

UNIT 4: Structured populations

Outline

Introduction

Example: biennial dandelions

Modeling approach

Constructing a model

Model dynamics

Life tables

Examples

Calculation details

Measuring growth rates

Life-table patterns

Survivorship

Fecundity

Other structured models

Stage structure

Regulated growth

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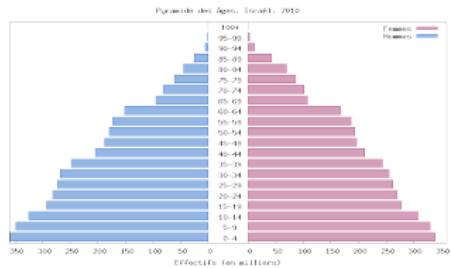
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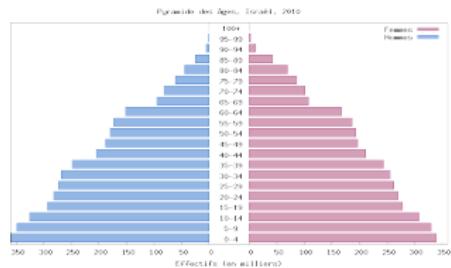
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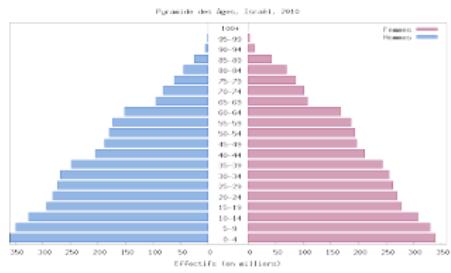
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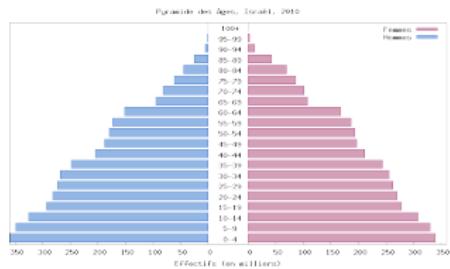
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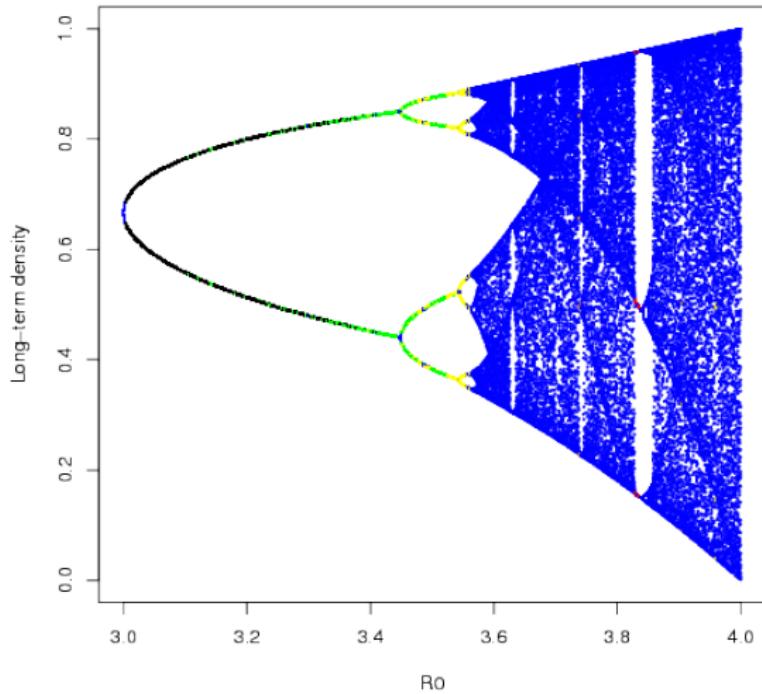
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Outline

Introduction

Example: biennial dandelions

Modeling approach

Constructing a model

Model dynamics

Life tables

Examples

Calculation details

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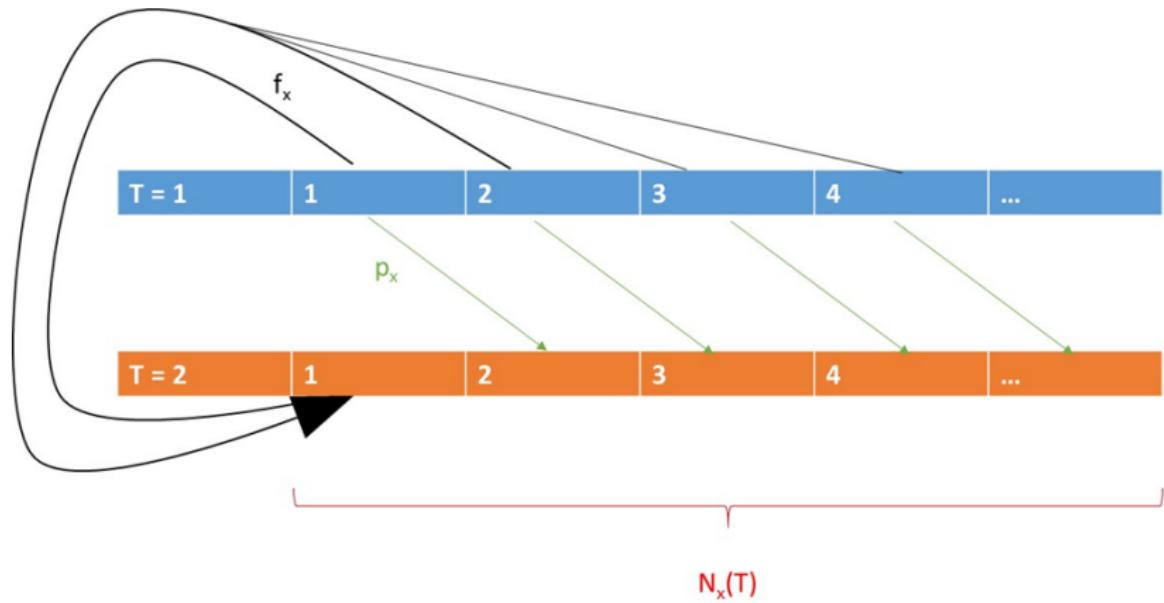
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Outline

Introduction

Example: biennial dandelions

Modeling approach

Constructing a model

Model dynamics

Life tables

Examples

Calculation details

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Outline

Introduction

Example: biennial dandelions

Modeling approach

Constructing a model

Model dynamics

Life tables

Examples

Calculation details

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Life-table patterns

Survivorship

Fecundity

Other structured models

Stage structure

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Outline

Introduction

Example: biennial dandelions

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Constructing a model

Model dynamics

Life tables

Examples

Calculation details

Measuring growth rates

Life-table patterns

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Other structured models

Stage structure

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Dandelion life table

x	f_x	p_x	ℓ_x	$\ell_x f_x$
1				
2				
R				

Dandelion life table (repeat)

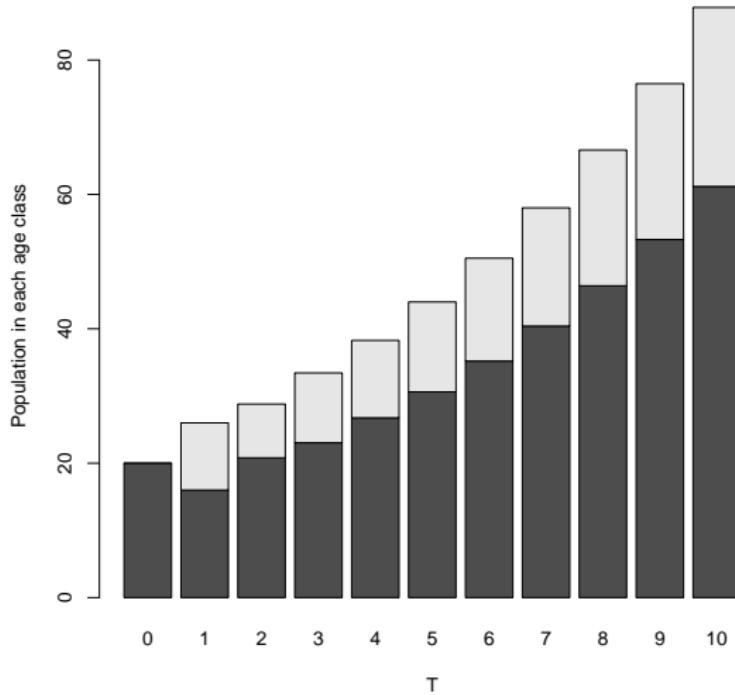
x	f_x	p_x	ℓ_x	$\ell_x f_x$
1	0.8	0.5		
2	0.8	0		
R				

Dandelion life table

x	f_x	p_x	ℓ_x	$\ell_x f_x$
1	0.8	0.5	1.000	0.800
2	0.8	0	0.500	0.400
R				1.200

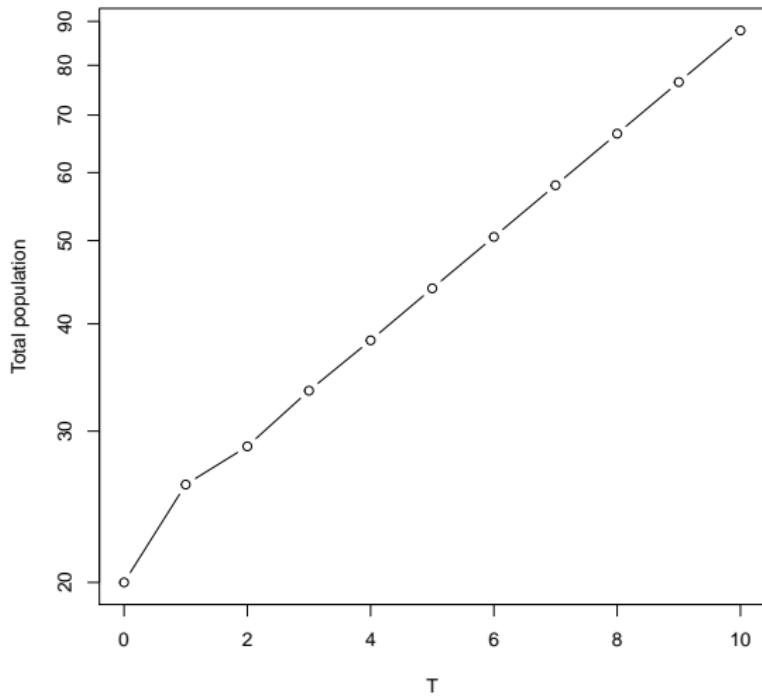
Dandelion dynamics

Dandelions from lecture

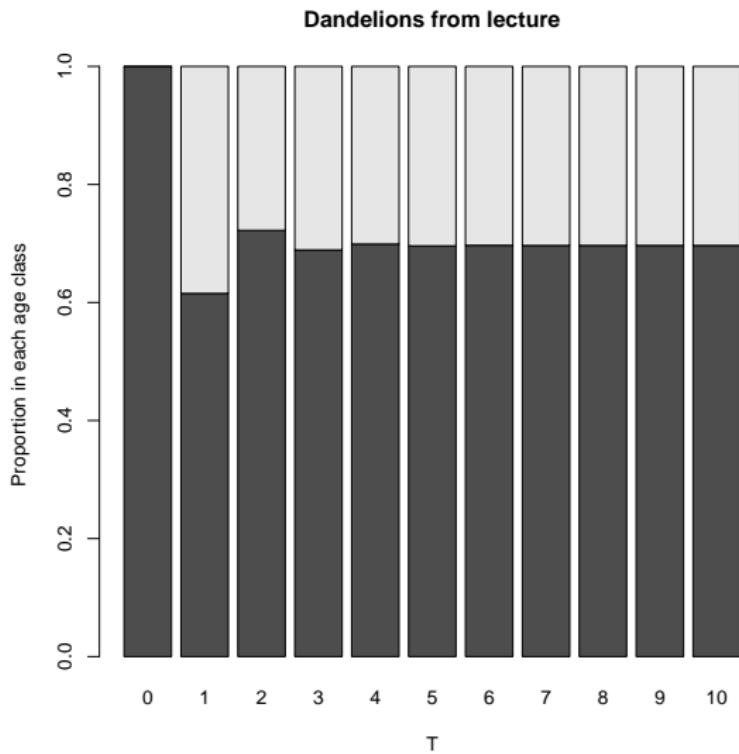


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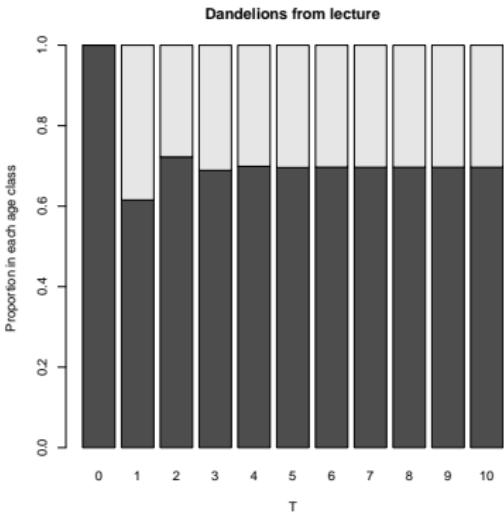
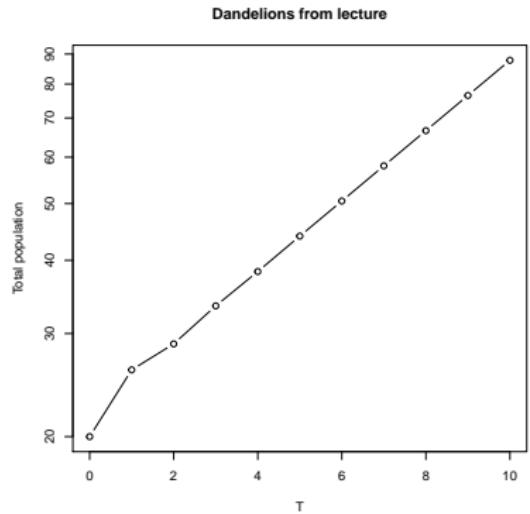
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Dandelion age dynamics



Dandelion dynamics



Squirrel example



Gray squirrel population example

x	f_x	p_x	ℓ_x	$\ell_x f_x$
1	0	0.25		
2	1.28	0.46		
3	2.28	0.77		
4	2.28	0.65		
5	2.28	0.67		
6	2.28	0.64		
7	2.28	0.88		
8	2.28	0.0		
R				

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 - ▶ * Older age groups seem to be grouped for fecundity.
 - ▶ * Strange pattern in survivorship; do we really believe nobody survives past the last year?
 - ▶ * Might be better to use a model where they keep track of 1 year, 2 year, and “adult” – not much harder.

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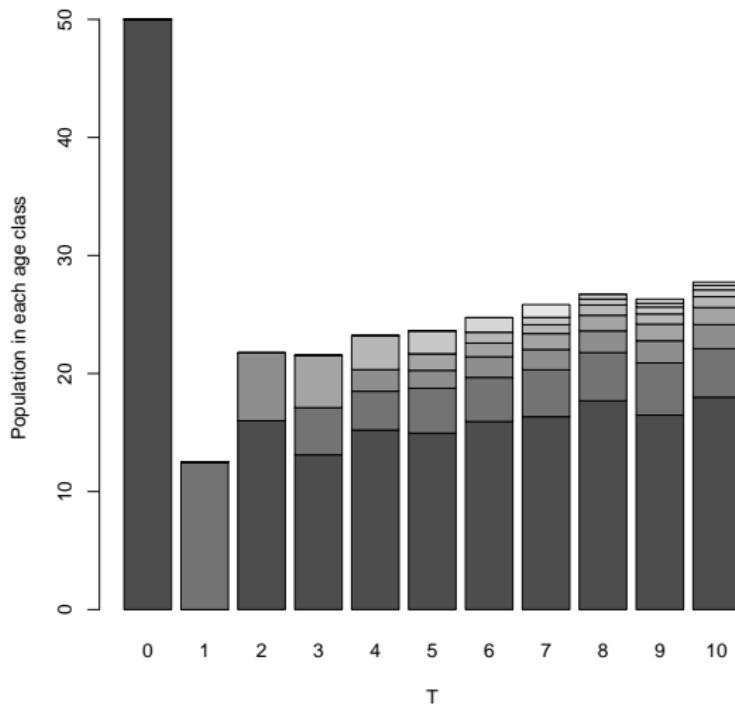
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Gray squirrel population example

x	f_x	p_x	ℓ_x	$\ell_x f_x$
1	0	0.25	1.000	0.000
2	1.28	0.46	0.250	0.320
3	2.28	0.77	0.115	0.262
4	2.28	0.65	0.089	0.202
5	2.28	0.67	0.058	0.131
6	2.28	0.64	0.039	0.088
7	2.28	0.88	0.025	0.056
8	2.28	0.0	0.022	0.050
R				1.109

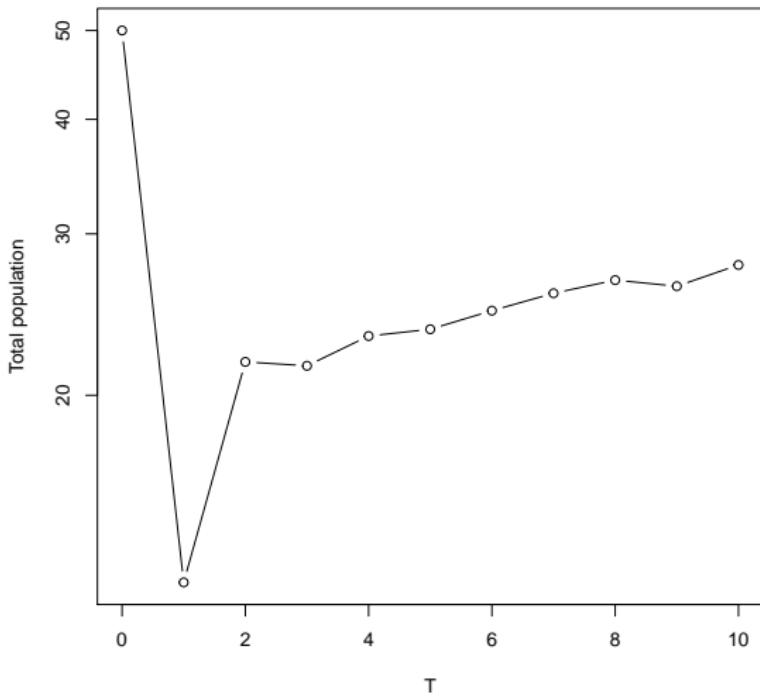
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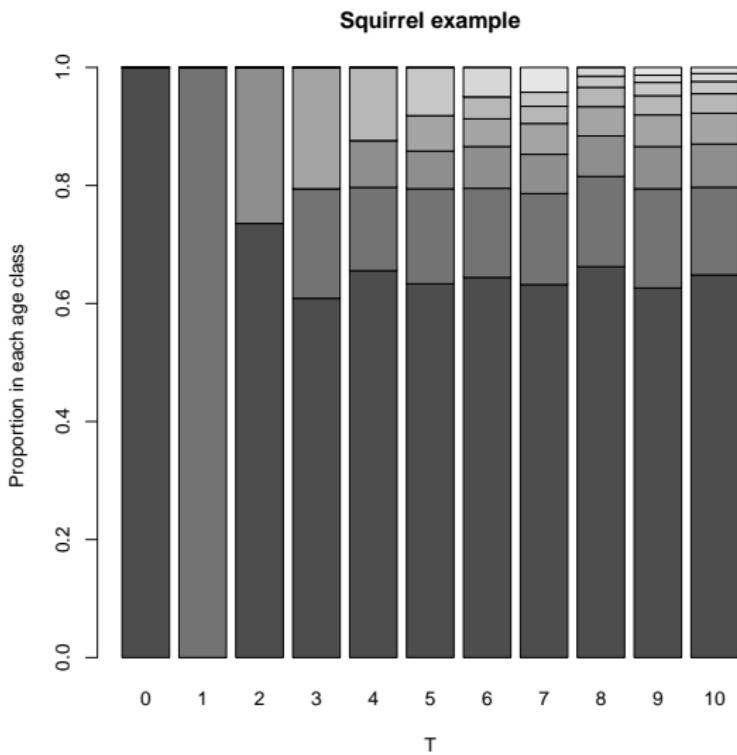


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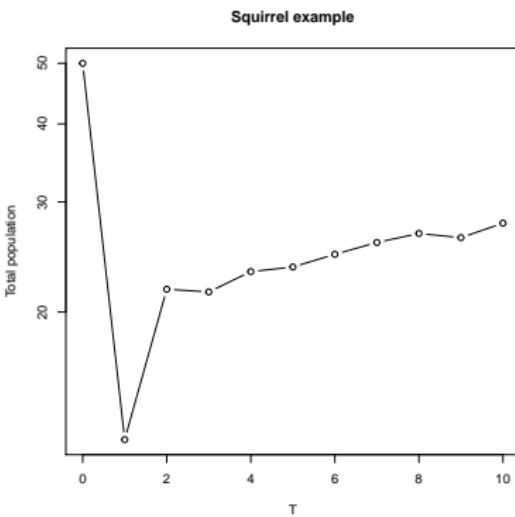
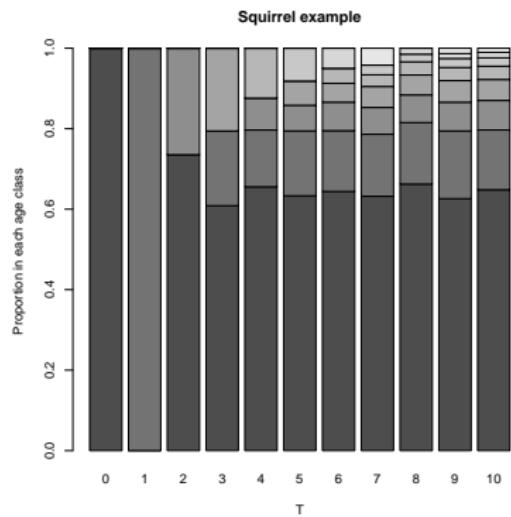
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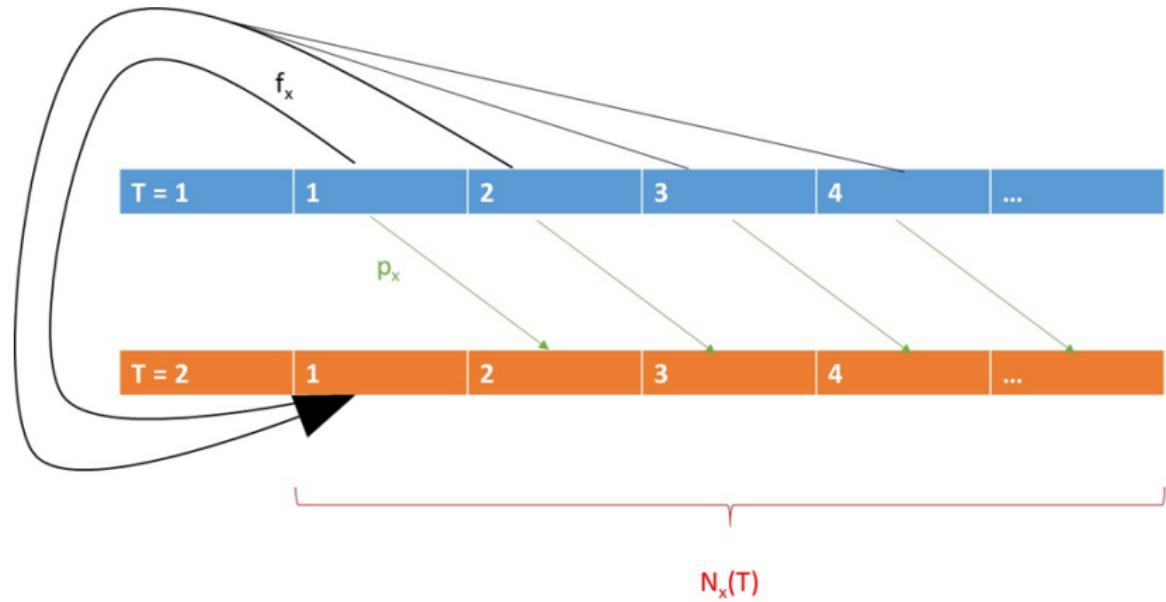
Gray squirrel age dynamics



Gray squirrel dynamics



The structured model (repeat)



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Dandelion life table

x	f_x	p_x	ℓ_x	$\ell_x f_x$
1	0.8	0.5	1.000	0.800
2	0.8	0	0.500	0.400
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Counting after reproduction

There are two different approaches

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- ▶ \mathcal{R} gives the number of individuals per individual *over a lifetime*
- ▶ Poll: What relationship do we expect for an annual population (individuals die every year)?
 - ▶ * $\mathcal{R} = \lambda$; each organism observed reproduces \mathcal{R} offspring on average, all in one time step
- ▶ Poll: For a long-lived population?
 - ▶ * The \mathcal{R} offspring are produced slowly, so population changes slowly
 - ▶ * λ should be closer to 1 than \mathcal{R} is.
 - ▶ * But on the same side (same answer about whether population is growing)

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Outline

Introduction

Example: biennial dandelions

Modeling approach

Constructing a model

Model dynamics

Life tables

Examples

Calculation details

Measuring growth rates

Life-table patterns

Survivorship

Fecundity

Other structured models

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Regulated growth

Outline

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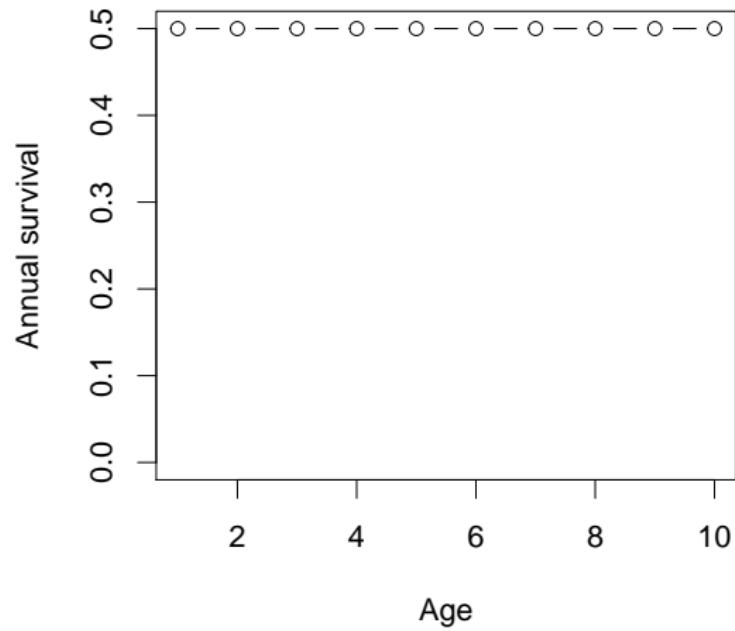
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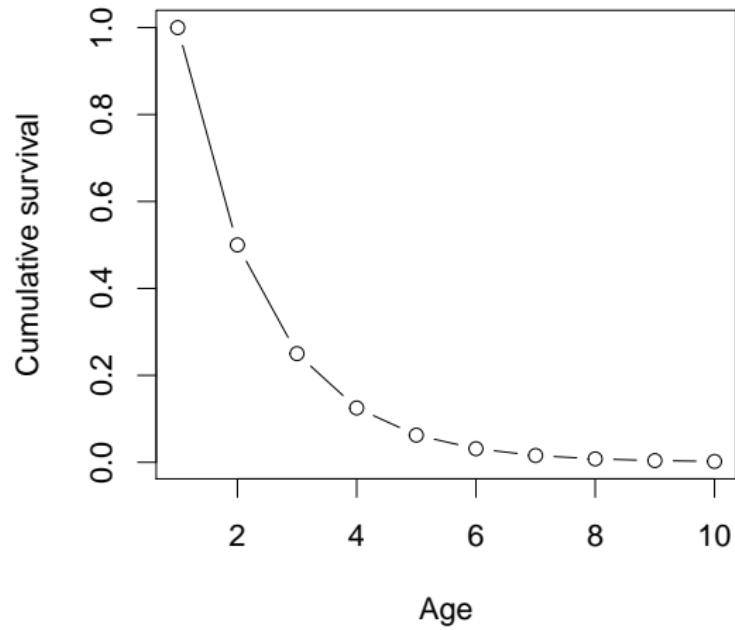
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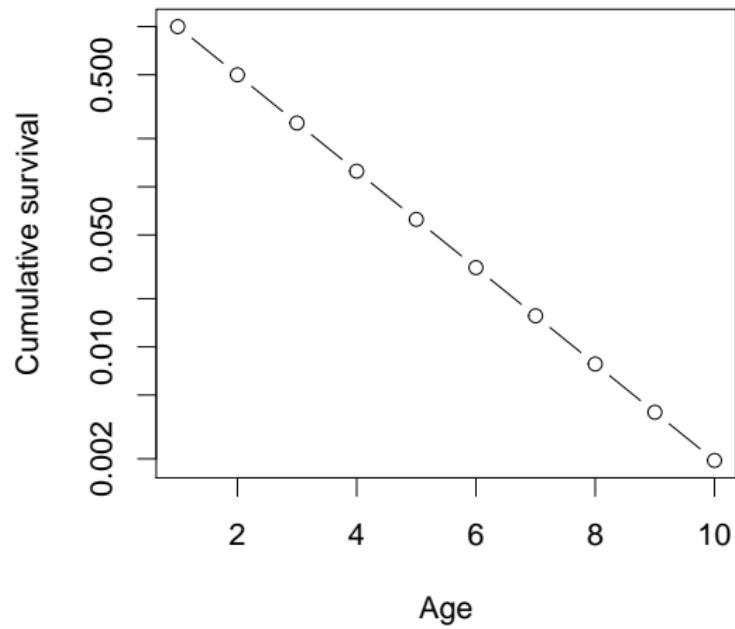
Constant survivorship



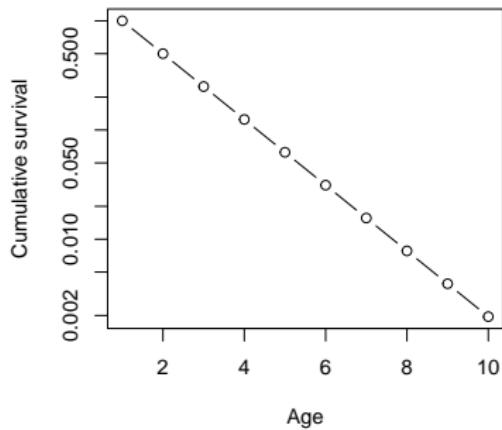
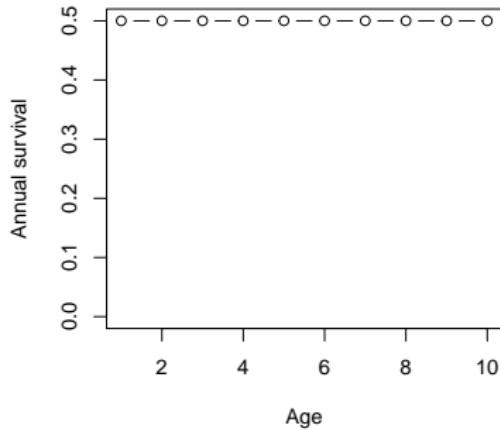
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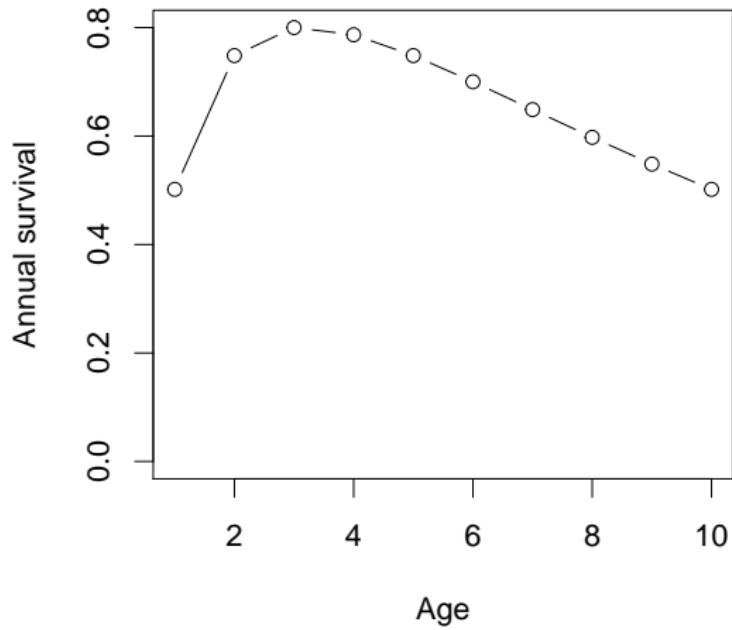
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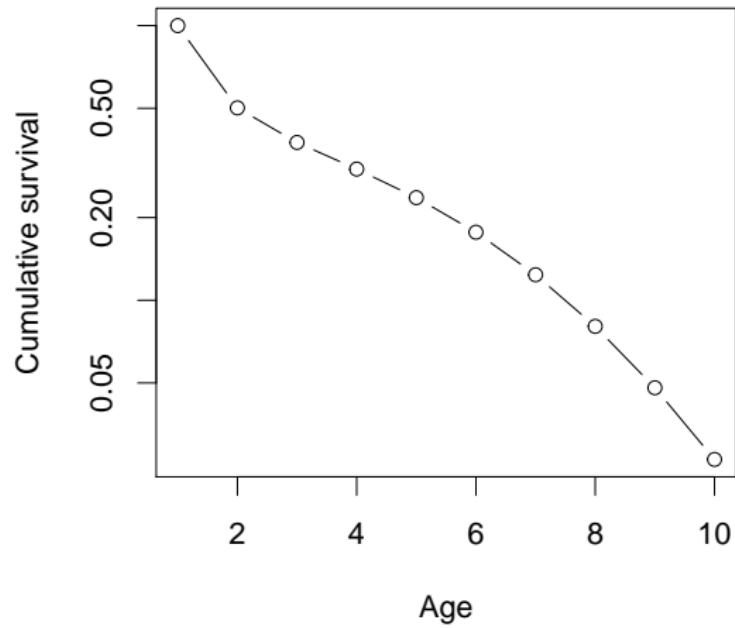
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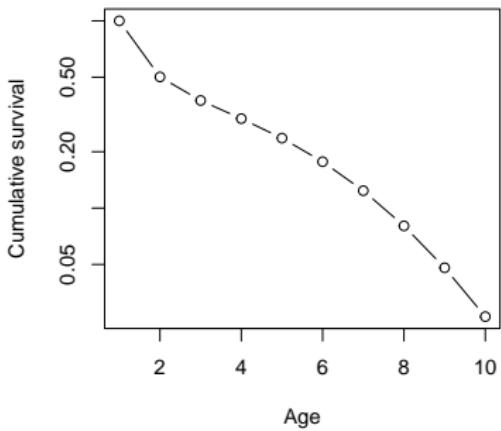
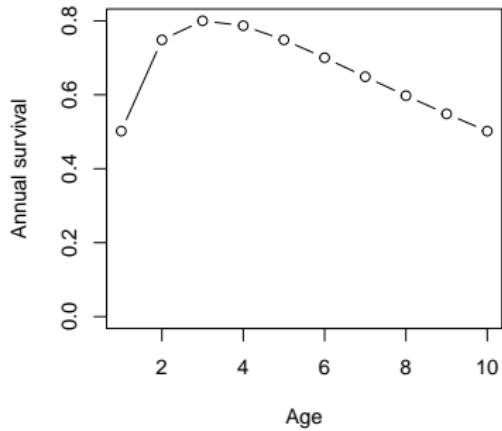
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Outline

Introduction

Example: biennial dandelions

Modeling approach

Constructing a model

Model dynamics

Life tables

Examples

Calculation details

Measuring growth rates

Life-table patterns

Survivorship

Fecundity

Other structured models

Stage structure

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Outline

Introduction

Example: biennial dandelions

Modeling approach

Constructing a model

Model dynamics

Life tables

Examples

Calculation details

Measuring growth rates

Life-table patterns

Survivorship

Fecundity

Other structured models

Stage structure

Regulated growth

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Outline

Introduction

Example: biennial dandelions

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Constructing a model

Model dynamics

Life tables

Examples

Calculation details

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Life-table patterns

Survivorship

Fecundity

Other structured models

Stage structure

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 - ▶ * Survival with recruitment (ie., to the next larger class of individuals)
- ▶ More complicated models are also possible

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Outline

Introduction

Example: biennial dandelions

Modeling approach

Constructing a model

Model dynamics

Life tables

Examples

Calculation details

Measuring growth rates

Life-table patterns

Survivorship

Fecundity

Other structured models

Stage structure

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