

## UNIT 2: Linear population models

# Outline

## Constructing models

- Dynamical models

- Examples

- A simple discrete-time model

- A simple continuous-time model

## Units and scaling

## Key parameters

- Discrete-time model

- Continuous-time model

- Links

## Growth and regulation

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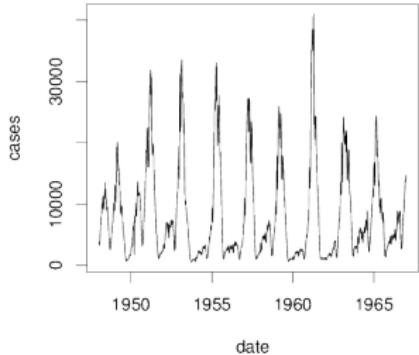
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- Models are what we use to link:



Measles reports from England and Wales



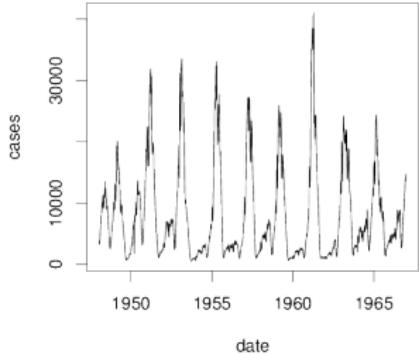
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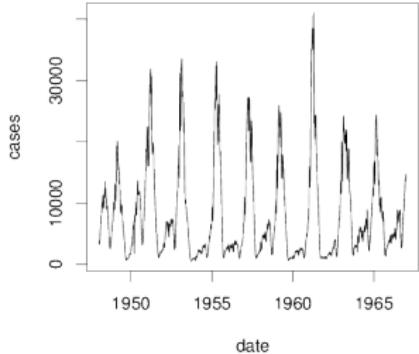
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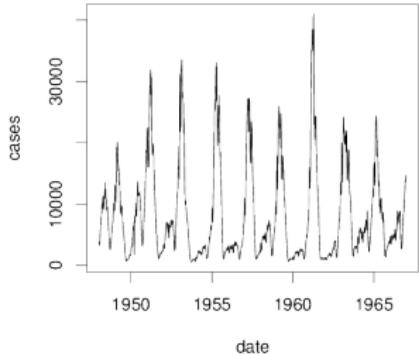
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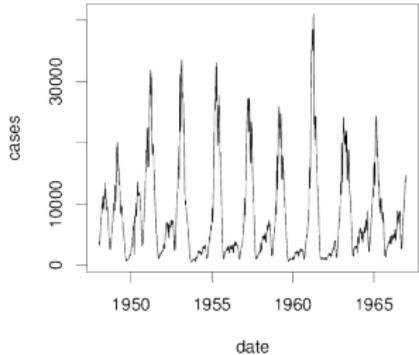
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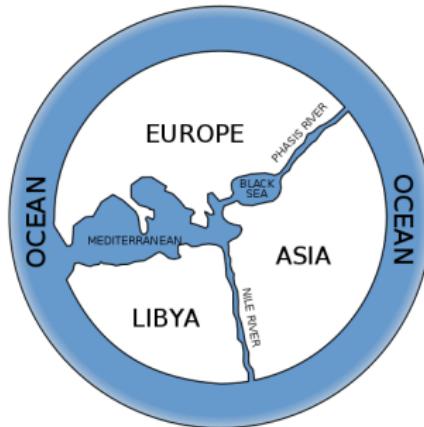


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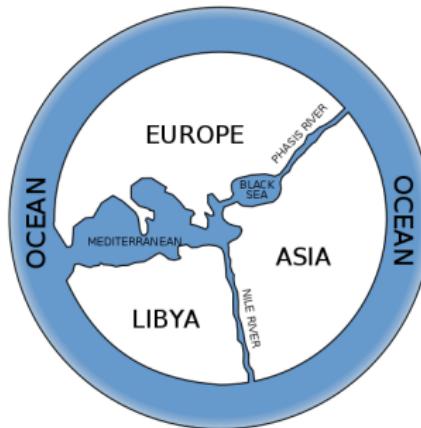
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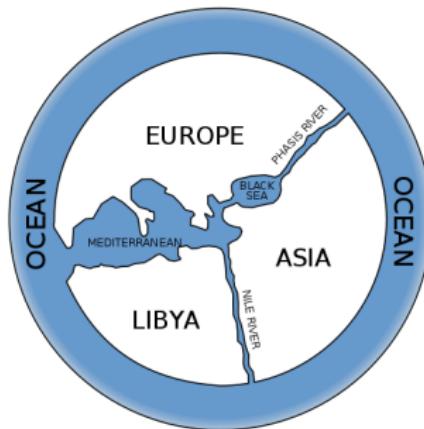
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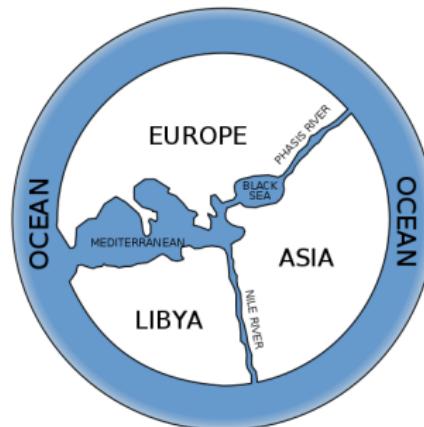
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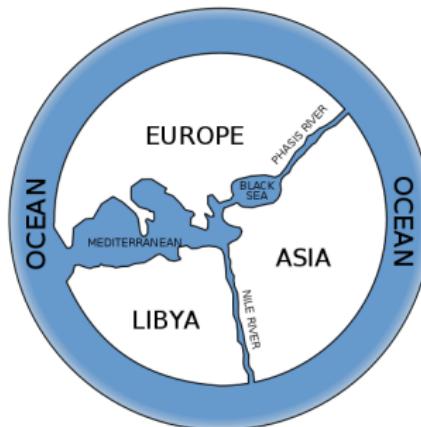
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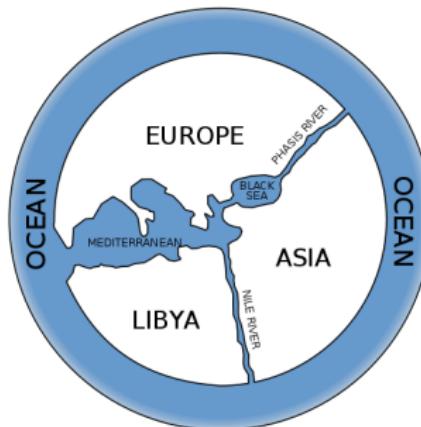
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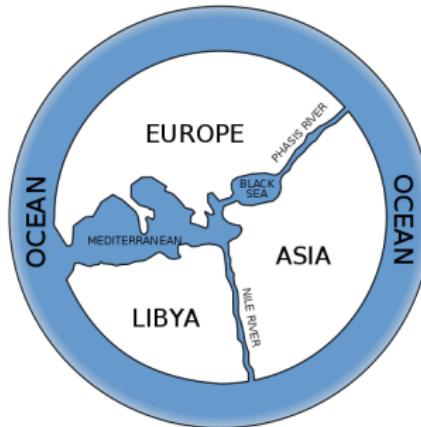
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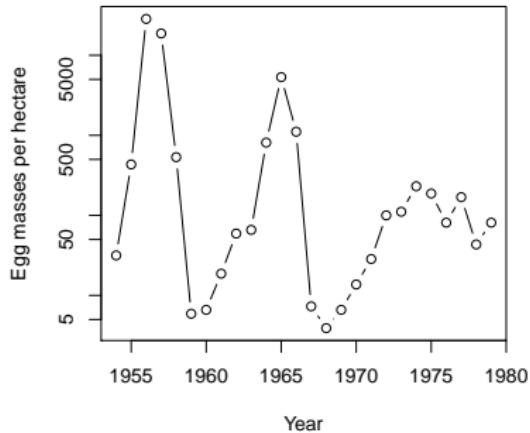
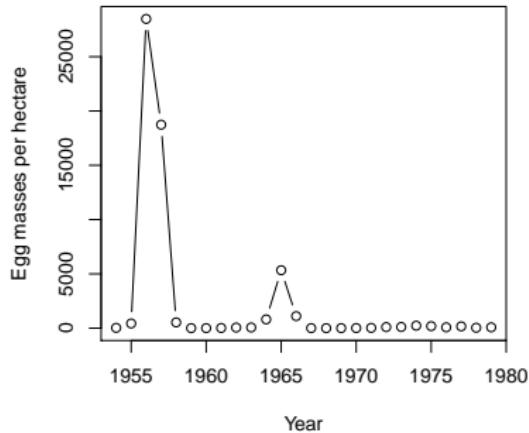


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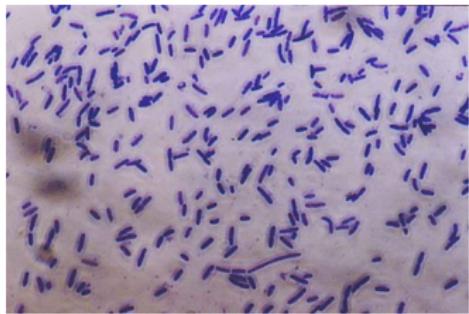
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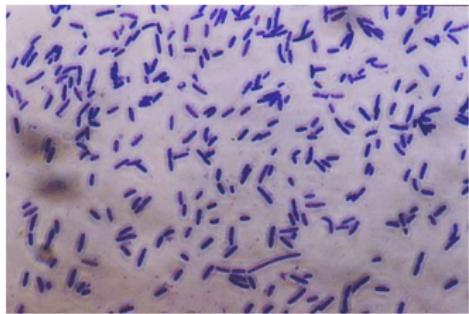
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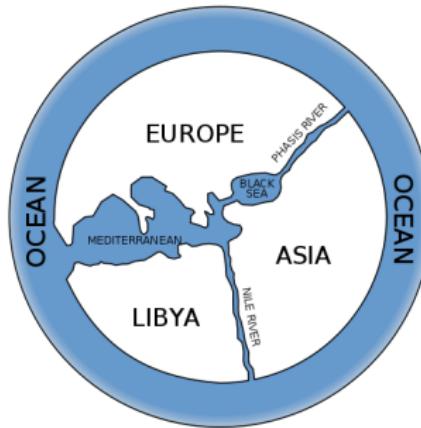
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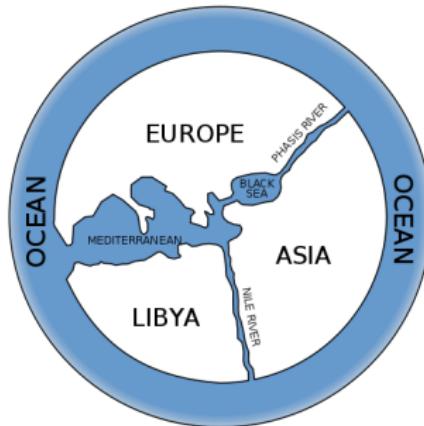
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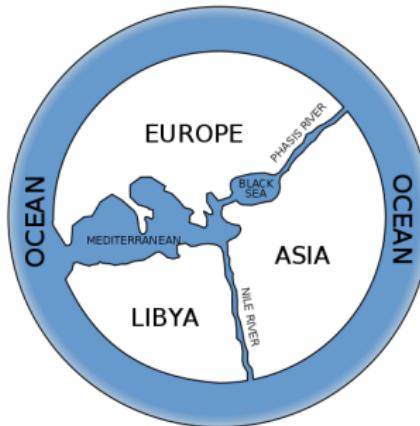
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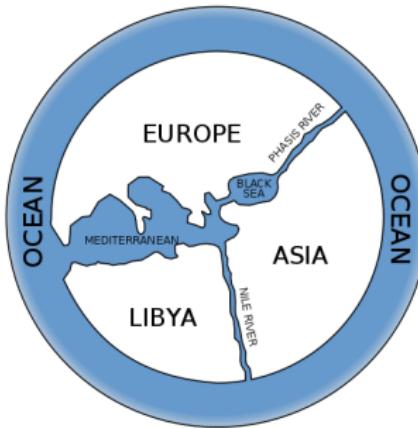
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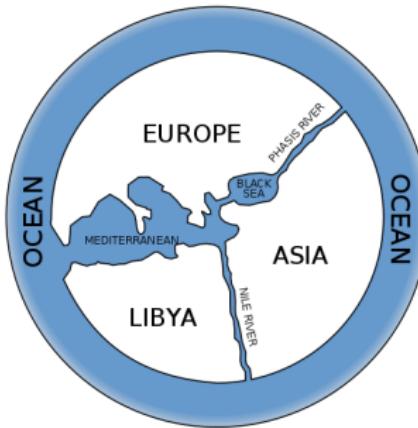
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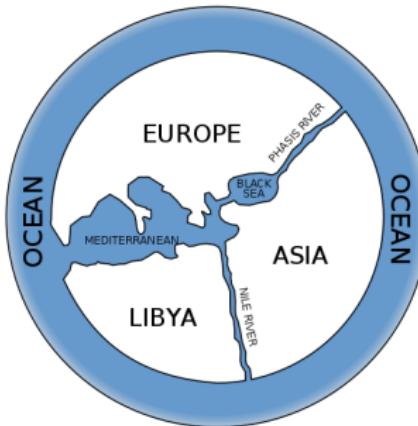
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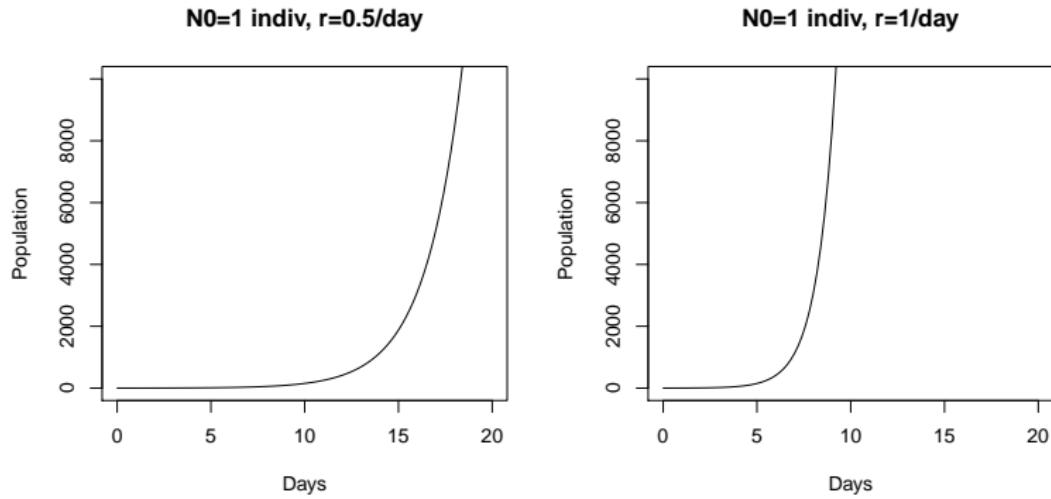
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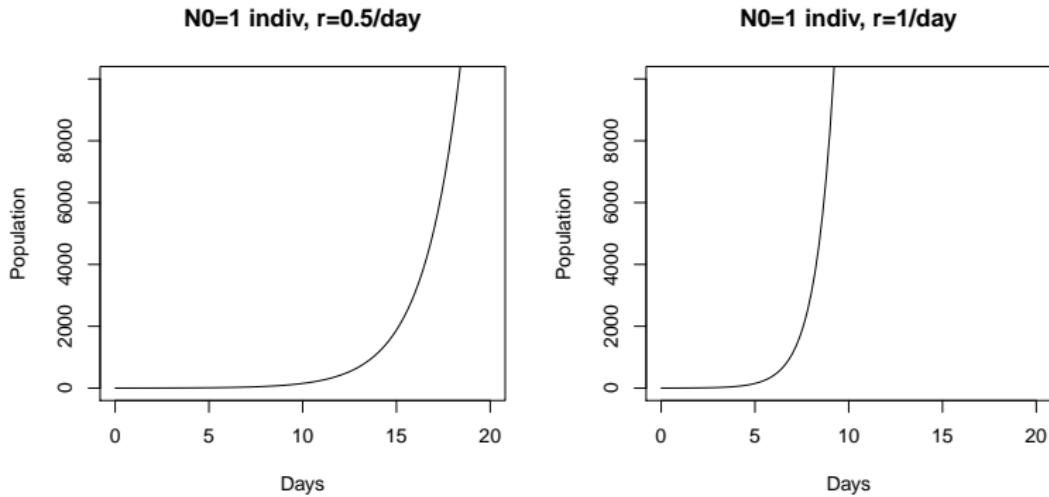
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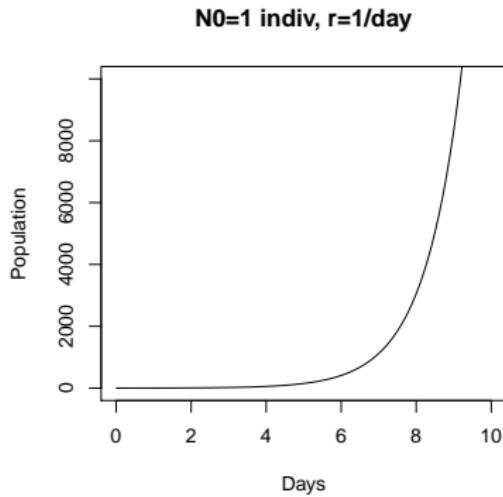
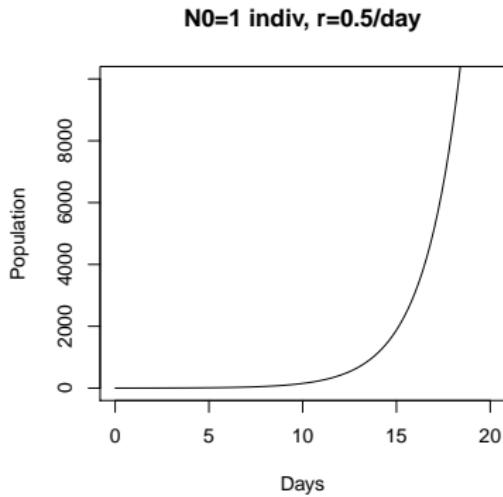
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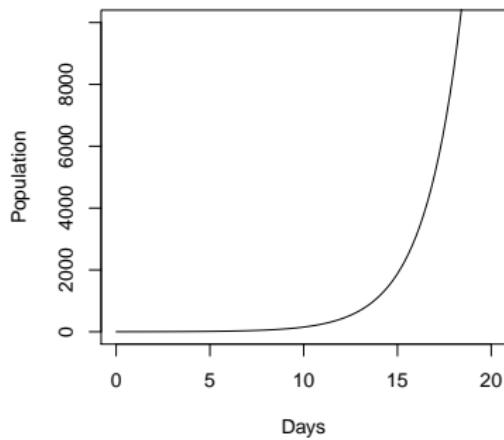


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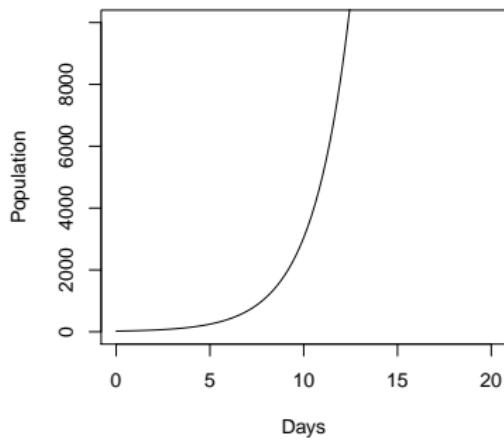


## Scaling population

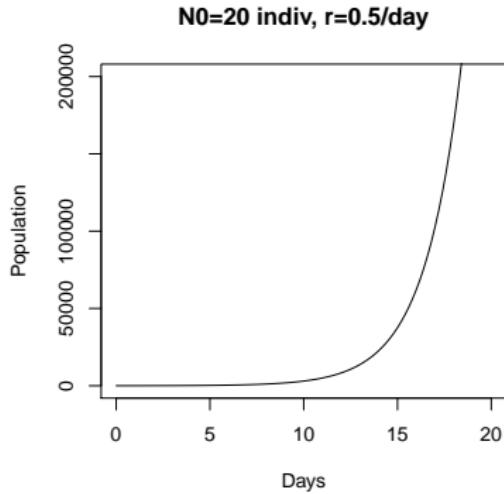
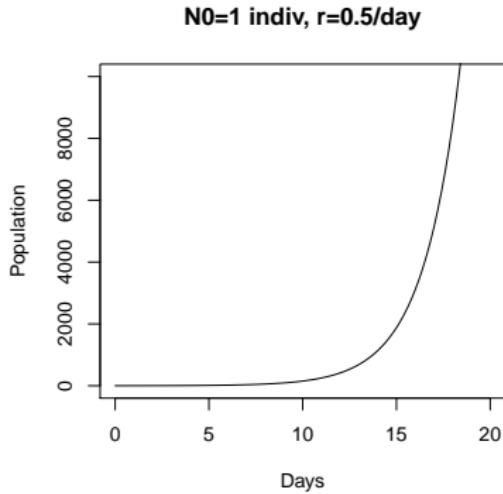
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Constructing models

Dynamical models

Examples

A simple discrete-time model

A simple continuous-time model

Units and scaling

**Key parameters**

Discrete-time model

**Continuous-time model**

Links

Growth and regulation

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