

BIO3SS3: Population Ecology

UNIT 1: Introduction

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Examples

Example populations

Dandelions

Gypsy moths

Bacteria

Coronavirus

Exponential growth

Log and linear scales

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 - ▶ HIV, rabies, ebola, influenza, ...

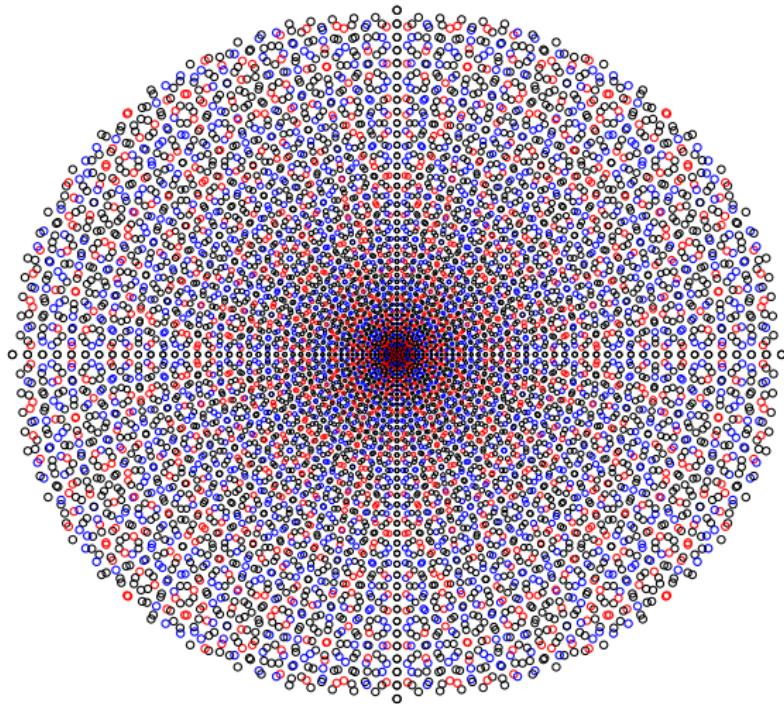
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Pythagorean triples



Which country?



Which country?



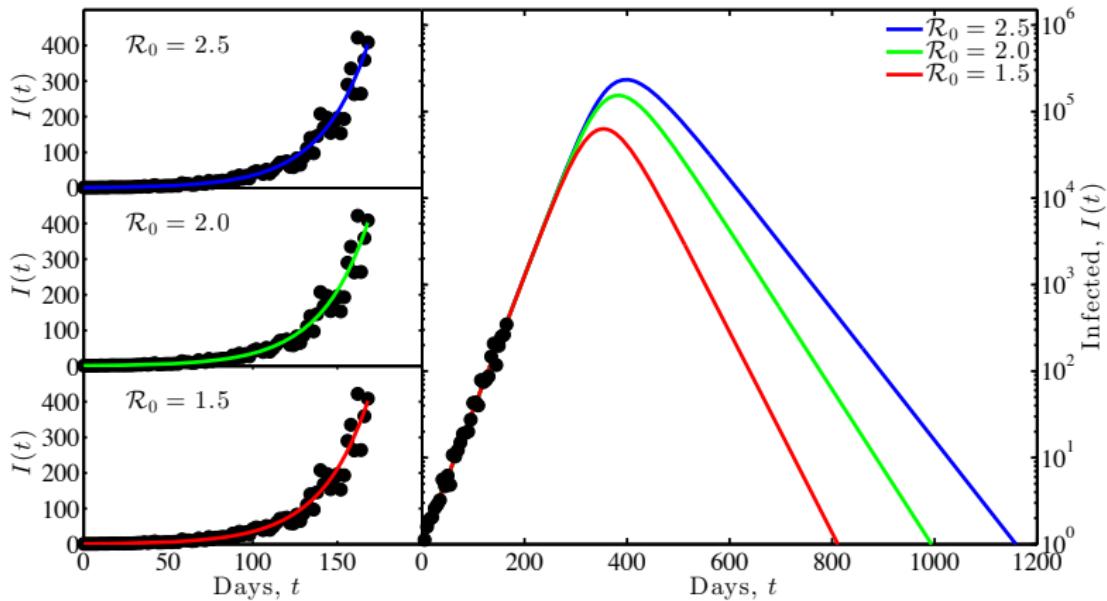
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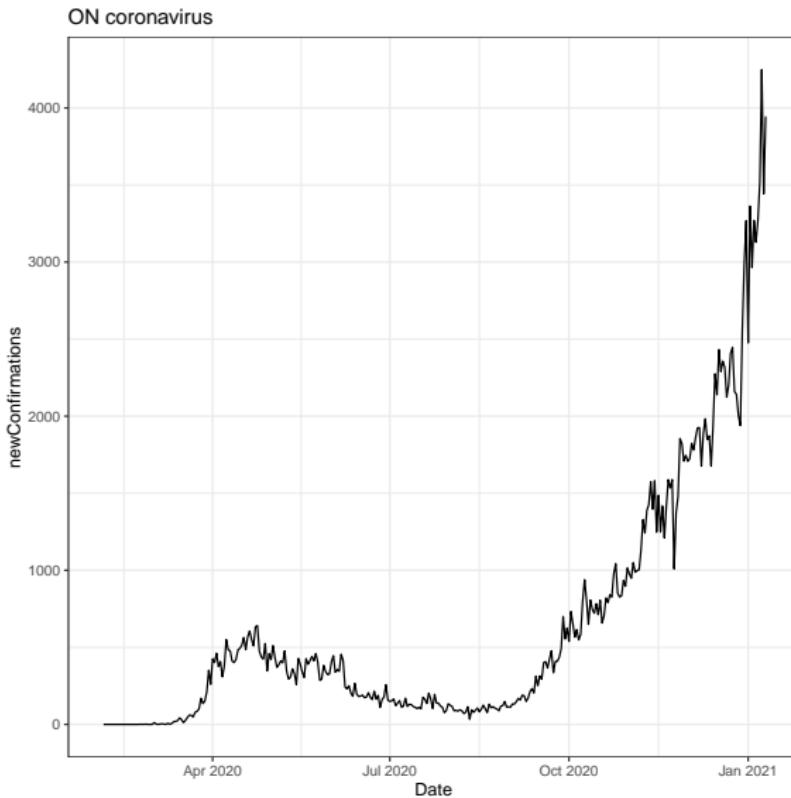
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Disease research



Disease research



TAs

- ▶ Steve Cygu

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Students

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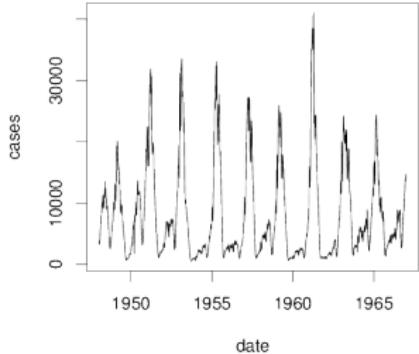
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Dynamical modeling

- ▶ Investigates the links between local, short-term processes, and large-scale, long-term outcomes



Measles reports from England and Wales

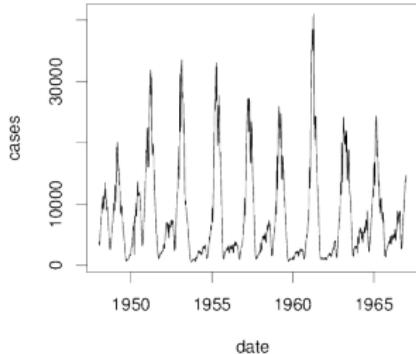


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- ▶ Allows us to explore what assumptions we're making, and how assumptions affect the link



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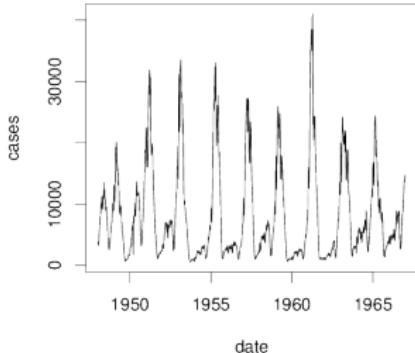


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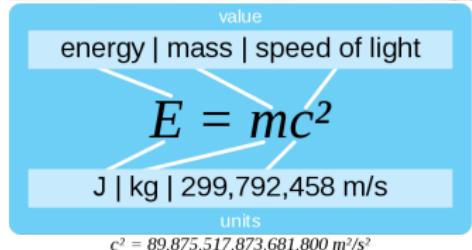
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Humans and abstract thought

- ▶ People are evolved to be concrete thinkers, not conceptual thinkers



Humans and abstract thought

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- ▶ A goal of this course is to build conceptual thinking skills



$$E = mc^2$$

value

energy | mass | speed of light

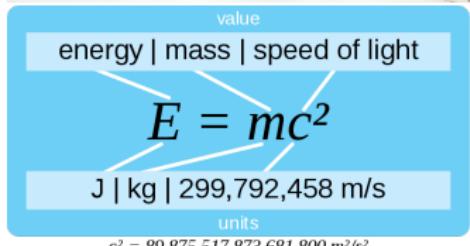
J | kg | 299,792,458 m/s

units

$c^2 = 89,875,517,873,681,800 \text{ m}^2/\text{s}^2$

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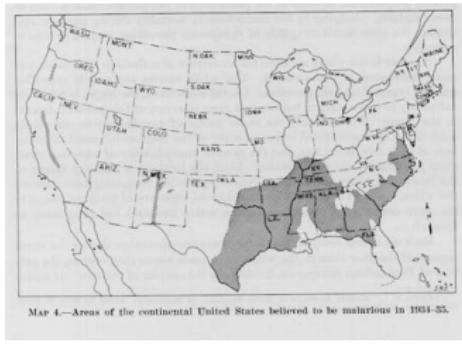
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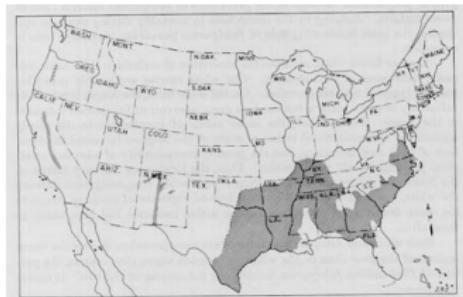
Malaria

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Malaria

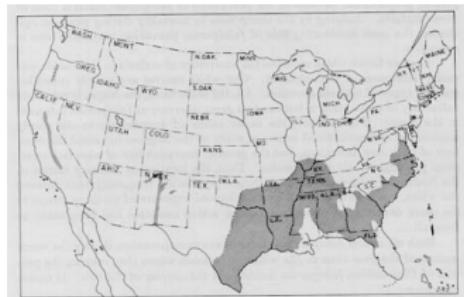
- ▶ A nasty, mosquito-borne disease
- ▶ In some places (e.g., the southeastern US), it has been eradicated almost by accident



MAP 4.—Areas of the continental United States believed to be malarious in 1951-55.

Malaria

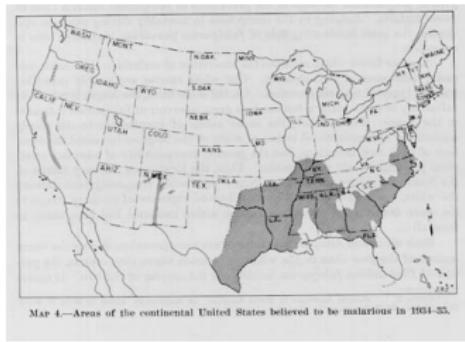
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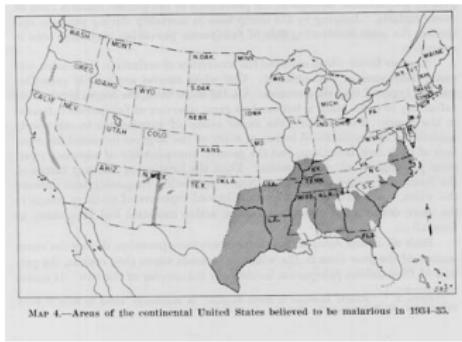
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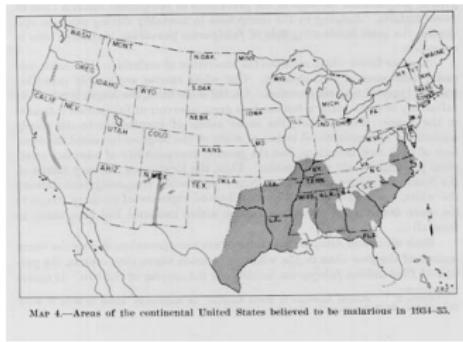
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Red squirrels

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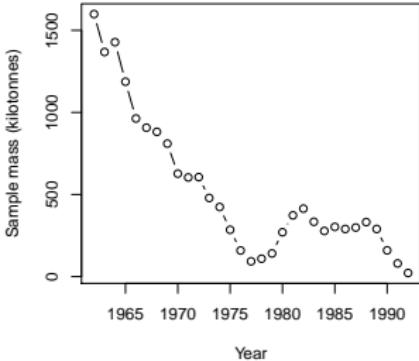
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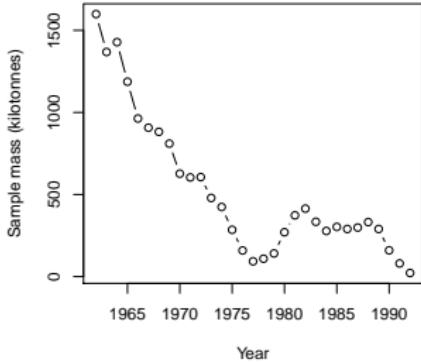
Cod fisheries

- ▶ Is the ocean too big for people to affect?



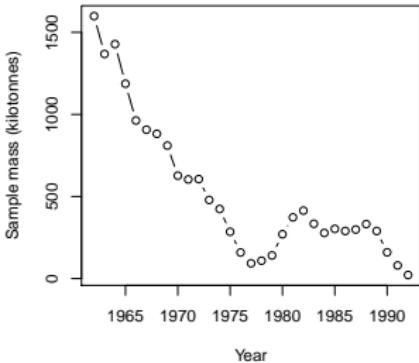
Cod fisheries

- ▶ Is the ocean too big for people to affect?
- ▶ What happened to the cod?



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Populations

- Poll: What population of organisms interests you?

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Dandelions

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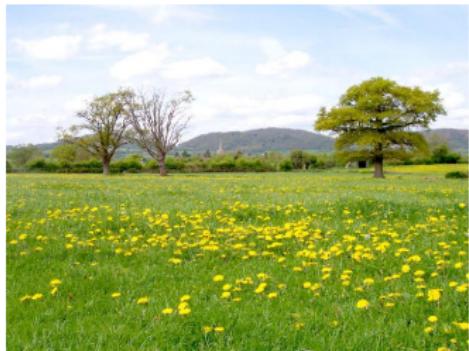
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- ▶ The spreadsheet is an implementation of a dynamical model!



Dandelions

- ▶ Start with one dandelion; it produces 100 seeds, of which only 4% survive to reproduce the next year.
- ▶ How many dandelions after 3 years?
 - ▶ * 64?
 - ▶ * 125?
 - ▶ See spreadsheet on resource page
- ▶ The spreadsheet is an implementation of a dynamical model!



Dynamical models

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Gypsy moths

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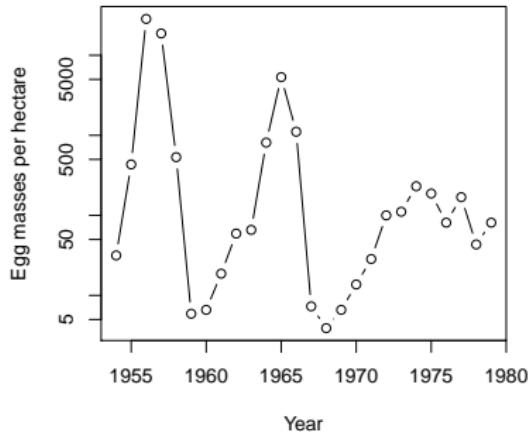
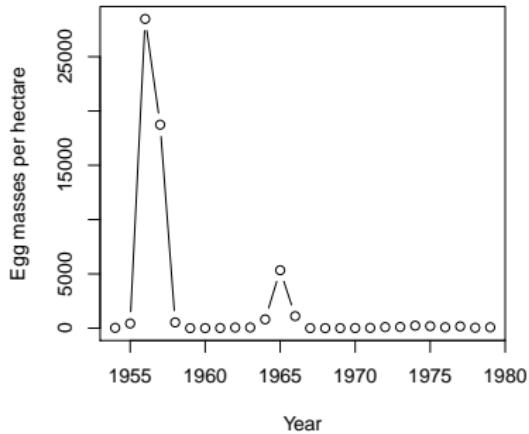


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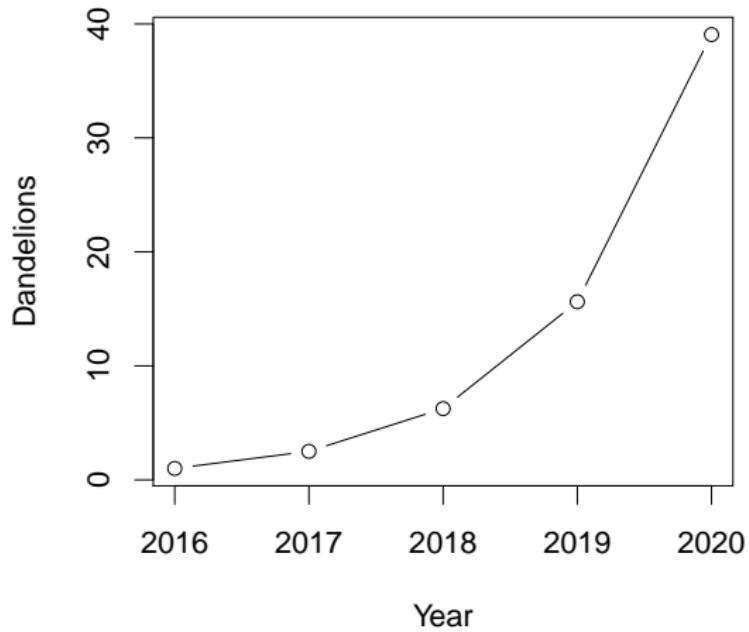
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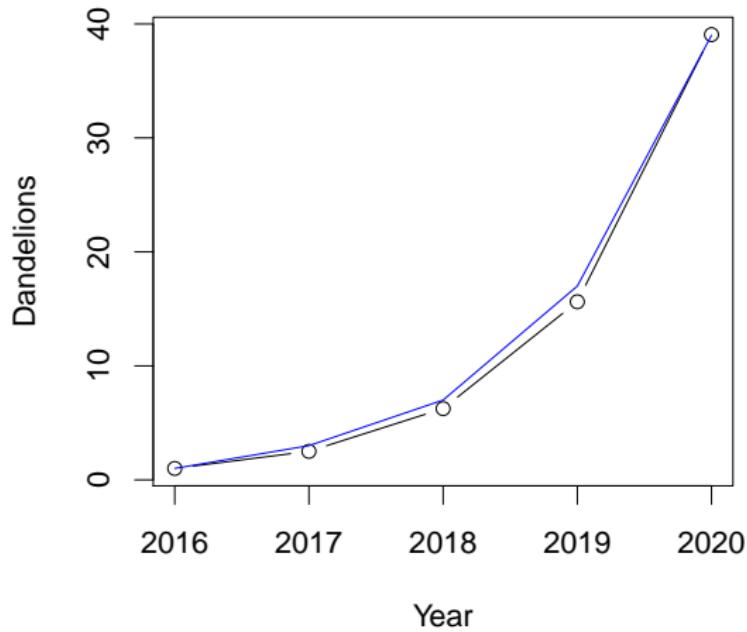
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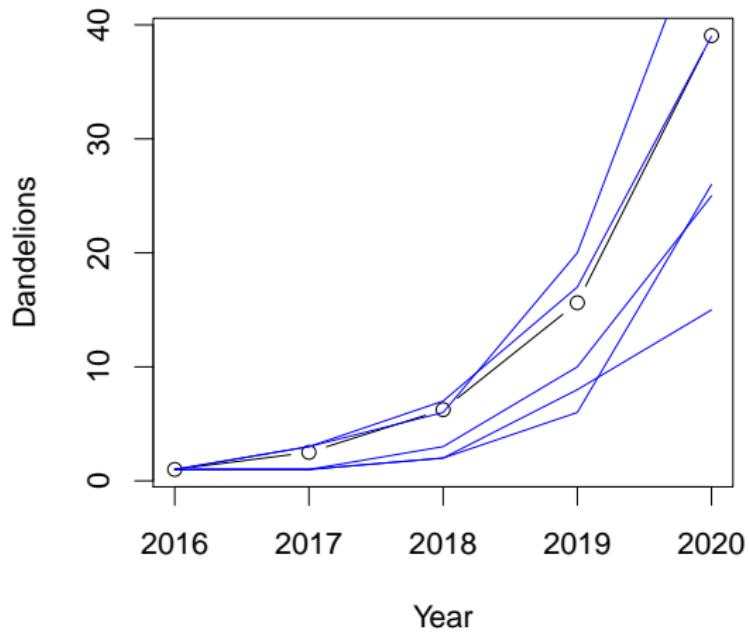
Stochastic model



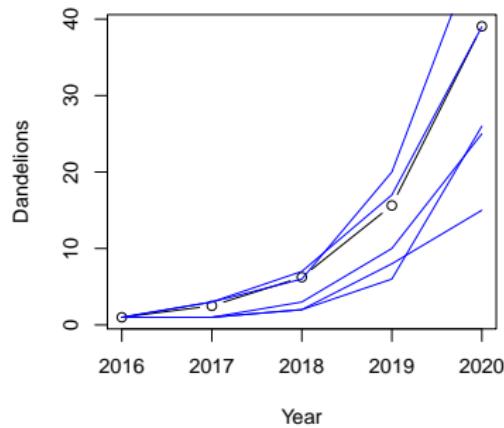
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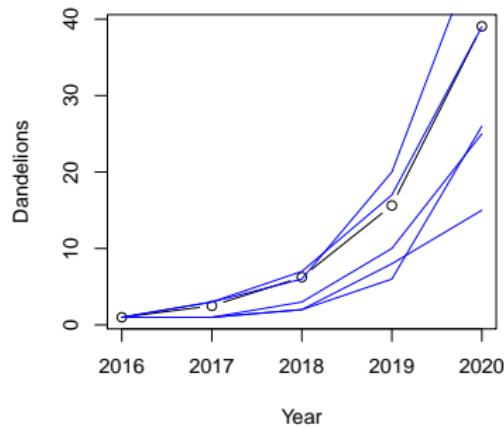


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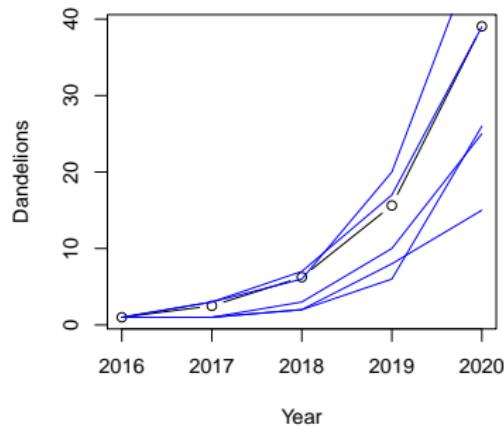
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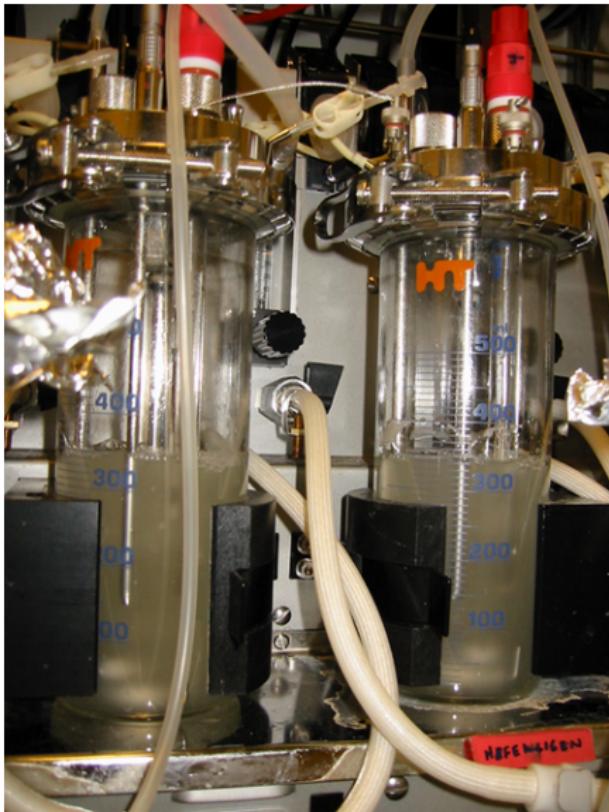
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Bacteria in a tank



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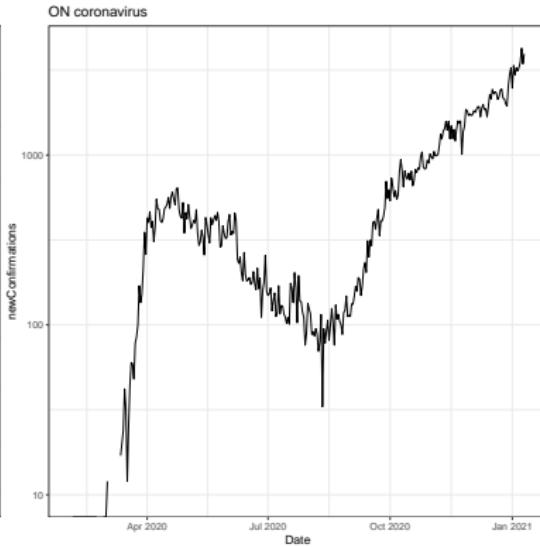
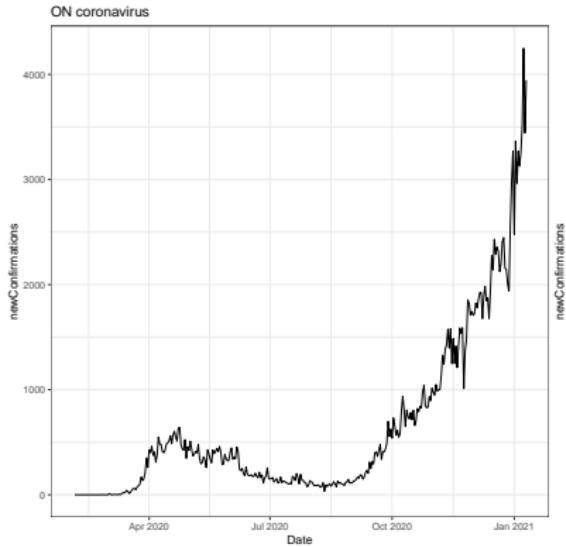
Coronavirus

Exponential growth

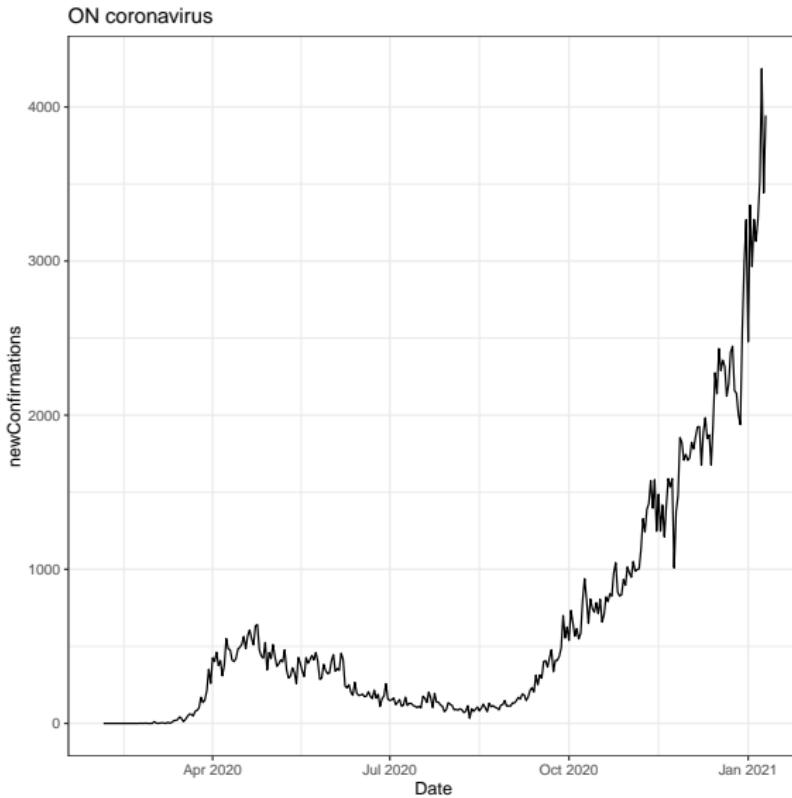
Log and linear scales

Time scales

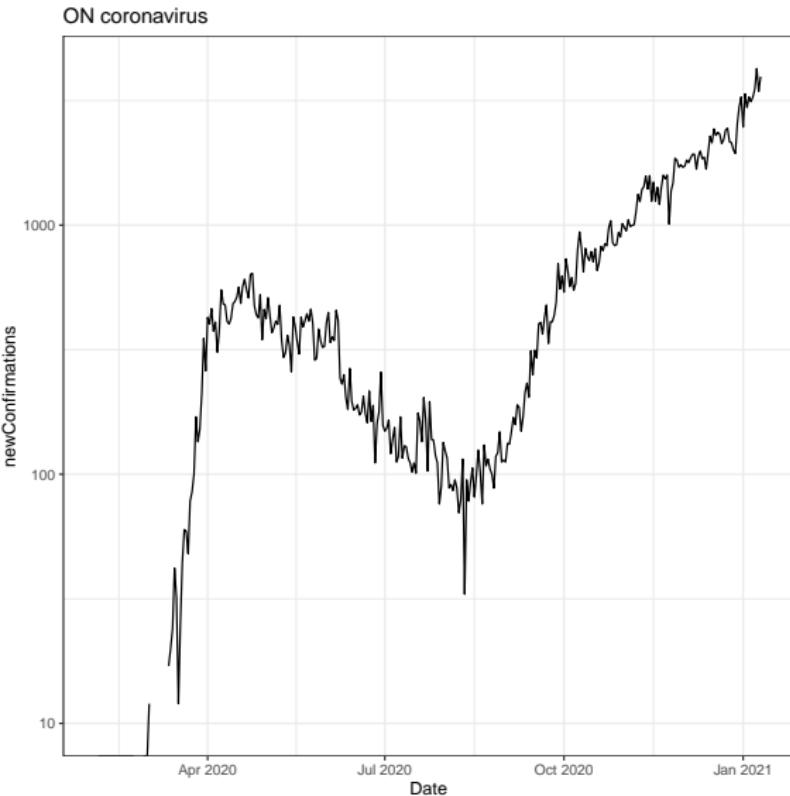
Coronavirus



Coronavirus (repeat)



Coronavirus (repeat)



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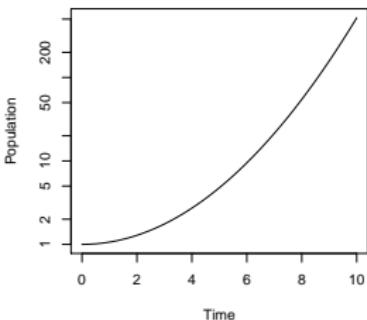
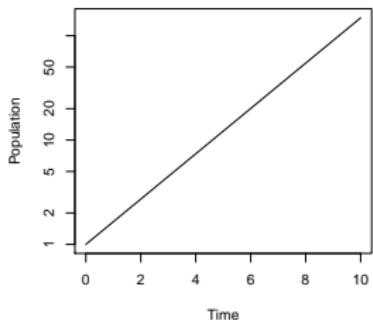
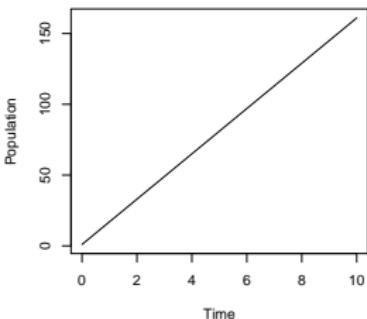
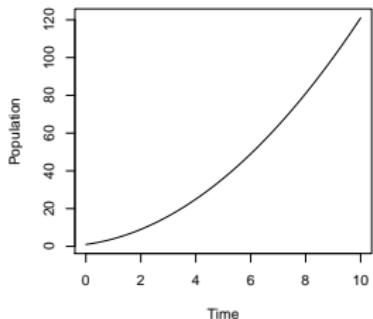
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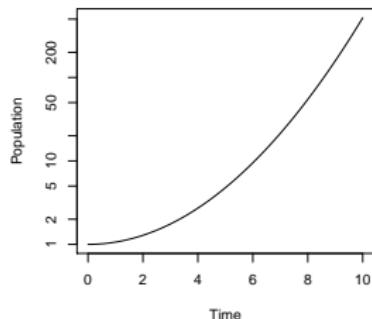
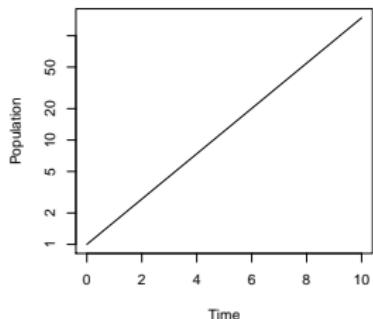
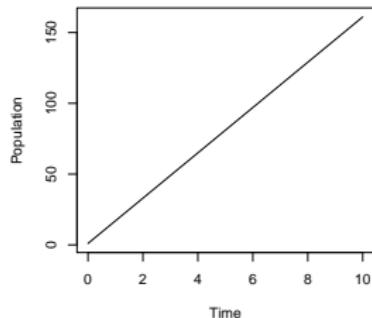
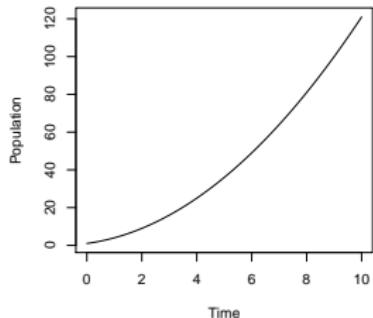
Exponential growth

► What is exponential growth?



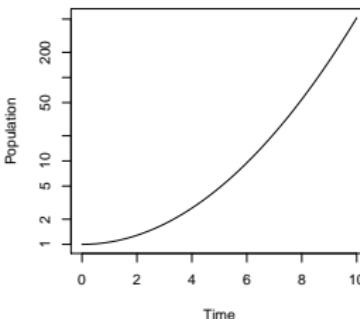
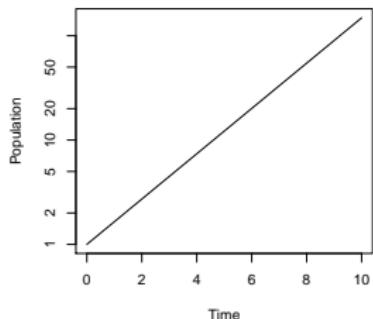
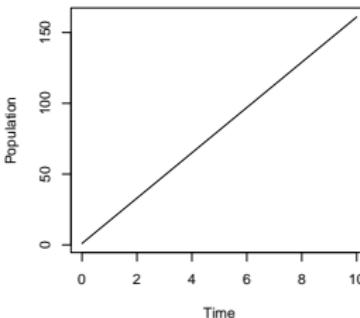
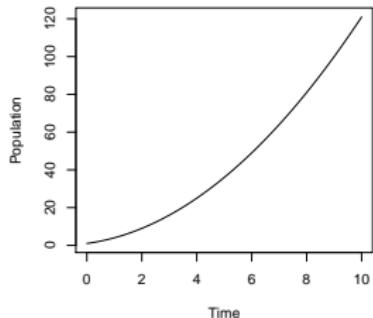
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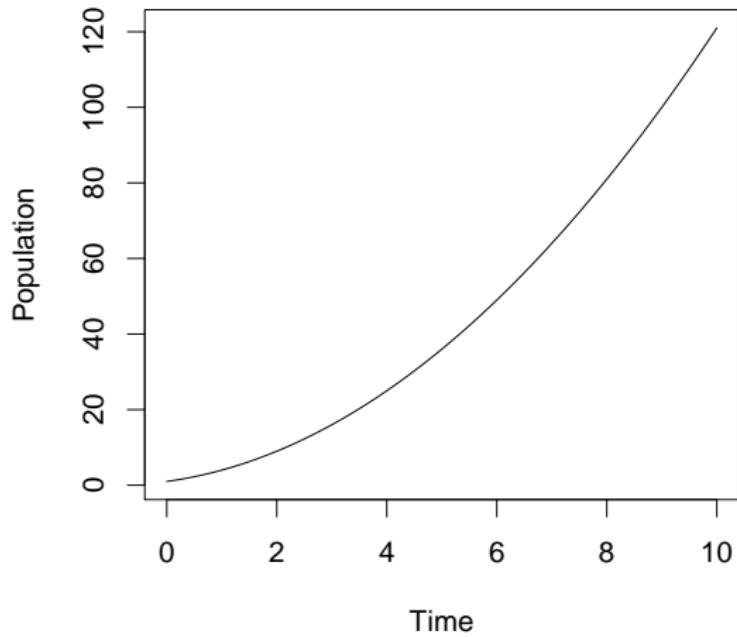


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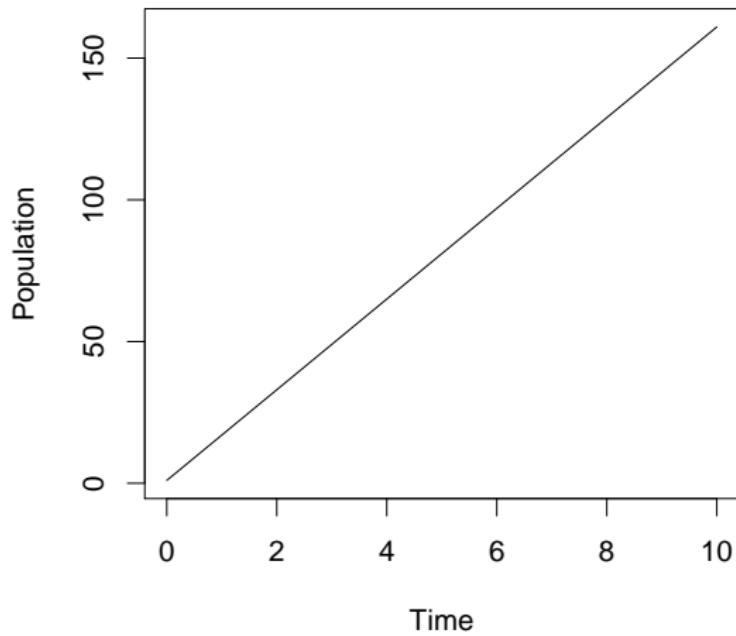
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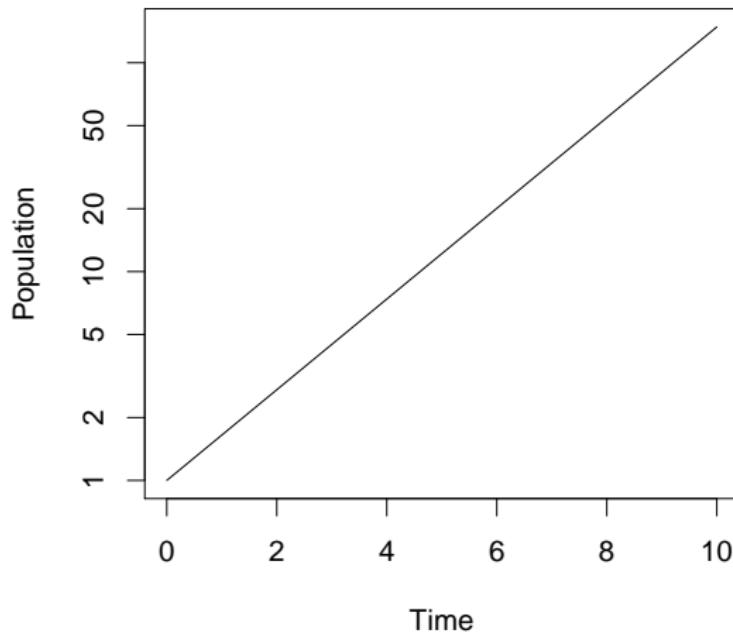
A (repeat)



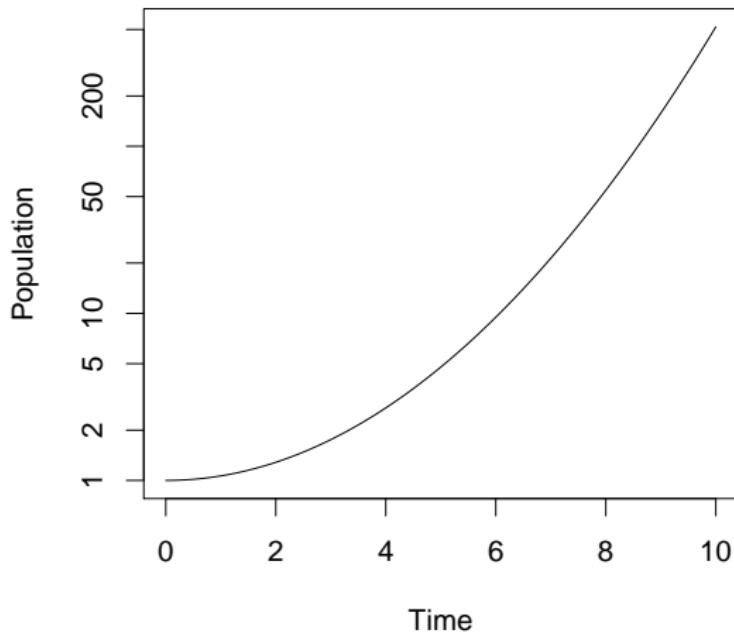
B (repeat)



C (repeat)

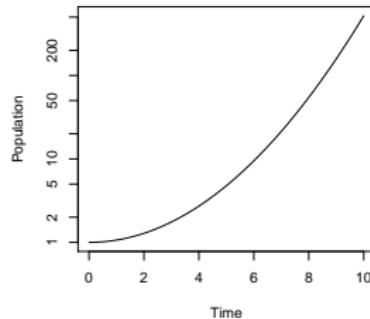
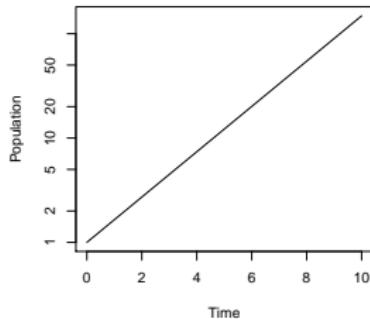
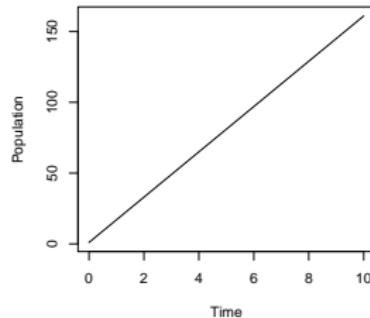
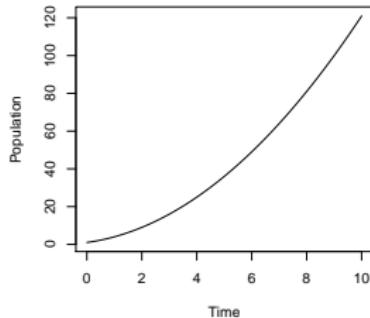


D (repeat)



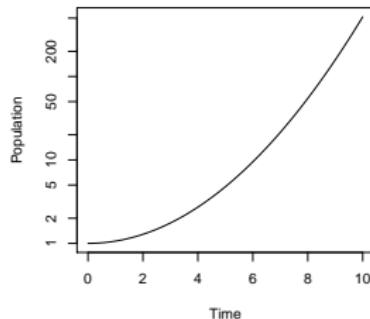
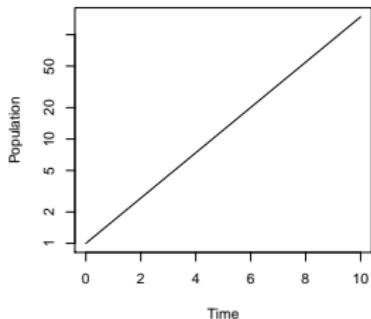
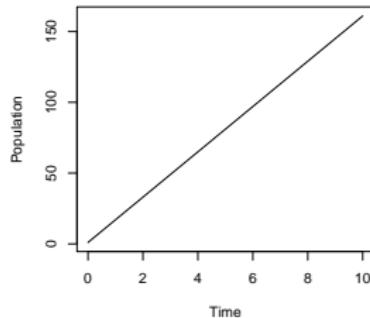
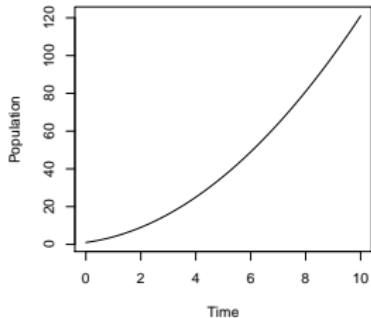
Exponential growth (repeat)

► Poll: What is exponential growth?



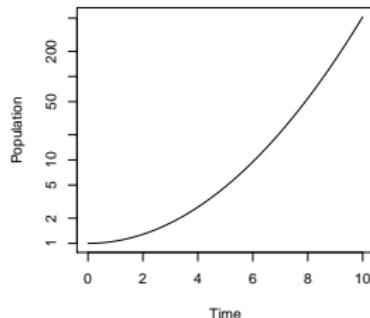
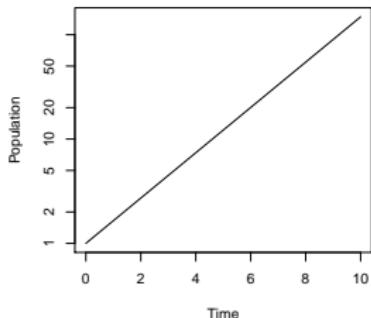
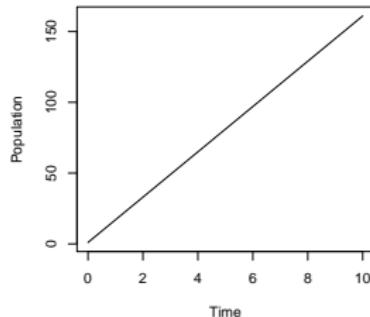
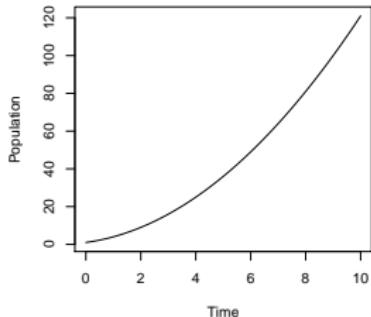
Exponential growth (repeat)

- ▶ Poll: What is exponential growth?
- ▶ Poll: Which of these is an example?

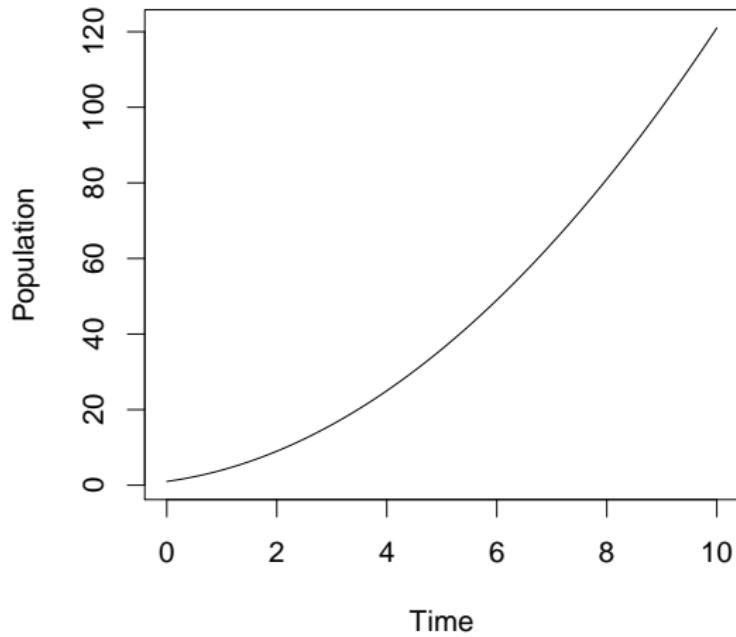


Exponential growth (repeat)

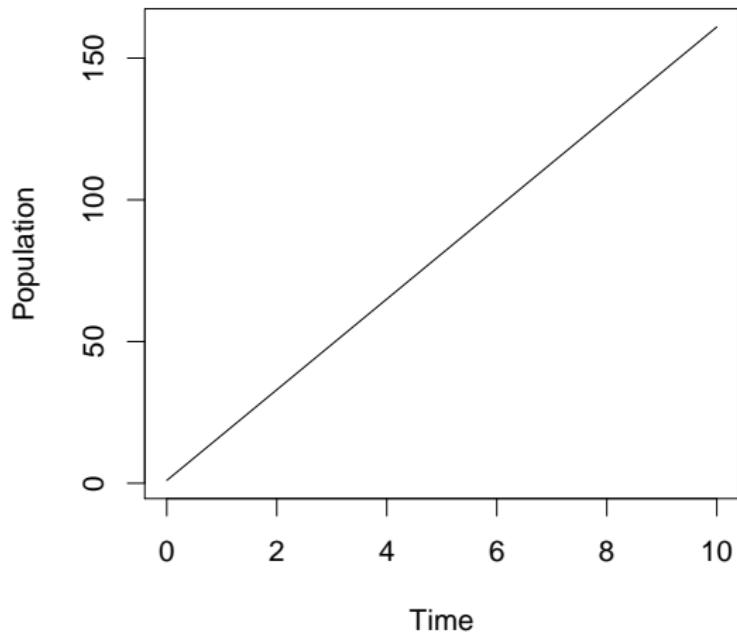
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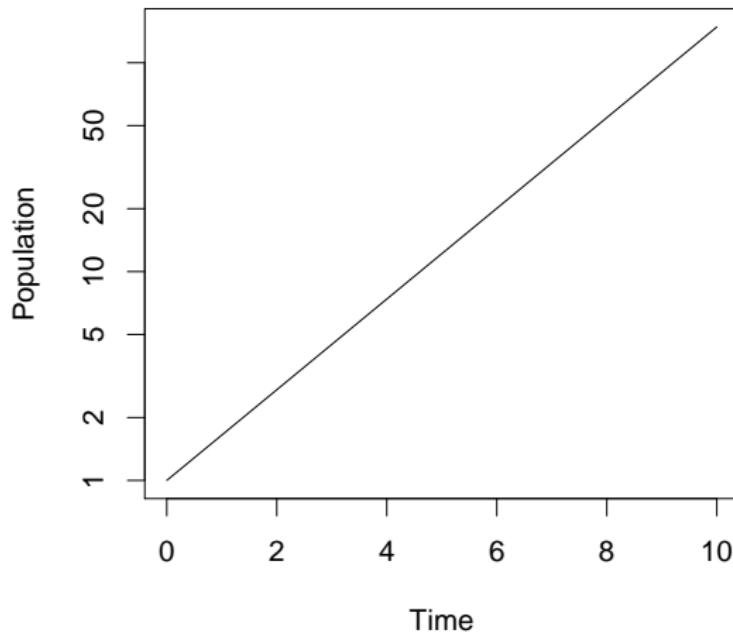
A (repeat)



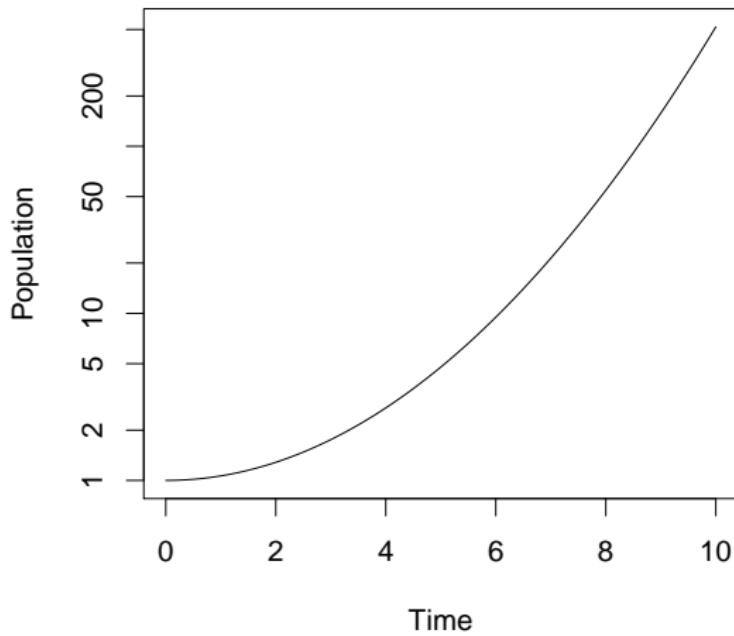
B (repeat)



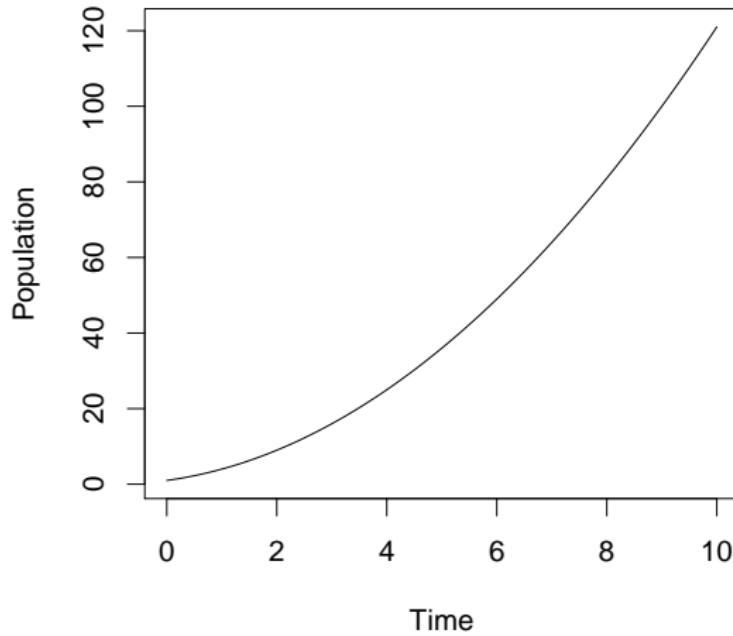
C (repeat)



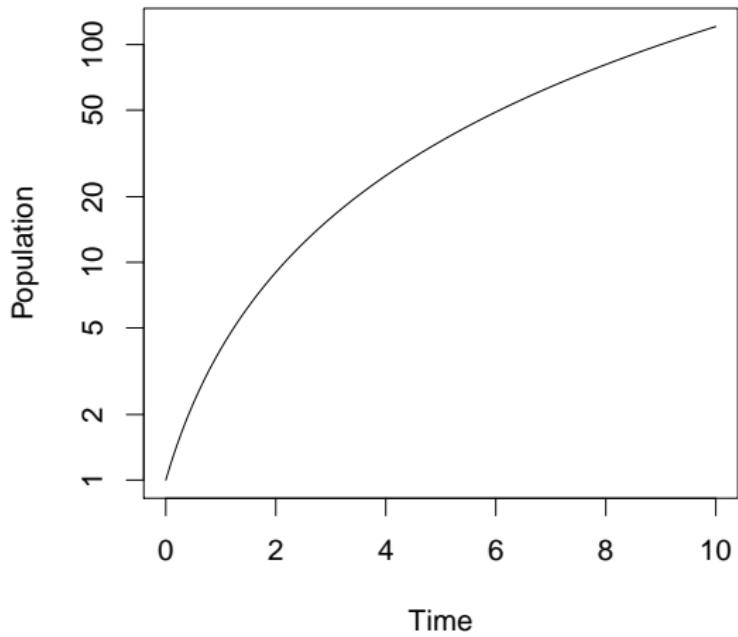
D (repeat)



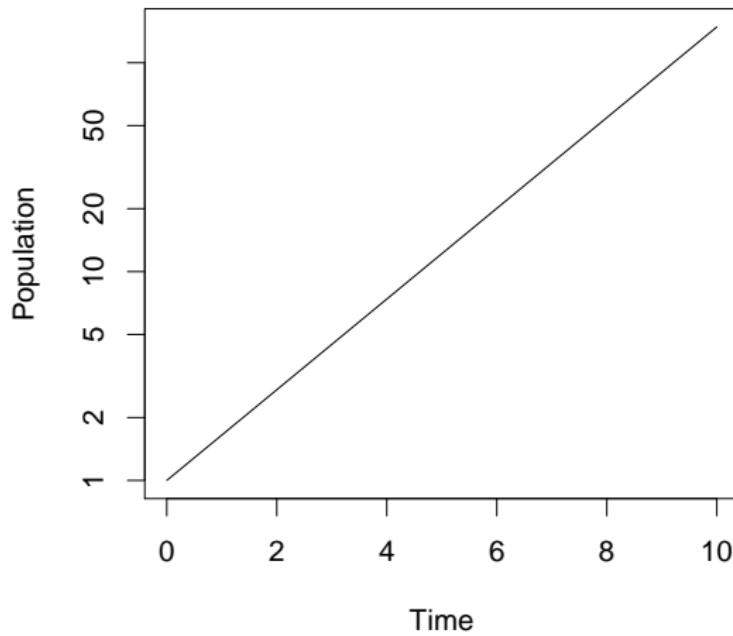
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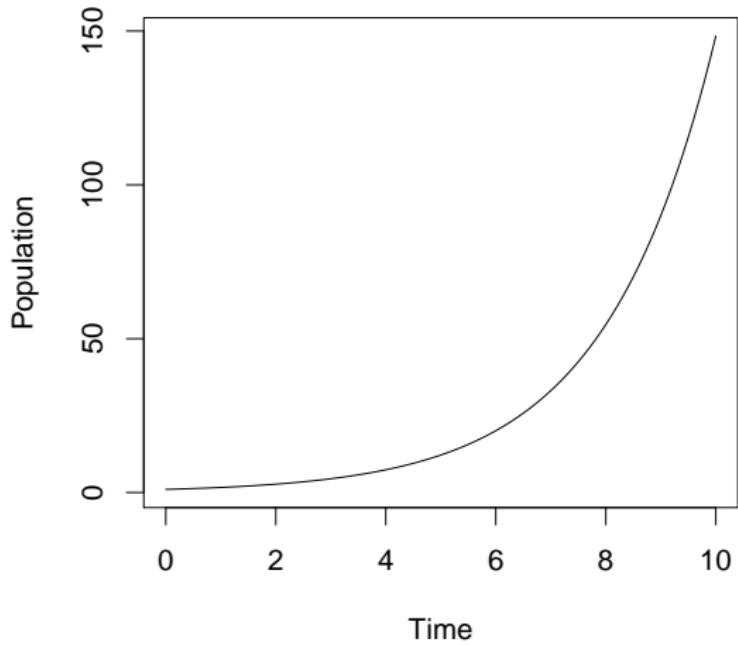
A on the log scale



C (repeat)



C on the linear scale



Types of growth

- arithmetic/linear:

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Exponential decline?

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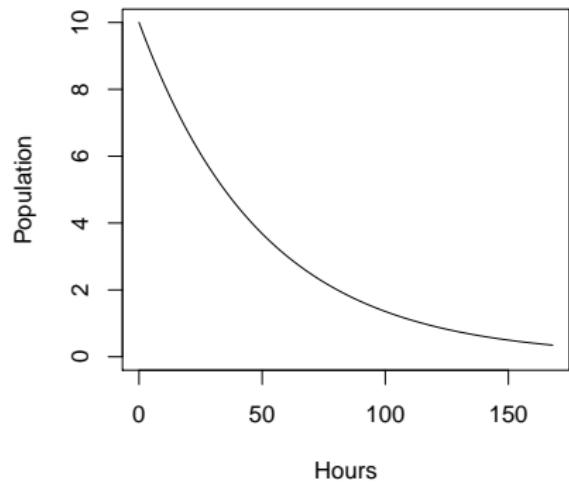
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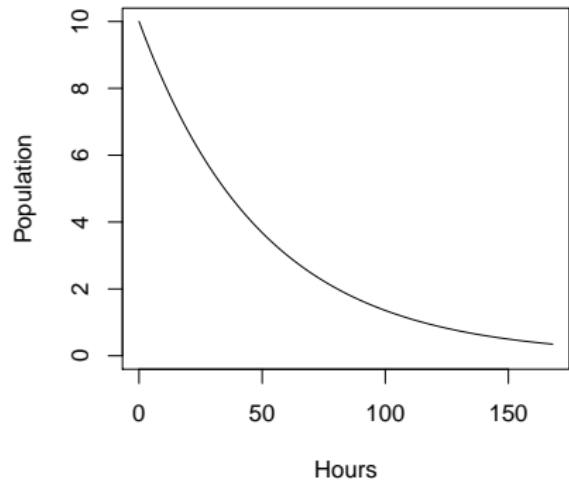
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Exponential decline



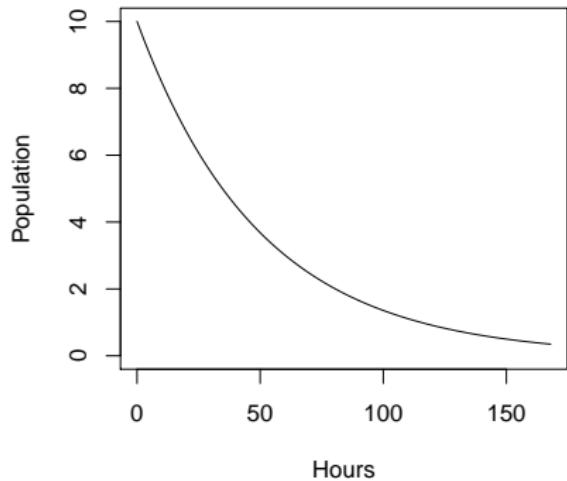
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Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

Example populations

Dandelions

Gypsy moths

Bacteria

Coronavirus

Exponential growth

Log and linear scales

Time scales

Scales of comparison

- Poll: 1 is to 10 as 10 is to what?

Scales of comparison

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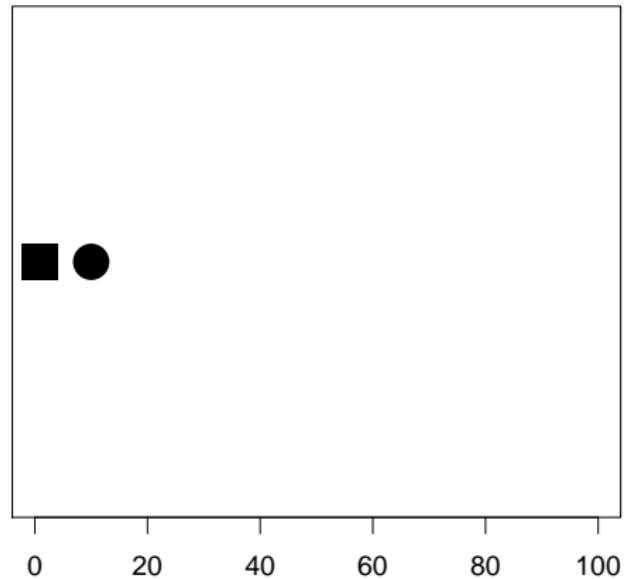
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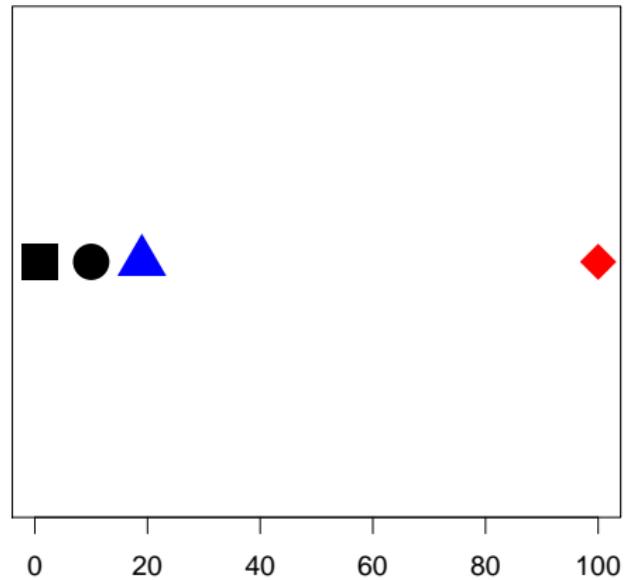
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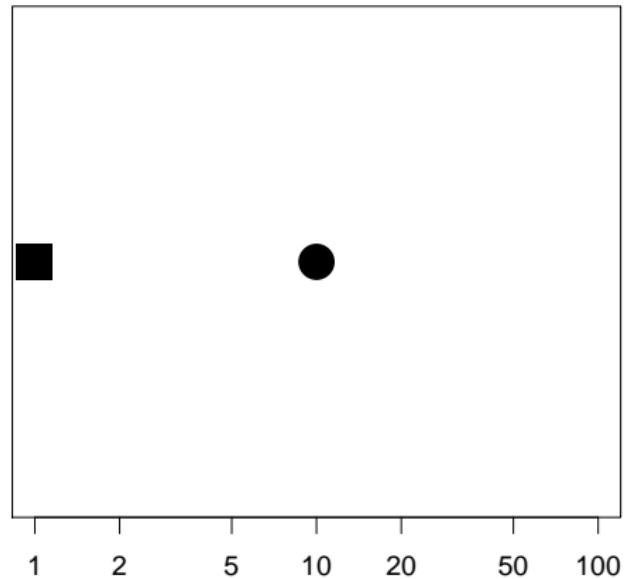
Scales of display



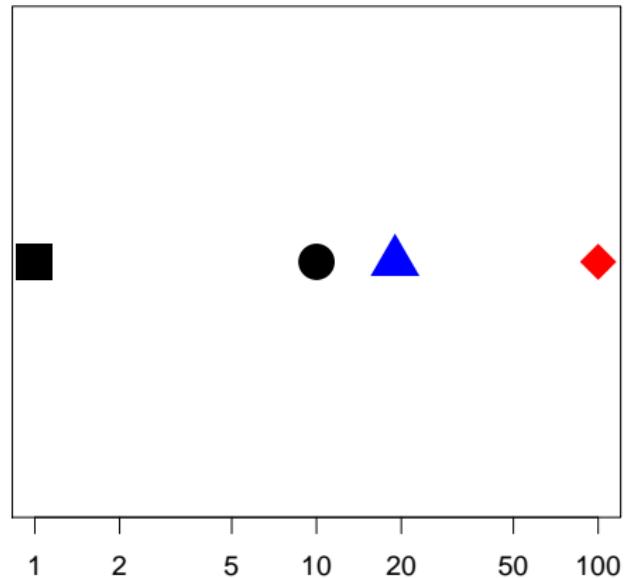
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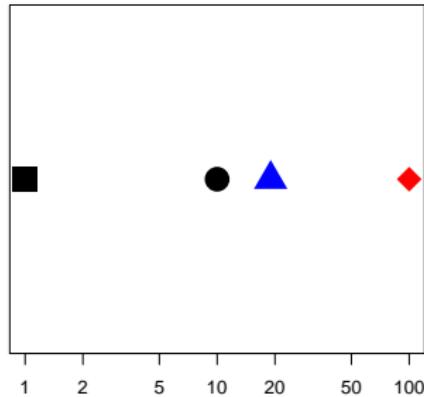
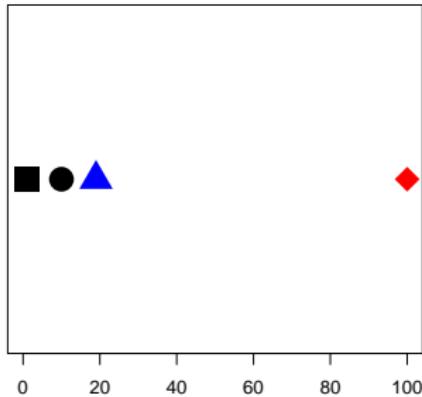
Scales of display



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Scales of display



There is only one log scale; it doesn't matter which base you use!

Canadian provinces

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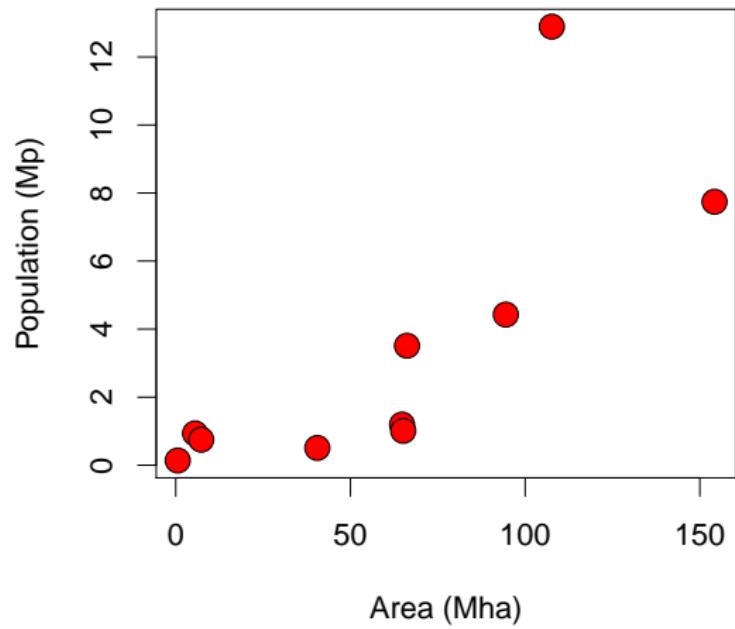
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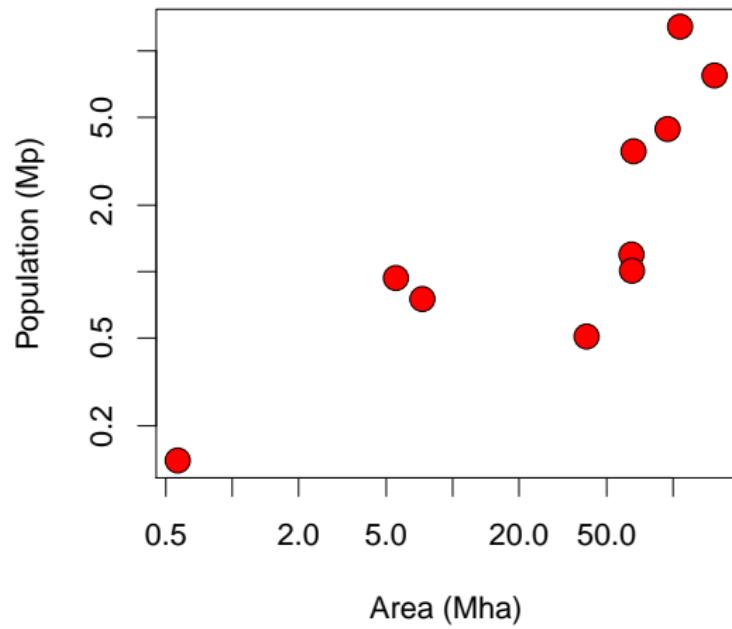
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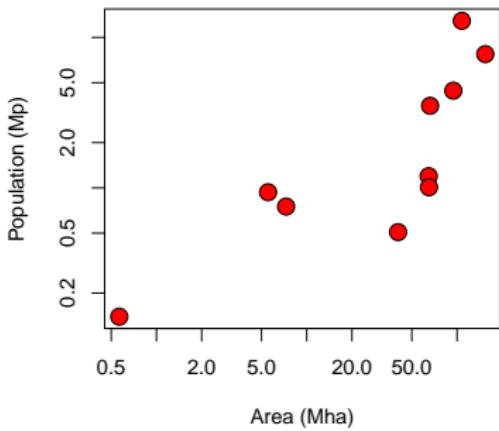
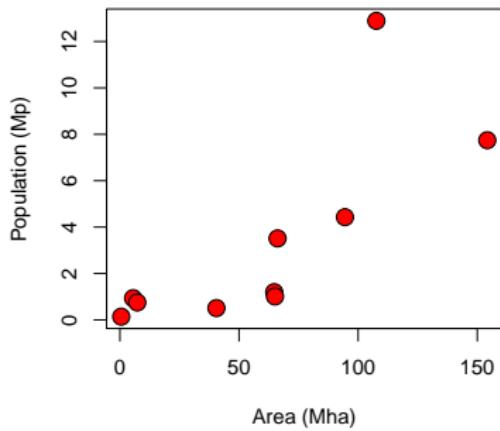
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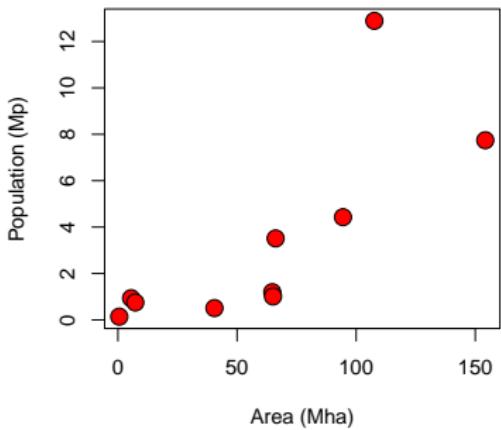
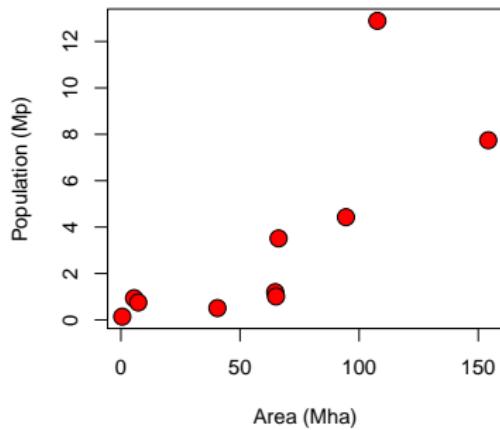
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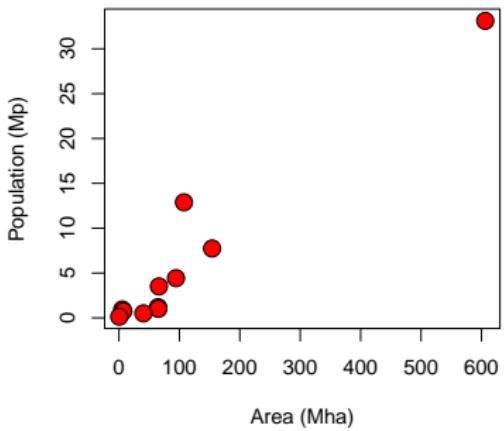
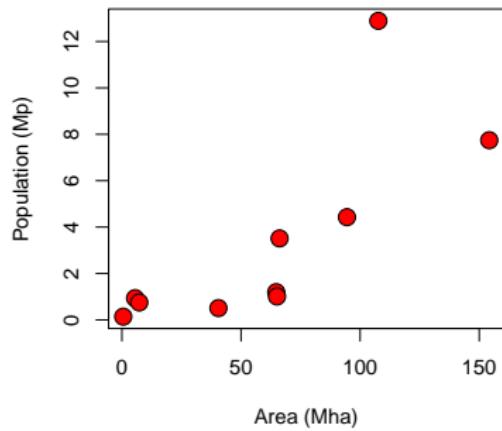
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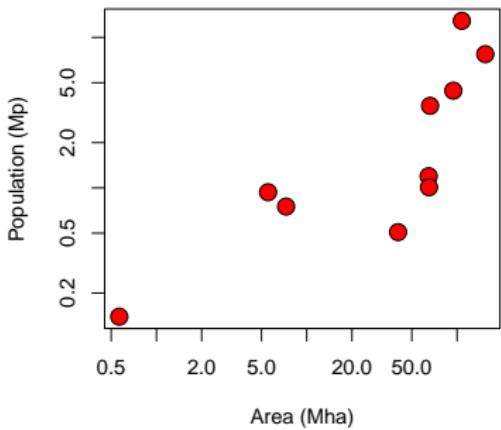
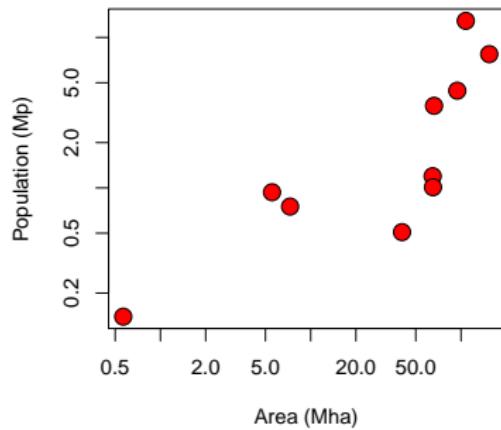
Canadian provinces plus Canada?



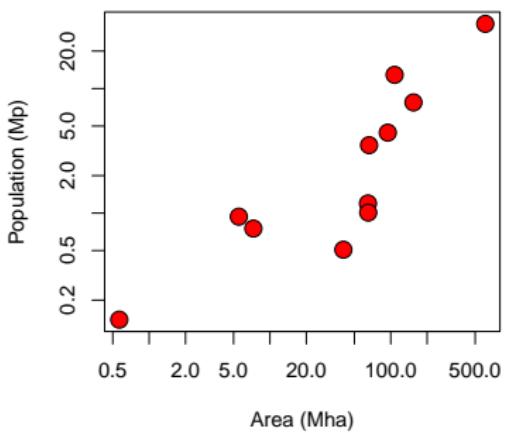
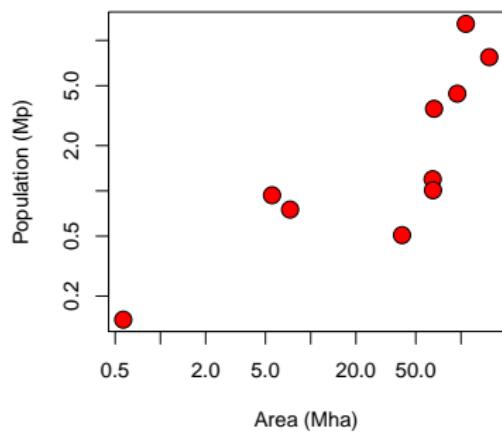
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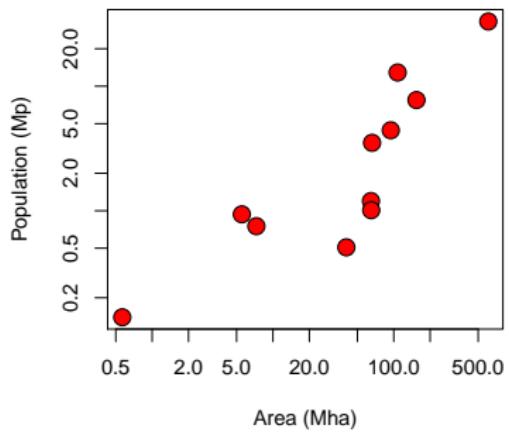
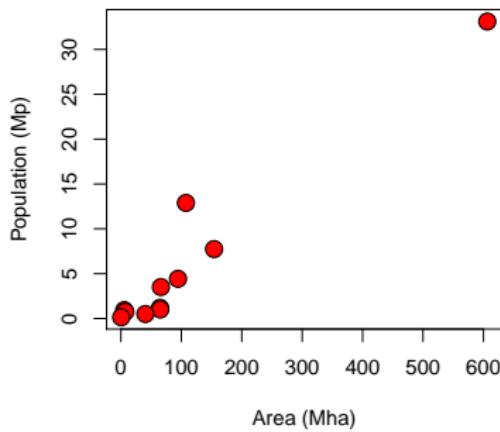
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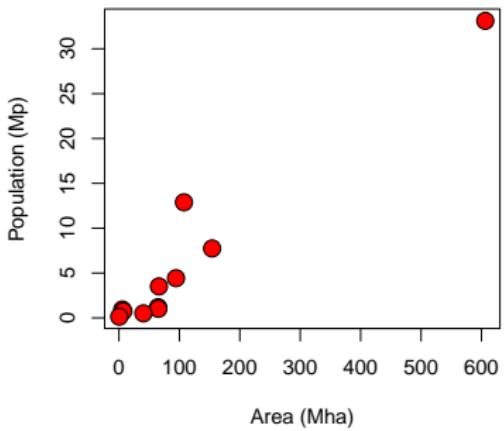
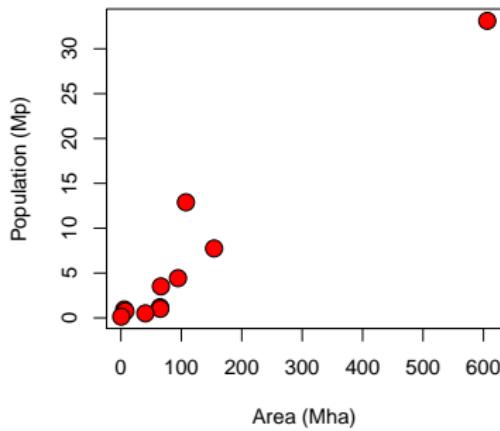
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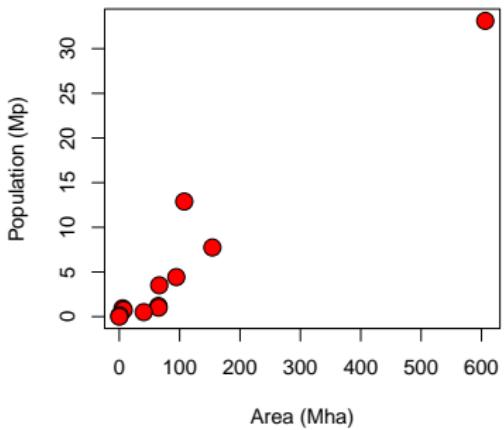
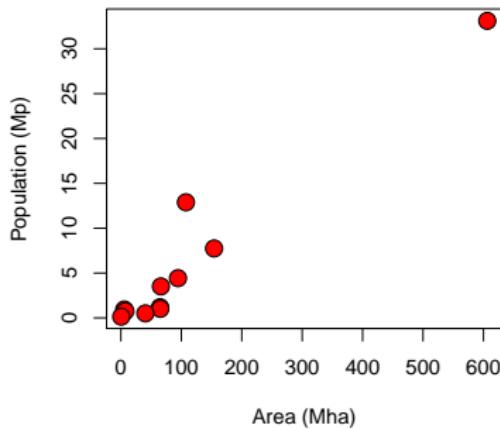
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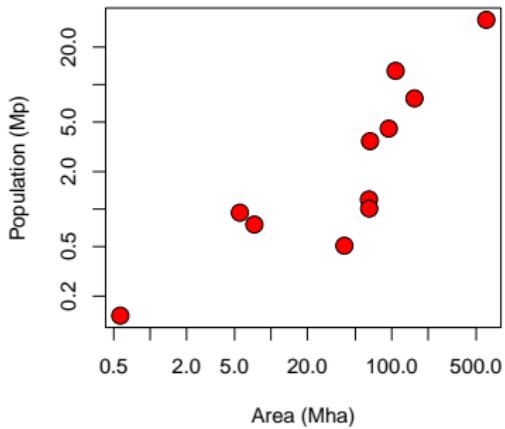
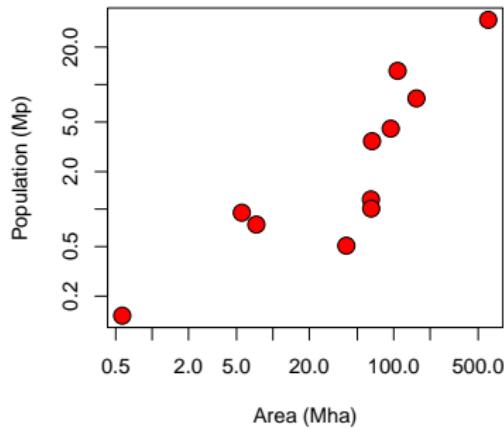
Canada plus Room 1105



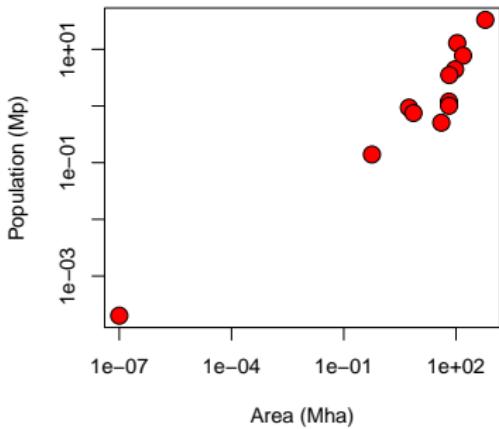
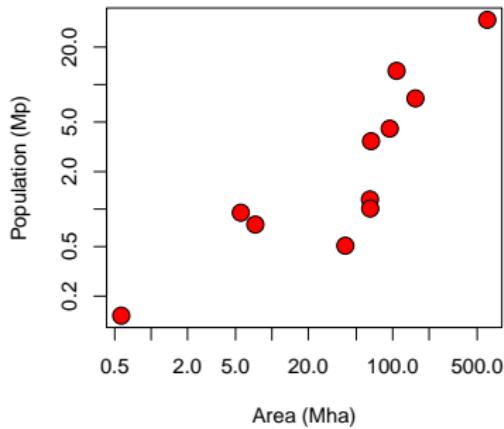
Canada plus Room 1105



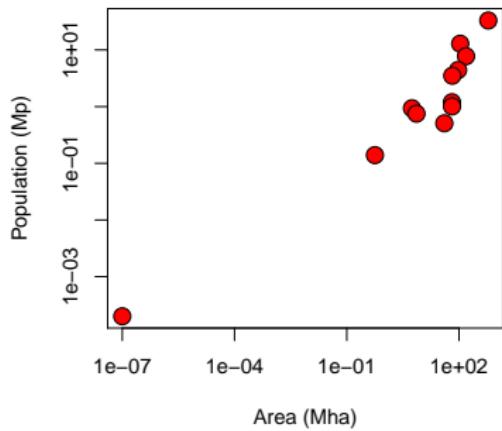
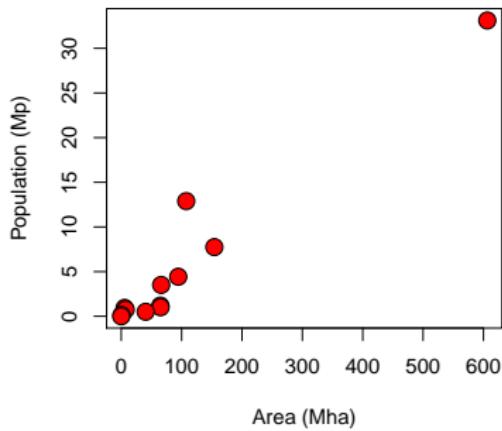
Canada plus room Room 1105?



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Predation comparison



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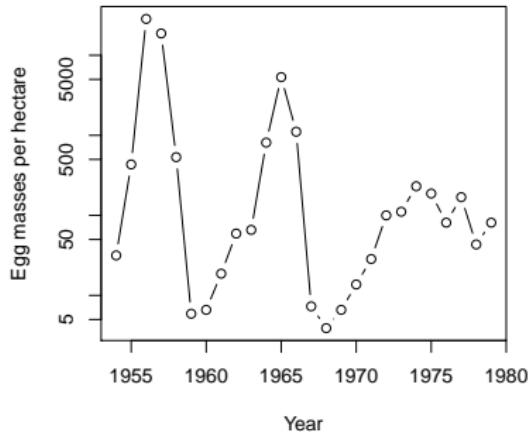
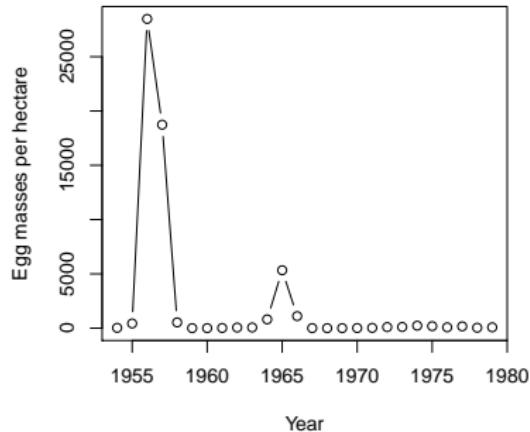
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Gypsy-moth example



Scales in population biology

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- ▶ The log scale looks at differences at the individual scale (per capita)

Scales in population biology

- ▶ The linear scale looks at differences at the population scale
- ▶ The log scale looks at differences at the individual scale (per capita)

Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

Example populations

Dandelions

Gypsy moths

Bacteria

Coronavirus

Exponential growth

Log and linear scales

Time scales

Speeding in Taiwan

- A life experience



Speeding in Taiwan

- ▶ A life experience
- ▶ Some clarifications



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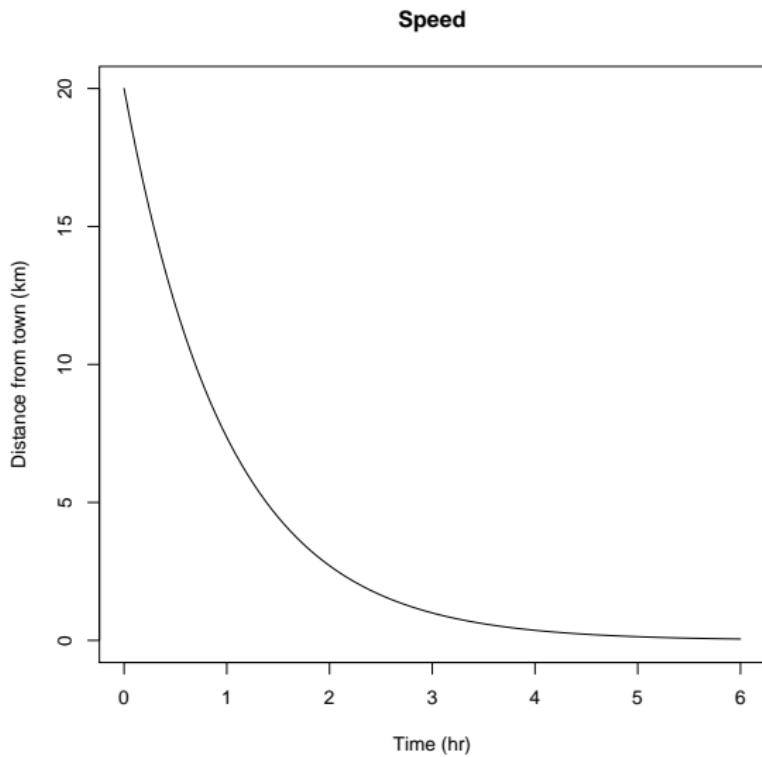
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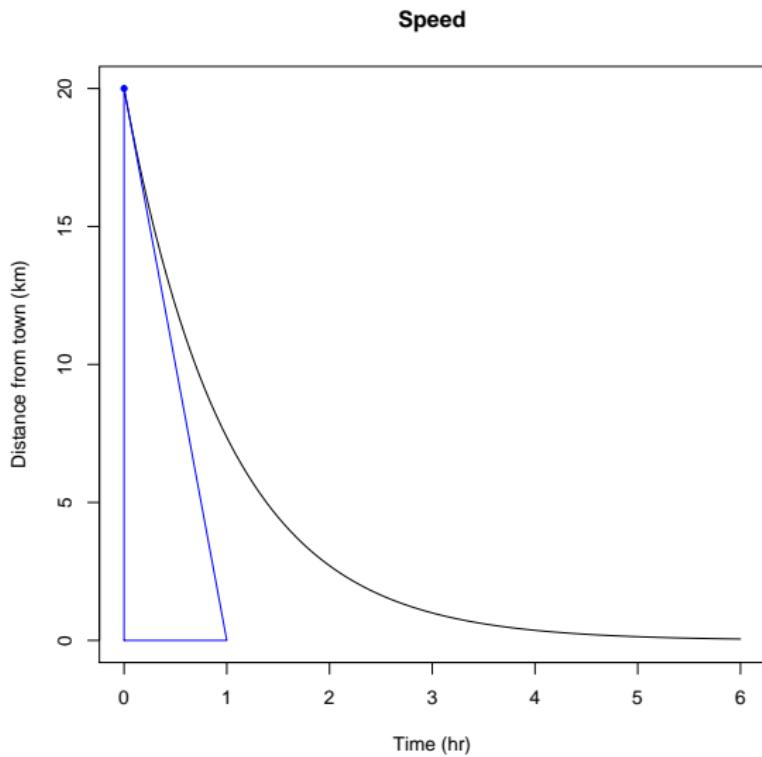
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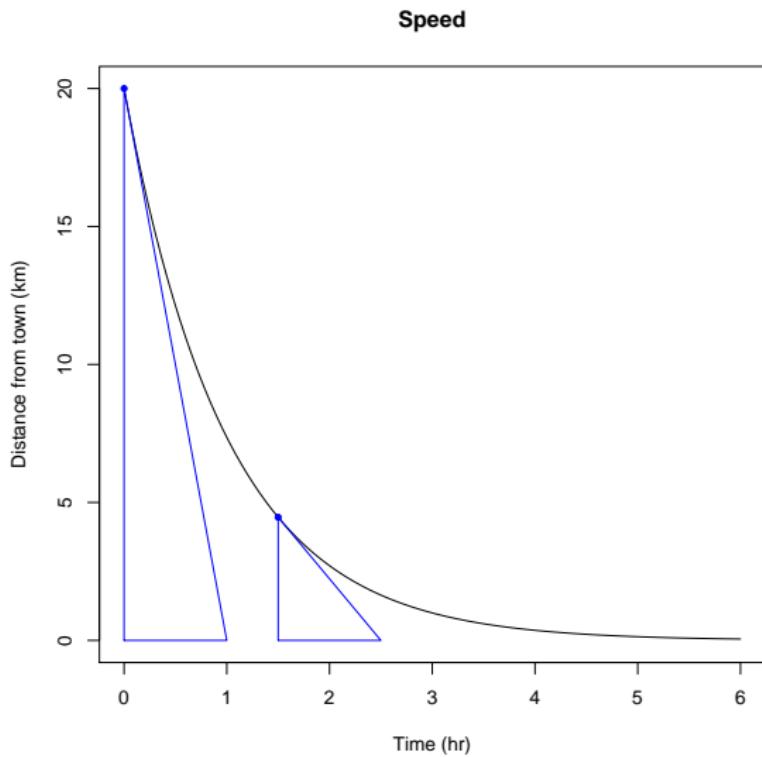
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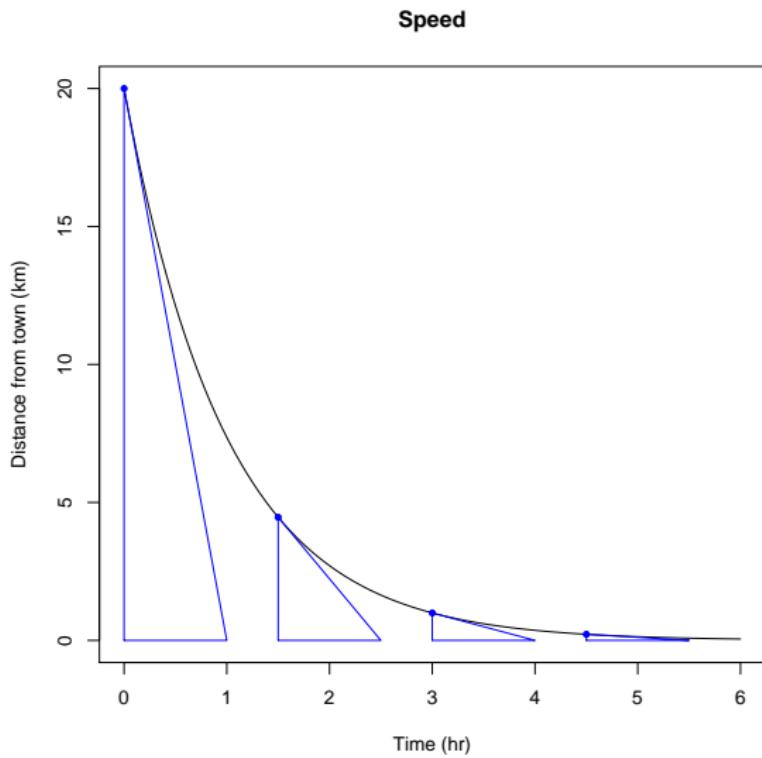
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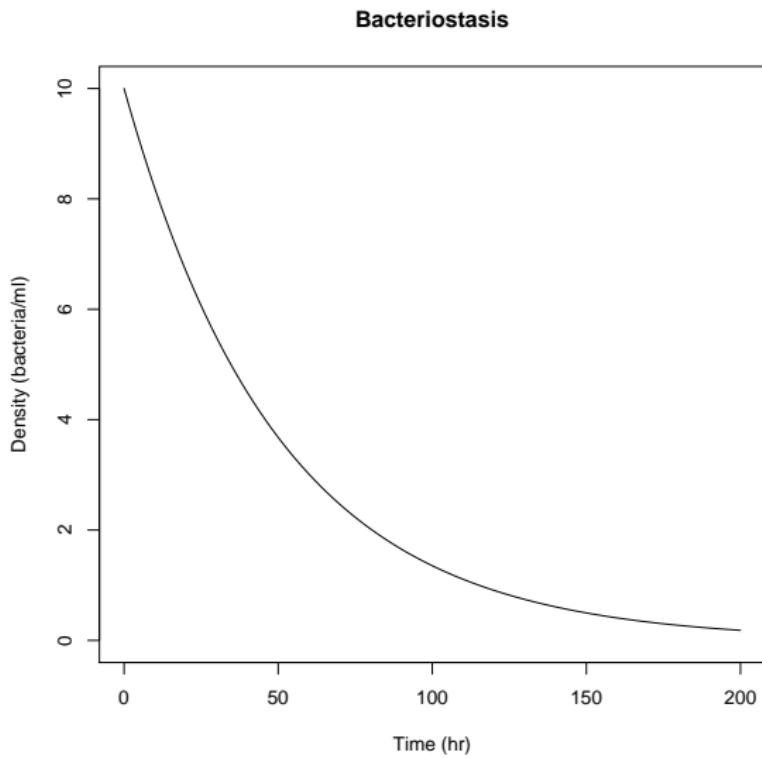
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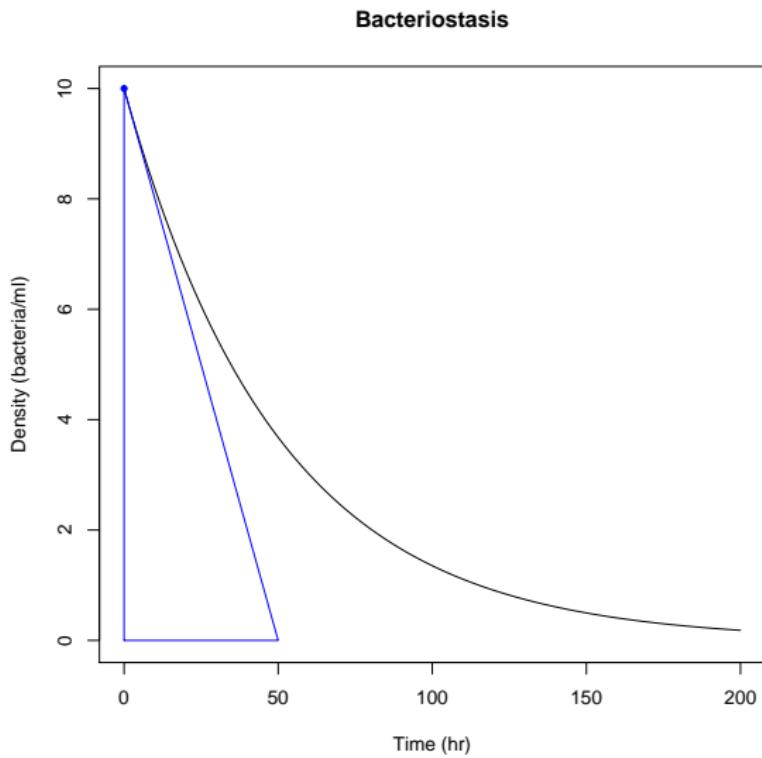
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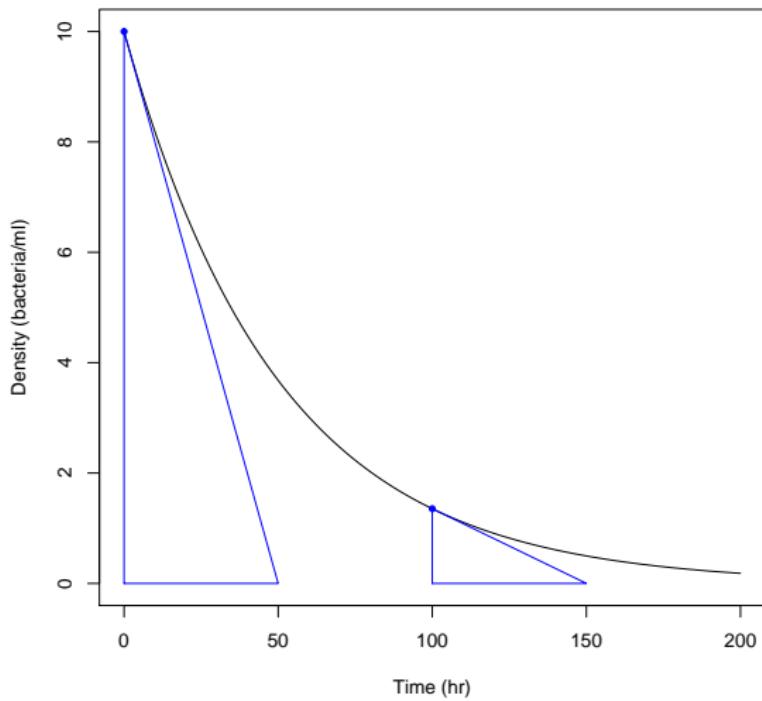


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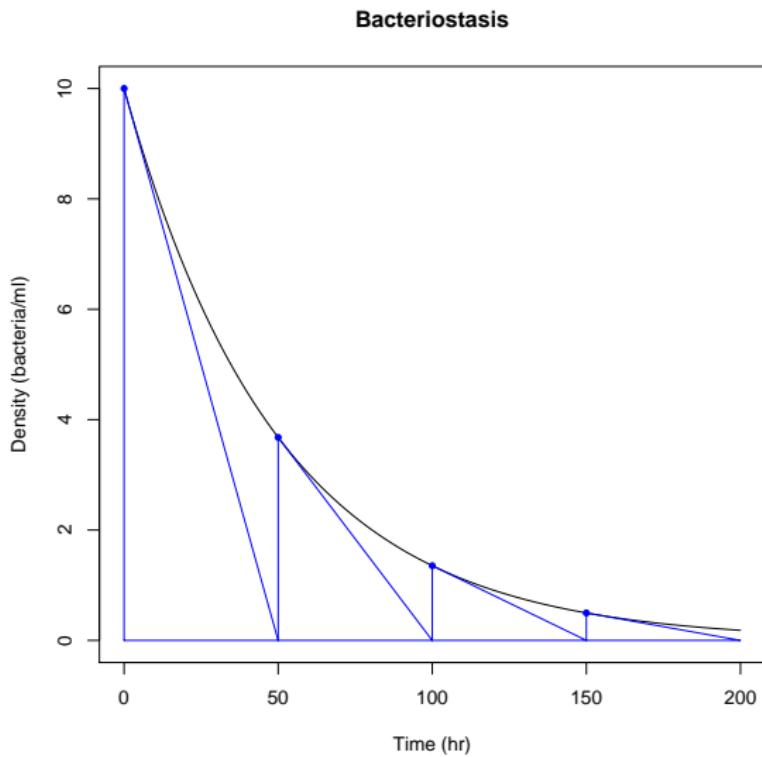


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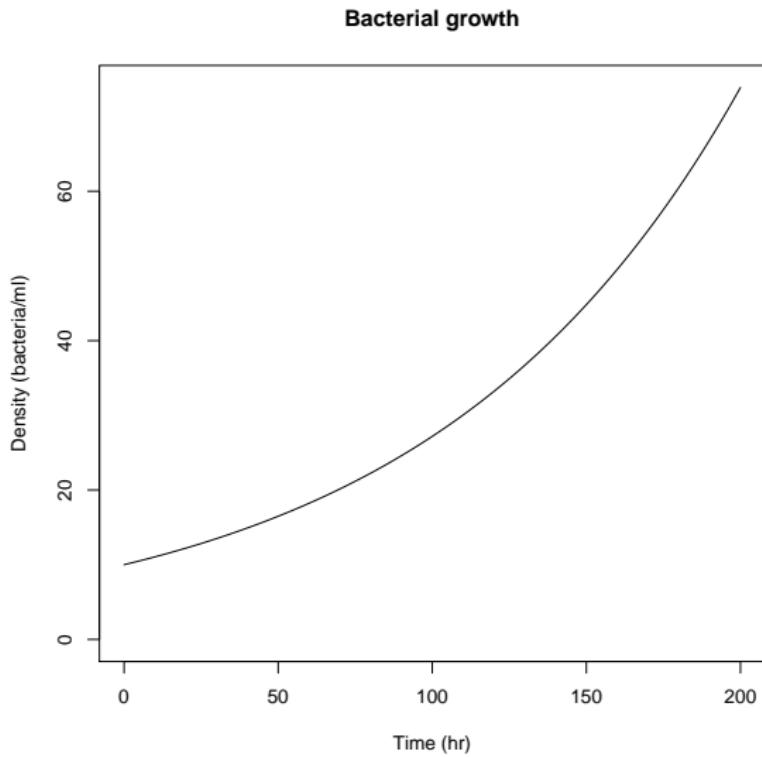
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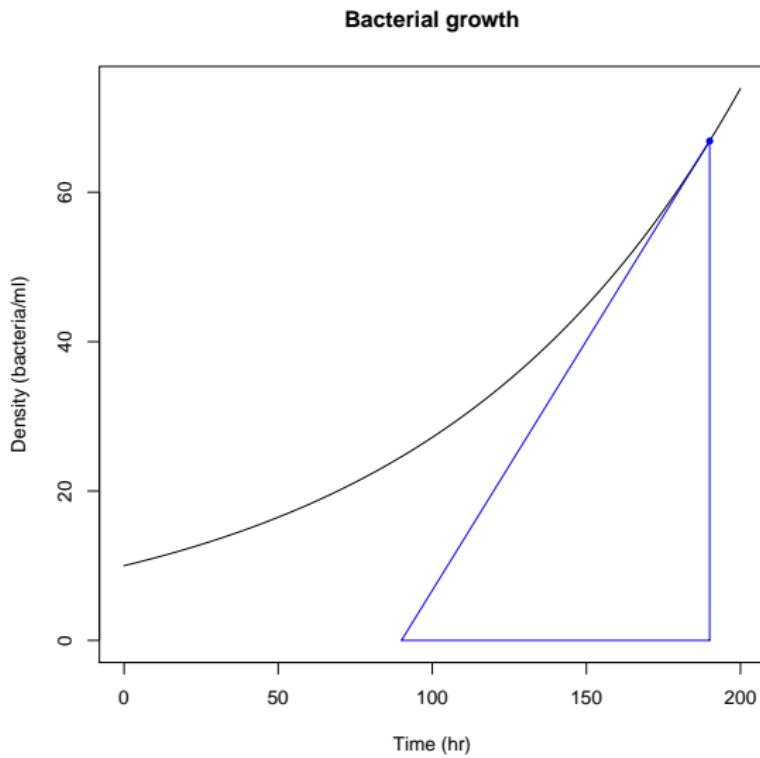
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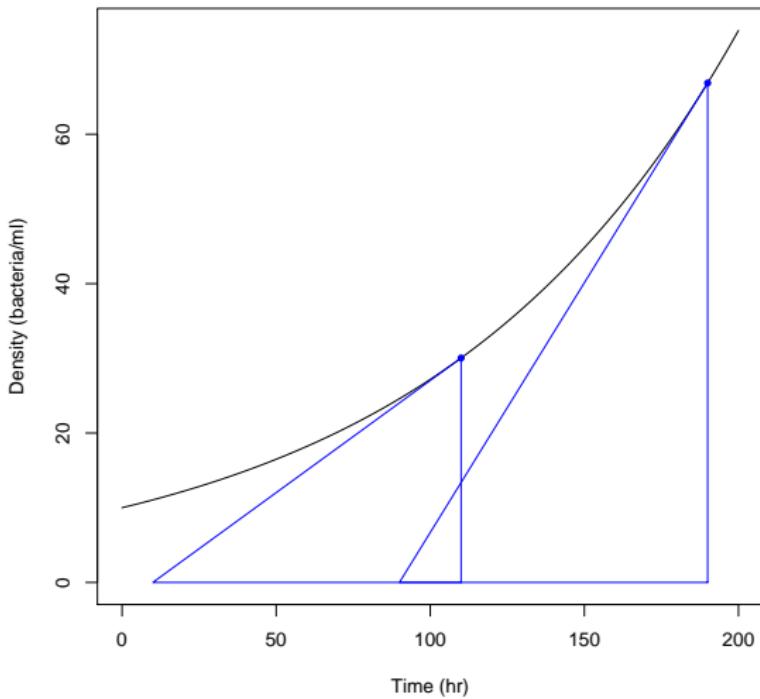


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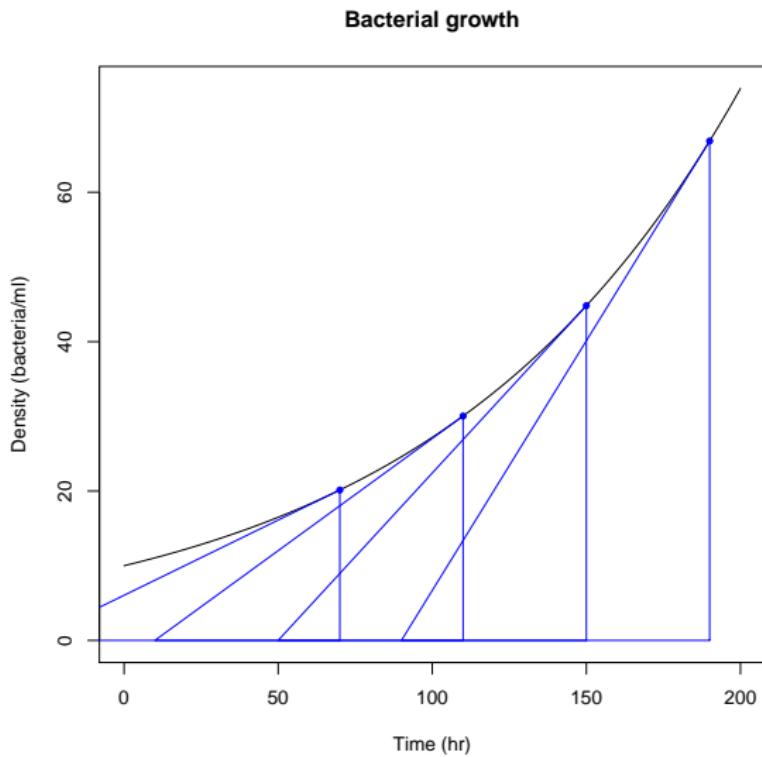


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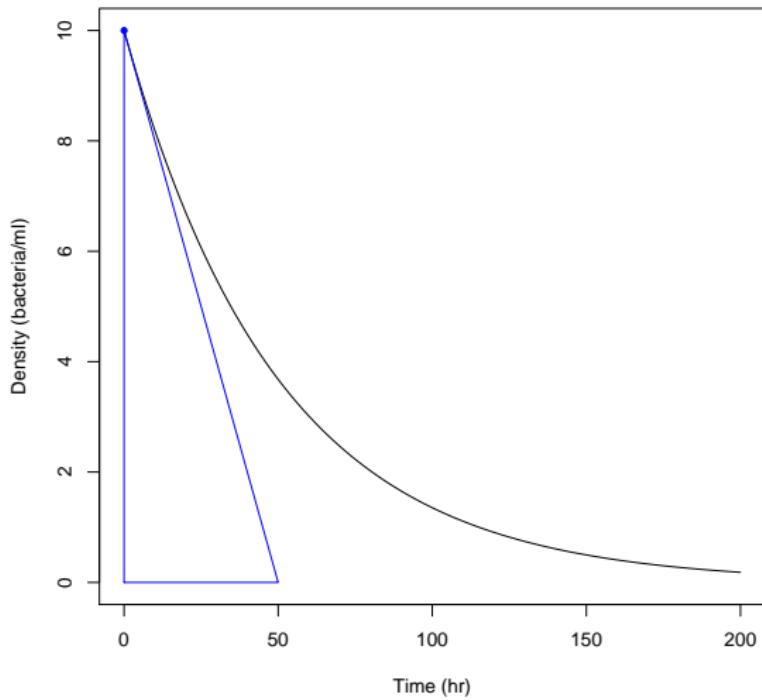
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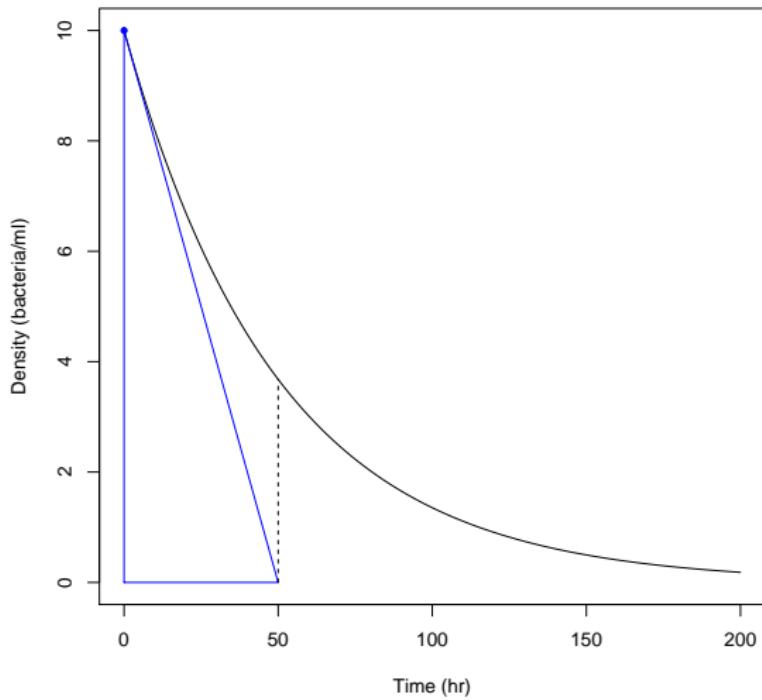
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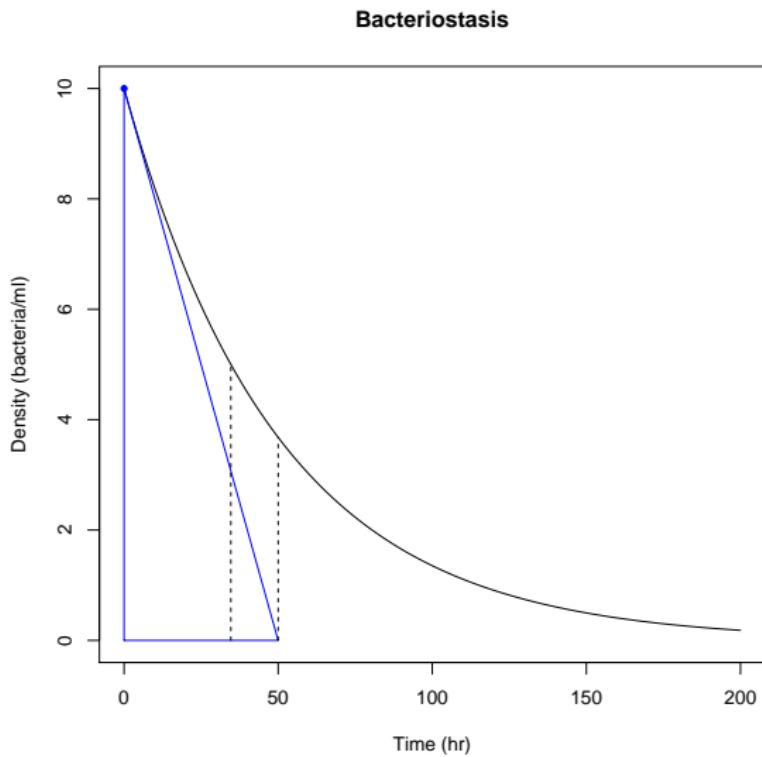
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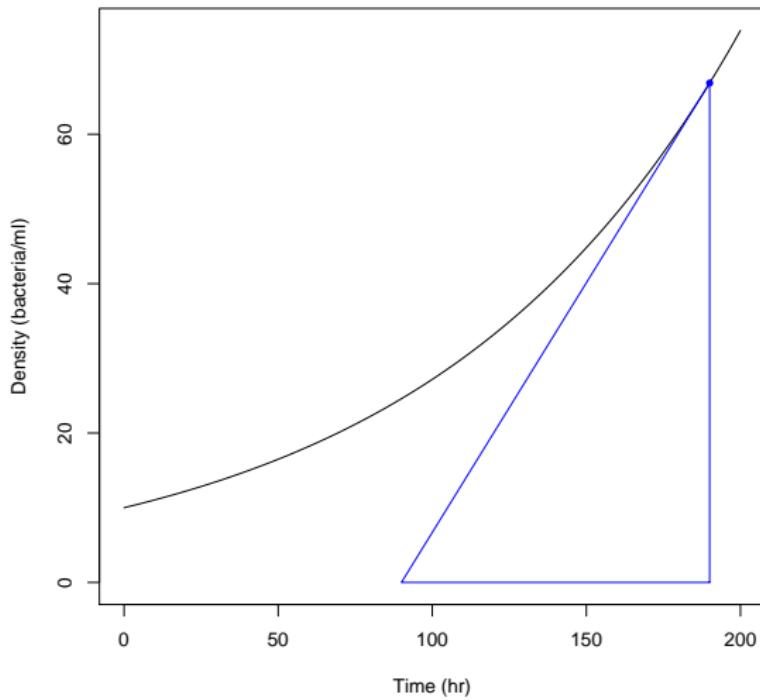
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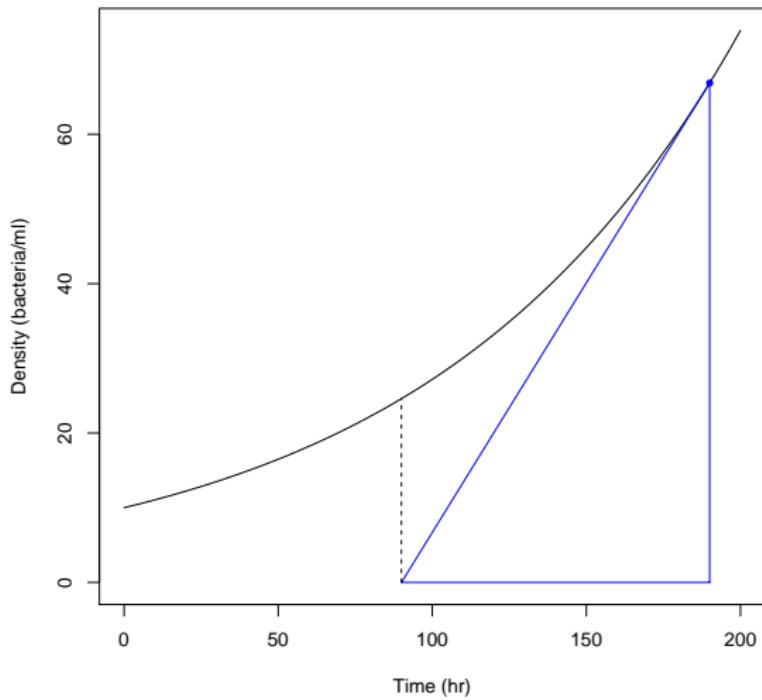
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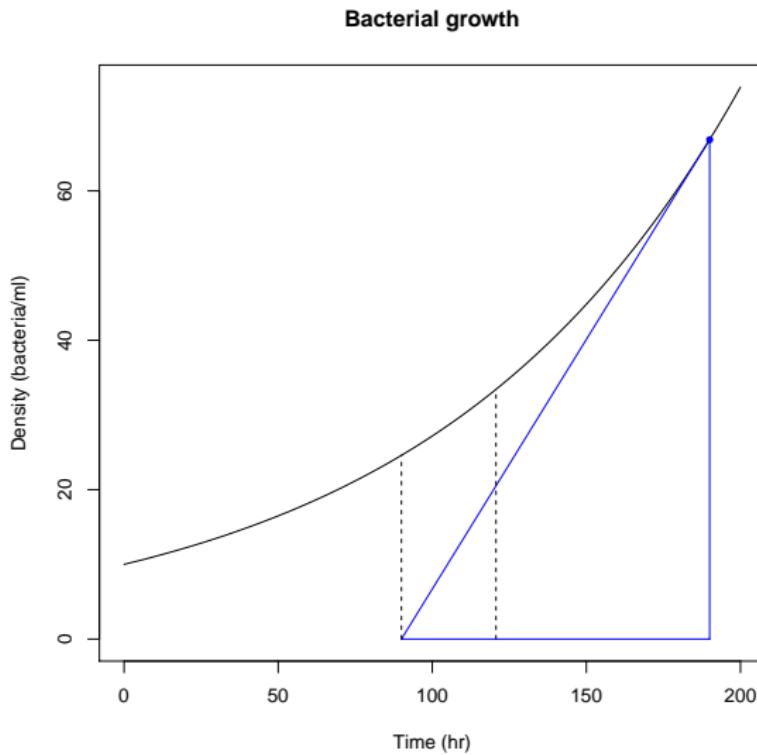
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