UNIT 3 Non-linear population models

Outline

Introduction

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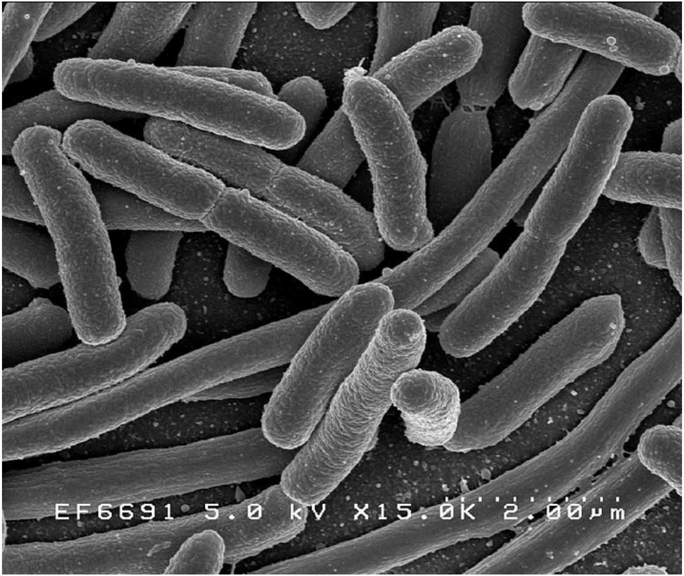
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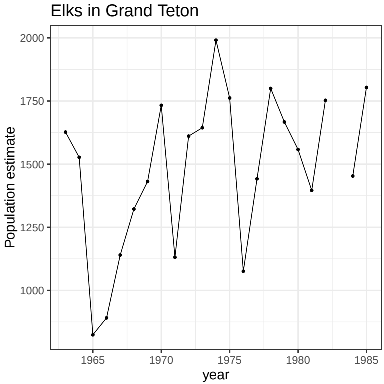
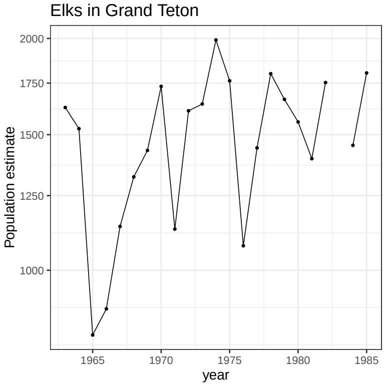
(preview)

Elk


Elks in Grand Teton
Population estimate
2000
1750
1500
1250
1000
1965
1970
1975
1980
1985
year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Population estimate
2000
1750
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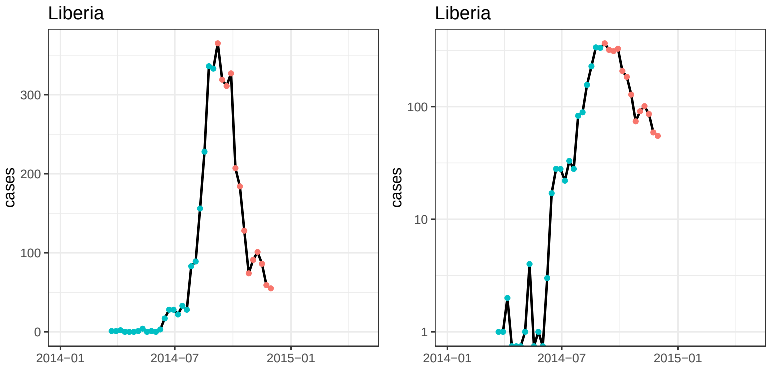
year

Ebola


Liberia
cases
300
200
100
0
2014−01
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Liberia
cases
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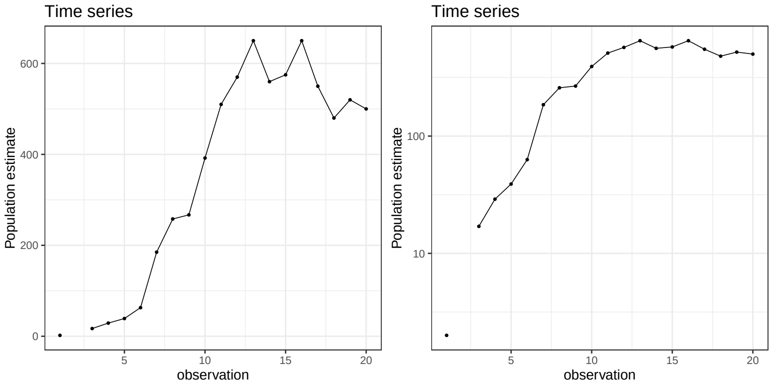
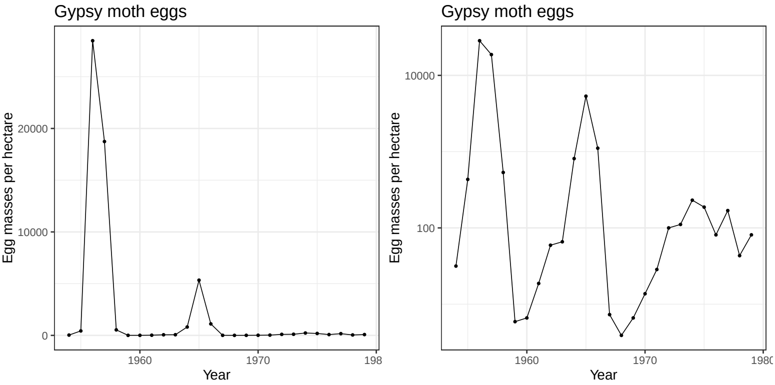
Paramecia (preview)



Gypsy moths (preview)



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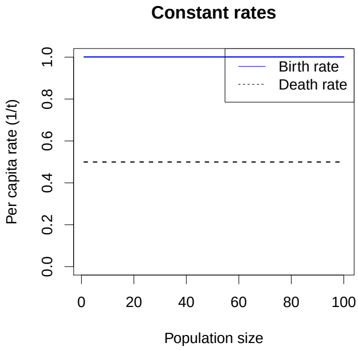

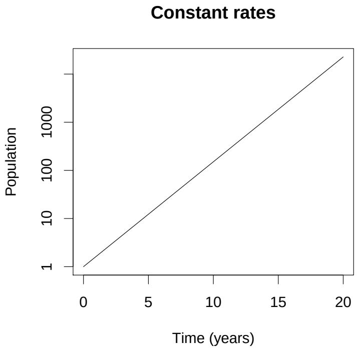

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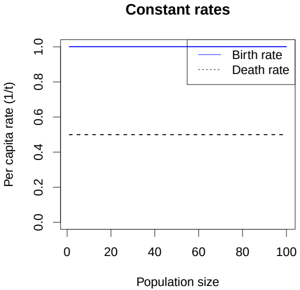

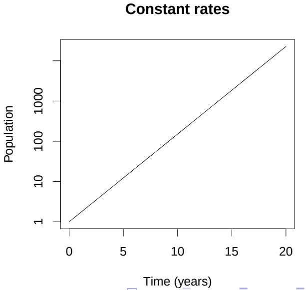

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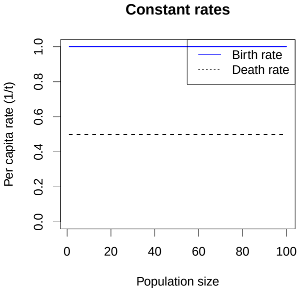

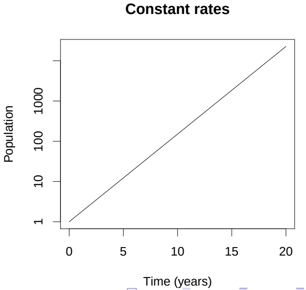

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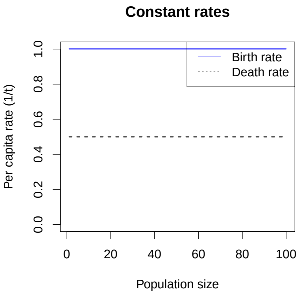

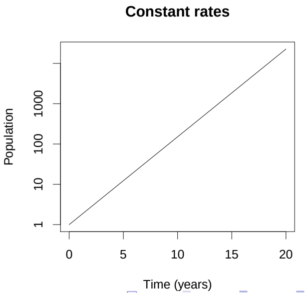

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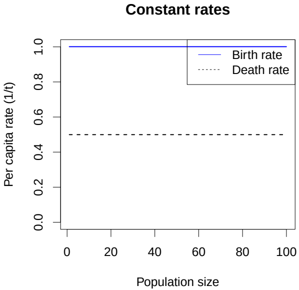

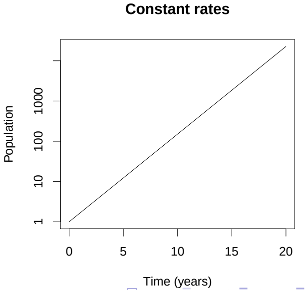

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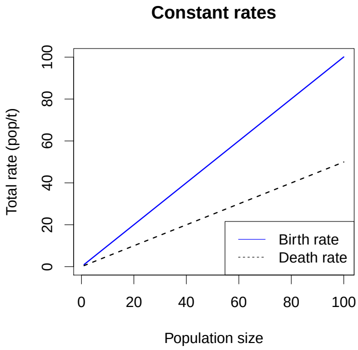

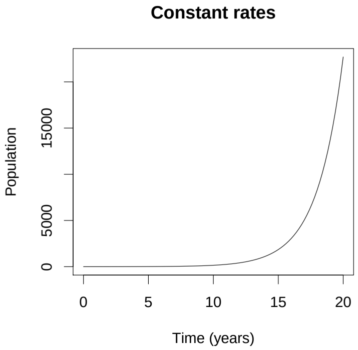

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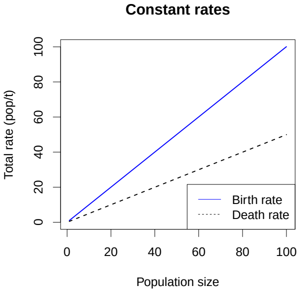

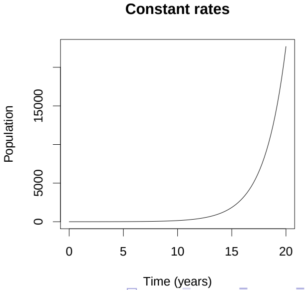

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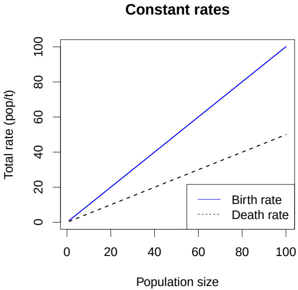

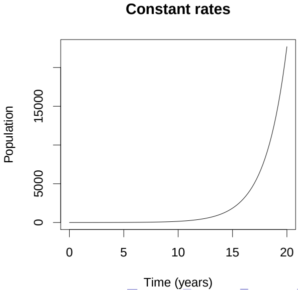

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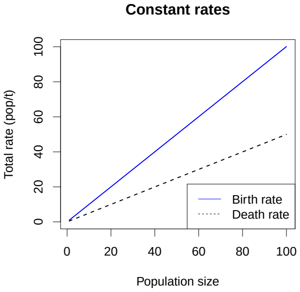

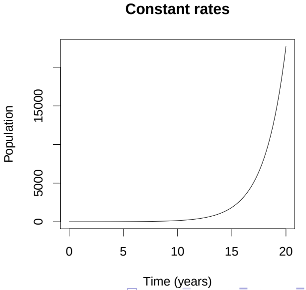

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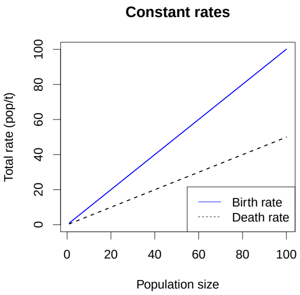

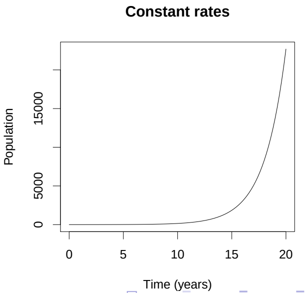

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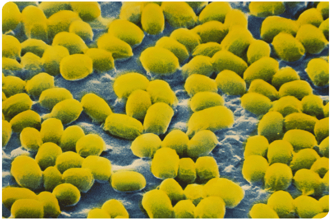

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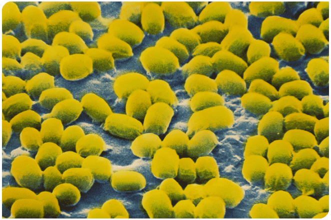

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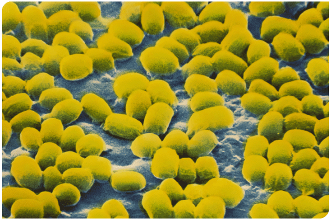

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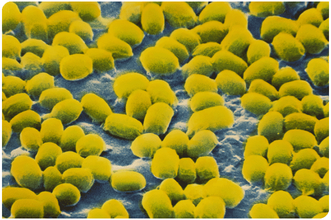

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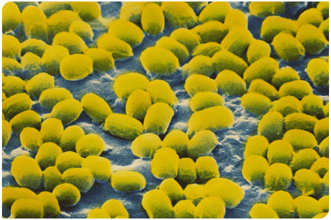

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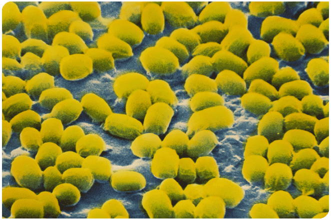

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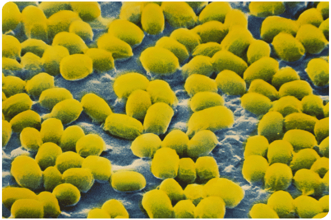

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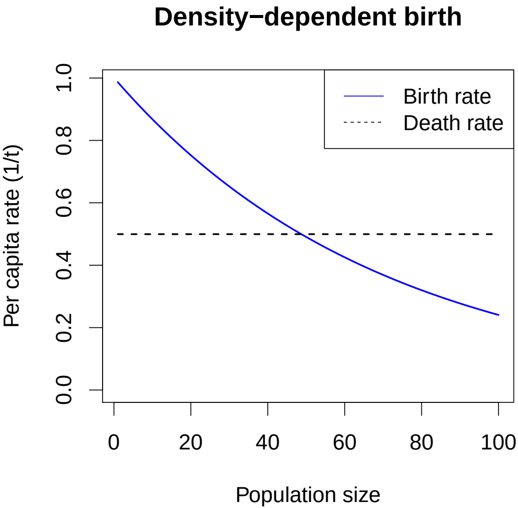

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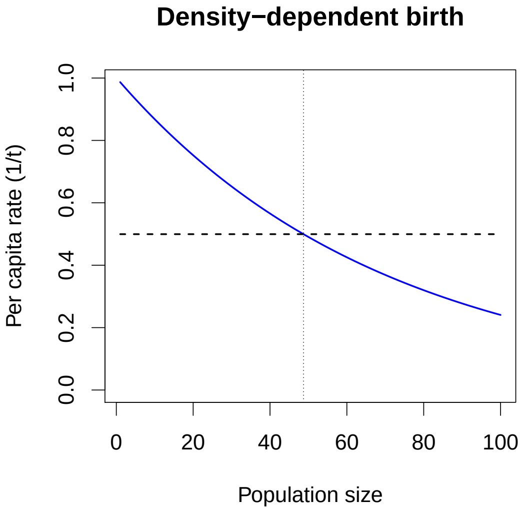

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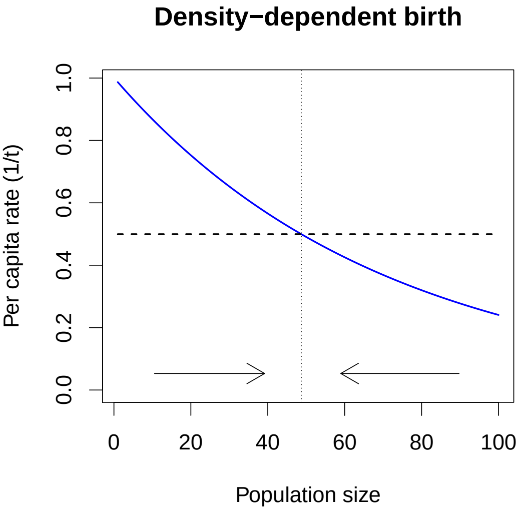

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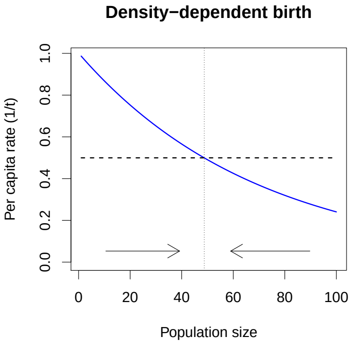

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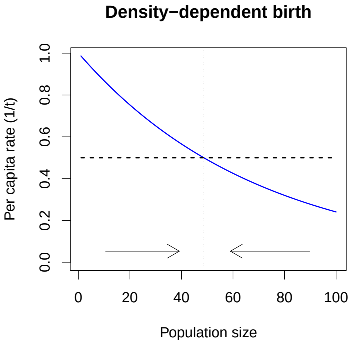

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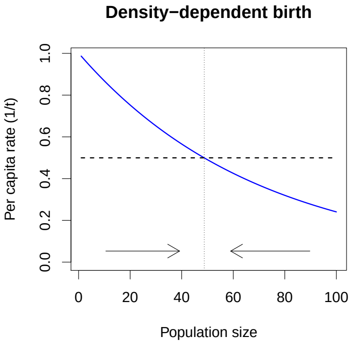

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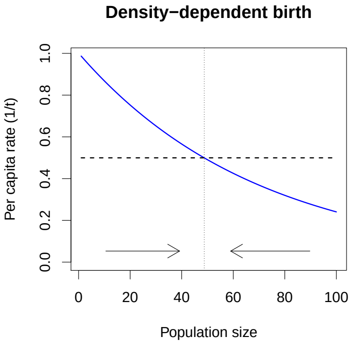

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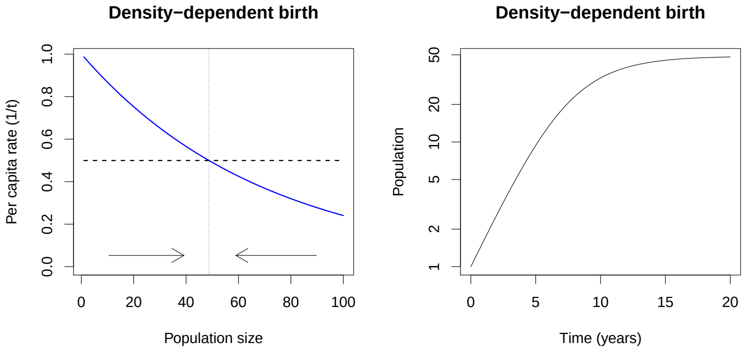

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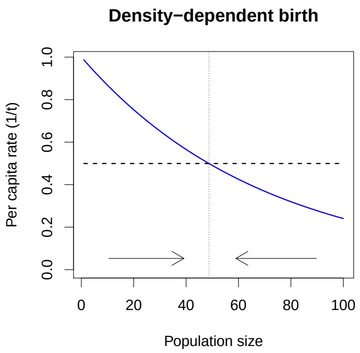

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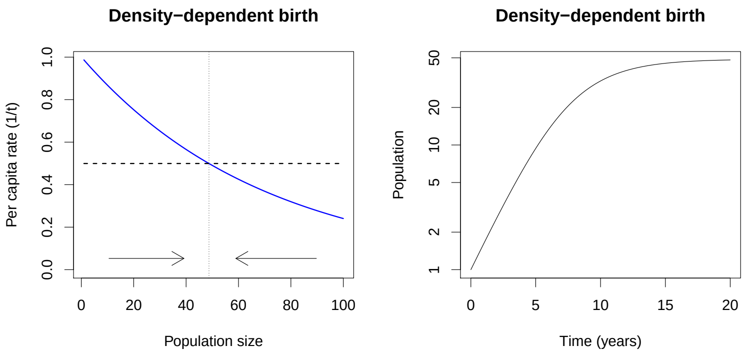
What will this model do? (repeat)



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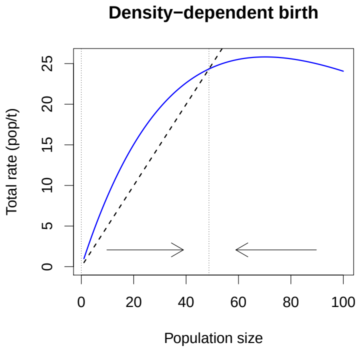

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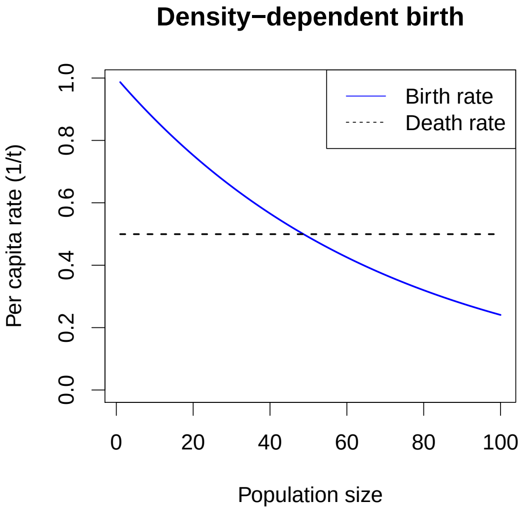

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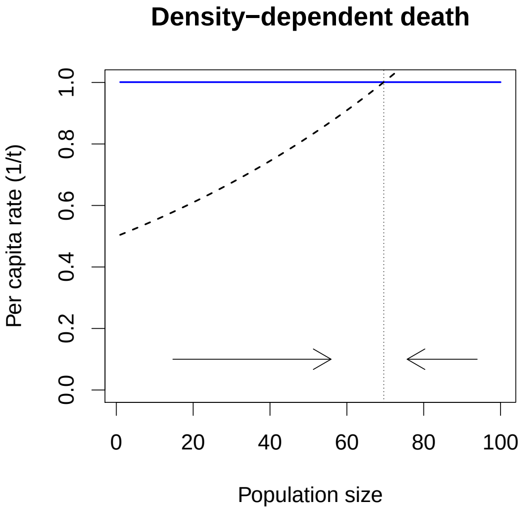

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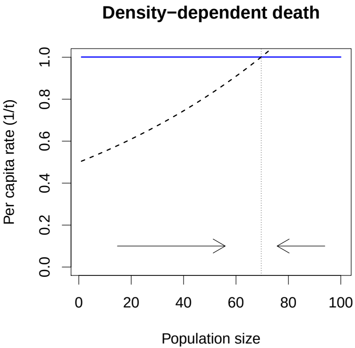

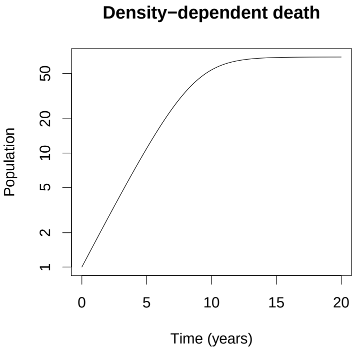

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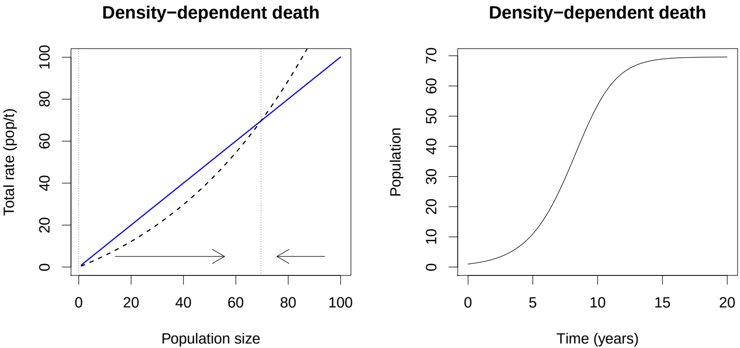

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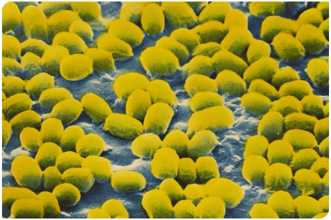

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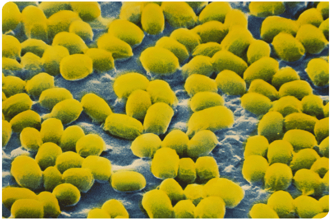

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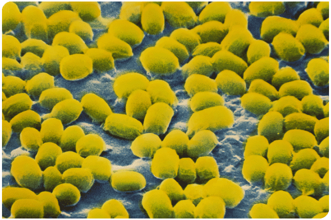

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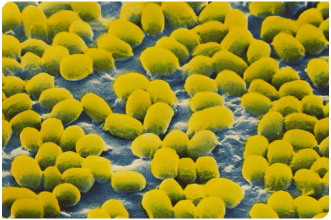

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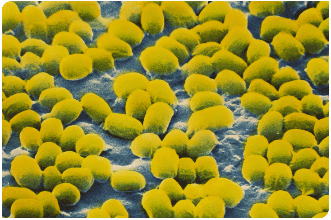

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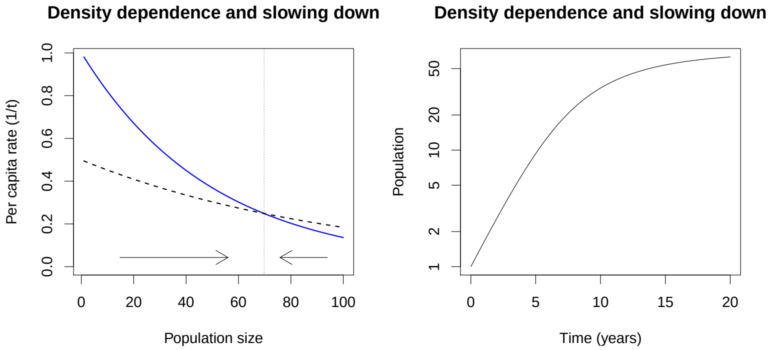

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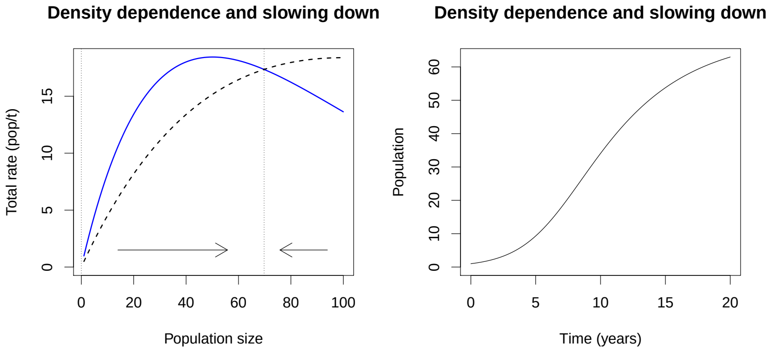

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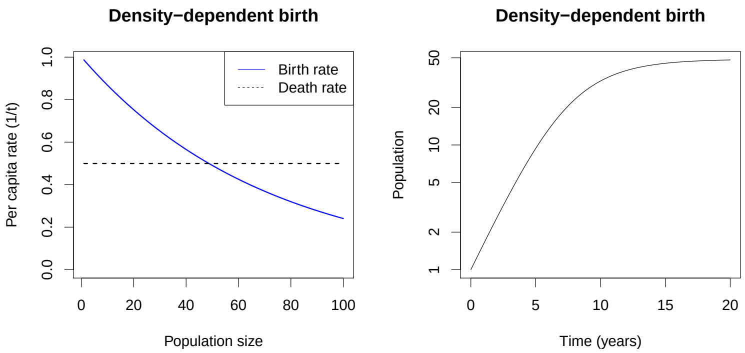

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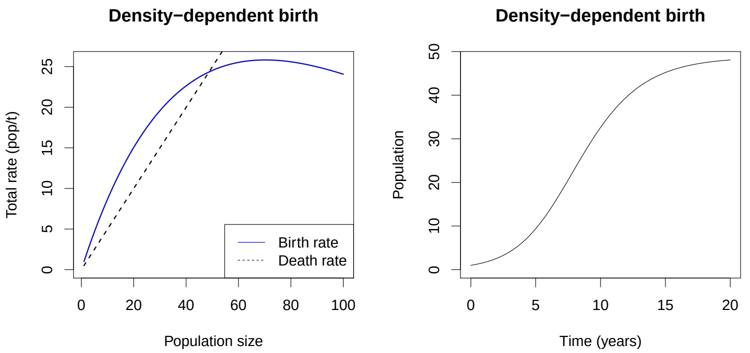

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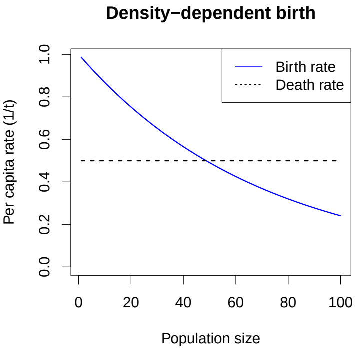

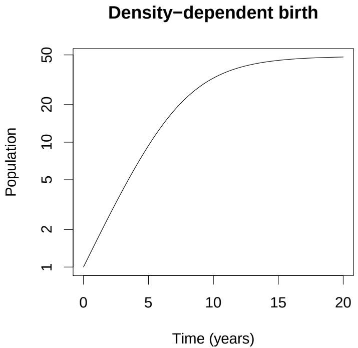

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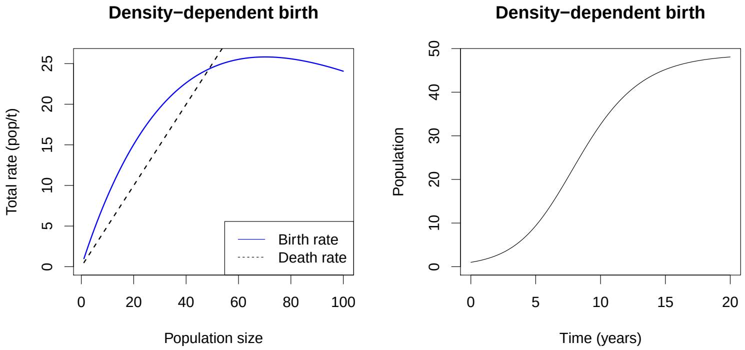

Outline

Equilibria and time scales

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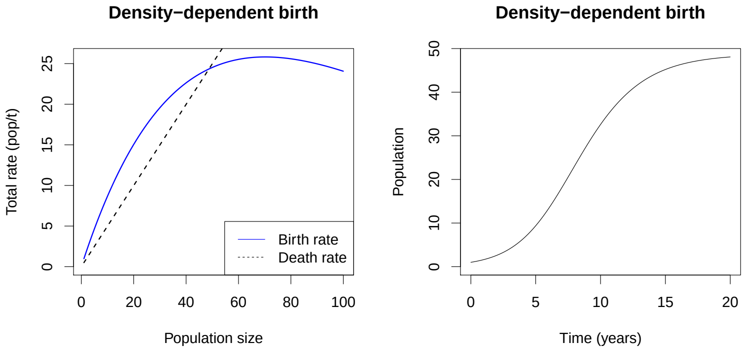

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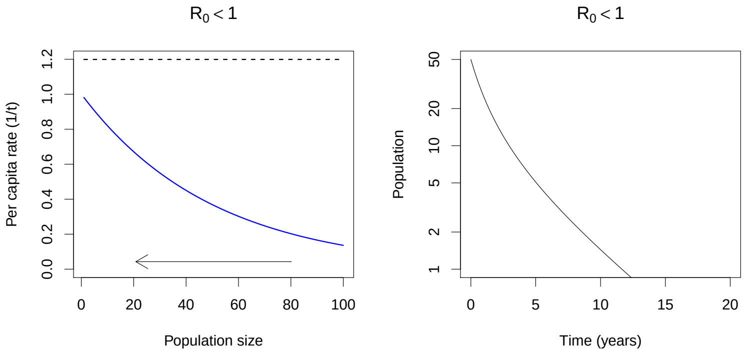

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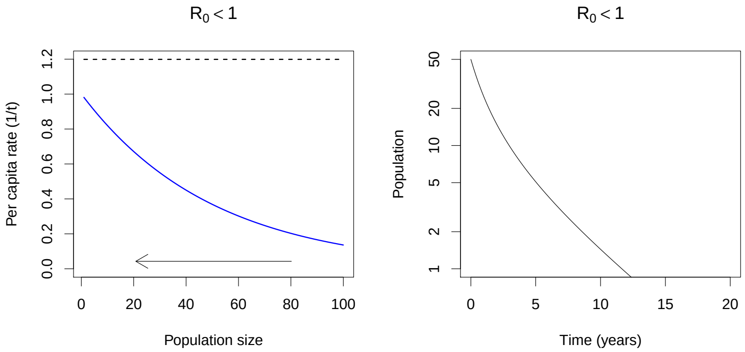

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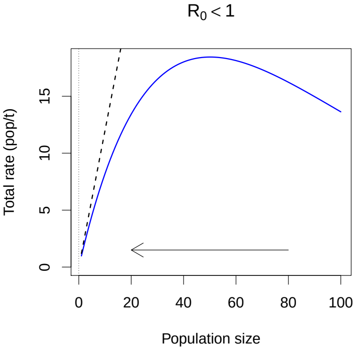

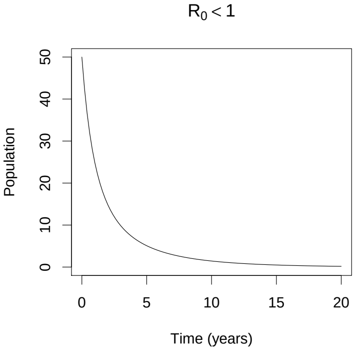

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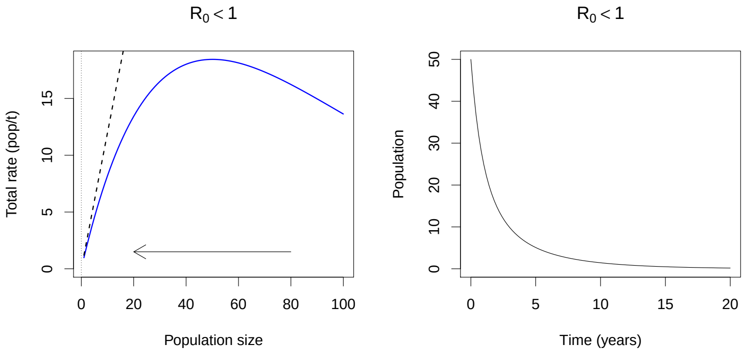

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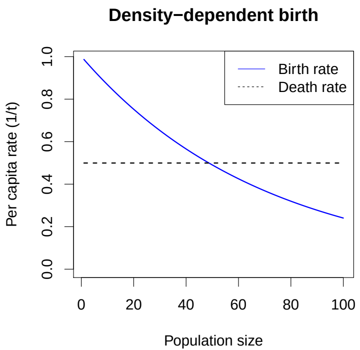

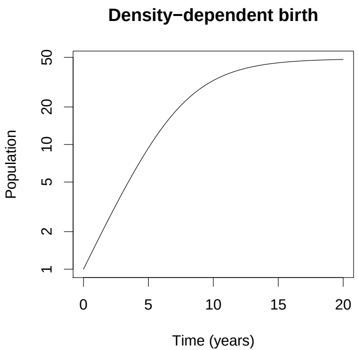

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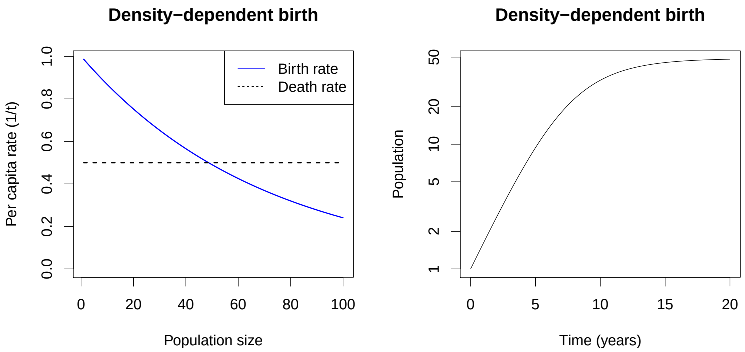

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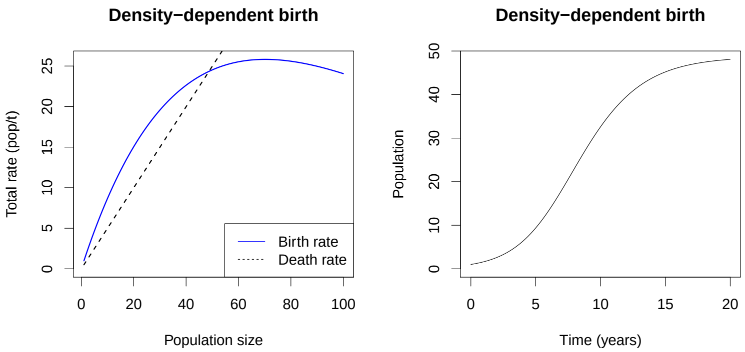

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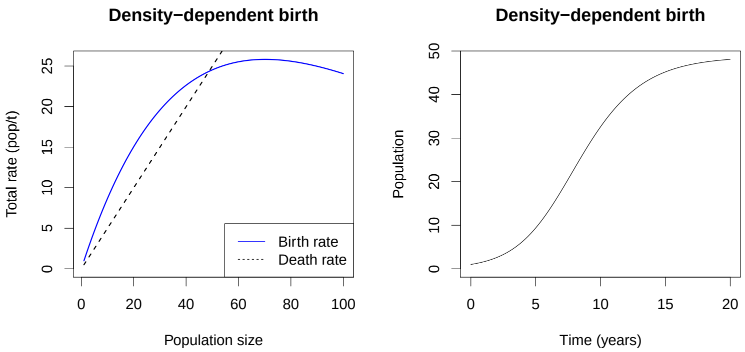

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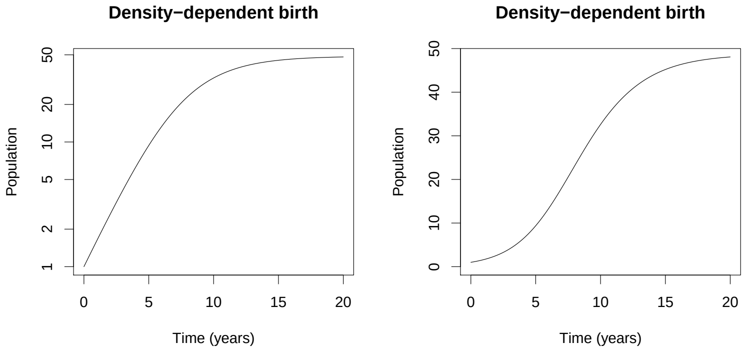

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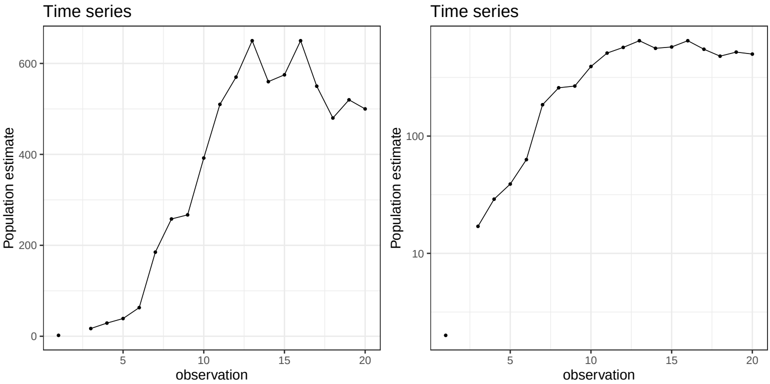
Paramecia



Time series
Population estimate
600
400
200
0
5
10
15
20
observation
Time series
Population estimate
100
10
5
10
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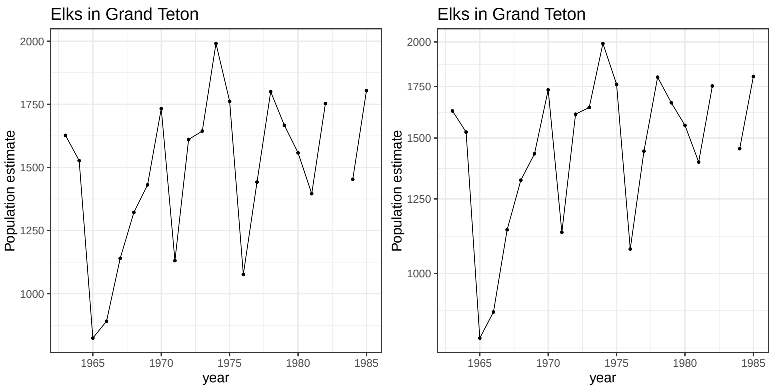
Elk



Elks in Grand Teton
Population estimate
2000
1750
1500
1250
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1965
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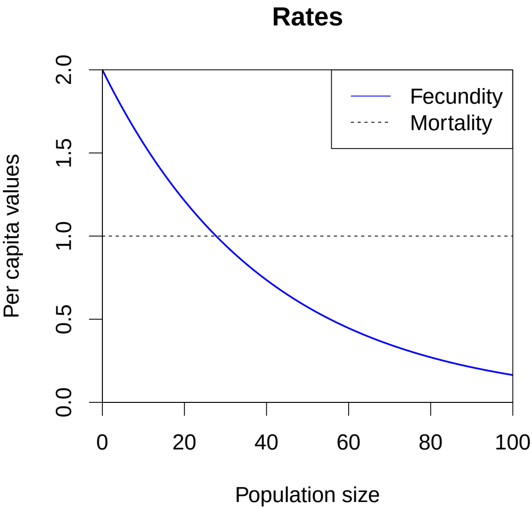

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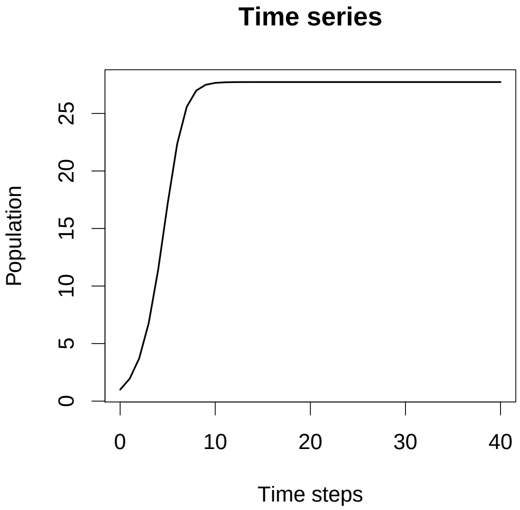

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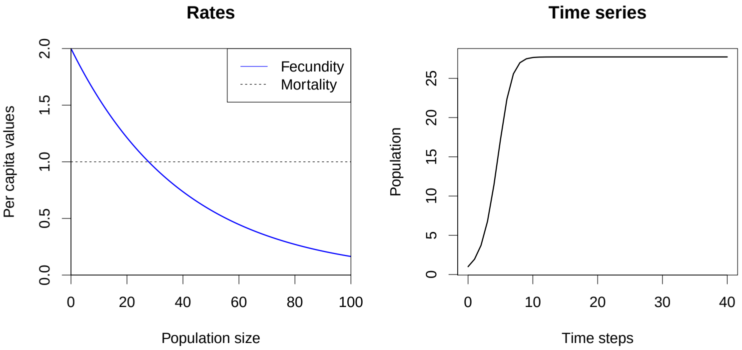
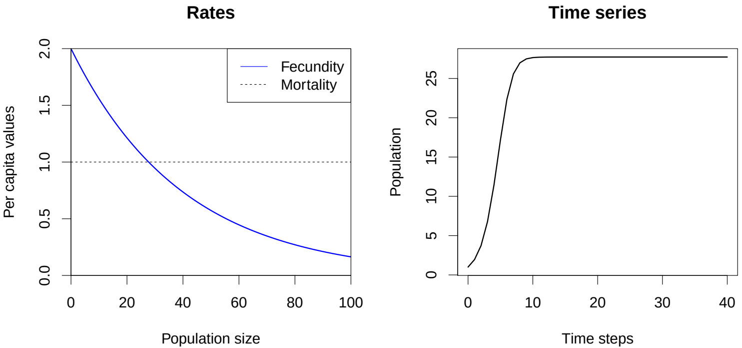
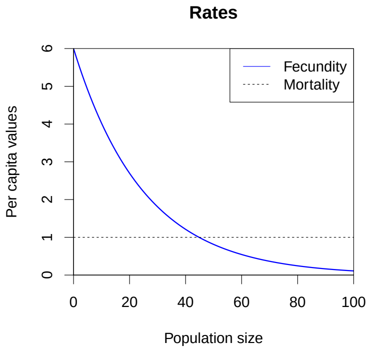


What dynamics do we get?

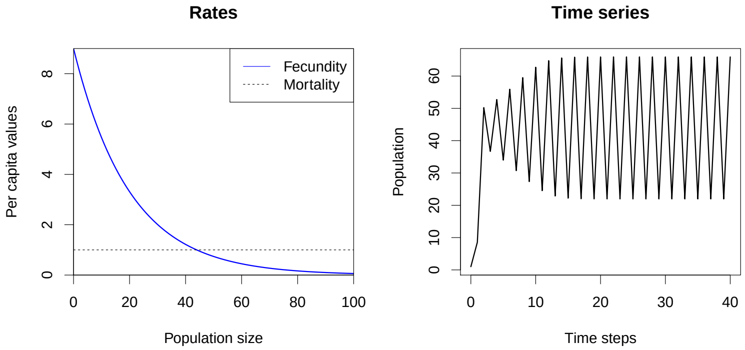
Simple dynamics (repeat)



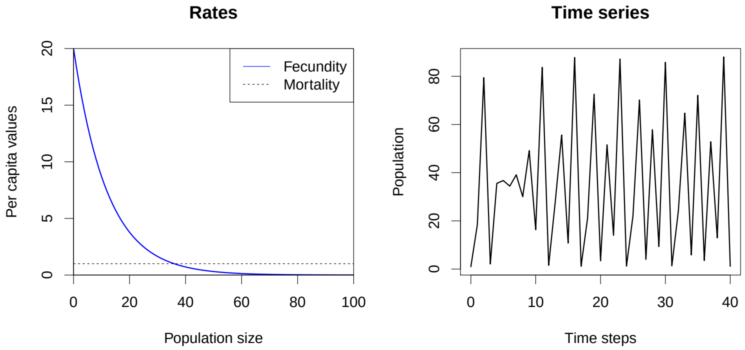
Damped oscillations



Persistent oscillations



Lots of other behaviours



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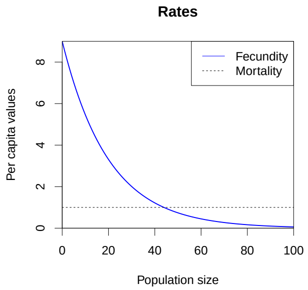

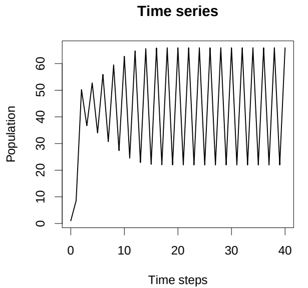

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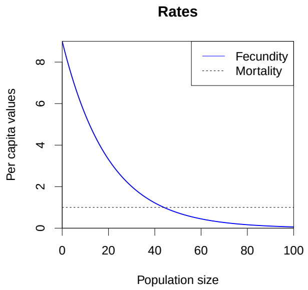

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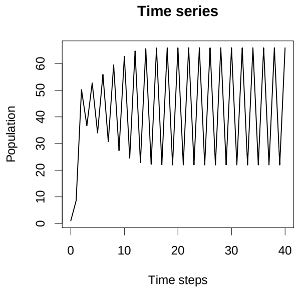

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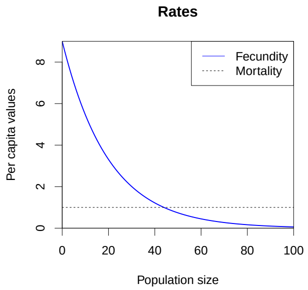

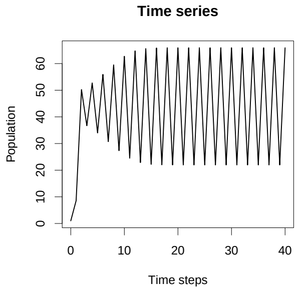

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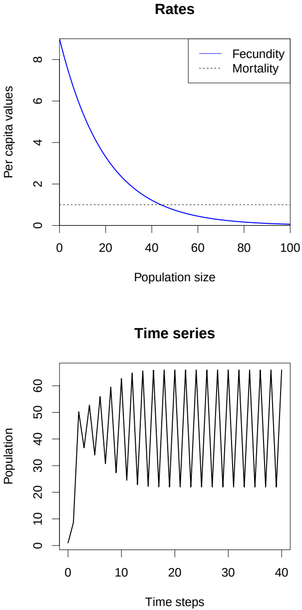
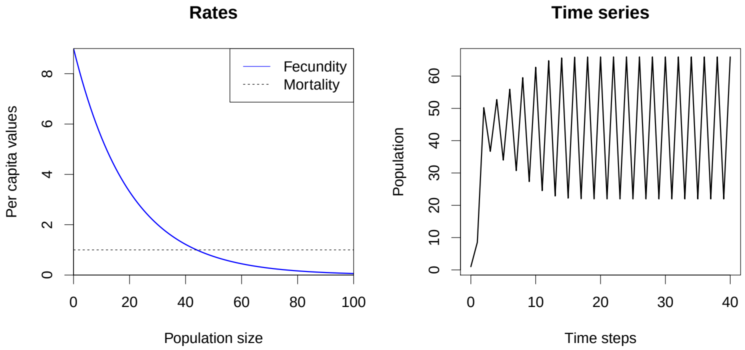


Interpreting complex behaviour (repeat)



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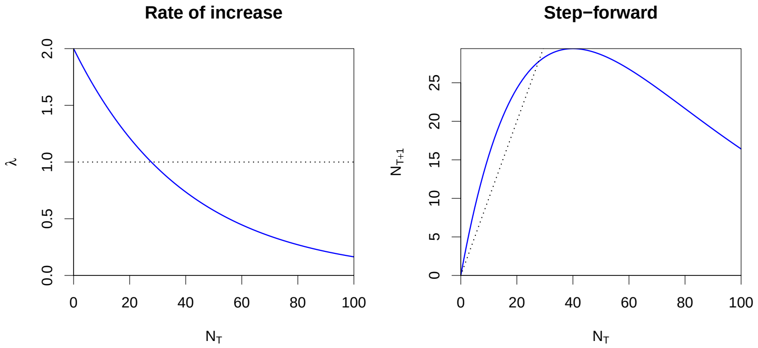

Turnover

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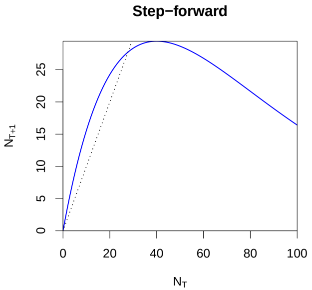

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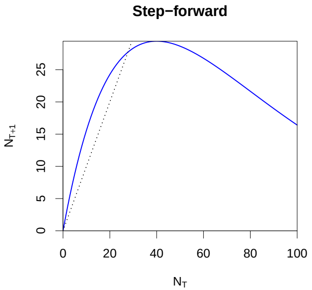

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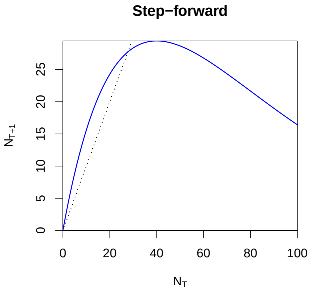

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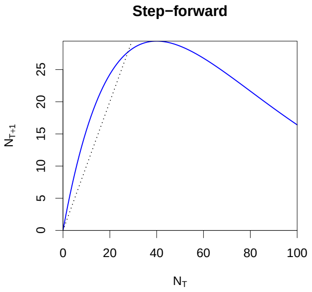
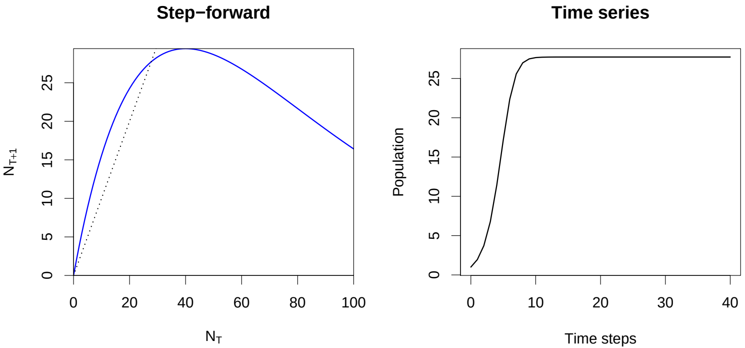
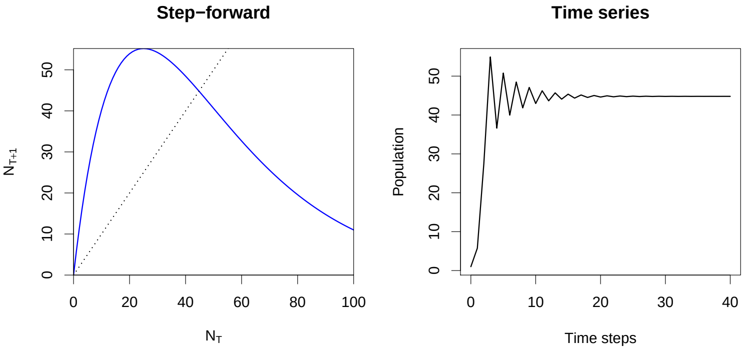


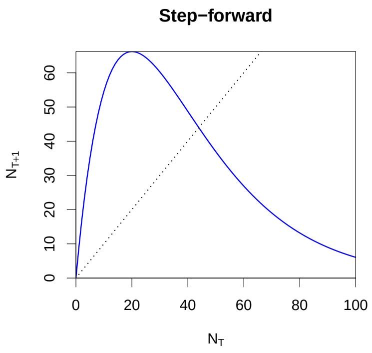
Simple dynamics



Damped oscillations



Persistent oscillations



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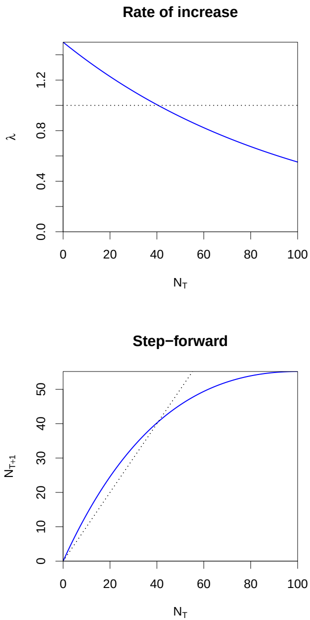

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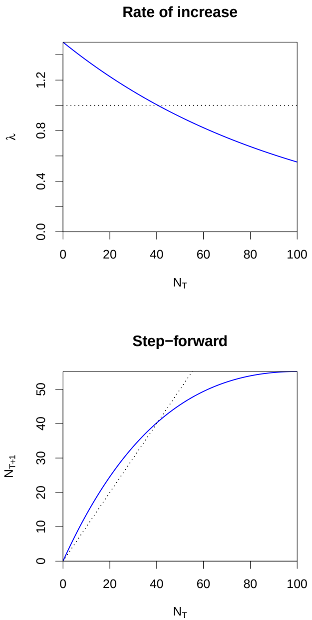

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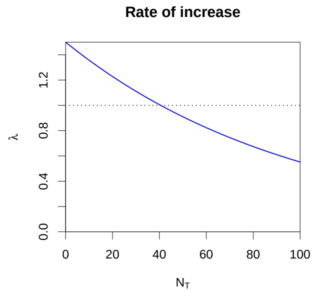

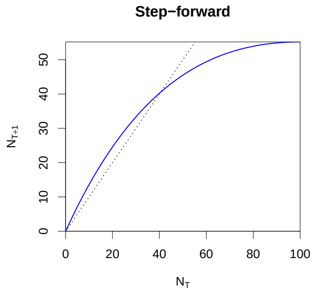

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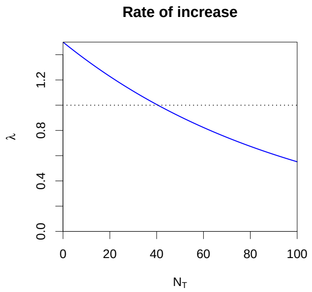
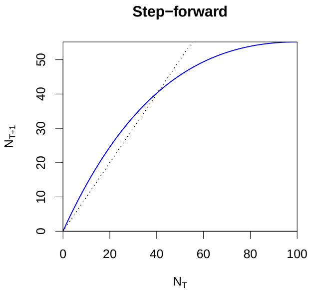
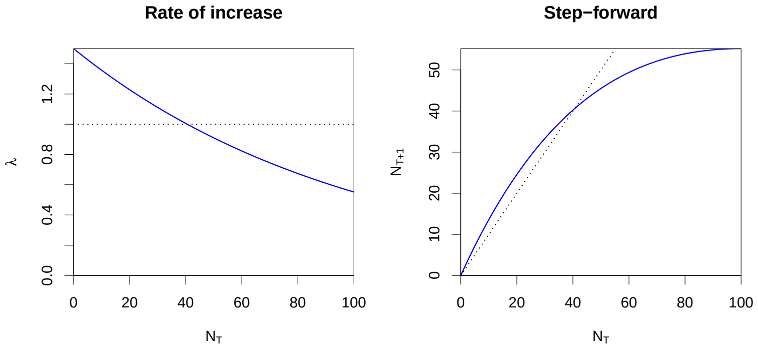




Contest regulation



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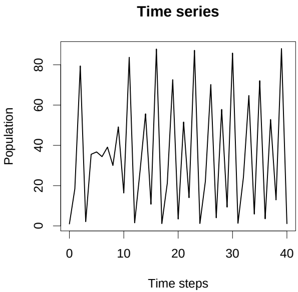

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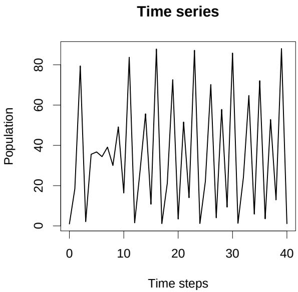

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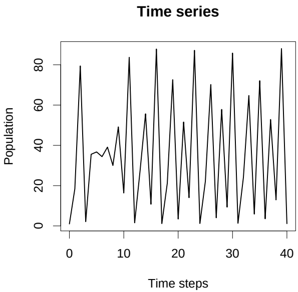

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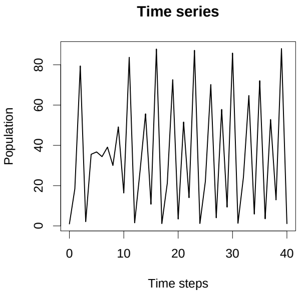

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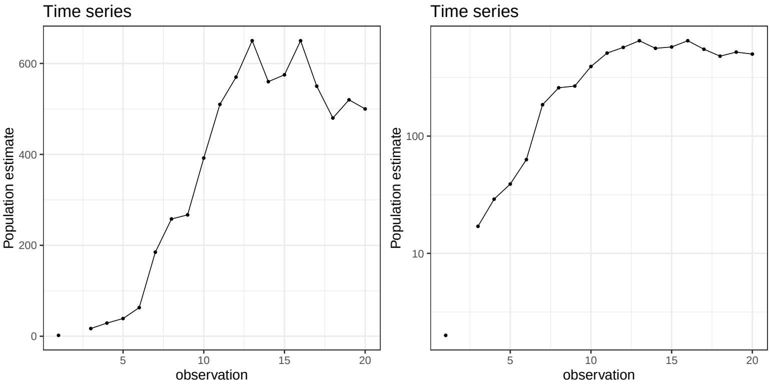

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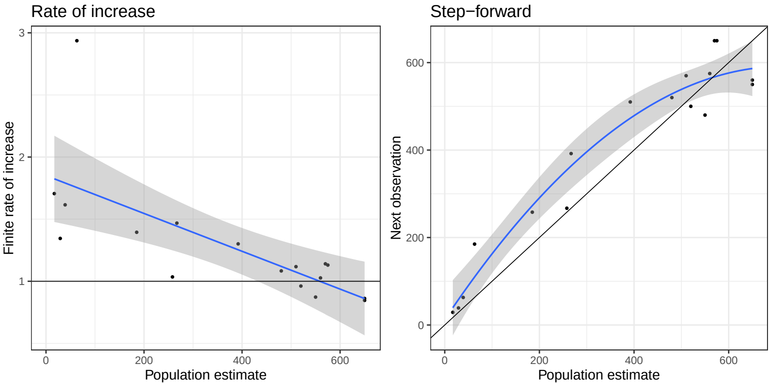
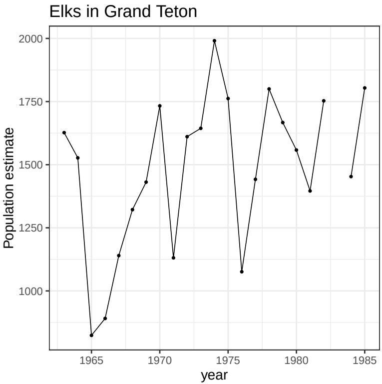


Rate of increase
Finite rate of increase
3
2
1
0
200
400
600
Population estimate
Step−forward
Next observation
600
400
200
0
0
200
400
600
Population estimate

Elk (repeat)



Elks in Grand Teton
Population estimate
2000
1750
1500
1250
1000
1965
1970
1975
1980
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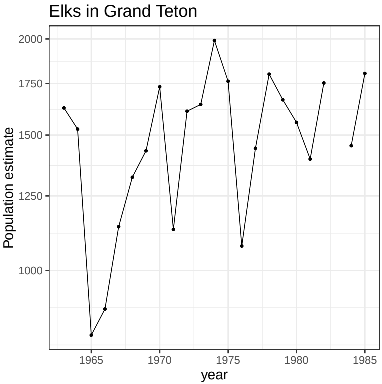
year



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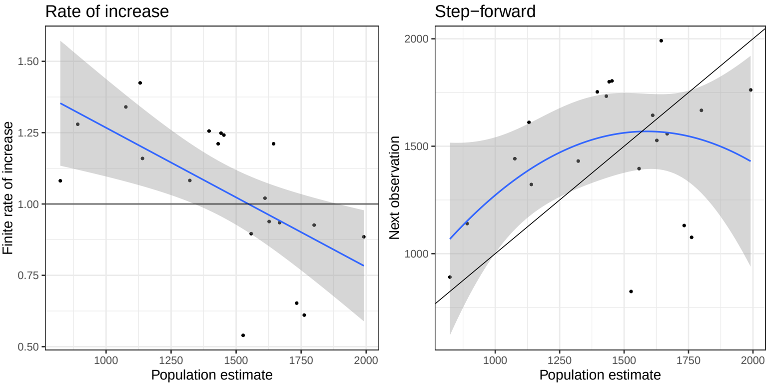

Rate of increase
Finite rate of increase
1.50
1.25
1.00
0.75
0.50
1000
1250
1500
1750
2000
Population estimate
Step−forward
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  - May be hard to see because of "noise" – i.e., other sources of variation
  - Cycles may be due to more complicated mechanisms

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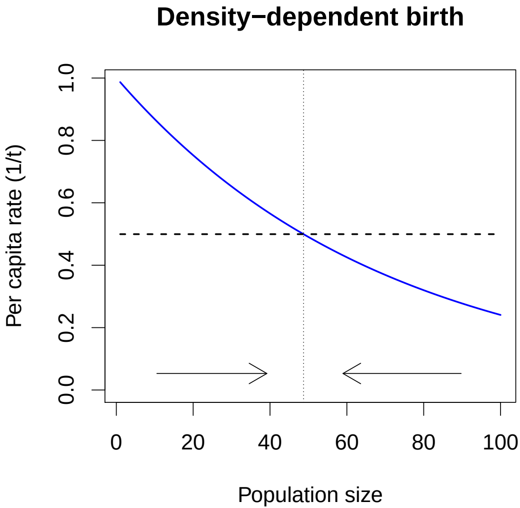

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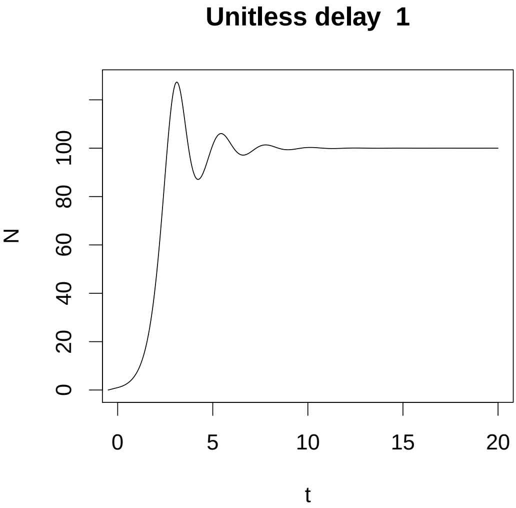

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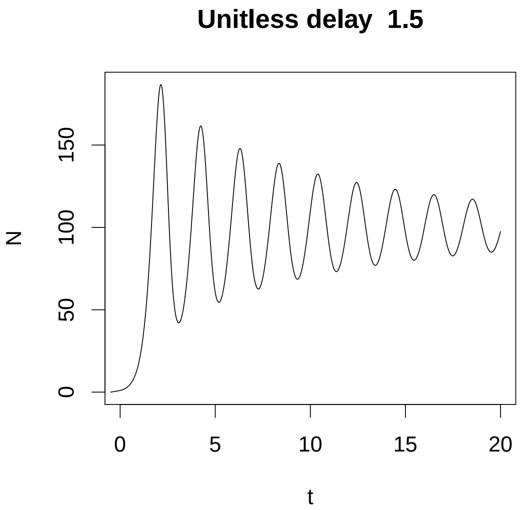

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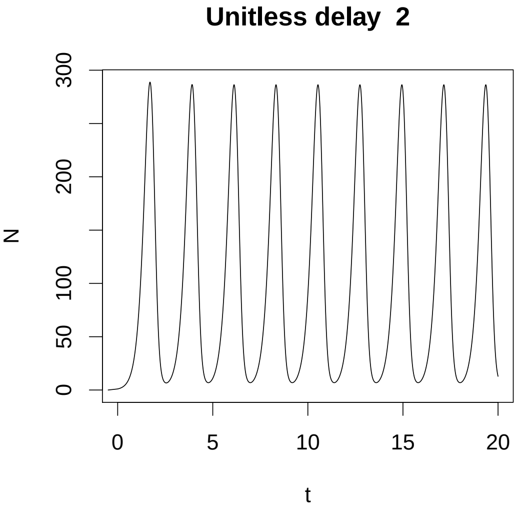

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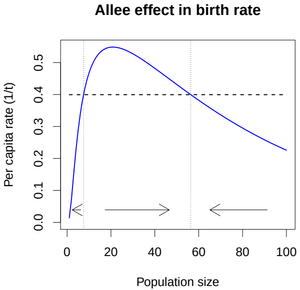

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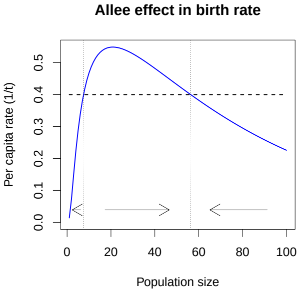

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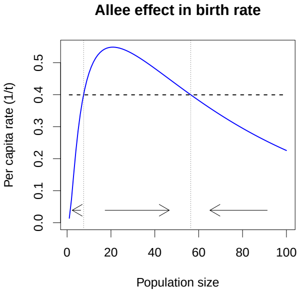

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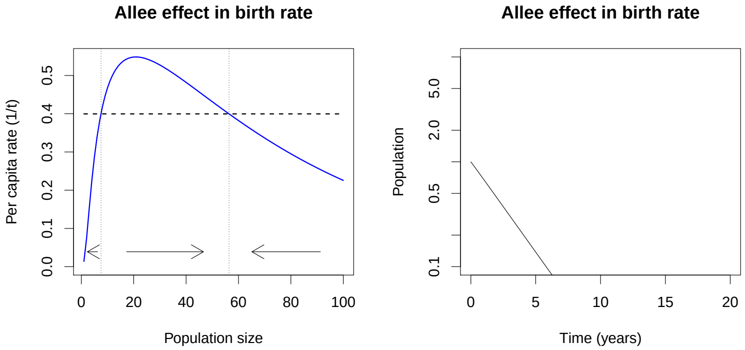

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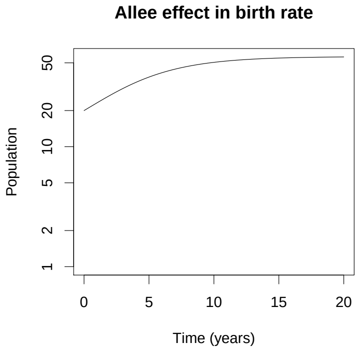

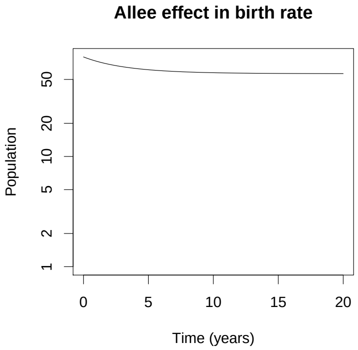

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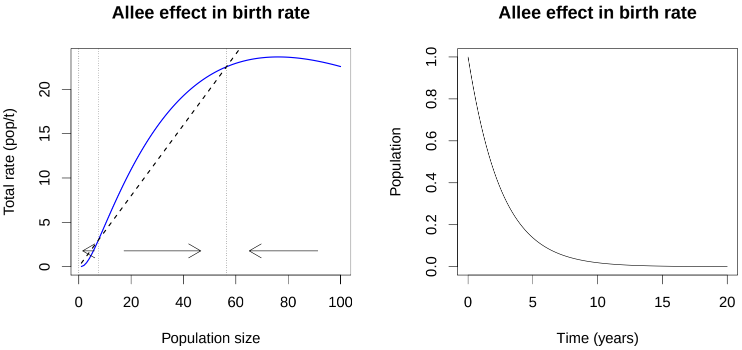

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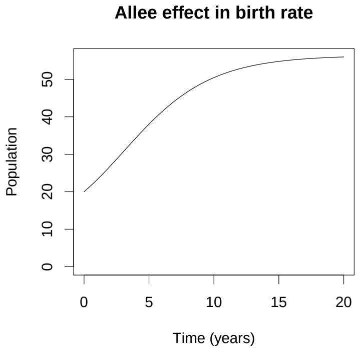
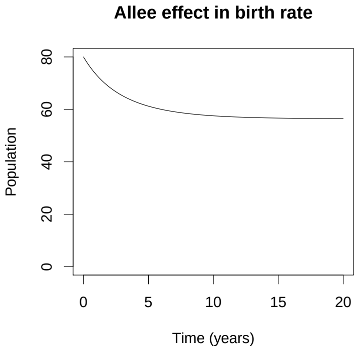
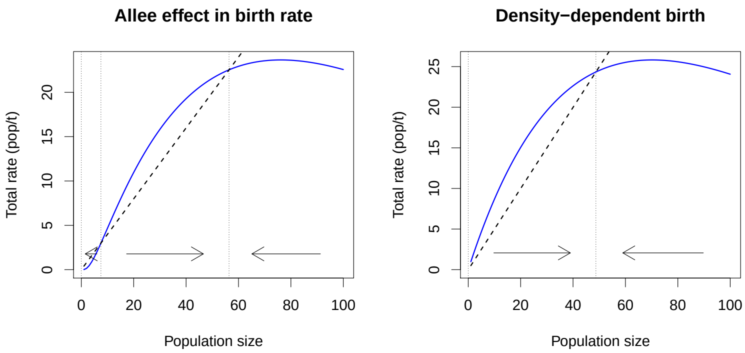




Population comparison (repeat)



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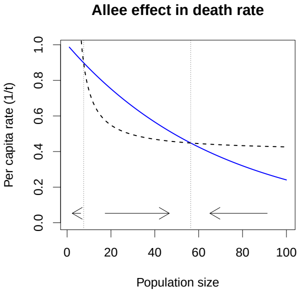

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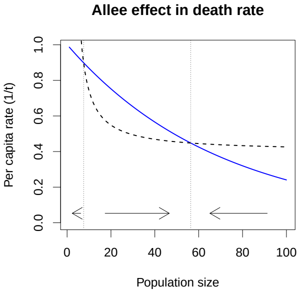

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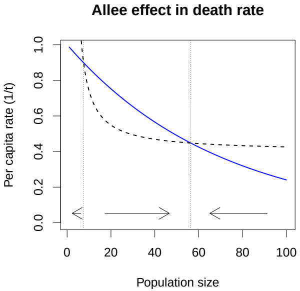

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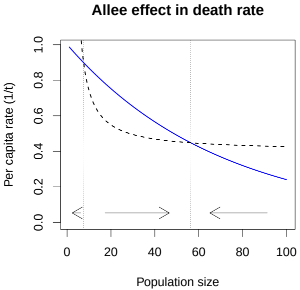

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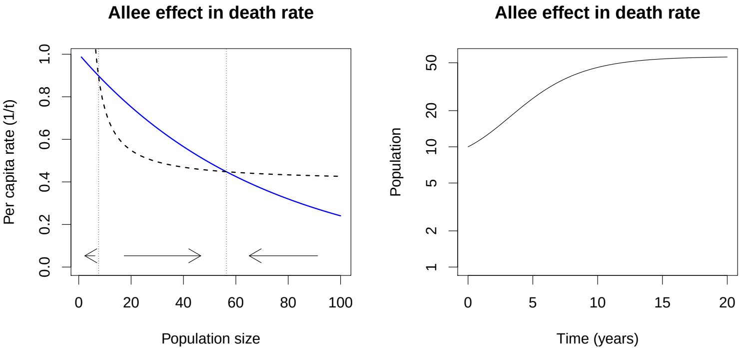

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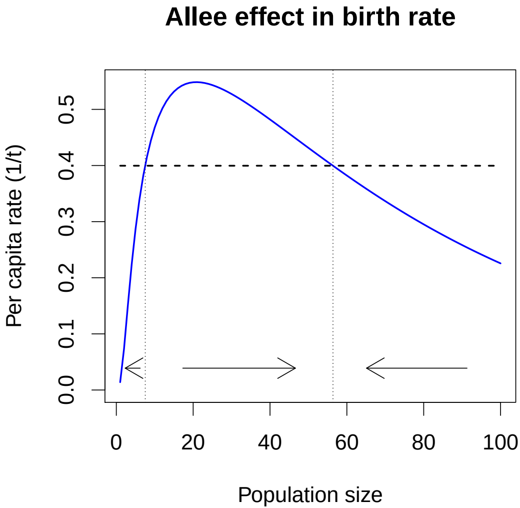
- When Allee effects are present, it's no longer true that a species that can't invade can't persist
  - * If $\mathcal{R}_0 < 1$ population can't invade, but if $\mathcal{R}_{\text{max}} > 1$ it can still persist

$\mathcal{R}_0$ and $\mathcal{R}_{max}$ (repeat)



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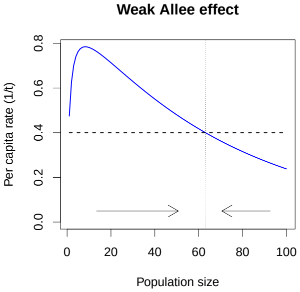

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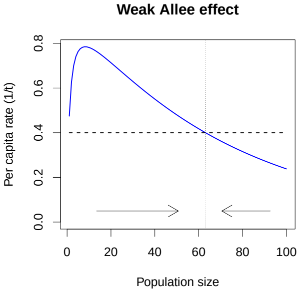

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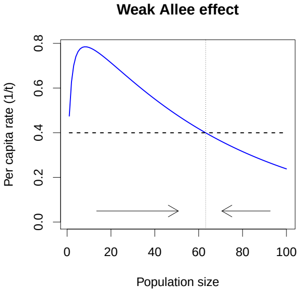

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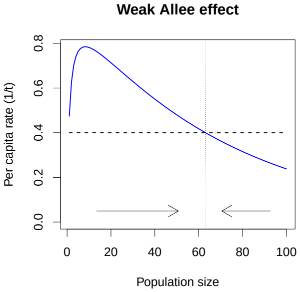

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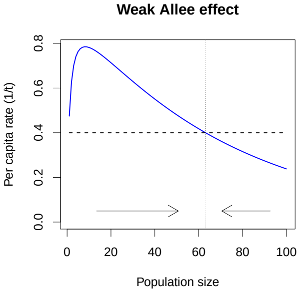

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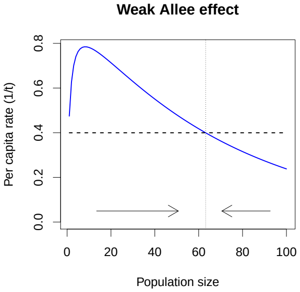
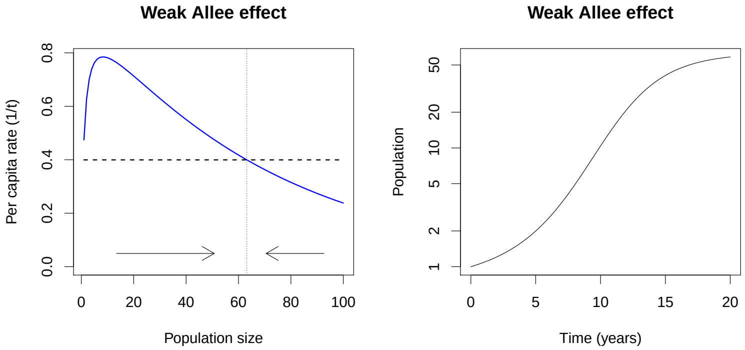


Individual perspective



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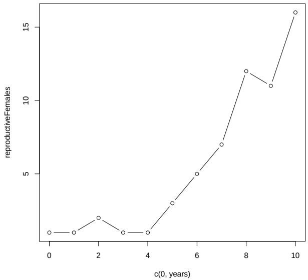

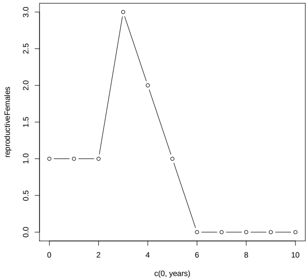

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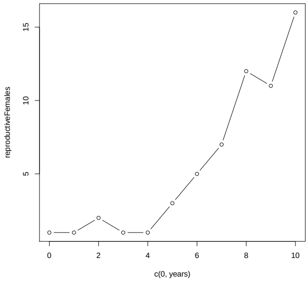

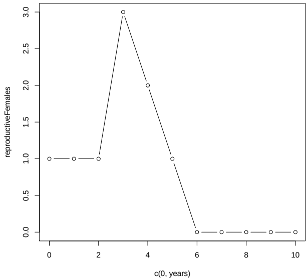

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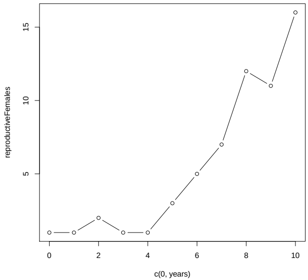

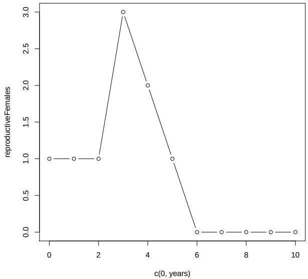

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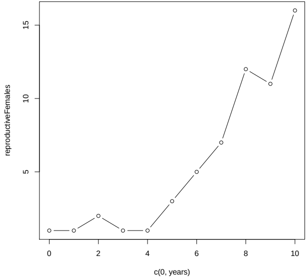

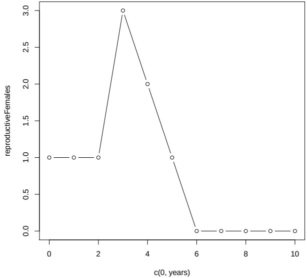

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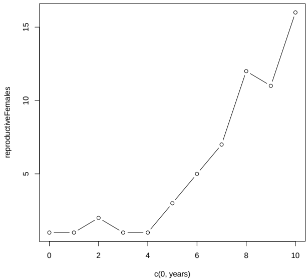

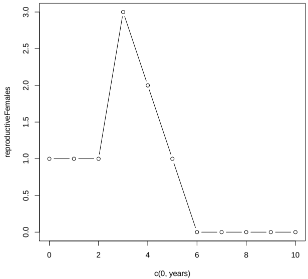

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