#### UNIT 8: Infectious disease

#### Outline

#### Introduction

Rate of spread

Single-epidemic model Epidemic size

Recurrent epidemic models Dynamics

Reproductive numbers and risk

Pathogen aggressiveness

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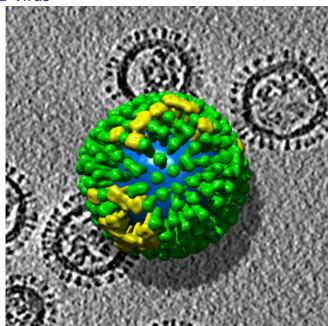
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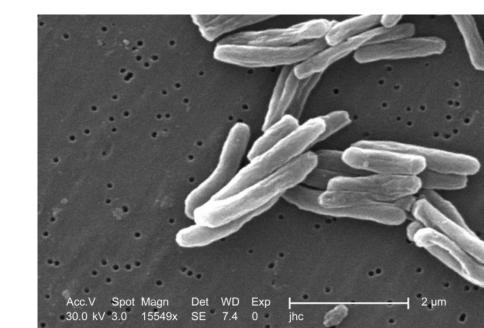
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# Tuberculosis bacilli



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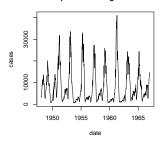
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Measles reports from England and Wales

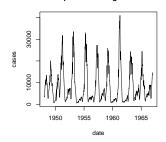


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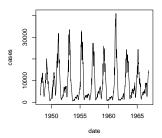


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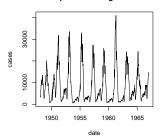


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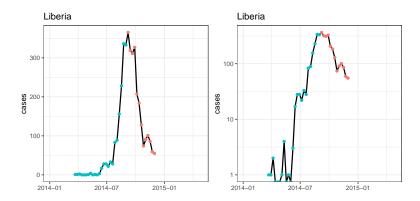
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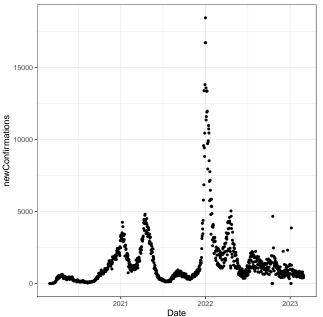
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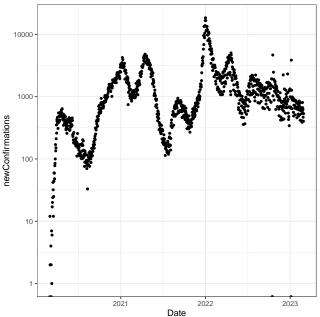
# Example: the West African Ebola epidemic



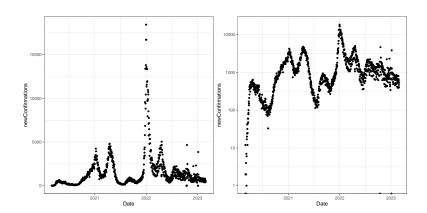
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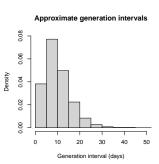
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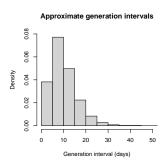
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#### Generation intervals

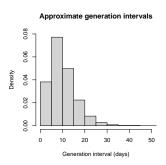
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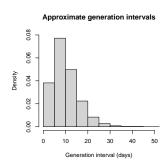
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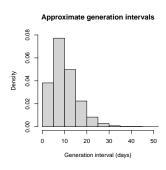
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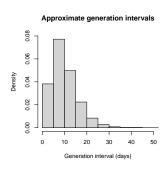
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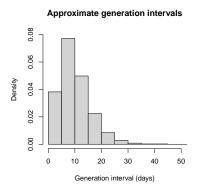


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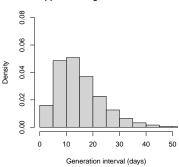


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#### Approximate generation intervals



► Which is more dangerous, a fast disease, or a slow disease?

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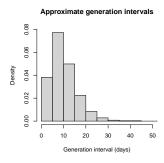
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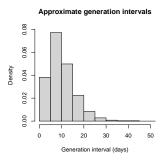
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# Approximate generation intervals 80 0 90 0 10 20 30 40 50 Generation interval (days)

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## Fighting Ebola



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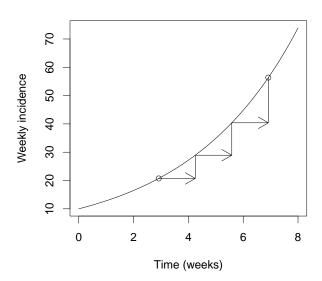
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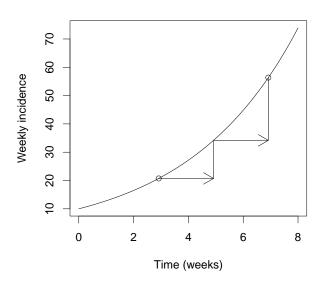
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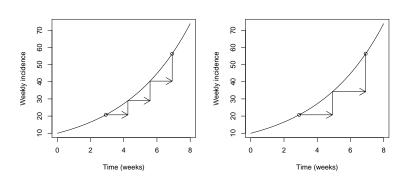
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#### Outline

Introduction

Rate of spread

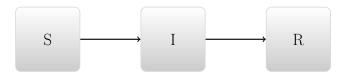
Single-epidemic model Epidemic size

Recurrent epidemic models Dynamics

Reproductive numbers and risk

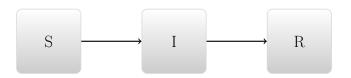
Pathogen aggressiveness

## Single-epidemic model



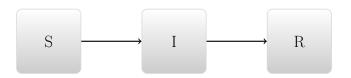
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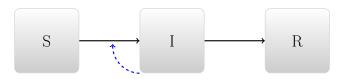


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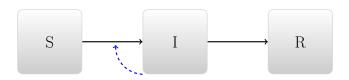
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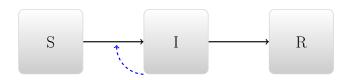
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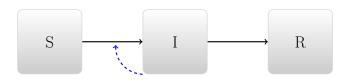
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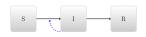


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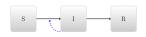
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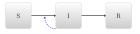
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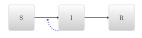


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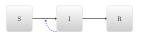
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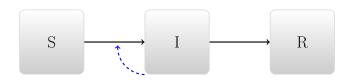
$$\frac{dI}{dt} = \beta \frac{SI}{N} - \gamma I$$

$$rac{dR}{dt} = \gamma I$$

$$N = S + I + R$$

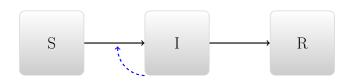


## Quantities



#### State variables

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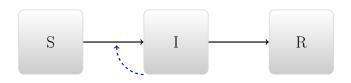
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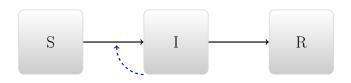
## Quantities (repeat)



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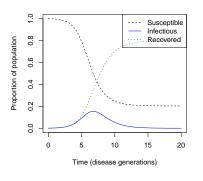
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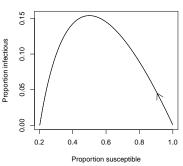


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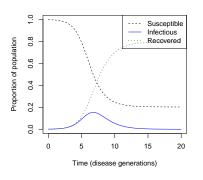
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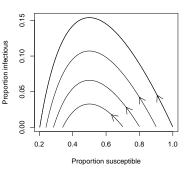
# Simulating the model (repeat)





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The effective reproductive number gives the number of new infections per infectious individual in a partially susceptible population:

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#### Outline

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Rate of spread

Single-epidemic model Epidemic size

Recurrent epidemic models Dynamics

Reproductive numbers and risk

Pathogen aggressiveness

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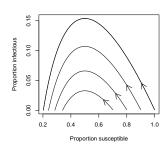
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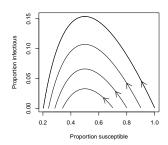
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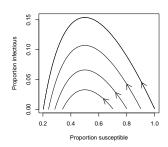


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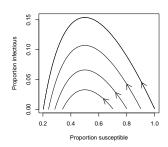
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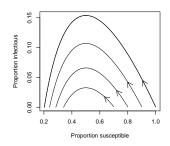
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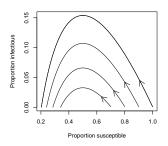
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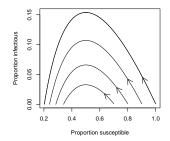
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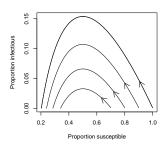
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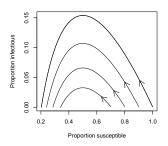
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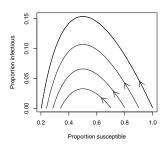
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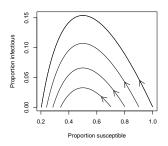
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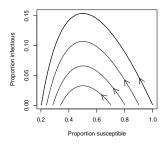
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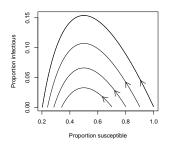


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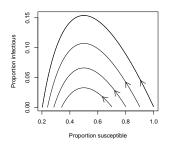
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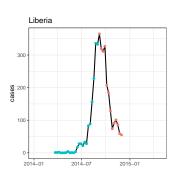


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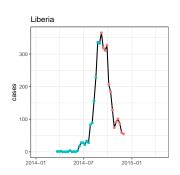
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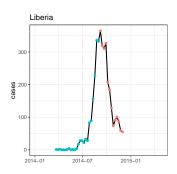
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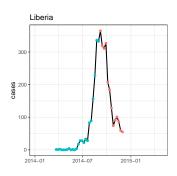
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### Outline

Introduction

Rate of spread

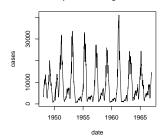
Single-epidemic model Epidemic size

Recurrent epidemic models
Dynamics

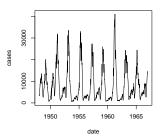
Reproductive numbers and risk

Pathogen aggressiveness

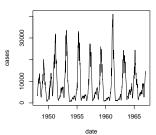
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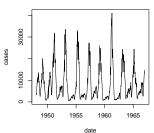
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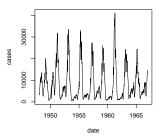
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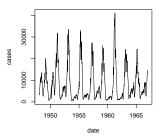
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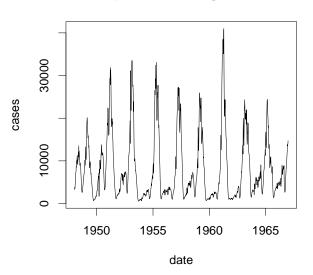
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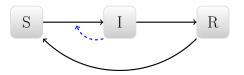


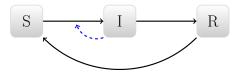
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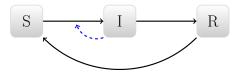
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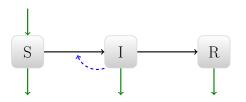




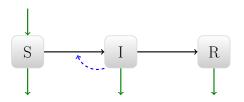
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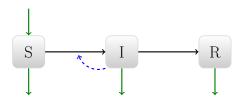
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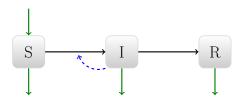


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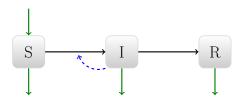


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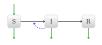
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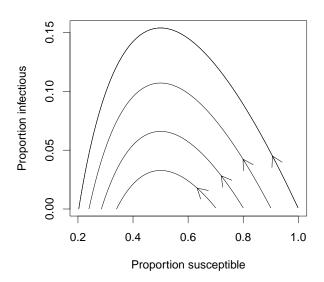
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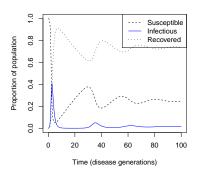
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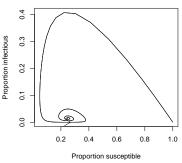
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# Single-epidemic model (repeat)







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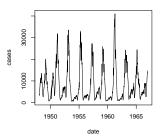
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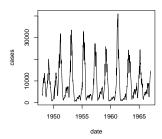
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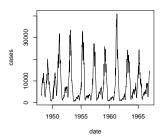


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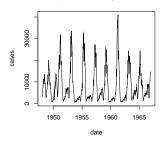


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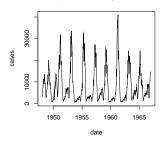


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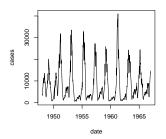
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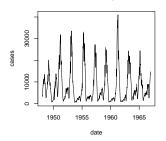
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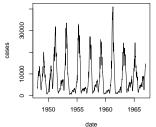
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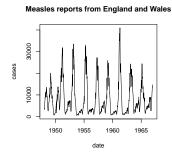
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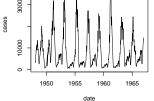


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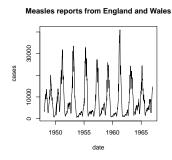
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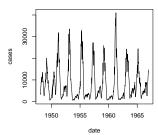
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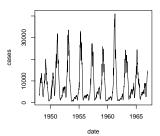


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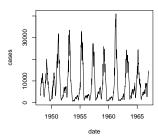
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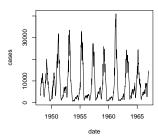
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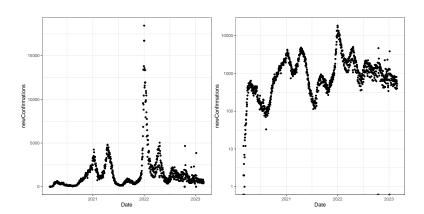
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#### **COVID** in Ontario



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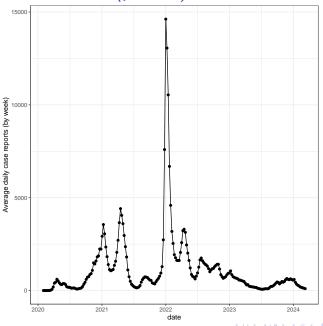
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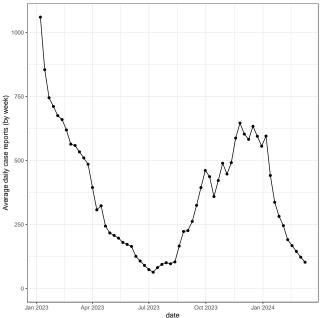
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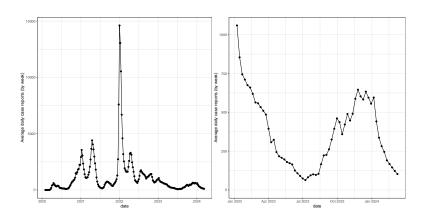
# Post-pandemic COVID (preview)



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# Post-pandemic COVID



#### Outline

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Rate of spread

Single-epidemic model Epidemic size

Recurrent epidemic models

Dynamics

Reproductive numbers and risk

Pathogen aggressiveness

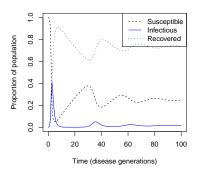
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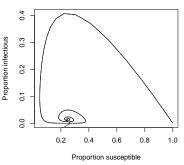
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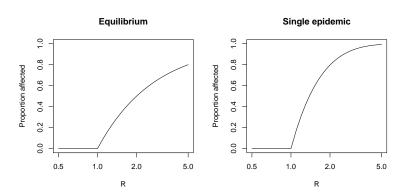
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# Reproductive numbers and risk (repeat)







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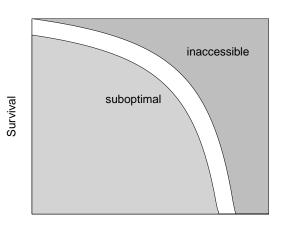
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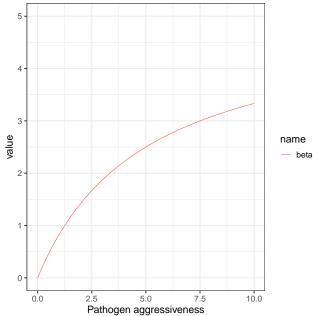
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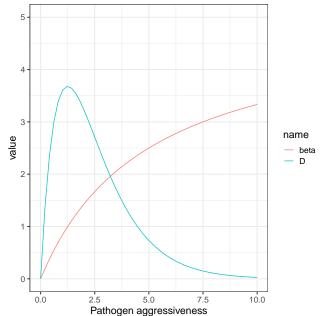
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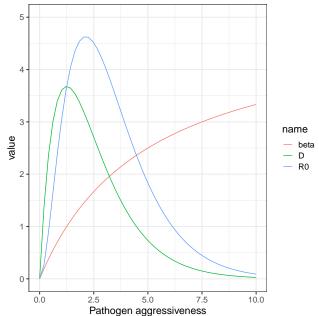
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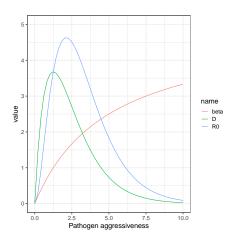


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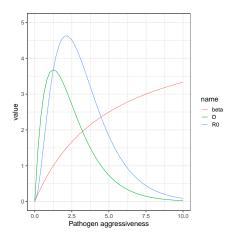


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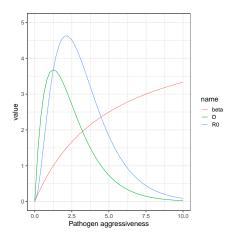




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