UNIT 3 Non-linear population models

Outline

Introduction

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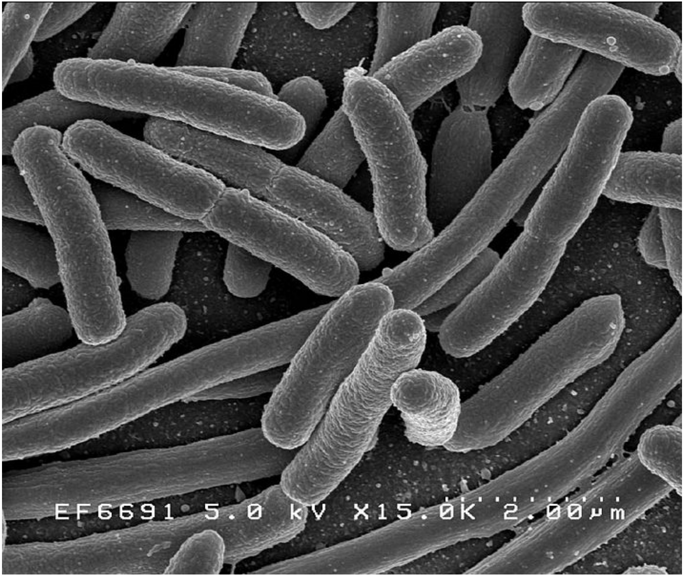
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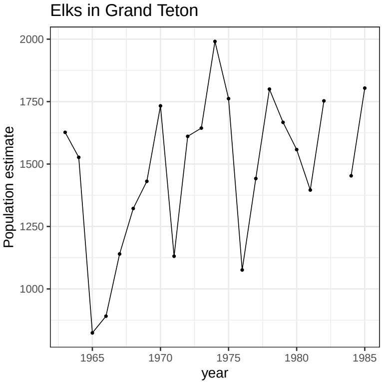
(preview)

Elk


Elks in Grand Teton
Population estimate
2000
1750
1500
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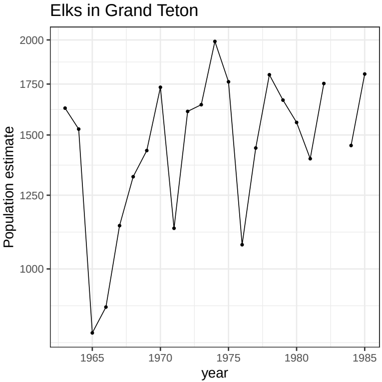
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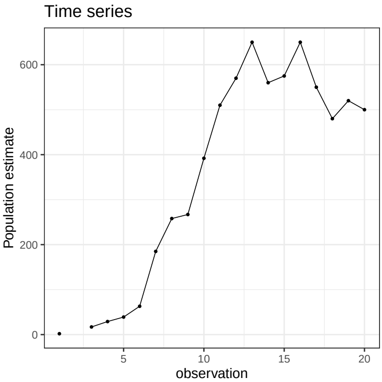
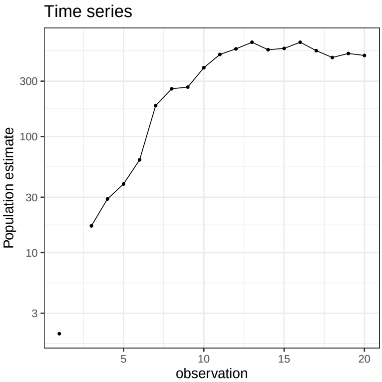
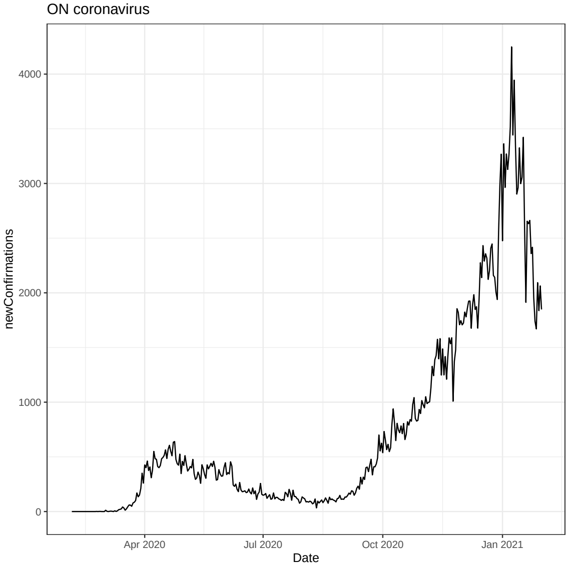
Paramecia (preview)





Coronavirus (preview)



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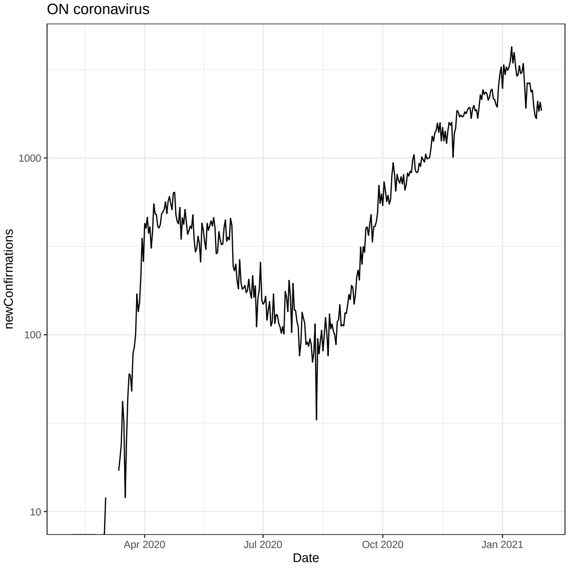

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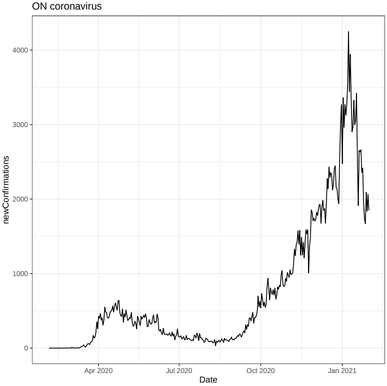
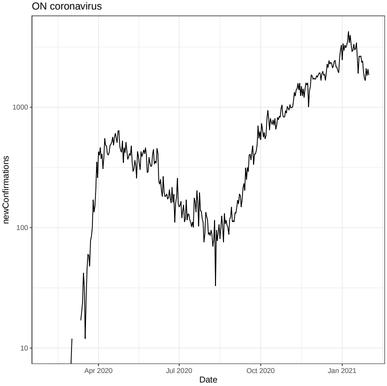
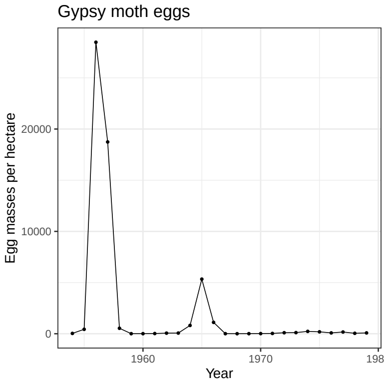
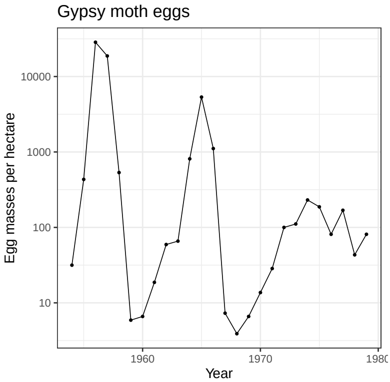




Gypsy moths (preview)





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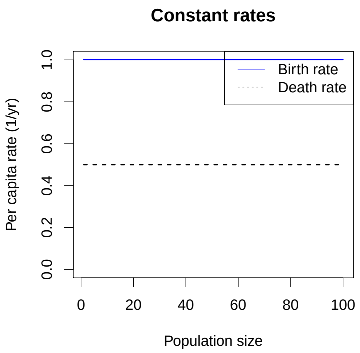

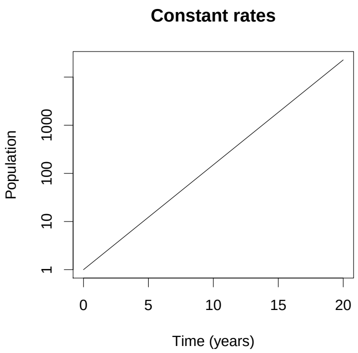

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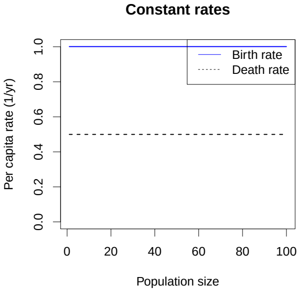

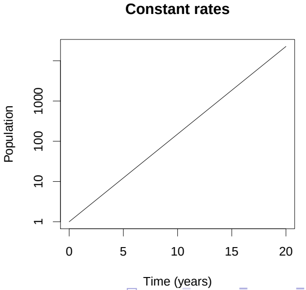

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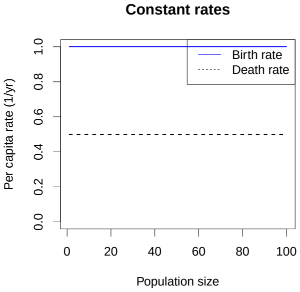

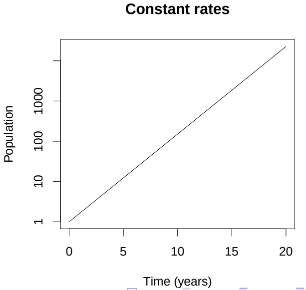

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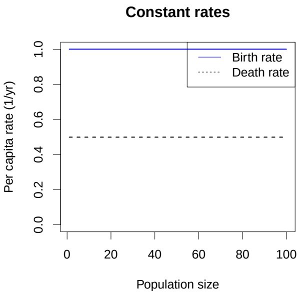

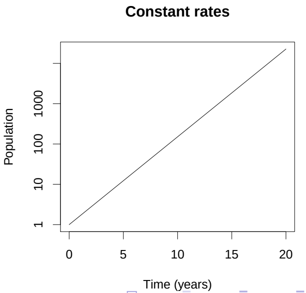

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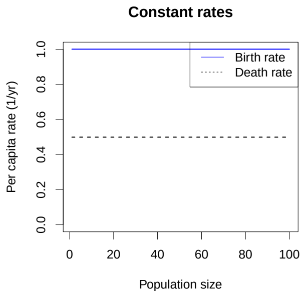

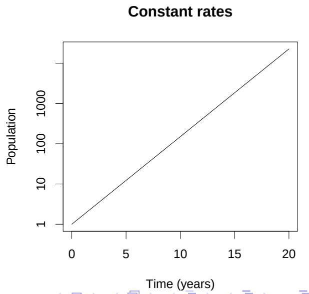

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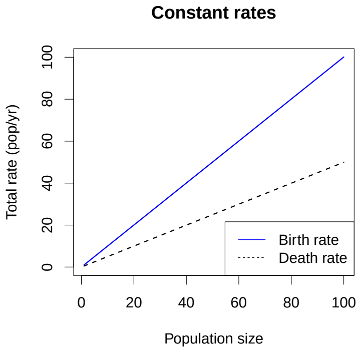

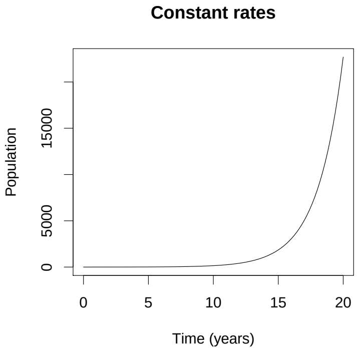

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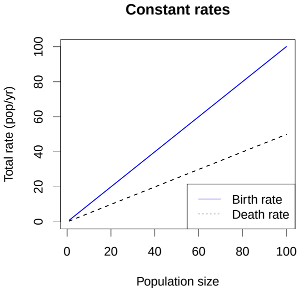

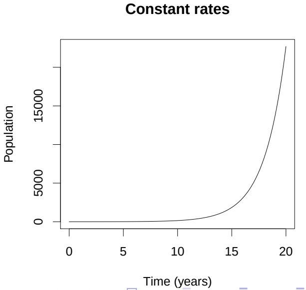

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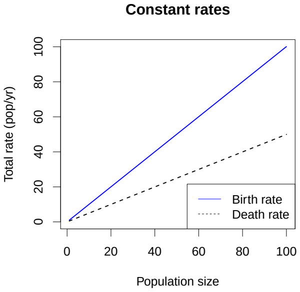

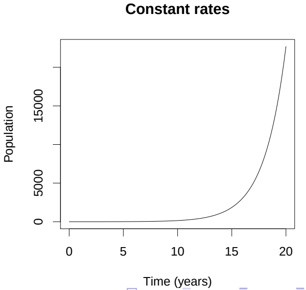

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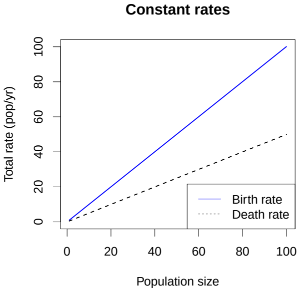

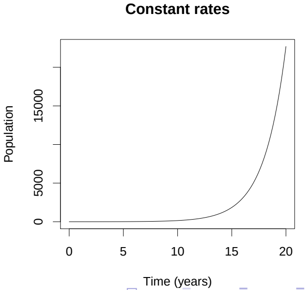

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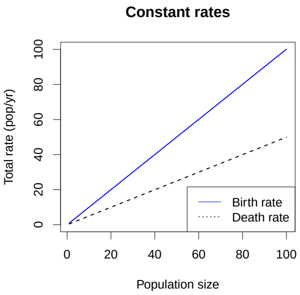

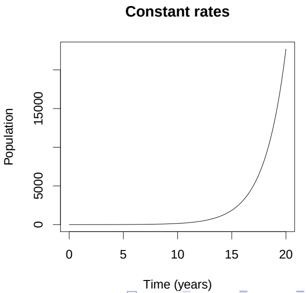

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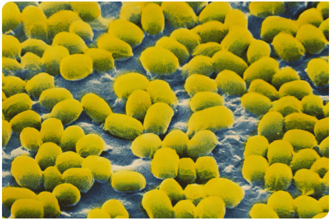

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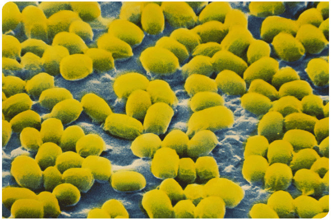

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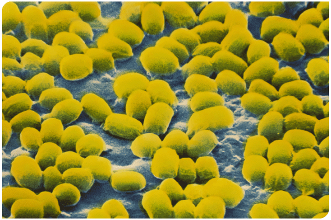

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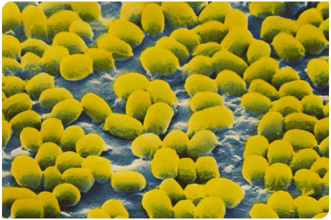

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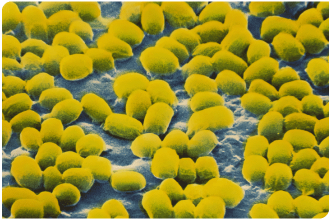

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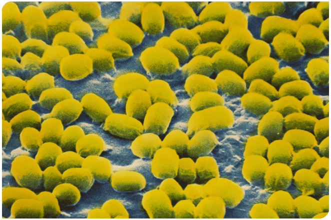

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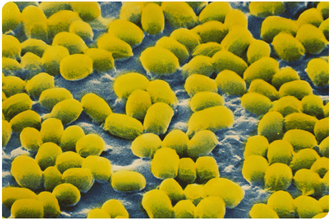

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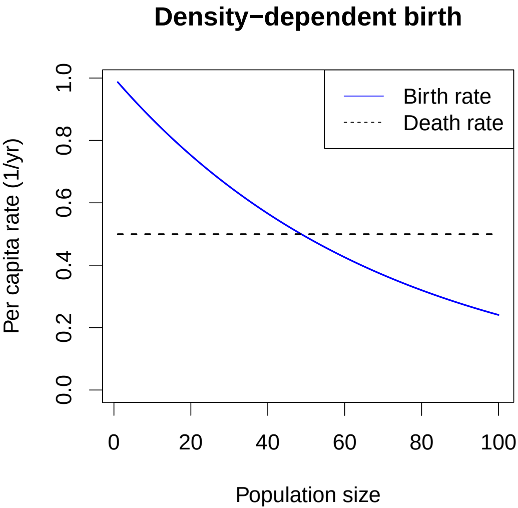

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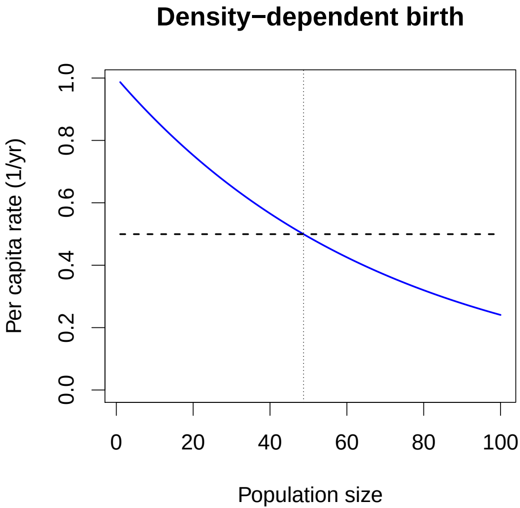

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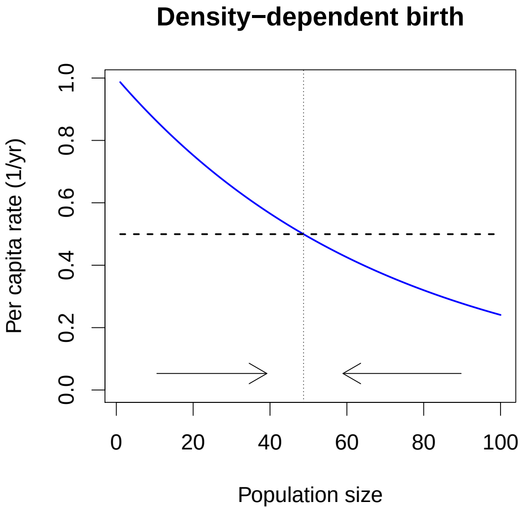

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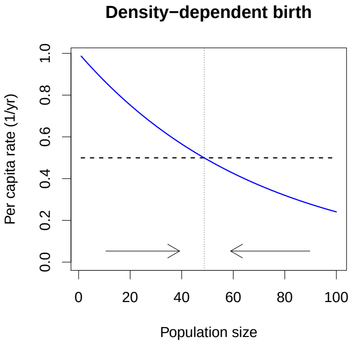

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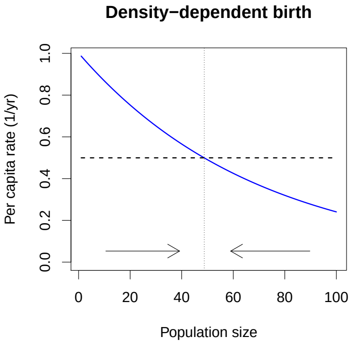

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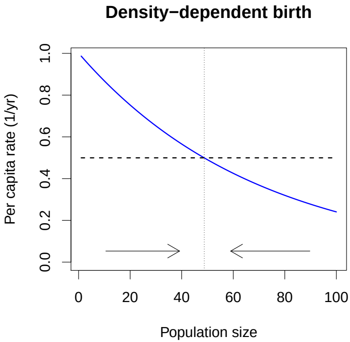

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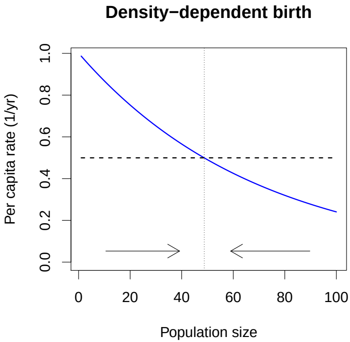

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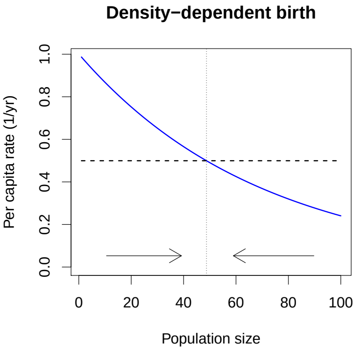

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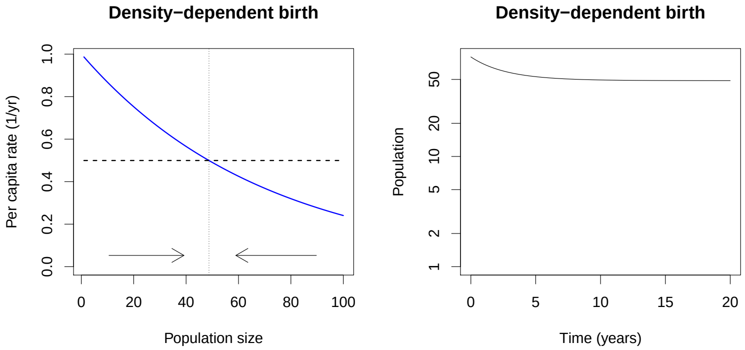

Examples


Density−dependent birth
Population
1
2
5
10
20
50
0
5
10
15
20
Time (years)


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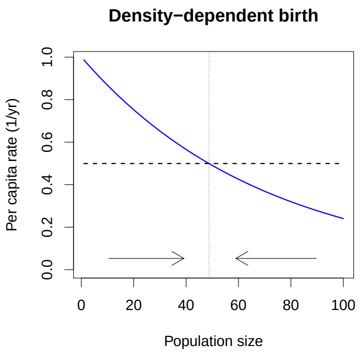

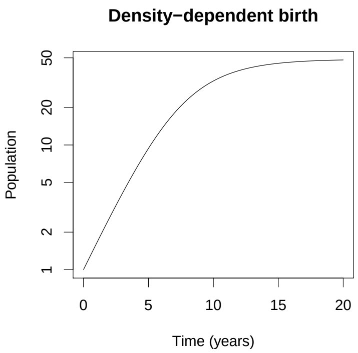

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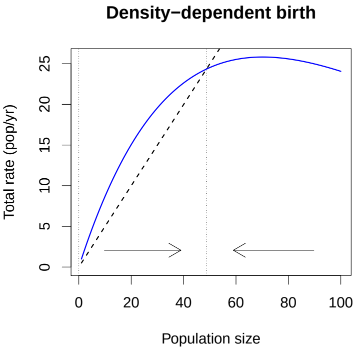

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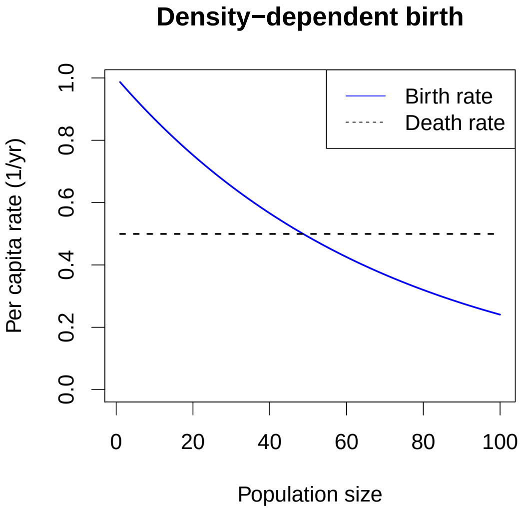

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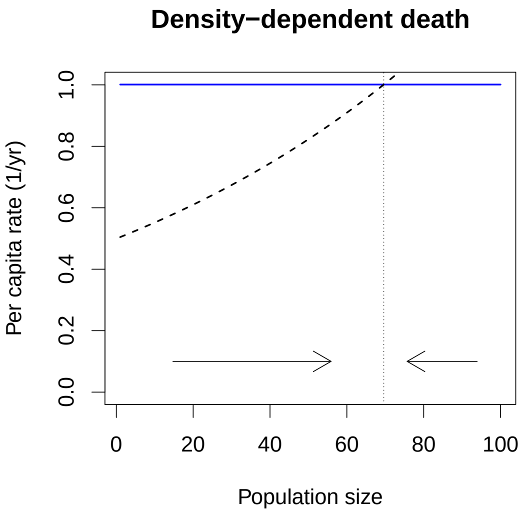

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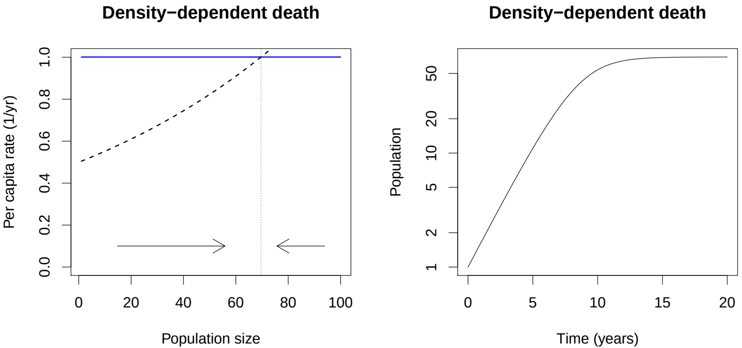

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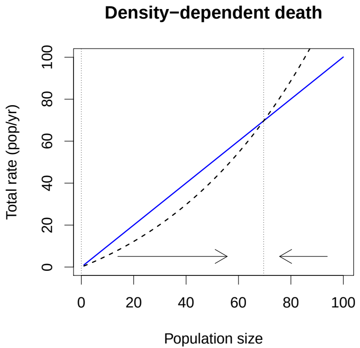

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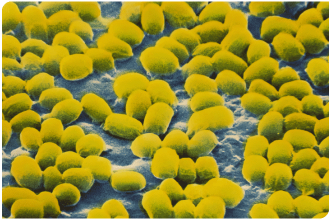

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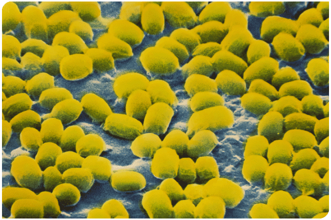

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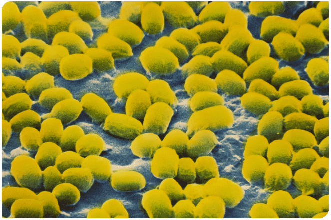

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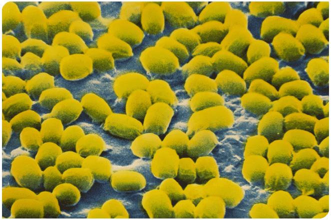

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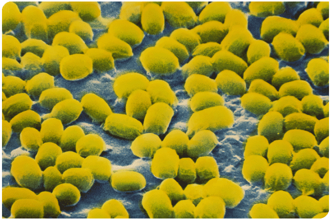

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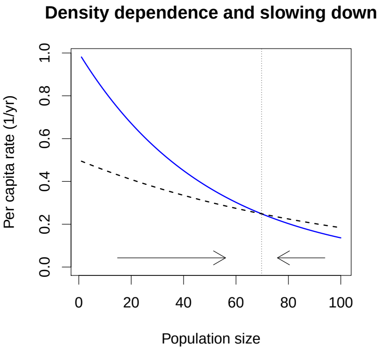

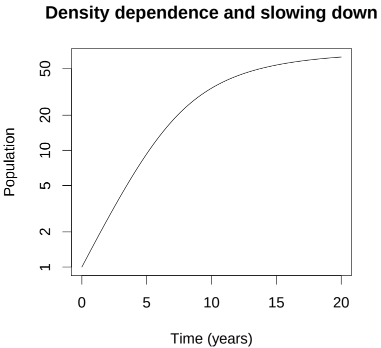

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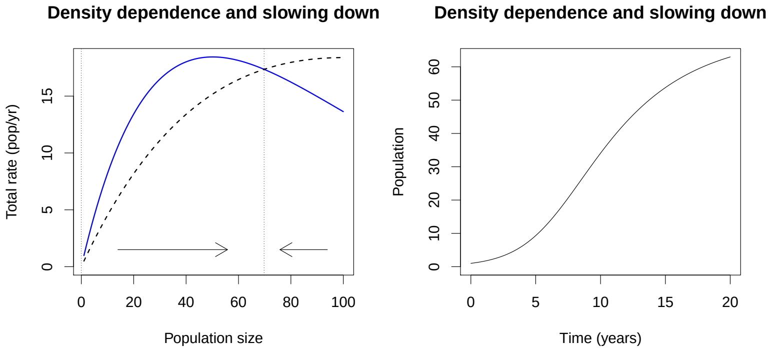

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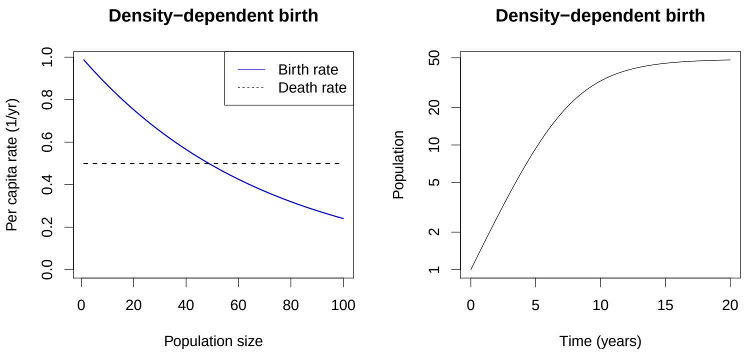

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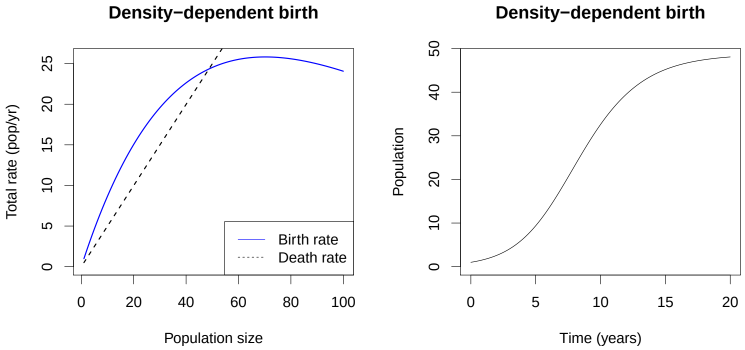

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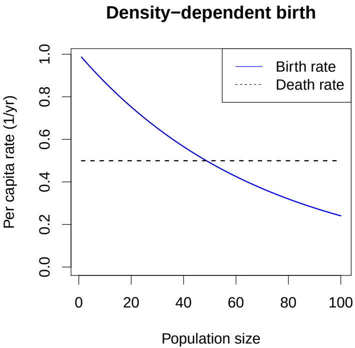

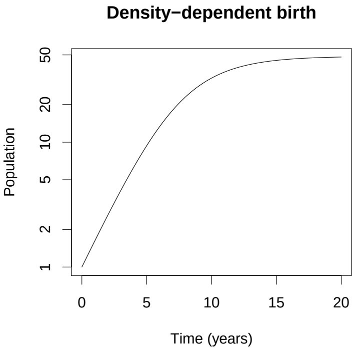

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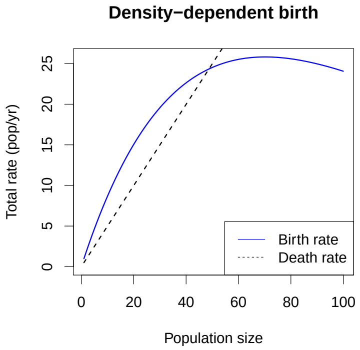

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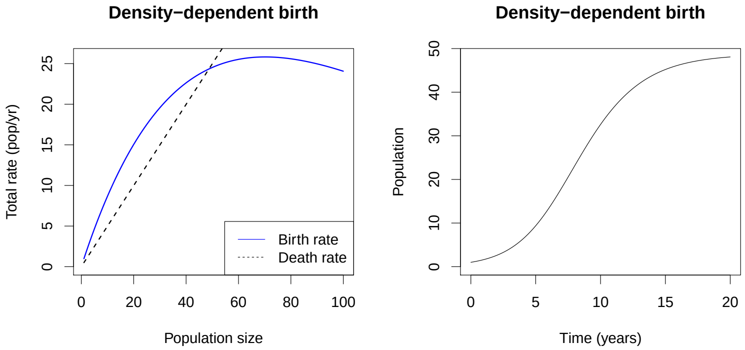

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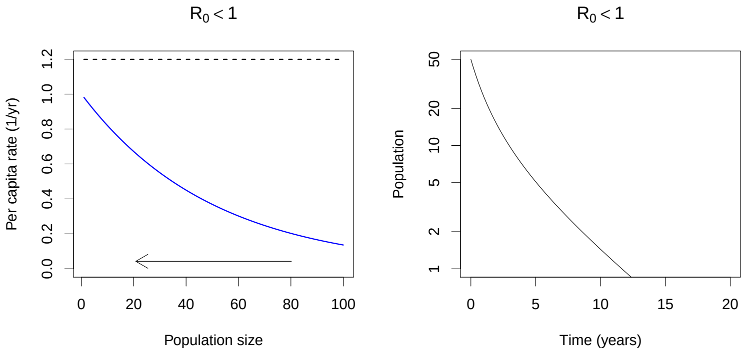

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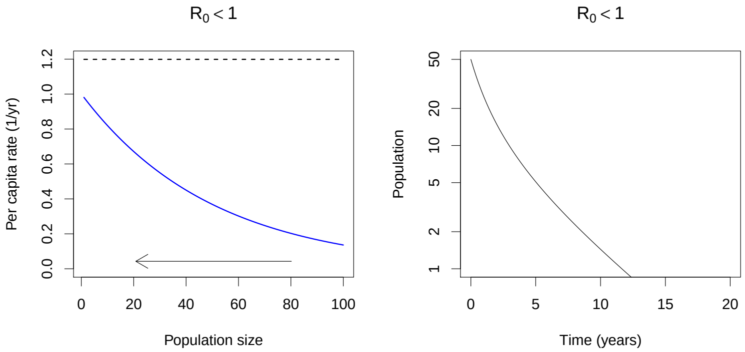

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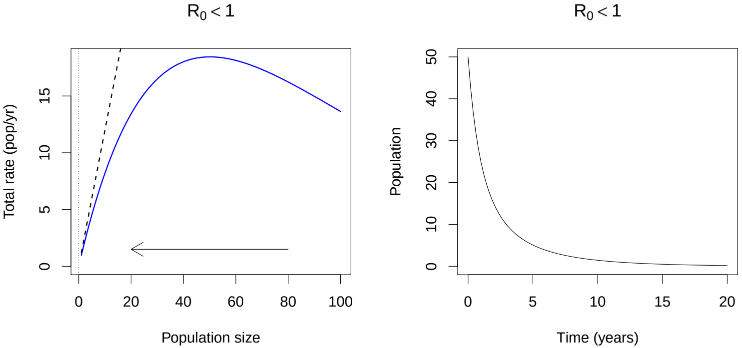

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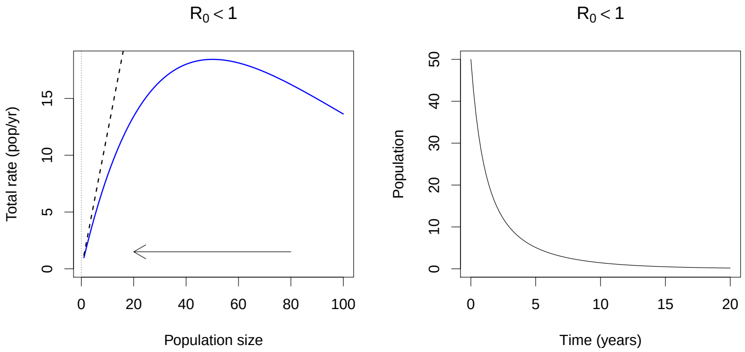

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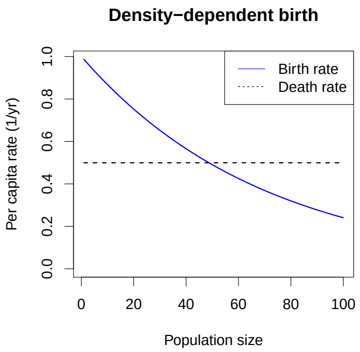

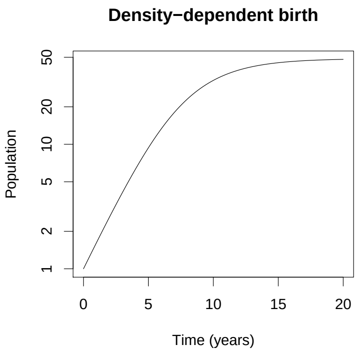

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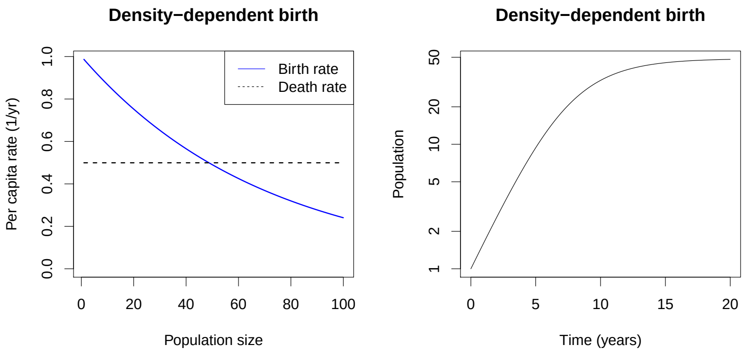

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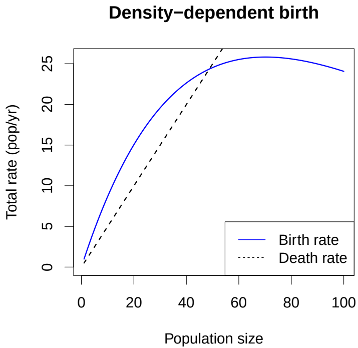

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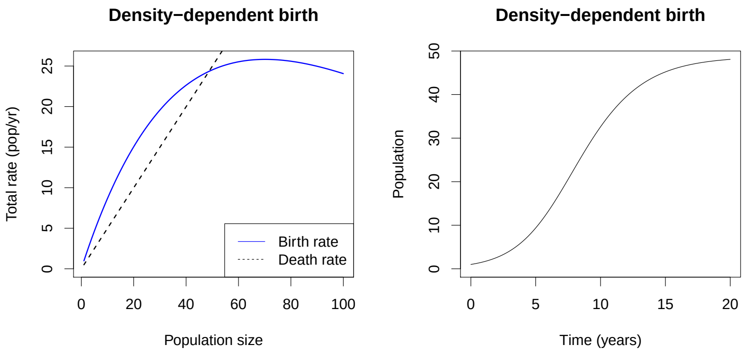

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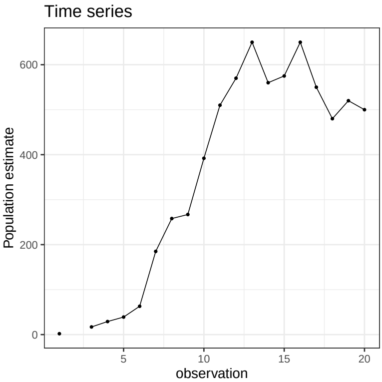
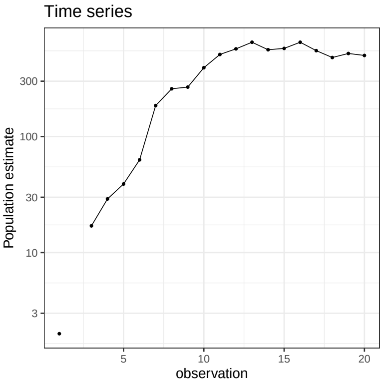


Time series
Population estimate
600
400
200
0
5
10
15
20
observation



Time series
Population estimate
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100
30
10
3
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10
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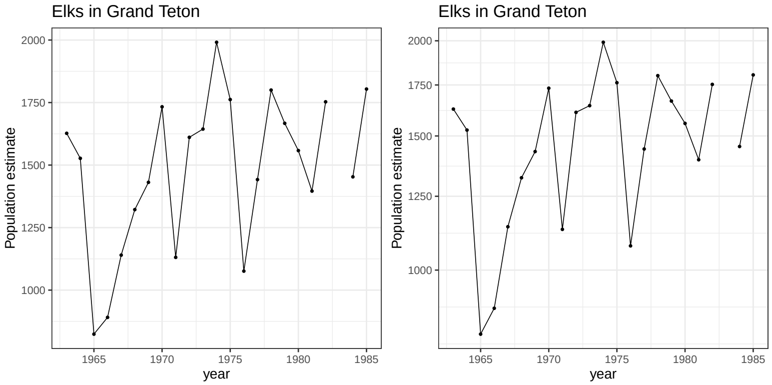
Elk



Elks in Grand Teton
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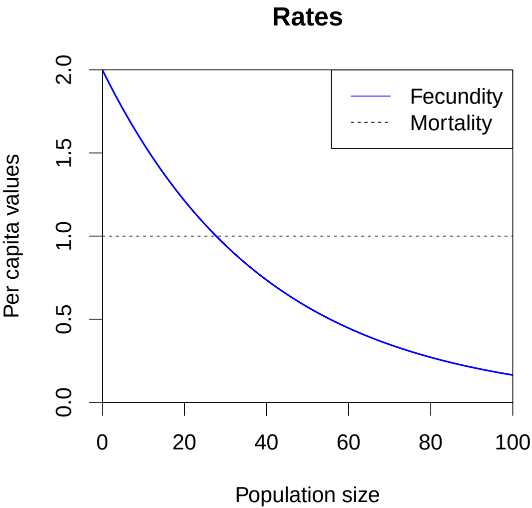

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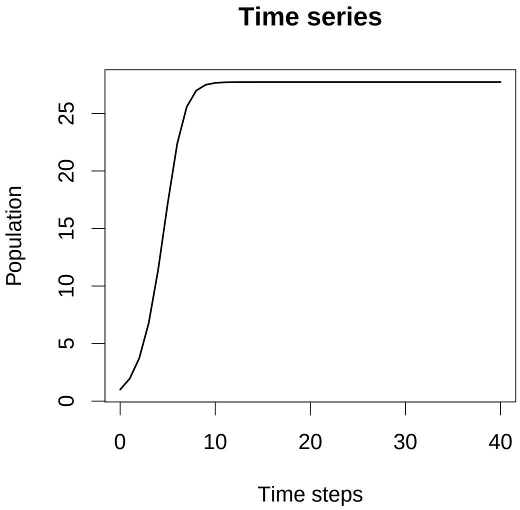
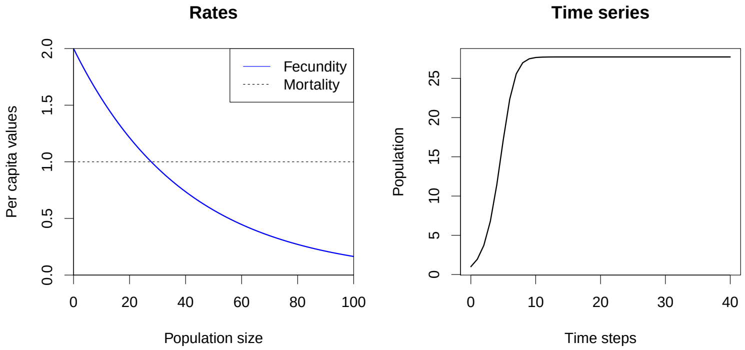
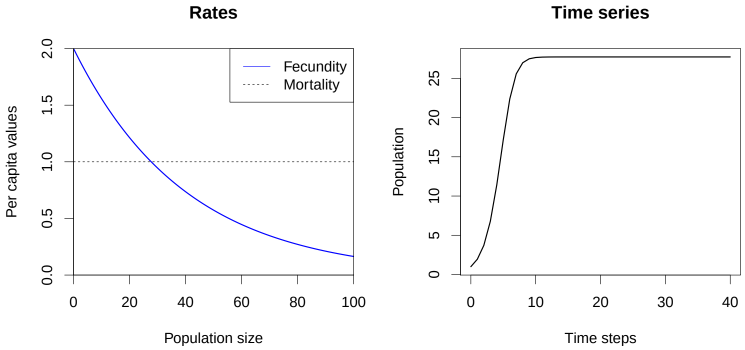


We expect simple dynamics



What dynamics do we get?

Simple dynamics (repeat)



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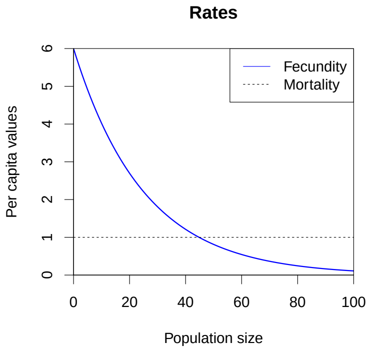

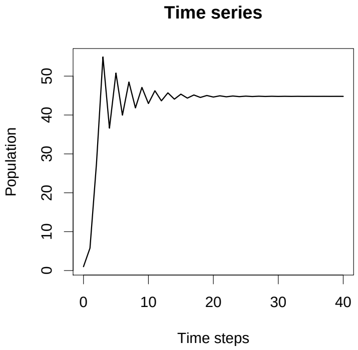

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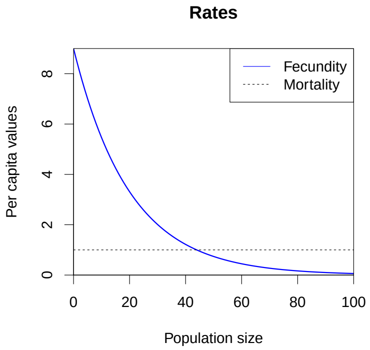
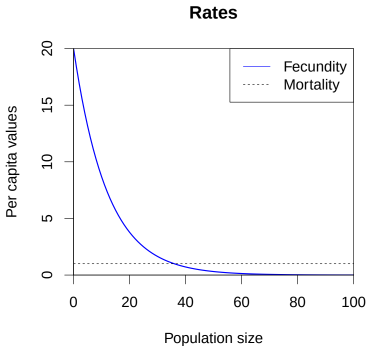


Lots of other behaviours



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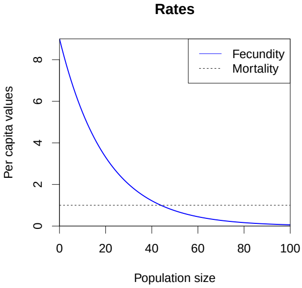

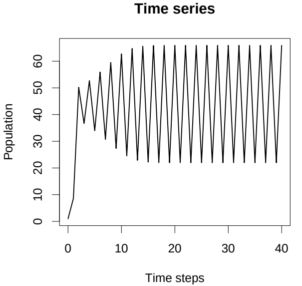

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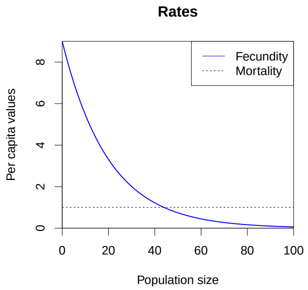

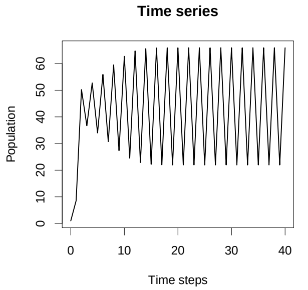

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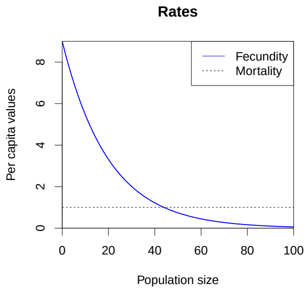

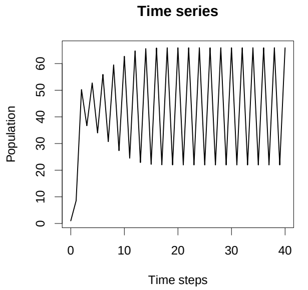

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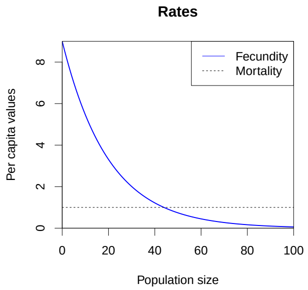

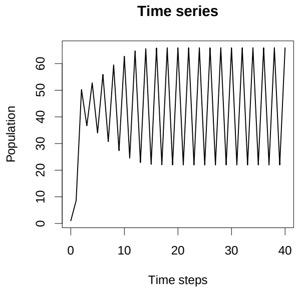

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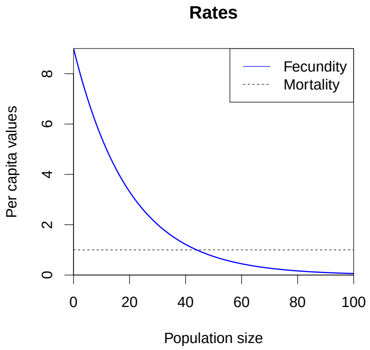

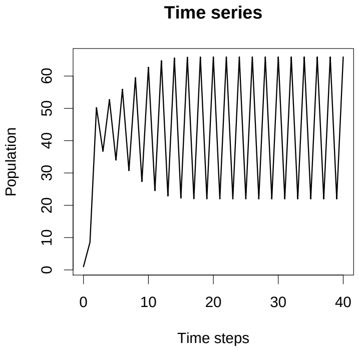

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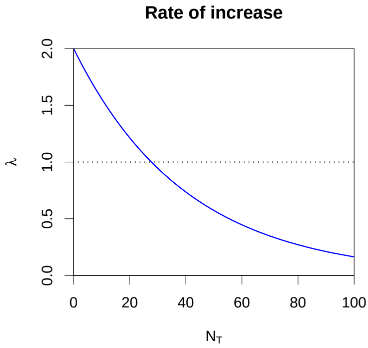

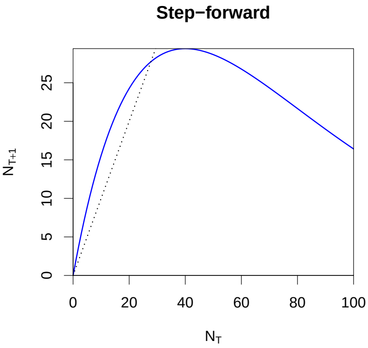

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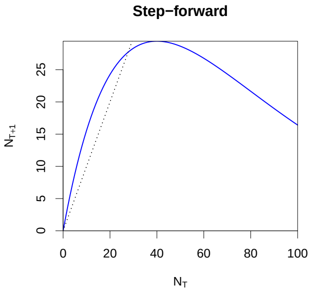

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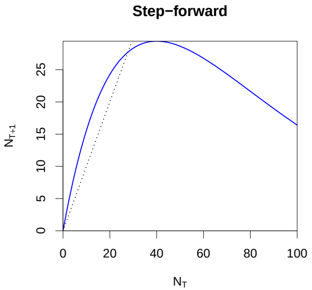

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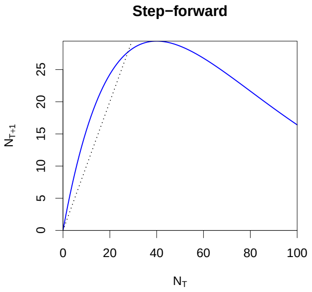

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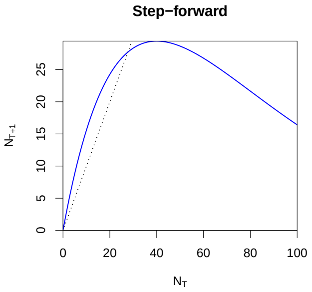

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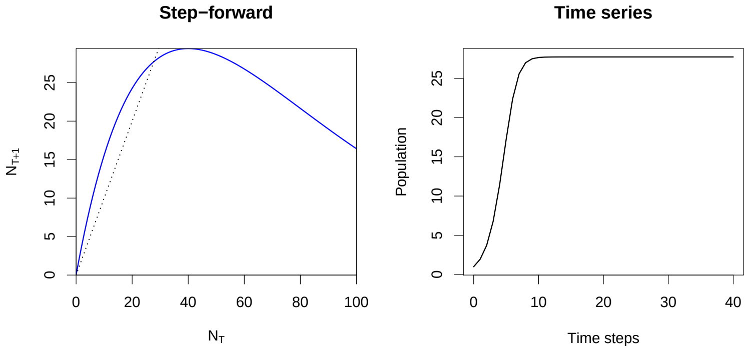

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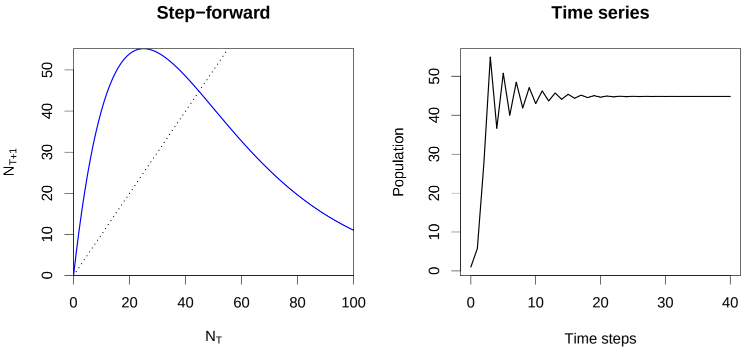

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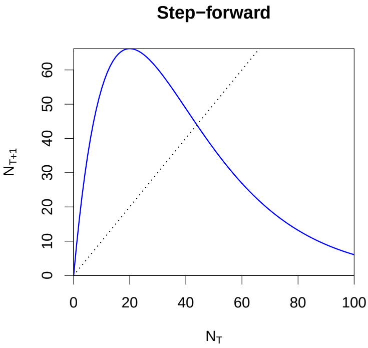

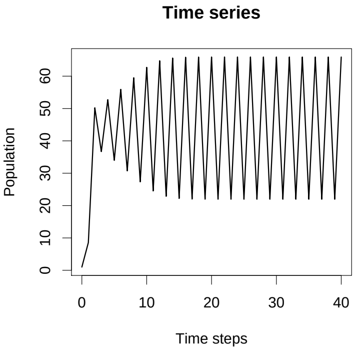

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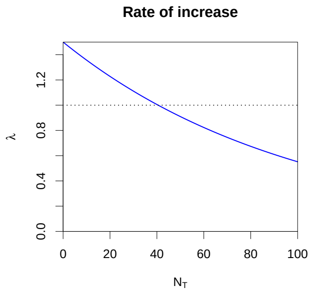

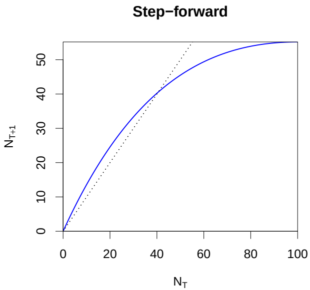

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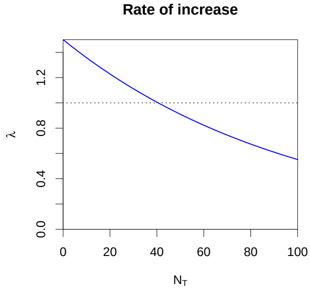

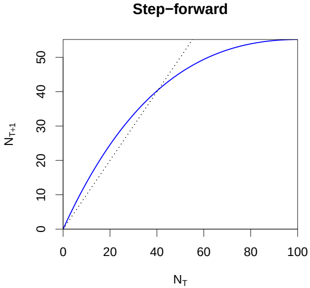

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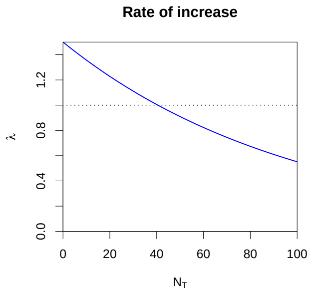

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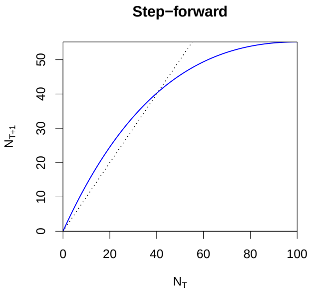

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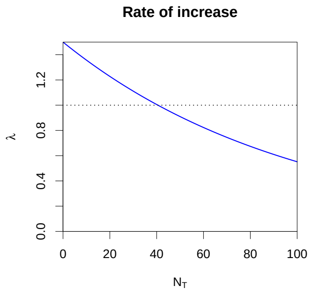

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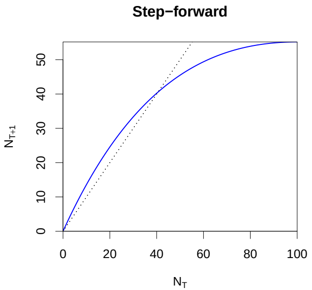
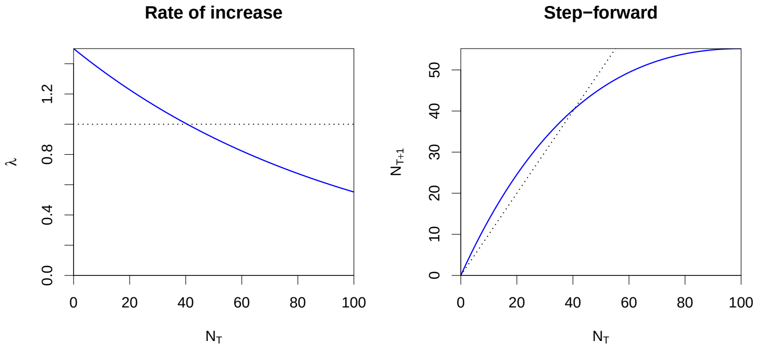


Contest regulation



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Complex behaviour from a simple model

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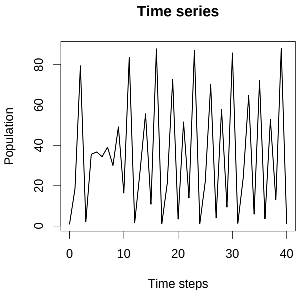

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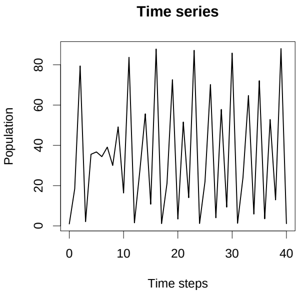

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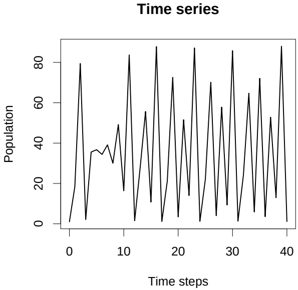

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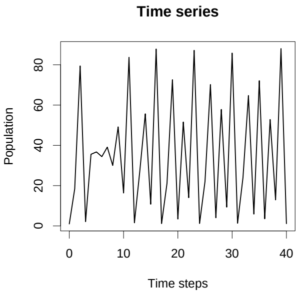

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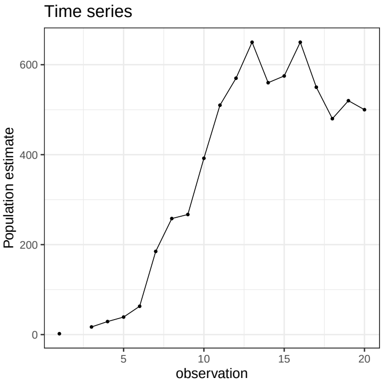
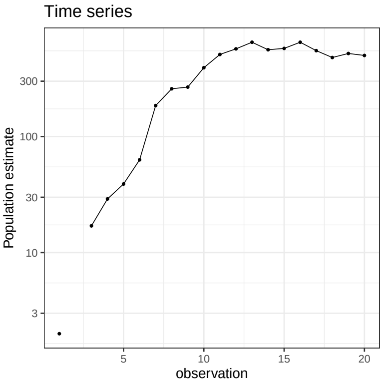
Paramecia (repeat)





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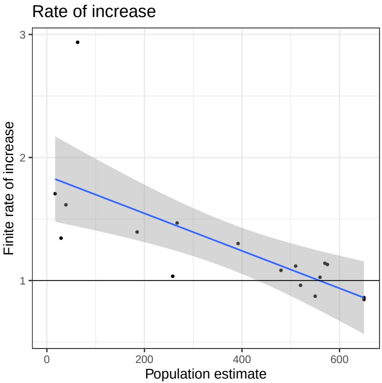
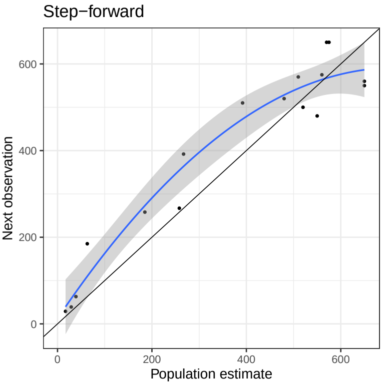
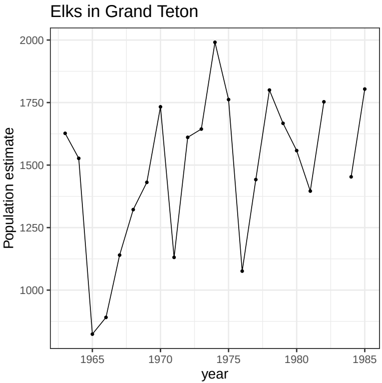


Rate of increase
Finite rate of increase
3
2
1
0
200
400
600
Population estimate



Step−forward
Next observation
600
400
200
0
0
200
400
600
Population estimate

Elk (repeat)


Elks in Grand Teton
Population estimate
2000
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1250
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1965
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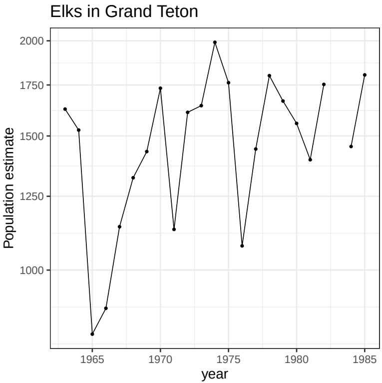
year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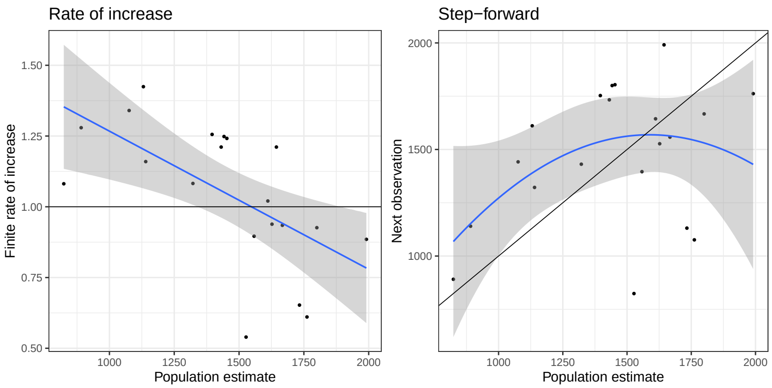

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1.50
1.25
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0.75
0.50
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1750
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Outline

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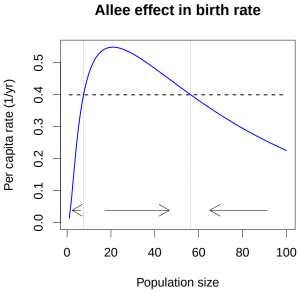

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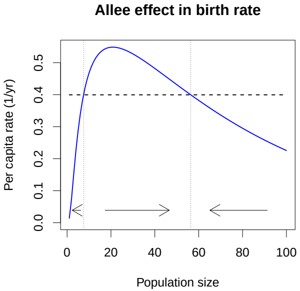

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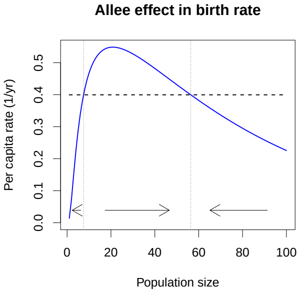

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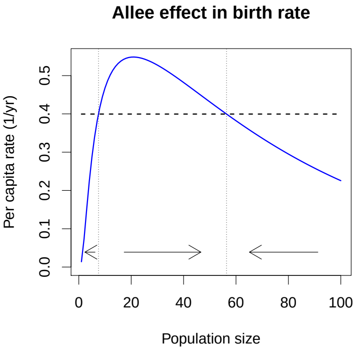

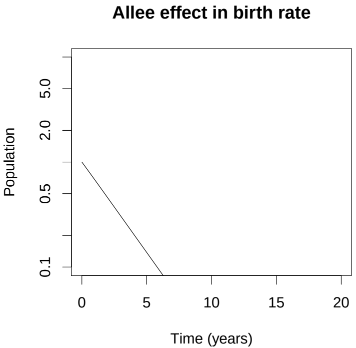

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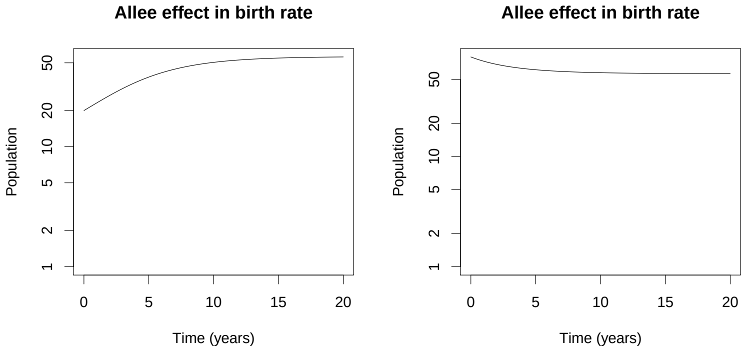

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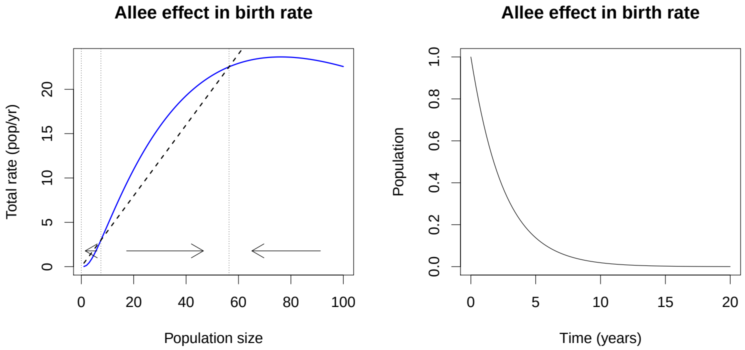

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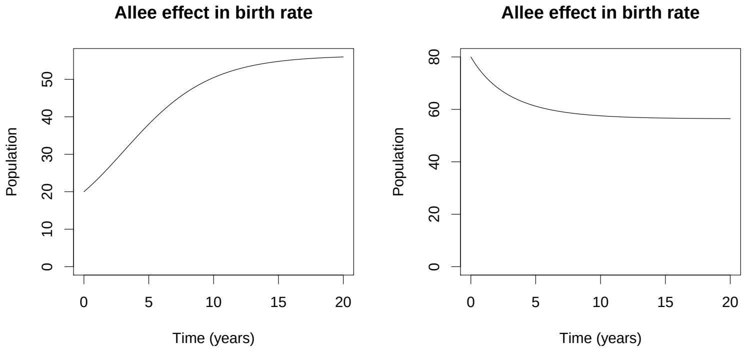
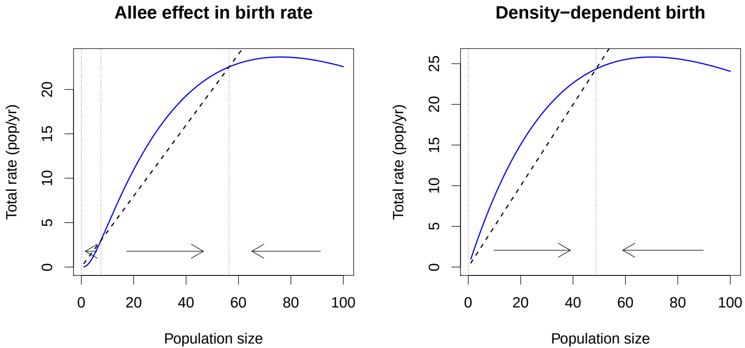


Population comparison (repeat)



Allee effect in death rate

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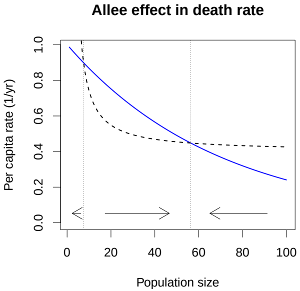

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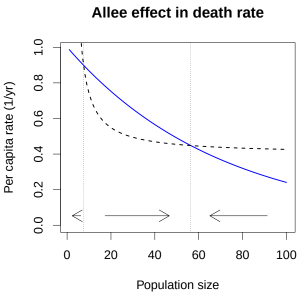

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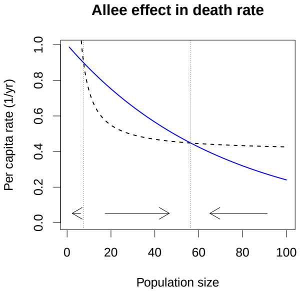

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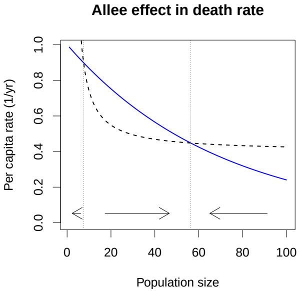

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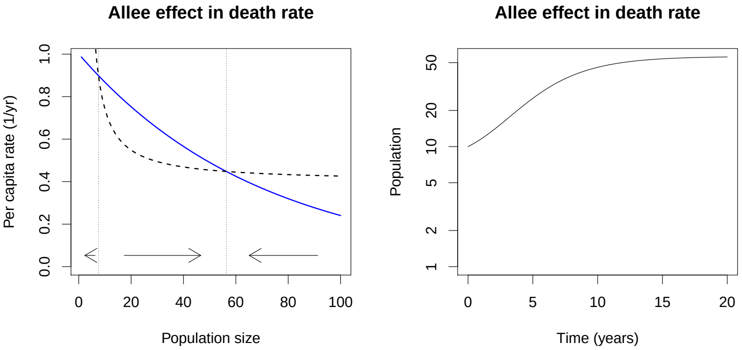

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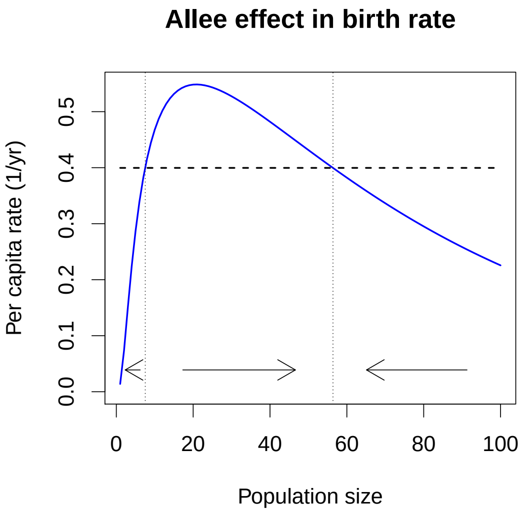

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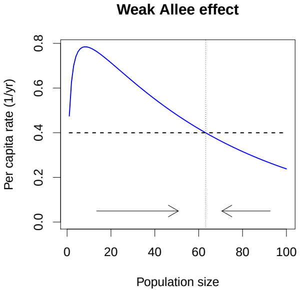

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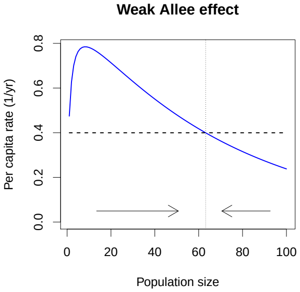

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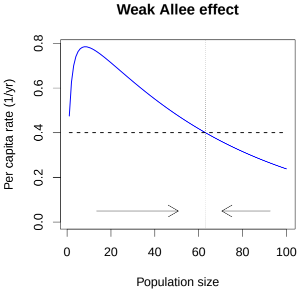

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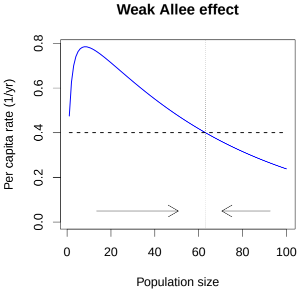

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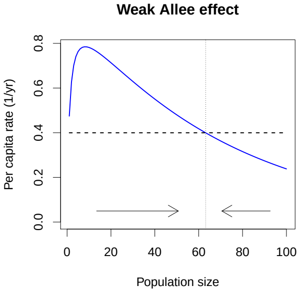

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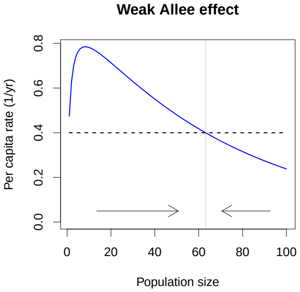
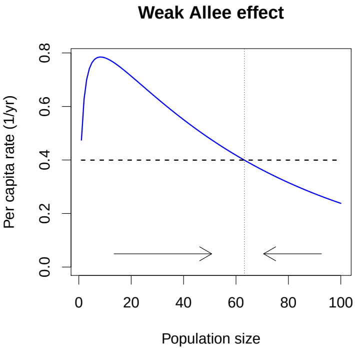
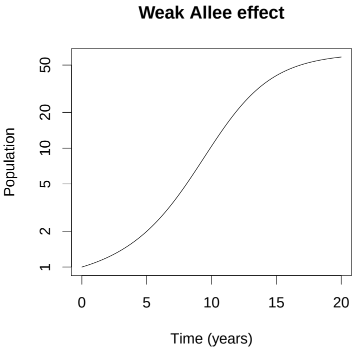


Individual perspective





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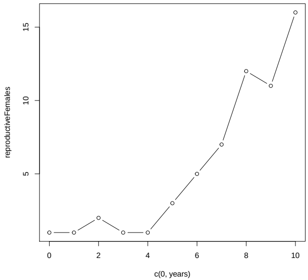

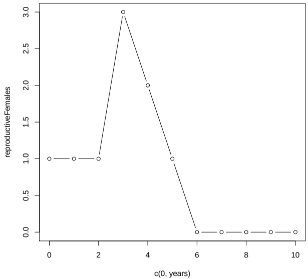

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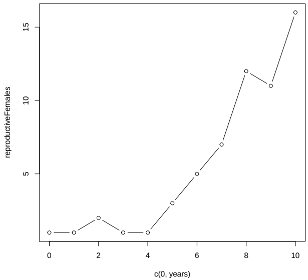

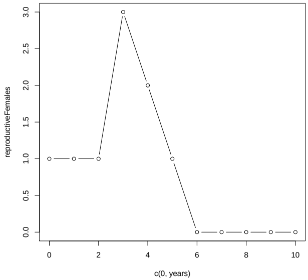

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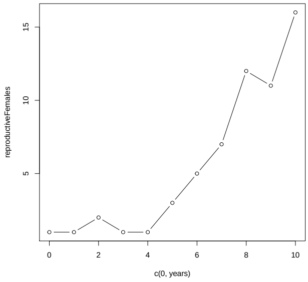

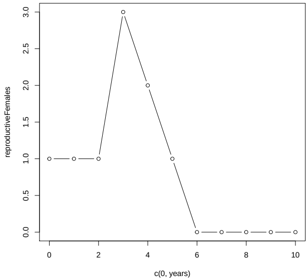

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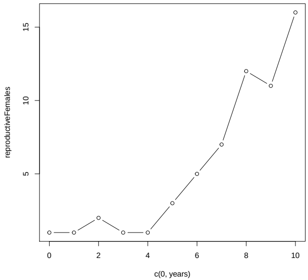

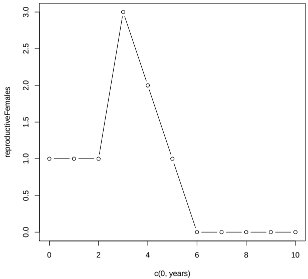

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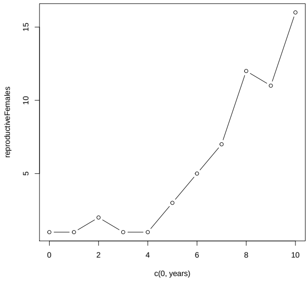

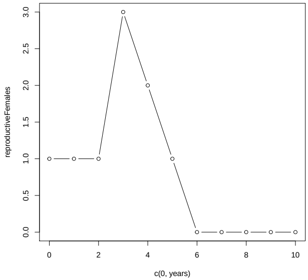

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