UNIT 2: Linear population models

Outline

Outline

Dynamical models

Tools to link scales

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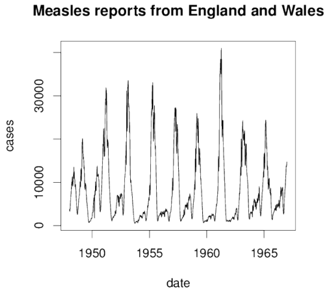

Dynamical models

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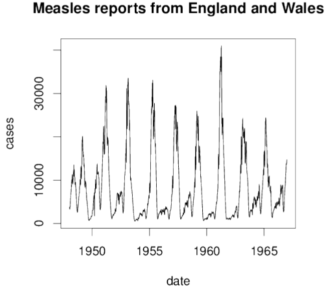

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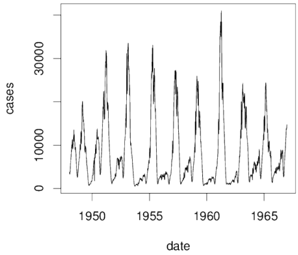

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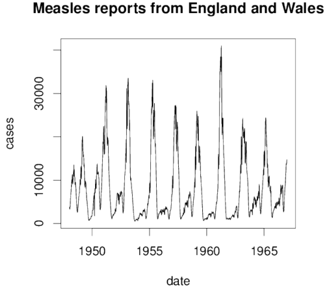

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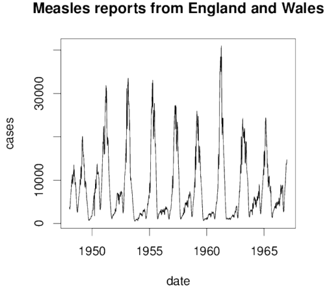

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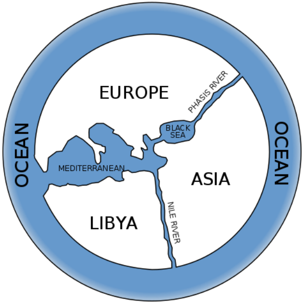

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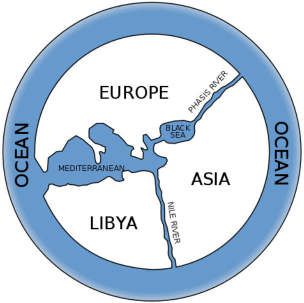

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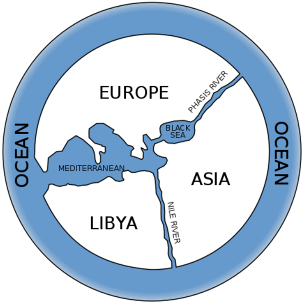

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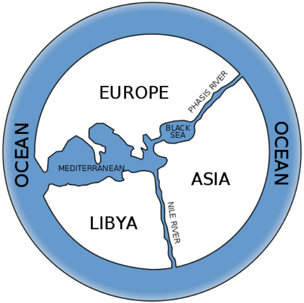

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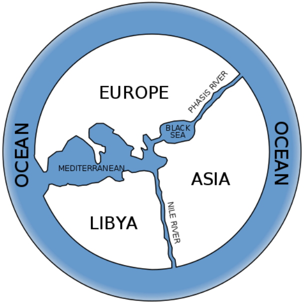

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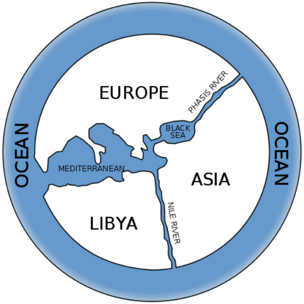

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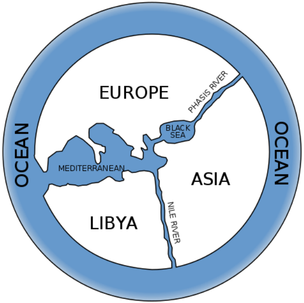

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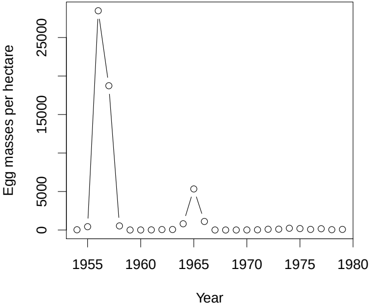

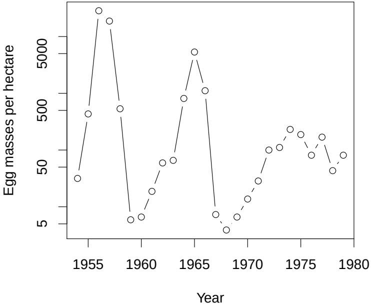

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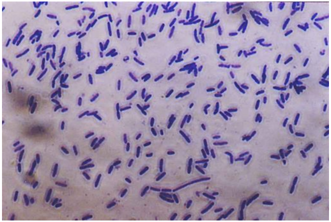

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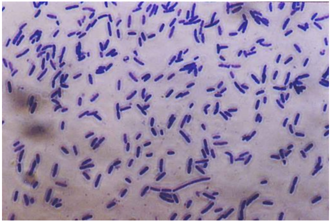

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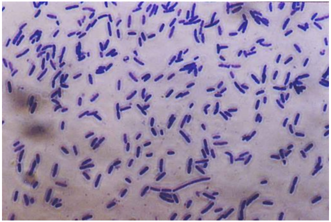

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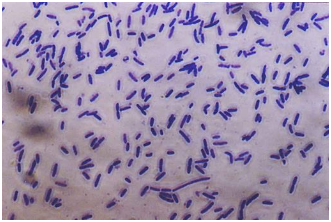

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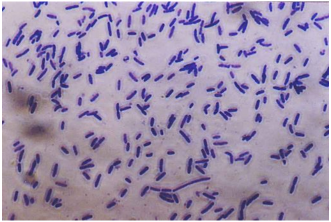

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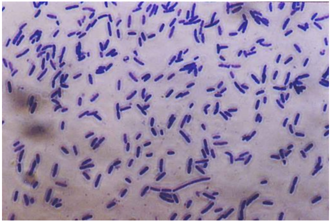

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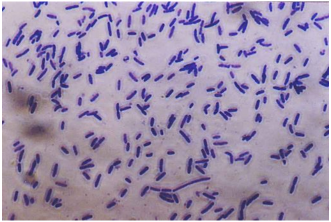

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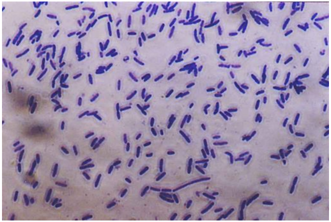

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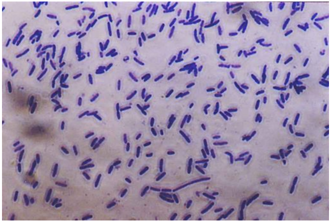

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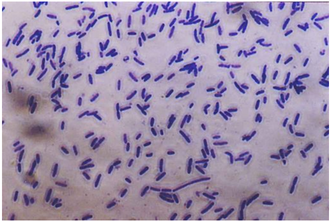

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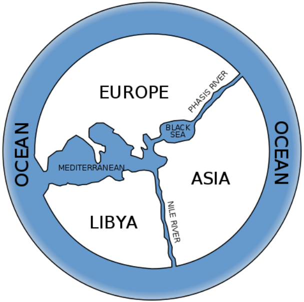

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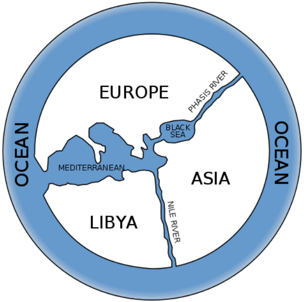

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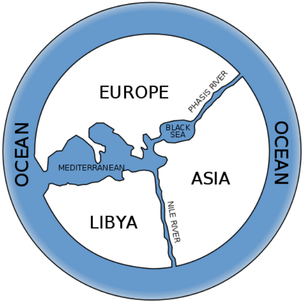

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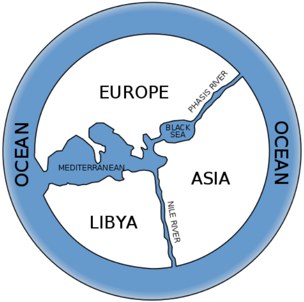

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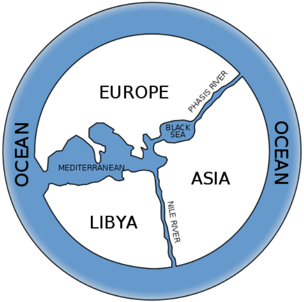

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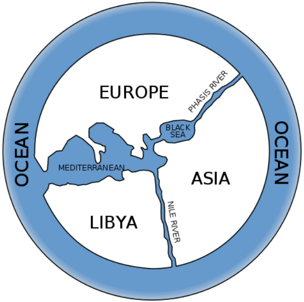

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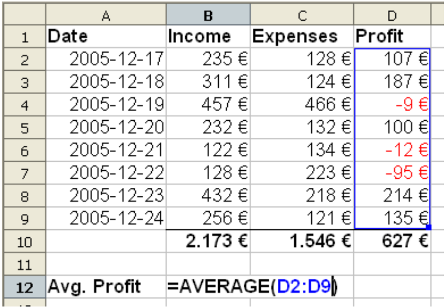

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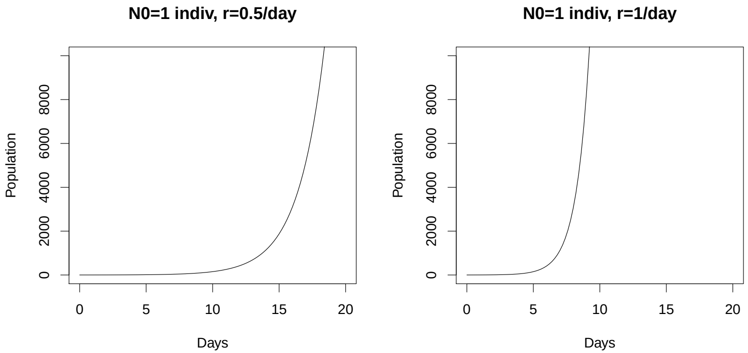
N0=1 indiv, r=0.5/day
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Population
0
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4000
6000
8000
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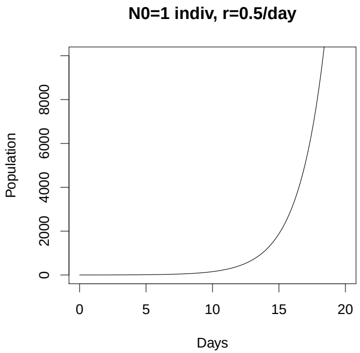

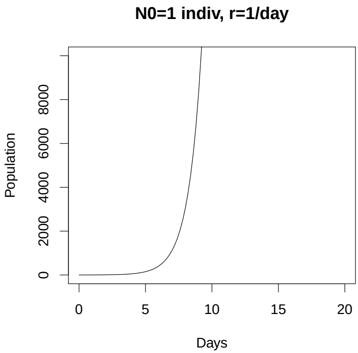

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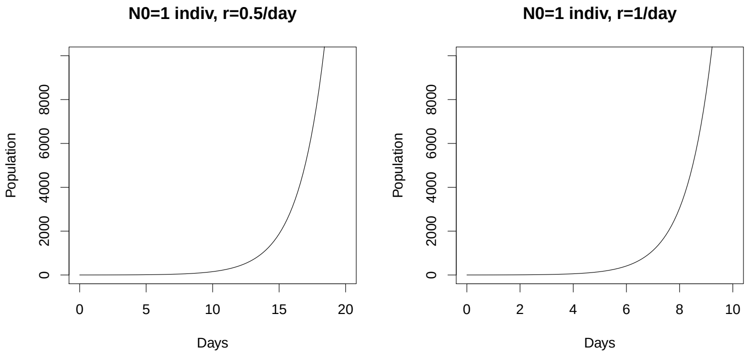

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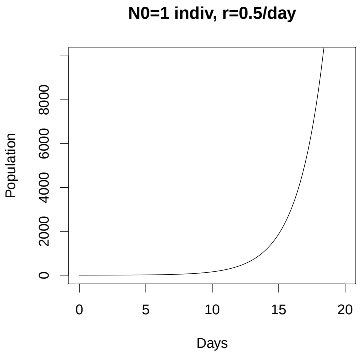

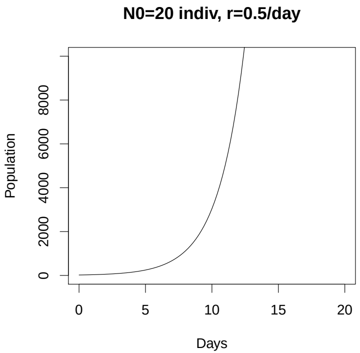

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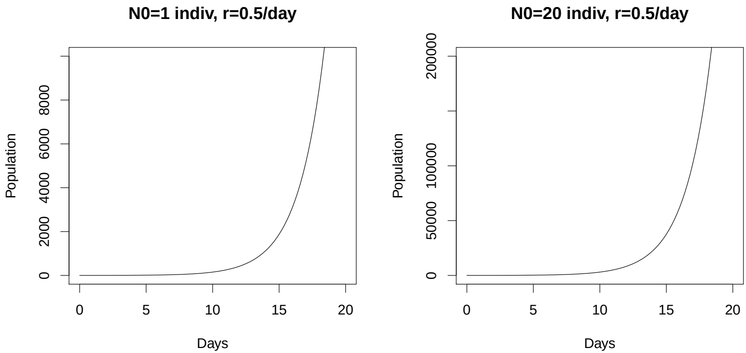

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    - * Roughly makes sense, and is also right
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