UNIT 3 Non-linear population models

Outline

Introduction

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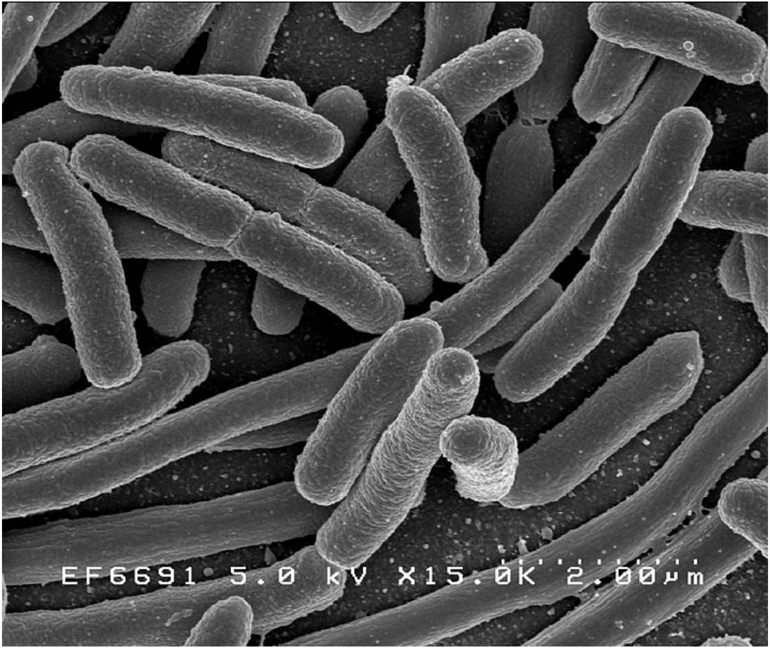
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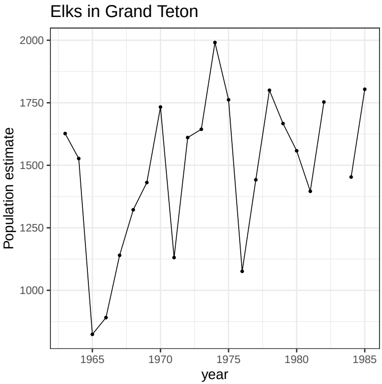
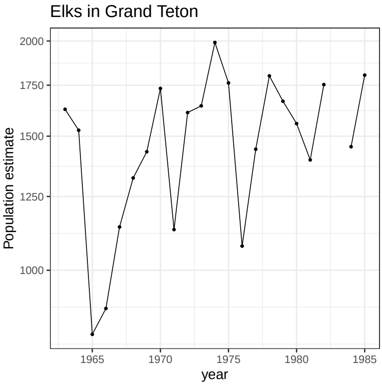


Elks in Grand Teton
Population estimate
year
2000
1750
1500
1250
1000
1965
1970
1975
1980
1985



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Population estimate
year
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1750
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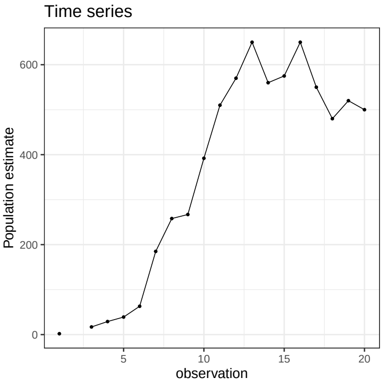
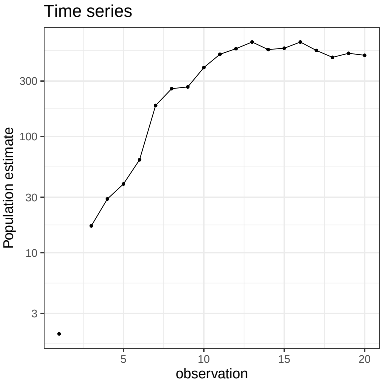
Paramecia

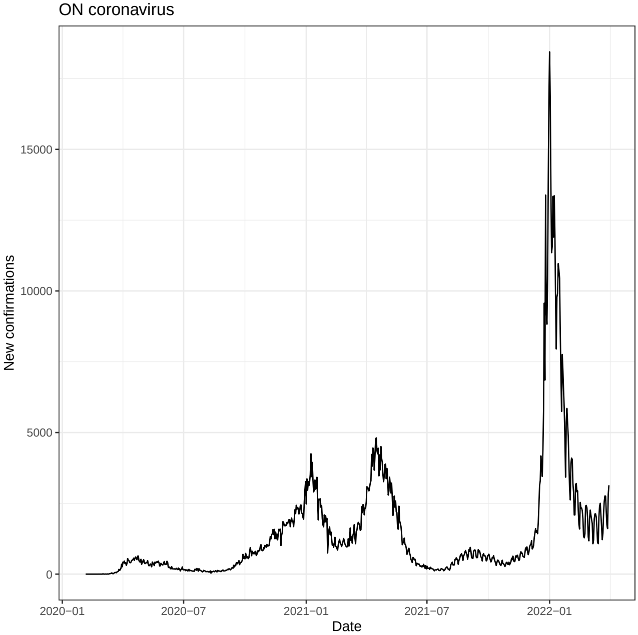


Time series
Population estimate
600
400
200
0
5
10
15
20
observation



Time series
Population estimate
300
100
30
10
3
5
10
15
20
observation

Coronavirus



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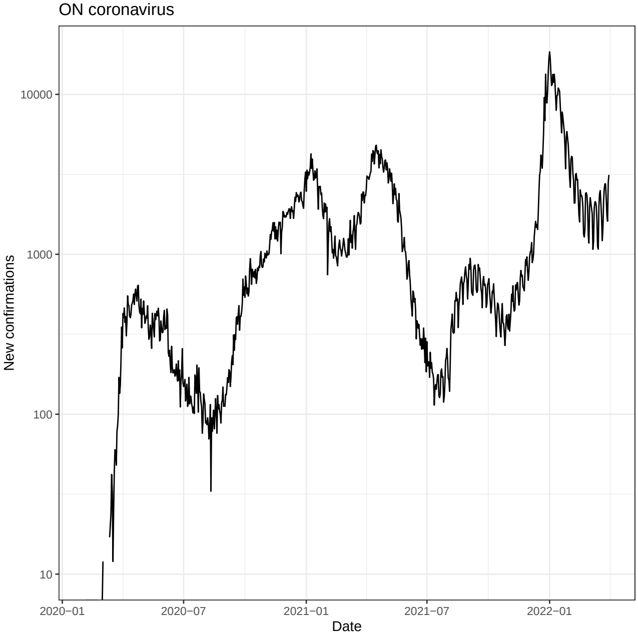

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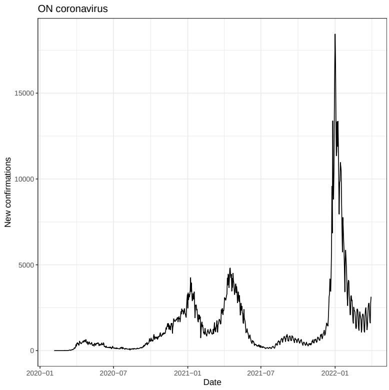
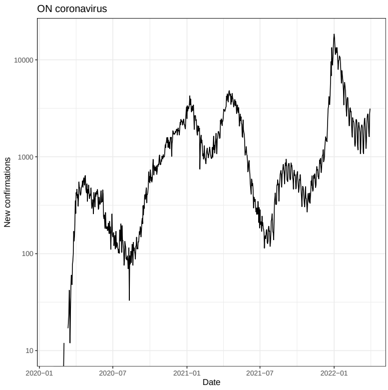
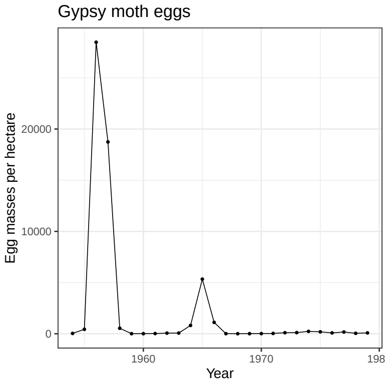
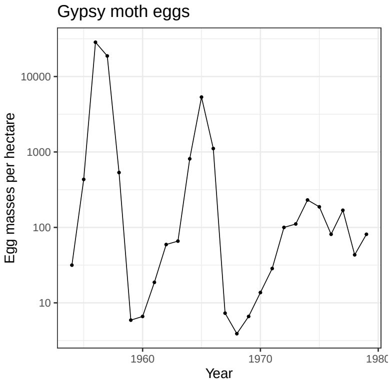




Gypsy moths



Gypsy moth eggs
Egg masses per hectare
20000
10000
0
1960
1970
198
Year



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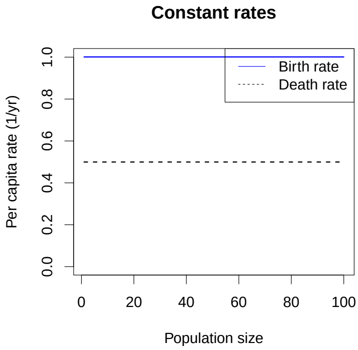
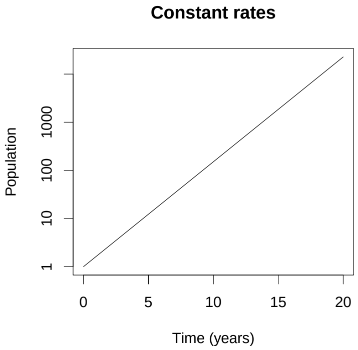
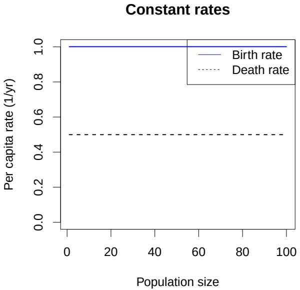
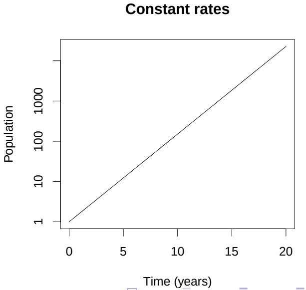




Individual perspective

- Per capita rate shows birth and death per individual





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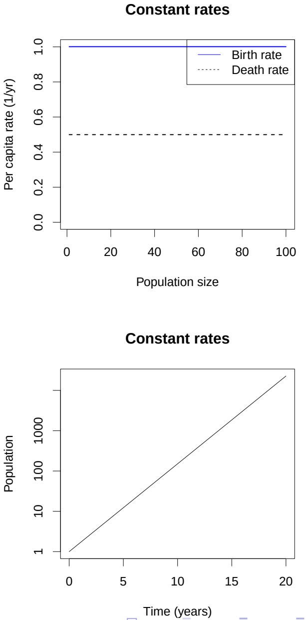

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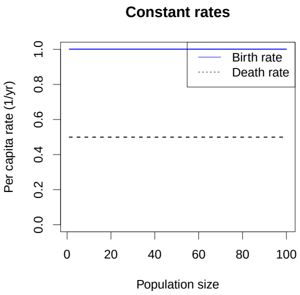

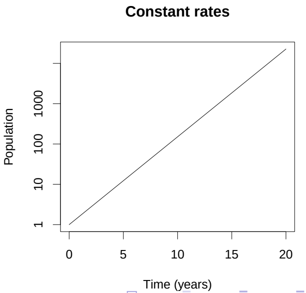

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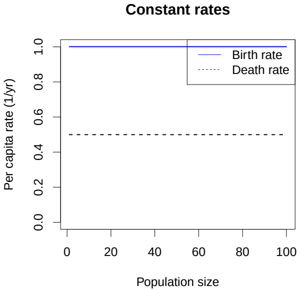
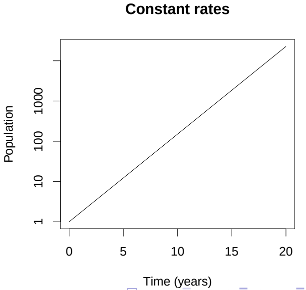
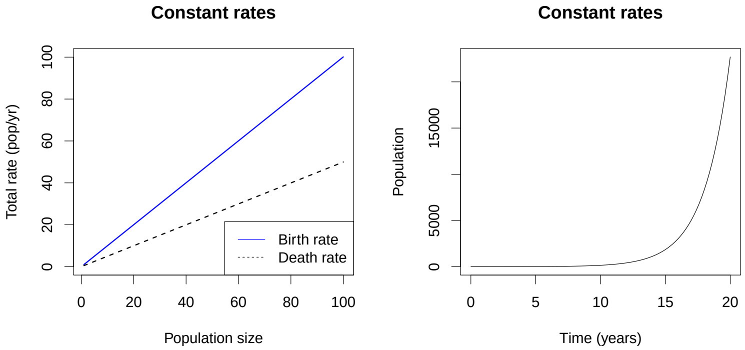




Population perspective



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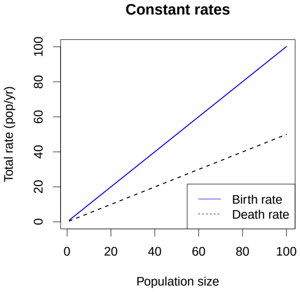

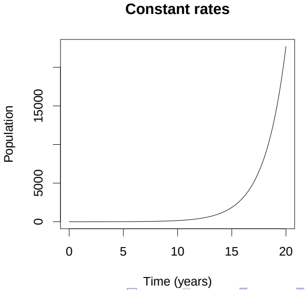

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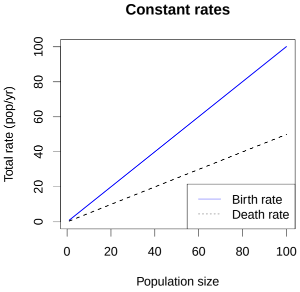

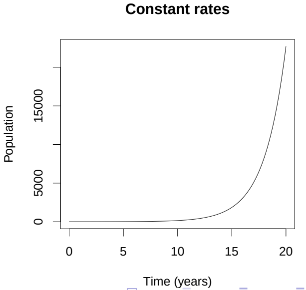

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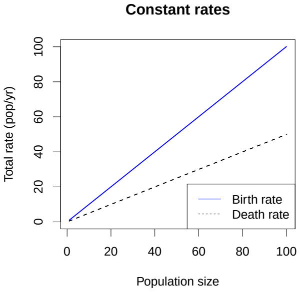

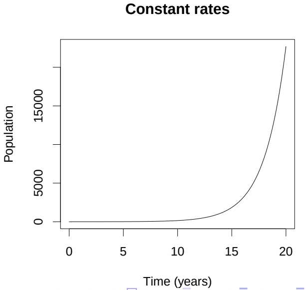

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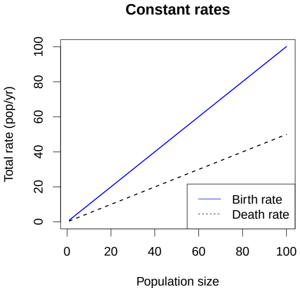
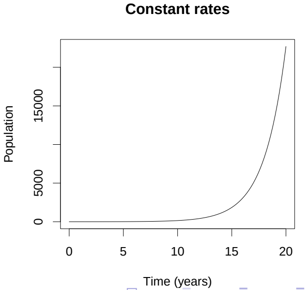
  - On the linear scale we see *additive* or *absolute* change





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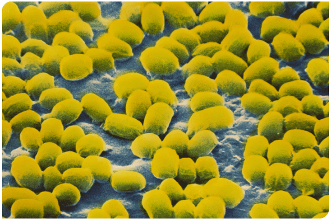

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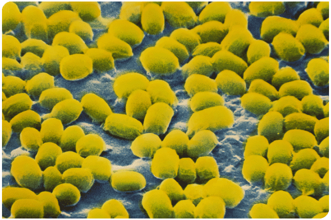

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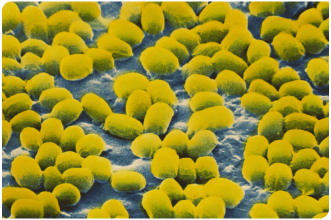

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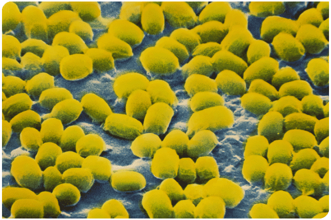

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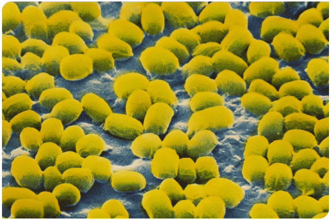

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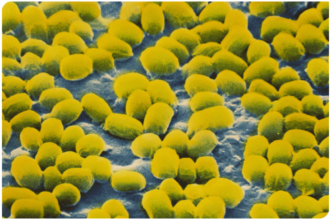

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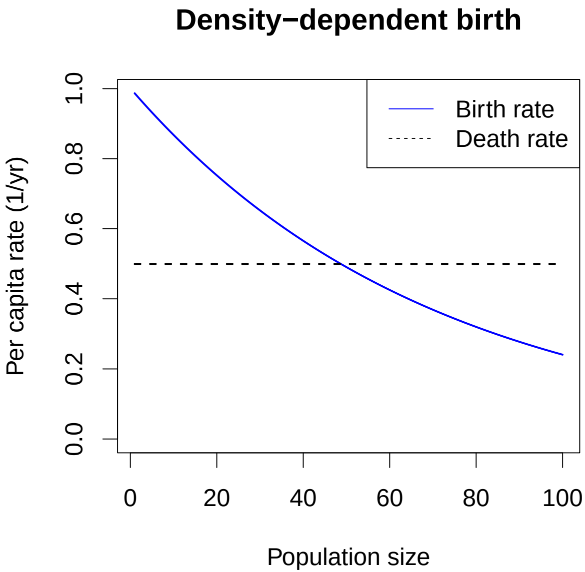
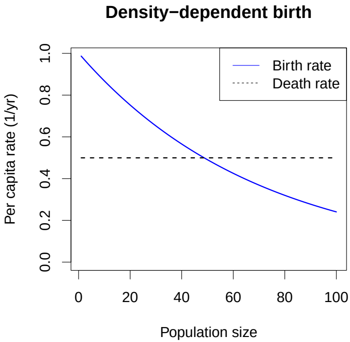
  - near zero?
  - near the equilibrium?
  - at a high value?

A model



Density−dependent birth
Birth rate
Death rate
Per capita rate (1/yr)
0.0
0.2
0.4
0.6
0.8
1.0
0
20
40
60
80
100
Population size

What is the characteristic scale?



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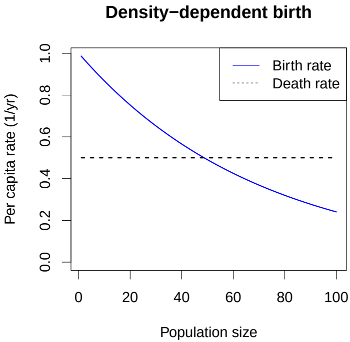

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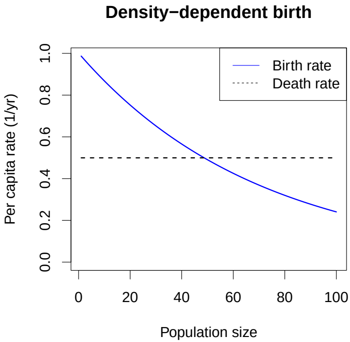

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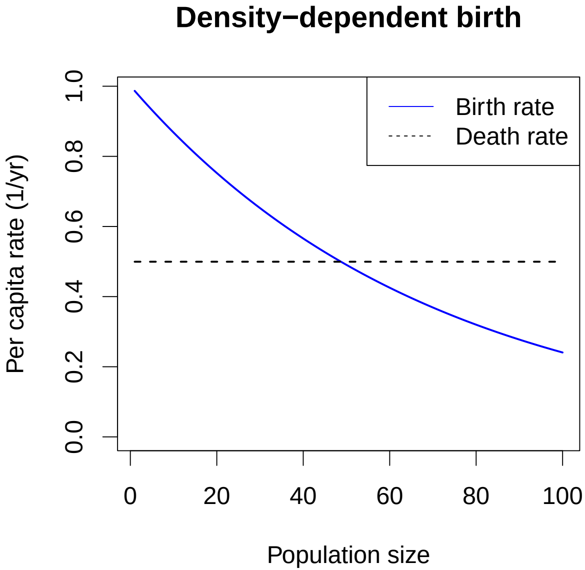

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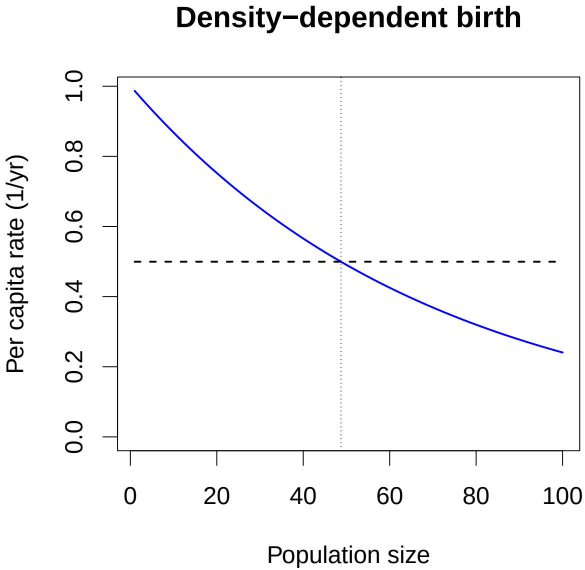

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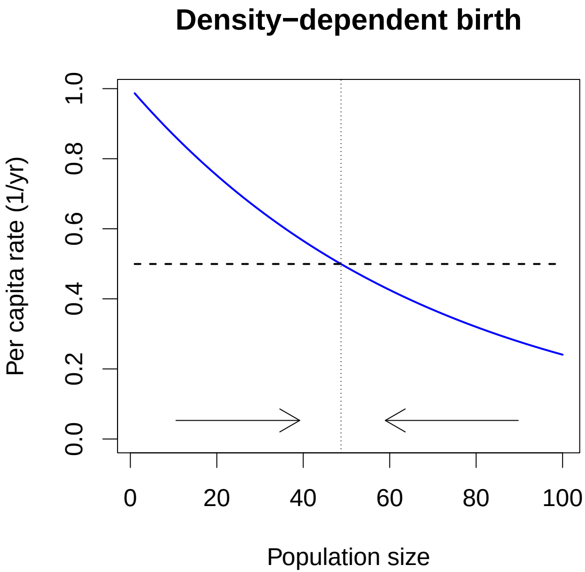

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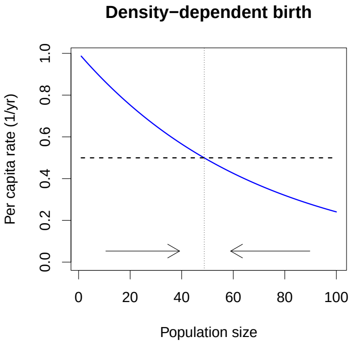
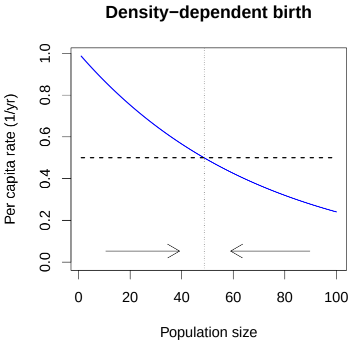


- Increase when population is below equilibrium

What will this model do?



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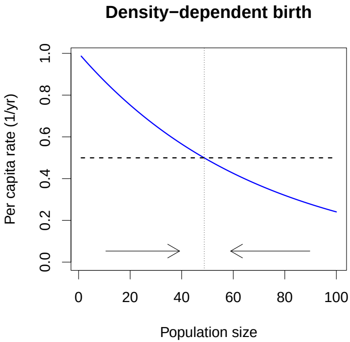

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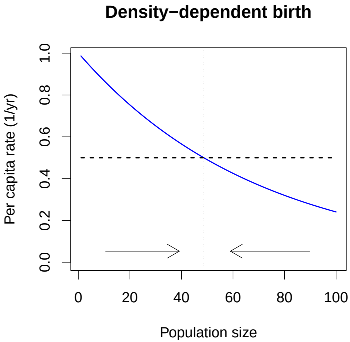

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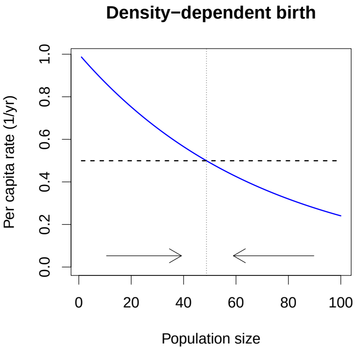

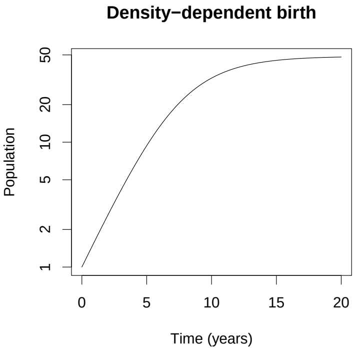

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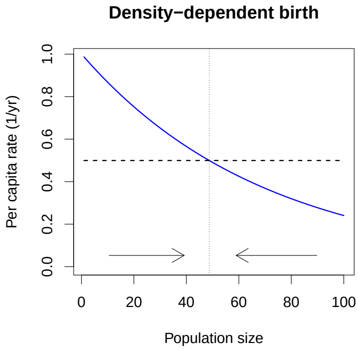

Examples

Density−dependent birth
Population
1
2
5
10
20
50
0
5
10
15
20
Time (years)

Density−dependent birth
Population
1
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5
10
20
50
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Outline

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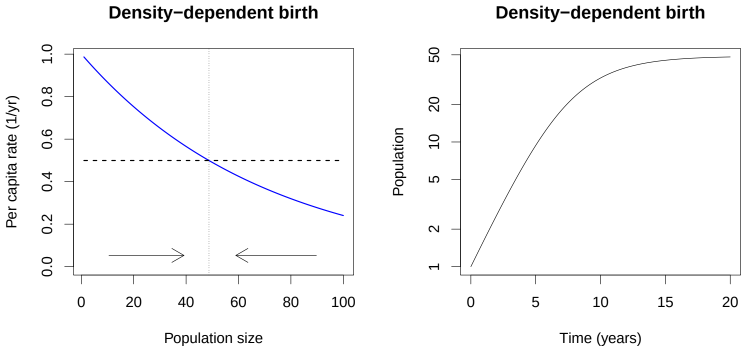

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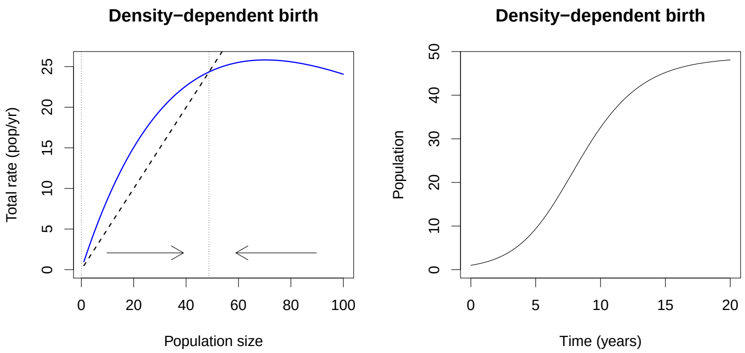

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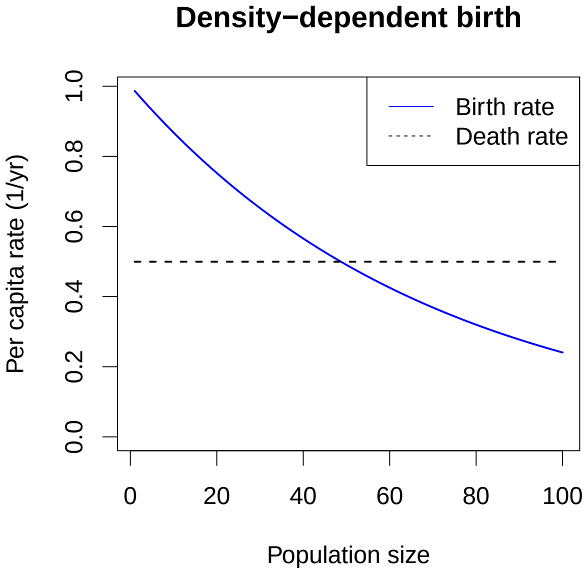
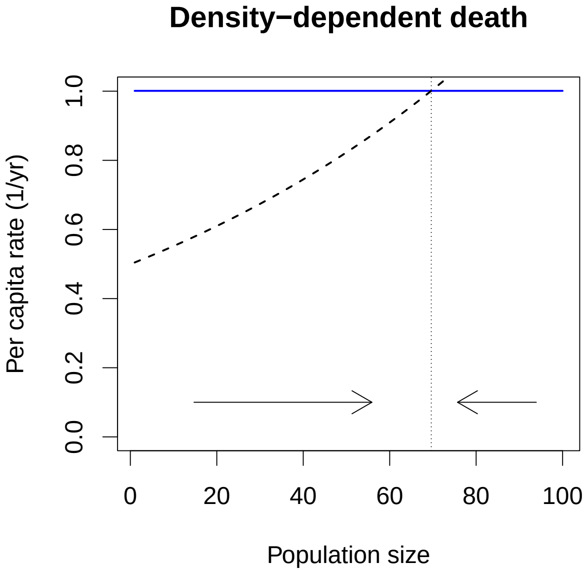


Increasing death rates



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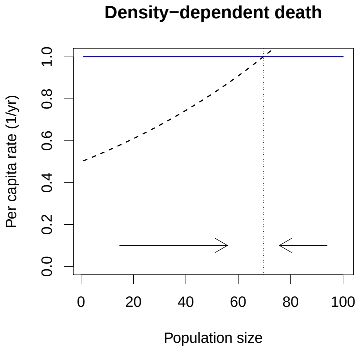

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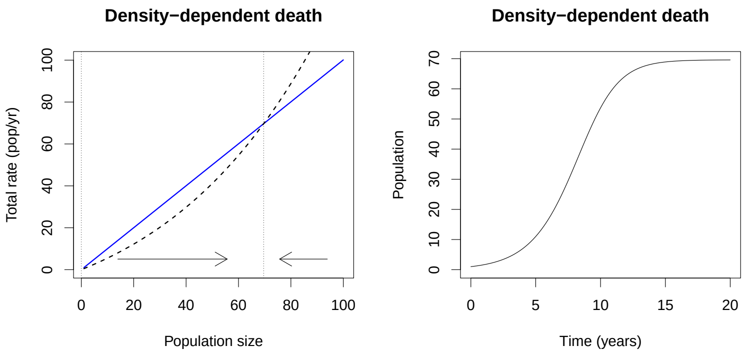

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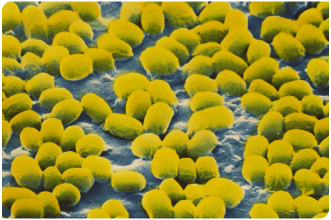

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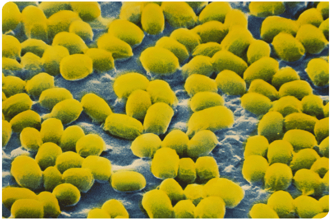

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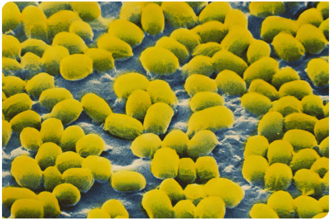

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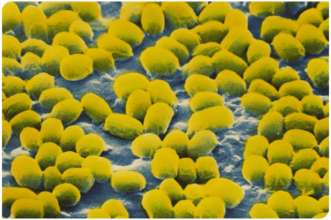

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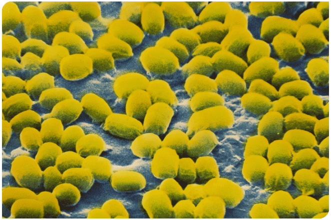

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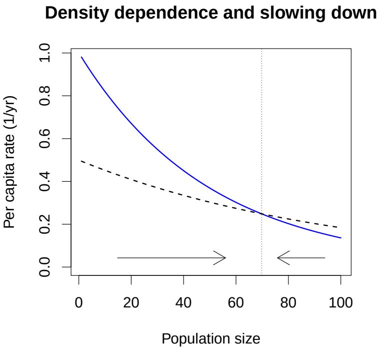
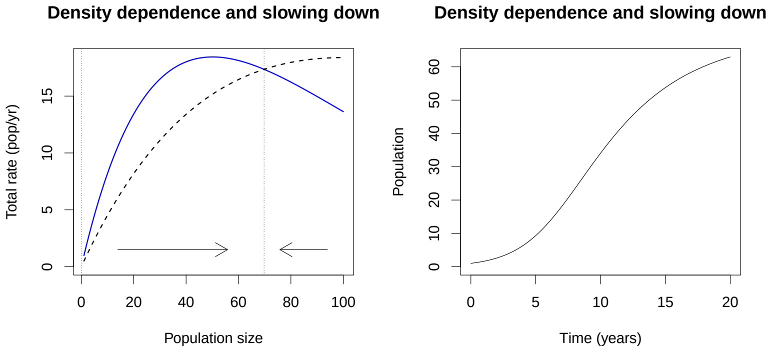


Population perspective



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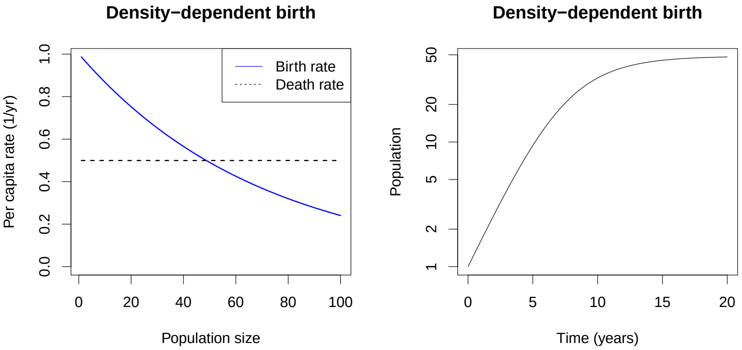

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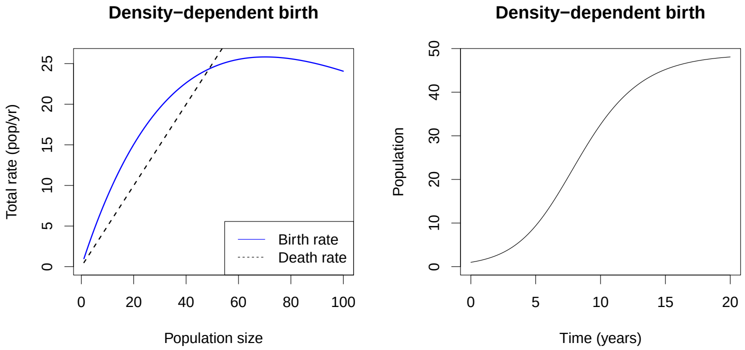

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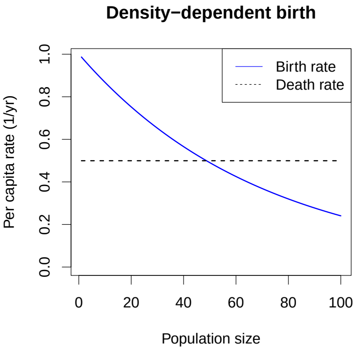

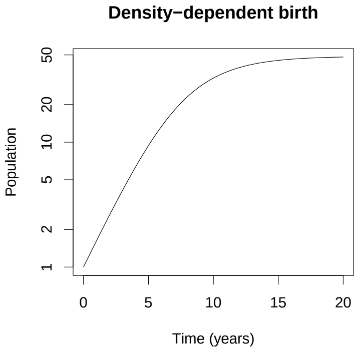

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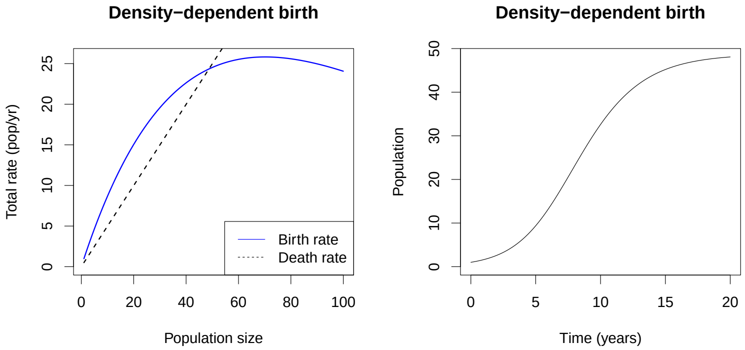

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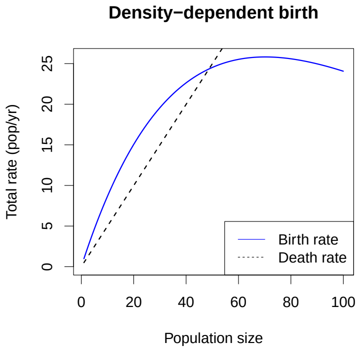

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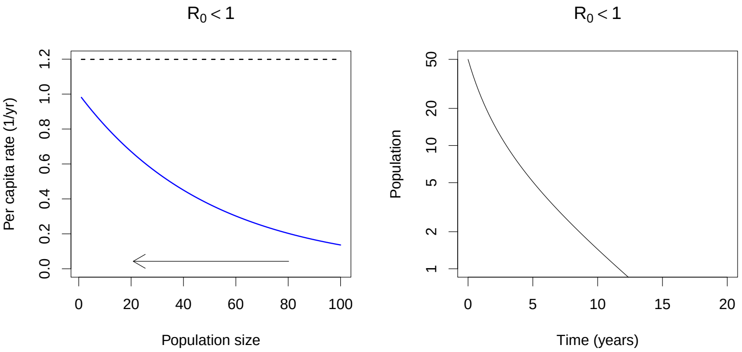

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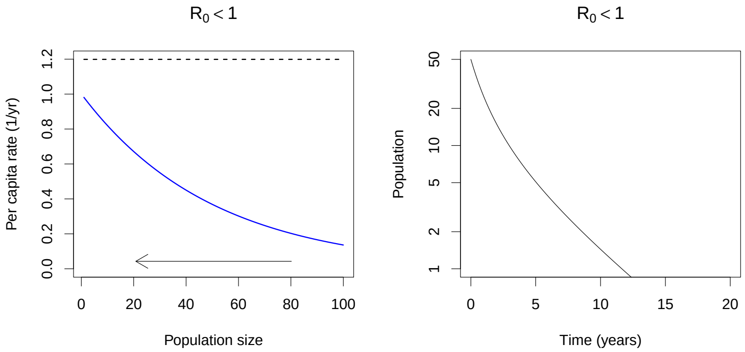

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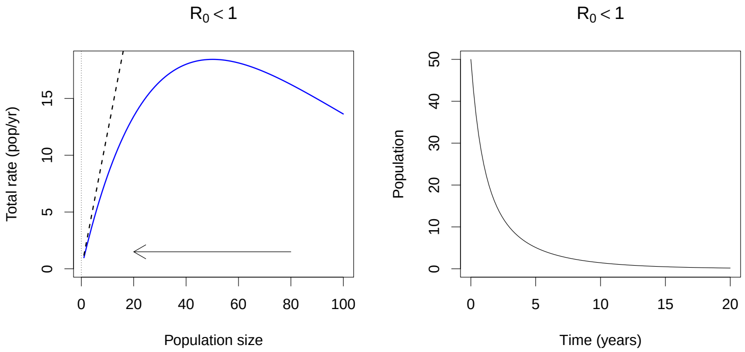

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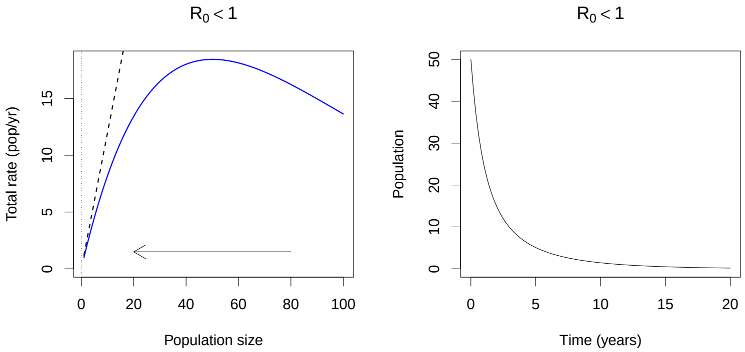

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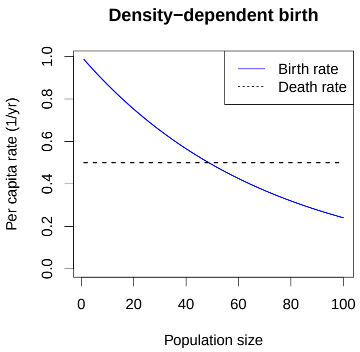

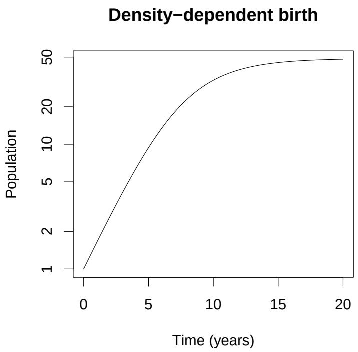

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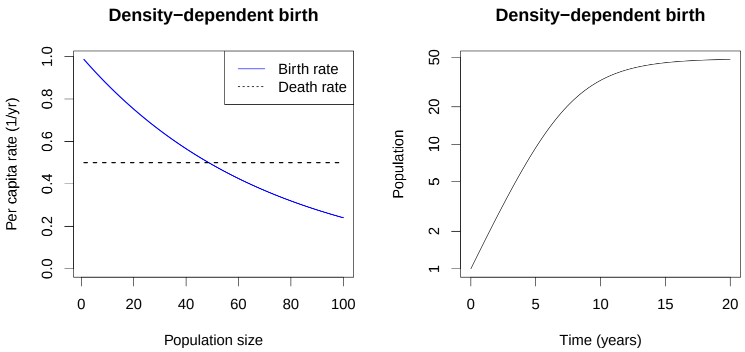

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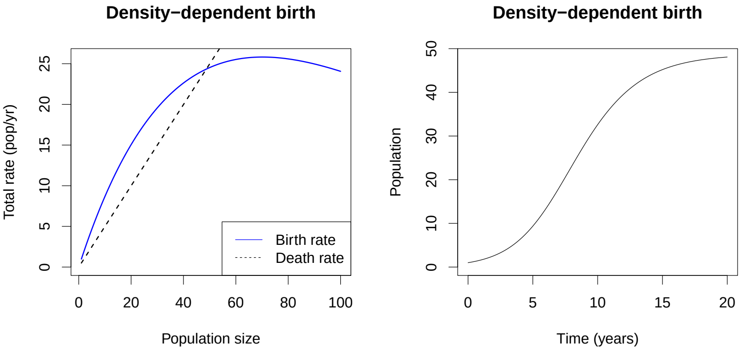

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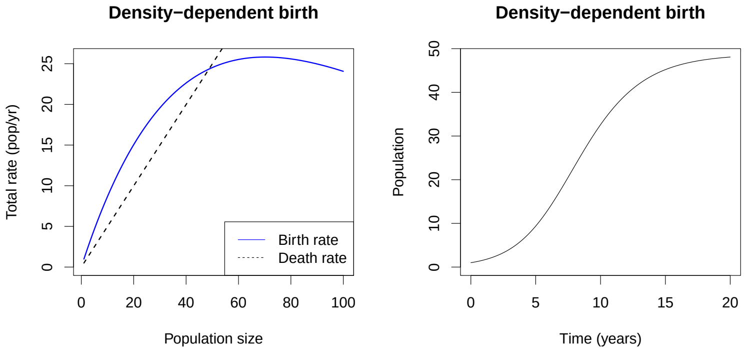

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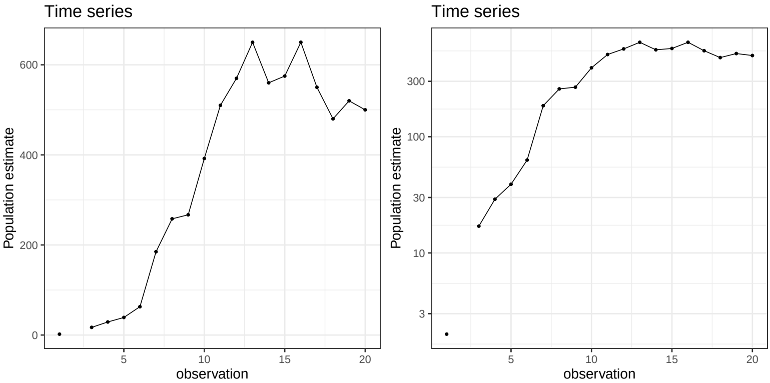
Time series
Population estimate
600
400
200
0
5
10
15
20
observation
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30
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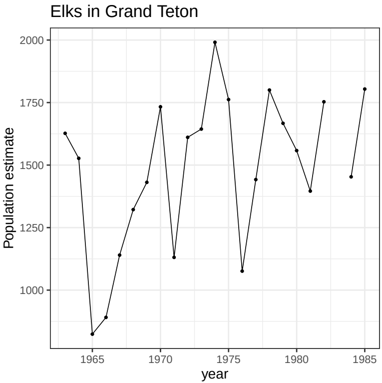
Elk



Elks in Grand Teton
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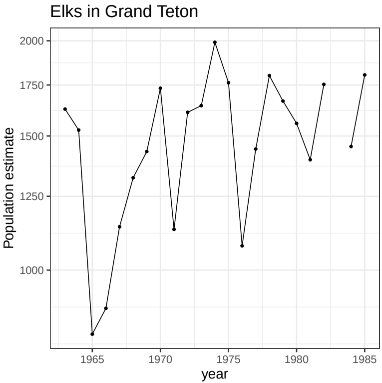

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Parameters

- What quantities describe the rules for this system?
  - * f_0 [1]
  - * p_0 [1]
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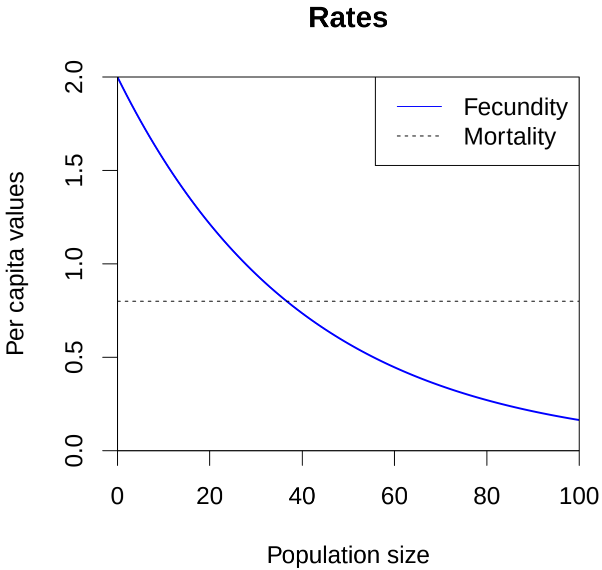

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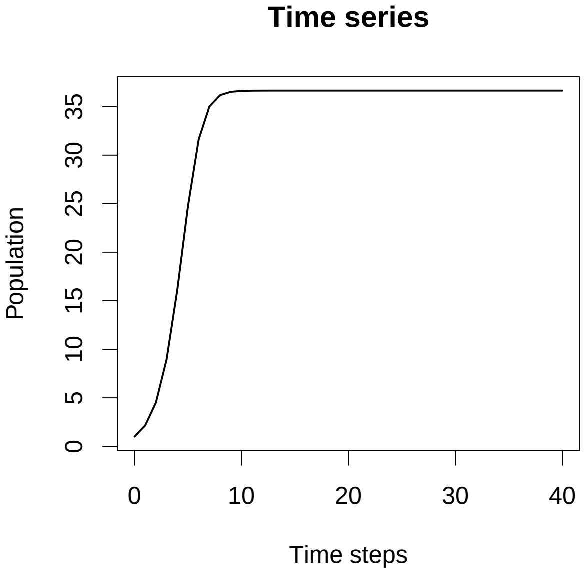
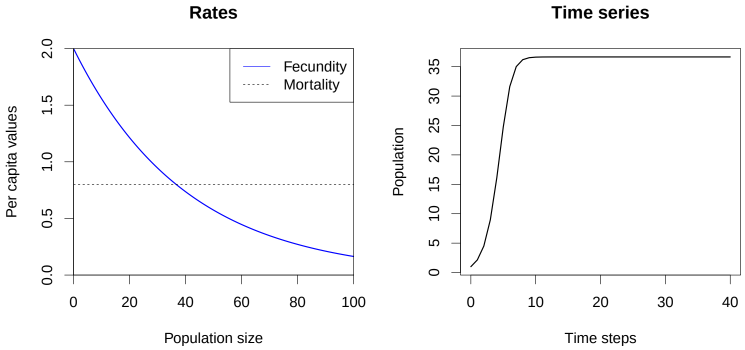


We expect simple dynamics



What dynamics do we get?

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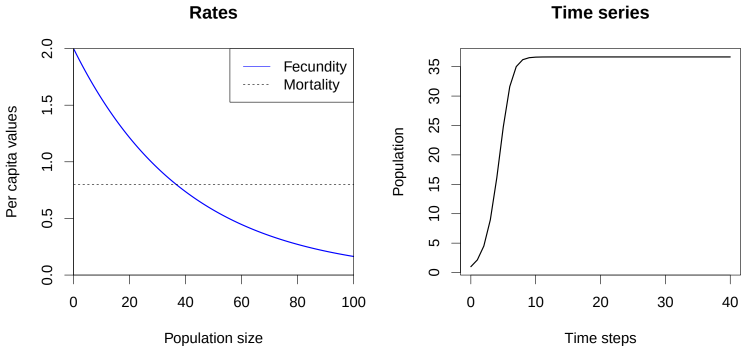

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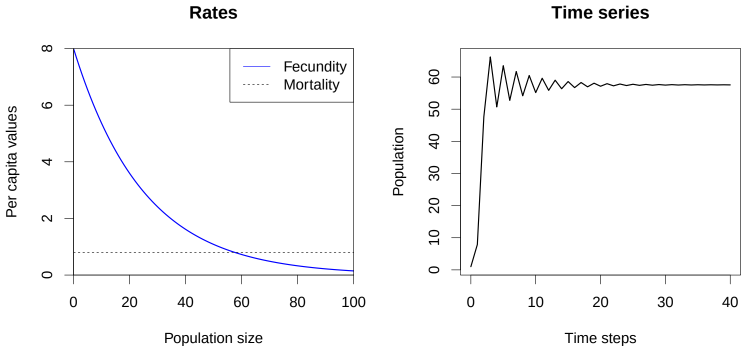

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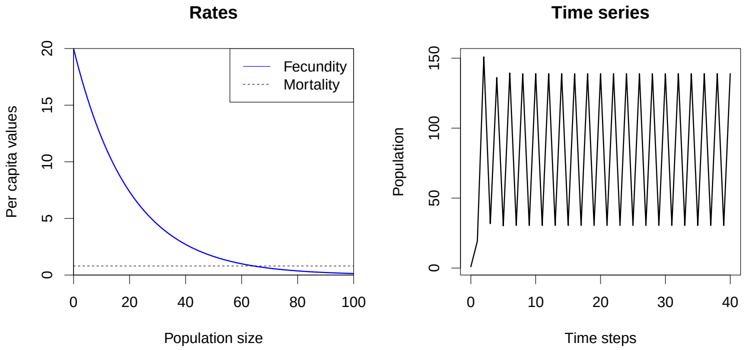
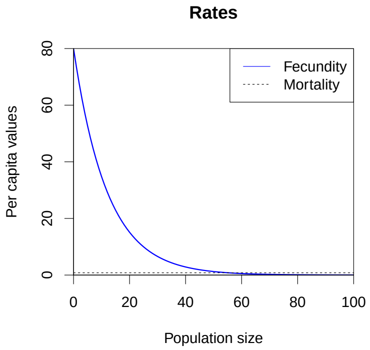
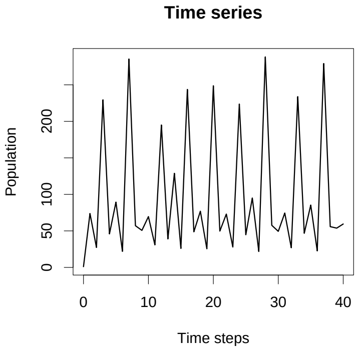


Lots of other behaviours





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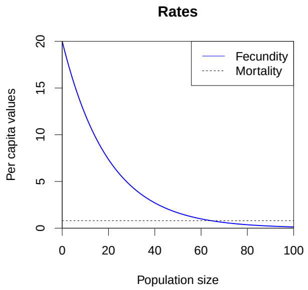

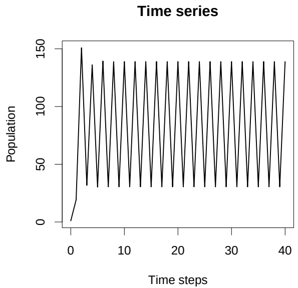

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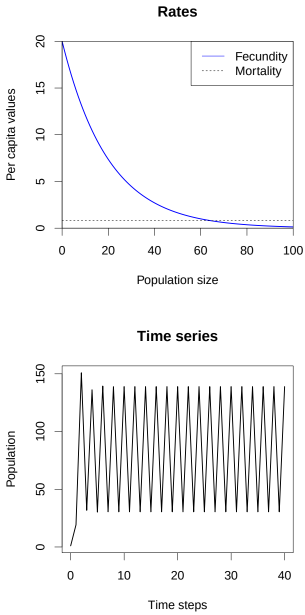

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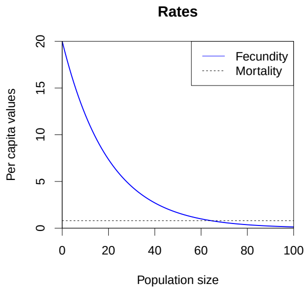

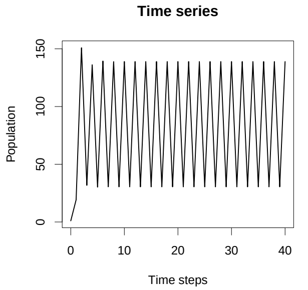

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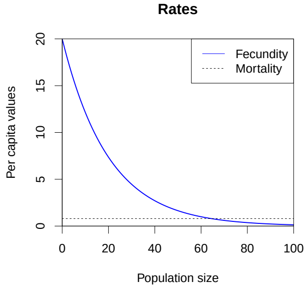

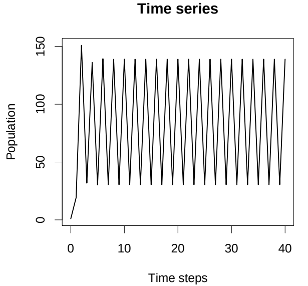

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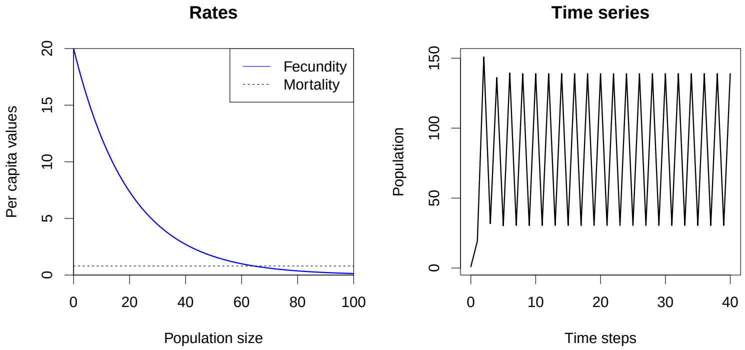

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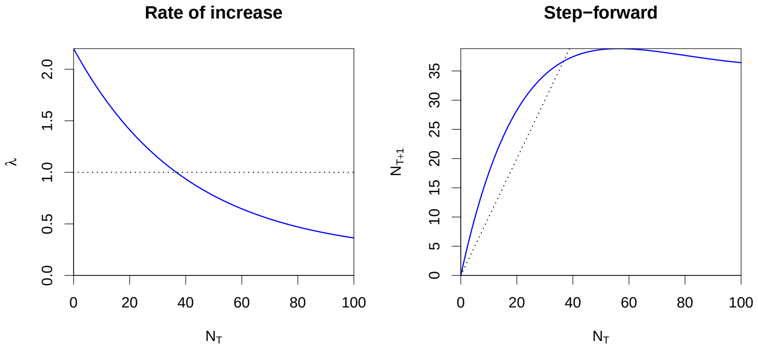

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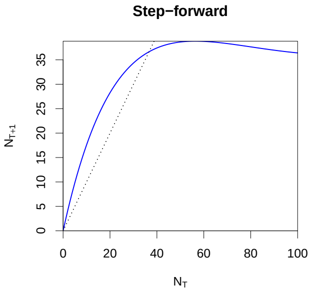

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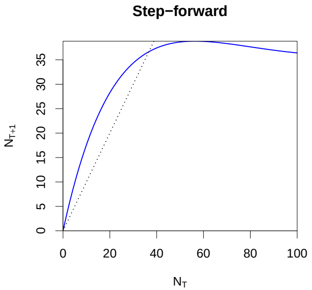

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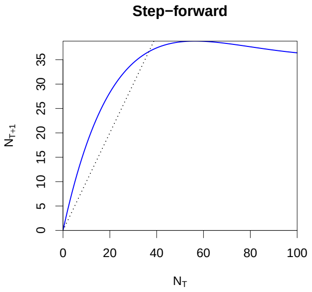

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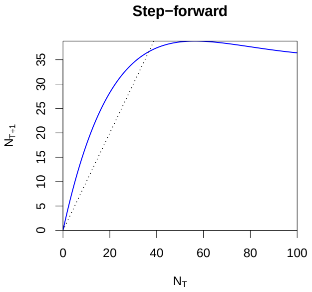

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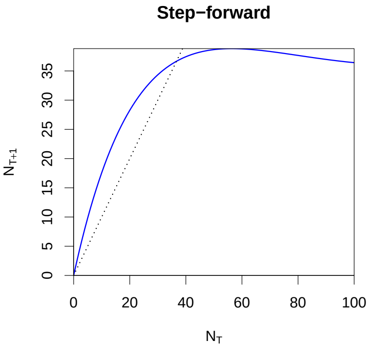

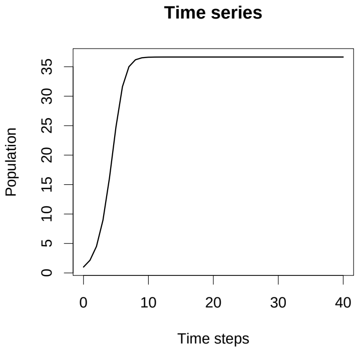

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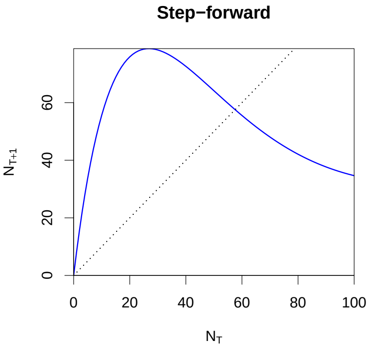

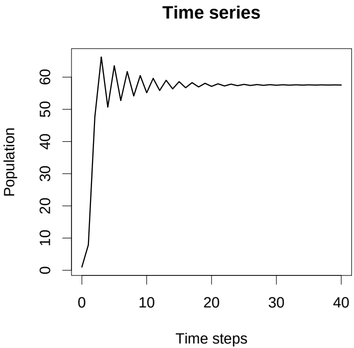

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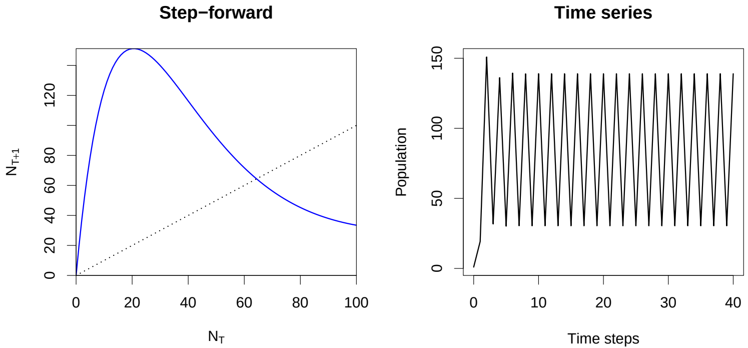

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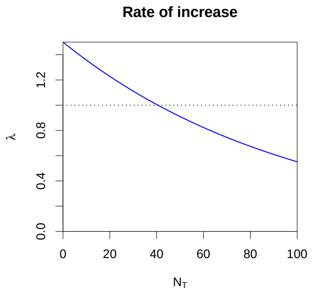

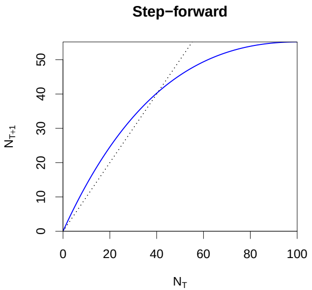

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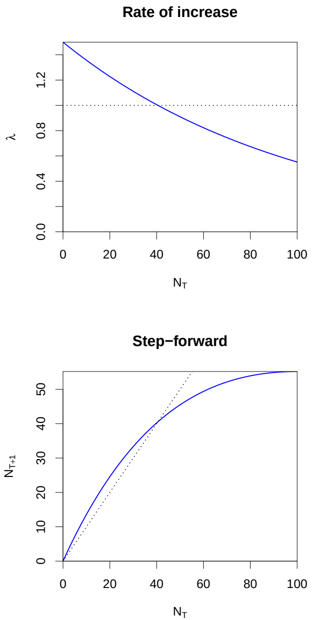

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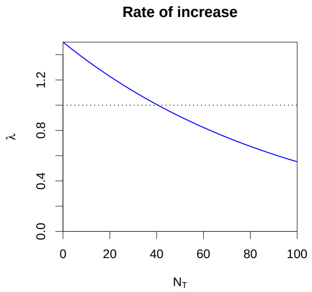

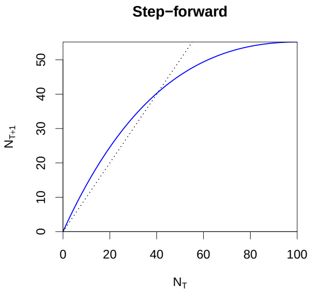

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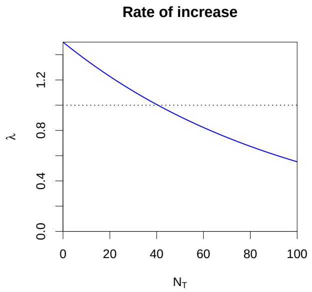

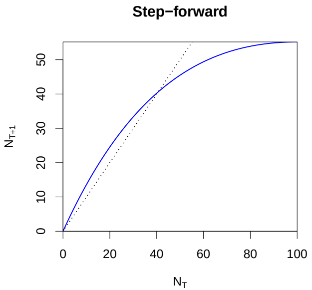

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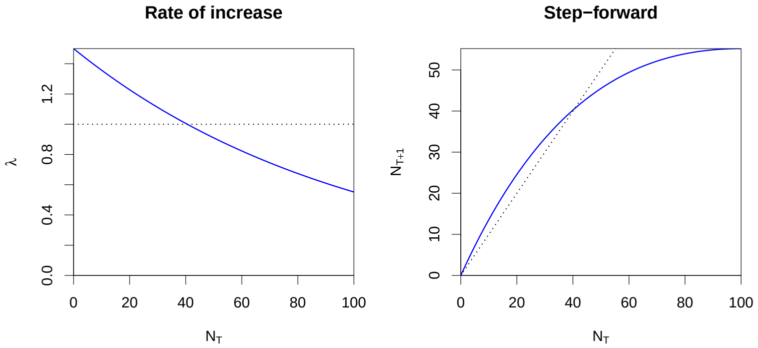

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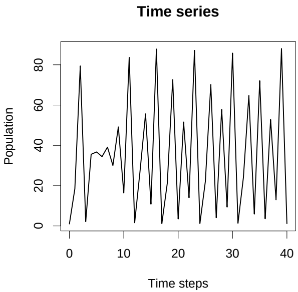

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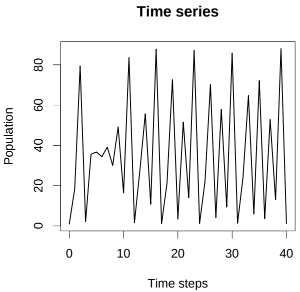

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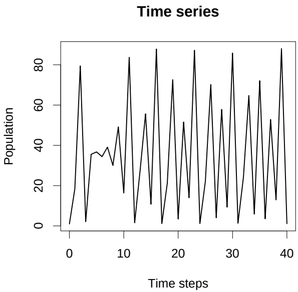

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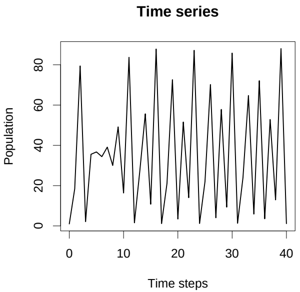

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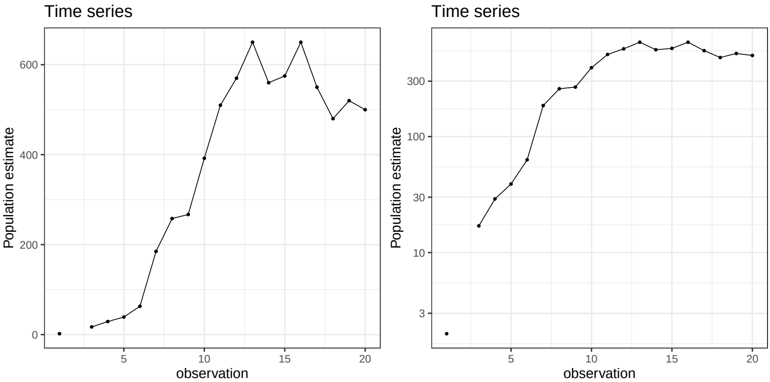
Time series
Population estimate
600
400
200
0
5
10
15
20
observation
Time series
Population estimate
300
100
30
10
3
5
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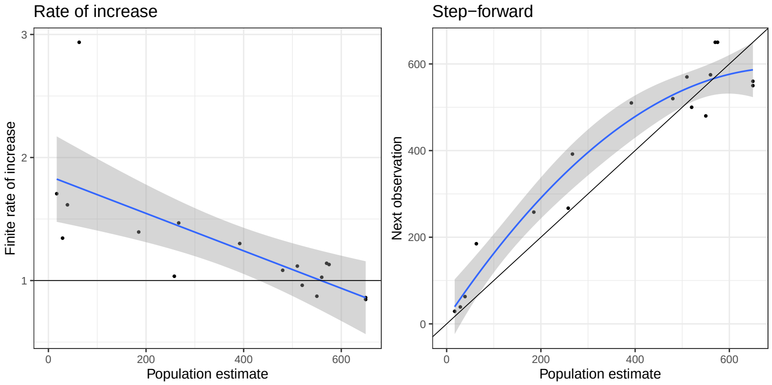
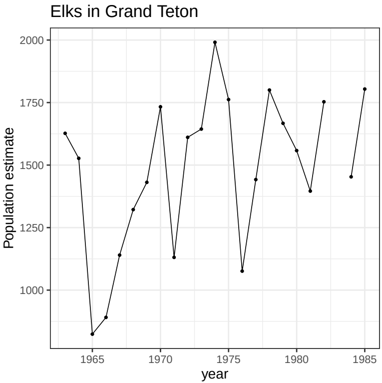

Rate of increase
Finite rate of increase
3
2
1
0
200
400
600
Population estimate
Step−forward
Next observation
600
400
200
0
0
200
400
600
Population estimate

Elk


Elks in Grand Teton
2000
1750
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1965
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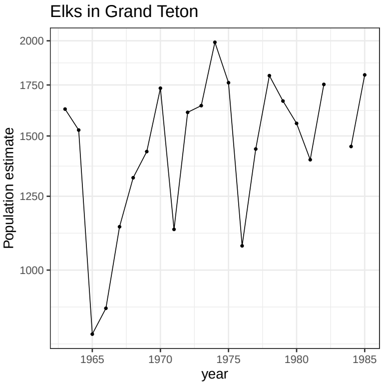
1985
year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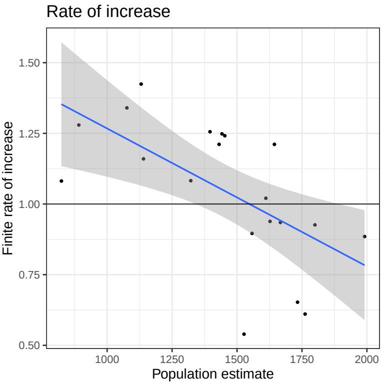

Rate of increase
Finite rate of increase
1.50
1.25
1.00
0.75
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1000
1250
1500
1750
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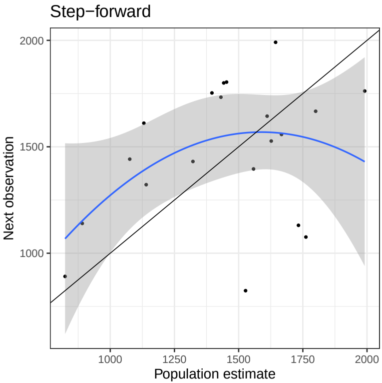

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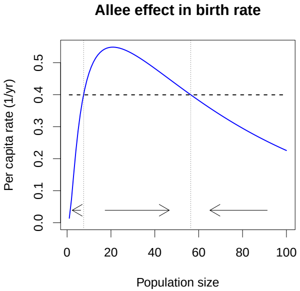

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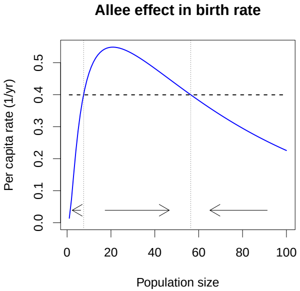

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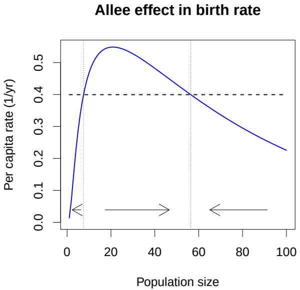

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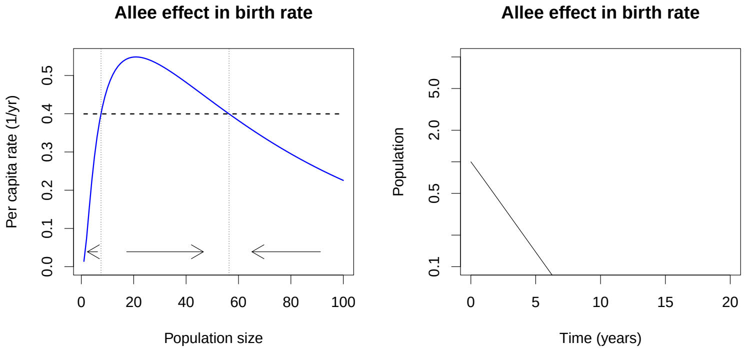

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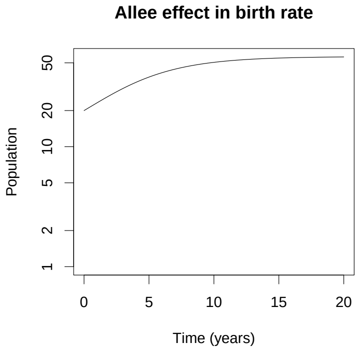

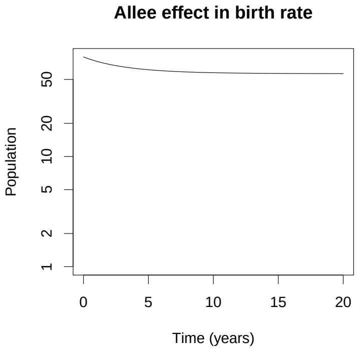

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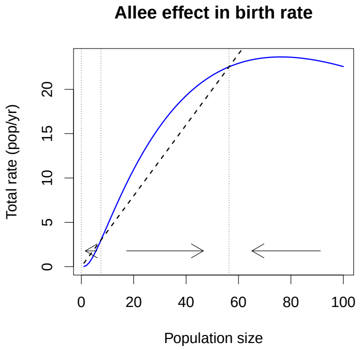

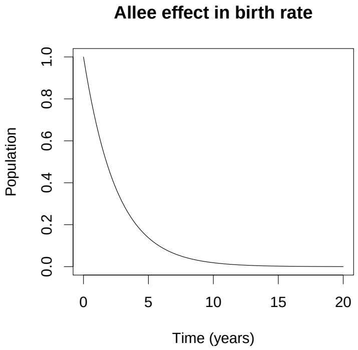

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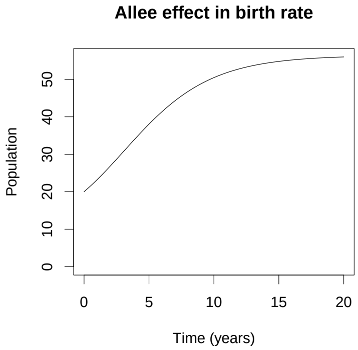
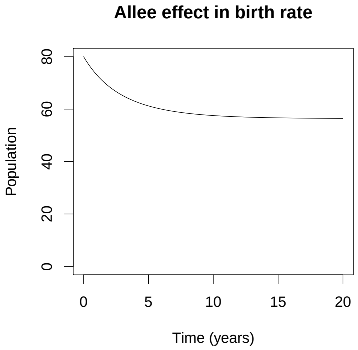
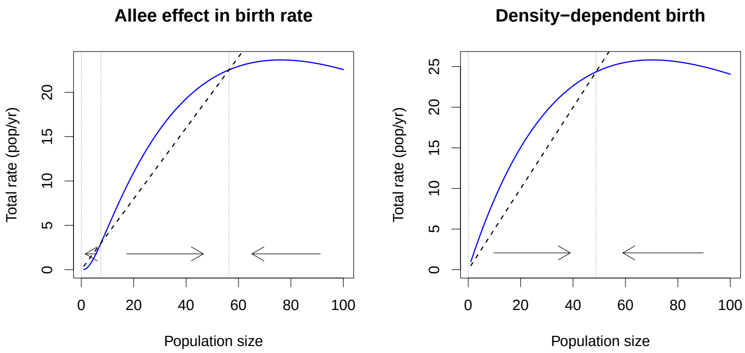




Population comparison



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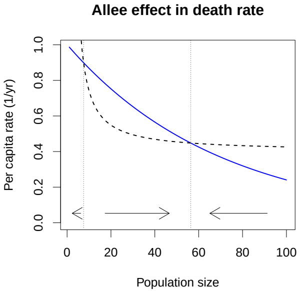

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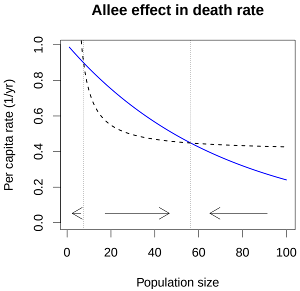

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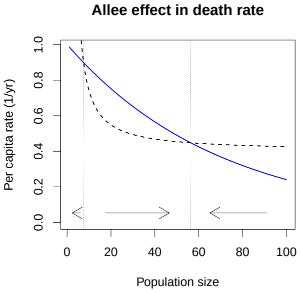

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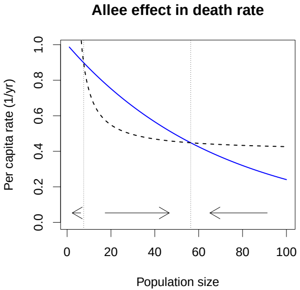

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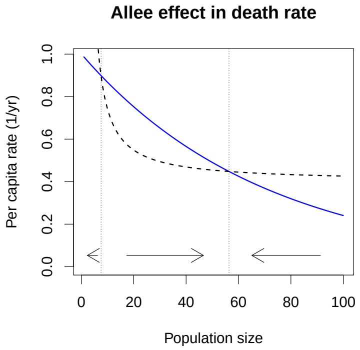

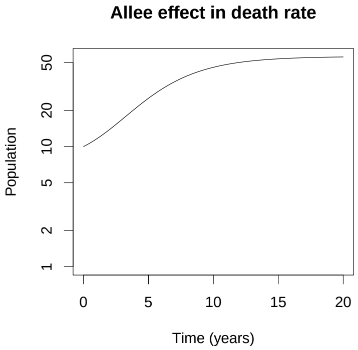

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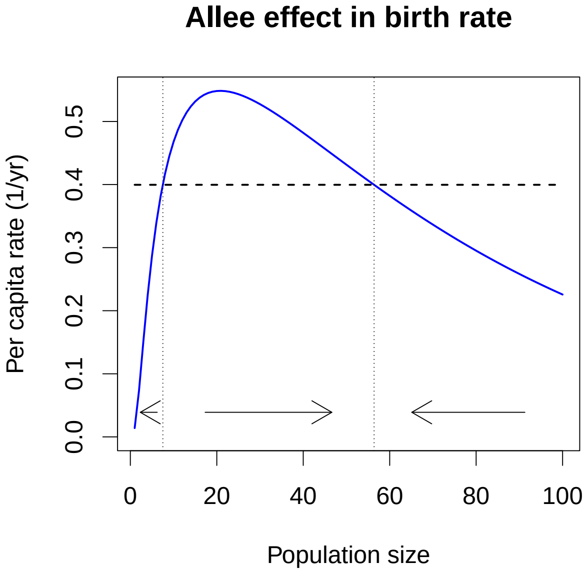

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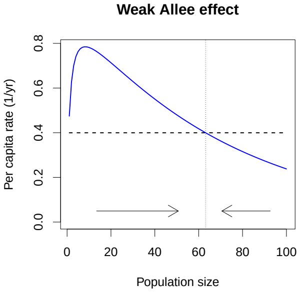

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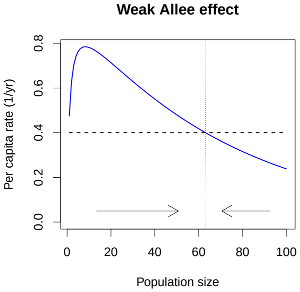

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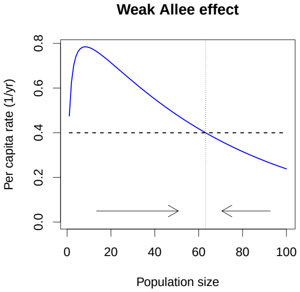

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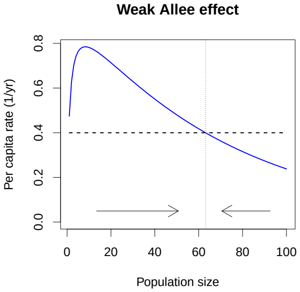

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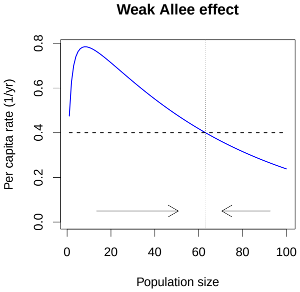

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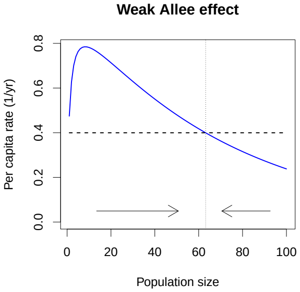

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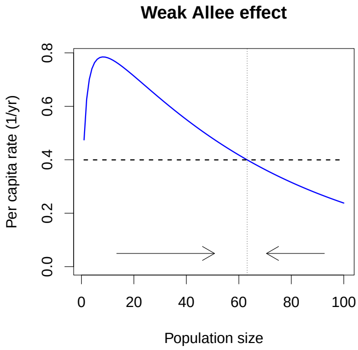

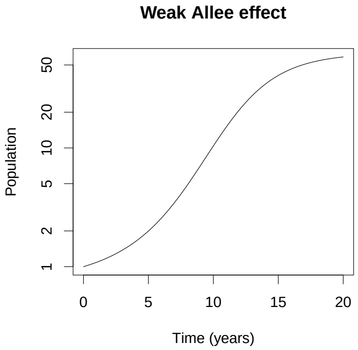

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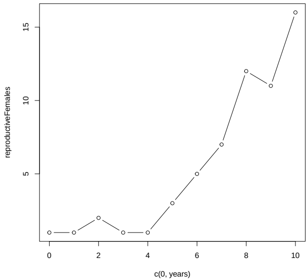

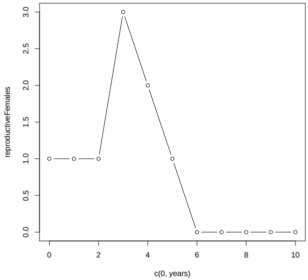

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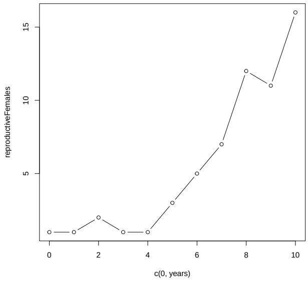

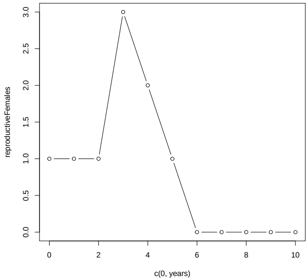

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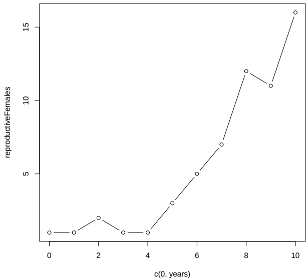

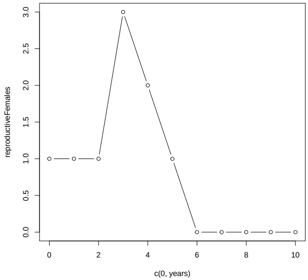

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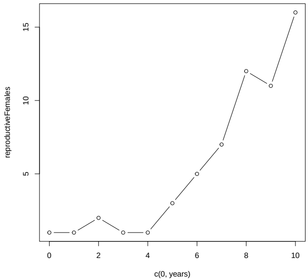

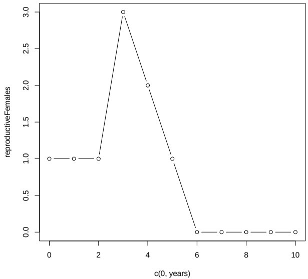

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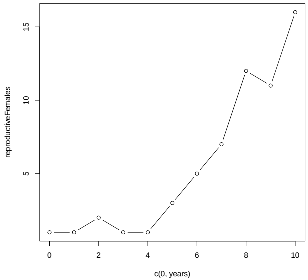

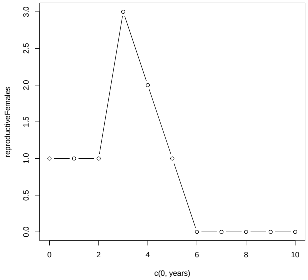

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