

UNIT 1: Introduction

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Examples

Example populations

Dandelions

Gypsy moths

Bacteria

Exponential growth

Log and linear scales

Time scales

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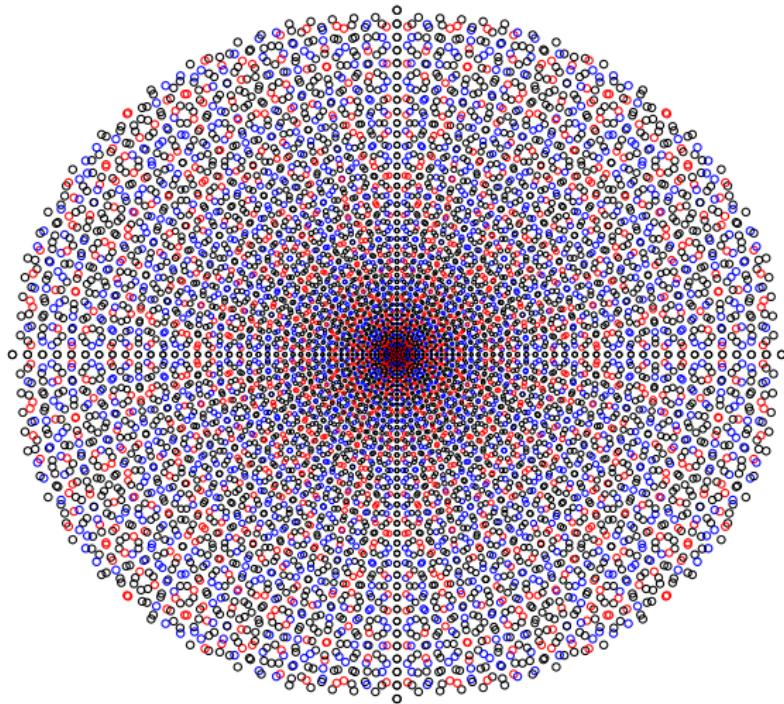
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Pythagorean triples



Which country?



Which country?



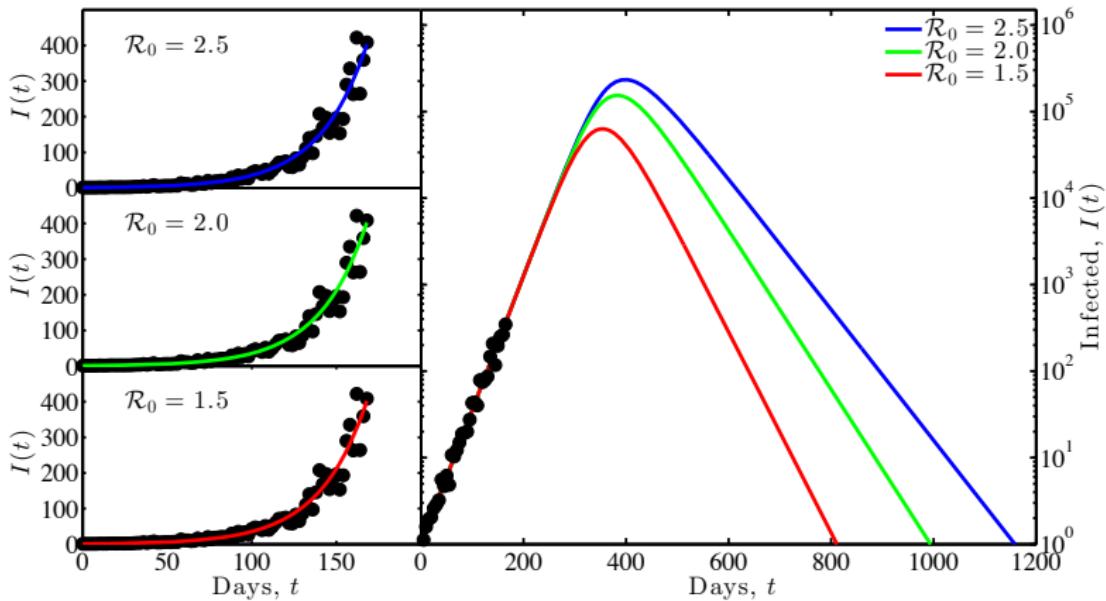
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Ebola research



TAs

- Morgan Kain

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Students

- ▶ Poll: What year are you in?

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- ▶ Ecology and population ecology
- ▶ Quantitative thinking

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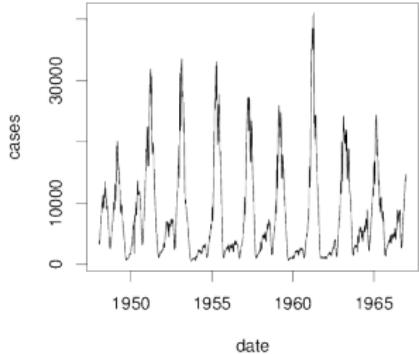
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Dynamical modeling

- ▶ Investigates the links between local, short-term processes, and large-scale, long-term outcomes



Measles reports from England and Wales

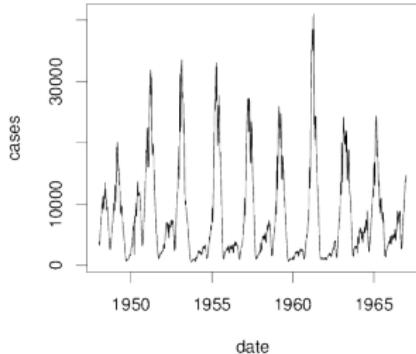


Dynamical modeling

- ▶ Investigates the links between local, short-term processes, and large-scale, long-term outcomes
- ▶ Allows us to explore what assumptions we're making, and how assumptions affect the link



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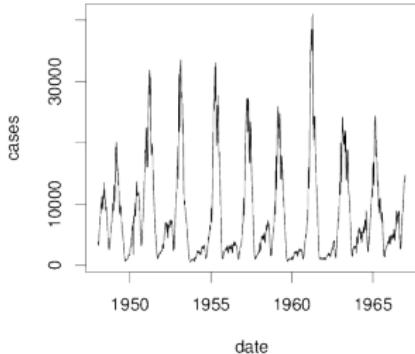


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Humans and abstract thought

- ▶ People are evolved to be concrete thinkers, not conceptual thinkers



$$E = mc^2$$

value

energy | mass | speed of light

J | kg | 299,792,458 m/s

units

$c^2 = 89,875,517,873,681,800 \text{ m}^2/\text{s}^2$

A diagram illustrating the components of the equation $E = mc^2$. The equation is centered at the bottom. Above it, three arrows point down from the words "value", "energy | mass | speed of light". Below the equation, three arrows point up from the units "J | kg | 299,792,458 m/s". At the very bottom, an arrow points up from the units "units" to the value equation $c^2 = 89,875,517,873,681,800 \text{ m}^2/\text{s}^2$.

Humans and abstract thought

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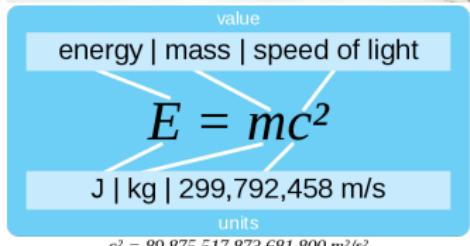
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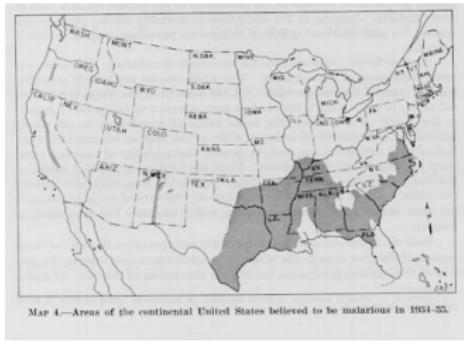
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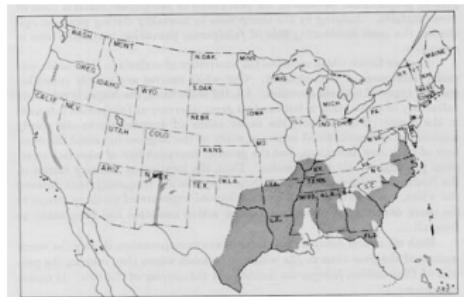
Malaria

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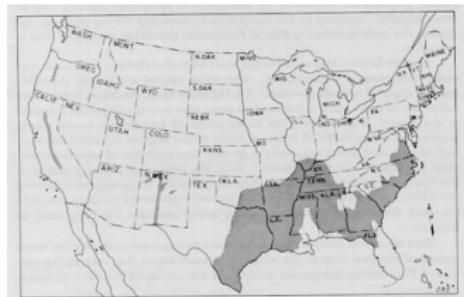
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MAP 4.—Areas of the continental United States believed to be malarious in 1951-55.

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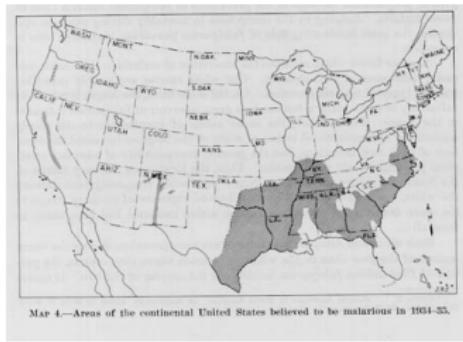
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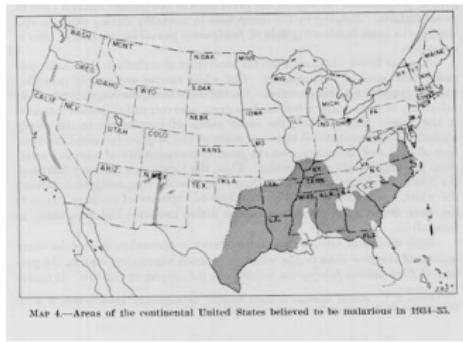
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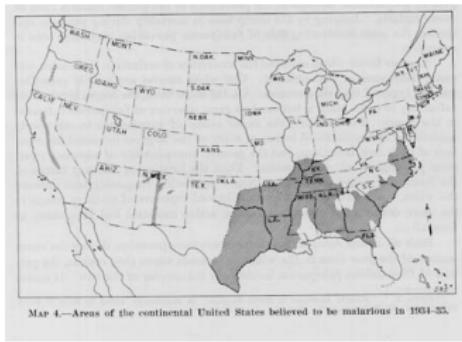
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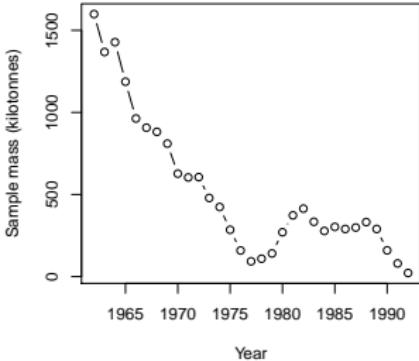
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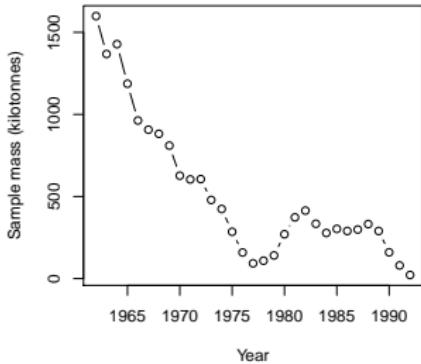
Cod fisheries

- ▶ Is the ocean too big for people to affect?



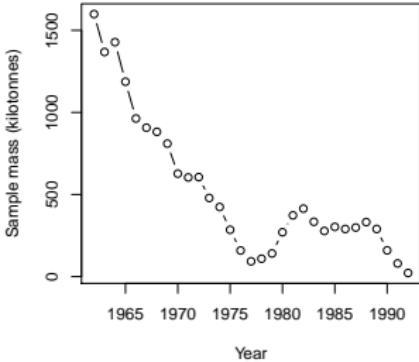
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Populations

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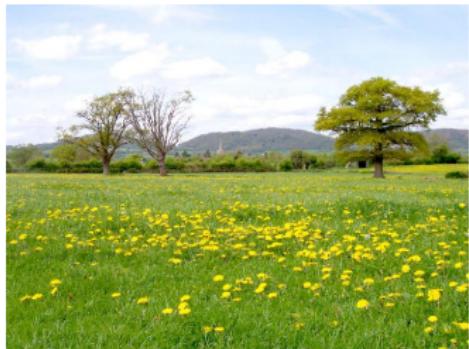
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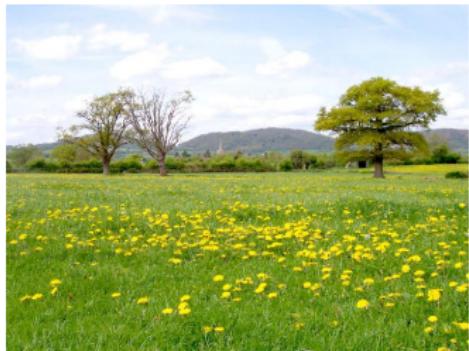
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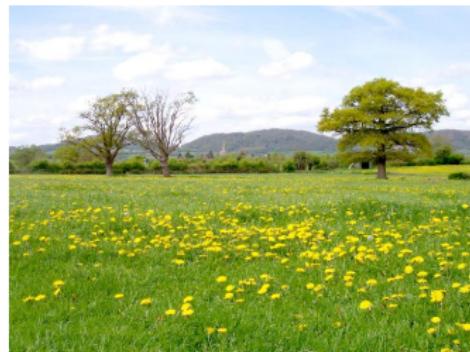
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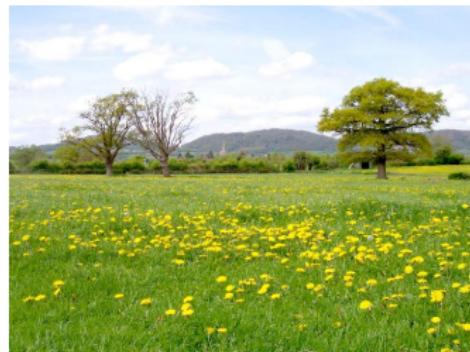
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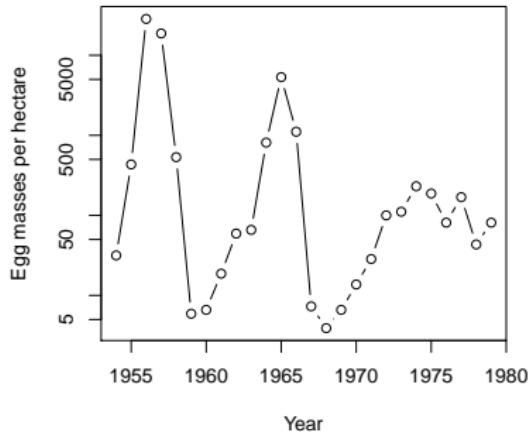
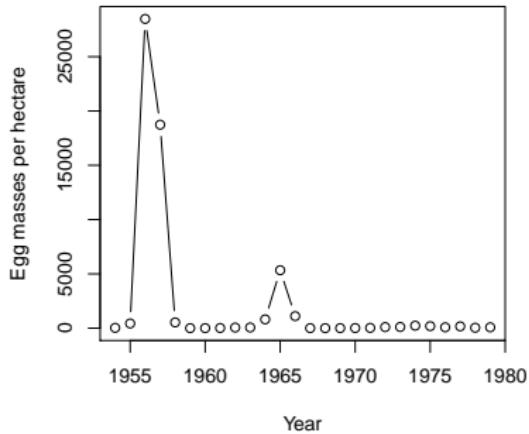


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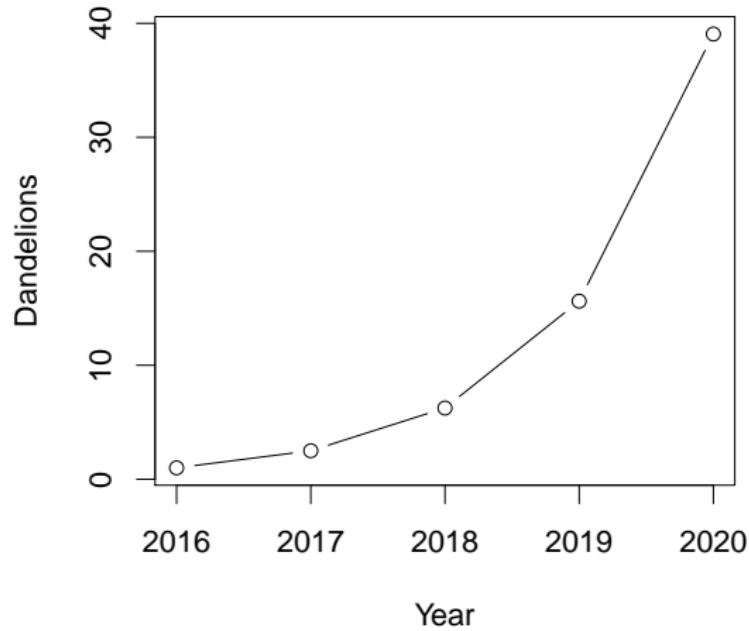
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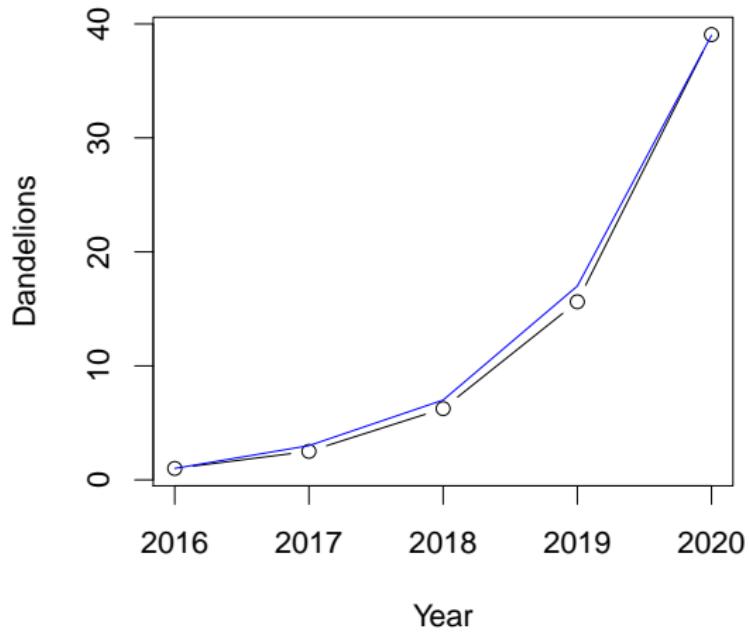
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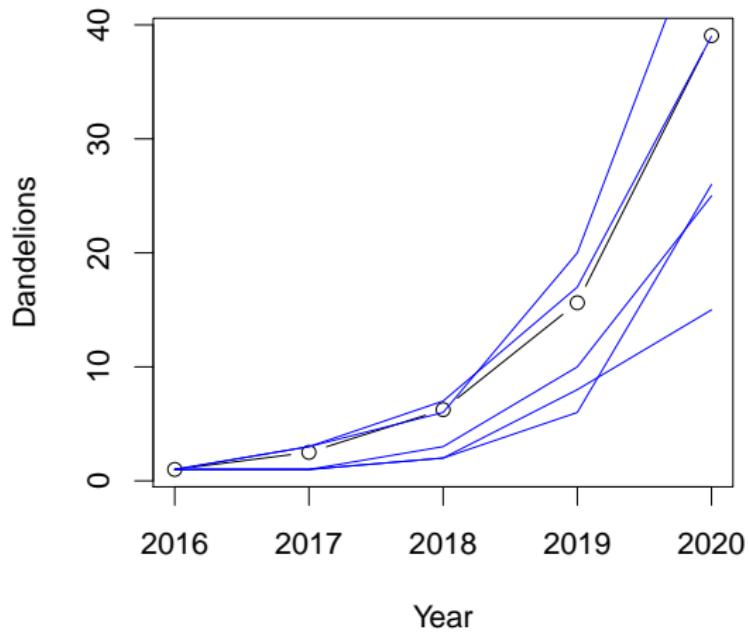


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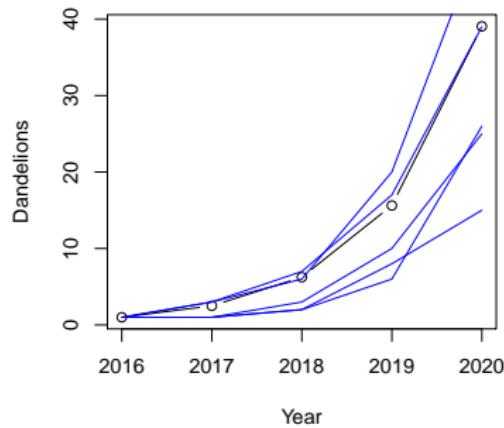
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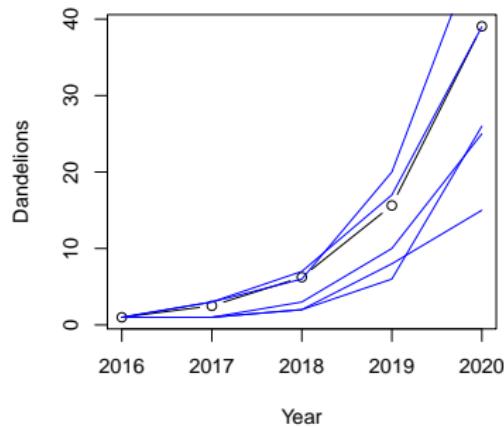


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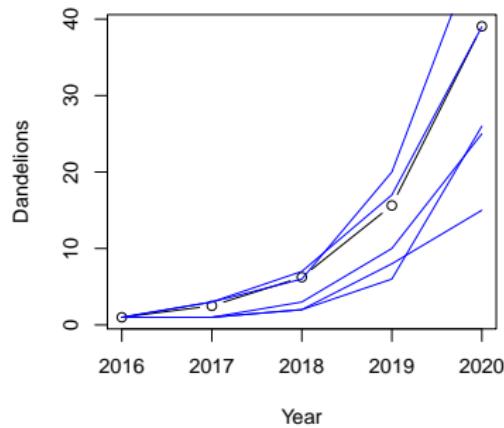
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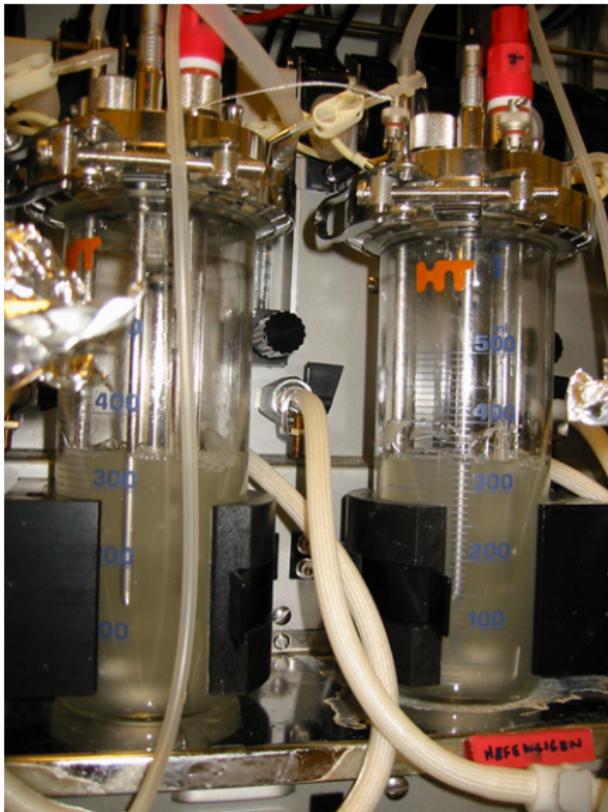
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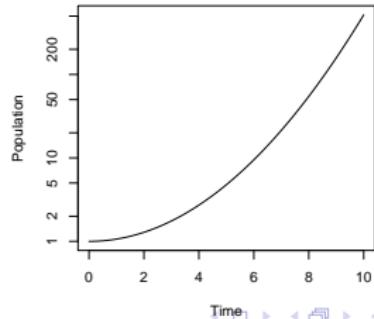
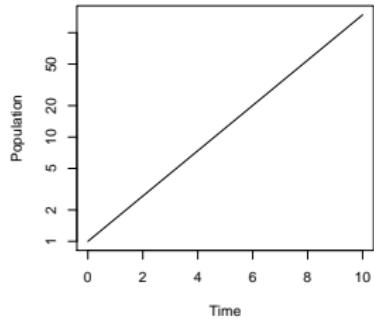
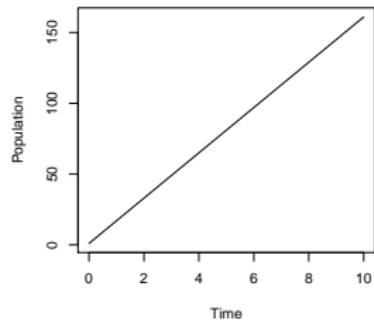
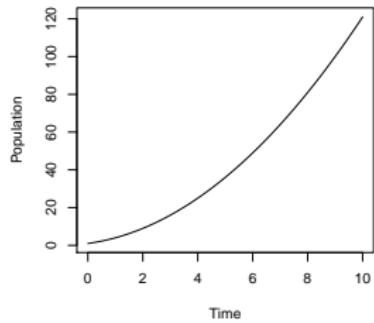
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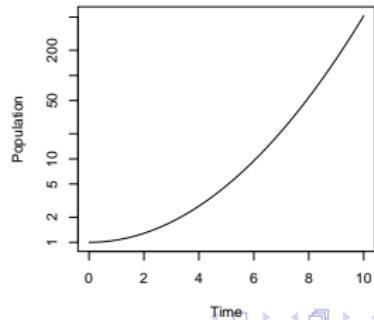
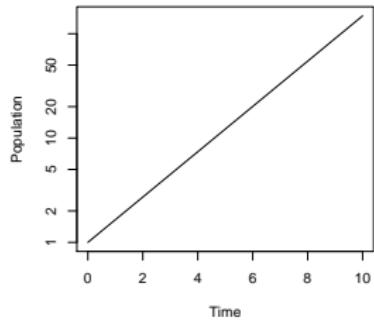
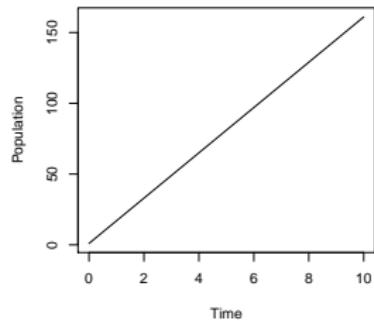
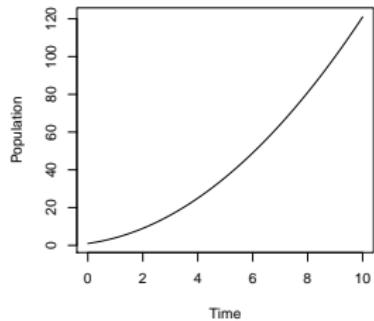
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► What is exponential growth?



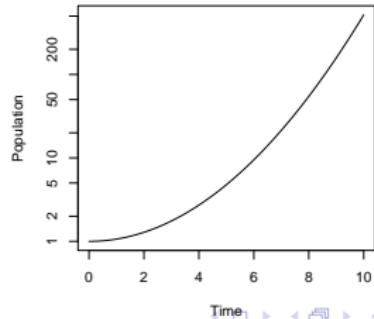
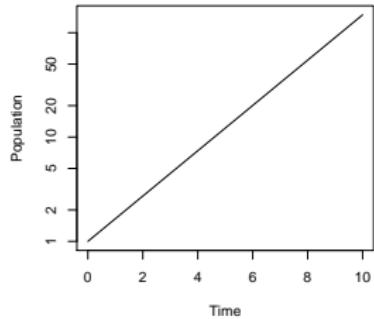
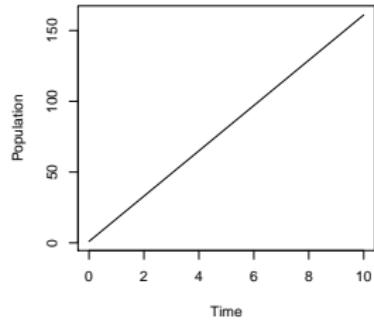
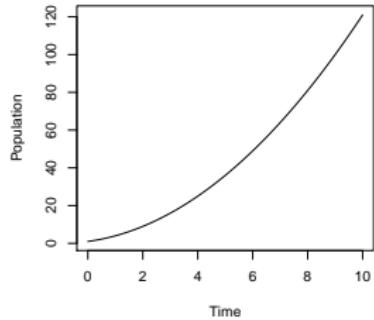
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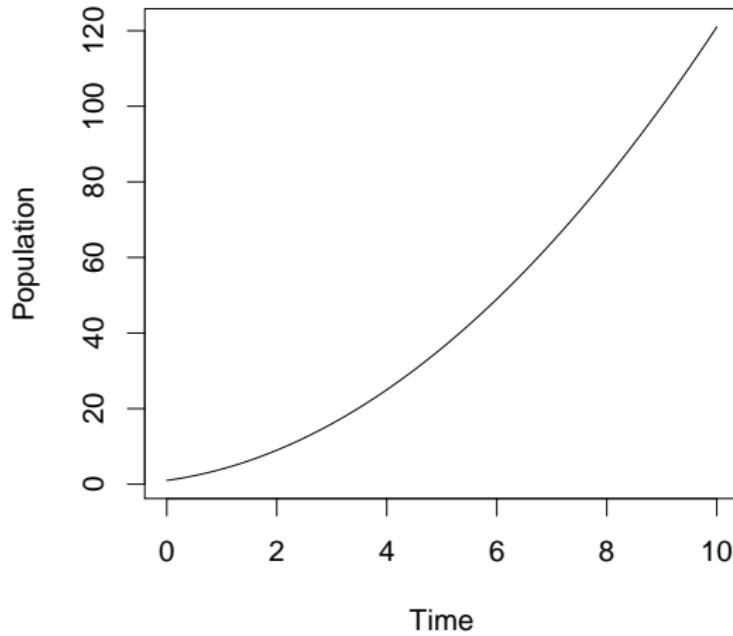


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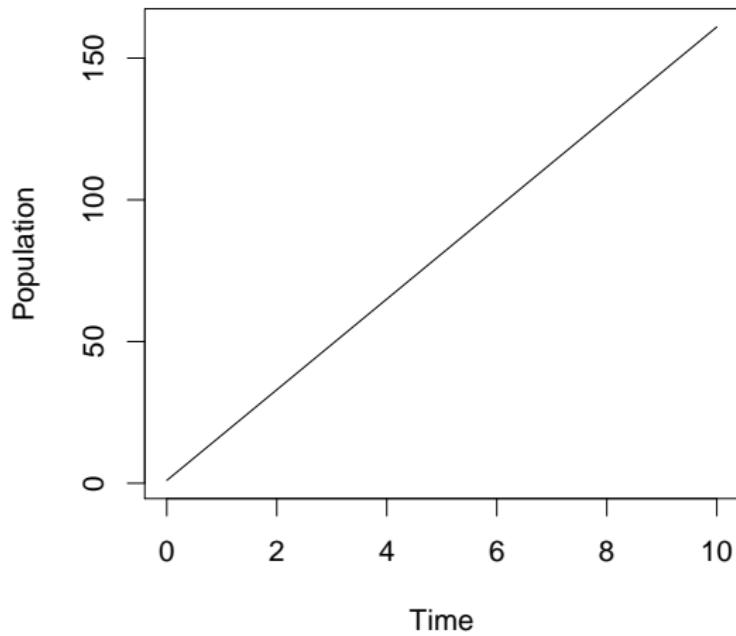
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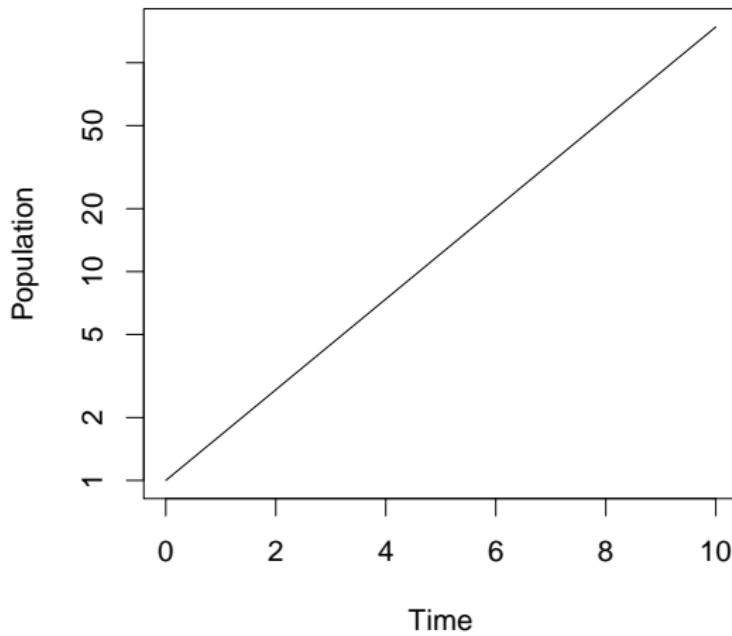
A (repeat)



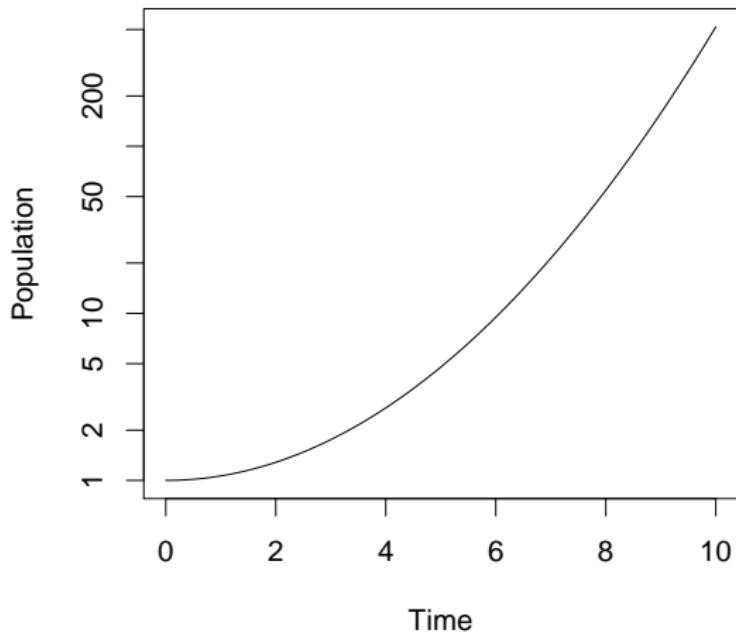
B (repeat)



C (repeat)

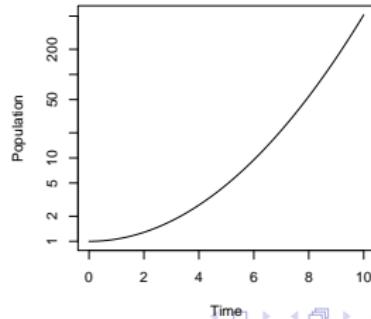
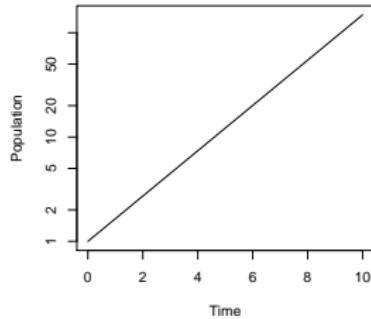
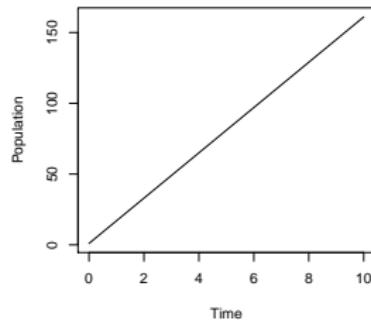
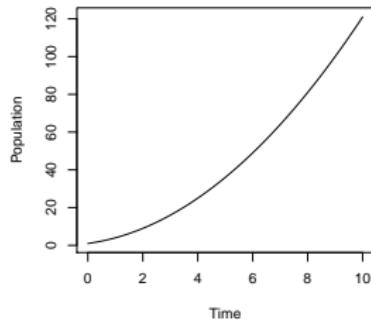


D (repeat)



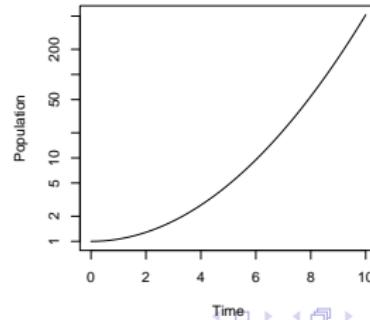
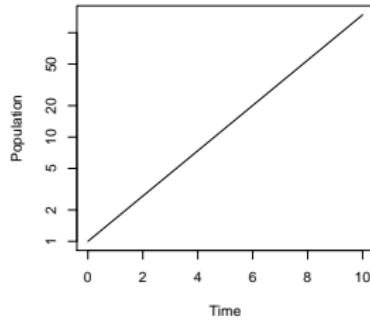
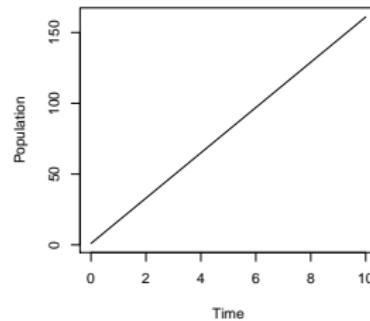
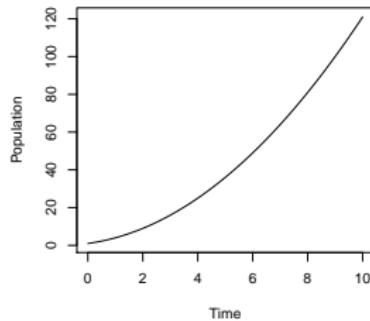
Exponential growth (repeat)

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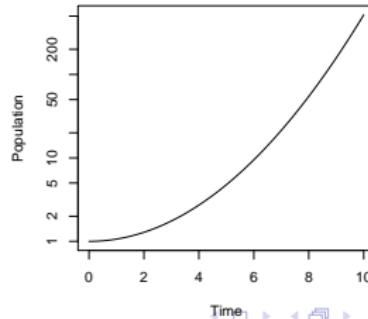
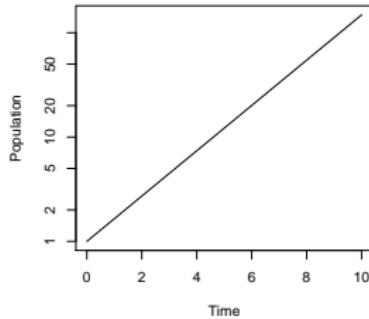
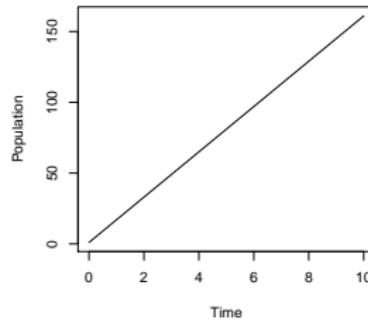
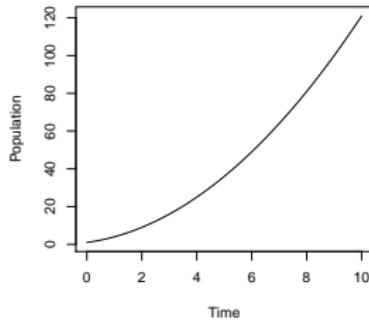
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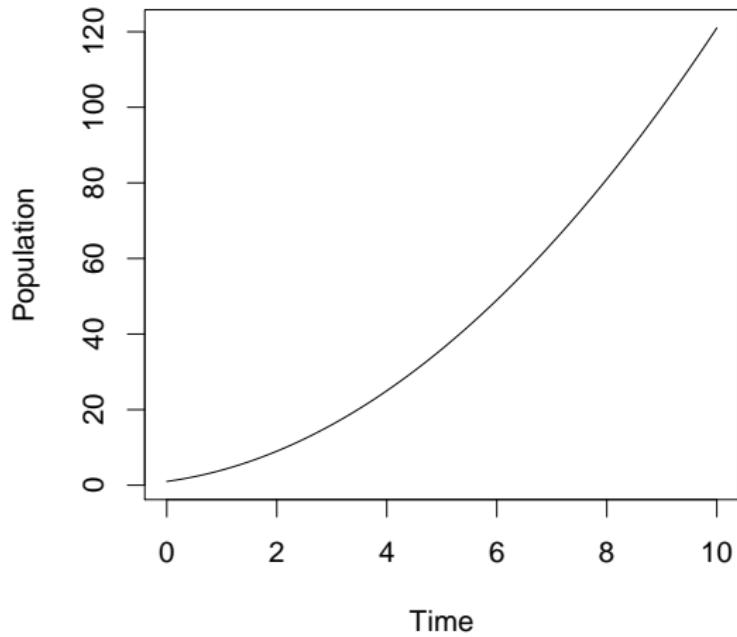


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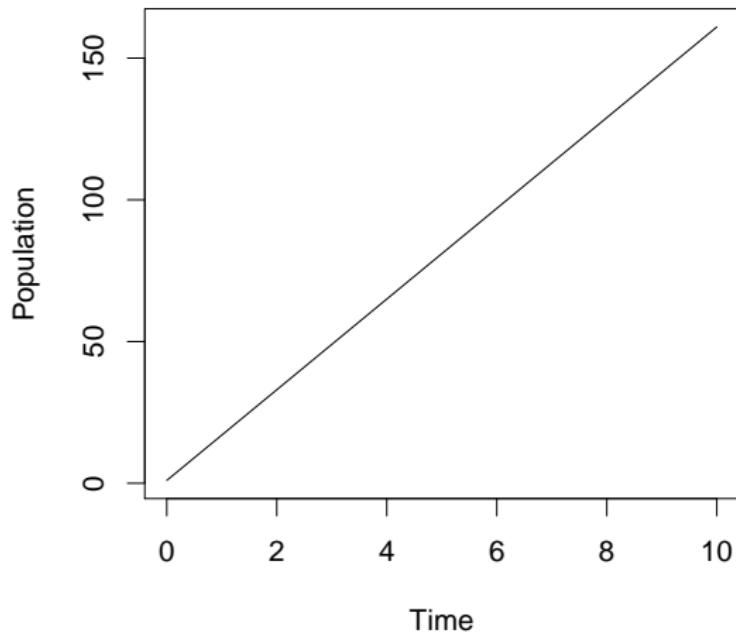
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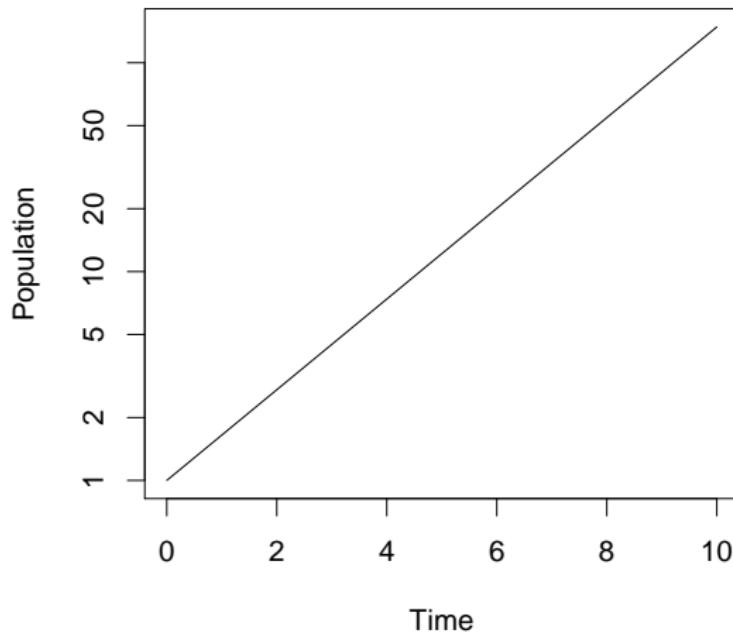
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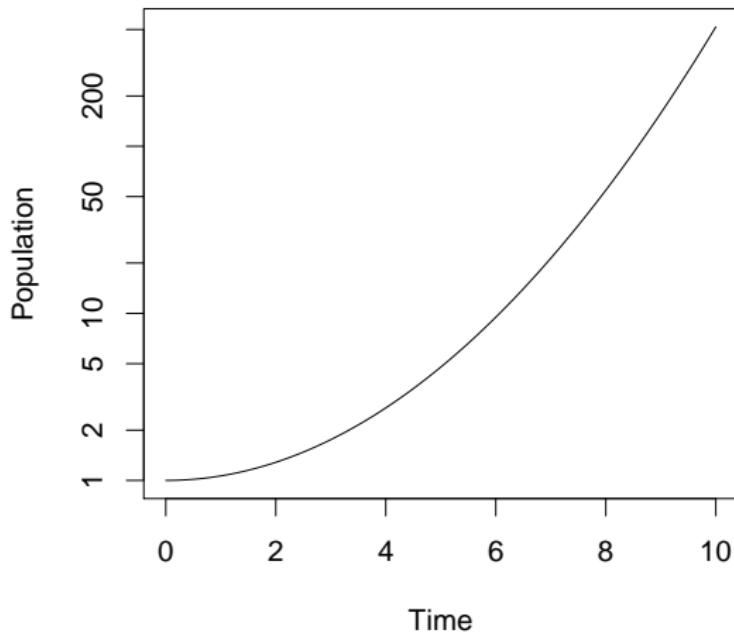
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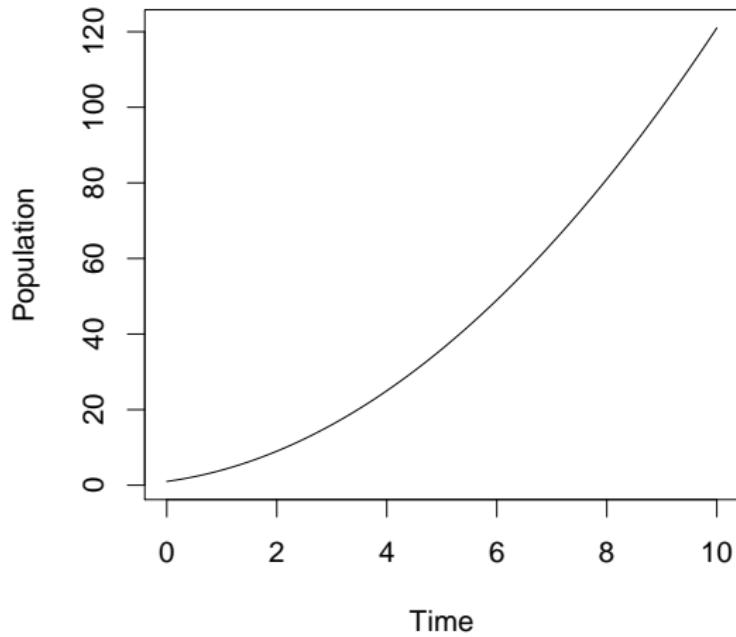
C (repeat)



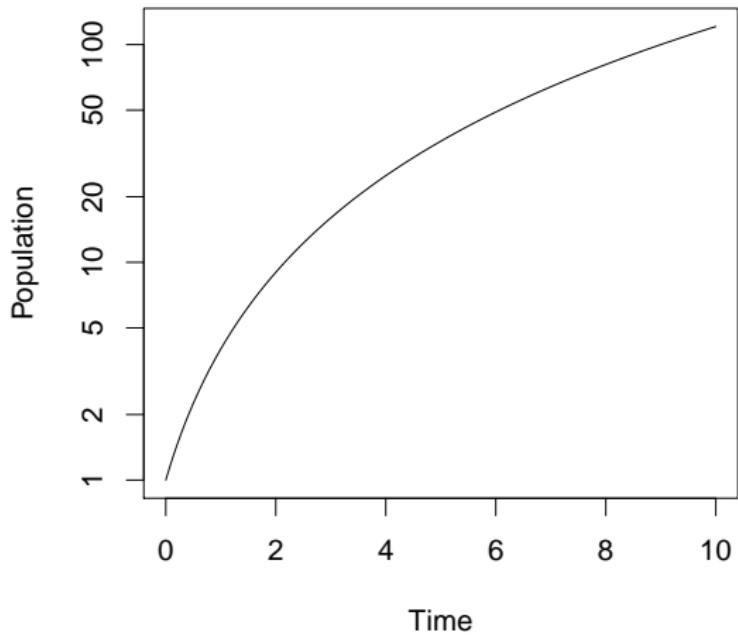
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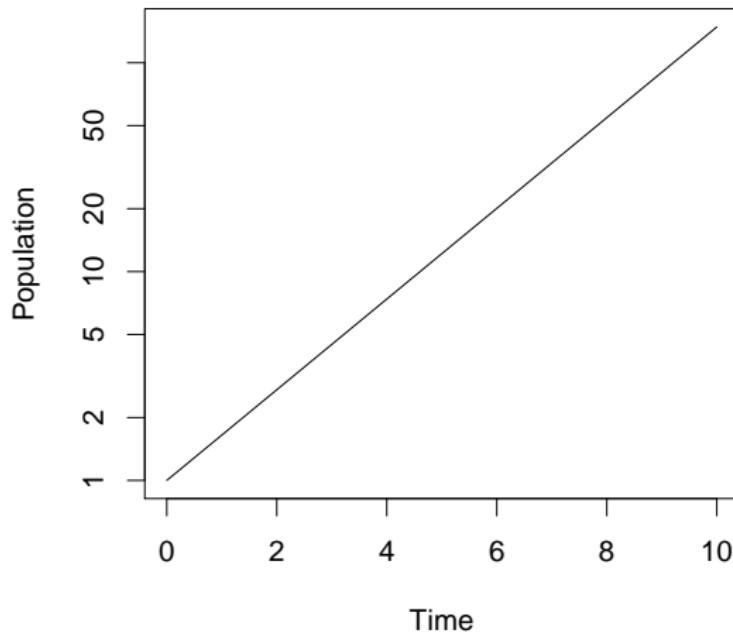
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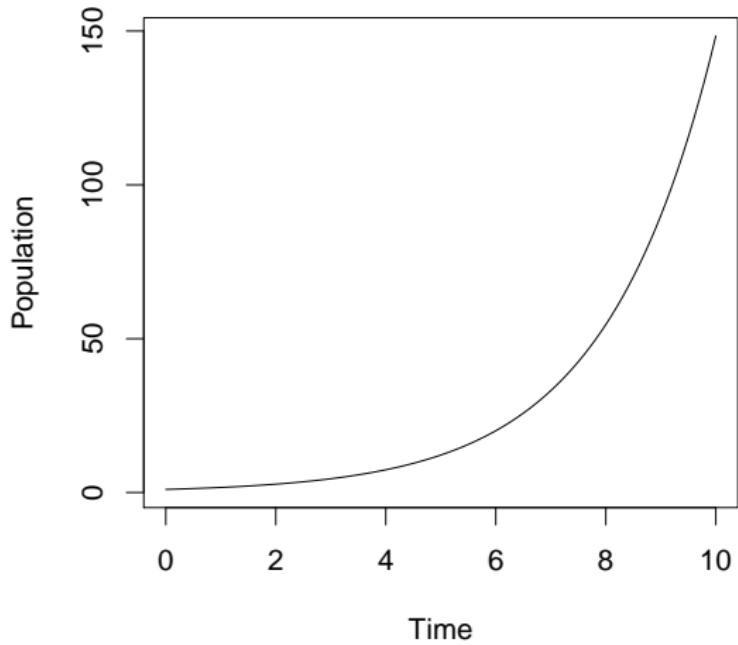
A on the log scale



C (repeat)



C on the linear scale



Types of growth

- arithmetic/linear:

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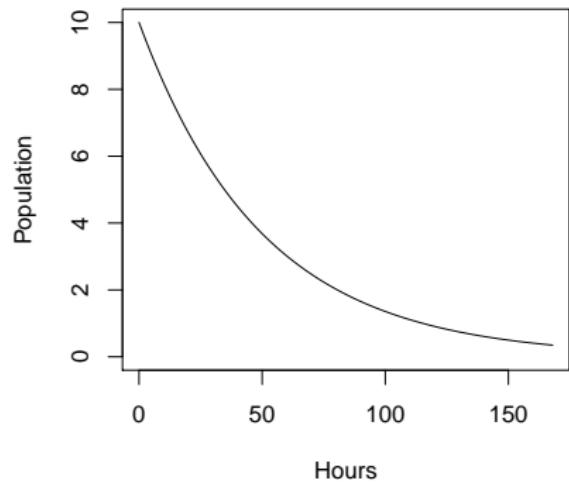
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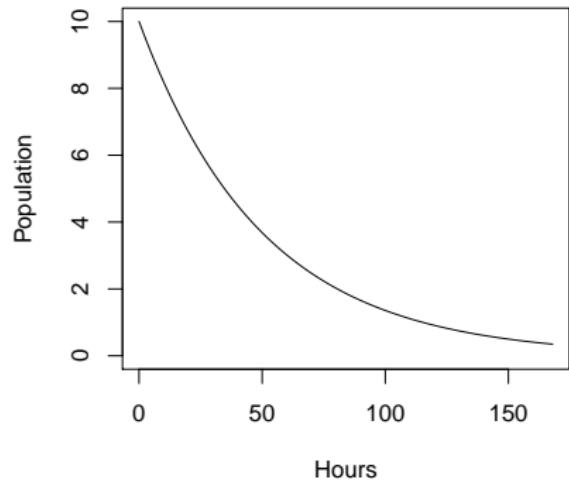
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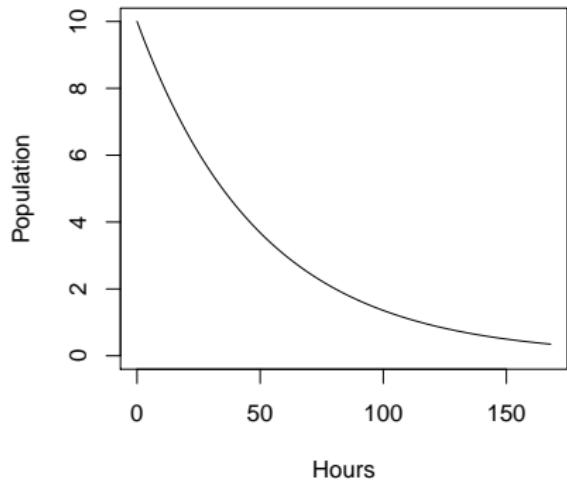
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Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

Example populations

Dandelions

Gypsy moths

Bacteria

Exponential growth

Log and linear scales

Time scales

Scales of comparison

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Scales of comparison

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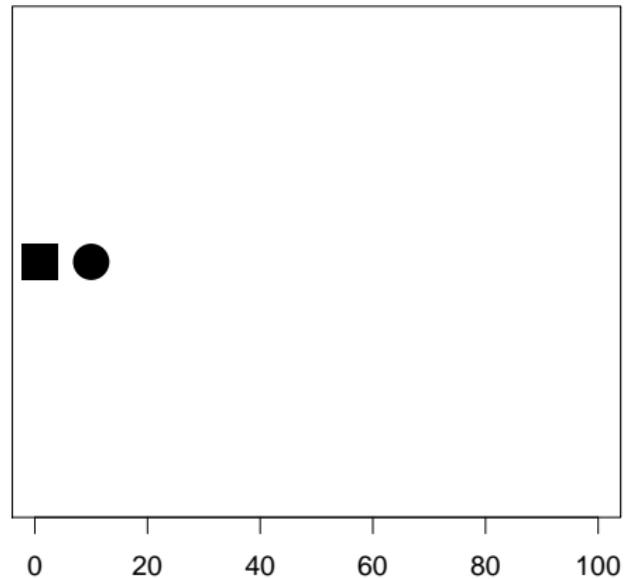
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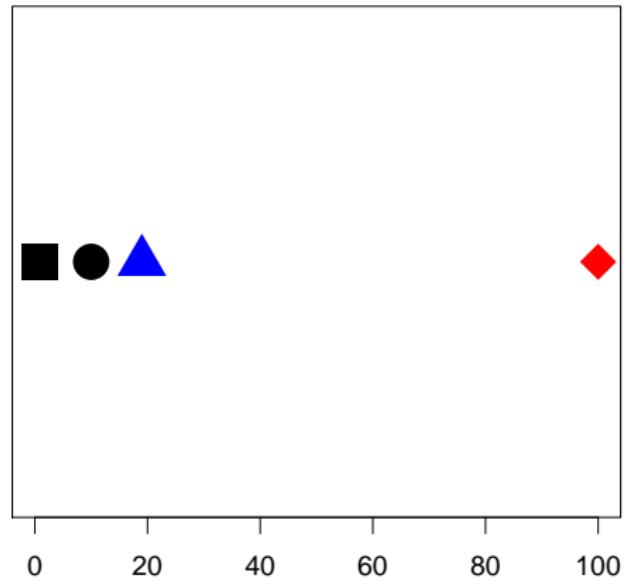
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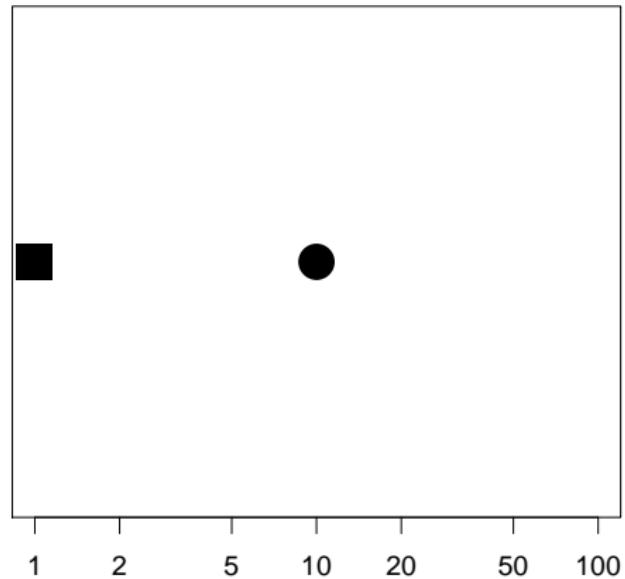
Scales of display



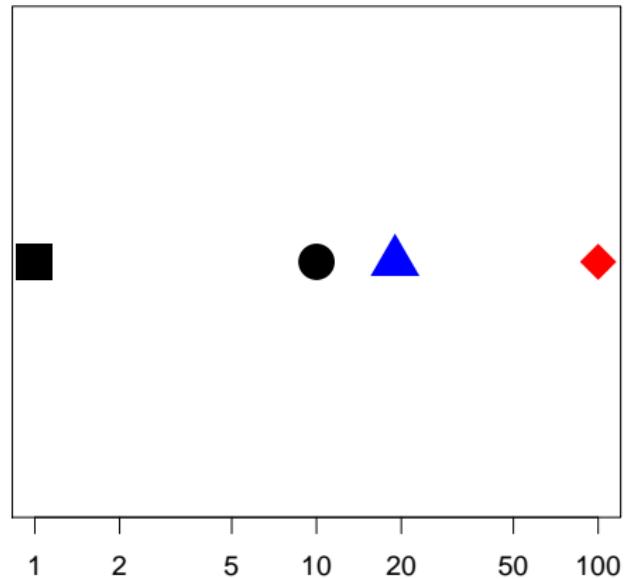
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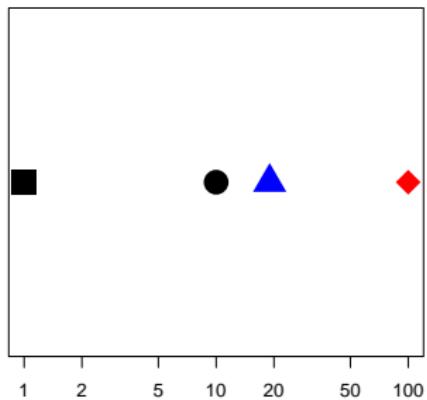
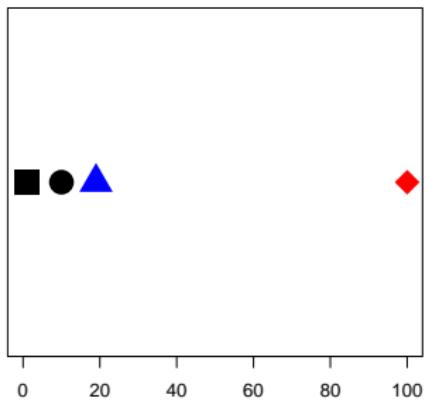
Scales of display



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There is only one log scale; it doesn't matter which base you use!

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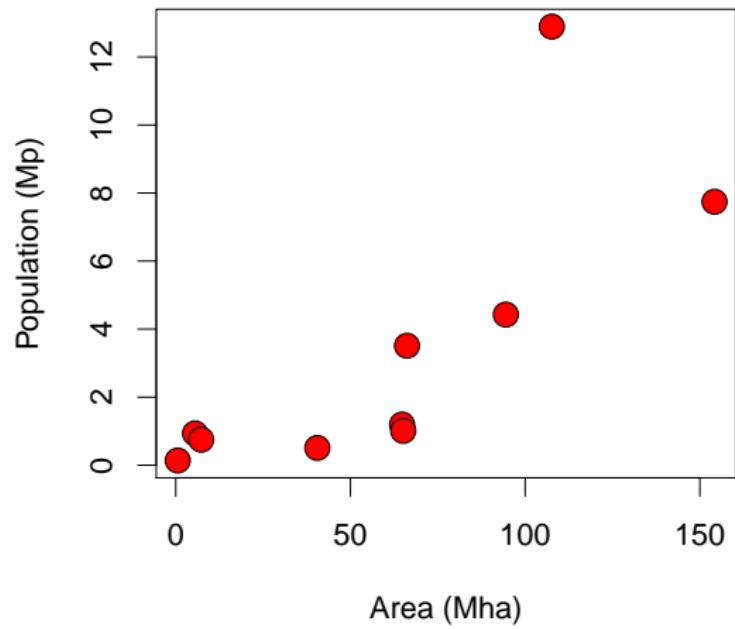
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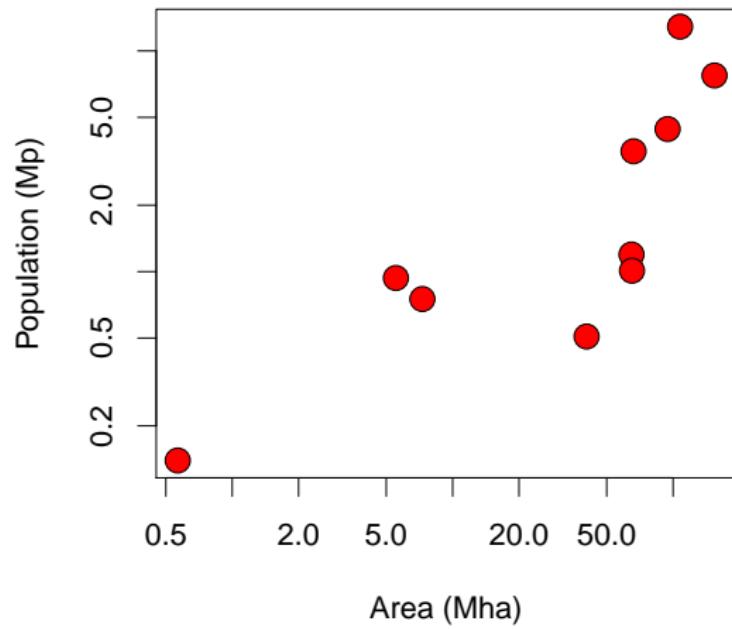
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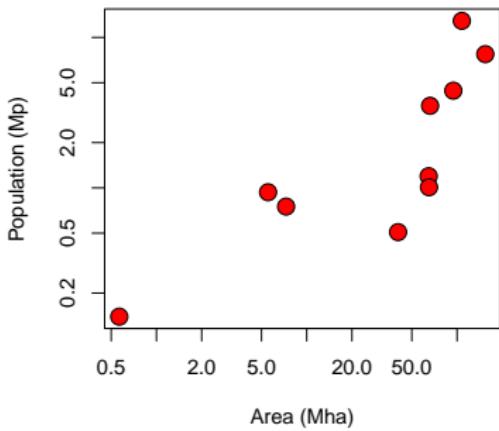
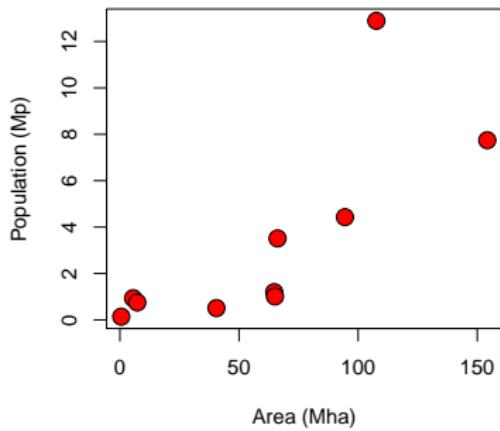
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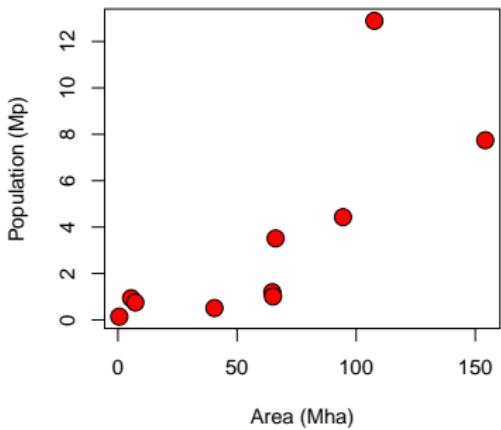
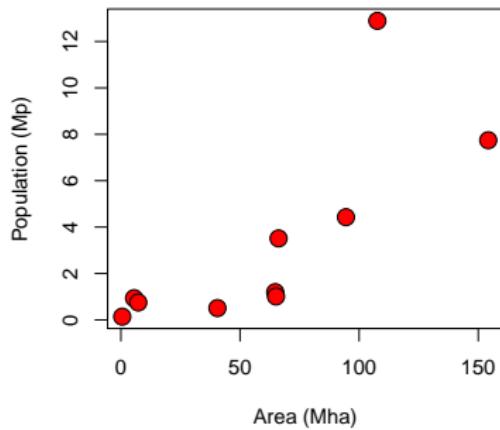
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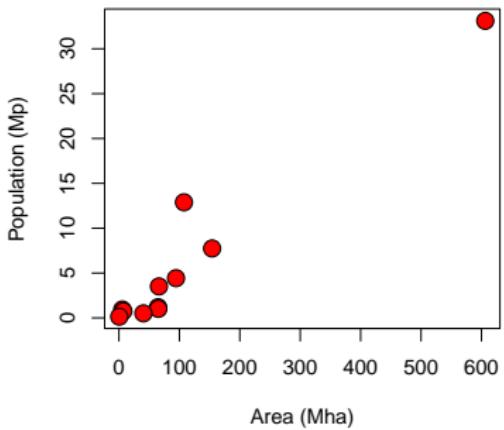
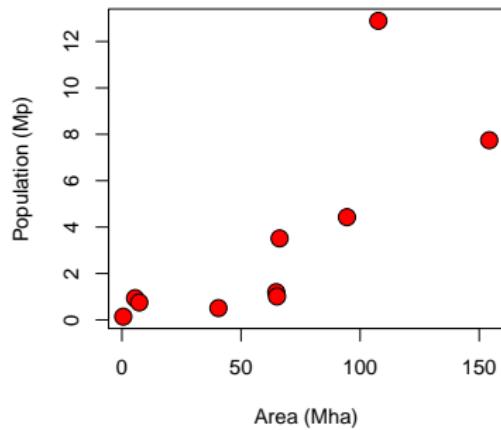
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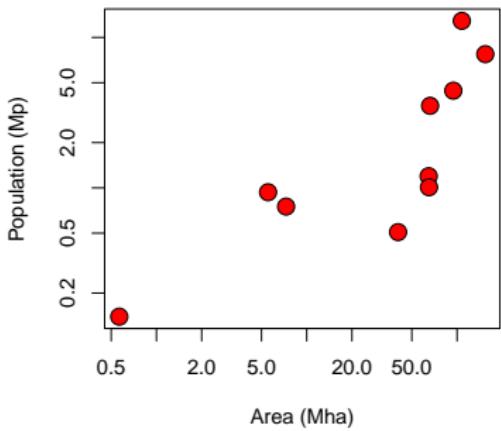
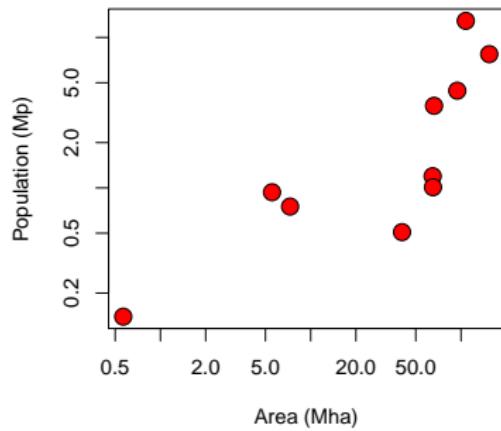
Canadian provinces plus Canada?



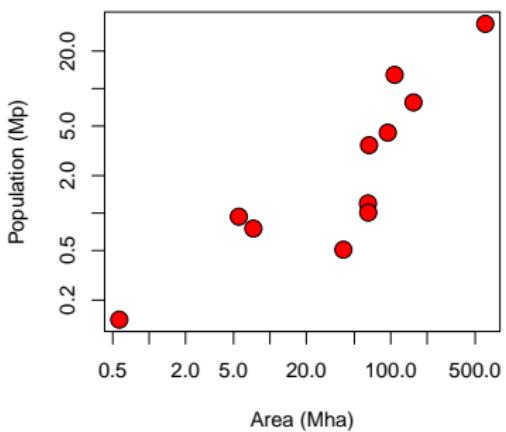
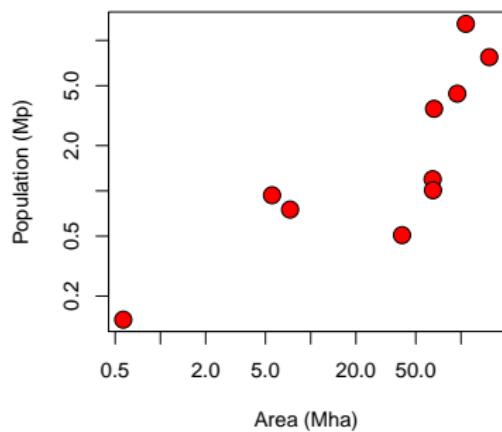
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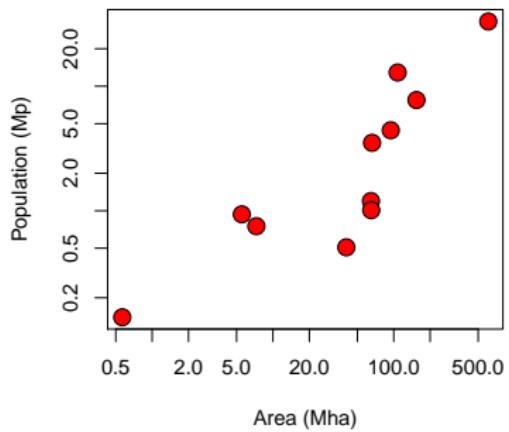
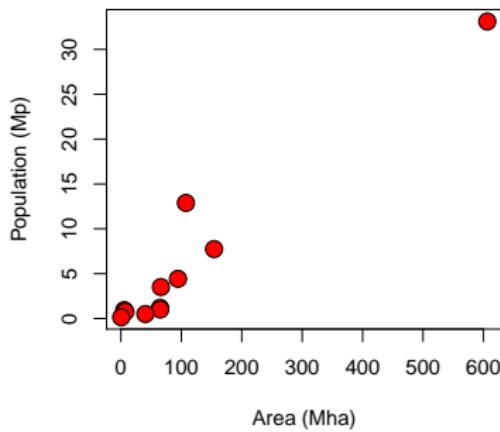
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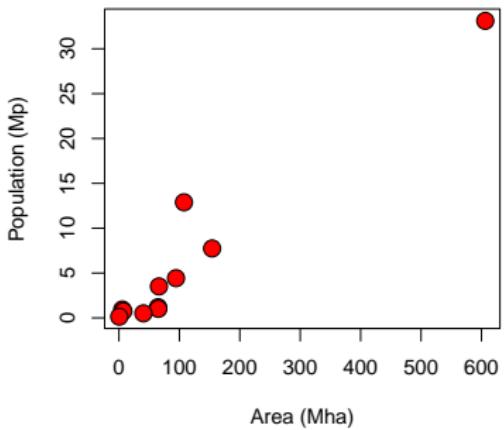
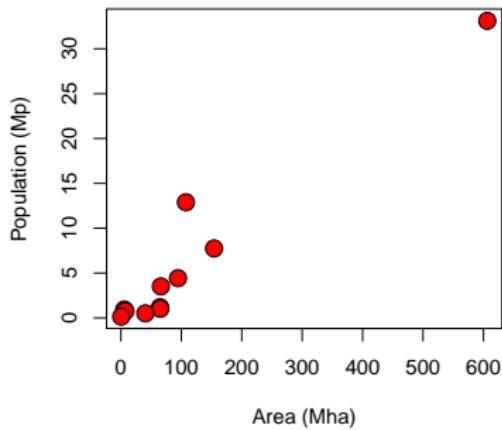
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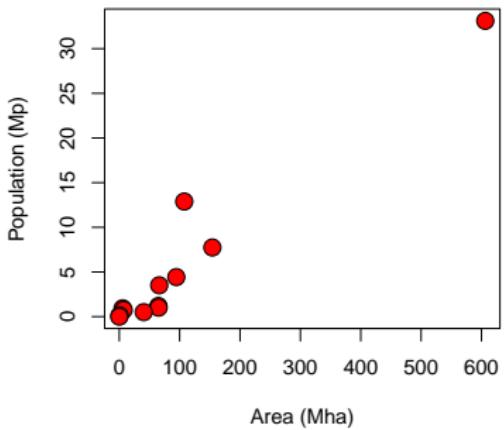
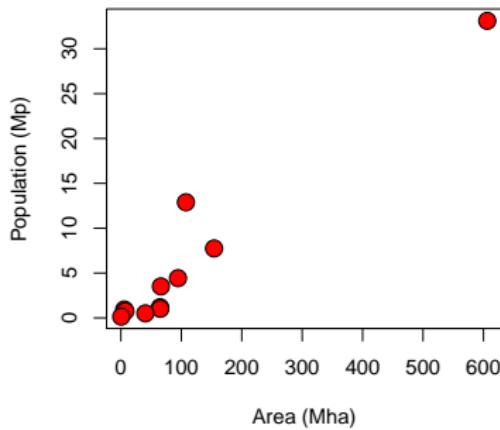
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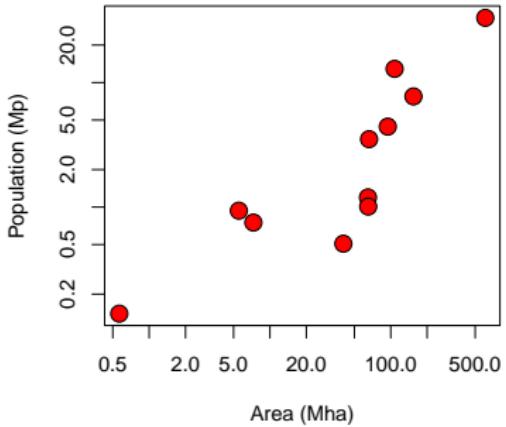
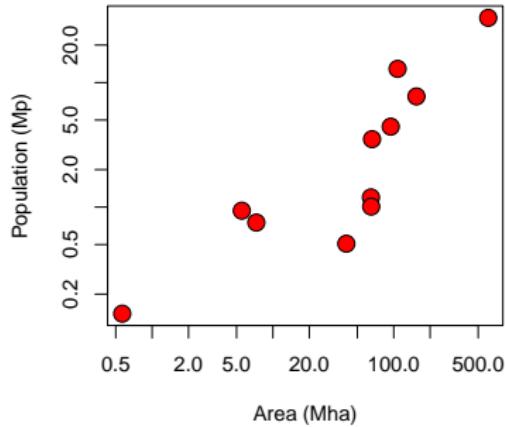
Canada plus Room 1102



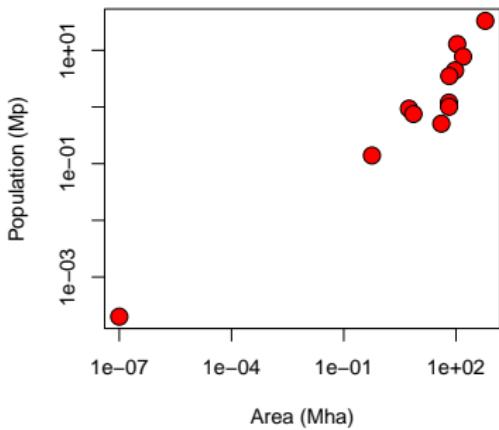
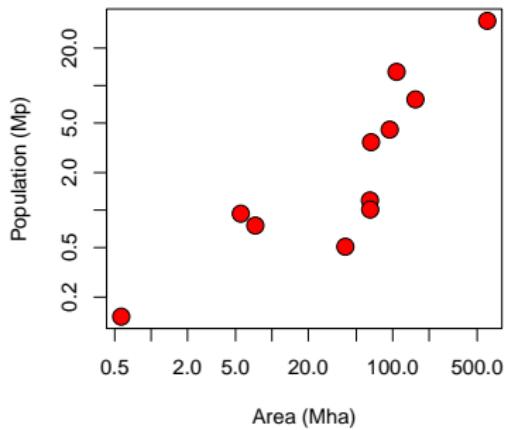
Canada plus Room 1102



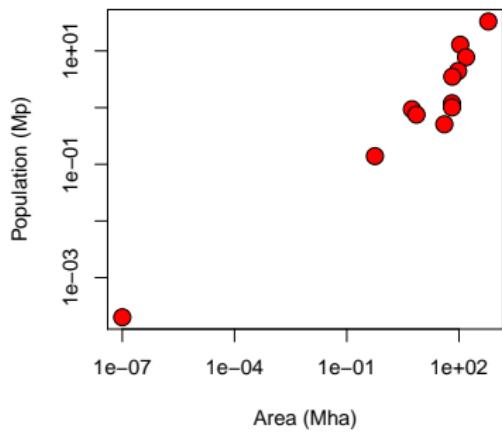
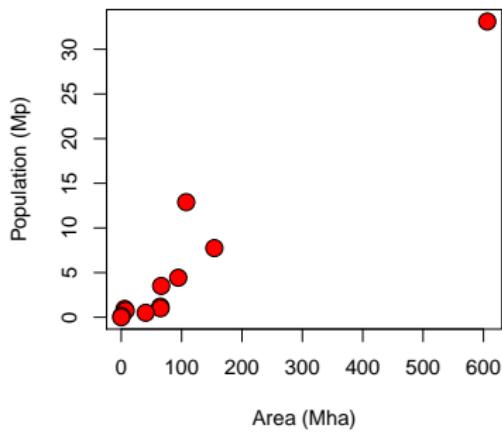
Canada plus room Room 1102?



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Predation comparison



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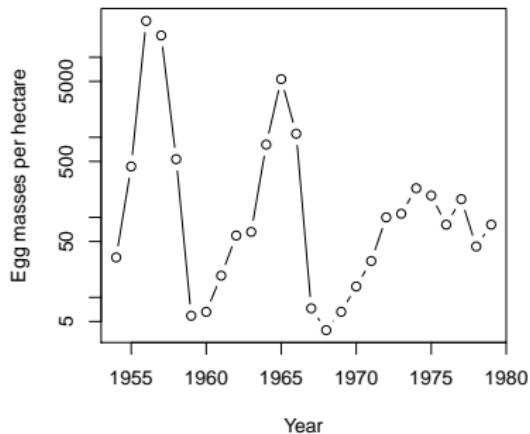
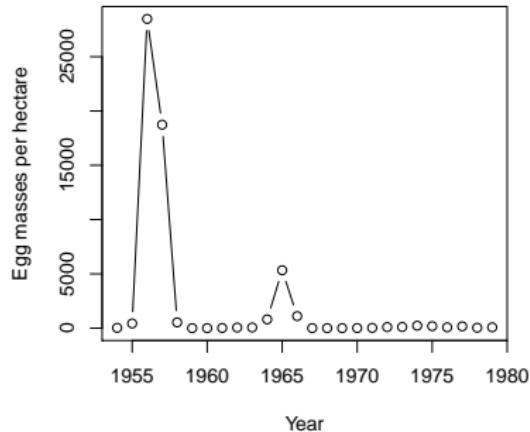
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Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

Example populations

Dandelions

Gypsy moths

Bacteria

Exponential growth

Log and linear scales

Time scales

Speeding in Taiwan

- A life experience



Speeding in Taiwan

- ▶ A life experience
- ▶ Some clarifications



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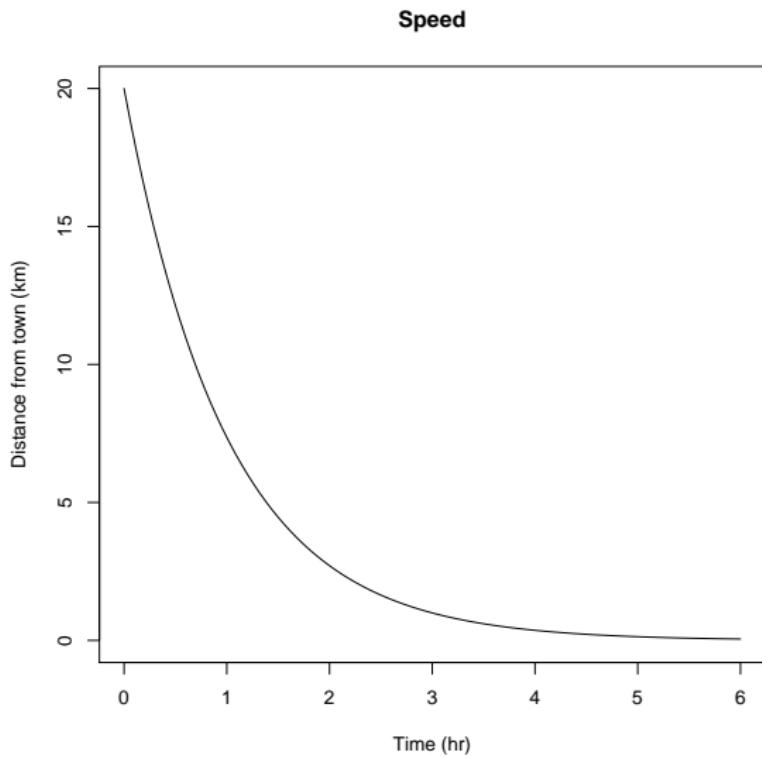
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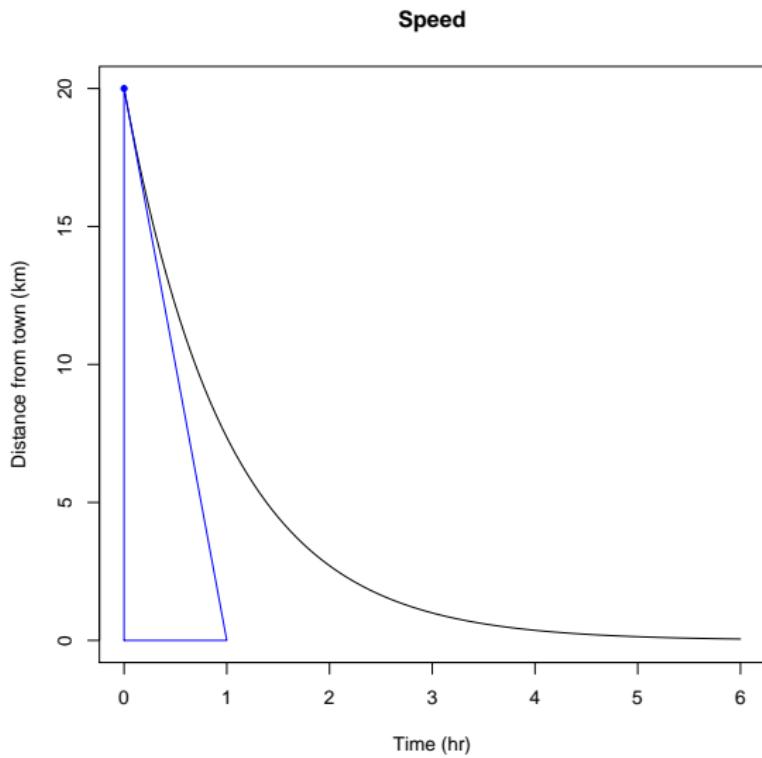
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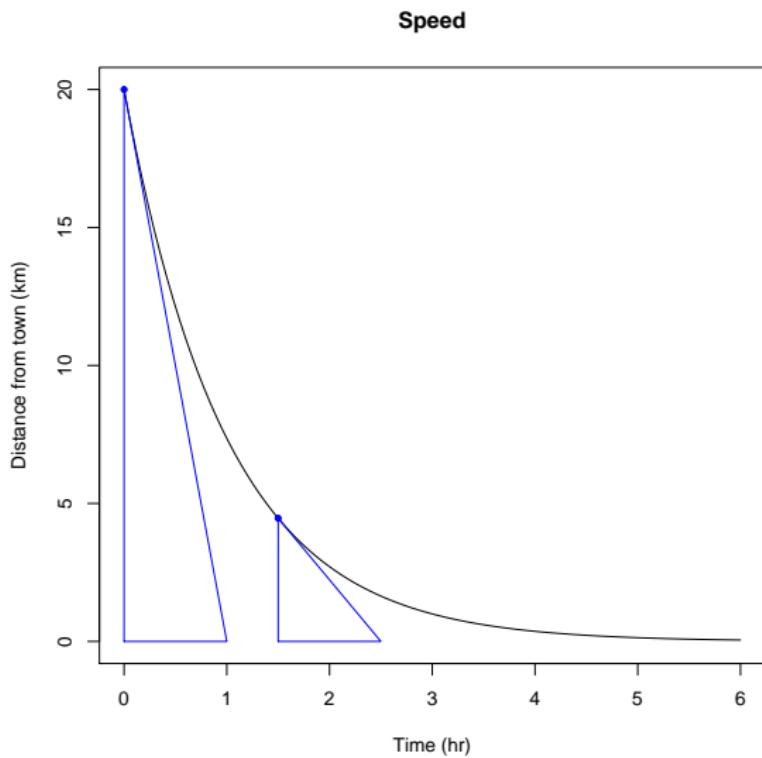
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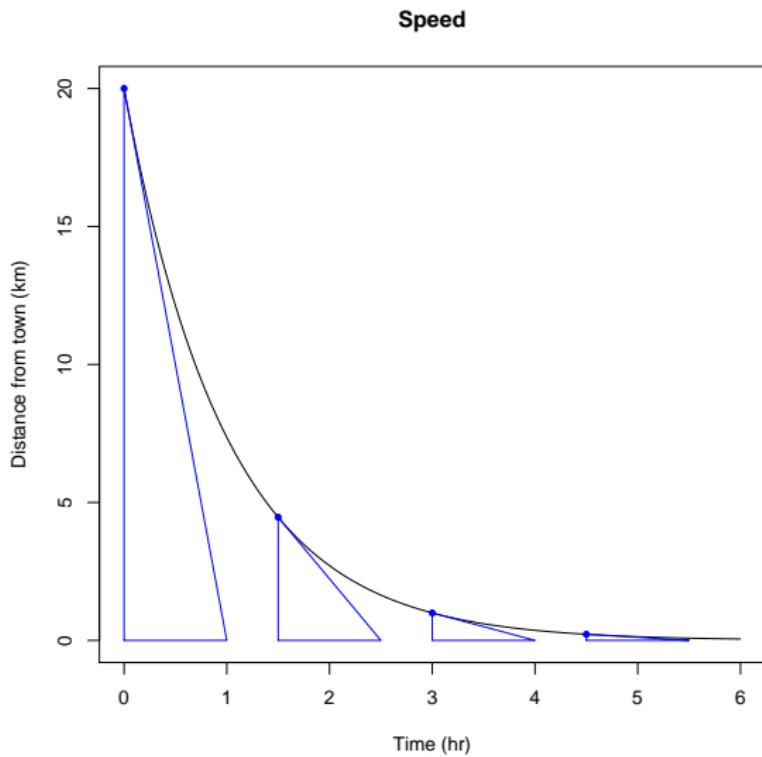
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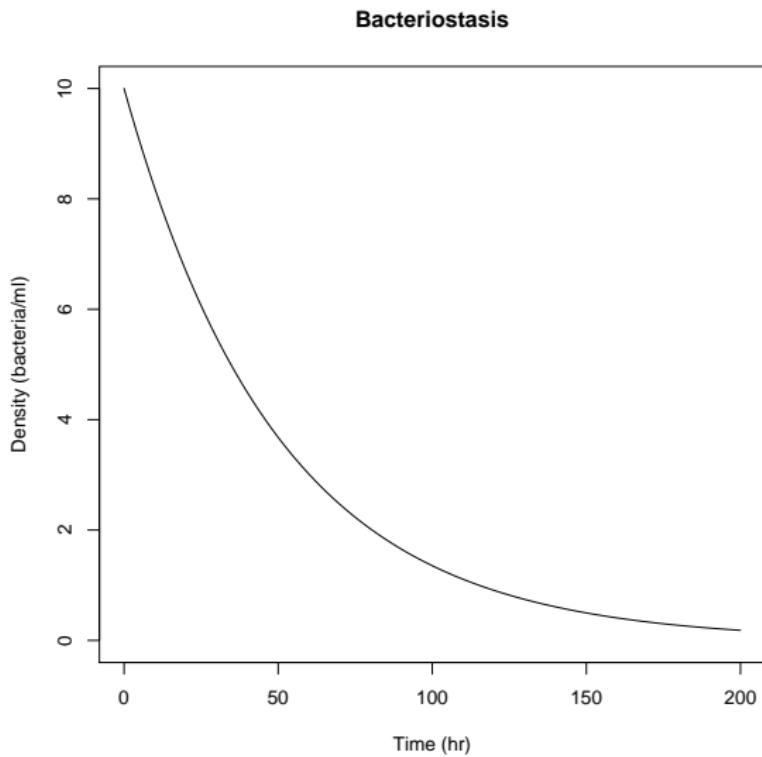
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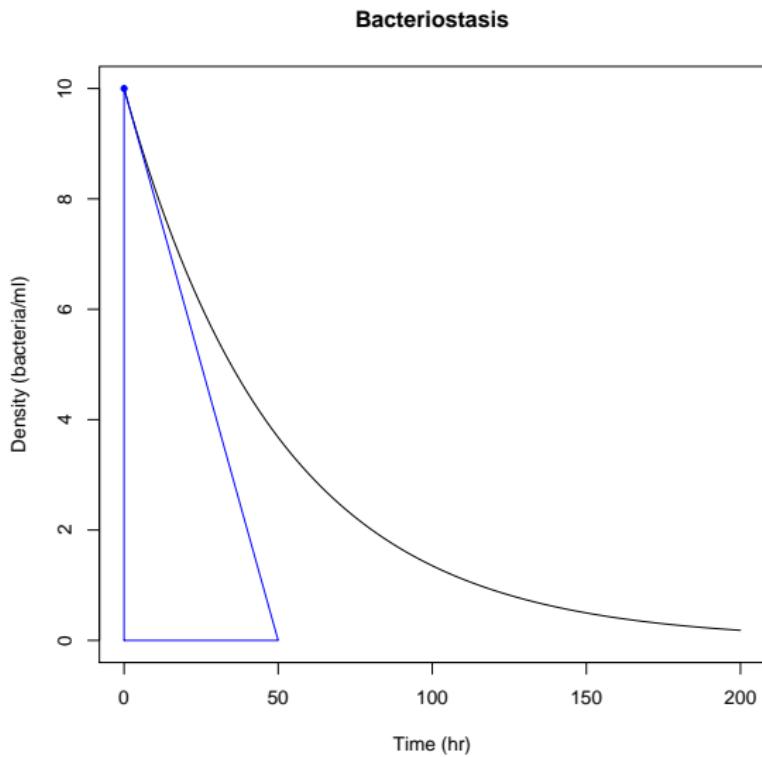
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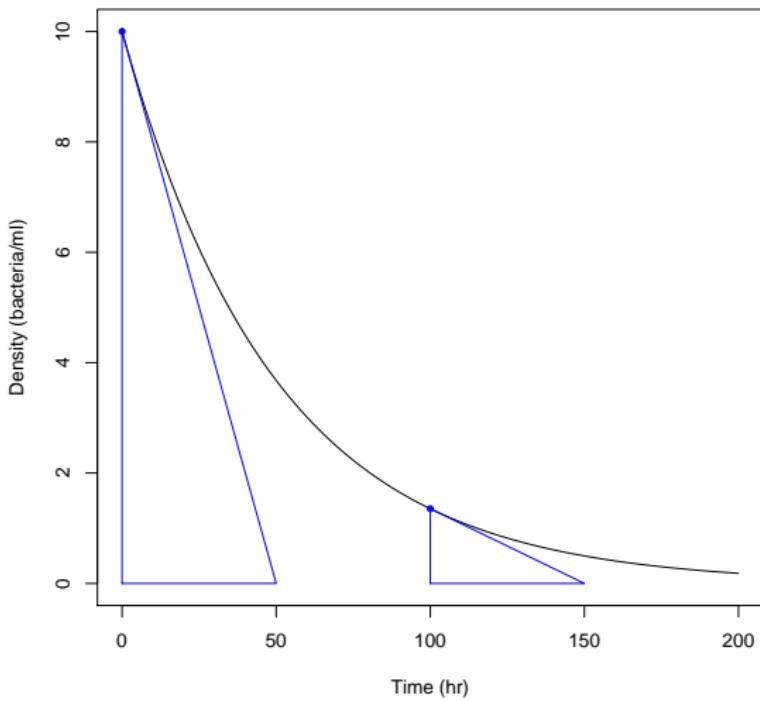


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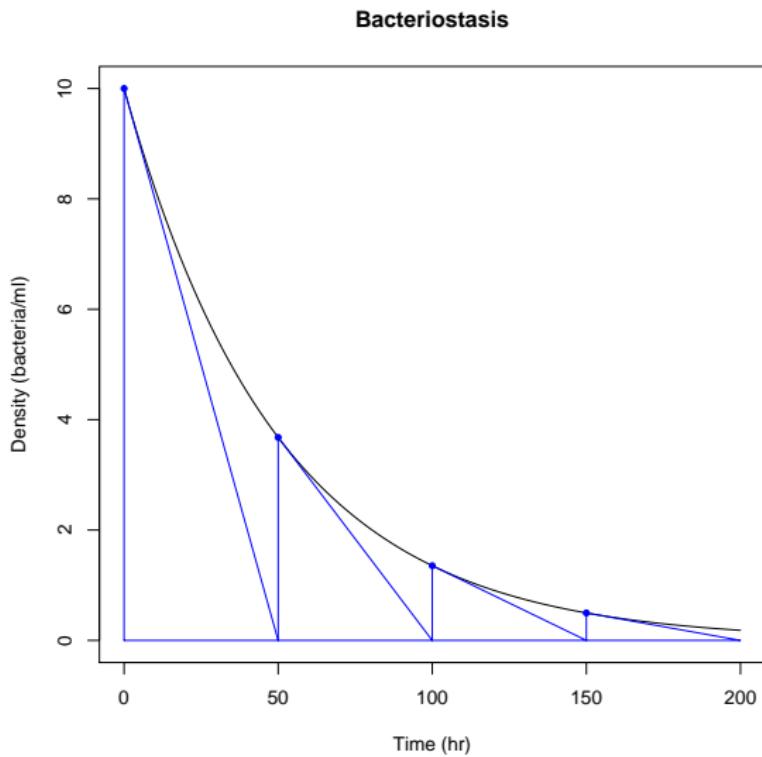


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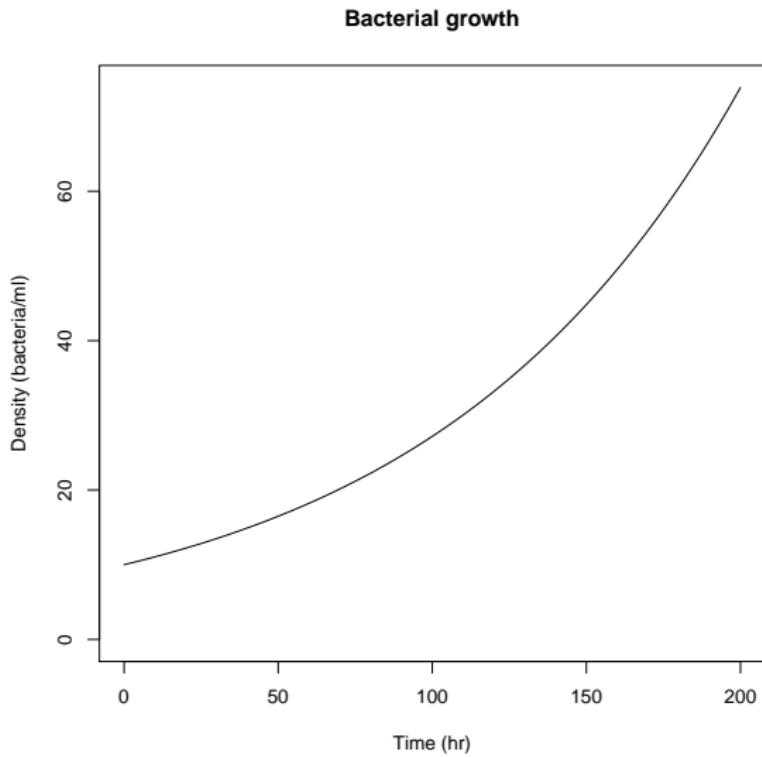
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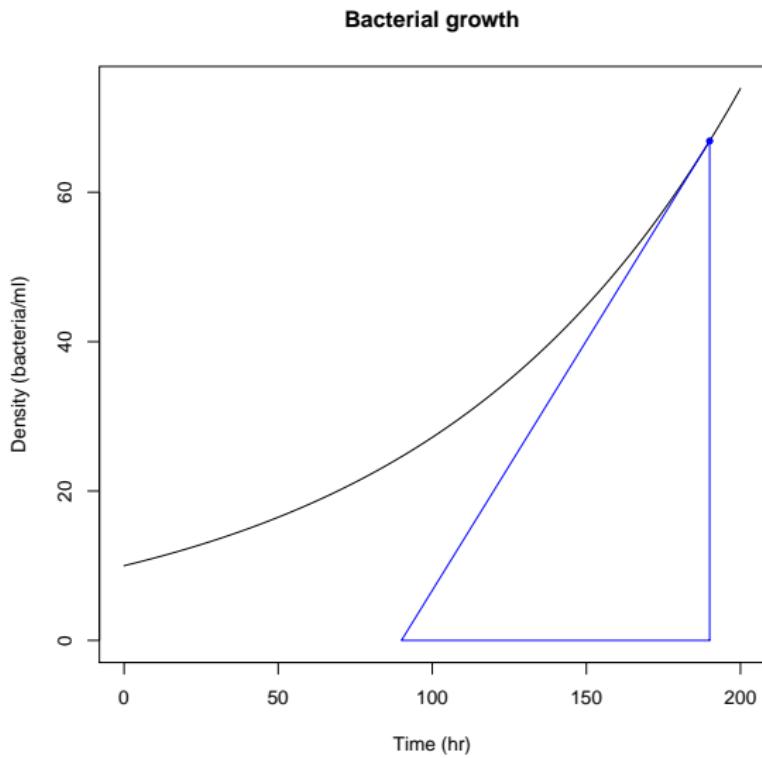
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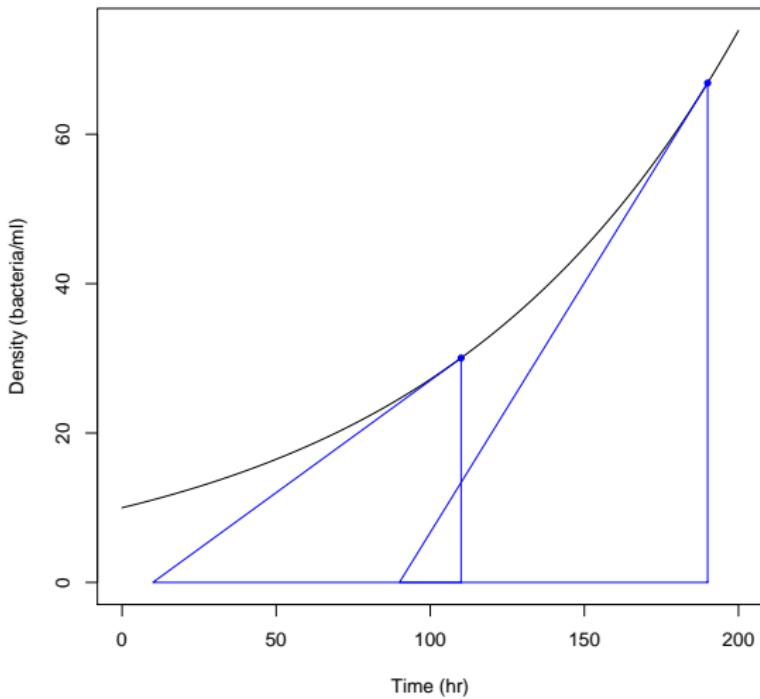


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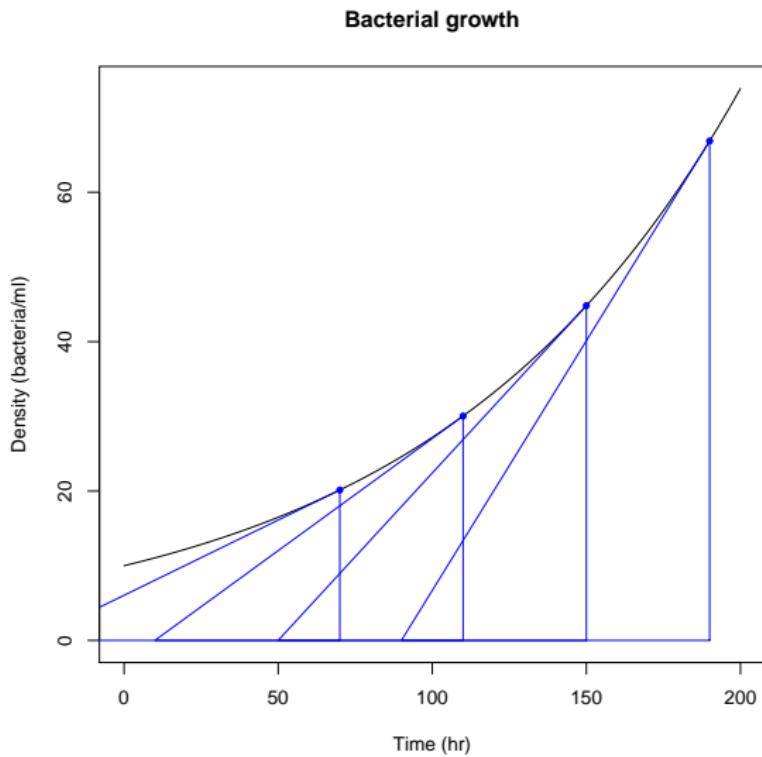


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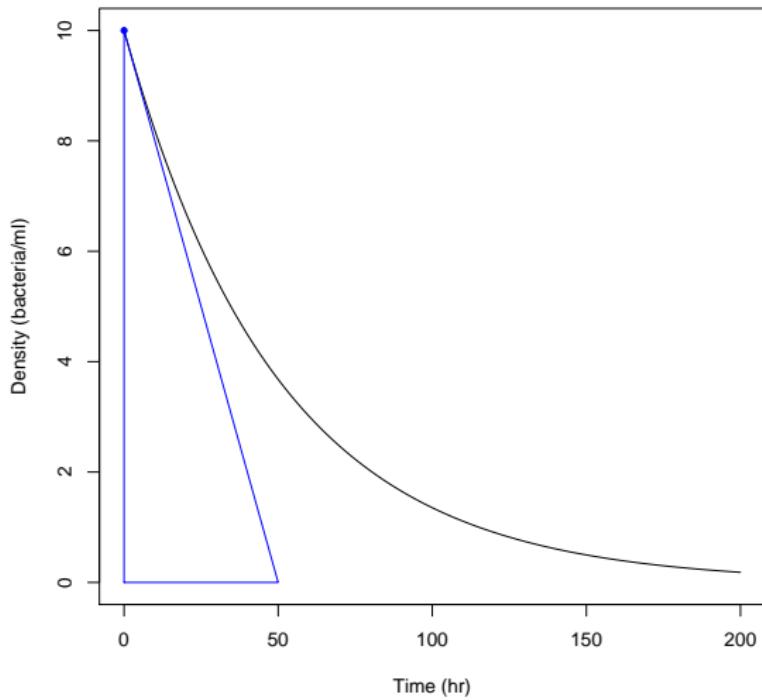
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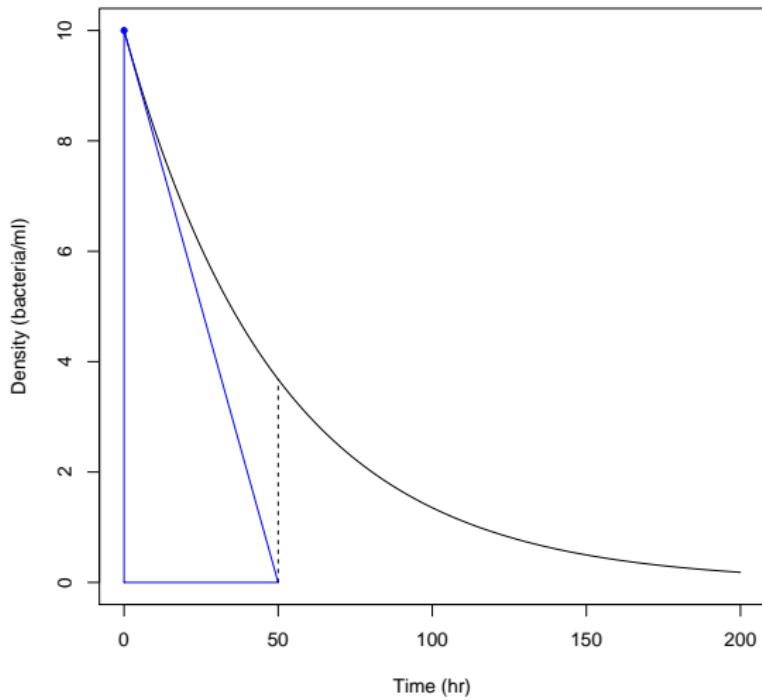
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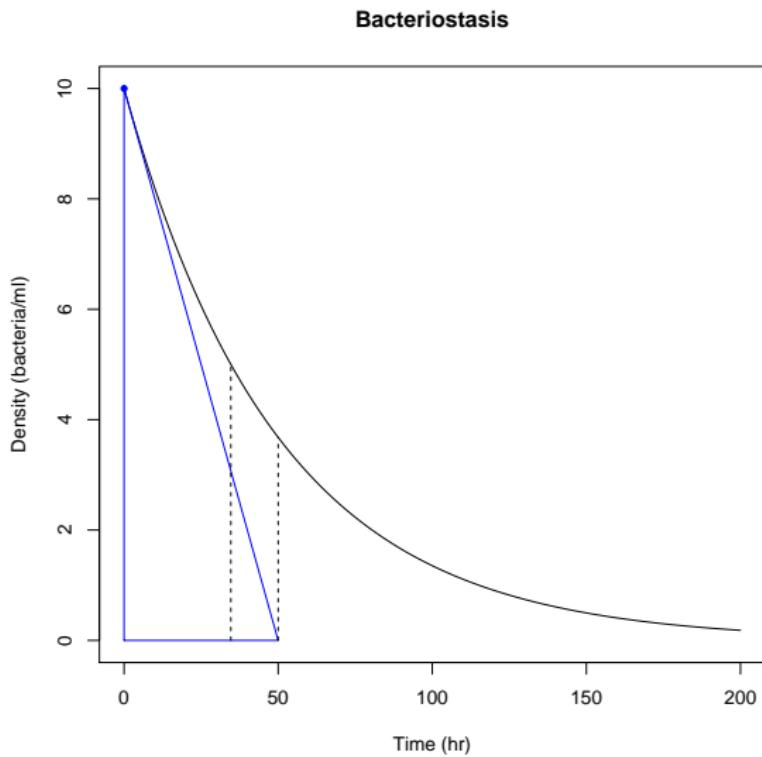
Bacteriostasis



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Characteristic time and half life



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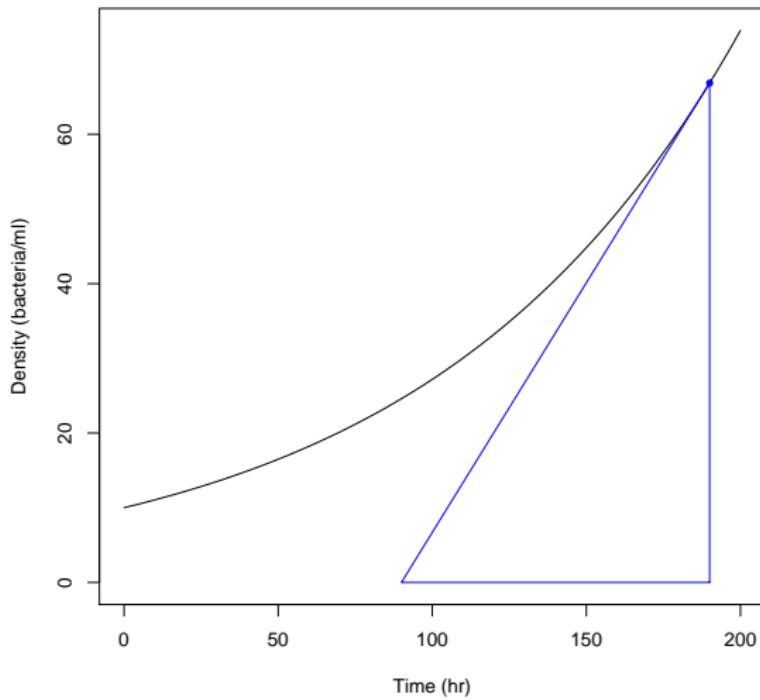
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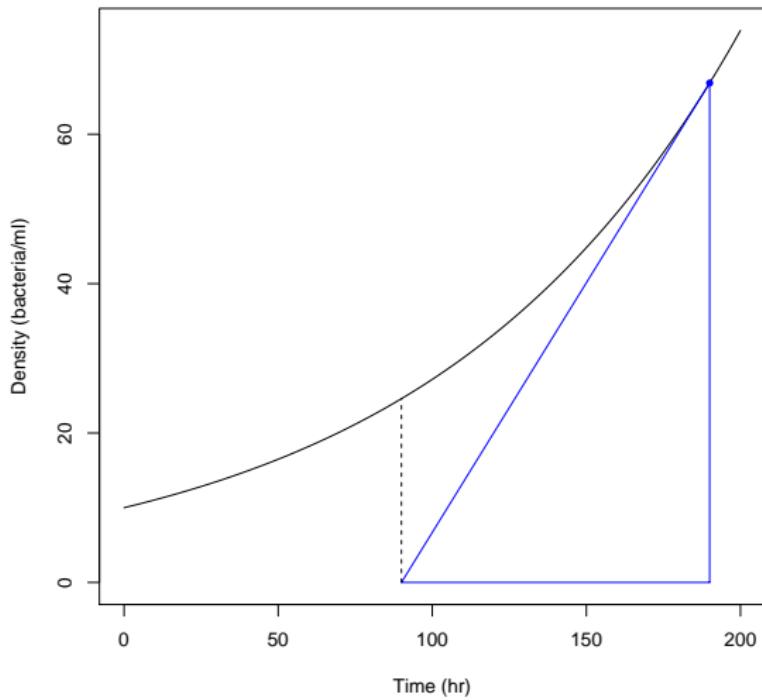
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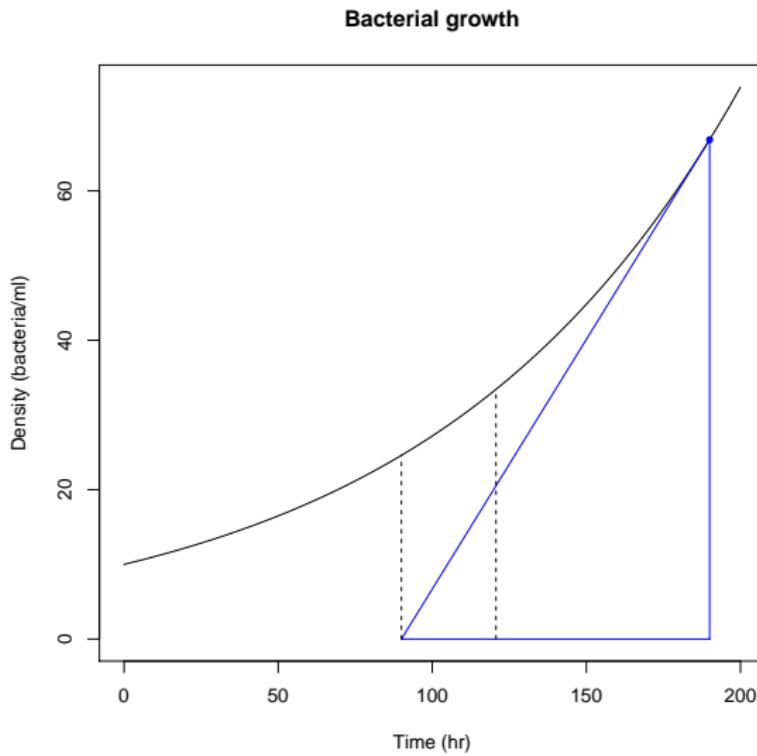
Bacterial growth



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