

# UNIT 8: Infectious disease

# Outline

## Introduction

Rate of spread

Single-epidemic model

Epidemic size

Recurrent epidemic models

Dynamics

Reproductive numbers and risk

Pathogen aggressiveness

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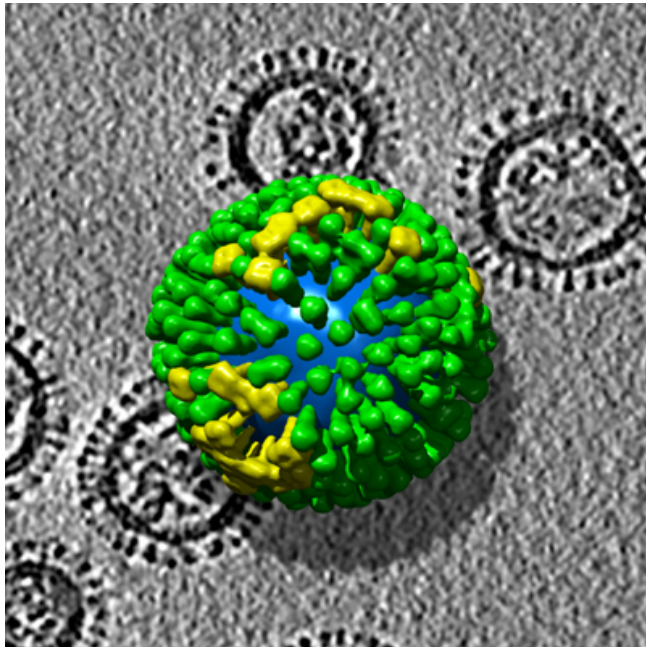
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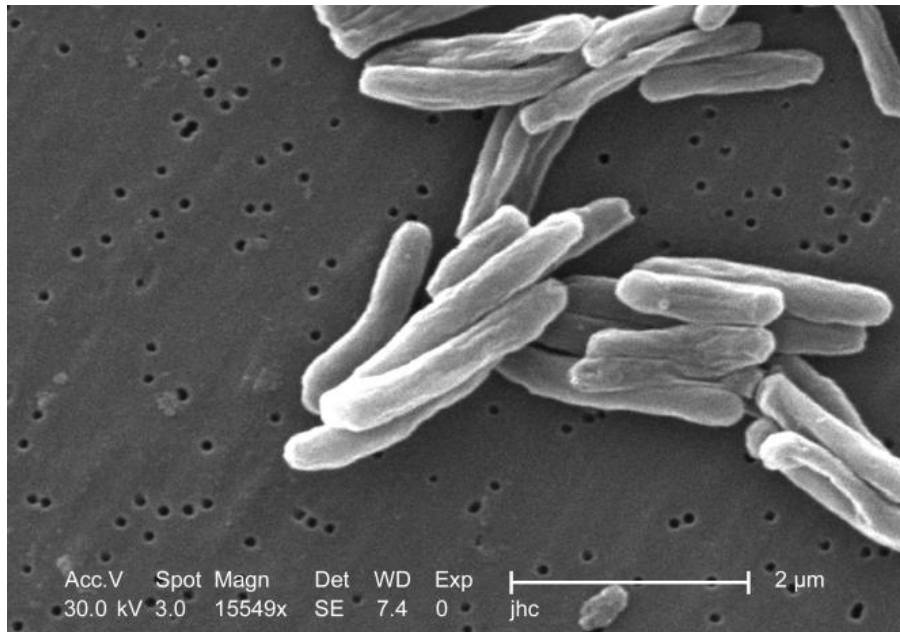
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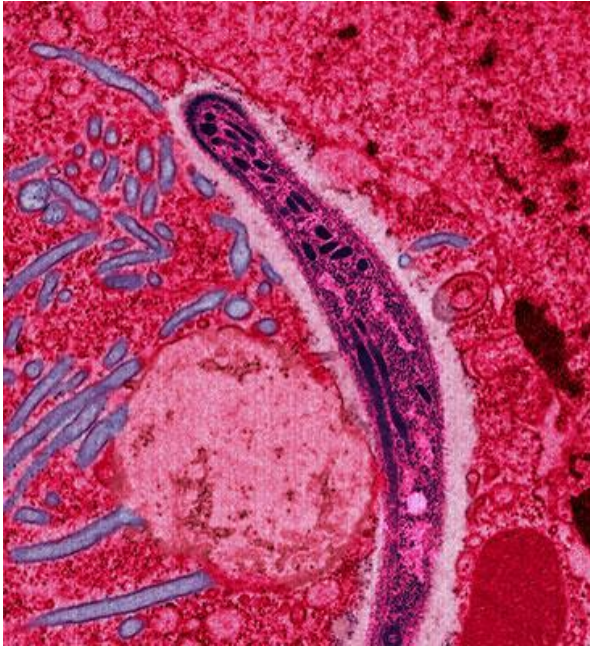
# Influenza virus



# Tuberculosis bacilli



# Malaria sporozoite





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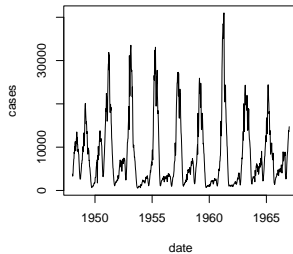
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**Measles reports from England and Wales**

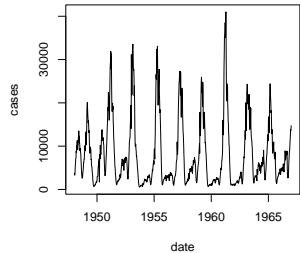


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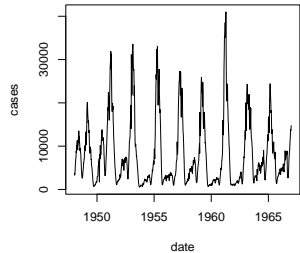


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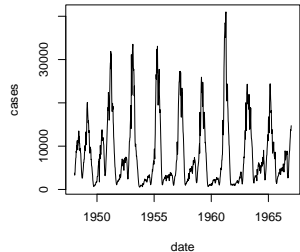


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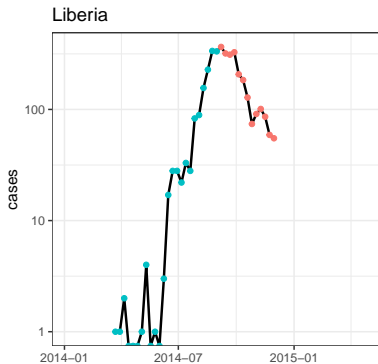
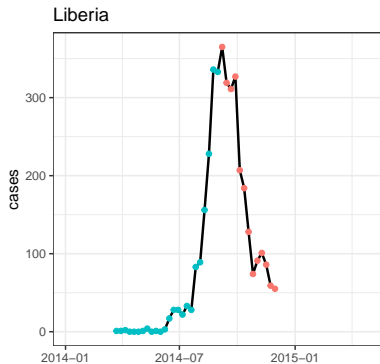
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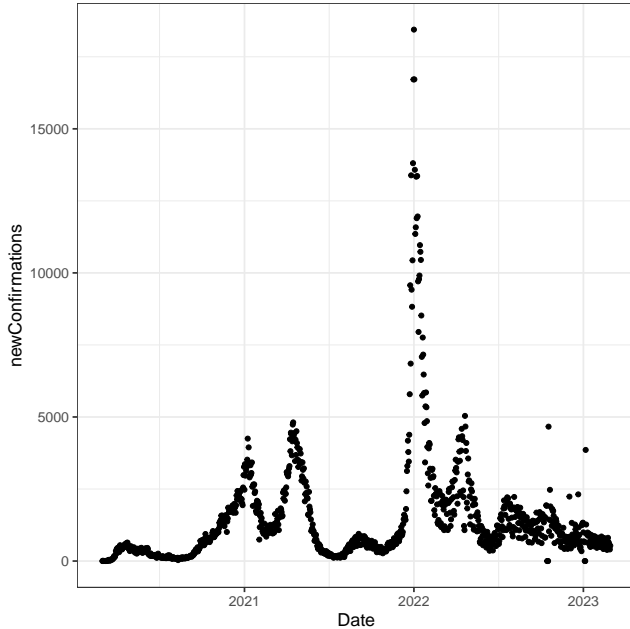
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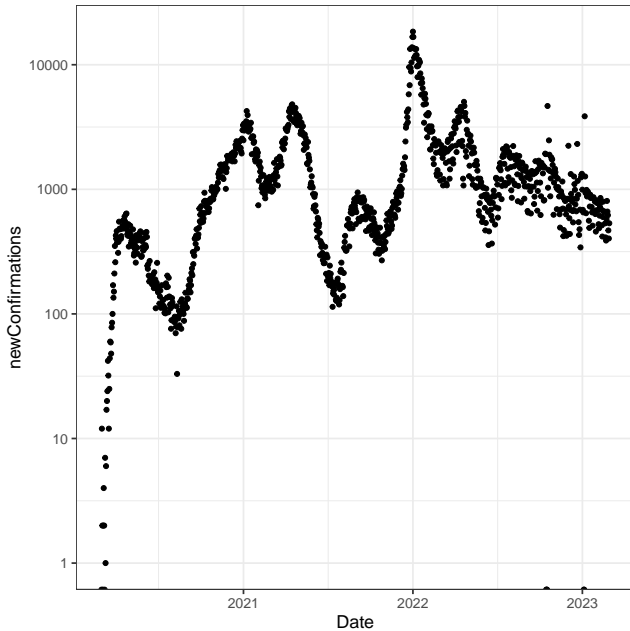
# Example: the West African Ebola epidemic



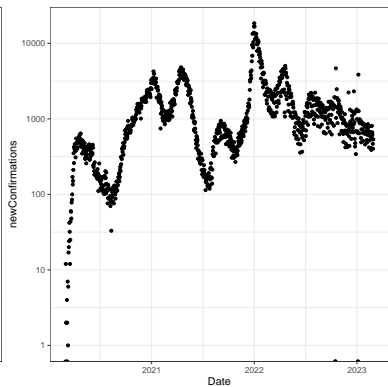
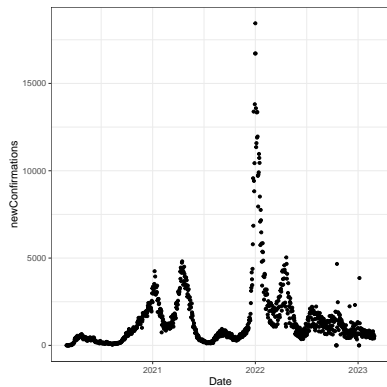
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    - ▶ \* Rate of death, recovery, or effective quarantine
- ▶ How do you think we estimate?
  - ▶ \* We estimate  $r$  from the population-level increase in disease
    - ▶ \* Then we use that to estimate  $b = d + r$
  - ▶ \* Models go both directions!
    - ▶ Individuals  $\leftrightarrow$  Populations

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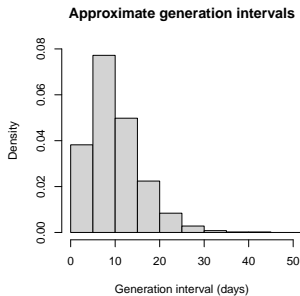
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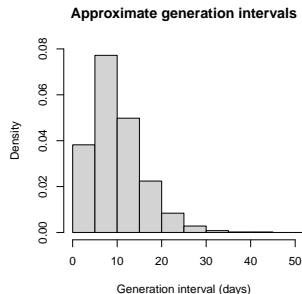
- Researchers try to estimate the *proportion* of transmission that happens for different **ages of infection**





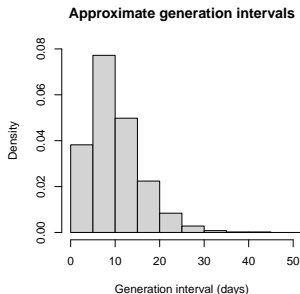
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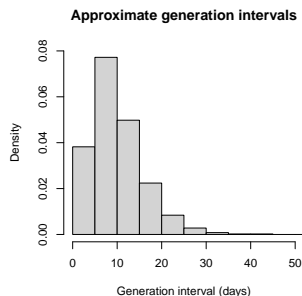
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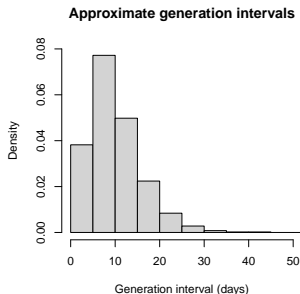
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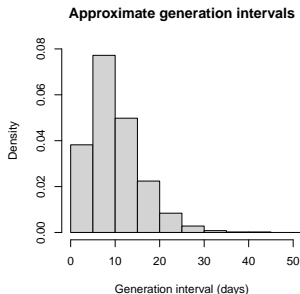
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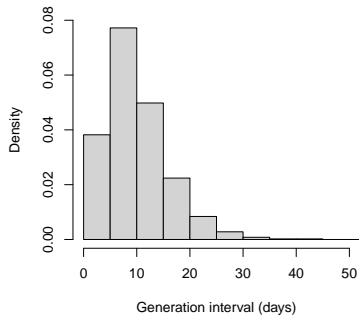
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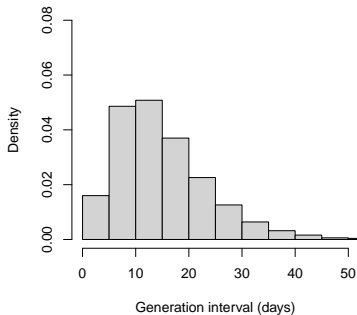


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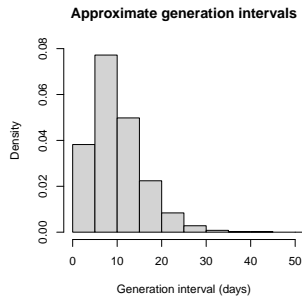


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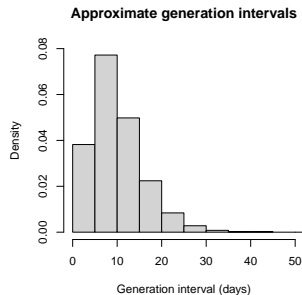
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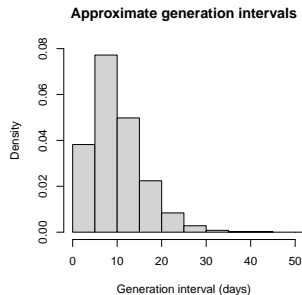
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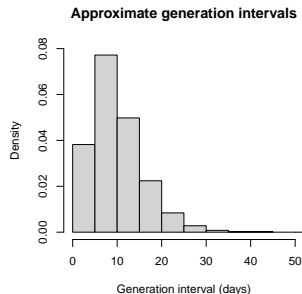
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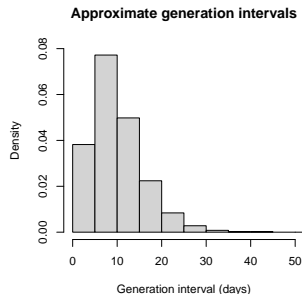
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# Fighting Ebola



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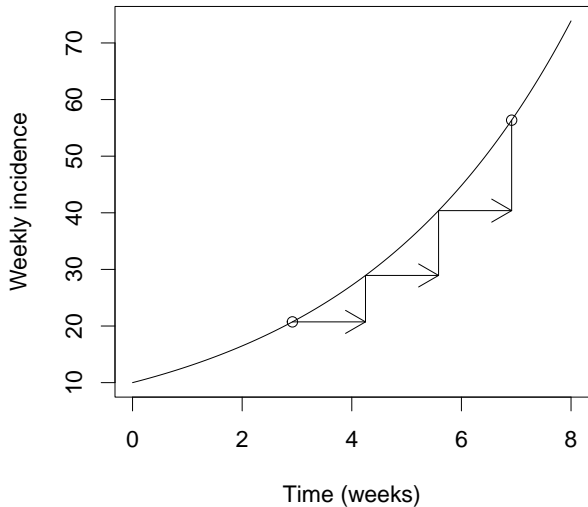
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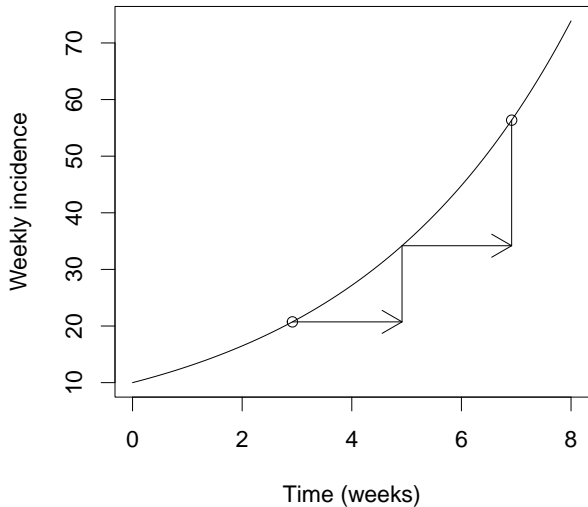
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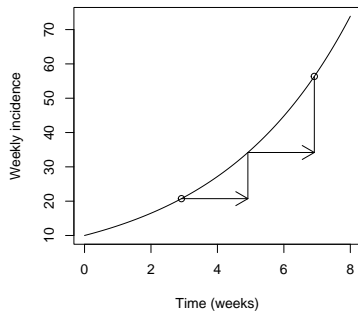
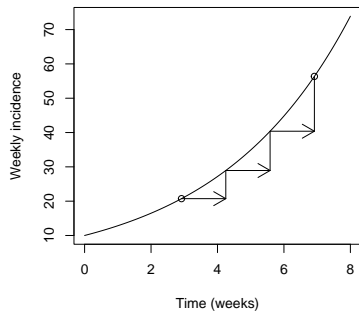
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- ▶  $\mathcal{R} = \exp(r\hat{G})$
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# Outline

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Rate of spread

Single-epidemic model

Epidemic size

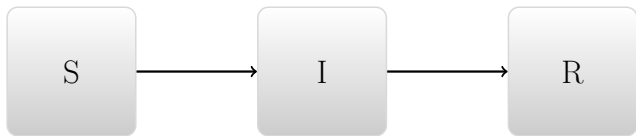
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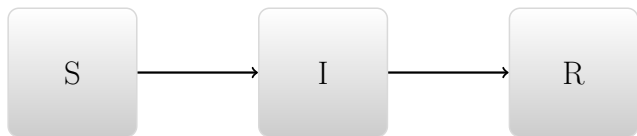
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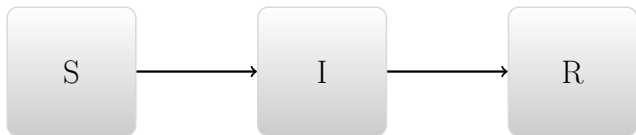
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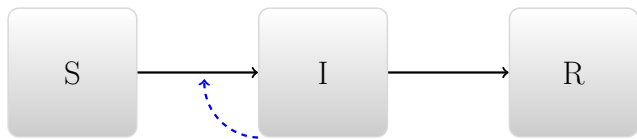
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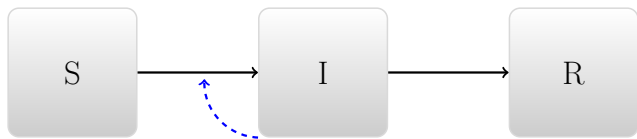
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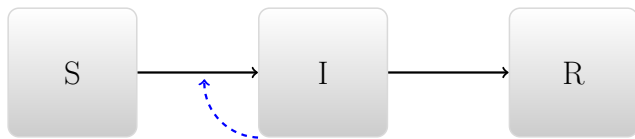
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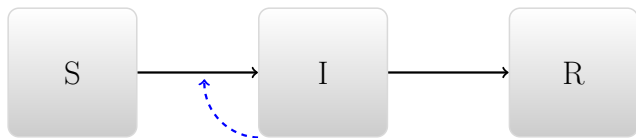


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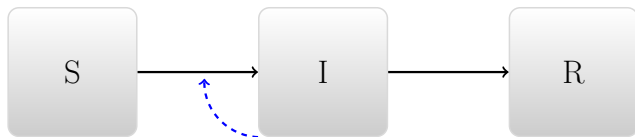
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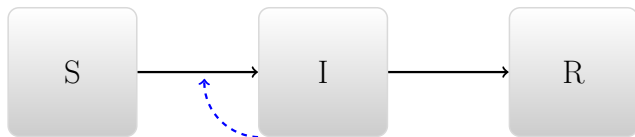


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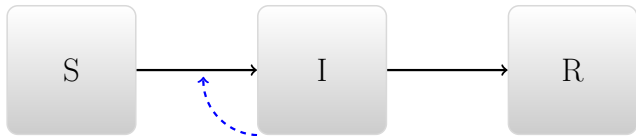
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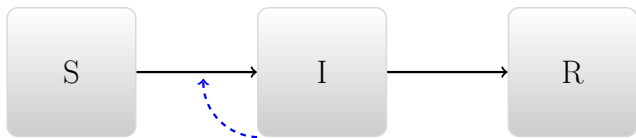
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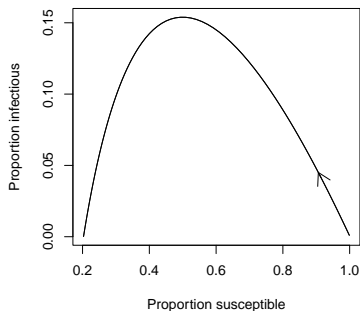
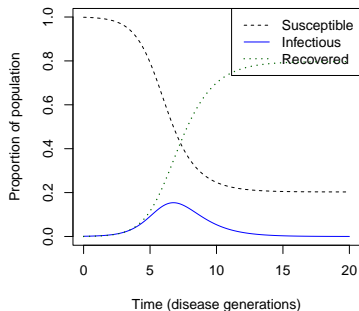
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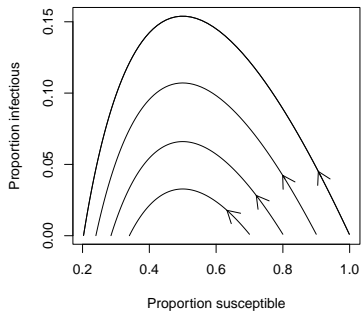
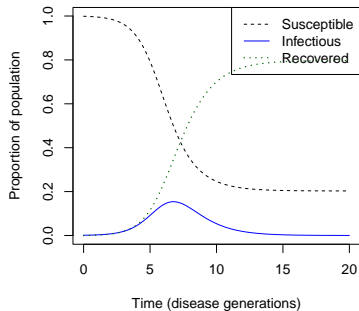
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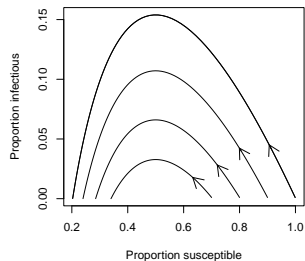
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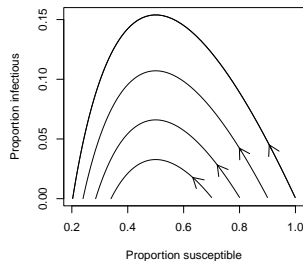
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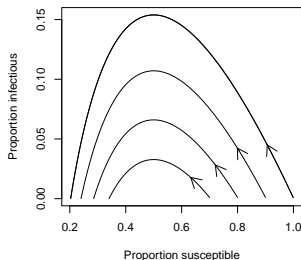
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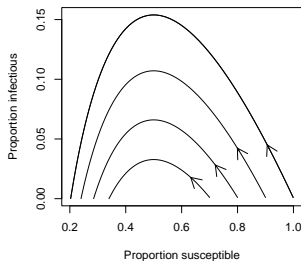


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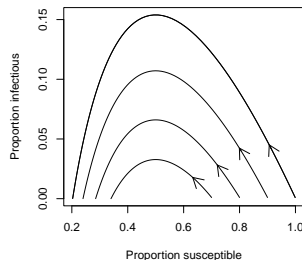
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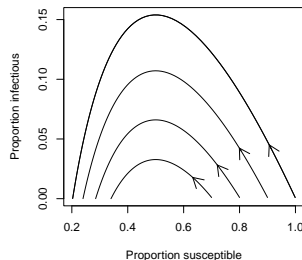
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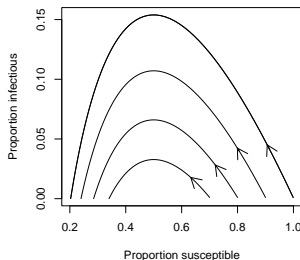
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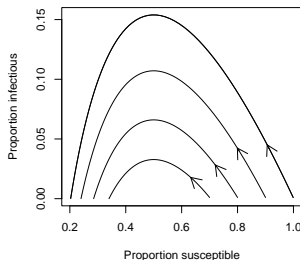
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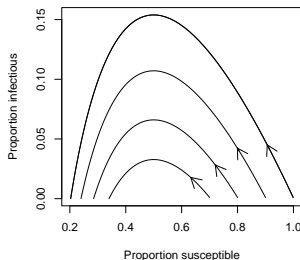
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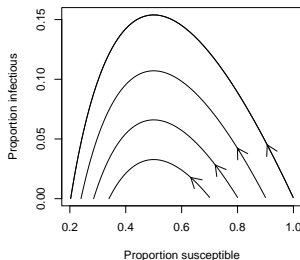
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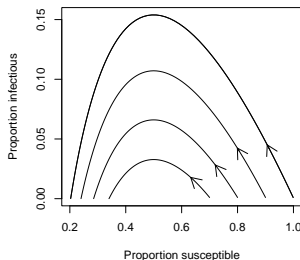
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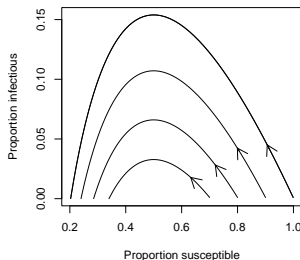
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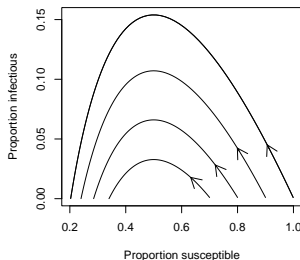
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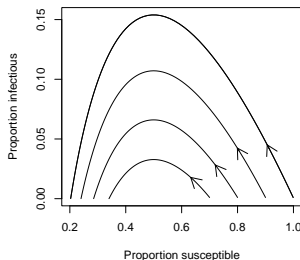
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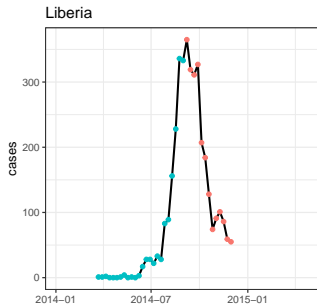
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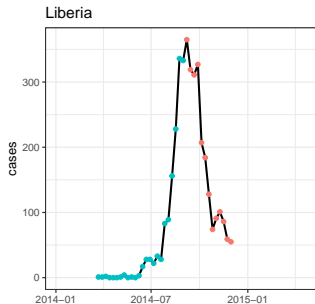
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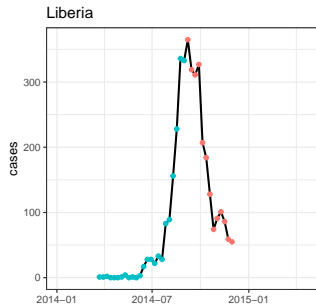
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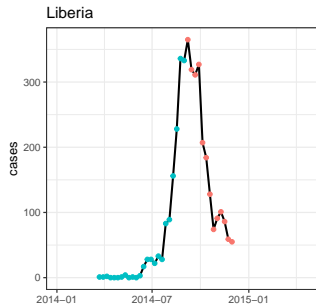
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Rate of spread

Single-epidemic model

Epidemic size

Recurrent epidemic models

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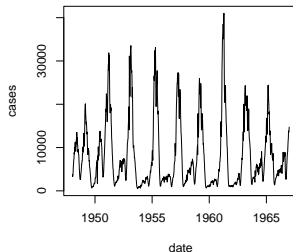
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**Measles reports from England and Wales**

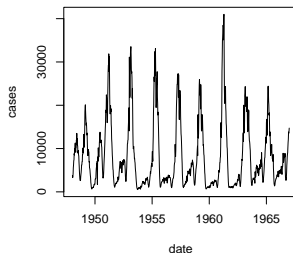


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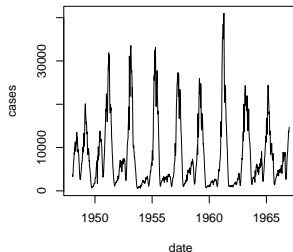
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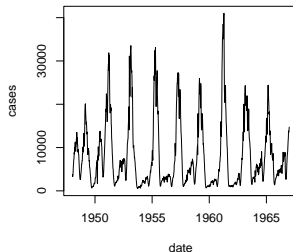
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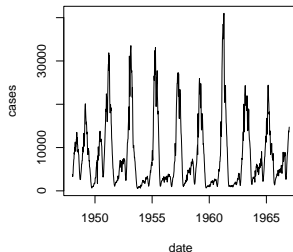
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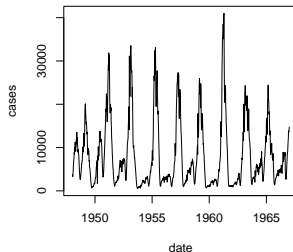
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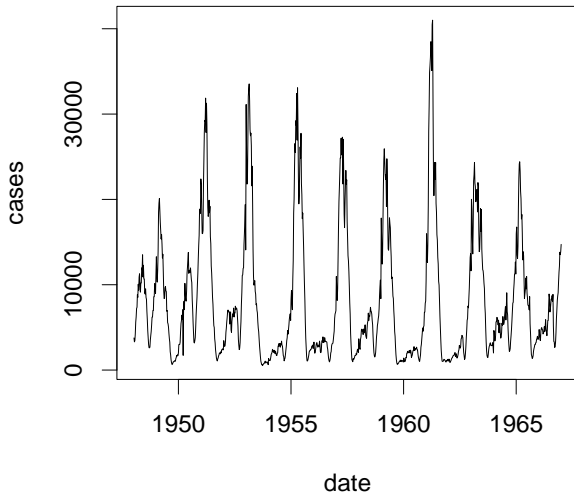
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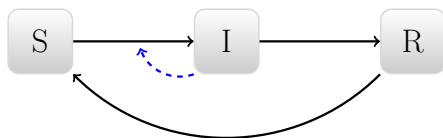


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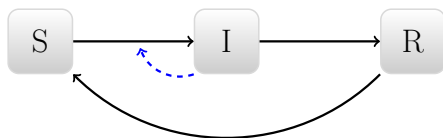
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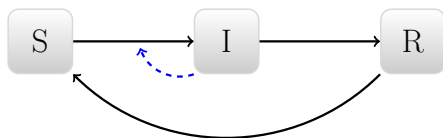


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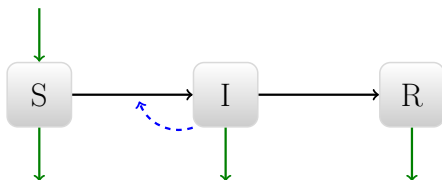
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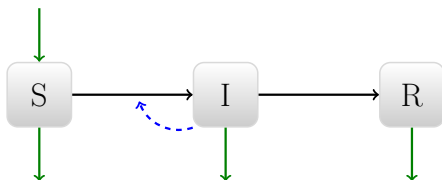


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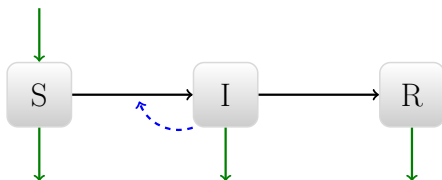


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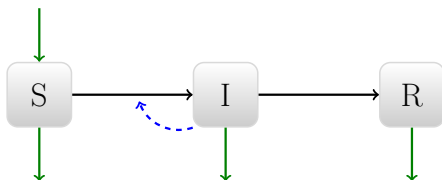
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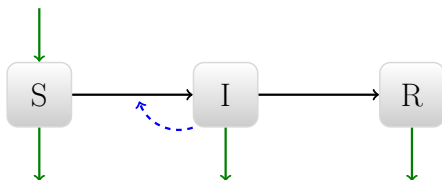
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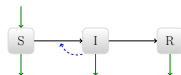


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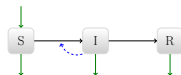
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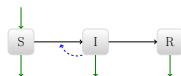
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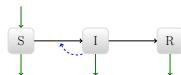
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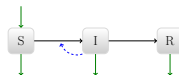
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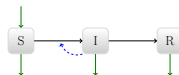
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- ▶ As susceptibles go up, what happens?
  - ▶ Per capita replenishment goes down
  - ▶ Infections required goes down

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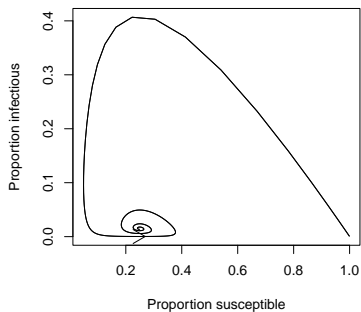
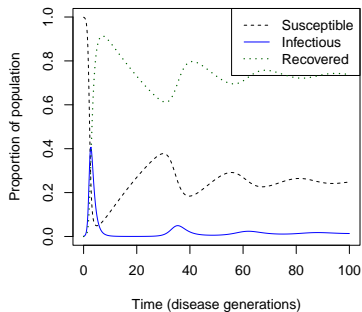
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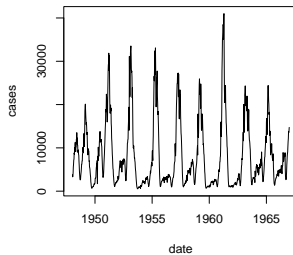
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**Measles reports from England and Wales**

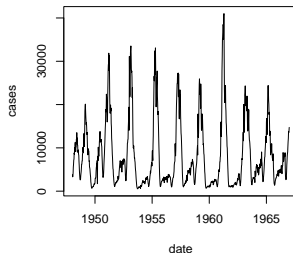


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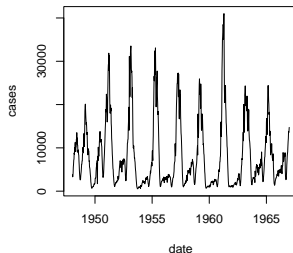
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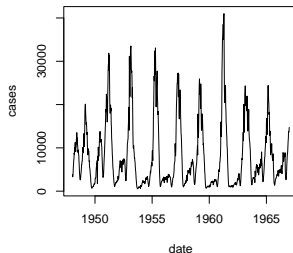
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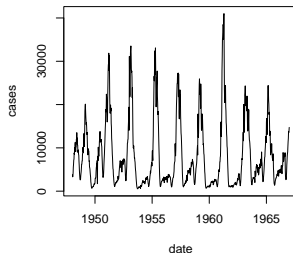
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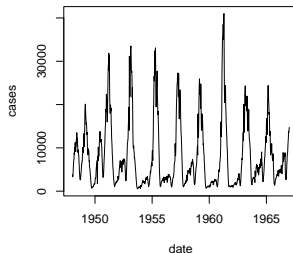


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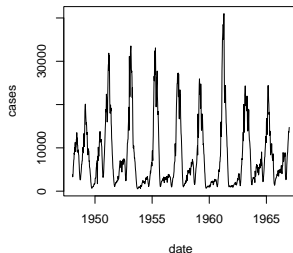
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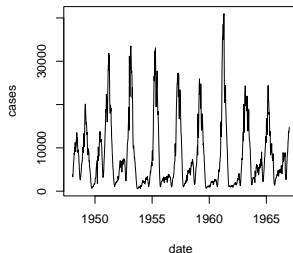
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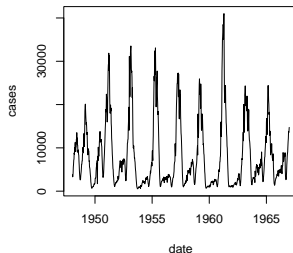
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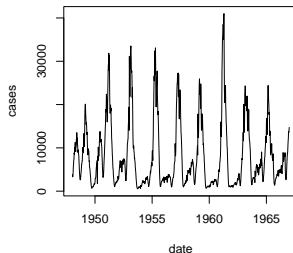


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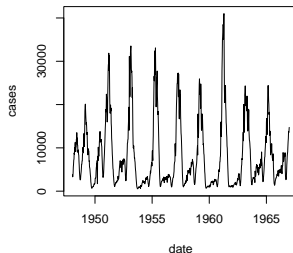
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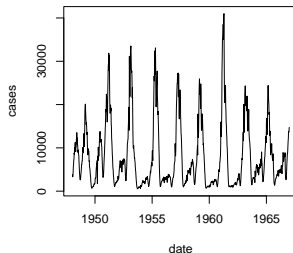
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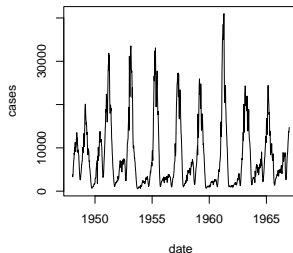
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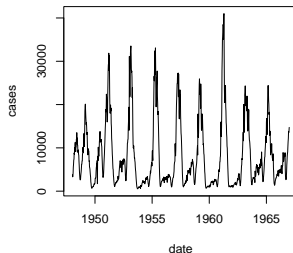
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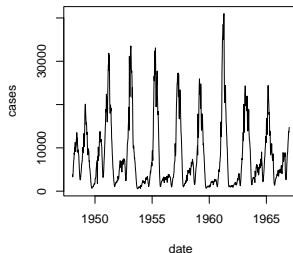
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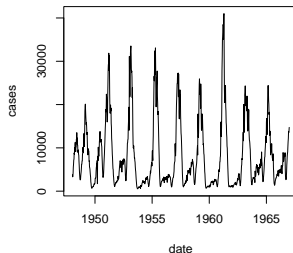


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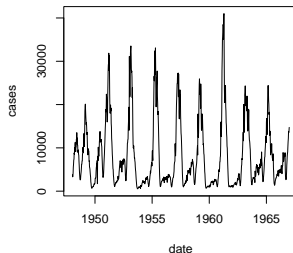
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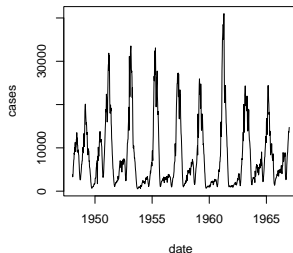
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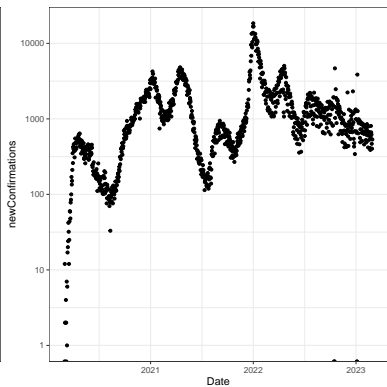
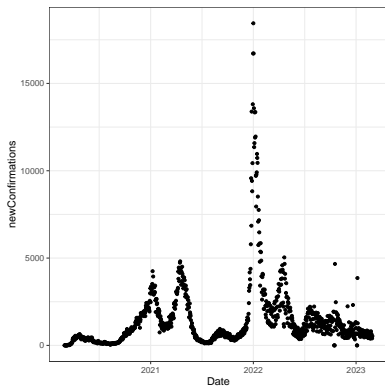
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# COVID in Ontario



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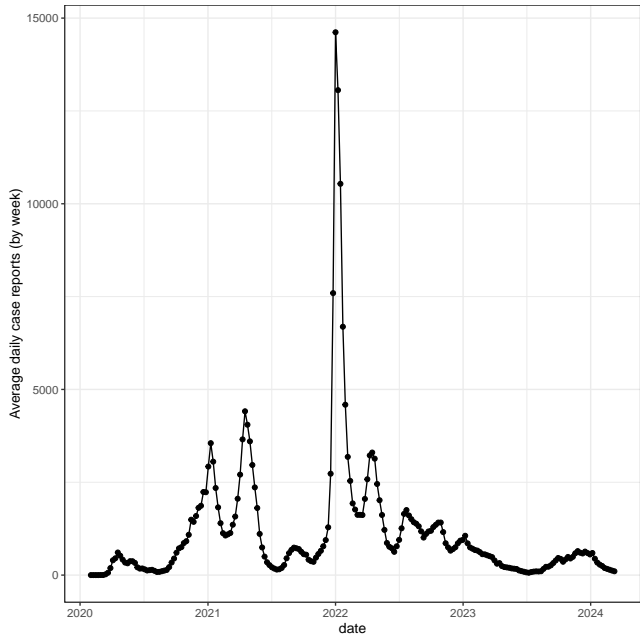
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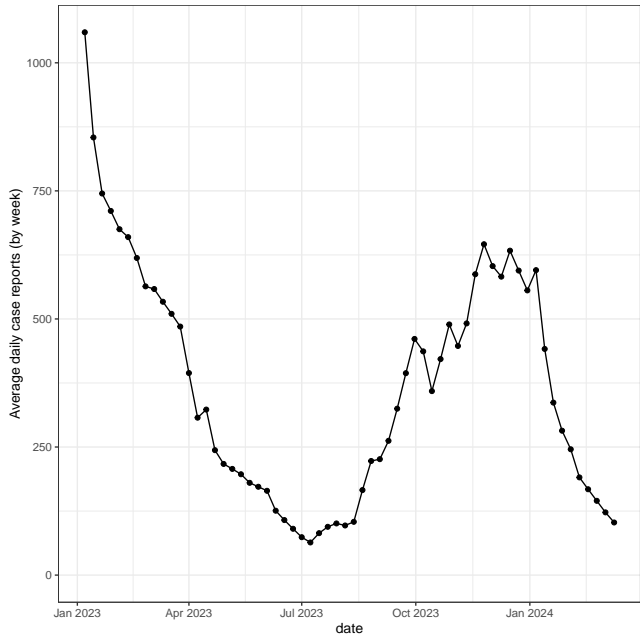
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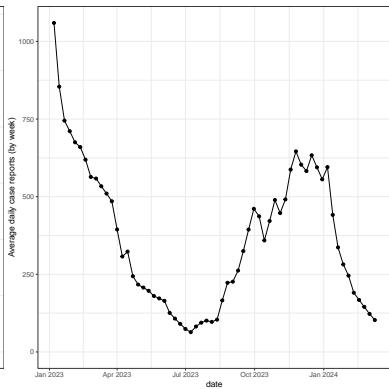
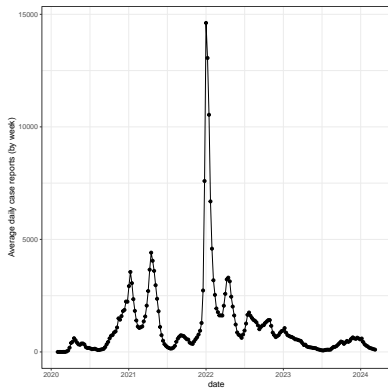
# Post-pandemic COVID (preview)



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# Outline

Introduction

Rate of spread

Single-epidemic model

Epidemic size

Recurrent epidemic models

Dynamics

Reproductive numbers and risk

Pathogen aggressiveness

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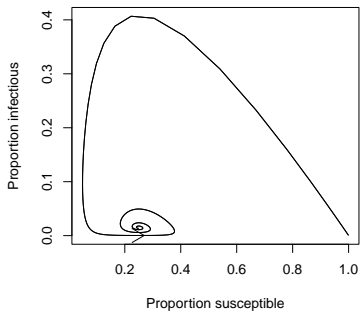
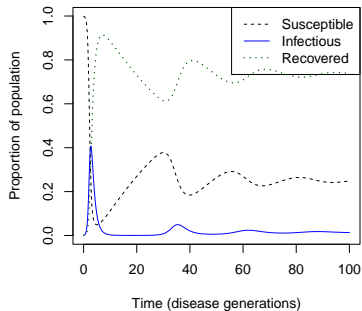
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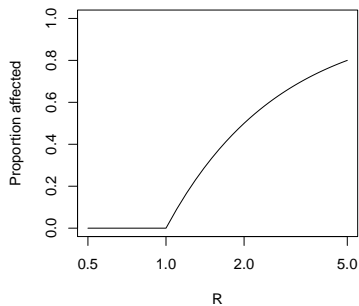


## Reproductive numbers and risk (repeat)

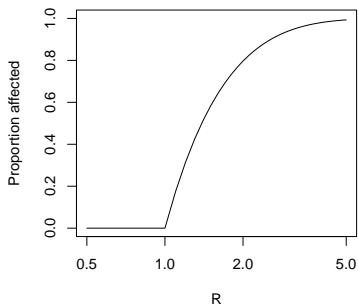


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**Single epidemic**



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Introduction

Rate of spread

Single-epidemic model

Epidemic size

Recurrent epidemic models

Dynamics

Reproductive numbers and risk

Pathogen aggressiveness

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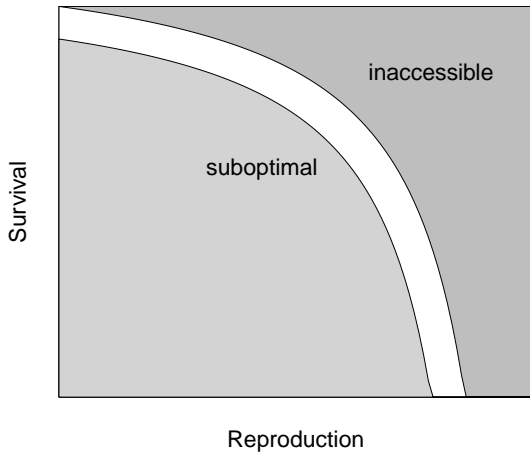
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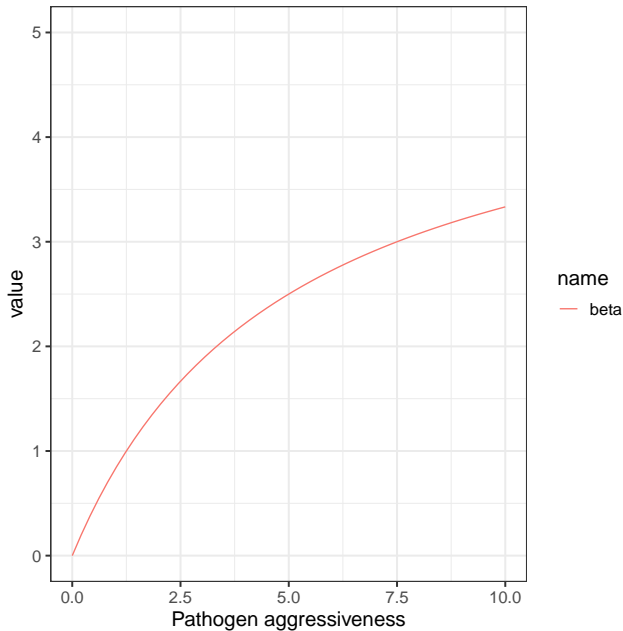
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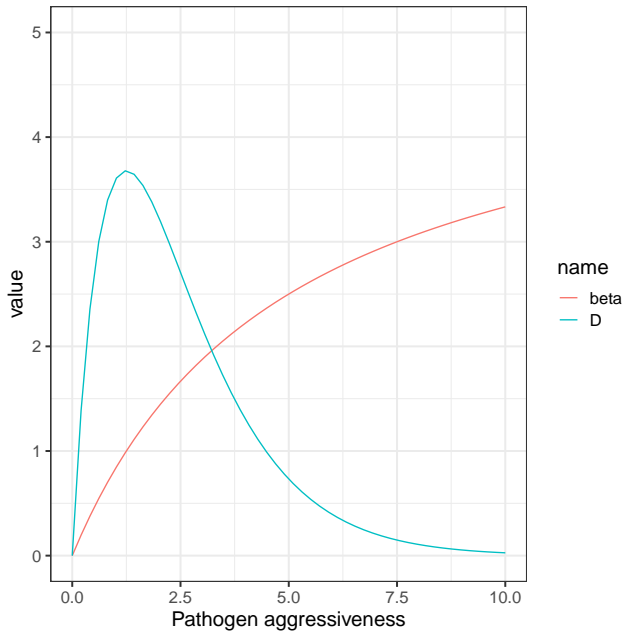
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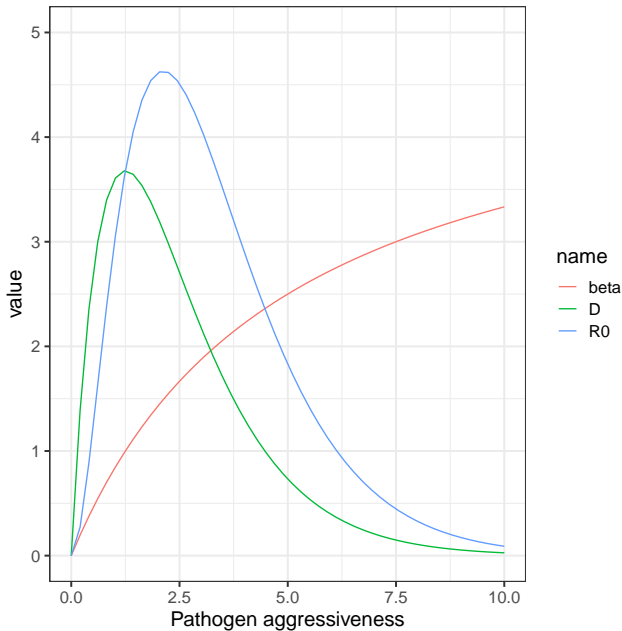
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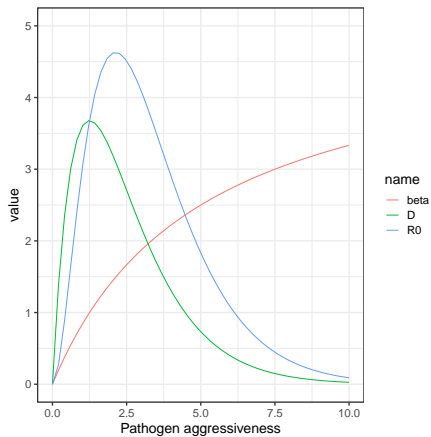
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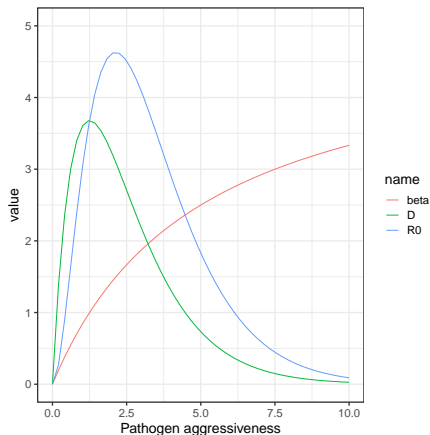


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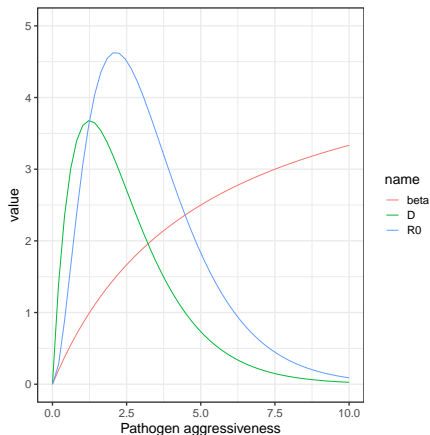
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