

UNIT 8: Infectious disease

Outline

Introduction

Rate of spread

Single-epidemic model

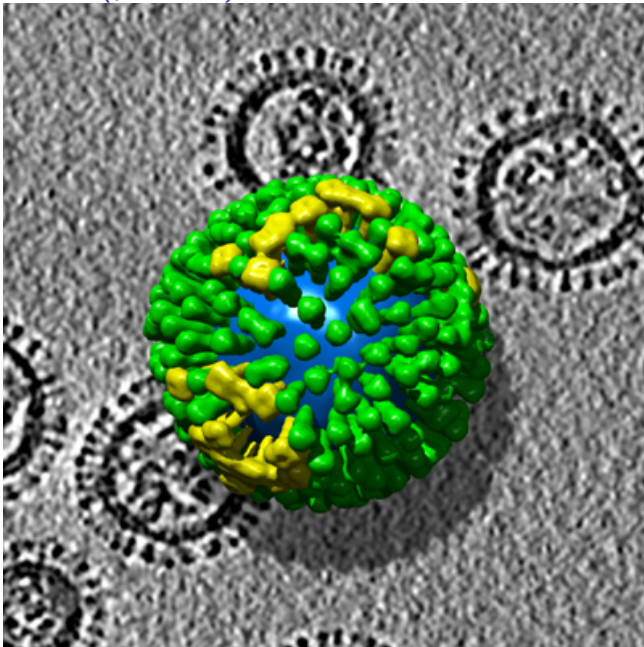
Infectious disease

- ▶ Extremely common
- ▶ Huge impacts on ecological interactions
- ▶ A form of exploitation, but doesn't fit well into our previous modeling framework
 - ▶ How many people are there?
 - ▶ How many influenza viruses are there?
 - ▶ How do they find each other?

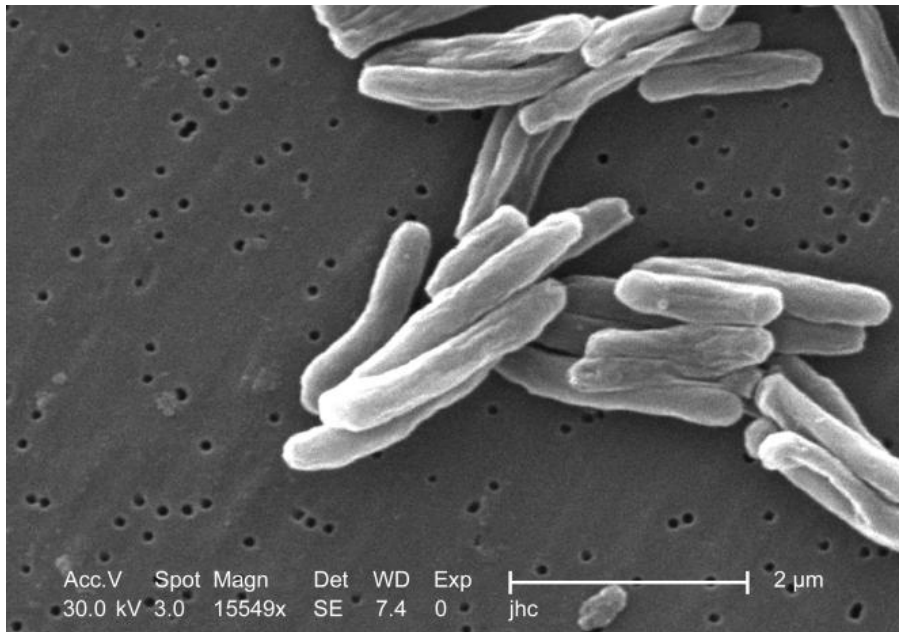
Disease agents

- ▶ Name an infectious agent that causes disease in humans.
- ▶ Disease agents vary tremendously:
 - ▶ Most **viruses** have just a handful of genes that allow them to hijack a cell and get it to make virus copies
 - ▶ * influenza virus, Ebola virus, HIV, SARS-CoV-2
 - ▶ **Bacteria** are independent, free-living cells with hundreds or thousands of chemical pathways
 - ▶ * Tuberculosis, anthrax, pertussis
 - ▶ **Eukaryotic** pathogens are nucleated cells who are more closely related to you than they are to bacteria
 - ▶ * Malaria, various worms

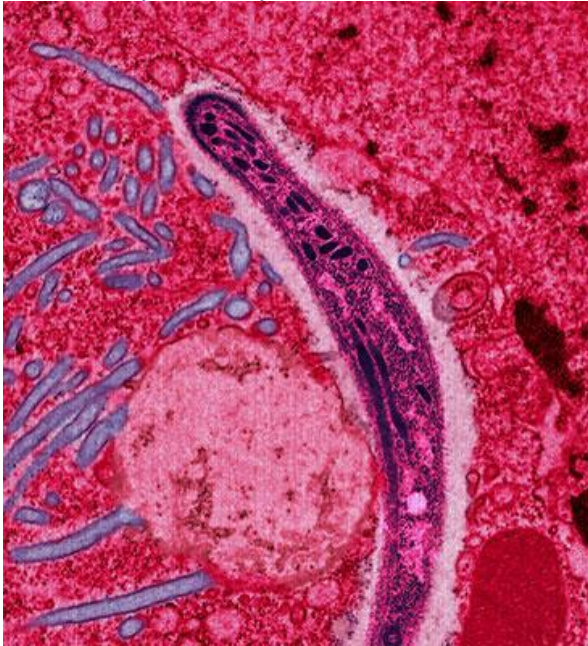
Influenza virus (present)



Tuberculosis bacilli (present)



Malaria sporozoite (present)



Microparasites

- ▶ For infections with small pathogens (viruses and bacteria), we don't attempt to count pathogens, but instead divide disease into stages
 - ▶ Latently infected (infected but not yet infectious)
 - ▶ Infectious
 - ▶ Recovered

Microparasite models

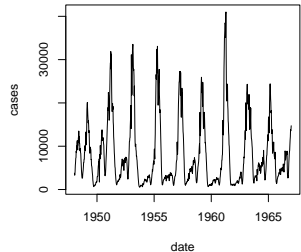
- ▶ We model microparasites by counting the number of hosts in various **states**:
 - ▶ **Susceptible** individuals can become infected
 - ▶ **Infectious** individuals are infected and can infect others
 - ▶ **Resistant** individuals are not infected and cannot become infected
- ▶ More complicated models include other states:
 - ▶ * Not yet infectious
 - ▶ * Severe infections
 - ▶ * Asymptomatic but infectious

Models as tools

- ▶ Models are the tools that we use to connect scales:
 - ▶ individuals to populations
 - ▶ single actions to trends through time



Measles reports from England and Wales



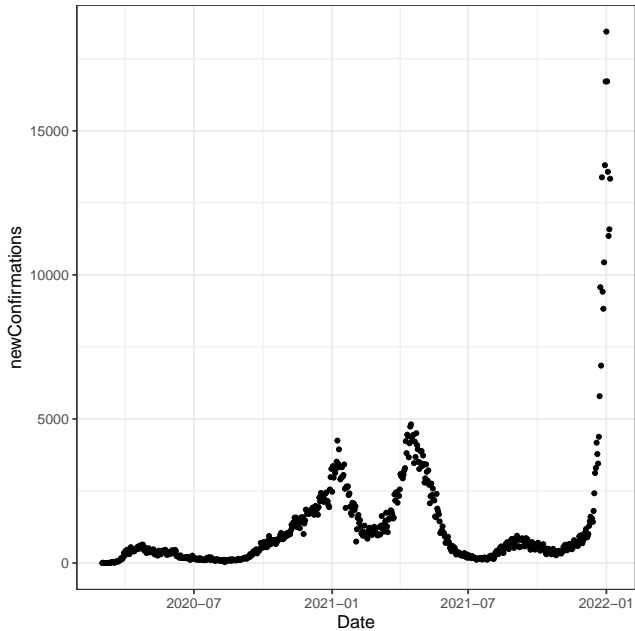
Outline

Introduction

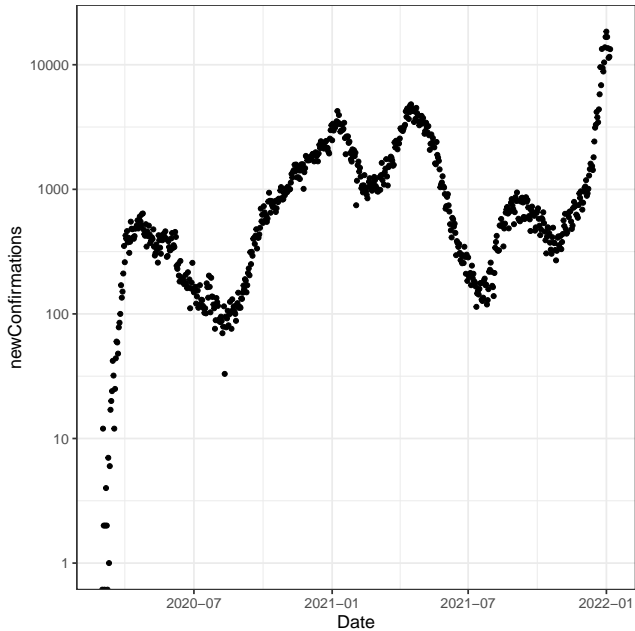
Rate of spread

Single-epidemic model

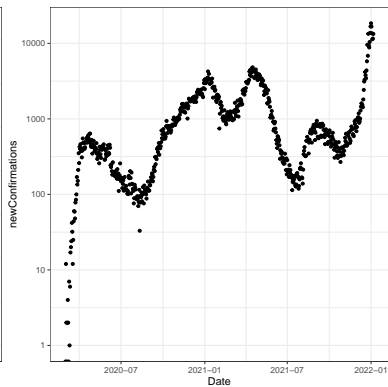
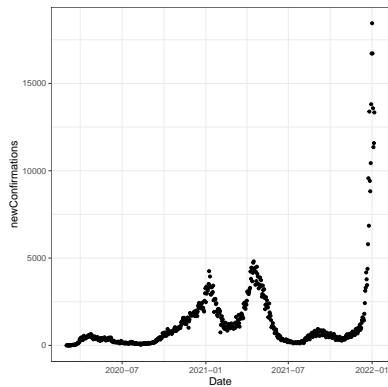
COVID in Ontario (repeat)



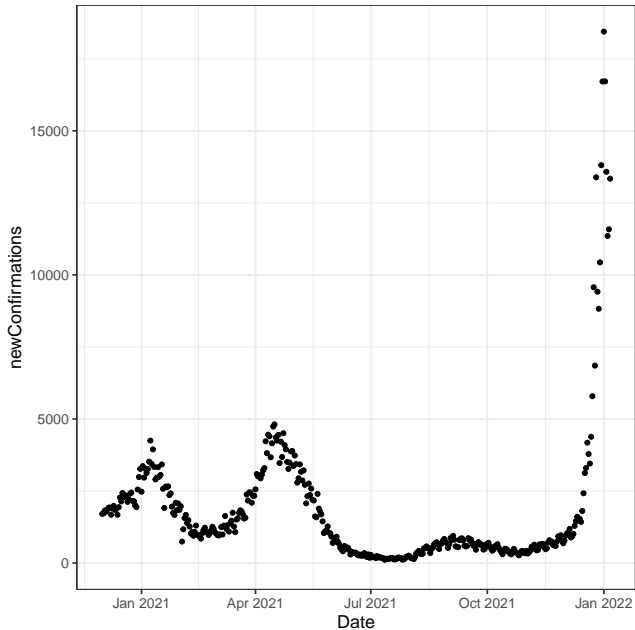
COVID in Ontario (repeat)



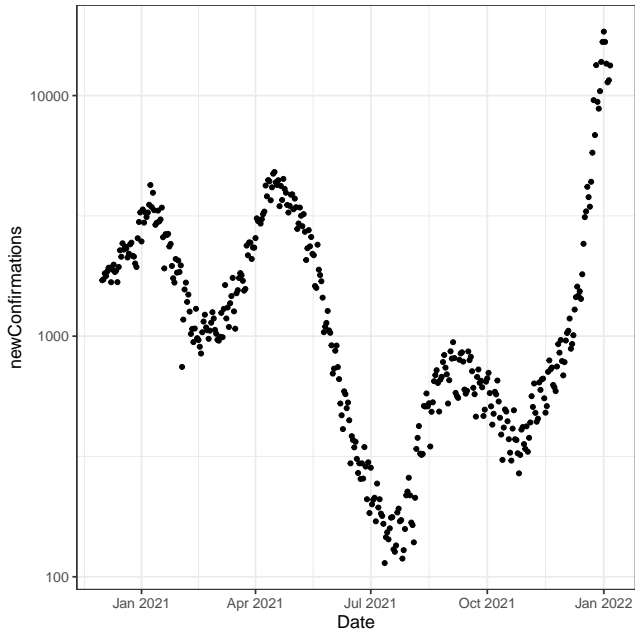
COVID in Ontario



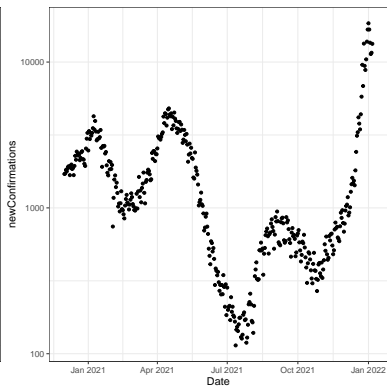
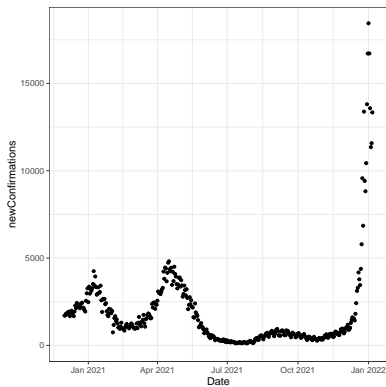
COVID in Ontario (repeat)



COVID in Ontario (repeat)



COVID in Ontario



Scales

- ▶ Which scale should we look at?
 - ▶ * Log scale is better for looking at trends
 - ▶ * Linear scale is better for looking at impacts

Population biology

- ▶ What quantities do we want to look at?
 - ▶ * Speed of exponential growth r
 - ▶ * Finite rate of increase λ
 - ▶ * Skipped this year
 - ▶ * Lifetime reproduction

Instantaneous rate of growth r

- ▶ What are the components?
 - ▶ * Birth rate
 - ▶ * Instantaneous rate of a case producing new cases
 - ▶ * $[\text{case}/(\text{case} \cdot \text{time})]$
 - ▶ * Death rate
 - ▶ * Virus-centered!
 - ▶ * Rate of death, recovery, or effective quarantine
- ▶ How do you think we estimate?
 - ▶ * We estimate r from the population-level increase in disease
 - ▶ * Then using that to estimate b
 - ▶ * Models go both directions!
 - ▶ Individuals \leftrightarrow Populations

Reproductive number \mathcal{R}

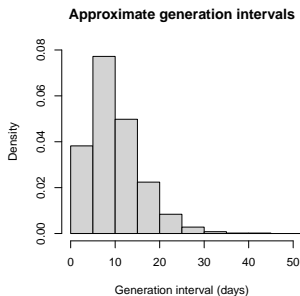
- ▶ What is it?
 - ▶ * Expected number of new cases per case over the lifetime of a case
- ▶ Why do we want this?
 - ▶ * An important measure of how hard the epidemic will be to stop
- ▶ How do we calculate it?
 - ▶ * $\mathcal{R} = b/d$; if we can estimate those

Example

- ▶ $r \approx 0.14/\text{day}$
- ▶ What is our estimate of \mathcal{R} ?
 - ▶ When average length of infection $L = 5$ day?
 - ▶ When average length of infection $L = 10$ day?

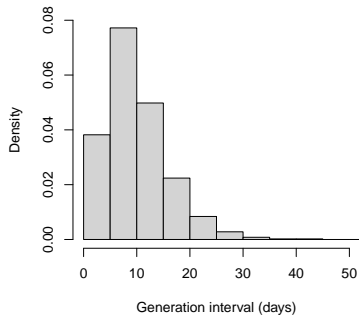
Generation intervals

- ▶ Researchers try to estimate the *proportion* of transmission that happens for different **ages of infection**
- ▶ How long from the time you are *infected* to the time you *infect someone else*?
- ▶ Analogous to a life table
- ▶ The effective generation time \hat{G} has units of time
 - ▶ And is hard to calculate, like λ in a structured model

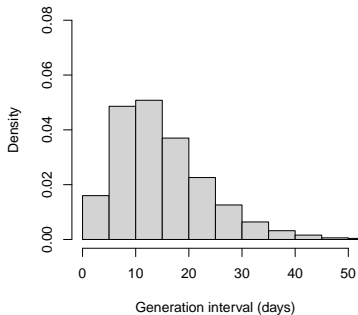


Generation intervals

Approximate generation intervals

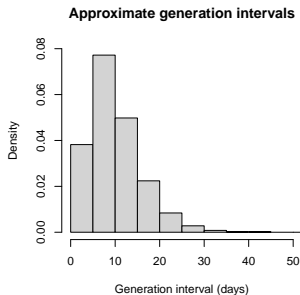


Approximate generation intervals



Speed and risk

- ▶ Which is more dangerous, a fast disease, or a slow disease?
 - ▶ How are we measuring speed?
 - ▶ How are we measuring danger?
 - ▶ *What do we already know?*



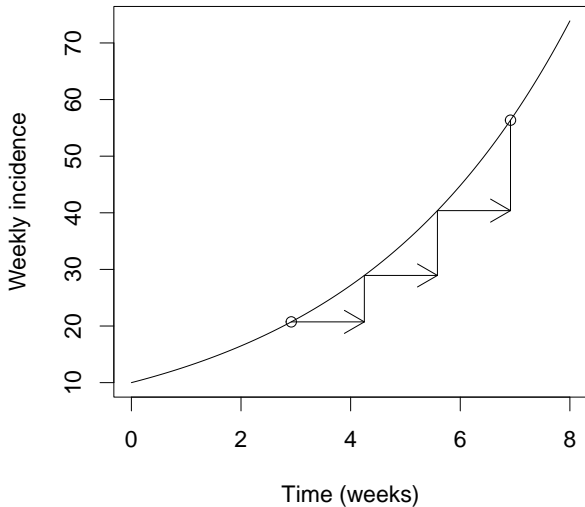
Fighting Ebola (present)



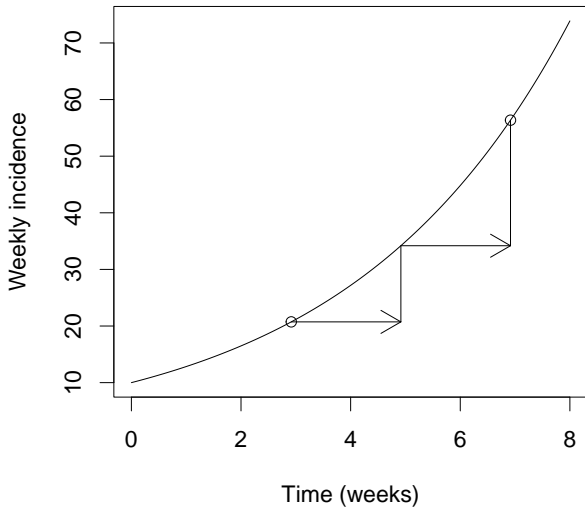
Generation time and risk

- ▶ If we know \mathcal{R} , what does the generation time tell us about r ?
 - ▶ * The faster the generations (small \hat{G}), the faster the exponential growth (large r)
- ▶ If we know r , what does the generation time tell us about \mathcal{R} ?
 - ▶ * The faster the generations (small \hat{G}), the the *smaller* the strength of the epidemic (small reproductive number \mathcal{R})
- ▶ $\mathcal{R} = \exp(r\hat{G})$

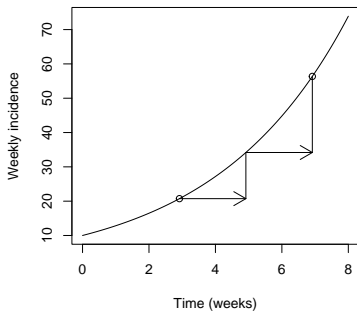
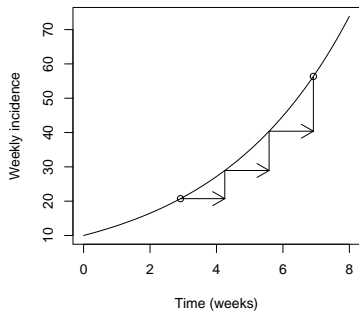
Generation time and risk (repeat)



Generation time and risk (repeat)



Generation time and risk



Generation time and risk

- ▶ $\mathcal{R} = \exp(r\hat{G})$
- ▶ An intuitive view:
 - ▶ Epidemic speed = Generation strength \times Generation speed
 - ▶ *Mathematically:* $r = \log(\mathcal{R}) * (1/\hat{G})$
- ▶ If we know generation speed, then a faster epidemic speed means:
 - ▶ * More strength required (greater \mathcal{R})
- ▶ If we know epidemic speed, a faster generation speed means
 - ▶ * Less strength required (smaller \mathcal{R})

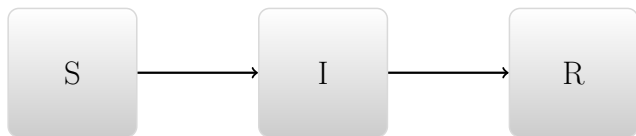
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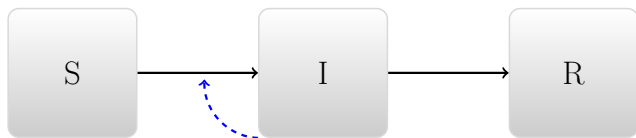
Single-epidemic model

Single-epidemic model



- ▶ Susceptible \rightarrow Infectious \rightarrow Recovered
- ▶ We also use N to mean the total population

Transition rates



- ▶ What factors govern movement through the boxes?
 - ▶ People get better independently
 - ▶ People get infected by infectious people

Conceptual modeling

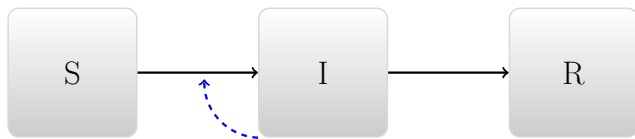
- ▶ What happens in the long term if we introduce an infectious individual?
 - ▶ * The *may be* an **epidemic**
 - an outbreak of disease
 - ▶ * Disease burns out
 - ▶ * Everyone winds up either recovered . . .
 - ▶ * or susceptible!



Interpreting

- ▶ Why might there not be an epidemic?
 - ▶ * If the disease can't spread well enough in the population
 - ▶ * Could depend on season, or immunity ...
 - ▶ * Demographic stochasticity: if we only start with one individual, we expect an element of chance
- ▶ Why doesn't everyone get infected?
 - ▶ * NOANS

Quantities



State variables

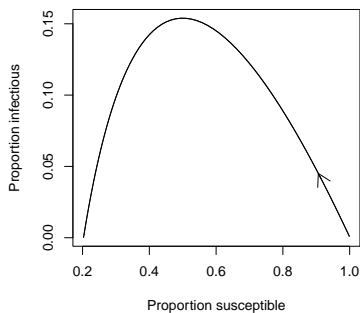
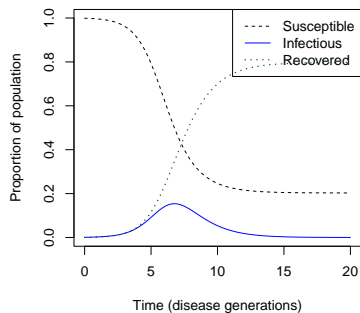
- ▶ S, I, R, N : [people] or [people/ha]

Quantities

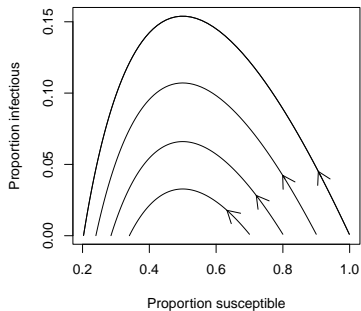
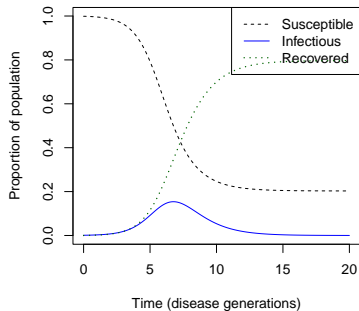
Parameters

- ▶ Susceptible people have **potentially effective** contacts at rate β (units [1/time])
 - ▶ These are contacts that would lead to infection if the person contacted is infectious
 - ▶ Total infection rate is $\beta I/N$, because I/N is the proportion of the population infectious
- ▶ Infectious people recover at *per capita* rate γ (units [1/time])
 - ▶ Total recovery rate is γI
 - ▶ Mean time infectious is $D = 1/\gamma$ (units [time])

Simulating the model (repeat)



Simulating the model



Basic reproductive number

- ▶ What *unitless* parameter can you make from the model above?
 - ▶ * $\mathcal{R}_0 = \beta D = \beta/\gamma$ is the **basic reproductive number**
 - ▶ * The *potential* number of infections caused by an average infectious individual
 - ▶ * That is: the number they would cause on average if everyone else were susceptible
 - ▶ * The product of the rate β (units [1/t]) and the duration D ([t])

Basic reproductive number

- ▶ What happens early in the epidemic if $\mathcal{R}_0 > 1$?
 - ▶ * Number of infected individuals grows exponentially
- ▶ What happens early in the epidemic if $\mathcal{R}_0 < 1$?
 - ▶ * Number of infected individuals does not grow (disease cannot invade)

Effective reproductive number

- ▶ The effective reproductive number gives the number of new infections per infectious individual in a partially susceptible population:
 - ▶ * $\mathcal{R}_{\text{eff}} = \mathcal{R}_0 S/N$
- ▶ Is the disease increasing or decreasing?
 - ▶ * It will increase when $\mathcal{R}_{\text{eff}} > 1$ (more than one case per case)
 - ▶ * This happens when $S/N > 1/\mathcal{R}_0$
- ▶ Why doesn't everyone get infected?
 - ▶ * When susceptibles are low enough $\mathcal{R}_{\text{eff}} < 1$
 - ▶ * When $\mathcal{R}_{\text{eff}} < 1$, the disease dies out on its own (less than one case per case)