UNIT 2: Linear population models

Outline

Constructing models
- Dynamical models
- Examples
- A simple discrete-time model
- A simple continuous-time model

Units and scaling

Key parameters
- Discrete-time model
- Continuous-time model
- Links

Growth and regulation

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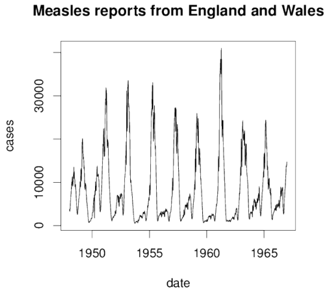

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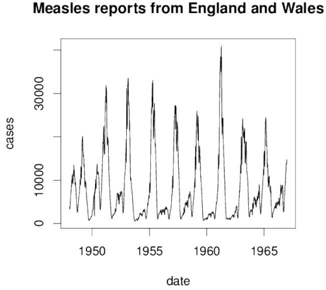

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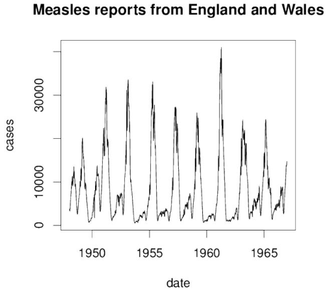

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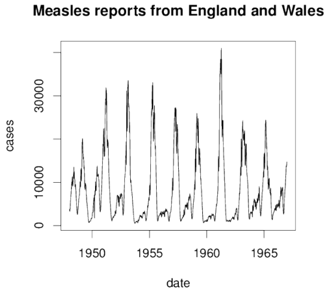

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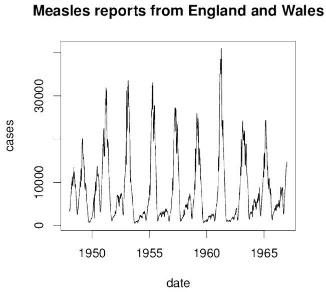

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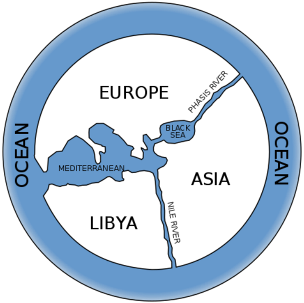

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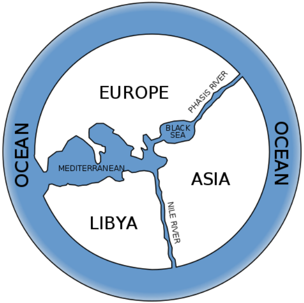

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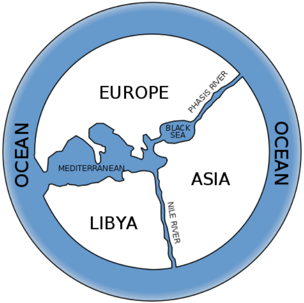

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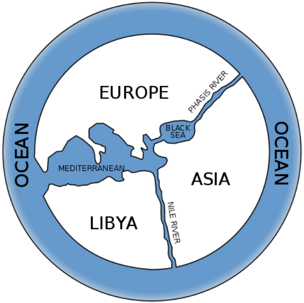

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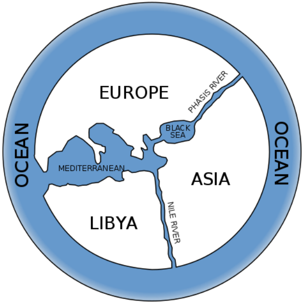

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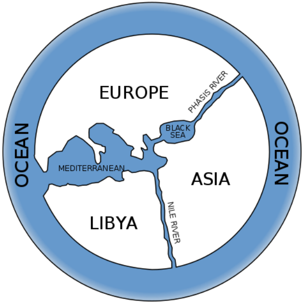

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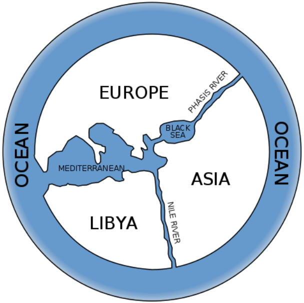

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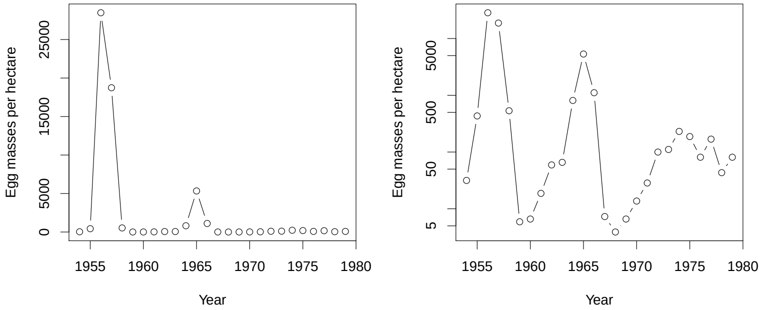

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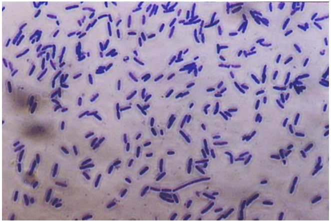

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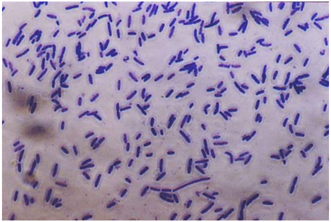

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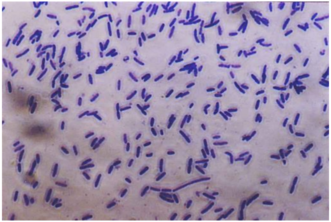

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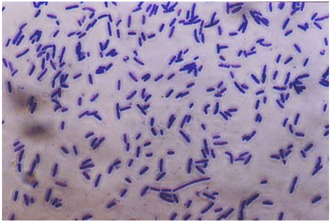

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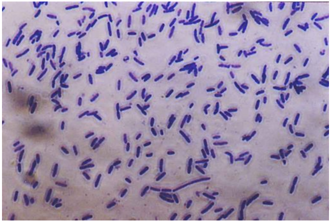

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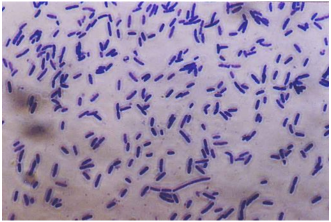

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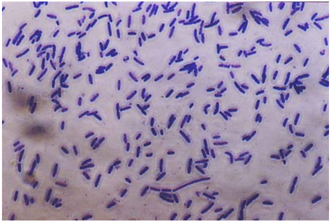

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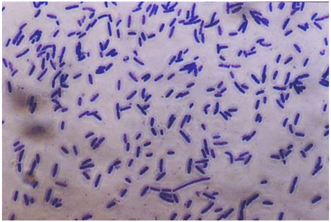

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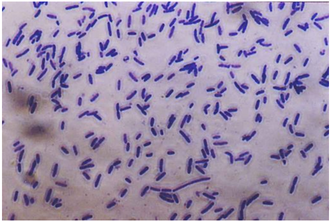

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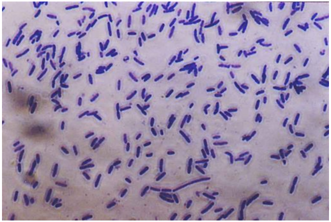

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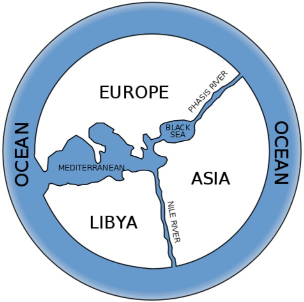

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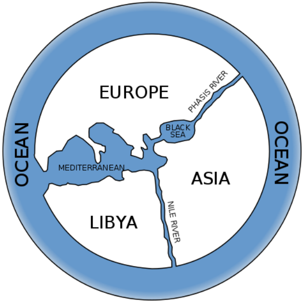

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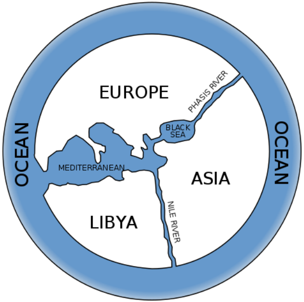

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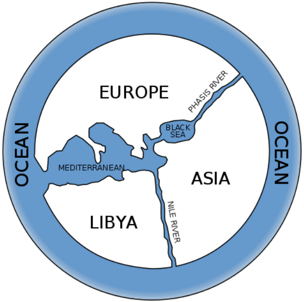

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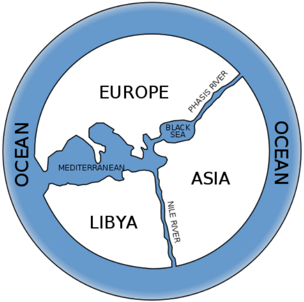

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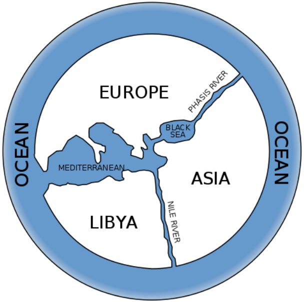

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	A	B	C	D
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3	2005-12-18	311 €	124 €	187 €
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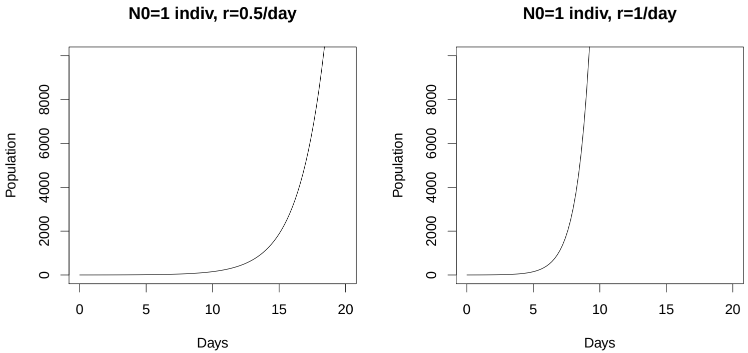

N0=1 indiv, r=0.5/day
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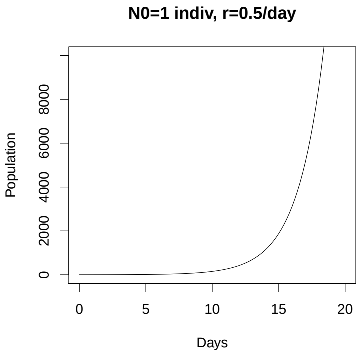

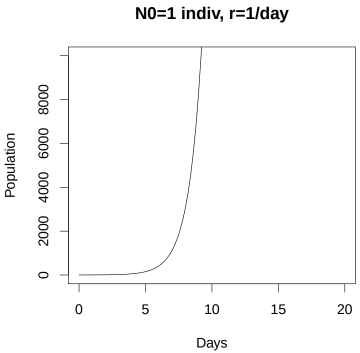

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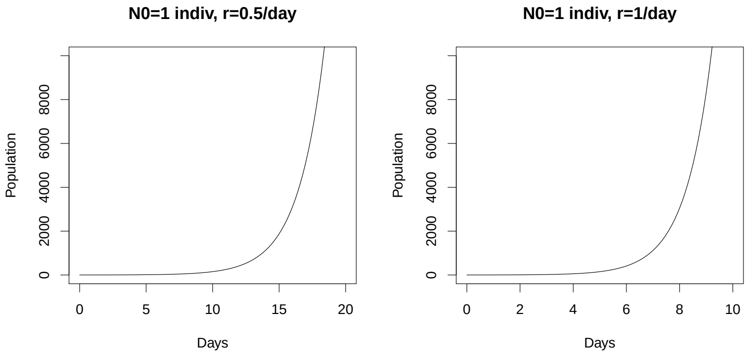

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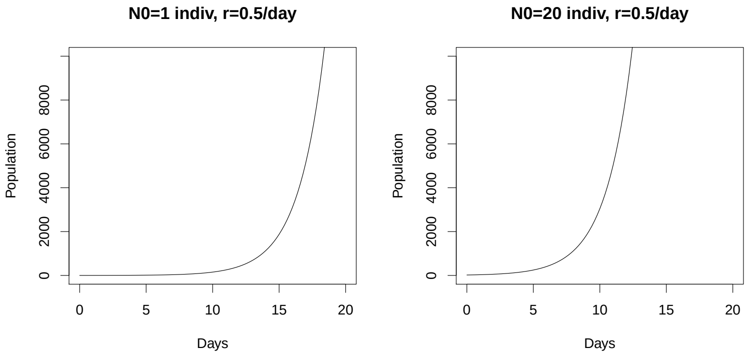

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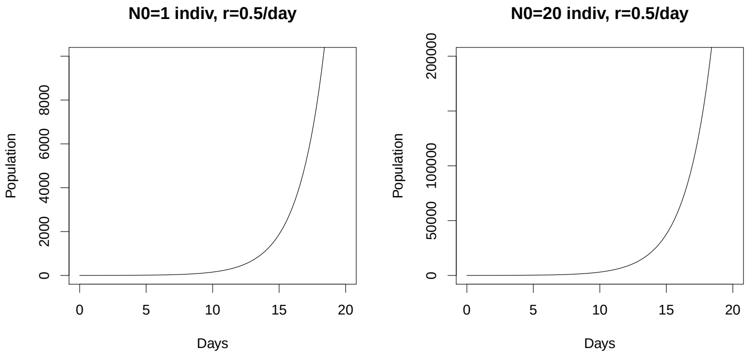

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