BIO3SS3: Population Ecology

UNIT 1: Introduction

Outline

- Course overview
  - Course structure
  - People
- Course content
  - Learning goals
  - Examples
- Example populations
  - Dandelions
  - Gypsy moths
  - Bacteria
- Exponential growth
  - Log and linear scales
  - Time scales

Outline

Communication

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Outline

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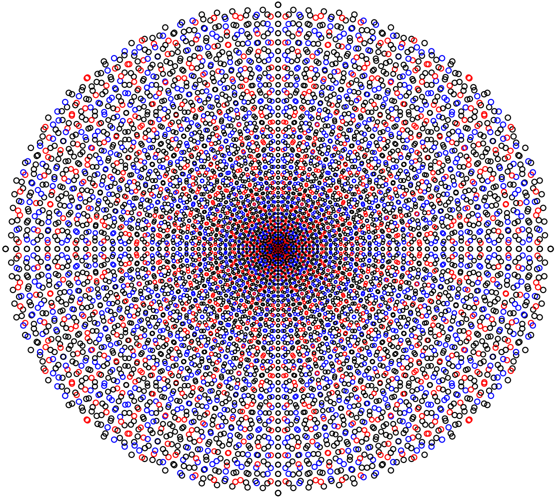
Pythagorean triples



Which country?

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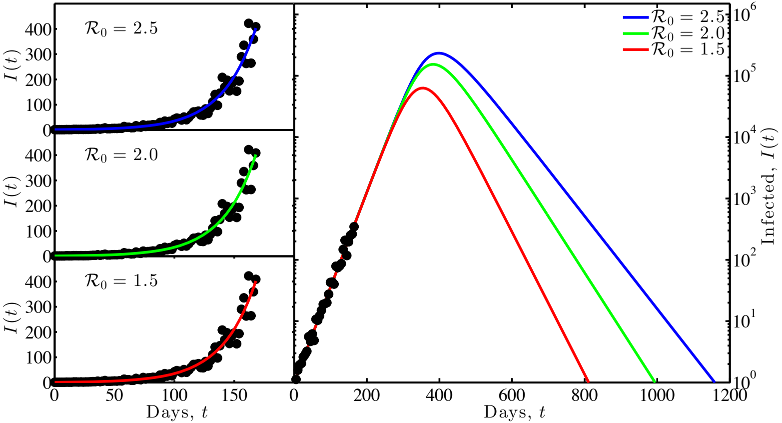
Disease research



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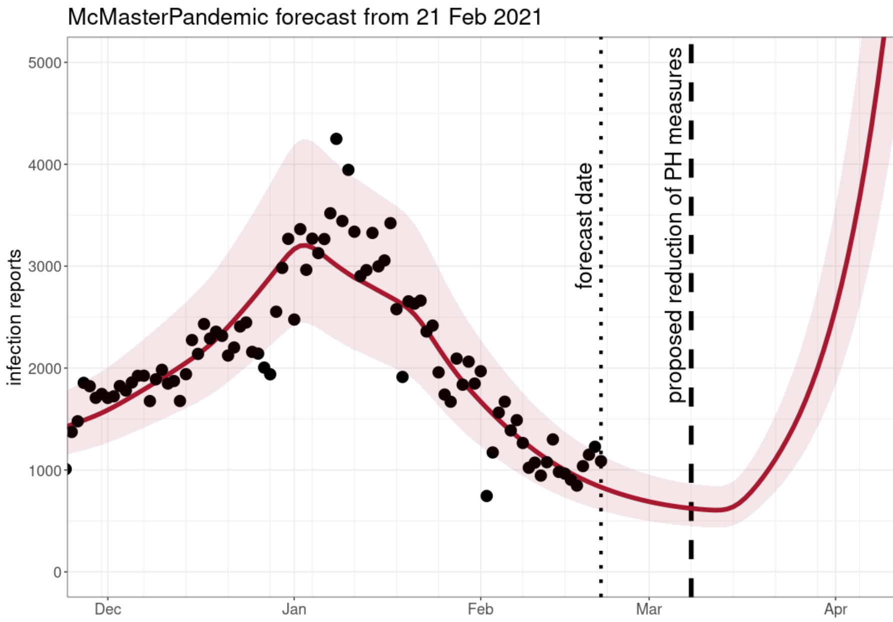

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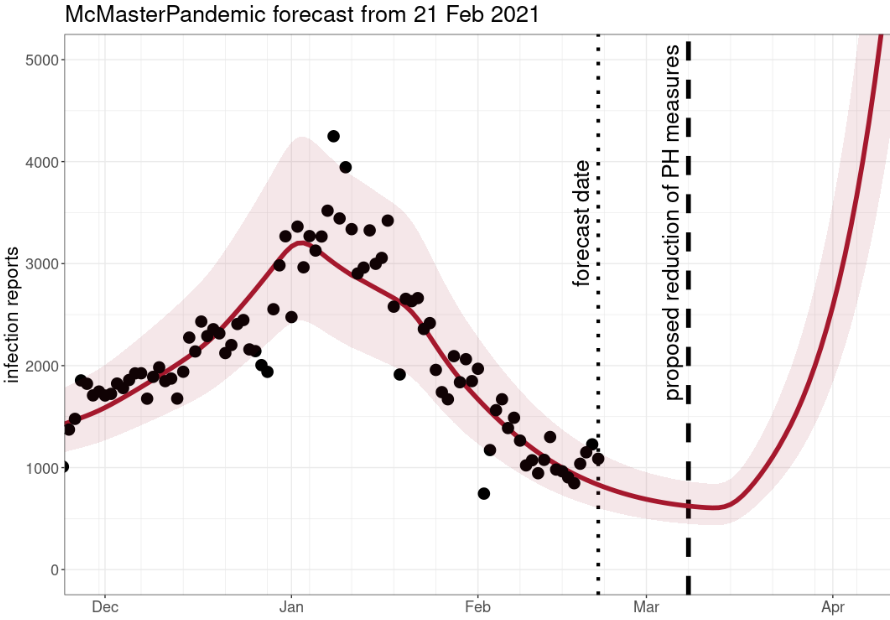

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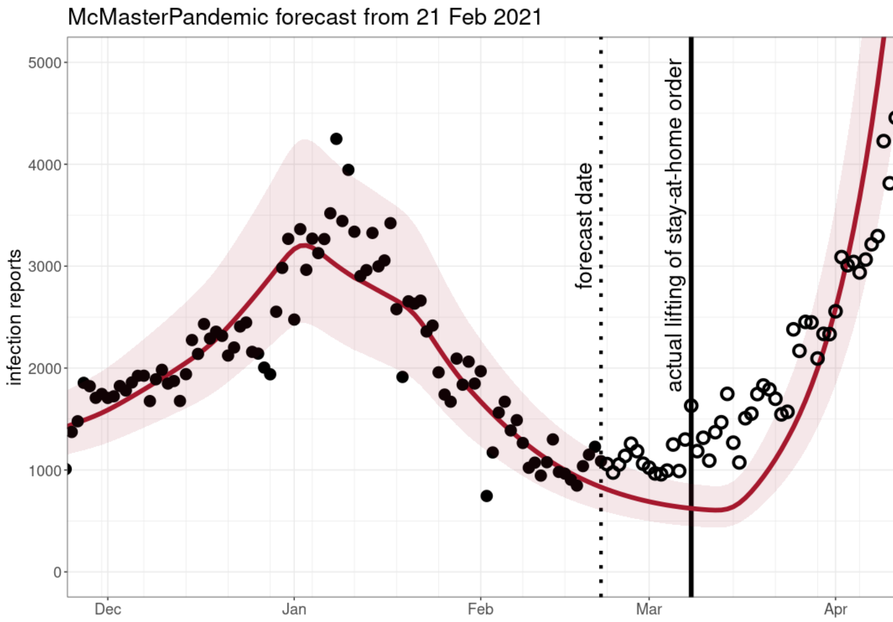

TAs

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Dynamical modeling

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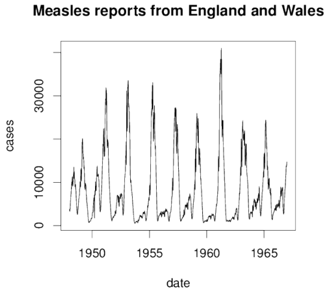

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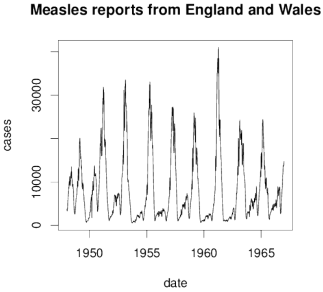

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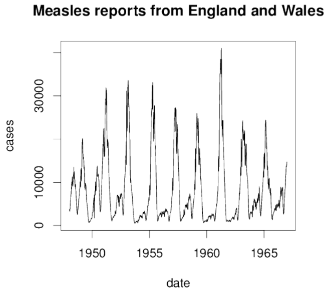

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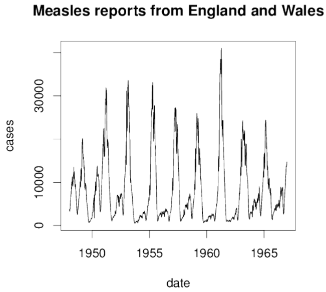

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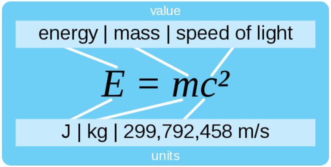
Humans and abstract thought

- People are evolved to be concrete thinkers, not conceptual thinkers



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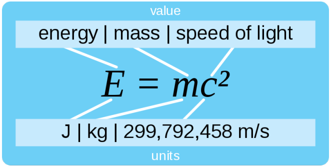

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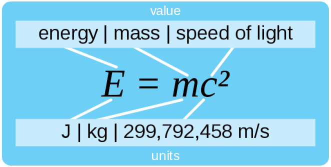

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Outline

Malaria

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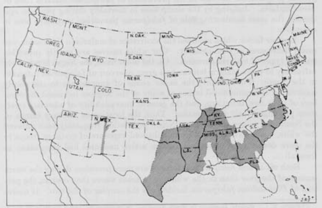

MAP 4.—Areas of the continental United States believed to be malarious in 1934–35.

Malaria

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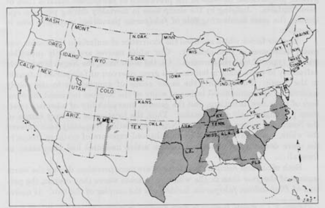

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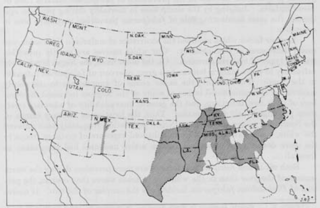

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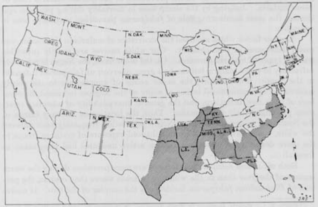

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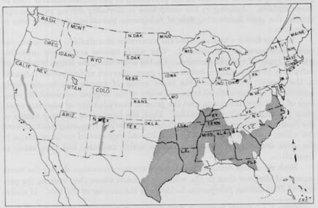

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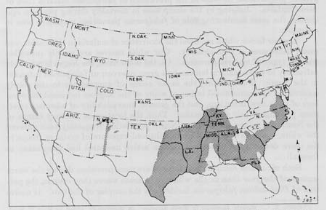

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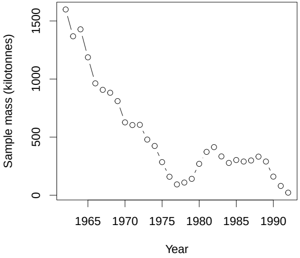
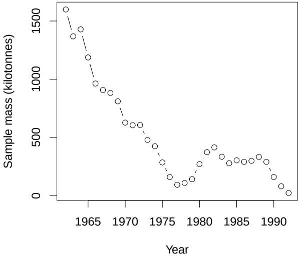
Cod fisheries

- Is the ocean too big for people to affect?



Cod fisheries

- Is the ocean too big for people to affect?
- What happened to the cod?



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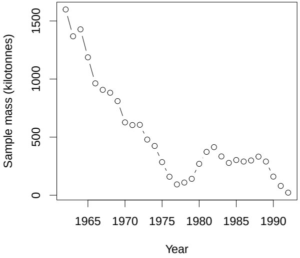

Populations

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Dandelions

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- The spreadsheet is an implementation of a dynamical model!

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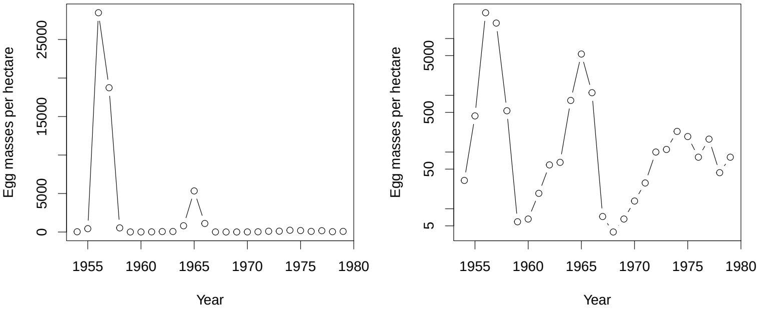
Gypsy moth populations



Moth calculation

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  - 10% of larvae mature into pupae
  - 50% of pupae mature into adults

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  - 50% of pupae mature into adults
  - 50% of adults survive to reproduce

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  - 10% of larvae mature into pupae
  - 50% of pupae mature into adults
  - 50% of adults survive to reproduce
  - All adults die after reproduction

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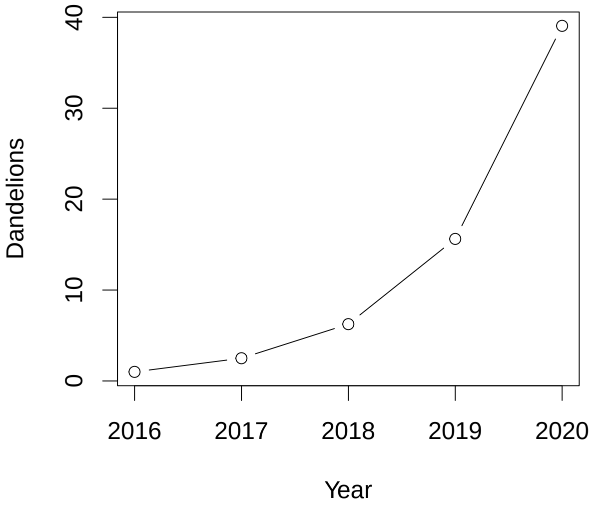

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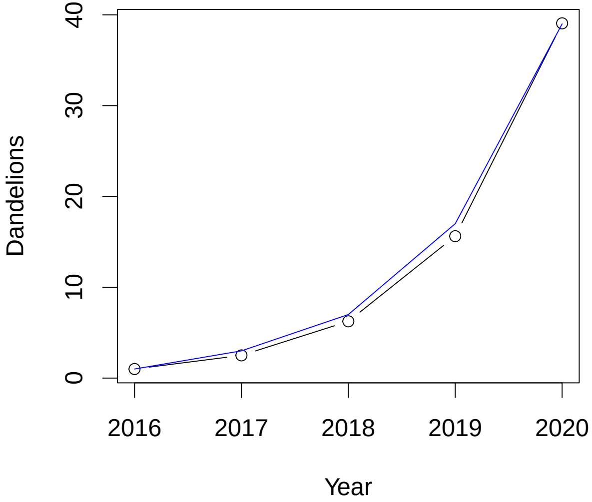

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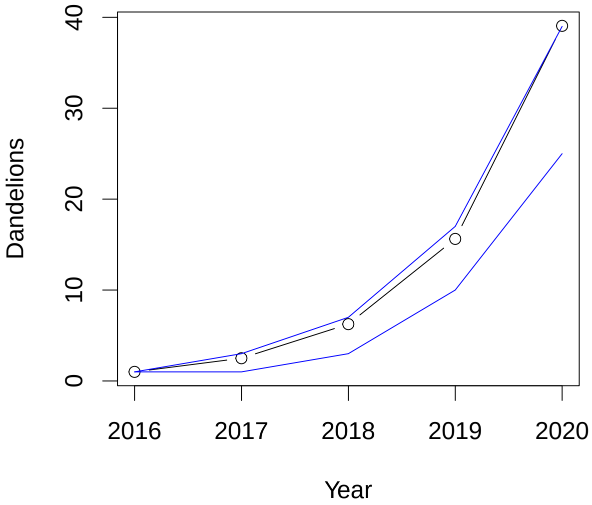

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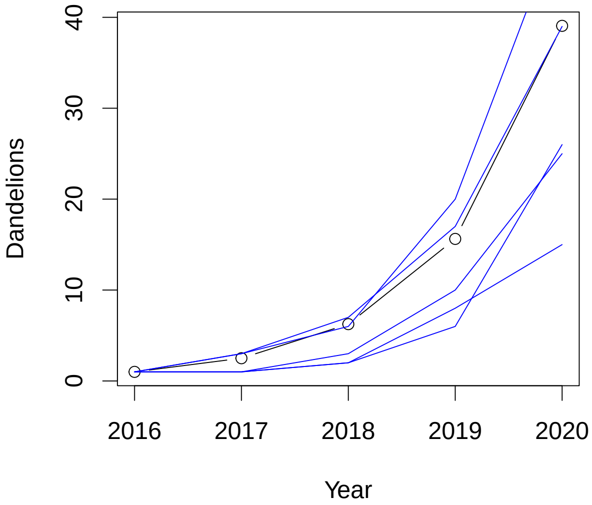

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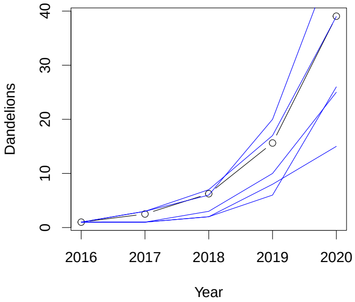

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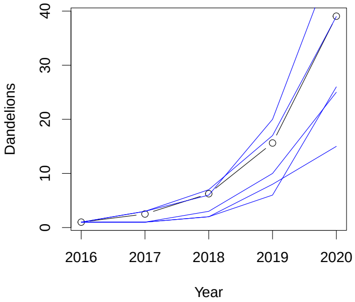

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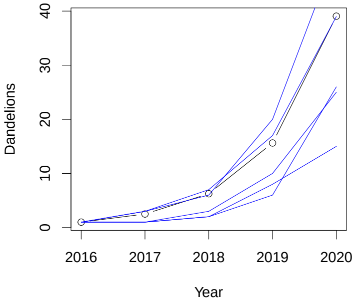

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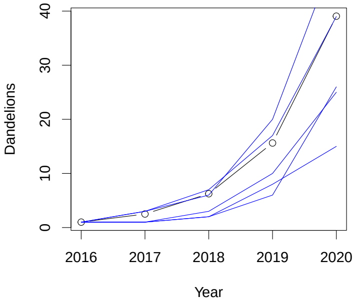

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Outline

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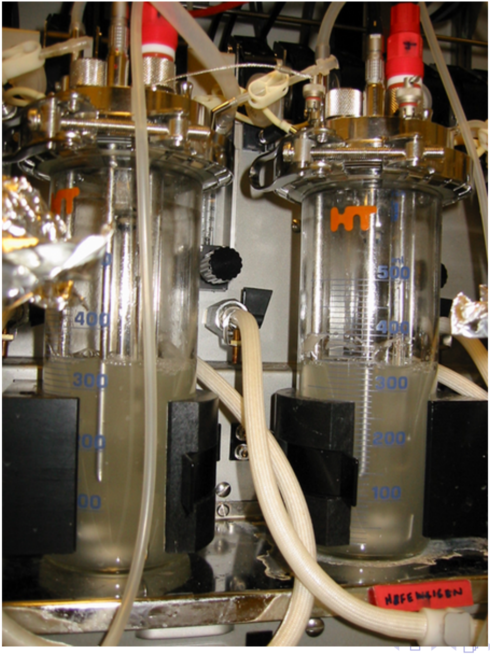

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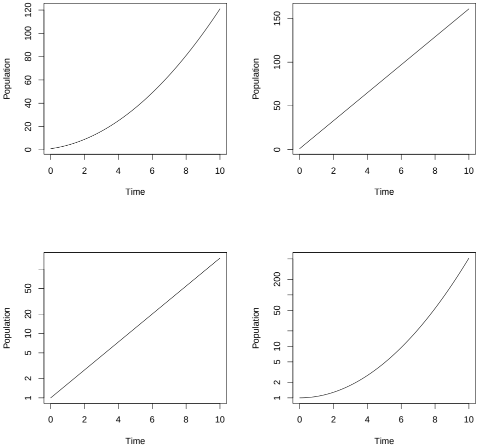

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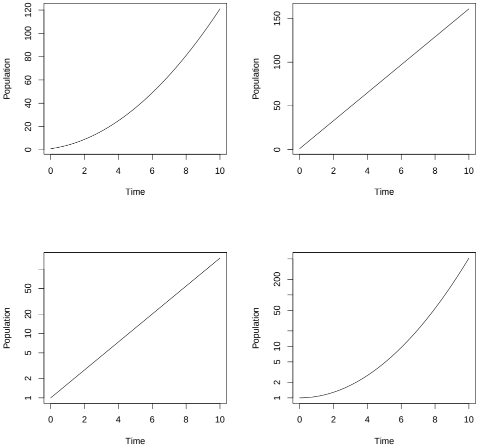

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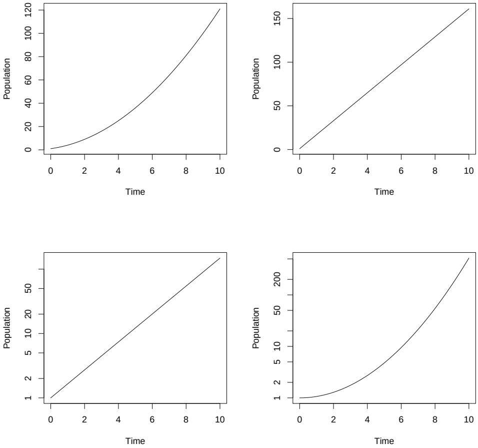

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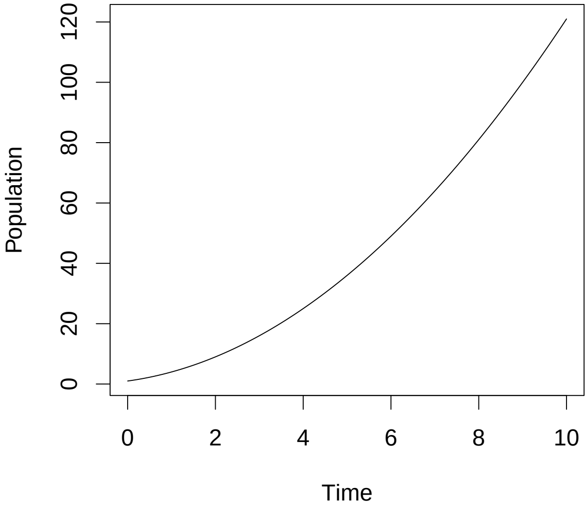

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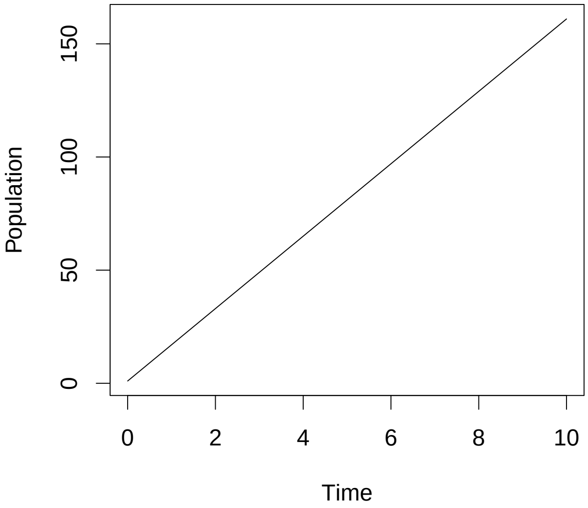

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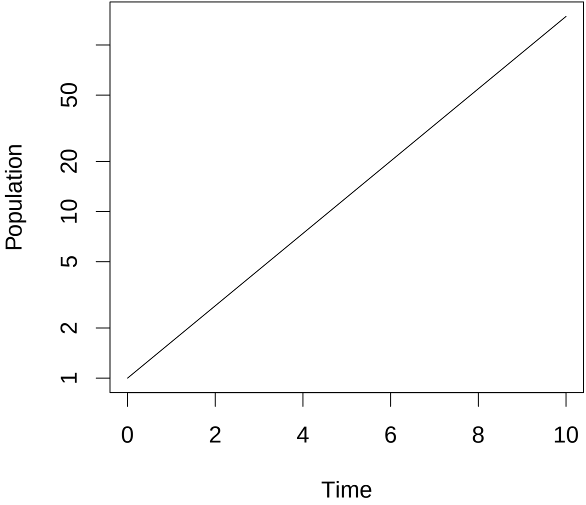

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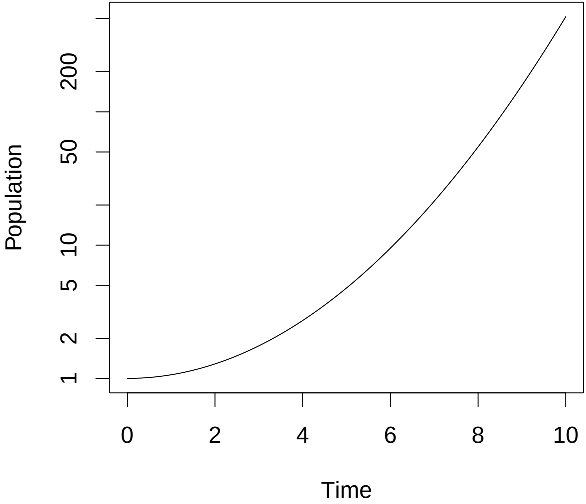

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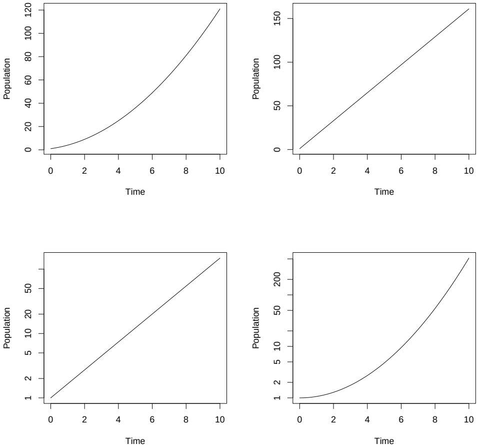

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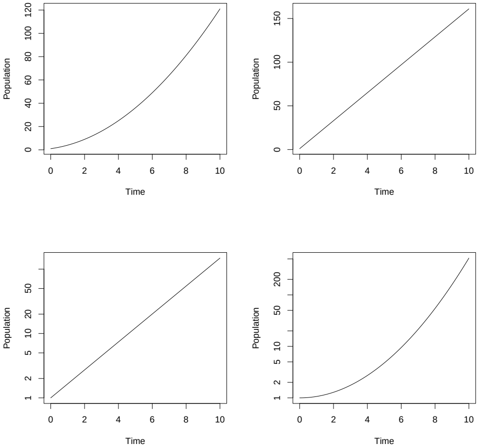

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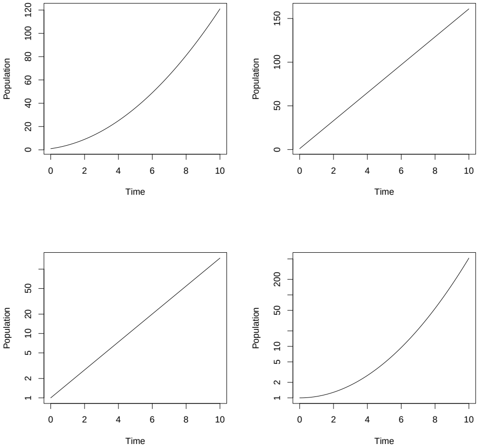

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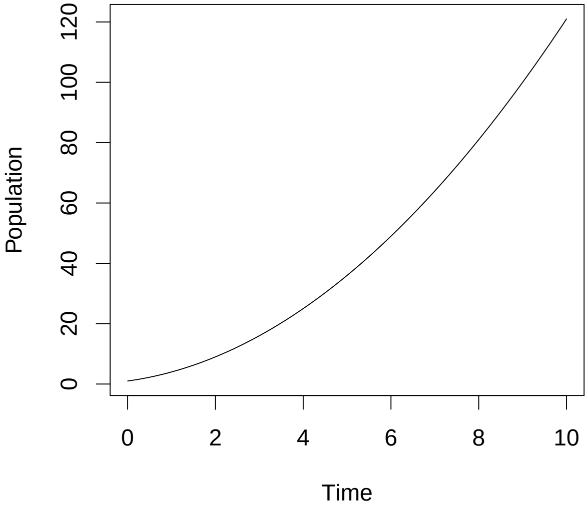

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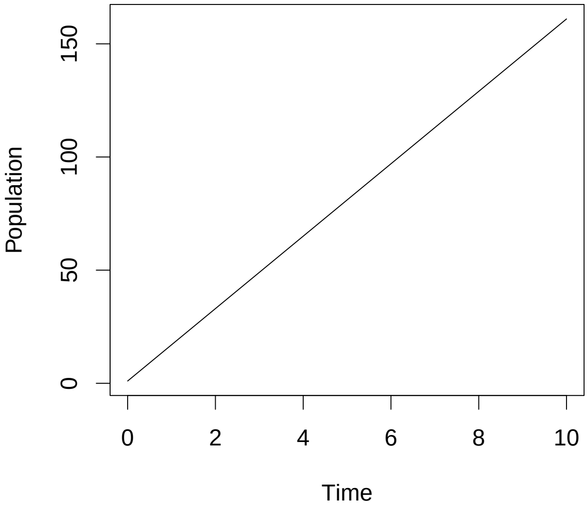

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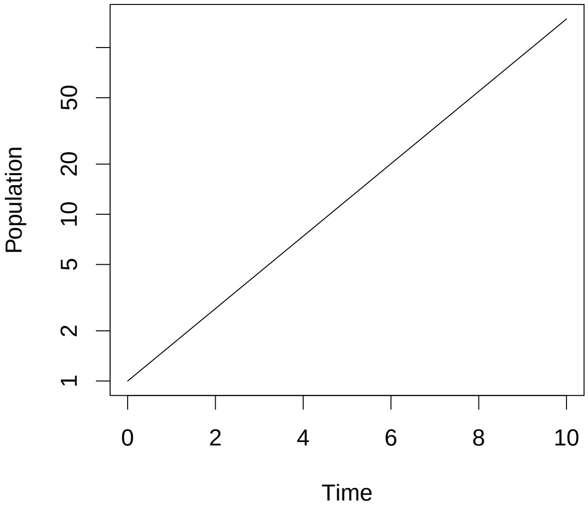

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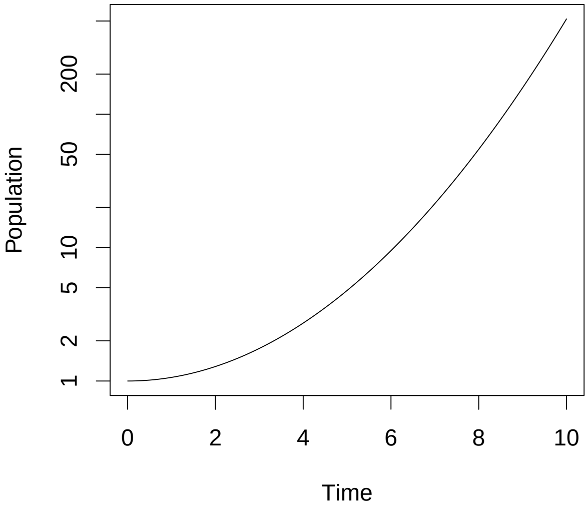

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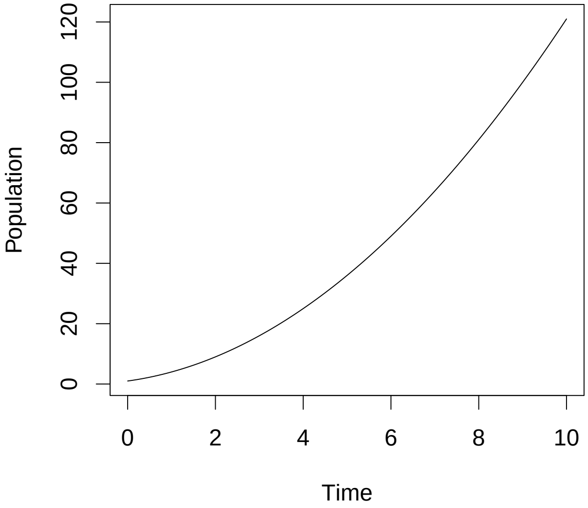
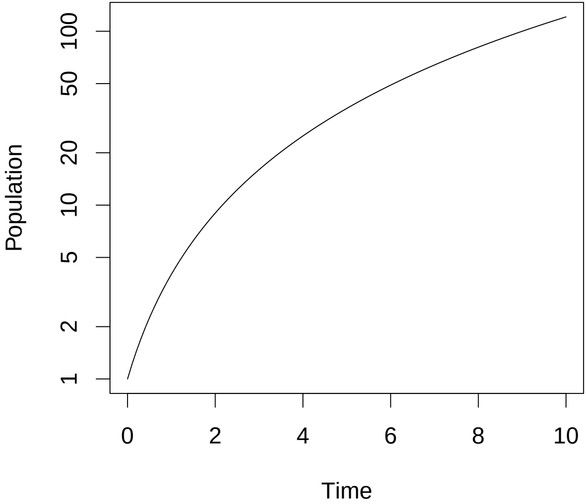


A on the log scale



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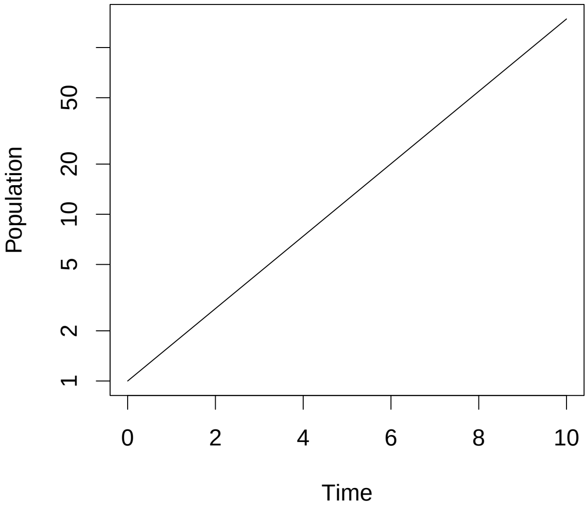

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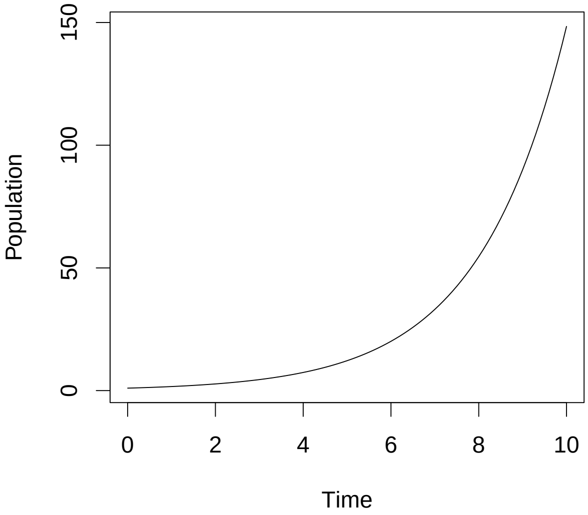

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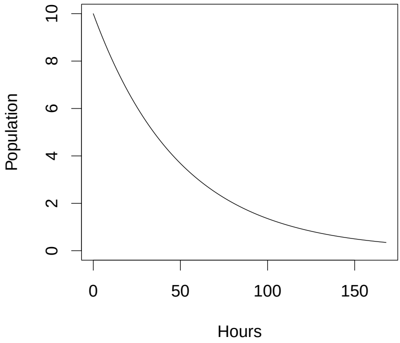
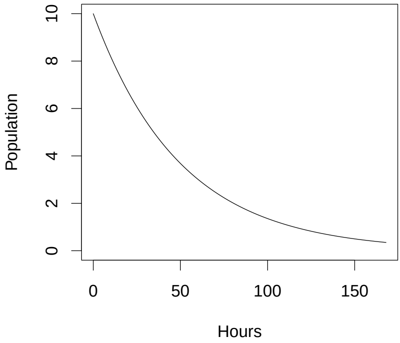


- Decline is proportional to size

Exponential decline



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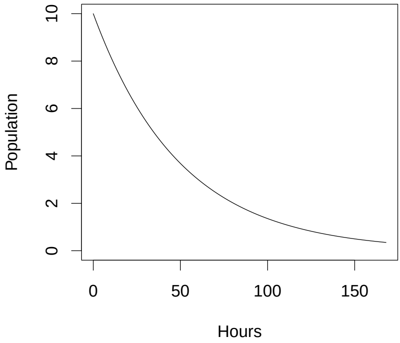

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Outline

Scales of comparison

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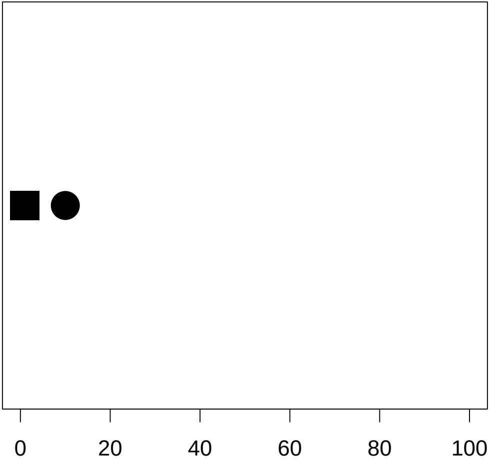

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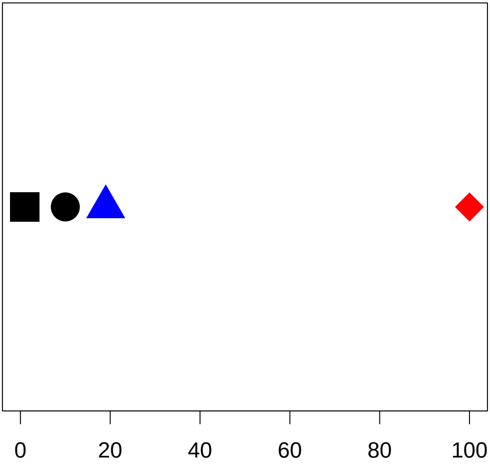

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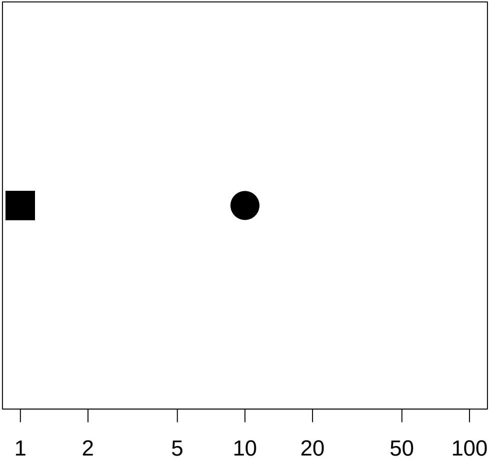

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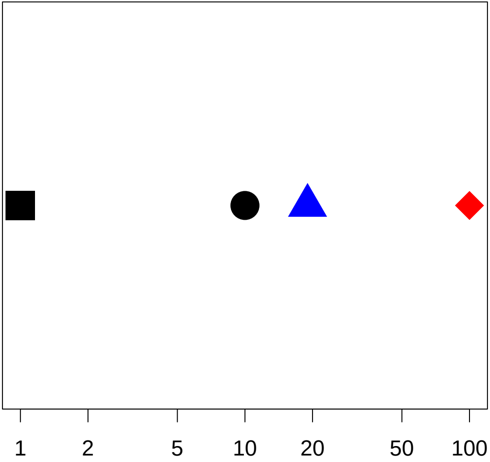

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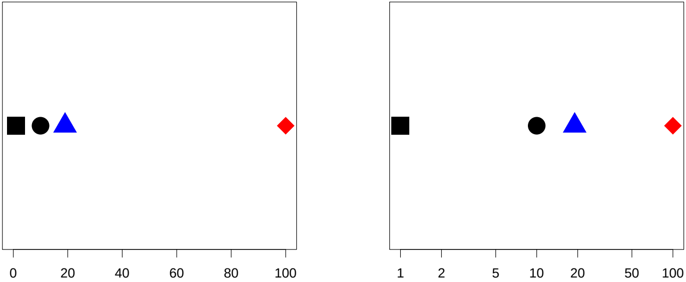

There is only one log scale; it doesn't matter which base you use!

Canadian provinces

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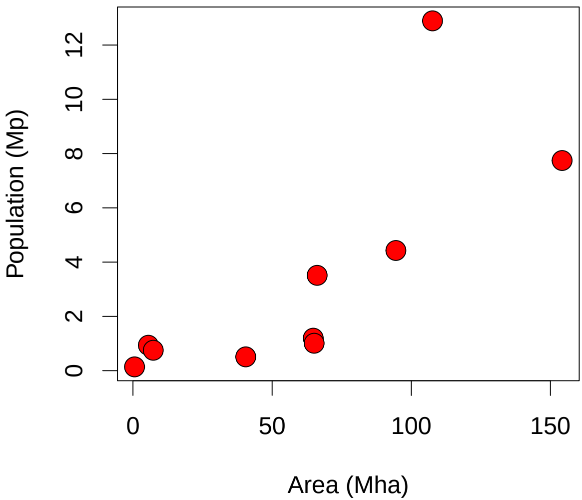

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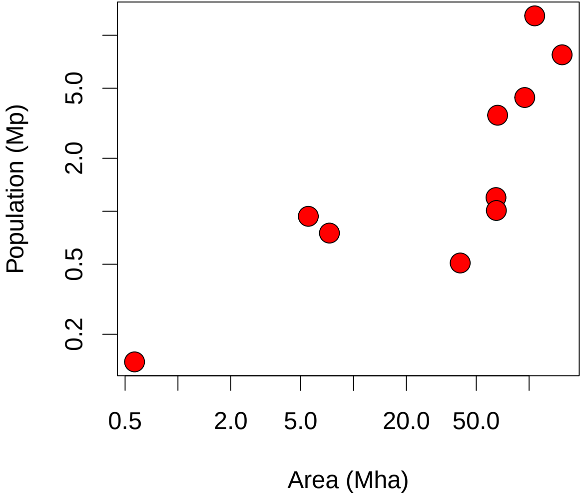

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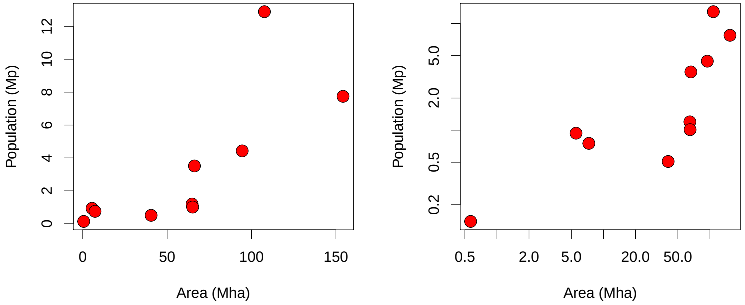
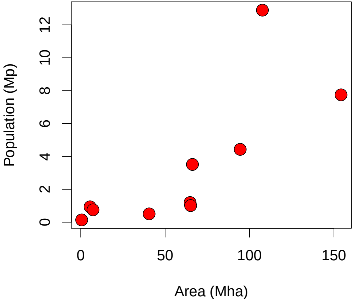
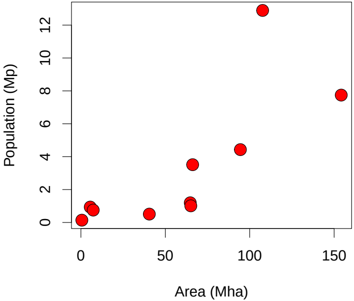


Canadian provinces plus Canada?





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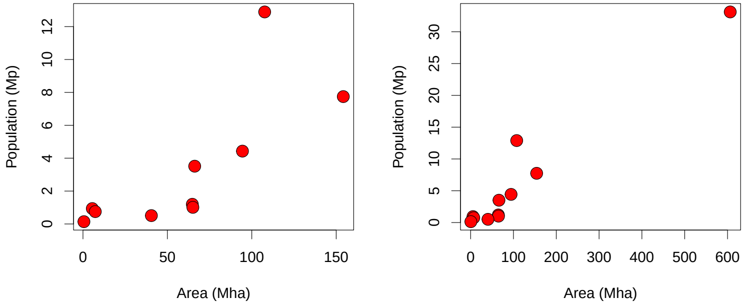

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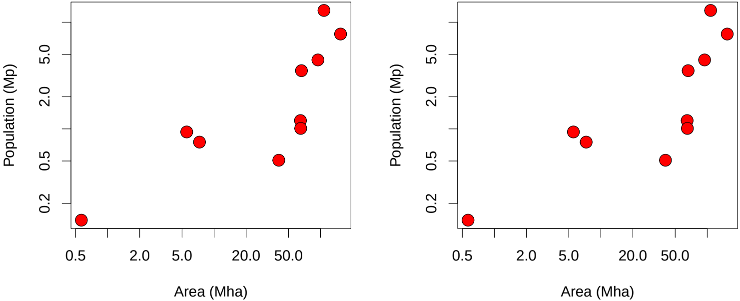

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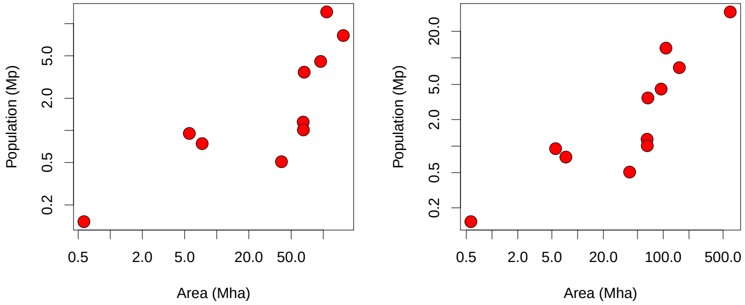

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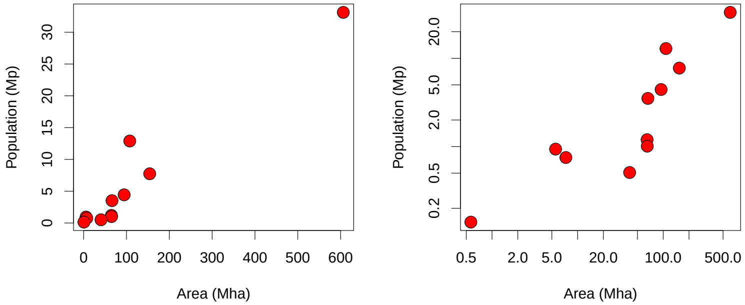
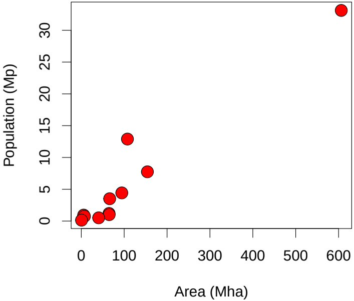
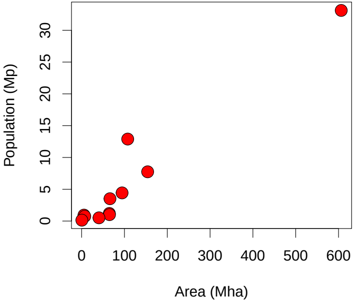


Canada plus this classroom





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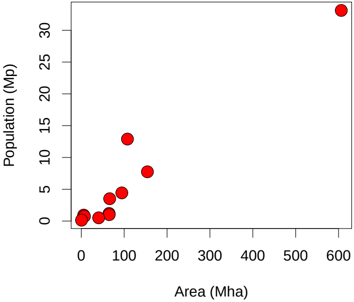

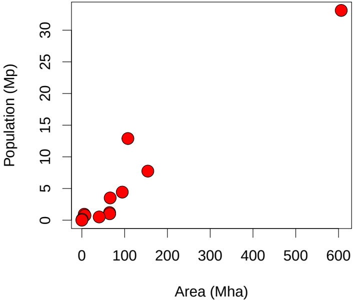

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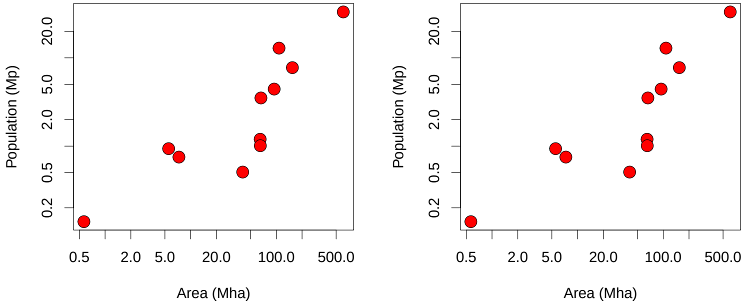

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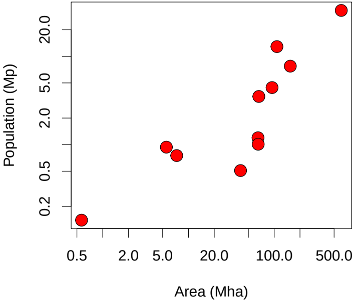

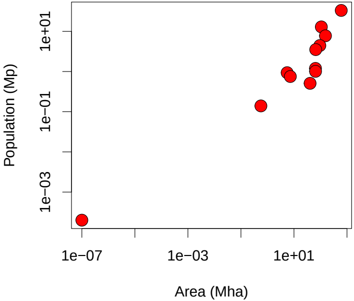

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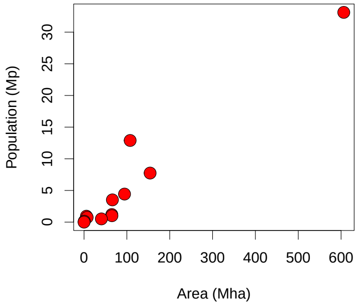

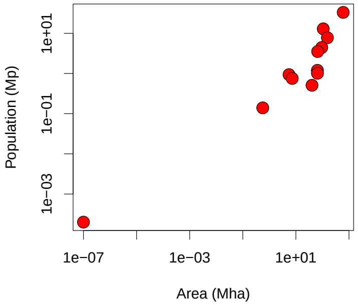

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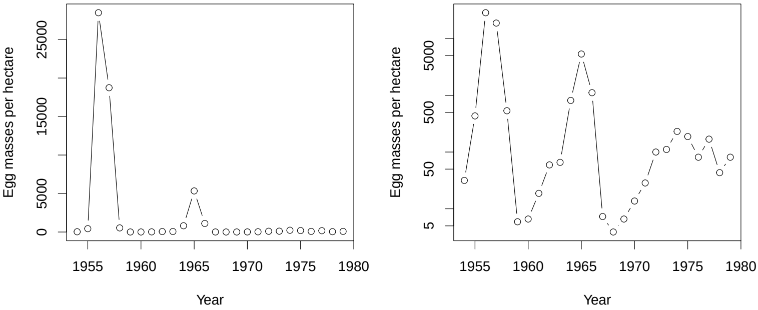
Gypsy-moth example



Scales in population biology

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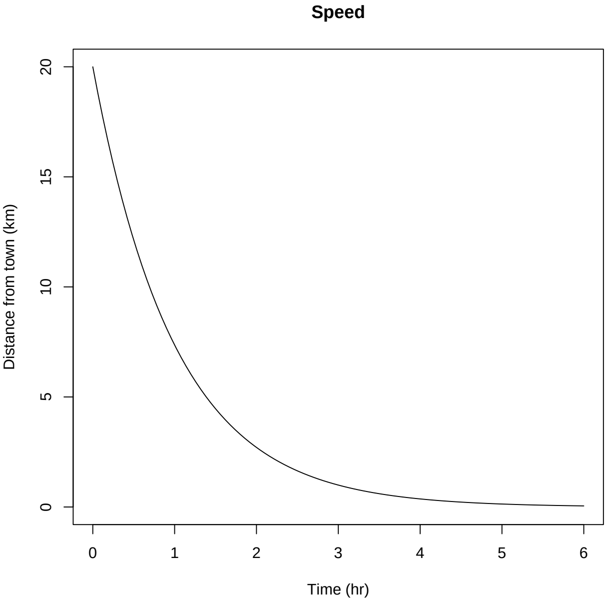

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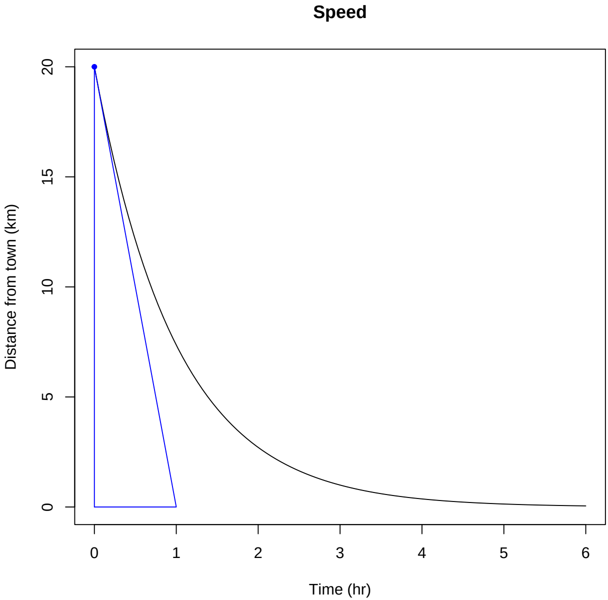

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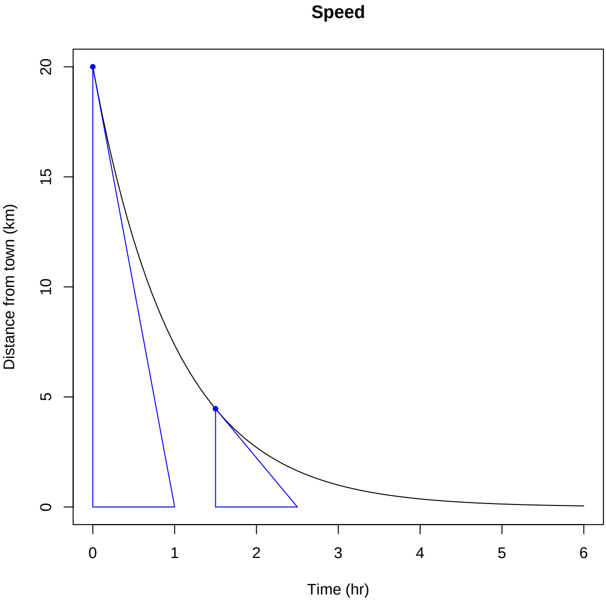

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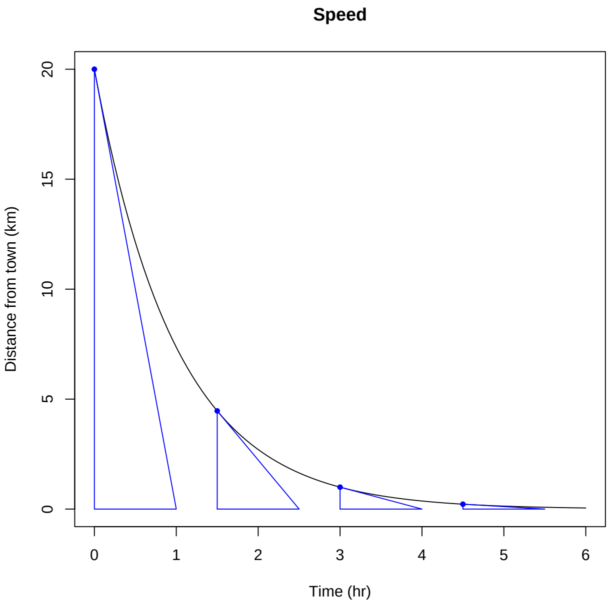

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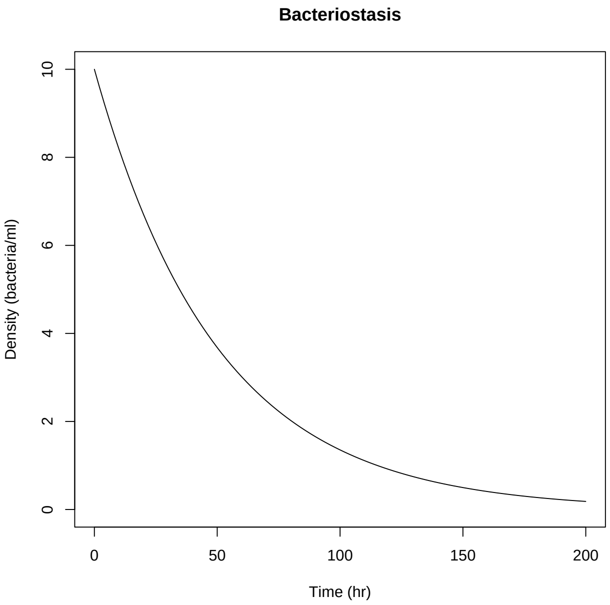

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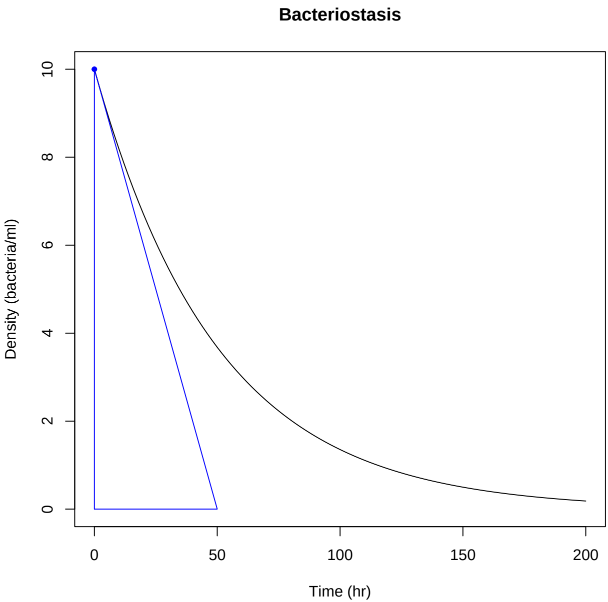
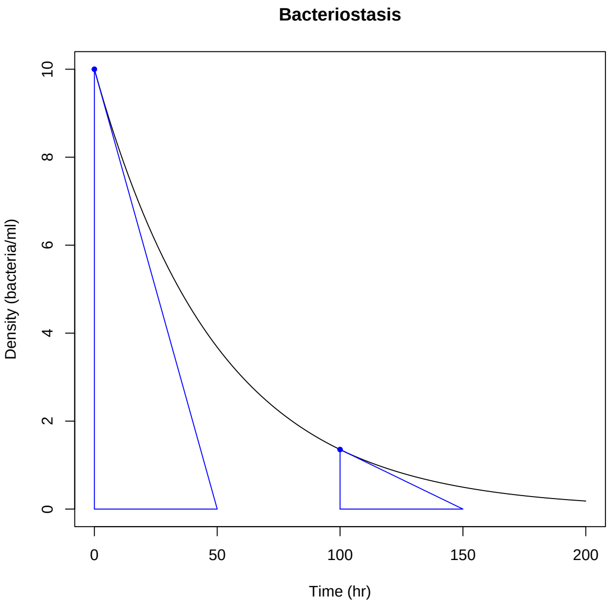


Characteristic times



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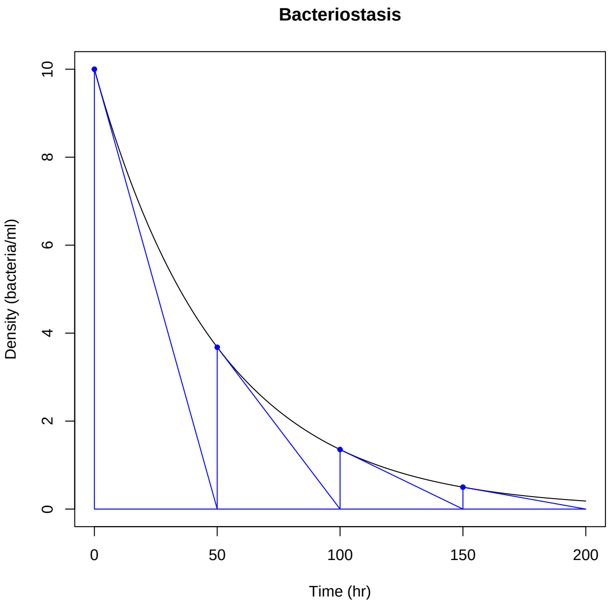

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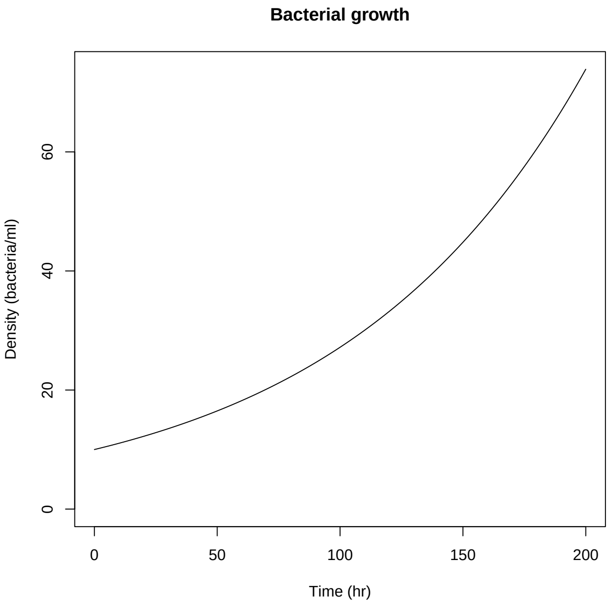

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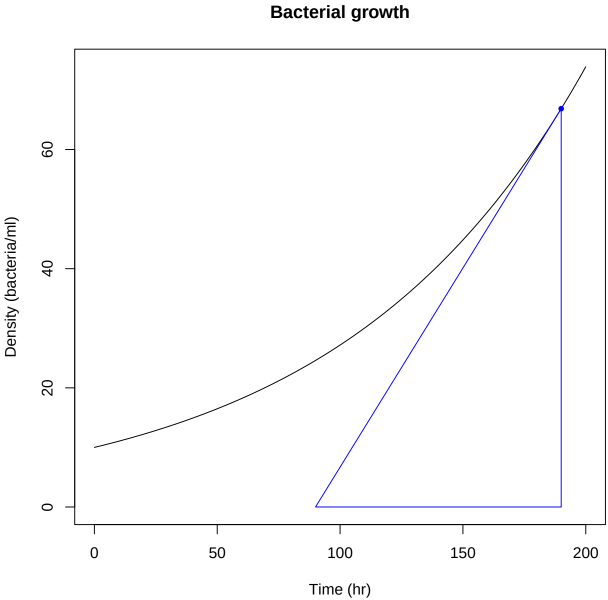

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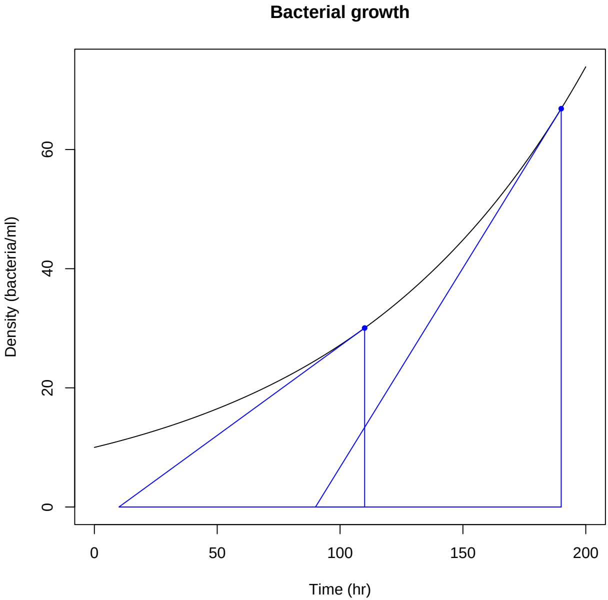

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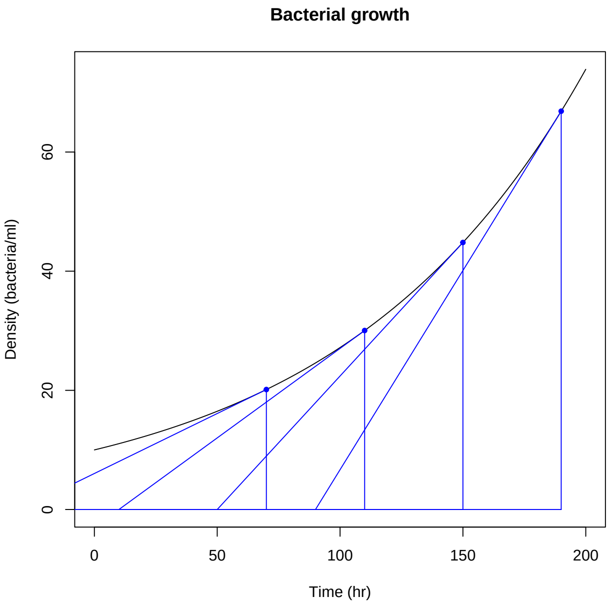

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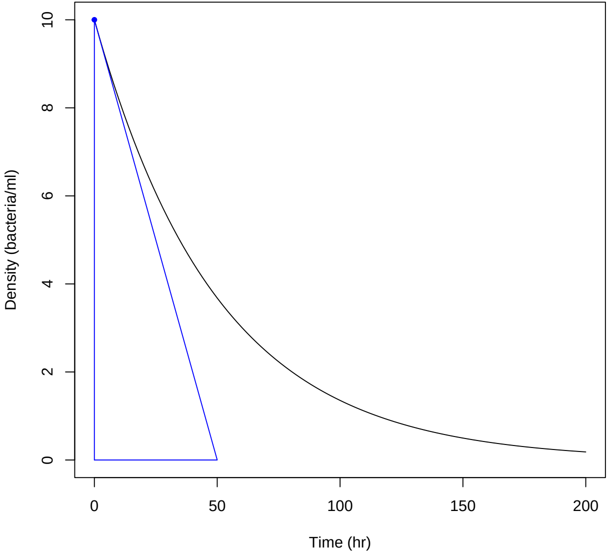
Bacteriostasis



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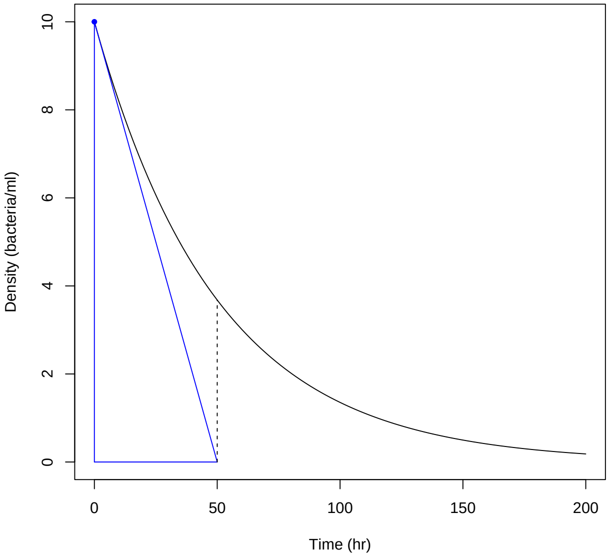
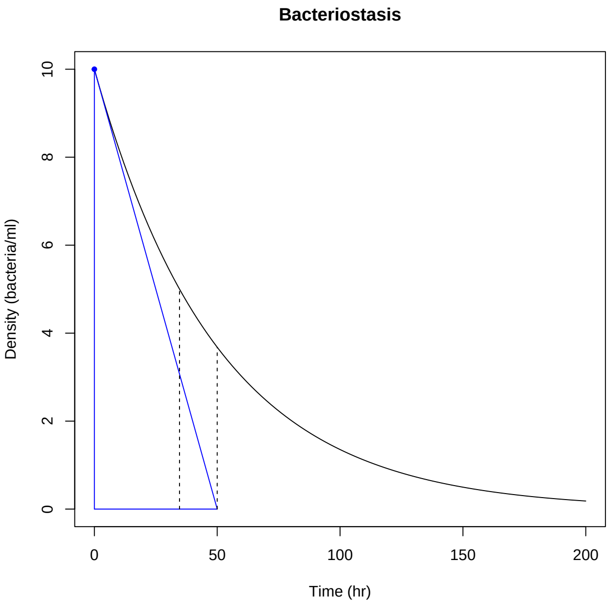
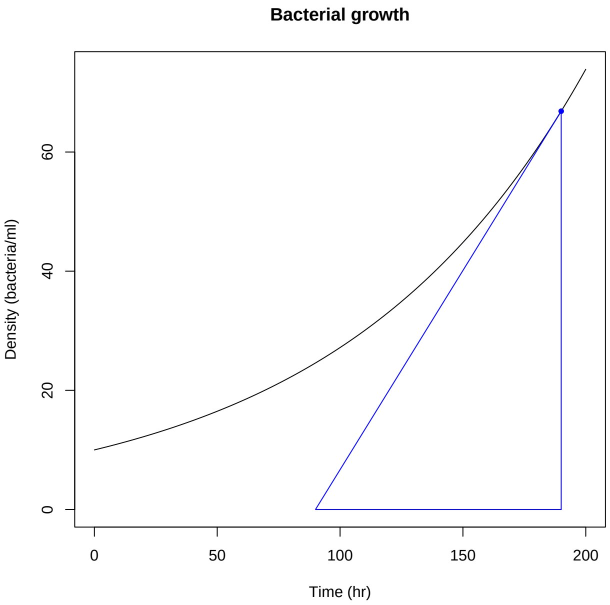


Characteristic time and half life





Bacterial growth
Density (bacteria/ml)
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60
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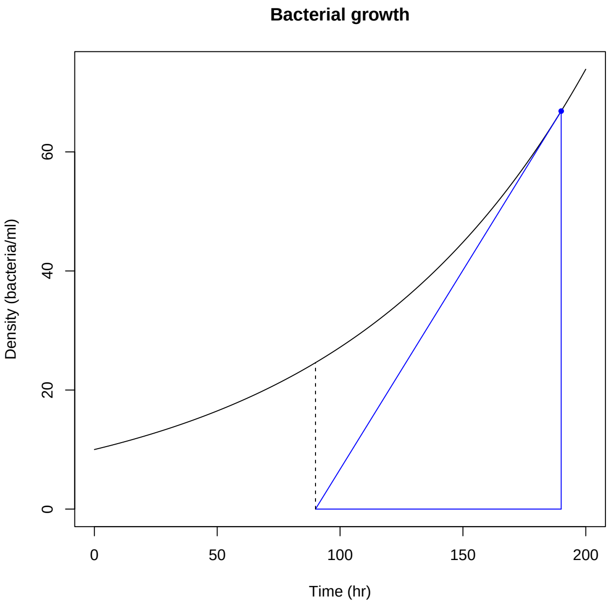
50
100
150
200
Time (hr)



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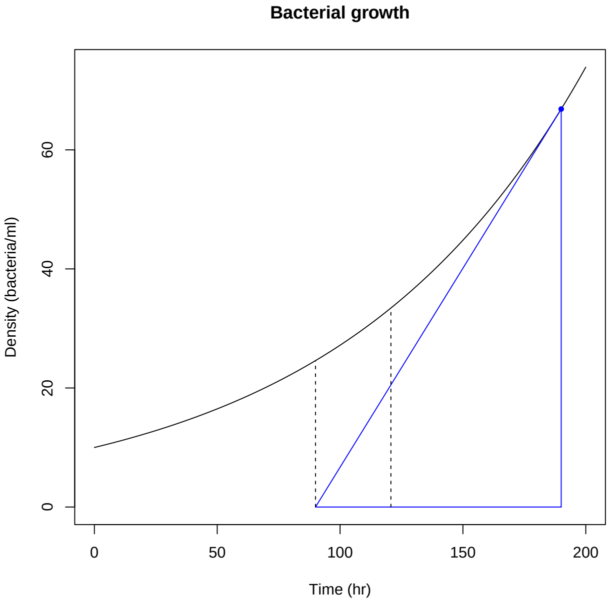

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