

BIO3SS3: Population Ecology

UNIT 1: Introduction

Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

Example populations

Dandelions

Gypsy moths

Bacteria

Exponential growth

Log and linear scales

Time scales

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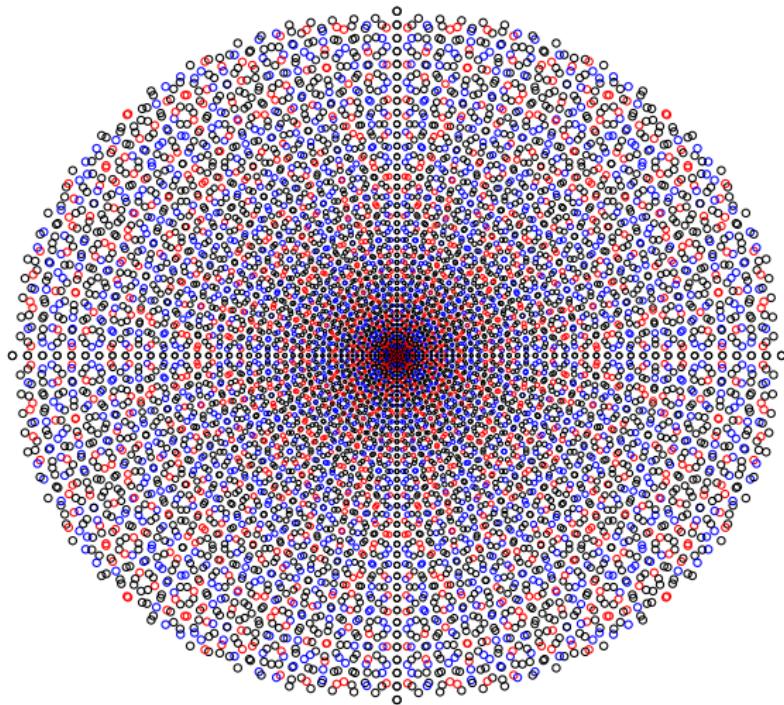
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Pythagorean triples



Which country?



Which country?



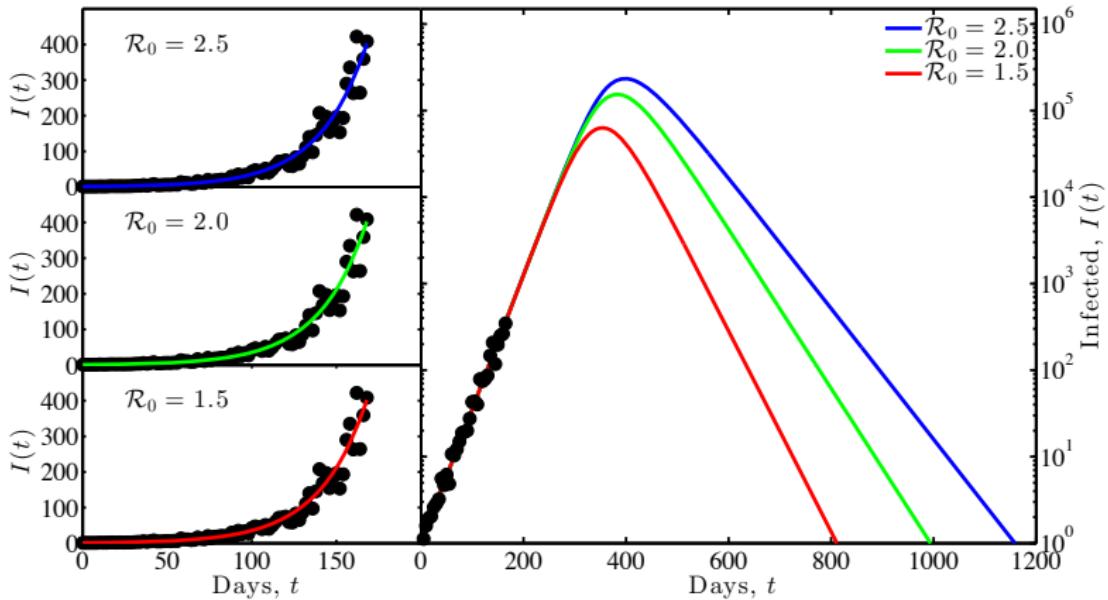
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Ebola research



TAs

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Course overview

Course structure

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Course content

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- ▶ Ecology and population ecology

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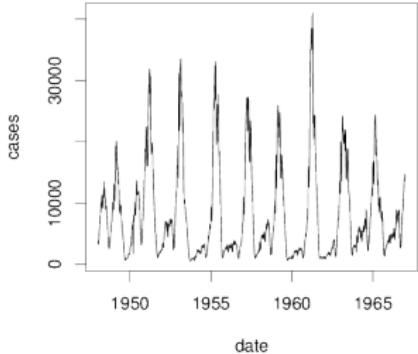
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Dynamical modeling

- ▶ Investigates the links between local, short-term processes, and large-scale, long-term outcomes



Measles reports from England and Wales

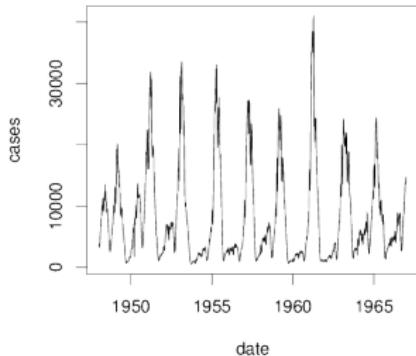


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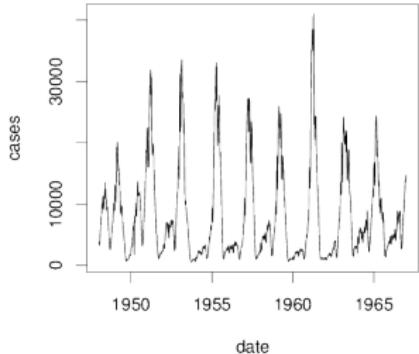


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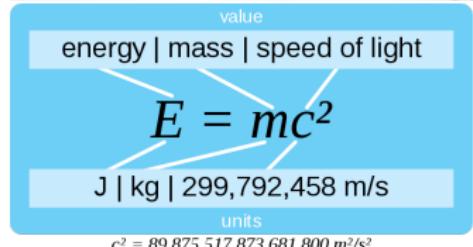
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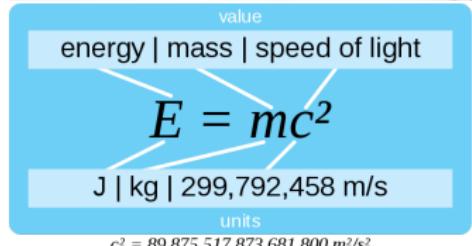
Humans and abstract thought

- ▶ People are evolved to be concrete thinkers, not conceptual thinkers



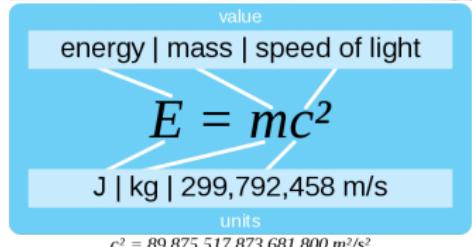
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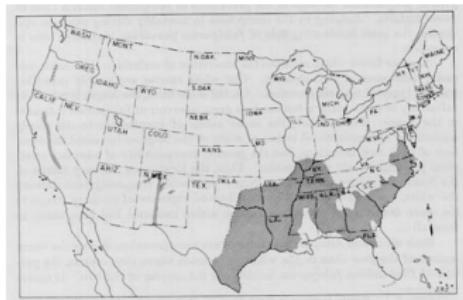
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Malaria

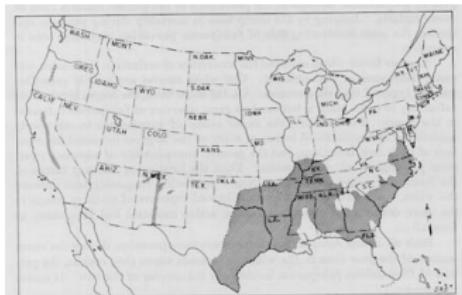
- A nasty, mosquito-borne disease



Map 4.—Areas of the continental United States believed to be malarious in 1931-33.

Malaria

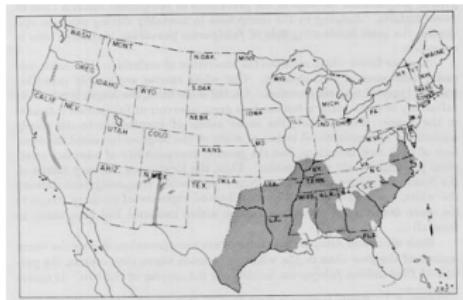
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- ▶ In some places (e.g., the southeastern US), it has been eradicated almost by accident



MAP 4.—Areas of the continental United States believed to be malarious in 1951-55.

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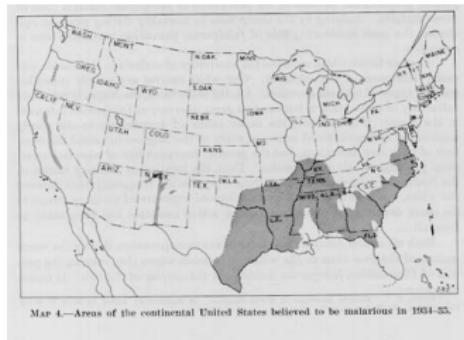
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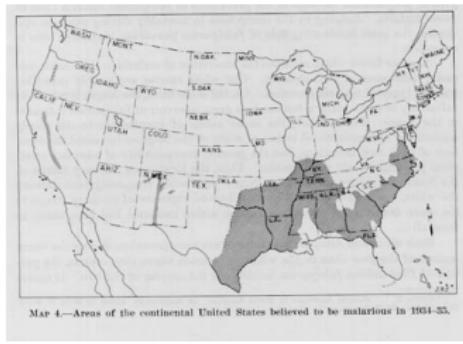
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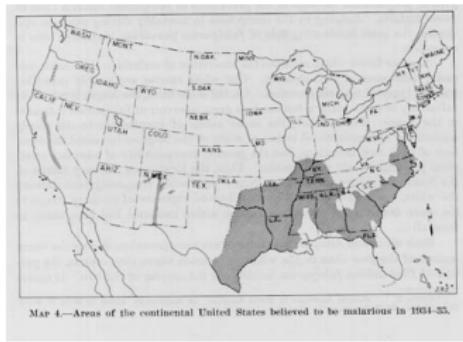
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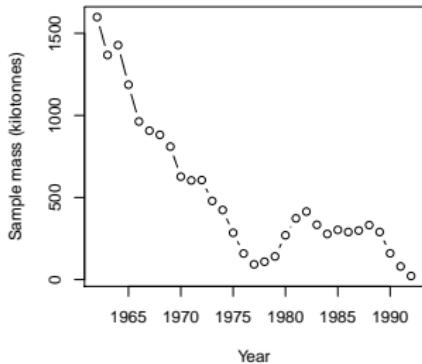
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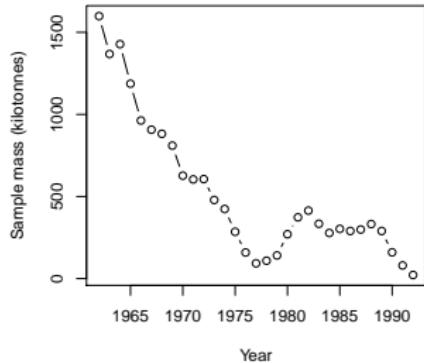
Cod fisheries

- ▶ Is the ocean too big for people to affect?



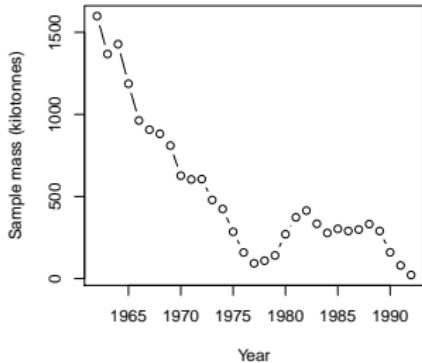
Cod fisheries

- ▶ Is the ocean too big for people to affect?
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Populations

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- Course structure

- People

Course content

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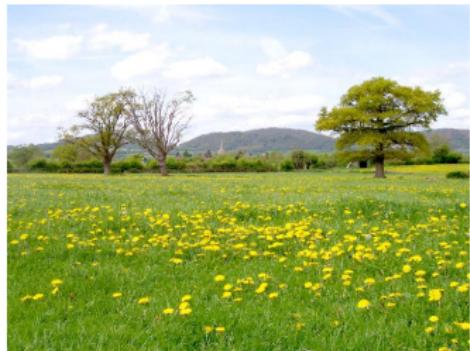
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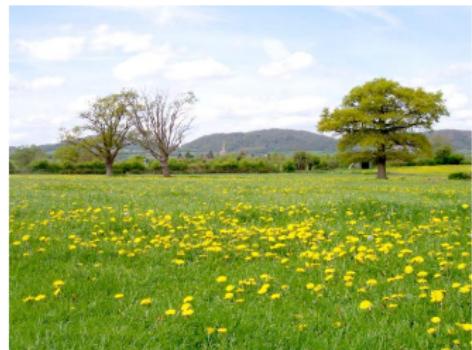
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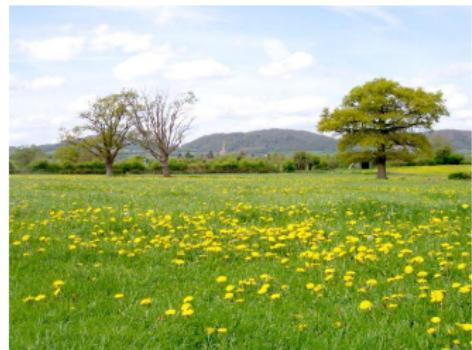
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Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

Example populations

Dandelions

Gypsy moths

Bacteria

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Gypsy moths

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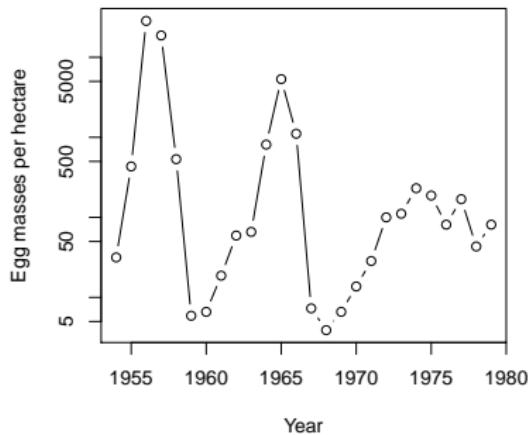
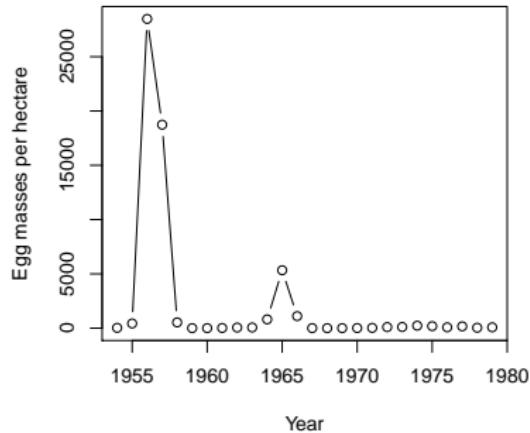


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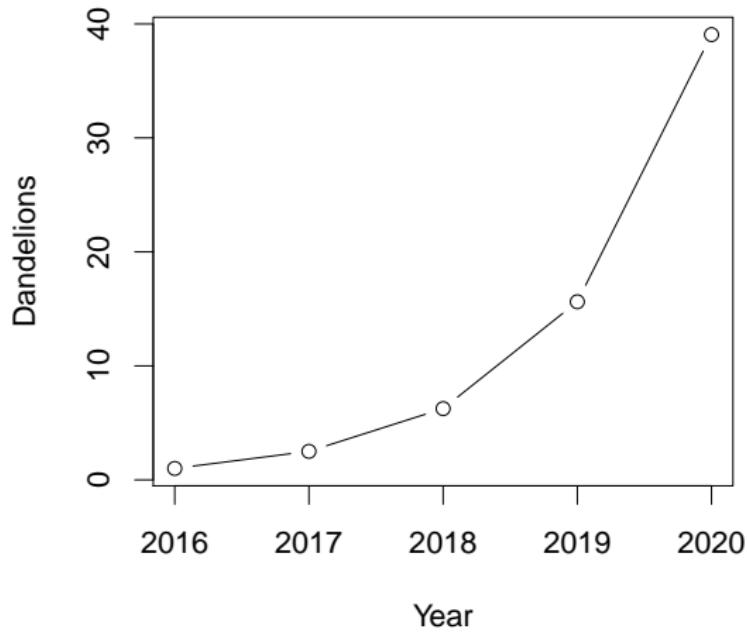
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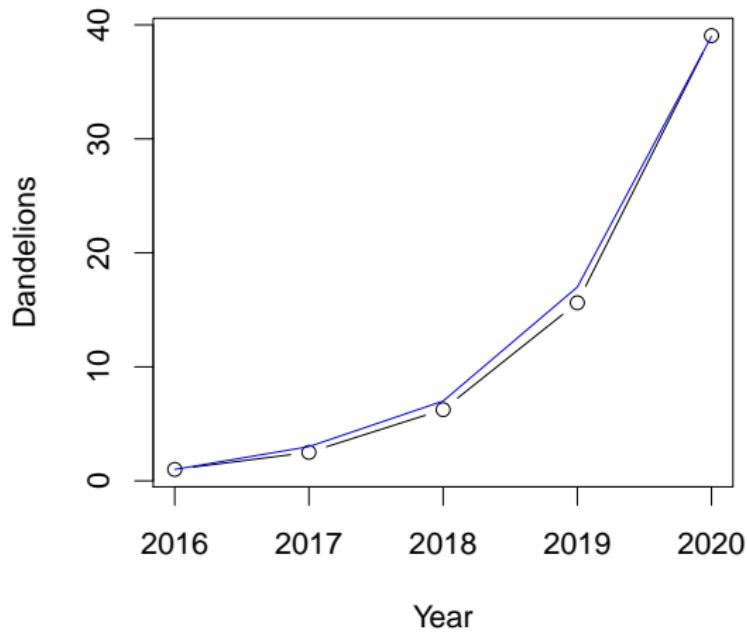
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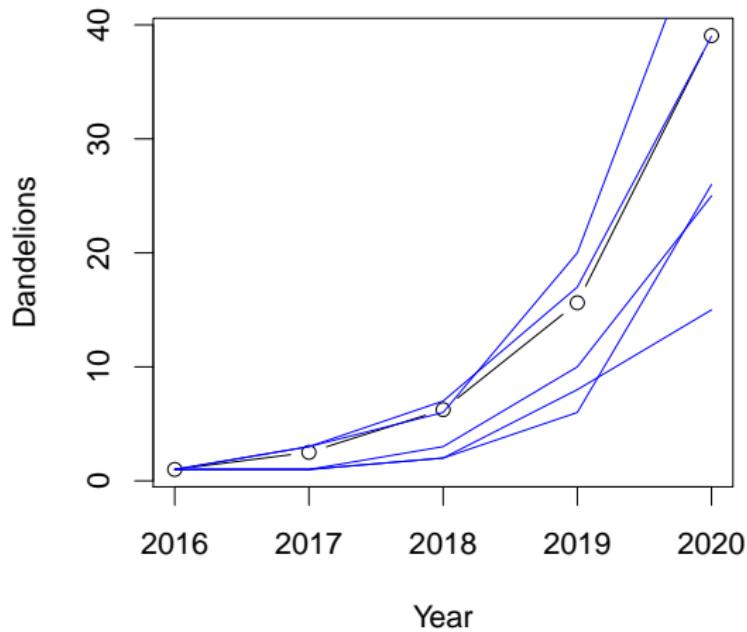
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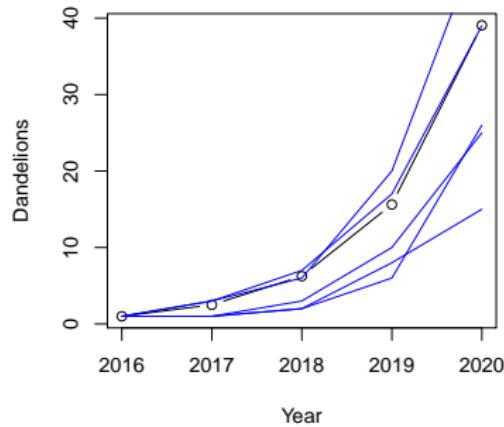
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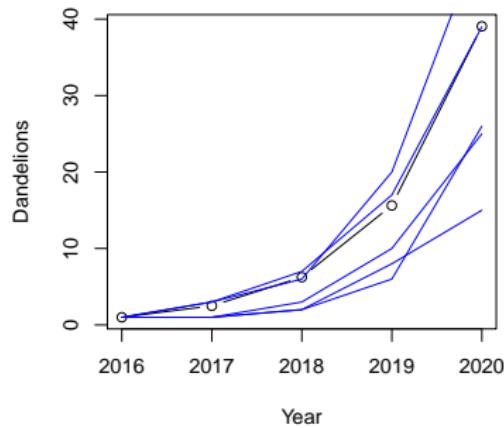


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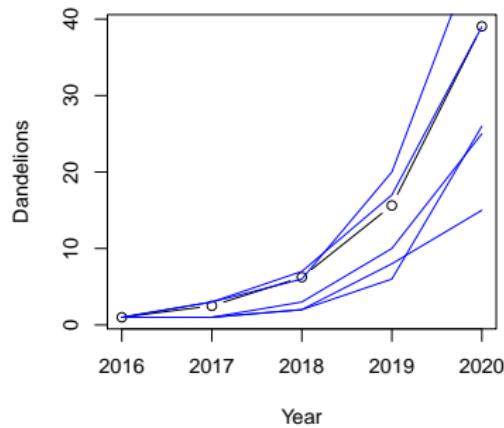
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Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

Example populations

Dandelions

Gypsy moths

Bacteria

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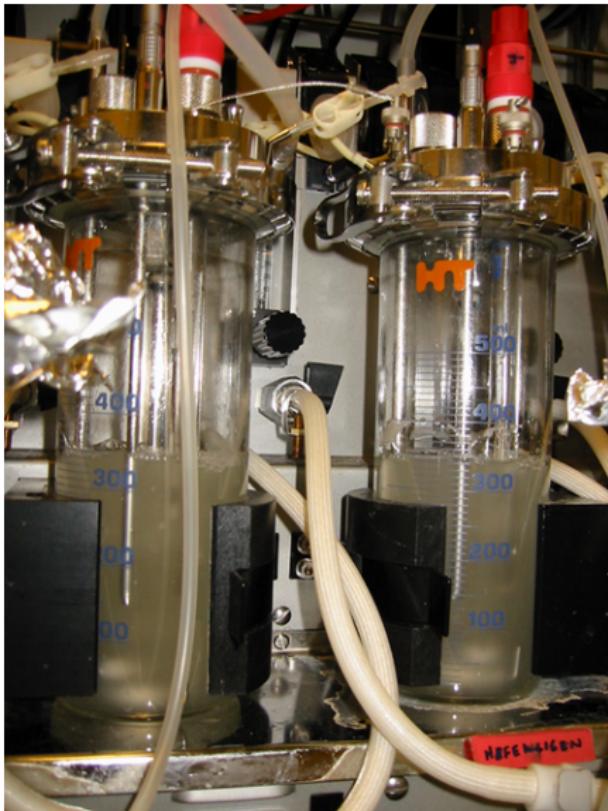
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Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

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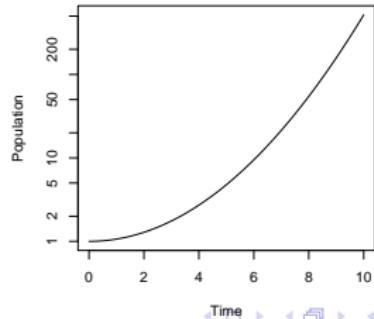
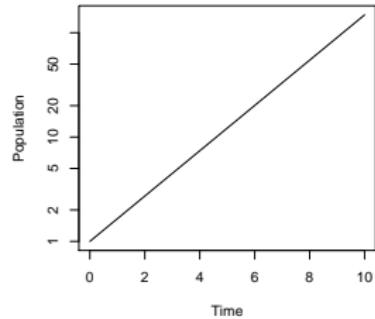
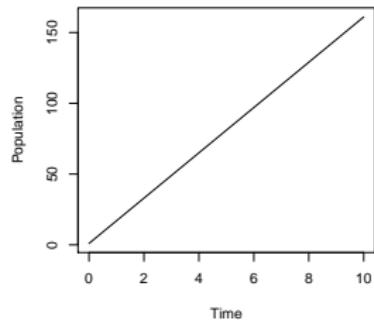
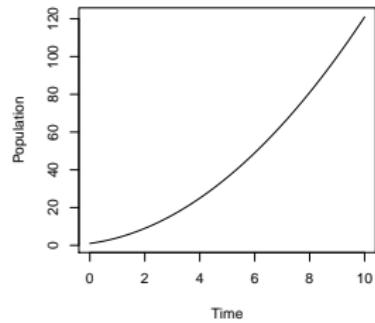
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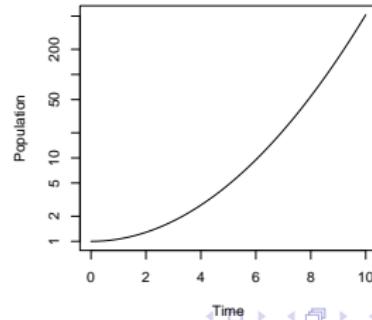
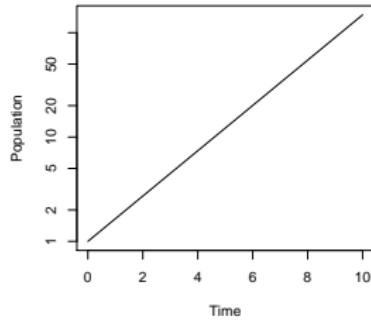
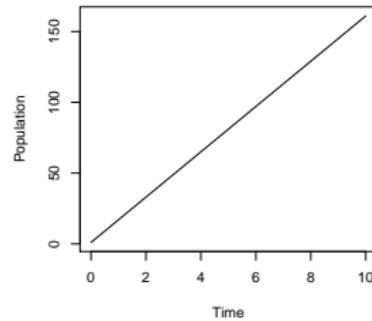
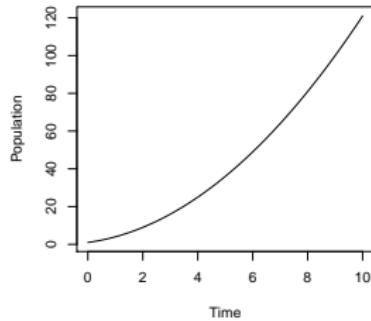
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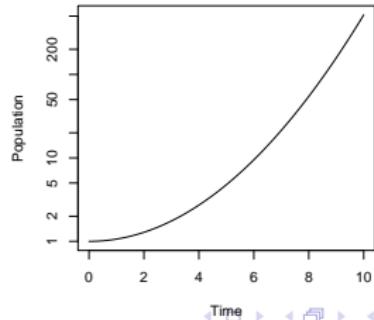
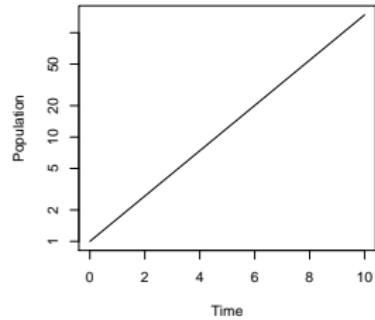
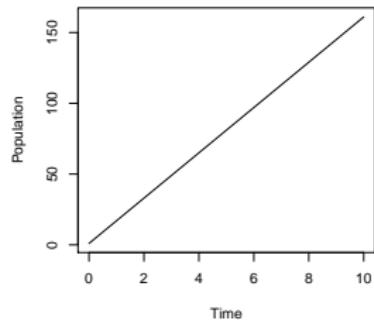
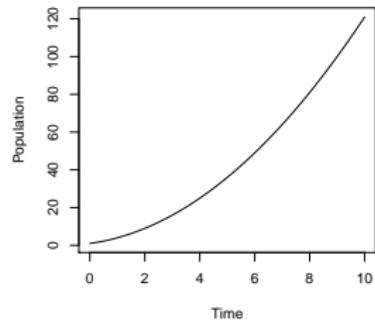
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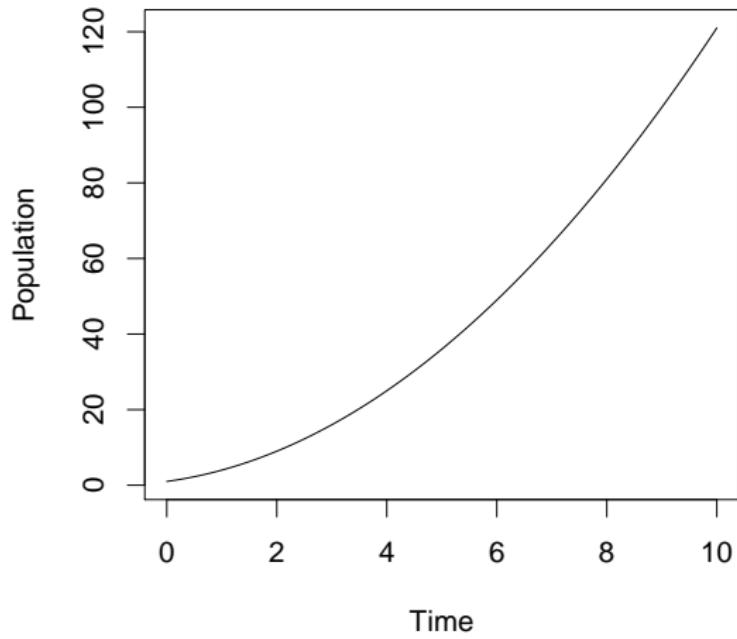


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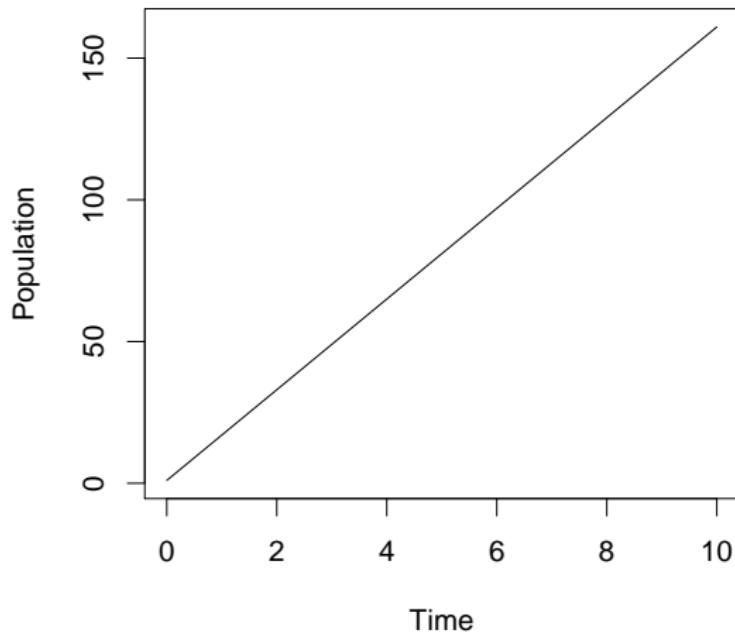
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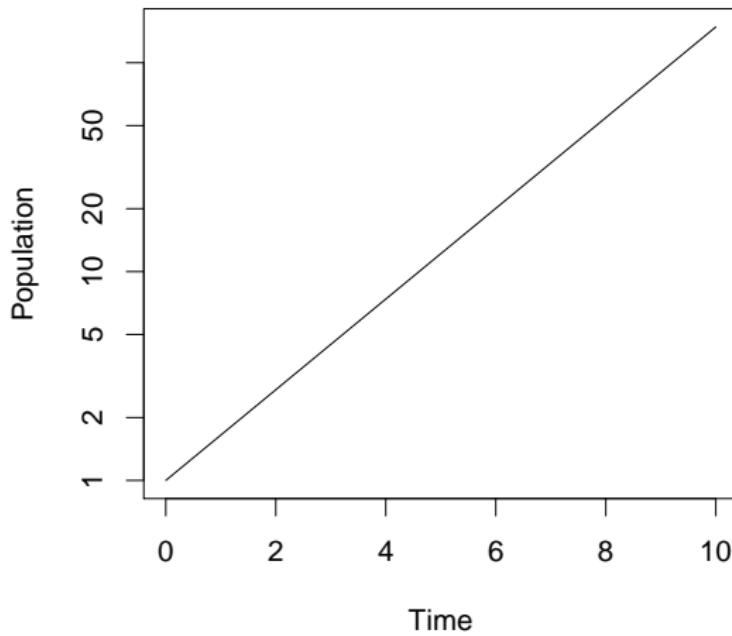
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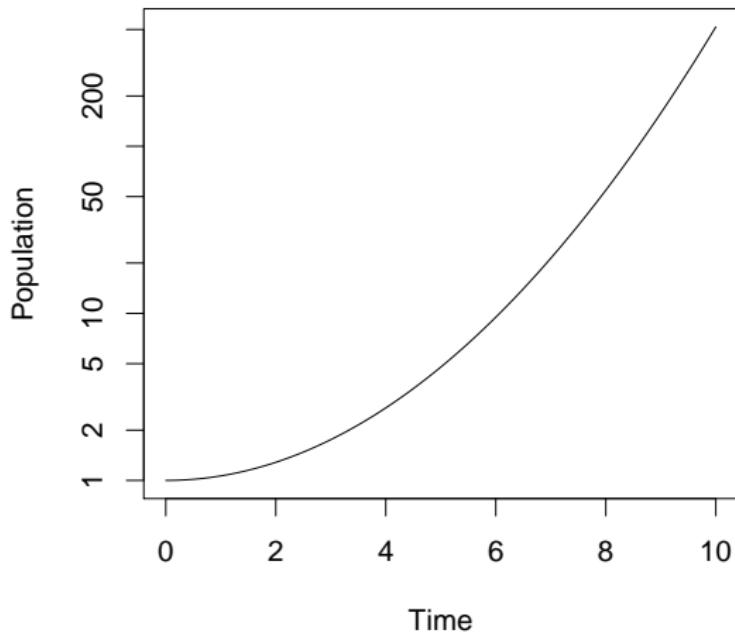
B (repeat)



C (repeat)

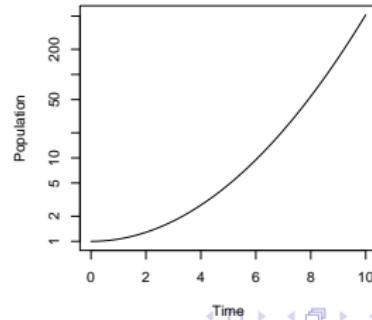
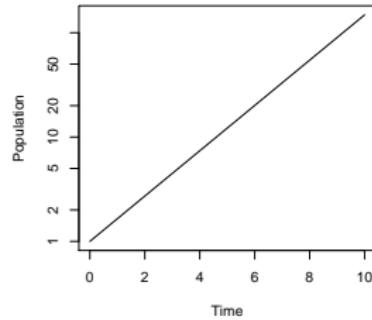
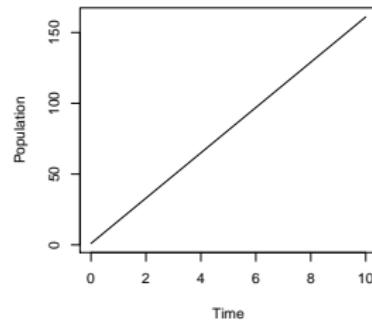
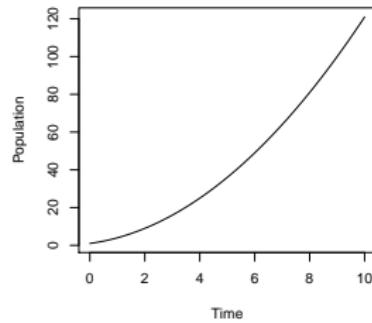


D (repeat)



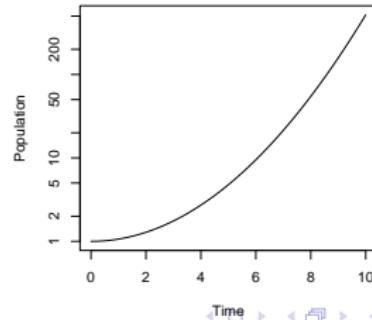
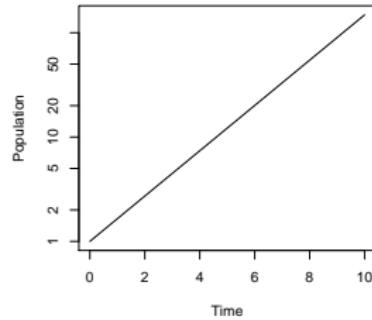
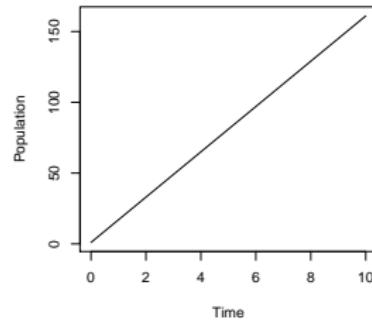
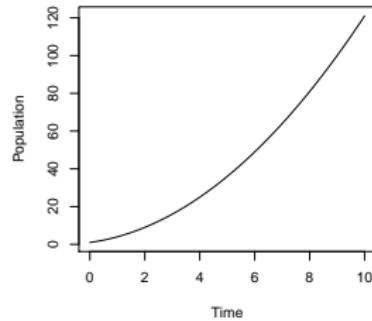
Exponential growth (repeat)

► Poll: What is exponential growth?



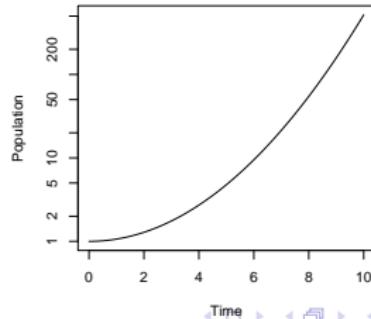
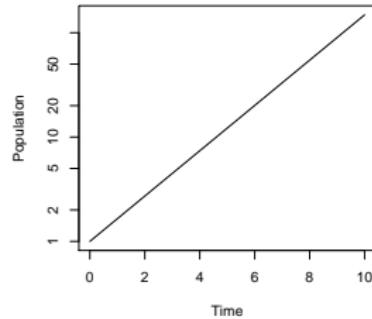
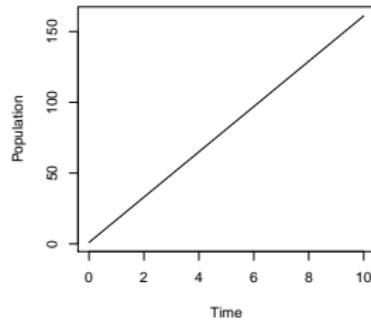
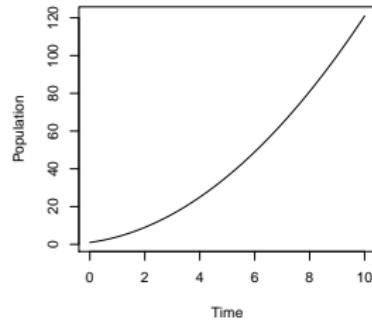
Exponential growth (repeat)

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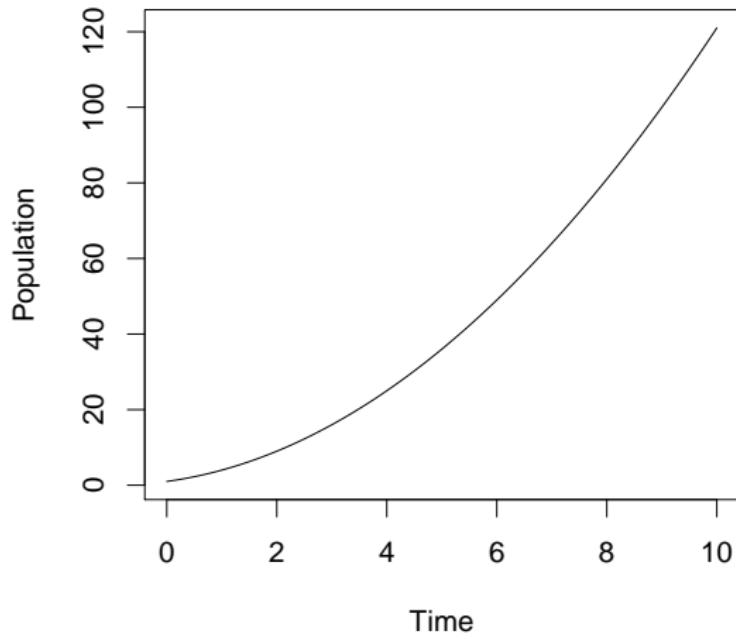


Exponential growth (repeat)

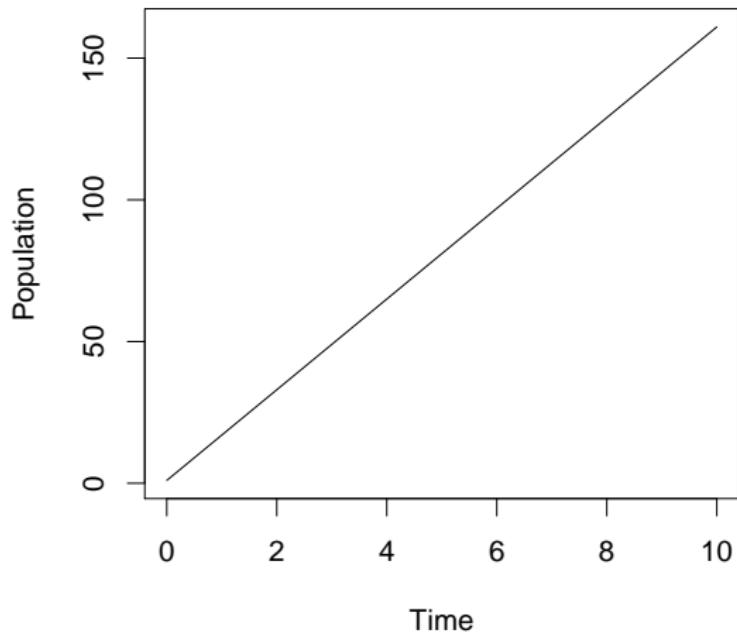
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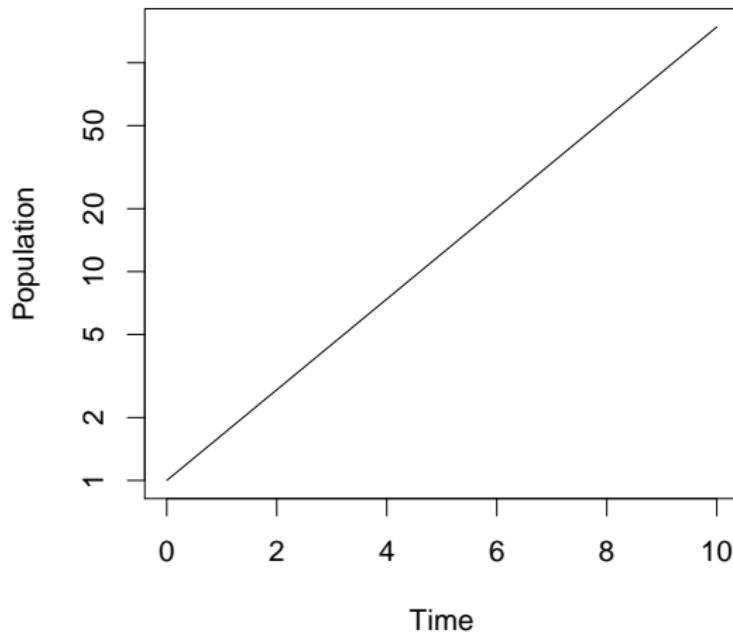
A (repeat)



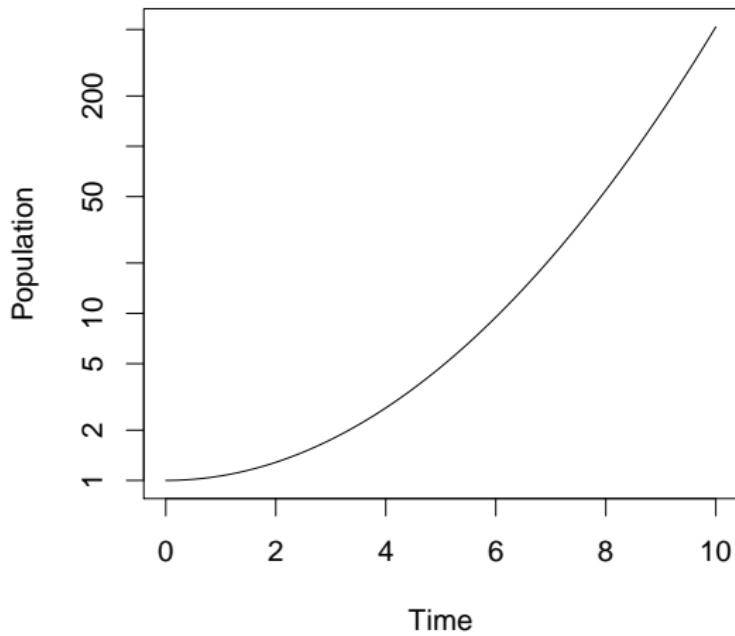
B (repeat)



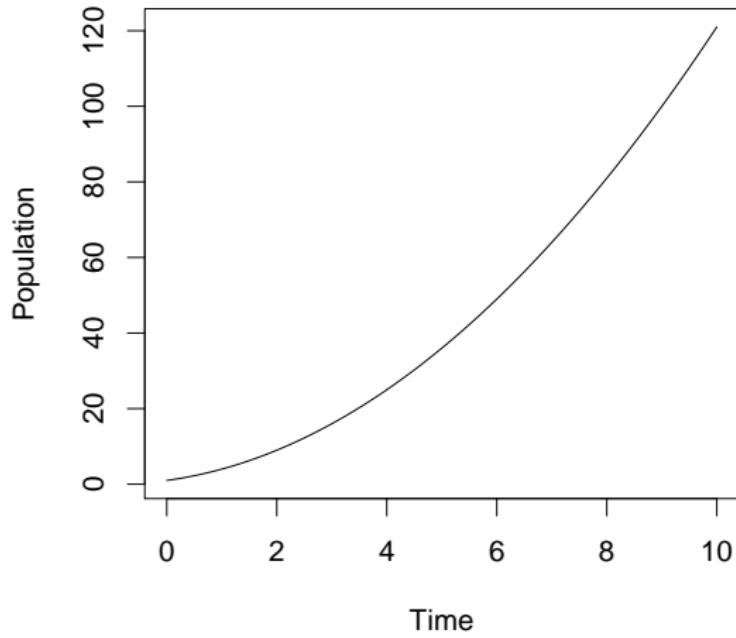
C (repeat)



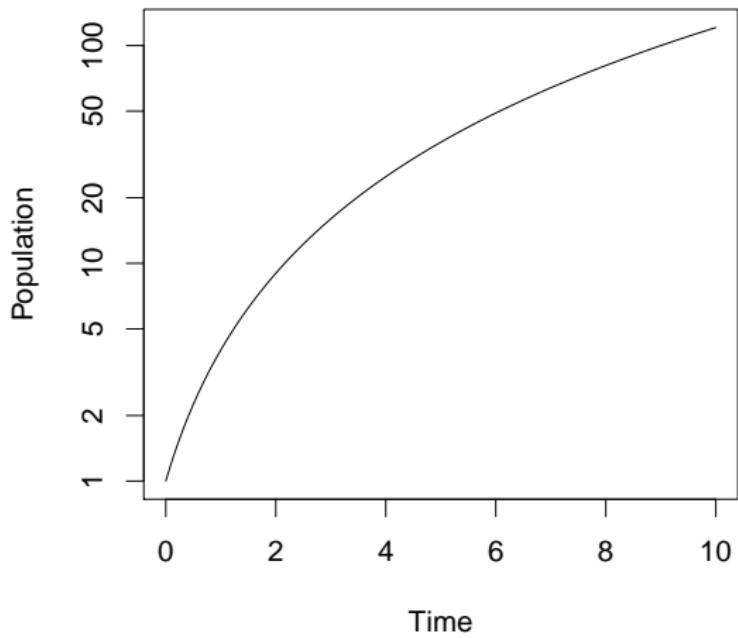
D (repeat)



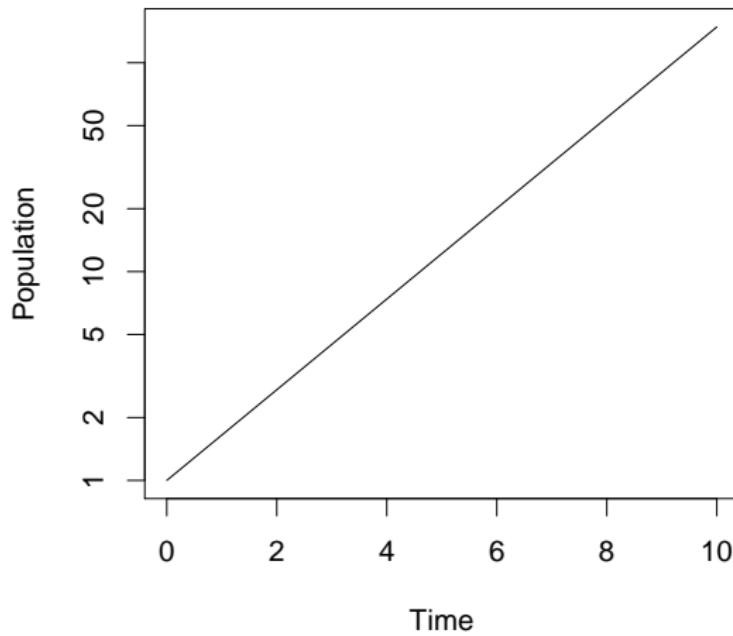
A (repeat)



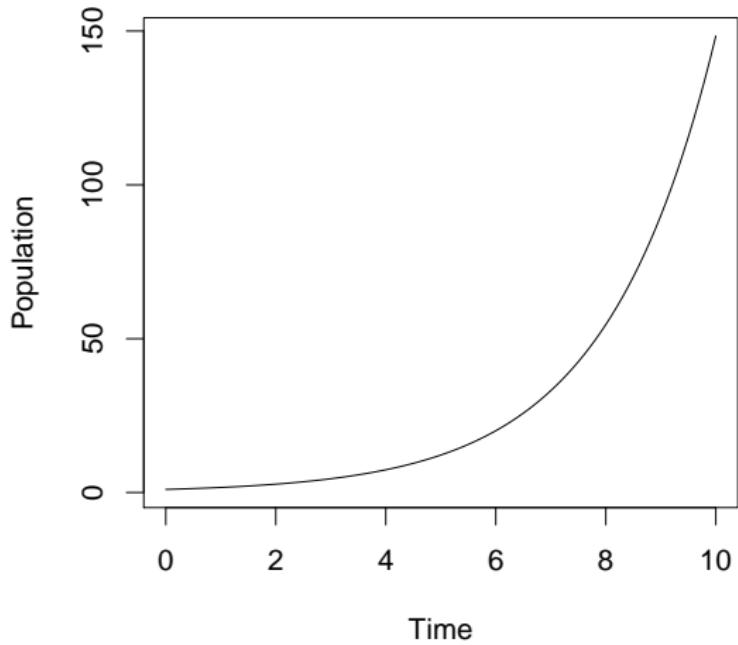
A on the log scale



C (repeat)



C on the linear scale



Types of growth

- arithmetic/linear:

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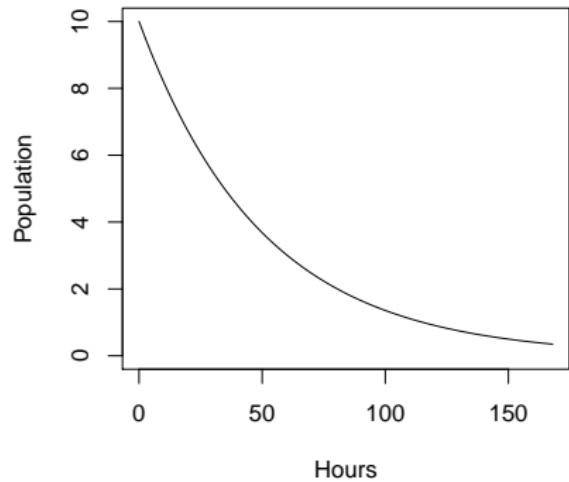
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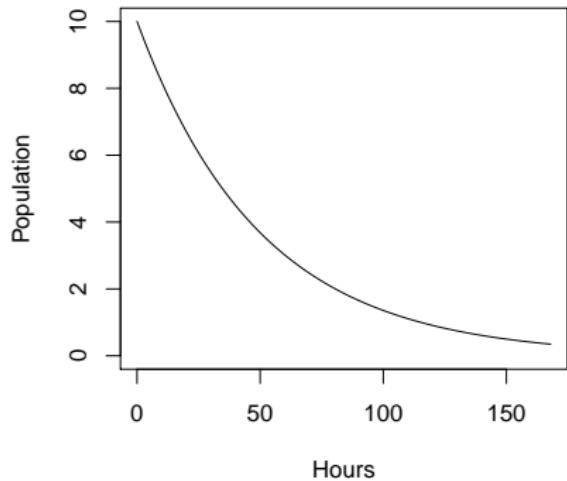
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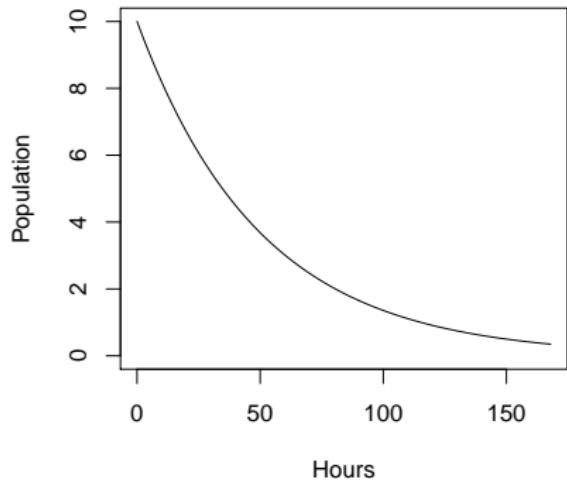
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Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

Example populations

Dandelions

Gypsy moths

Bacteria

Exponential growth

Log and linear scales

Time scales

Scales of comparison

- ▶ Poll: 1 is to 10 as 10 is to what?

Scales of comparison

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Scales of comparison

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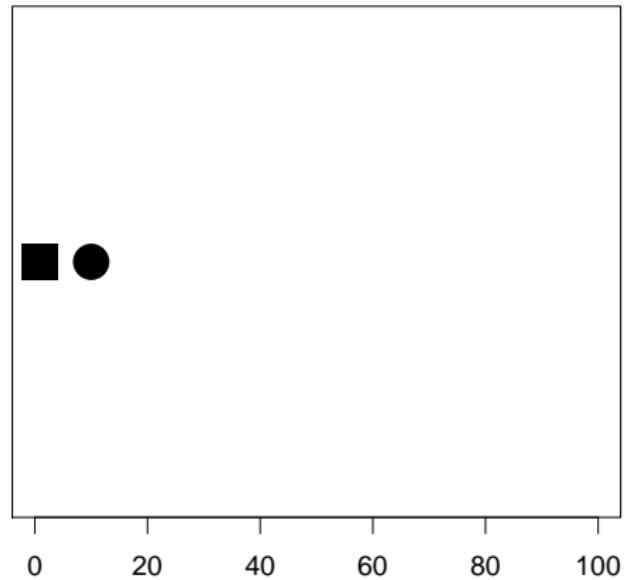
Scales of comparison

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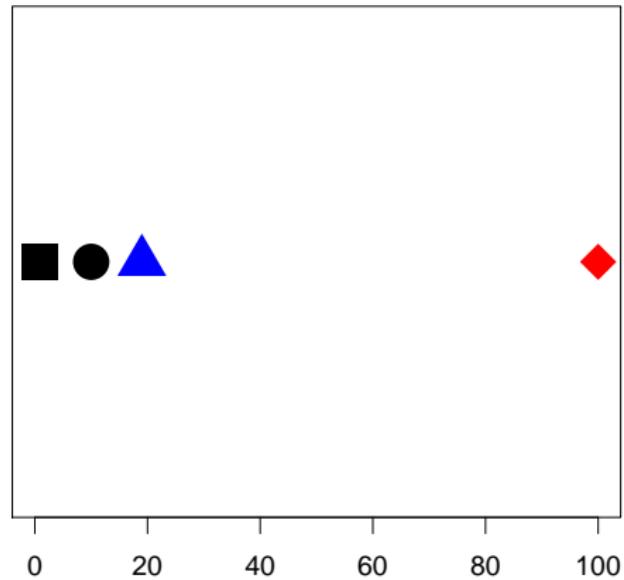
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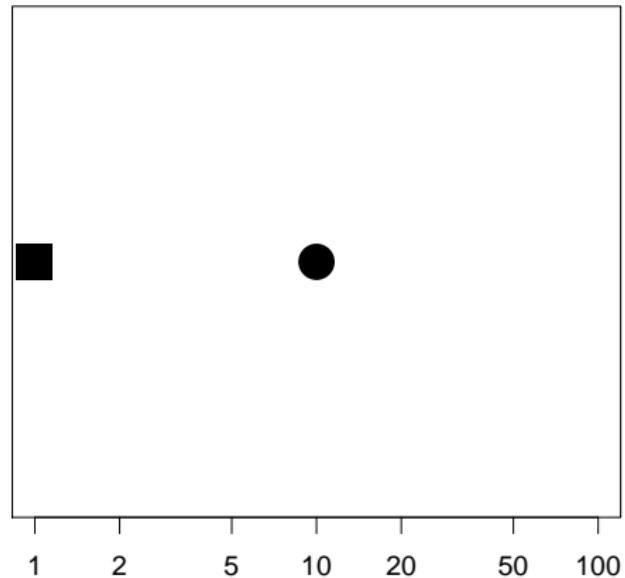
Scales of display



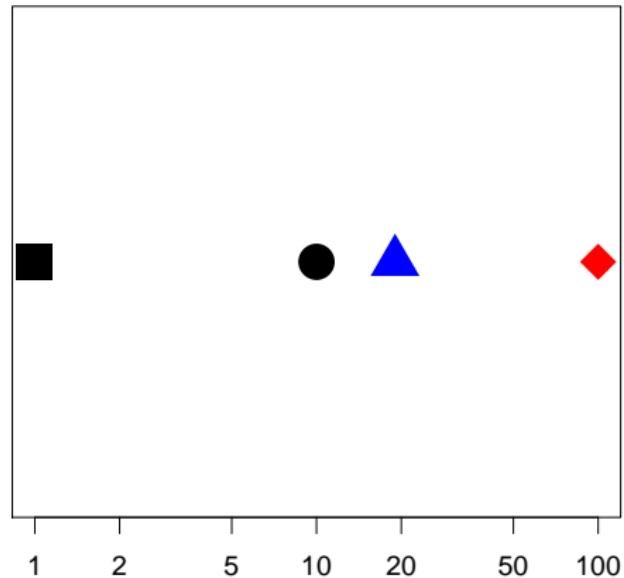
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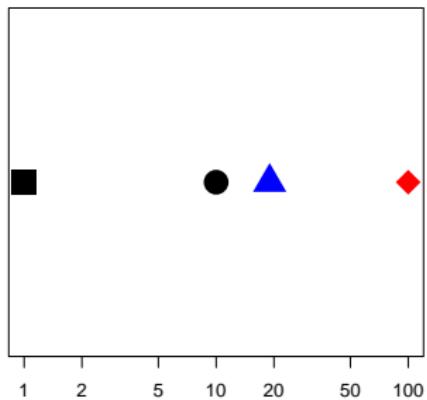
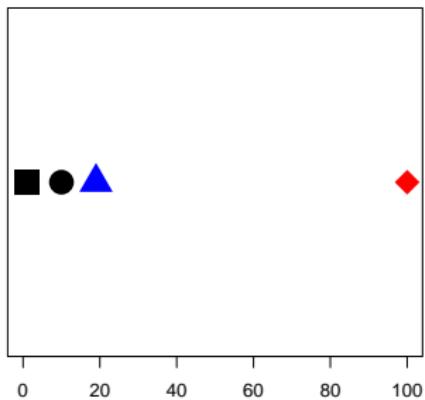
Scales of display



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Scales of display



There is only one log scale; it doesn't matter which base you use!

Canadian provinces

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Canadian provinces

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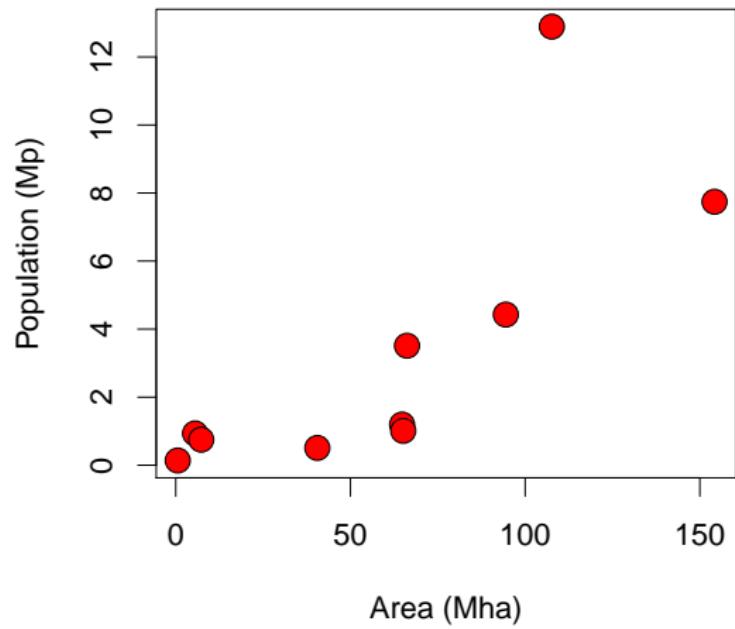
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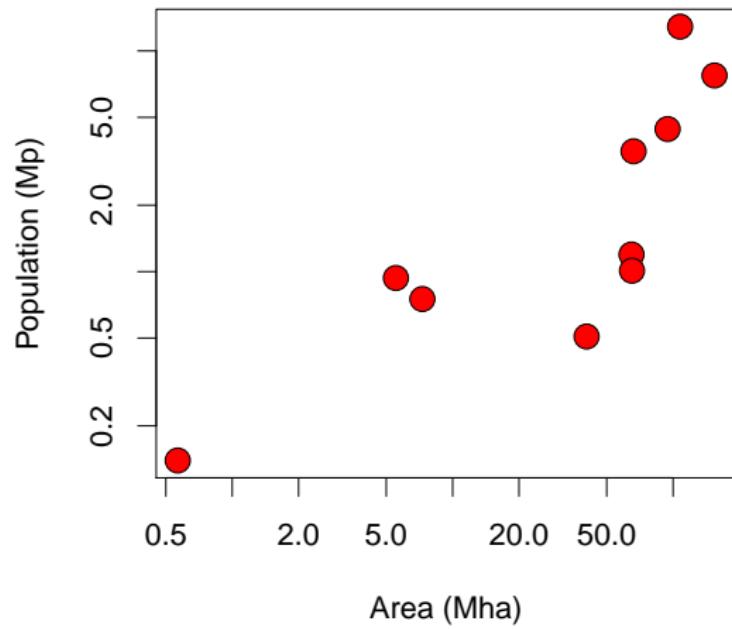
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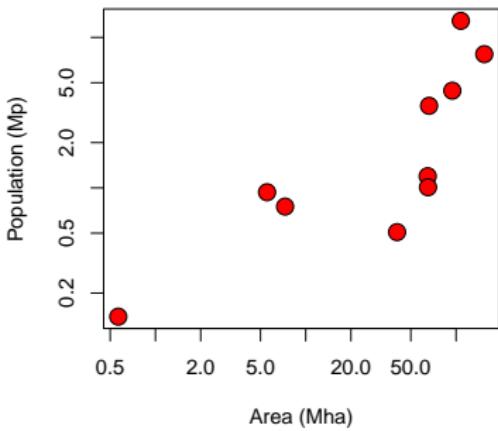
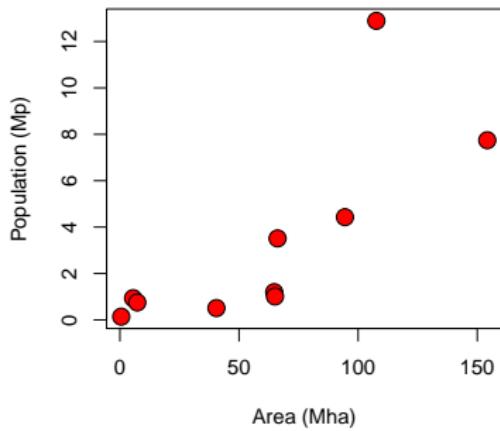
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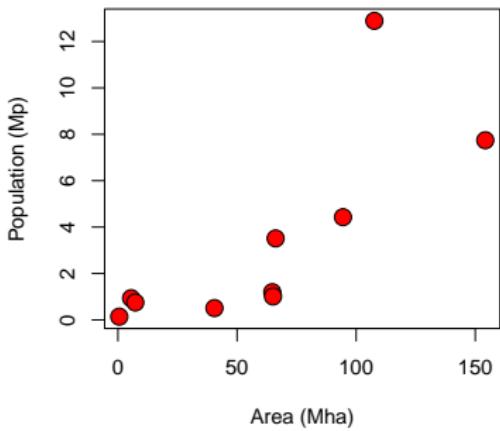
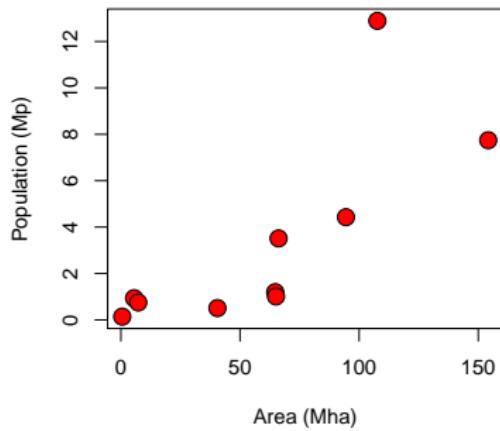
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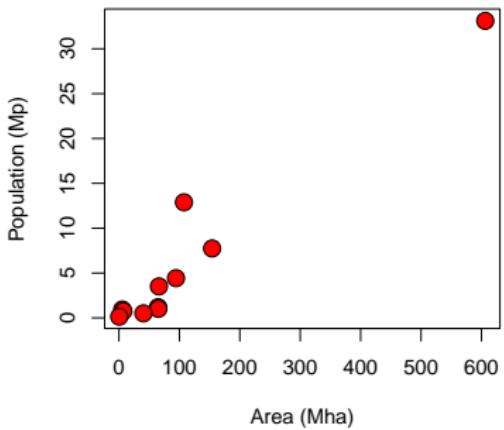
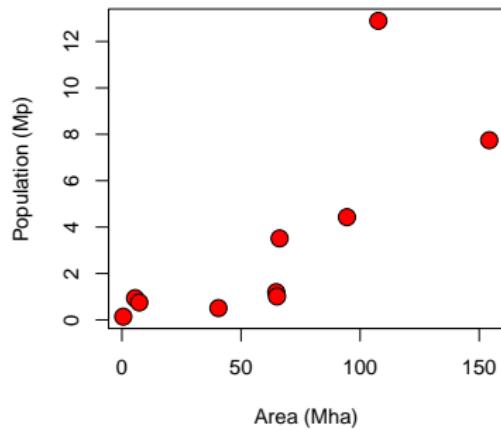
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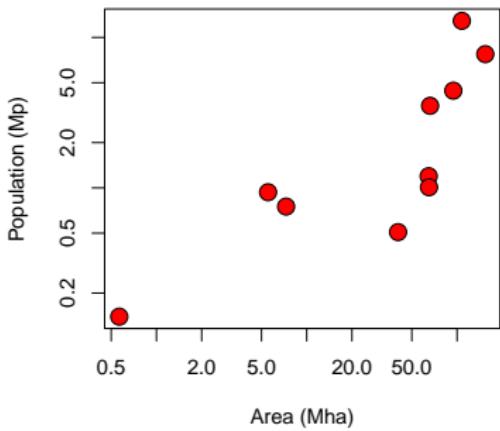
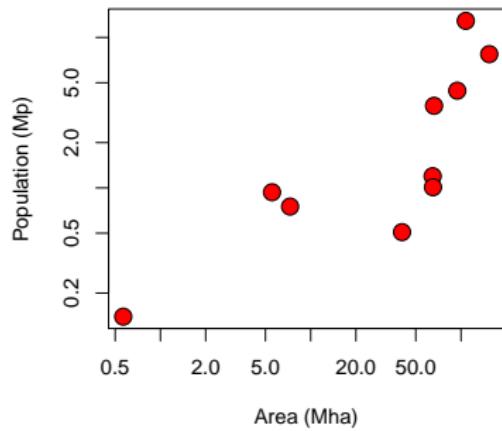
Canadian provinces plus Canada?



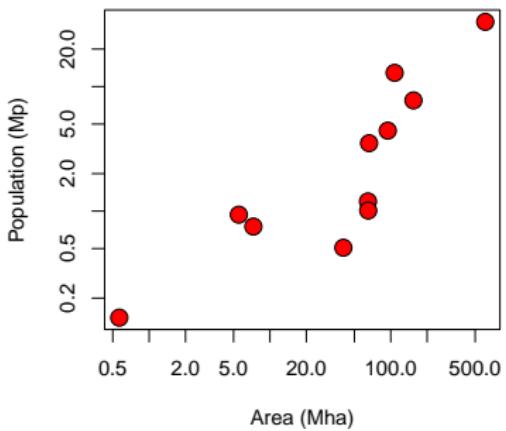
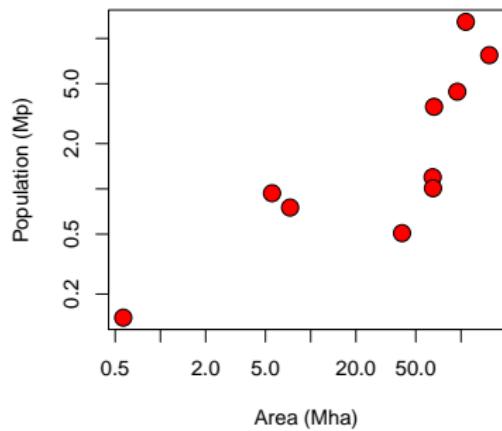
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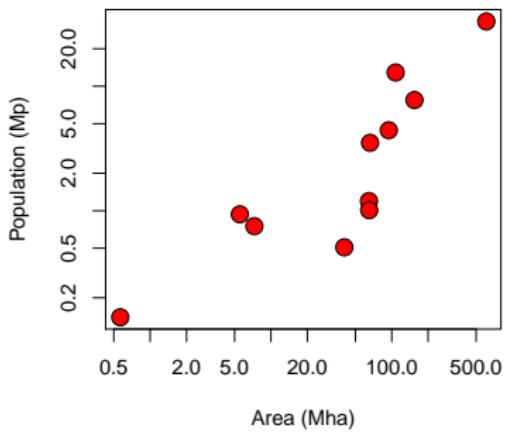
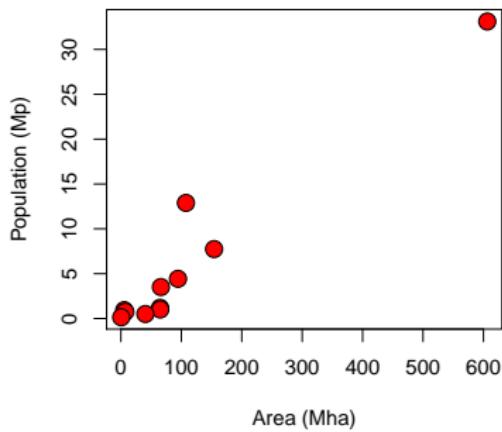
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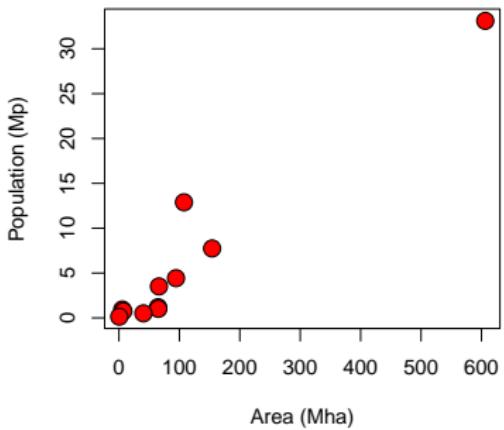
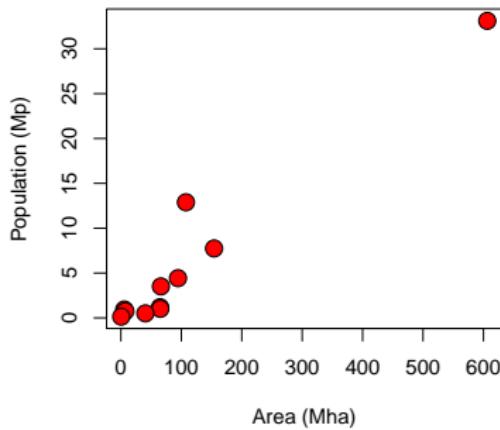
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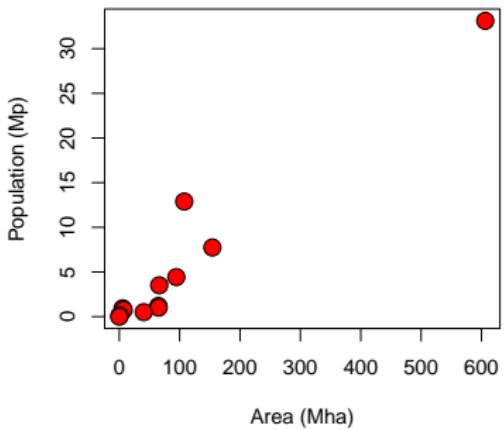
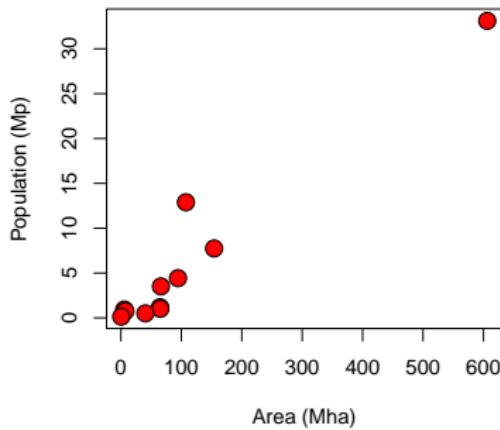
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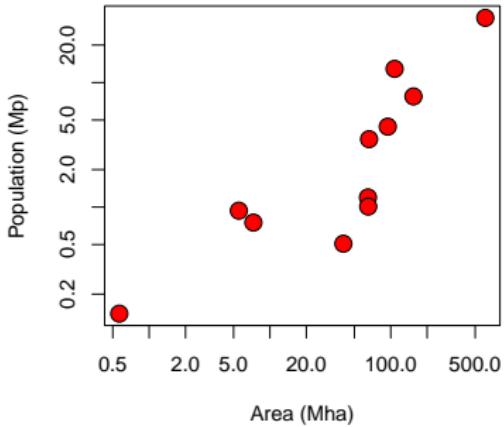
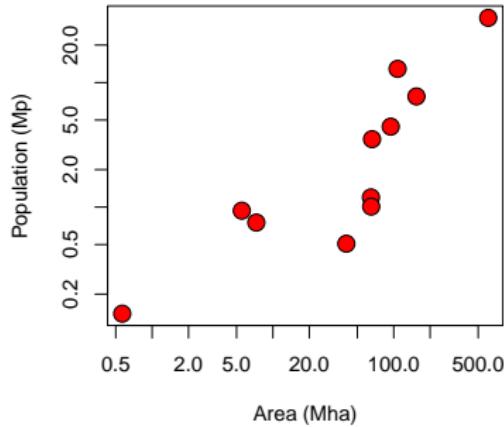
Canada plus Room 1105



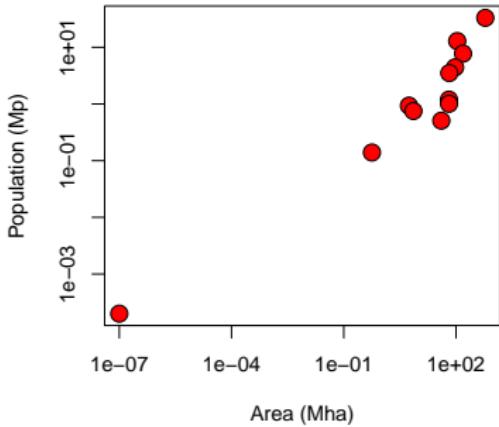
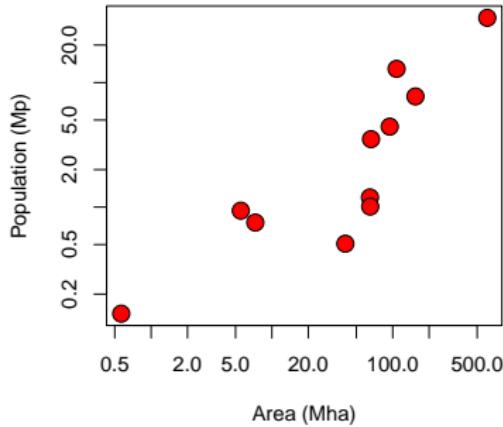
Canada plus Room 1105



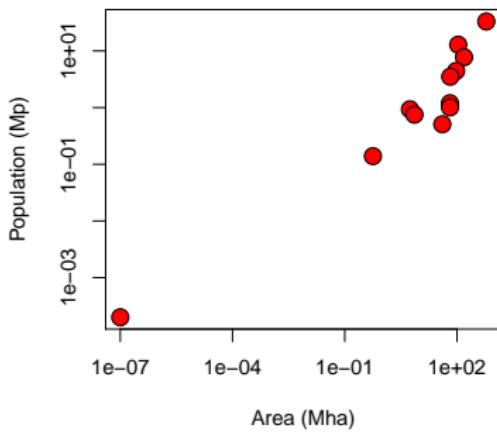
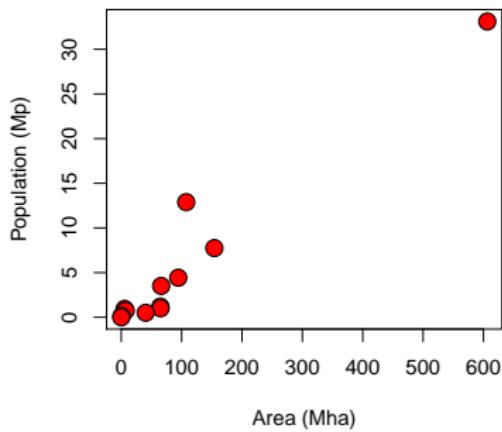
Canada plus room Room 1105?



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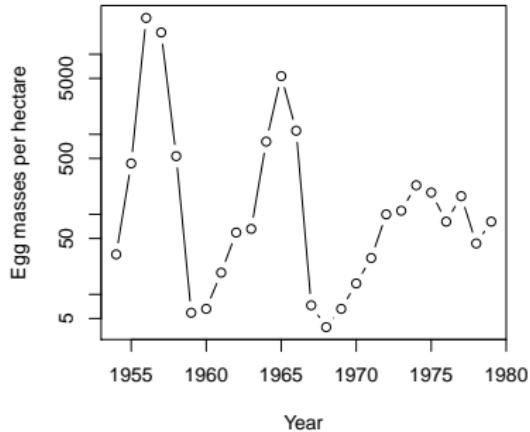
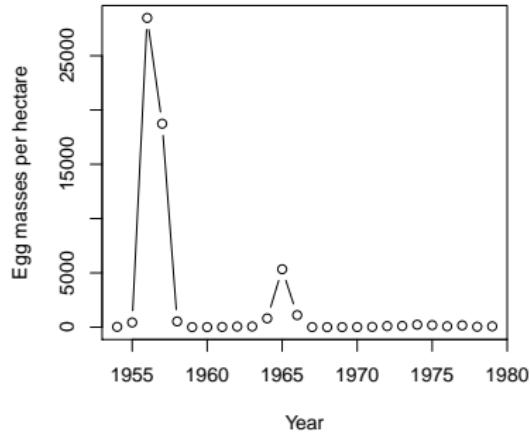
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Gypsy-moth example



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Outline

Course overview

- Course structure

- People

Course content

- Learning goals

- Examples

Example populations

- Dandelions

- Gypsy moths

- Bacteria

Exponential growth

- Log and linear scales

- Time scales

Speeding in Taiwan

- A life experience



Speeding in Taiwan

- ▶ A life experience
- ▶ Some clarifications



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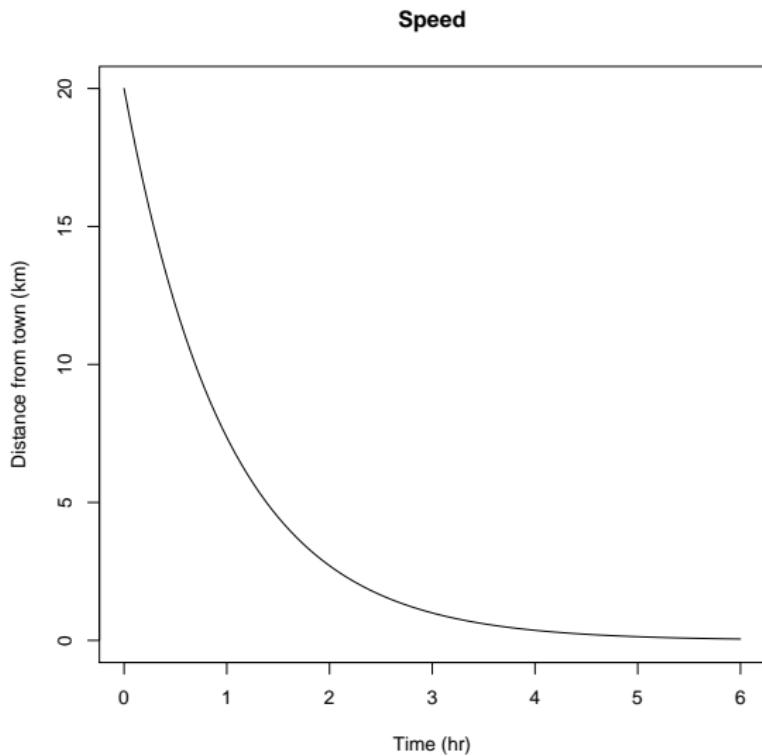
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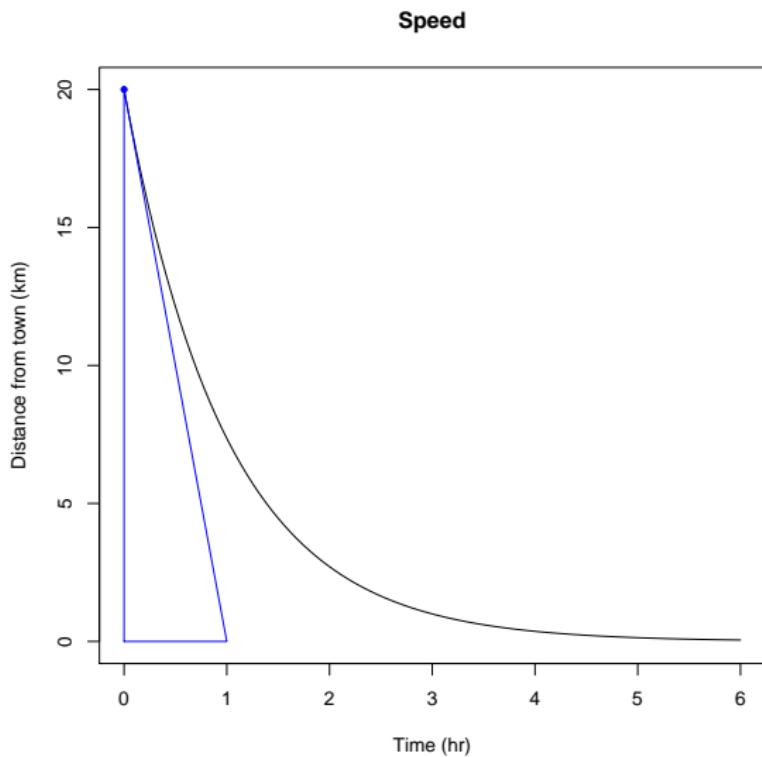
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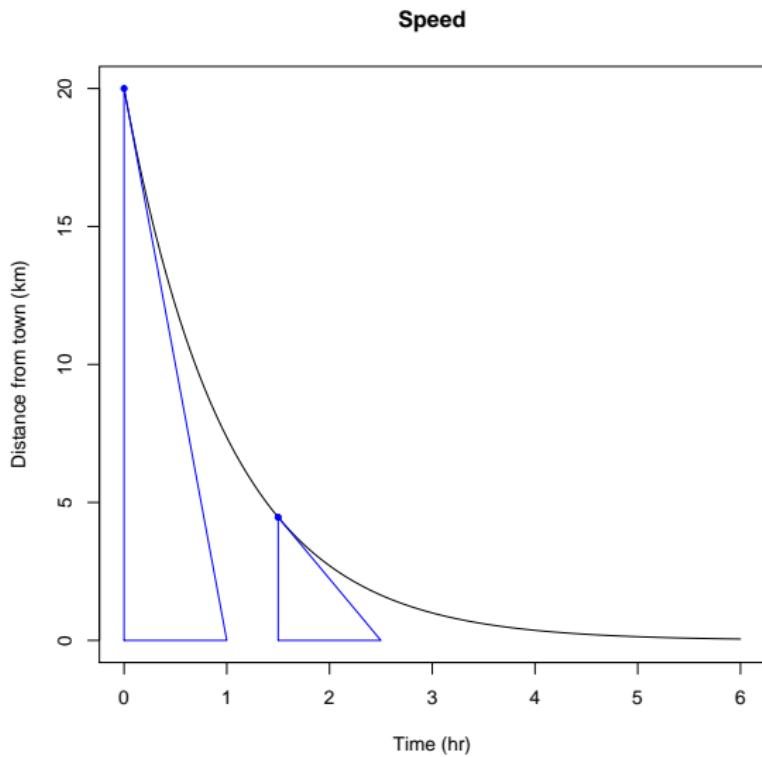
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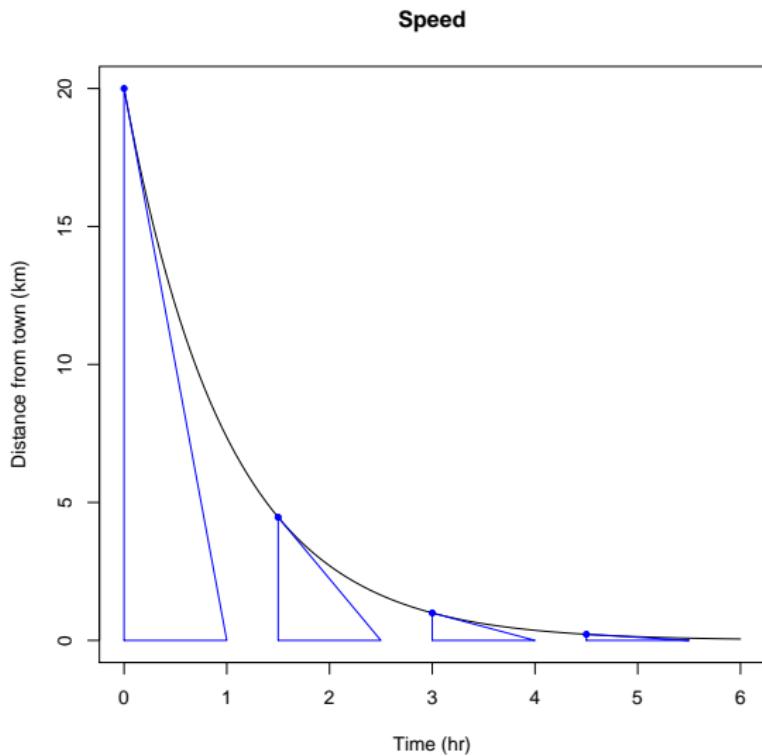
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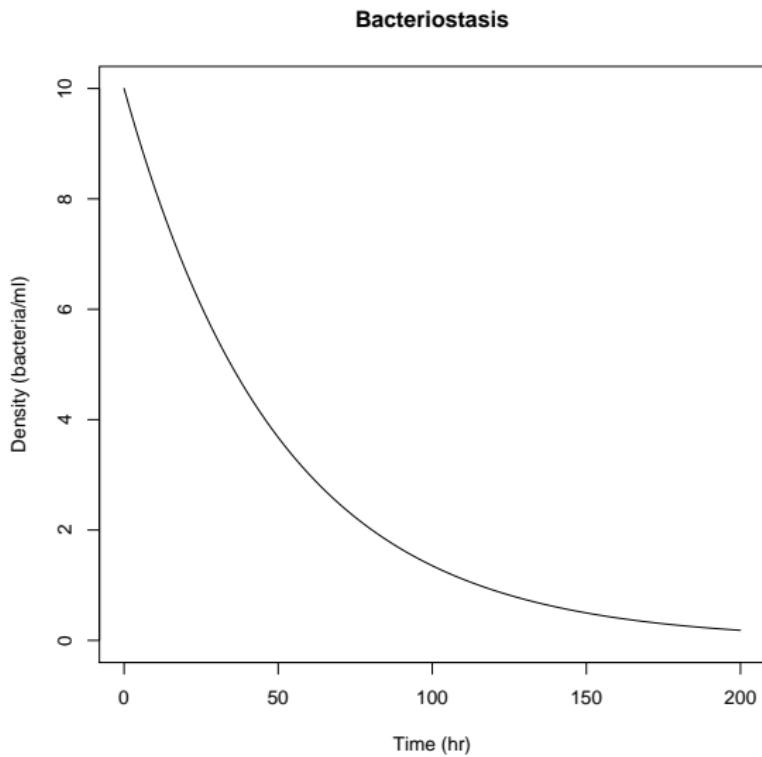
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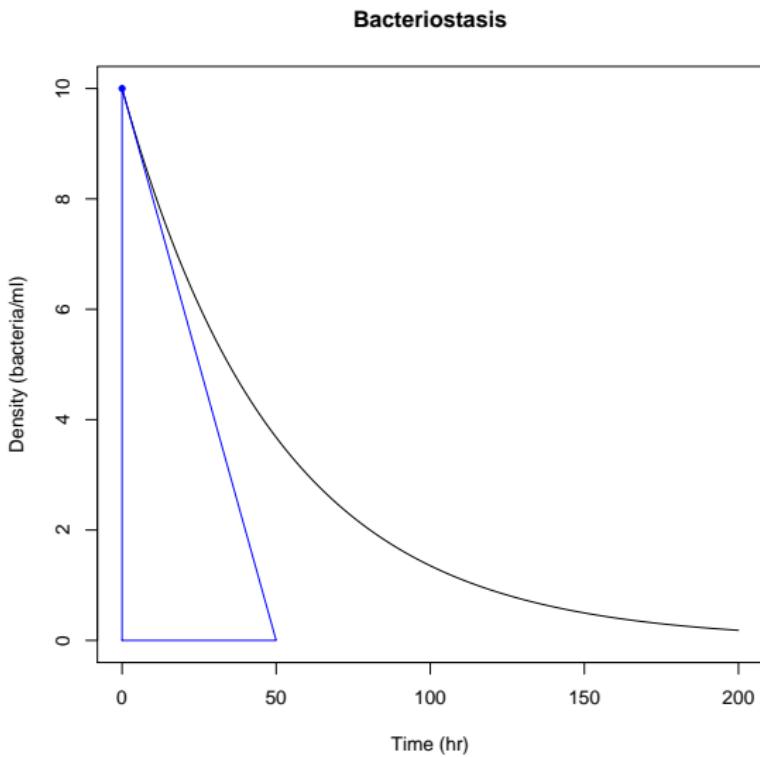
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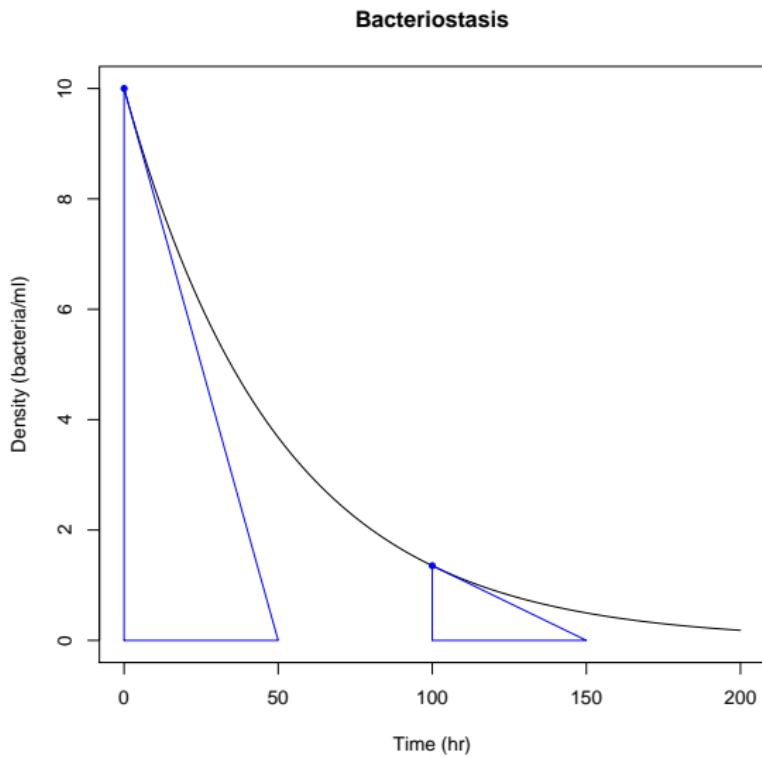
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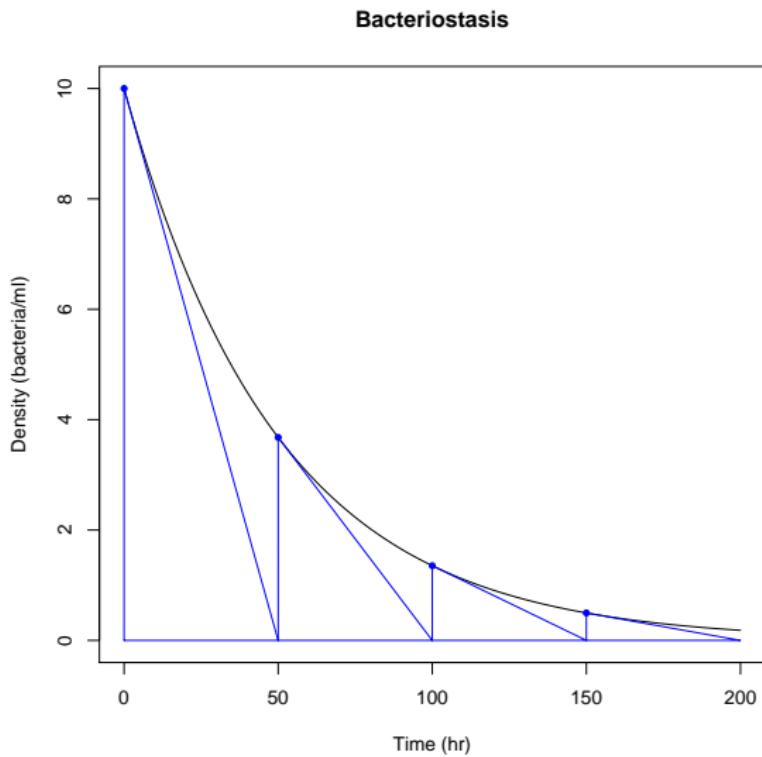
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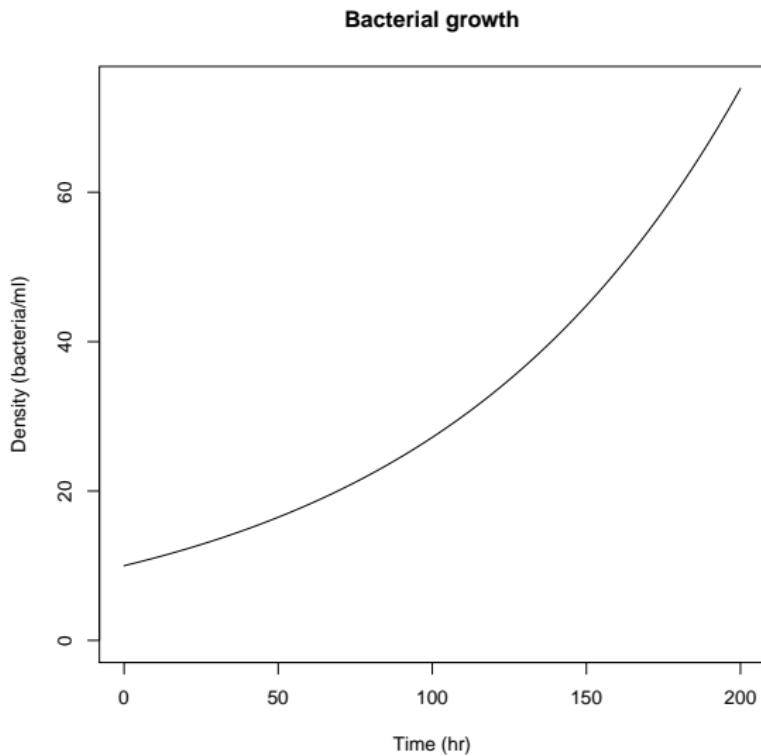
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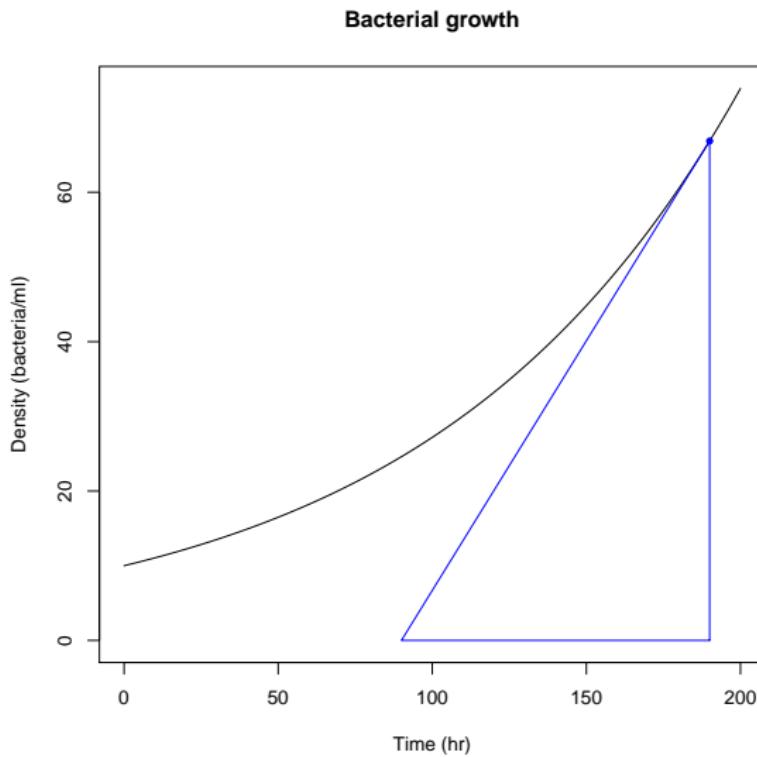
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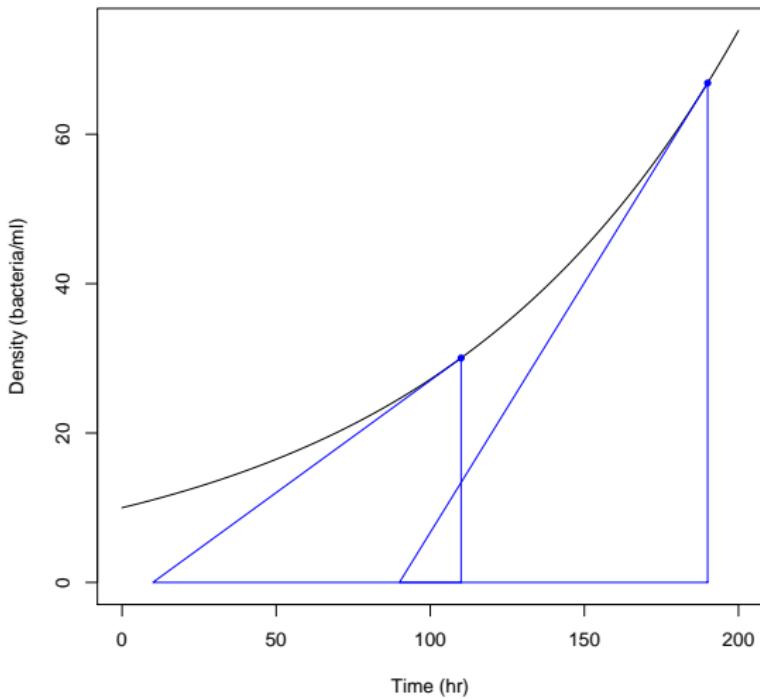


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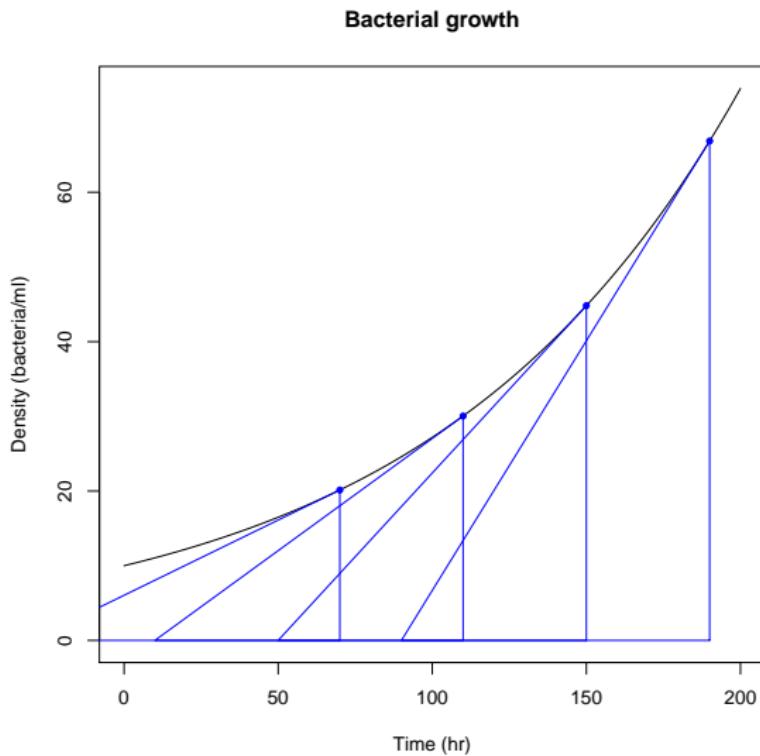


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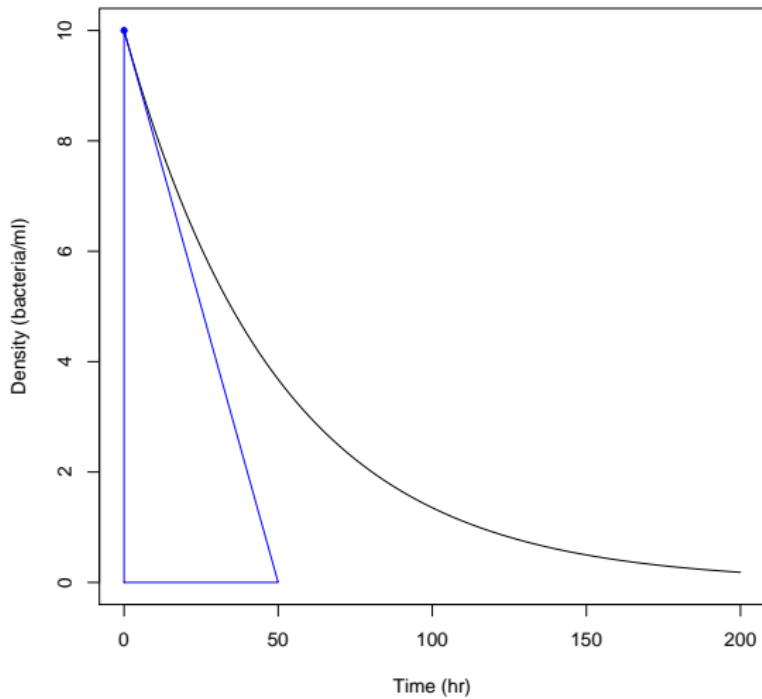
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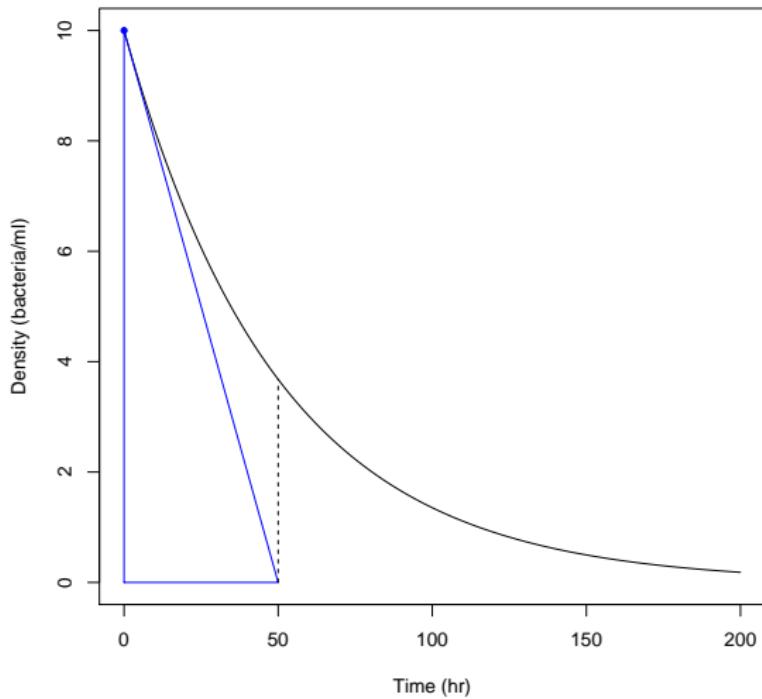
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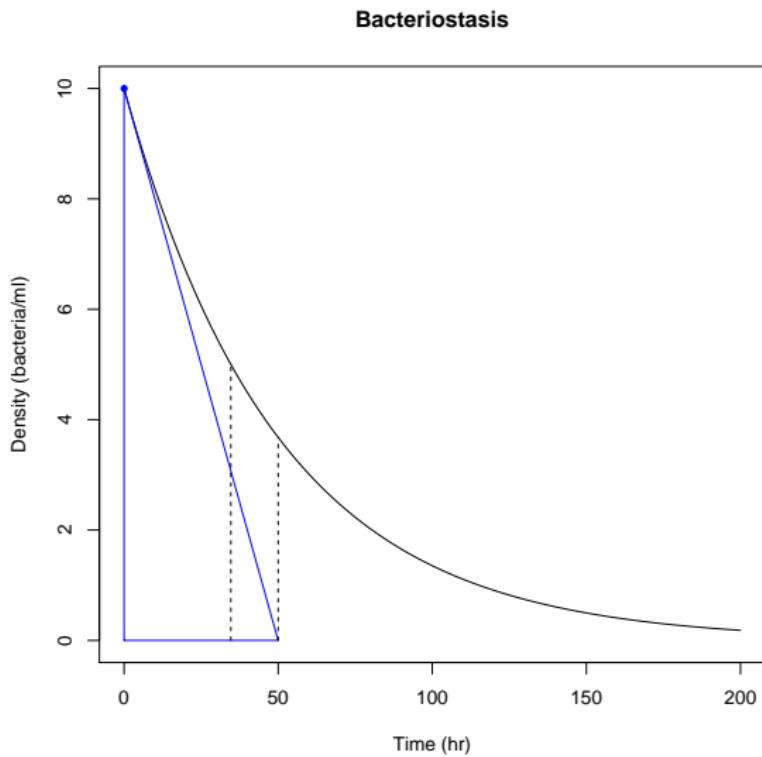
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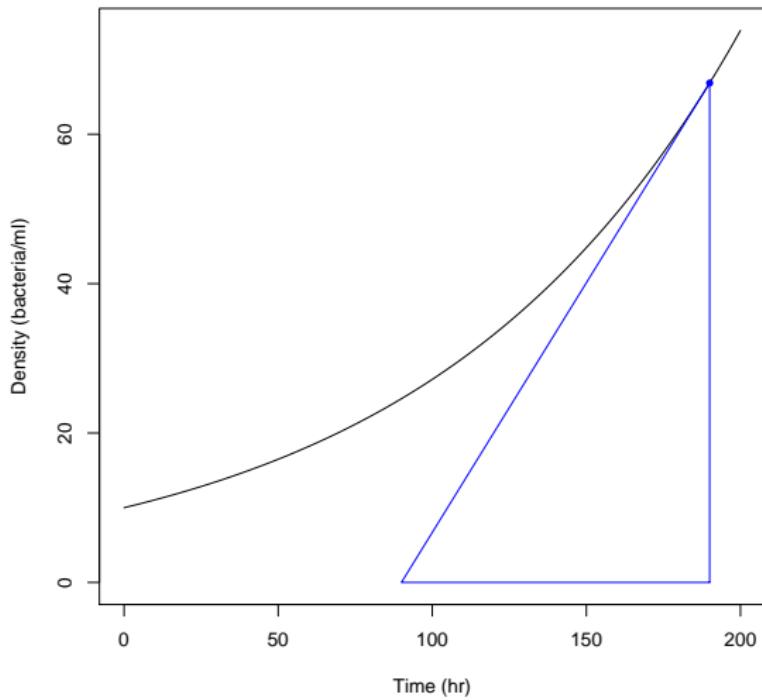
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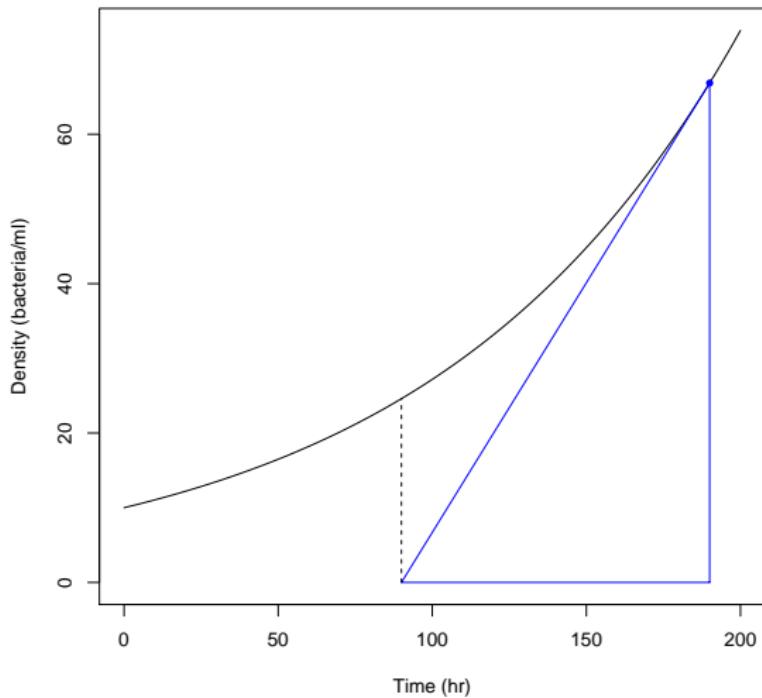
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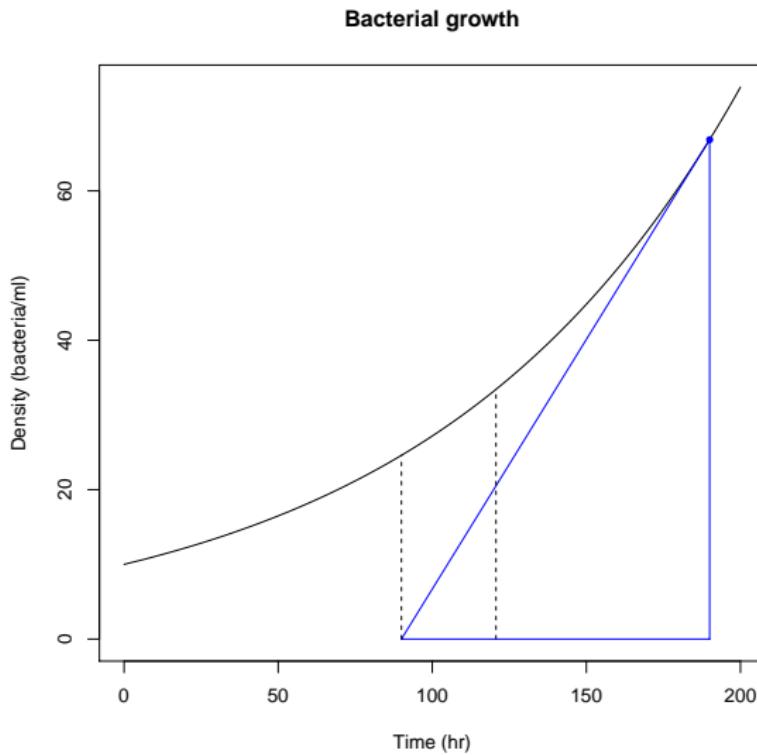
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