UNIT 3 Non-linear population models

Outline

Introduction
- Population Examples

Continuous-time regulation
- A simple, continuous-time model
- Simulating model behaviour
- Equilibria

Discrete-time regulation
- A simple, discrete-time model
- Simulating this system
- Interpreting complex behaviour

Small populations
- Allee effects
- Stochastic effects

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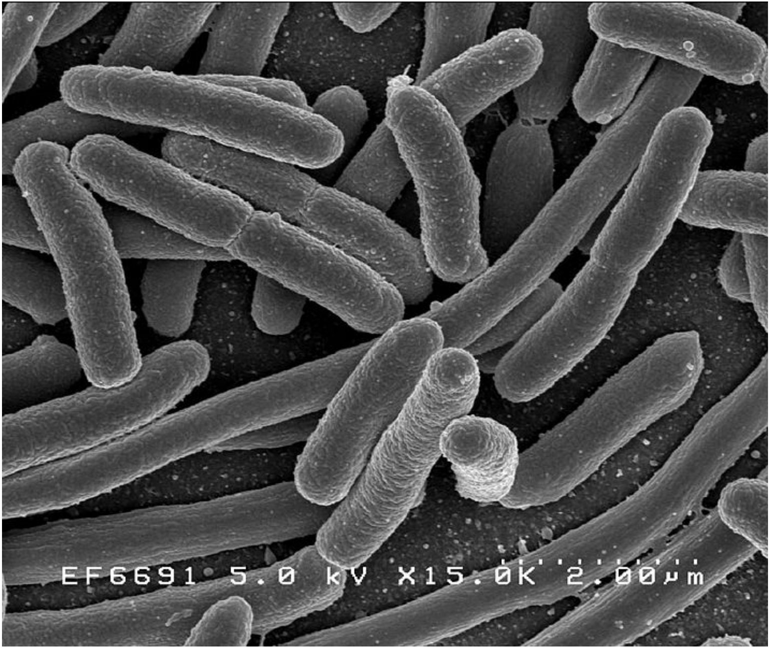
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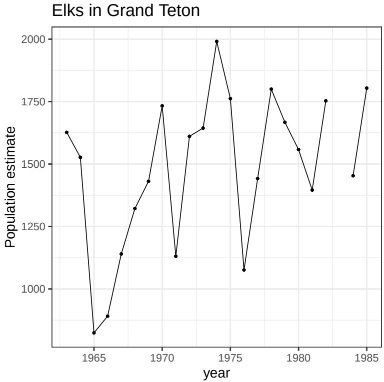
Elks in Grand Teton
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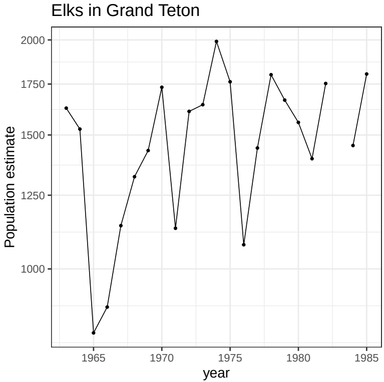
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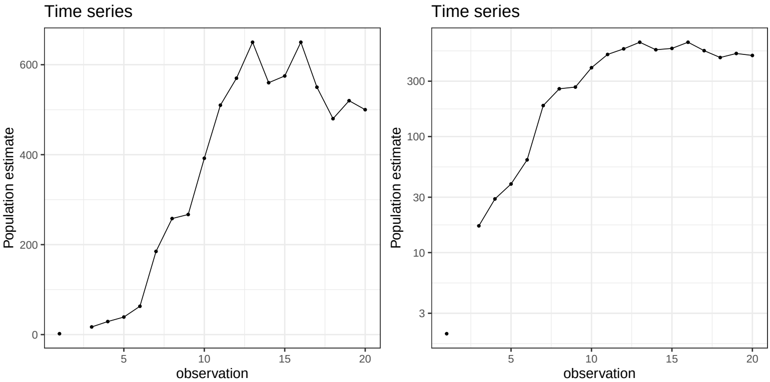
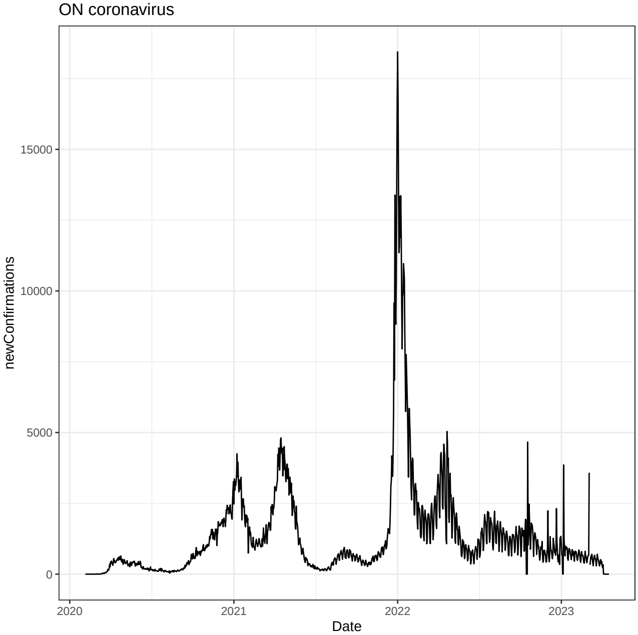
Paramecia (preview)



Coronavirus (preview)



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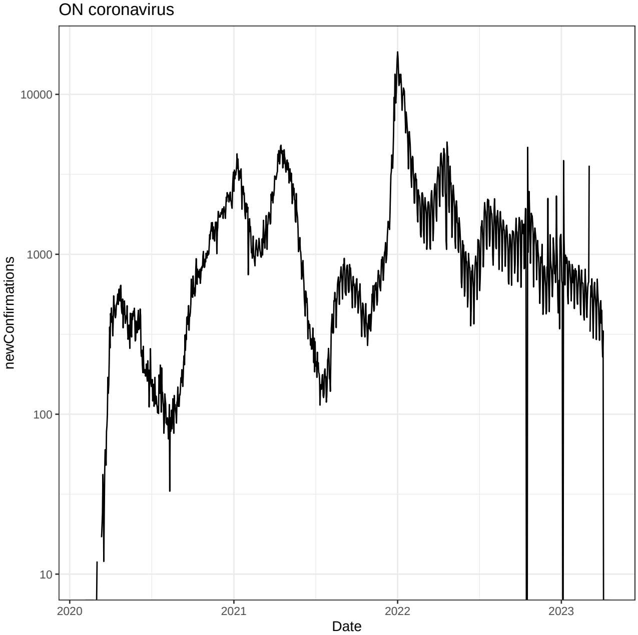

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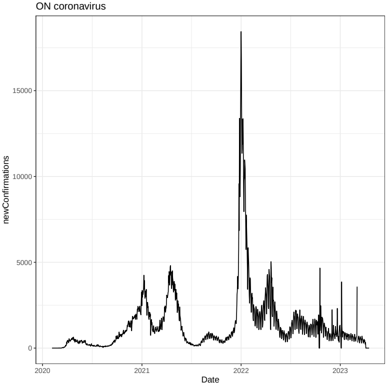
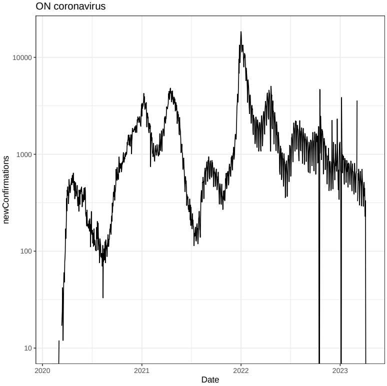
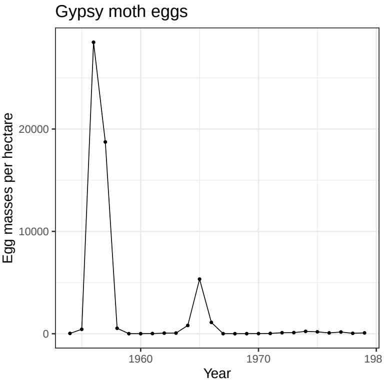
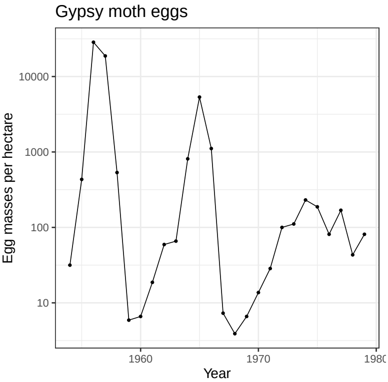




Gypsy moths (preview)





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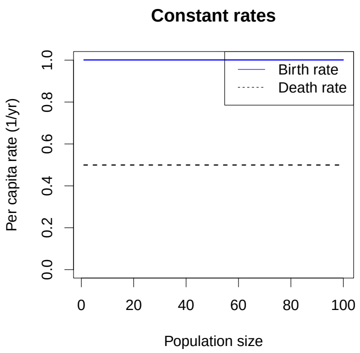

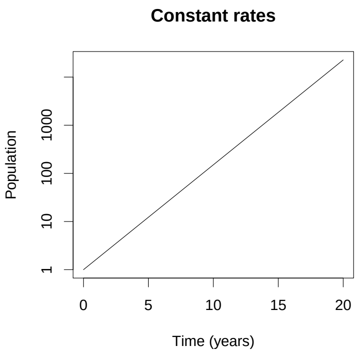

Individual perspective

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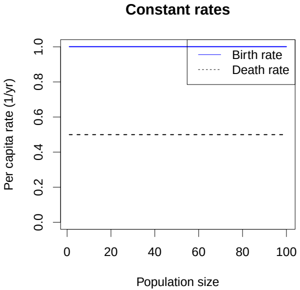

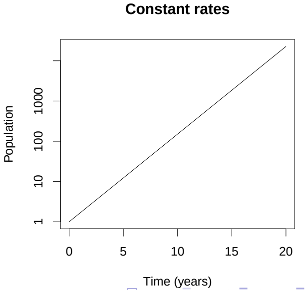

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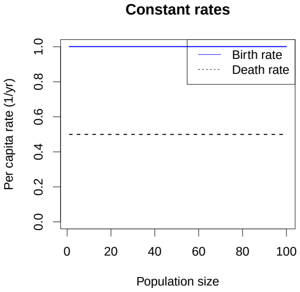

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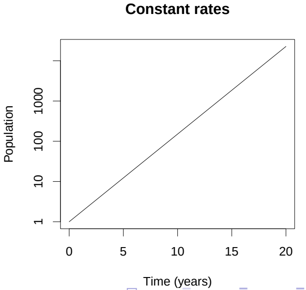

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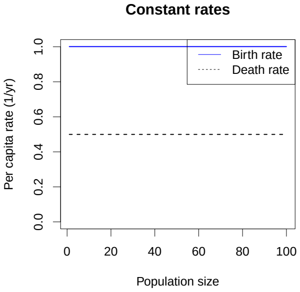

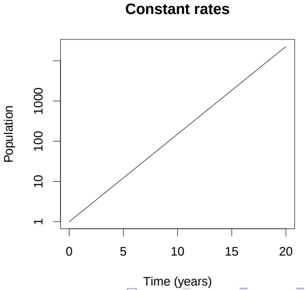

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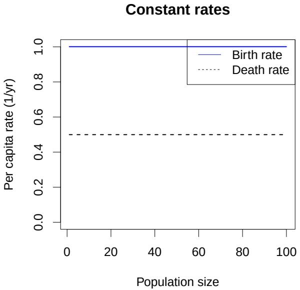

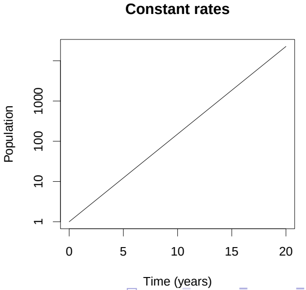

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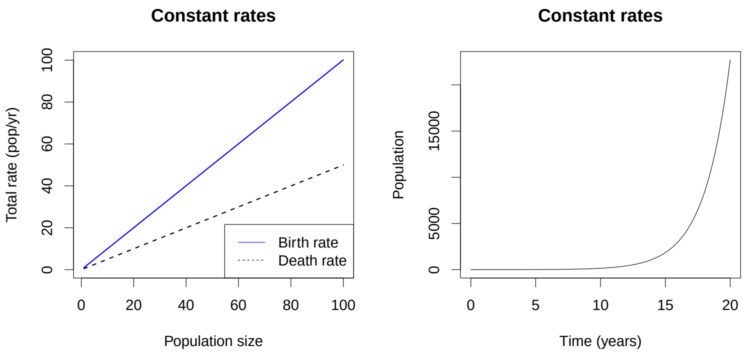

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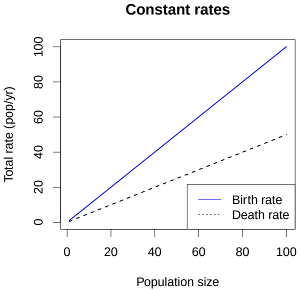

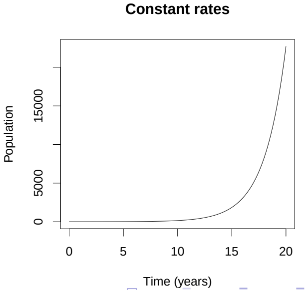

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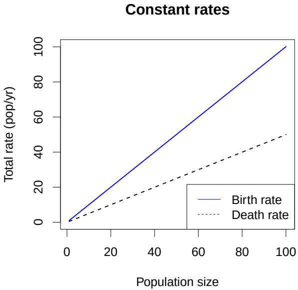

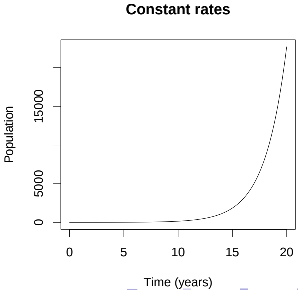

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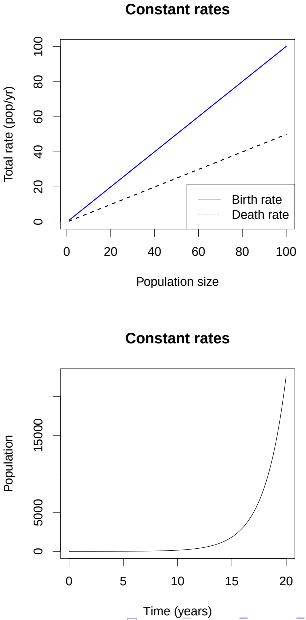

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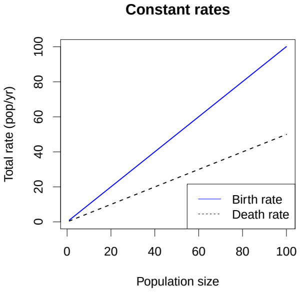

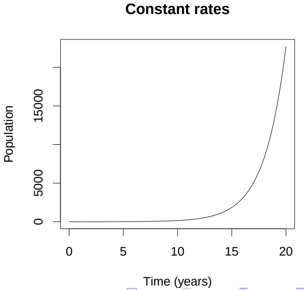

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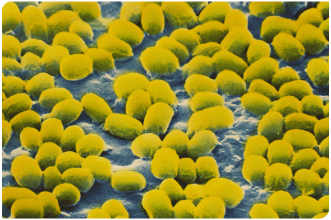

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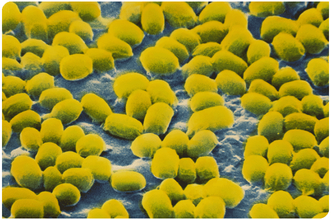

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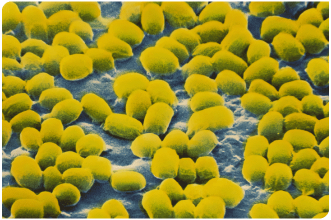

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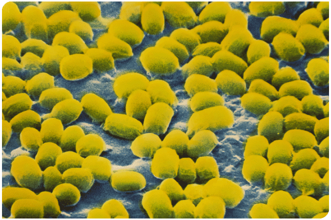

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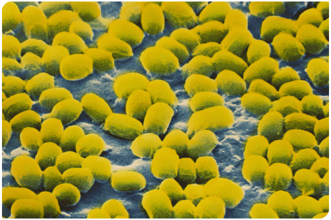

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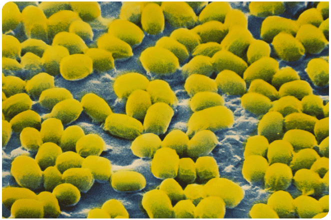

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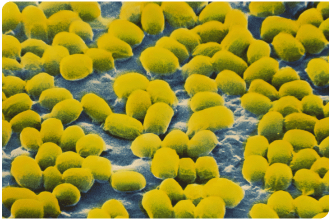

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Outline

Introduction
- Population Examples

Continuous-time regulation
- A simple, continuous-time model
- Simulating model behaviour
- Equilibria

Discrete-time regulation
- A simple, discrete-time model
- Simulating this system
- Interpreting complex behaviour

Small populations
- Allee effects
- Stochastic effects

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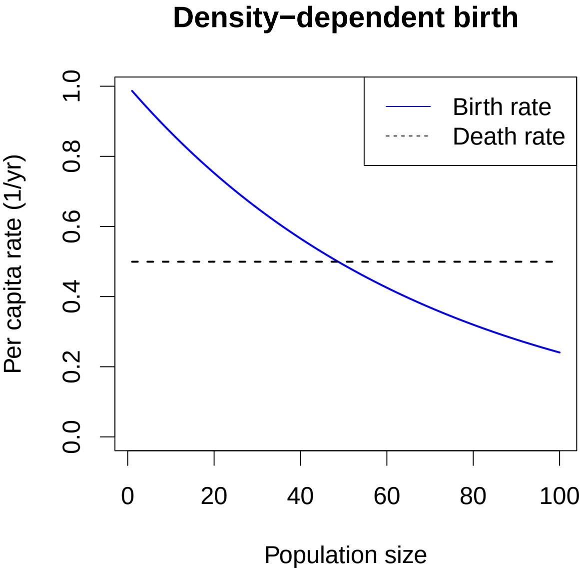
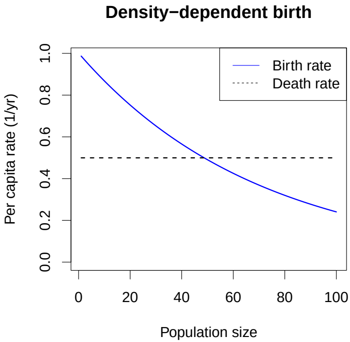
A model



Density−dependent birth
Birth rate
Death rate
Per capita rate (1/yr)
0.0
0.2
0.4
0.6
0.8
1.0
0
20
40
60
80
100
Population size

What is the characteristic scale?



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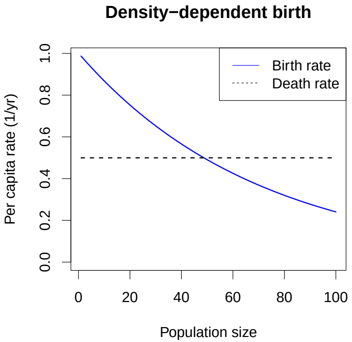

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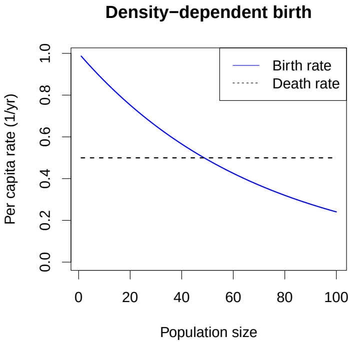

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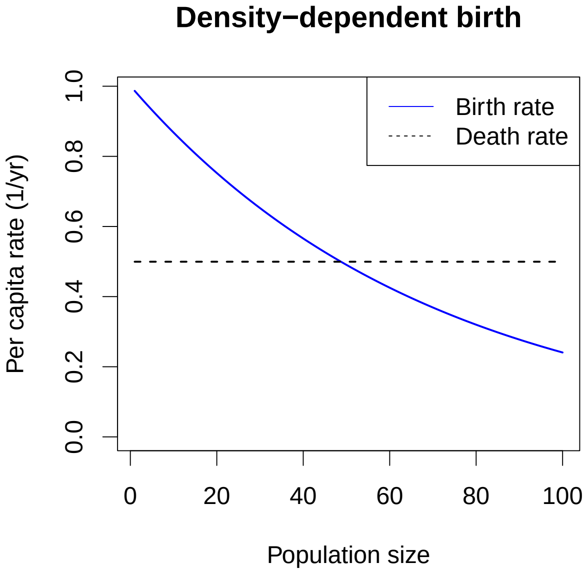

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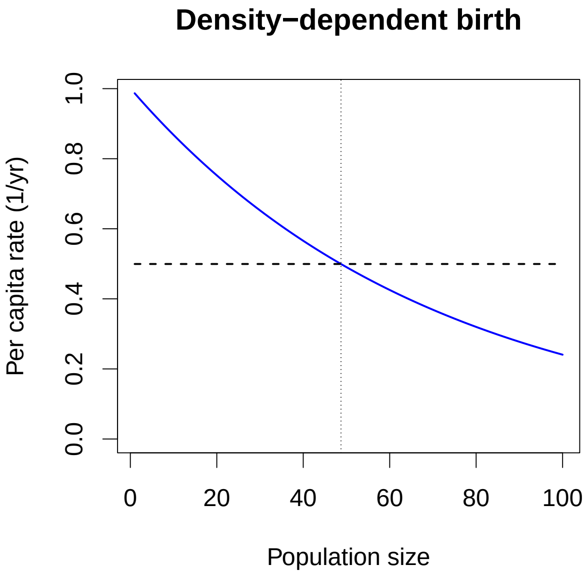

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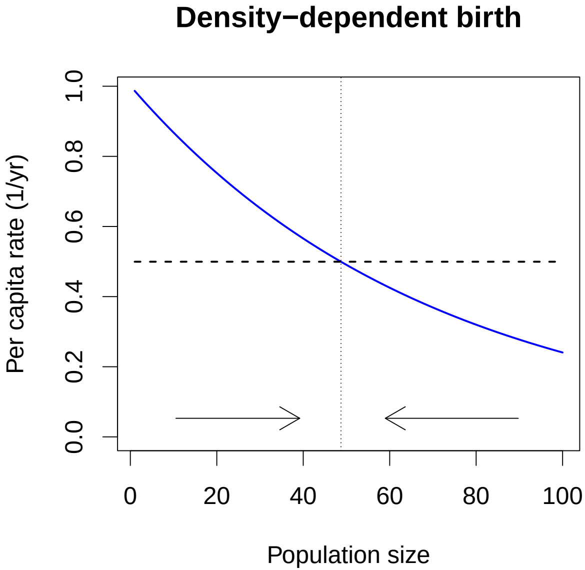

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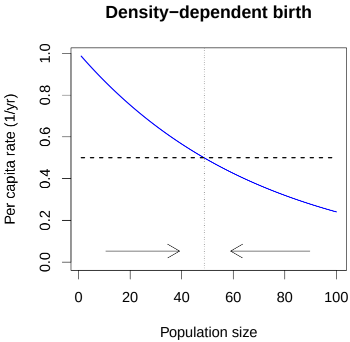

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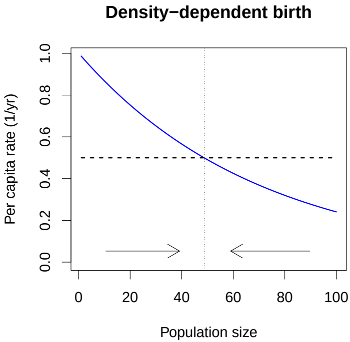

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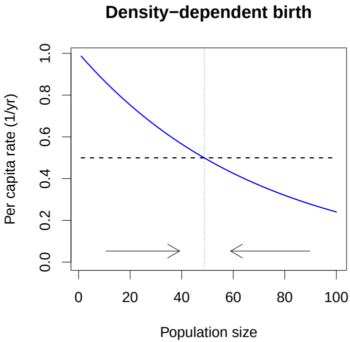

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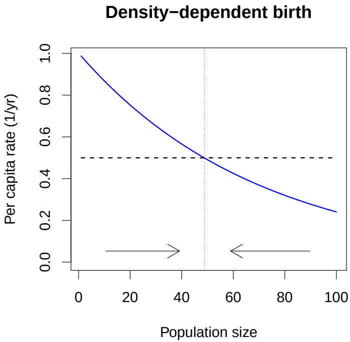

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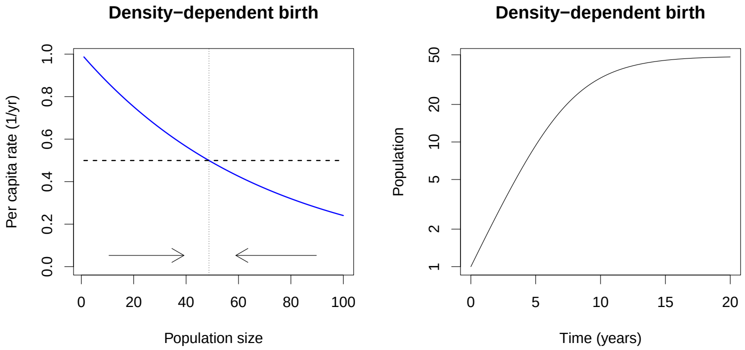

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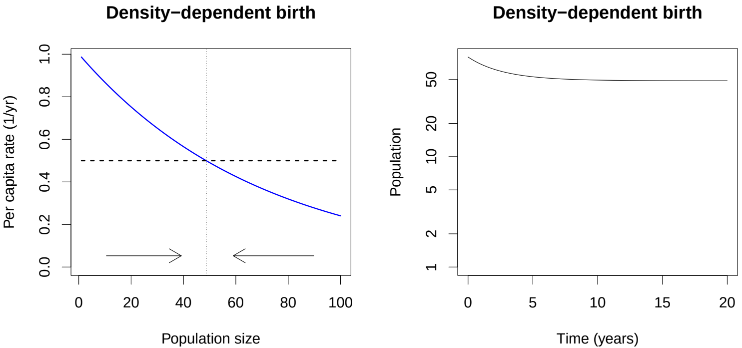

Examples


Density−dependent birth
Population
Time (years)
0
5
10
15
20
1
2
5
10
20
50
Density−dependent birth
Population
Time (years)
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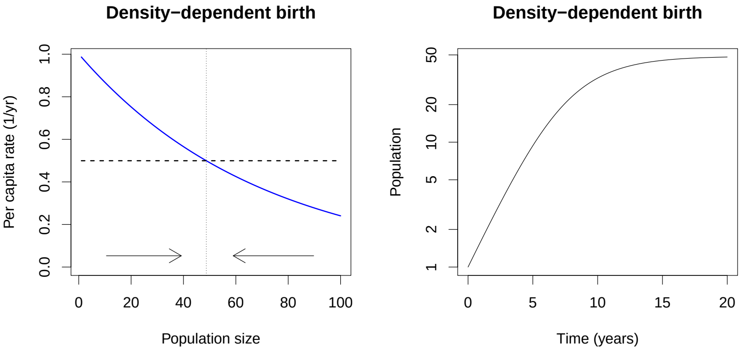

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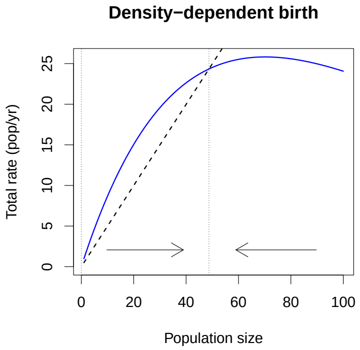

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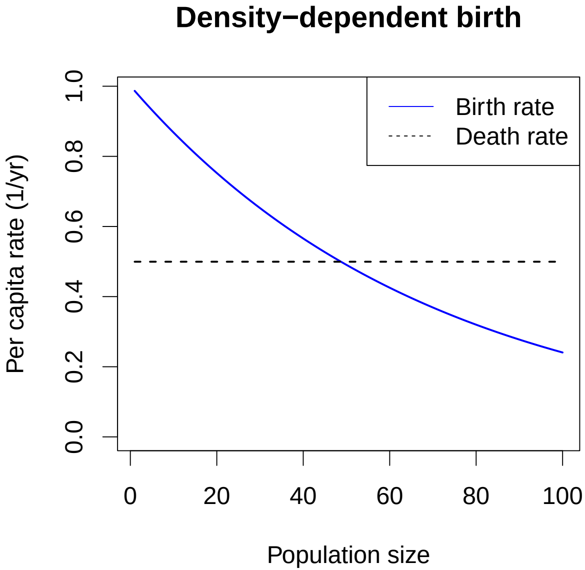

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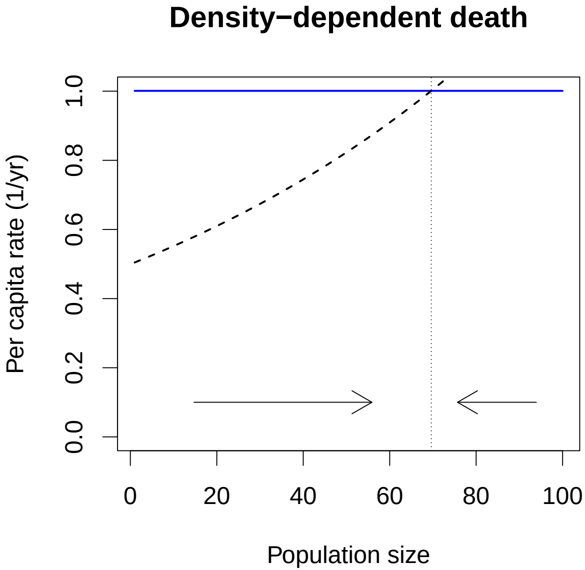

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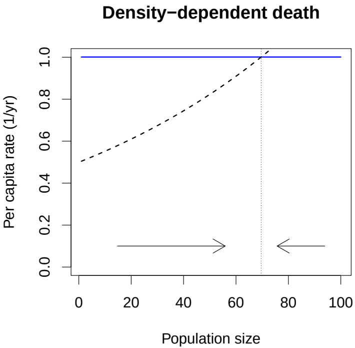

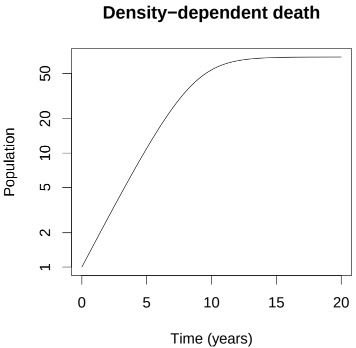

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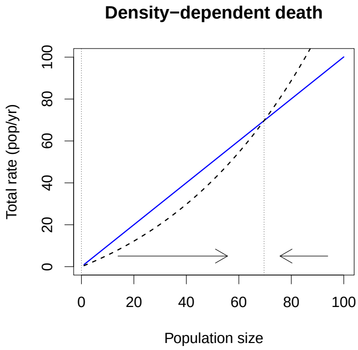

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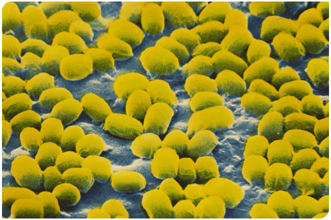

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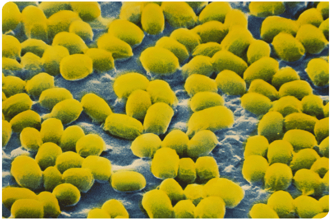

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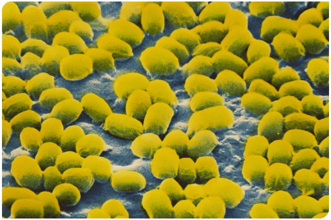

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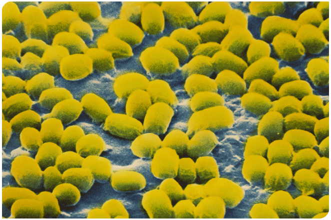

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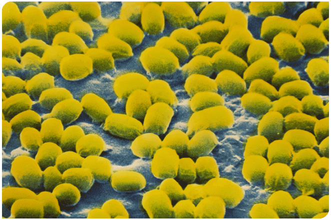

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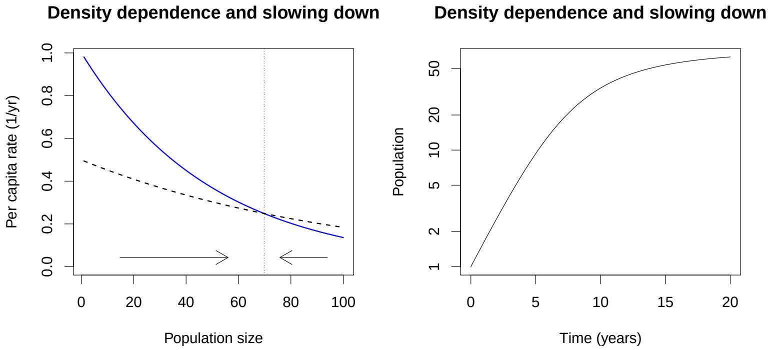

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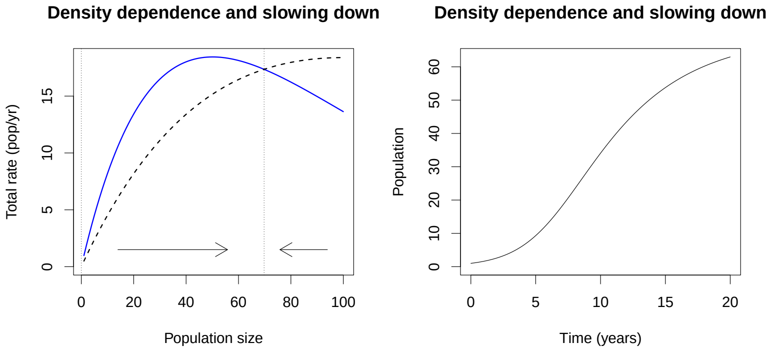

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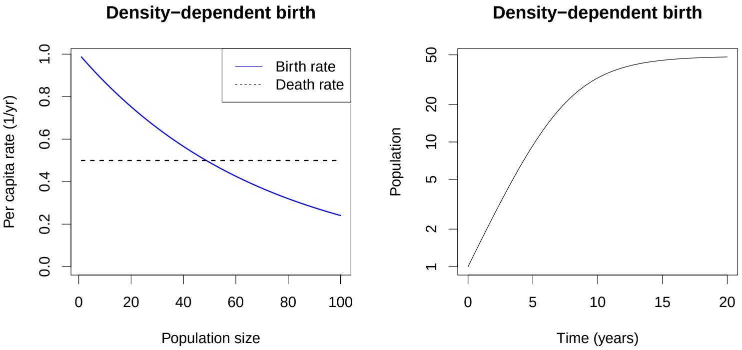

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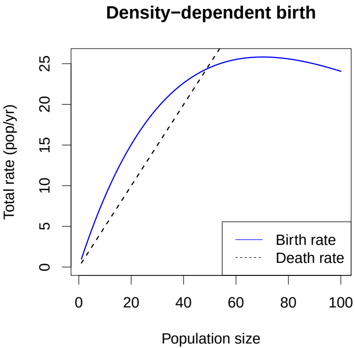

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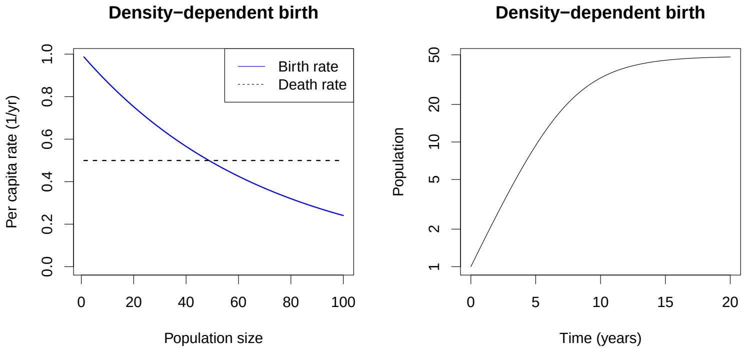

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Stable and unstable equilibria

- Aren't equilibria always stable?
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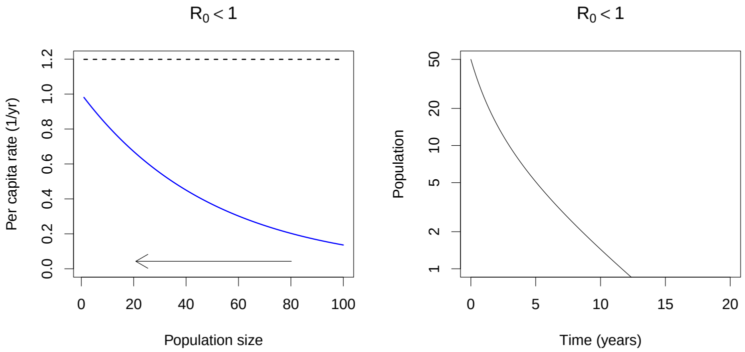

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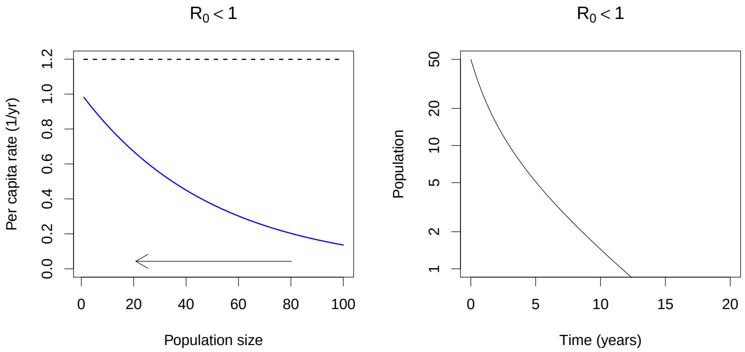

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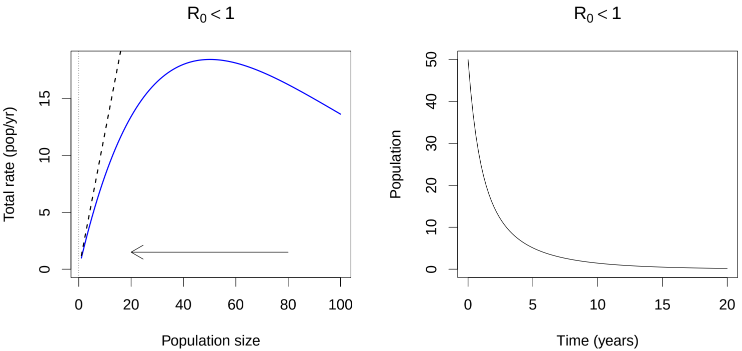

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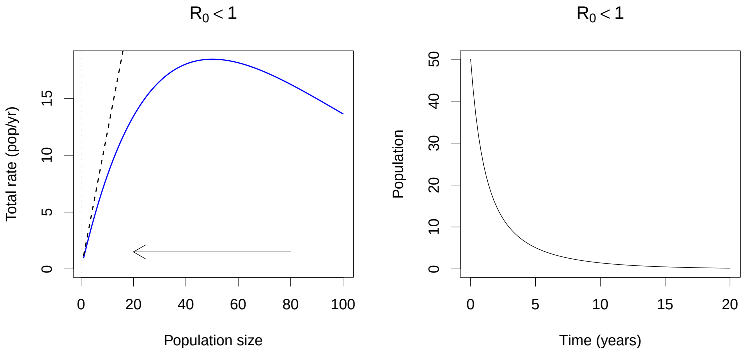

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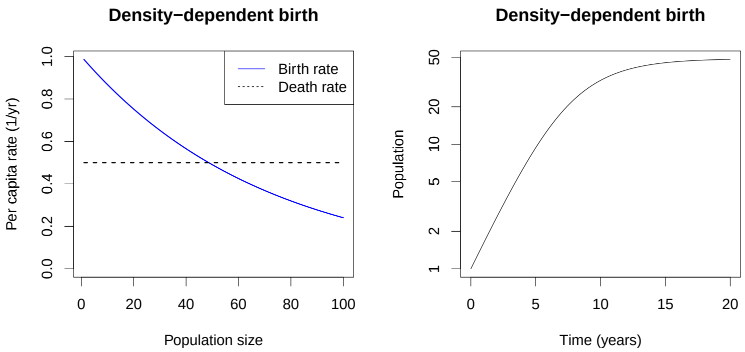

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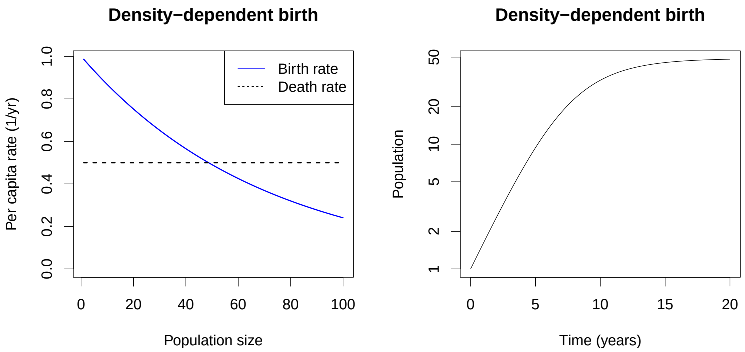

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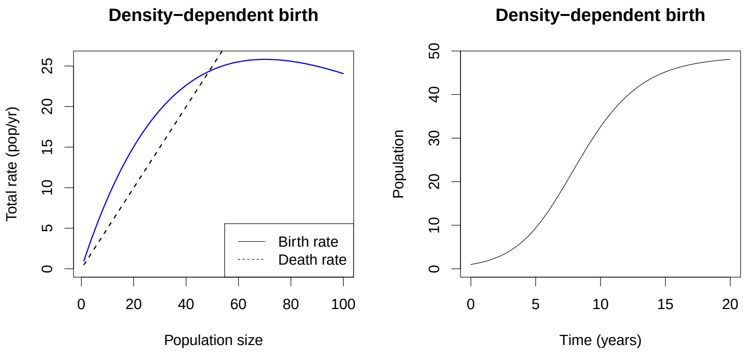

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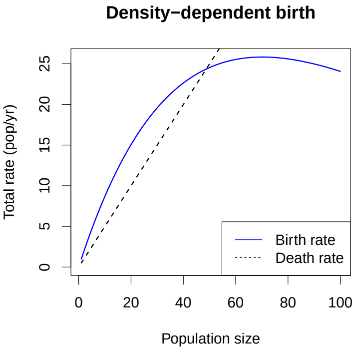

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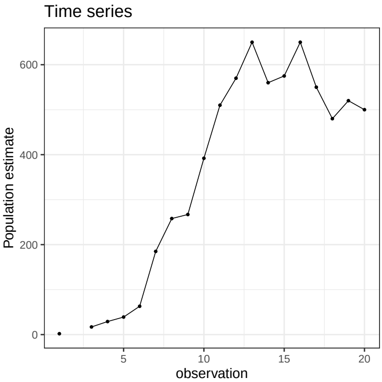
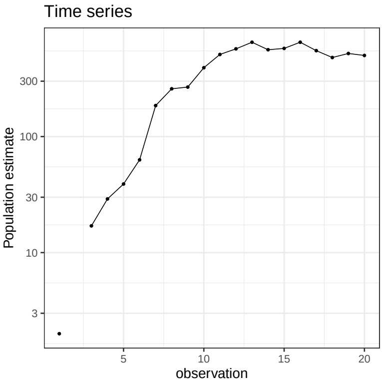
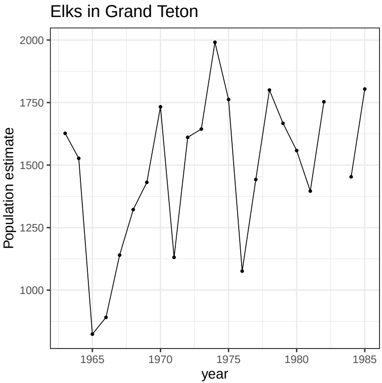




Elk (repeat)



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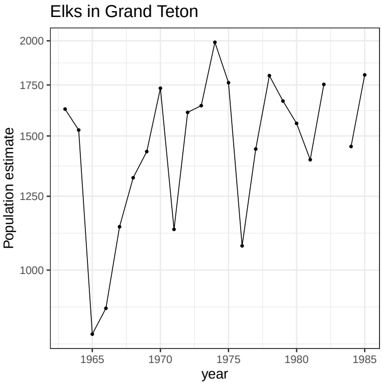

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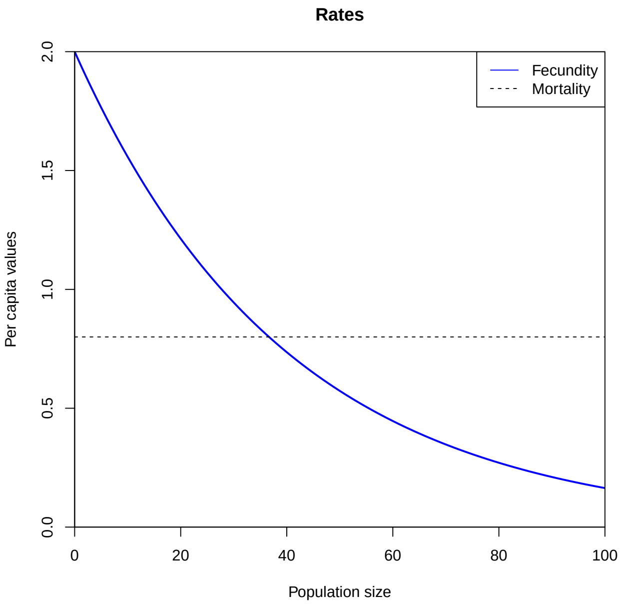

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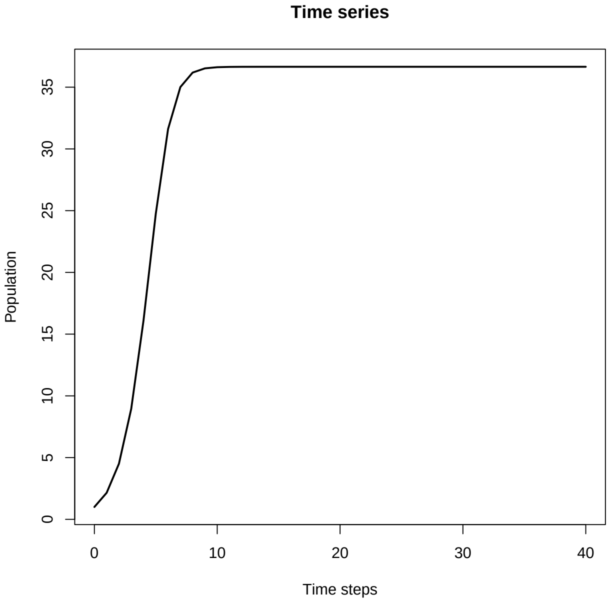
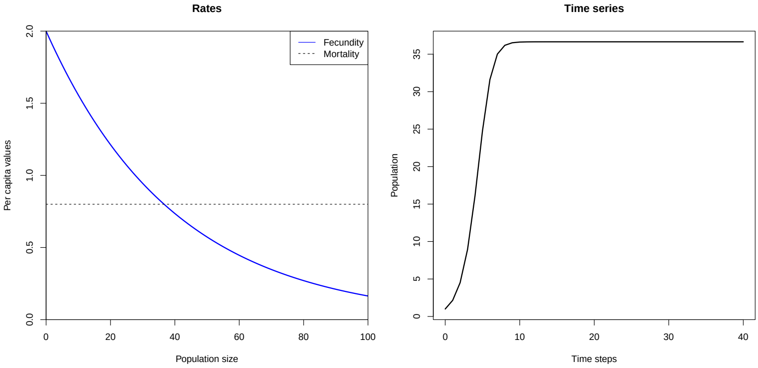


We expect simple dynamics



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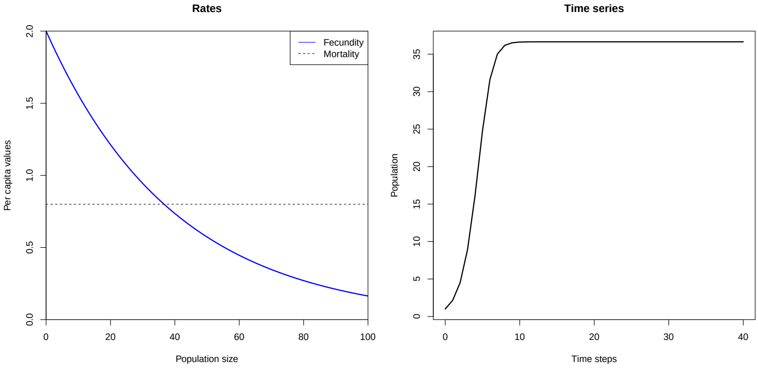

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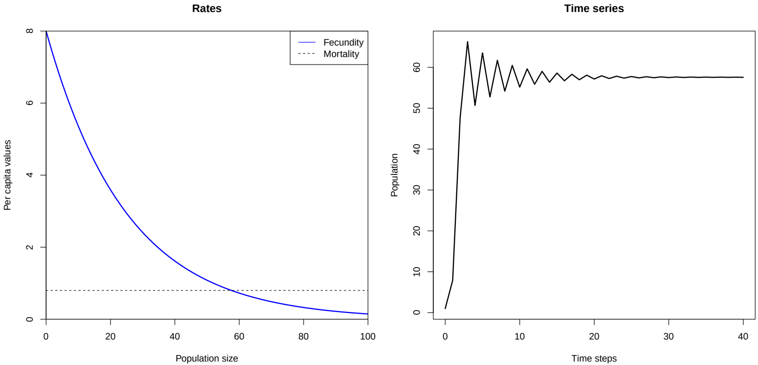

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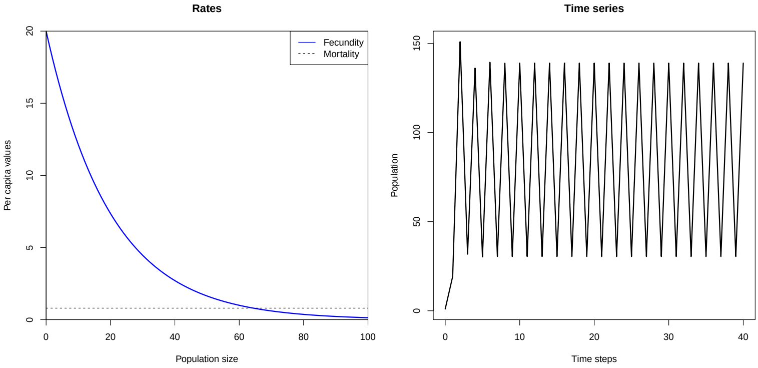
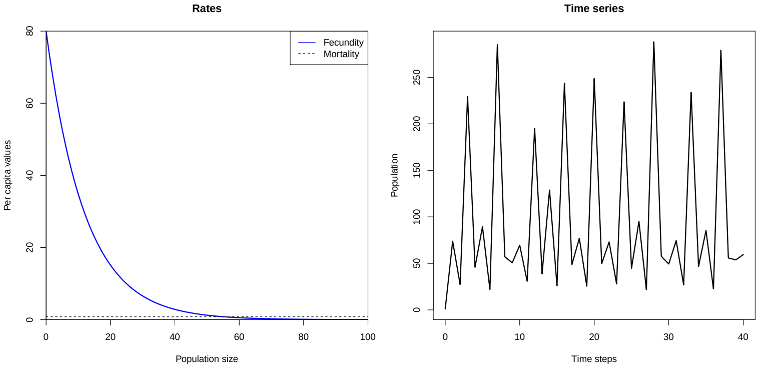


Lots of other behaviours



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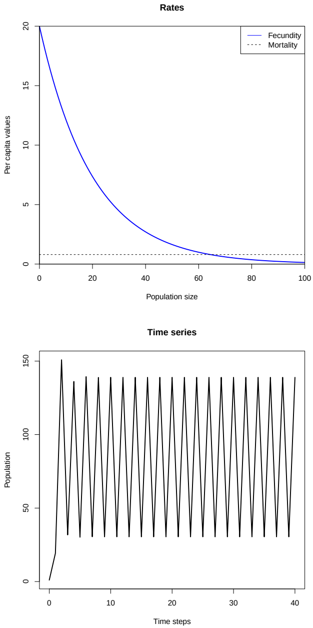

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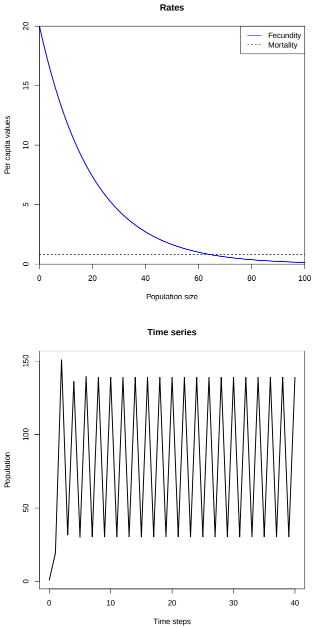

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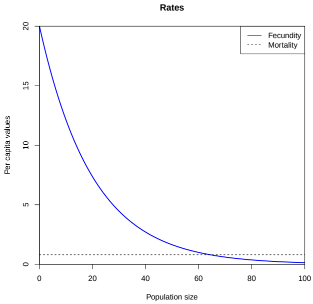

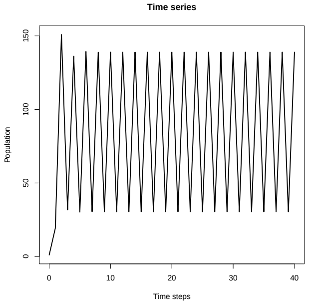

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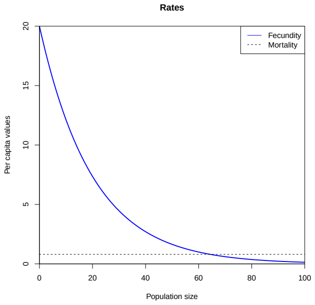

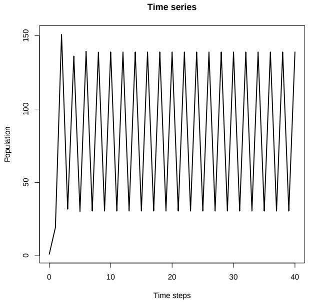

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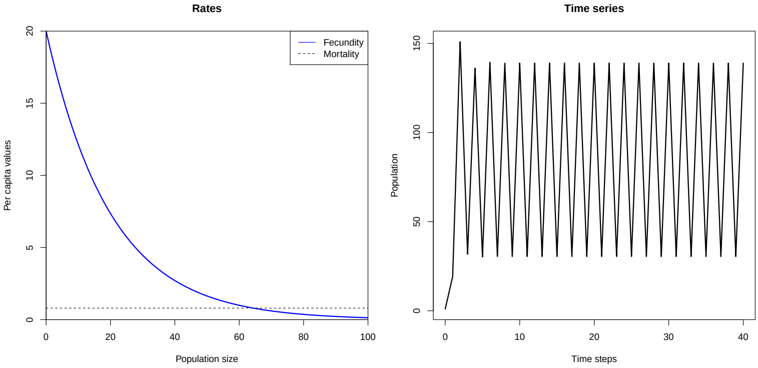

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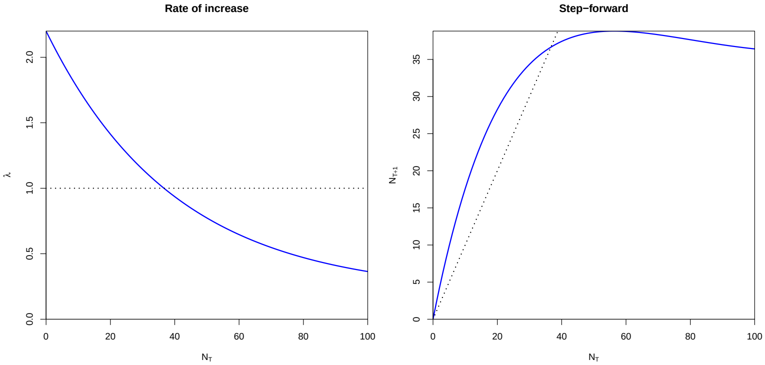

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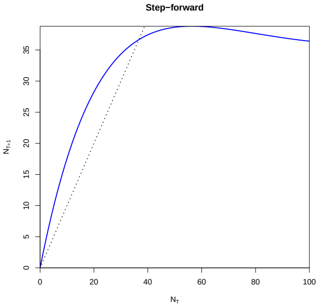

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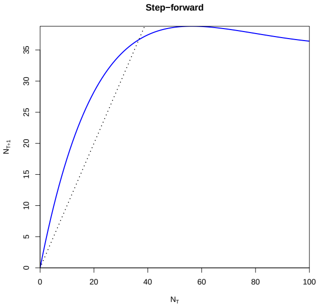

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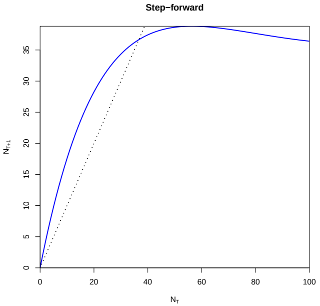

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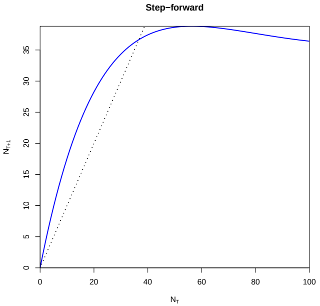

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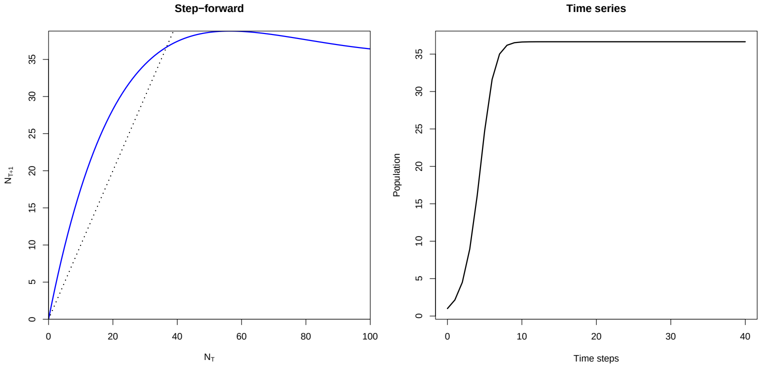

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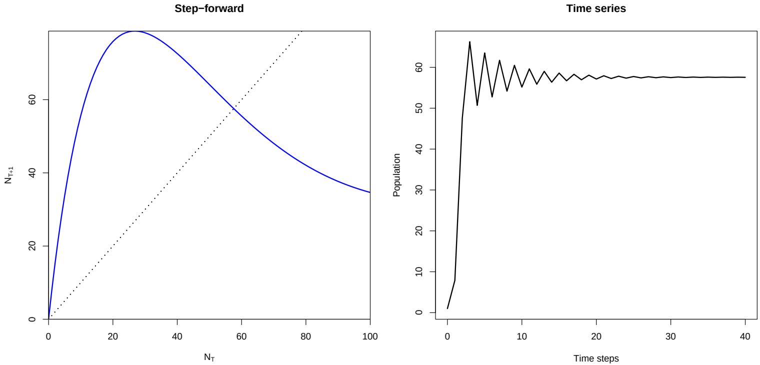

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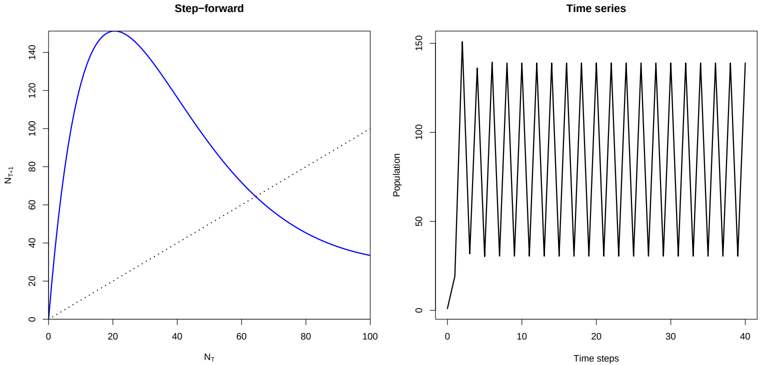

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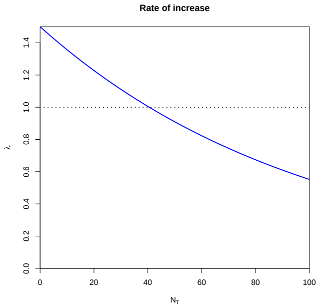

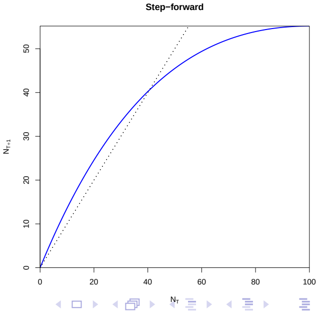

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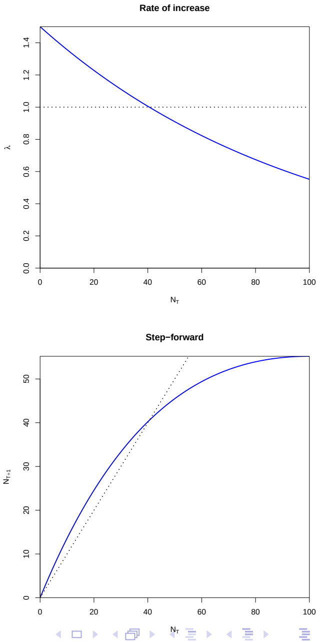

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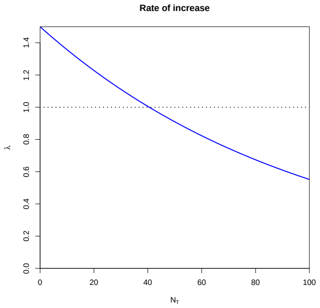

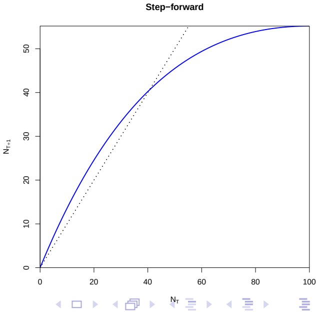

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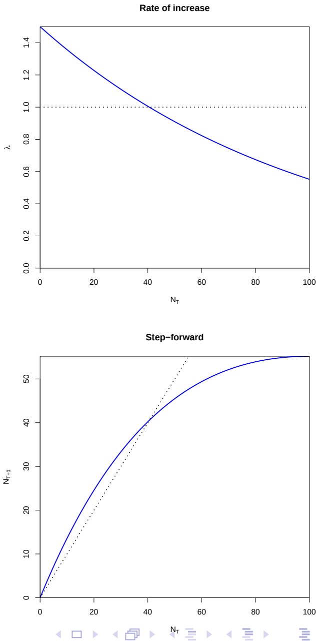
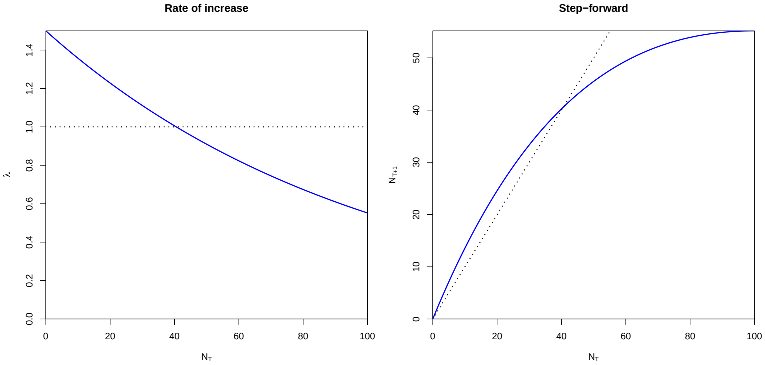


Contest regulation



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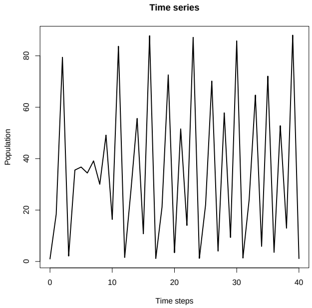

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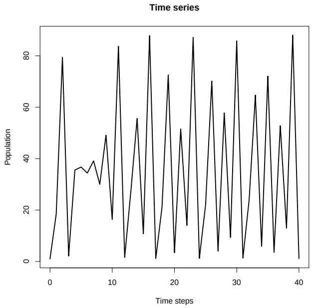

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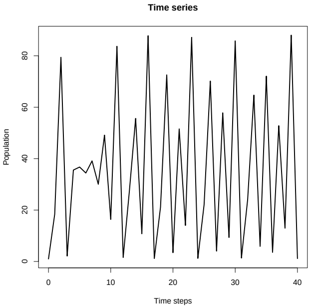

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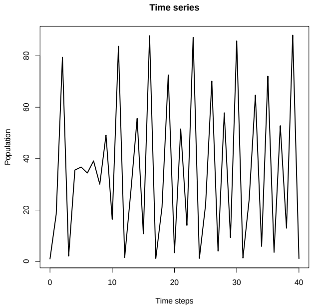

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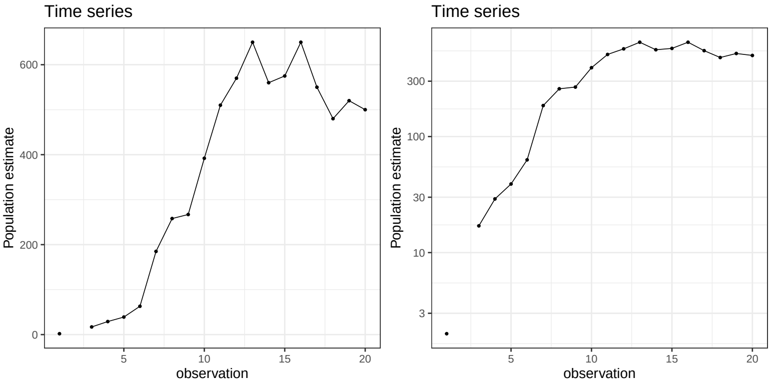

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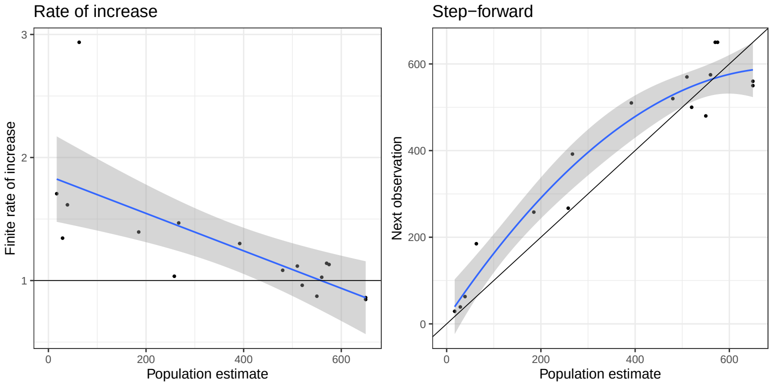
Rate of increase
Finite rate of increase
3
2
1
0
200
400
600
Population estimate
Step−forward
Next observation
600
400
200
0
0
200
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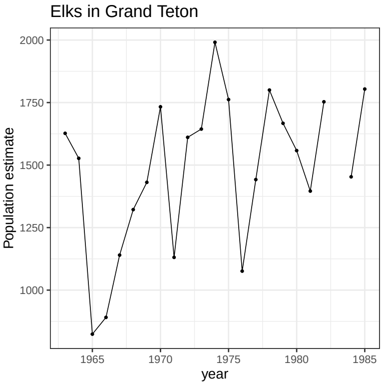
Population estimate

Elk (repeat)


Elks in Grand Teton
Population estimate
2000
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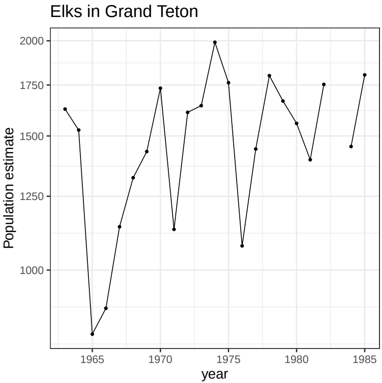
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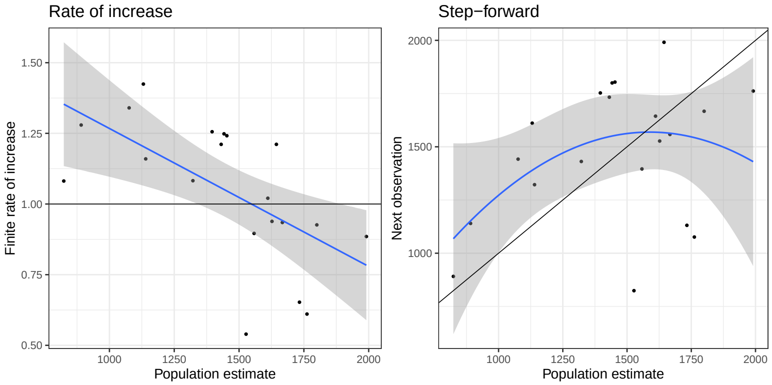

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Finite rate of increase
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1.25
1.00
0.75
0.50
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1500
1750
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- Population Examples

Continuous-time regulation
- A simple, continuous-time model
- Simulating model behaviour
- Equilibria

Discrete-time regulation
- A simple, discrete-time model
- Simulating this system
- Interpreting complex behaviour

Small populations
- Allee effects
- Stochastic effects

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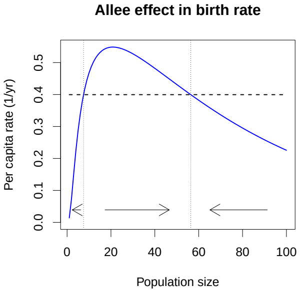

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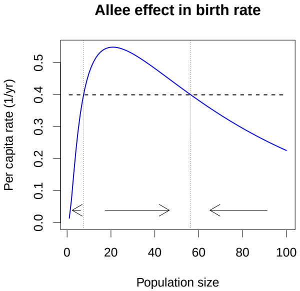

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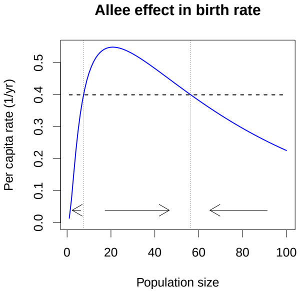

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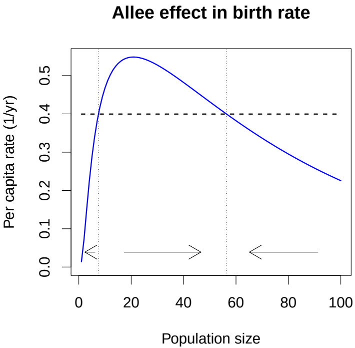

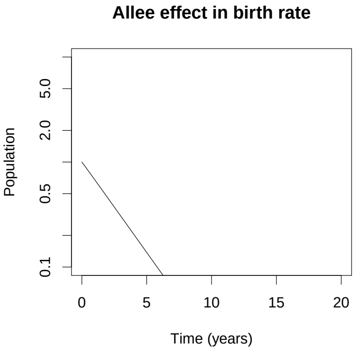

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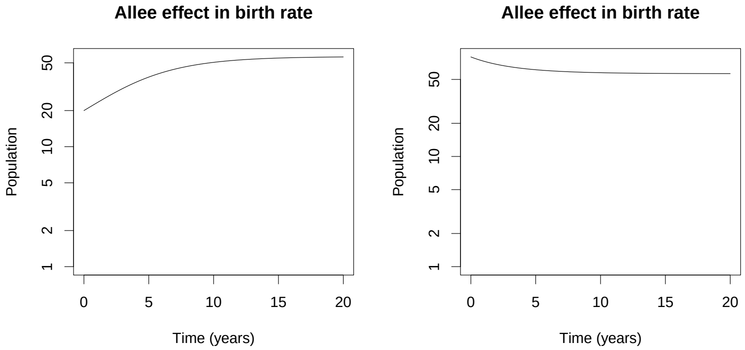

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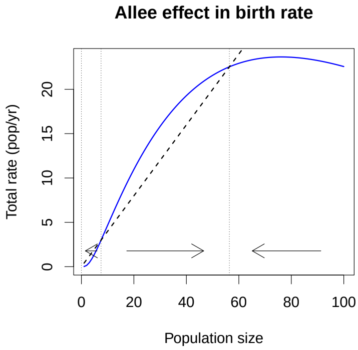

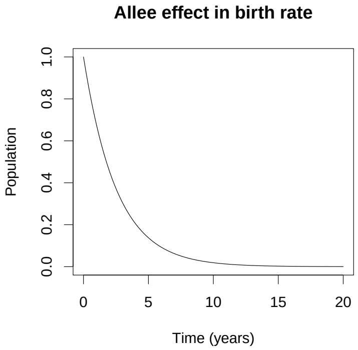

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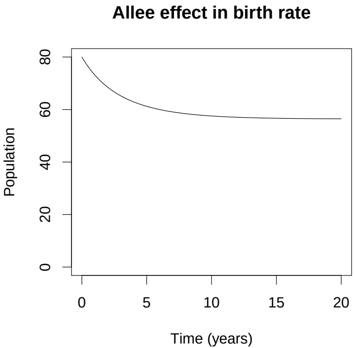
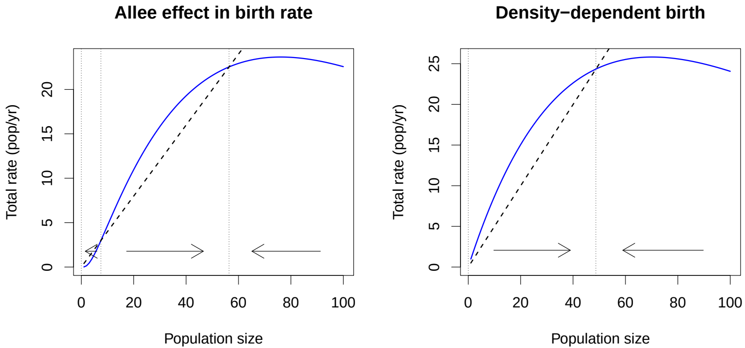


Population comparison (repeat)



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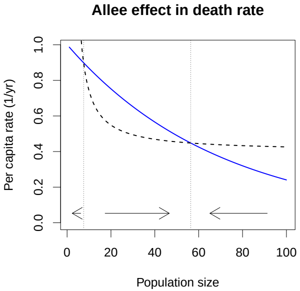

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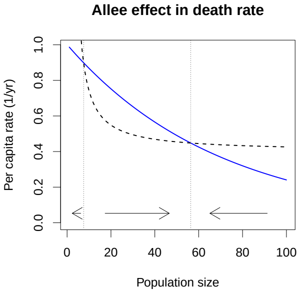

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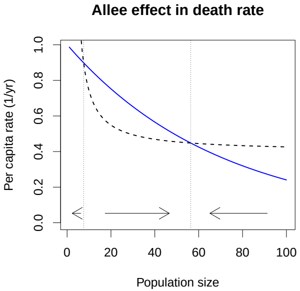

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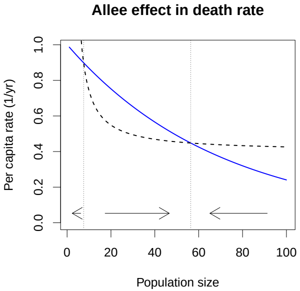

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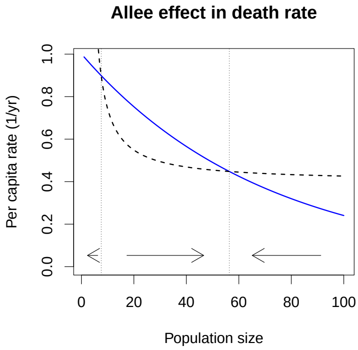

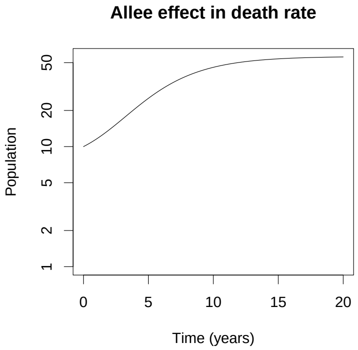

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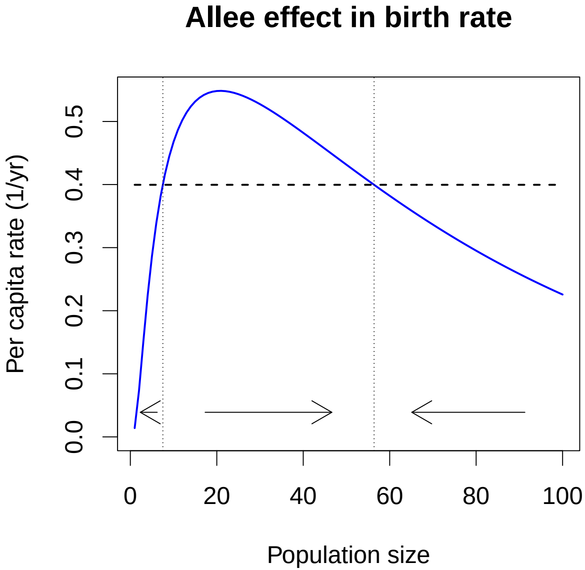
$\mathcal{R}_0$ and $\mathcal{R}_{max}$ (repeat)



Weak Allee effects

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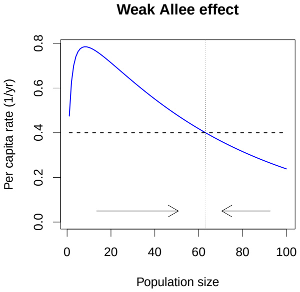

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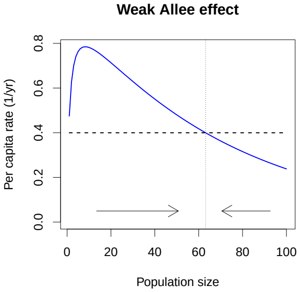

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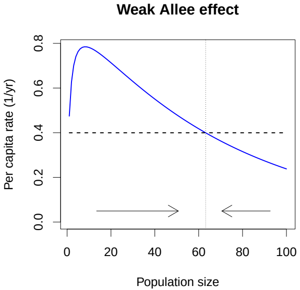

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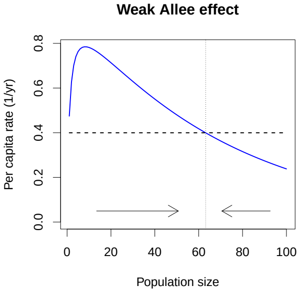

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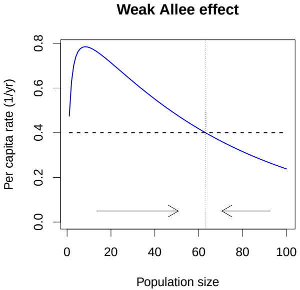

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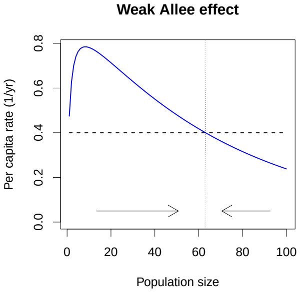

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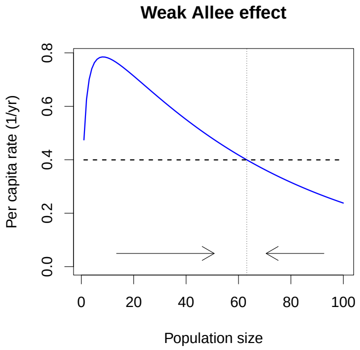

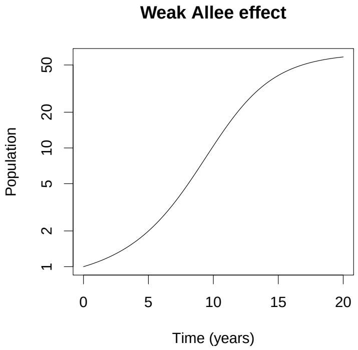

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Outline

Introduction
- Population Examples

Continuous-time regulation
- A simple, continuous-time model
- Simulating model behaviour
- Equilibria

Discrete-time regulation
- A simple, discrete-time model
- Simulating this system
- Interpreting complex behaviour

Small populations
- Allee effects
- Stochastic effects

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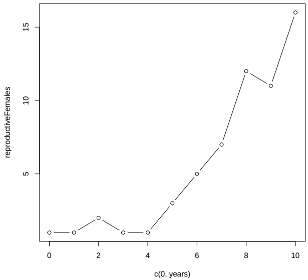

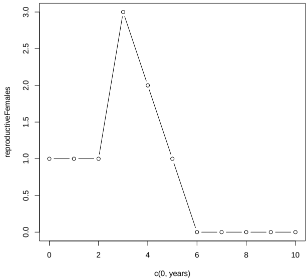

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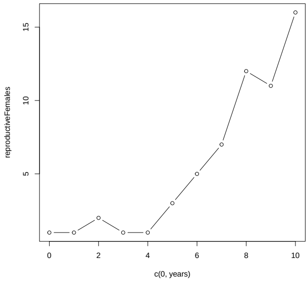

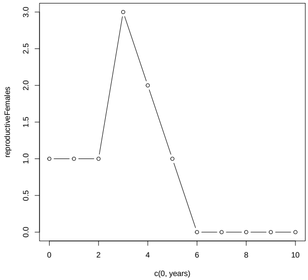

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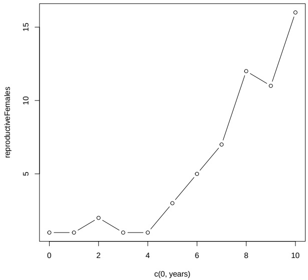

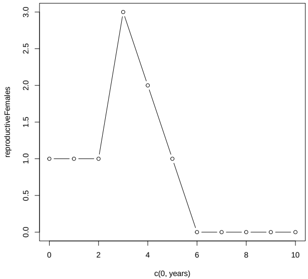

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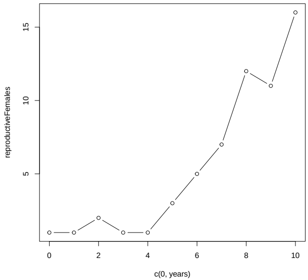

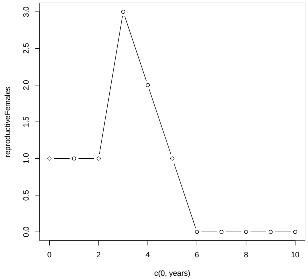

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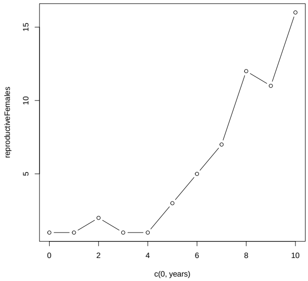

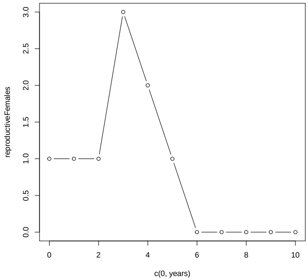

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