UNIT 3 Non-linear population models

Outline

Introduction
- Population Examples

Continuous-time regulation
- A simple, continuous-time model
- Simulating model behaviour
- Equilibria and time scales

Discrete-time regulation
- A simple, discrete-time model
- Simulating this system
- Interpreting complex behaviour

Delayed regulation

Small populations and stochasticity
- Allee effects
- Stochastic effects

Introduction

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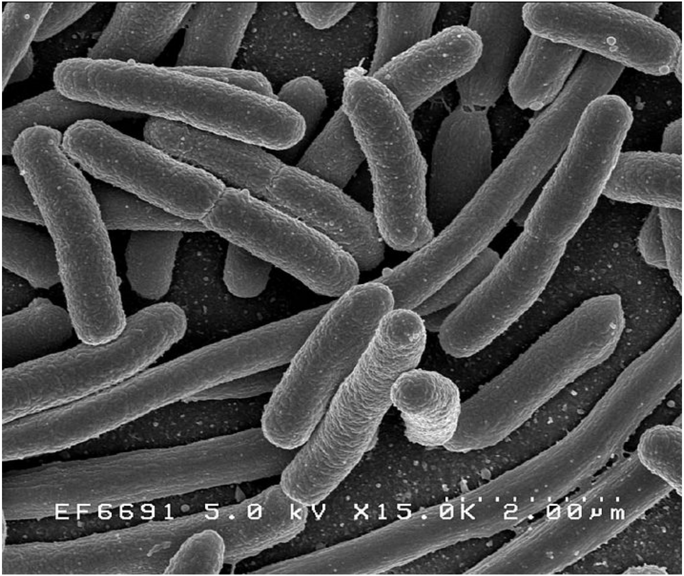
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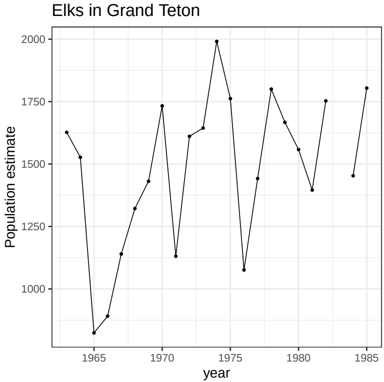
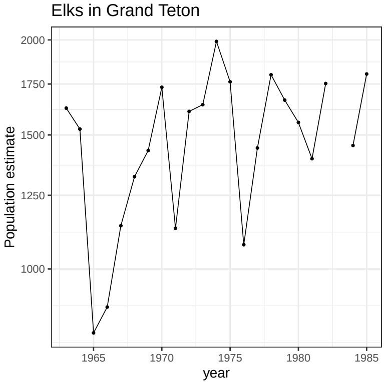
(preview)

Elk


Elks in Grand Teton
2000
1750
1500
1250
1000
Population estimate
1965
1970
1975
1980
1985
year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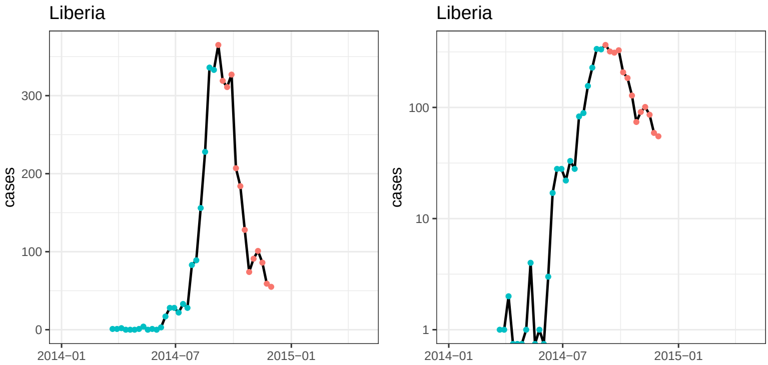
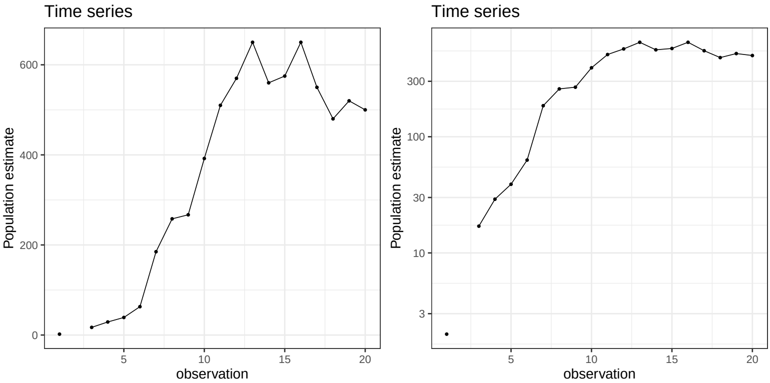
Ebola

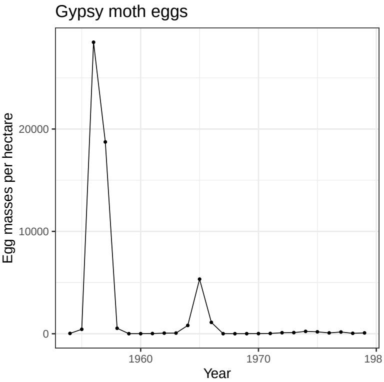


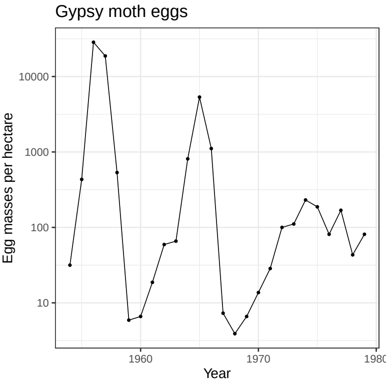
Liberia
cases
300
200
100
0
2014−01
2014−07
2015−01
Liberia
cases
100
10
1
2014−01
2014−07
2015−01

Paramecia (preview)



Gypsy moths (preview)





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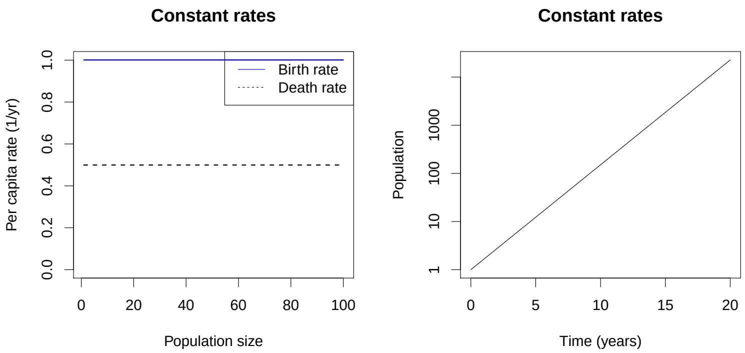

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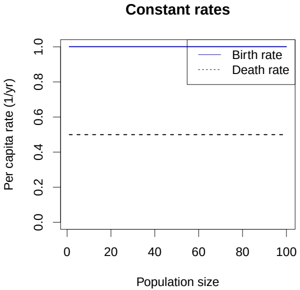

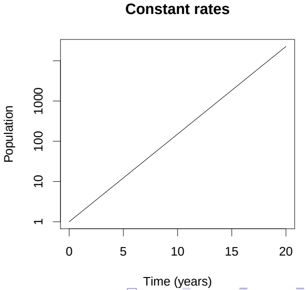

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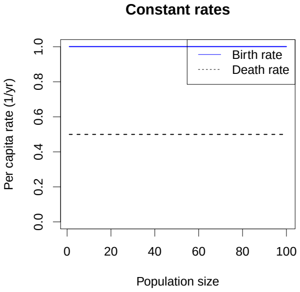

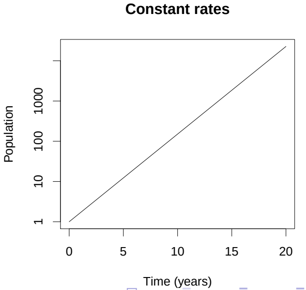

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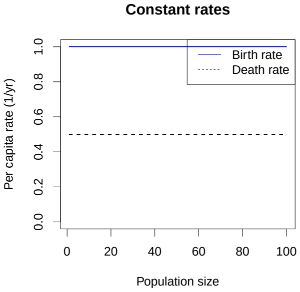

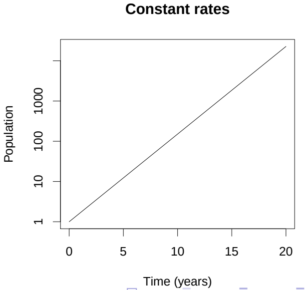

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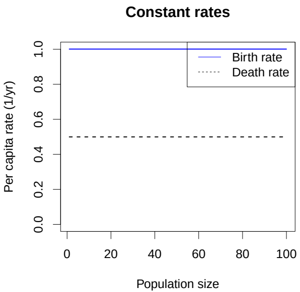

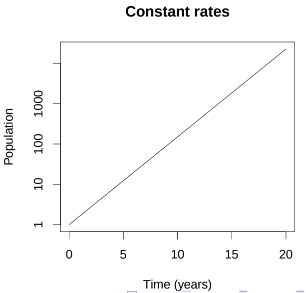

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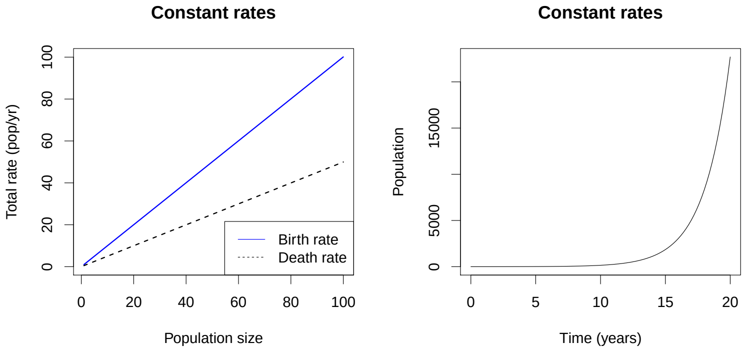

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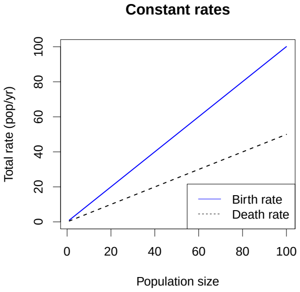

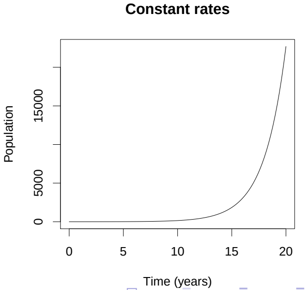

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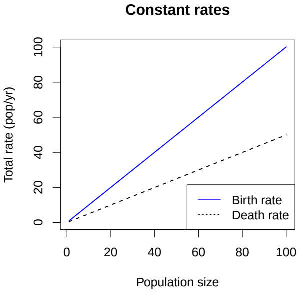

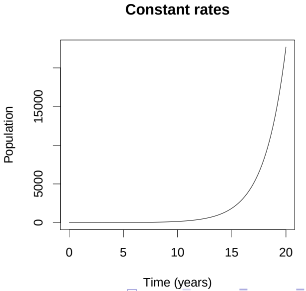

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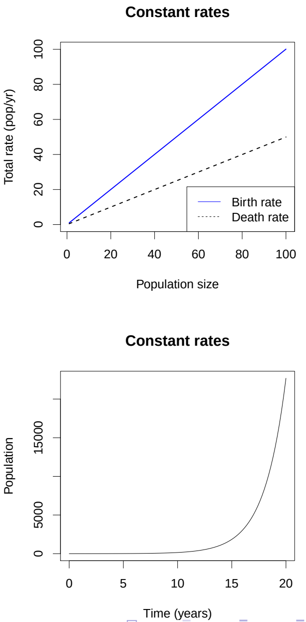

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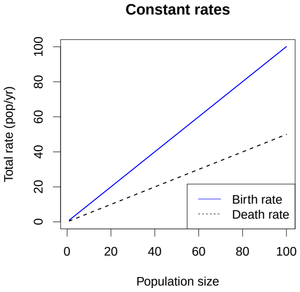

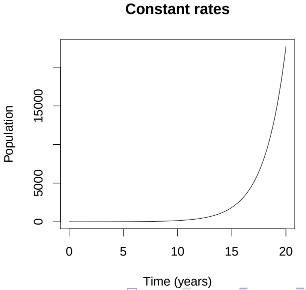

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- **Recruitment** is when an organism moves from one life stage to another:
  - Seed $\rightarrow$seedling $\rightarrow$sapling $\rightarrow$tree
  - Egg $\rightarrow$larva $\rightarrow$pupa $\rightarrow$moth
- In simple continuous-time population models, recruitment is included in birth:
  - b is the rate at which adults produce new adults; or seeds produce new seeds – we have to "close the loop"

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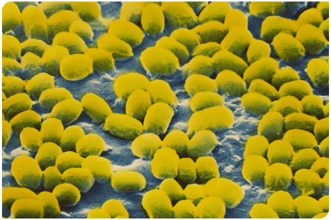

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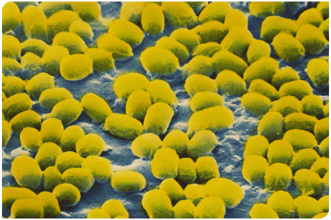

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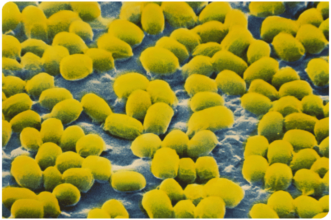

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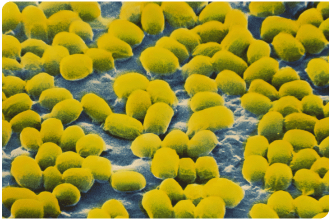

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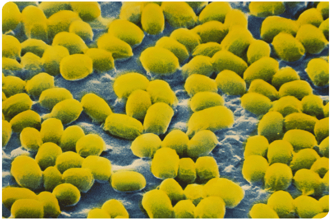

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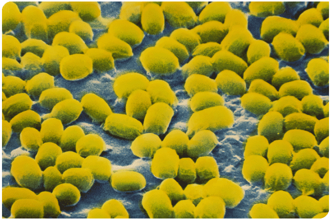

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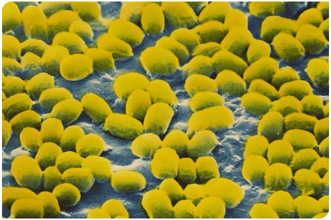

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Outline

Introduction
- Population Examples

Continuous-time regulation
- A simple, continuous-time model
- Simulating model behaviour
- Equilibria and time scales

Discrete-time regulation
- A simple, discrete-time model
- Simulating this system
- Interpreting complex behaviour

Delayed regulation

Small populations and stochasticity
- Allee effects
- Stochastic effects

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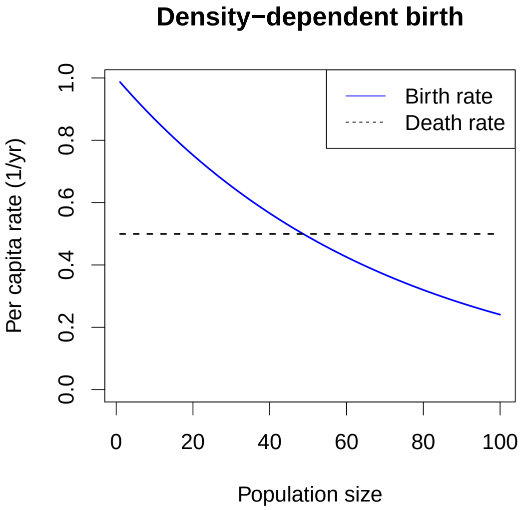

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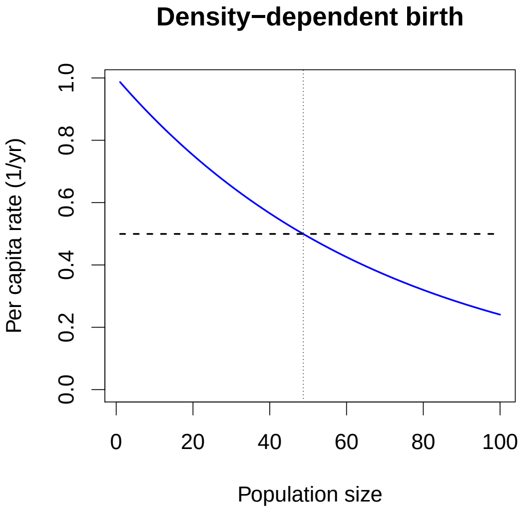

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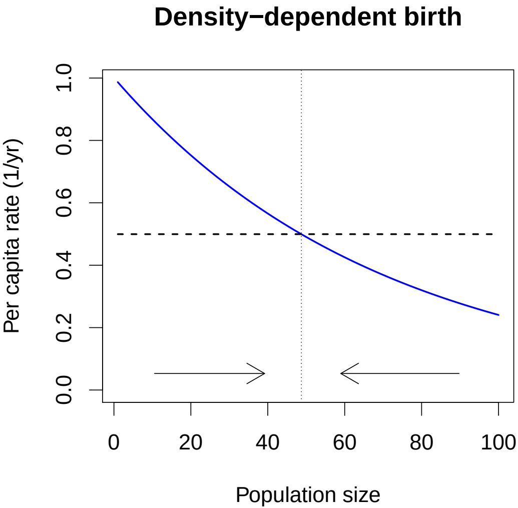

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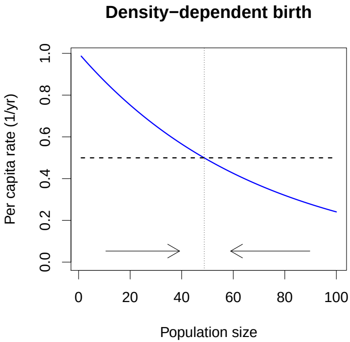

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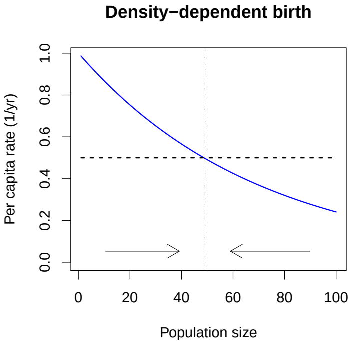

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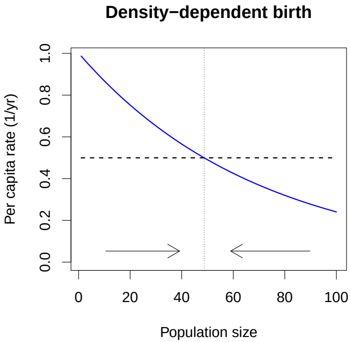

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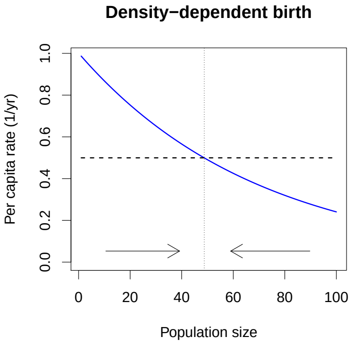

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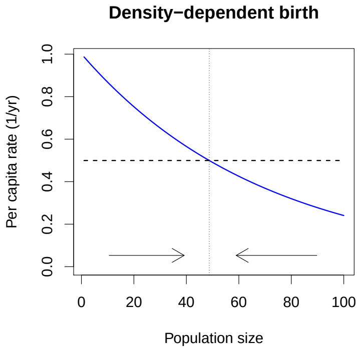
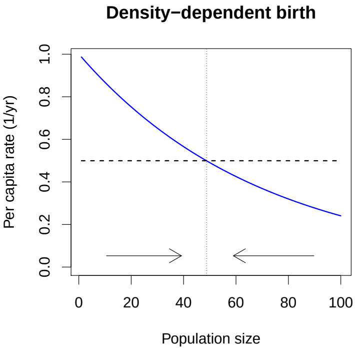
Low starting population example



High starting population example



Examples

Density−dependent birth
Population
Time (years)
0
5
10
15
20
1
2
5
10
20
50

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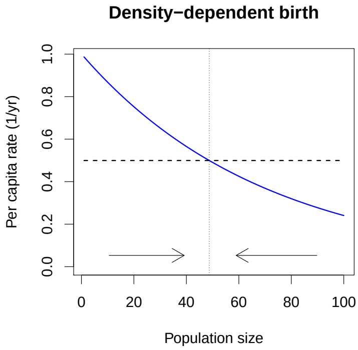
What will this model do? (repeat)



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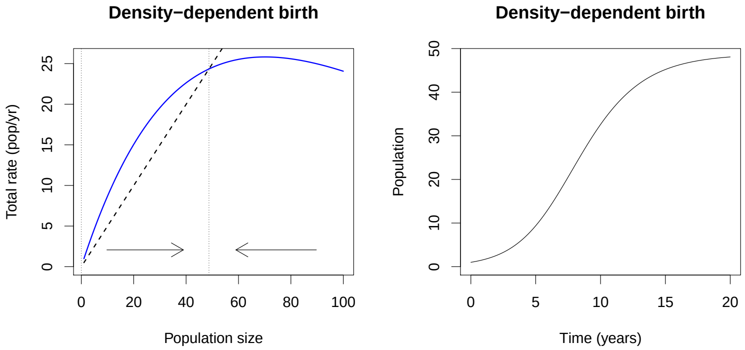

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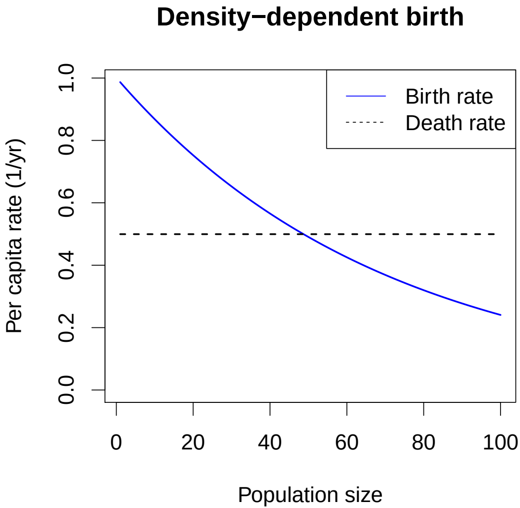

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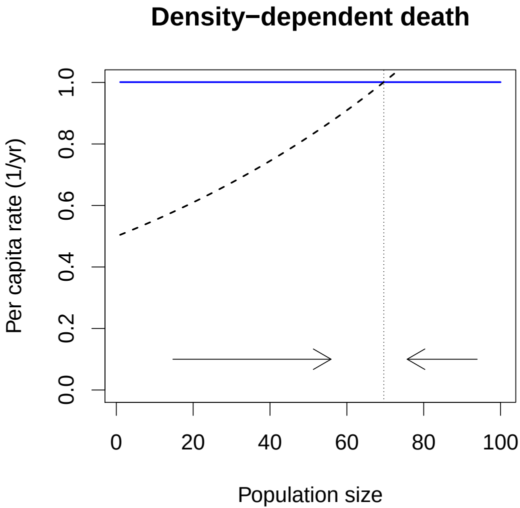

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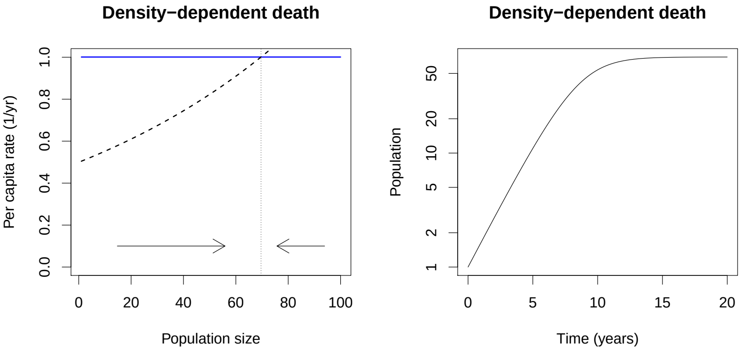

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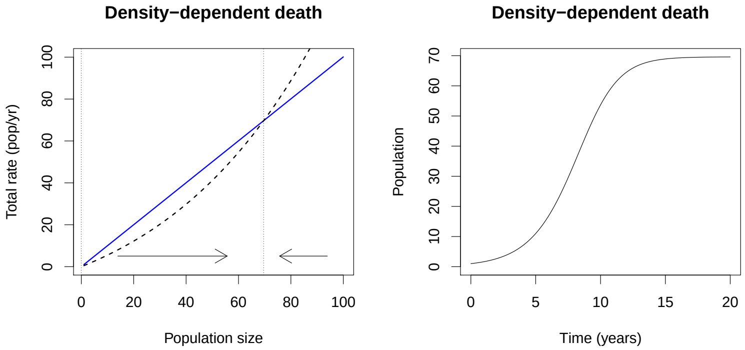

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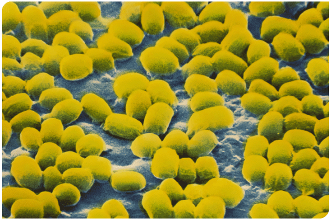

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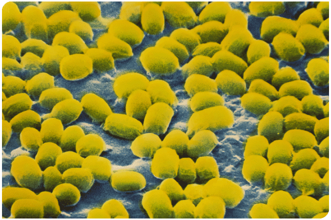

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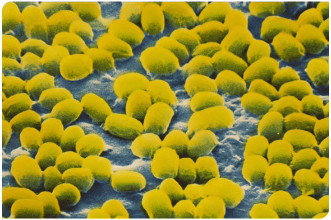

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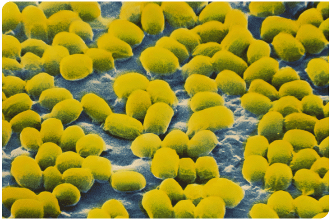

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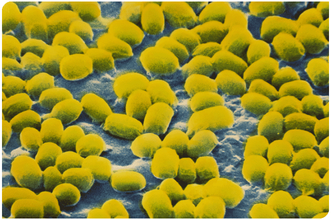

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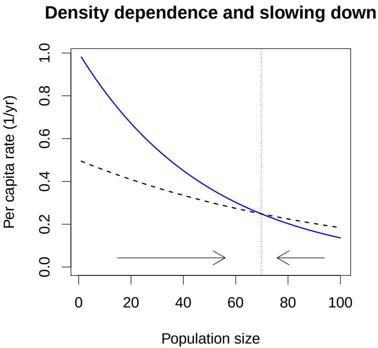

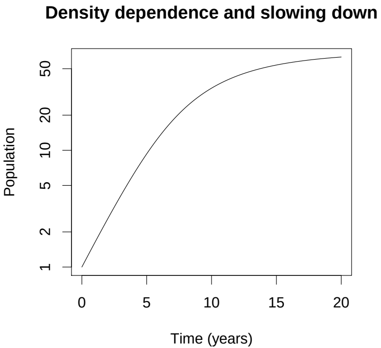

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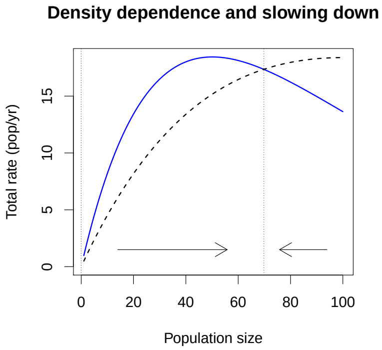

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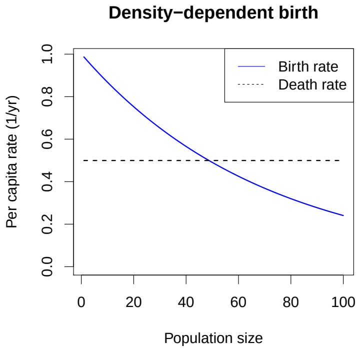

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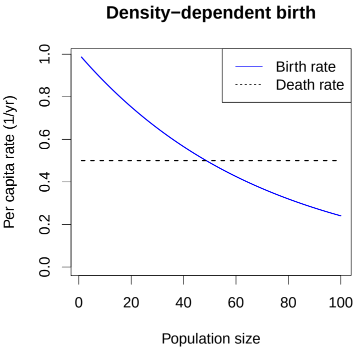

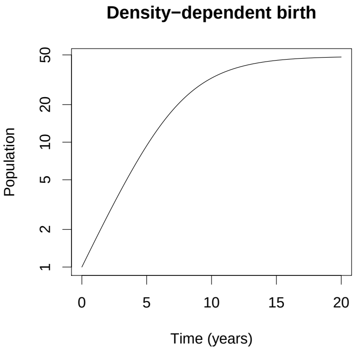

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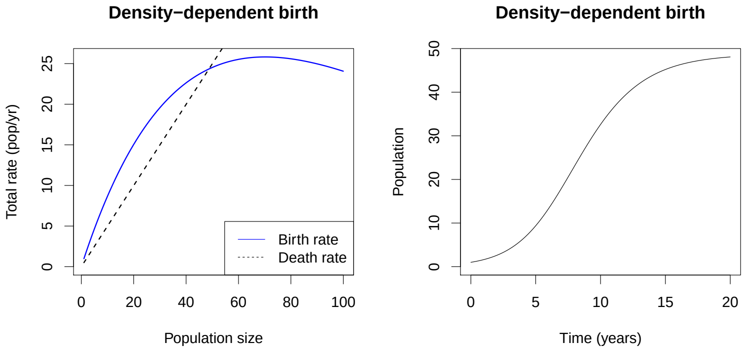

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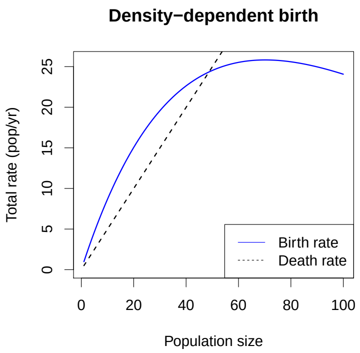

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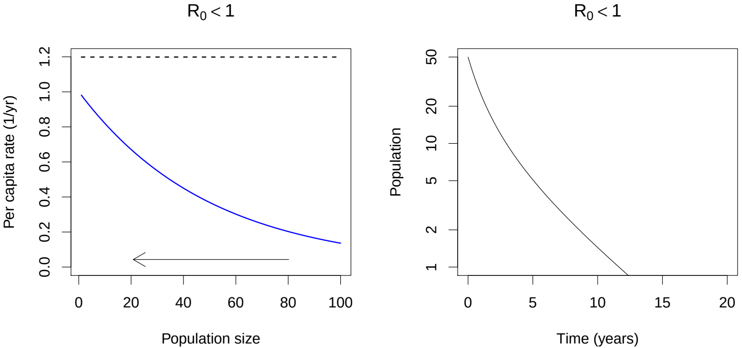

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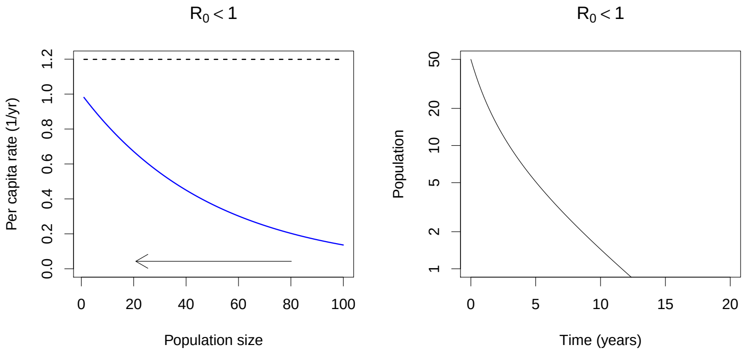

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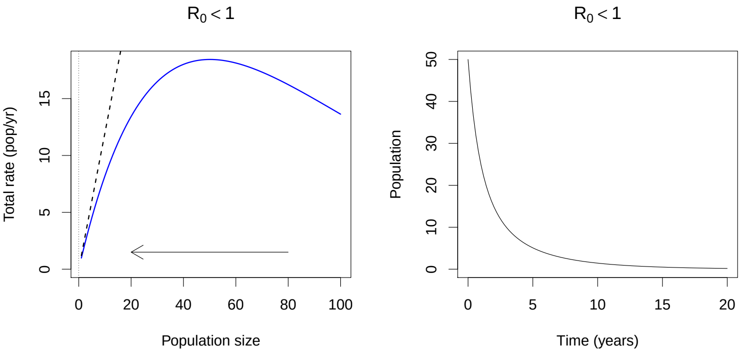

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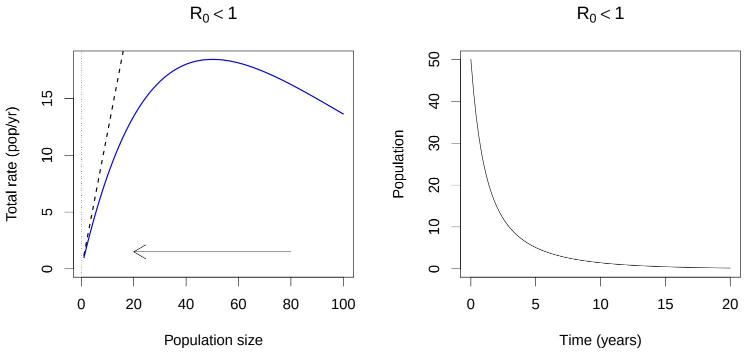

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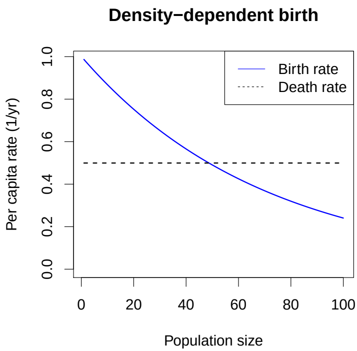

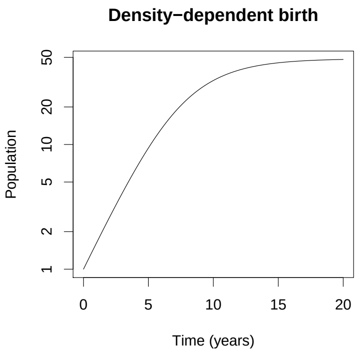

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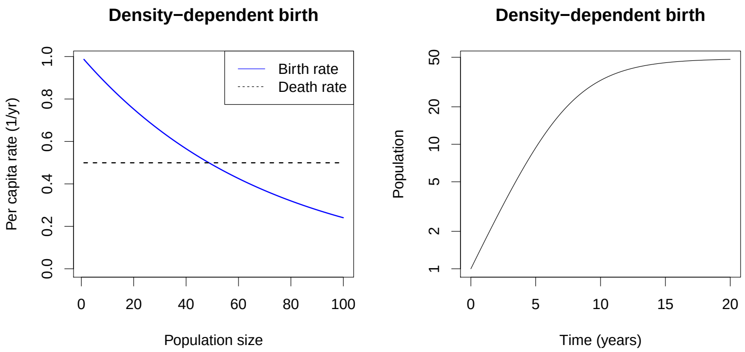

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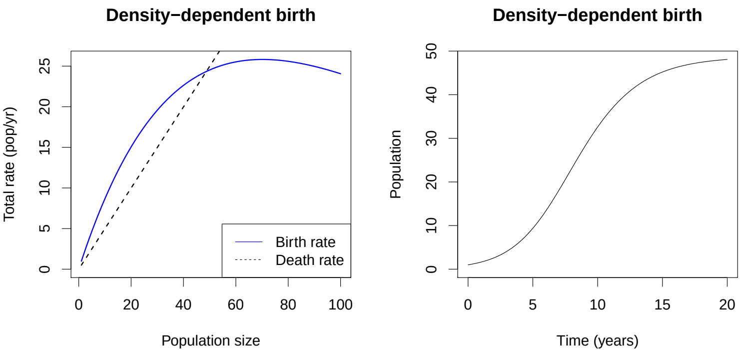

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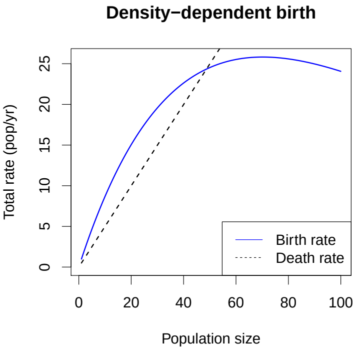

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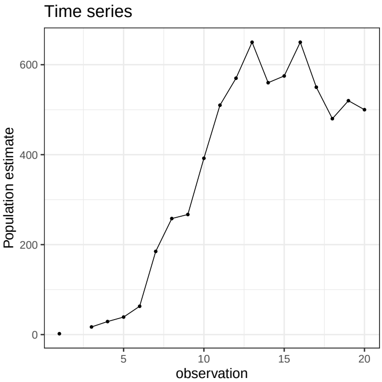
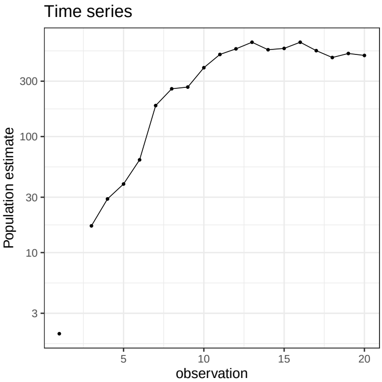


Time series
Population estimate
600
400
200
0
5
10
15
20
observation



Time series
Population estimate
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30
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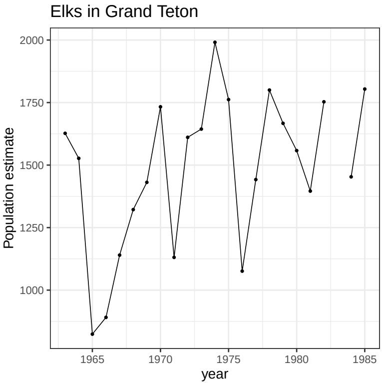
Elk



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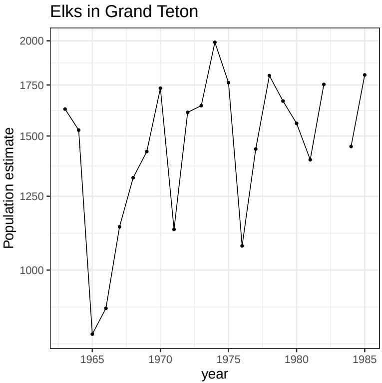

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Equilibria and time scales

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Interpreting complex behaviour

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Small populations and stochasticity
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- A simple, discrete-time model
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- Interpreting complex behaviour

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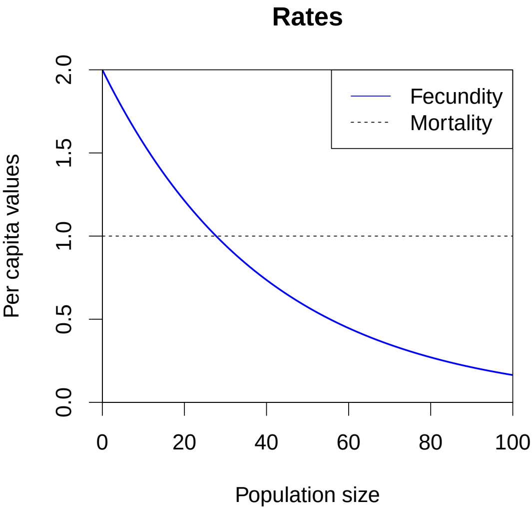

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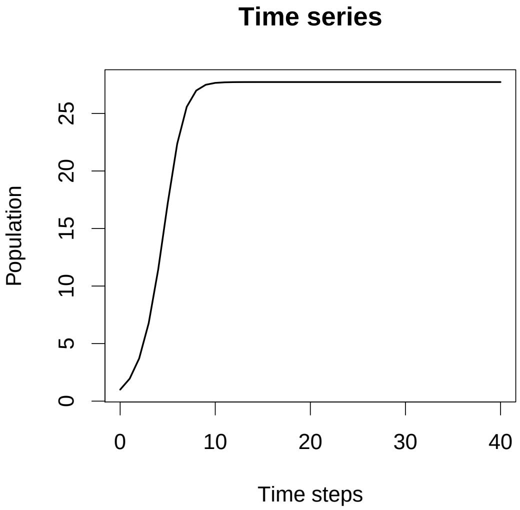
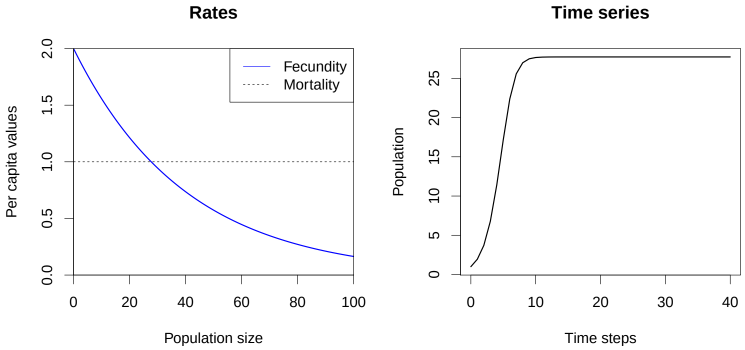


We expect simple dynamics



What dynamics do we get?

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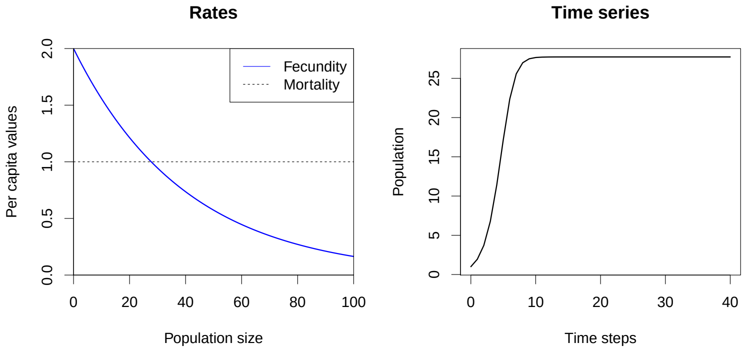

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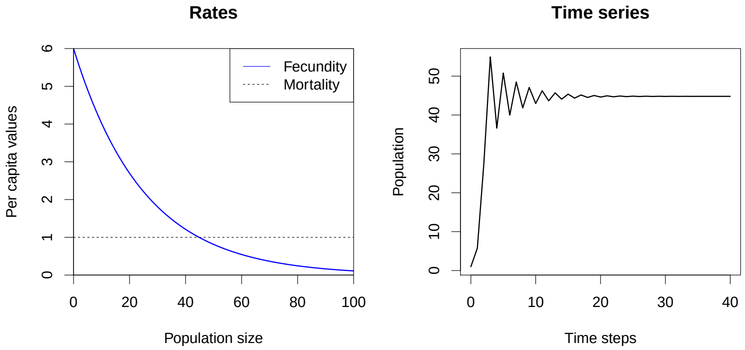

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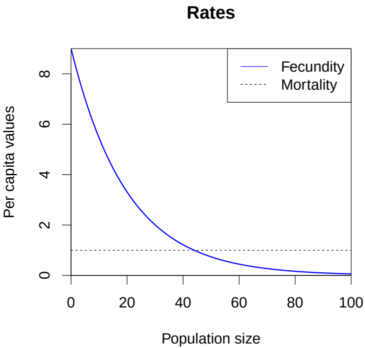
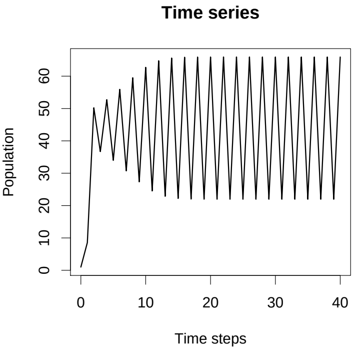
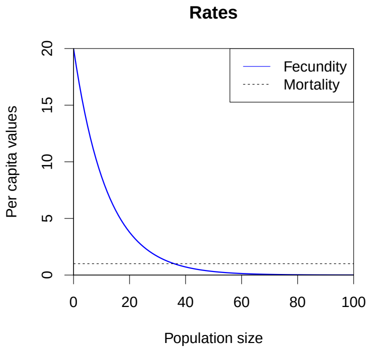




Lots of other behaviours



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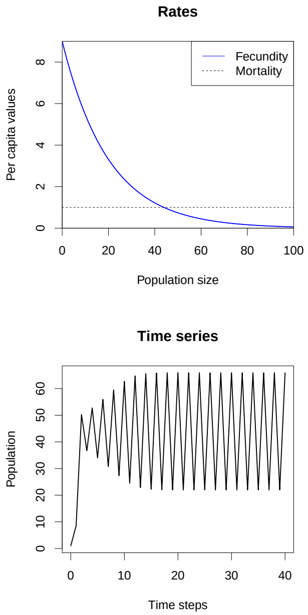

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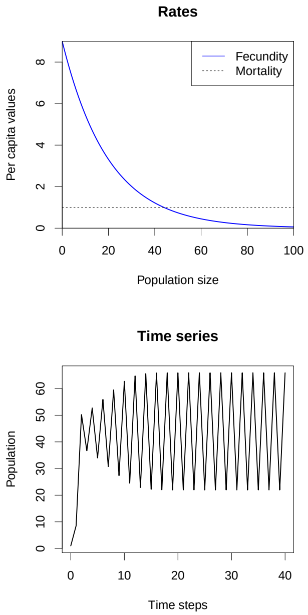

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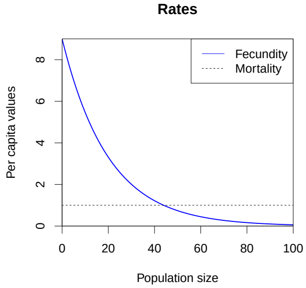

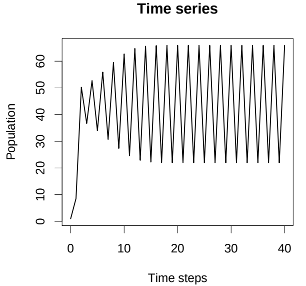

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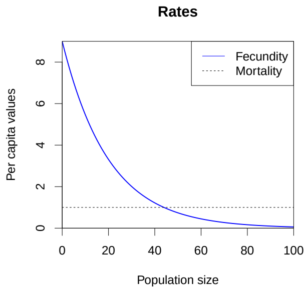

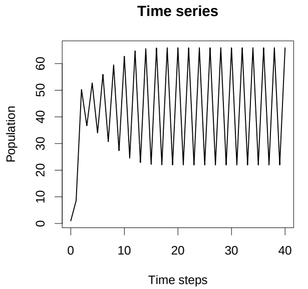

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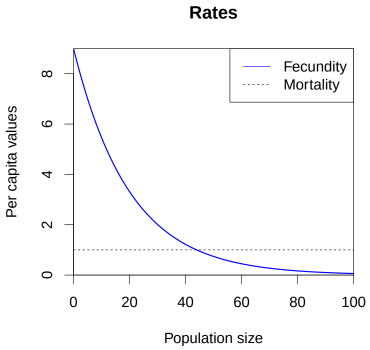

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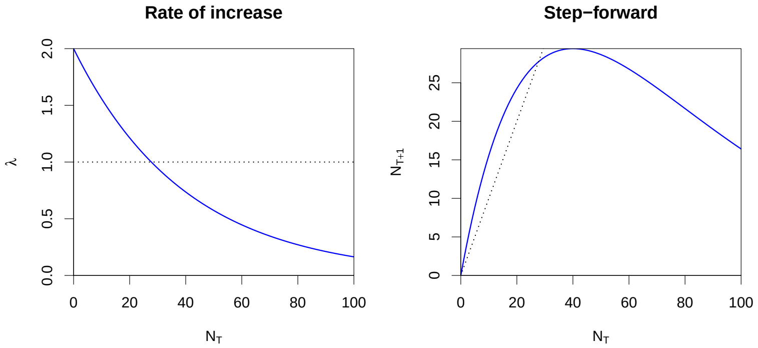

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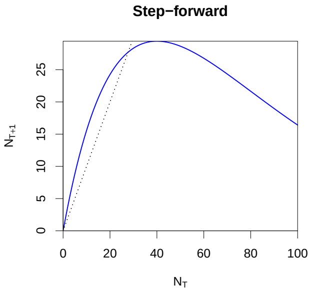

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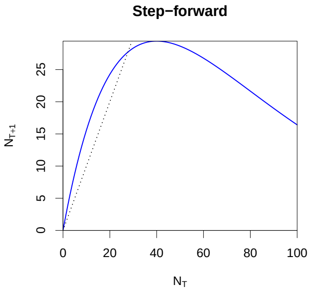

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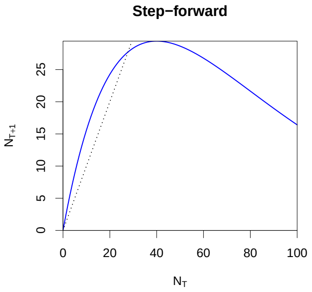

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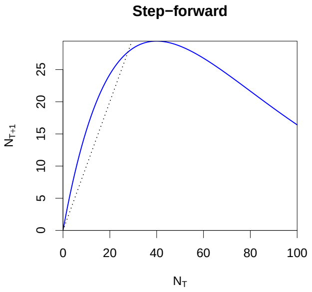

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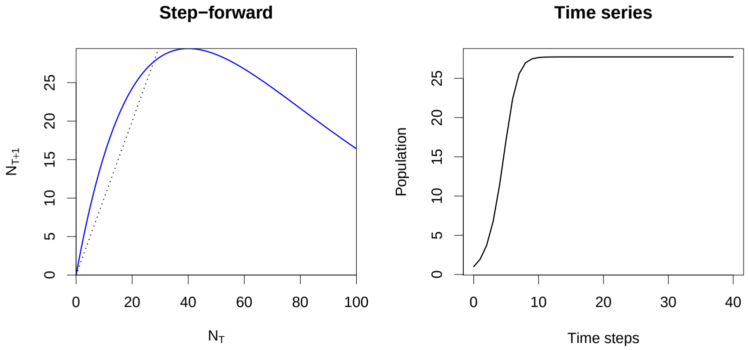

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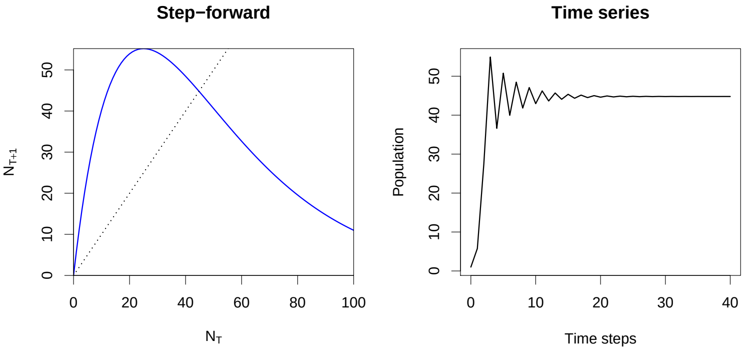

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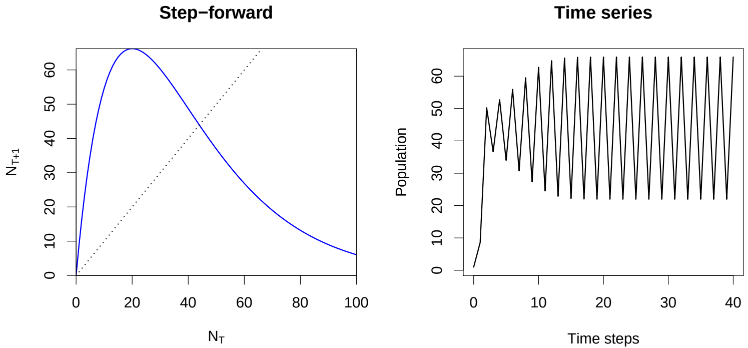

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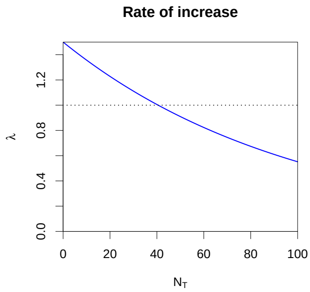

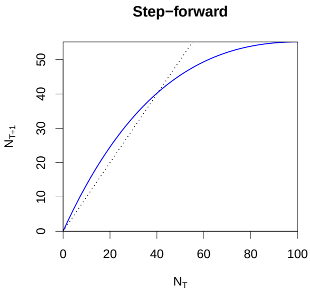

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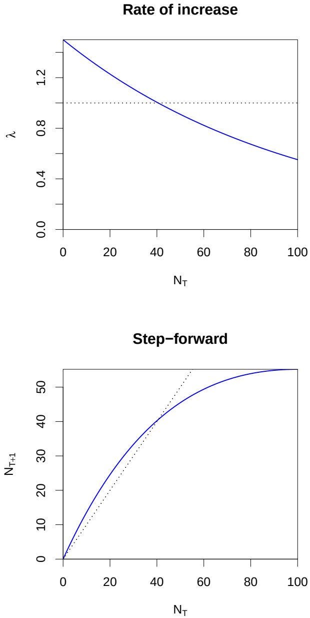

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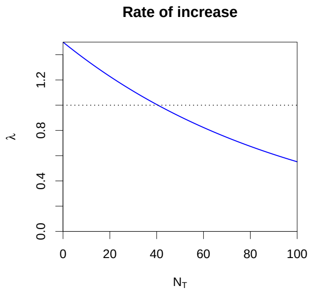

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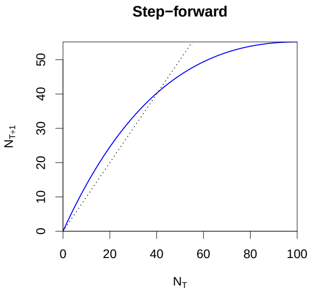

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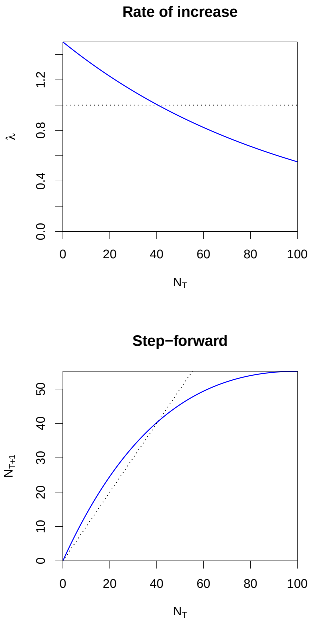

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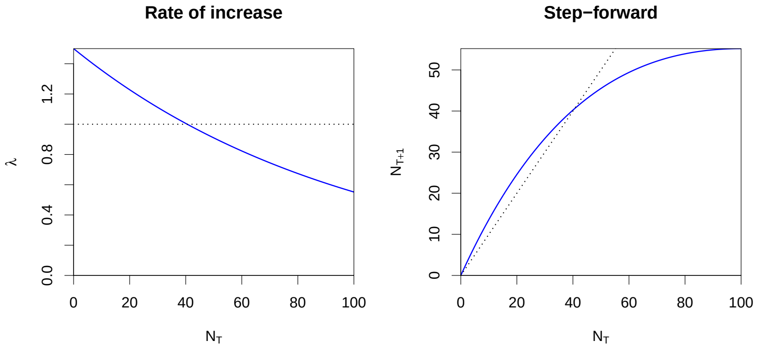

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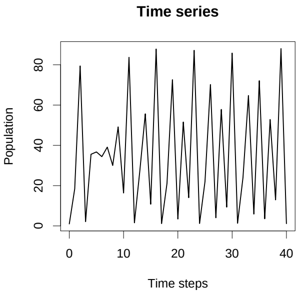

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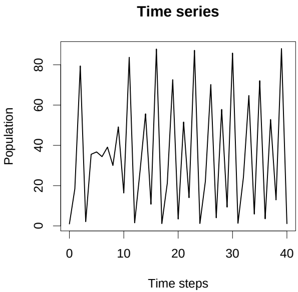

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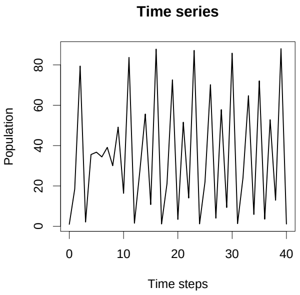

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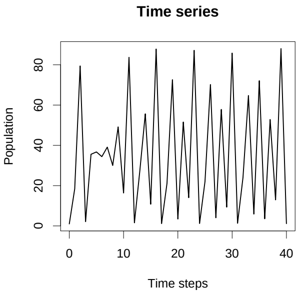
- People always tend to look for specific reasons, but sometimes the changes we observe are just natural dynamics



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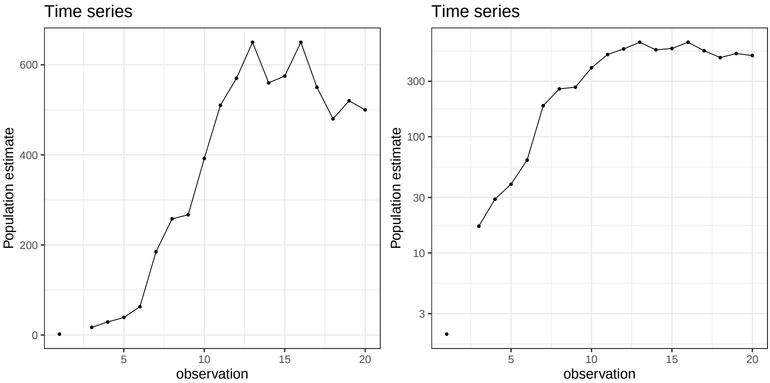
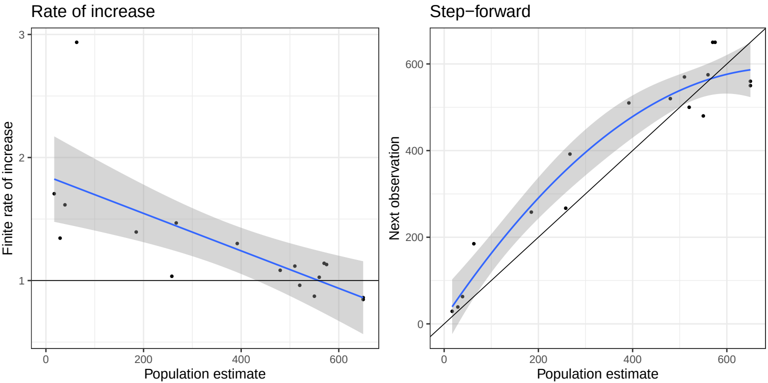
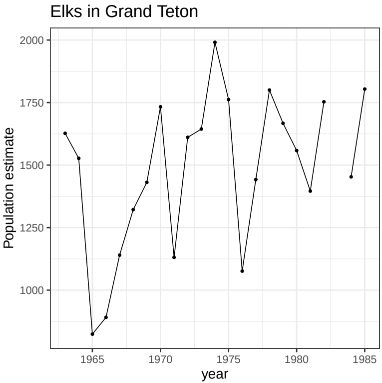


Paramecia



Rate of increase
Finite rate of increase
3
2
1
0
200
400
600
Population estimate
Step−forward
Next observation
600
400
200
0
0
200
400
600
Population estimate

Elk (repeat)


Elks in Grand Teton
Population estimate
2000
1750
1500
1250
1000
1965
1970
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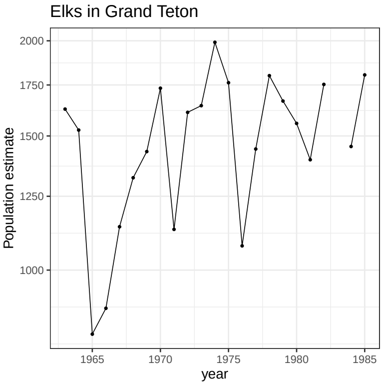
1985
year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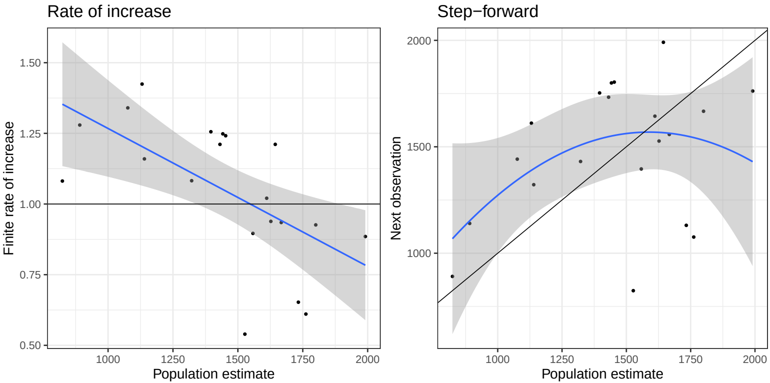
Rate of increase
Finite rate of increase
1.50
1.25
1.00
0.75
0.50
1000
1250
1500
1750
2000
Population estimate
Step−forward
Next observation
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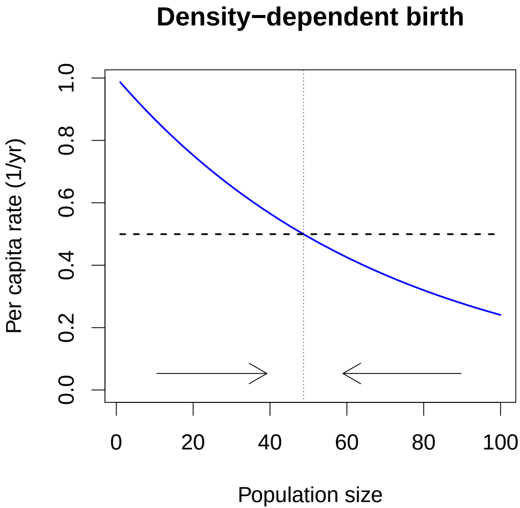
Arrows with time delay



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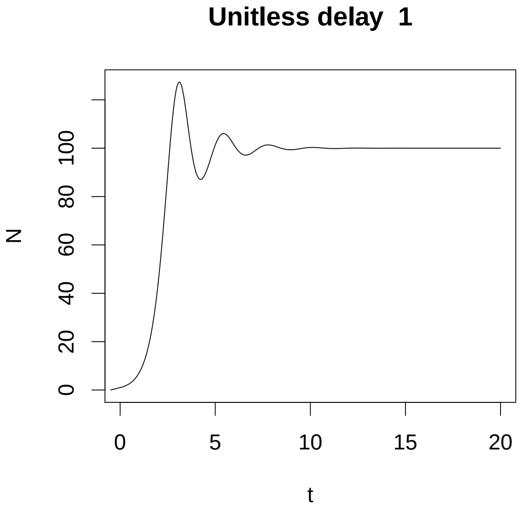

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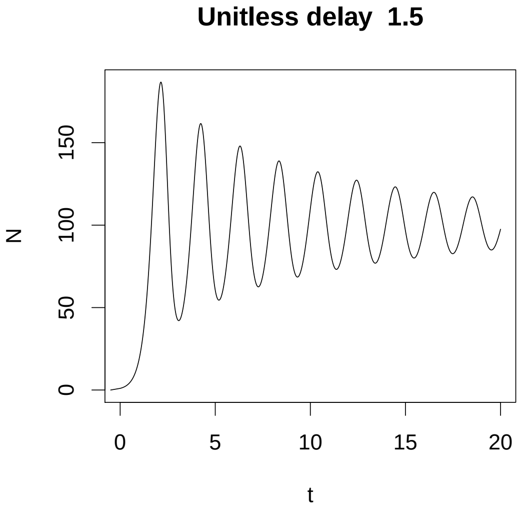

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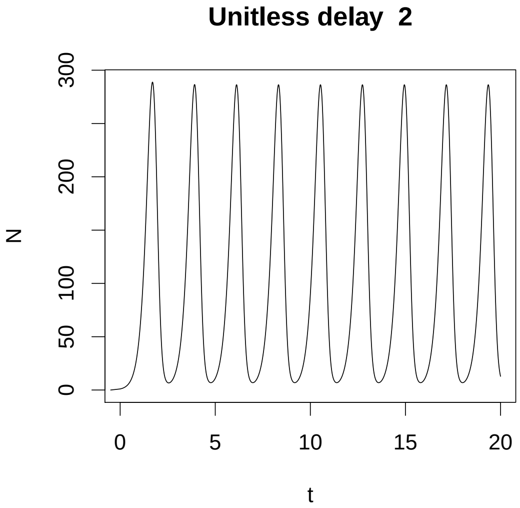

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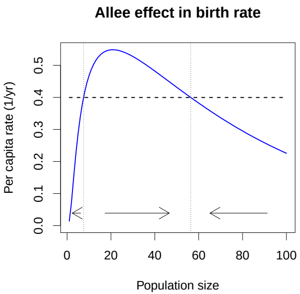

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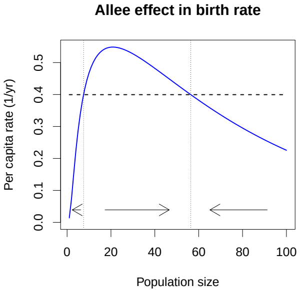

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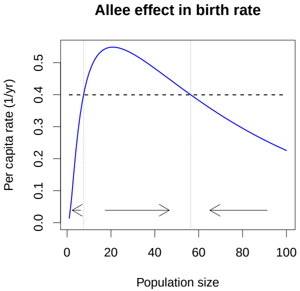

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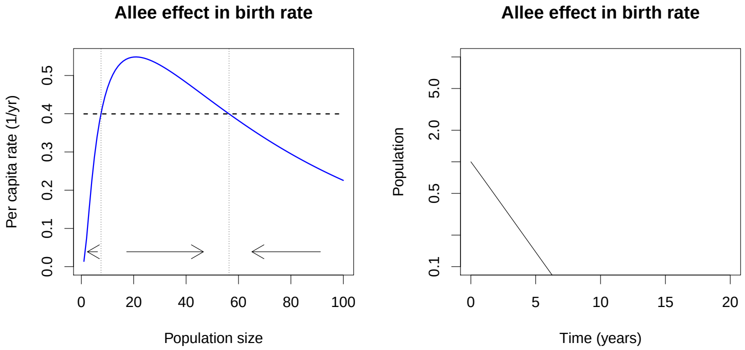

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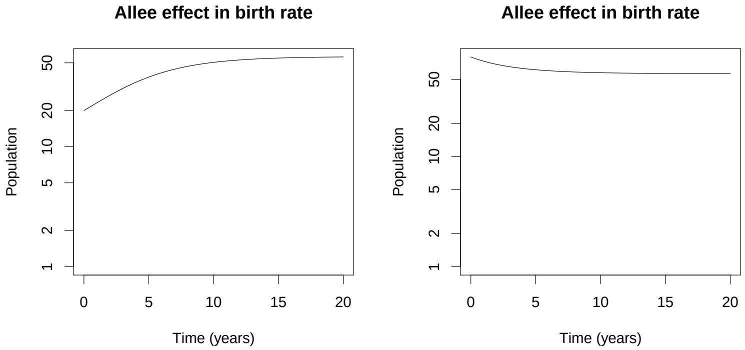

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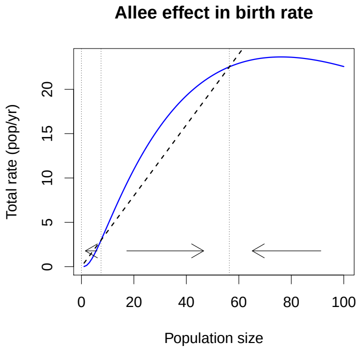

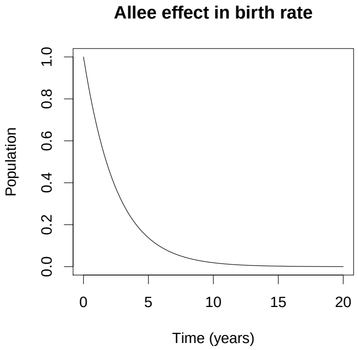

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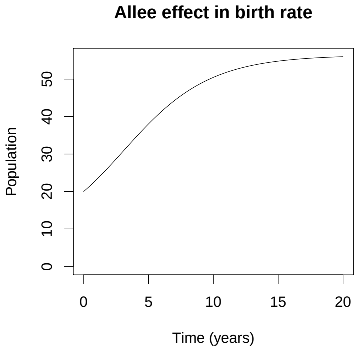
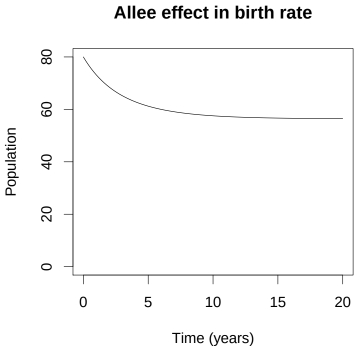
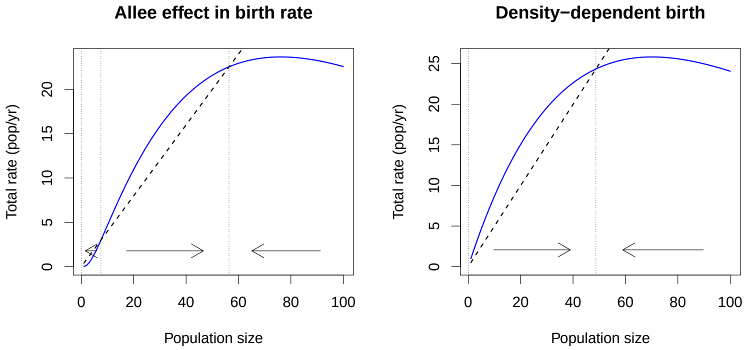




Population comparison (repeat)



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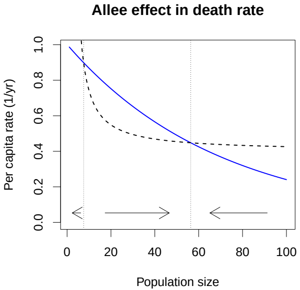

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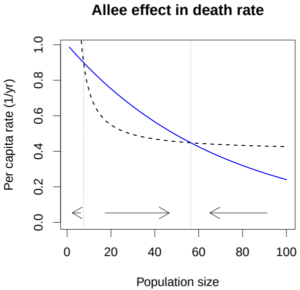

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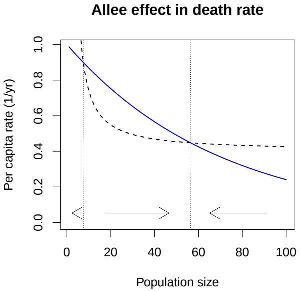

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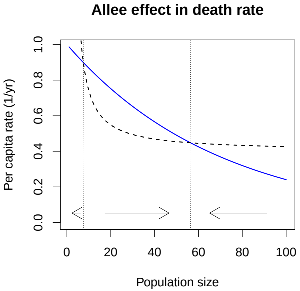

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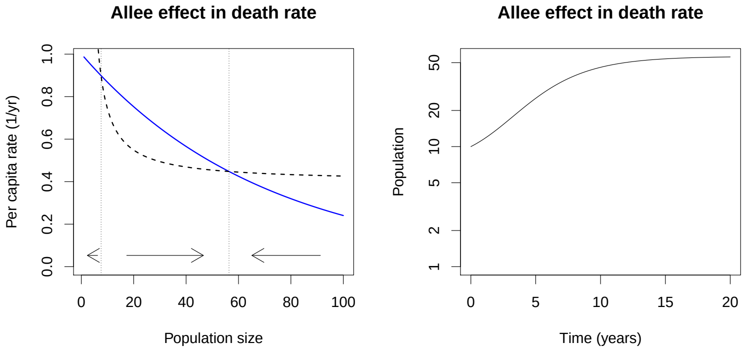

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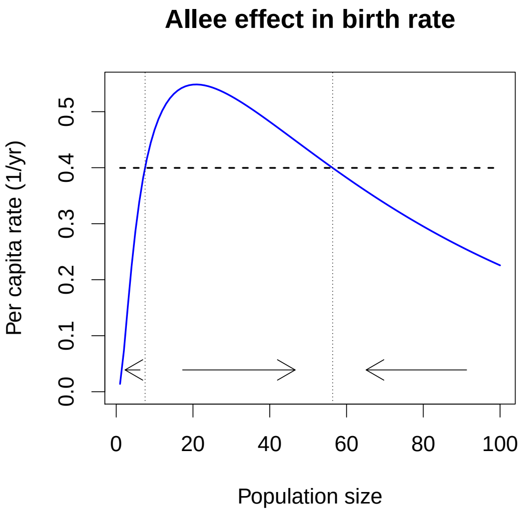

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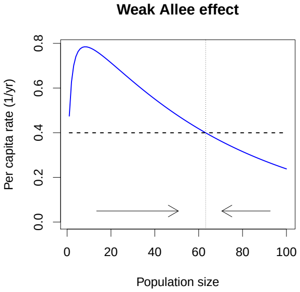

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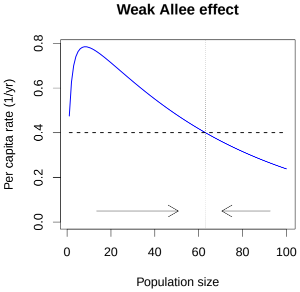

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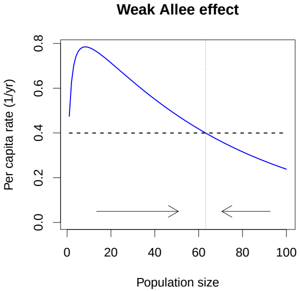

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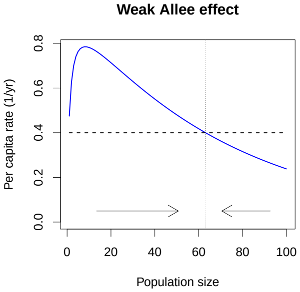

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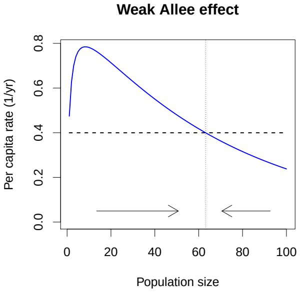

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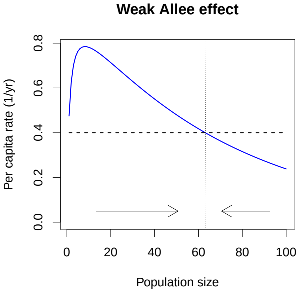

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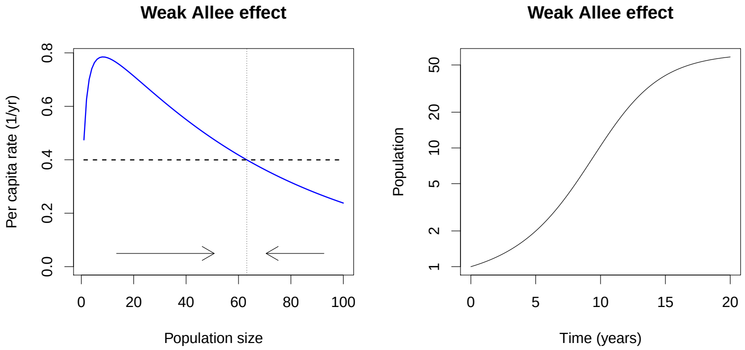

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Outline

Introduction
- Population Examples

Continuous-time regulation
- A simple, continuous-time model
- Simulating model behaviour
- Equilibria and time scales

Discrete-time regulation
- A simple, discrete-time model
- Simulating this system
- Interpreting complex behaviour

Delayed regulation

Small populations and stochasticity
- Allee effects
- Stochastic effects

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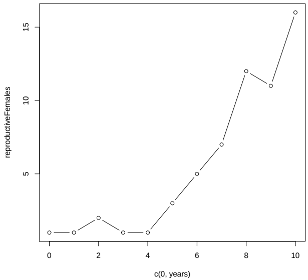

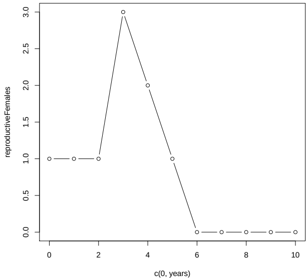

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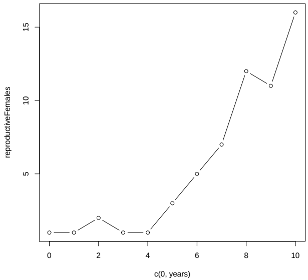

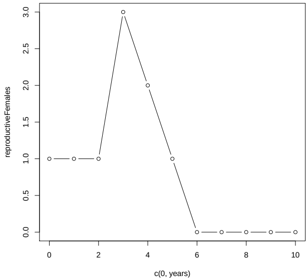

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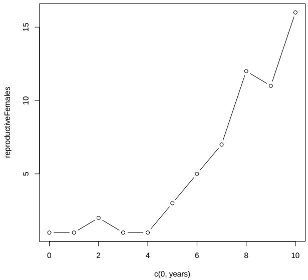

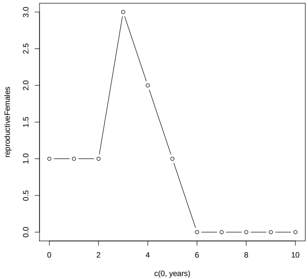

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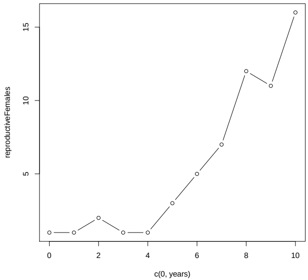

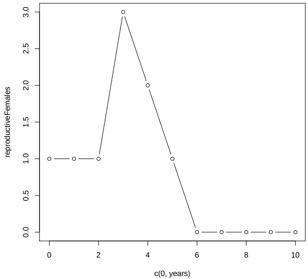

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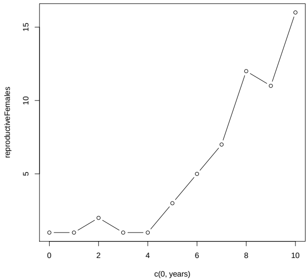

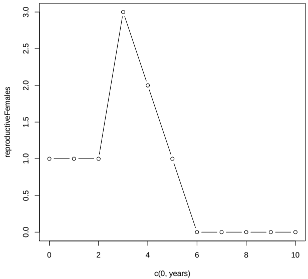

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