

Bio 3SS Math review

Notation (*e* is cool)

- In this course \log or \log_e means the *natural* logarithm (written *ln* in the textbooks)
 - If I mean \log_{10} , I will say so
- $\exp()$ is a synonym for “*e* to the power of”

Logarithms

- Multiplying on the original scale is equivalent to adding on the log scale
 - $\log(ab) = \log(a) + \log(b)$
- Division is the opposite of multiplication and subtraction is the opposite of addition:
 - $\log(a/b) \equiv \log\left(\frac{a}{b}\right) = \log(a) - \log(b)$
- Exponentiation is to multiplication like multiplication is to addition:
 - $\log(\lambda^k) = k \log(\lambda)$

Complementarity

- \log_e and \exp are complementary functions
 - $\exp(\log_e(x)) \equiv e^{\log_e(x)} = x$
 - $\log_e(\exp(x)) \equiv \log_e(e^x) = x$

Exponentiation

- The rules for \exp are exactly complementary to those for \log
 - $\exp(a + b) = \exp(a) \cdot \exp(b)$
 - $\exp(a - b) = \exp(a) / \exp(b)$
 - $\exp(k \log_e(\lambda)) = \lambda^k$
- Remember: you can't take the log or exponent of something with units

Problems

- You should be able to use these rules to solve simple problems, algebraically, or on your calculator
 - If $\lambda^{20} = 0.1$, what is λ ?
- Also, word problems
 - If a population declines by 90% in 20 generations, what is its reproductive number per generation?

Algebra

- If $\lambda^{20} = 0.1$, what is λ ?
 - **Answer:** $20 \log_e(\lambda) = \log_e(0.1)$
 - **Answer:** $\log_e(\lambda) = \log_e(0.1)/20$
 - **Answer:** $\lambda = \exp(\log_e(0.1)/20)$
 - **Answer:** $\lambda = 0.89$
- If $\exp(rt) = 5$, and $r = 0.1/\text{yr}$, what is t ?
 - **Answer:** $rt = \log_e(5)$
 - **Answer:** $t = \frac{\log_e(5)}{0.1/\text{yr}}$
 - **Answer:** $t = 16 \text{ yr}$

Units are our friends

- Keep track of units at all times
- Use units to confirm that your answers make sense
 - Or to find quick ways of getting the answer
- Get used to manipulating and cancelling units
- $36 \text{ mpg} = ?? \text{ L}/100\text{km}$
 - **Answer:** $\frac{\text{gal}}{36 \text{ mi}} \cdot \frac{\text{mi}}{1.6 \text{ km}} \cdot \frac{3.8 \text{ L}}{\text{gal}} \cdot \frac{100 \text{ km}}{100 \text{ km}}$
 - **Answer:** $6.6 \text{ L}/100\text{km}$