BIO3SS3: Population Ecology

UNIT 1: Introduction

Outline

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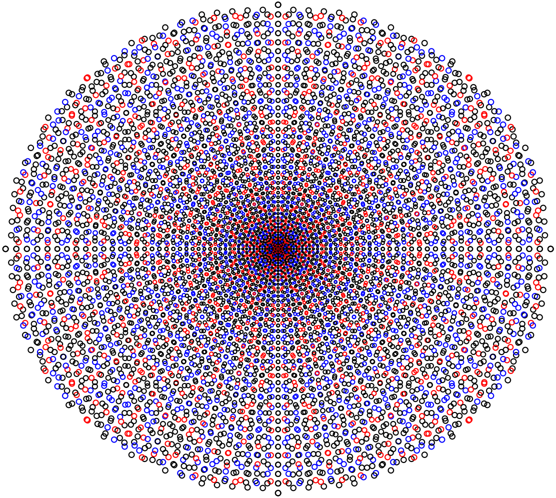
Pythagorean triples



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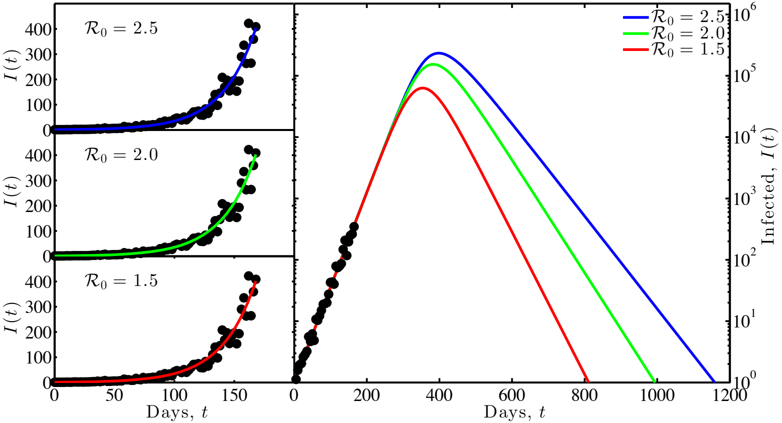
Disease research



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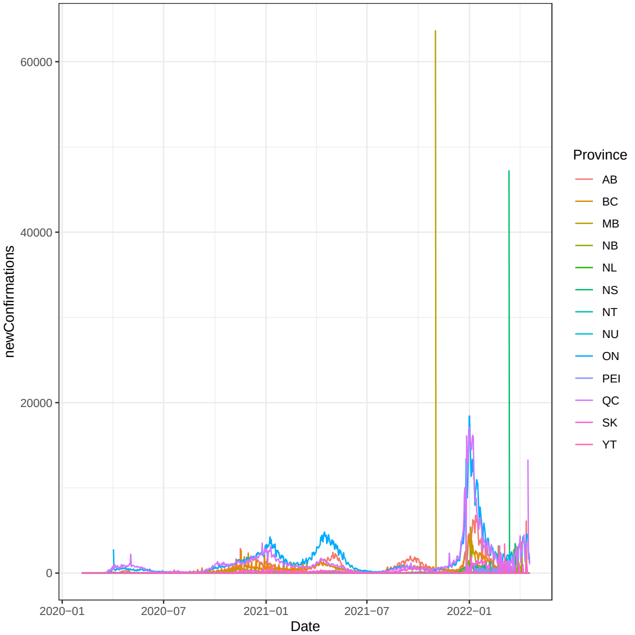

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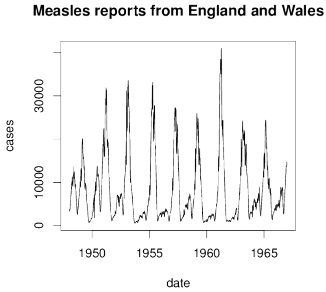
Dynamical modeling

- Investigates the links between local, short-term processes, and large-scale, long-term outcomes



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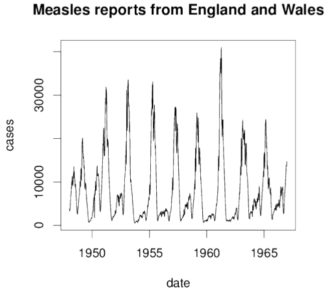

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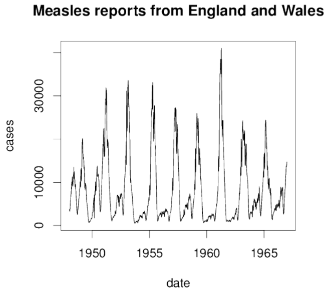

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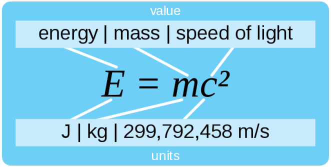
Humans and abstract thought

- People are evolved to be concrete thinkers, not conceptual thinkers



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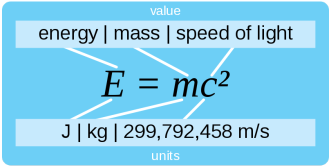

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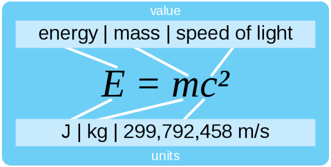

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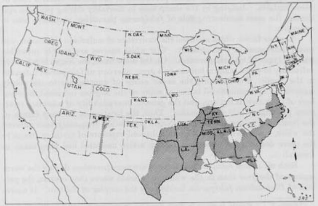

MAP 4.—Areas of the continental United States believed to be malarious in 1934–35.

Malaria

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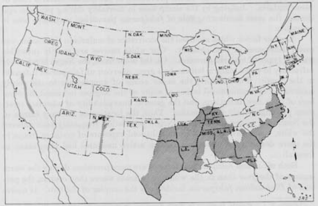

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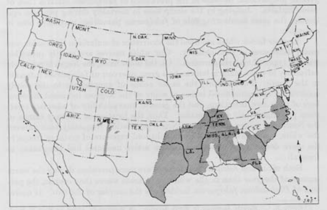

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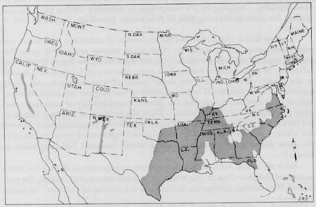

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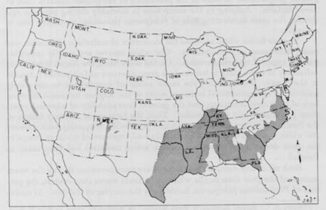

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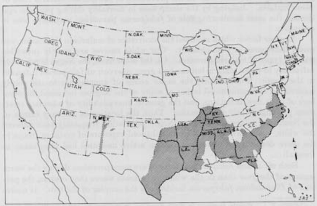

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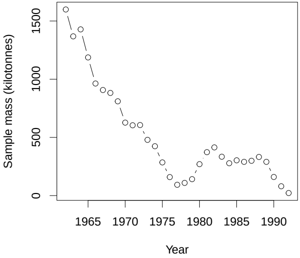

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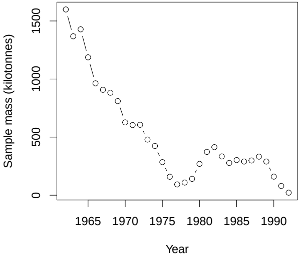

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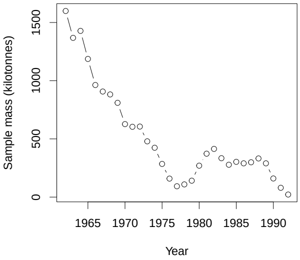

Populations

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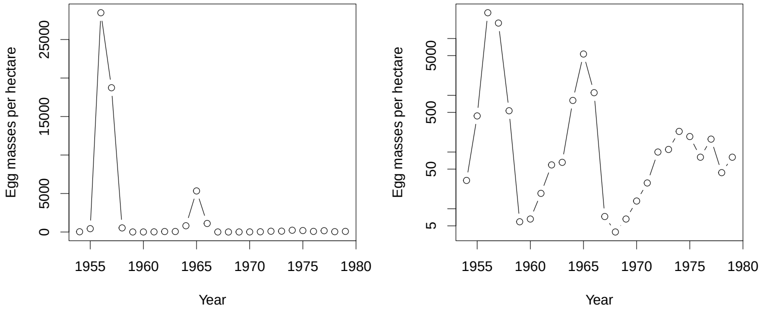
Gypsy moth populations



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  - 50% of pupae mature into adults

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  - 50% of pupae mature into adults
  - 50% of adults survive to reproduce

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Moth calculation (preview)

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  - The average reproductive female lays 600 eggs
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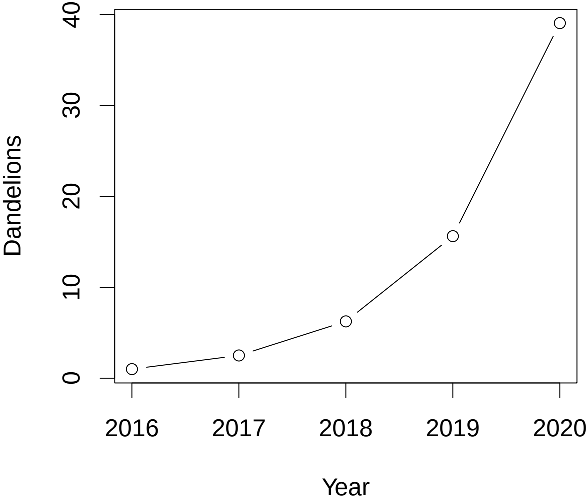

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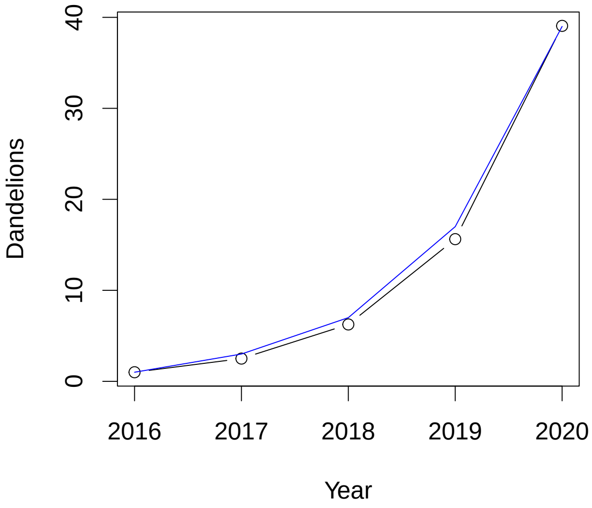

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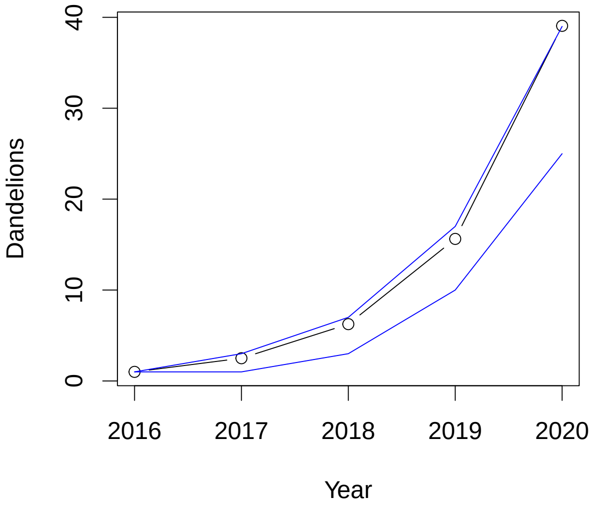

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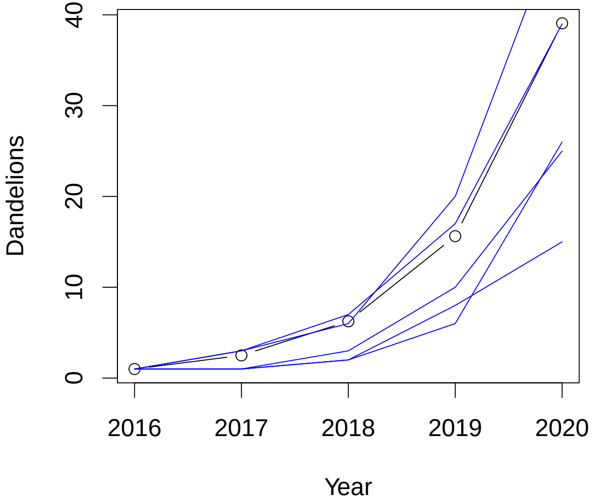

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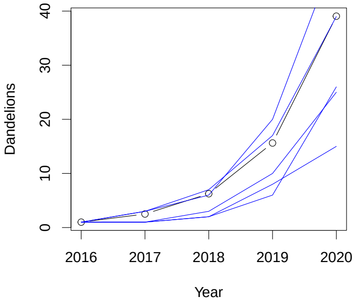

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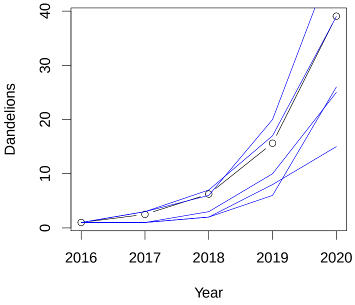

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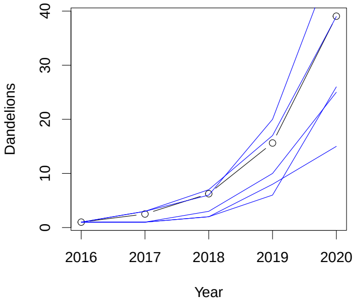

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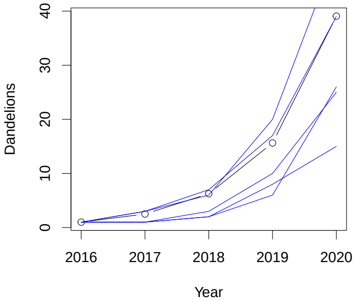

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Outline

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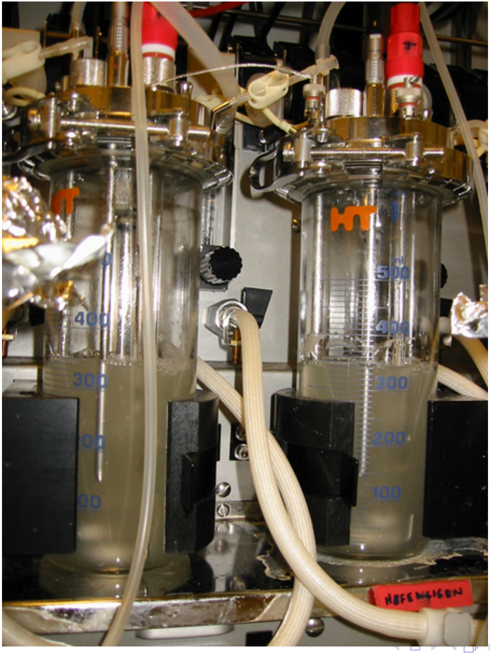
Bacteria in a tank



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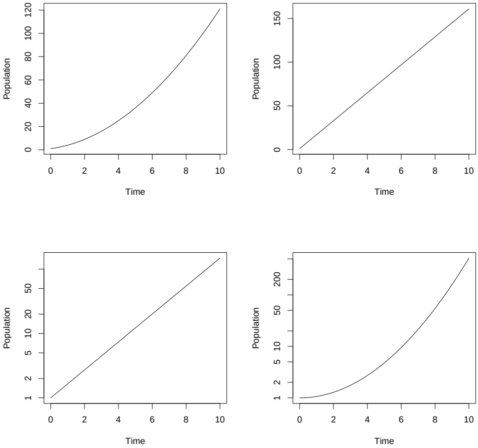
Exponential growth

- What is exponential growth?



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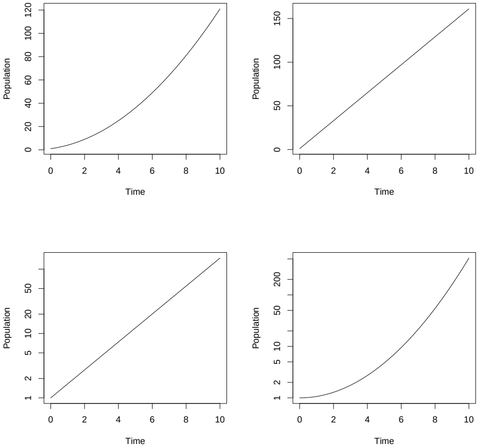

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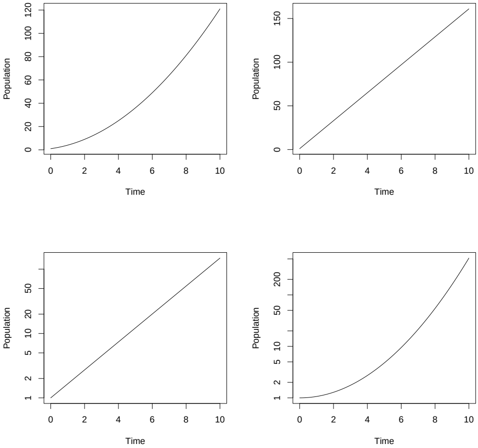

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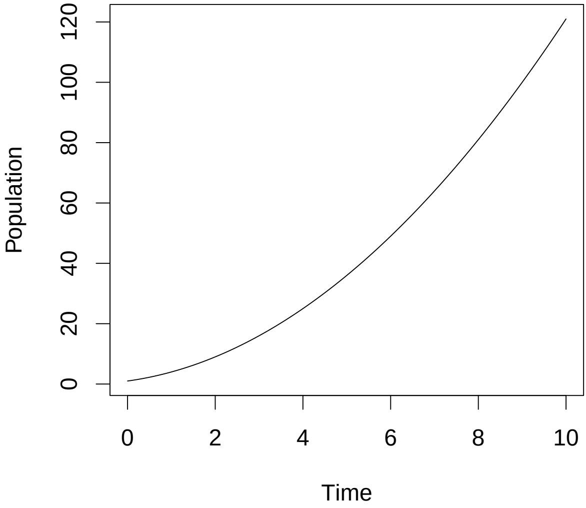

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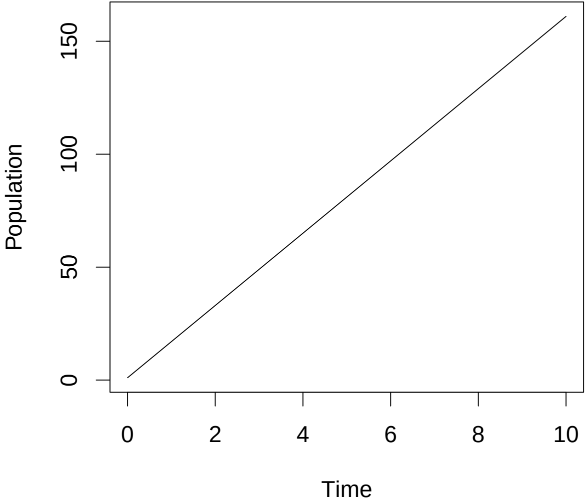
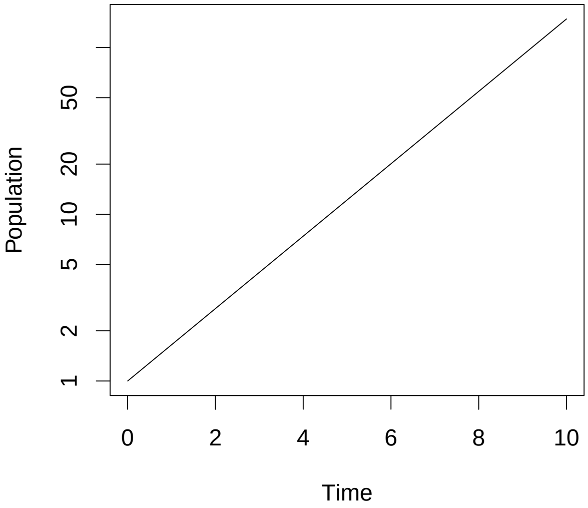
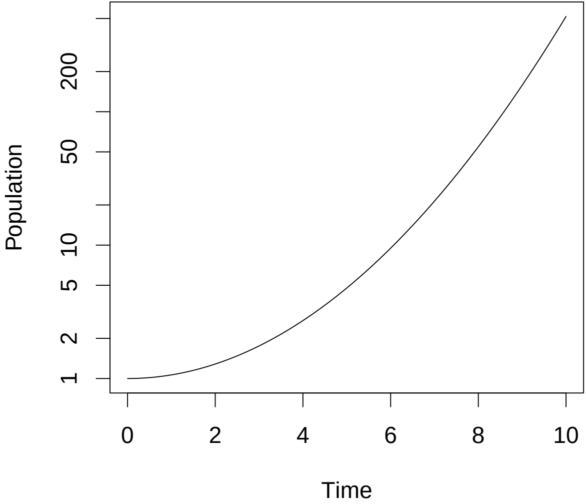


C (repeat)



D (repeat)



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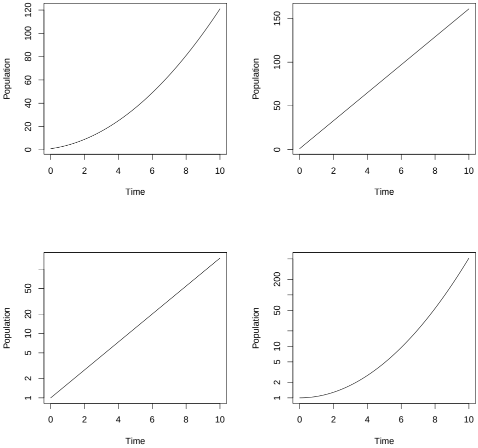

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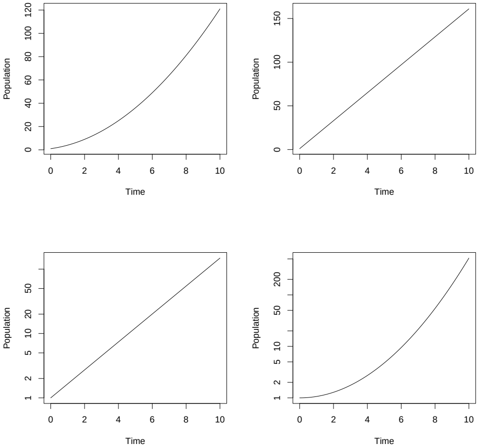

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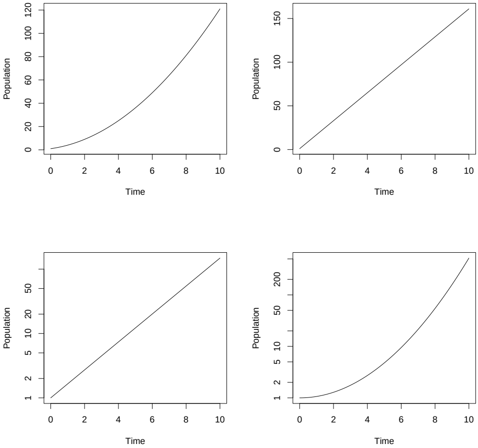

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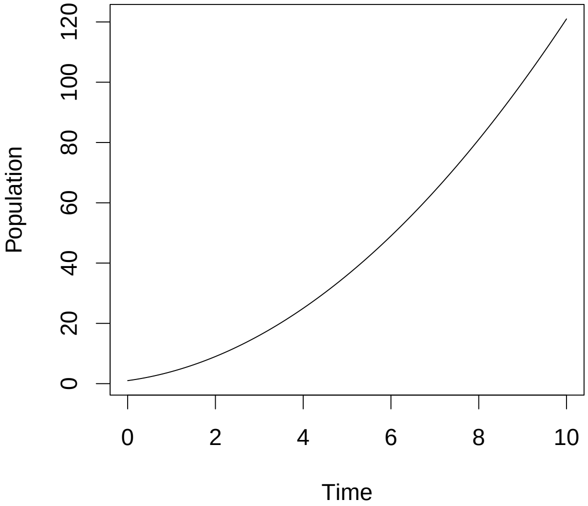

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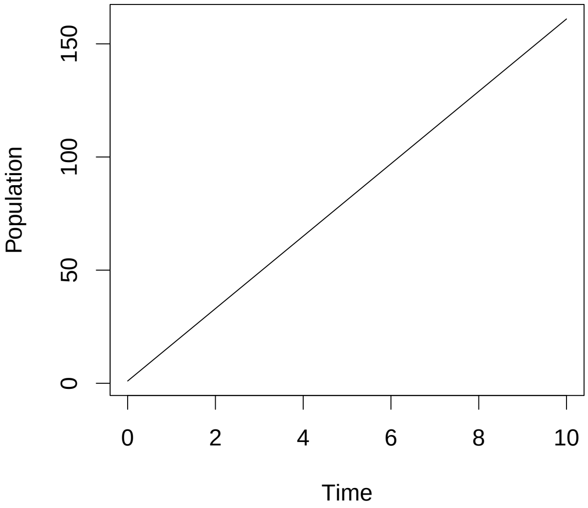

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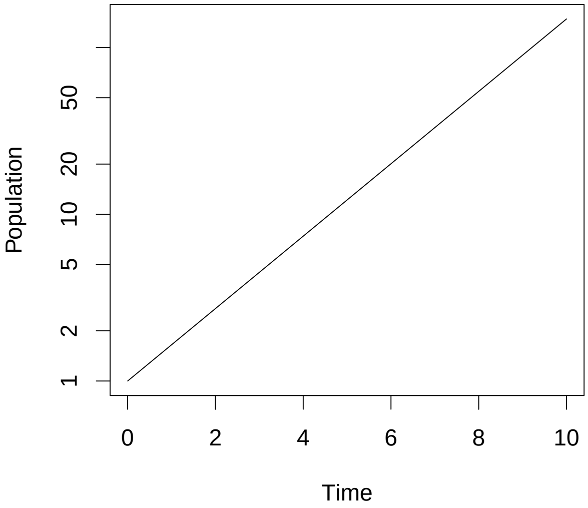

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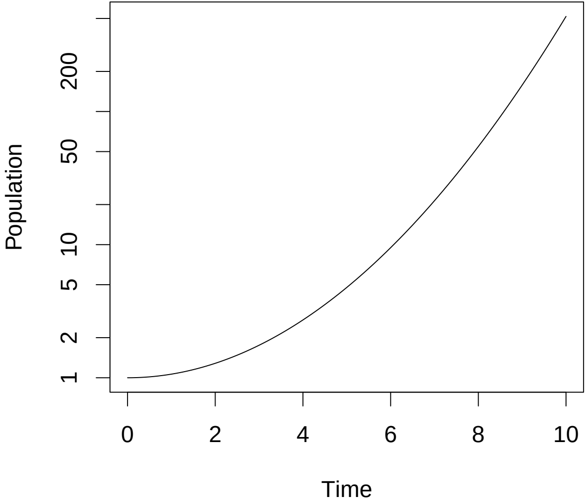

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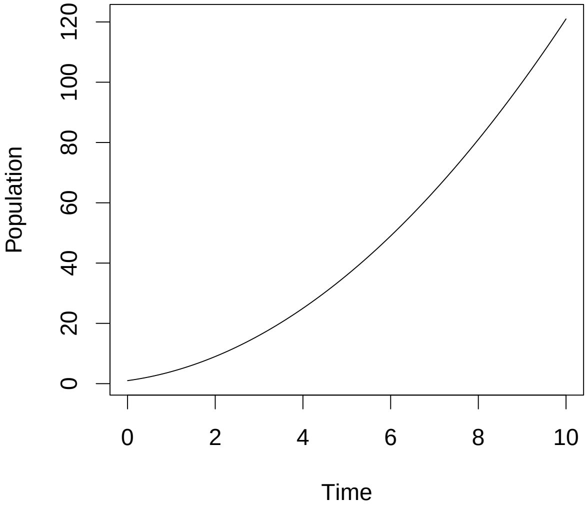
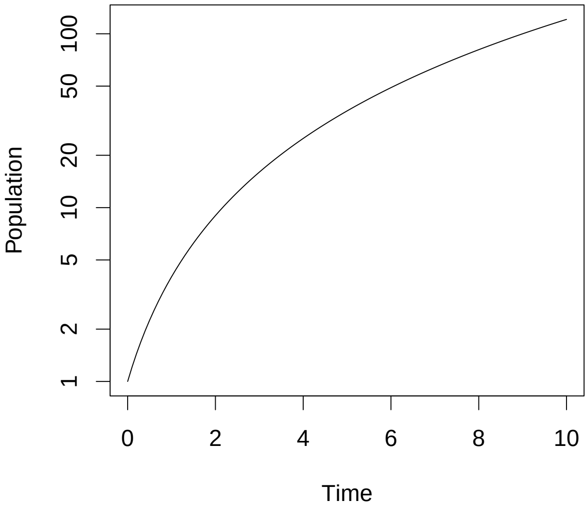


A on the log scale



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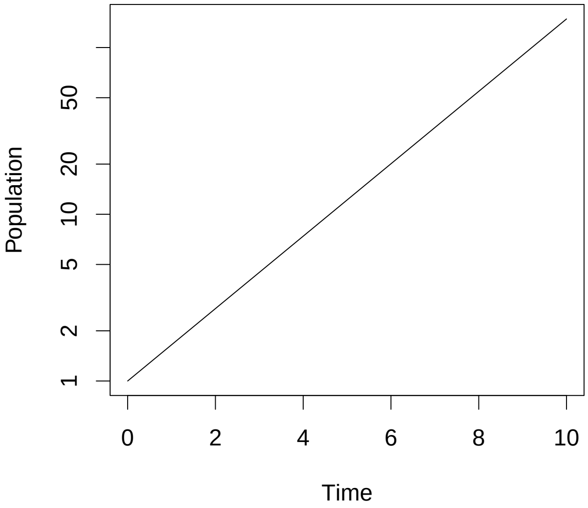
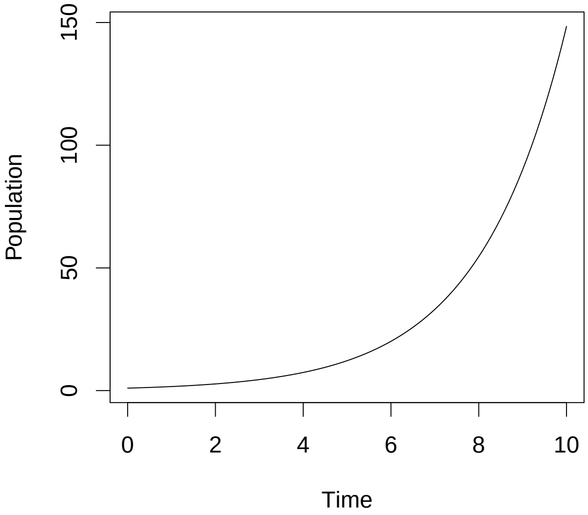


C on the linear scale



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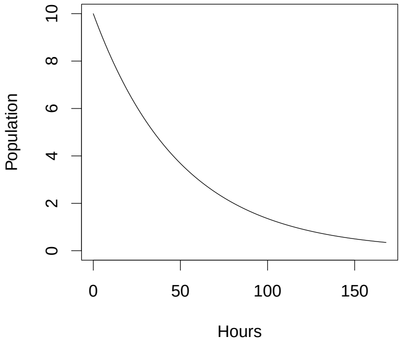

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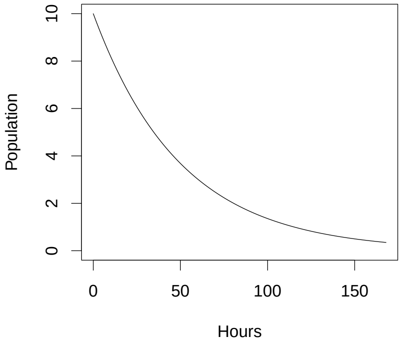

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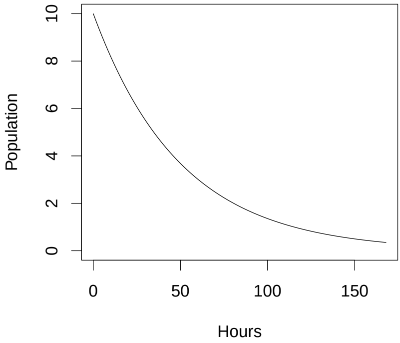

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Outline

Scales of comparison

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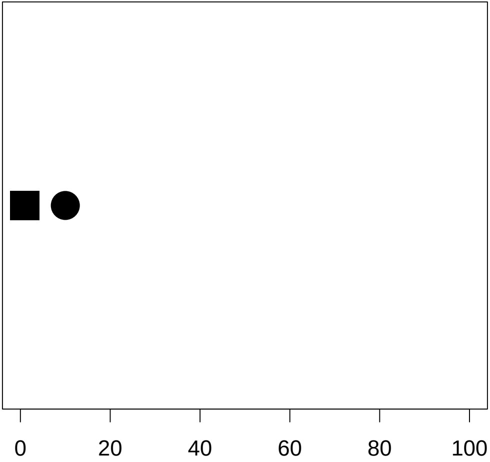

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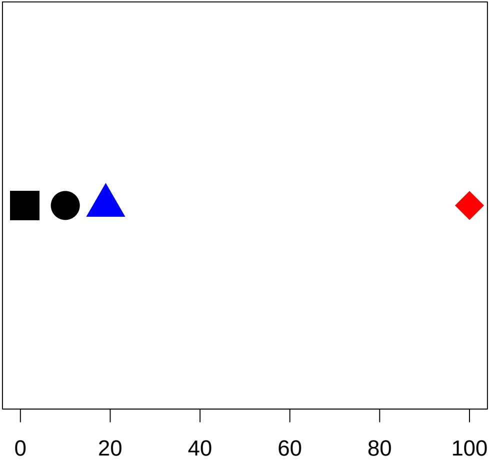

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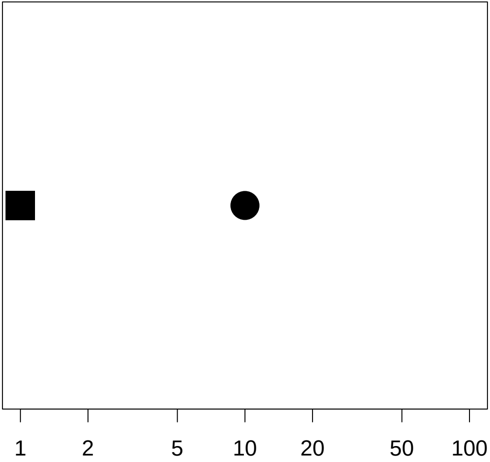

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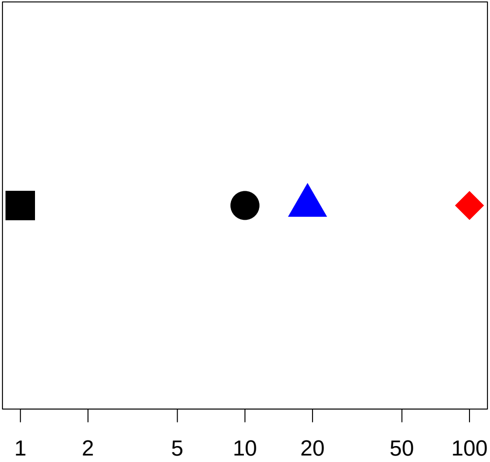

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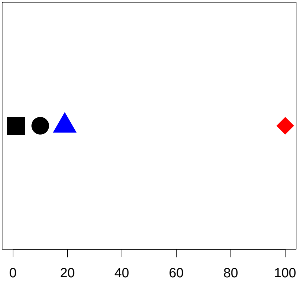

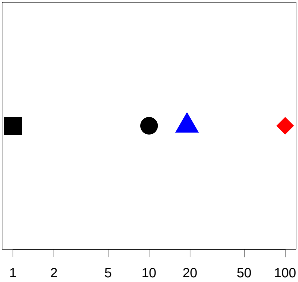

There is only one log scale; it doesn't matter which base you use!

Canadian provinces

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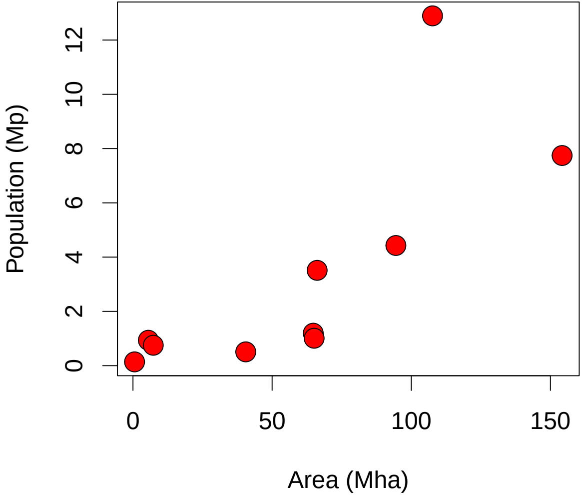

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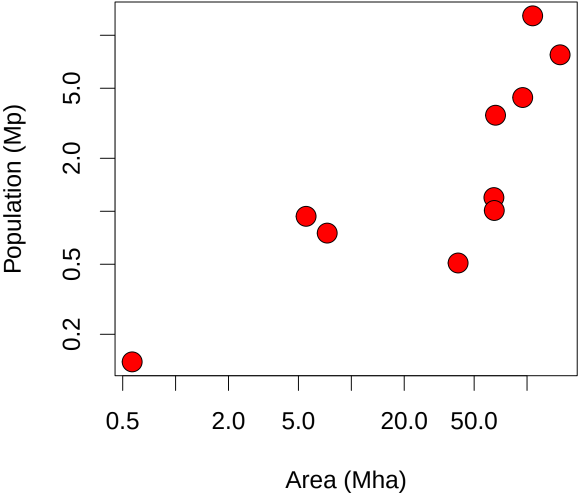

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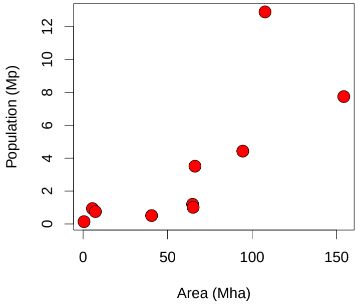

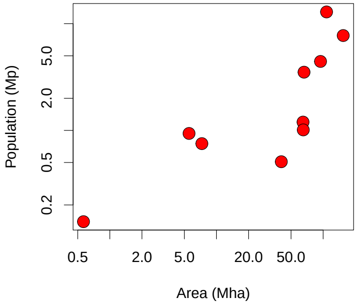

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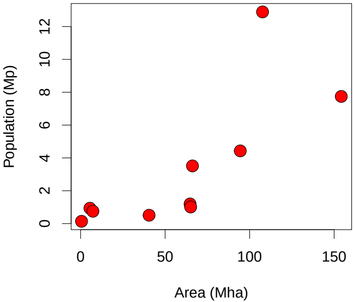

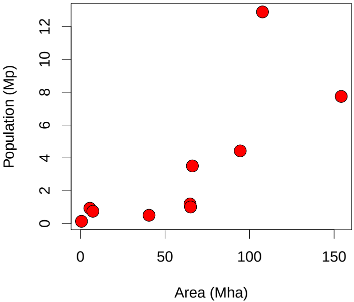

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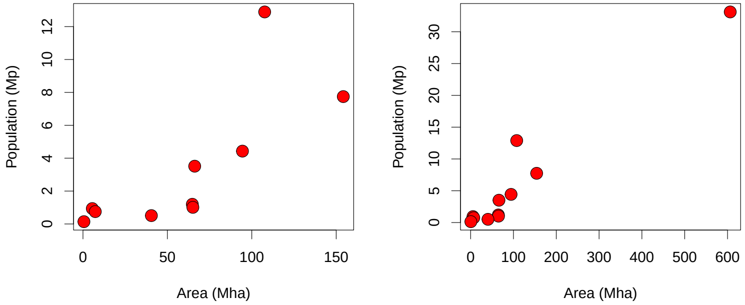

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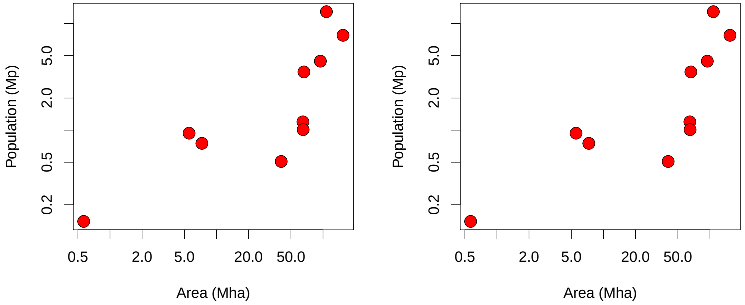

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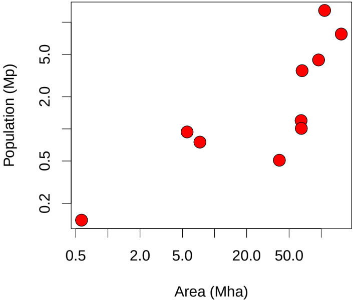

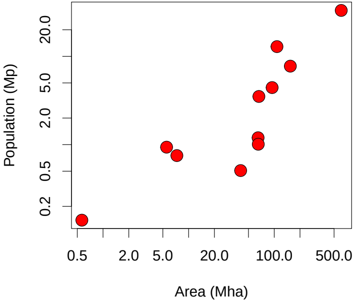

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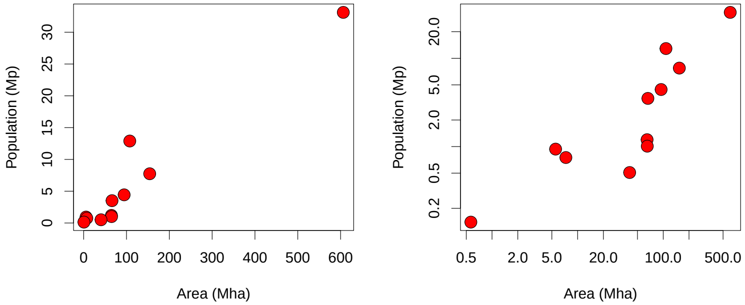

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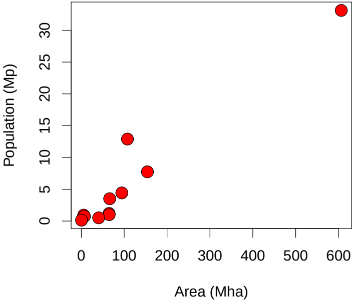

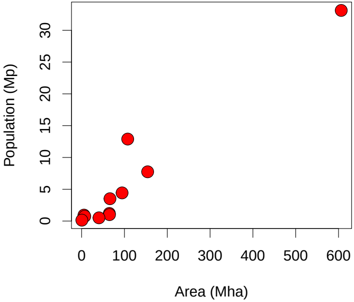

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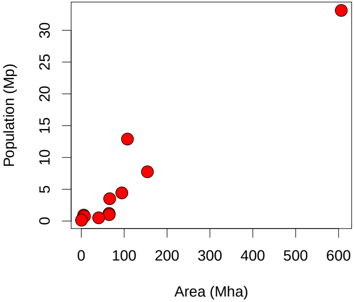

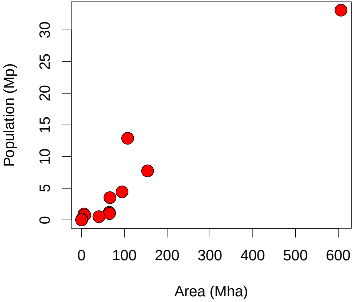

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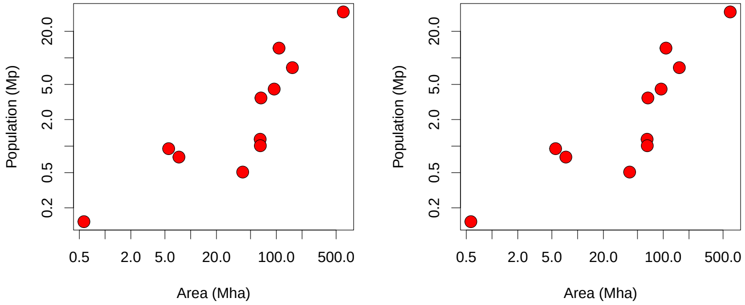

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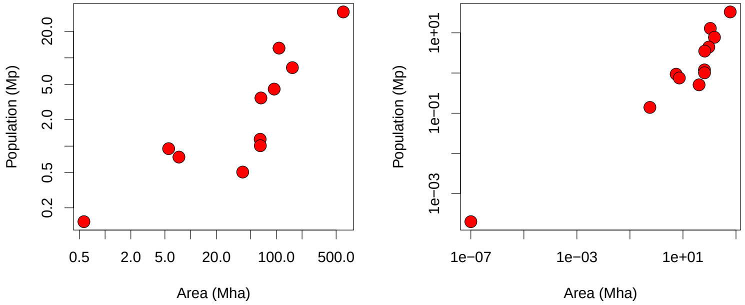

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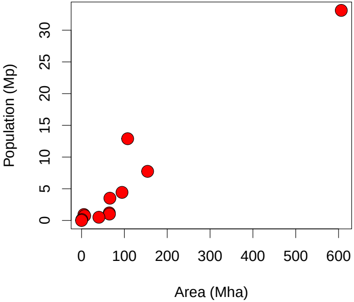

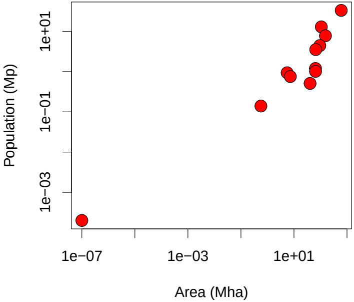

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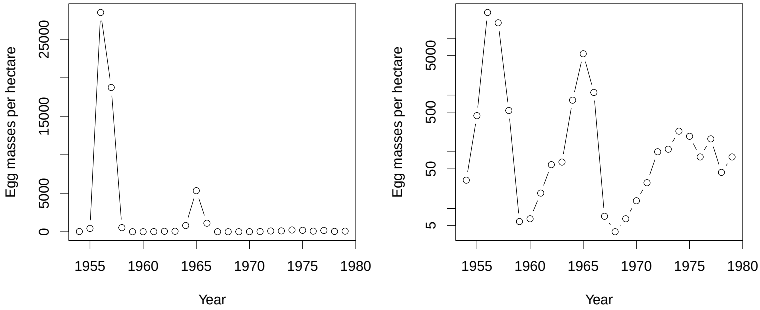

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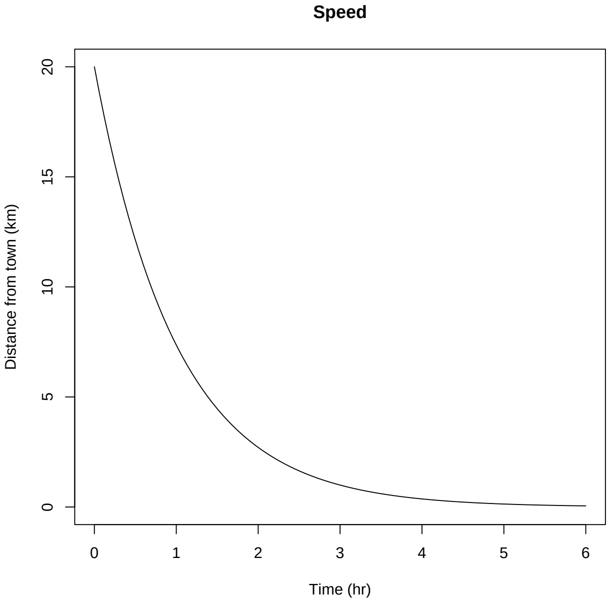

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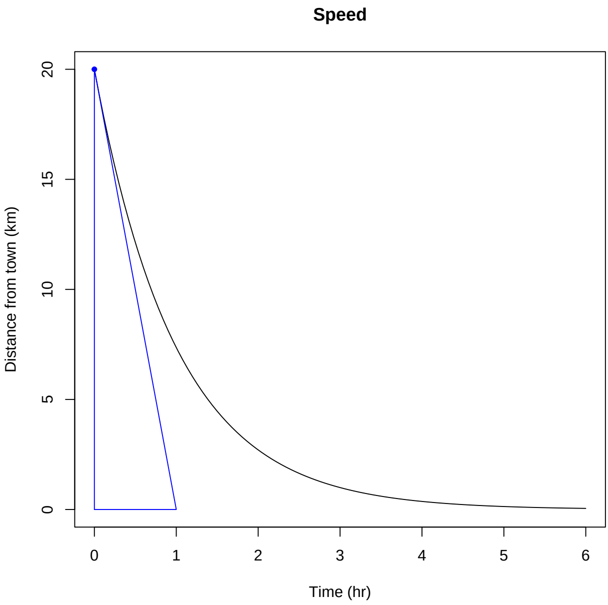

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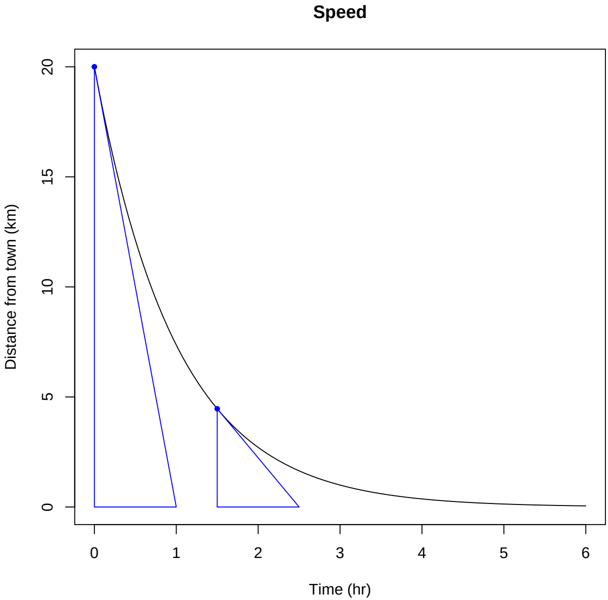

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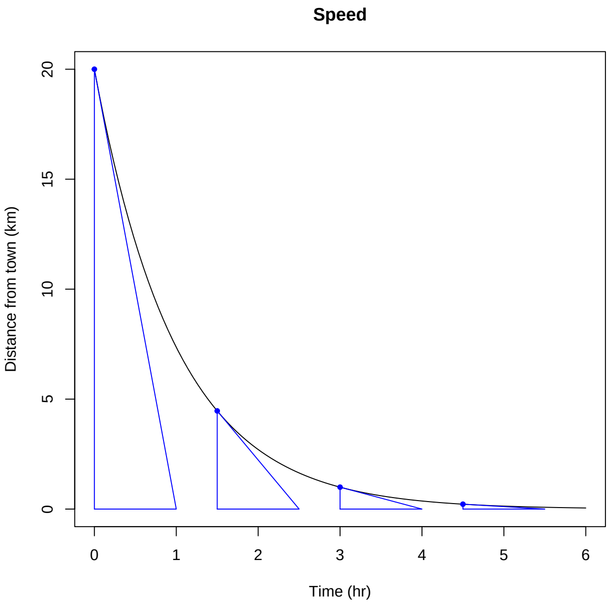

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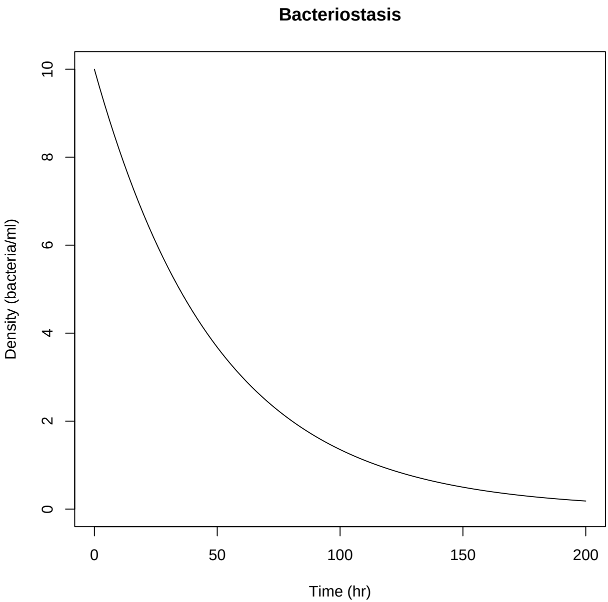

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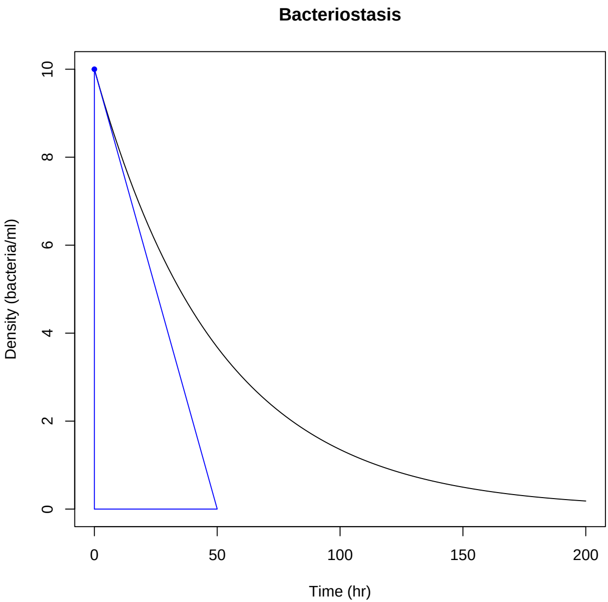

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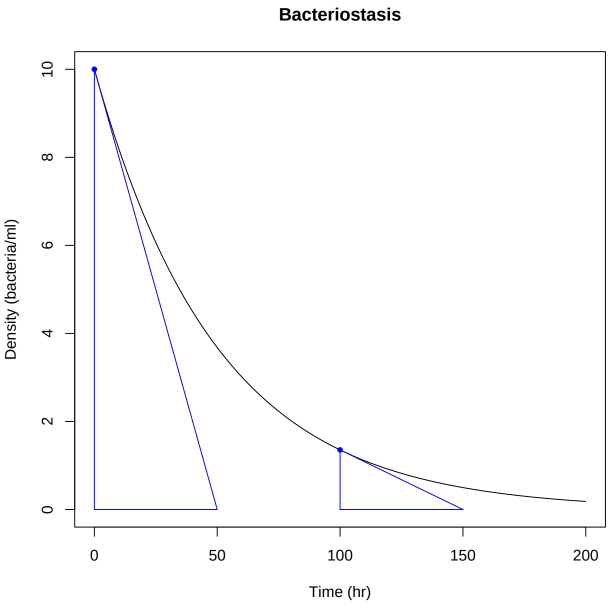

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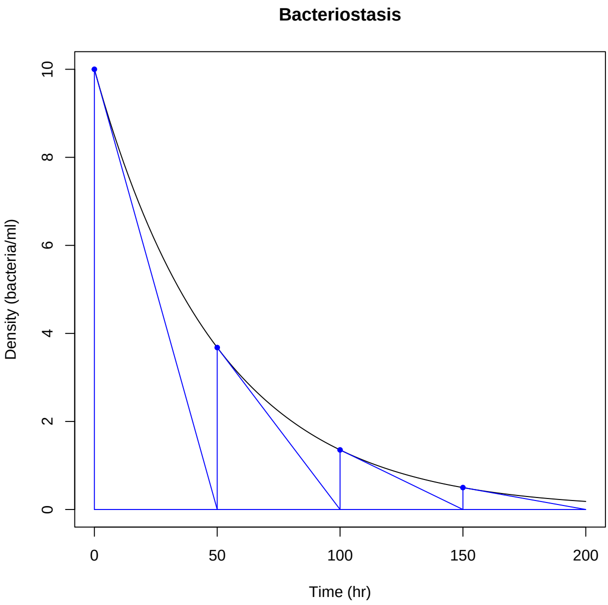

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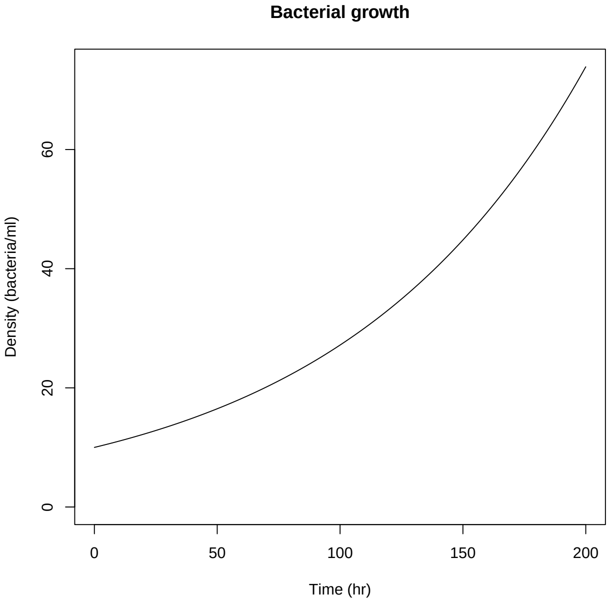

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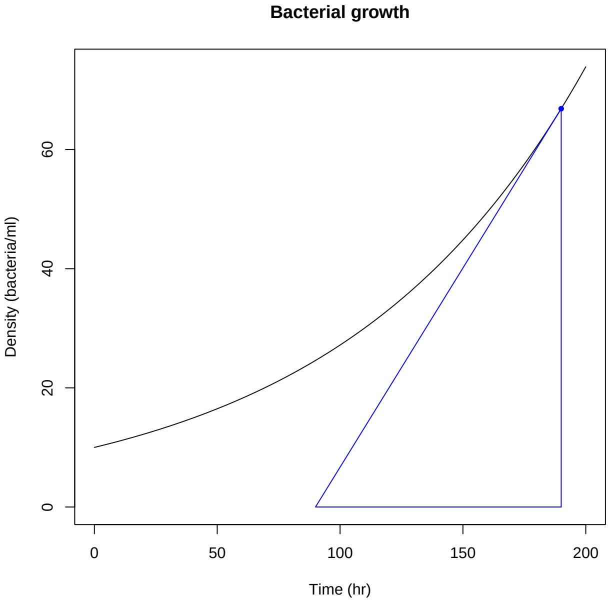

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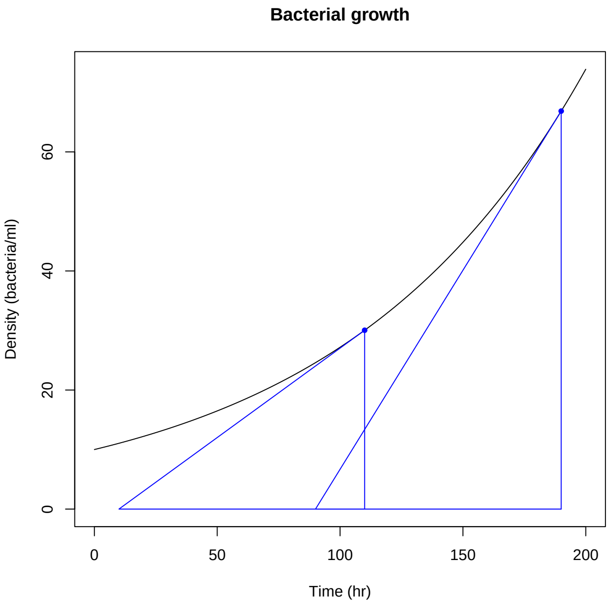

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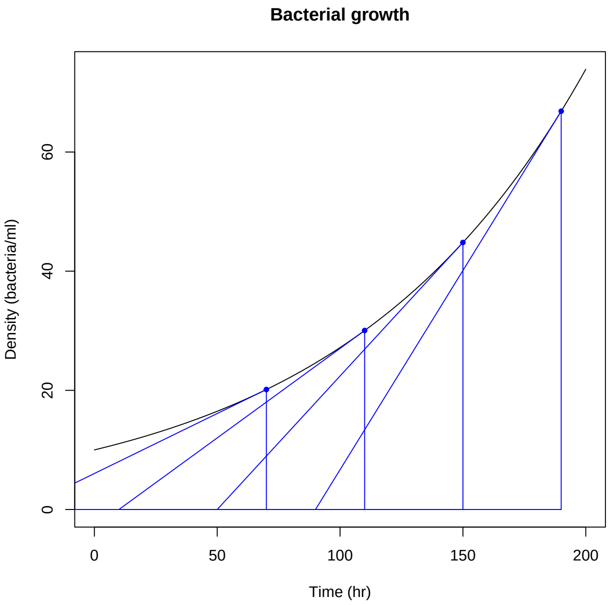

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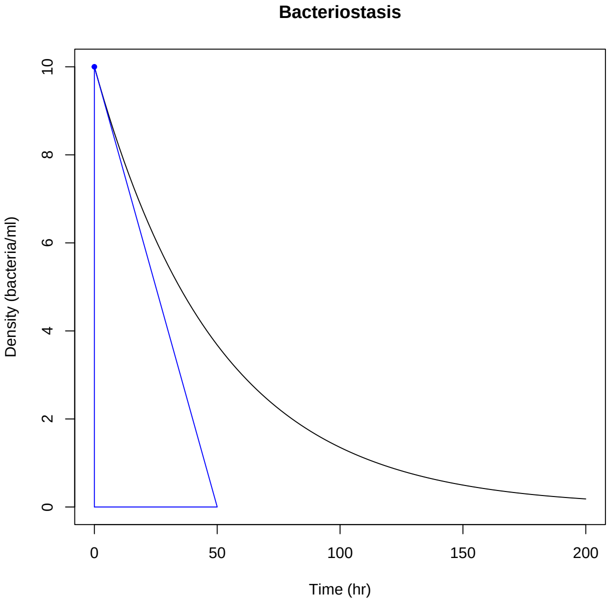
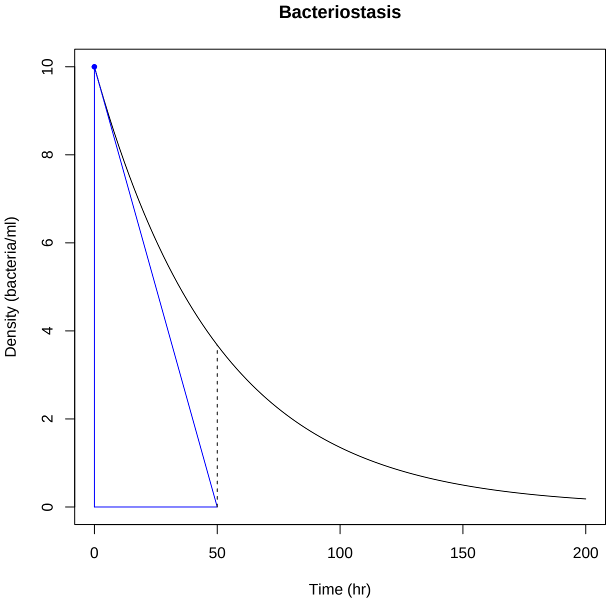

Bacteriostasis
Density (bacteria/ml)
0
2
4
6
8
10
0
50
100
150
200
Time (hr)



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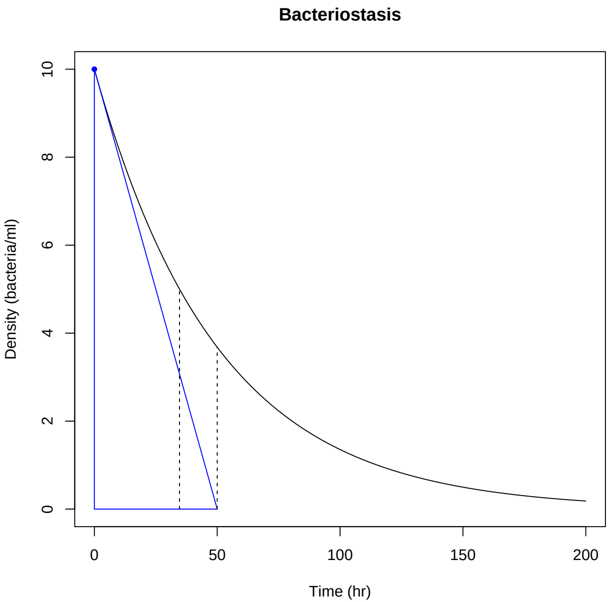

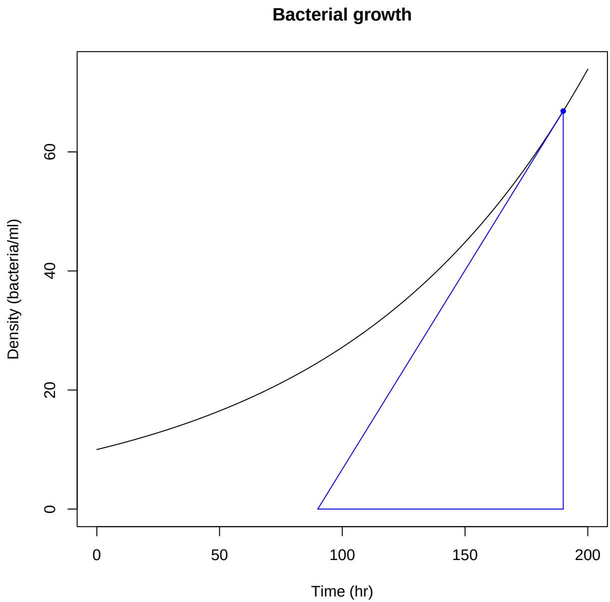
Bacterial growth
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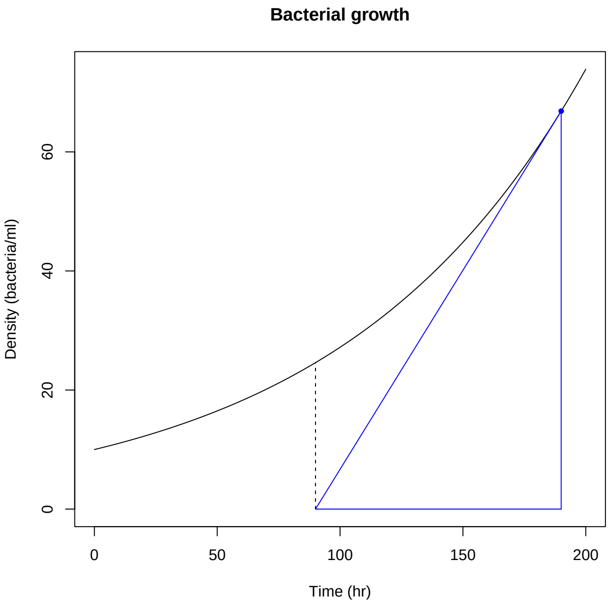
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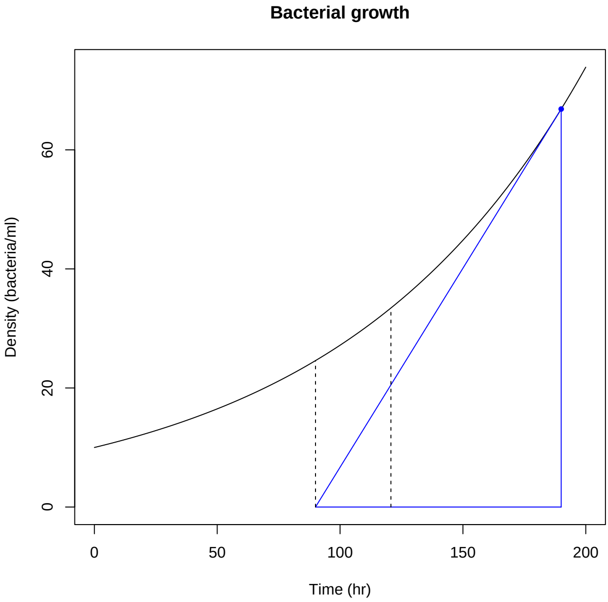

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