

UNIT 3: Structured populations

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Subsection 1

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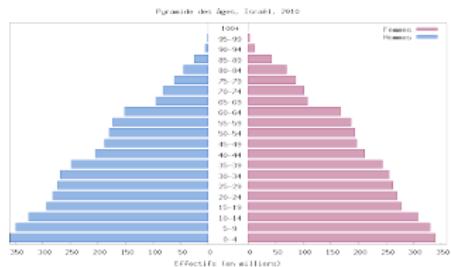
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Subsection 2

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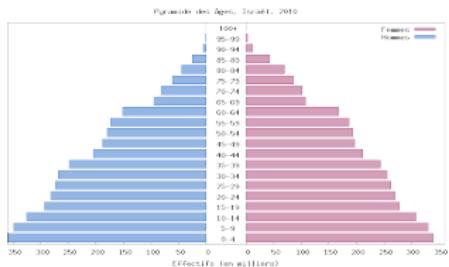
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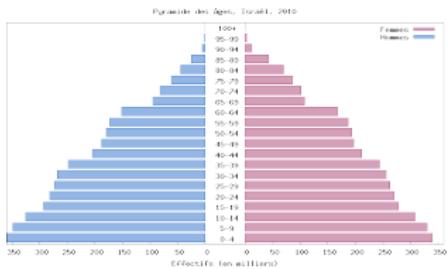
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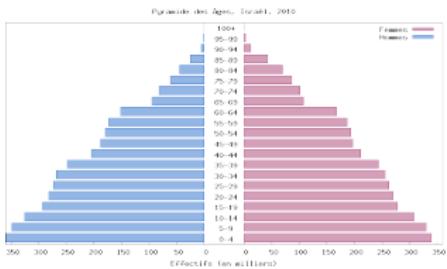
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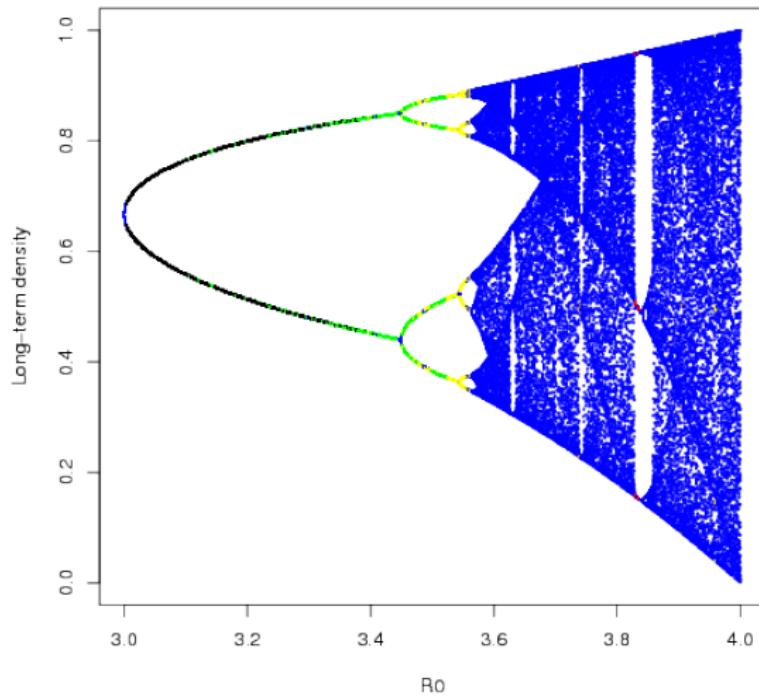
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Discrete vs. continuous time

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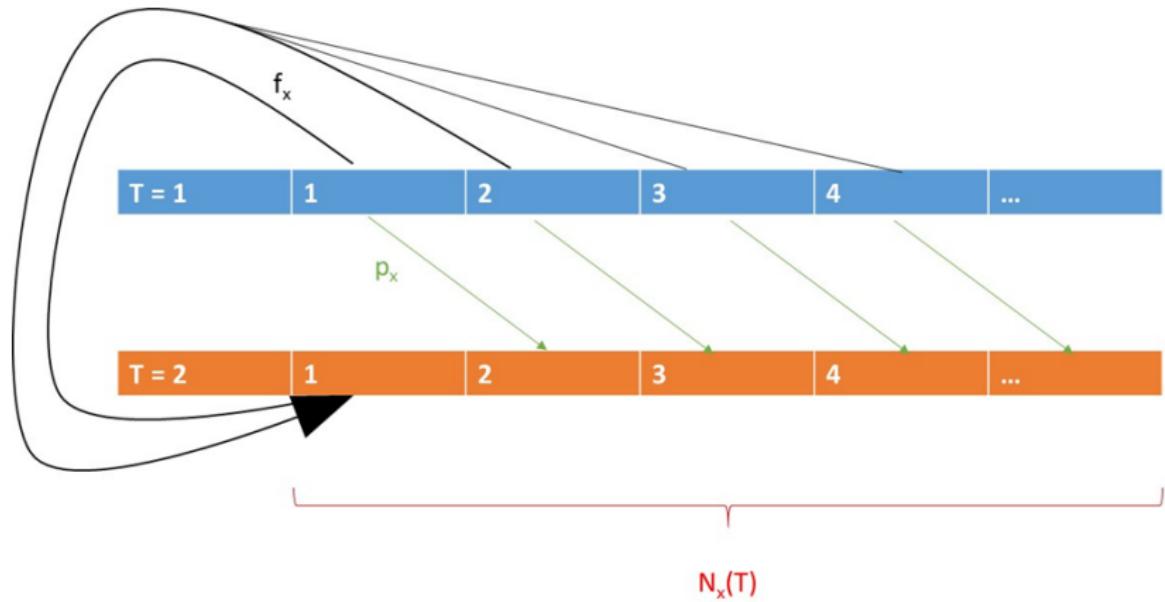
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Subsection 1

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Subsection 1

Examples

Dandelion example



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Dandelion life table

x	f_x	p_x	ℓ_x	$\ell_x f_x$
1				
2				
R				

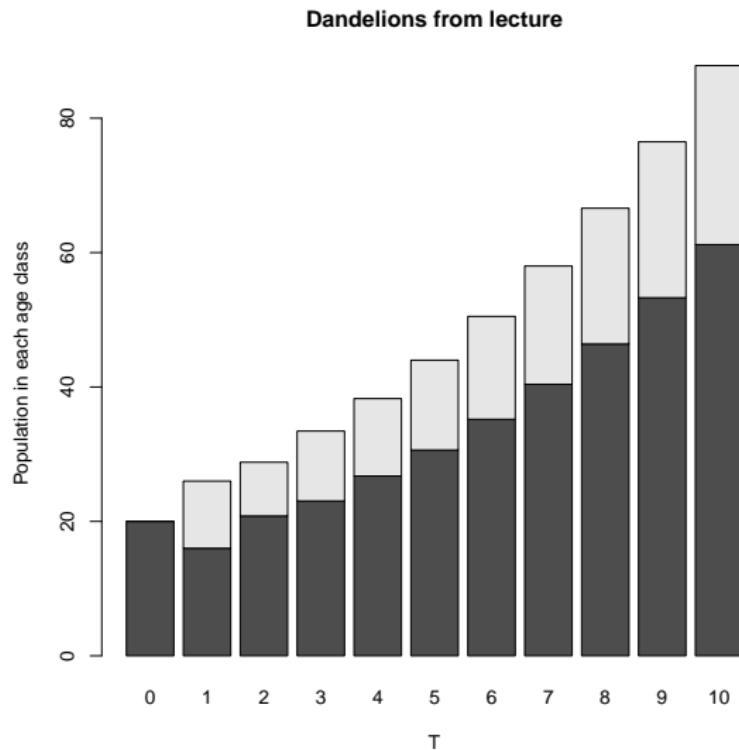
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x	f_x	p_x	ℓ_x	$\ell_x f_x$
1	0.8	0.5		
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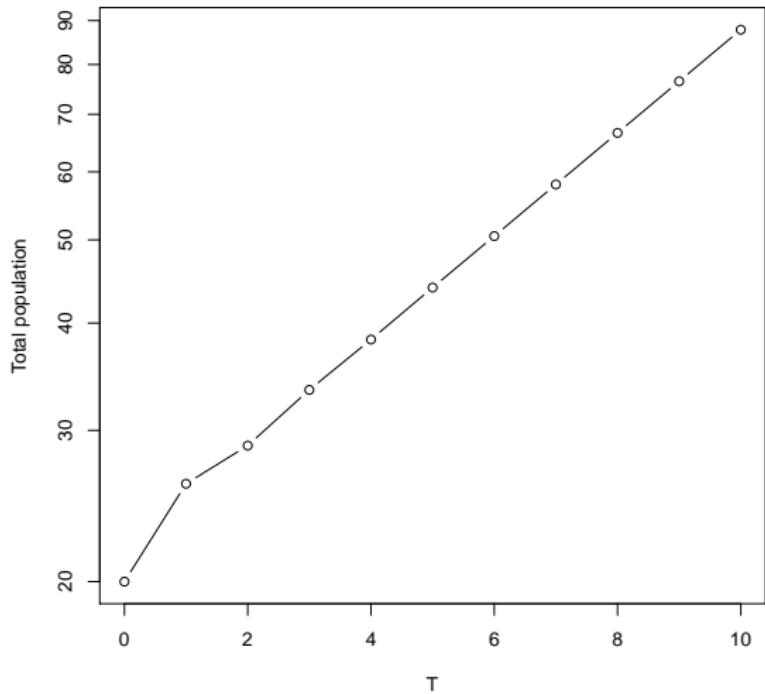
x	f_x	p_x	ℓ_x	$\ell_x f_x$
1	0.8	0.5	1.000	0.800
2	0.8	0	0.500	0.400
R				1.200

Dandelion dynamics

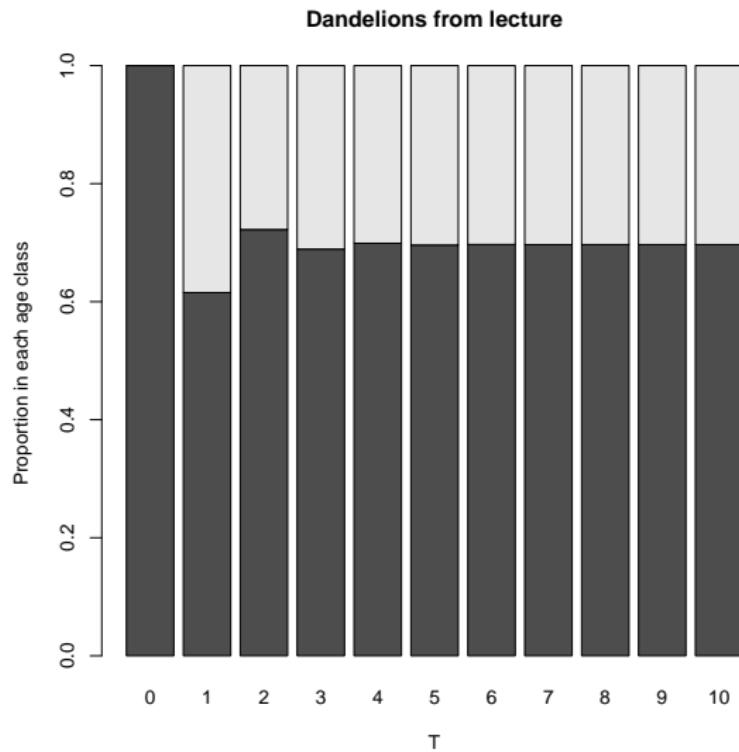


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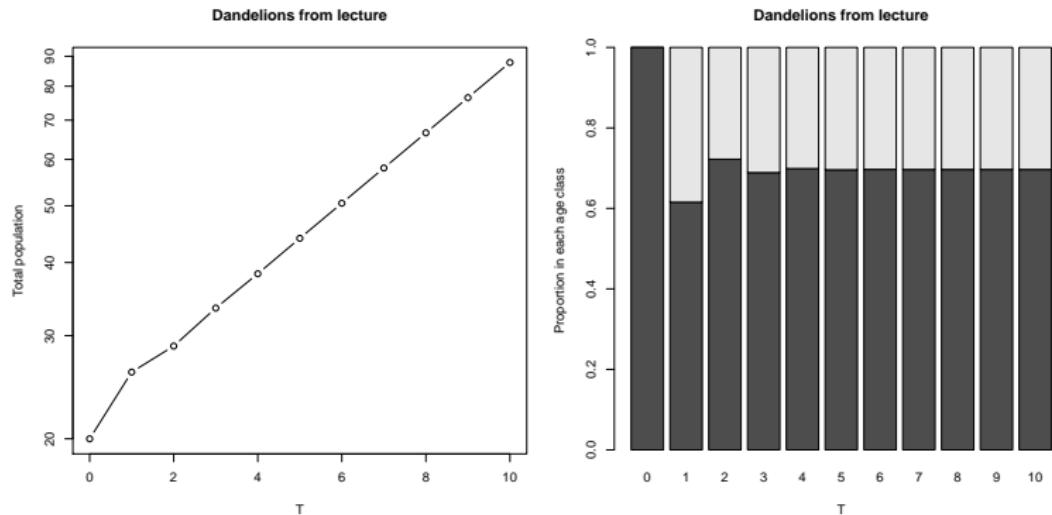
Dandelions from lecture



Dandelion dynamics



Dandelion dynamics



Squirrel example



Gray squirrel population example

x	f_x	p_x	ℓ_x	$\ell_x f_x$
1	0	0.25		
2	1.28	0.46		
3	2.28	0.77		
4	2.28	0.65		
5	2.28	0.67		
6	2.28	0.64		
7	2.28	0.88		
8	2.28	0.0		
R				

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 - ▶ * Might be better to use a model where they keep track of 1 year, 2 year, and "adult" – not much harder.

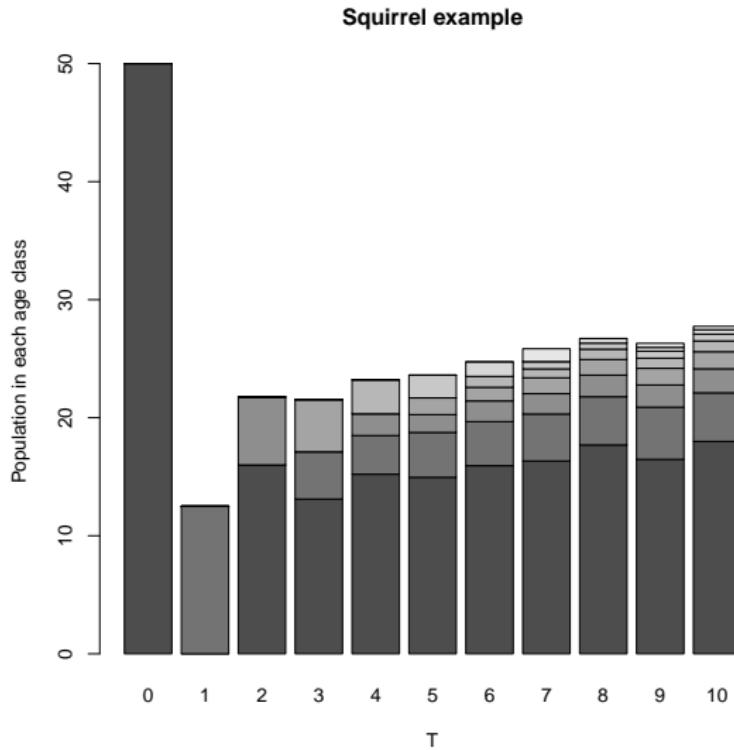
Squirrel observations

- ▶ Poll: Do you notice anything strange about the squirrel life table?
 - ▶ * Older age groups seem to be grouped for fecundity.
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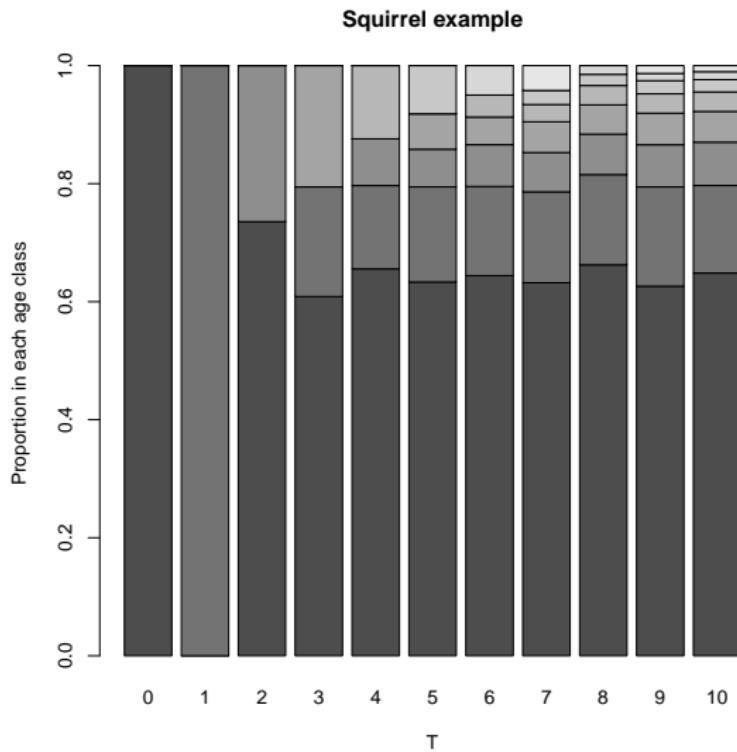
Gray squirrel population example

x	f_x	p_x	ℓ_x	$\ell_x f_x$
1	0	0.25	1.000	0.000
2	1.28	0.46	0.250	0.320
3	2.28	0.77	0.115	0.262
4	2.28	0.65	0.089	0.202
5	2.28	0.67	0.058	0.131
6	2.28	0.64	0.039	0.088
7	2.28	0.88	0.025	0.056
8	2.28	0.0	0.022	0.050
R				1.109

Gray squirrel dynamics

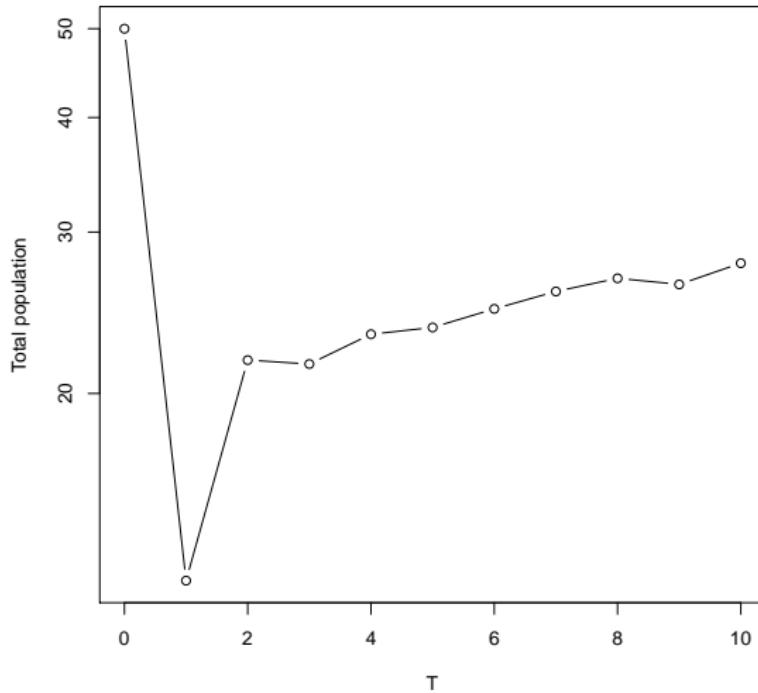


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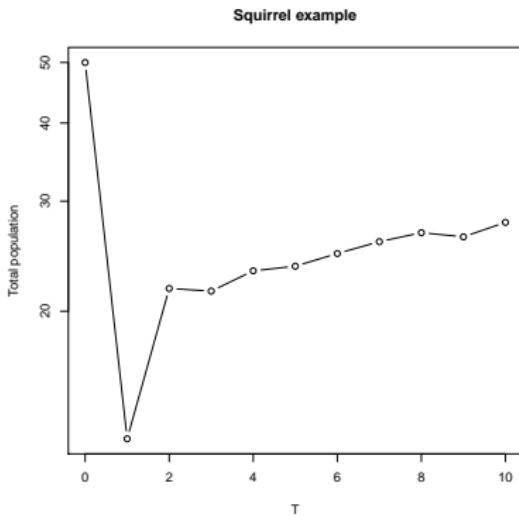
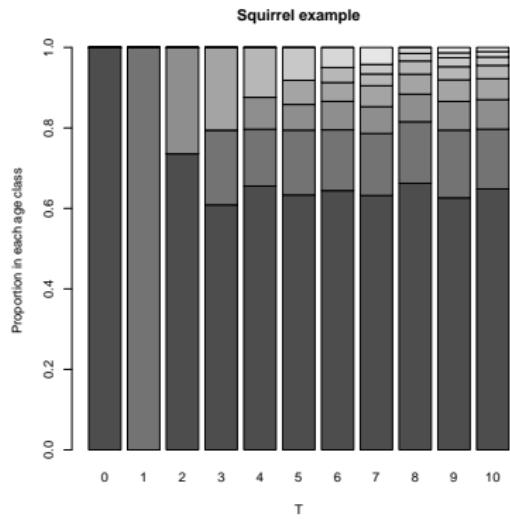


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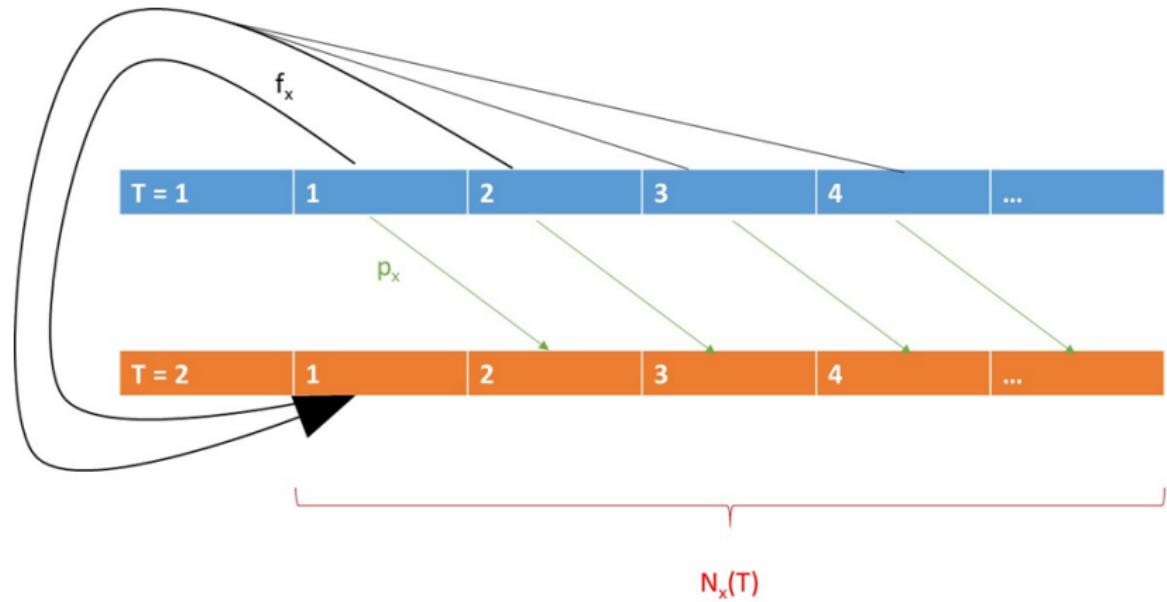
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The structured model



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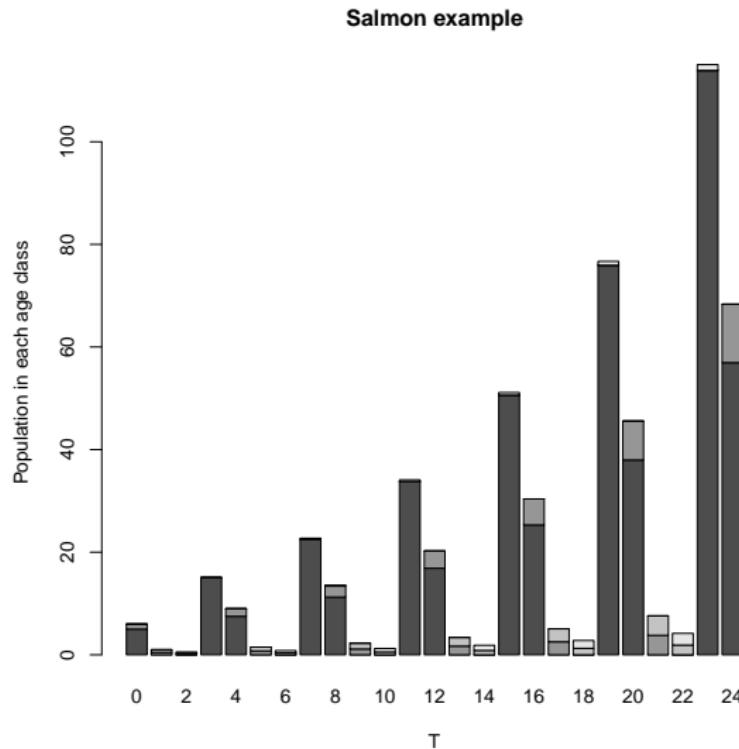
Salmon example

x	f_x	p_x	ℓ_x	$\ell_x f_x$
1	0	0.2		
2	0	0.6		
3	0	0.8		
4	10	0		
R				

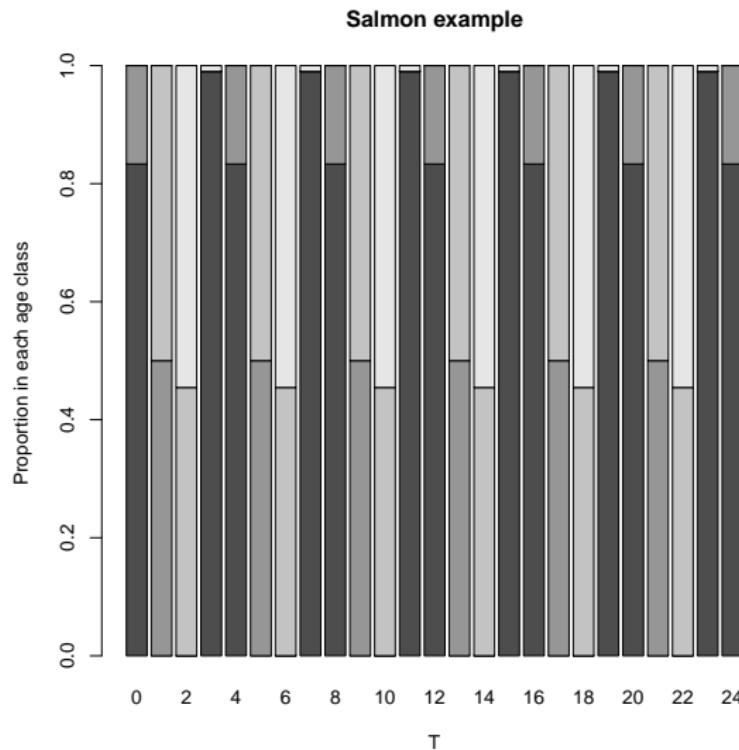
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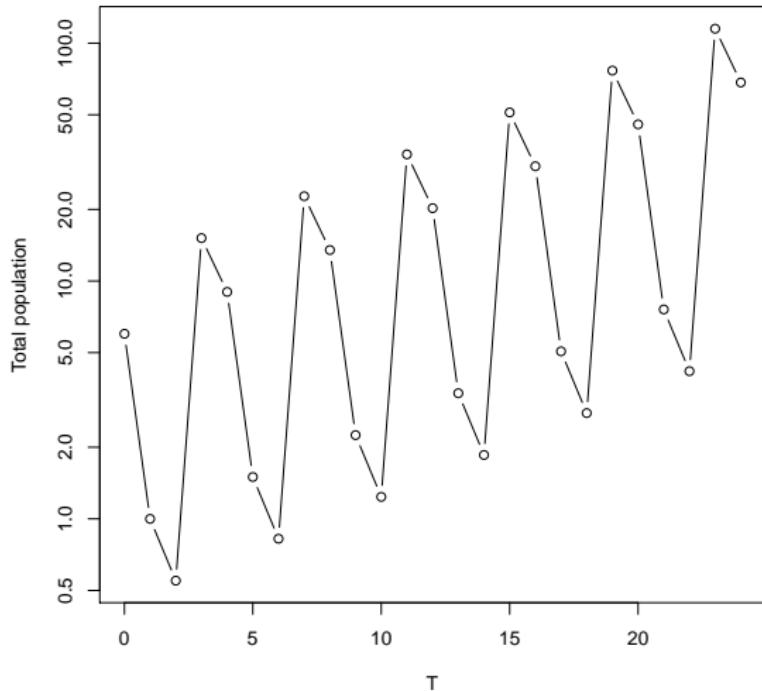


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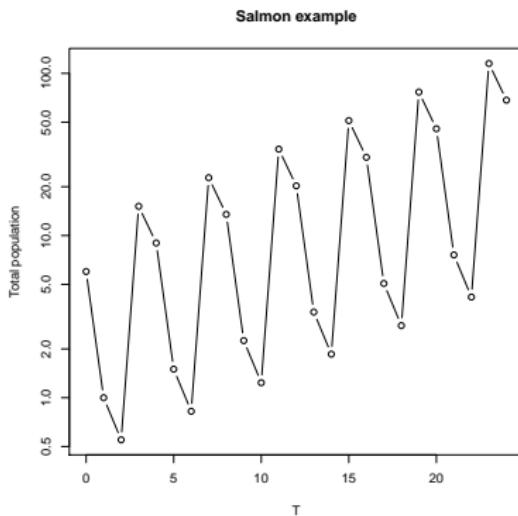
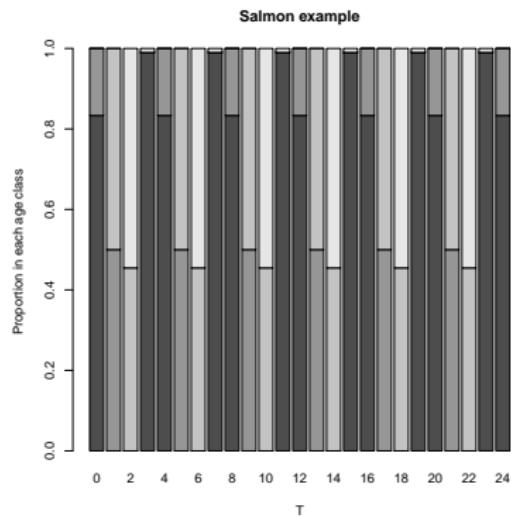


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Subsection 2

Calculation details

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Dandelion life table

x	f_x	p_x	ℓ_x	$\ell_x f_x$
1	0.8	0.5	1.000	0.800
2	0.8	0	0.500	0.400
R				1.200

Counting after reproduction

x	f_x	p_x	ℓ_x	$\ell_x f_x$
1	0.8	0.01	1.000	0.800
2	40	0	0.010	0.400
R				1.200

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Subsection 3

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Time scales

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Subsection 1

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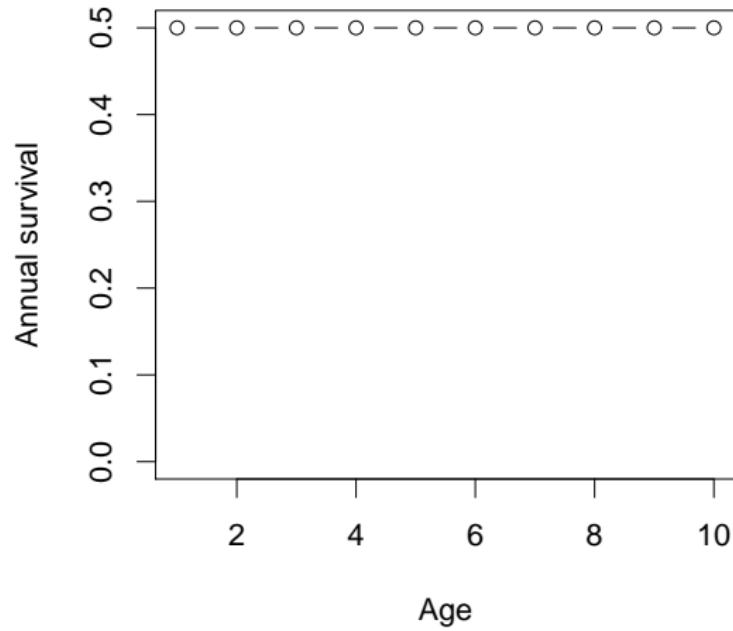
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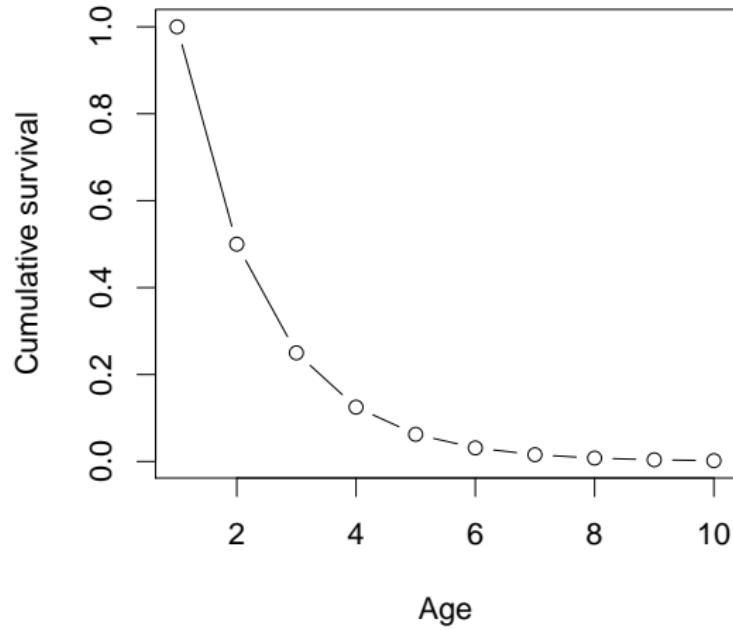
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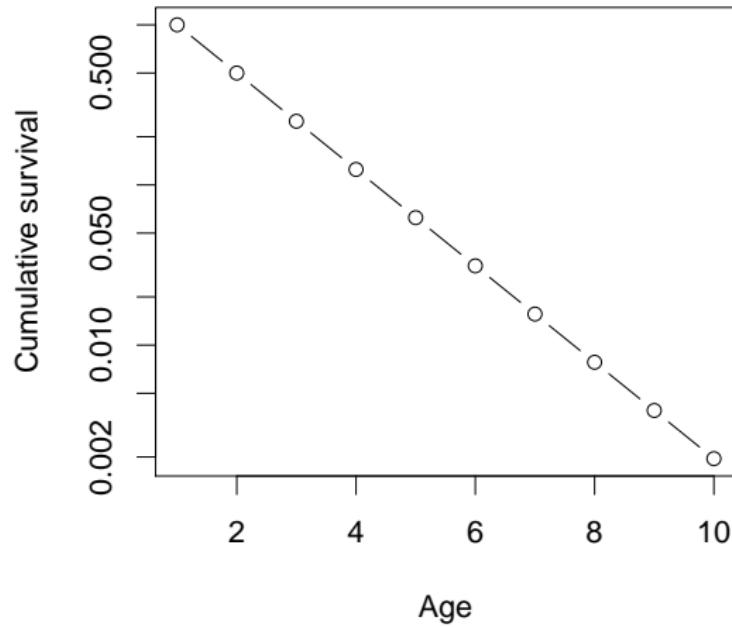
Constant survivorship



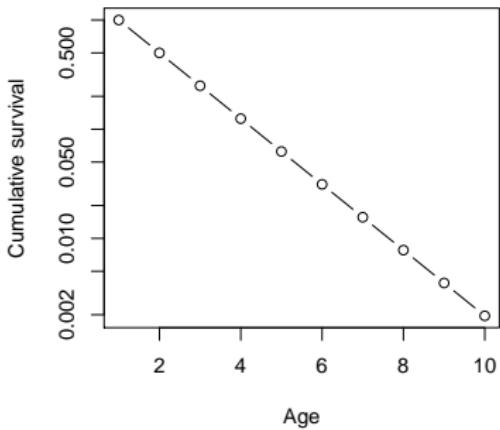
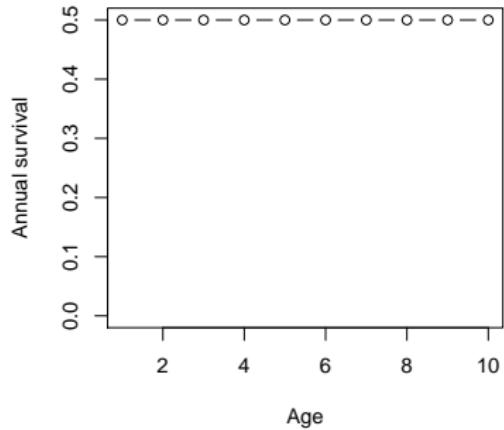
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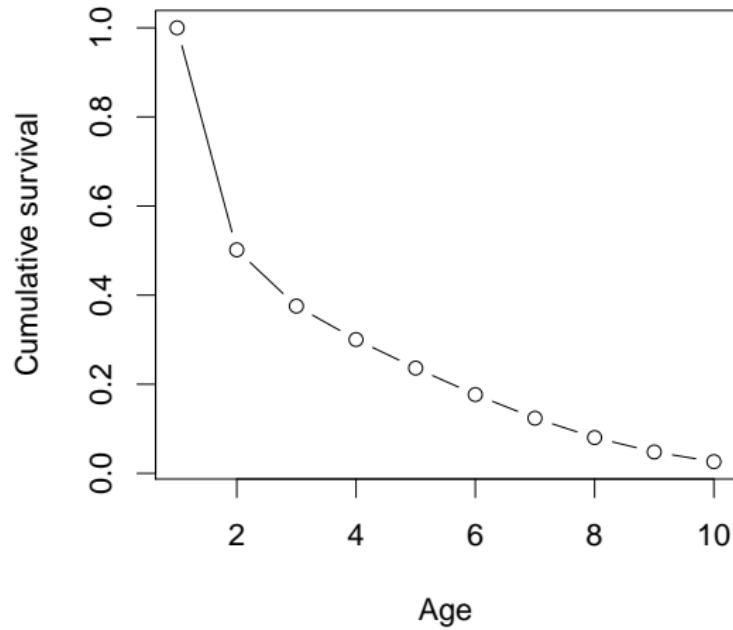
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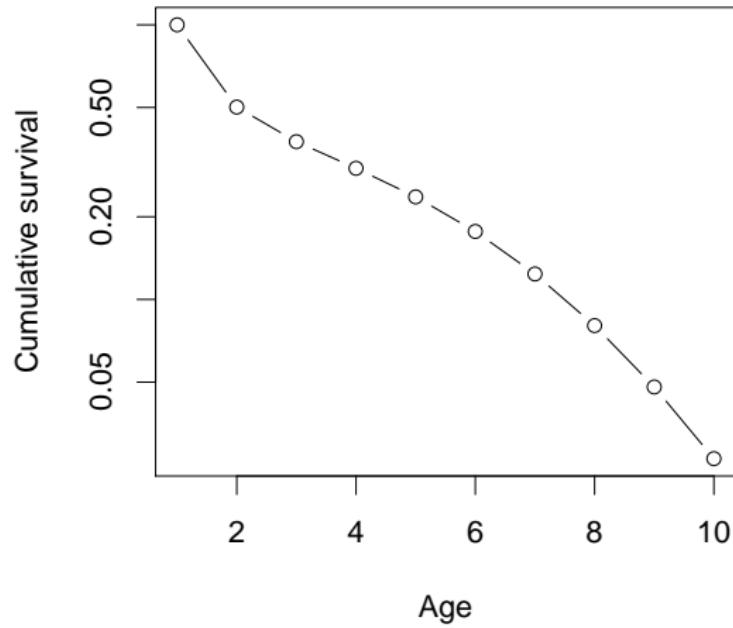
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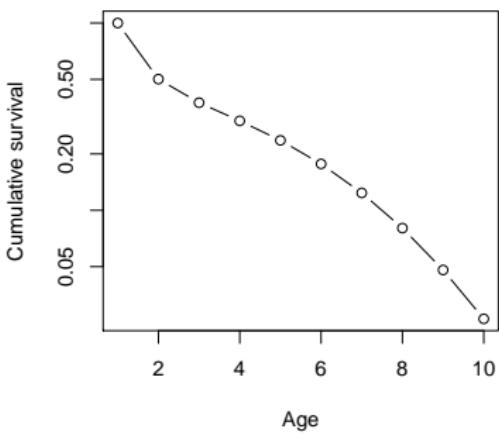
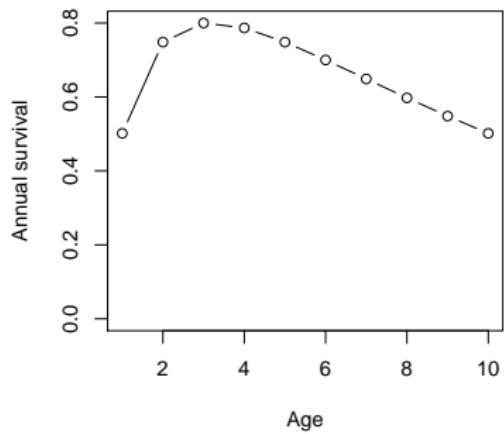
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Subsection 2

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► <http://www.gapminder.org/population/tool/>

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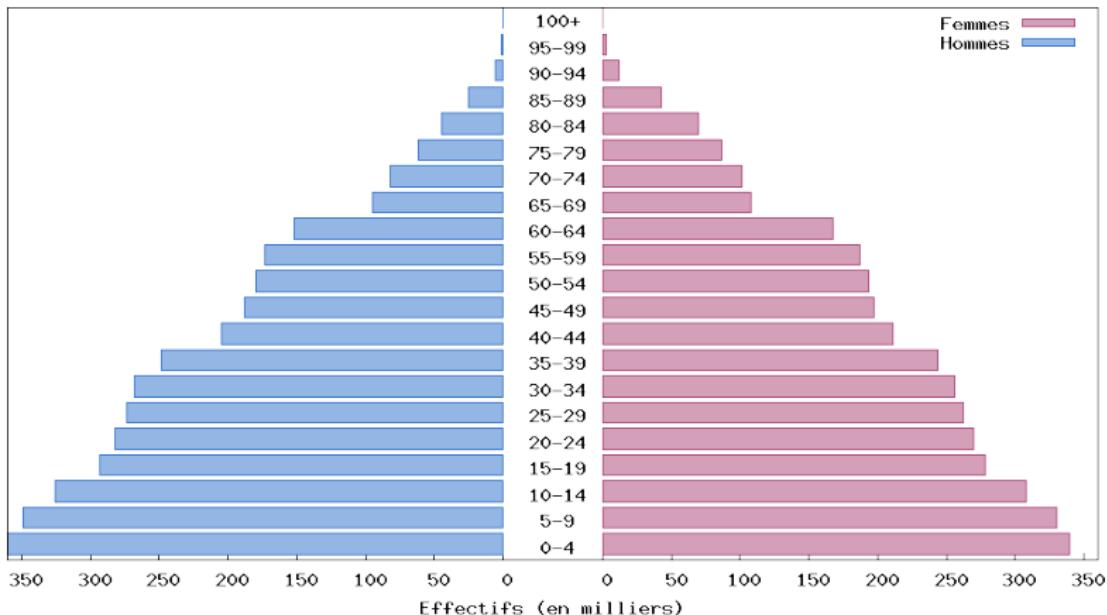
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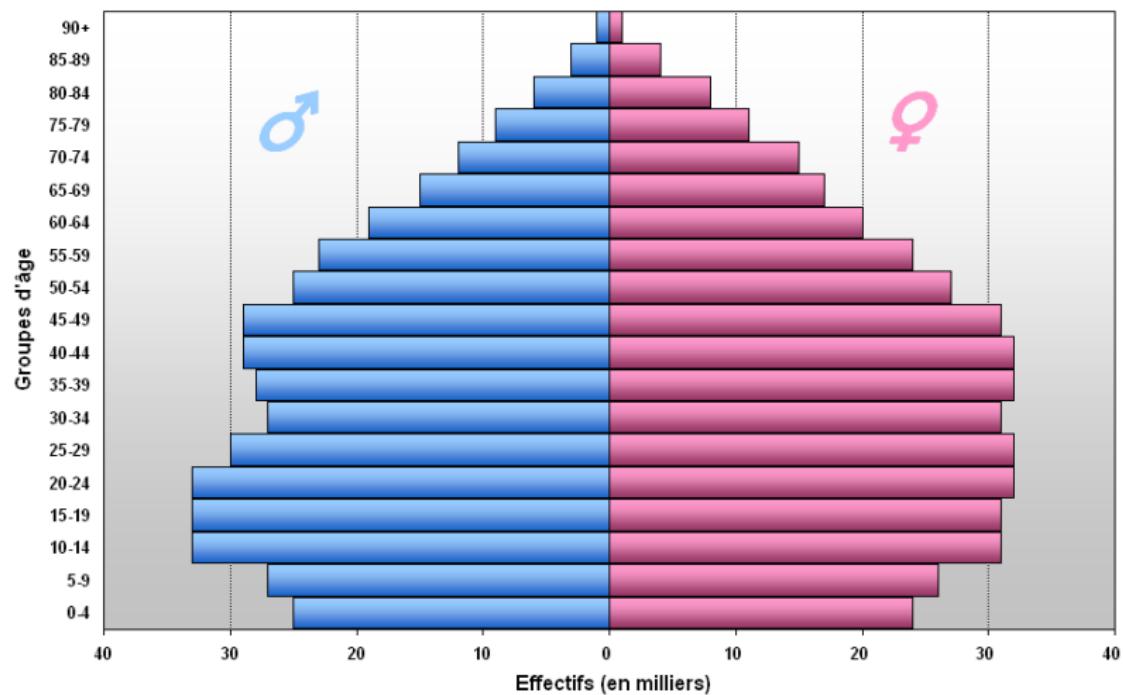
Age distributions

Pyramide des âges, Israël, 2010



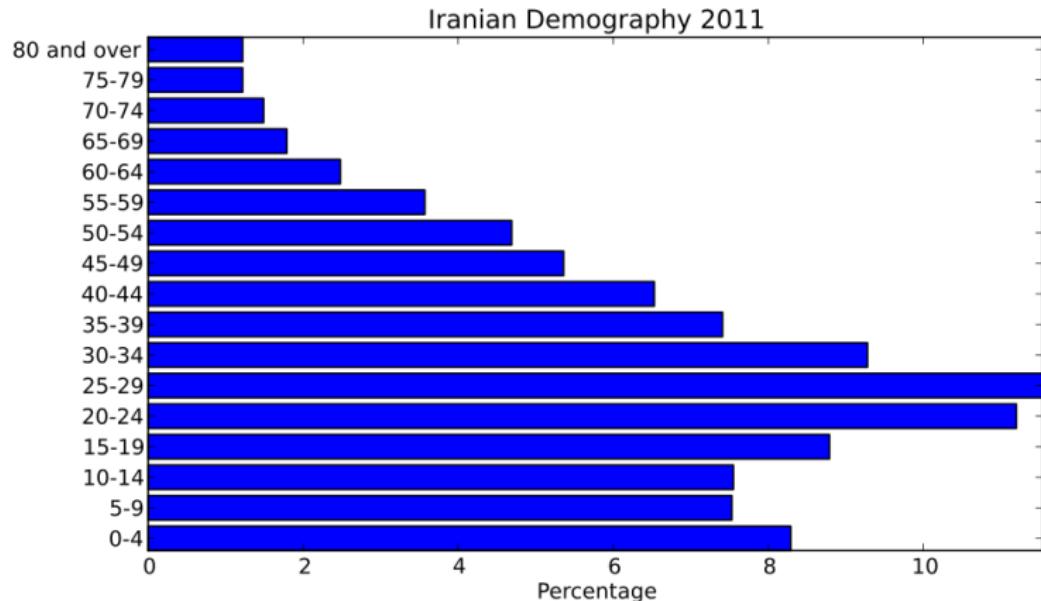
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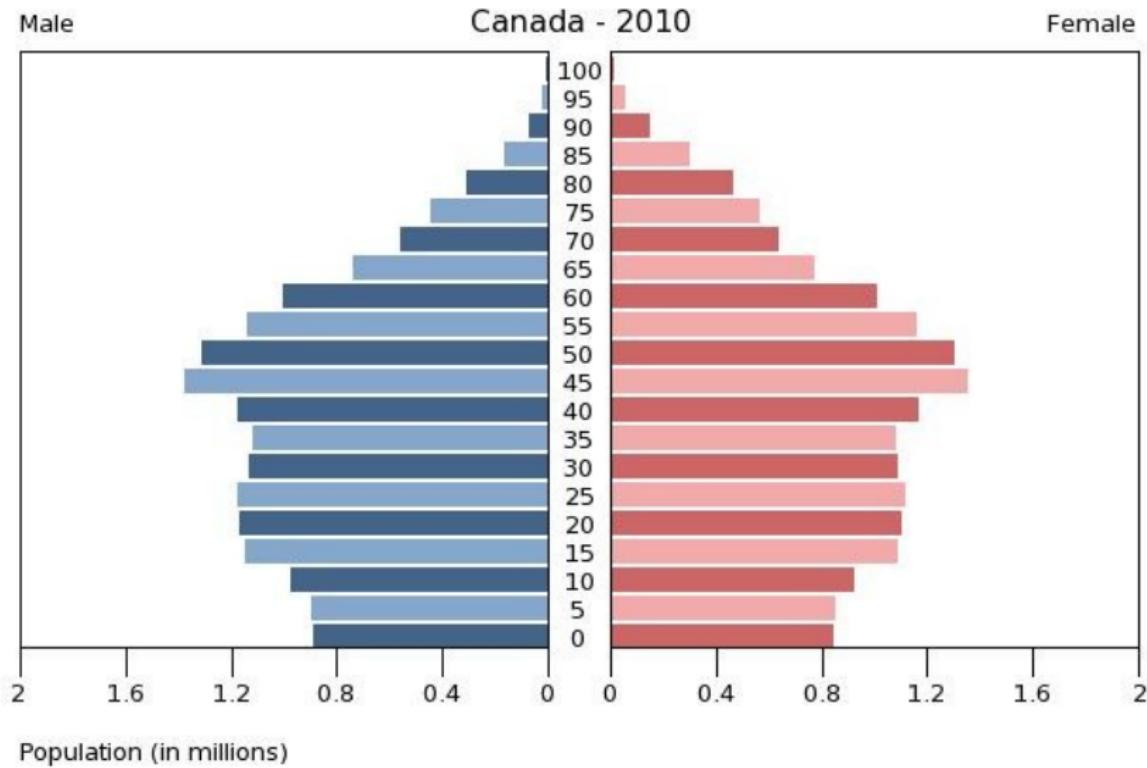


Source: Organisation des Nations Unies (World Population Prospects: The 2004 Revision)

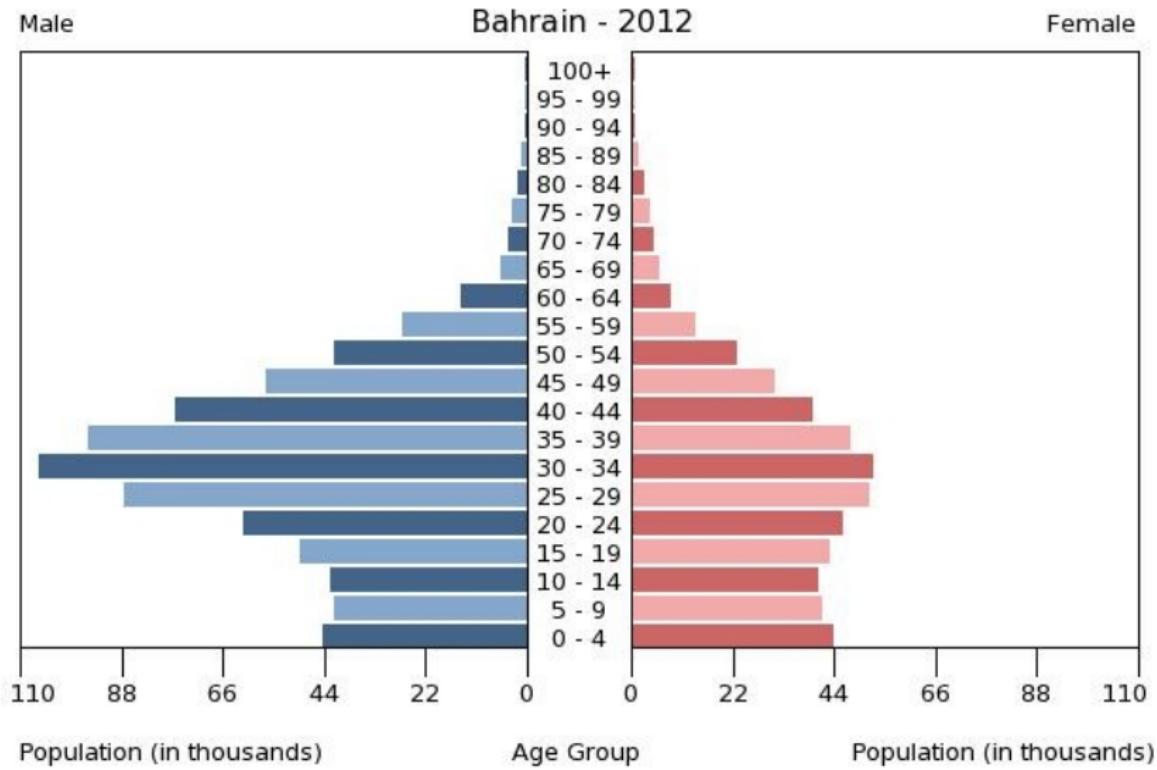
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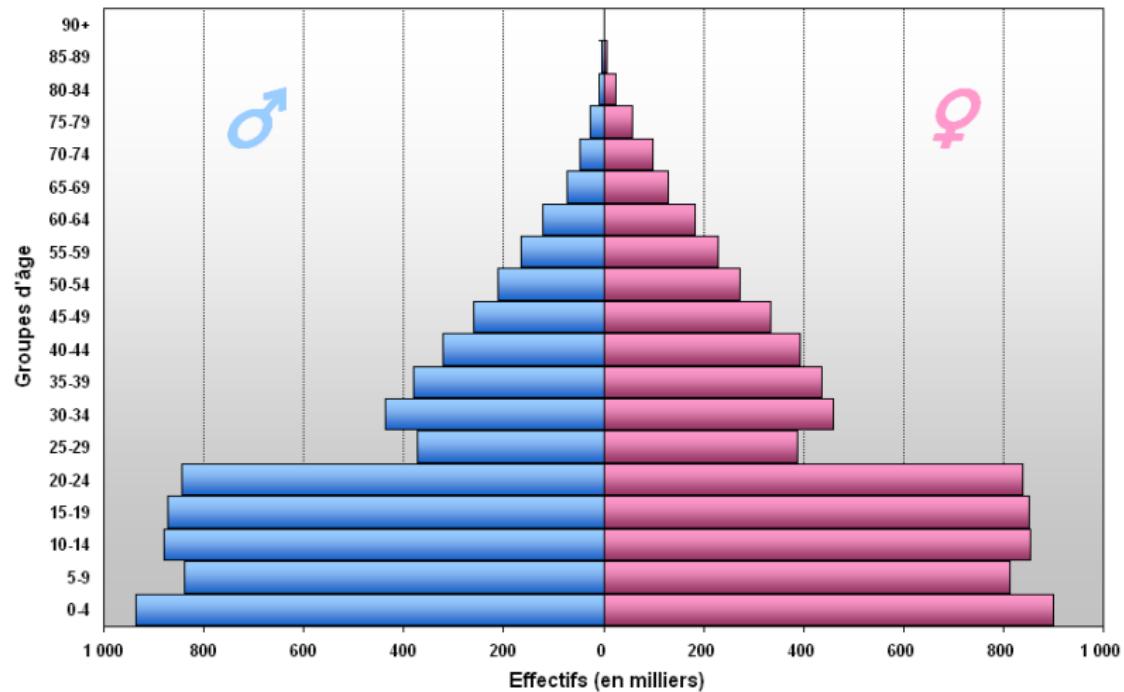


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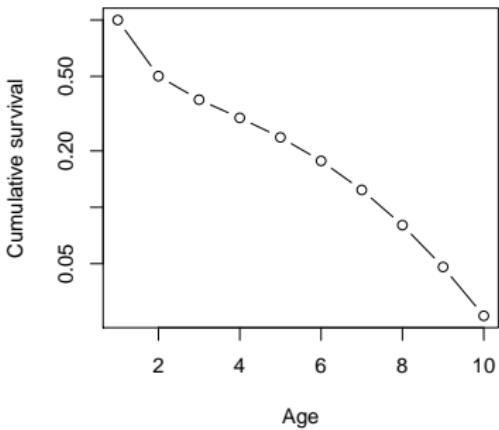
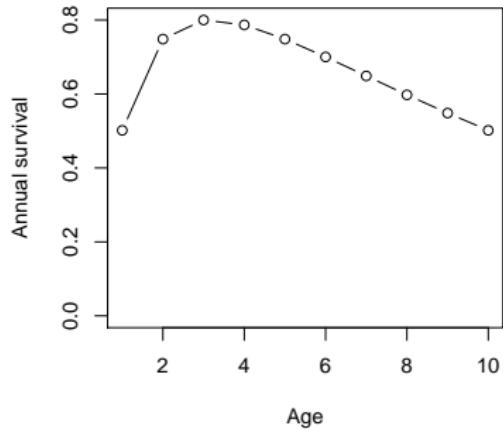
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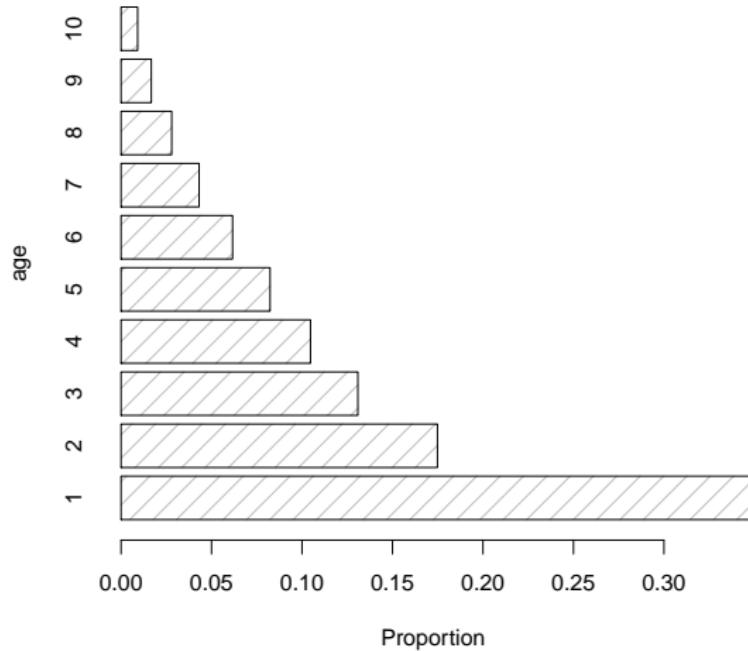
Changing survivorship



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Stable age distribution

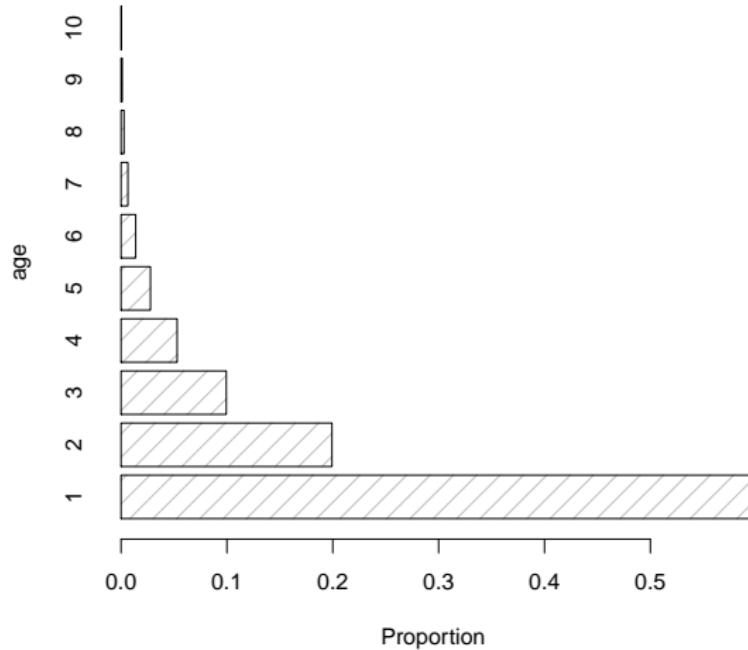
$\lambda = 1$



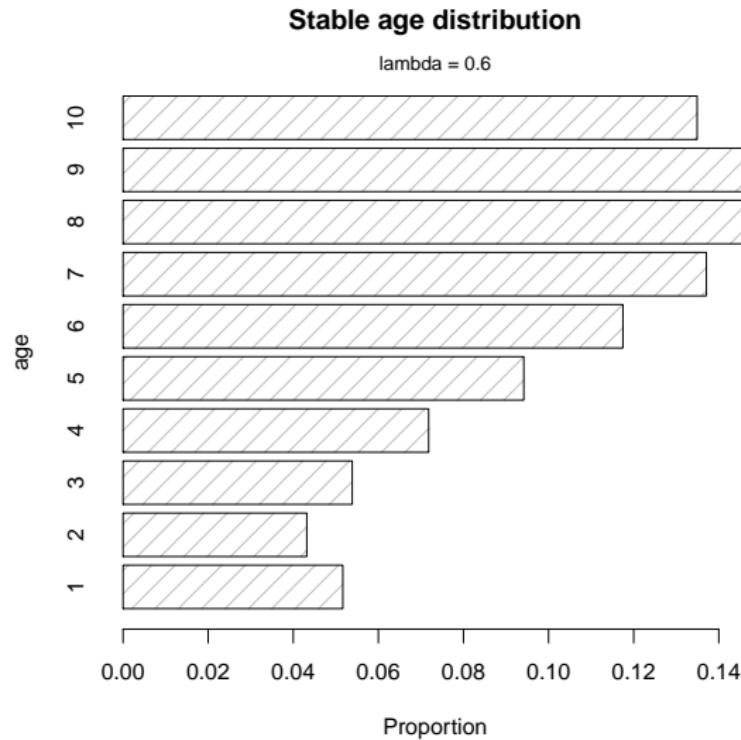
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Outline

Introduction

Example: biennial dandelions

Modeling approach

Constructing a model

Model dynamics

Life tables

Examples

Calculation details

Measuring growth rates

Life-table patterns

Survivorship

Fecundity

Age distributions

Other structured models

Stage structure

Regulated growth

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