UNIT 2: Linear population models

Outline

Outline

Dynamical models

Tools to link scales

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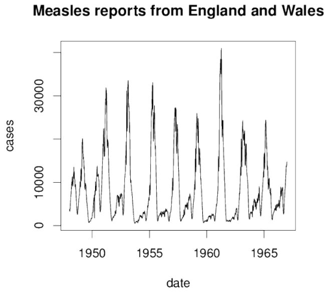

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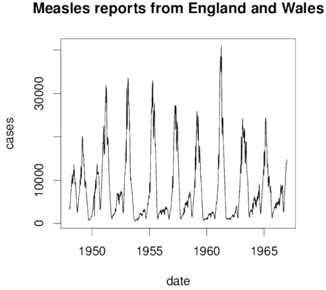

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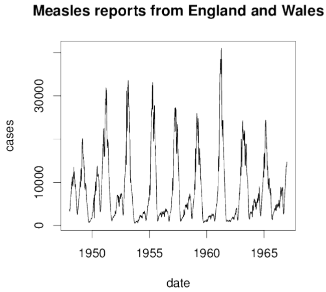

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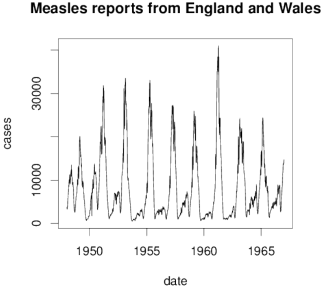

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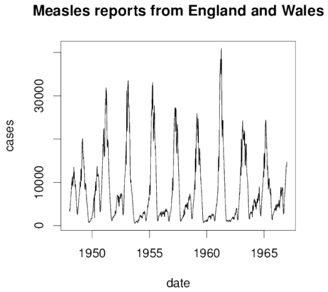

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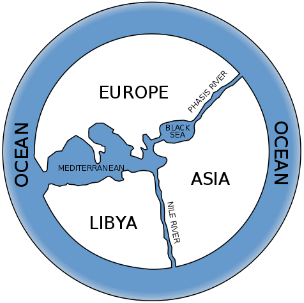

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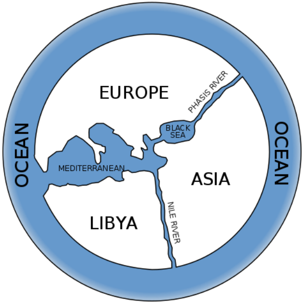

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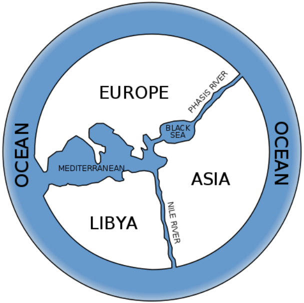

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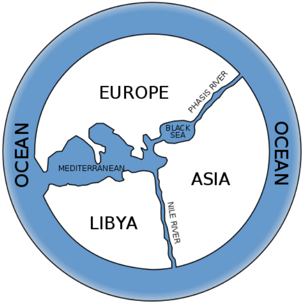

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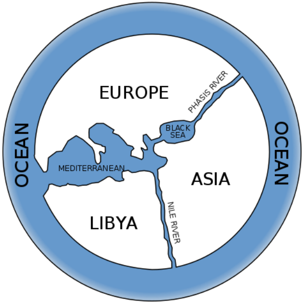

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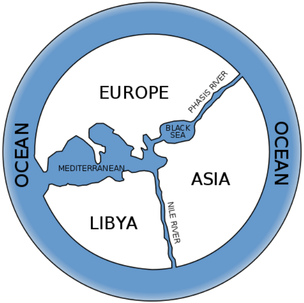

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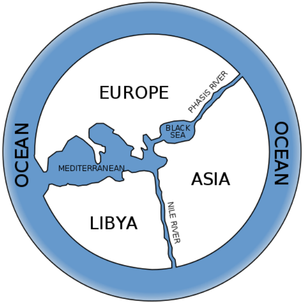

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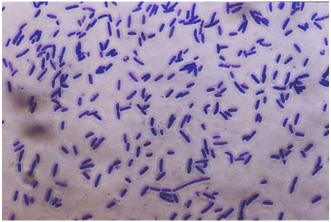

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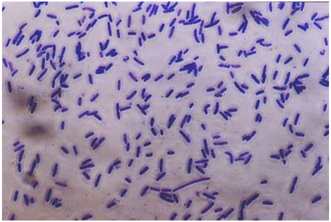

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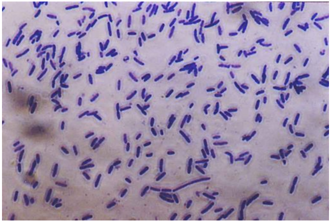

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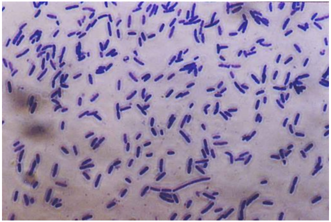

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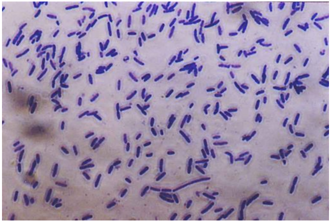

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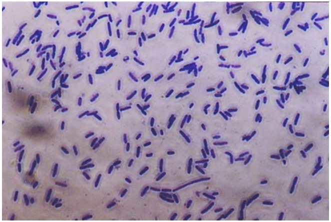

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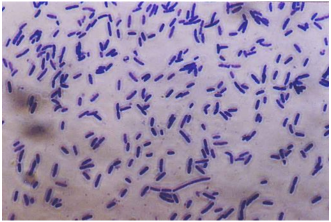

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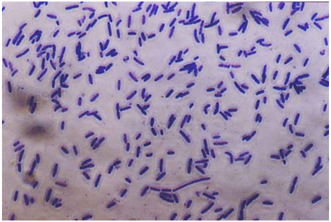

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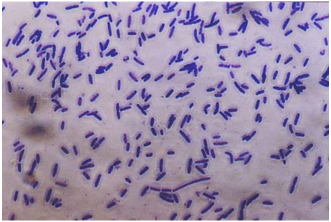

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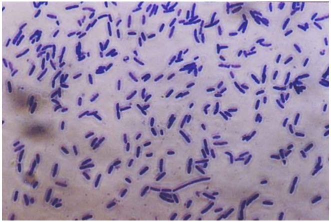

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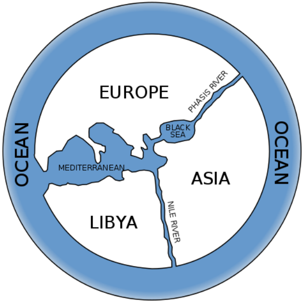

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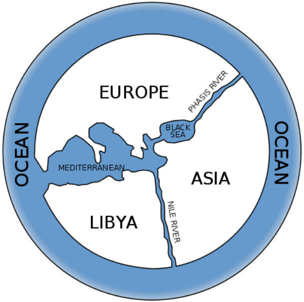

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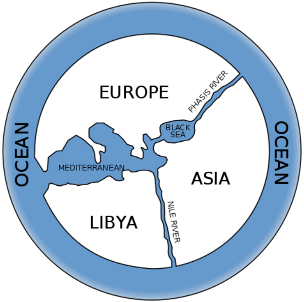

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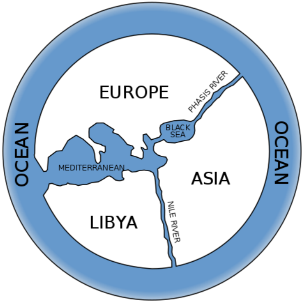

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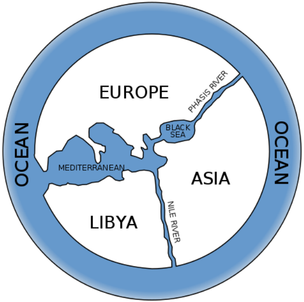

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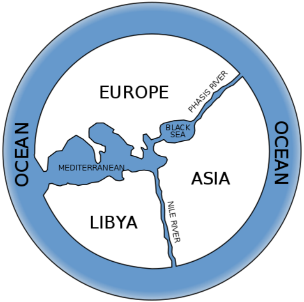

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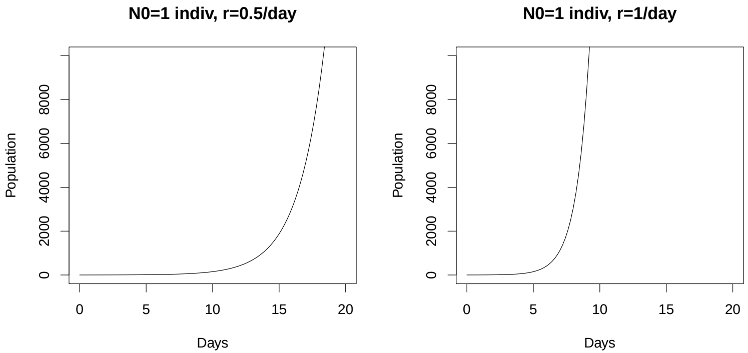

N0=1 indiv, r=0.5/day
Population
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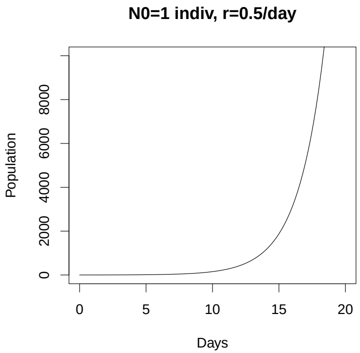

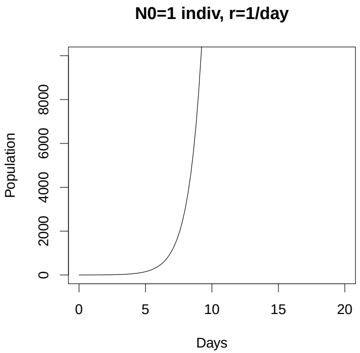

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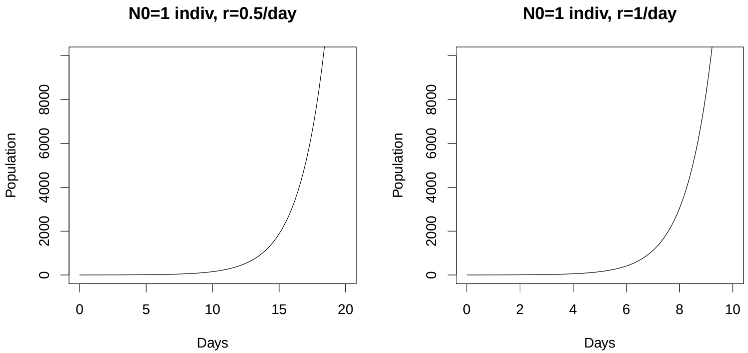

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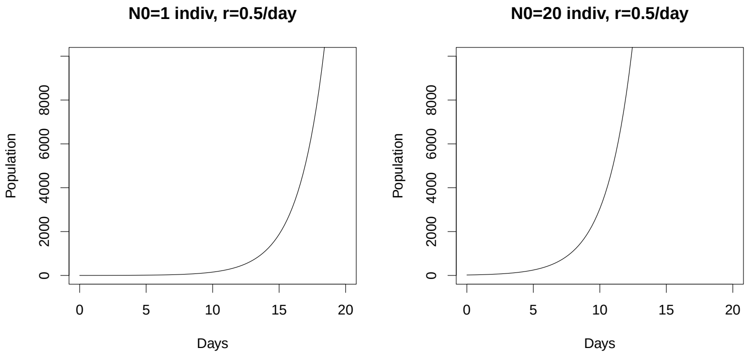

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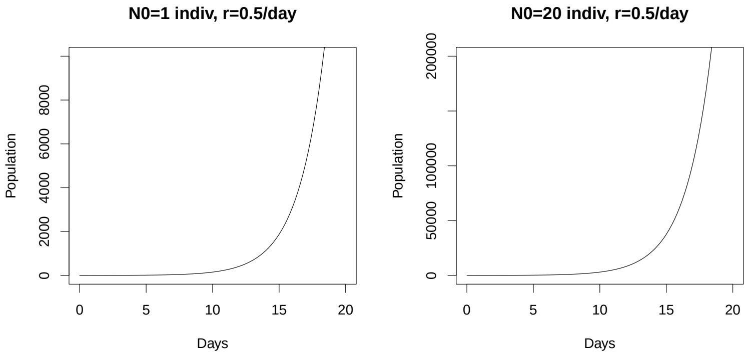

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