

BIO3SS3: Population Ecology

UNIT 1: Introduction

Outline

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Learning goals

Examples

Example populations

Dandelions

Gypsy moths

Bacteria

Exponential growth

Log and linear scales

Time scales

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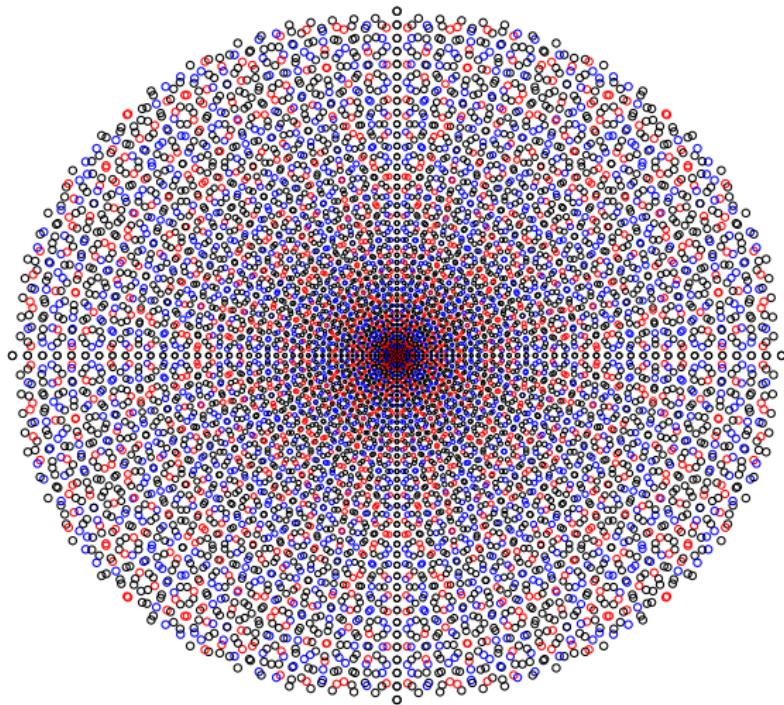
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Pythagorean triples



Which country?



Which country?



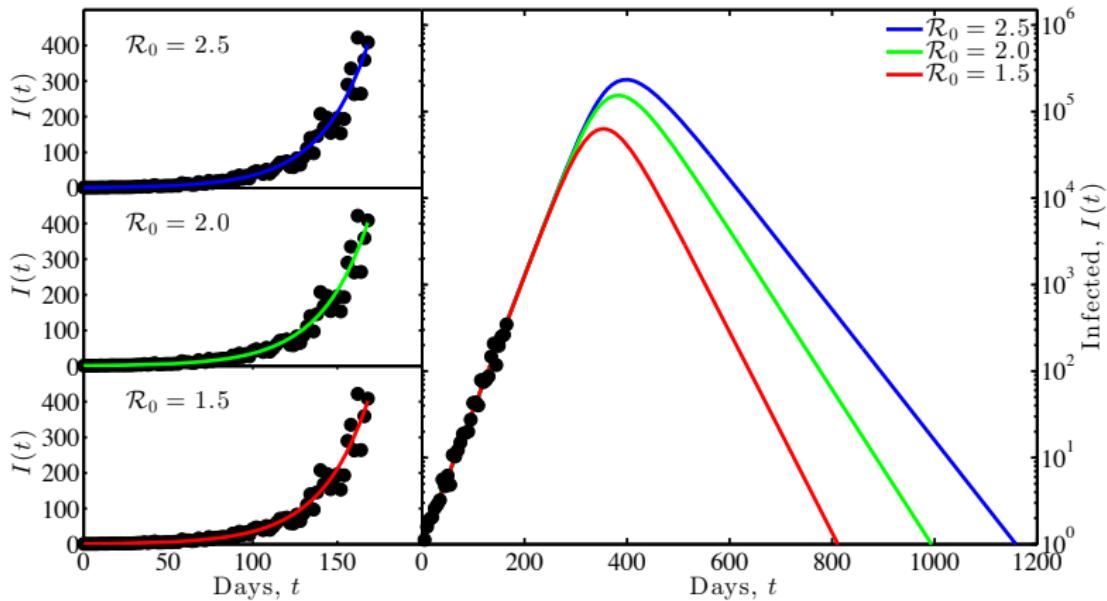
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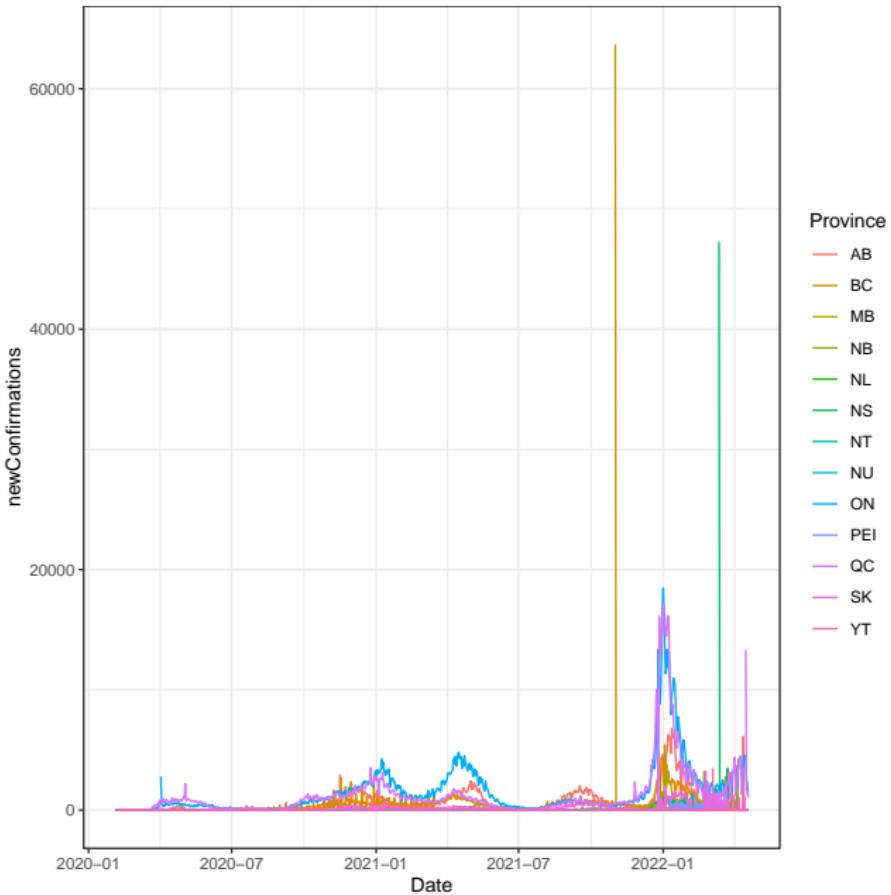
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Disease research



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Students

- ▶ Poll: What year are you in?

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- ▶ Ecology and population ecology

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- Poll: What is ecology?

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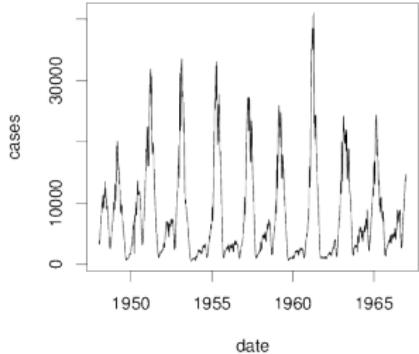
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Dynamical modeling

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Measles reports from England and Wales

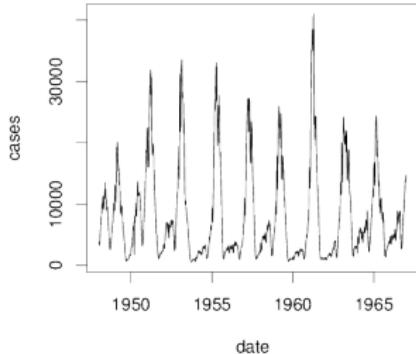


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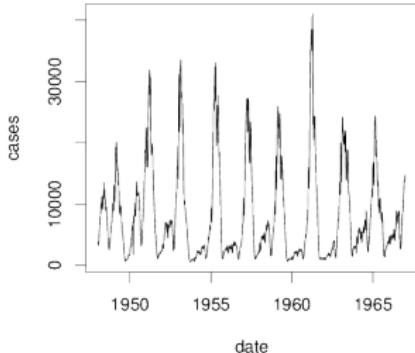


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Humans and abstract thought

- ▶ People are evolved to be concrete thinkers, not conceptual thinkers



$$E = mc^2$$

value

energy | mass | speed of light

J | kg | 299,792,458 m/s

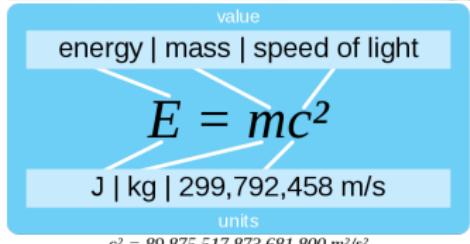
units

$c^2 = 89,875,517,873,681,800 \text{ m}^2/\text{s}^2$

A diagram illustrating the components of the equation $E = mc^2$. The equation is centered at the bottom. Above it, three arrows point from boxes containing "value" and "energy/mass/speed of light" to the m and c^2 terms. Below the equation, three arrows point from boxes containing "units" and "J/kg/299,792,458 m/s" to the E and c^2 terms. The entire diagram is contained within a light blue rectangular frame.

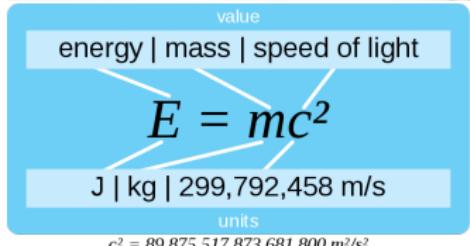
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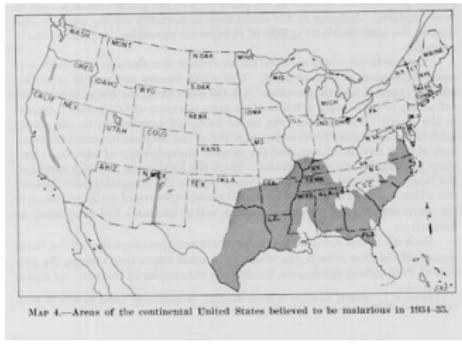
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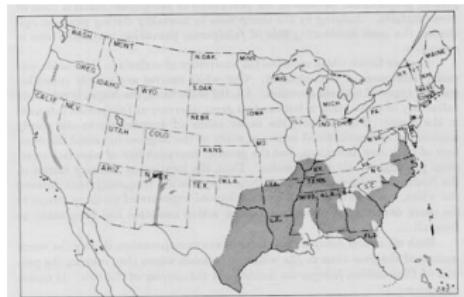
Malaria

- A nasty, mosquito-borne disease



Malaria

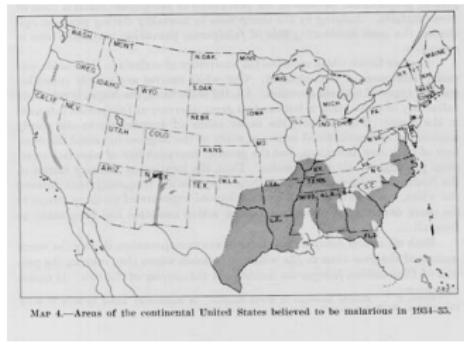
- ▶ A nasty, mosquito-borne disease
- ▶ In some places (e.g., the southeastern US), it has been eradicated almost by accident



MAP 4.—Areas of the continental United States believed to be malarious in 1951-55.

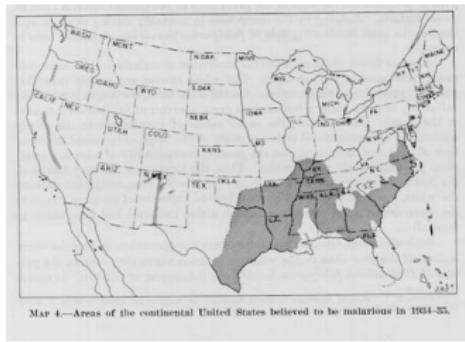
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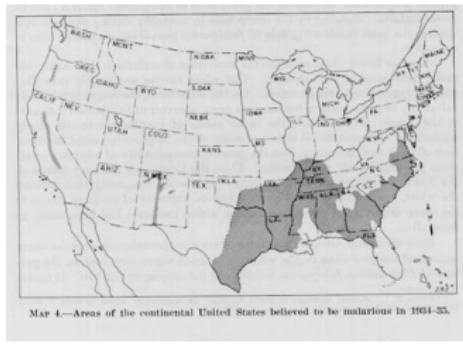
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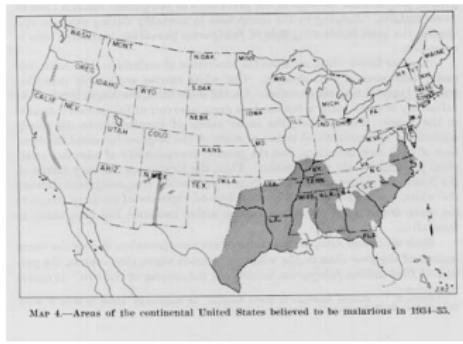
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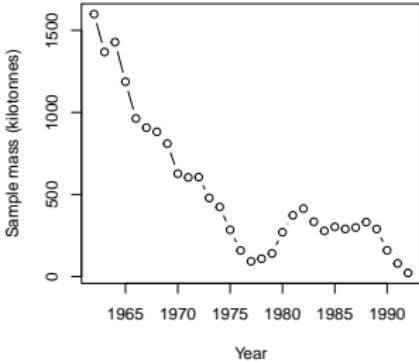
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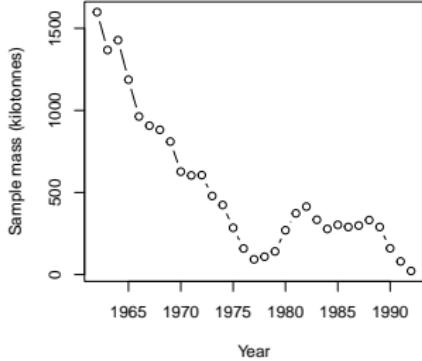
Cod fisheries

- ▶ Is the ocean too big for people to affect?



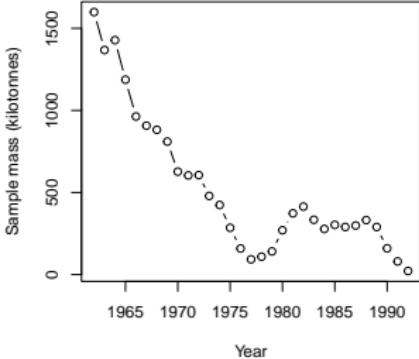
Cod fisheries

- ▶ Is the ocean too big for people to affect?
- ▶ What happened to the cod?



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Populations

- Poll: What population of organisms interests you?

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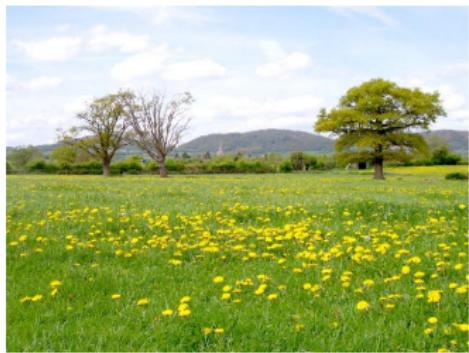
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Gypsy moths

- ▶ A pest species that feeds on deciduous trees
- ▶ Introduced to N. America from Europe 150 years ago
- ▶ Capable of wide-scale defoliation

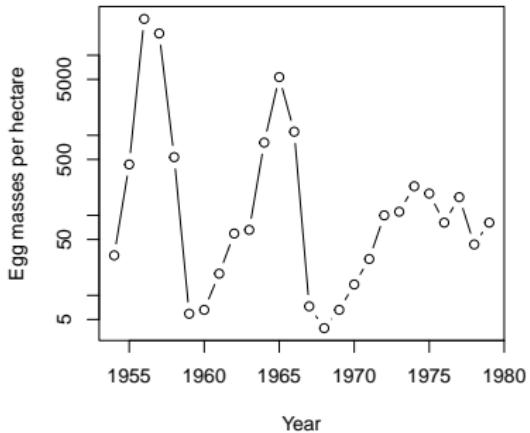
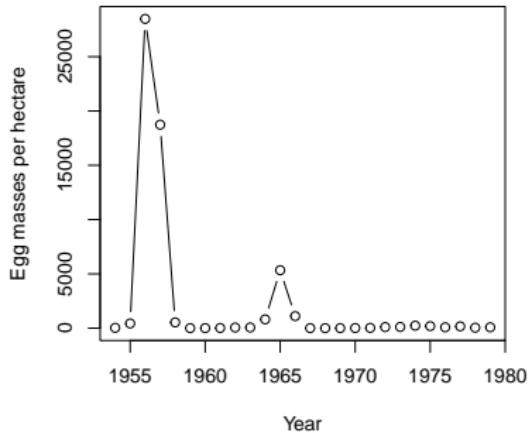


Gypsy moths

- ▶ A pest species that feeds on deciduous trees
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Gypsy moth populations



Moth calculation (preview)

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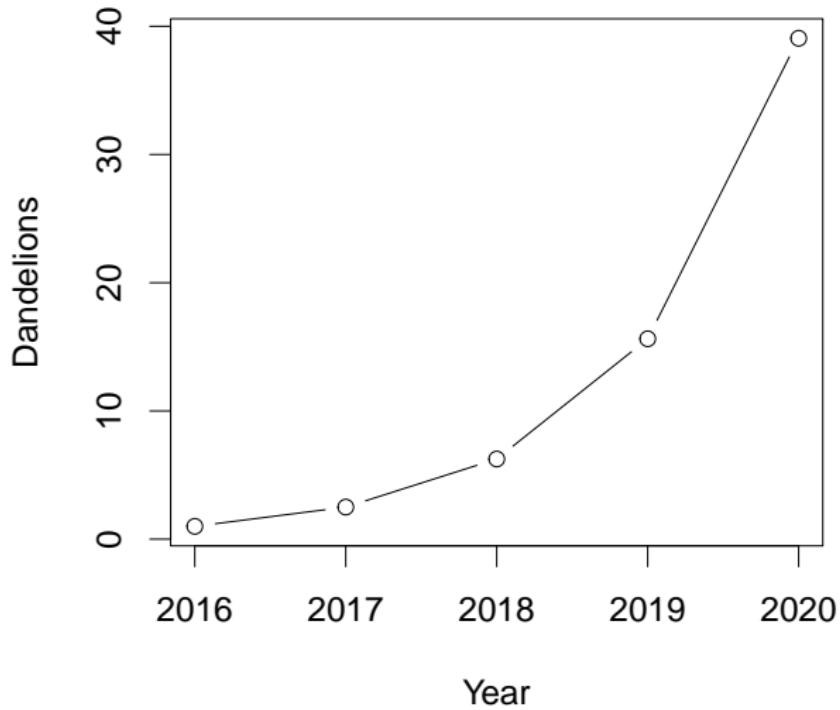
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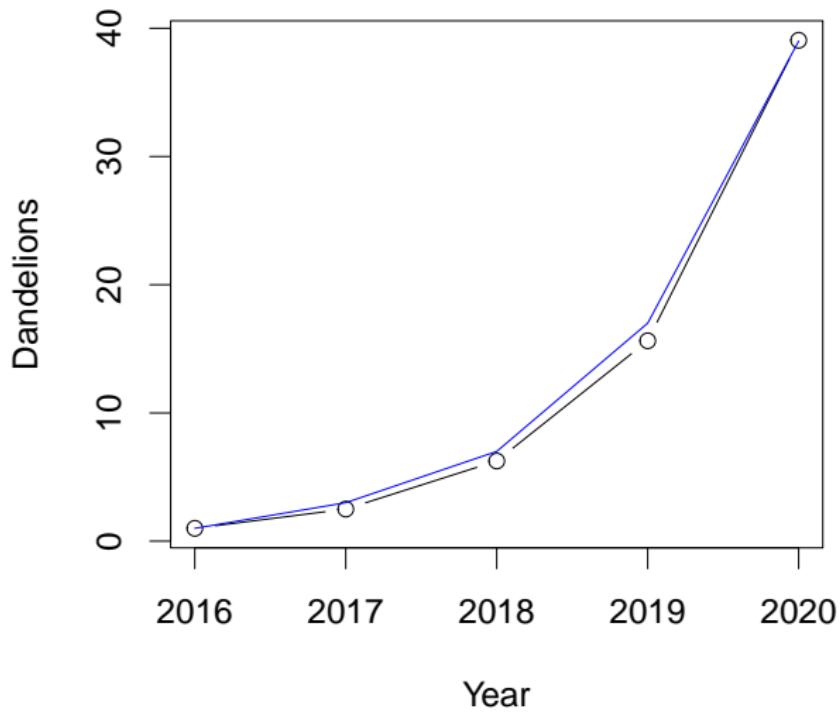
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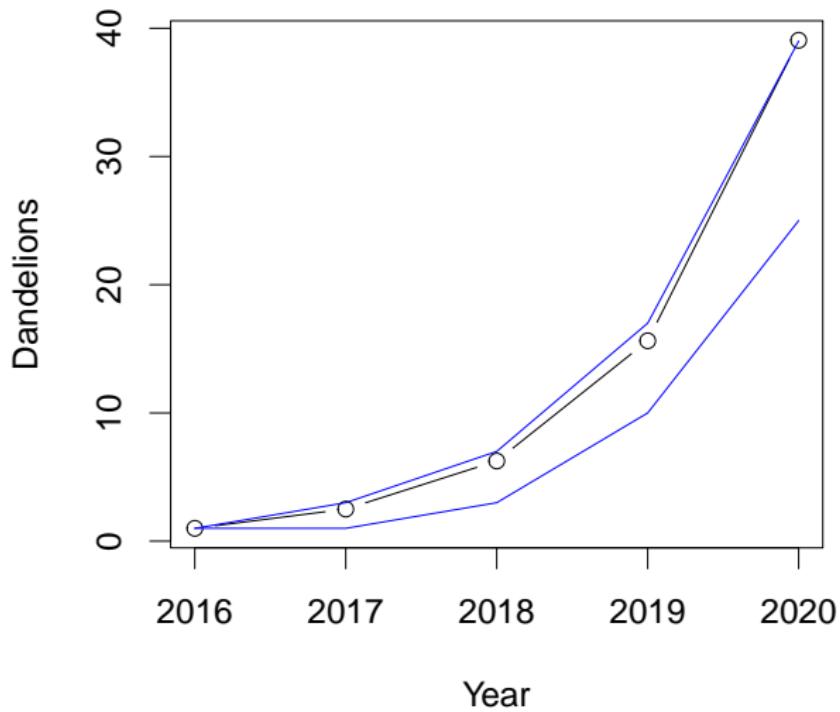
Stochastic model



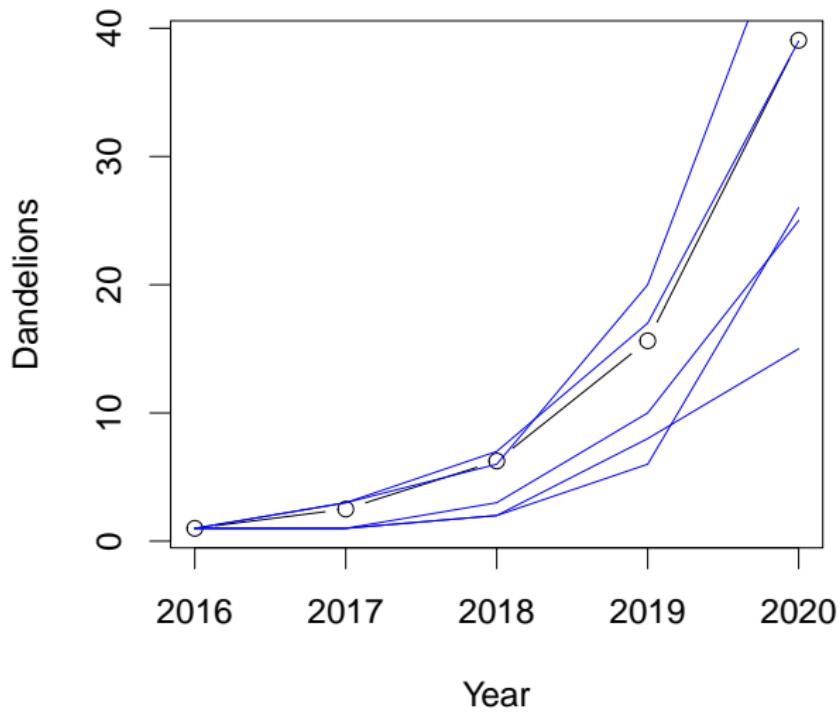
Stochastic model



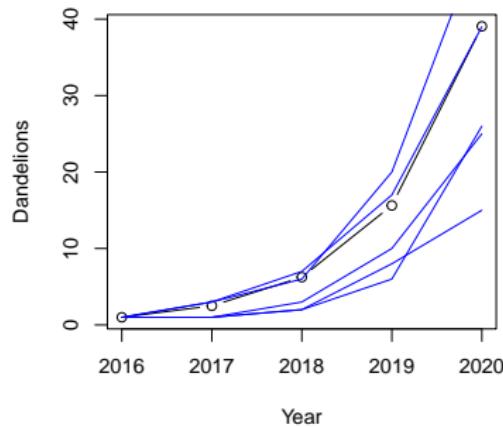
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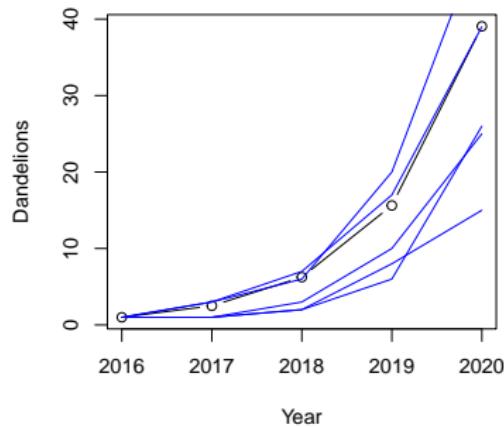


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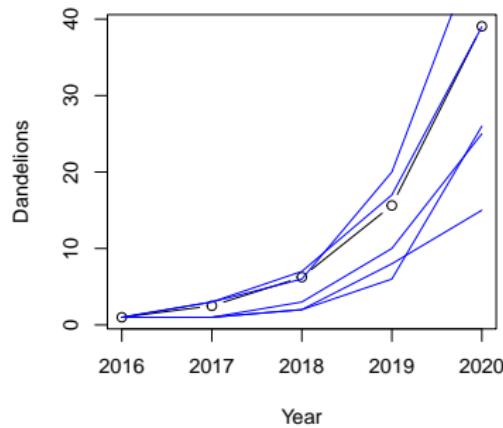
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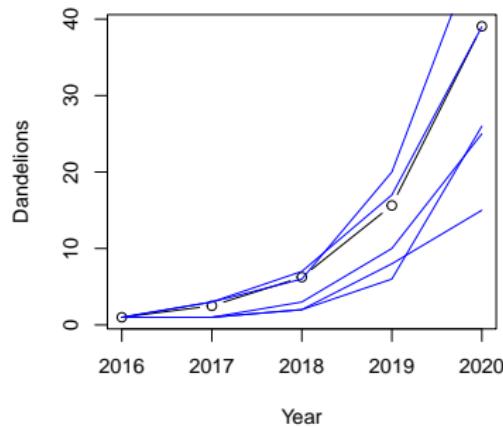
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Outline

Course overview

Course structure

People

Course content

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Examples

Example populations

Dandelions

Gypsy moths

Bacteria

Exponential growth

Log and linear scales

Time scales

Bacteria

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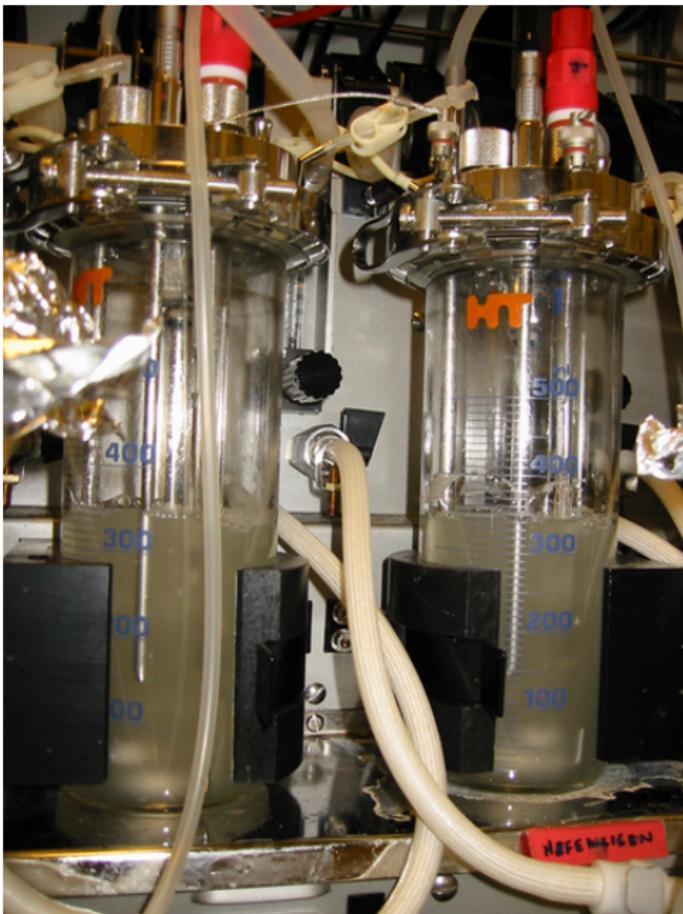
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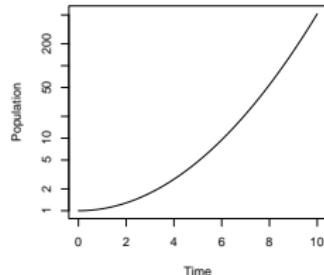
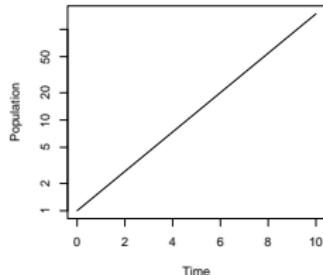
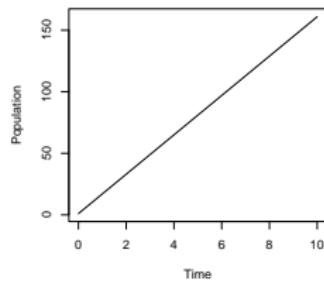
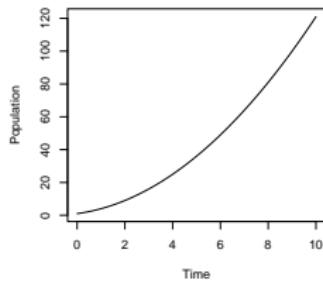
Exponential growth

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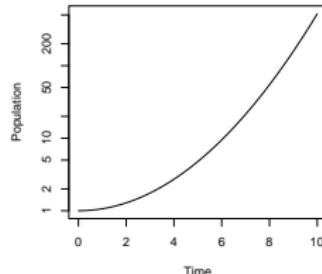
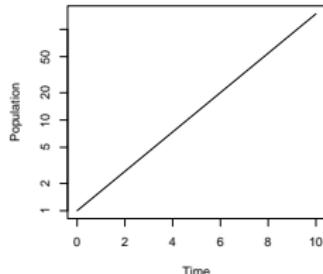
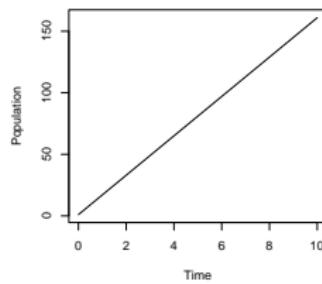
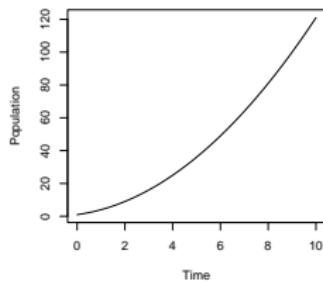
Exponential growth

► What is exponential growth?



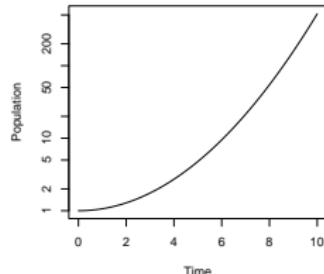
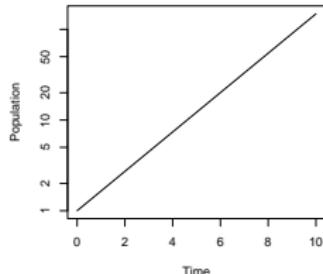
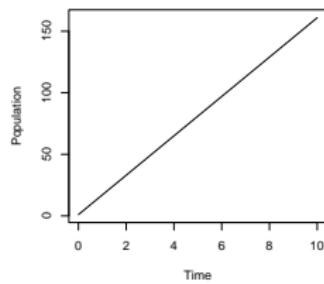
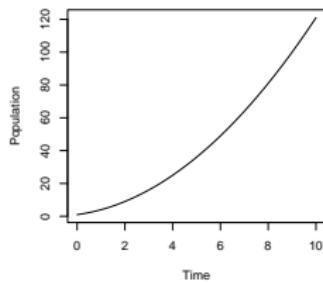
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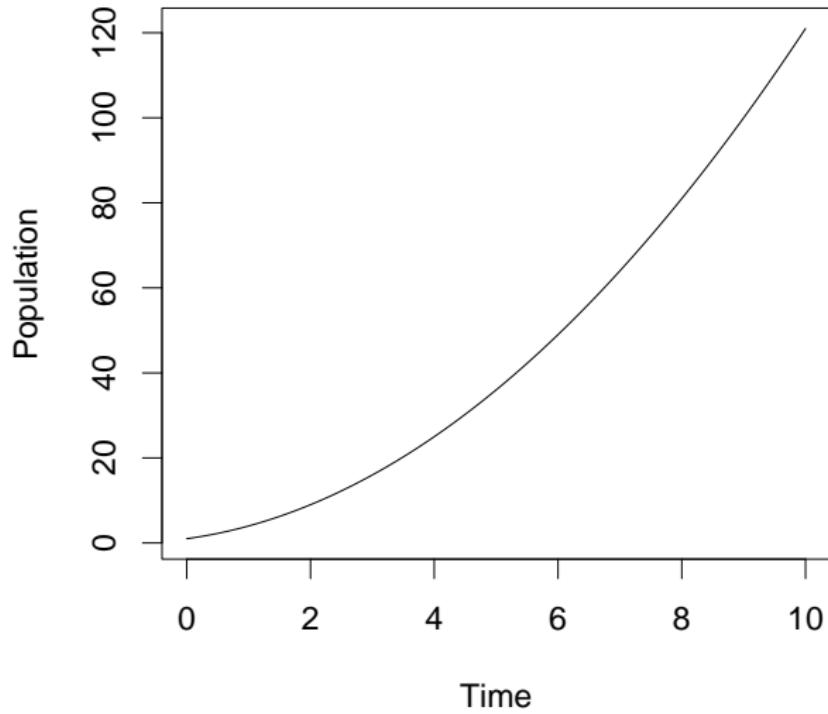


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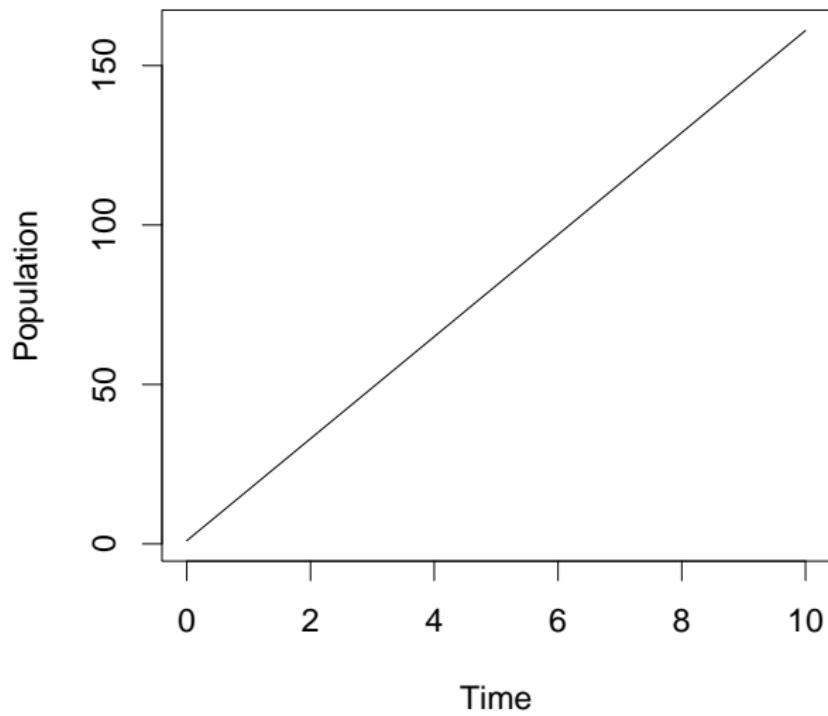
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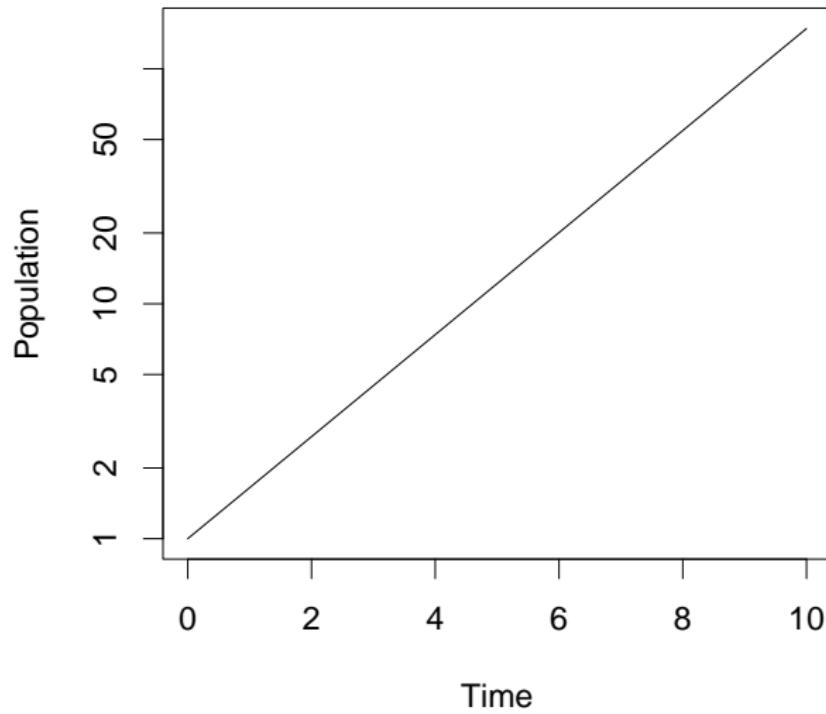
A (repeat)



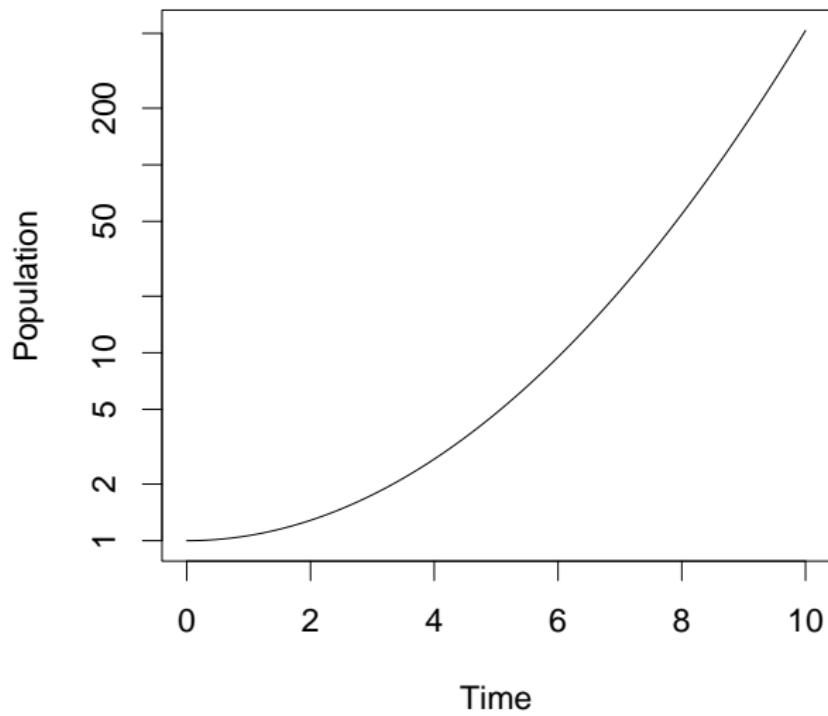
B (repeat)



C (repeat)

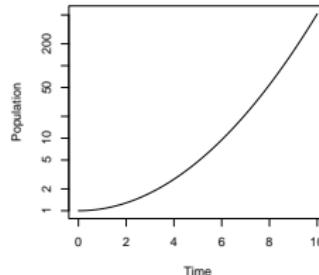
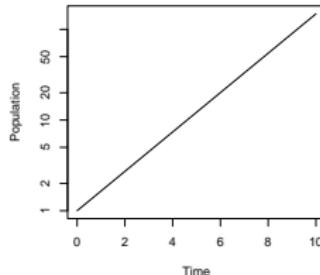
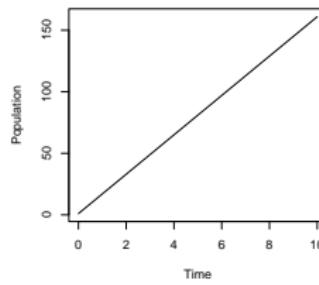
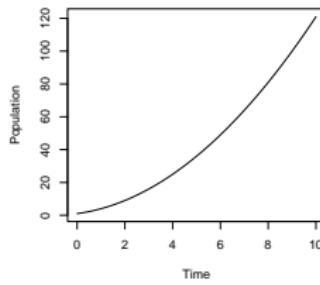


D (repeat)



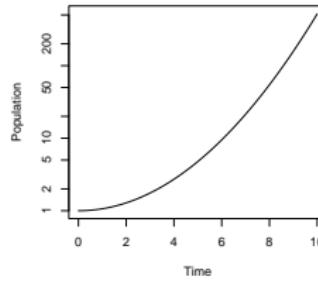
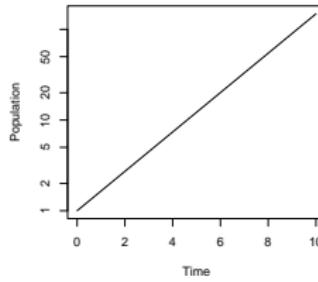
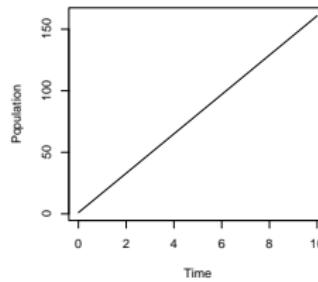
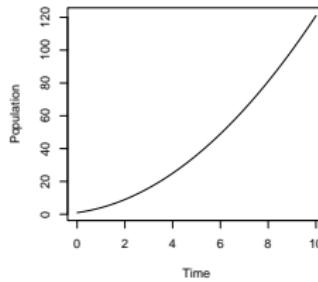
Exponential growth (repeat)

► Poll: What is exponential growth?



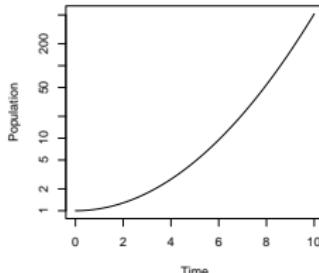
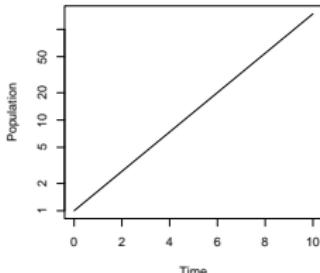
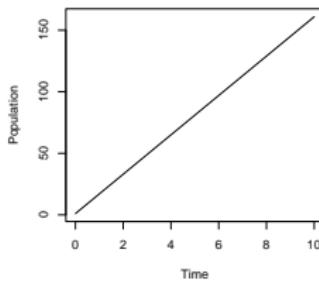
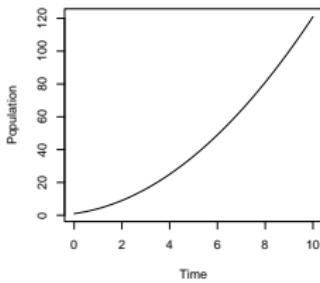
Exponential growth (repeat)

- ▶ Poll: What is exponential growth?
- ▶ Poll: Which of these is an example?

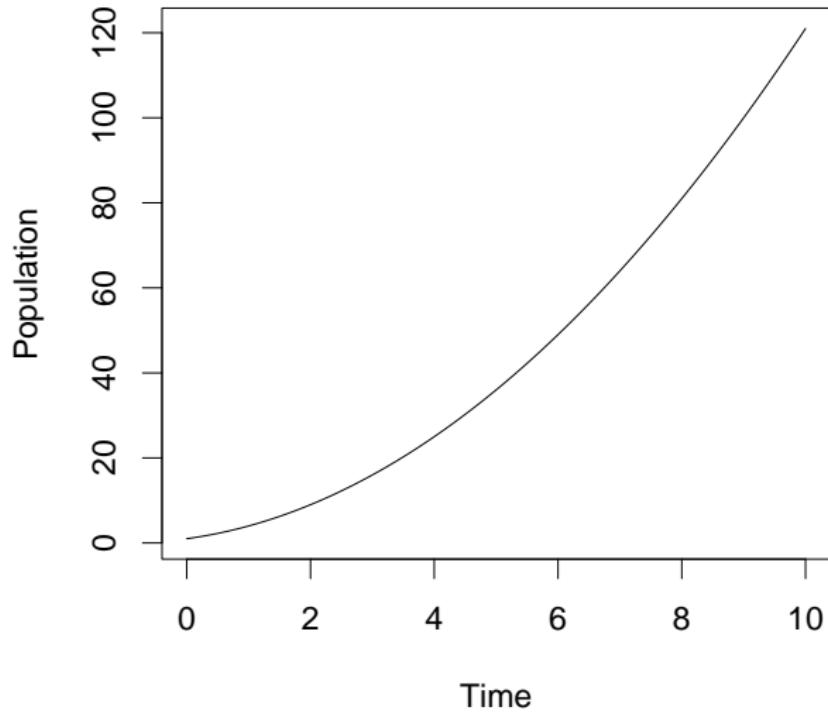


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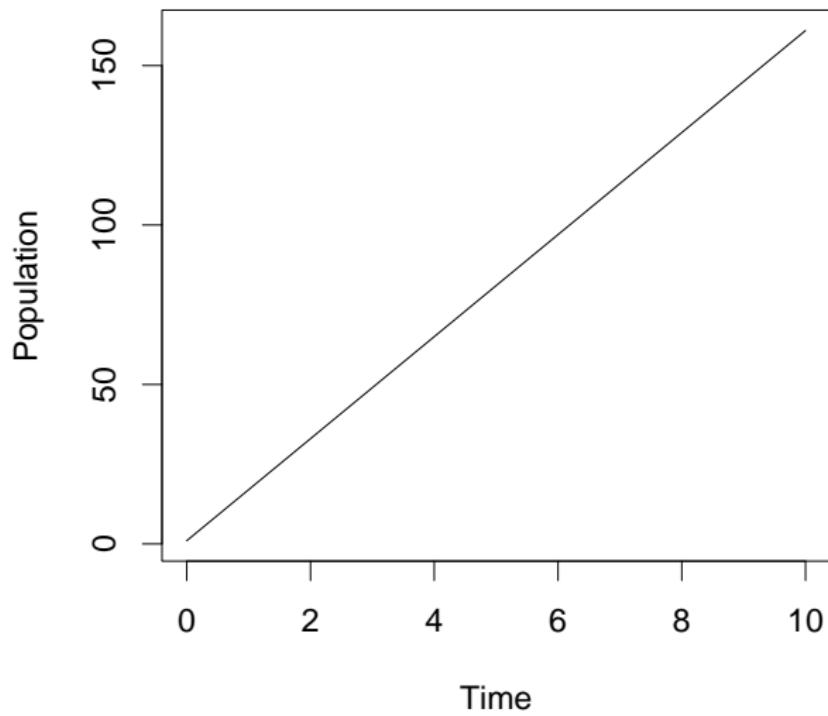
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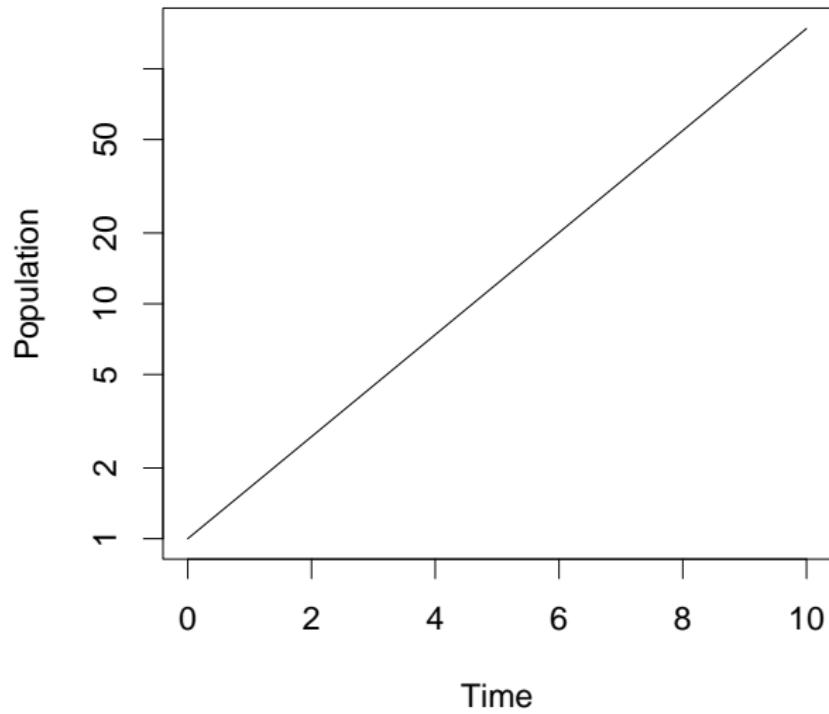
A (repeat)



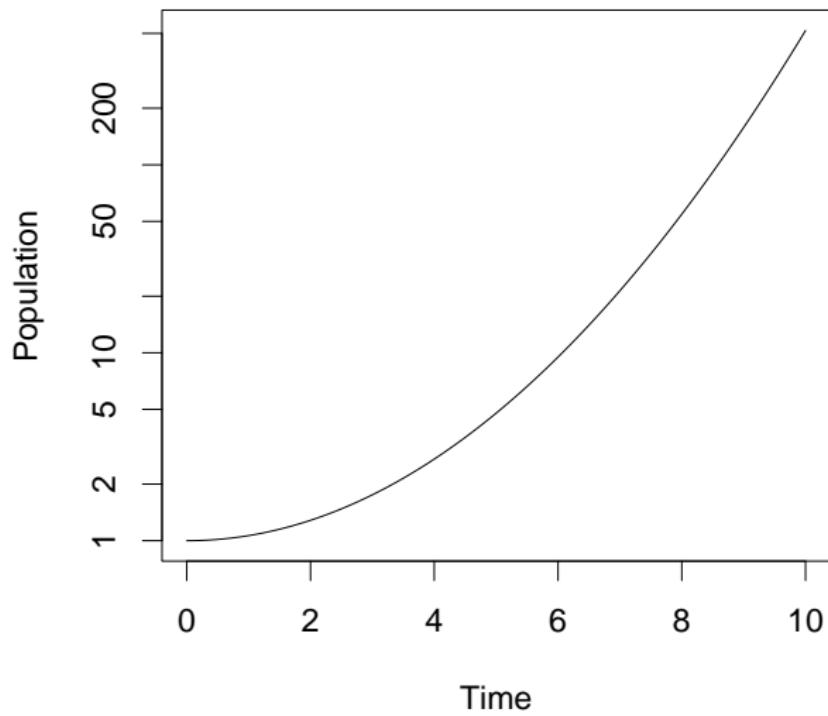
B (repeat)



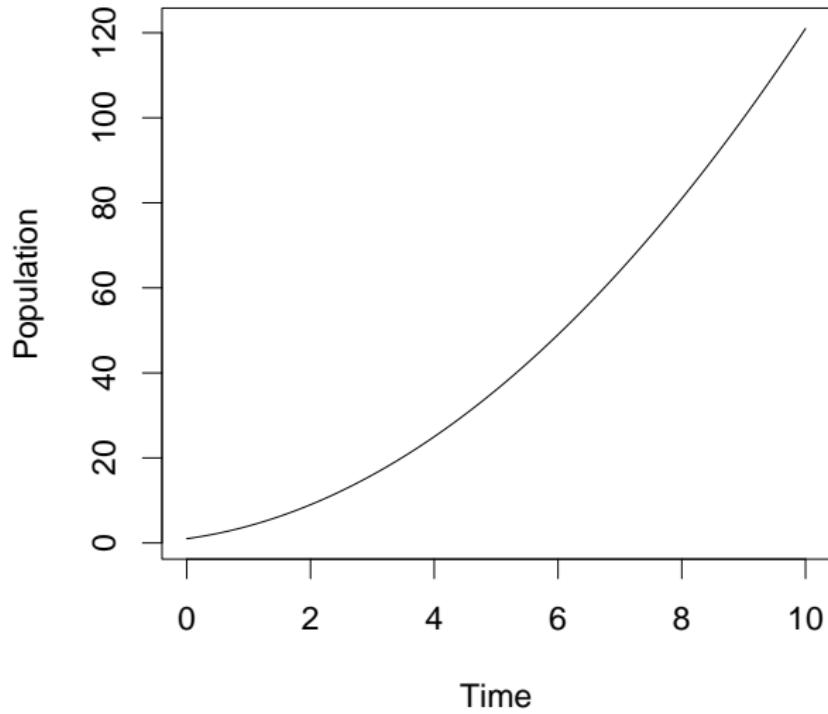
C (repeat)



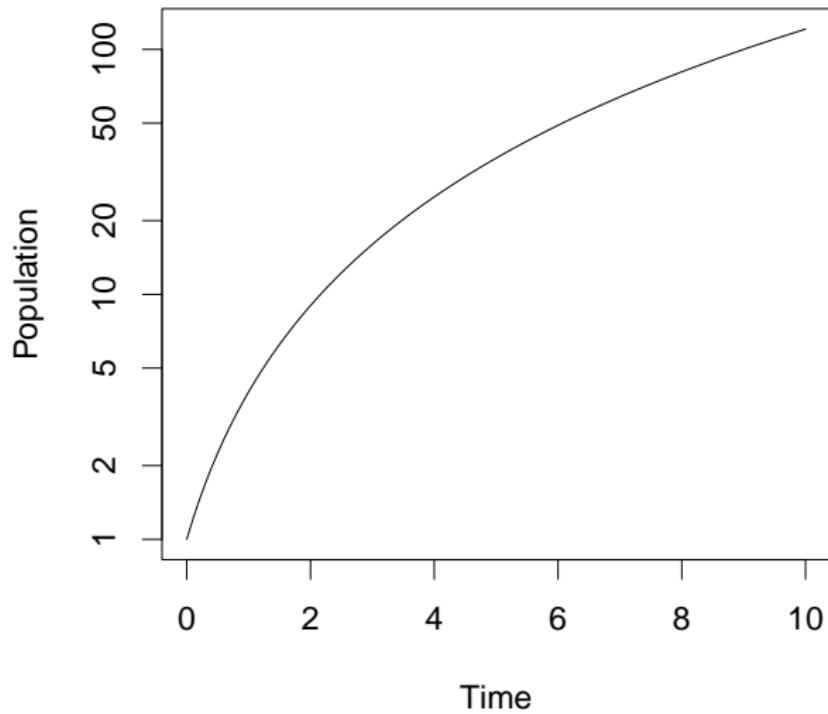
D (repeat)



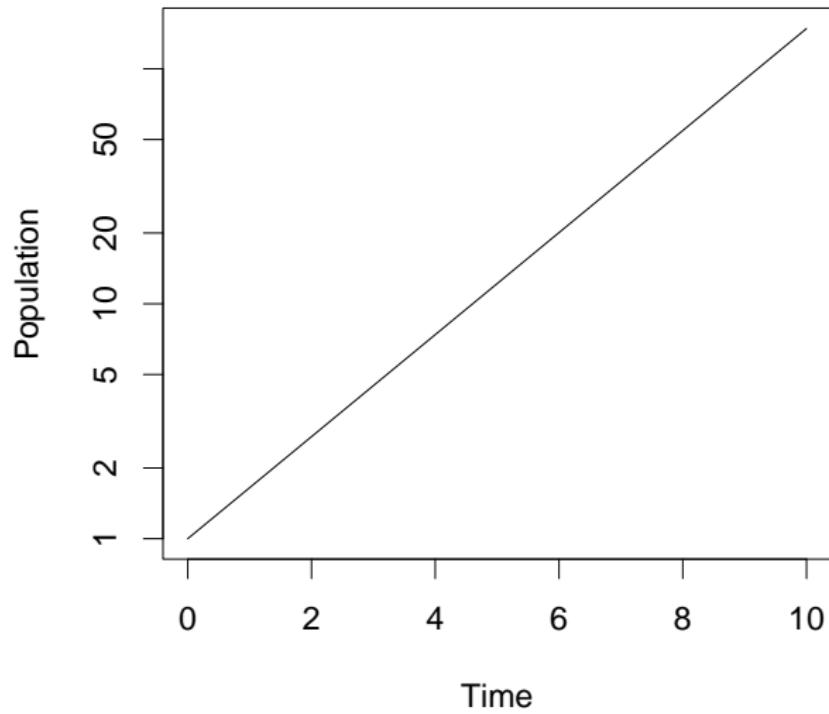
A (repeat)



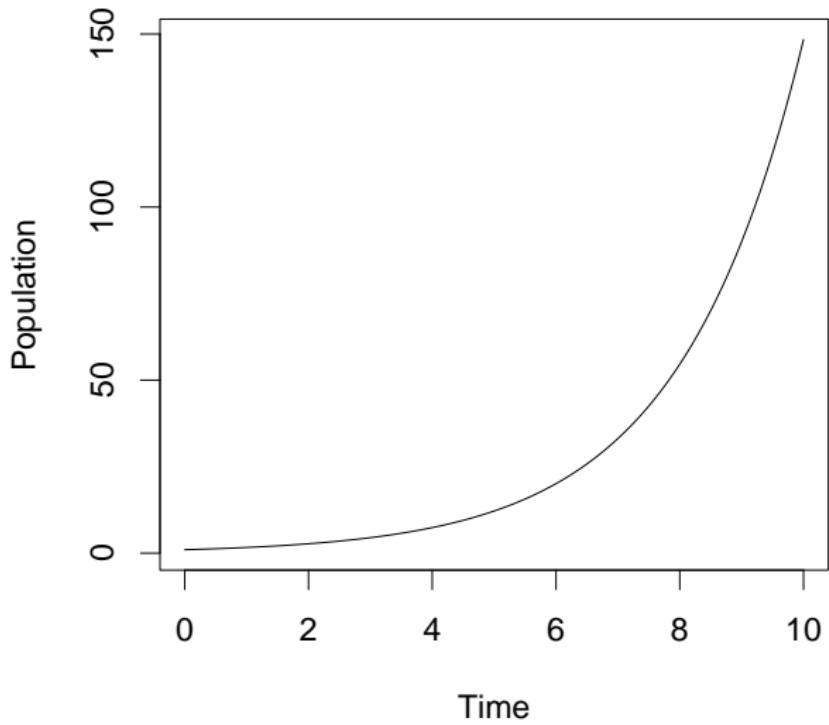
A on the log scale



C (repeat)



C on the linear scale



Types of growth

- arithmetic/linear:

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- ▶ Poll: What is exponential decline?

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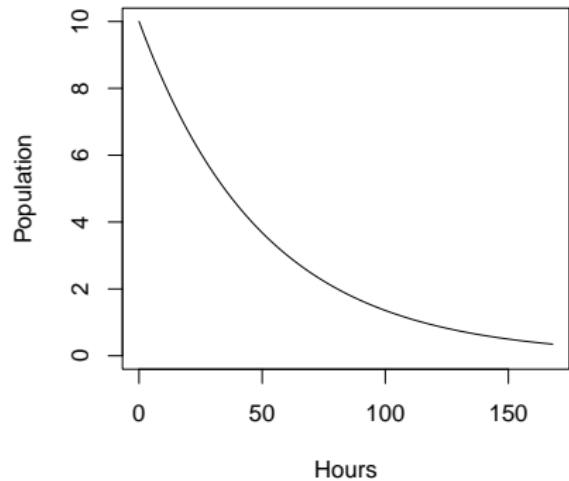
Exponential decline?

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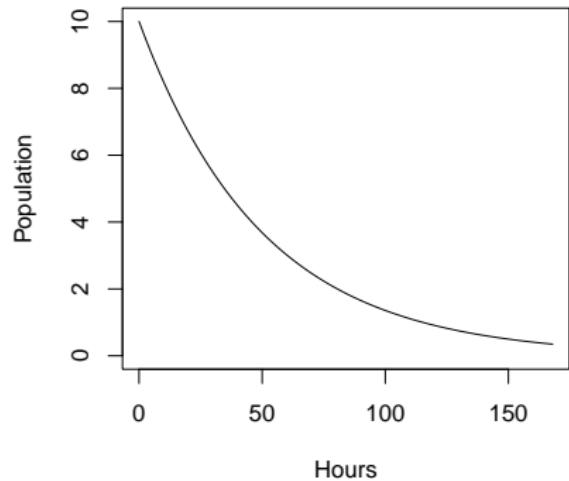
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Exponential decline



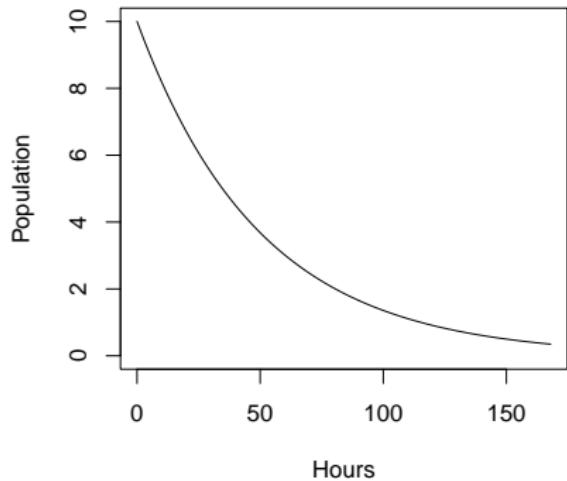
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Time scales

Scales of comparison

- Poll: 1 is to 10 as 10 is to what?

Scales of comparison

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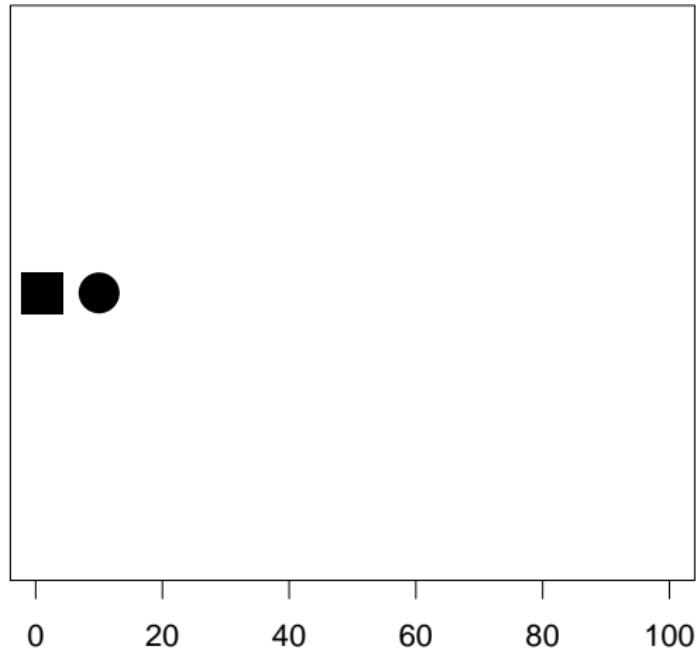
Scales of comparison

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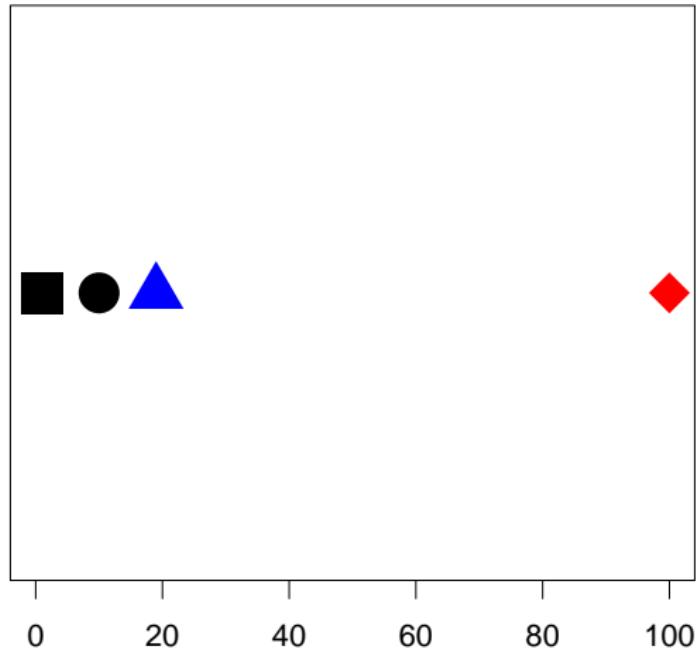
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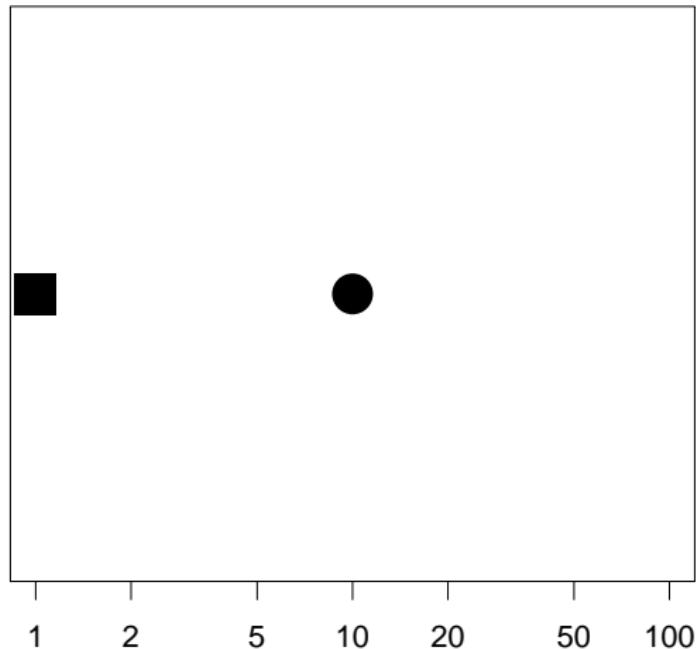
Scales of display



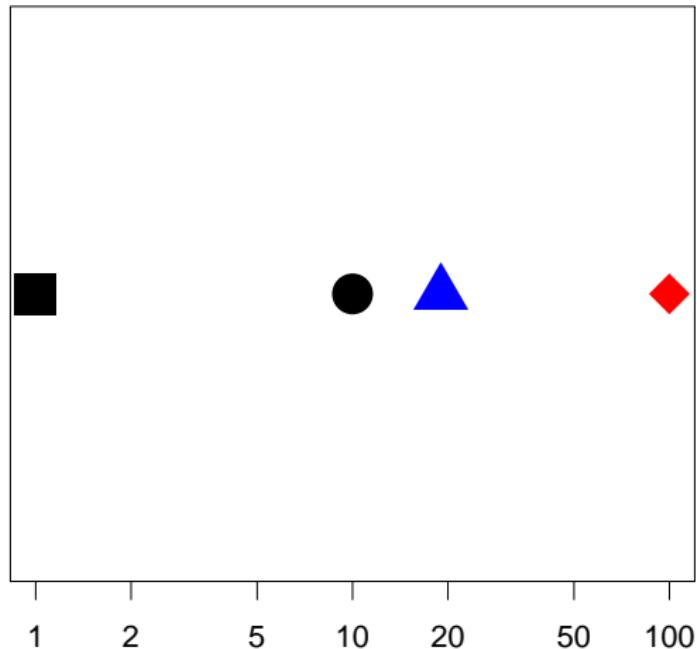
Scales of display



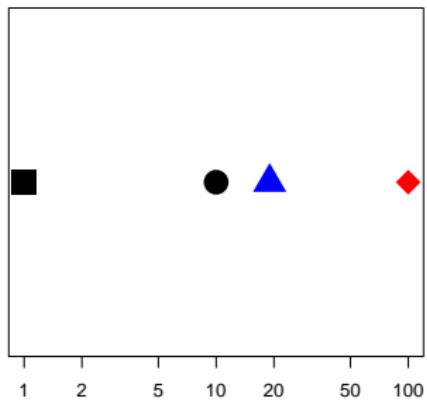
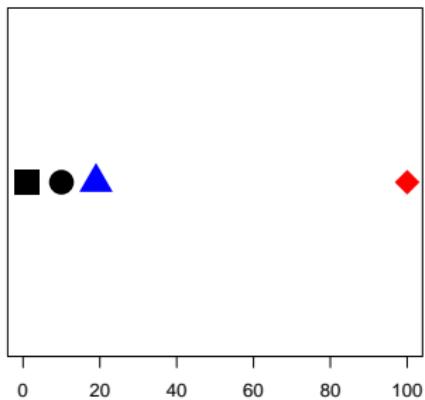
Scales of display



Scales of display



Scales of display



There is only one log scale; it doesn't matter which base you use!

Canadian provinces

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Canadian provinces

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- ▶ Poll: Which Canadian province is the most unusual in terms of area?

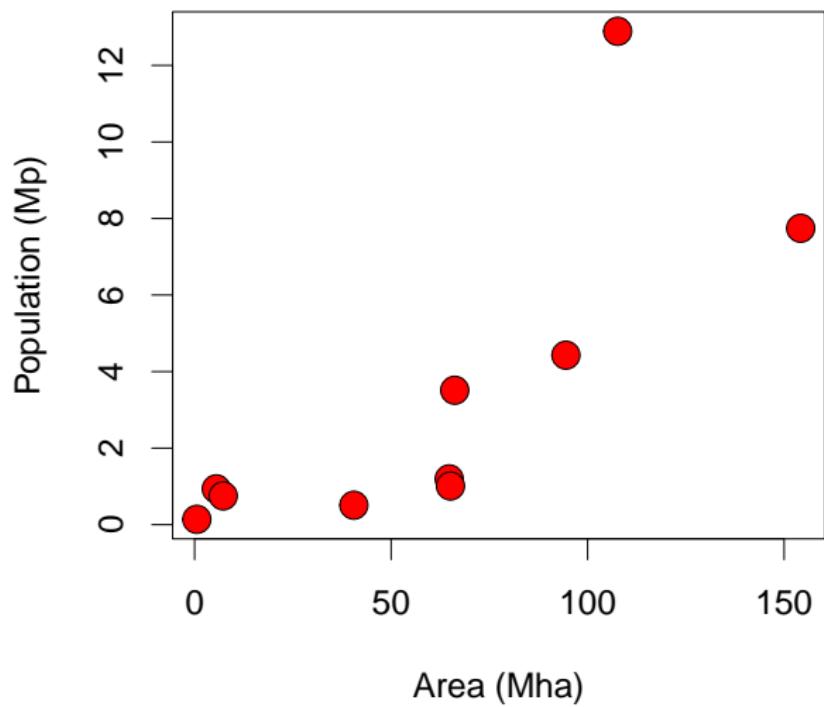
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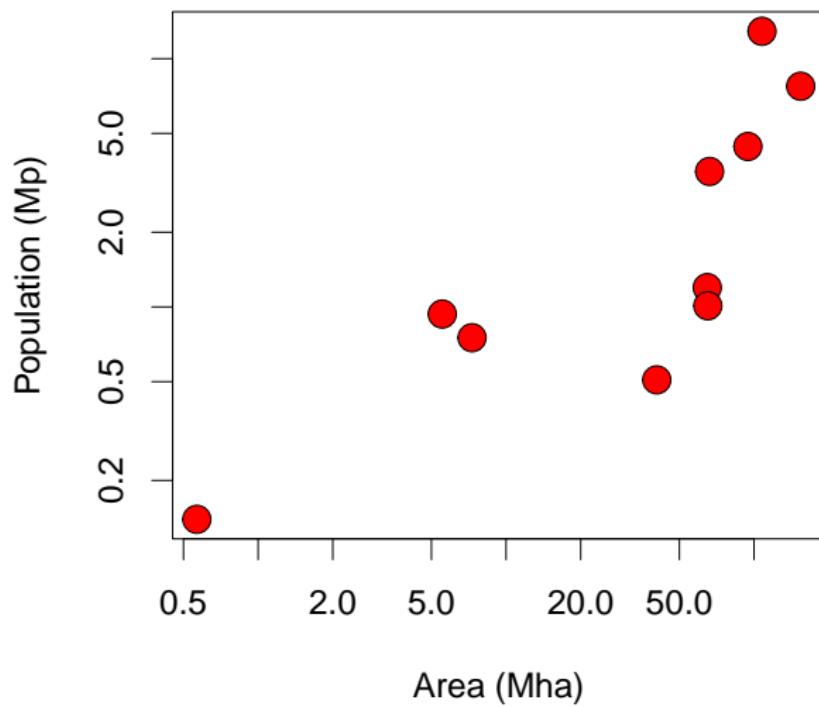
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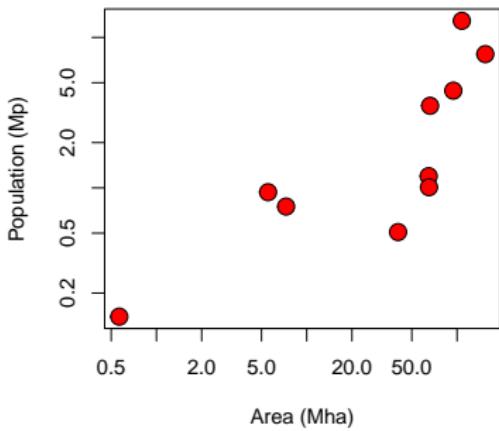
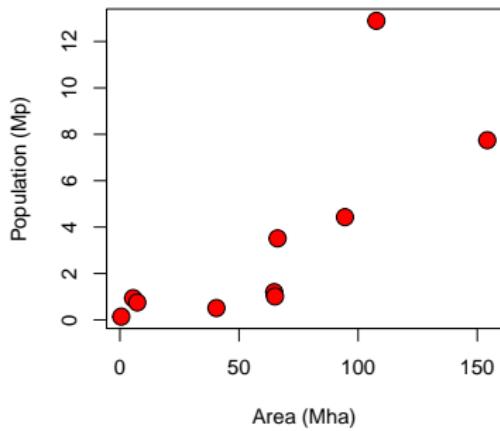
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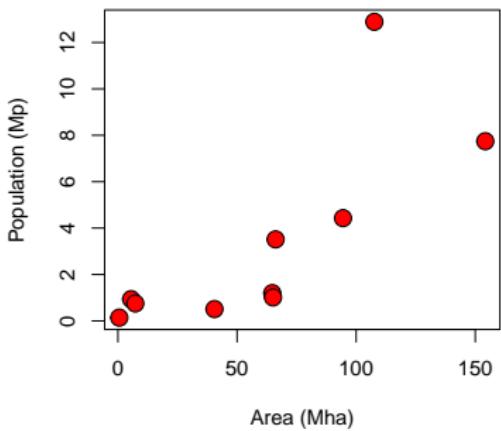
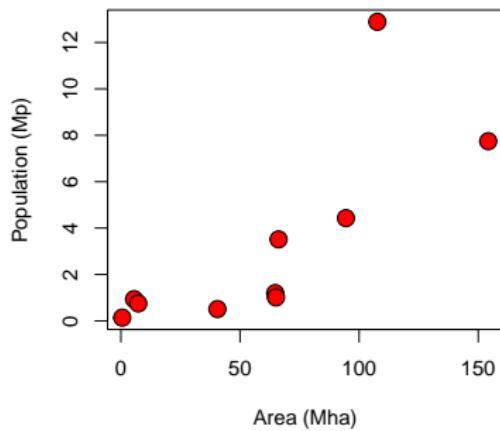
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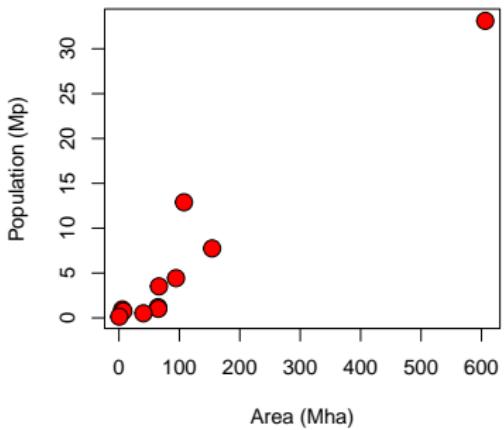
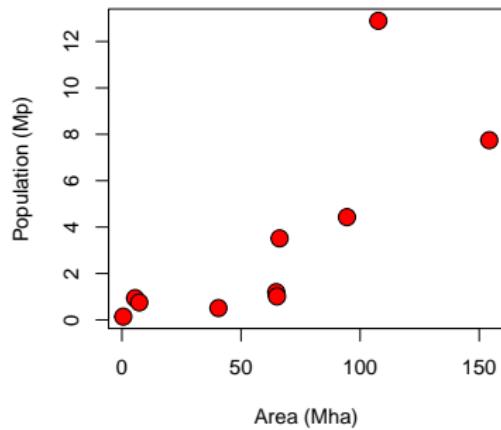
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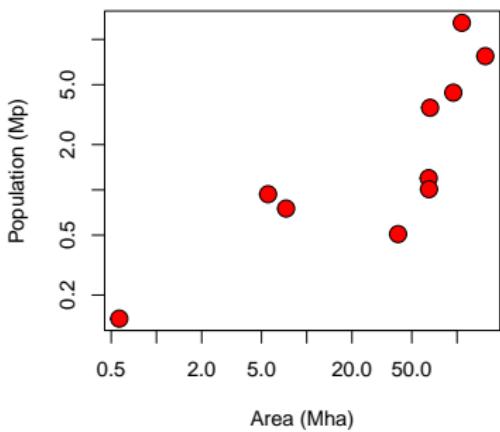
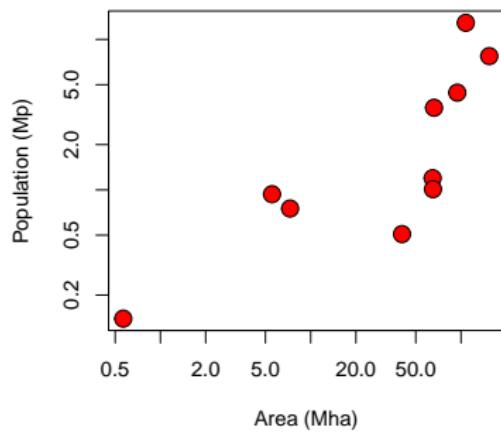
Canadian provinces plus Canada?



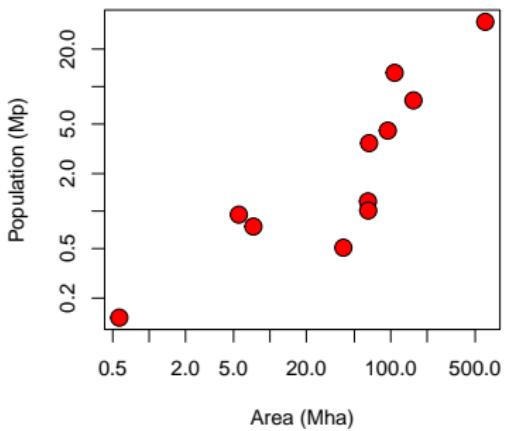
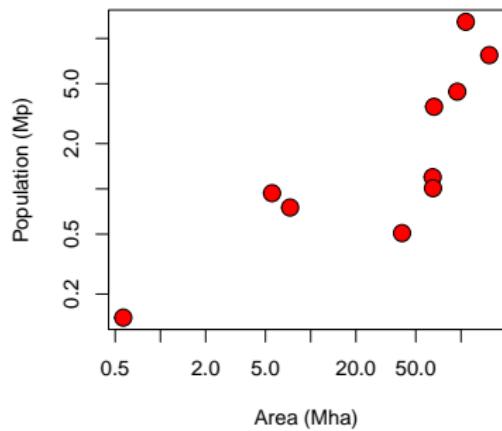
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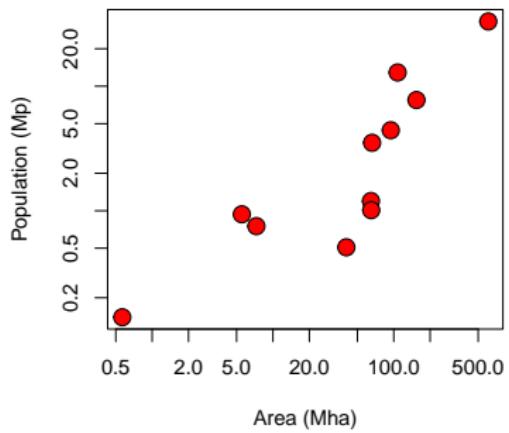
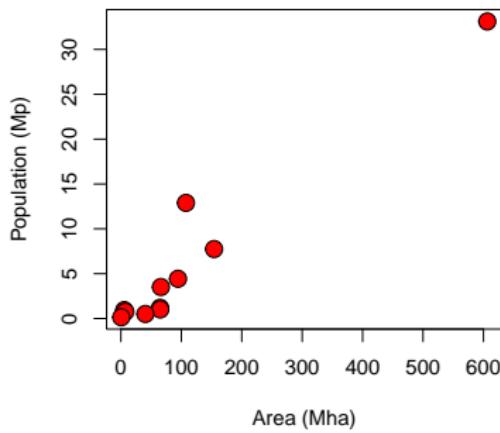
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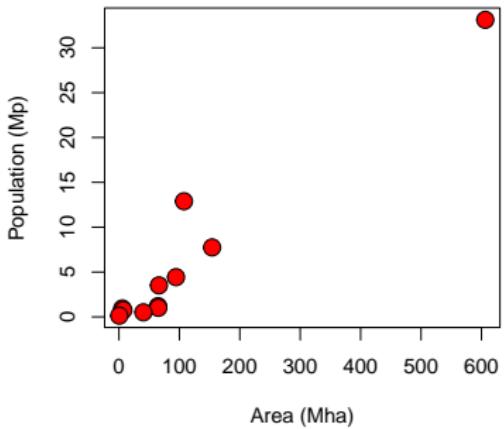
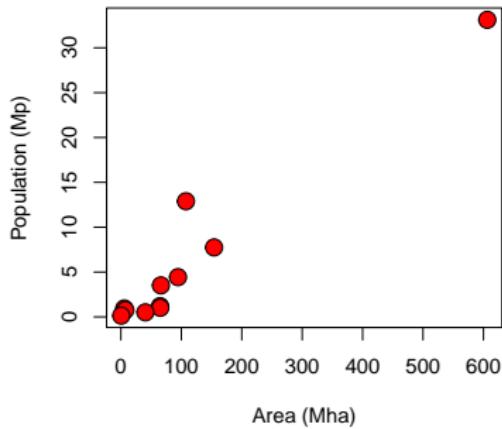
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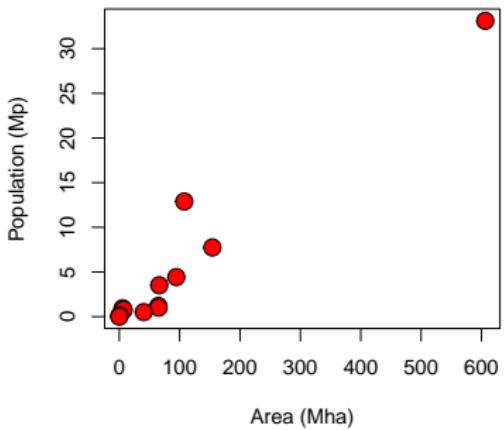
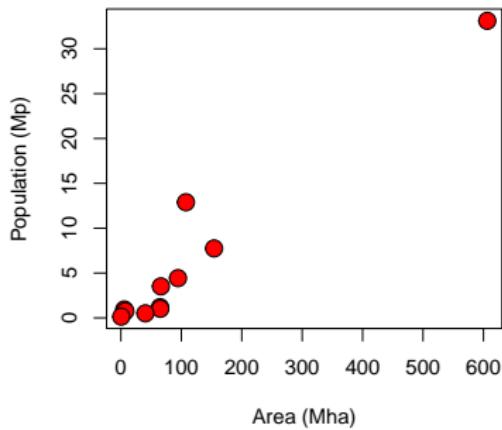
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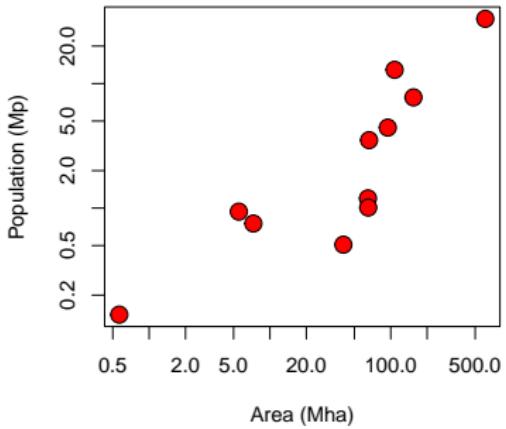
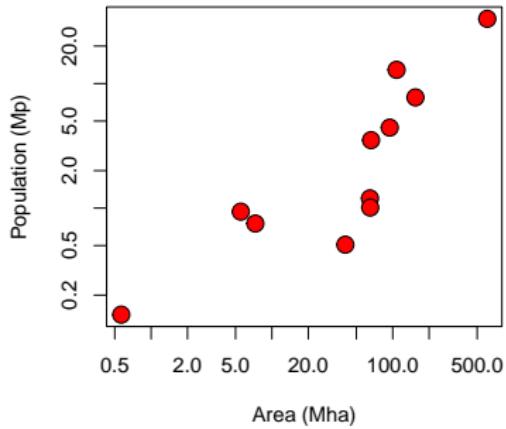
Canada plus Room 137



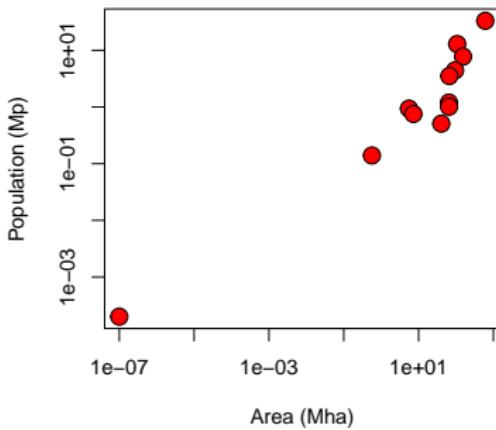
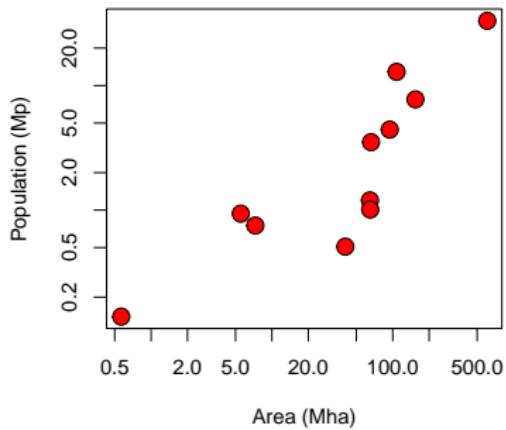
Canada plus Room 137



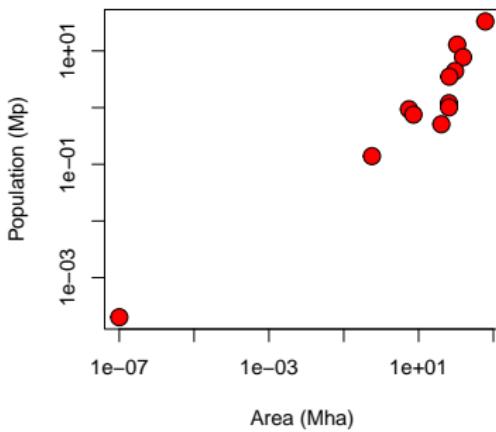
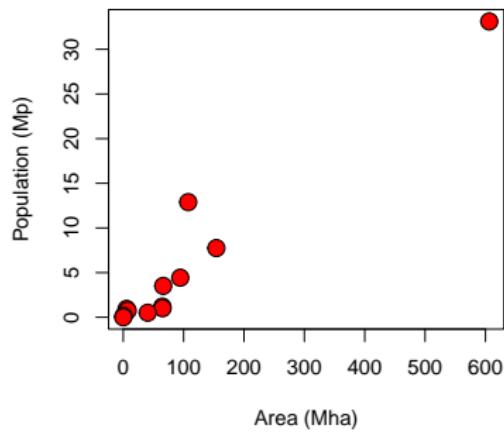
Canada plus room Room 137?



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Canada plus room Room 137



Predation comparison



Predation comparison

- A 300 lb lion is attacking a 600 lb buffalo!



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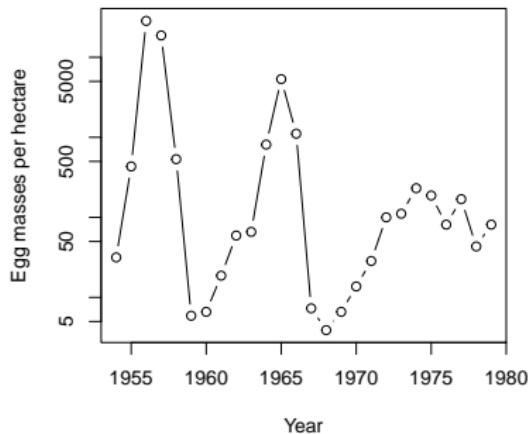
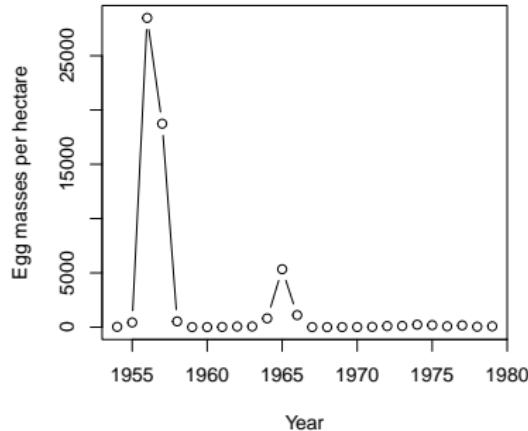
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Gypsy-moth example



Scales in population biology

- The linear scale looks at differences at the population scale

Scales in population biology

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- ▶ The log scale looks at differences at the individual scale (per capita)

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Speeding in Taiwan

- A life experience



Speeding in Taiwan

- ▶ A life experience
- ▶ Some clarifications



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Speeding in Taiwan

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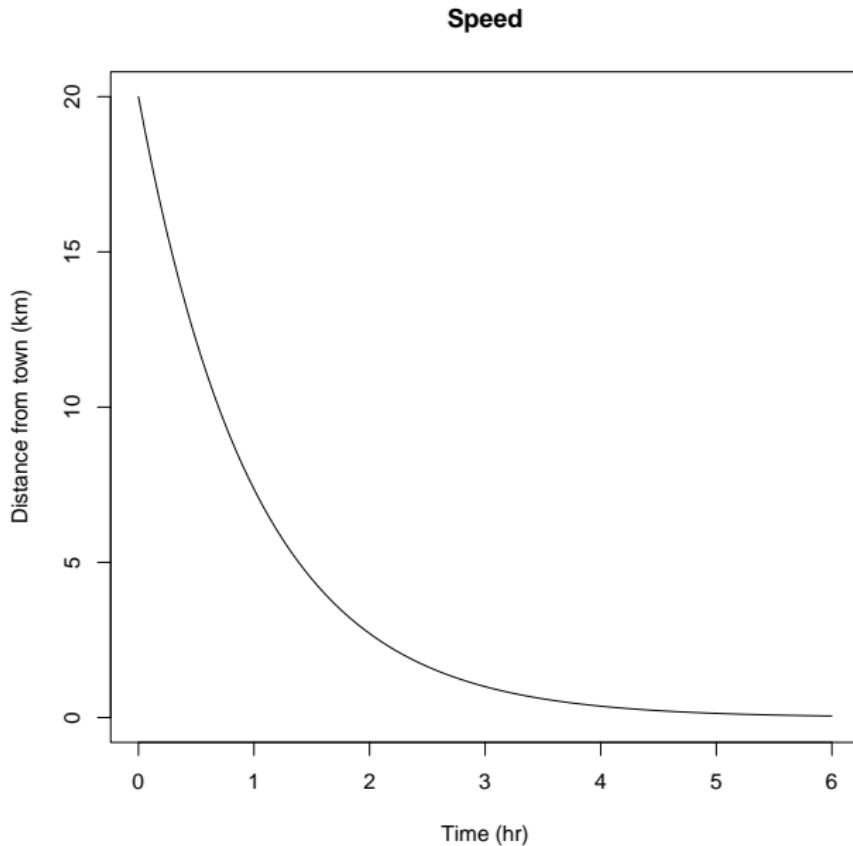
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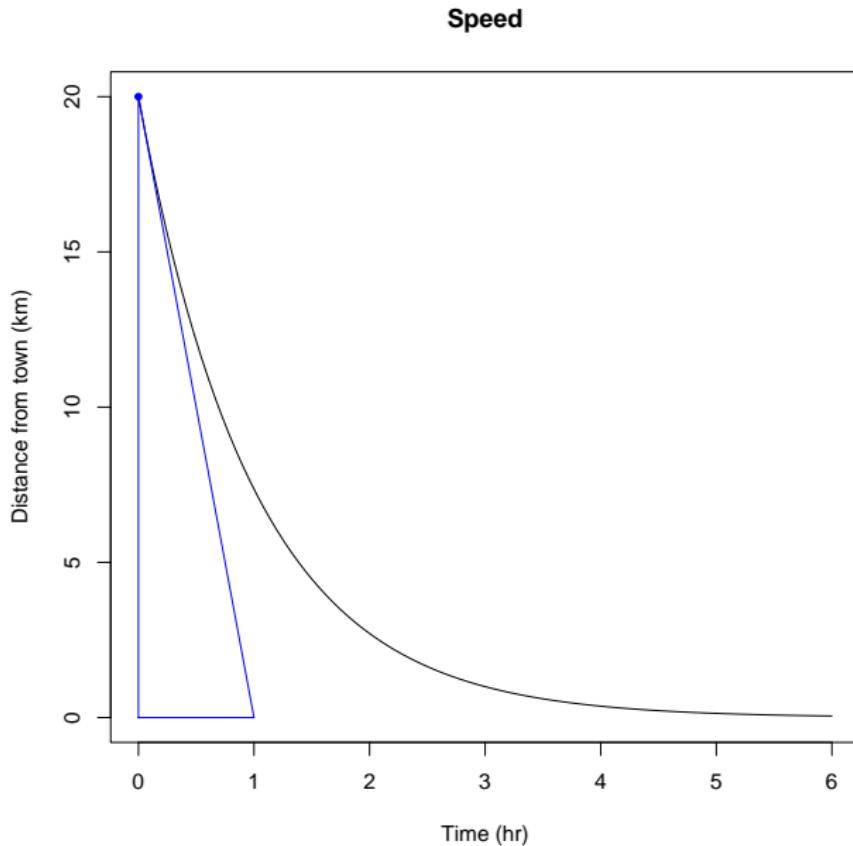
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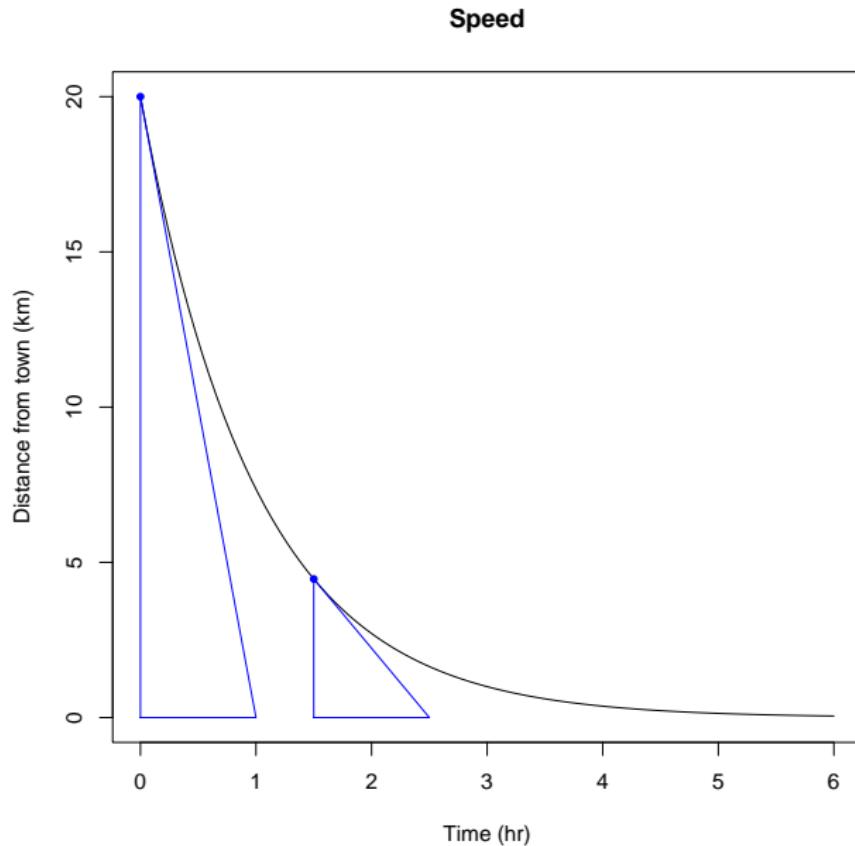
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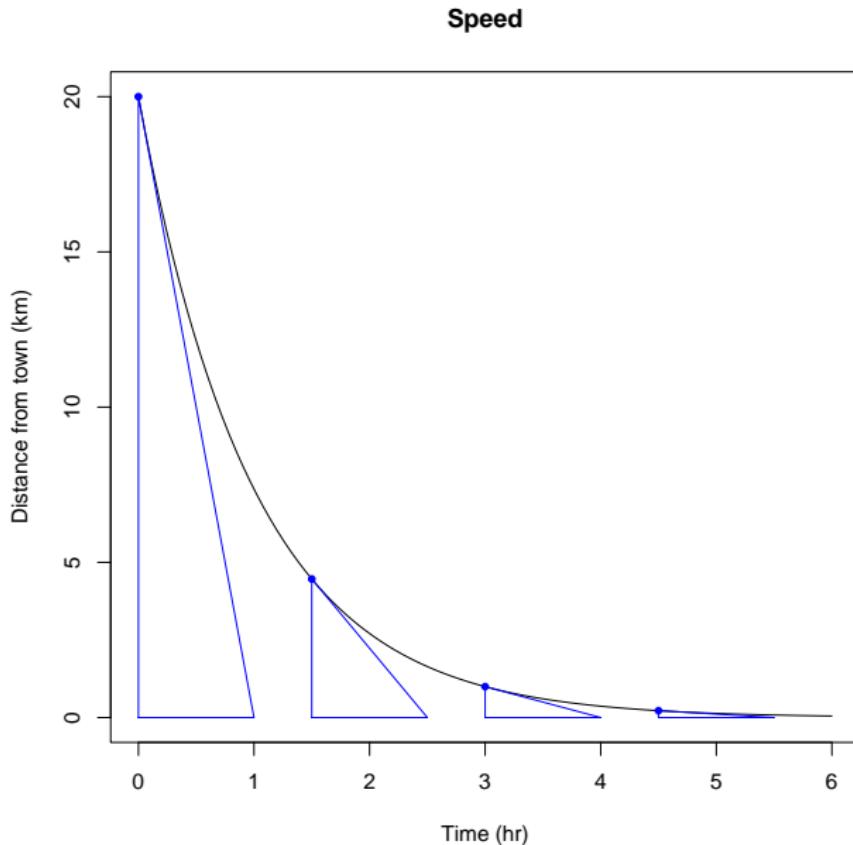
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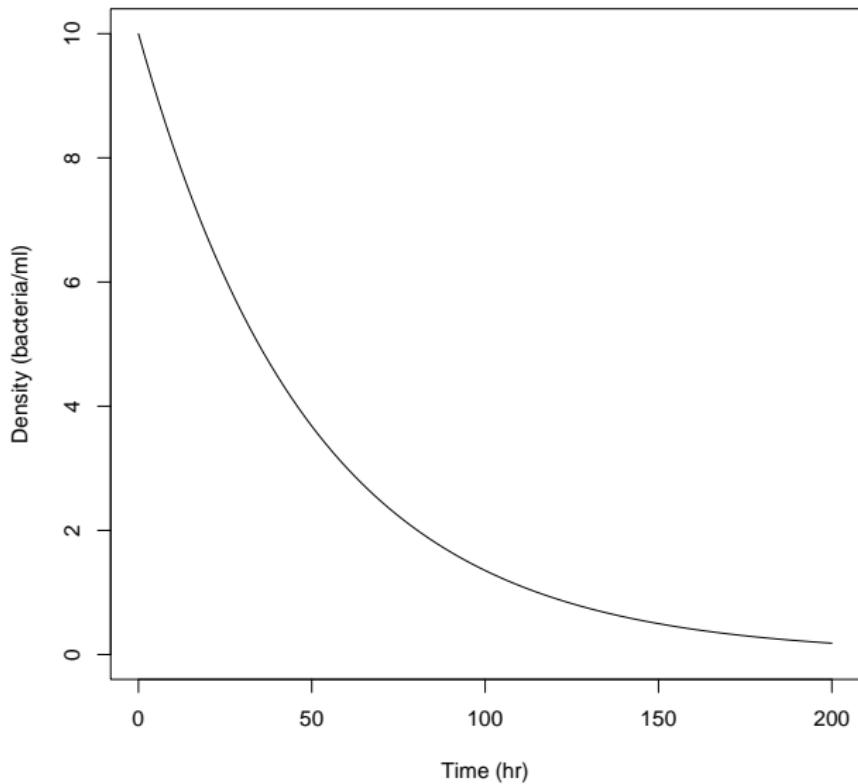
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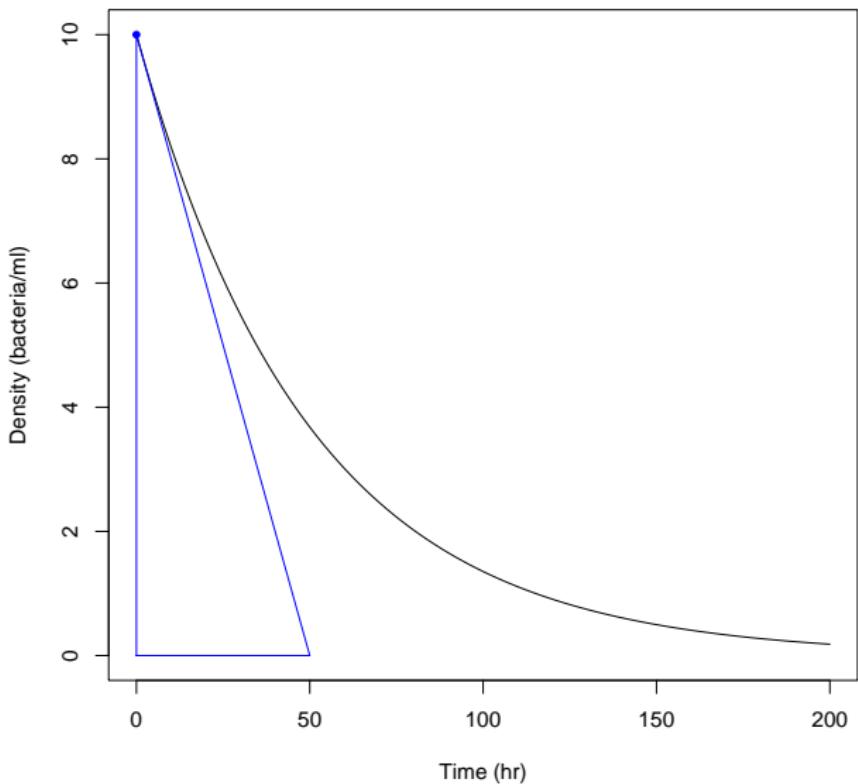
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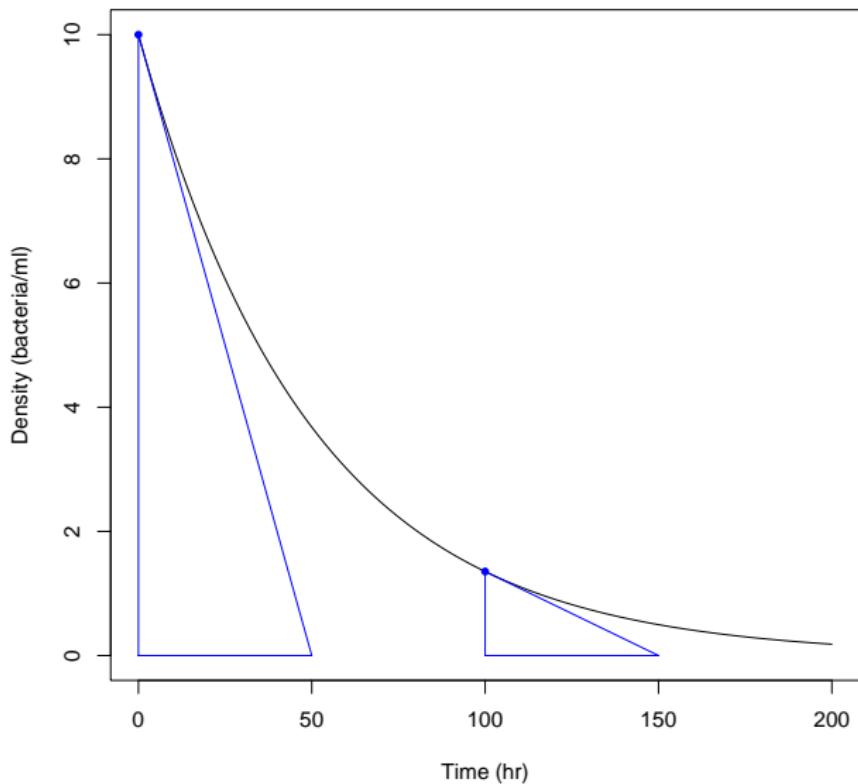
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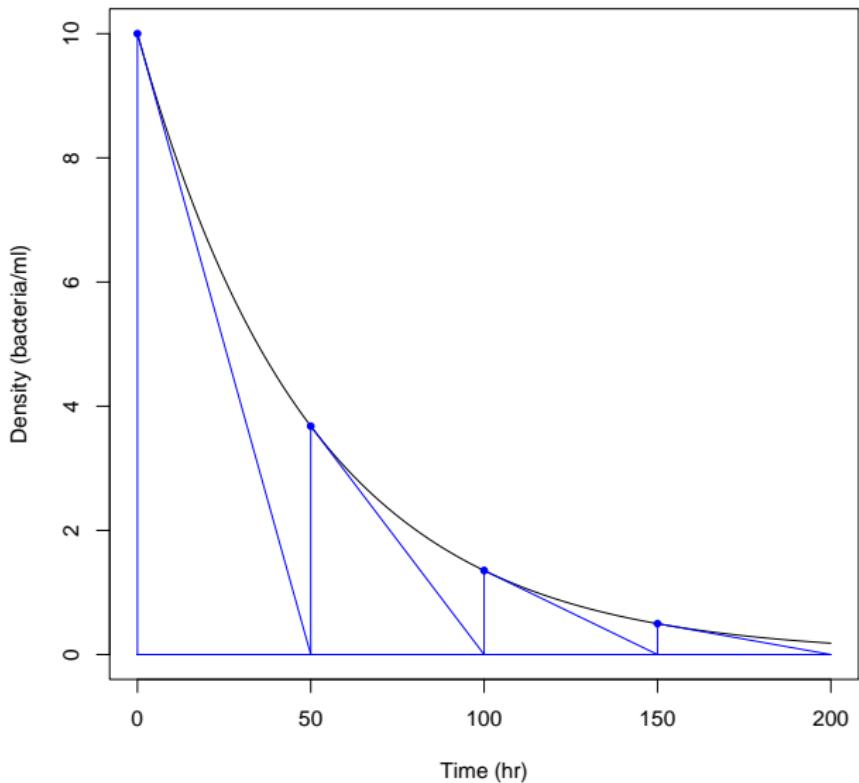
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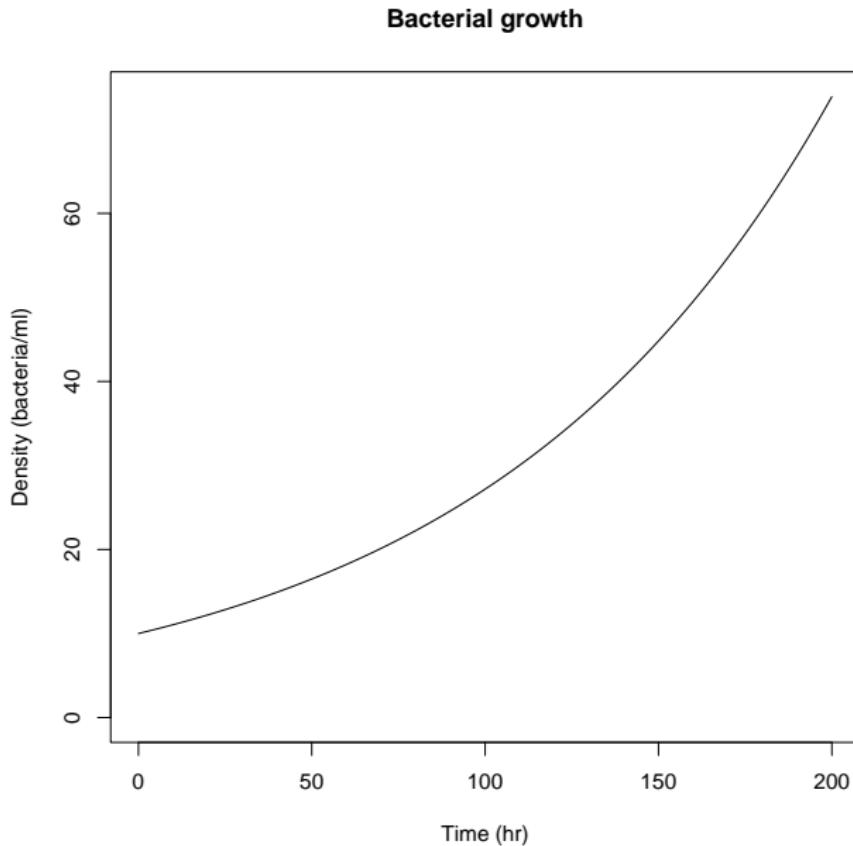
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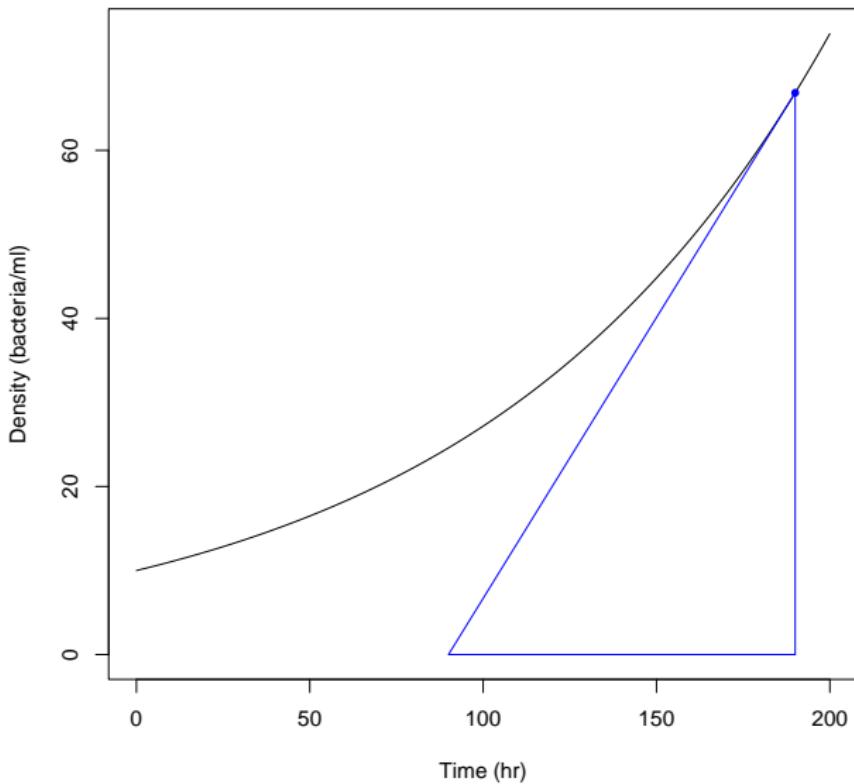
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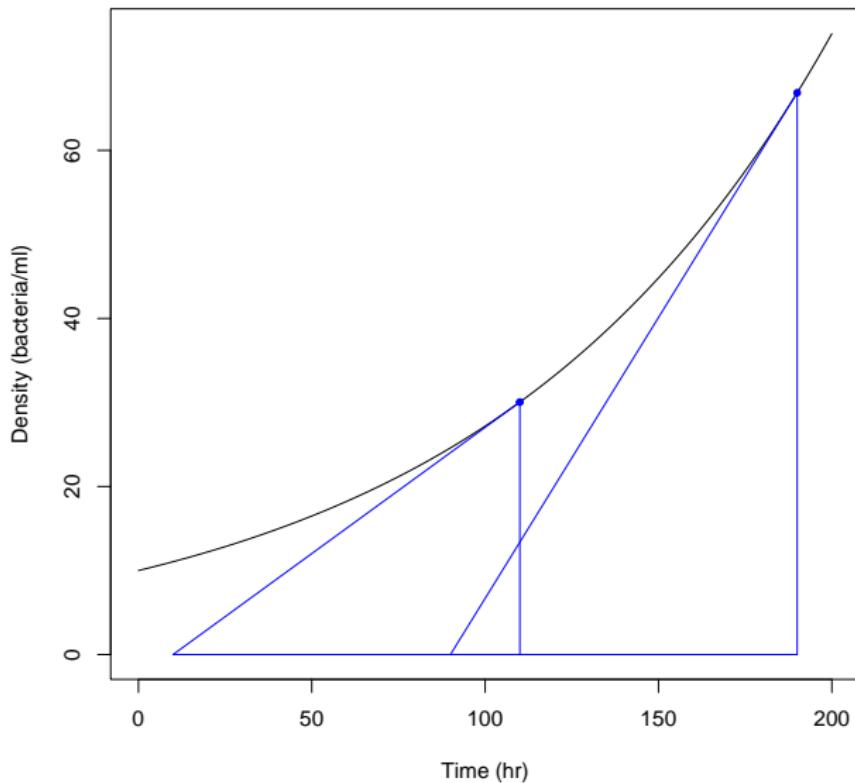
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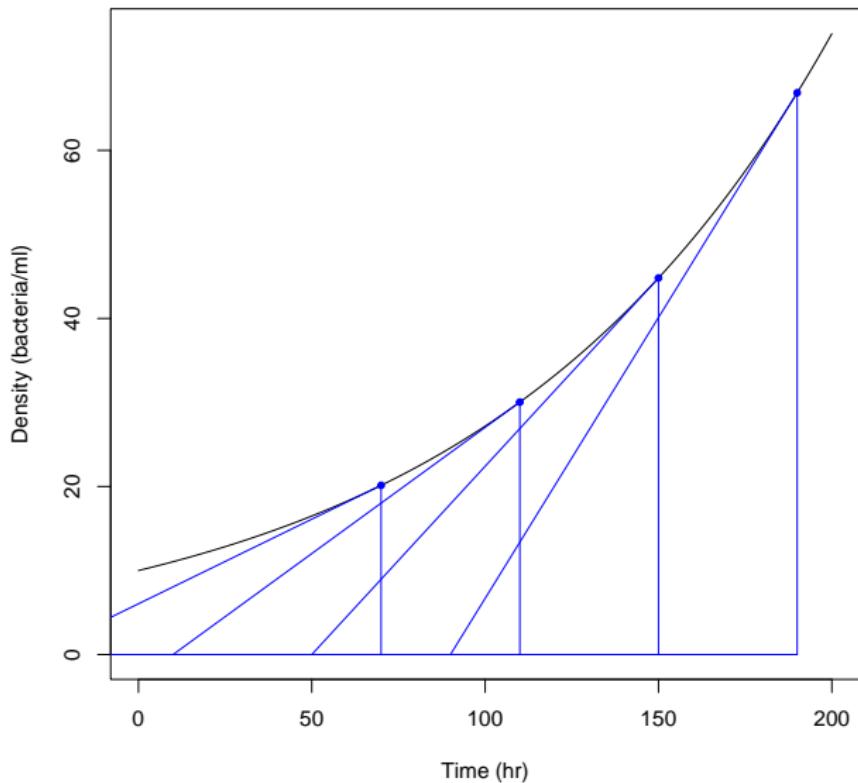
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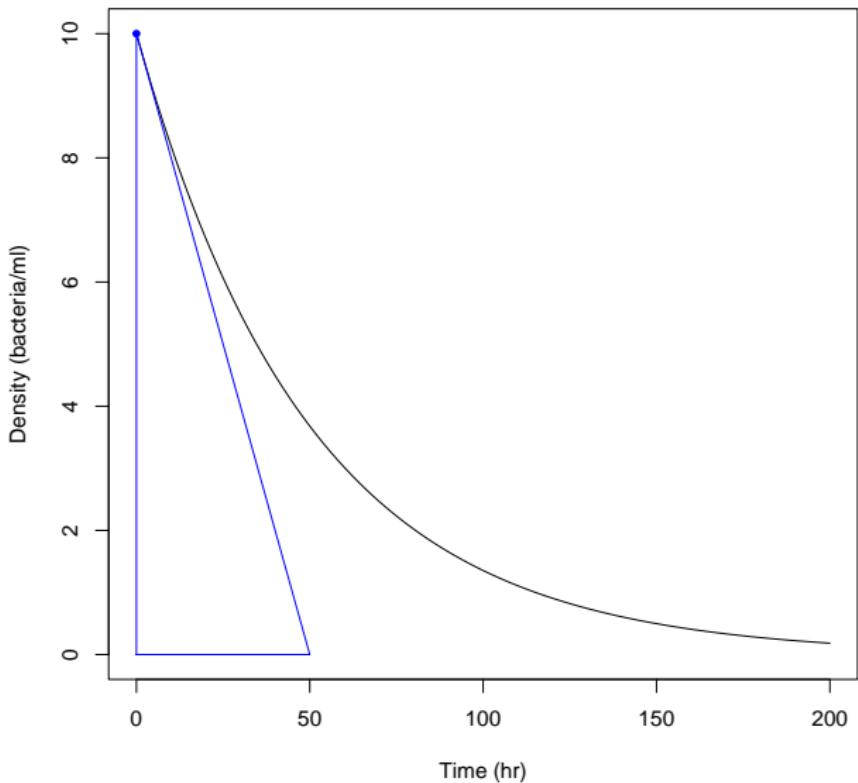
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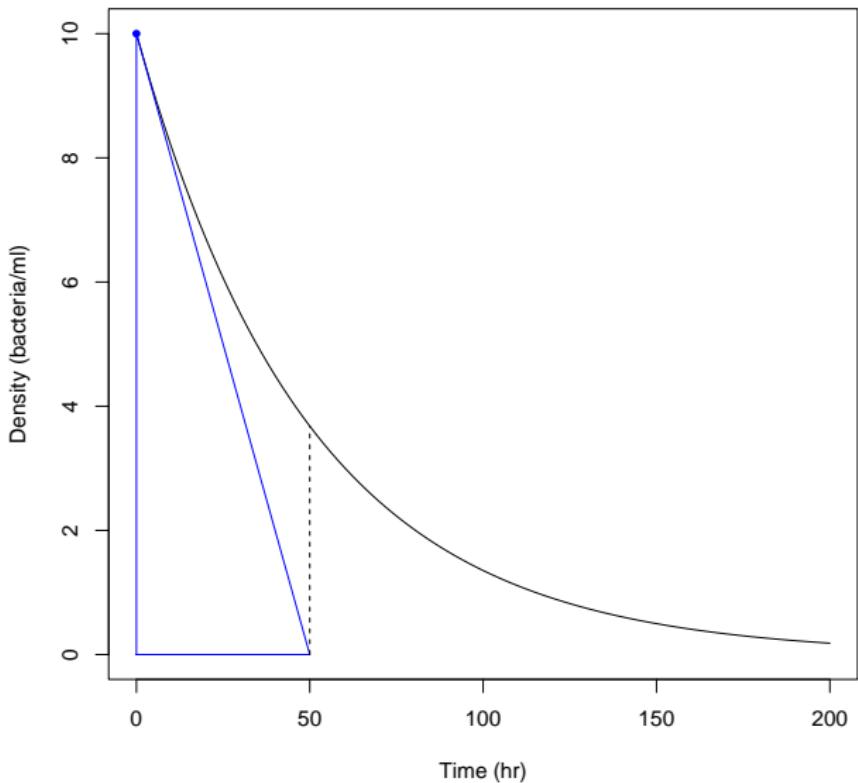
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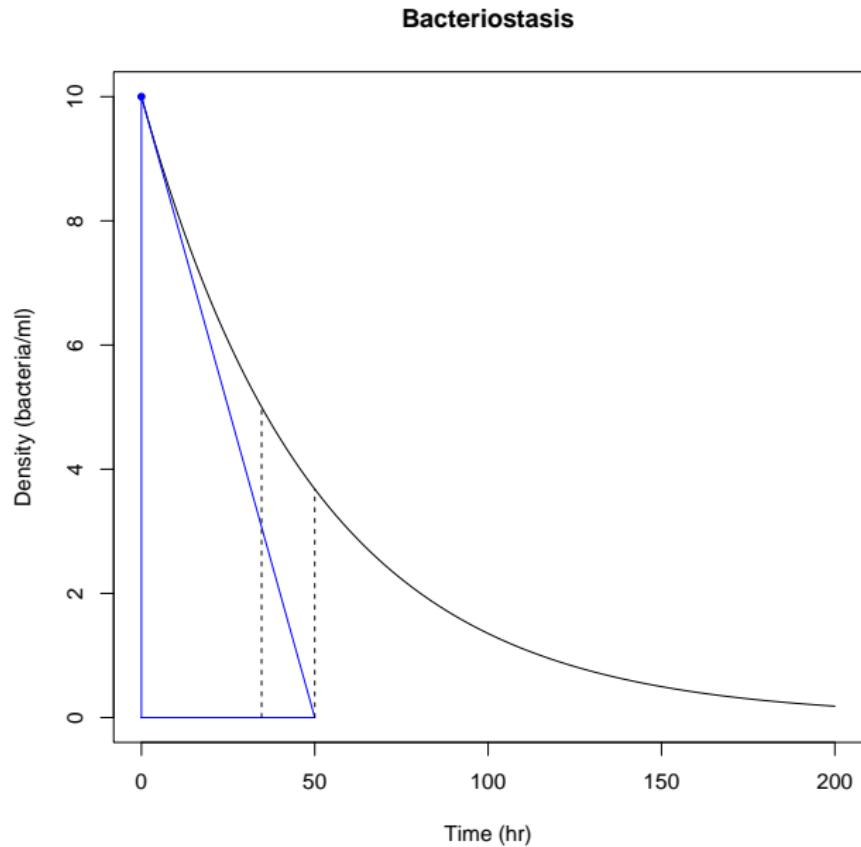
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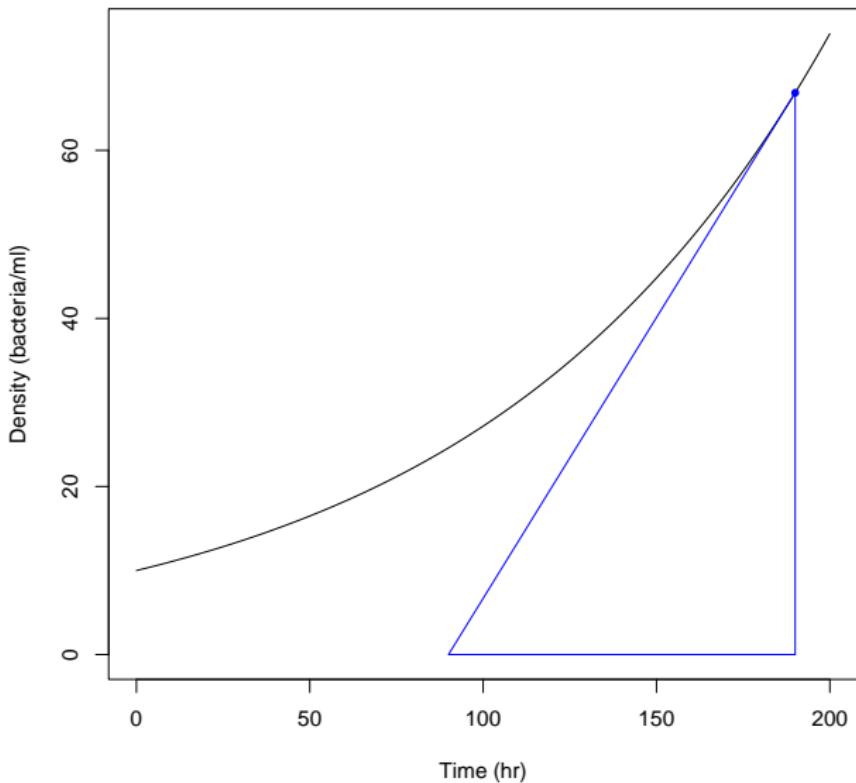
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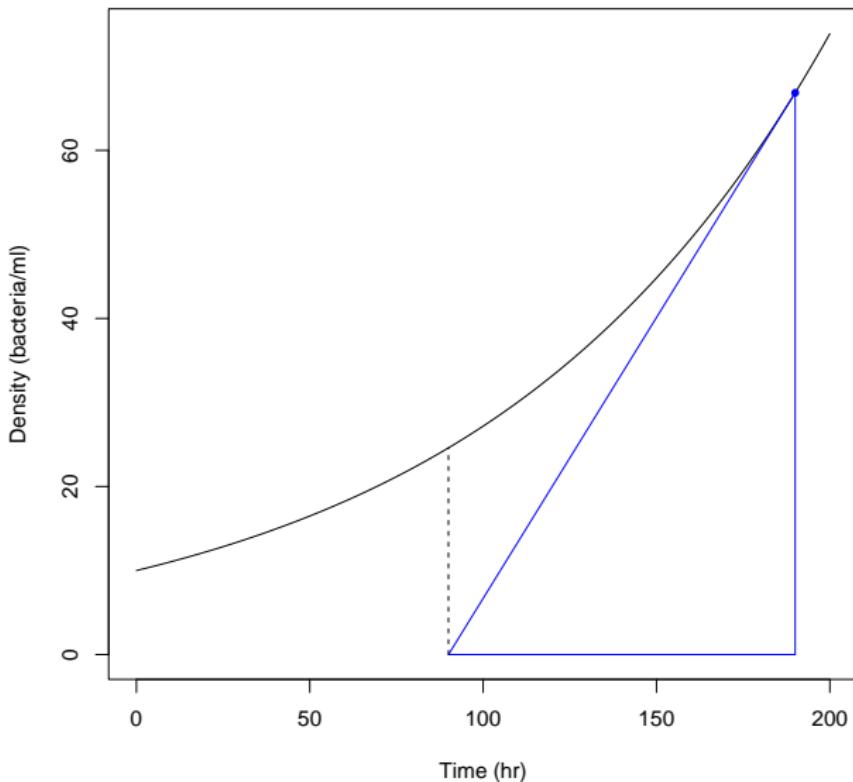
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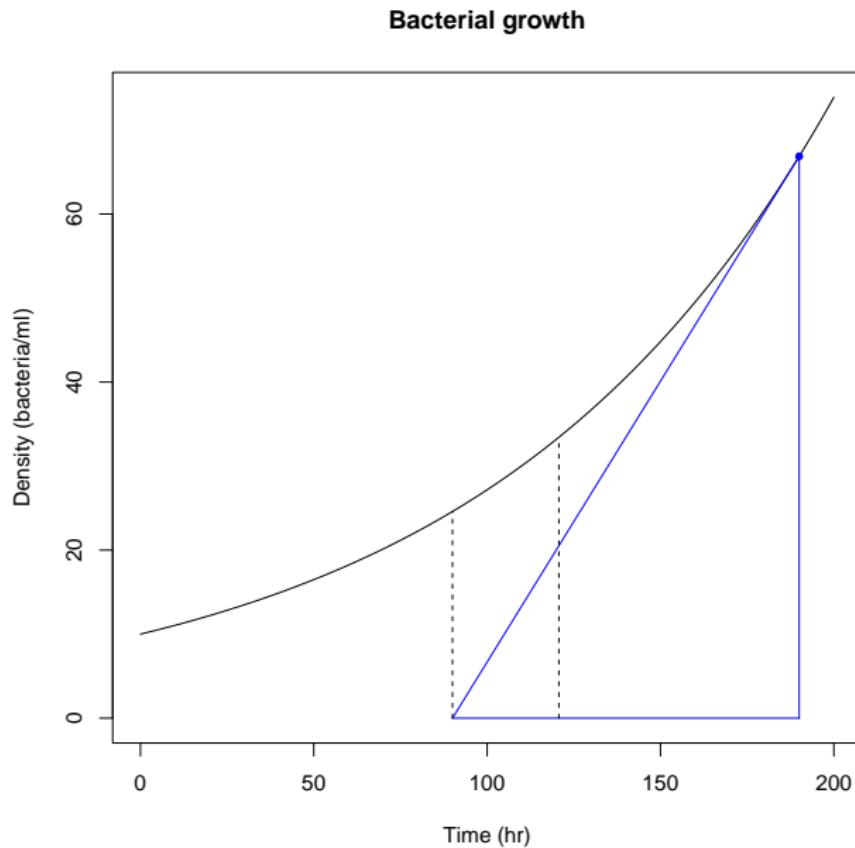
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