

UNIT 1: Introduction

Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

Example populations

Dandelions

Gypsy moths

Bacteria

Exponential growth

Log and linear scales

Time scales

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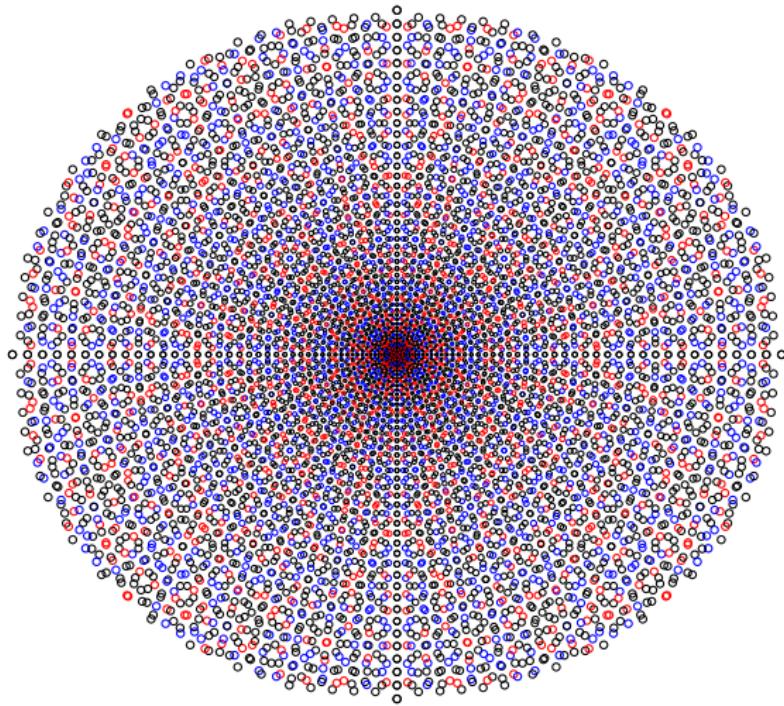
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Pythagorean triples



Which country?



Which country?



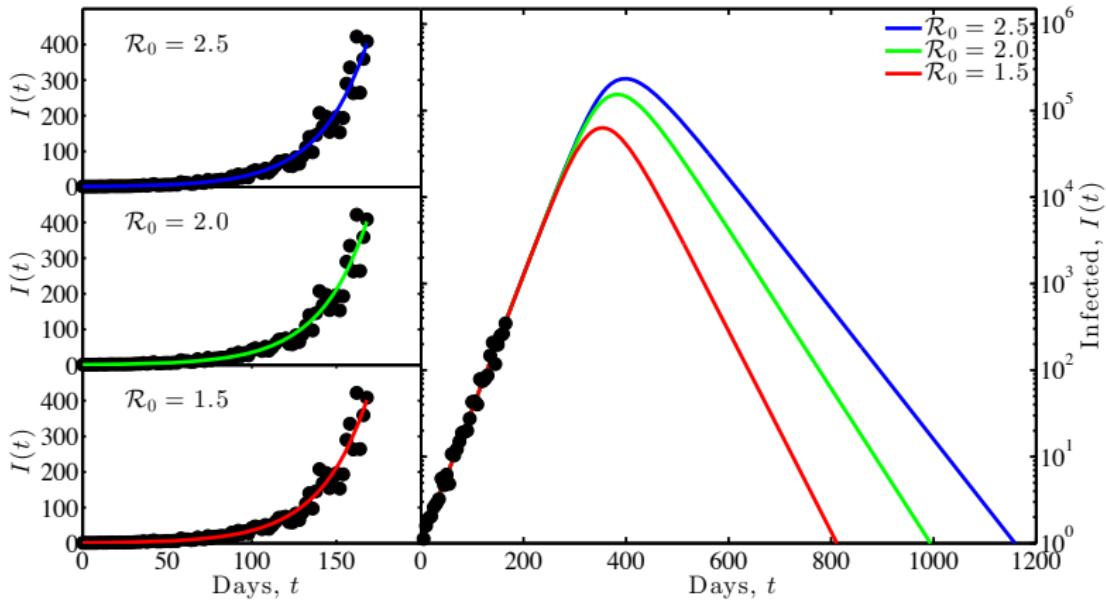
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Ebola research



TAs

- Morgan Kain



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Students

- ▶ Poll: What year are you in?

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- ▶ Poll: What is ecology?

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- ▶ My answer

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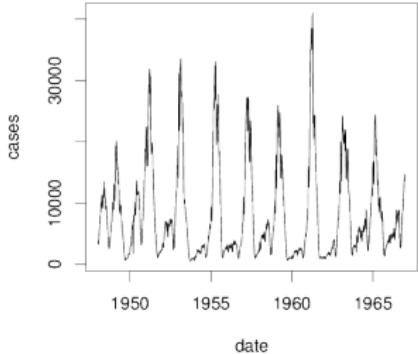
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Dynamical modeling

- ▶ Investigates the links between local, short-term processes, and large-scale, long-term outcomes



Measles reports from England and Wales

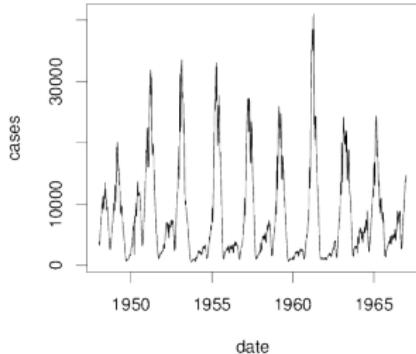


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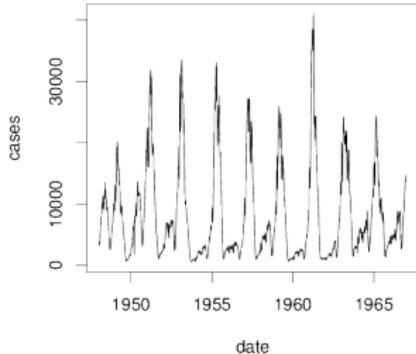


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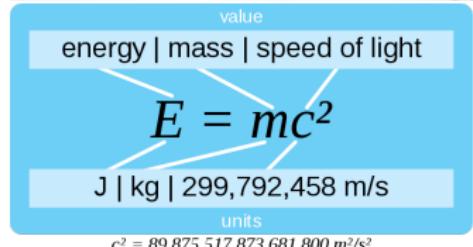
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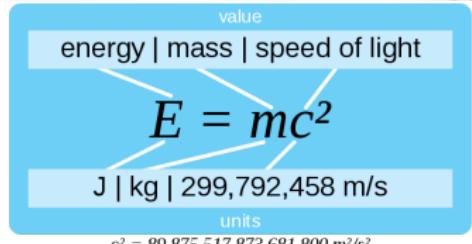
Humans and abstract thought

- ▶ People are evolved to be concrete thinkers, not conceptual thinkers



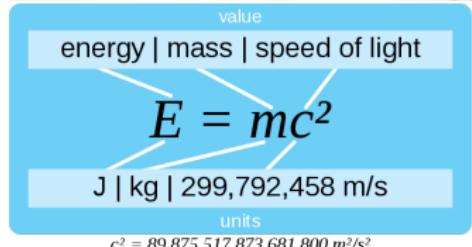
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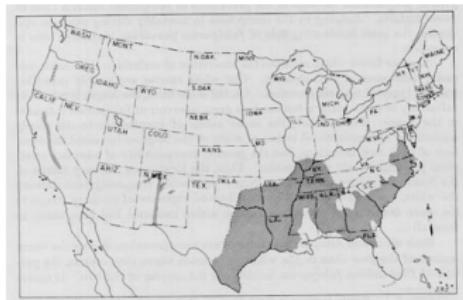
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Malaria

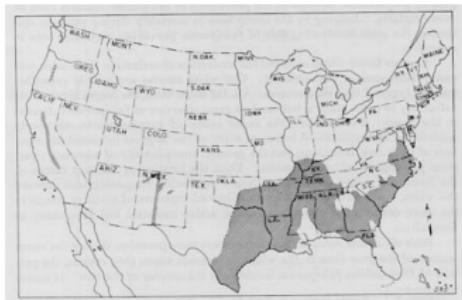
- A nasty, mosquito-borne disease



Map 4.—Areas of the continental United States believed to be malarious in 1931-33.

Malaria

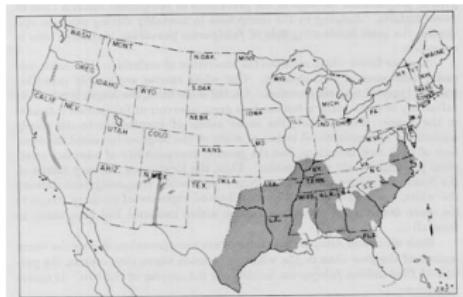
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MAP 4.—Areas of the continental United States believed to be malarious in 1951-55.

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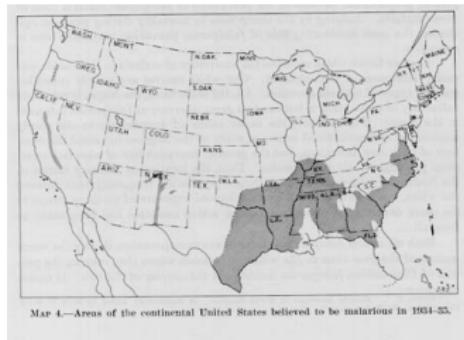
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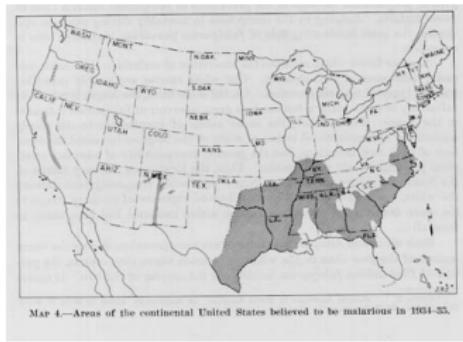
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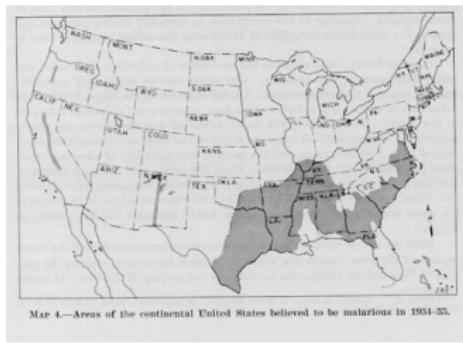
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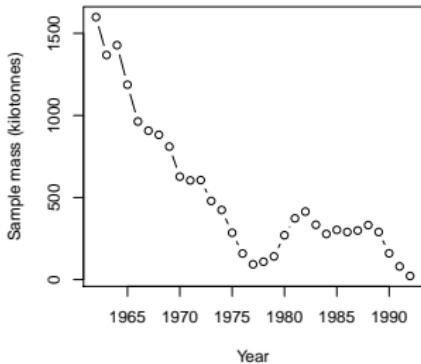
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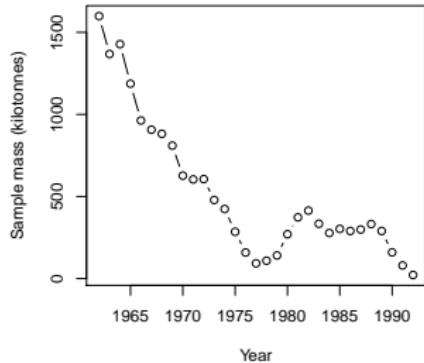
Cod fisheries

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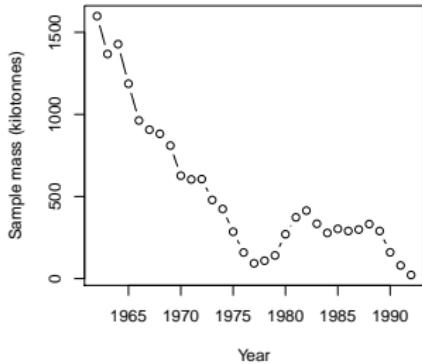
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- ▶ Is the ocean too big for people to affect?
- ▶ What happened to the cod?



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Populations

- ▶ **Poll:** What population of organisms interests you?

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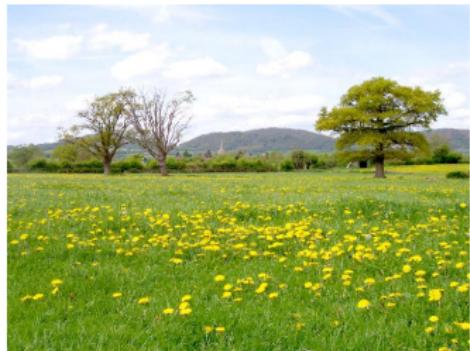
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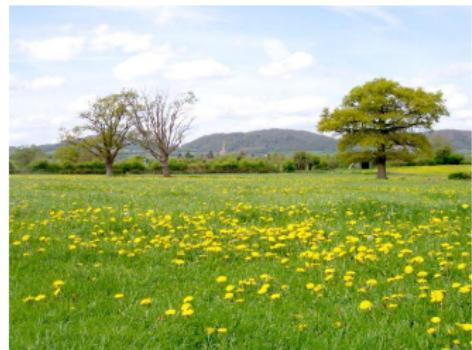
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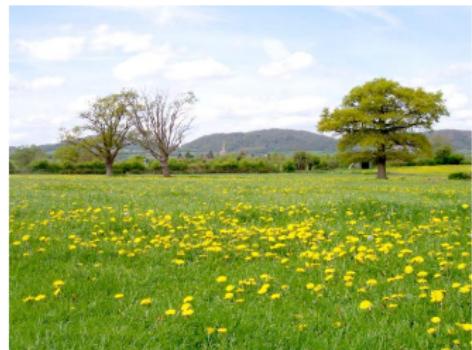


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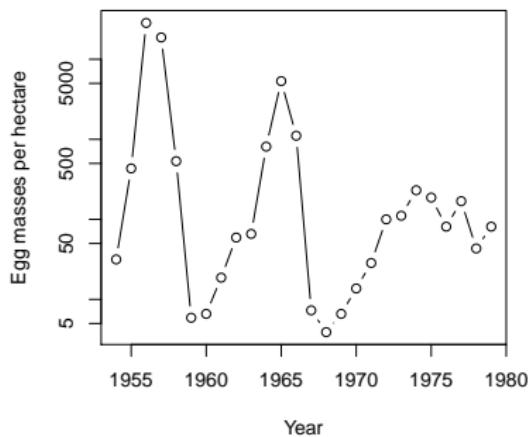
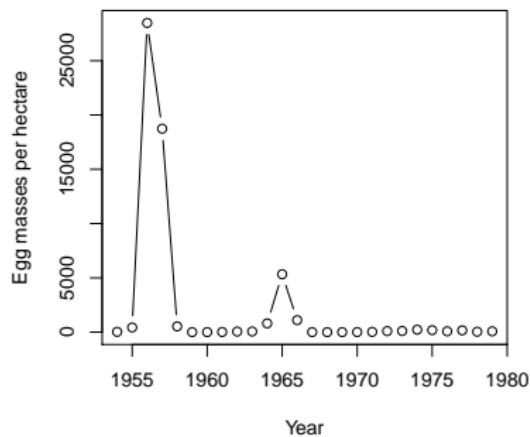


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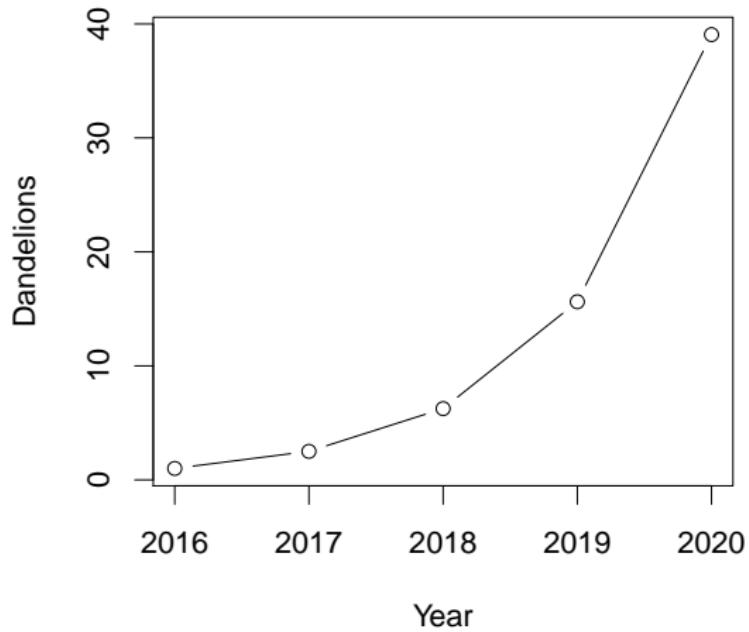
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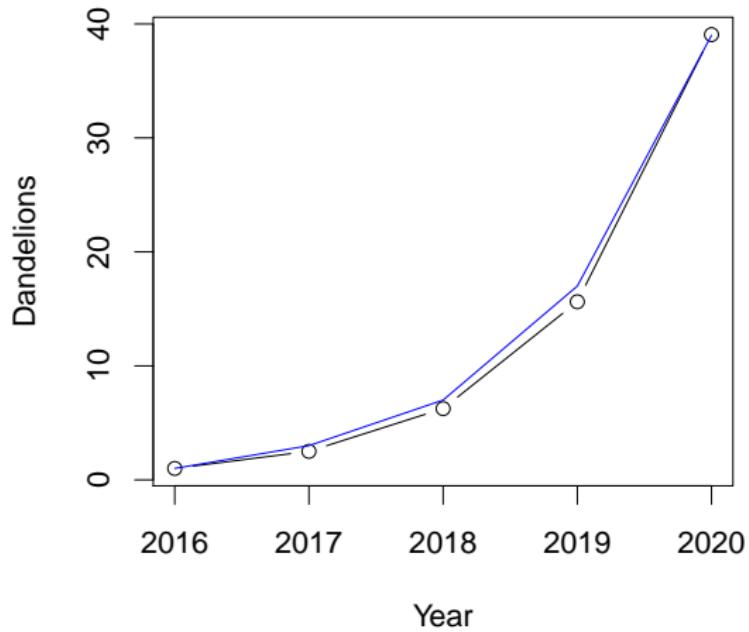
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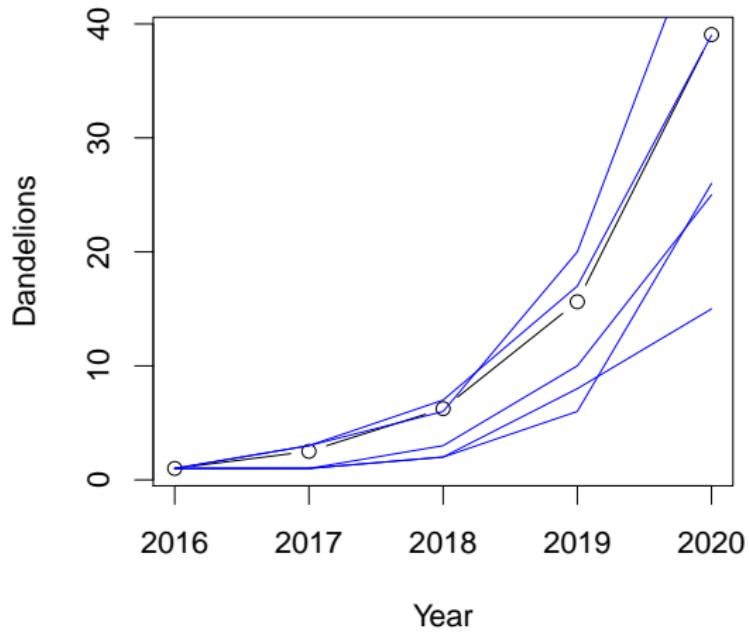
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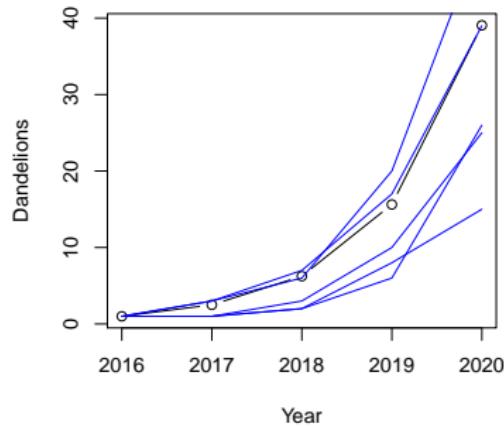
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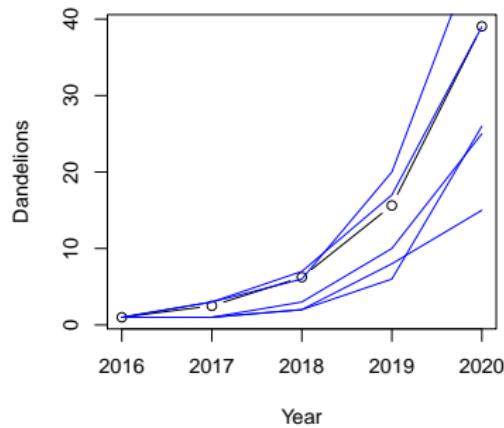


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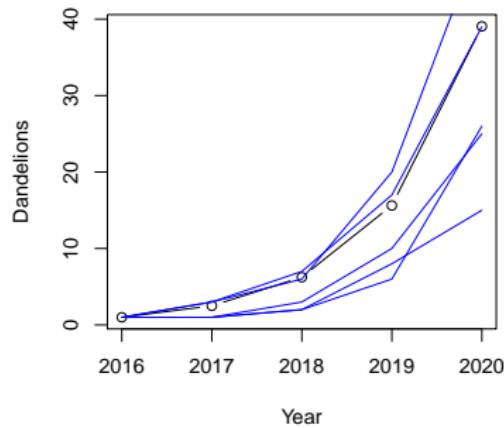
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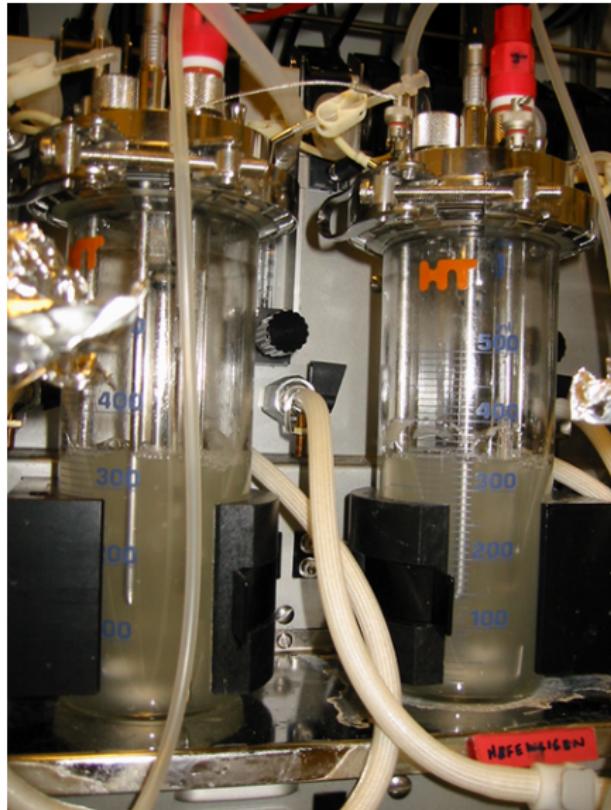
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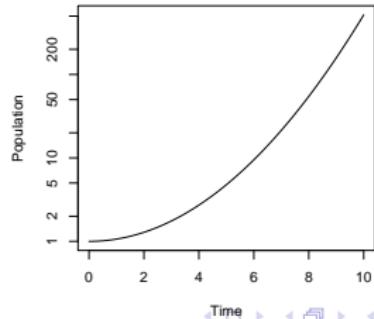
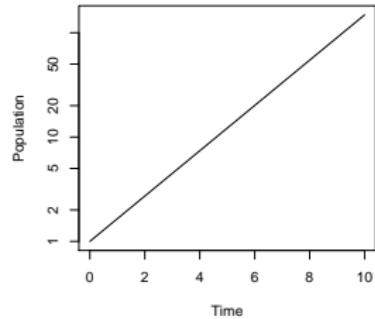
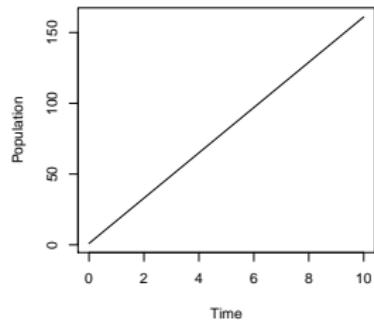
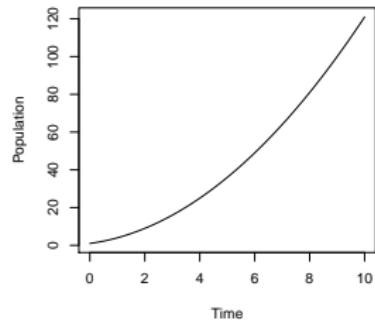
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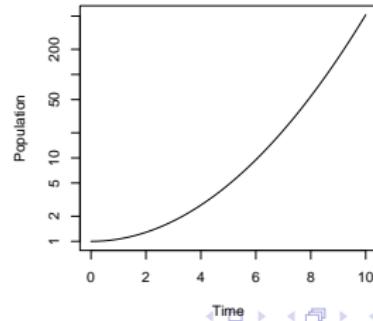
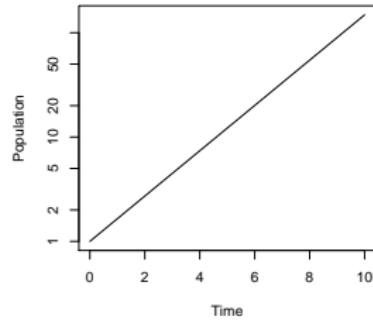
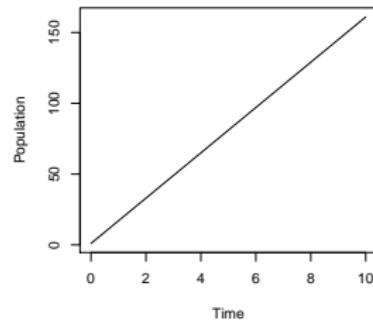
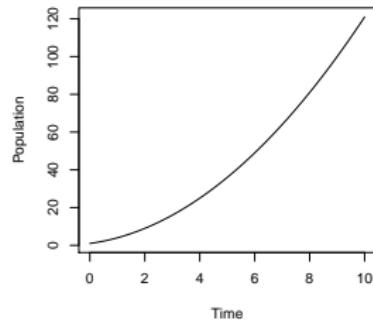
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- What is exponential growth?



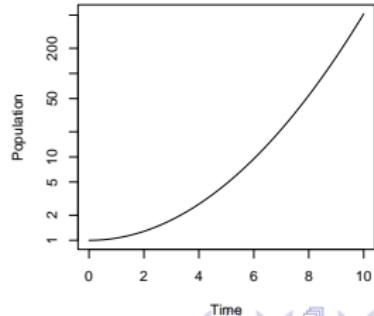
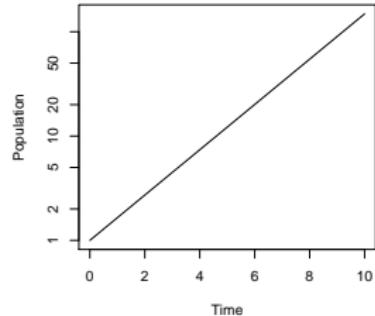
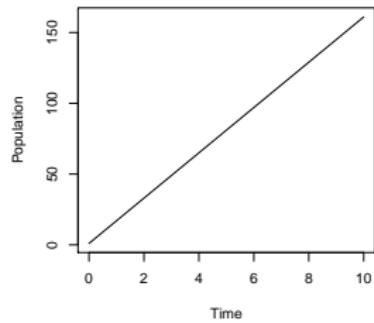
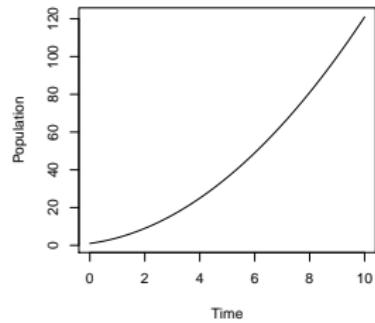
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- ▶ What is exponential growth?
- ▶ Which of these is an example?

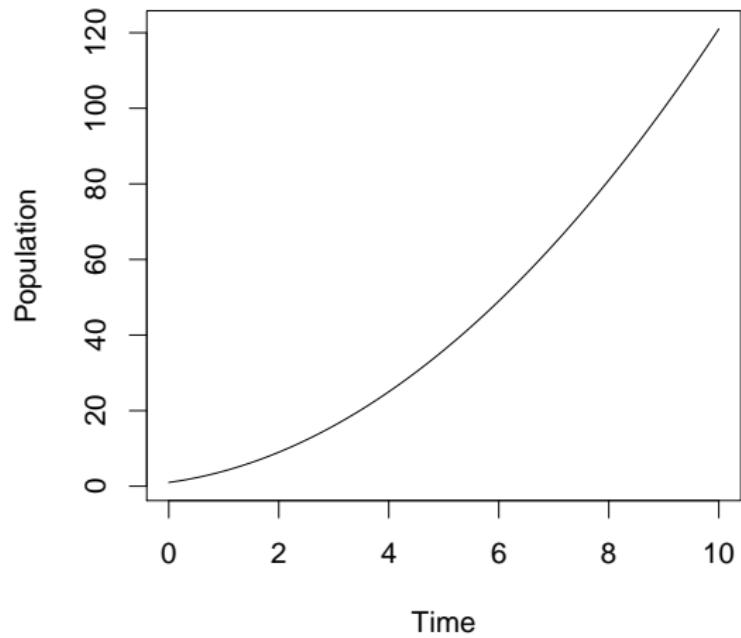


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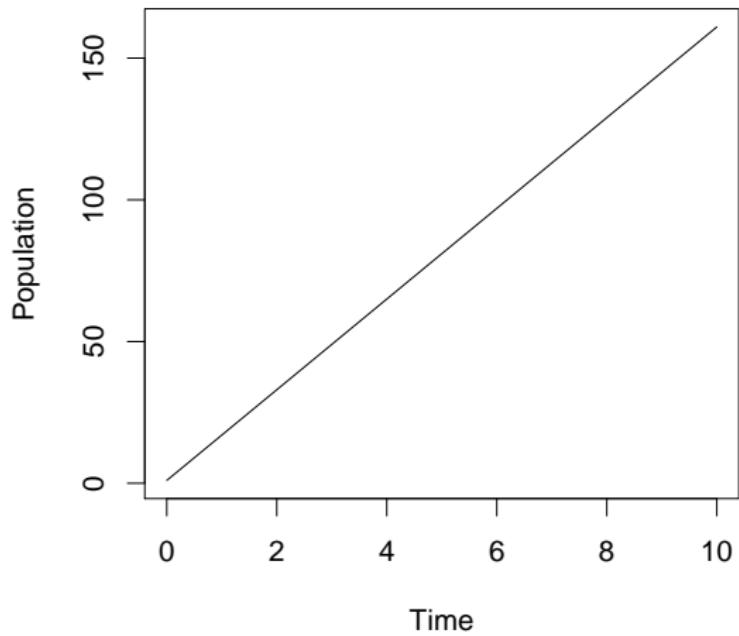
- ▶ What is exponential growth?
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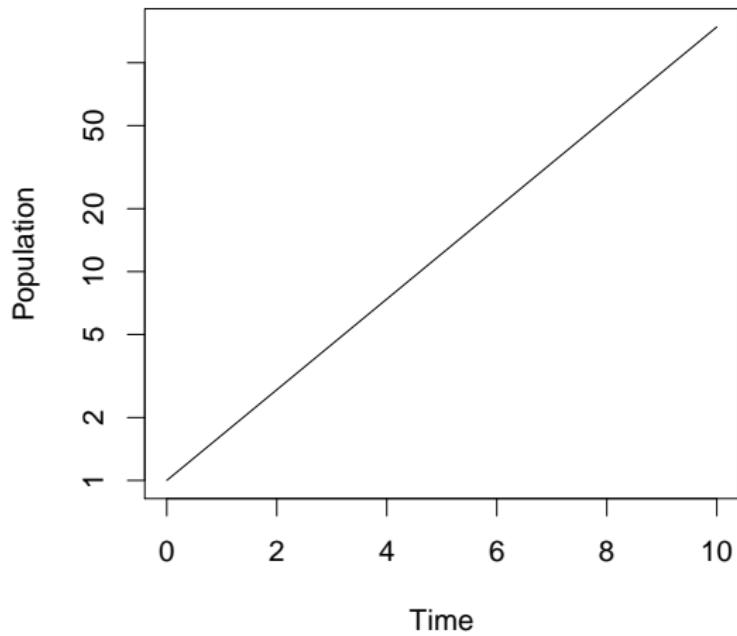
A



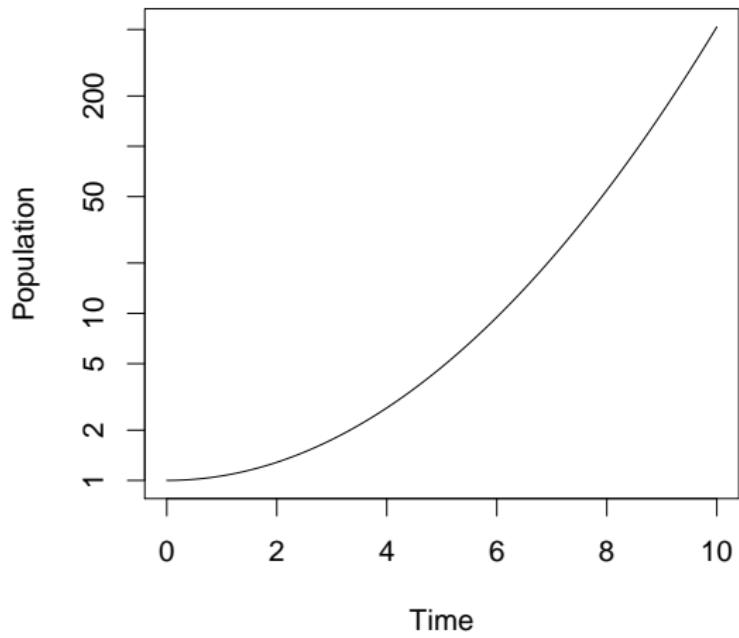
B



C

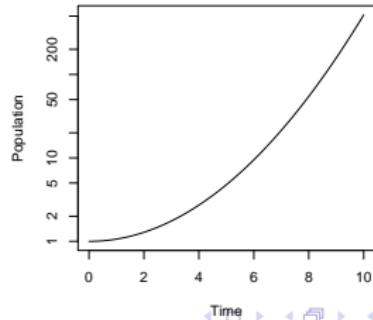
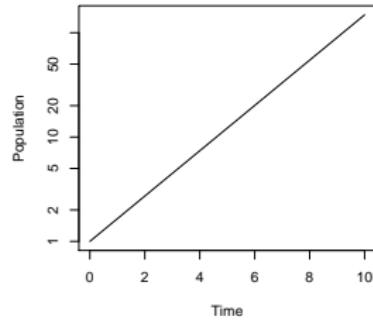
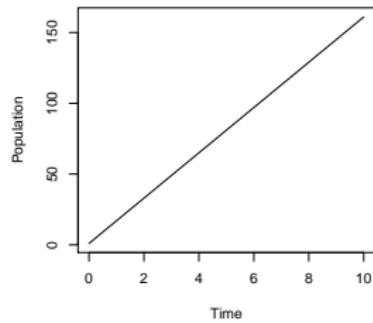
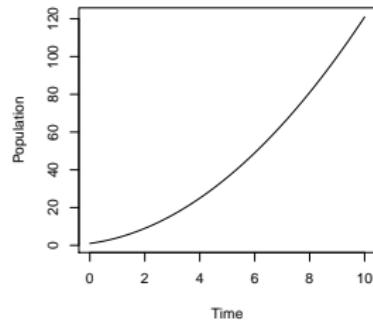


D



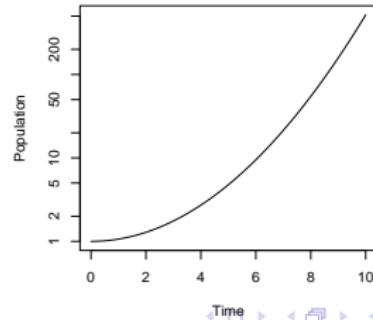
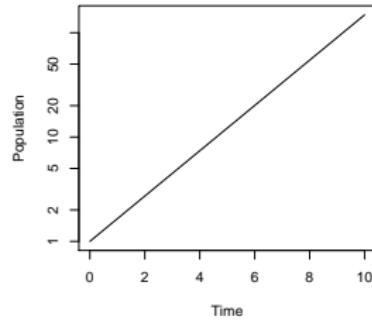
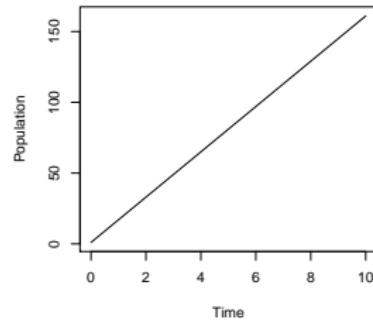
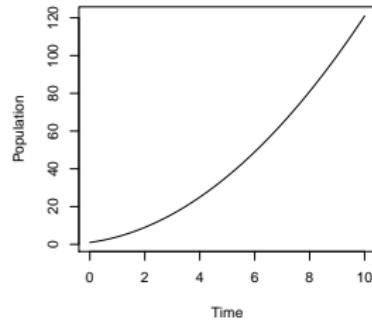
Exponential growth

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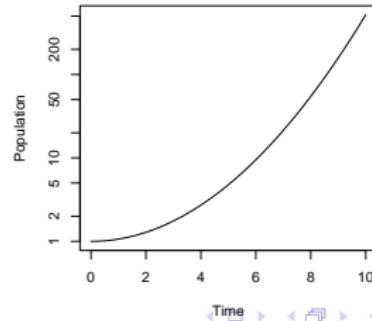
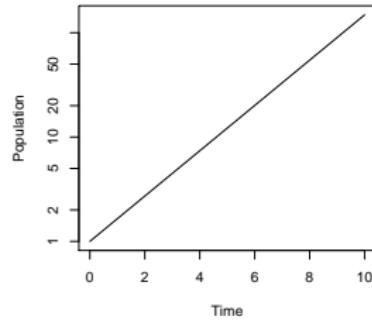
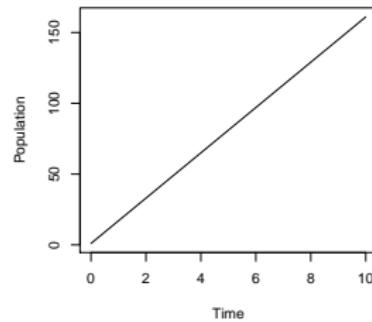
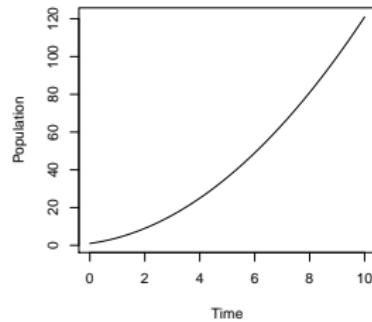
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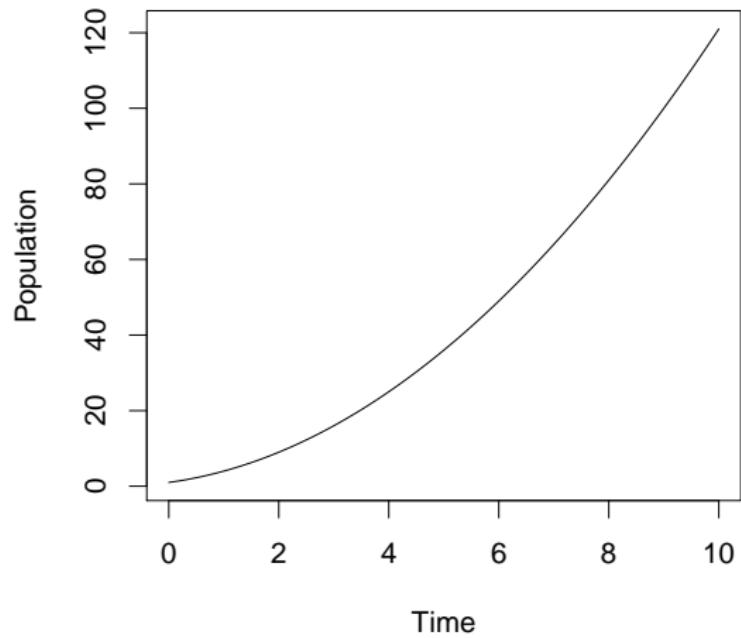


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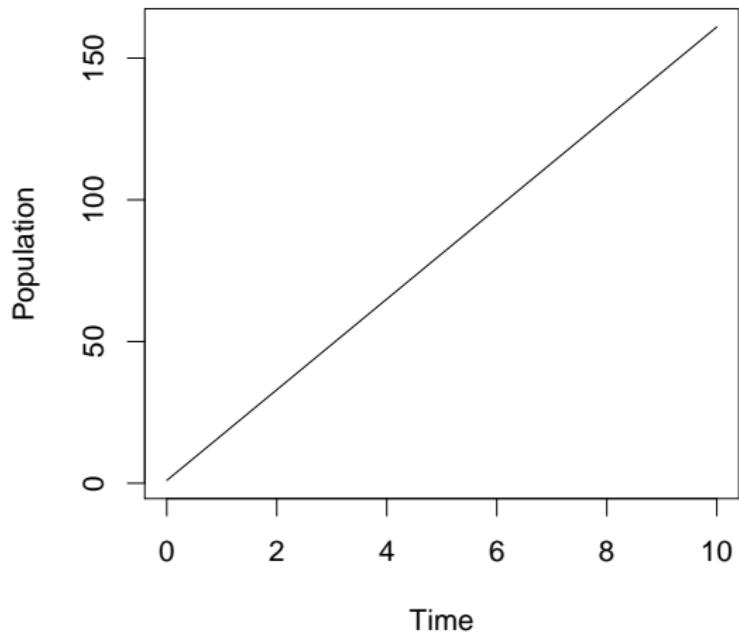
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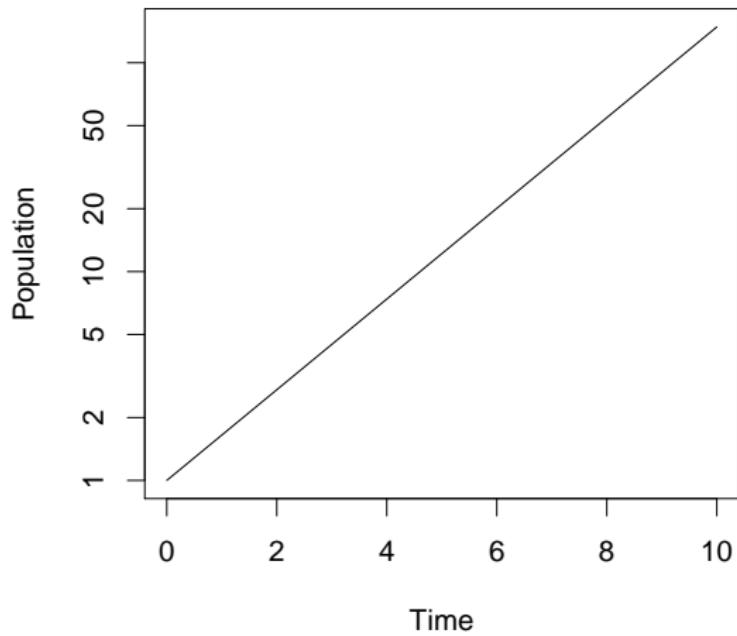
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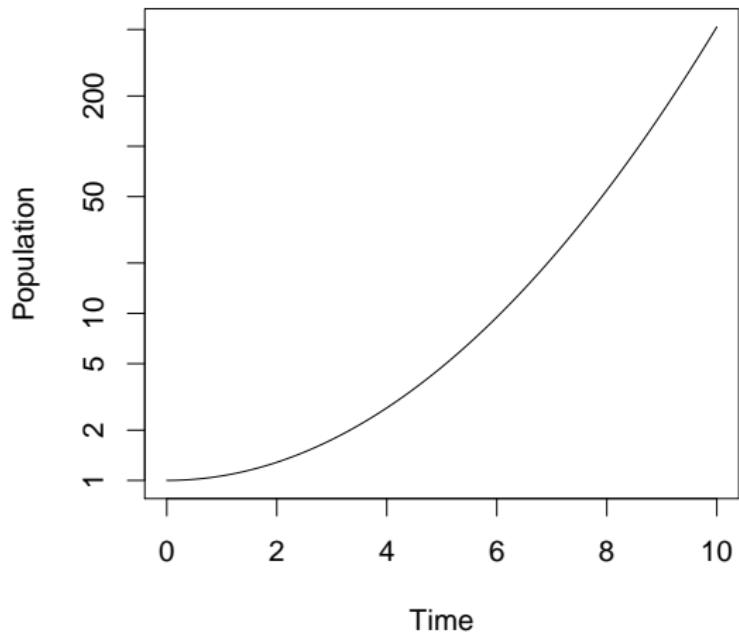
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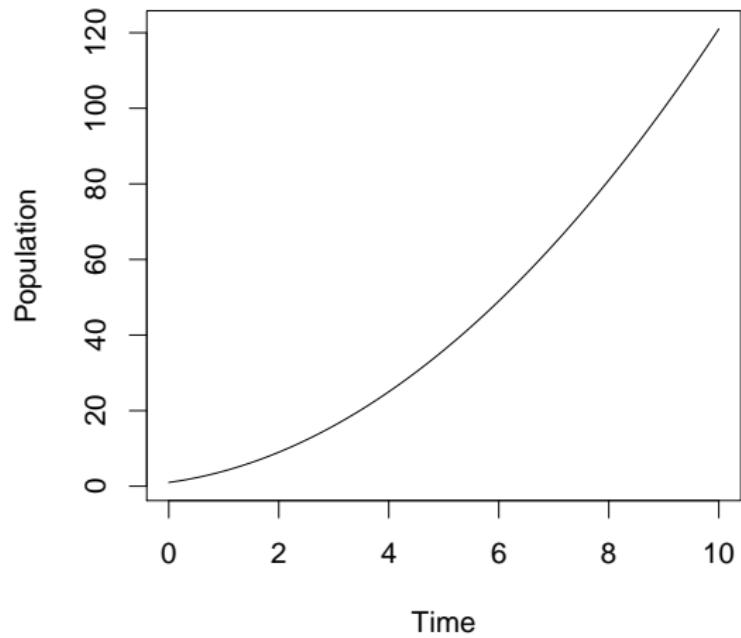
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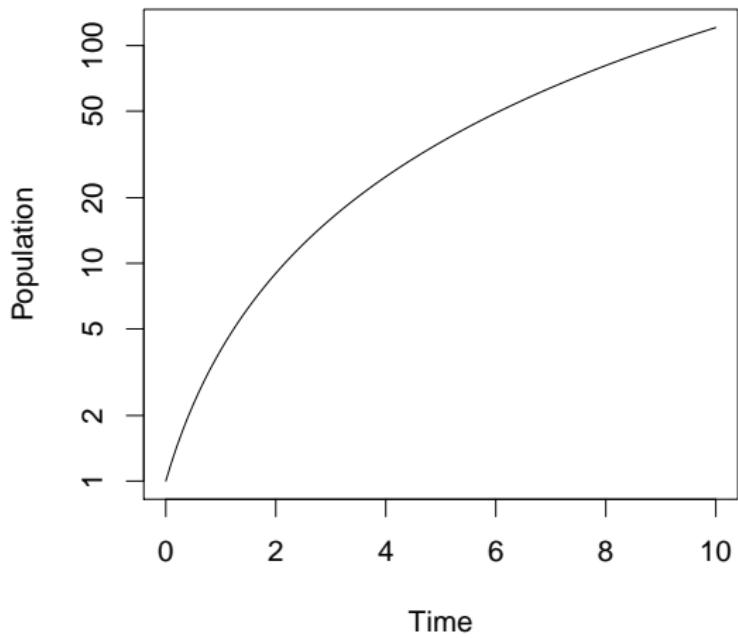
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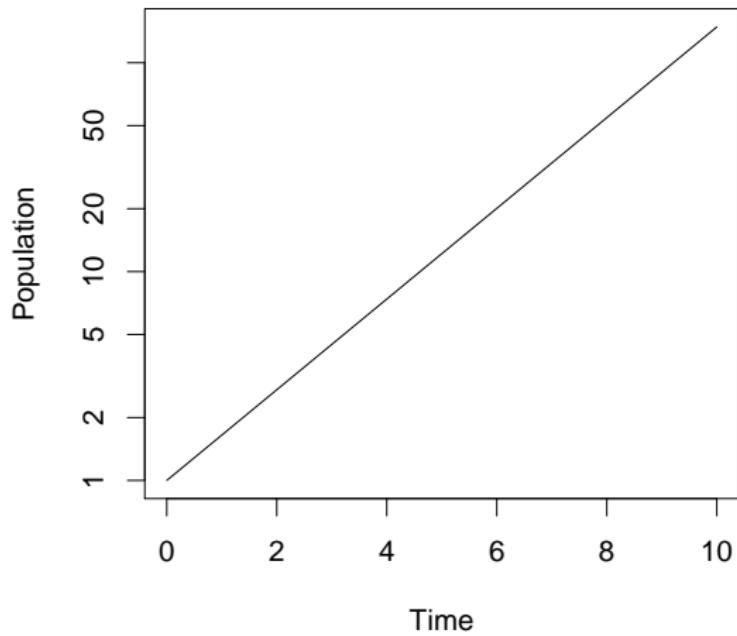
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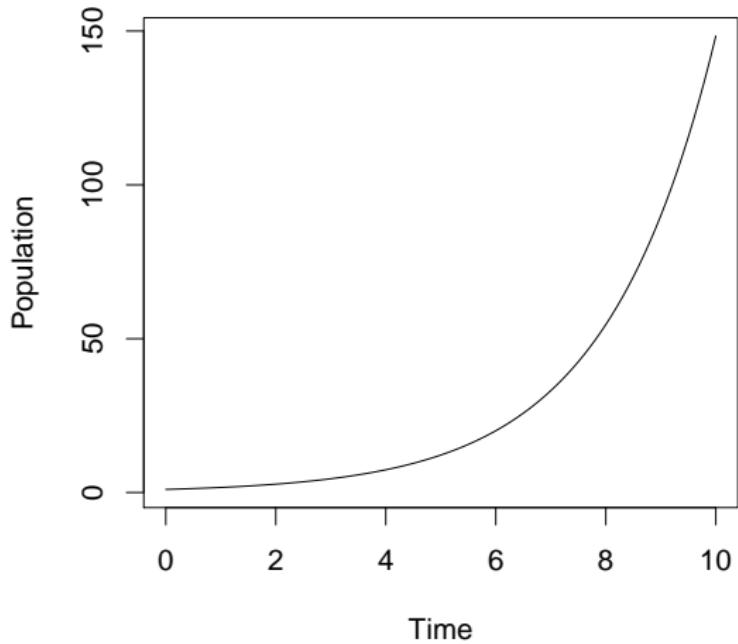
A on the log scale



C



C on the linear scale



Types of growth

- arithmetic/linear:

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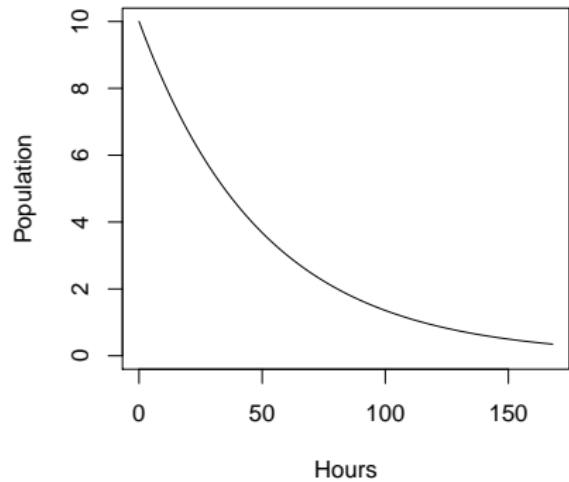
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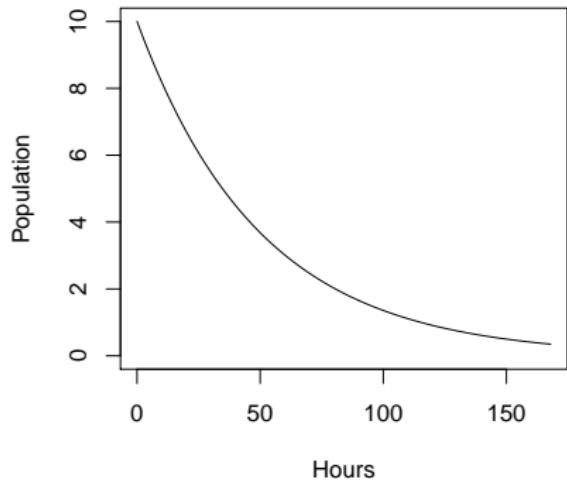
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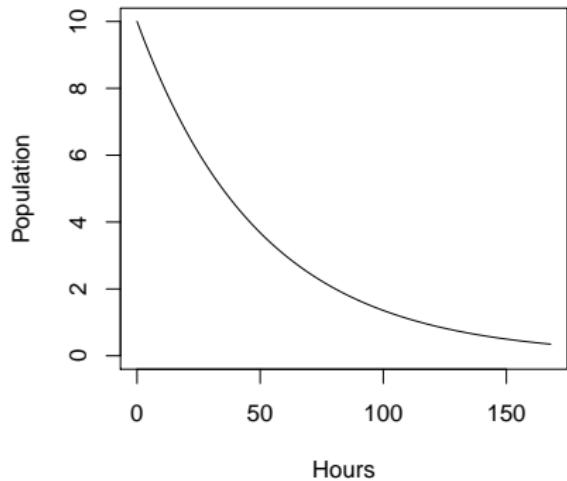
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Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

Example populations

Dandelions

Gypsy moths

Bacteria

Exponential growth

Log and linear scales

Time scales

Scales of comparison

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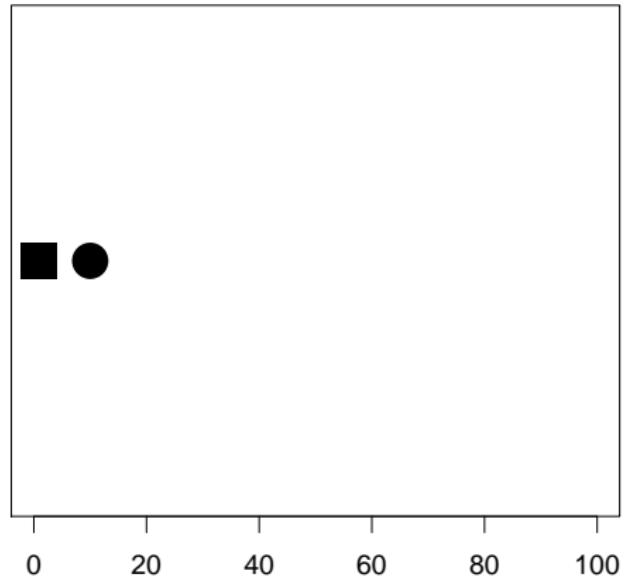
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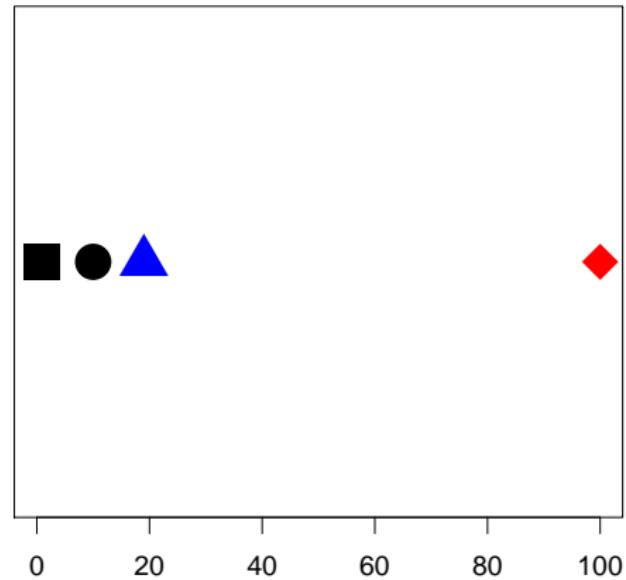
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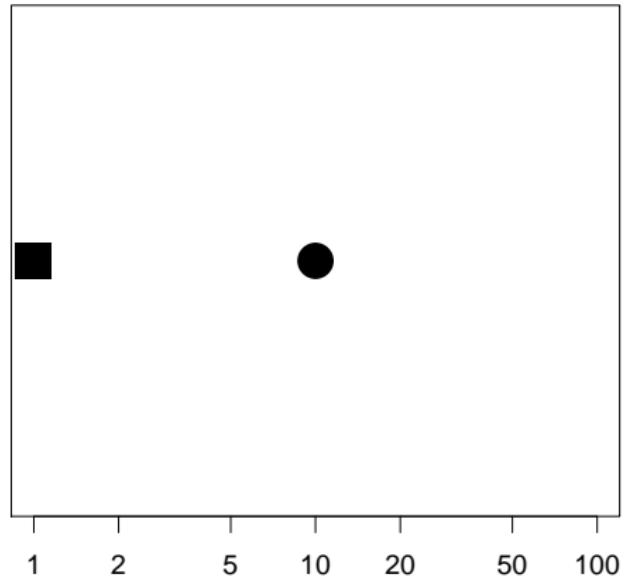
Scales of display



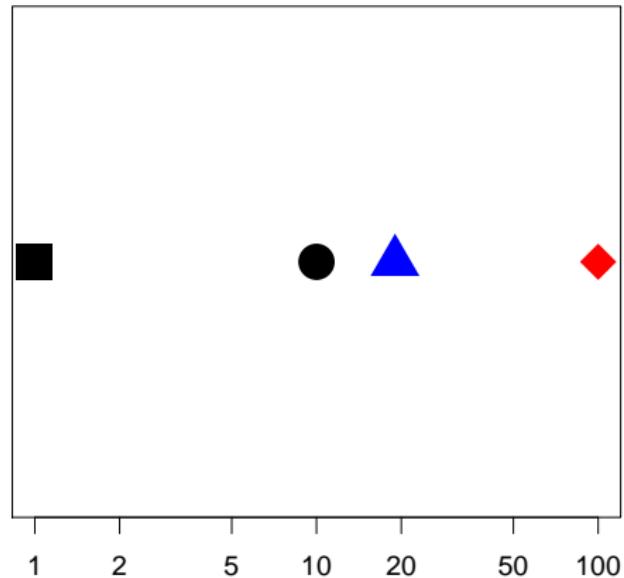
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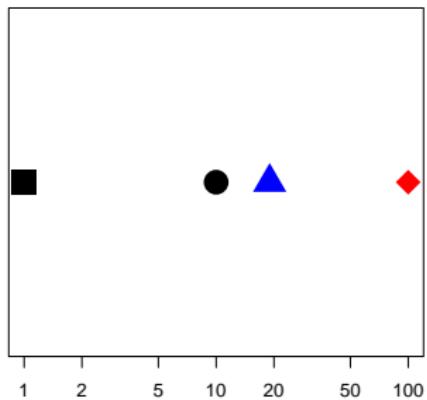
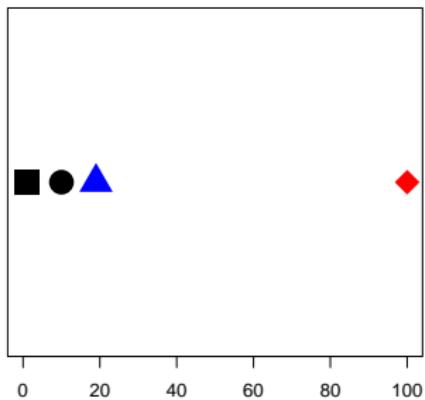
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There is only one log scale; it doesn't matter which base you use!

Canadian provinces

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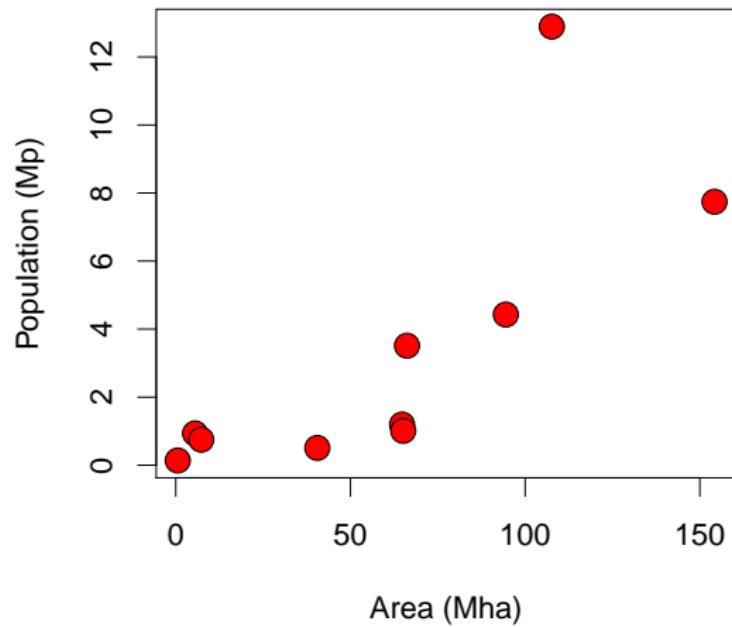
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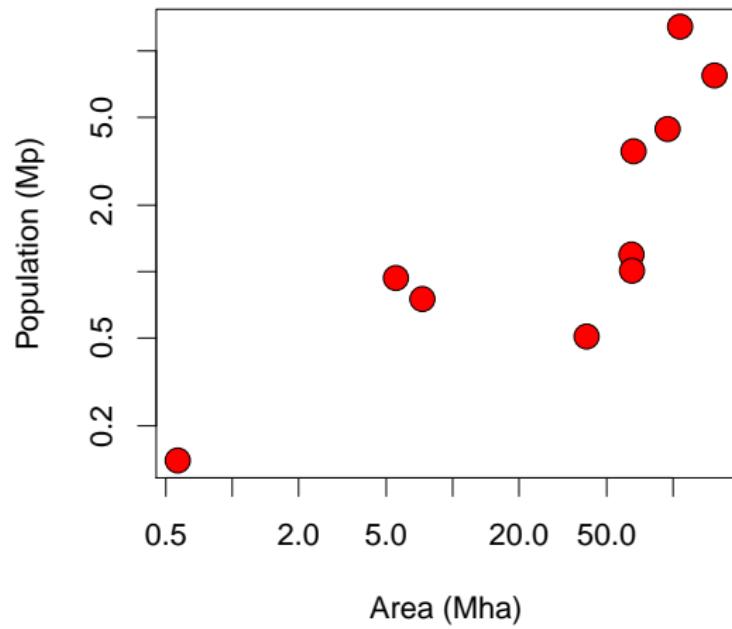
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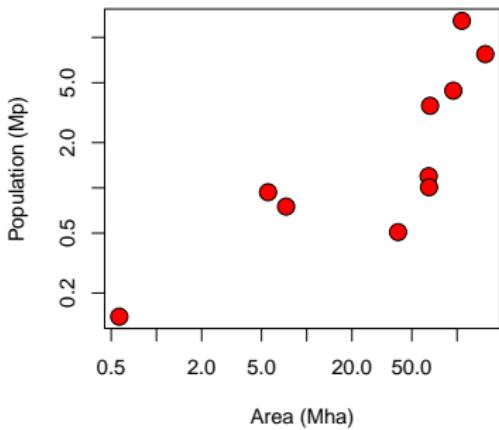
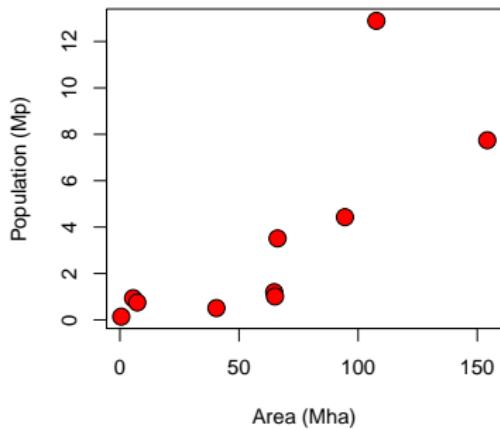
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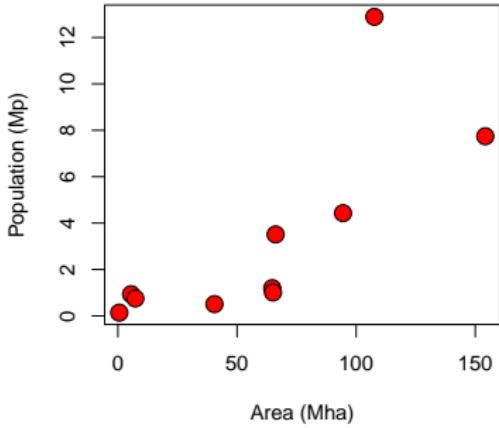
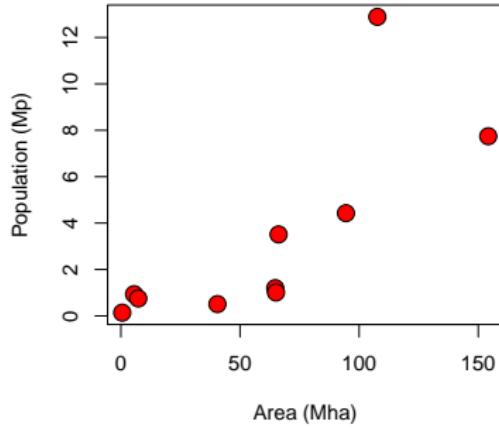
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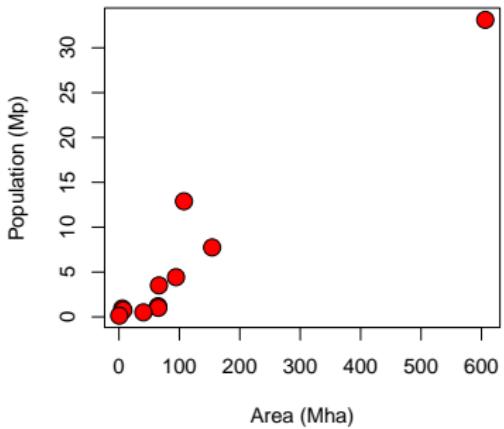
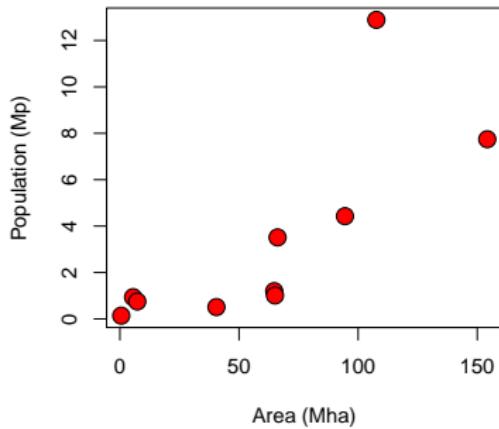
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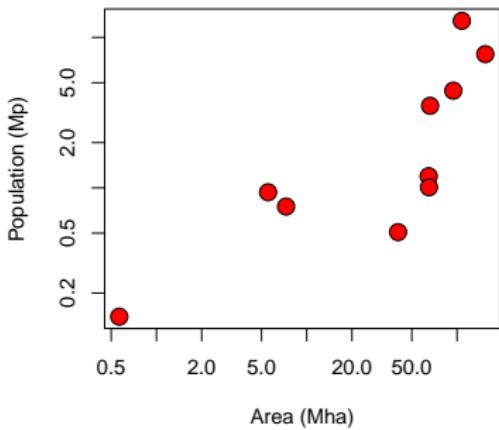
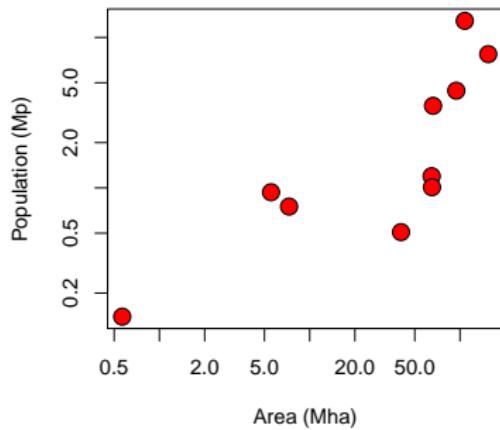
Canadian provinces plus Canada?



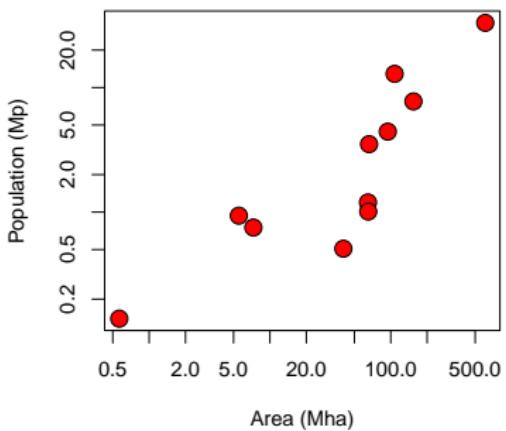
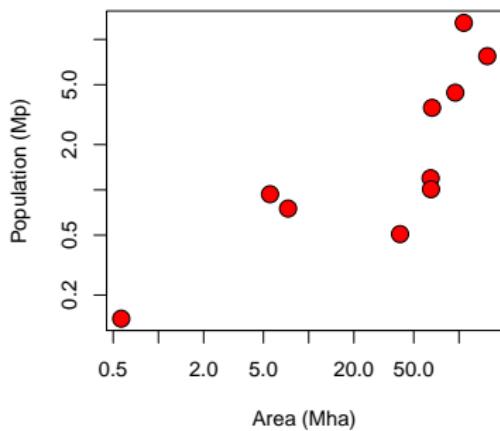
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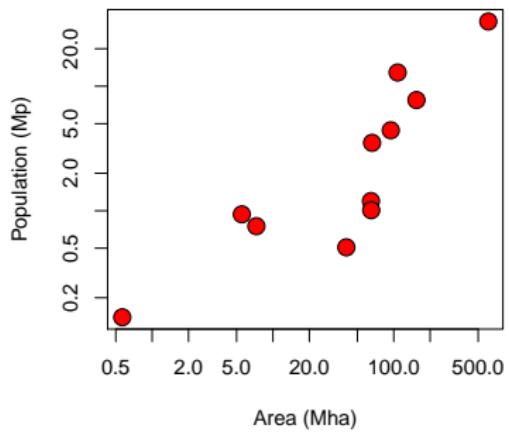
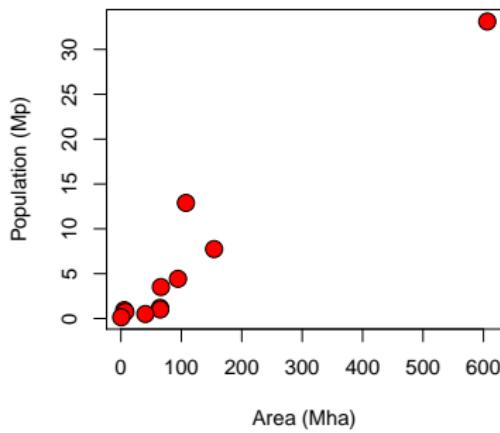
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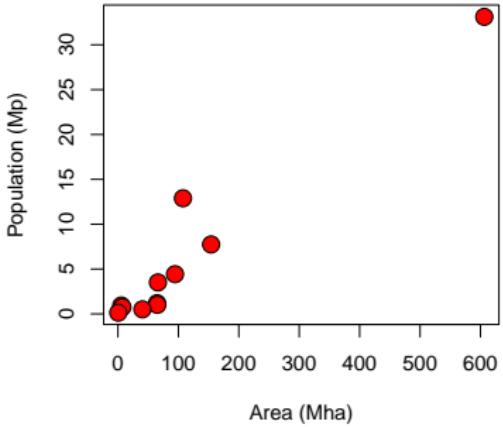
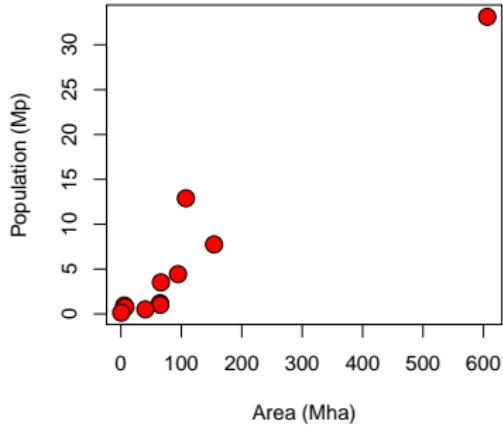
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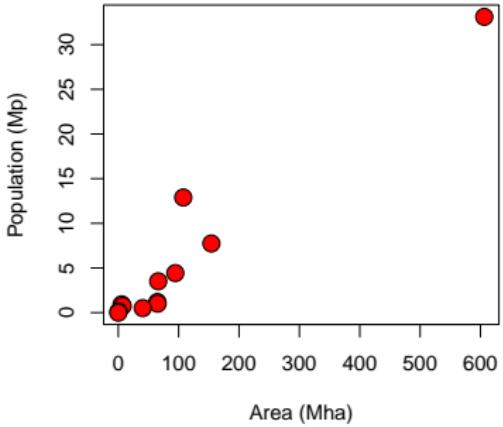
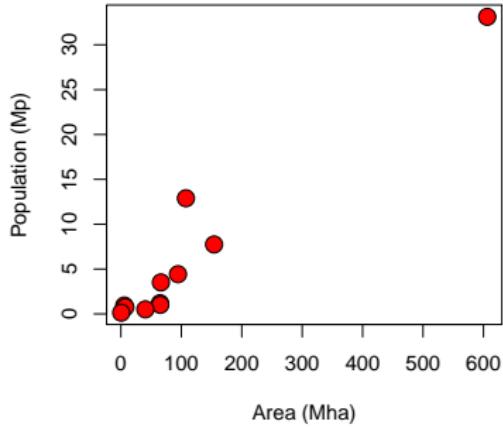
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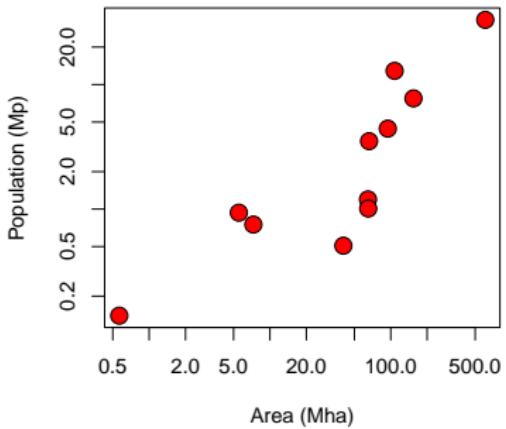
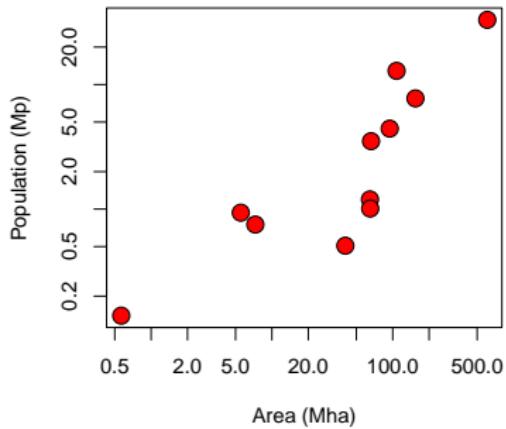
Canada plus B135



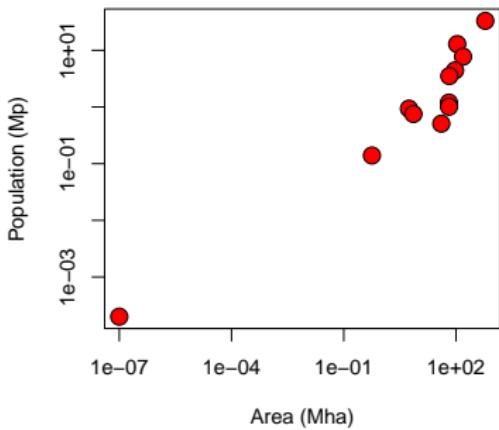
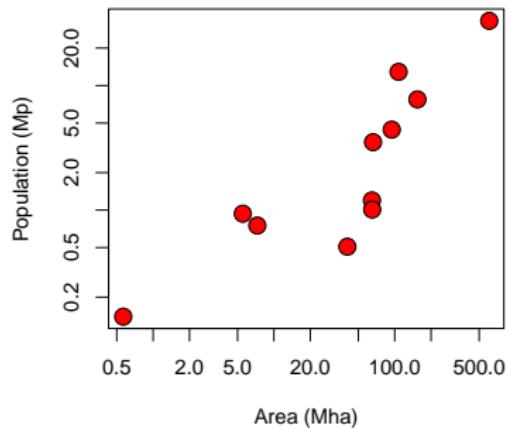
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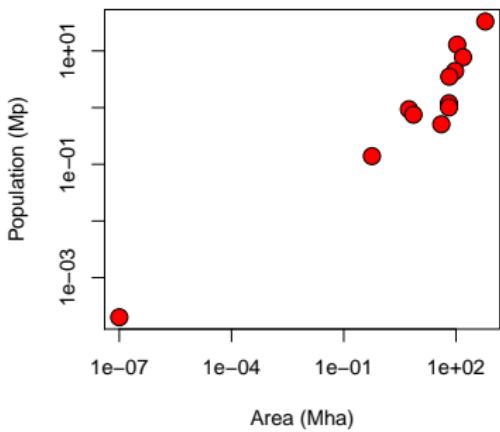
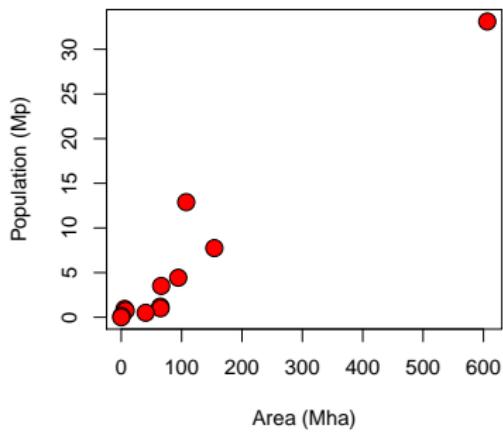
Canada plus room 1105?



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Predation comparison



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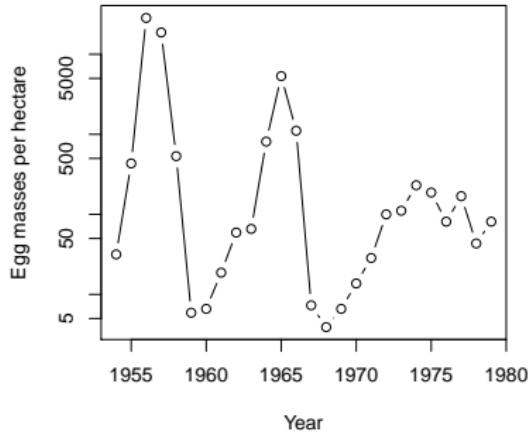
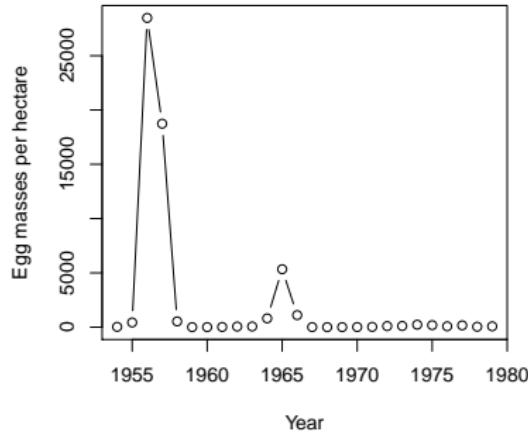
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Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

Example populations

Dandelions

Gypsy moths

Bacteria

Exponential growth

Log and linear scales

Time scales

Speeding in Taiwan

- A life experience



Speeding in Taiwan

- ▶ A life experience
- ▶ Some clarifications



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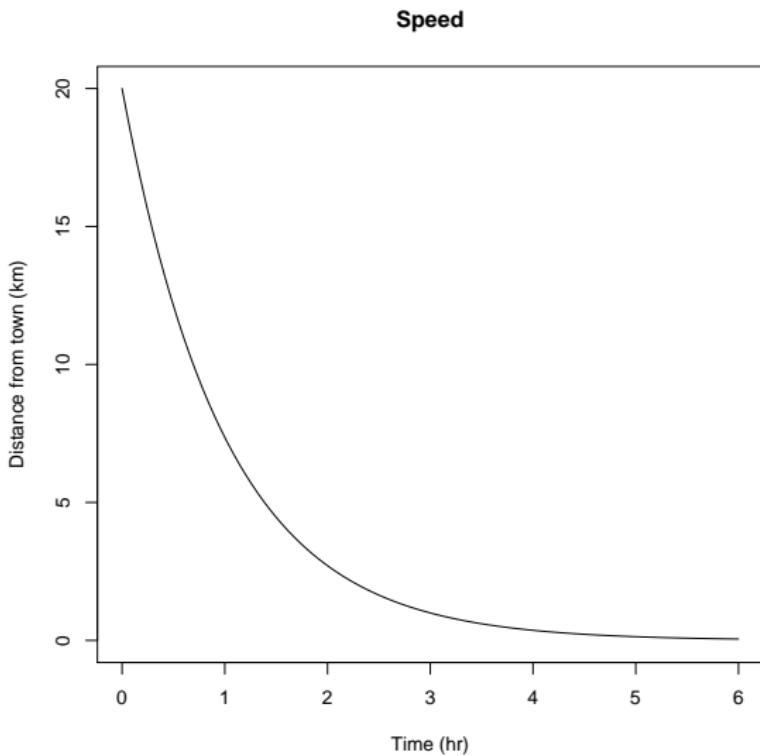
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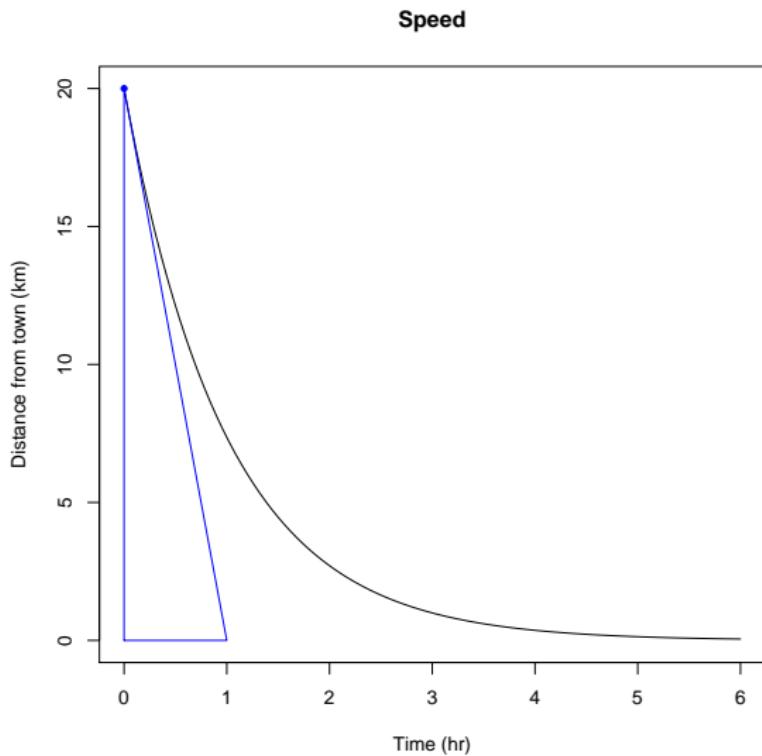
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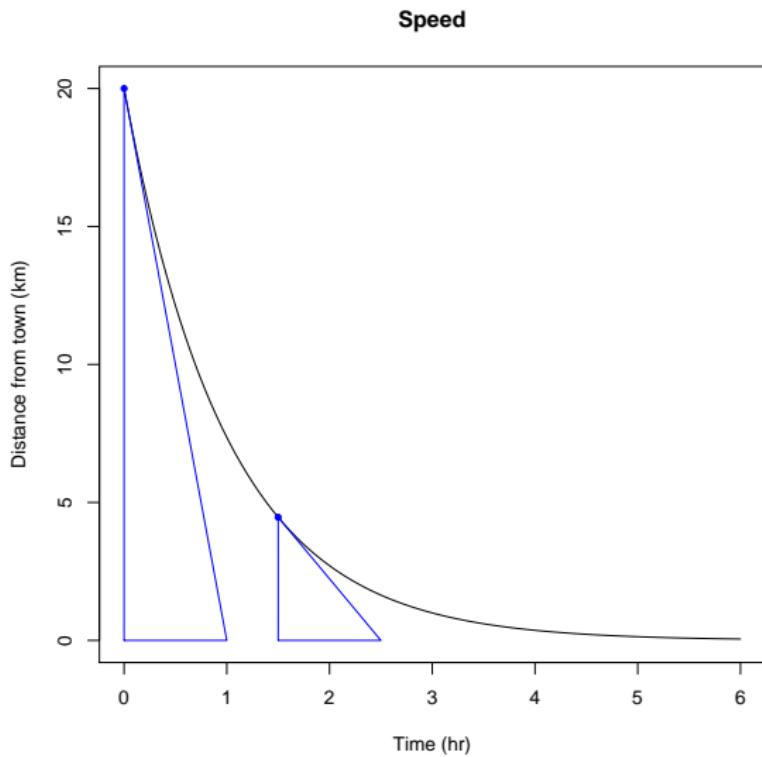
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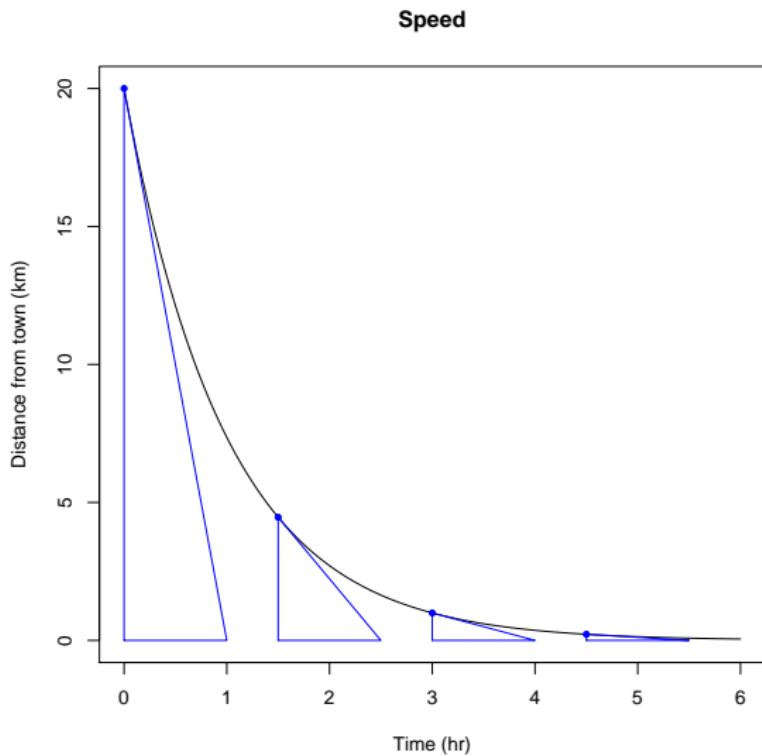
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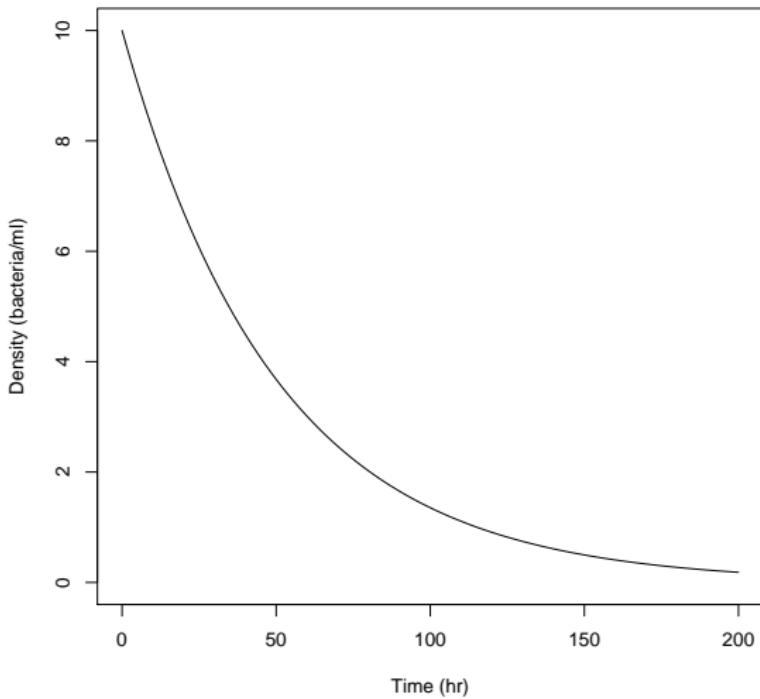
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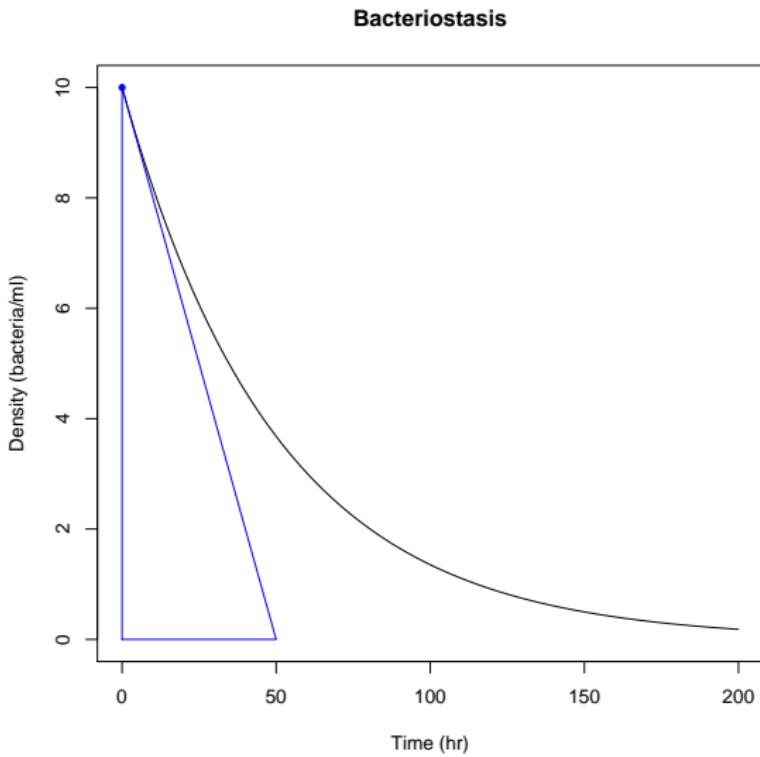
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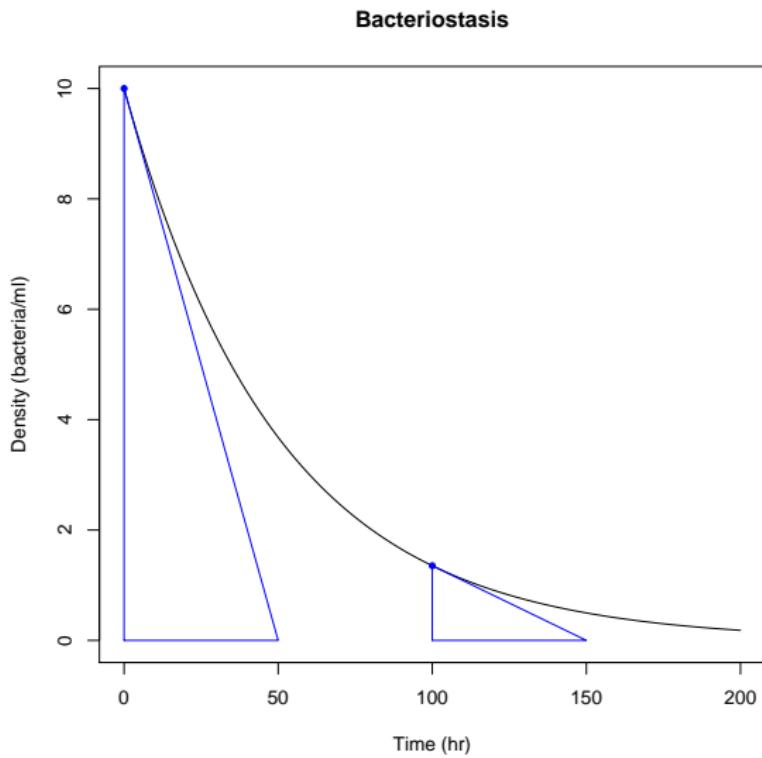
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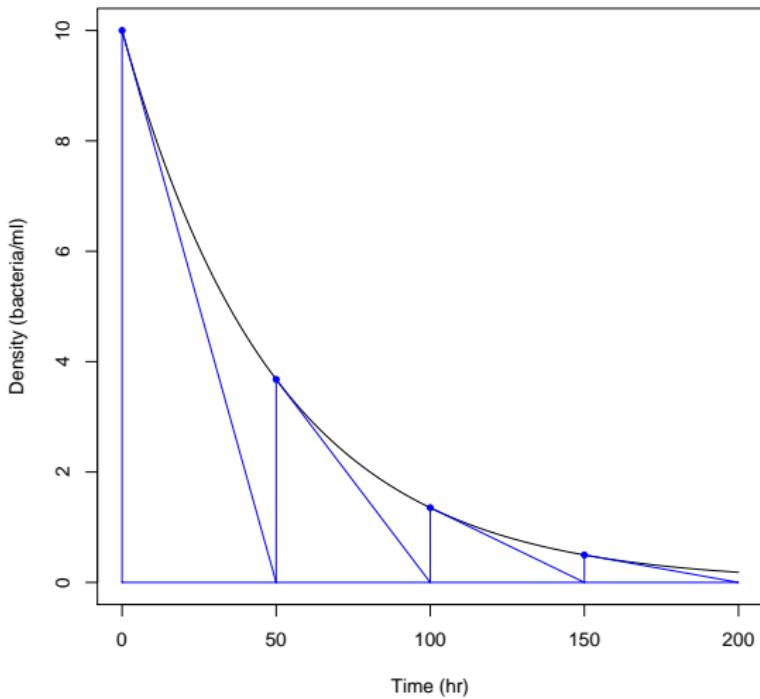


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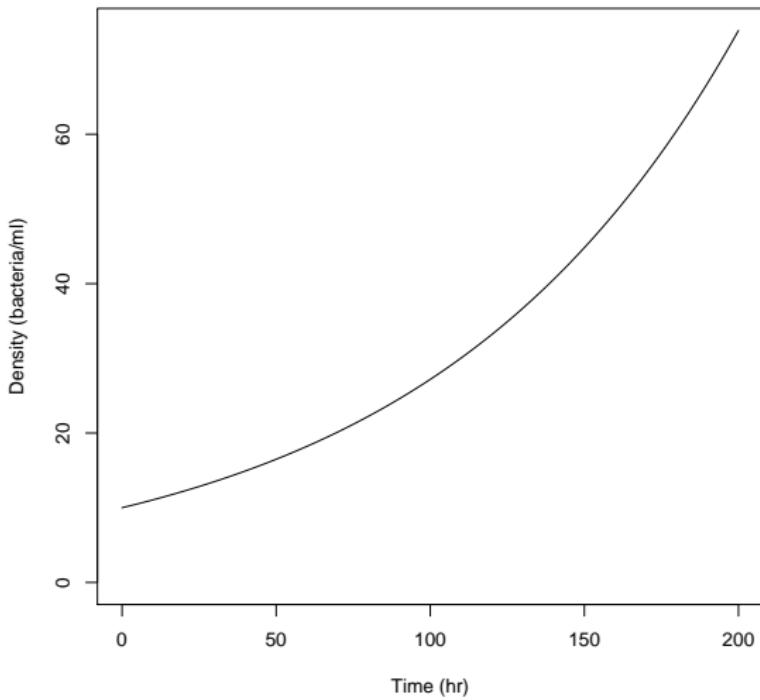
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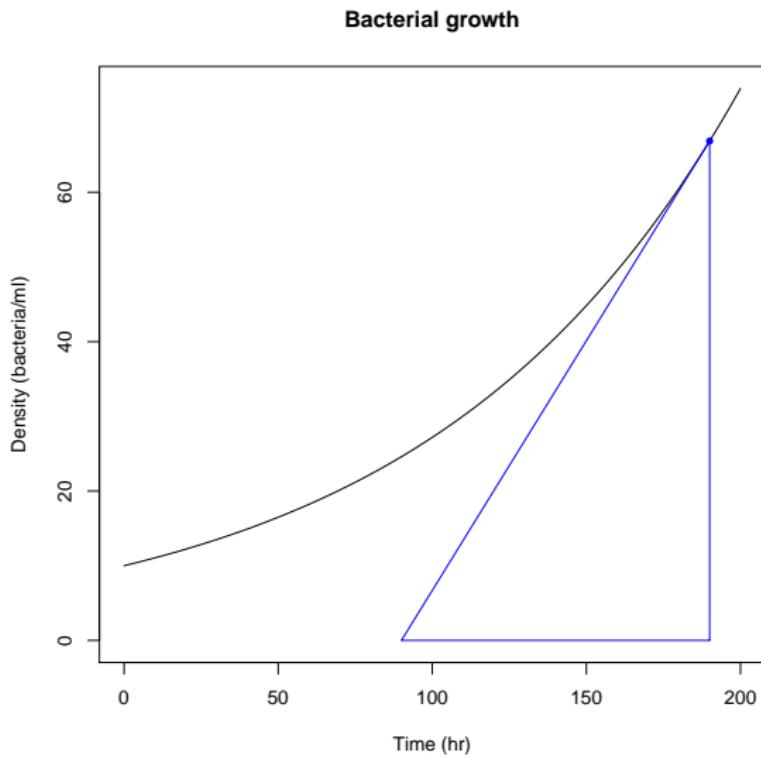
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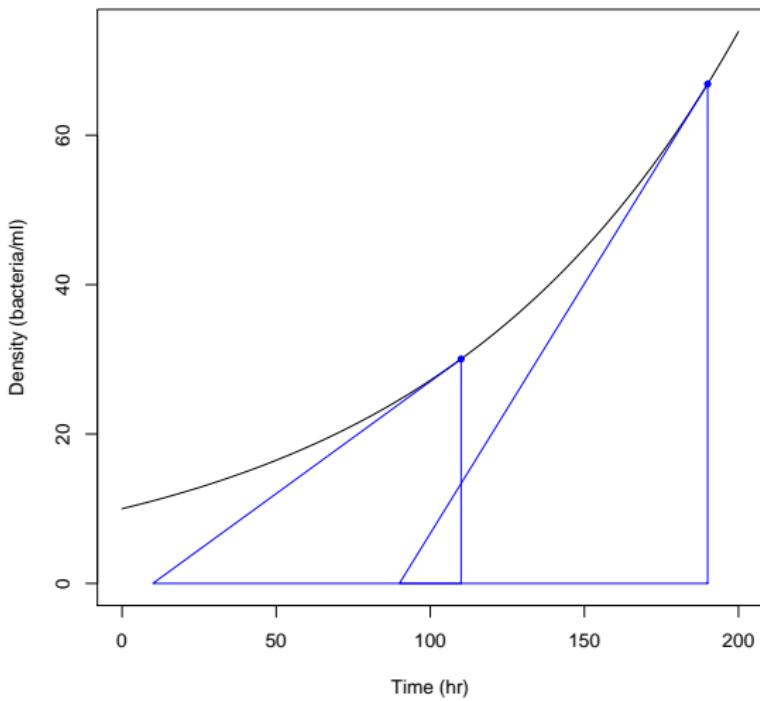


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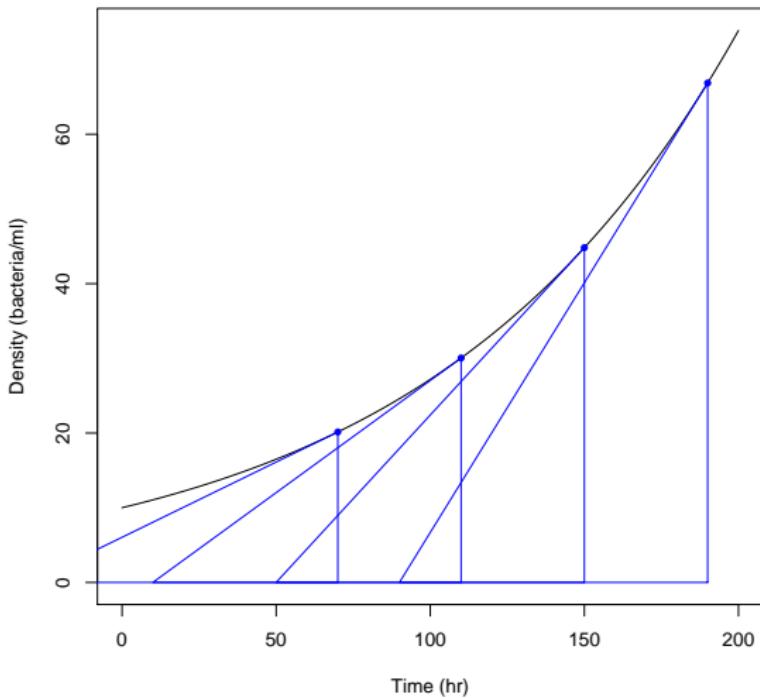
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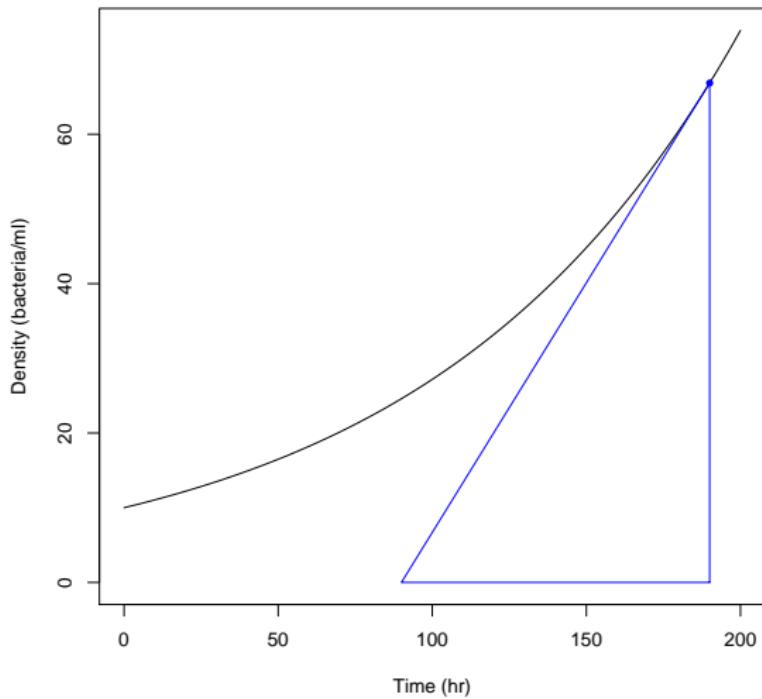
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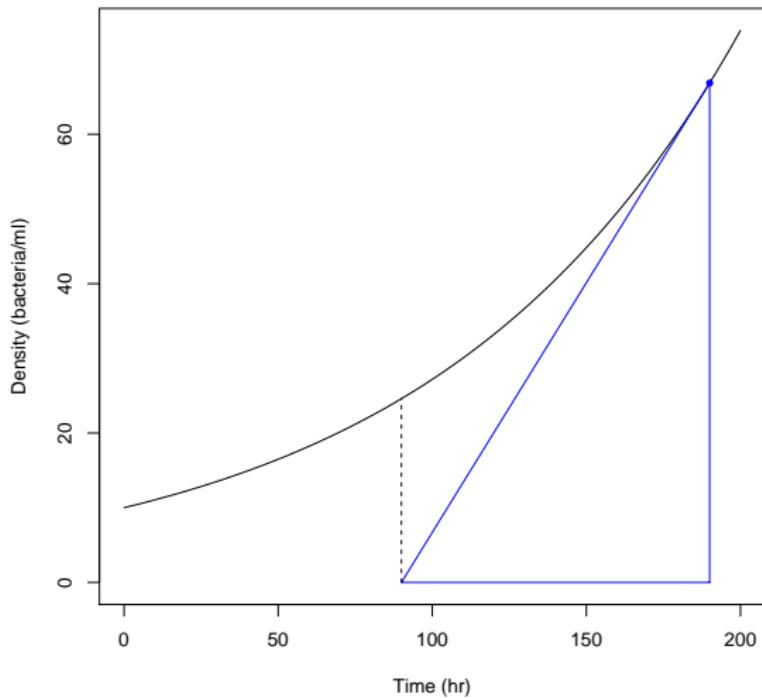
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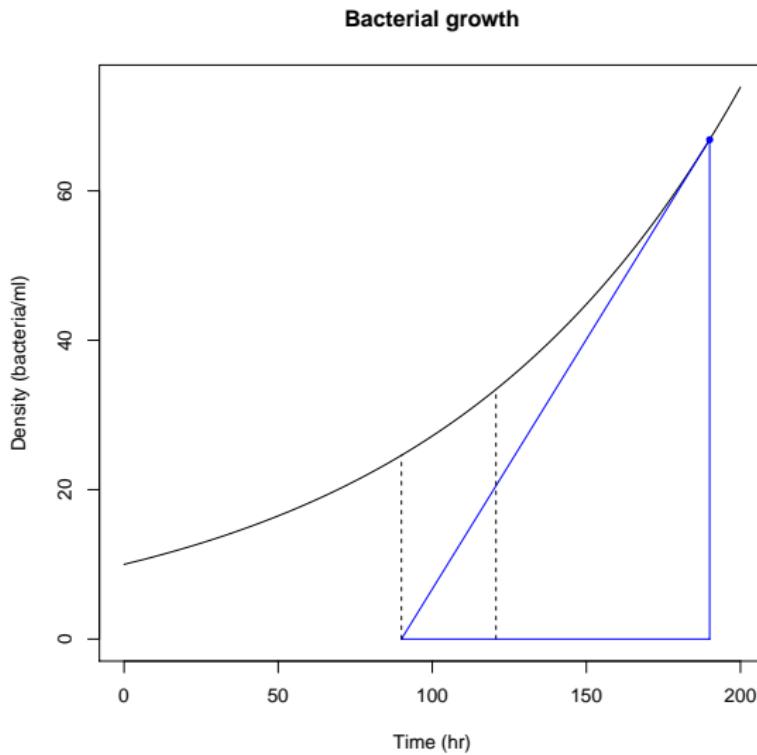
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Characteristic time and doubling time



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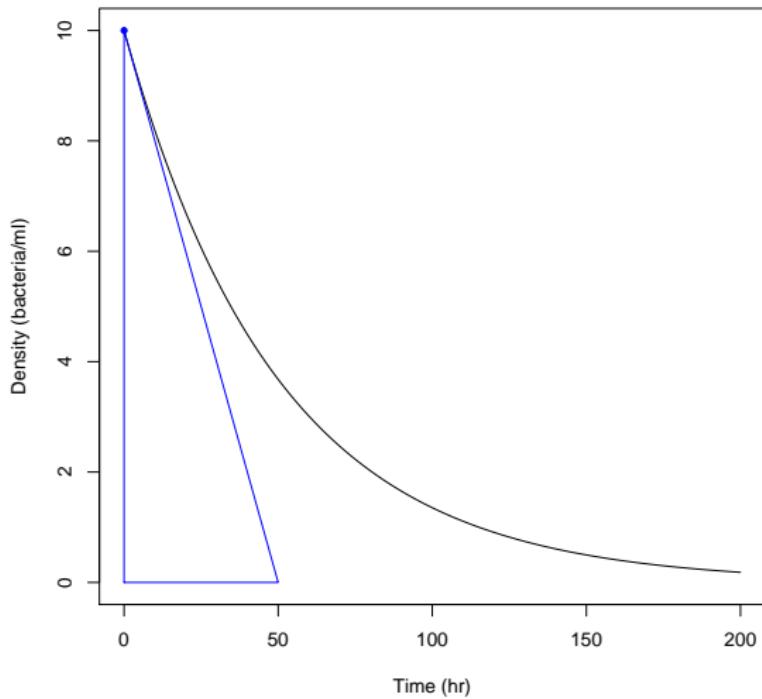
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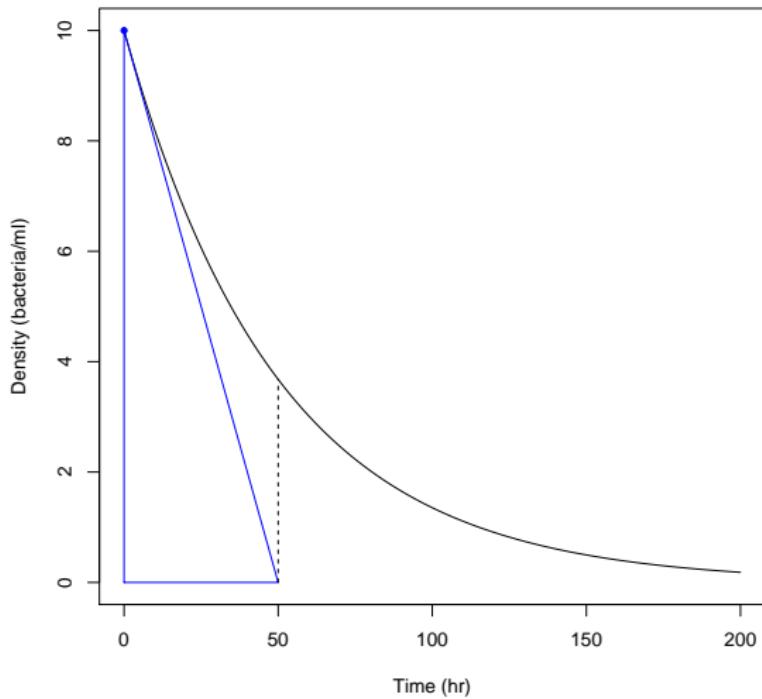
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