

UNIT 1: Introduction

Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

Example populations

Dandelions

Gypsy moths

Bacteria

Exponential growth

Log and linear scales

Time scales

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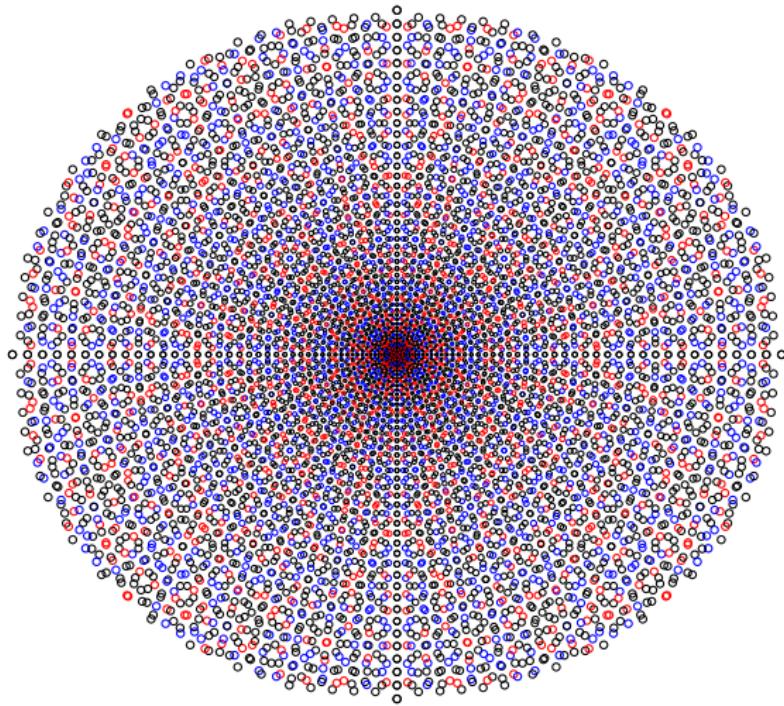
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Pythagorean triples



Which country?



Which country?



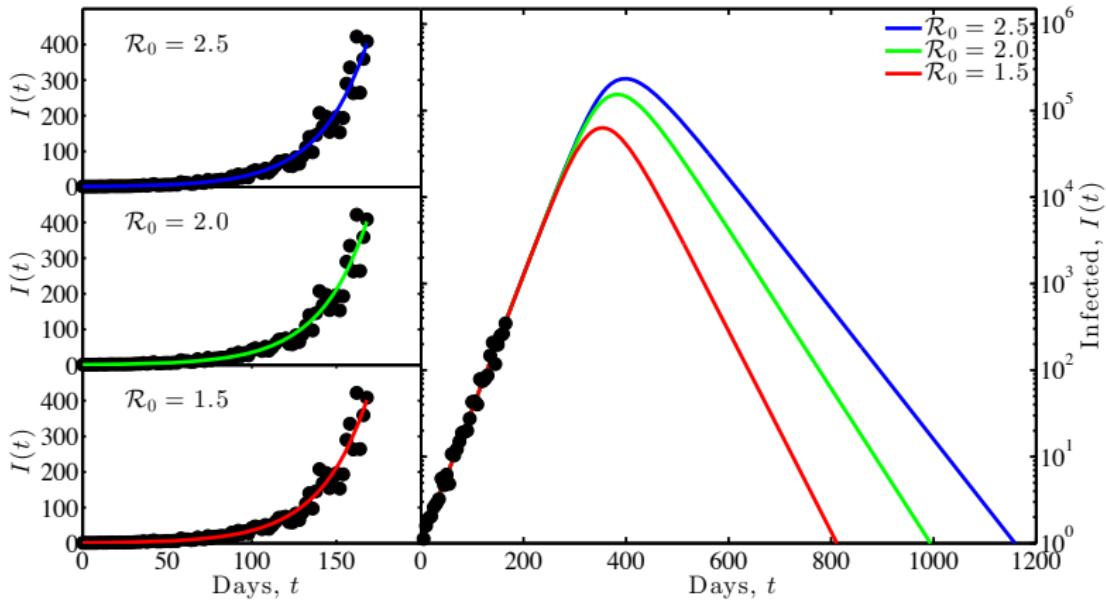
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Ebola research



TAs

- Morgan Kain



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 - ▶ Current: ecology and evolution of infectious diseases



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Students

- ▶ Poll: What year are you in?

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- ▶ Poll: What is ecology?

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- ▶ My answer

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 - ▶ * We use *dynamical models* to link from the individual level to the population level

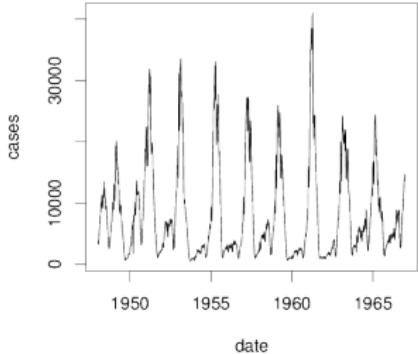
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Dynamical modeling

- ▶ Investigates the links between local, short-term processes, and large-scale, long-term outcomes



Measles reports from England and Wales

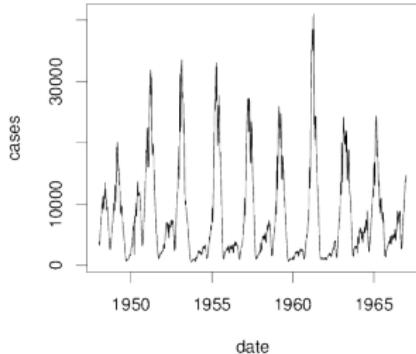


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- ▶ Investigates the links between local, short-term processes, and large-scale, long-term outcomes
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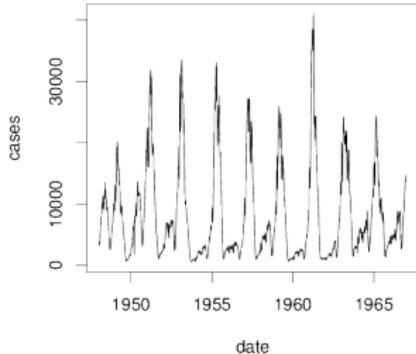


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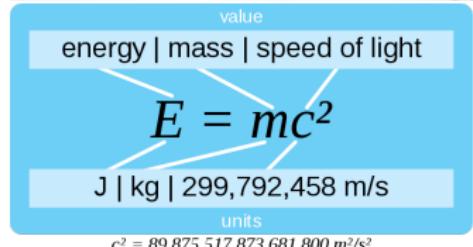
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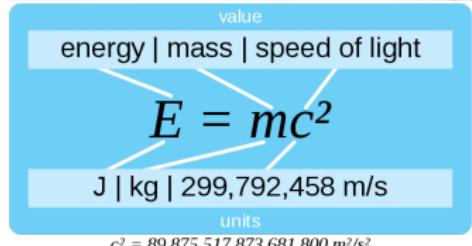
Humans and abstract thought

- ▶ People are evolved to be concrete thinkers, not conceptual thinkers



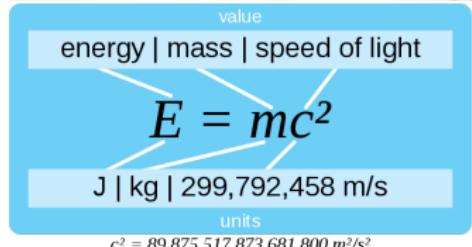
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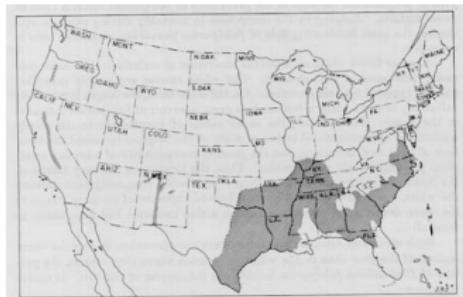
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Malaria

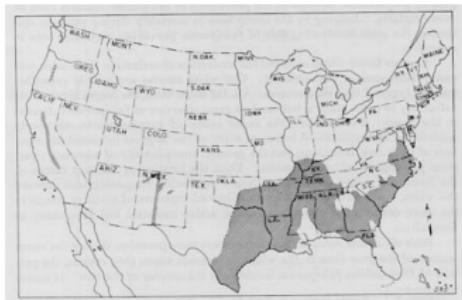
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Map 4.—Areas of the continental United States believed to be malarious in 1931-33.

Malaria

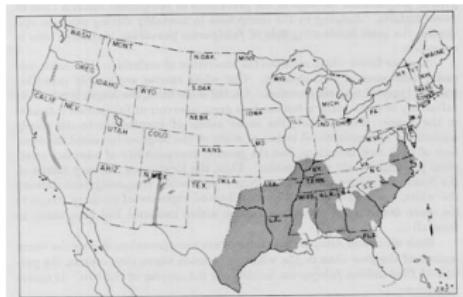
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MAP 4.—Areas of the continental United States believed to be malarious in 1951-55.

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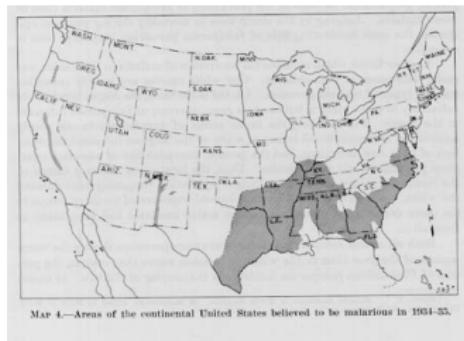
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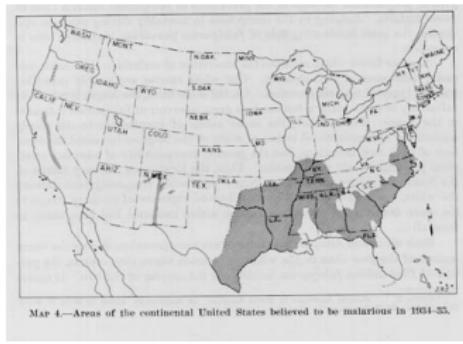
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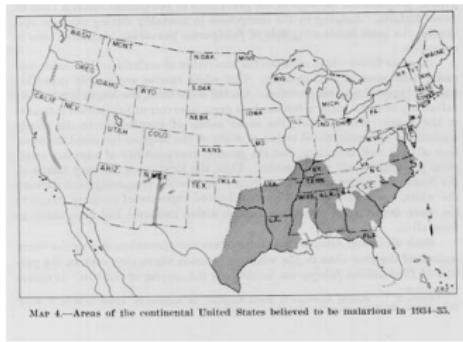
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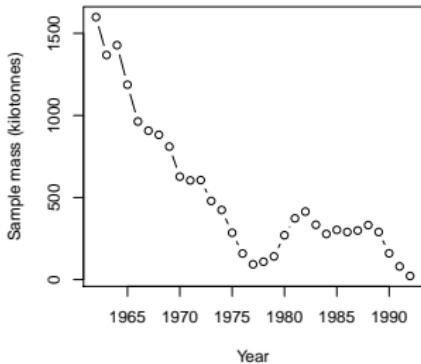
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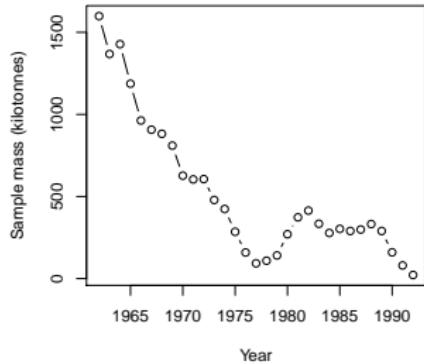
Cod fisheries

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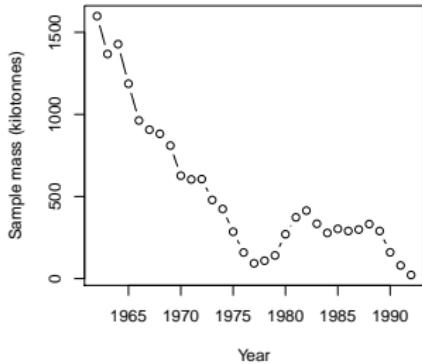
Cod fisheries

- ▶ Is the ocean too big for people to affect?
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Populations

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Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

Example populations

Dandelions

Gypsy moths

Bacteria

Exponential growth

Log and linear scales

Time scales

Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

Example populations

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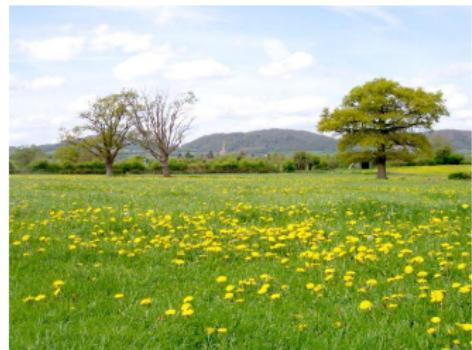
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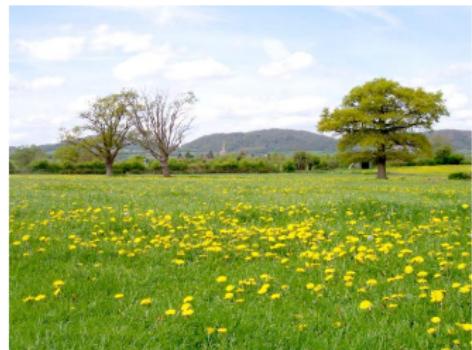


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Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

Example populations

Dandelions

Gypsy moths

Bacteria

Exponential growth

Log and linear scales

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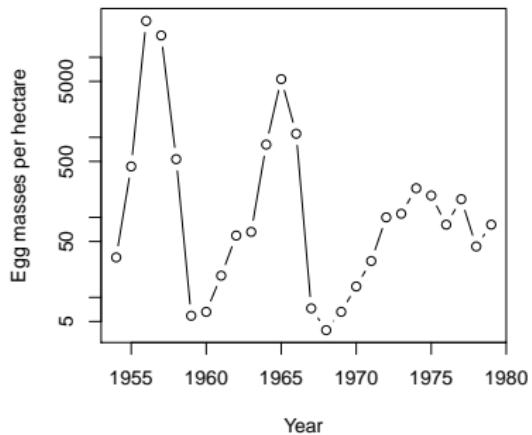
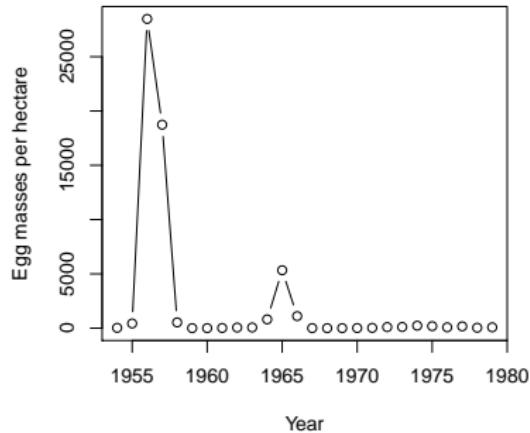


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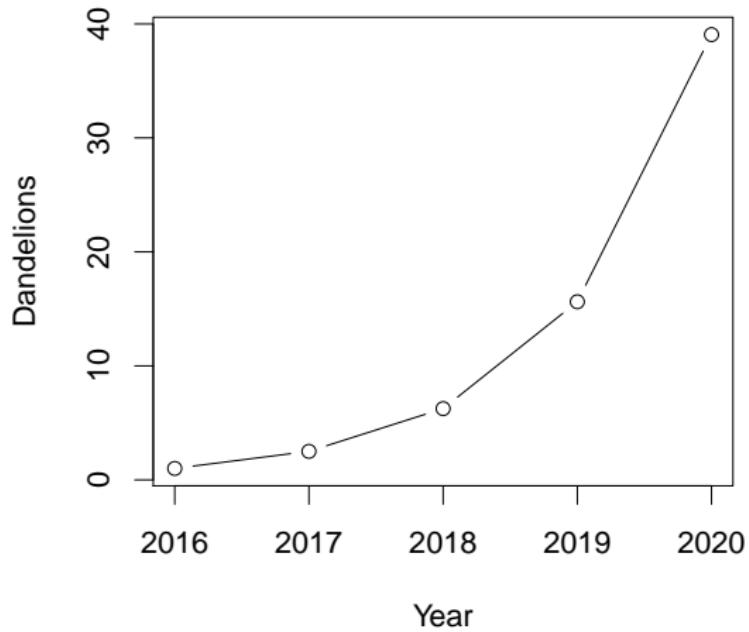
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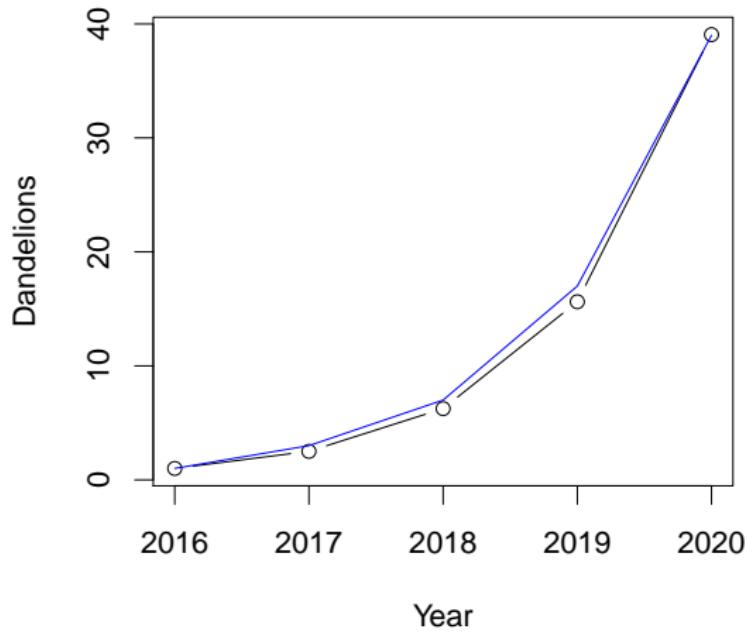
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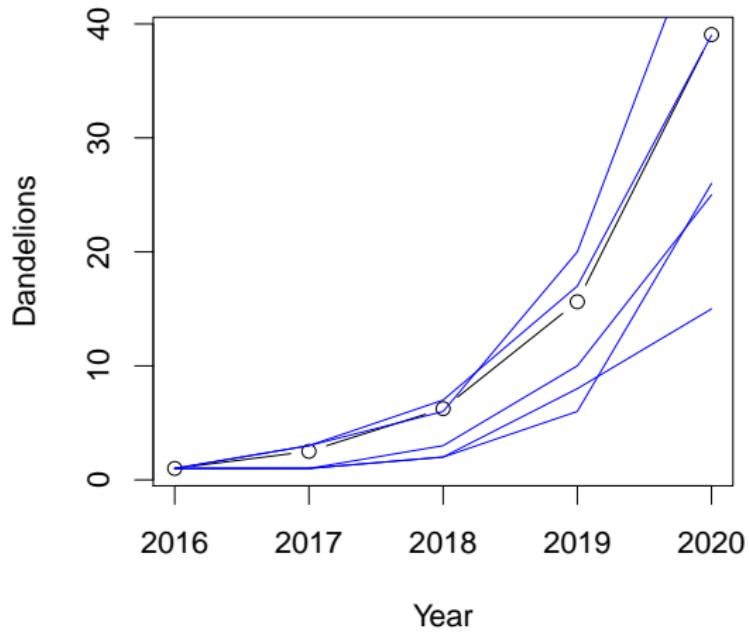
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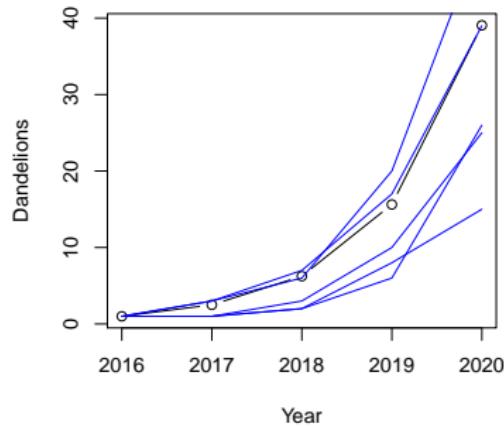
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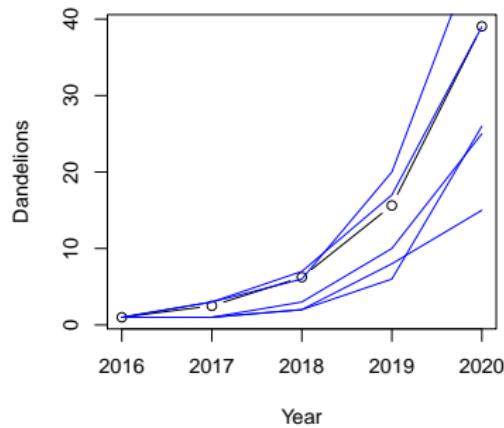


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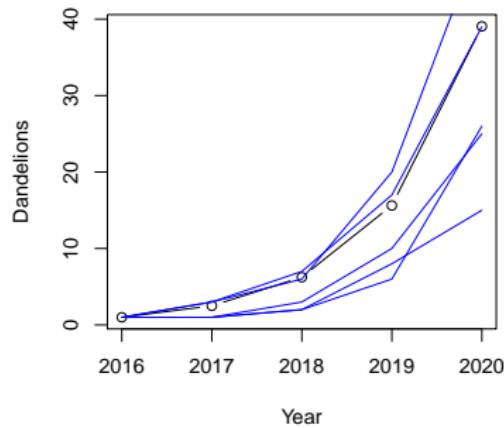
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Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

Example populations

Dandelions

Gypsy moths

Bacteria

Exponential growth

Log and linear scales

Time scales

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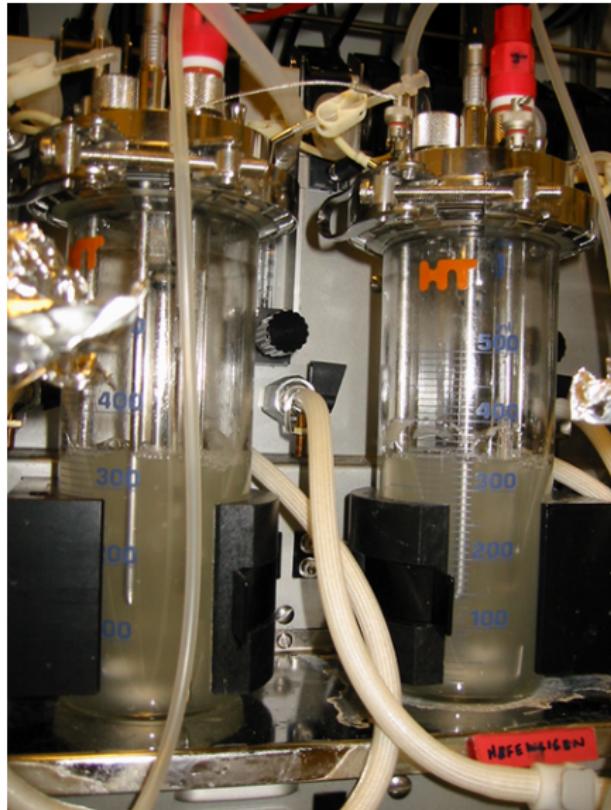
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Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

Example populations

Dandelions

Gypsy moths

Bacteria

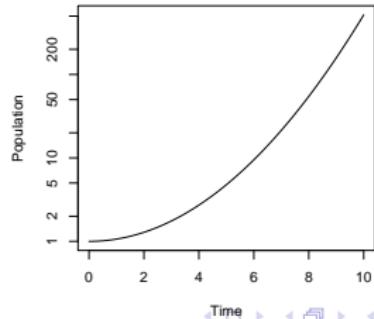
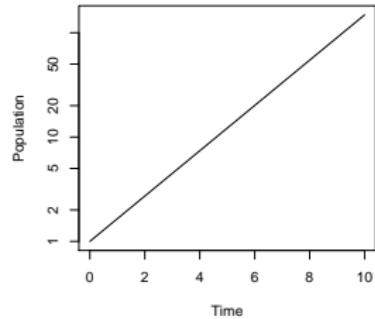
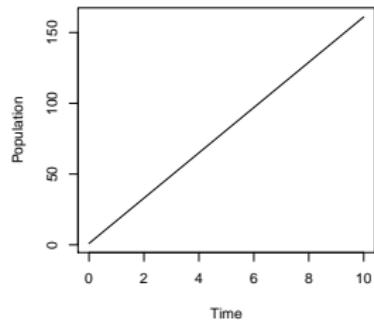
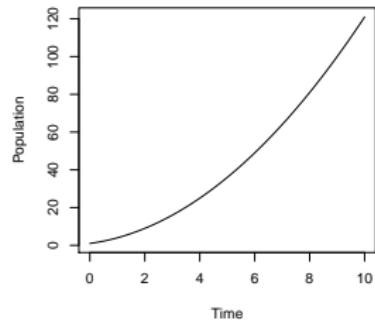
Exponential growth

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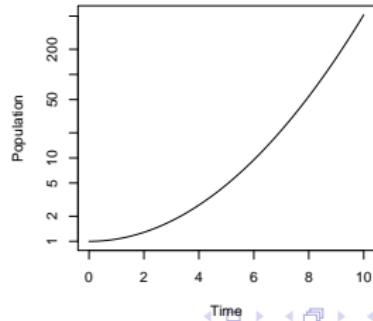
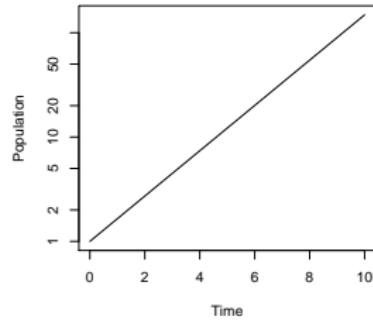
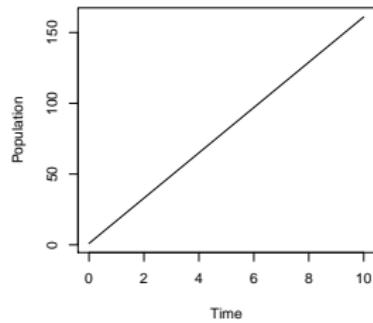
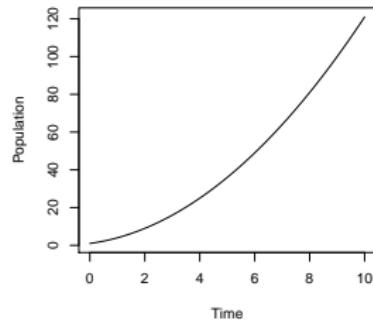
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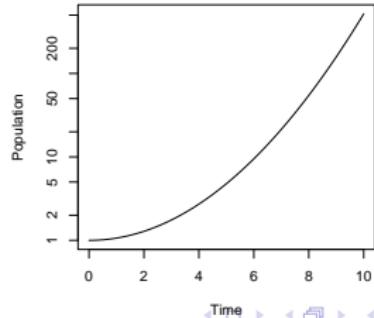
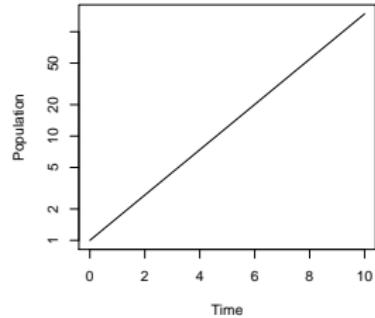
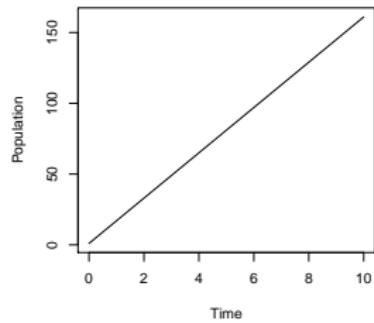
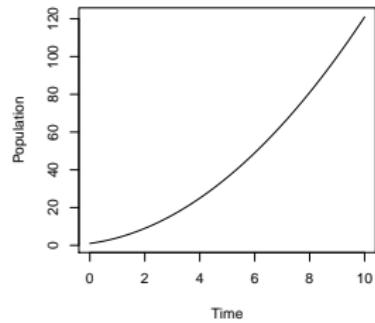
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- ▶ What is exponential growth?
- ▶ Which of these is an example?

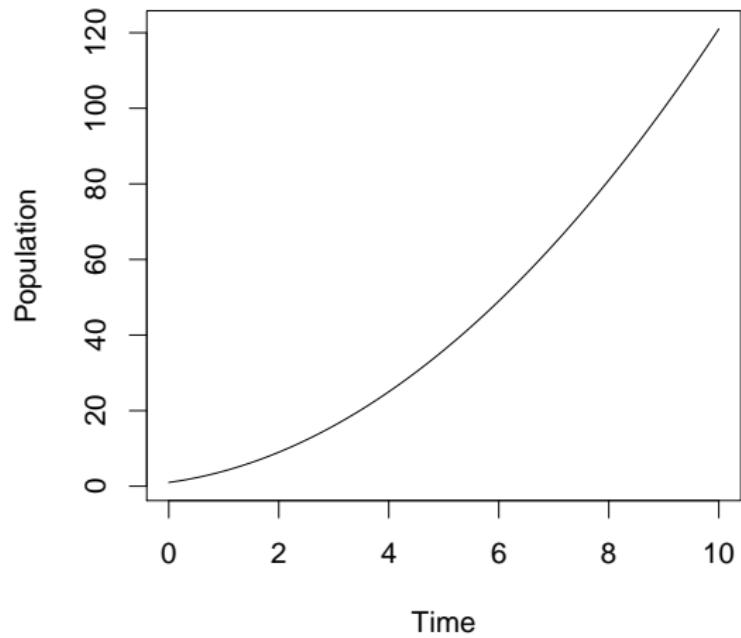


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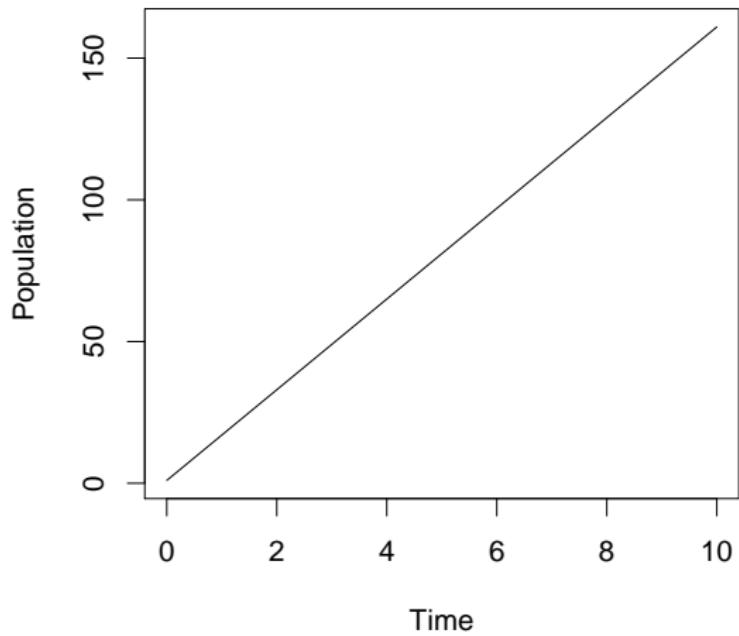
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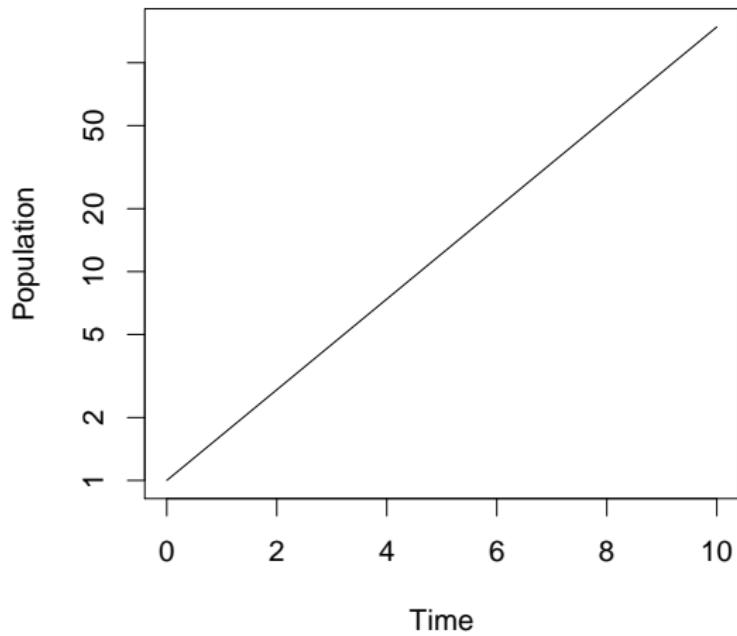
A



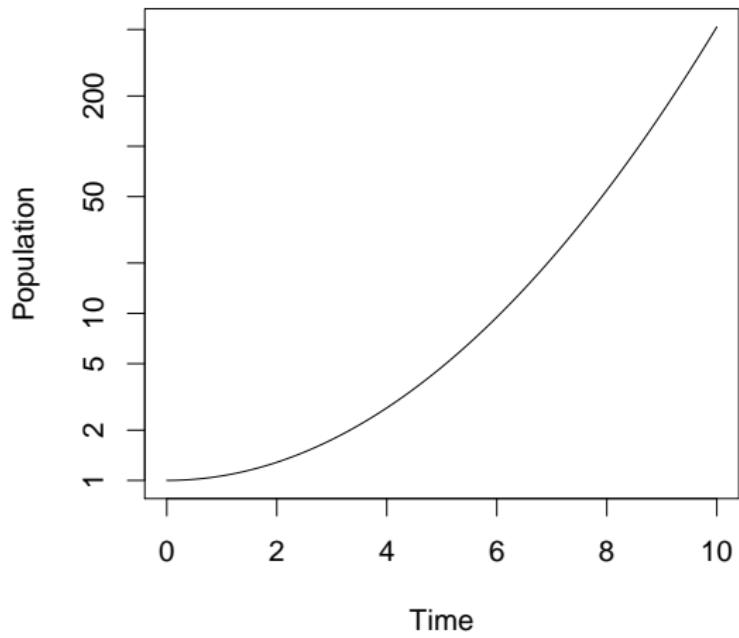
B



C

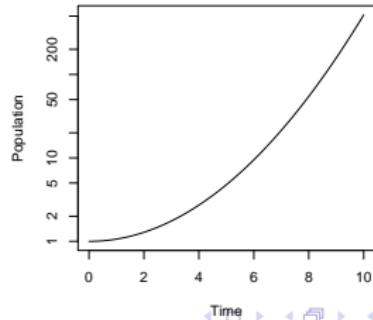
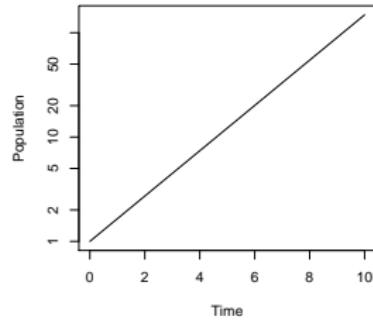
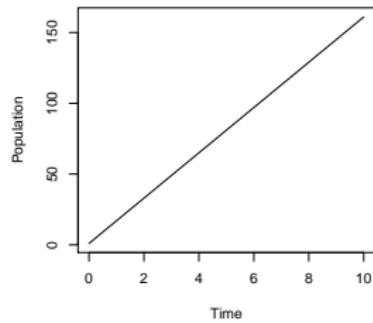
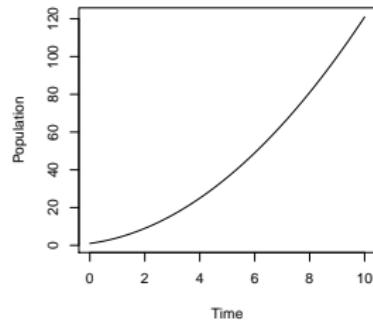


D



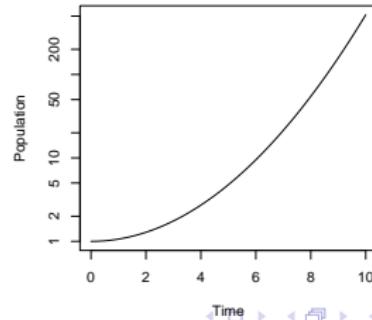
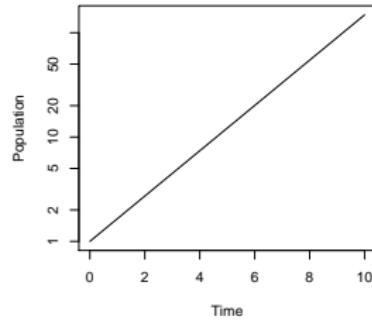
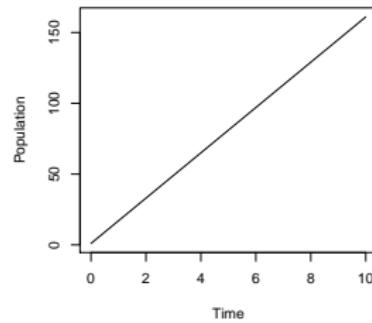
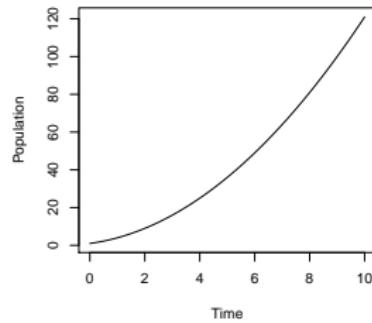
Exponential growth

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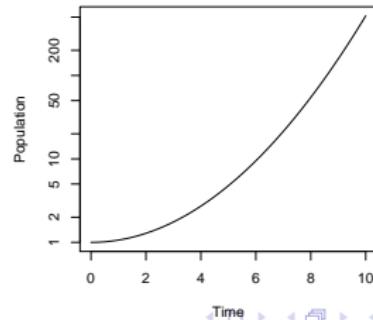
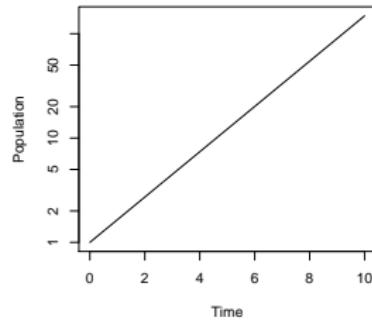
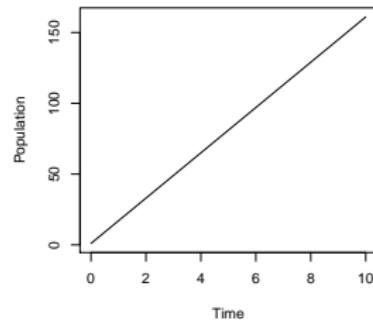
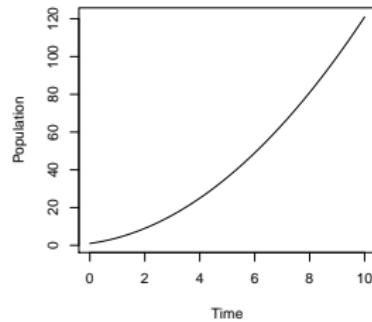
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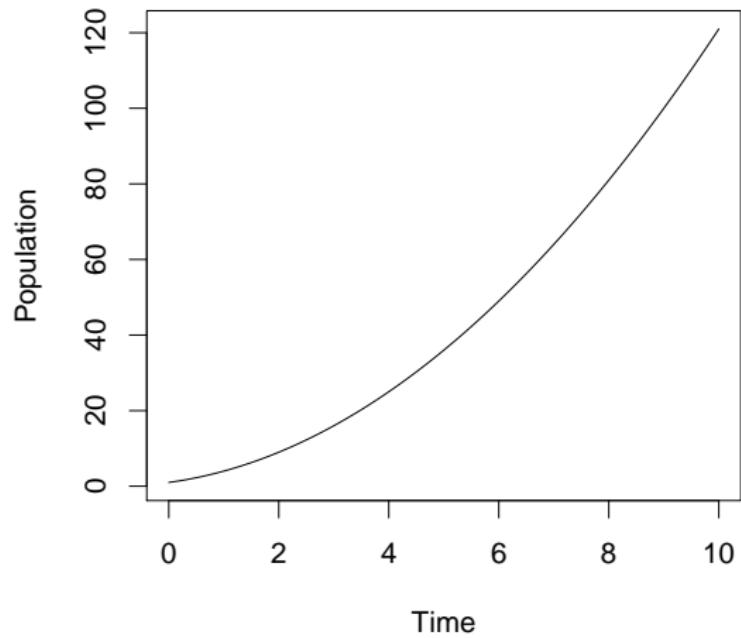


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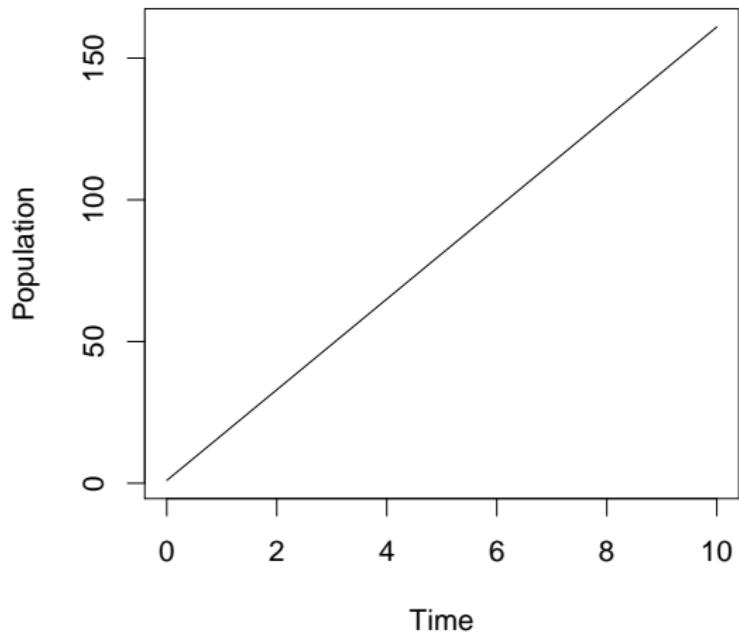
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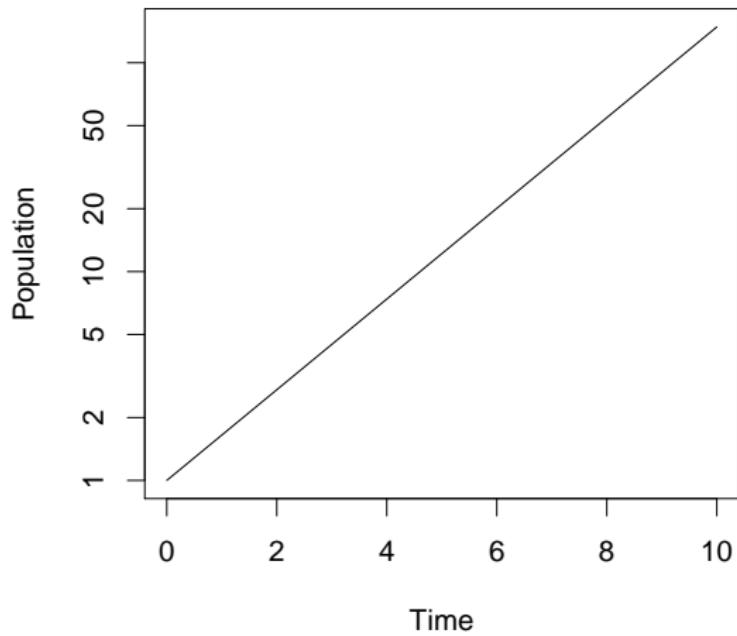
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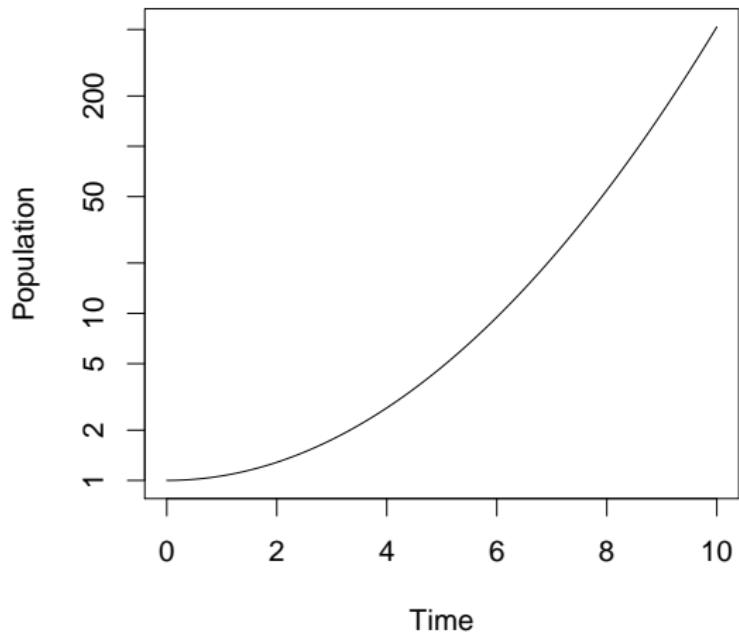
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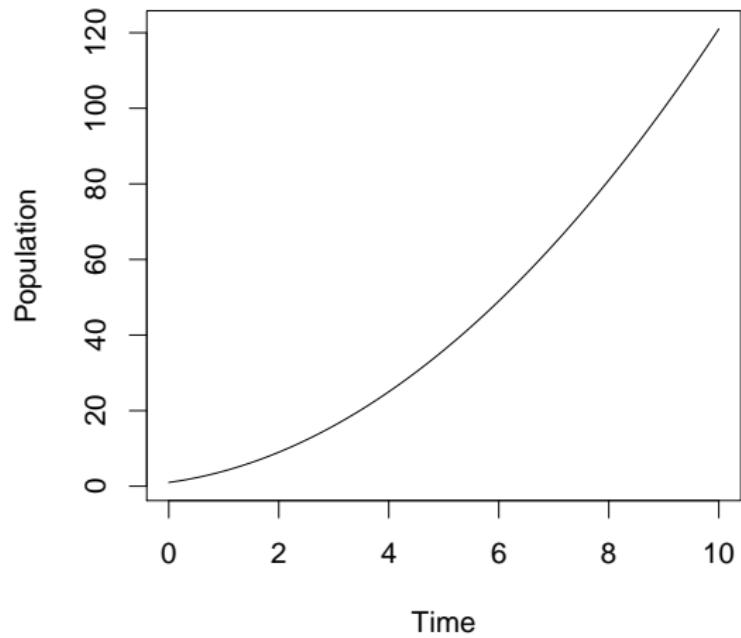
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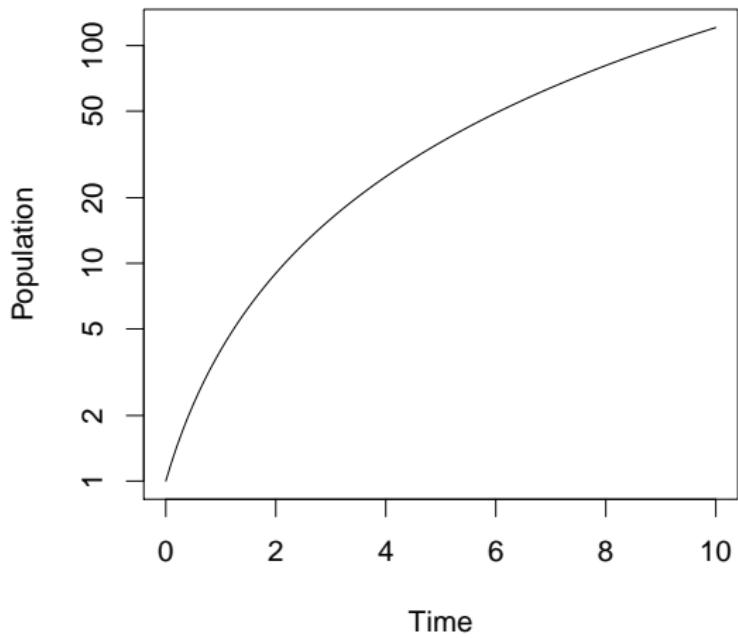
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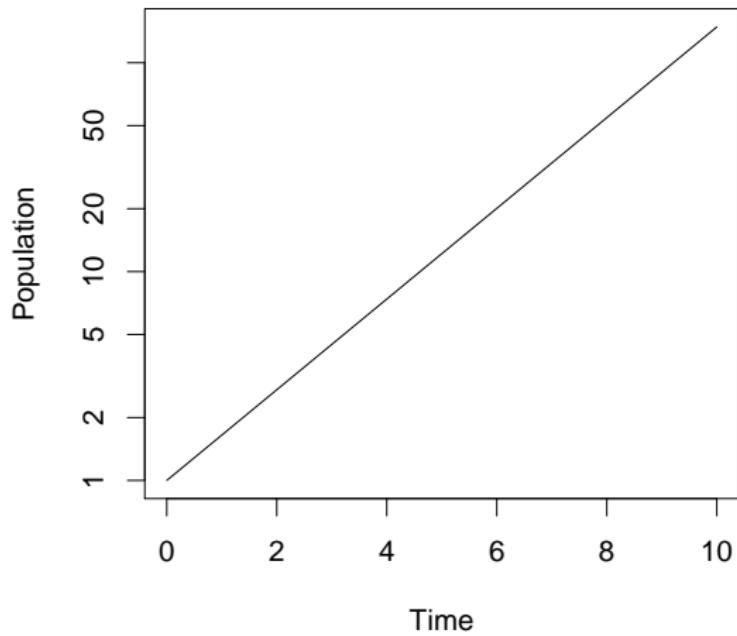
A



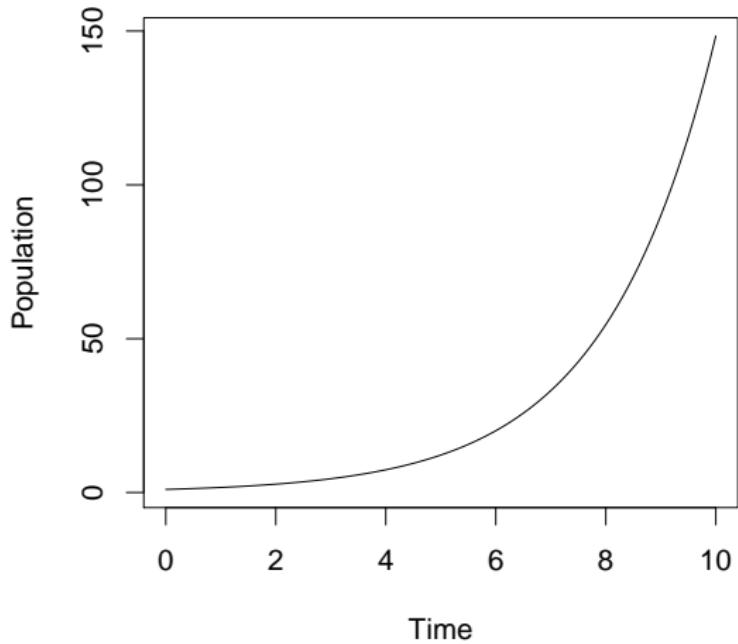
A on the log scale



C



C on the linear scale



Types of growth

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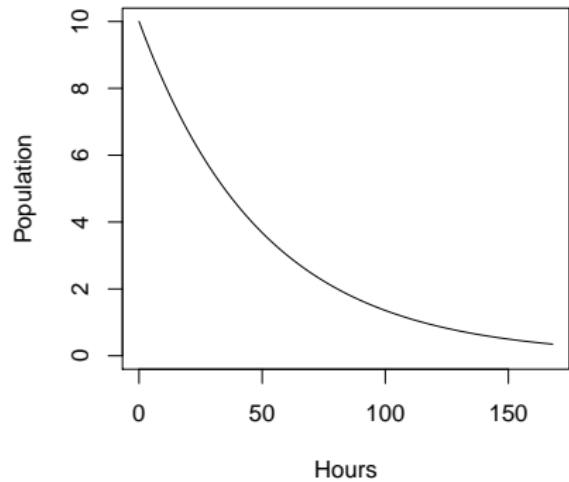
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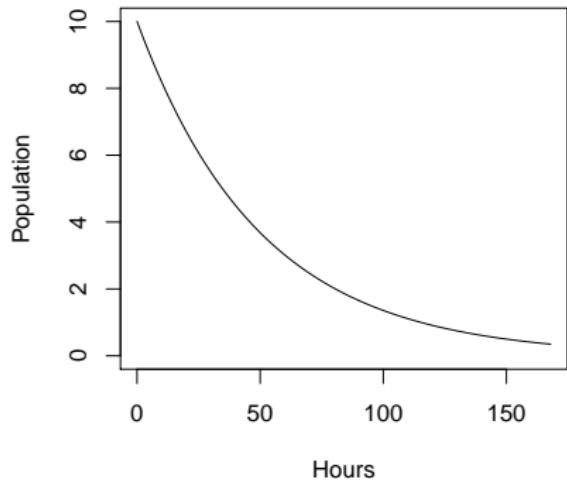
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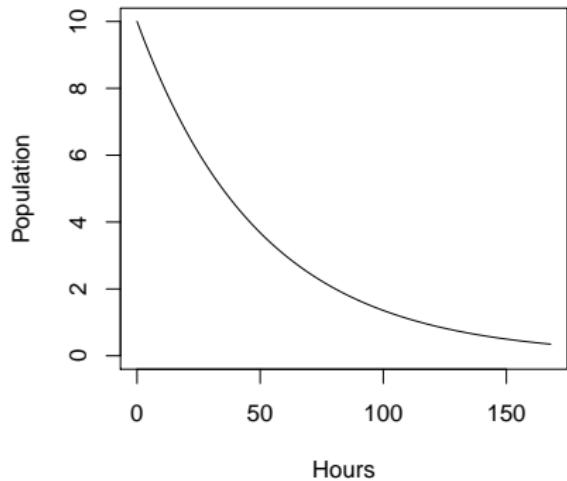
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Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

Example populations

Dandelions

Gypsy moths

Bacteria

Exponential growth

Log and linear scales

Time scales

Scales of comparison

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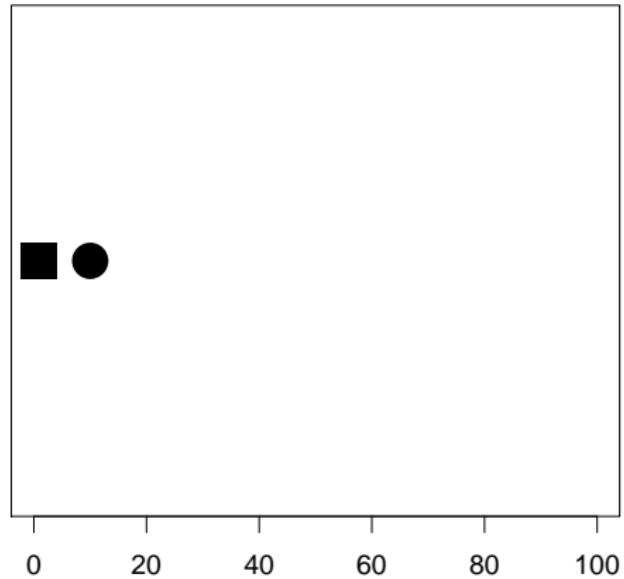
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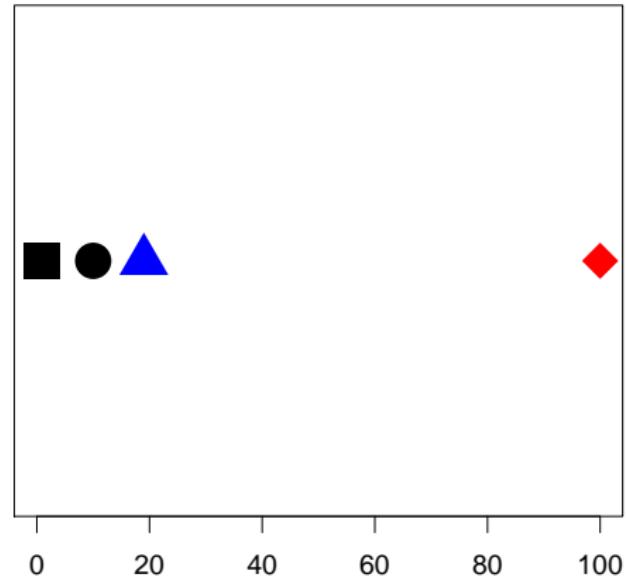
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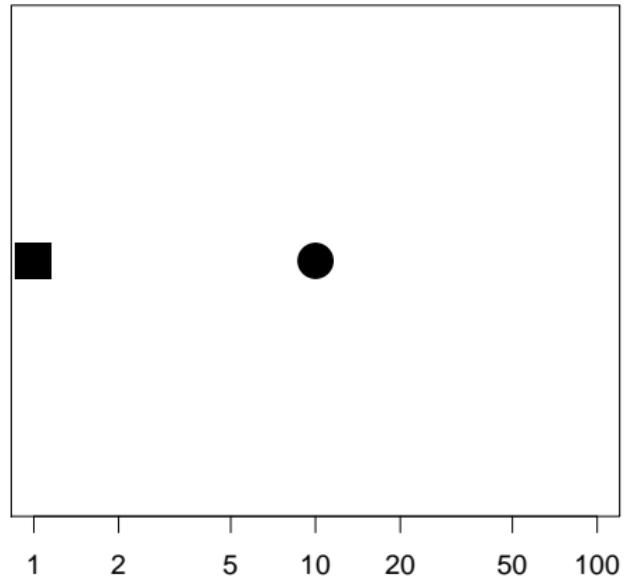
Scales of display



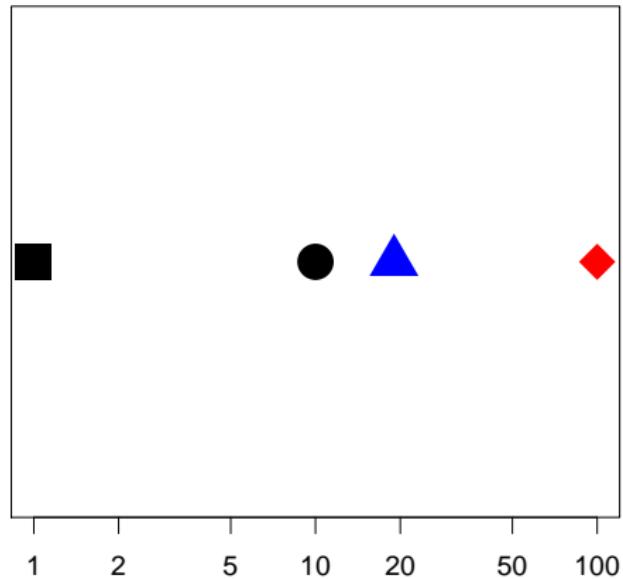
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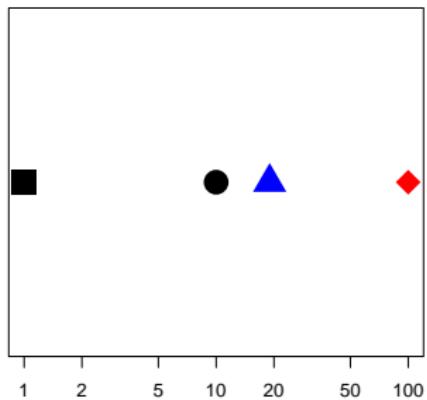
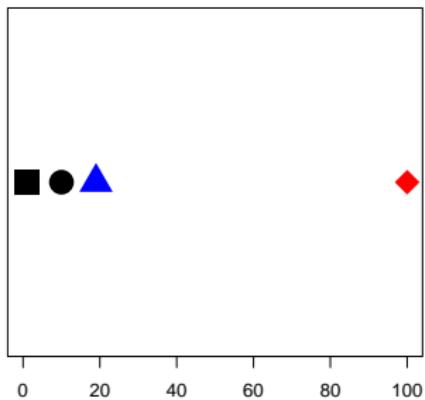
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There is only one log scale; it doesn't matter which base you use!

Canadian provinces

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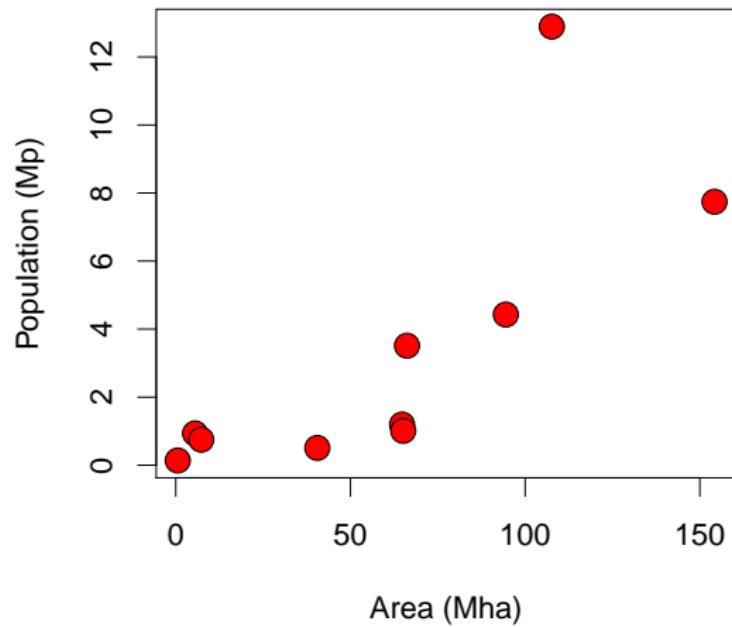
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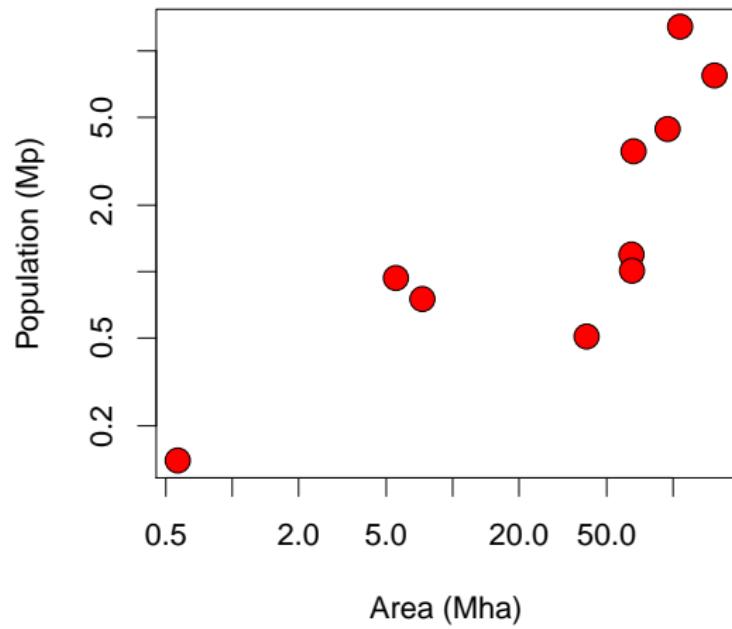
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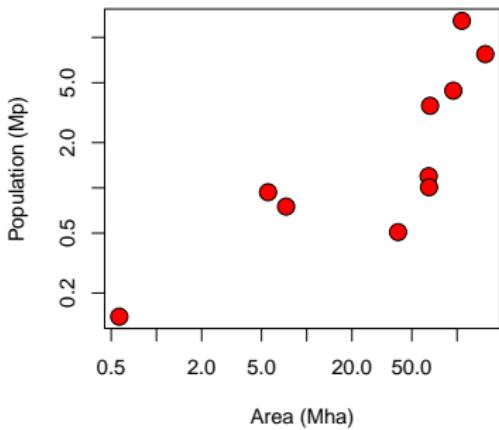
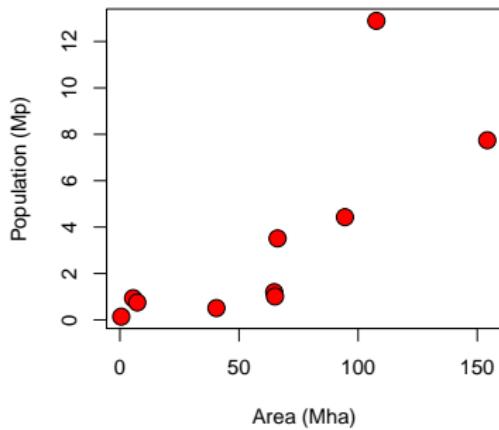
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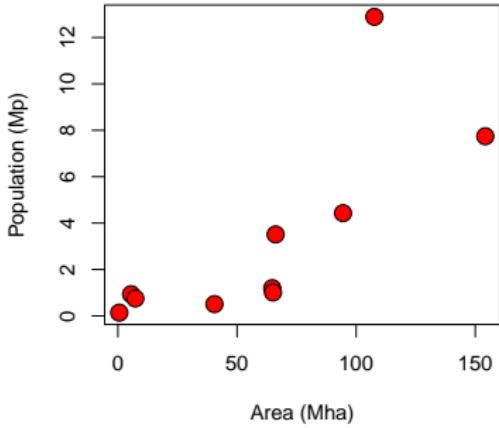
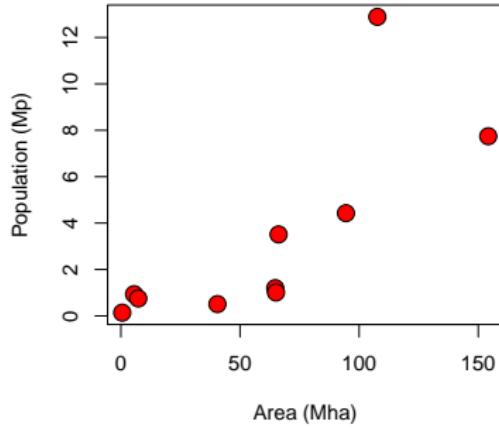
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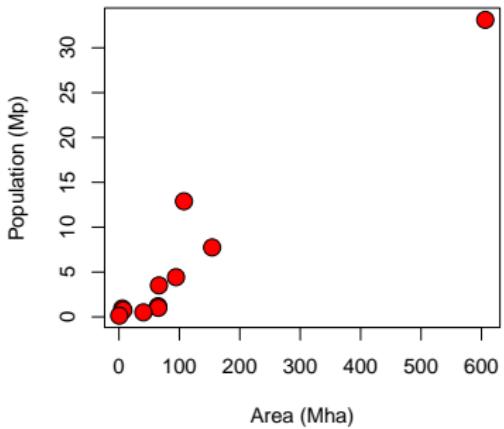
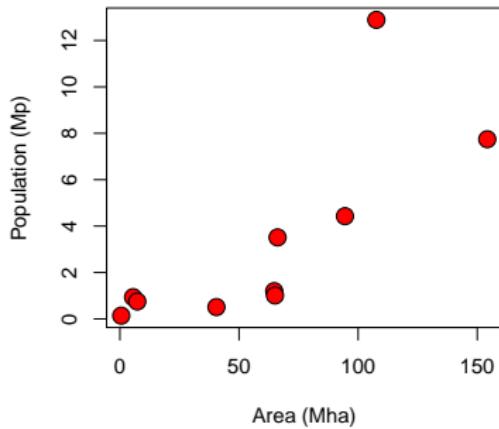
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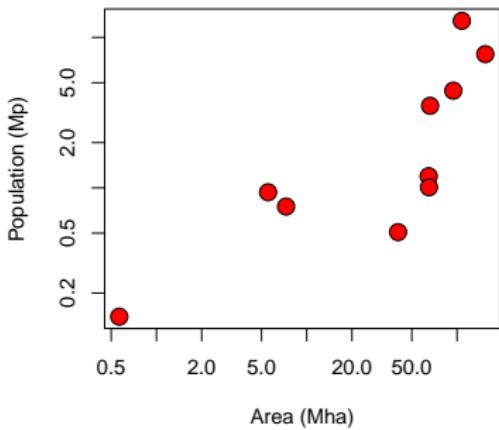
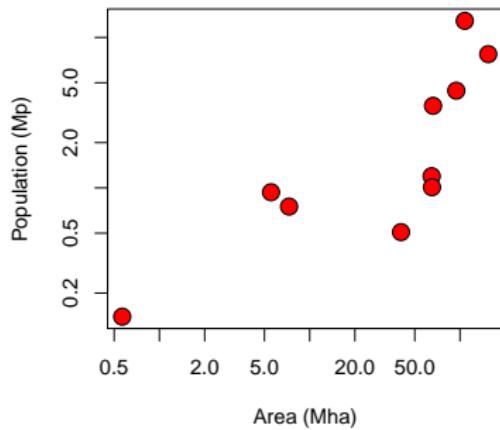
Canadian provinces plus Canada?



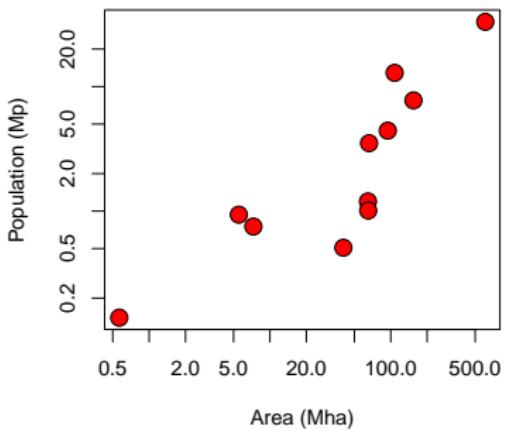
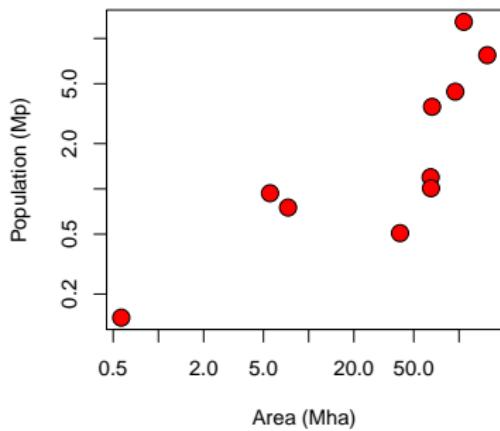
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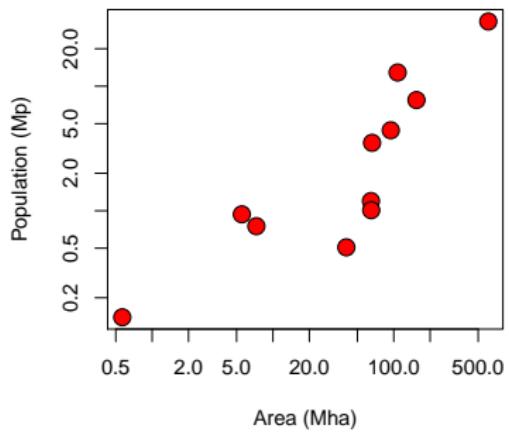
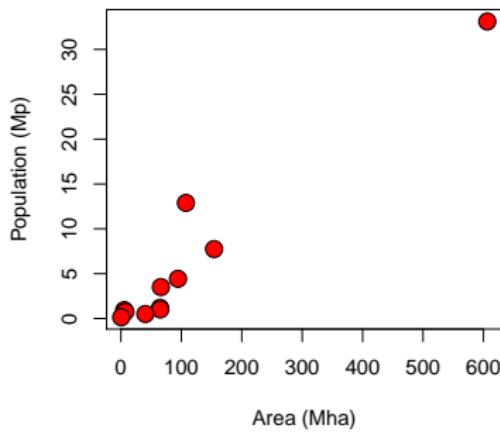
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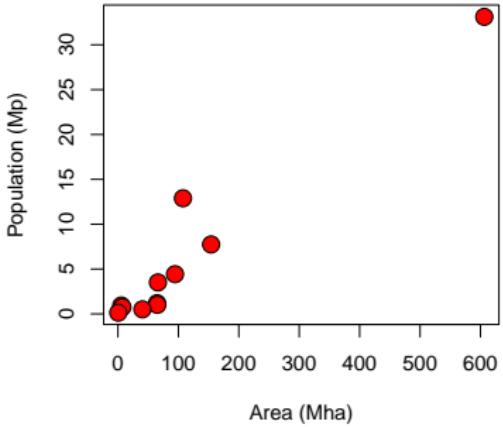
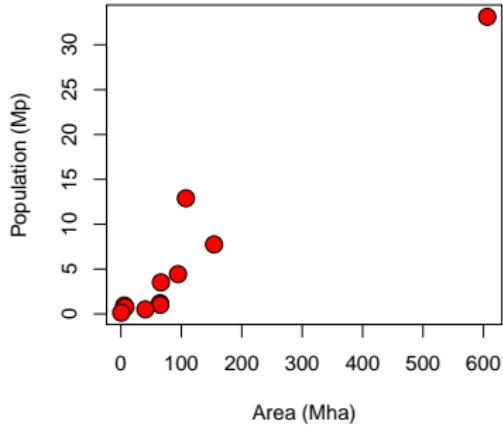
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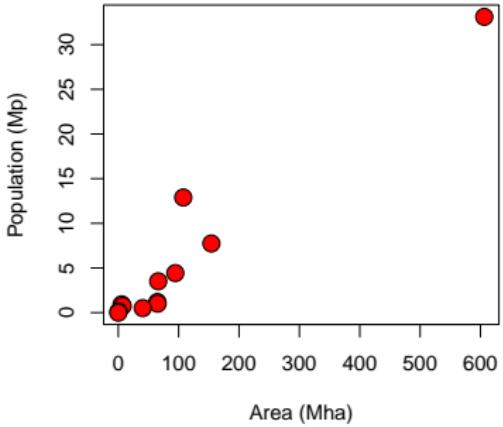
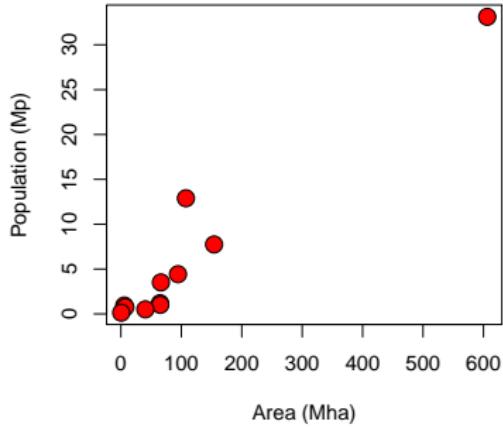
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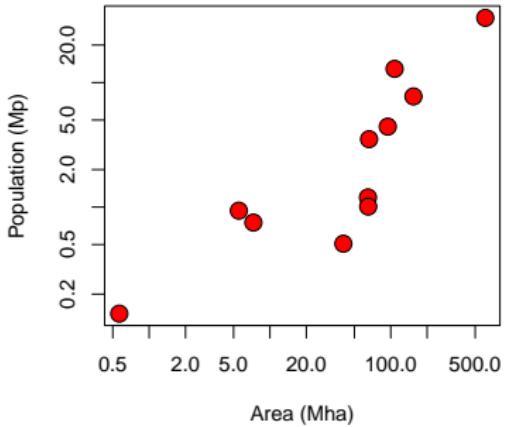
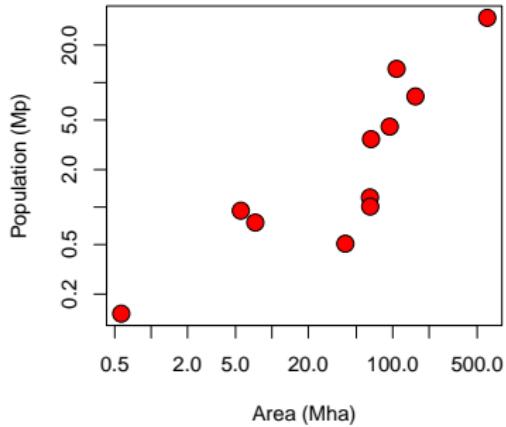
Canada plus B135



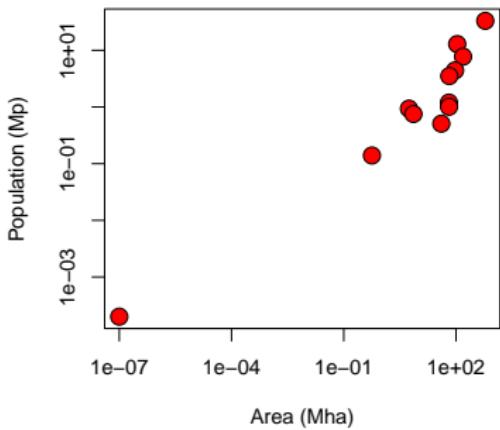
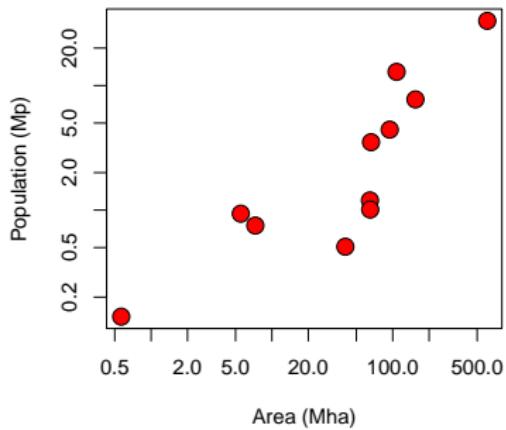
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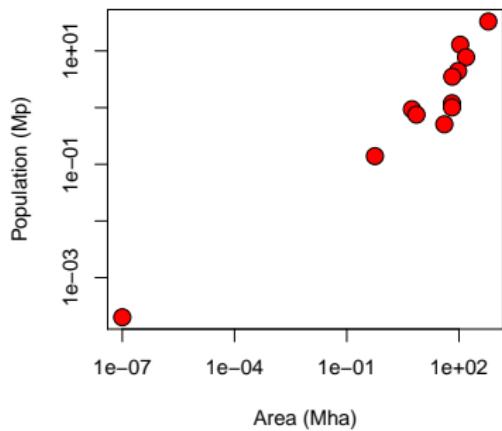
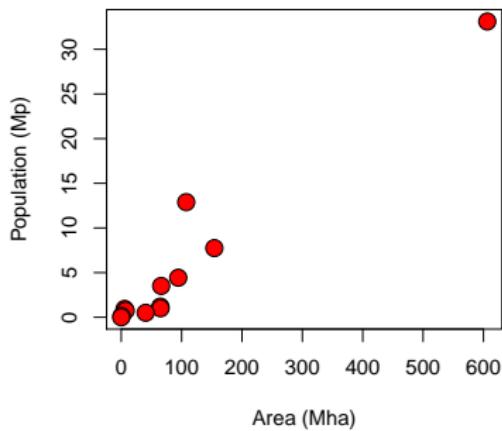
Canada plus room B135?



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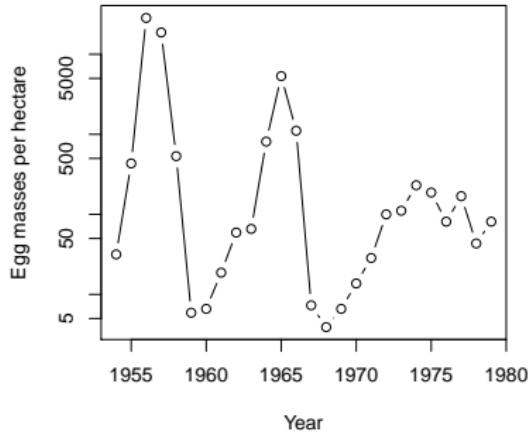
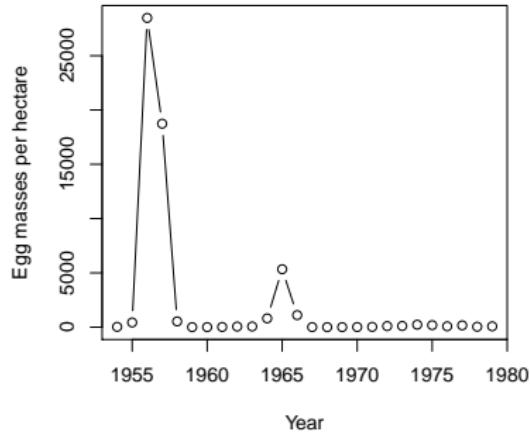
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Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

Example populations

Dandelions

Gypsy moths

Bacteria

Exponential growth

Log and linear scales

Time scales

Speeding in Taiwan

- A life experience



Speeding in Taiwan

- ▶ A life experience
- ▶ Some clarifications



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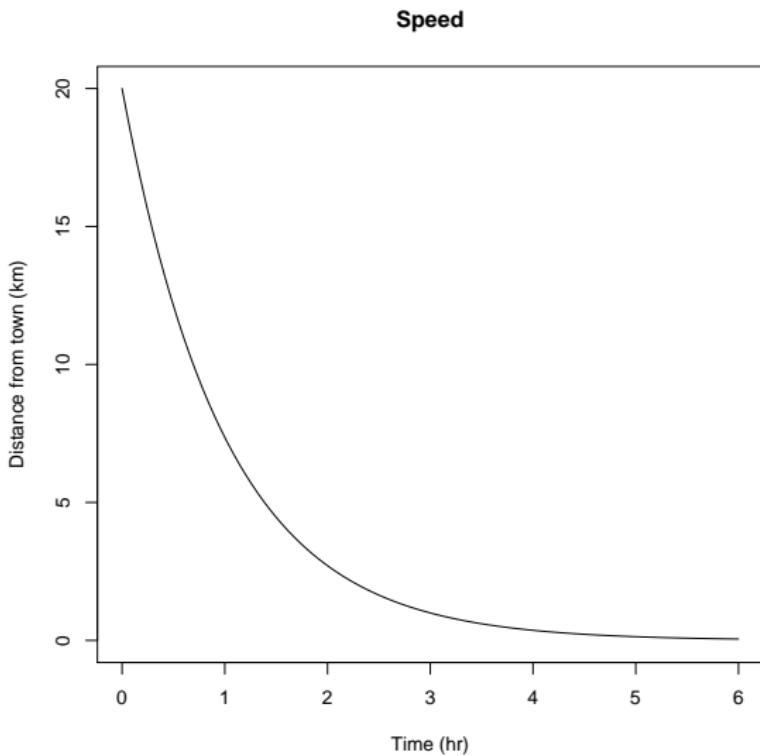
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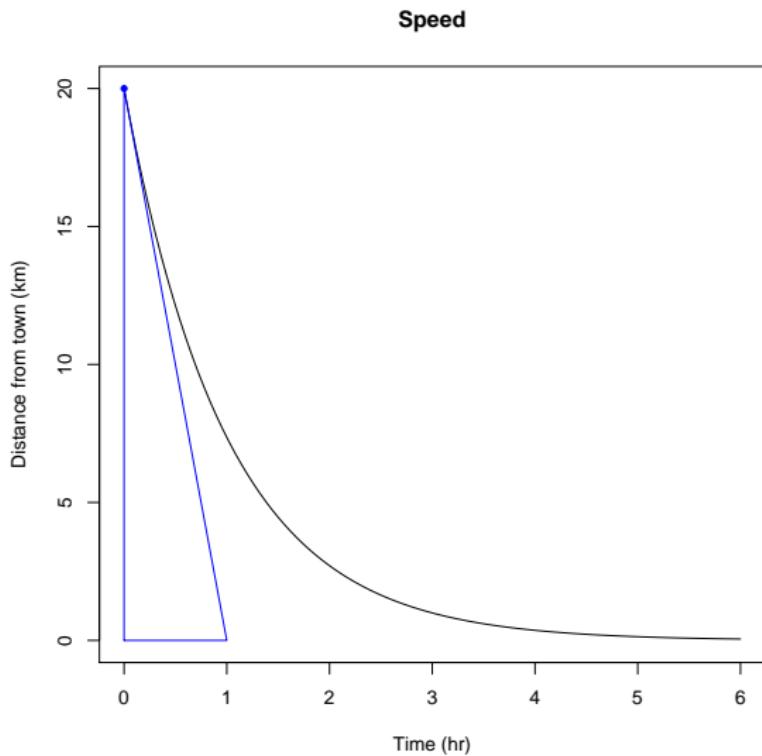
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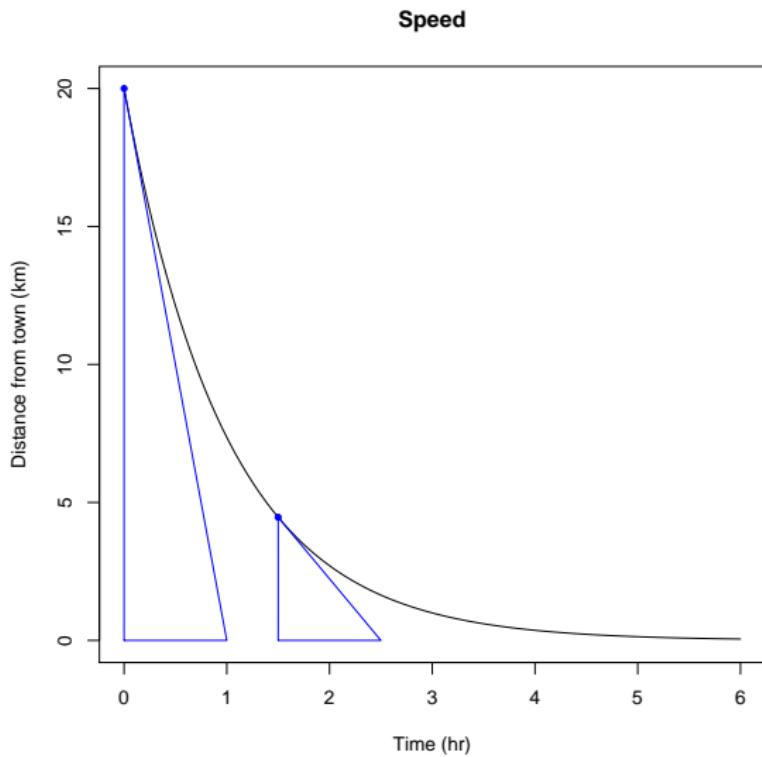
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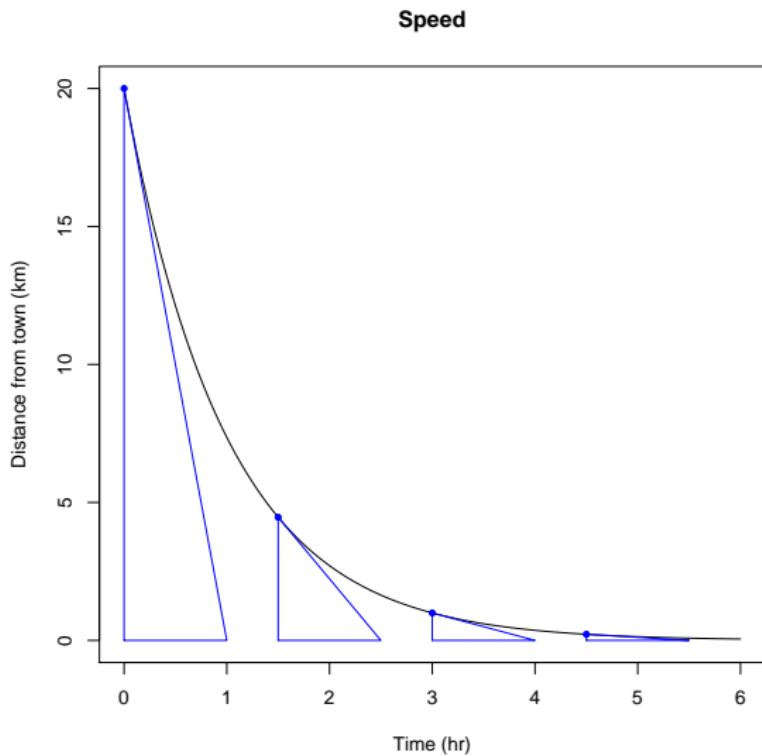
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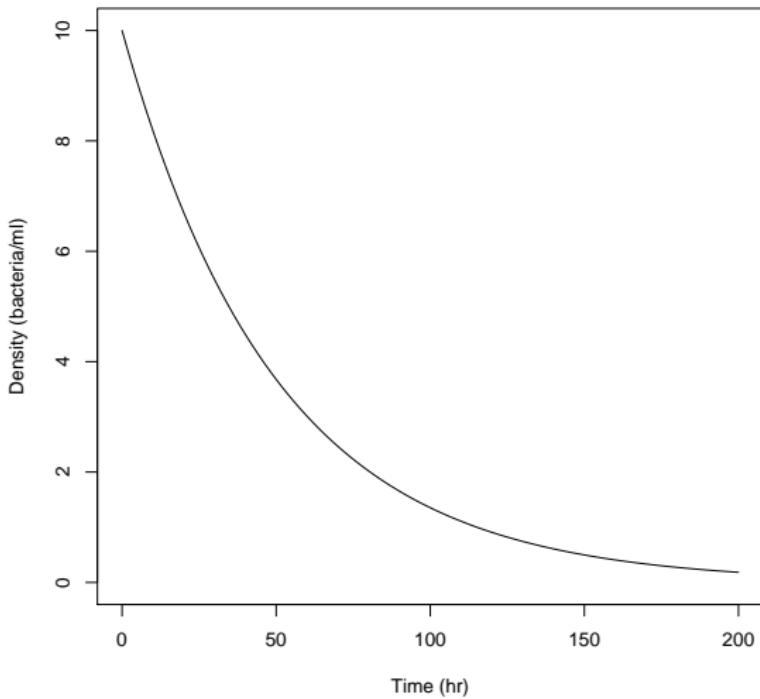
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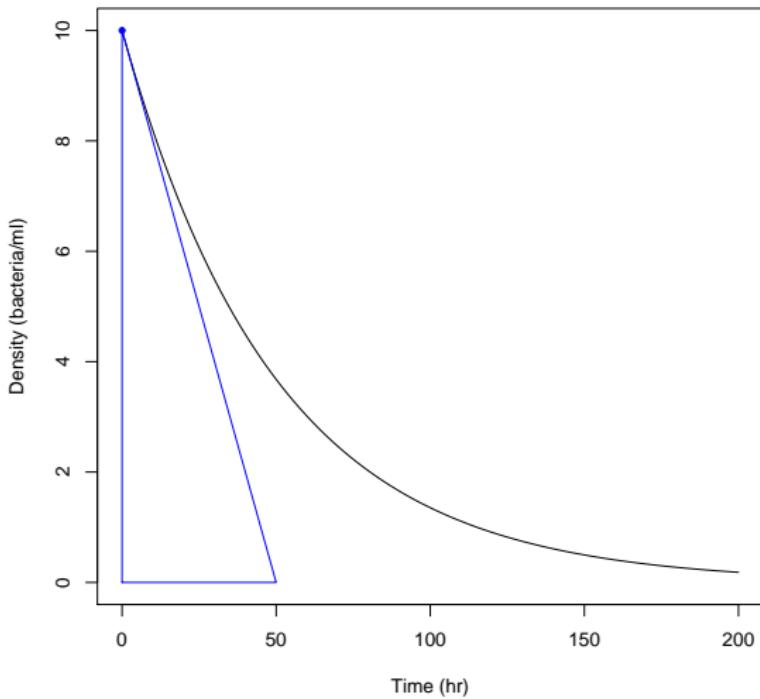
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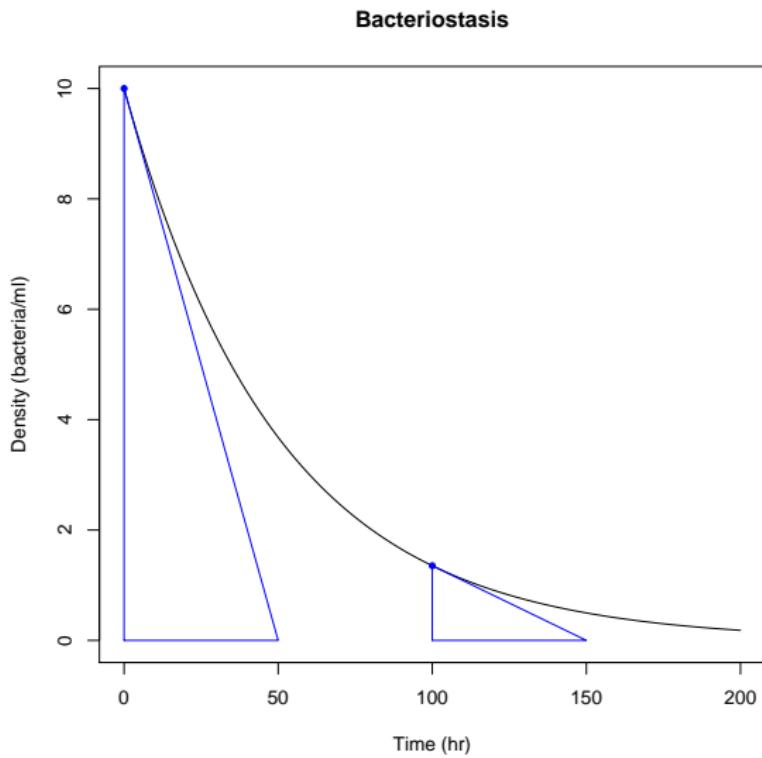


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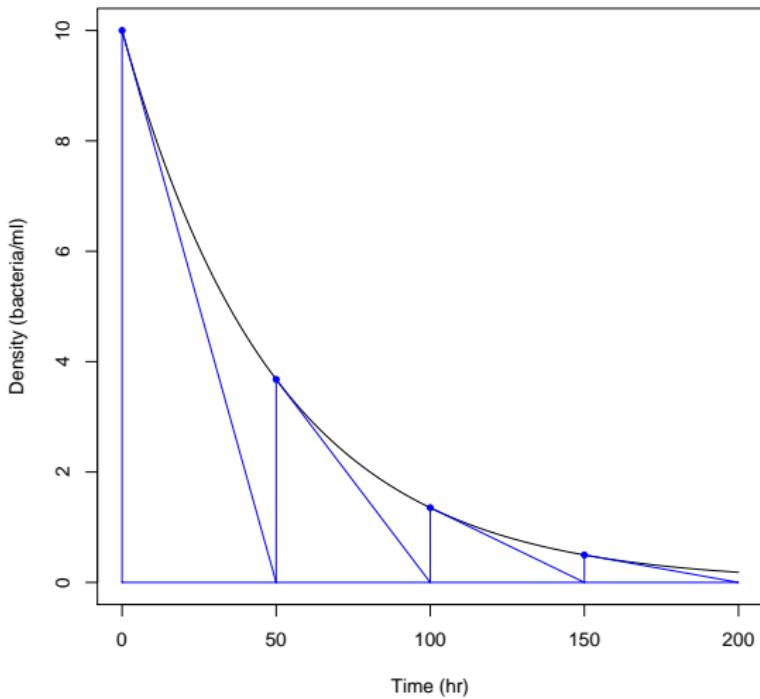


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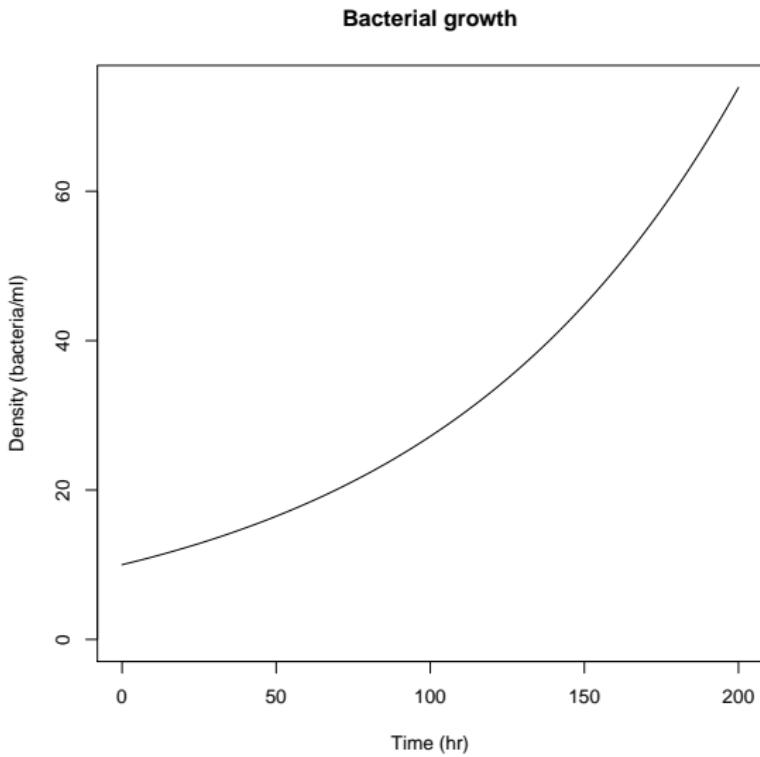
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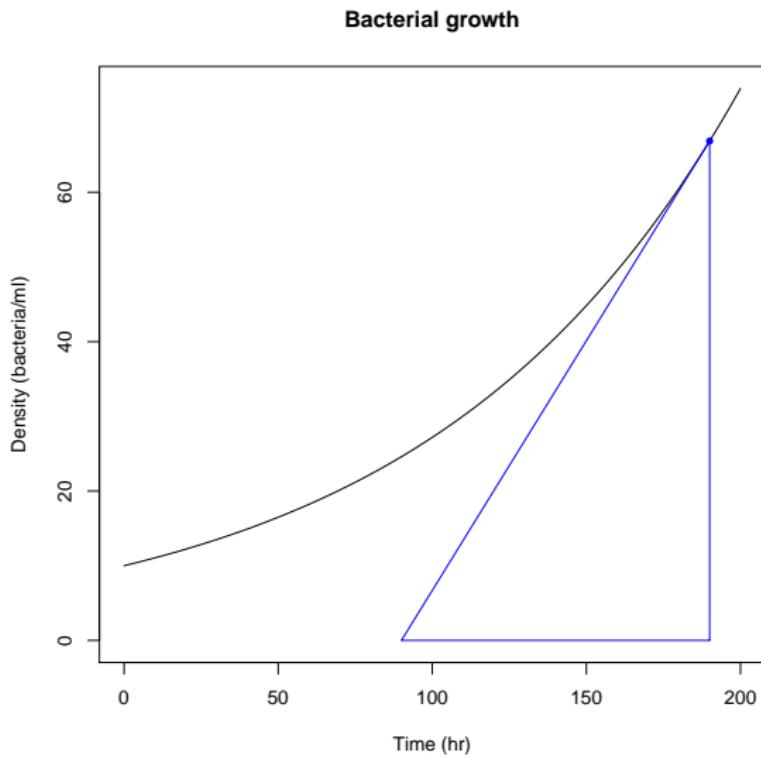
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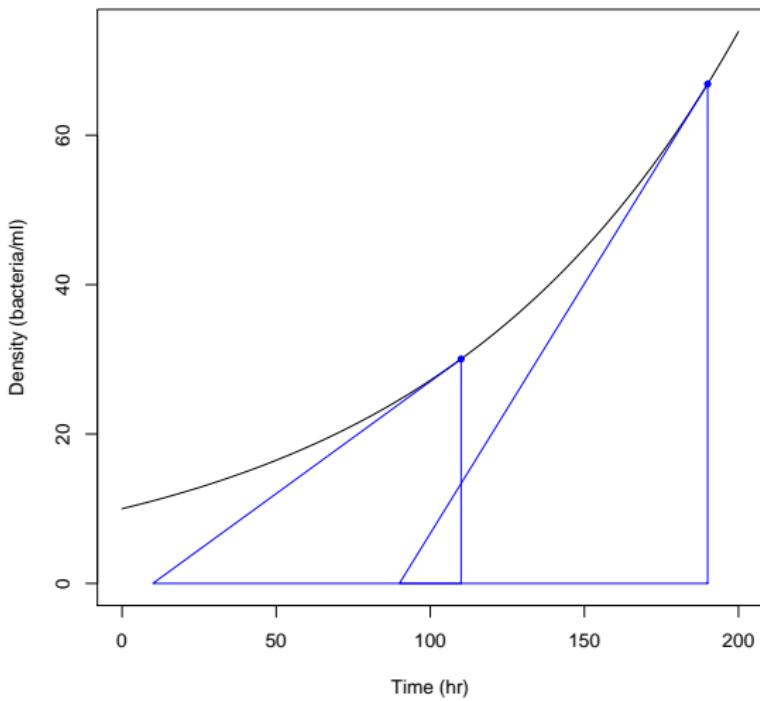


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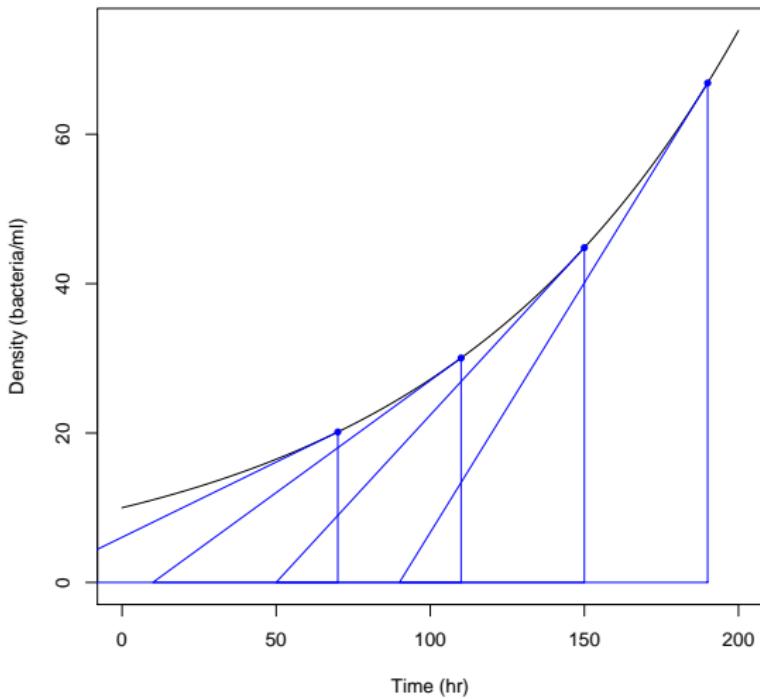
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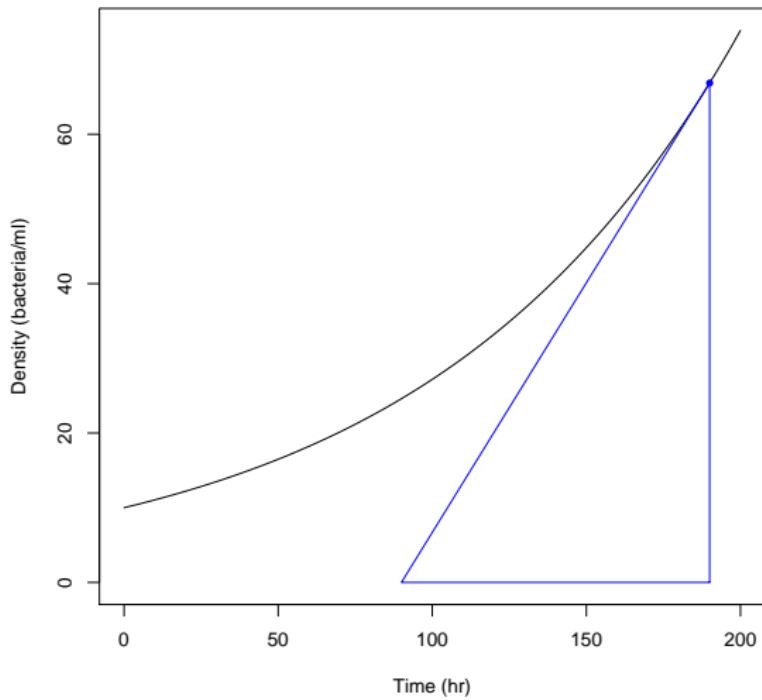
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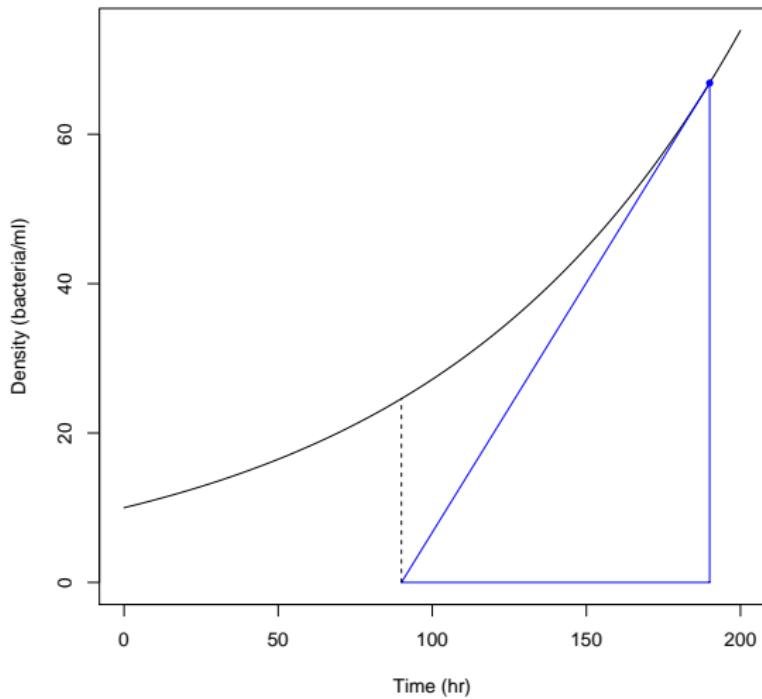
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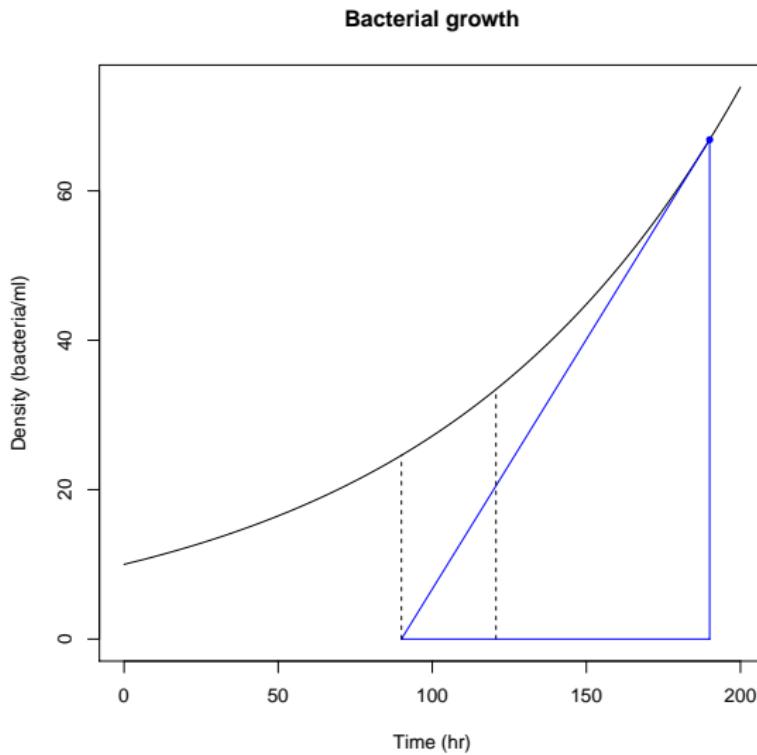
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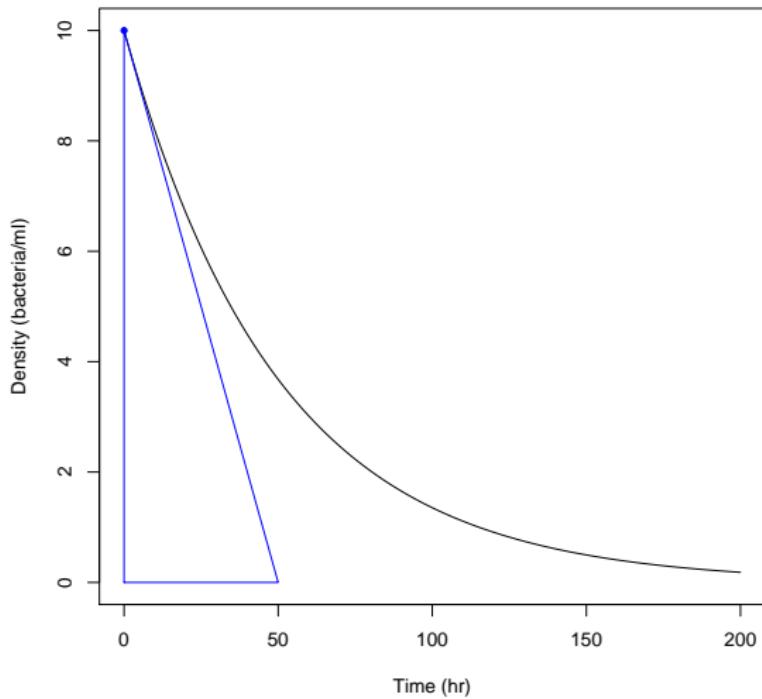
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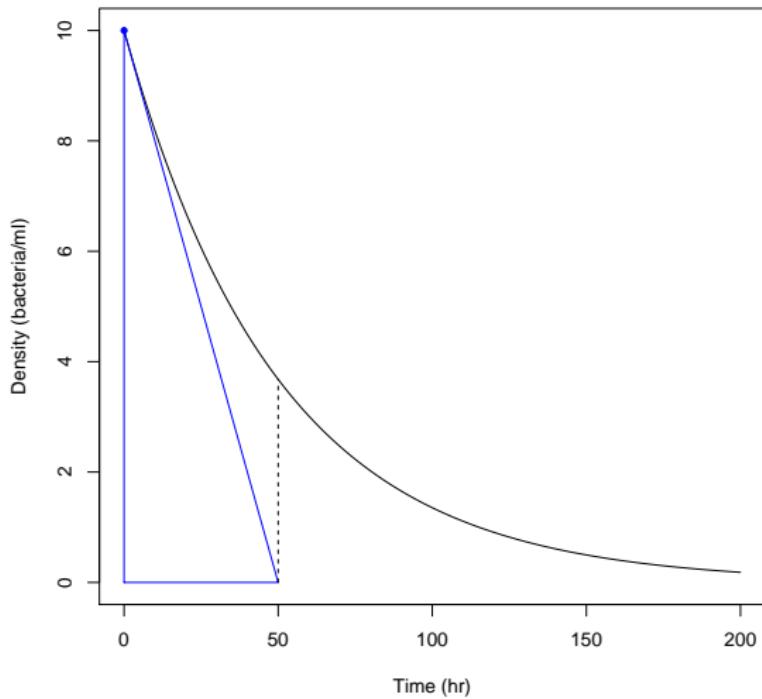
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