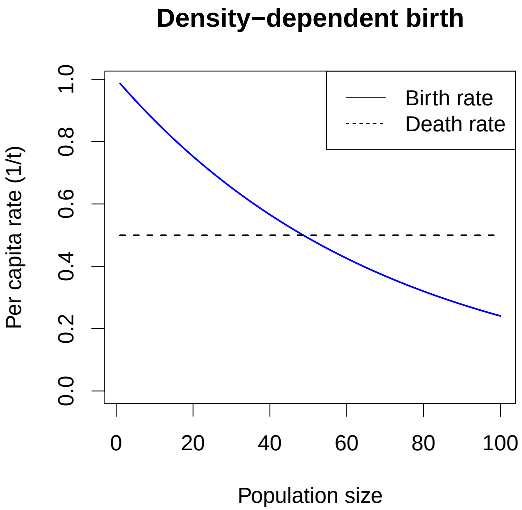

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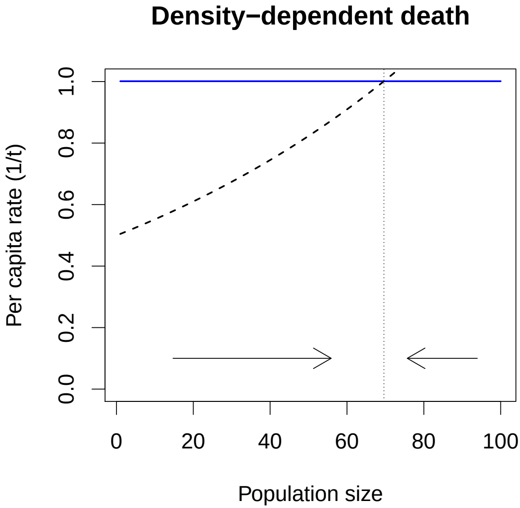

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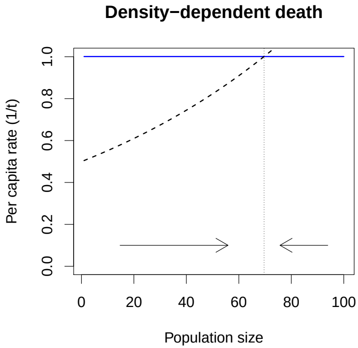

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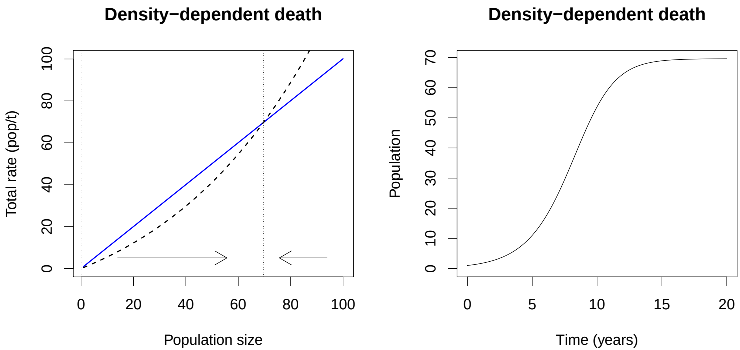

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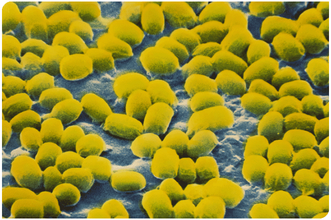

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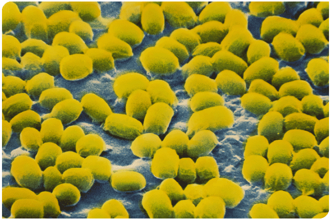

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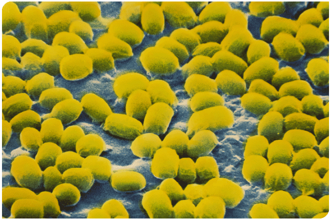

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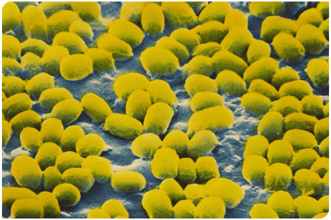

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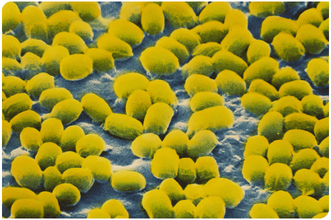

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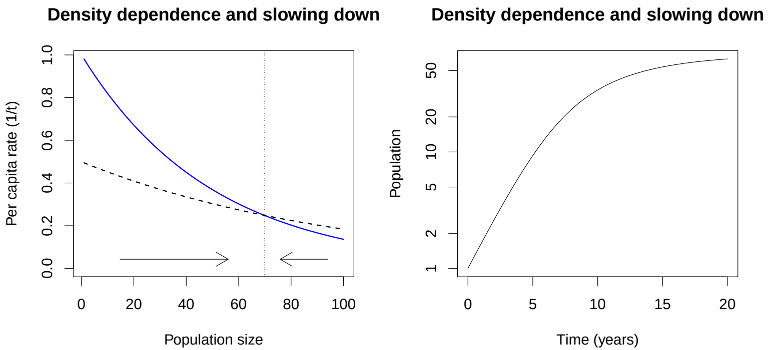

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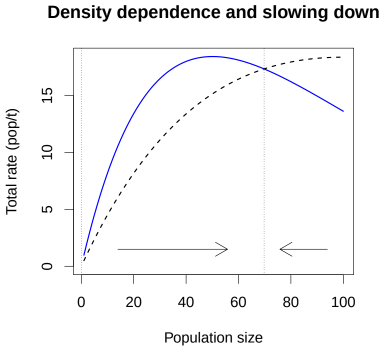

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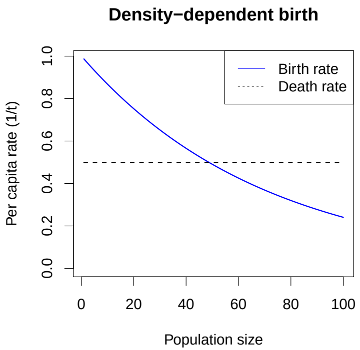

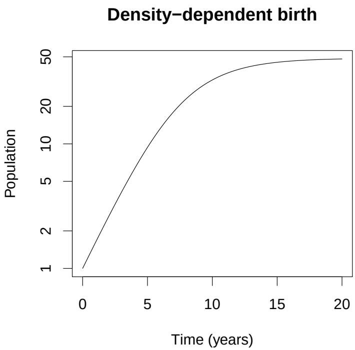

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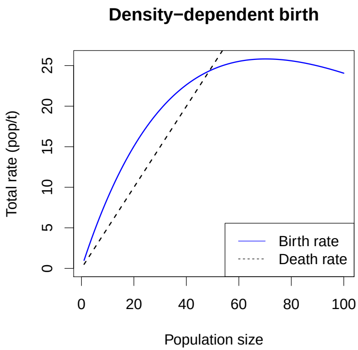

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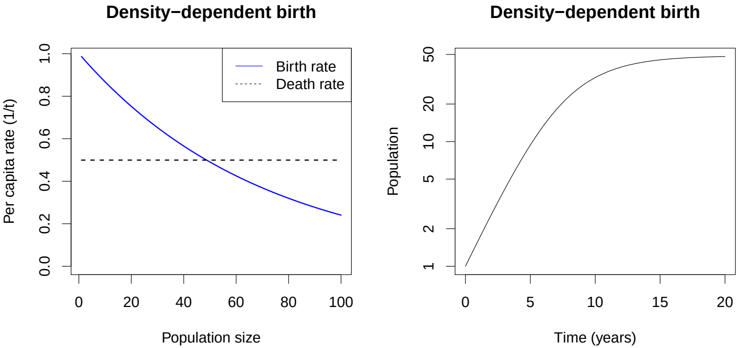

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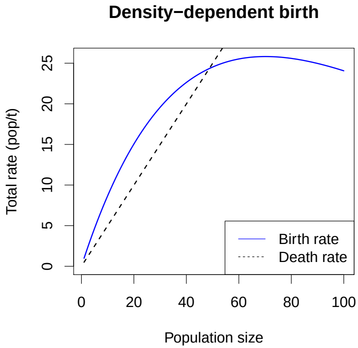

Subsection 3

Equilibria and time scales

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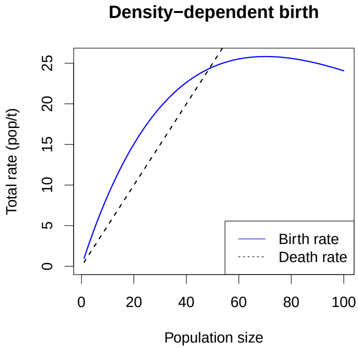

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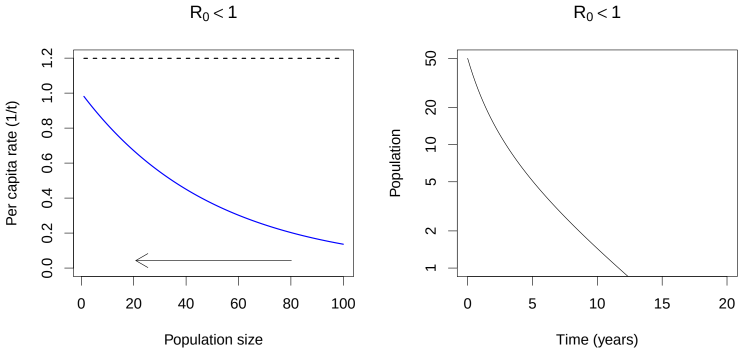

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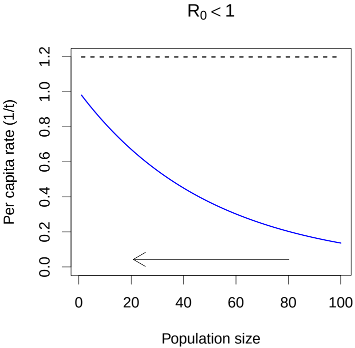

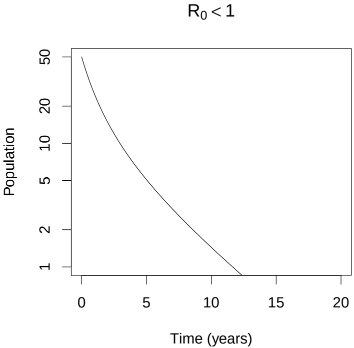

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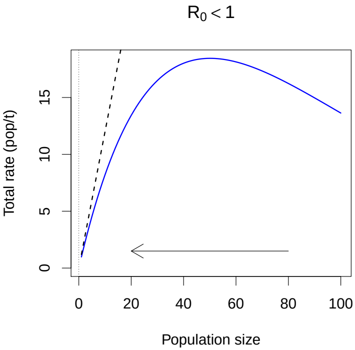

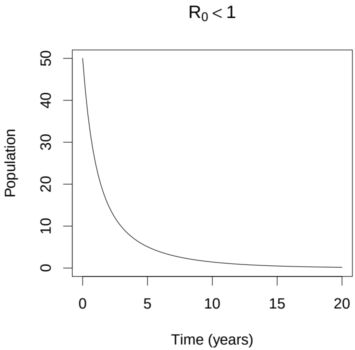

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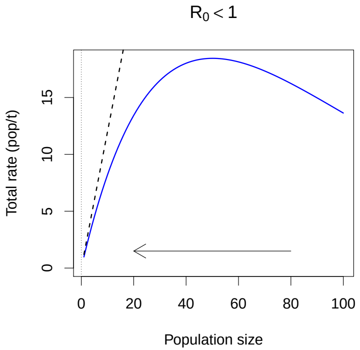

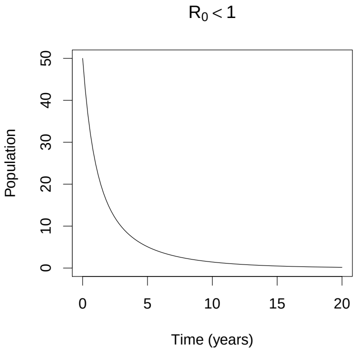

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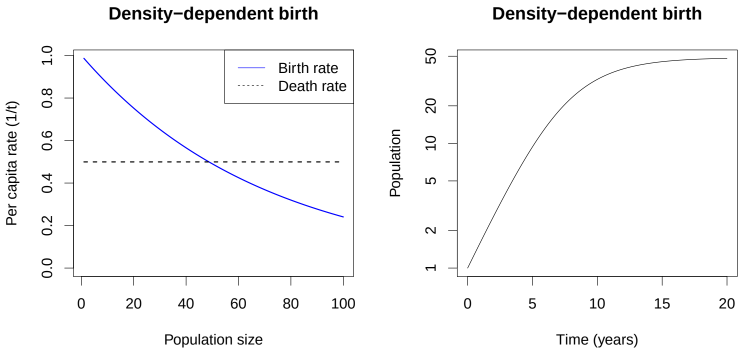

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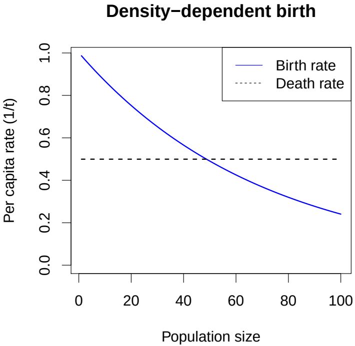

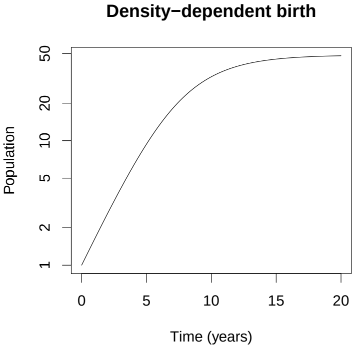

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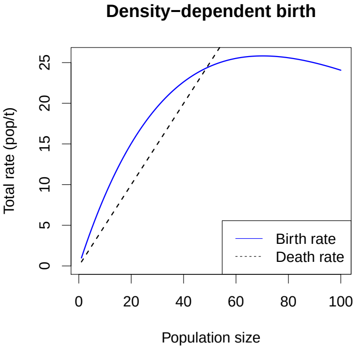

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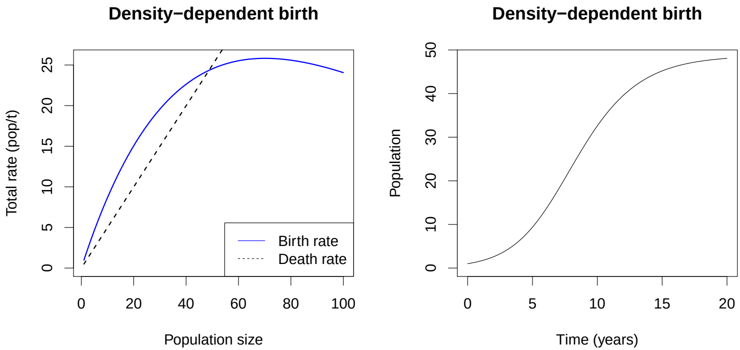

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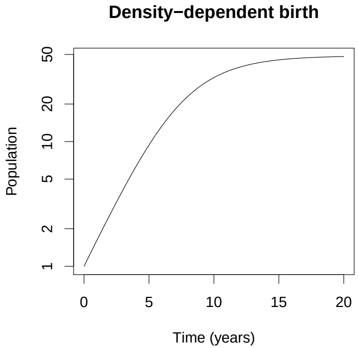
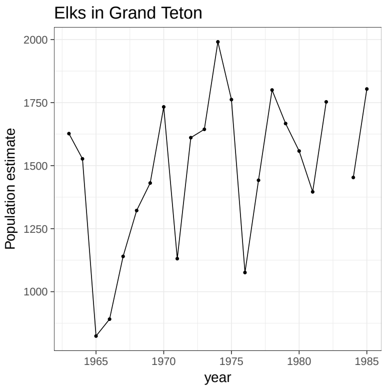
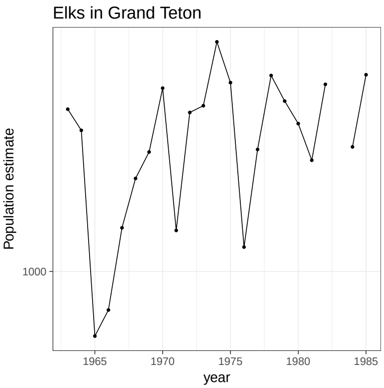


Elk



Elks in Grand Teton
Population estimate
2000
1750
1500
1250
1000
1965
1970
1975
1980
1985
year



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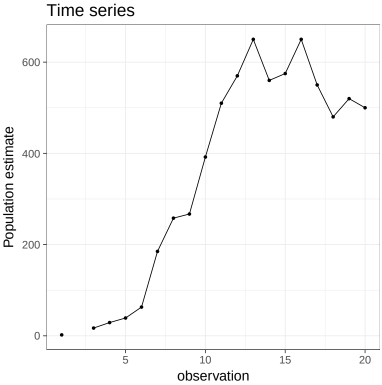
Time series
Population estimate
0
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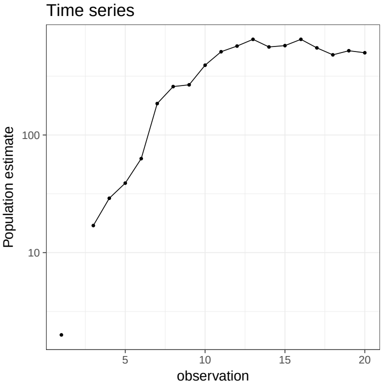
400
600
5
10
15
20
observation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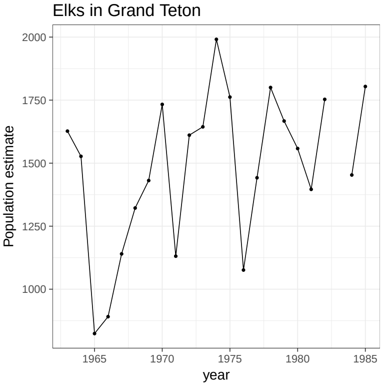
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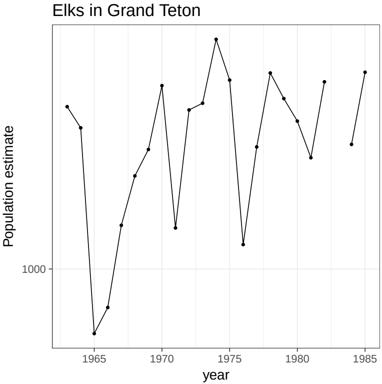
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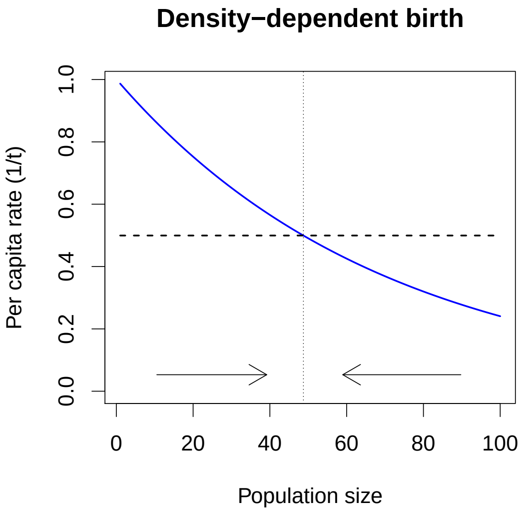

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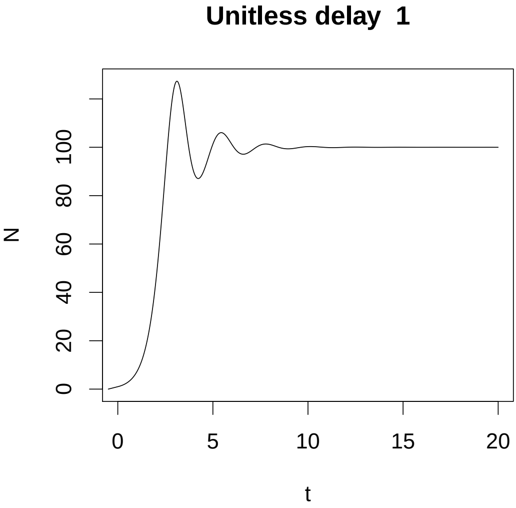

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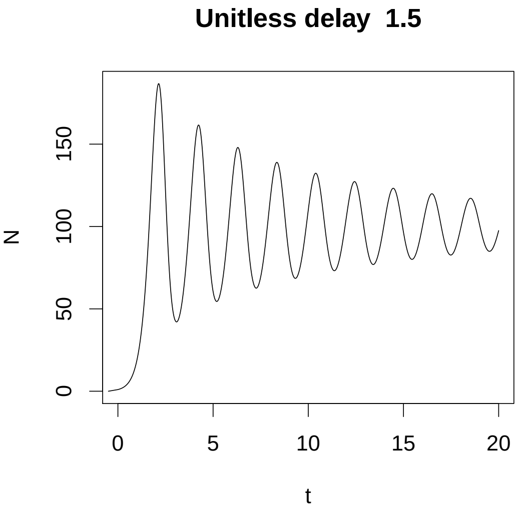

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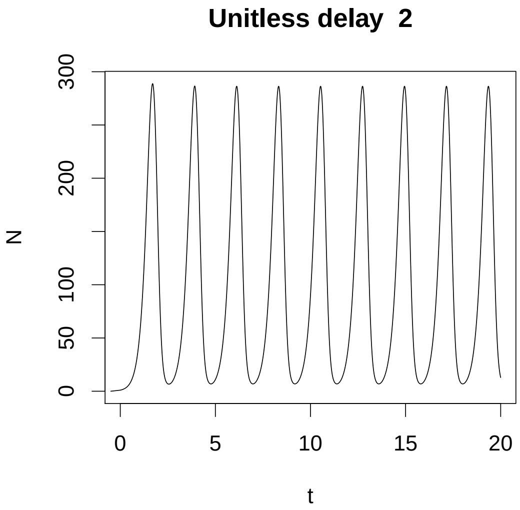

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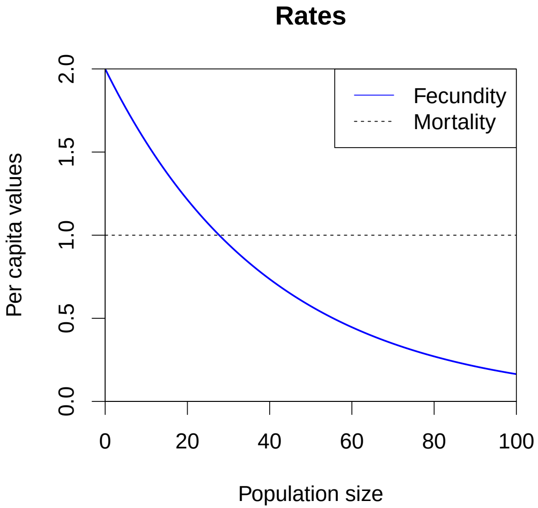

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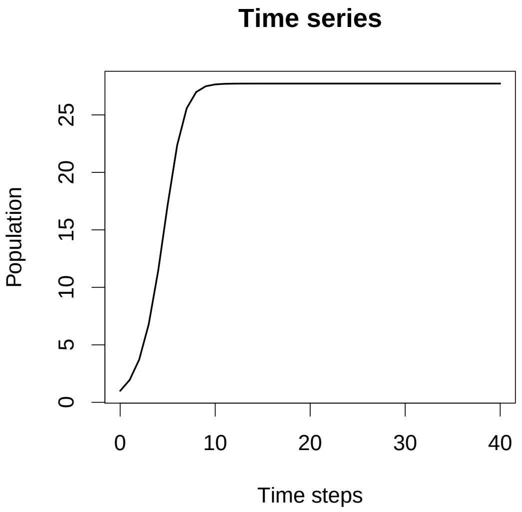
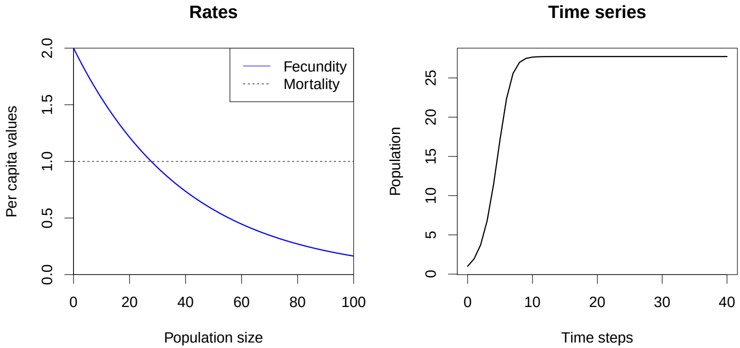
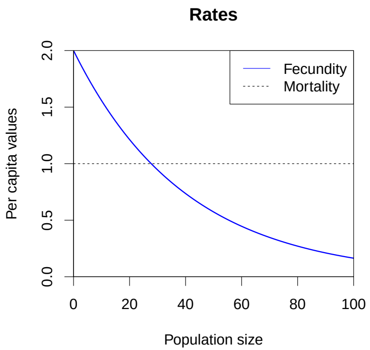


We expect simple dynamics

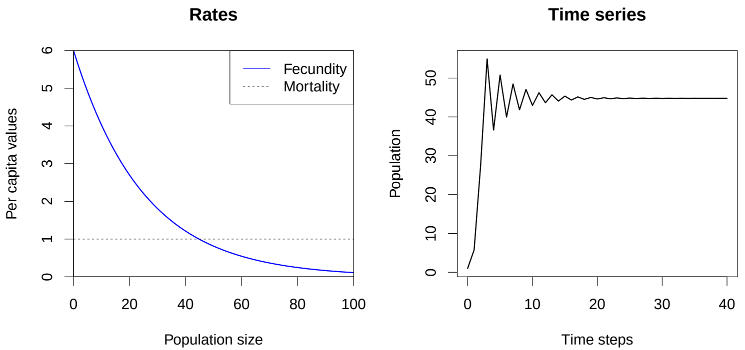


What dynamics do we get?

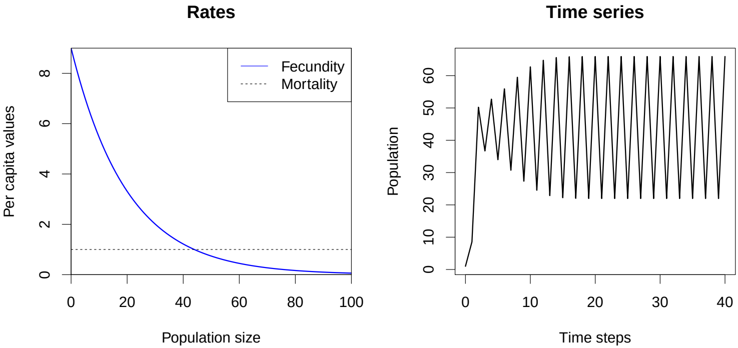
Simple dynamics



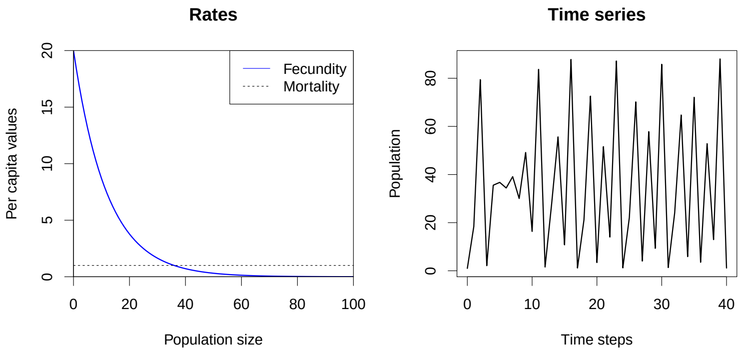
Damped oscillations



Persistent oscillations



Lots of other behaviours



Subsection 3

Interpreting complex behaviour

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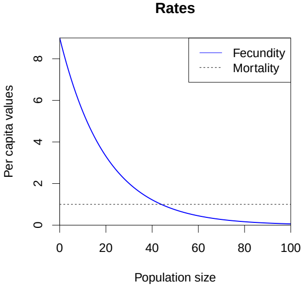

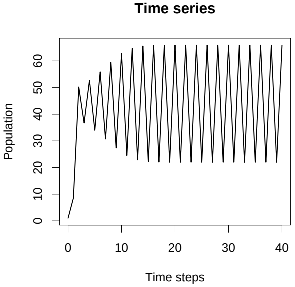

Interpreting complex behaviour

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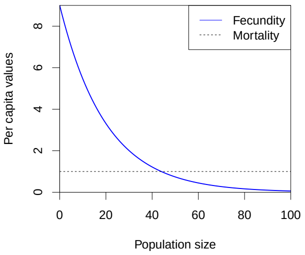
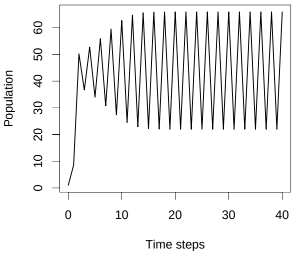


Time series



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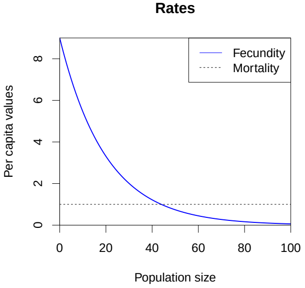

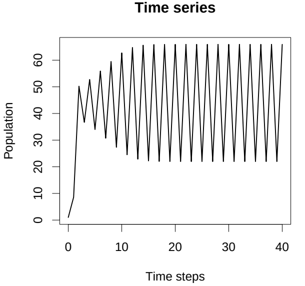

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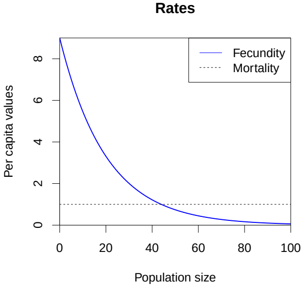

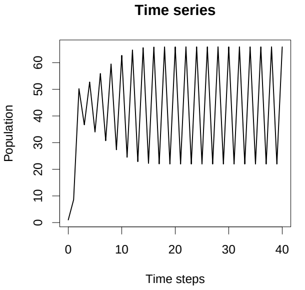

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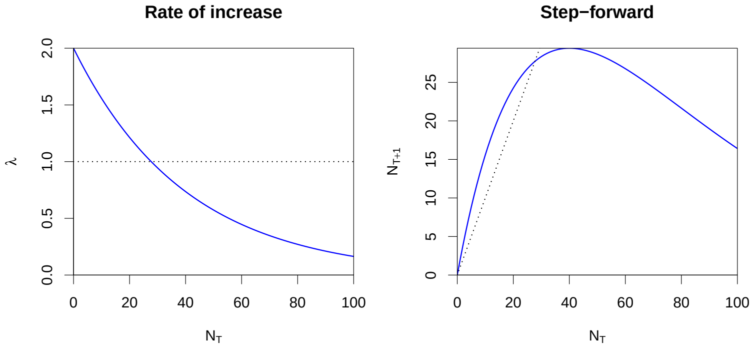

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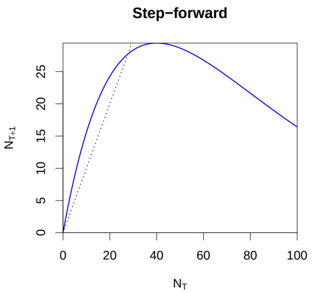

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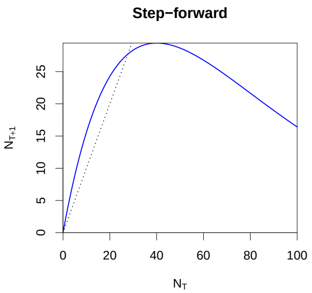

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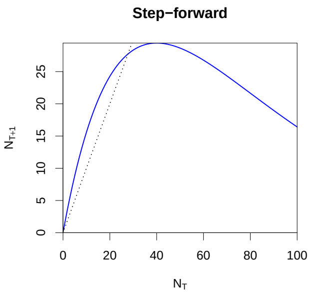

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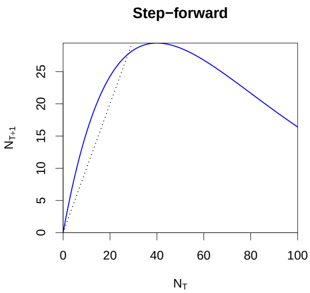

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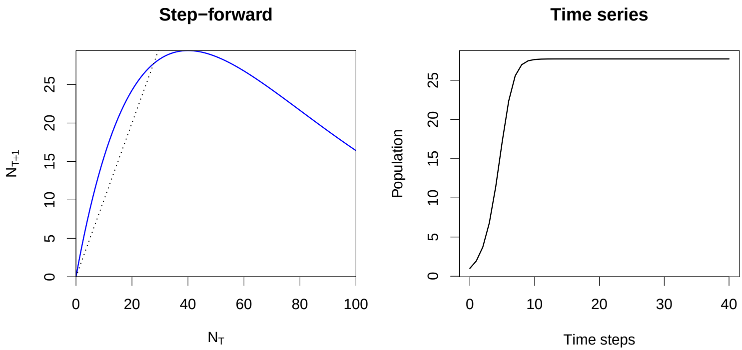

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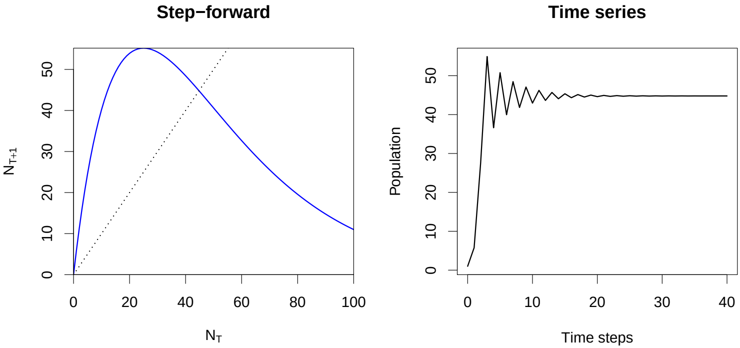

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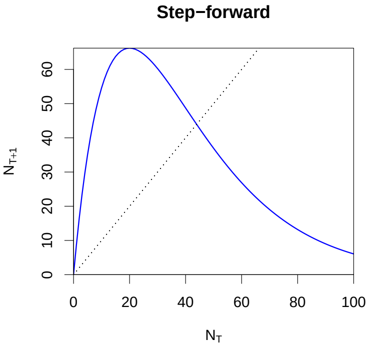

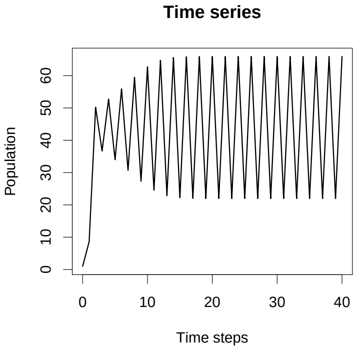

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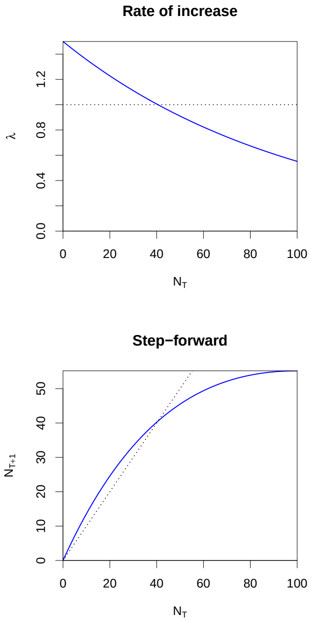

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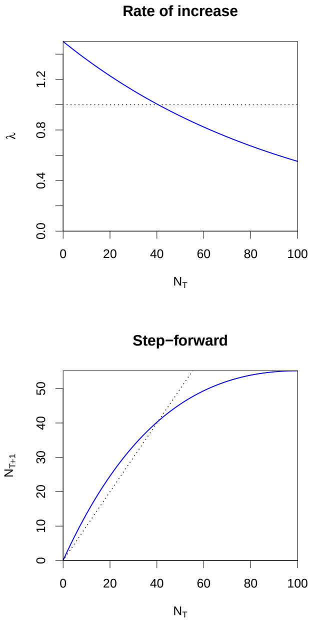

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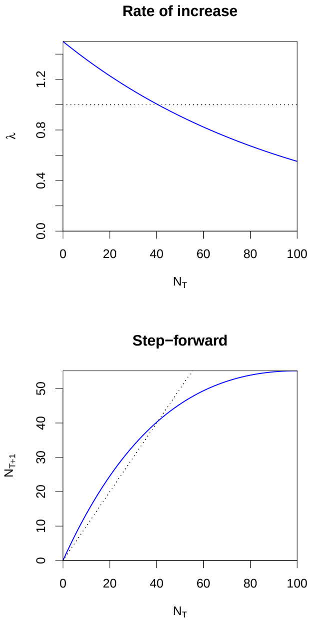

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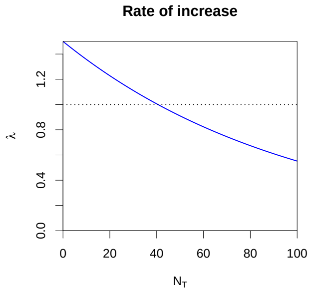
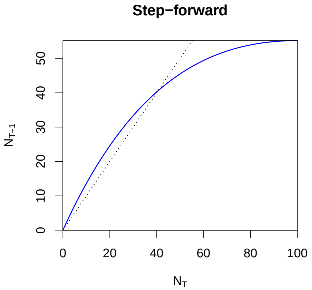
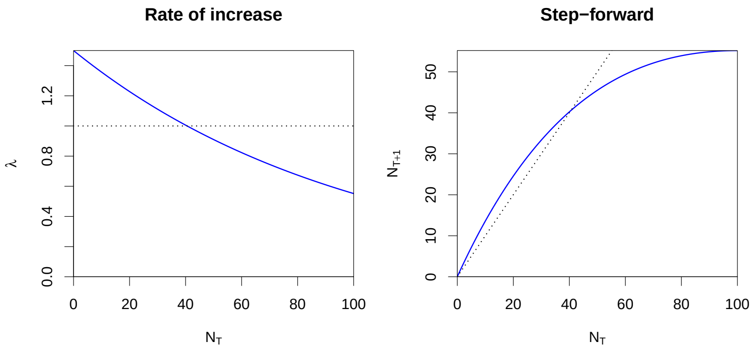




Contest regulation



Songbirds

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- Poll: Which is more likely to work out as a scramble?
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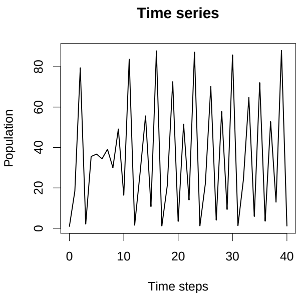

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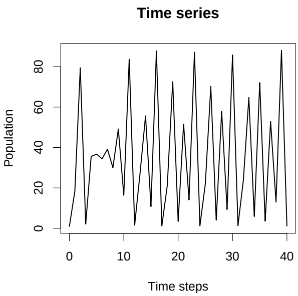

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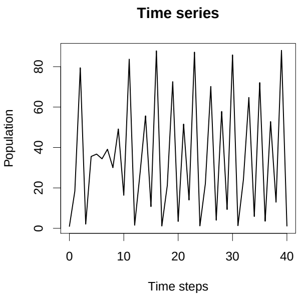

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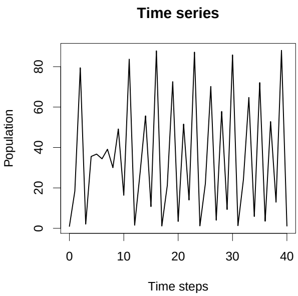

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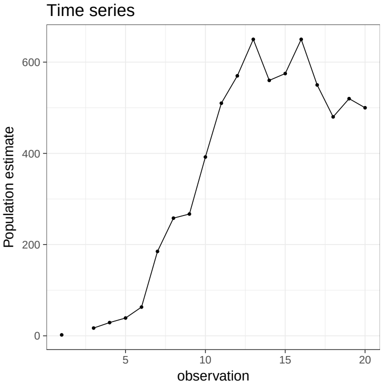
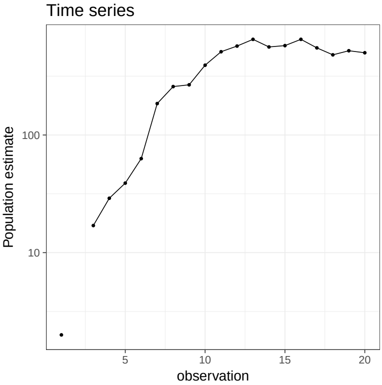
Paramecia


Time series
Population estimate
600
400
200
0
5
10
15
20
observation


Time series
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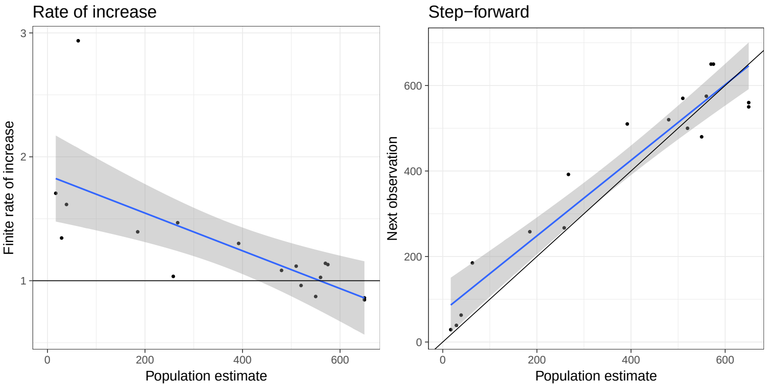
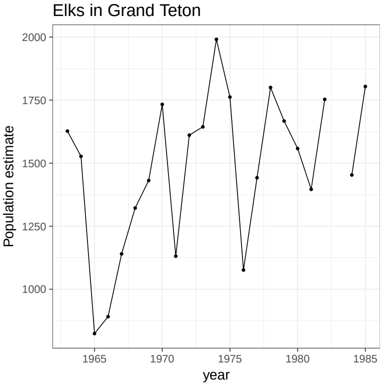
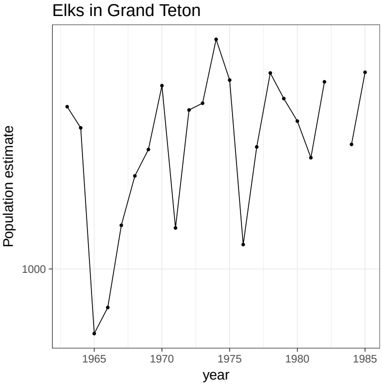

Rate of increase
Finite rate of increase
3
2
1
0
200
400
600
Population estimate
Step−forward
Next observation
600
400
200
0
0
200
400
600
Population estimate

Elk


Elks in Grand Teton
2000
1750
1500
1250
1000
Population estimate
1965
1970
1975
1980
1985
year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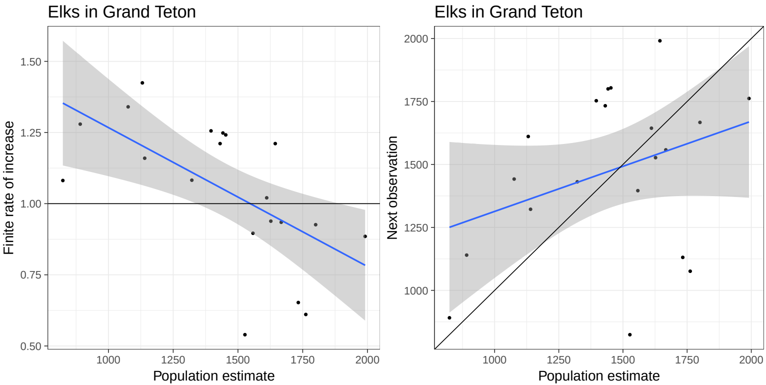

Elks in Grand Teton
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1.00
0.75
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Population estimate
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Subsection 1

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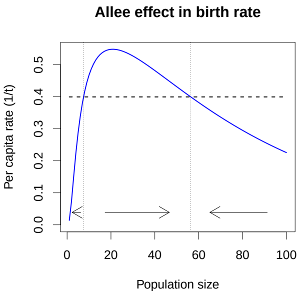

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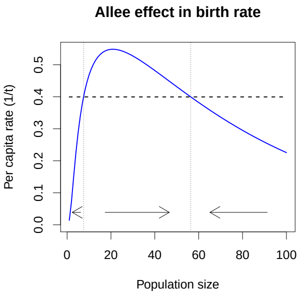

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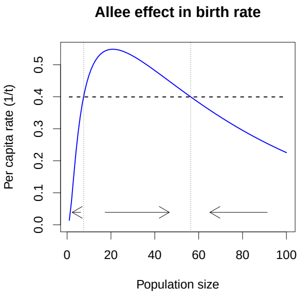

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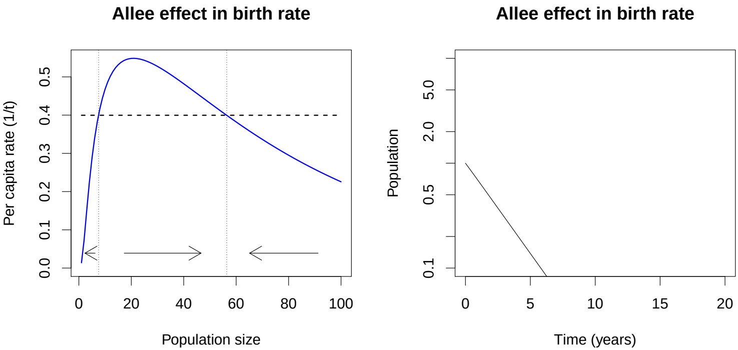

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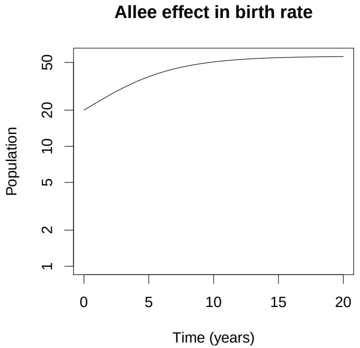

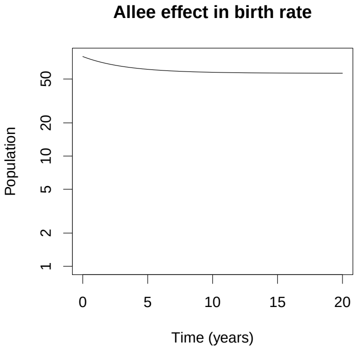

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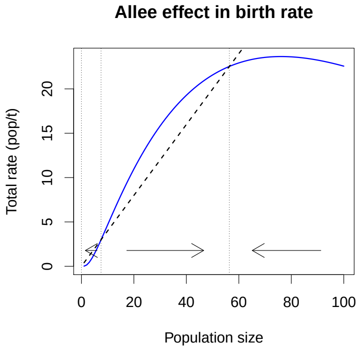

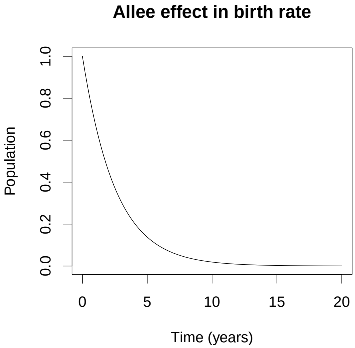

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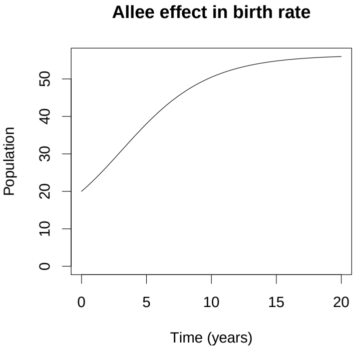
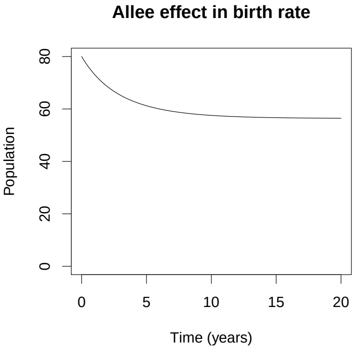
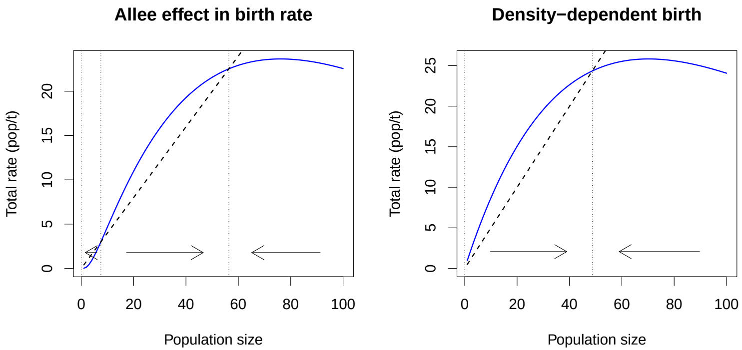




Population comparison



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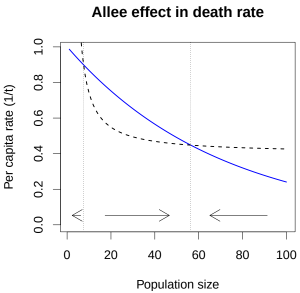

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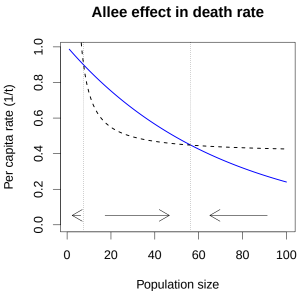

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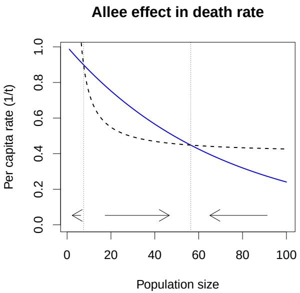

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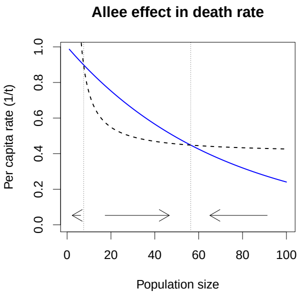

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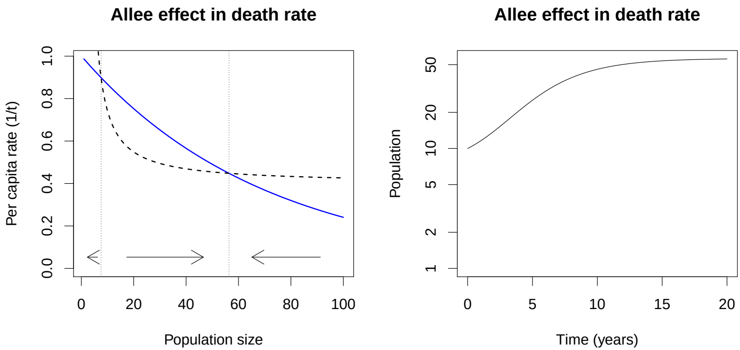

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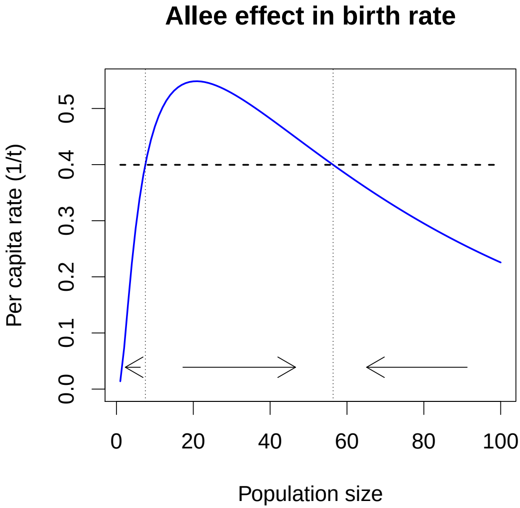

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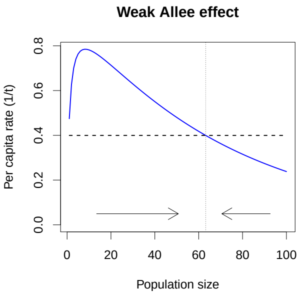

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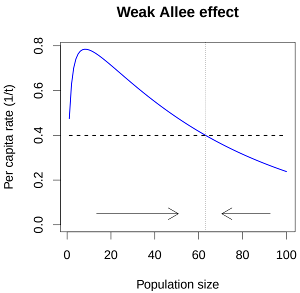

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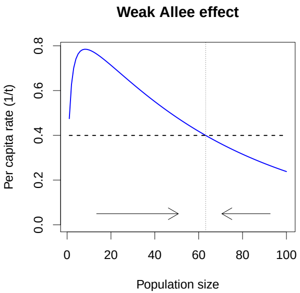

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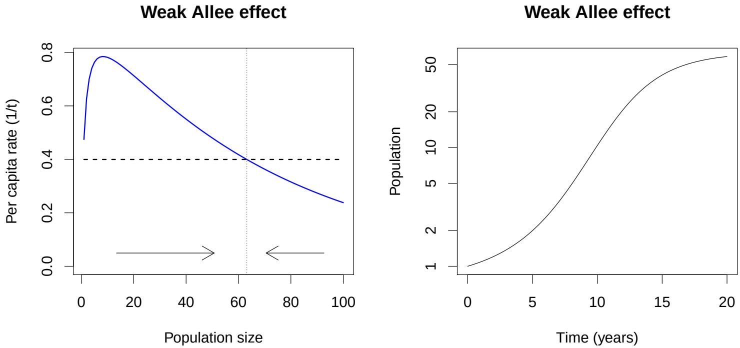

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Subsection 2

Stochastic effects

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Simulations

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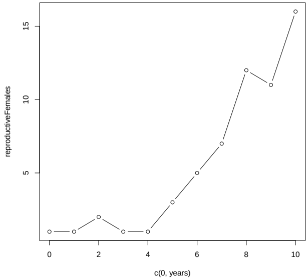

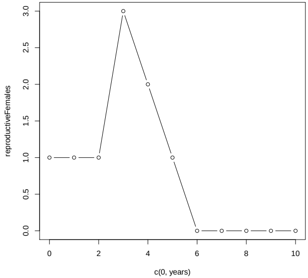

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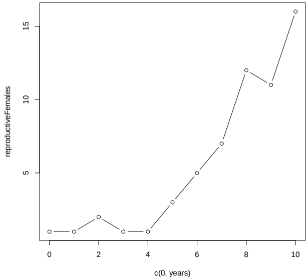

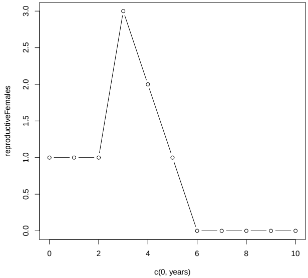

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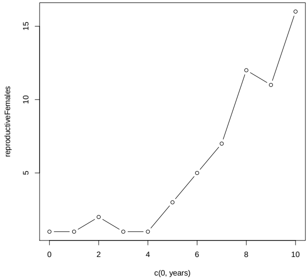

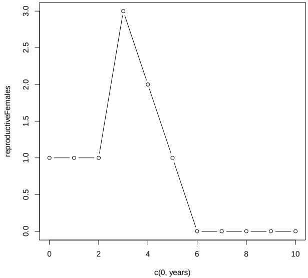

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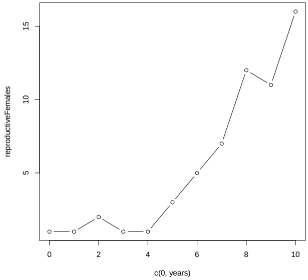

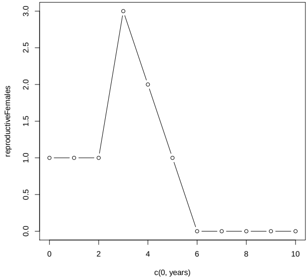

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