

UNIT 2: Linear population models

Outline

Constructing models

- Dynamical models

- Examples

- A simple discrete-time model

- A simple continuous-time model

Units and scaling

Key parameters

- Discrete-time model

- Continuous-time model

- Links

Growth and regulation

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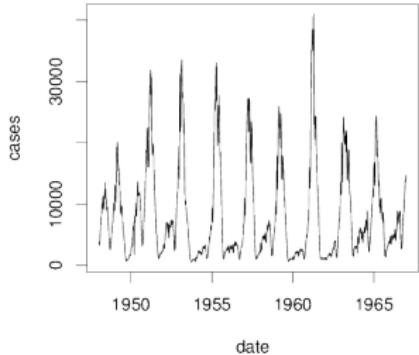
Dynamical models

Tools to link scales

- Models are what we use to link:



Measles reports from England and Wales



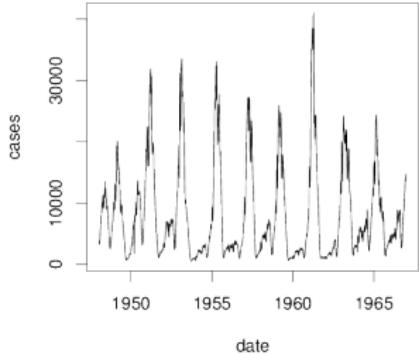
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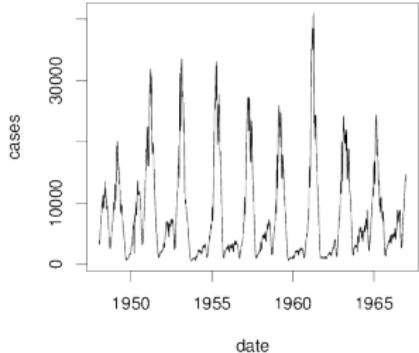
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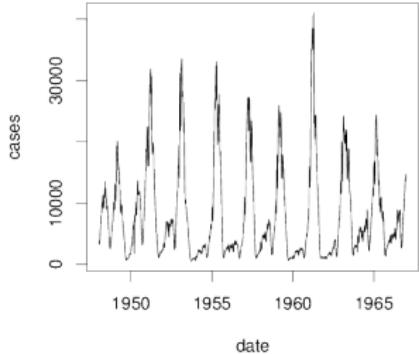
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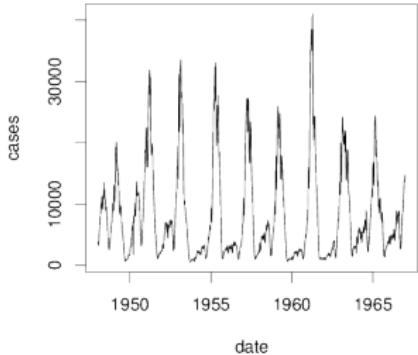
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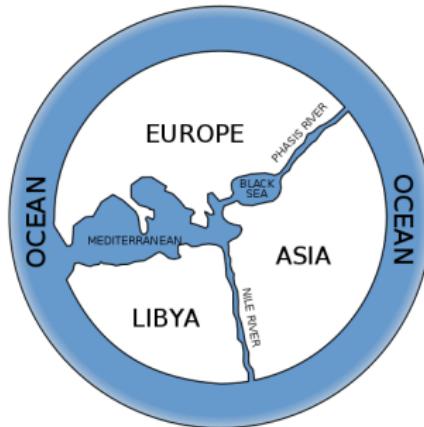


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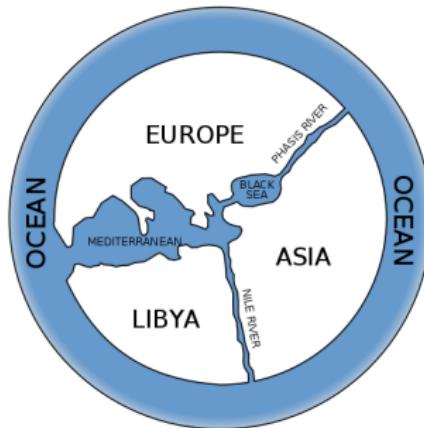
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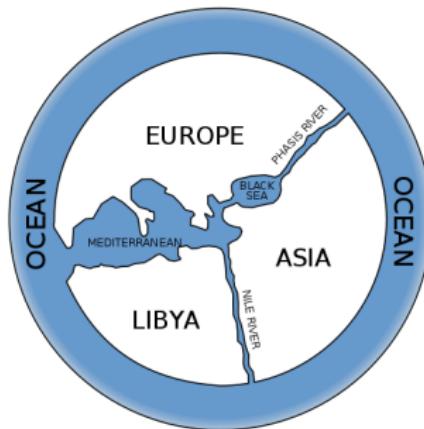
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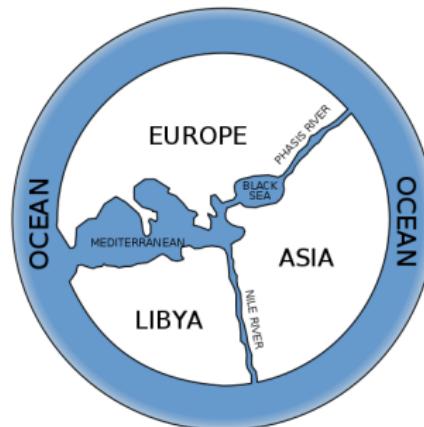
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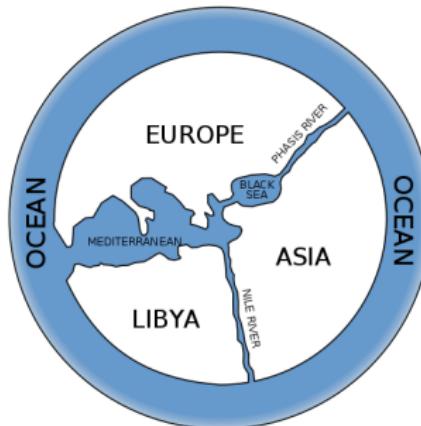
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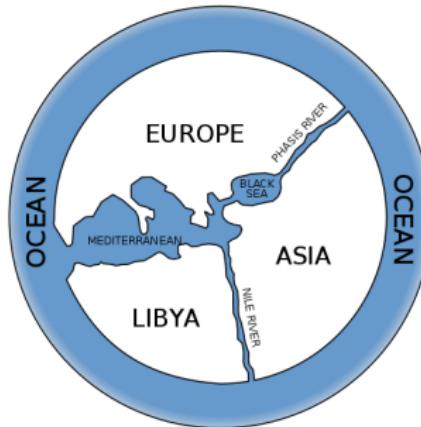
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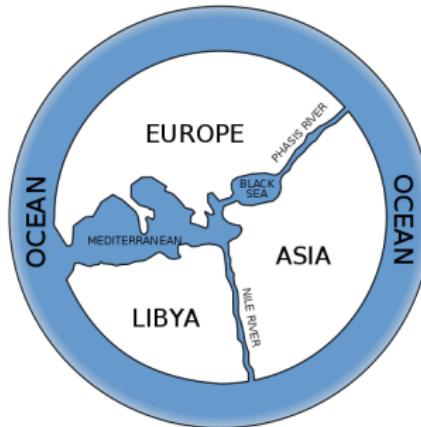
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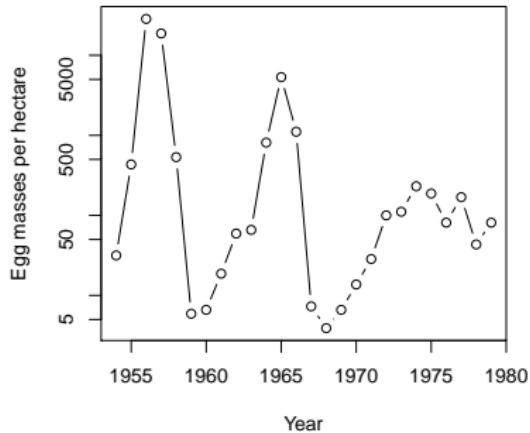
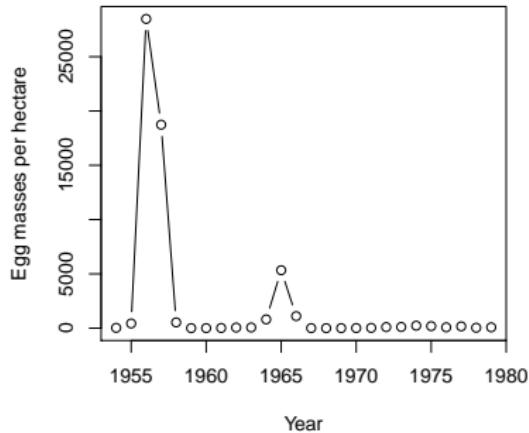


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Gypsy moth populations (repeat)



Moth example

- Poll: State variable



Moth example

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Moth example

- ▶ Poll: State variable
 - ▶ * Number of moths/ha



Moth example

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Moth example

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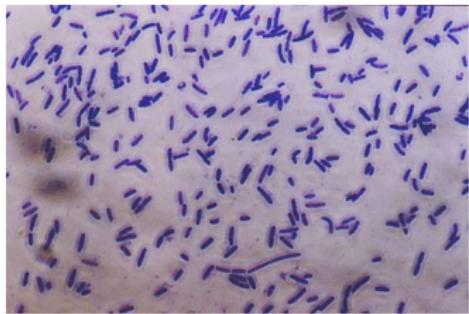
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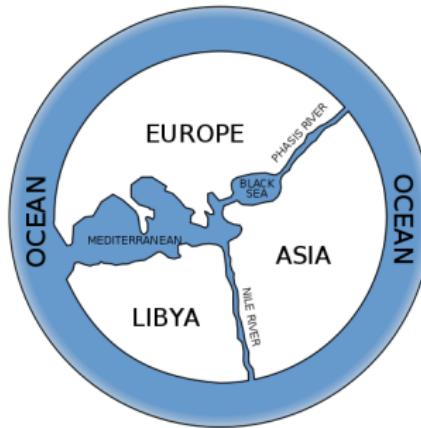
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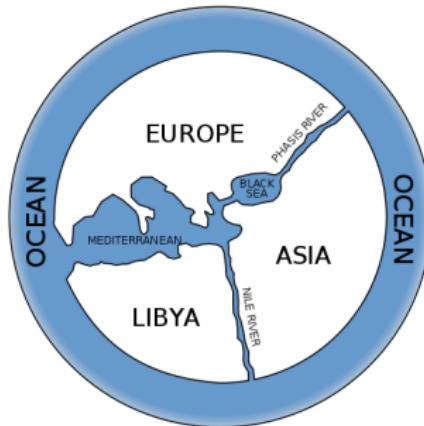
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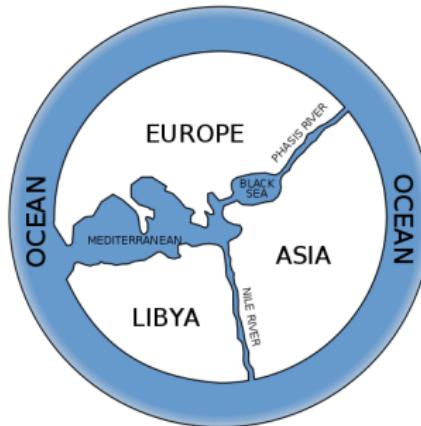
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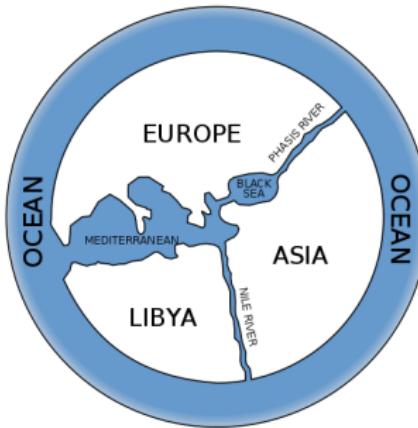
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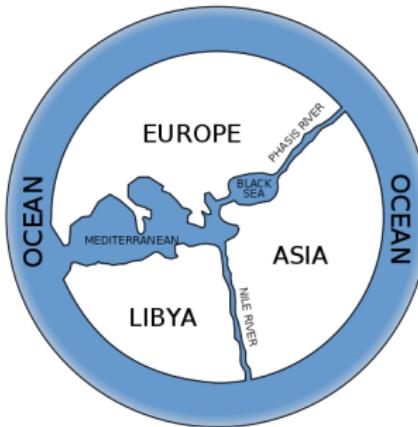
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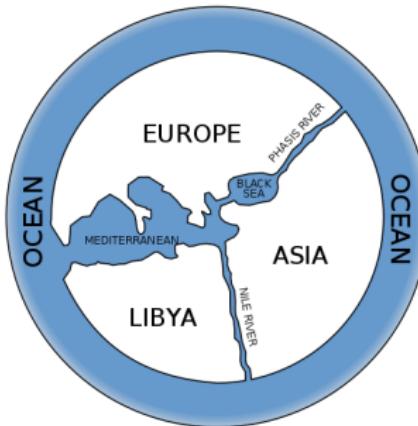
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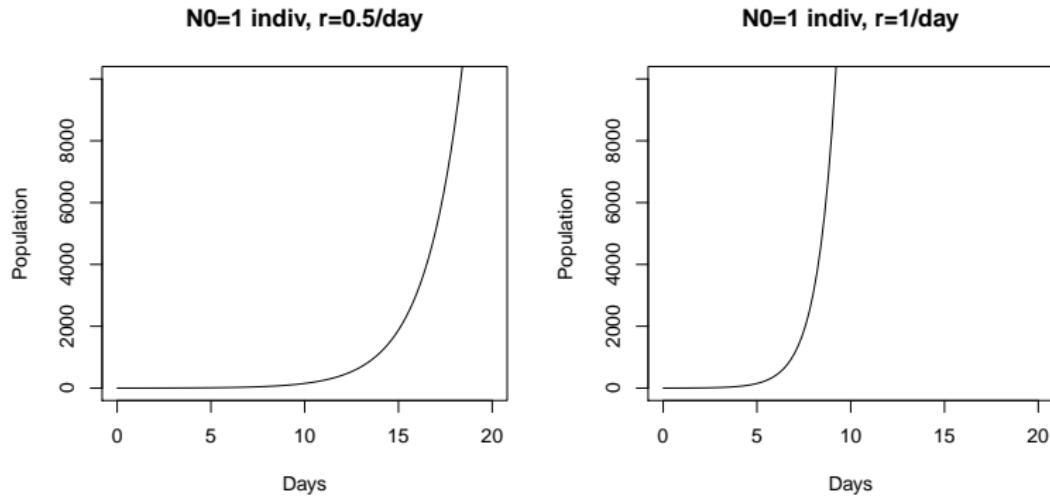
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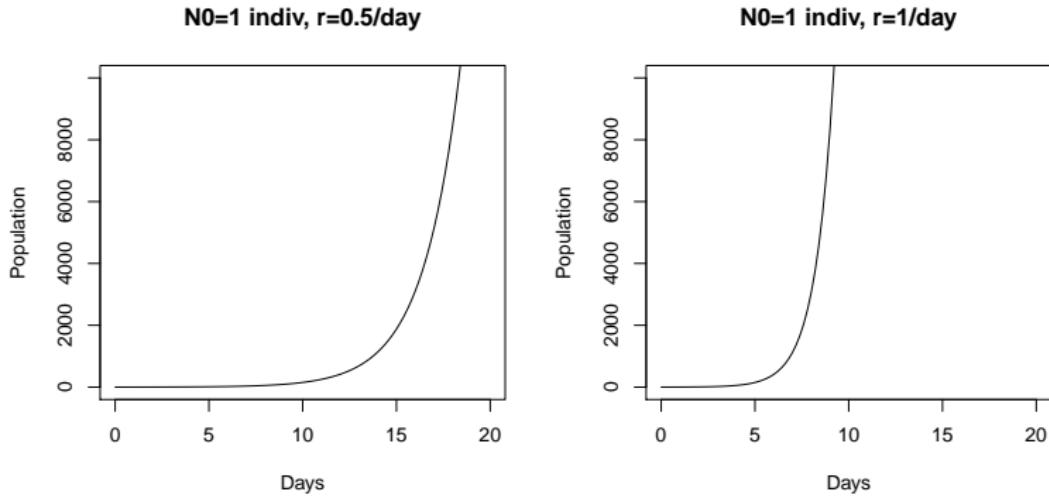
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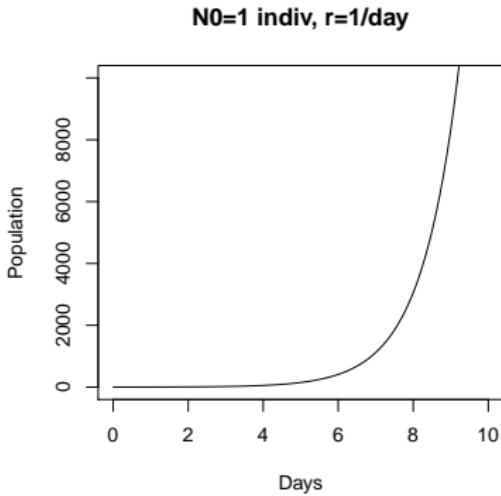
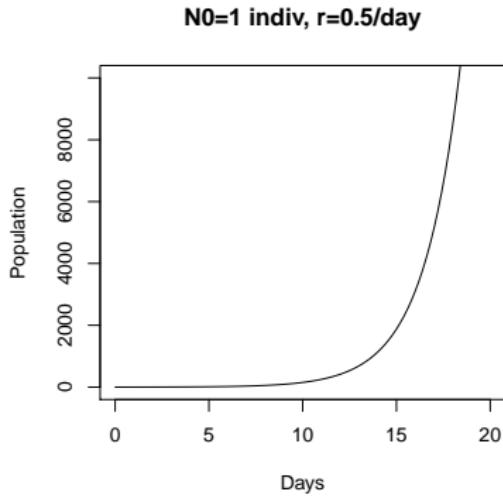
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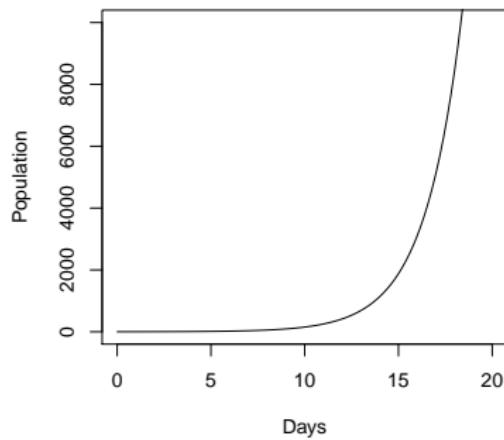


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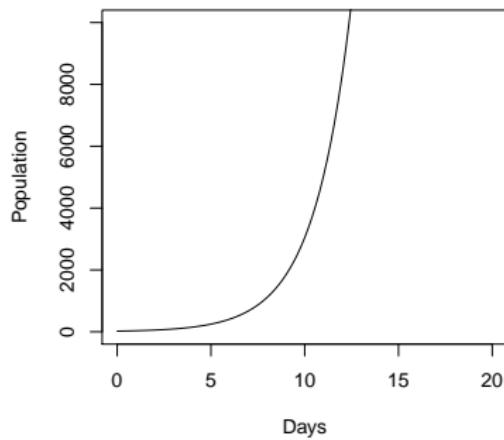


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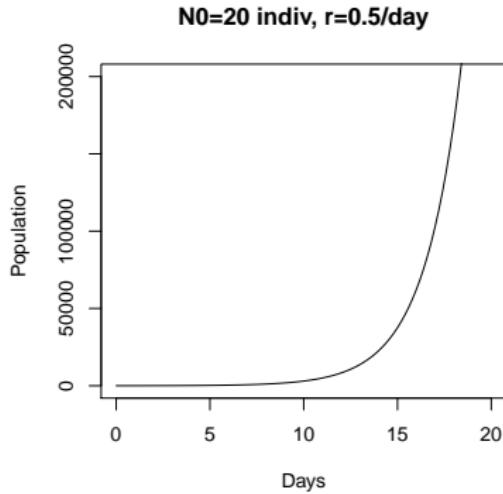
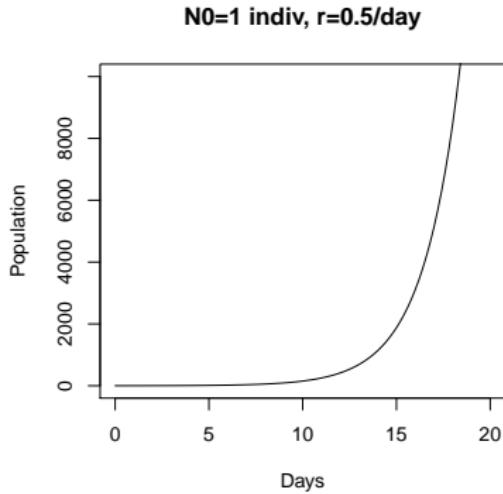
$N_0=1$ indiv, $r=0.5/\text{day}$



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Outline

Constructing models

Dynamical models

Examples

A simple discrete-time model

A simple continuous-time model

Units and scaling

Key parameters

Discrete-time model

Continuous-time model

Links

Growth and regulation

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