UNIT 2: Linear population models

Outline

Constructing models
- Dynamical models
- Examples
- A simple discrete-time model
- A simple continuous-time model

Units and scaling

Key parameters
- Discrete-time model
- Continuous-time model
- Links

Growth and regulation

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Dynamical models

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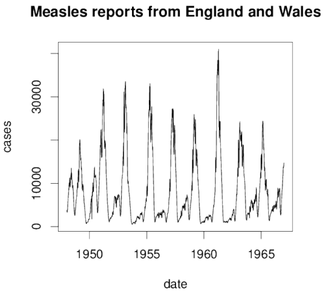

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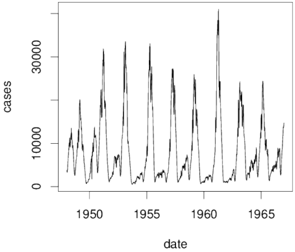

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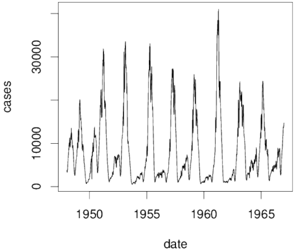

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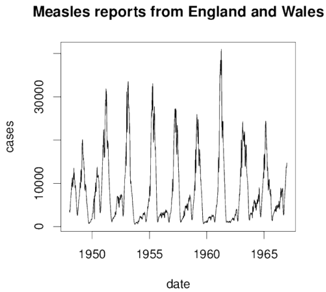

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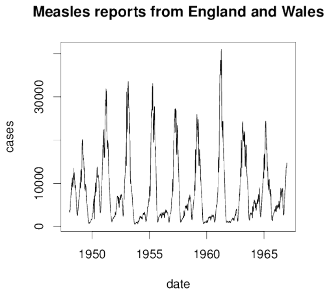

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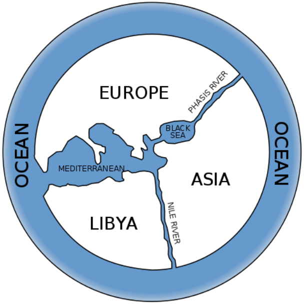

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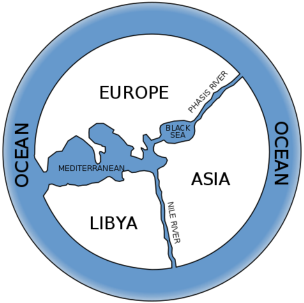

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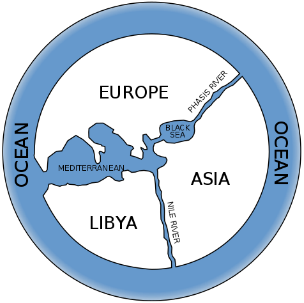

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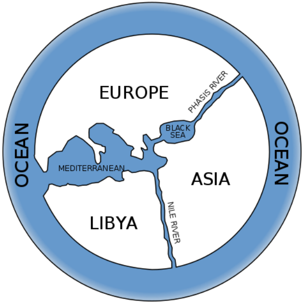

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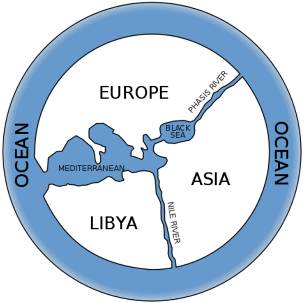

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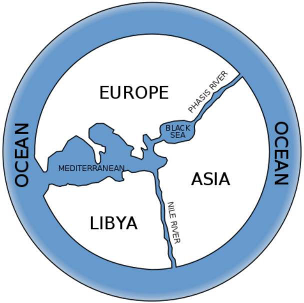

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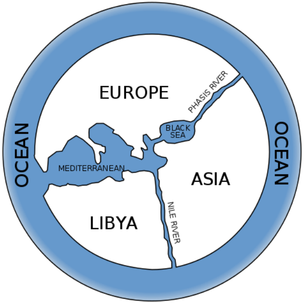

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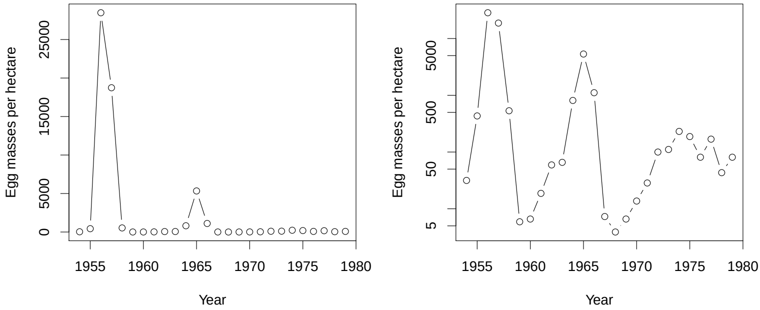

Moth example

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Bacteria

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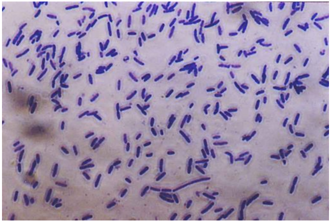

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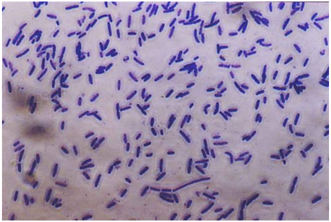

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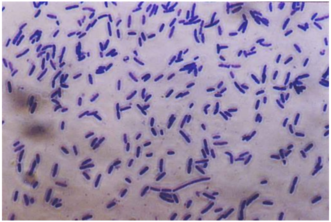

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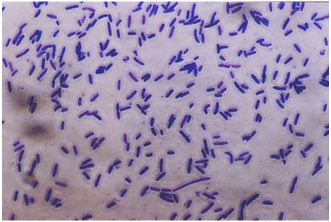

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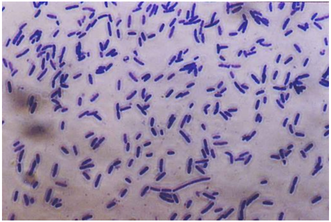

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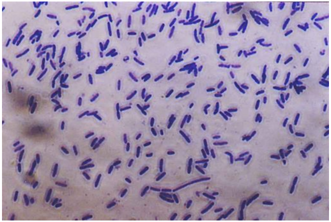

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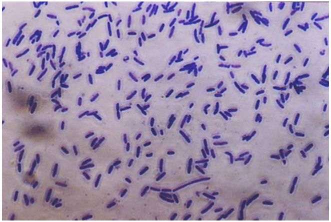

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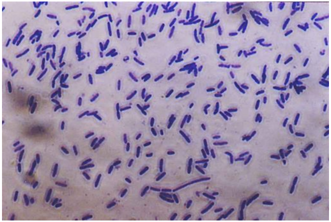

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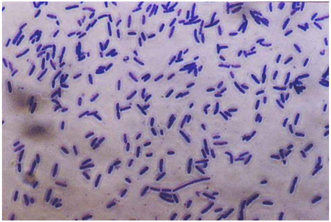

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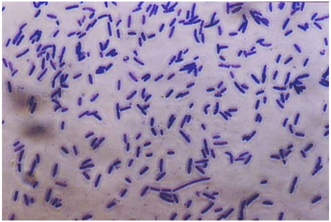

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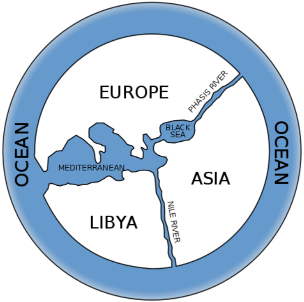

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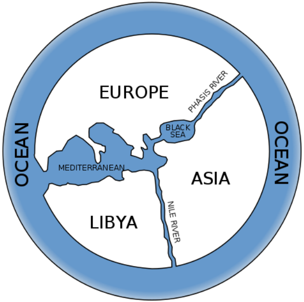

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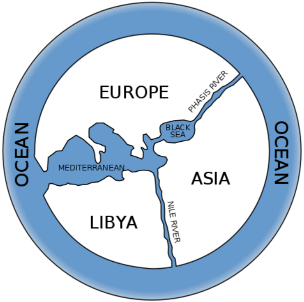

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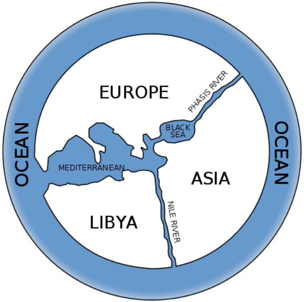

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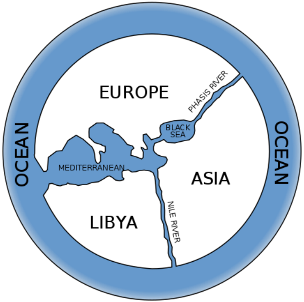

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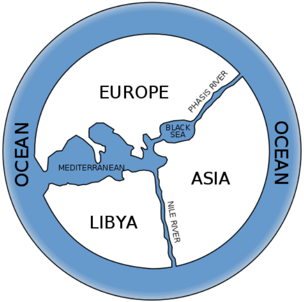

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- Dynamical models
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- A simple continuous-time model

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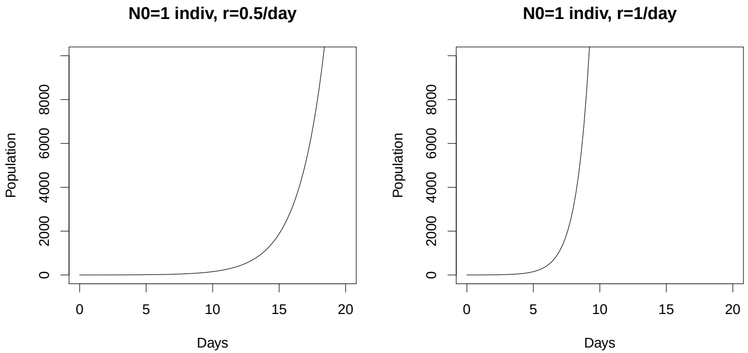
N0=1 indiv, r=0.5/day
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- Poll: How many seconds are there in a day?
  - * $\frac{60\,\text{sec}}{\text{min}} \cdot \frac{60\,\text{min}}{\text{hr}} \cdot \frac{24\,\text{hr}}{\text{day}}$
  - * $86400\ \text{sec}/\,\text{day}$

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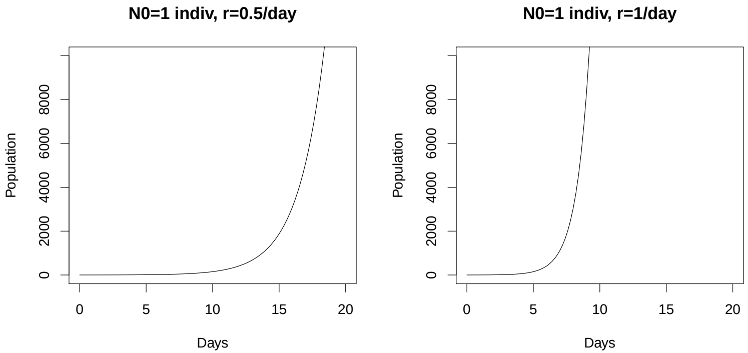

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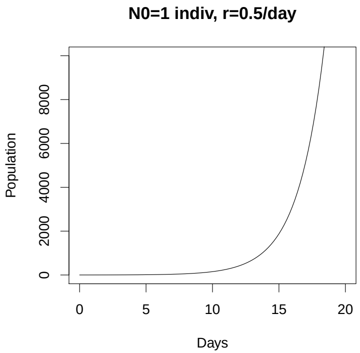

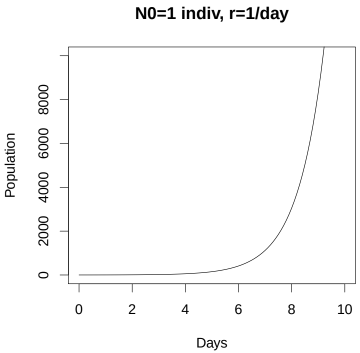

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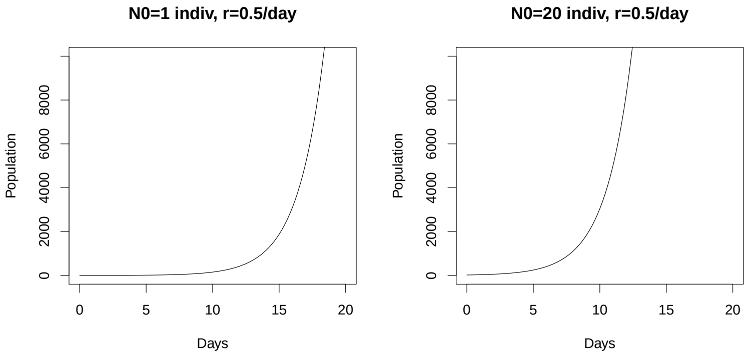

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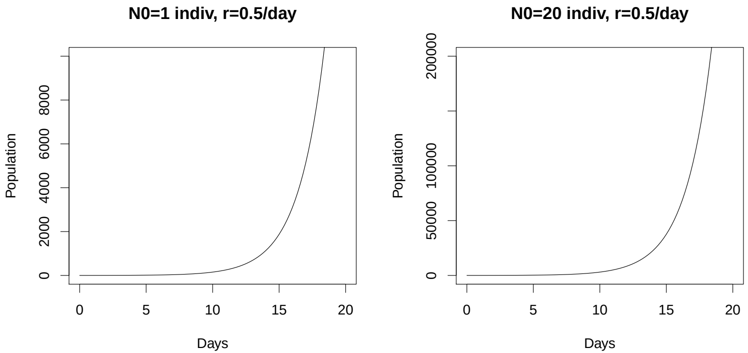

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Key parameters
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Example: Human population growth

- In the last 50,000 years, the population of modern humans has increased from about 1000 to about 7 billion
- What value of r does this correspond to? If we use a time step of 20-year generations, what value of λ does it correspond to?
  - * $N(t) = N(0)\exp(rt)$
    - * $r = \log_e(N/N(0))/t$
    - * $r = \log_e(7000000000/1000)/50000\,\mathrm{yr} = 0.0003/\,\mathrm{yr}$
  - * $N_T = N_0\lambda^T$
    - * $T = t/\Delta t = 50000\,\mathrm{yr}/20\,\mathrm{yr} = 2500$
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