UNIT 3 Non-linear population models

Outline

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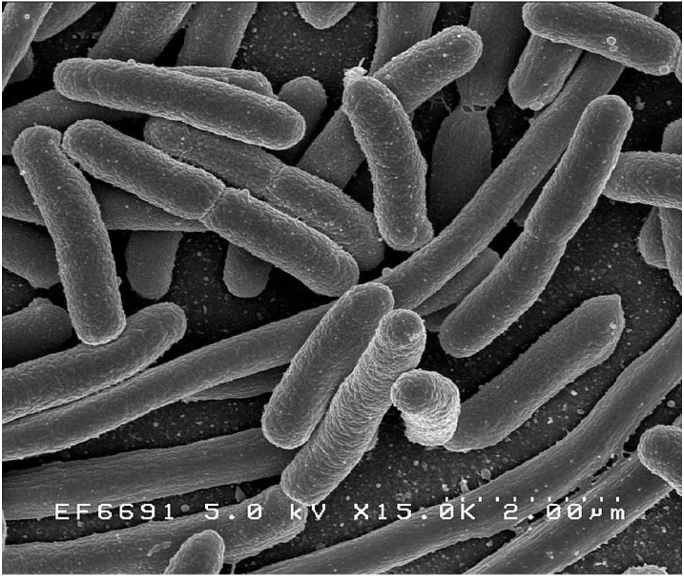
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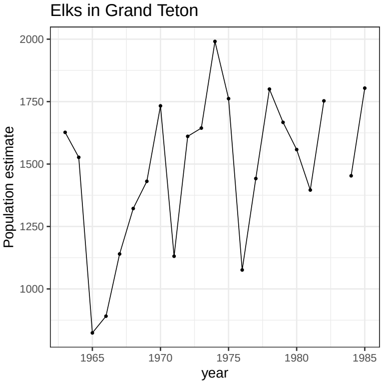
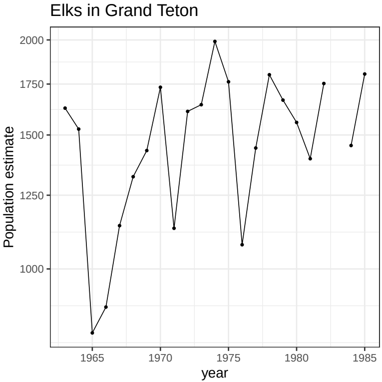
(preview)

Elk



Elks in Grand Teton
Population estimate
2000
1750
1500
1250
1000
1965
1970
1975
1980
1985
year



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2000
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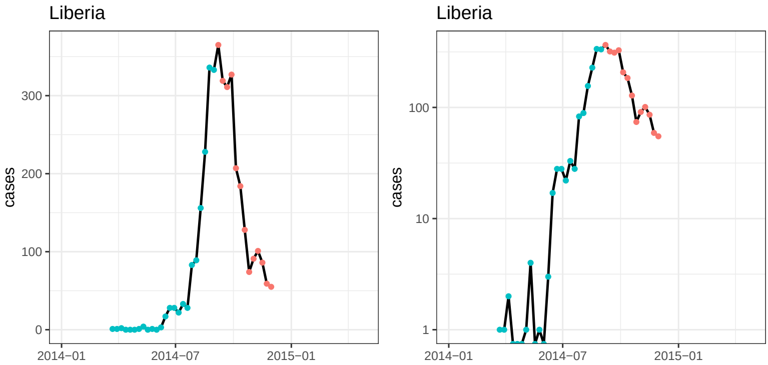
Ebola


Liberia
cases
300
200
100
0
2014−01
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2015−01
Liberia
cases
100
10
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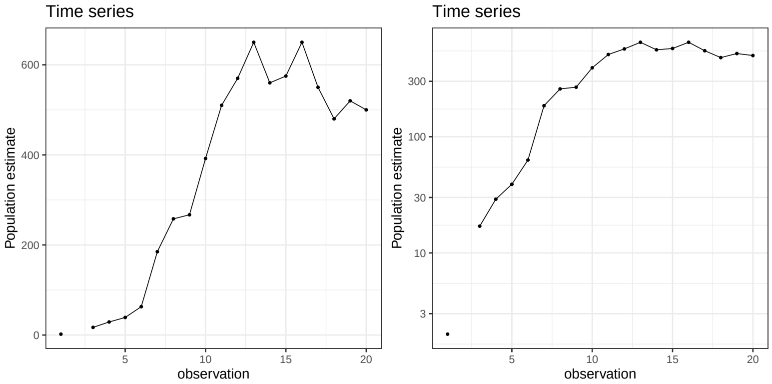
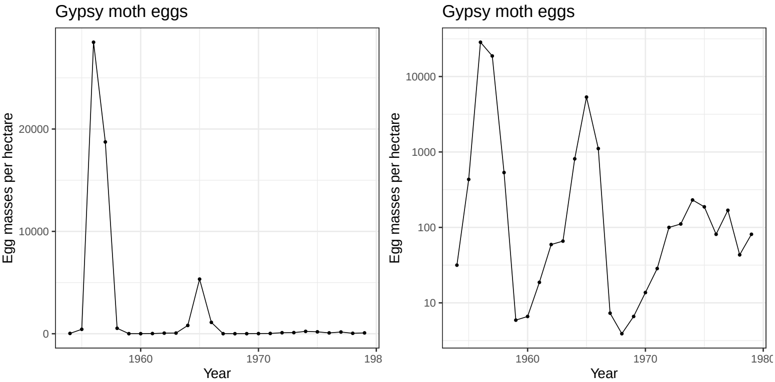
2014−07
2015−01

Paramecia (preview)



Gypsy moths (preview)



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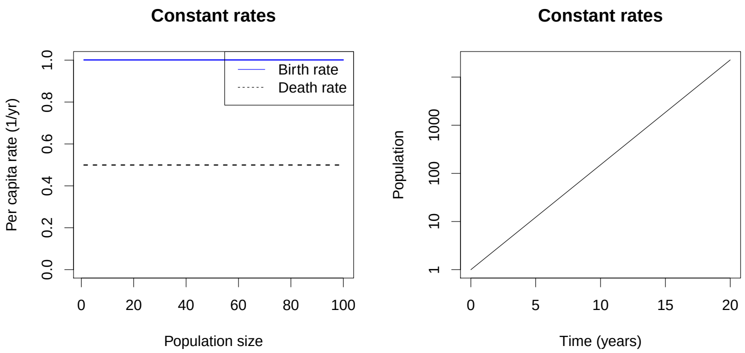

Individual perspective

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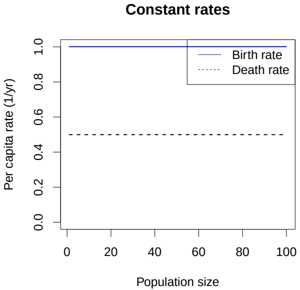

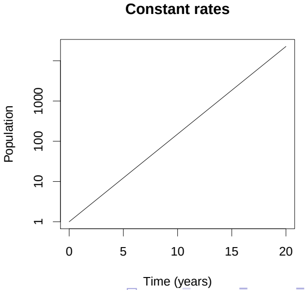

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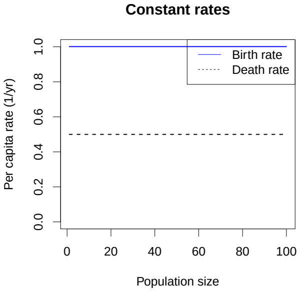

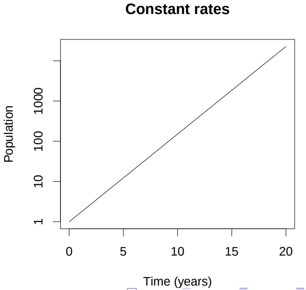

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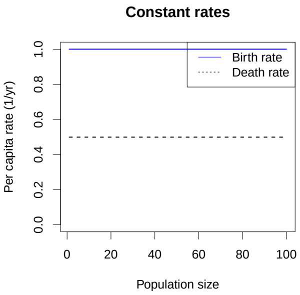

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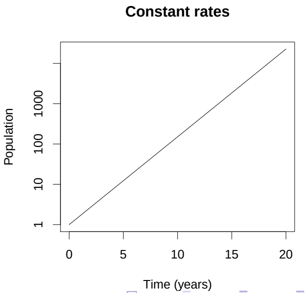

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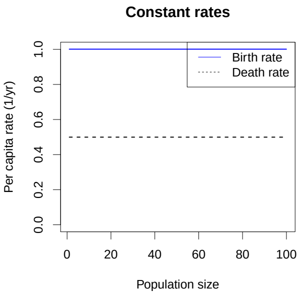

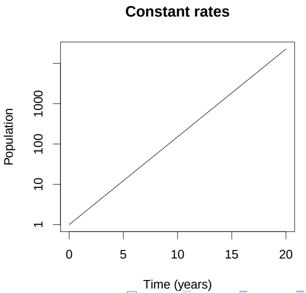

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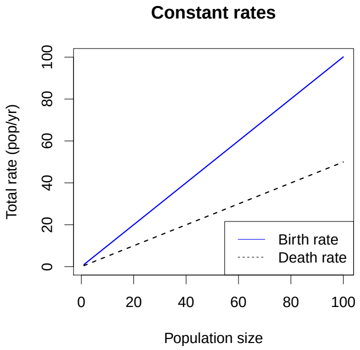

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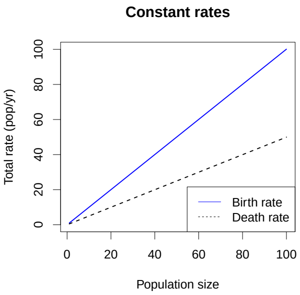

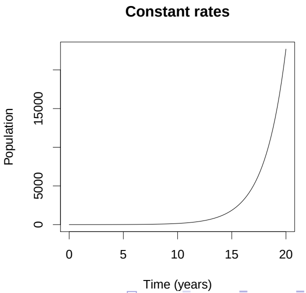

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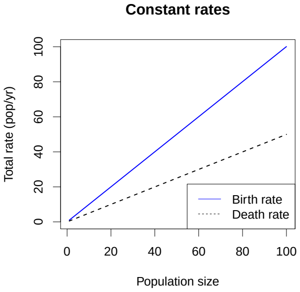

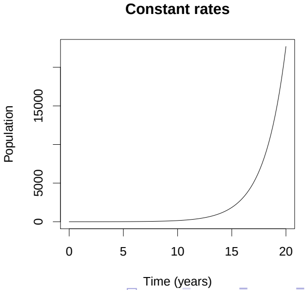

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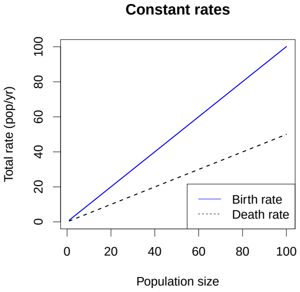

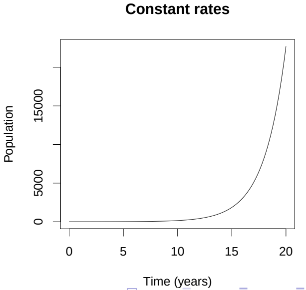

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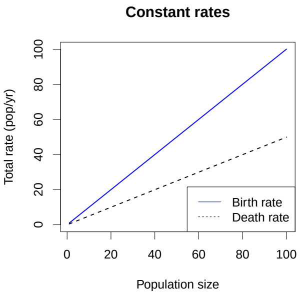

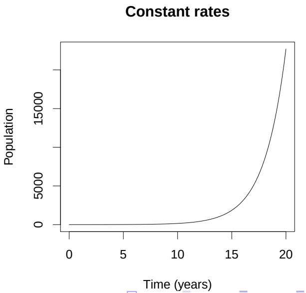

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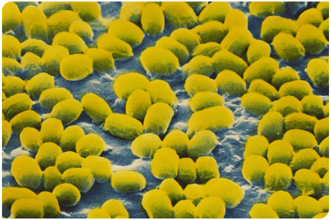

Death rates

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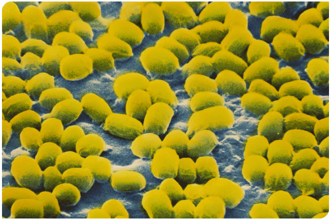

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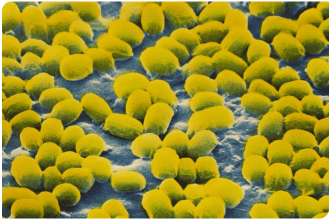

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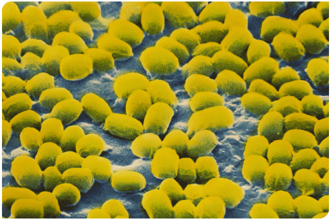

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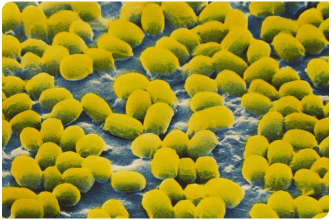

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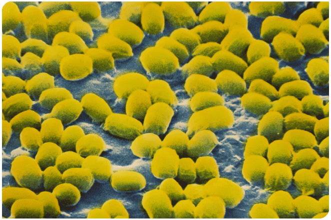

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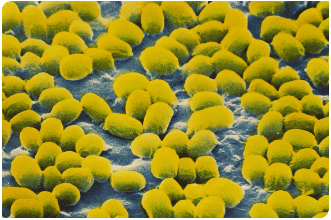

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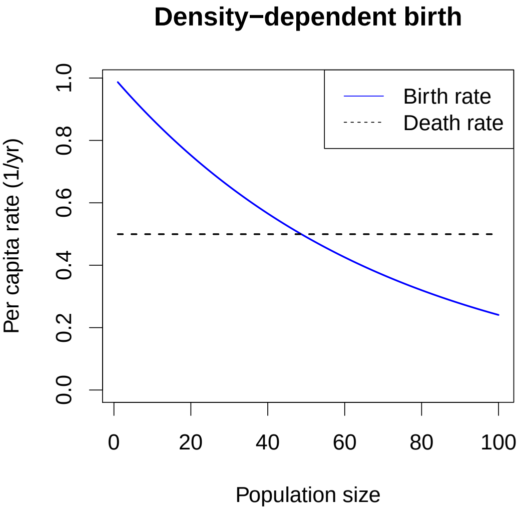

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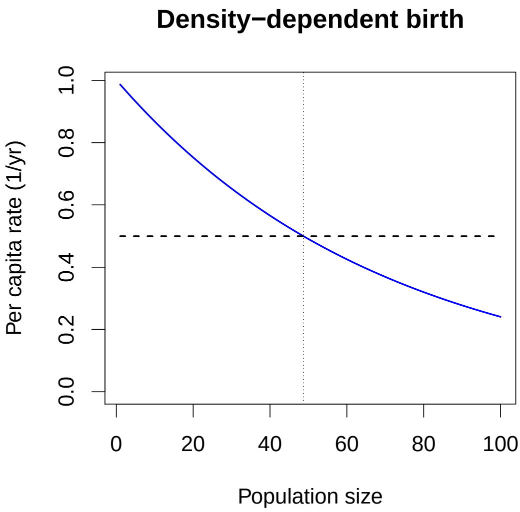

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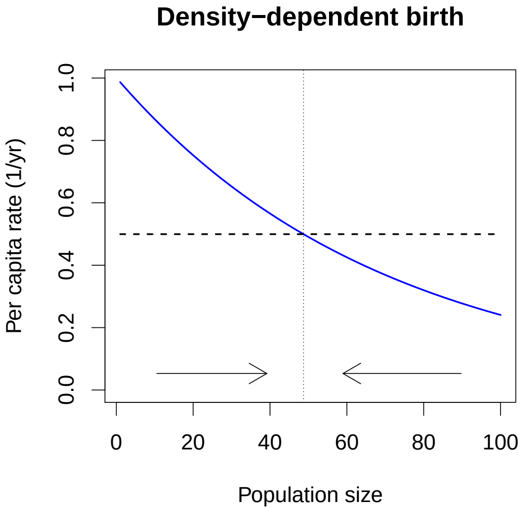

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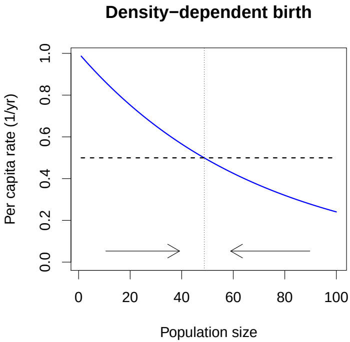

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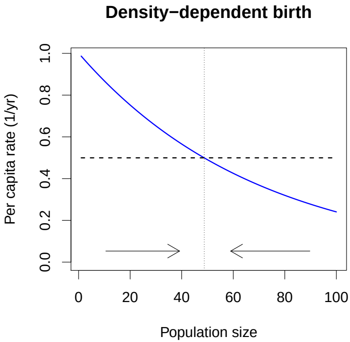

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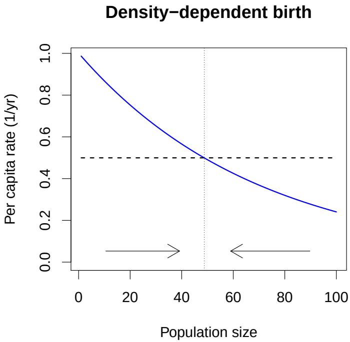

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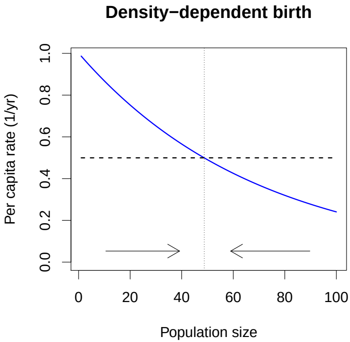

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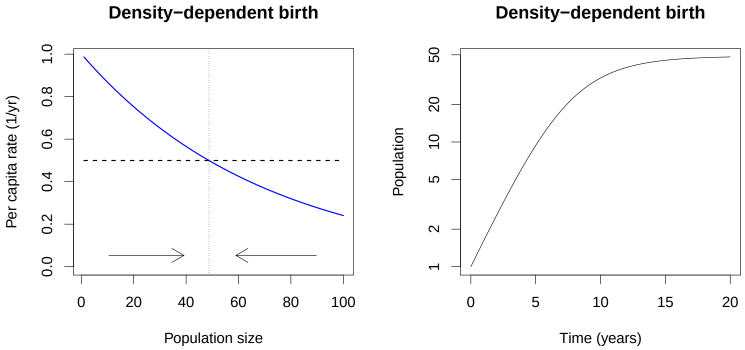

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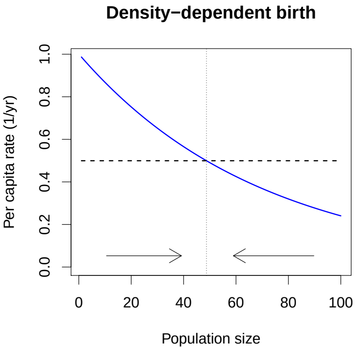

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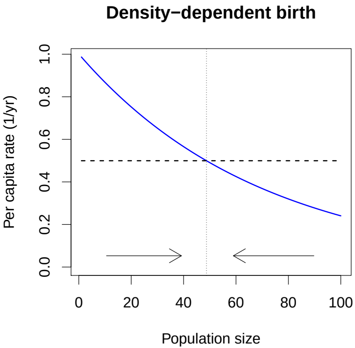
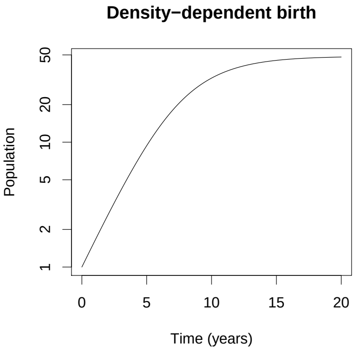
  - corresponds to the dynamics we visualize on a log-scale graph of the population

What will this model do? (repeat)





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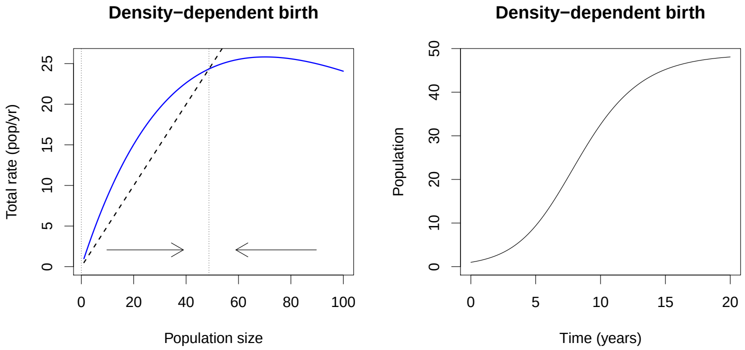

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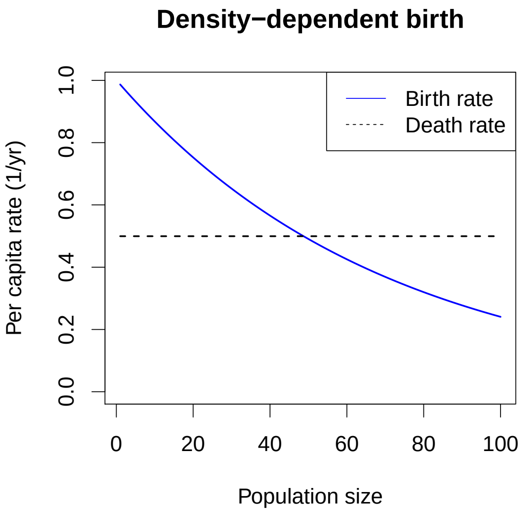
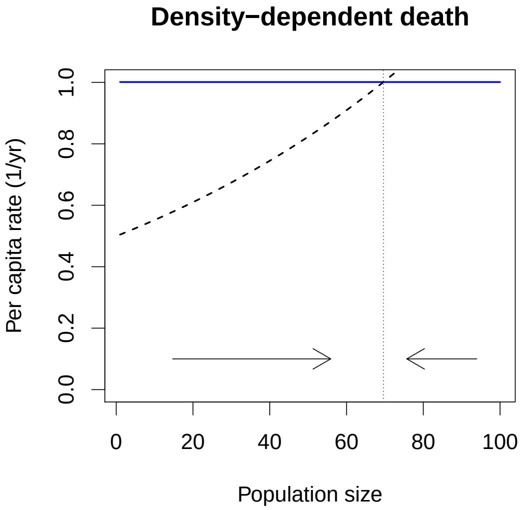


Increasing death rates



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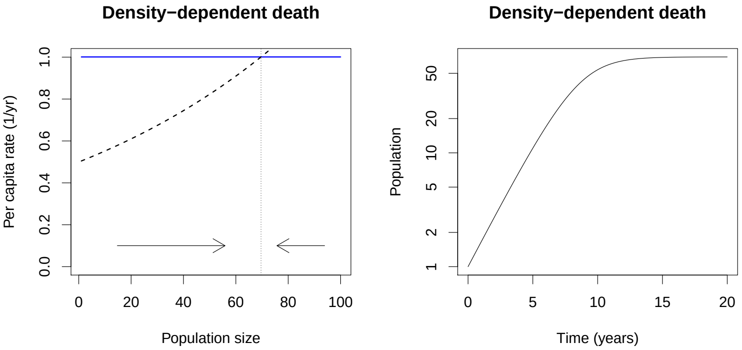

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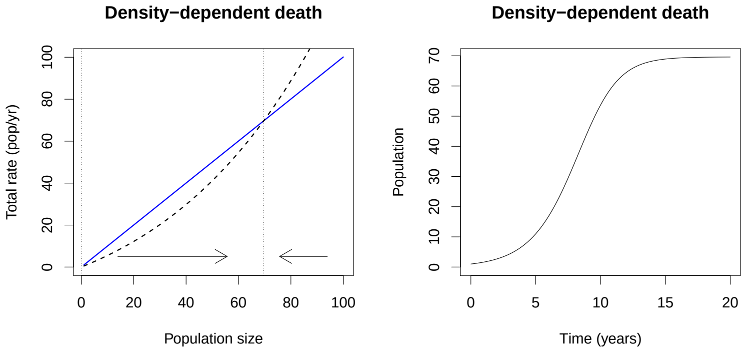

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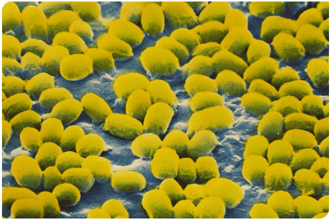

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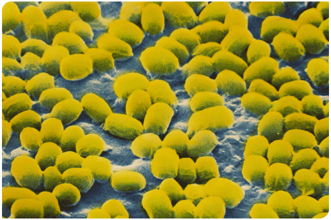

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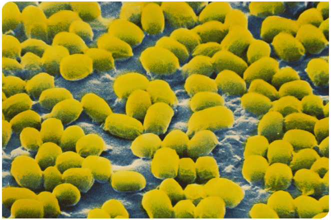

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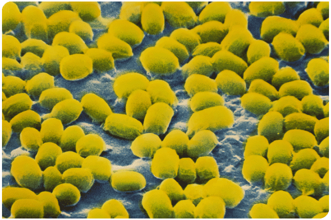

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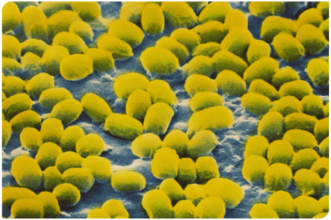

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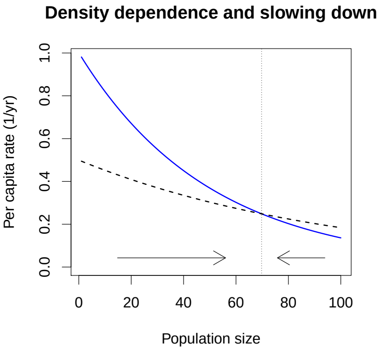

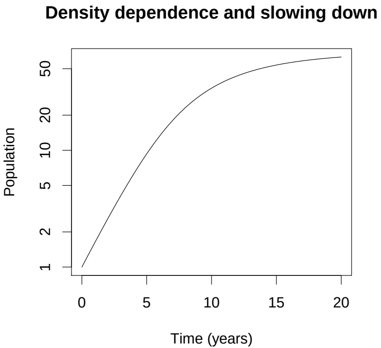

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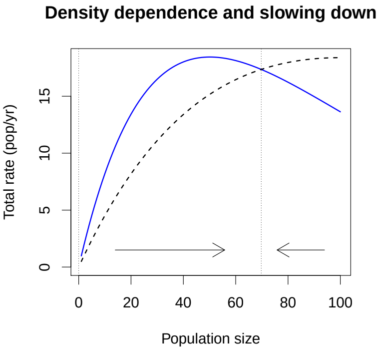

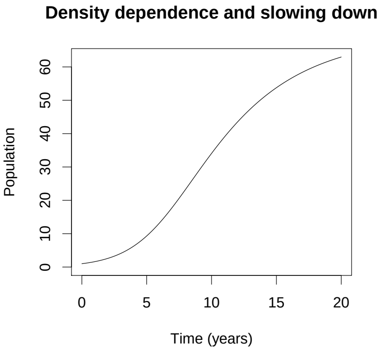

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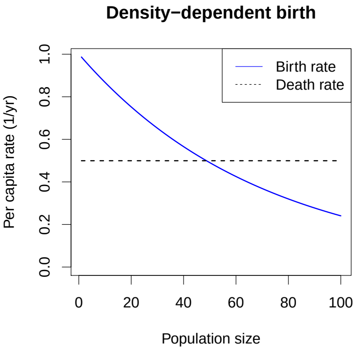

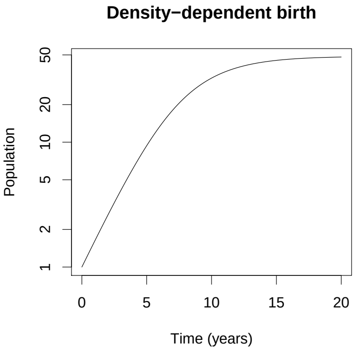

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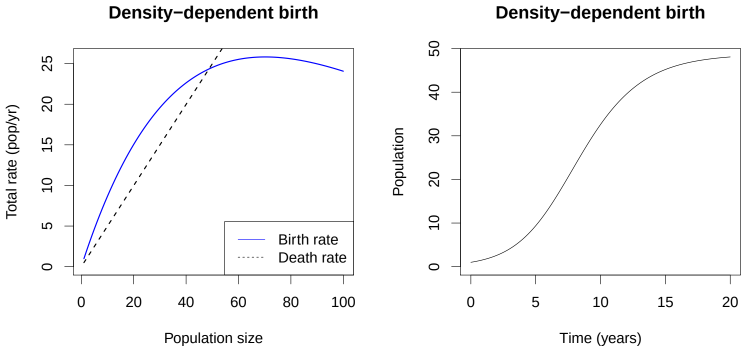

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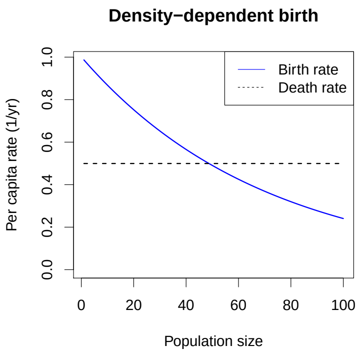

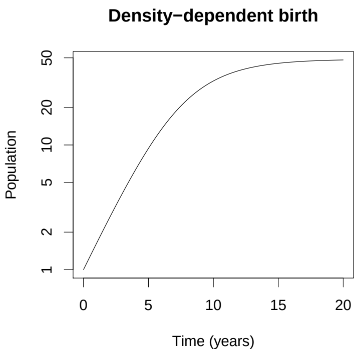

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- We say a species can "invade" a system if its rate of change is positive when the population is small.
- In other words, population can invade if the extinction equilibrium is not stable
- In this conceptual model, this is the same as saying $b(0) > d(0)$
- Which is the same as saying $\mathcal{R}_0 > 1$

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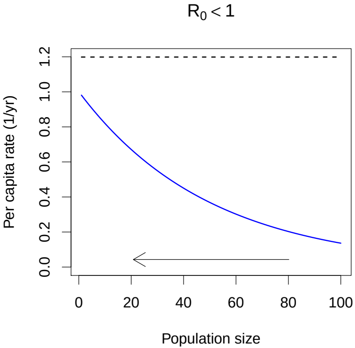

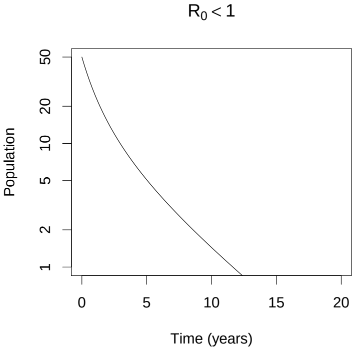

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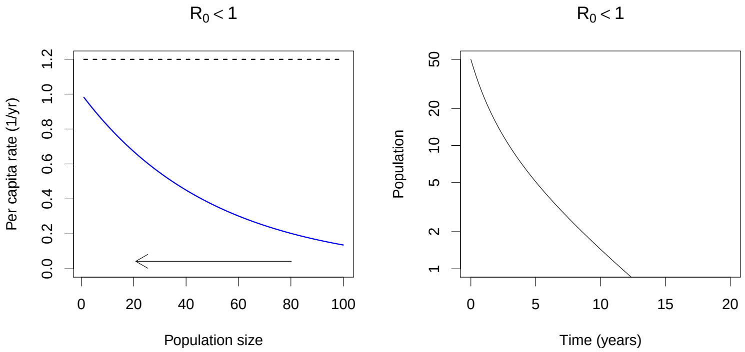

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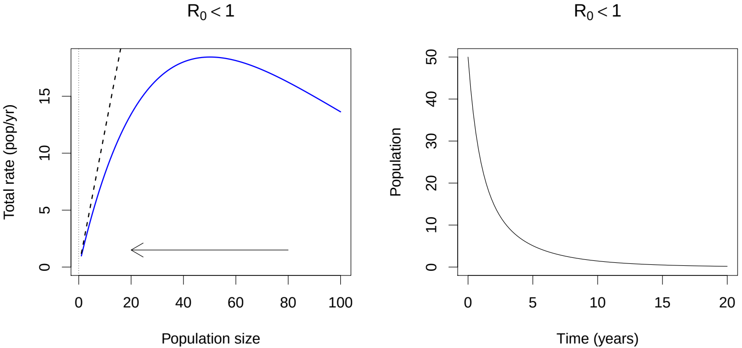

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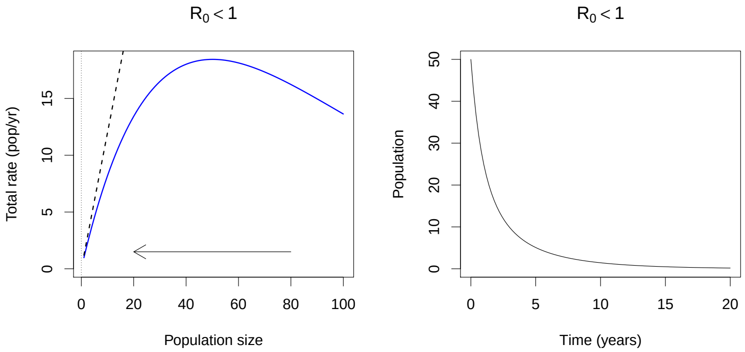

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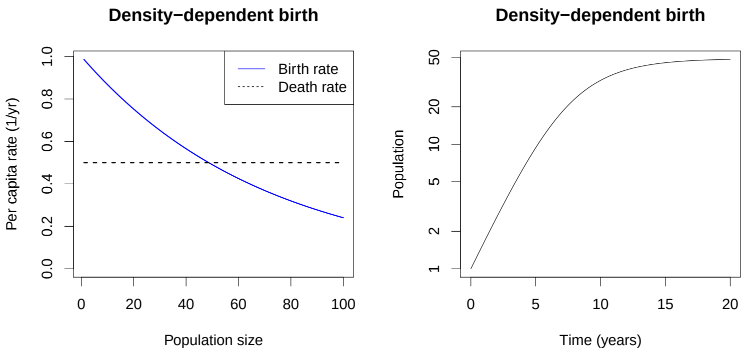

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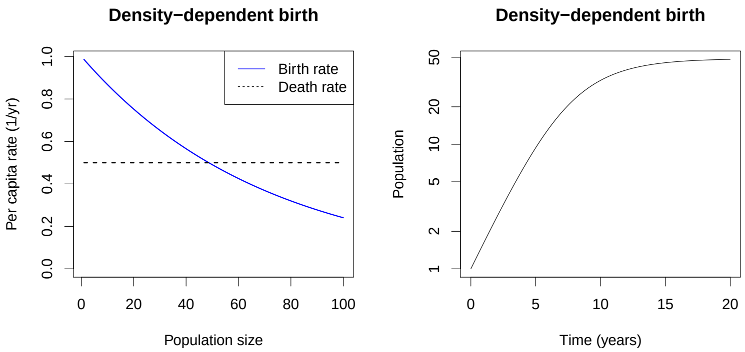

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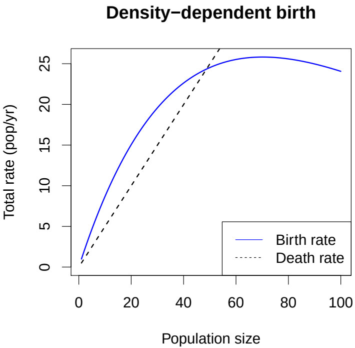

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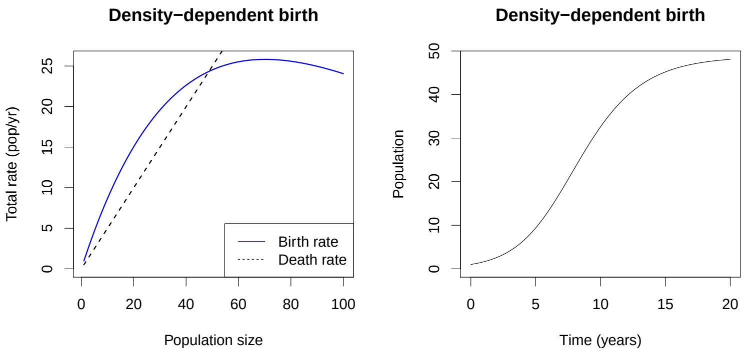

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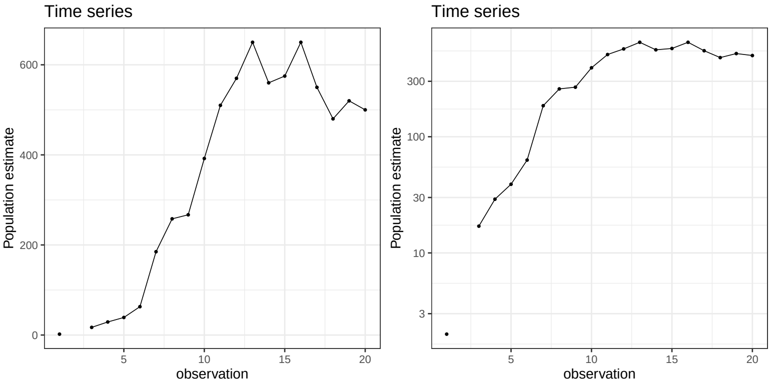

Time series
Population estimate
600
400
200
0
5
10
15
20
observation
Time series
Population estimate
300
100
30
10
3
5
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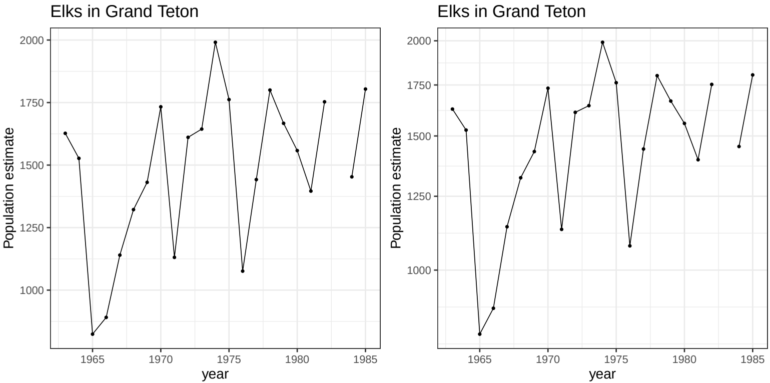
Elk



Elks in Grand Teton
Population estimate
2000
1750
1500
1250
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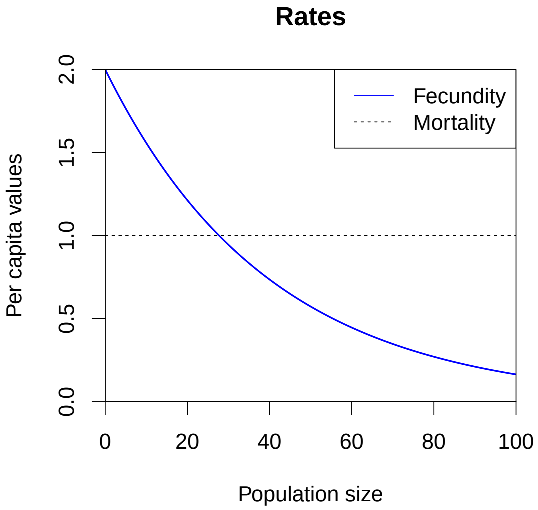

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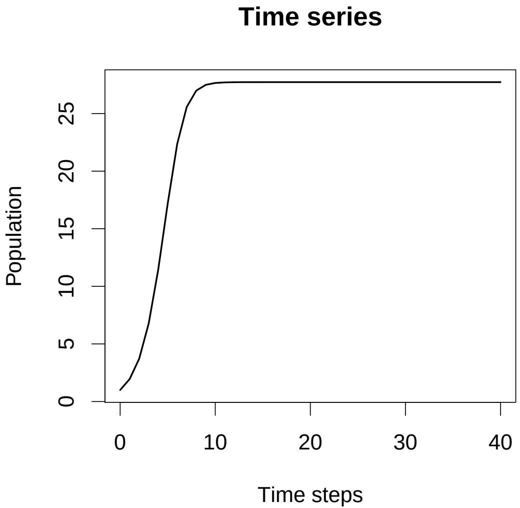

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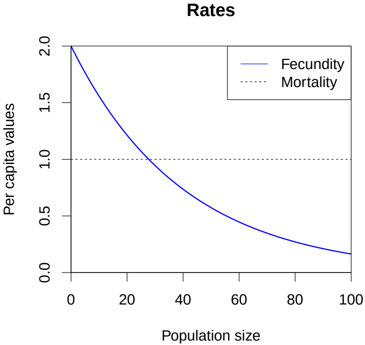
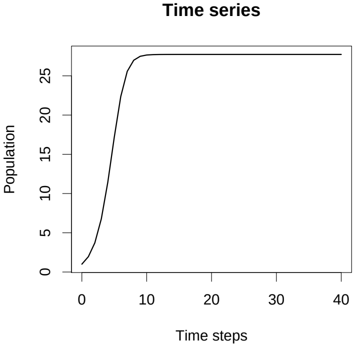
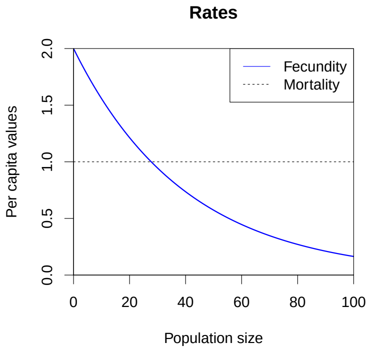


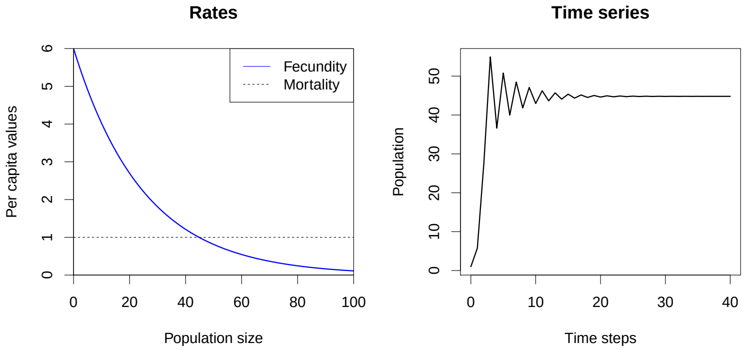


What dynamics do we get?

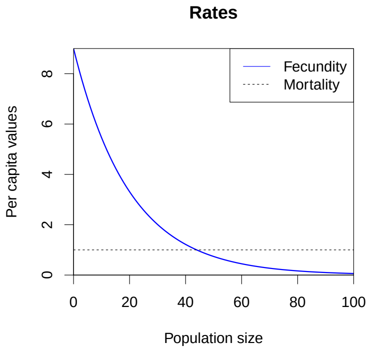
Simple dynamics (repeat)



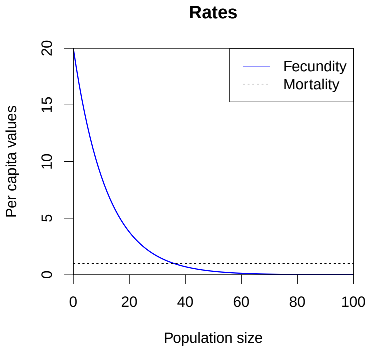
Damped oscillations



Persistent oscillations



Lots of other behaviours



Outline

Interpreting complex behaviour

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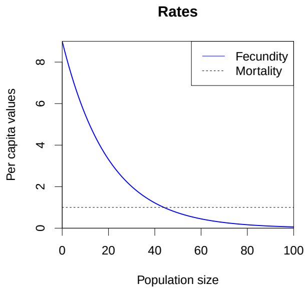

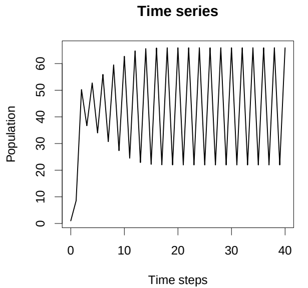

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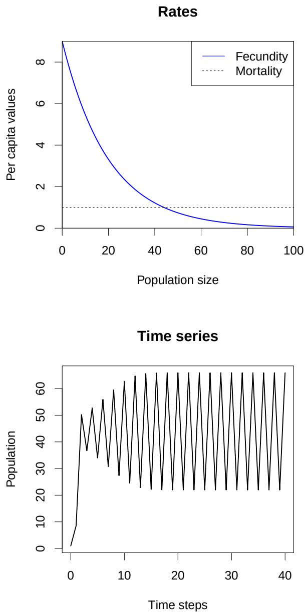

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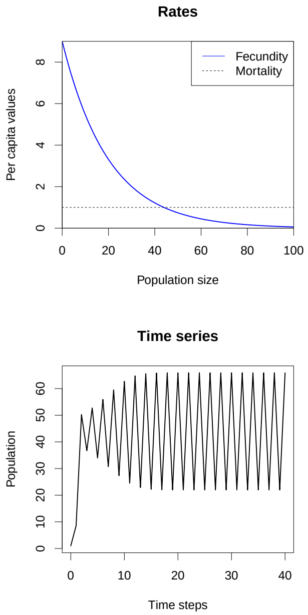

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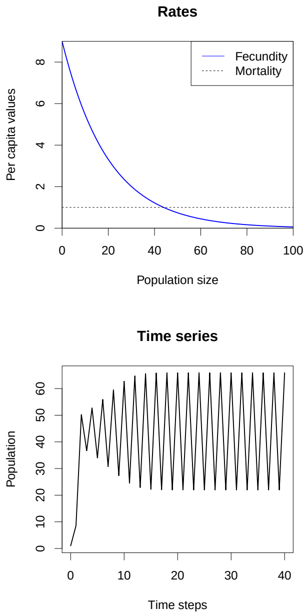
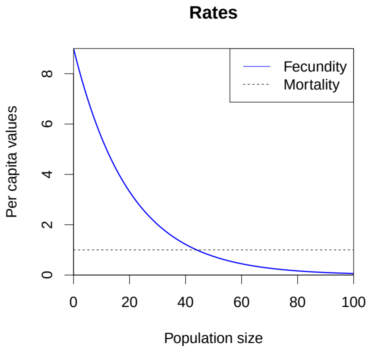


Interpreting complex behaviour (repeat)



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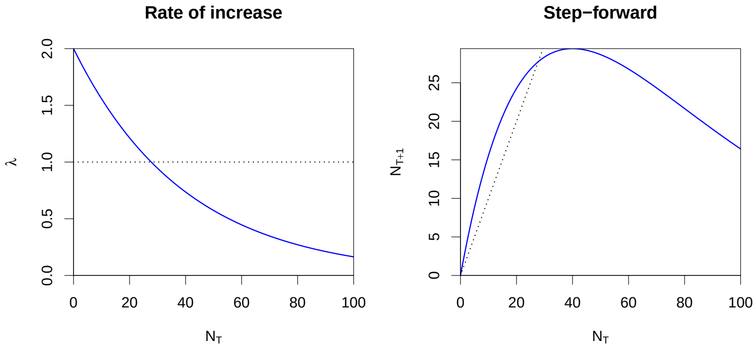

Turnover

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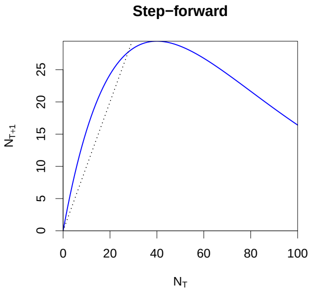

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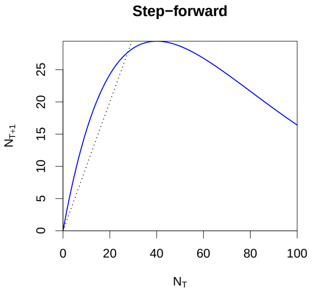

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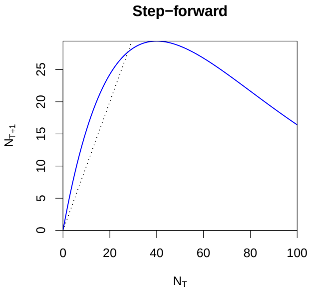

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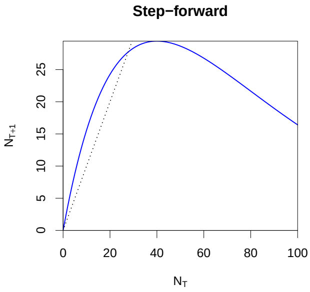
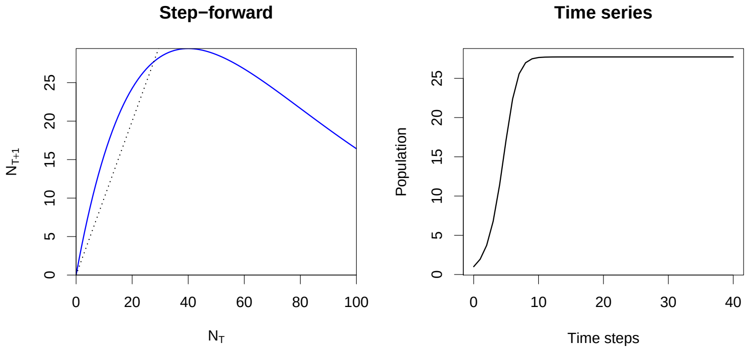


Simple dynamics



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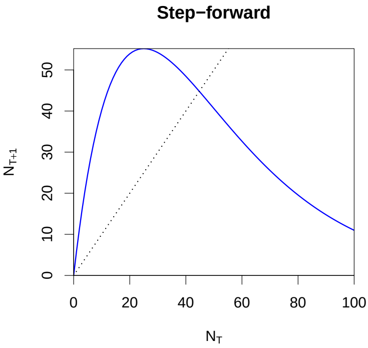

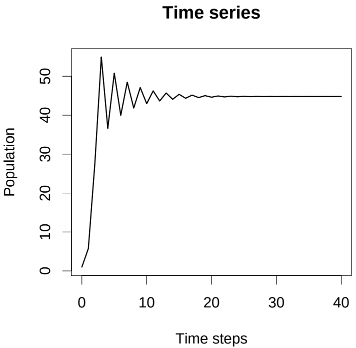

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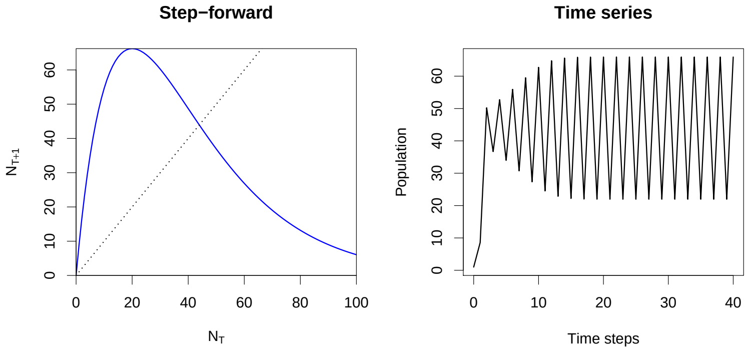

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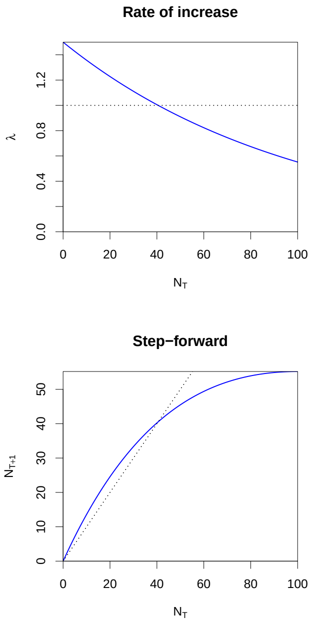

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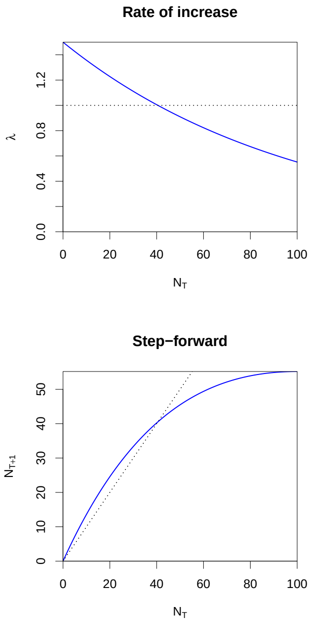

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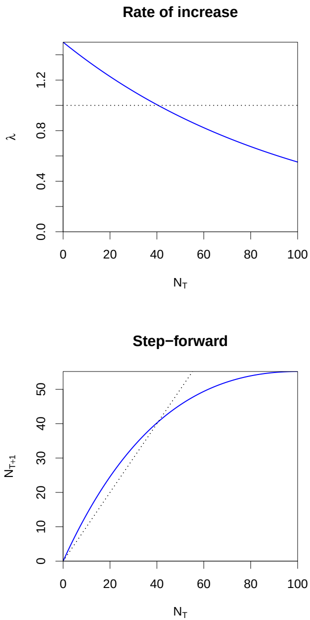

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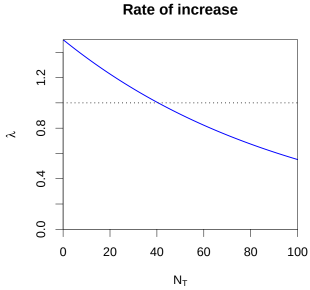

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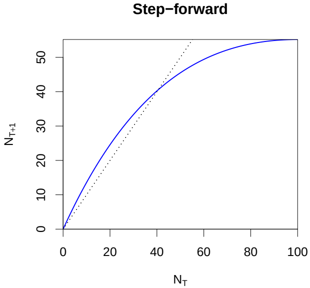
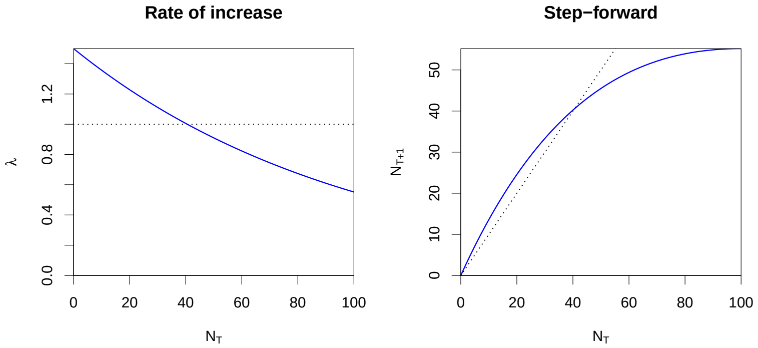


Contest regulation



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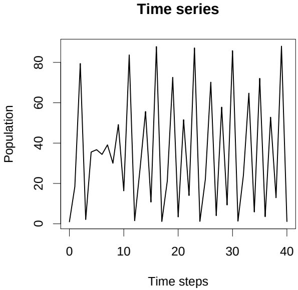

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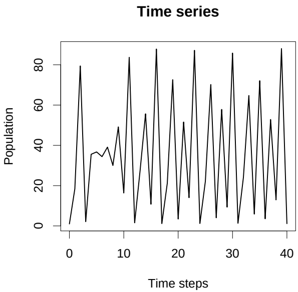

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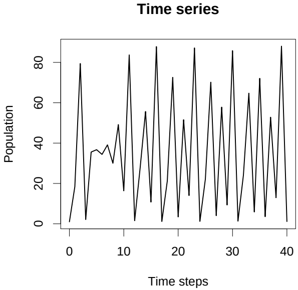

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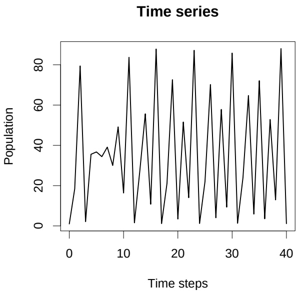

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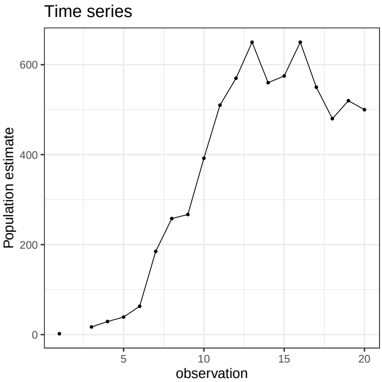

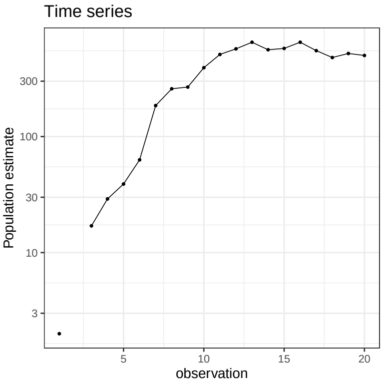

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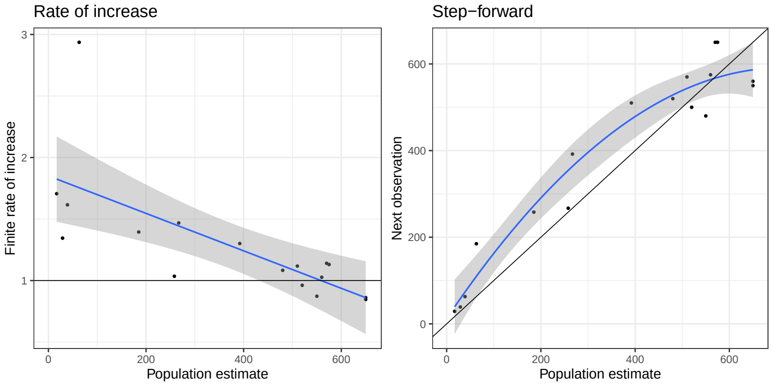
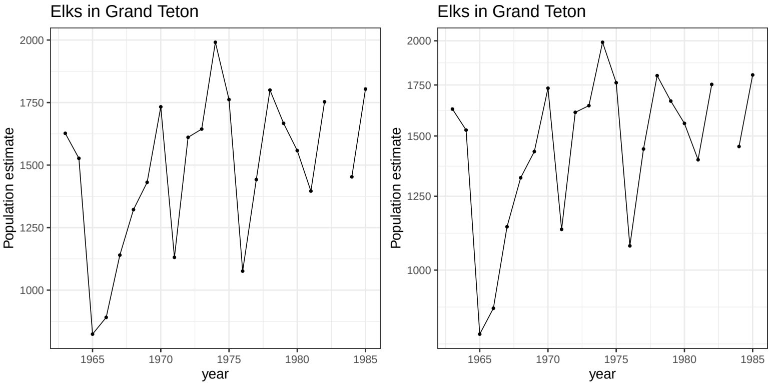

Rate of increase
Finite rate of increase
3
2
1
0
200
400
600
Population estimate
Step−forward
Next observation
600
400
200
0
0
200
400
600
Population estimate

Elk (repeat)



Elks in Grand Teton
Population estimate
2000
1750
1500
1250
1000
1965
1970
1975
1980
1985
year
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1750
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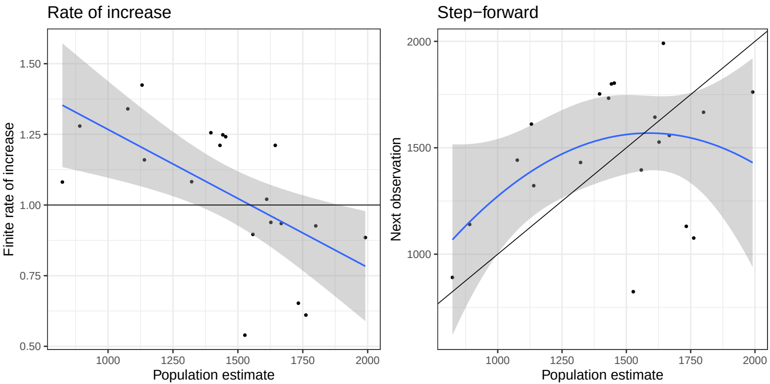
Elk



Rate of increase
Finite rate of increase
1.50
1.25
1.00
0.75
0.50
1000
1250
1500
1750
2000
Population estimate
Step−forward
Next observation
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1500
1000
1000
1250
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1750
2000
Population estimate

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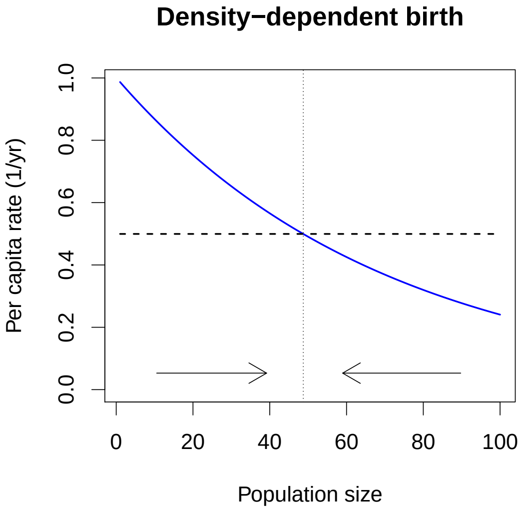
Arrows with time delay



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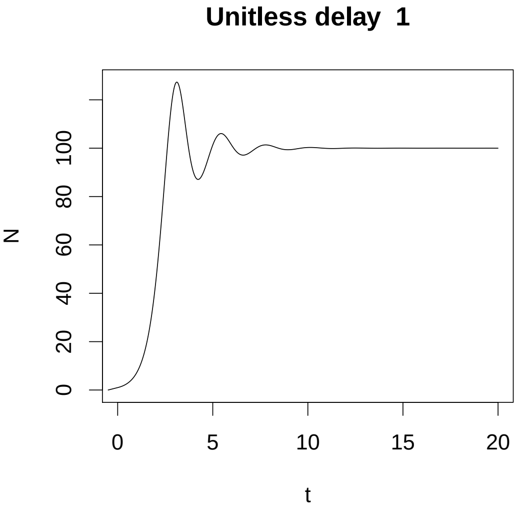

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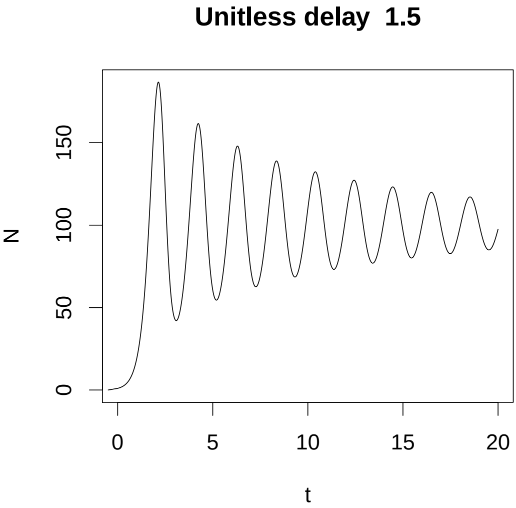

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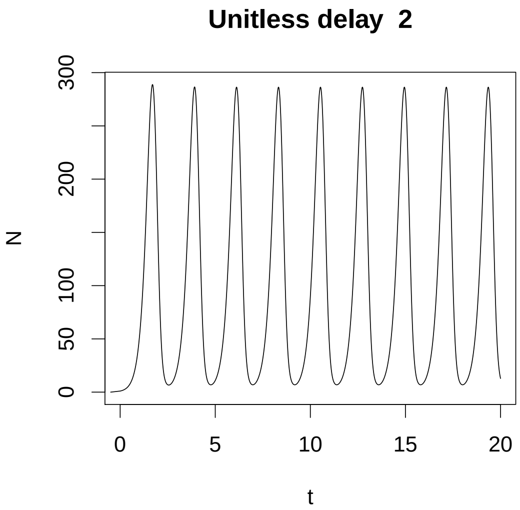

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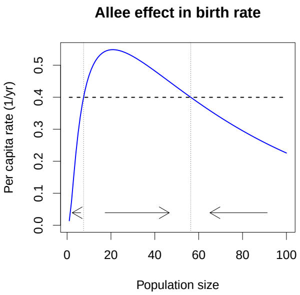

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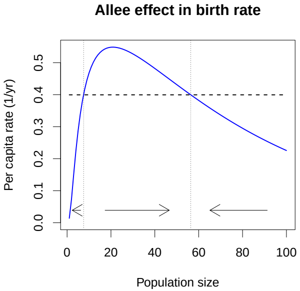

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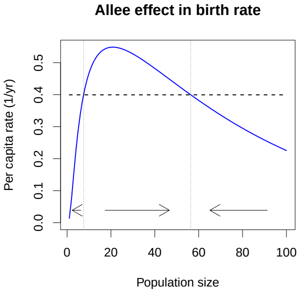

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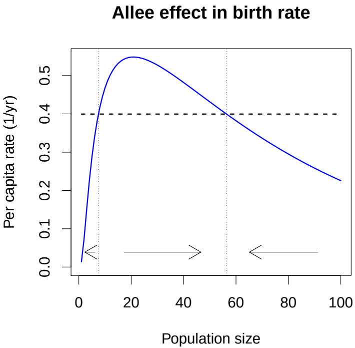

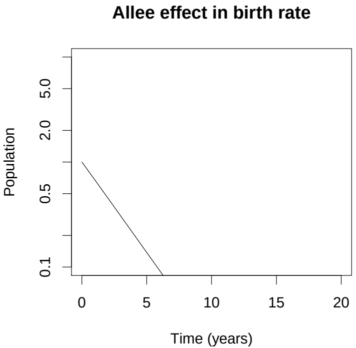

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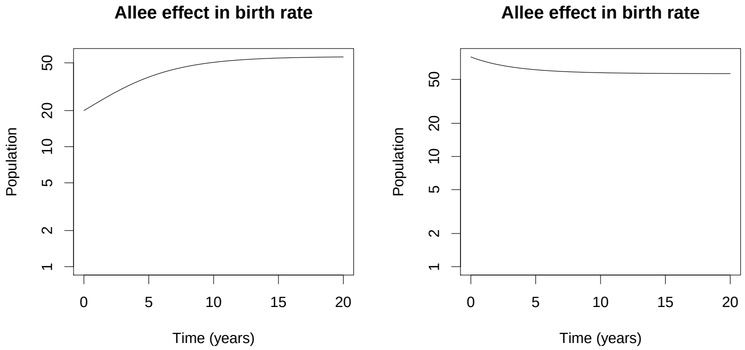

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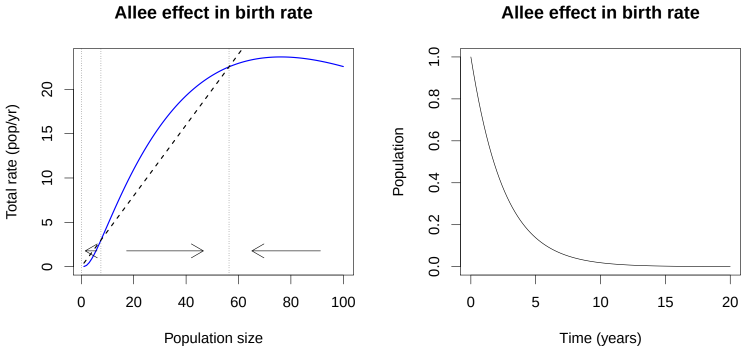

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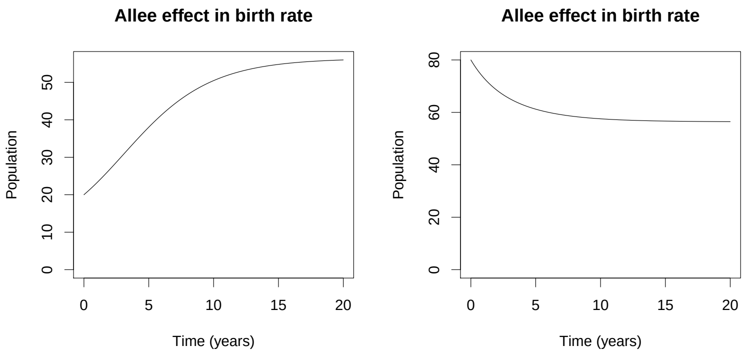
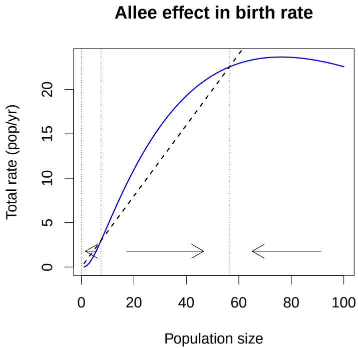
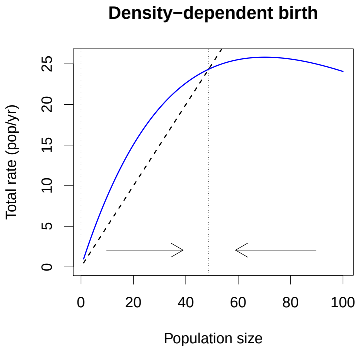


Population comparison (repeat)





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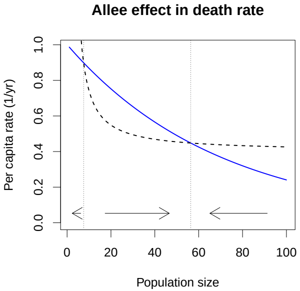

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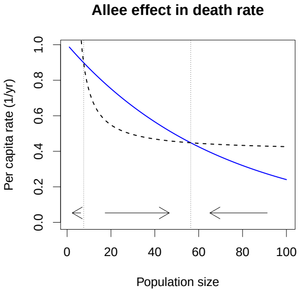

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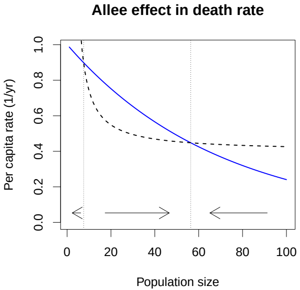

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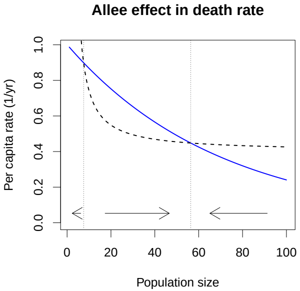

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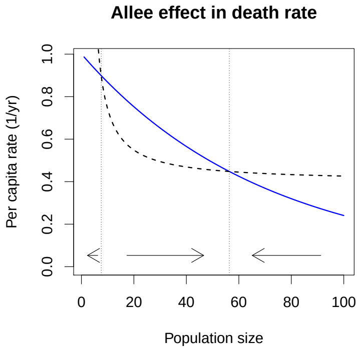

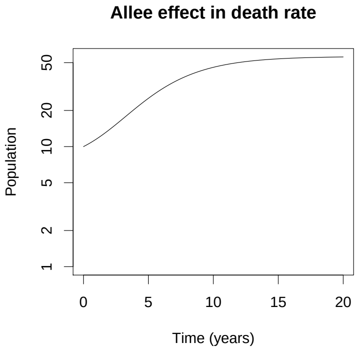

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Invasion

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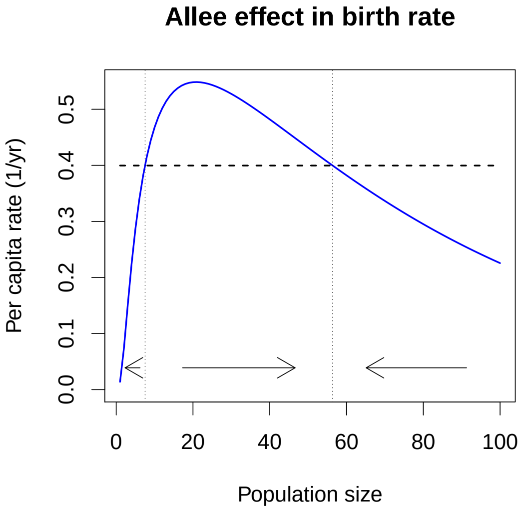

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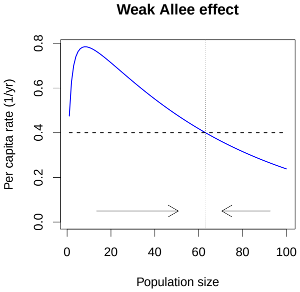

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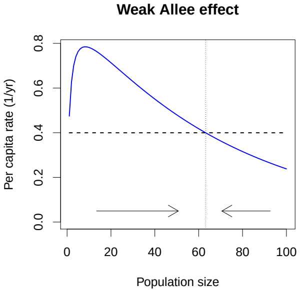

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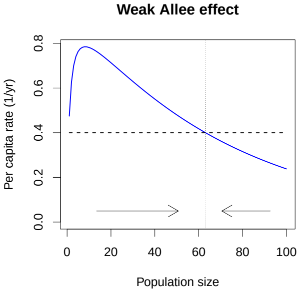

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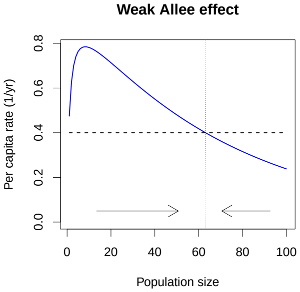

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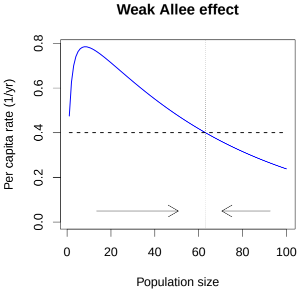

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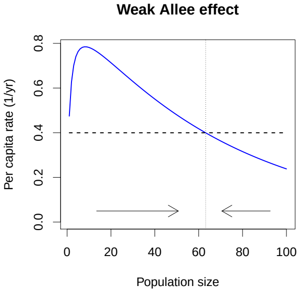
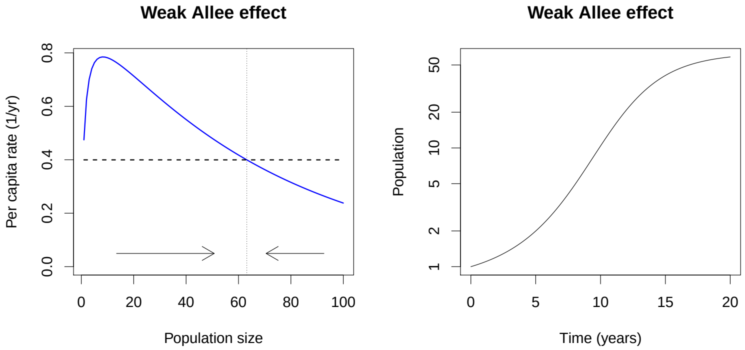


Individual perspective



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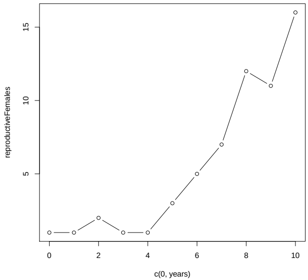

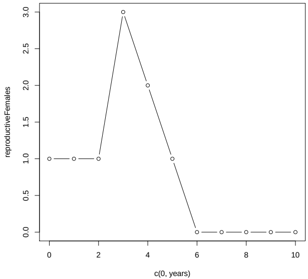

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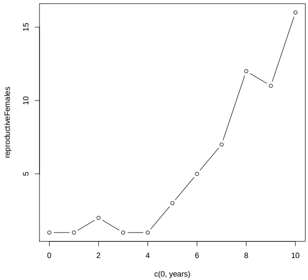

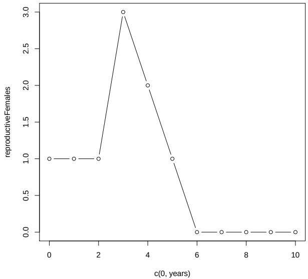

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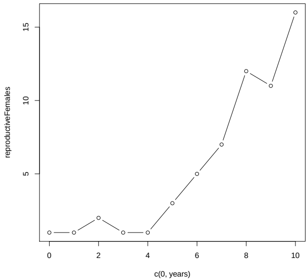

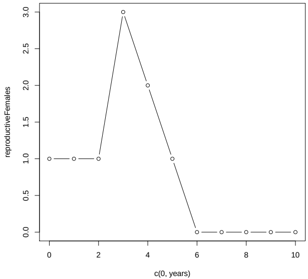

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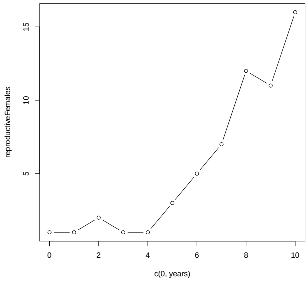

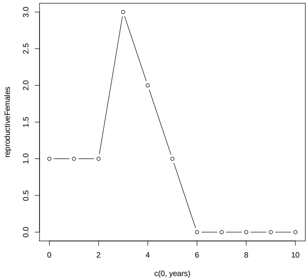

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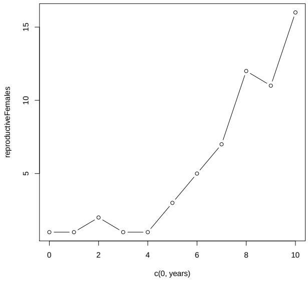

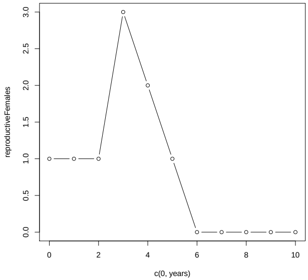

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