UNIT 4: Structured populations

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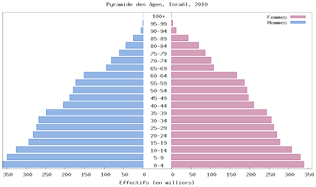

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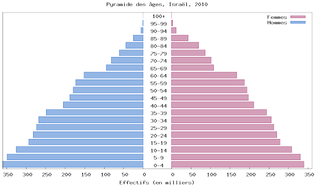

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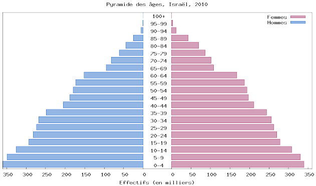

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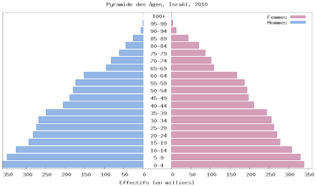

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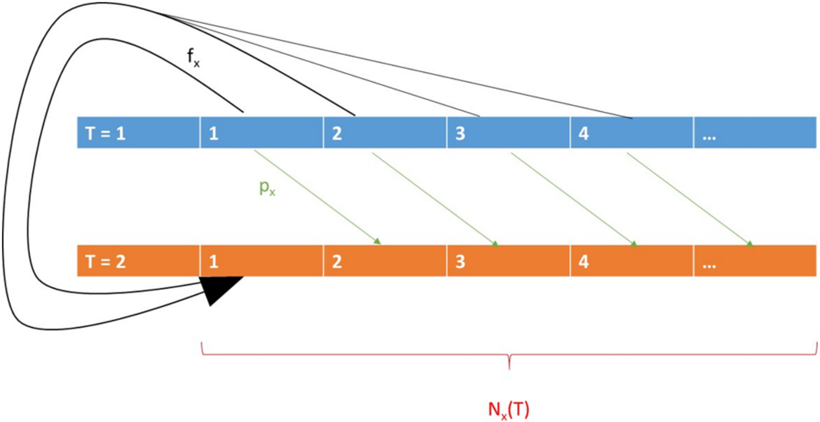

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- What does the life table look like?

Dandelion life table

x	f_x	p_x	ℓ_x	$\ell_x f_x$
1				
2				
R				

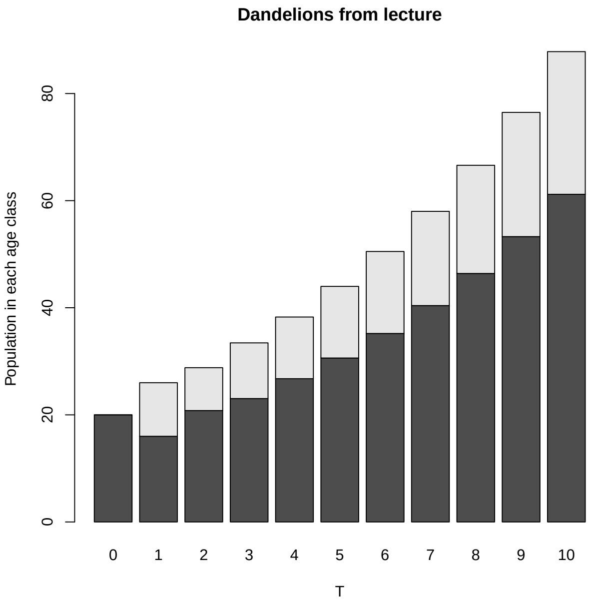
Dandelion life table (repeat)

x	f_x	p_x	ℓ_x	$\ell_x f_x$
1	0.8	0.5		
2	0.8	0		
R				

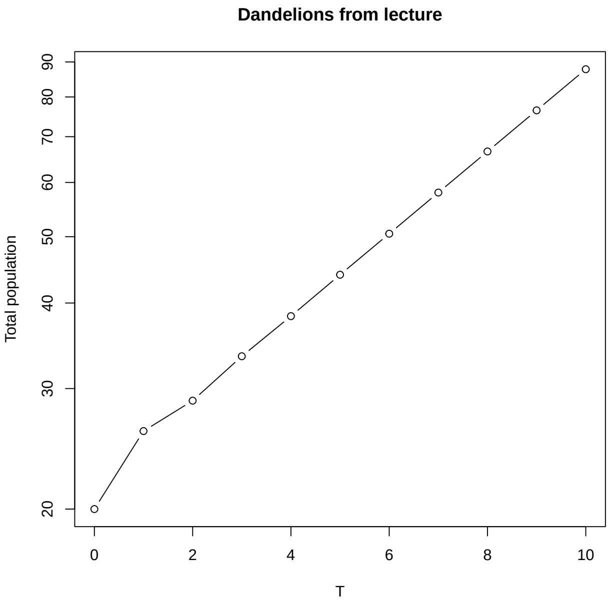
Dandelion life table

x	f_x	p_x	ℓ_x	$\ell_x f_x$
1	0.8	0.5	1.000	0.800
2	0.8	0	0.500	0.400
R				1.200

Dandelion dynamics



Dandelion population dynamics



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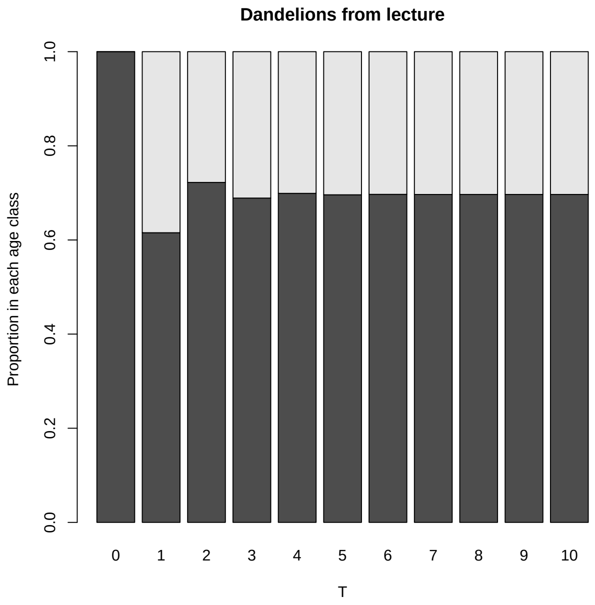

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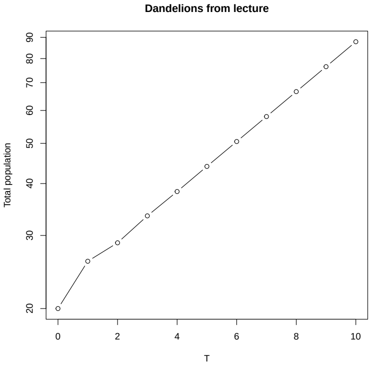

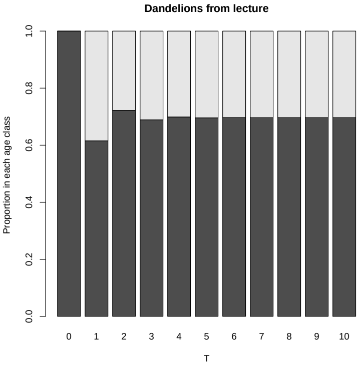

Squirrel example

Gray squirrel population example

x	f_x	p_x	ℓ_x	$\ell_x f_x$
1	0	0.25		
2	1.28	0.46		
3	2.28	0.77		
4	2.28	0.65		
5	2.28	0.67		
6	2.28	0.64		
7	2.28	0.88		
8	2.28	0.0		
R				

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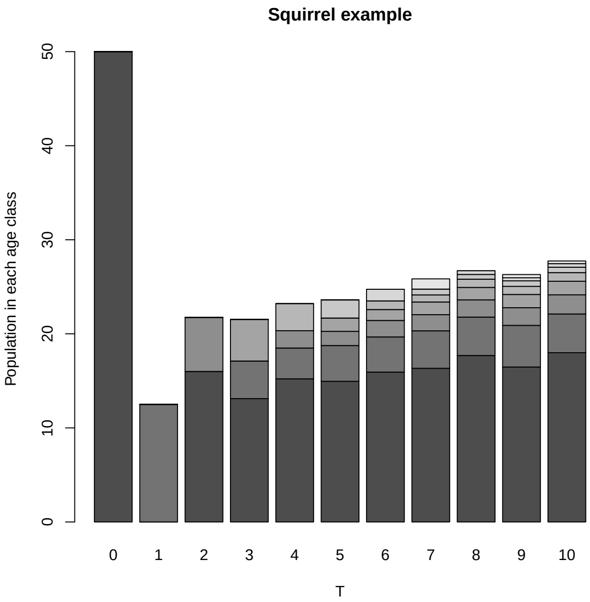
x	f_x	p_x	ℓ_x	$\ell_x f_x$
1	0	0.25	1.000	0.000
2	1.28	0.46	0.250	0.320
3	2.28	0.77	0.115	0.262
4	2.28	0.65	0.089	0.202
5	2.28	0.67	0.058	0.131
6	2.28	0.64	0.039	0.088
7	2.28	0.88	0.025	0.056
8	2.28	0.0	0.022	0.050
R				1.109

Gray squirrel dynamics



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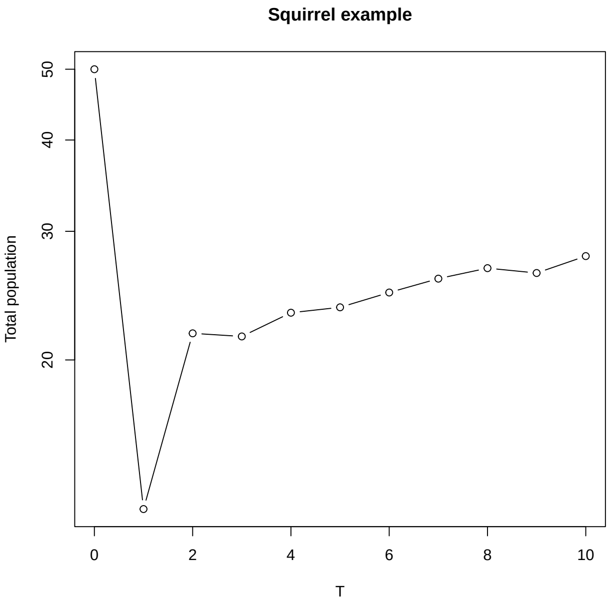

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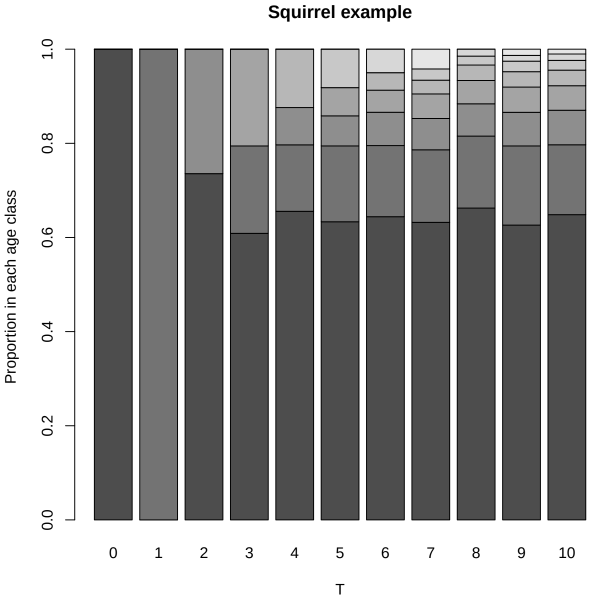

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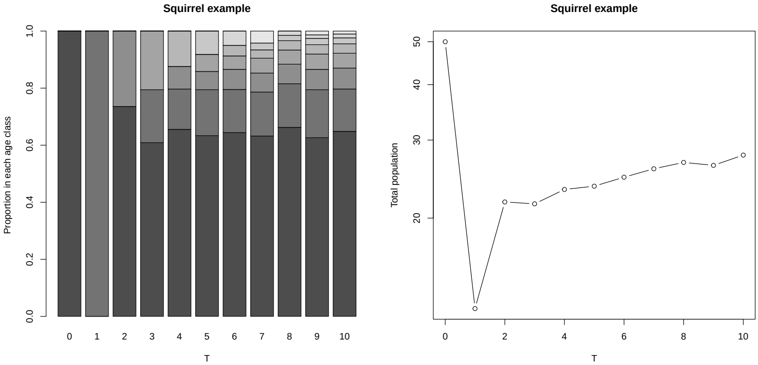
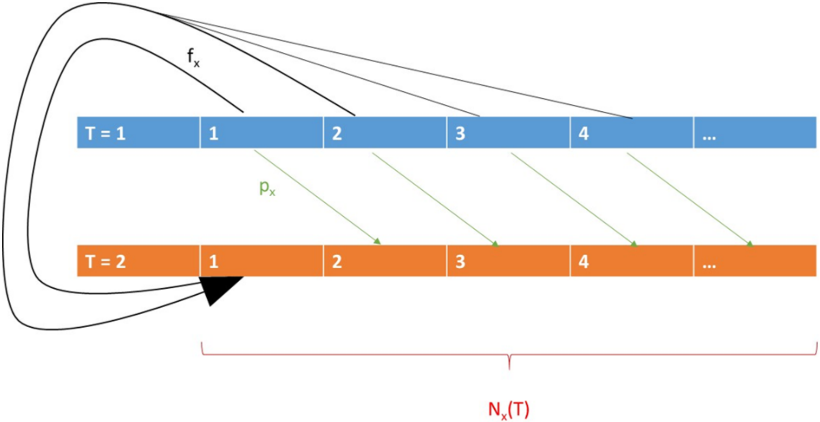


The structured model (repeat)



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- An alternative perspective is m_x: the total number of offspring per reproducing individual of age x

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Dandelion life table

x	f_x	p_x	ℓ_x	$\ell_x f_x$
1	0.8	0.5	1.000	0.800
2	0.8	0	0.500	0.400
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Counting after reproduction

x	f_x	p_x	ℓ_x	$\ell_x f_x$
1	0.8	0.01	1.000	0.800
2	40	0	0.010	0.400
R				1.200

There are two different approaches to the third age class: if we assume that we count the two-year old adults ($x = 3$), we can write $p_2 = 0.5$; $f_3 = 0$, and get the same answer (with one extra row that has zero contribution).

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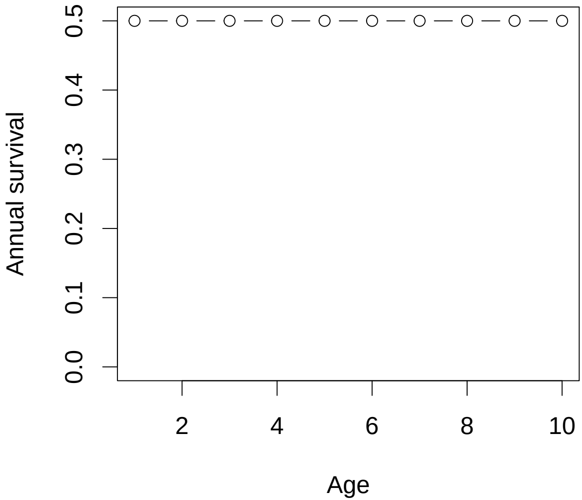

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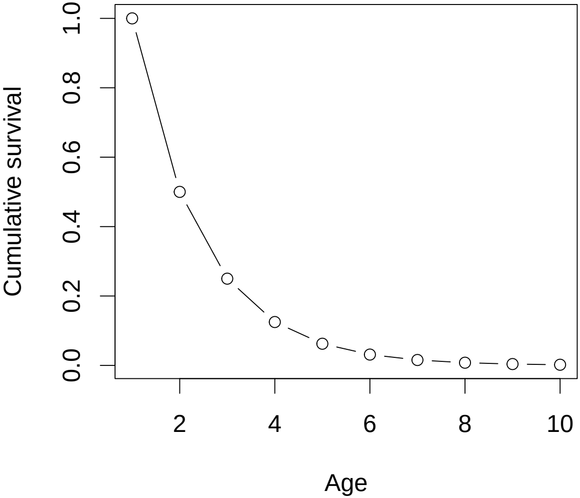

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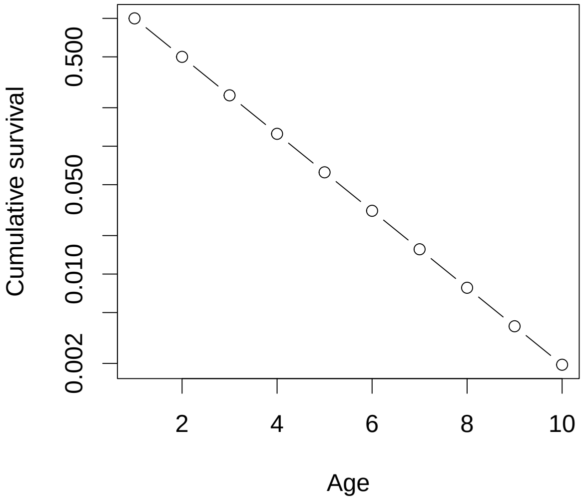

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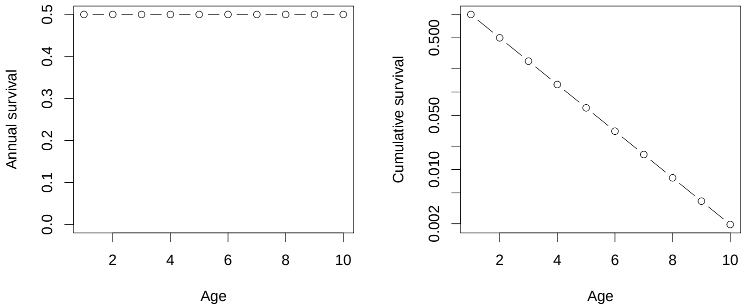

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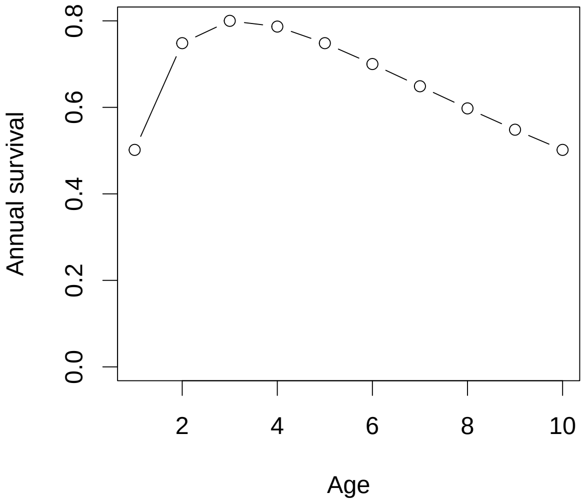

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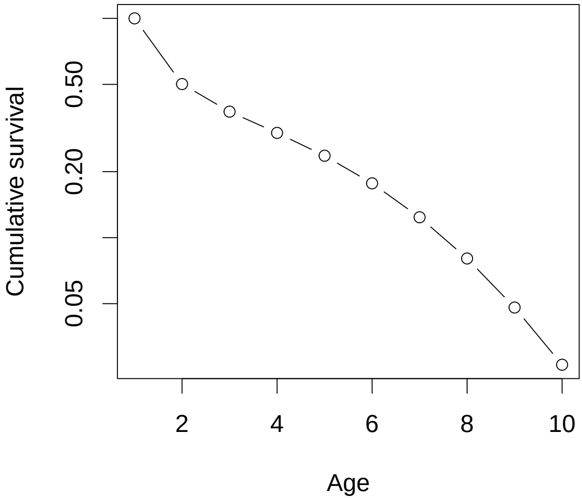

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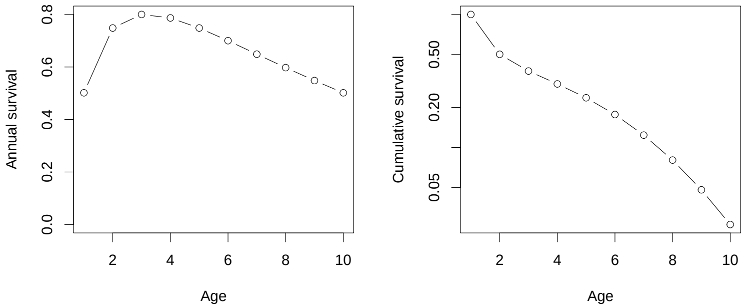

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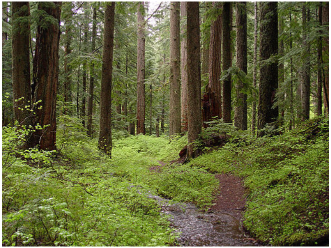

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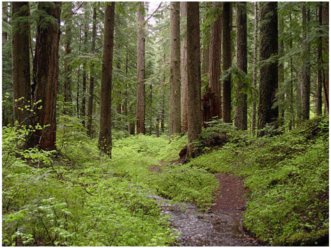

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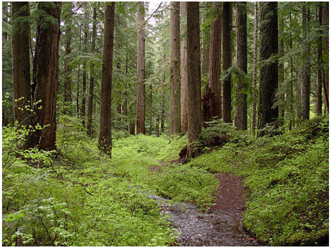

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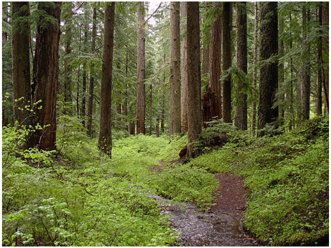

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