

# UNIT 1: Introduction

# Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

Example populations

Dandelions

Gypsy moths

Bacteria

Exponential growth

Log and linear scales

Time scales

# Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

Example populations

Dandelions

Gypsy moths

Bacteria

Exponential growth

Log and linear scales

Time scales

# Communication

- ▶ Lecture notes for each section will be available the afternoon before you need them
  - ▶ Check AtL frequently for announcements and new information
  - ▶ All info will also be on the course resource page
    - ▶ <http://bio3ss.github.io/>
- ▶ The professor is Jonathan Dushoff
  - ▶ dushoff@mcmaster.ca
  - ▶ Office hours will be announced
  - ▶ Or ask questions on facebook group
    - ▶ Bio 3SS Winter 2018
    - ▶ I am not your friend

## Expectations of professor

- ▶ Start and end on time
- ▶ Focus on conceptual understanding
- ▶ Make clear what terminology and facts must be learned
- ▶ Open to questions – both in class (within reason) and at office hours
- ▶ Responsive to questions on class forums (Facebook and AtL)

# Expectations of students

- ▶ Start and end on time
- ▶ Don't talk while other students are talking, or while I am responding to student questions
- ▶ If you must talk at other times, be unobtrusive
- ▶ Don't use the internet for non-class activities
- ▶ Attend the lecture, and the mandatory tutorials

## Texts

- ▶ The primary text for this course is the lecture notes
- ▶ You will be given readings, which will be posted to ATL
- ▶ You are required to have an Ecology textbook
  - ▶ Molles and Cahill, Second Canadian edition is recommended
  - ▶ If you would like to use a different textbook, let your TA know, so we can attempt to provide readings.

# Structure of presentation

- ▶ Required material will be clearly outlined in the notes
  - ▶ \* This is an answer: it was omitted from the notes for discussion purposes, you should probably write it in
  - ▶ *This is a comment: I omitted from the notes because I thought it wasn't necessary for you to study. If you write it in, make a note to yourself that it's a comment.*
- ▶ Required terminology will be presented in **bold**
- ▶ General ideas and approaches presented in class may also be required; you should take notes on these in your own words

## Taking notes

- ▶ You will do best if you take notes
  - ▶ You should know by now what works for you
  - ▶ Or else that you need to keep working on it
- ▶ If a new concept is making sense to you right now, write something that will help you remember
- ▶ If there's something specific I think you all need to write down, I will write it for you (or mark it as an answer)

# Polling

- ▶ You can obtain extra credit by responding to in-class polls
  - ▶ Text from your cell phone, or answer on the web
- ▶ Poll: Why are you taking this class?

# Outline

Course overview

Course structure

**People**

Course content

Learning goals

Examples

Example populations

Dandelions

Gypsy moths

Bacteria

Exponential growth

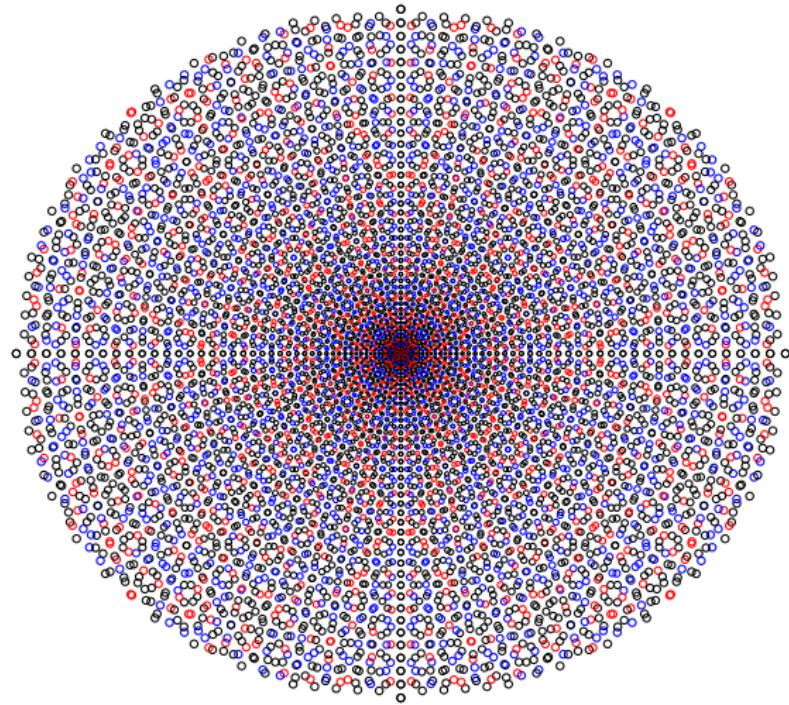
Log and linear scales

Time scales

# Dushoff

- ▶ Loves math
- ▶ Lived in four countries
- ▶ Studies evolution and spread of infectious diseases
  - ▶ HIV, rabies, ebola, influenza, ...
  - ▶ See notes for more info.

# *Pythagorean triples*



*Which country?*



# *Which country?*



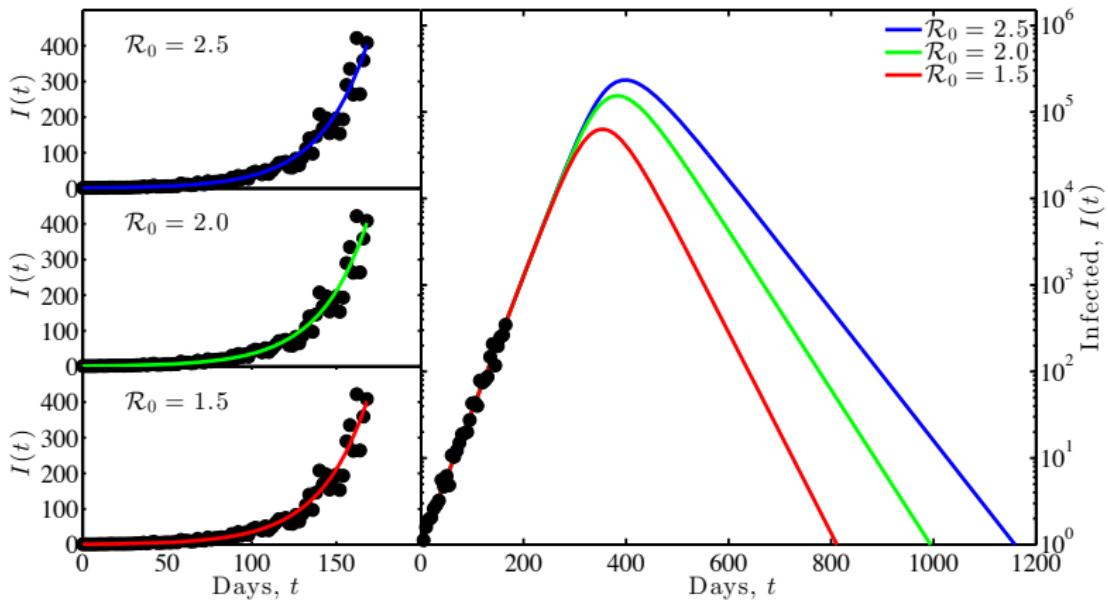
# Which country?



*Which country?*



## Ebola research



# TAs

- ▶ Morgan Kain

- ▶ Current: ecology and evolution of infectious diseases
- ▶ Forest ecology, animal behaviour
  - ▶ Focused on statistical approaches



- ▶ Michael Li

- ▶ Research: Disease forecasting and control
- ▶ Interests: Math, science, business, politics, philosophy.
  - ▶ How everything is connected



# Students

- ▶ Poll: What year are you in?
- ▶ Poll: What kind of career are you aiming for?

# Outline

## Course overview

Course structure

People

## Course content

Learning goals

Examples

## Example populations

Dandelions

Gypsy moths

Bacteria

## Exponential growth

Log and linear scales

Time scales

# Outline

## Course overview

Course structure

People

Course content

## Learning goals

Examples

## Example populations

Dandelions

Gypsy moths

Bacteria

## Exponential growth

Log and linear scales

Time scales

# Learning goals

- ▶ Ecology and population ecology
- ▶ Quantitative thinking
- ▶ Dynamical modeling

- ▶ Poll: What is ecology?
- ▶ My answer
  - ▶ \* The study of how organisms interact with each other and with the environment
  - ▶ \* Ecology is not environmentalism

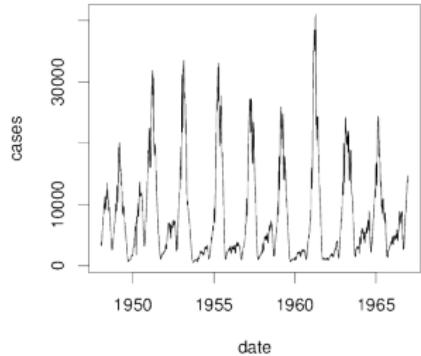
- ▶ Poll: What is population ecology?
- ▶ My answer
  - ▶ \* The study of how organisms interact with each other and with the environment at the population scale
  - ▶ \* Larger spatial scale, longer temporal scale
  - ▶ \* We use *dynamical models* to link from the individual level to the population level

# Dynamical modeling

- ▶ Investigates the links between local, short-term processes, and large-scale, long-term outcomes
- ▶ Allows us to explore what assumptions we're making, and how assumptions affect the link



Measles reports from England and Wales



# Math

- ▶ Population ecology uses math
  - ▶ Math is a critical tool for linking processes to outcomes
  - ▶ Math will play a central role in the course
- ▶ We will keep it *simple*
  - ▶ But we understand that simple does not always mean easy
- ▶ Review the math supplement

# Humans and abstract thought

- ▶ People are evolved to be concrete thinkers, not conceptual thinkers
- ▶ A goal of this course is to build conceptual thinking skills



value

energy | mass | speed of light

$$E = mc^2$$

J | kg | 299,792,458 m/s

units

$$c^2 = 89,875,517,873,681,800 \text{ m}^2/\text{s}^2$$

# Outline

## Course overview

Course structure

People

Course content

Learning goals

## Examples

## Example populations

Dandelions

Gypsy moths

Bacteria

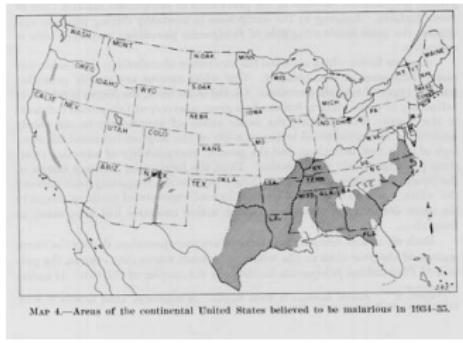
## Exponential growth

Log and linear scales

Time scales

# Malaria

- ▶ A nasty, mosquito-borne disease
- ▶ In some places (e.g., the southeastern US), it has been eradicated almost by accident
  - ▶ Mosquitoes are still present
- ▶ In other places it persists at high levels despite concerted efforts at elimination
- ▶ *What factors determine when and where malaria spreads?*



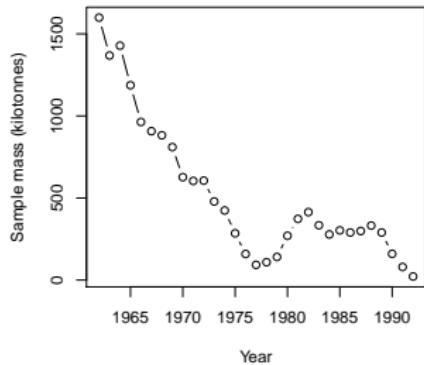
# Red squirrels

- ▶ Red squirrels are rapidly disappearing from England
  - ▶ Loss of suitable habitat?
  - ▶ Competition from gray squirrels introduced from North America?
  - ▶ Diseases carried by gray squirrels?



# Cod fisheries

- ▶ Is the ocean too big for people to affect?
- ▶ What happened to the cod?



# Populations

- ▶ Poll: What population of organisms interests you?

# Dandelions

- ▶ Start with one dandelion; it produces 100 seeds, of which only 4% survive to reproduce the next year.
  - ▶ How many dandelions after 3 years?
    - ▶ \* 64?
    - ▶ \* 125?



# Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

Example populations

Dandelions

Gypsy moths

Bacteria

Exponential growth

Log and linear scales

Time scales

# Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

Example populations

Dandelions

Gypsy moths

Bacteria

Exponential growth

Log and linear scales

Time scales

# Dandelions

- ▶ Start with one dandelion; it produces 100 seeds, of which only 4% survive to reproduce the next year.
- ▶ How many dandelions after 3 years?
  - ▶ \* 64?
  - ▶ \* 125?
  - ▶ See spreadsheet on resource page
- ▶ The spreadsheet is an implementation of a dynamical model!



# Dynamical models

- ▶ Make rules about how things change on a small scale
- ▶ Assumptions should be clear enough to allow you to calculate or simulate population-level results
- ▶ Challenging and clarifying assumptions is a key advantage of models

# Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

**Example populations**

Dandelions

Gypsy moths

Bacteria

Exponential growth

Log and linear scales

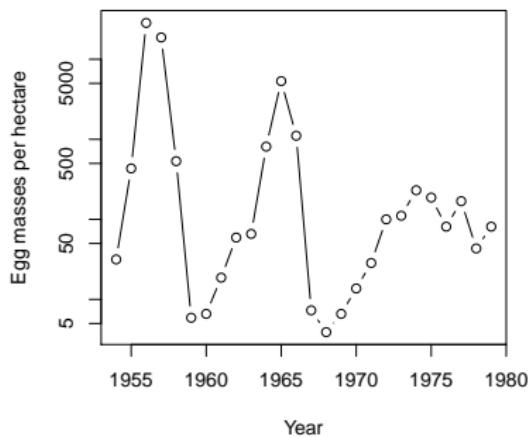
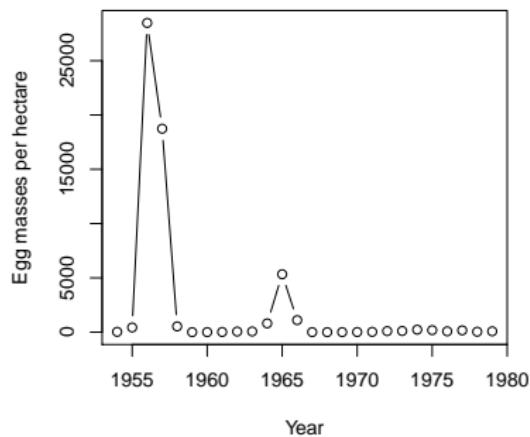
Time scales

# Gypsy moths

- ▶ A pest species that feeds on deciduous trees
- ▶ Introduced to N. America from Europe 150 years ago
- ▶ Capable of wide-scale defoliation



# Gypsy moth fopulations



## *Moth calculation*

- ▶ Researchers studying a gypsy moth population make the following estimates:
  - ▶ The average reproductive female lays 600 eggs
  - ▶ 10% of eggs hatch into larvae
  - ▶ 10% of larvae mature into pupae
  - ▶ 50% of pupae mature into adults
  - ▶ 50% of adults survive to reproduce
  - ▶ All adults die after reproduction
- ▶ Poll: What happens if we start with 10 moths?
  - ▶ \* We end up with 15 moths
  - ▶ \* On average

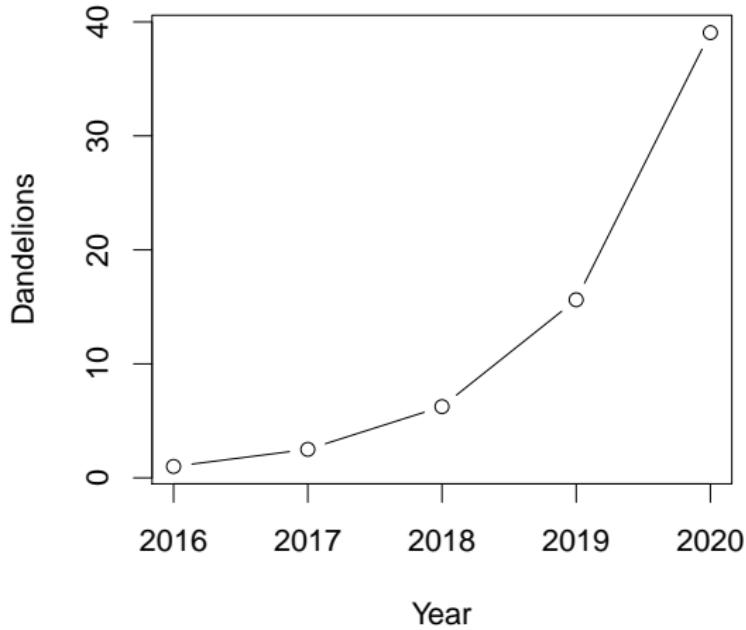
# Moth calculation

- ▶ Researchers studying a gypsy moth population make the following estimates:
  - ▶ The average reproductive female lays 600 eggs
    - ▶ \* Assume half are female
  - ▶ 10% of eggs hatch into larvae
  - ▶ 10% of larvae mature into pupae
  - ▶ 50% of pupae mature into adults
  - ▶ 50% of adults survive to reproduce
  - ▶ All adults die after reproduction
- ▶ What happens if we start with 10 moths?
  - ▶ \* If 5 are female, we end up with an average of 7.5 moths

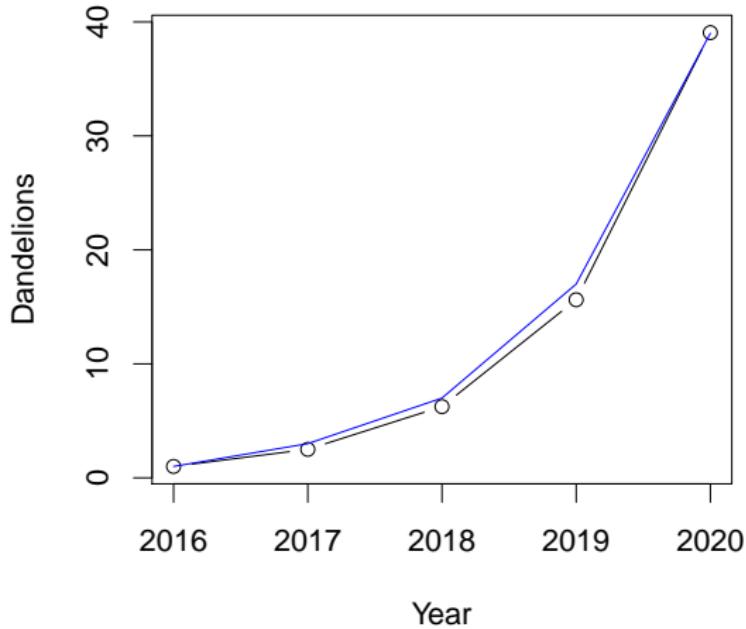
## Stochastic version

- ▶ Obviously, we will not get *exactly* 7.5 moths.
- ▶ If we consider moths as individuals, we need a **stochastic** model
- ▶ What do we mean by stochastic?
  - ▶ \* The model has randomness, to reflect details that we can't measure in advance, or can't predict

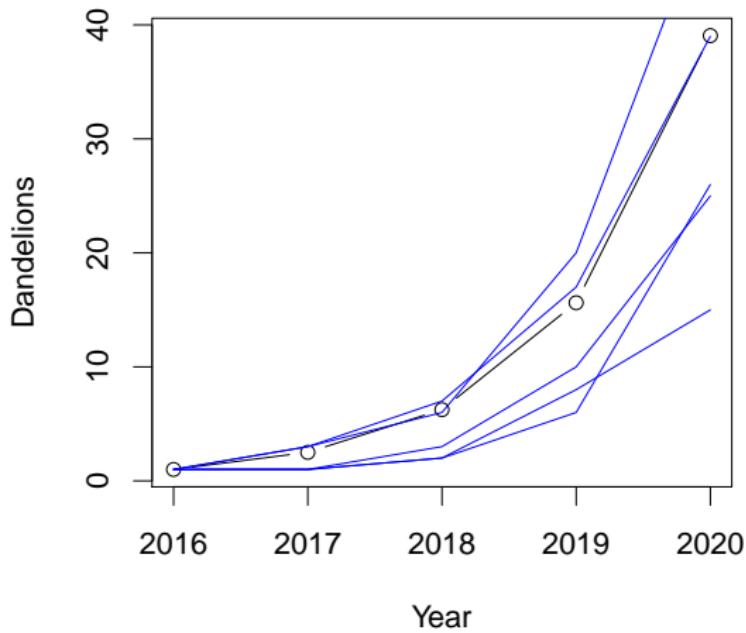
## Stochastic model



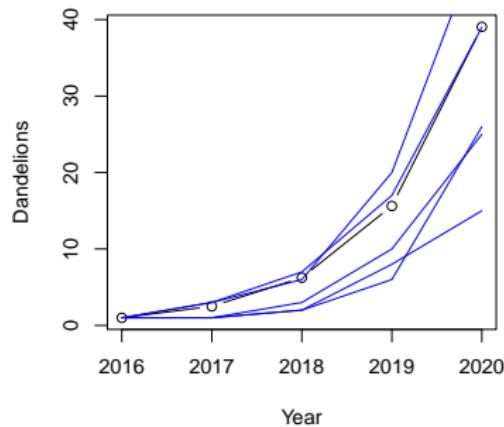
## Stochastic model



## Stochastic model



# Stochastic model



- ▶ A stochastic model has randomness in the model.
- ▶ If we run it again with the same parameters and starting conditions, we get a different answer

# Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

Example populations

Dandelions

Gypsy moths

Bacteria

Exponential growth

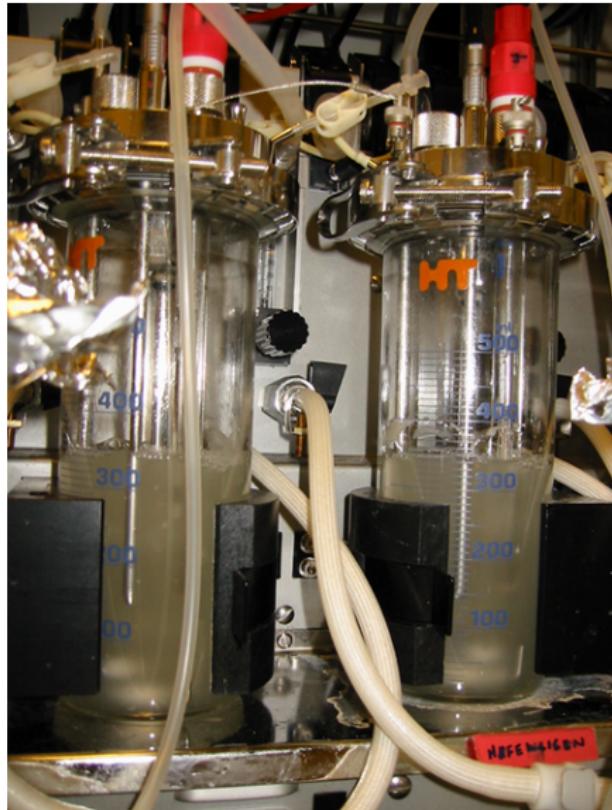
Log and linear scales

Time scales

# Bacteria

- ▶ Imagine we have some bacteria growing in a big tank, constantly dividing and dying:
  - ▶ They divide (forming two bacteria from one) at a rate of 0.04/ hr
  - ▶ They wash out of the tank at a rate of 0.02/ hr
  - ▶ They die at a rate of 0.01/ hr
- ▶ Rates are **per capita** (i.e., per individual) and **instantaneous** (they describe what is happening at each moment of time)
- ▶ We start with 10 bacteria/ml
  - ▶ How many do we have after 1 hr?
  - ▶ What about after 1 day?

# Bacteria in a tank



## Bacteria, rescaled

- ▶ Imagine we have some bacteria growing in a big tank:
  - ▶ They divide (forming two bacteria from one) at a rate of 0.96/day
  - ▶ They wash out of the tank at a rate of 0.48/day
  - ▶ They die at a rate of 0.24/day
- ▶ If we start with 10 bacteria/ml, how many do we have after 1 day?

# Units

- ▶ When we attach units to a quantity, the meaning is concrete
  - ▶  $0.24/\text{day}$  *must* mean exactly the same thing as  $0.01/\text{hr}$
  - ▶ The two questions above *must* have the same answer

## Bacterostasis

- ▶ What if we add an agent to the tank that makes the birth and death rates nearly zero?
- ▶ Now the bacteria are merely washing out at the rate of 0.02/hr
- ▶ If we start with 10 bacteria/ml, how many do we have after:
  - ▶ Poll: 1 hr?
  - ▶ Poll: 1 wk?

# Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

Example populations

Dandelions

Gypsy moths

Bacteria

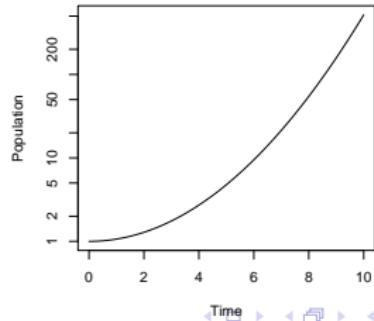
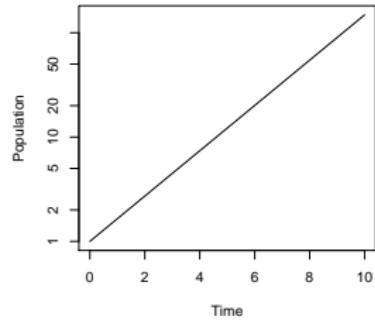
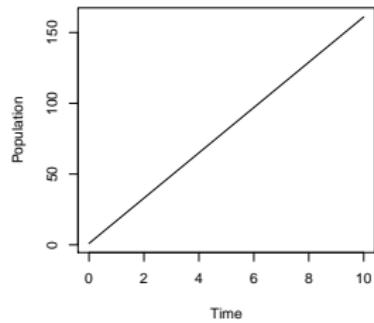
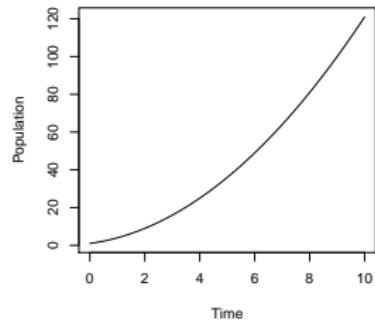
Exponential growth

Log and linear scales

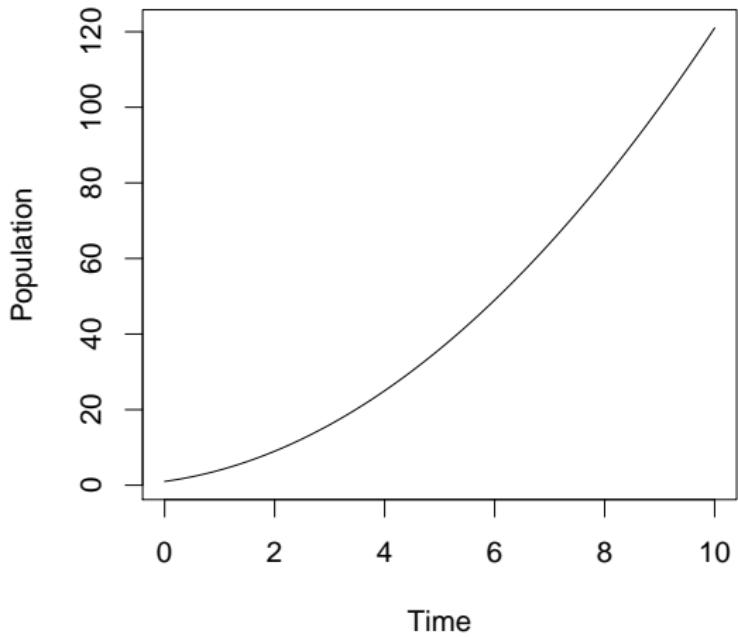
Time scales

# Exponential growth

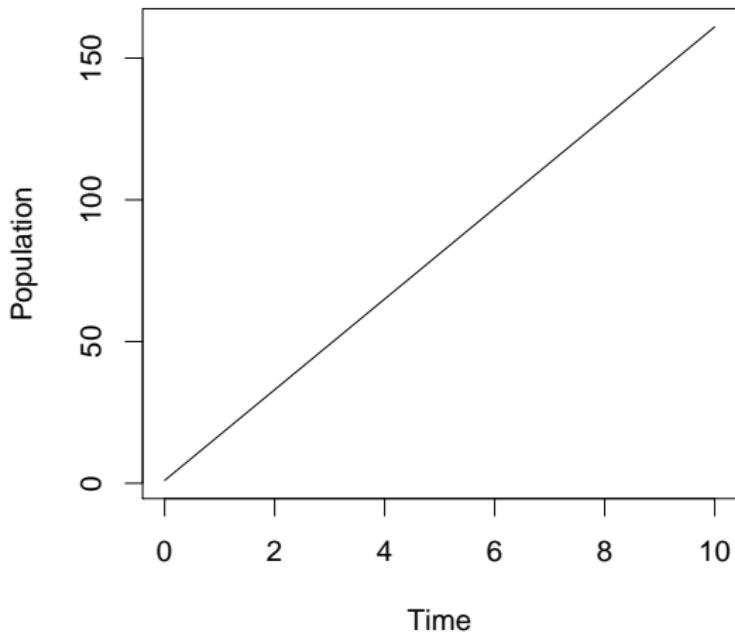
- ▶ What is exponential growth?
- ▶ Which of these is an example?



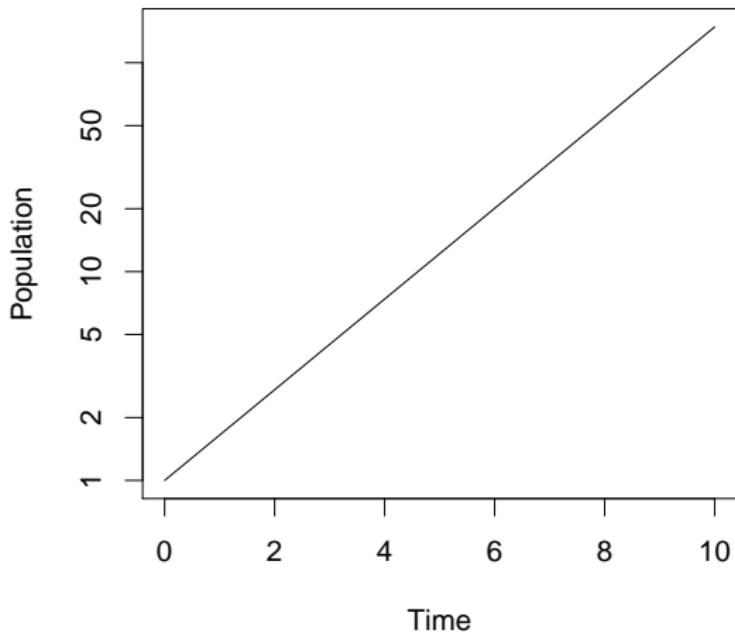
A



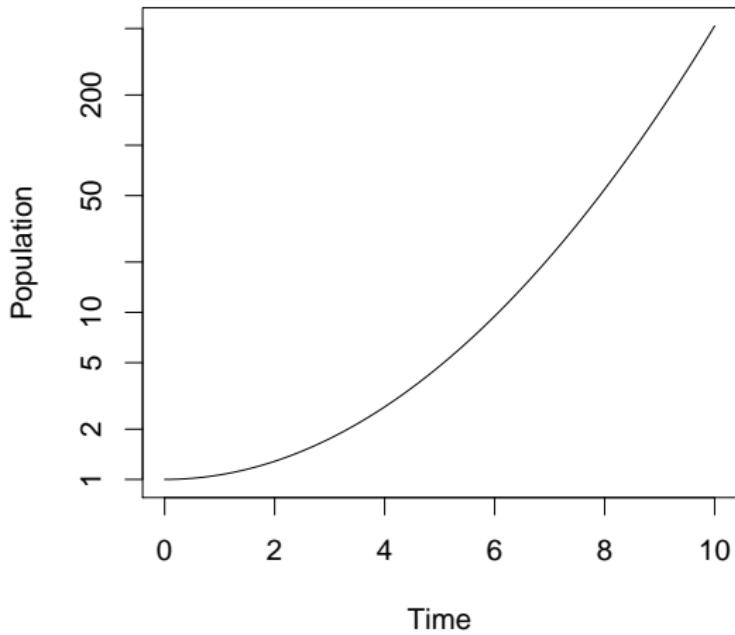
B



C

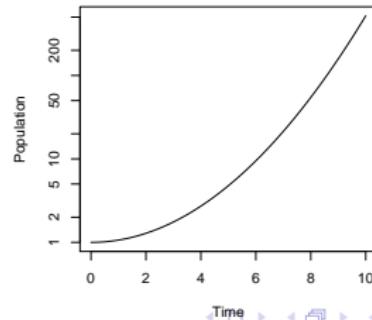
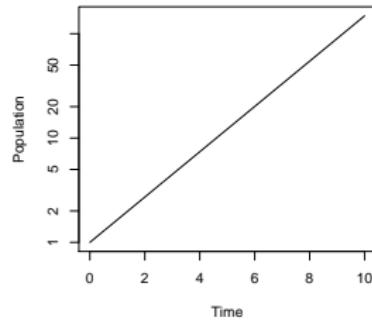
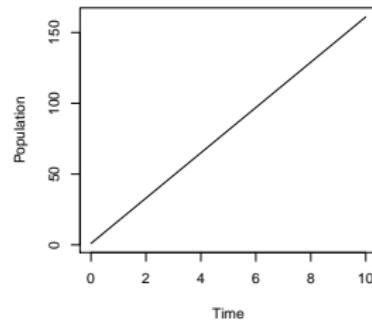
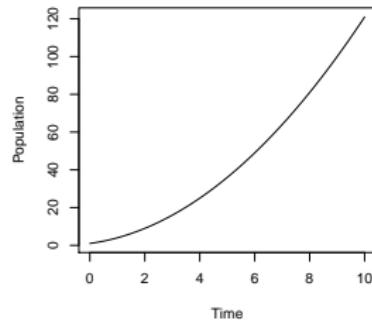


D

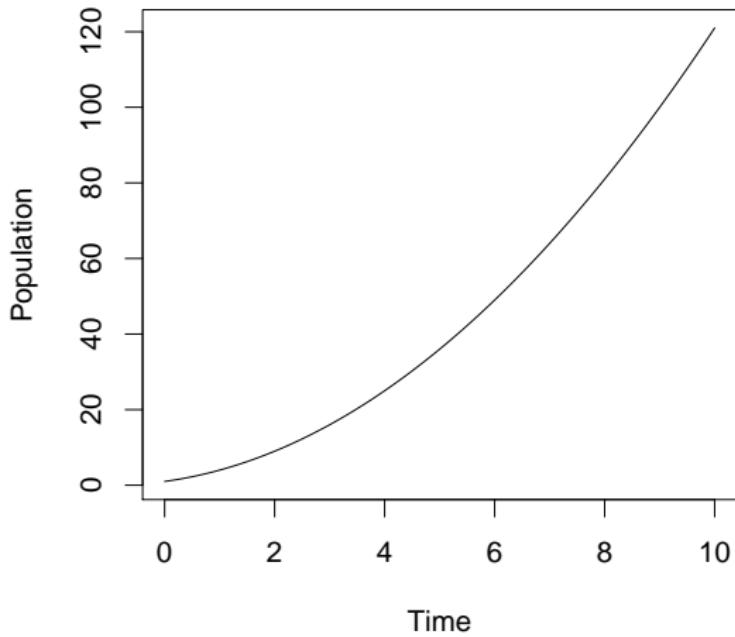


# Exponential growth

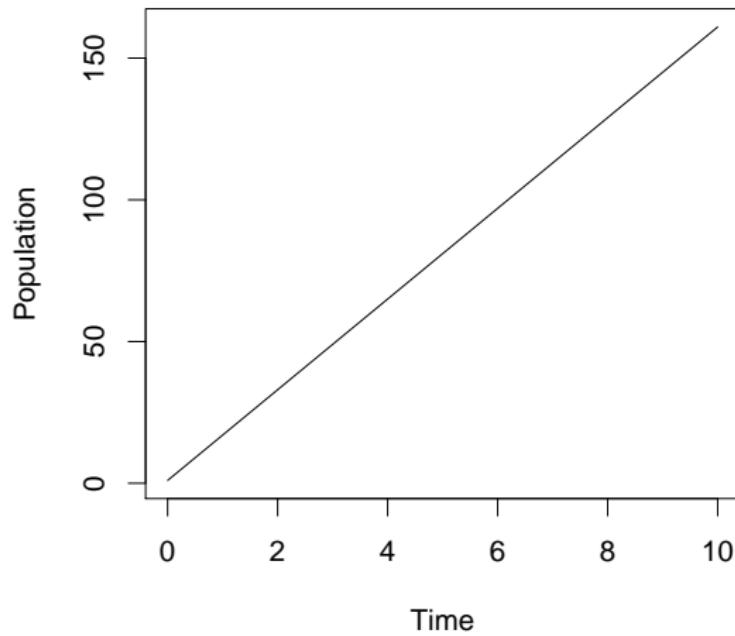
- ▶ Poll: What is exponential growth?
- ▶ Poll: Which of these is an example?



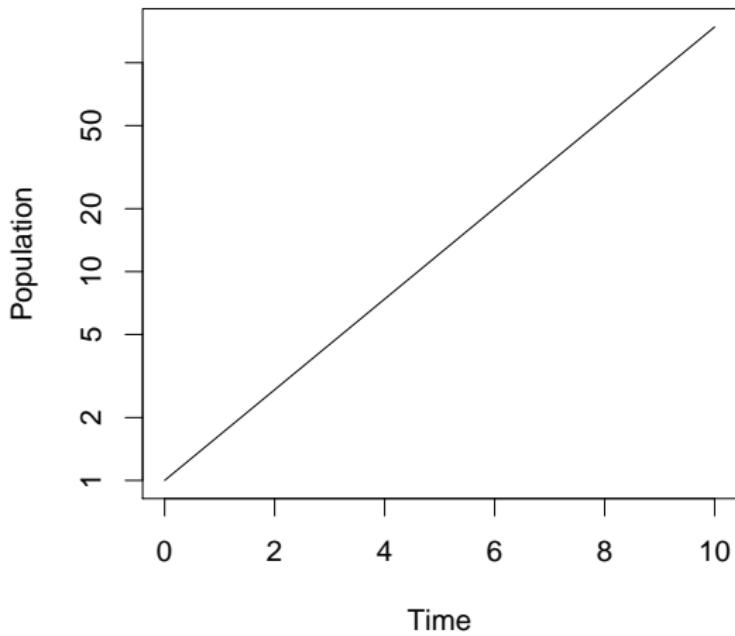
A



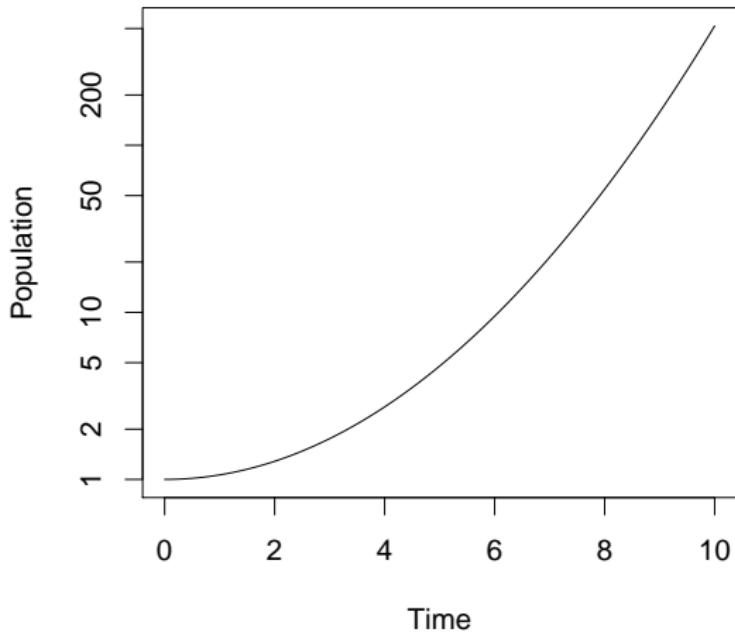
B



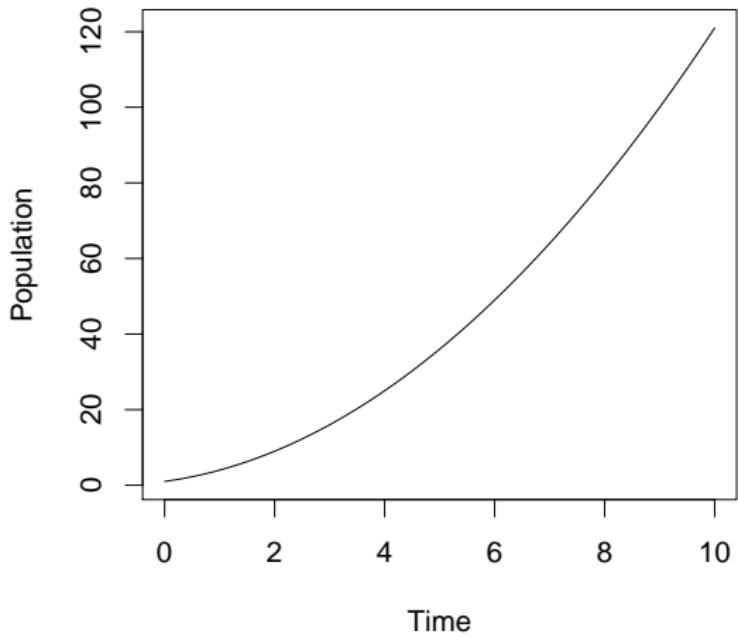
C



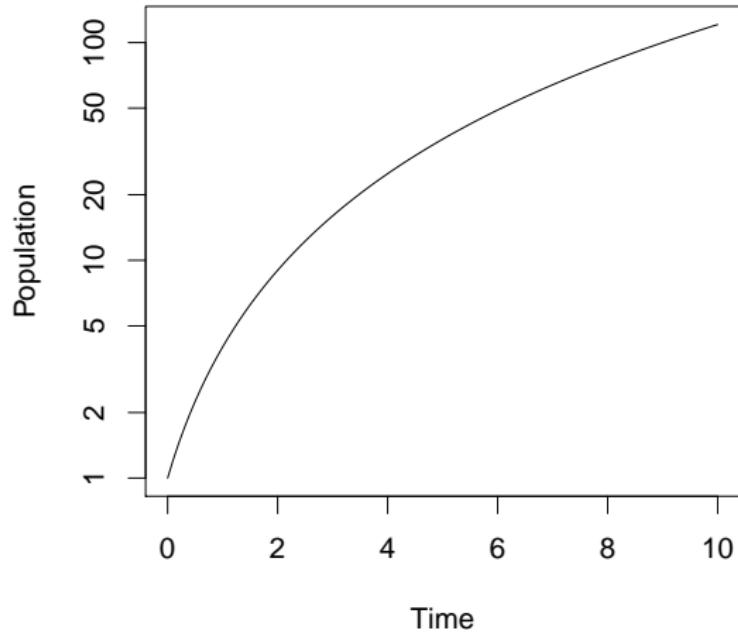
D



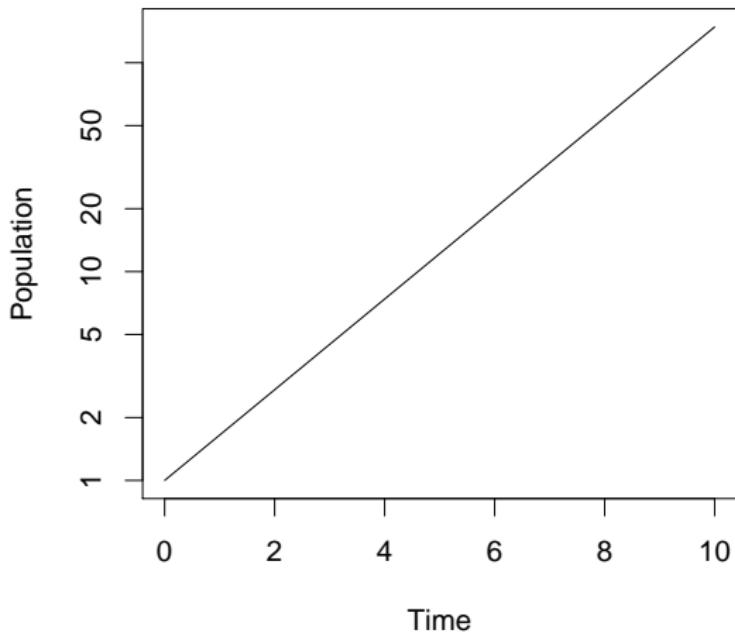
A



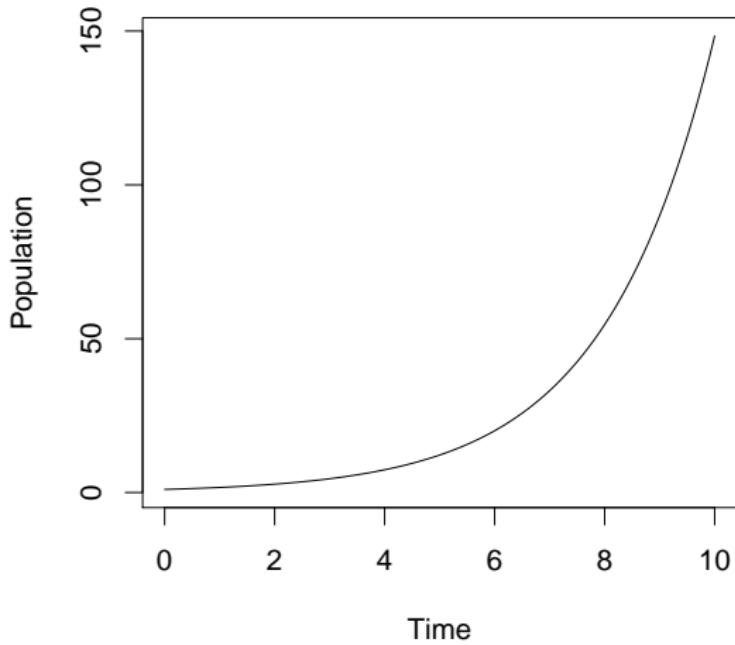
## A on the log scale



C



## C on the linear scale



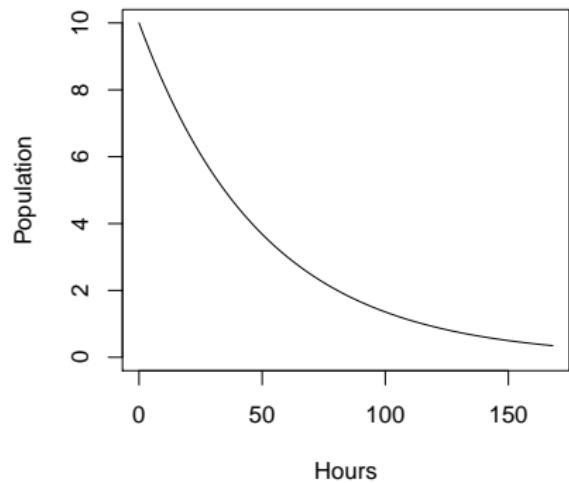
# Types of growth

- ▶ arithmetic/linear:
  - ▶ \* *Add a fixed amount in a given time interval*
  - ▶ \* *Total growth rate is constant*
- ▶ geometric/exponential:
  - ▶ \* *Multiply by a fixed amount in a given time interval*
  - ▶ \* *Per-capita growth is constant*
  - ▶ \* *Only C is exponential, mathematically speaking.*
- ▶ other:
  - ▶ Many possibilities, we may discuss some later

# Exponential decline?

- ▶ Poll: What does exponential decline look like?
  - ▶ \* Decline is proportional to size
  - ▶ \* Declines more and more *slowly* (on linear scale)

## Exponential decline



- ▶ Decline is proportional to size
- ▶ Declines more and more slowly (on linear scale)

# Terminology

- ▶ Sometimes people distinguish
  - ▶ **arithmetic** from **linear** growth, or
  - ▶ **geometric** from **exponential** growth
- ▶ Based on:
  - ▶ \* **discrete** vs. **continuous** time
- ▶ We won't worry much about this.

# Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

Example populations

Dandelions

Gypsy moths

Bacteria

Exponential growth

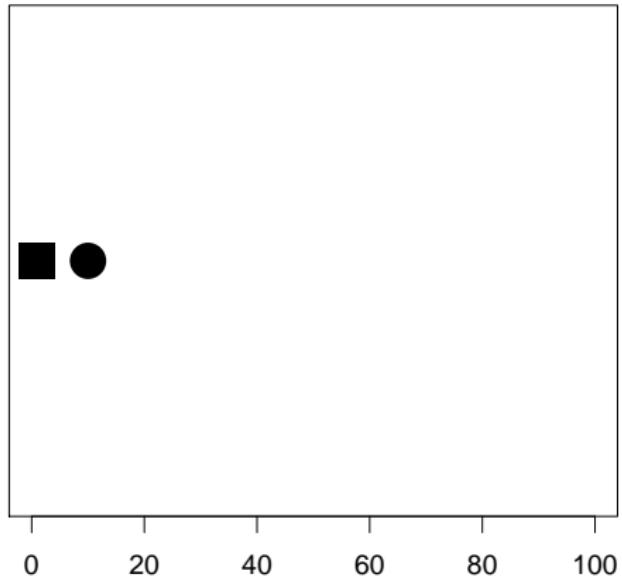
Log and linear scales

Time scales

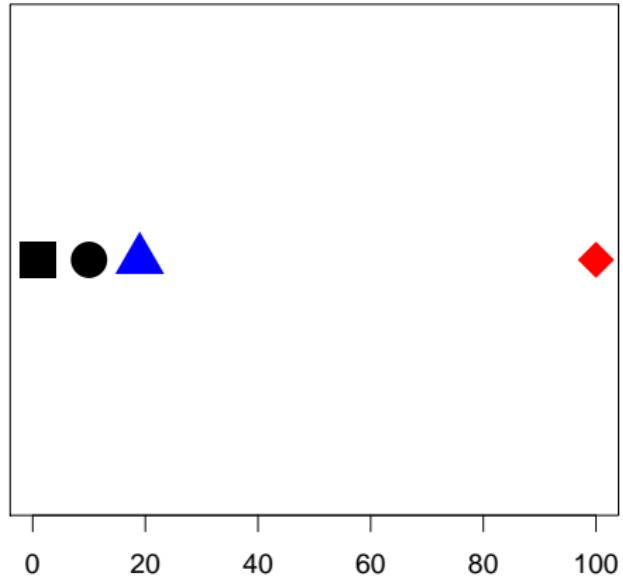
## Scales of comparison

- ▶ Poll: 1 is to 10 as 10 is to what?
  - ▶ \* If you said 100, you are thinking multiplicatively
  - ▶ \* If you said 19, you are thinking additively

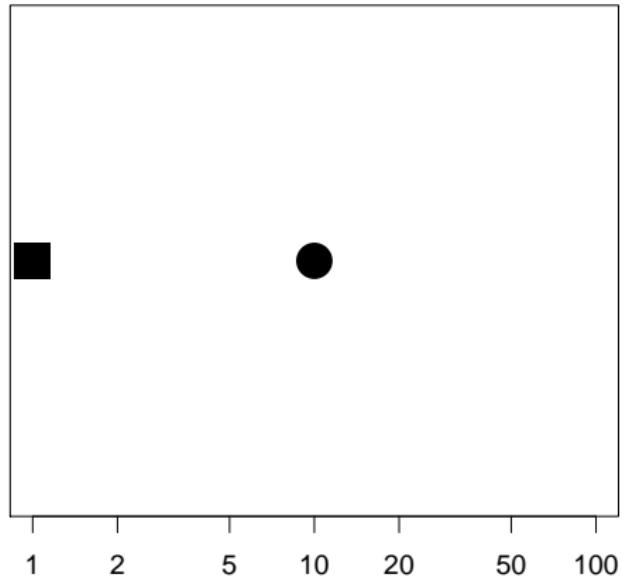
## *Scales of display*



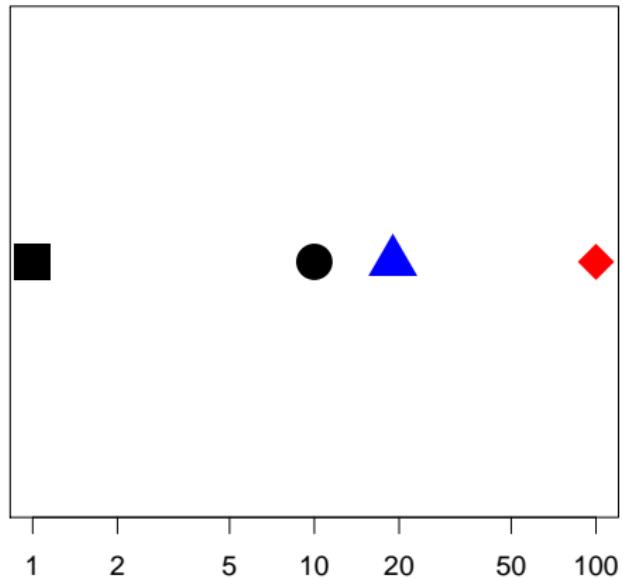
## *Scales of display*



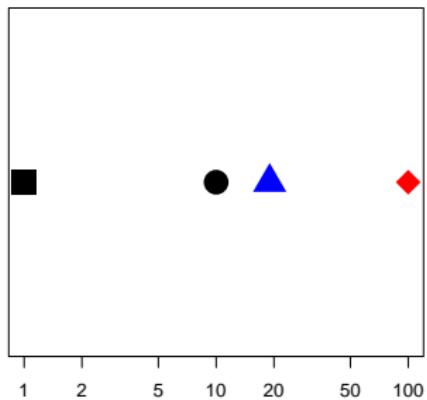
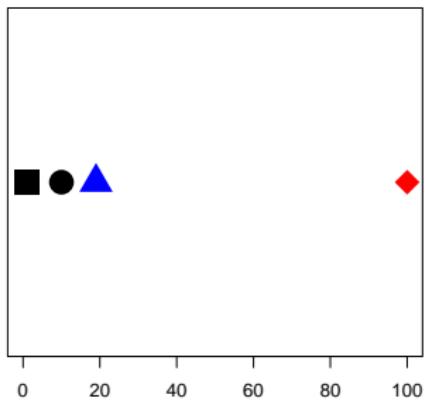
## *Scales of display*



## *Scales of display*



## Scales of display

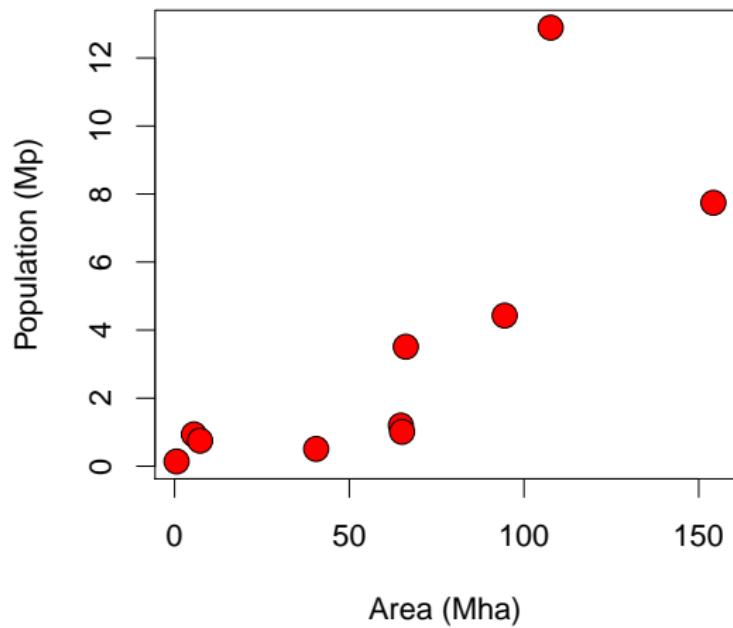


There is only one log scale; it doesn't matter which base you use!

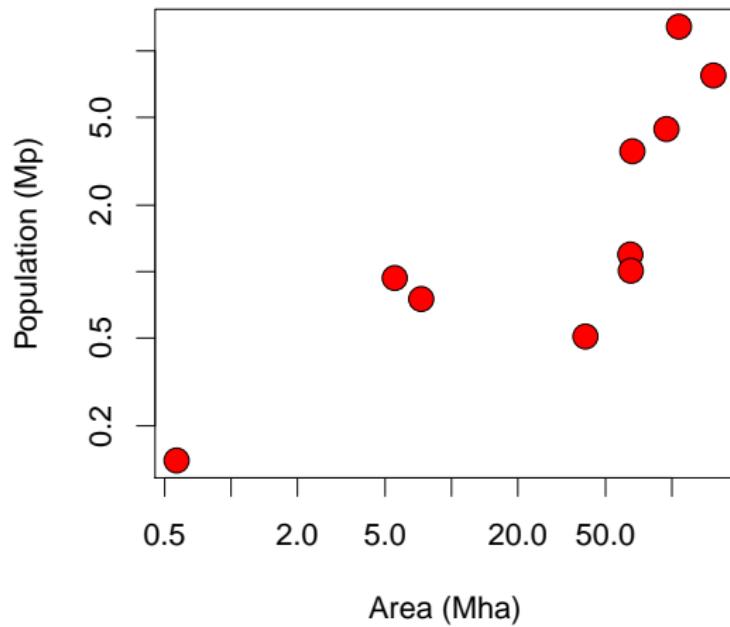
# Canadian provinces

- ▶ How many people know the Canadian provinces song?
- ▶ Poll: Which Canadian province is the most unusual in terms of area?
- ▶ Poll: Which Canadian province is the most unusual in terms of population?

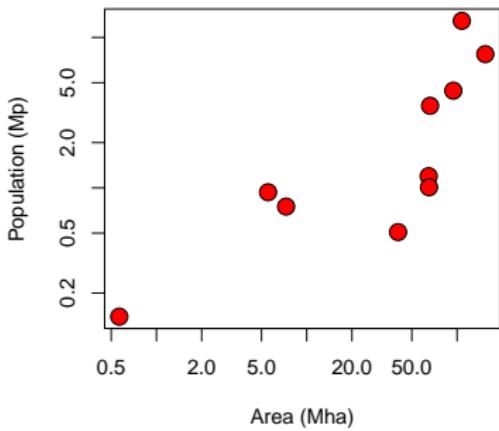
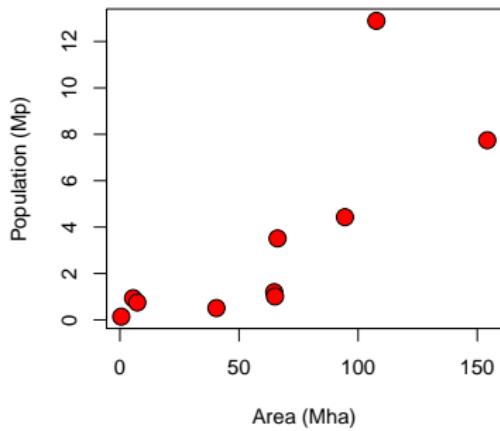
## *Canadian provinces*



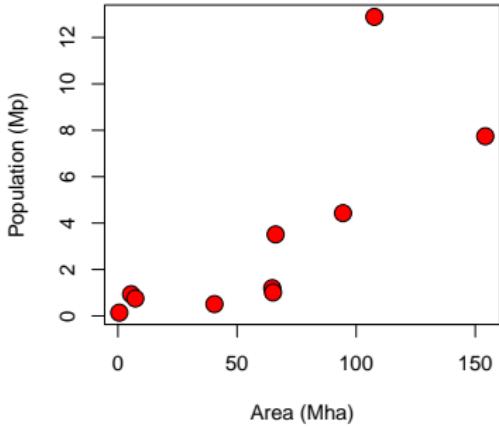
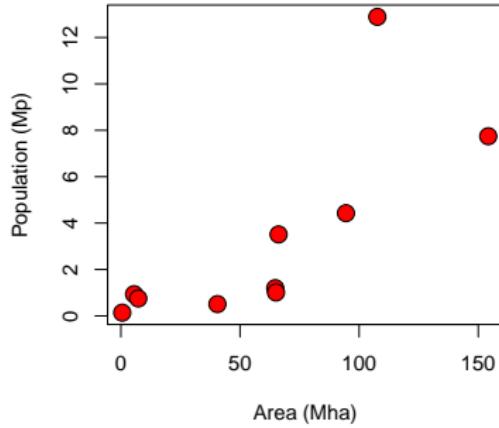
## Canadian provinces



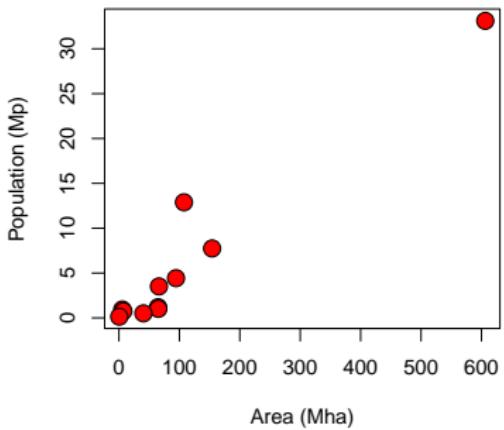
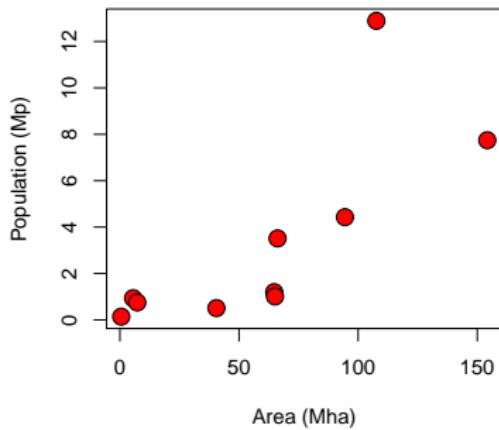
# Canadian provinces



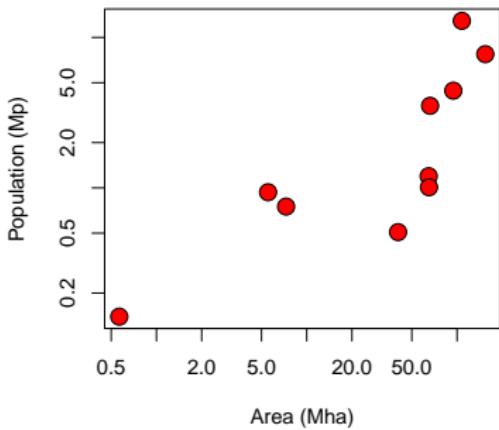
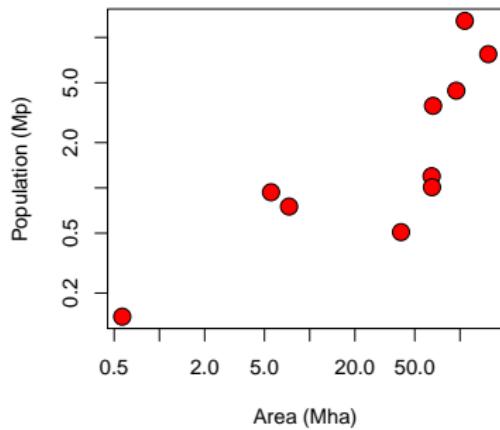
# *Canadian provinces plus Canada?*



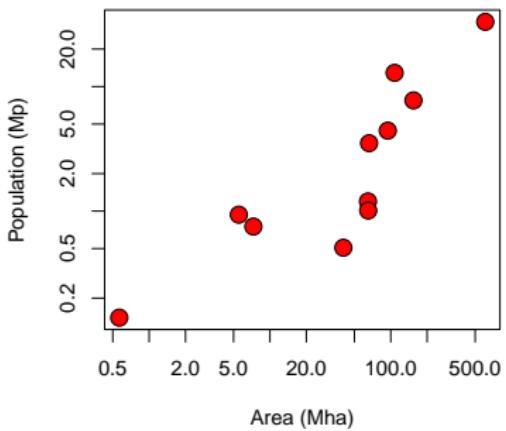
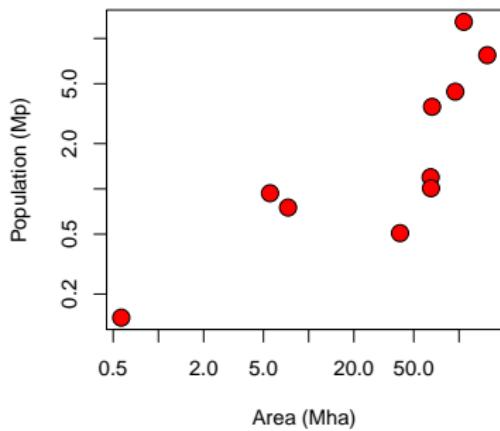
# *Canadian provinces plus Canada*



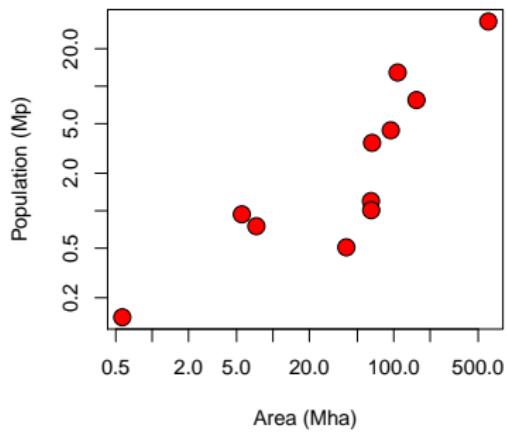
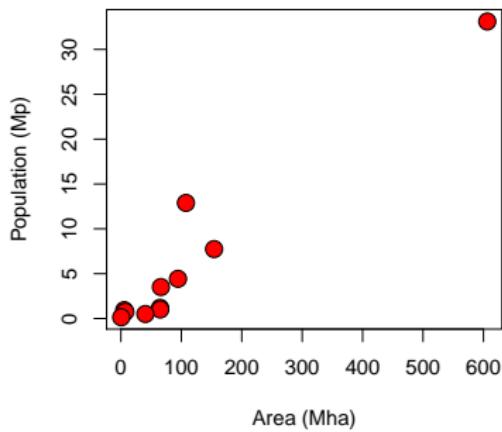
# *Canadian provinces plus Canada?*



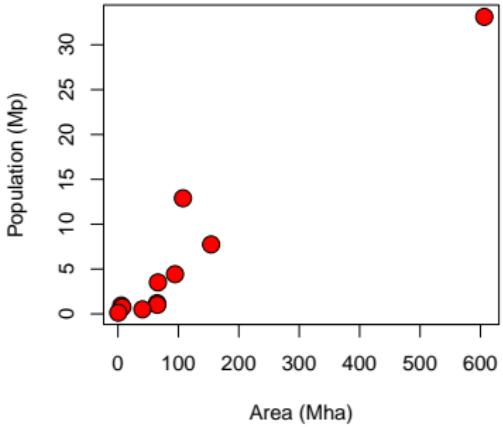
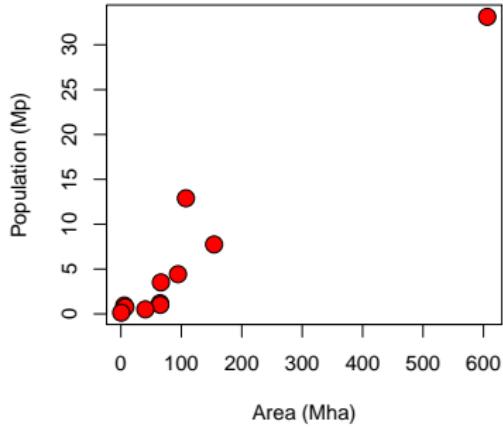
# *Canadian provinces plus Canada*



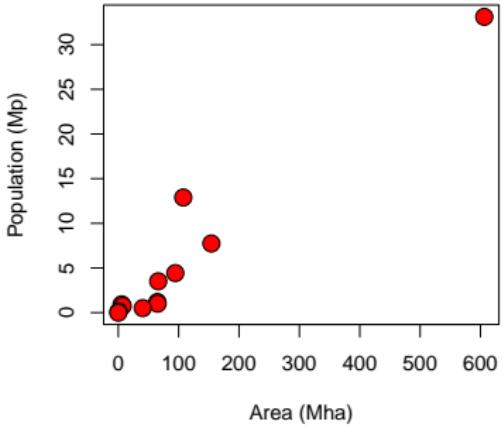
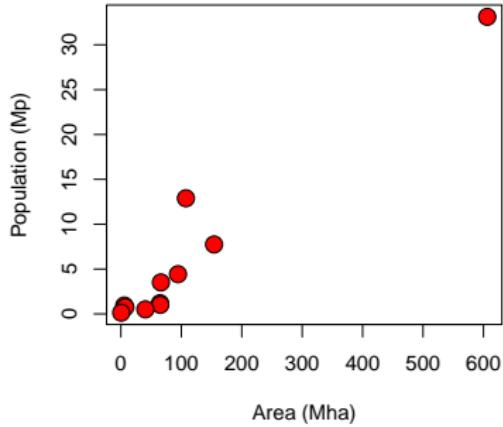
## Canadian provinces plus Canada



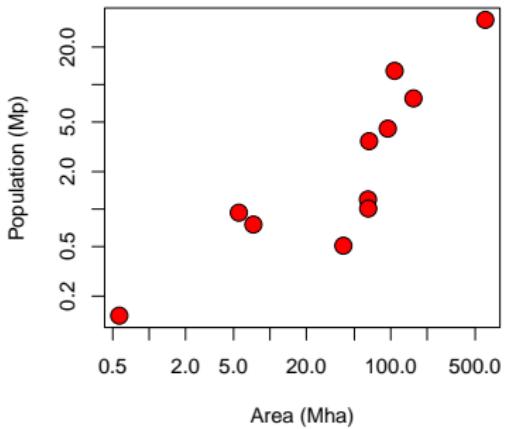
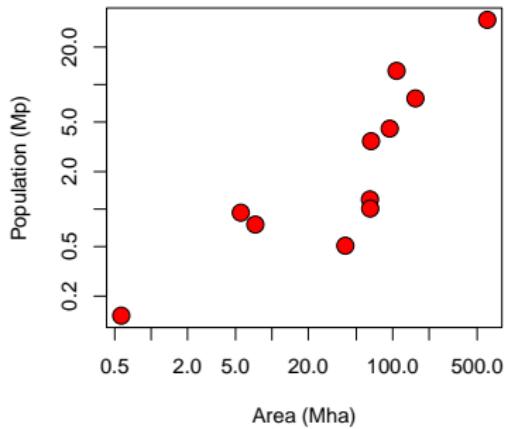
# *Canada plus B135*



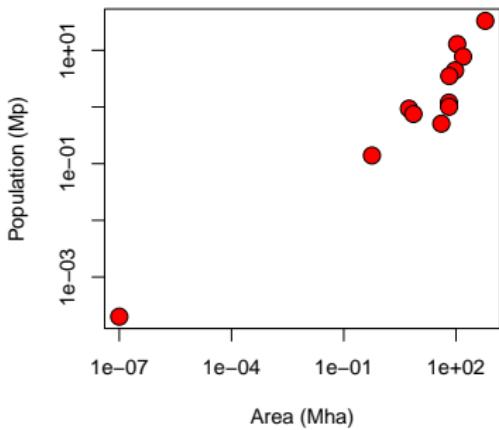
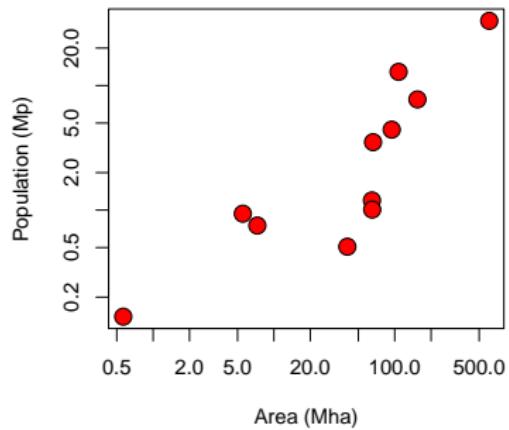
# *Canada plus B135*



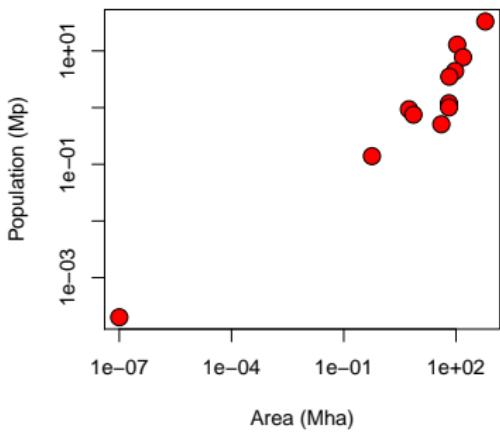
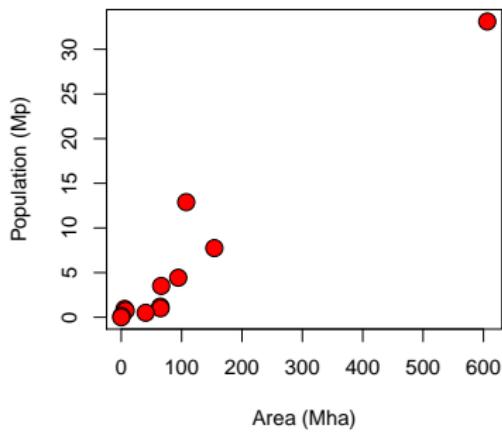
# *Canada plus room 1105?*



# *Canada plus room 1105*



# Canada plus room 1105



## Predation comparison



# Predation comparison

- ▶ A 300 lb lion is attacking a 600 lb buffalo!
- ▶ Poll: This is analogous to a 15 lb red fox attacking:
  - ▶ A 30 lb beaver (twice as heavy)?
  - ▶ A 315 lb elk (500 lbs heavier)?



## Different scales

- ▶ The log scale and linear scale provide different ways of looking at the same data
- ▶ Equally valid
- ▶ What are some advantages of each?

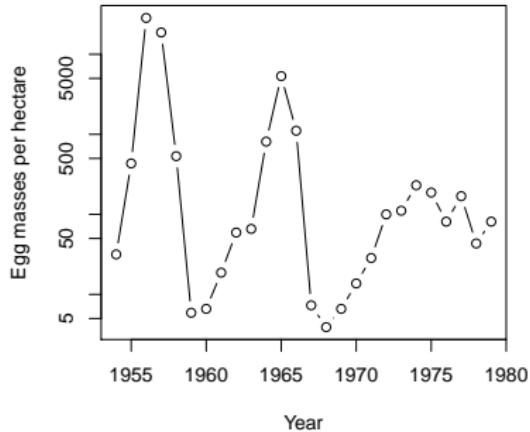
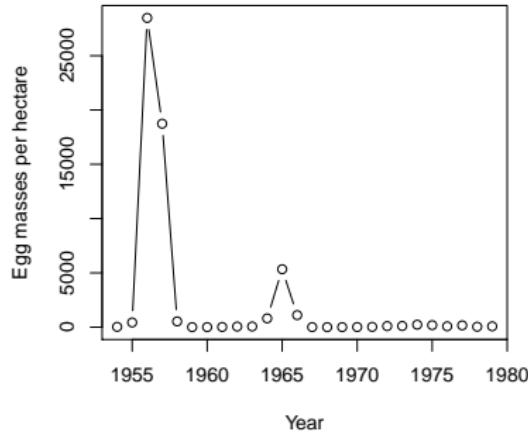
## Advantages of arithmetic view

- ▶ \* When there is no natural zero (or the natural zero is irrelevant)
  - ▶ \* Often the case for time or geography
- ▶ \* When zeroes (or negative numbers) can occur
- ▶ \* When we are interested in adding things up

## Advantages of geometric view

- ▶ \* When comparing physical quantities, or quantities with natural units
- ▶ \* When comparing proportionally

## Gypsy-moth example



## Scales in population biology

- ▶ The linear scale looks at differences at the population scale
- ▶ The log scale looks at differences at the individual scale (per capita)

# Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

Example populations

Dandelions

Gypsy moths

Bacteria

Exponential growth

Log and linear scales

Time scales

# *Speeding in Taiwan*

- ▶ A life experience
- ▶ Some clarifications
  - ▶ I was reading the sign wrong
  - ▶ I didn't actually know how to say speed
  - ▶ The whole thing never happened



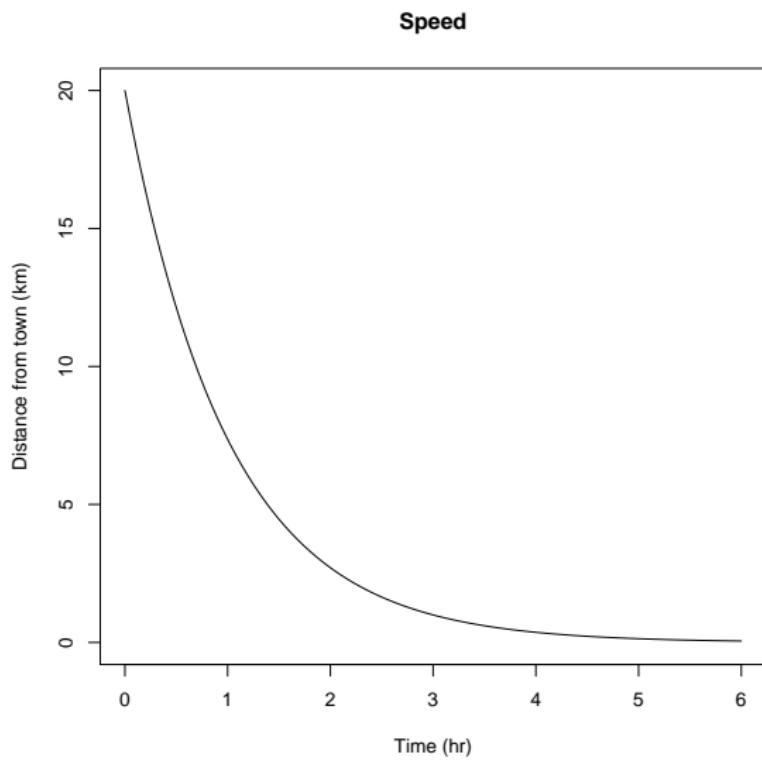
# Speeding in Taiwan

- ▶ Moral:
  - ▶ Units (km is *not* a speed)
  - ▶ Exponential decay
- ▶ Imagine now that I follow the signs exactly and unrealistically.
- ▶ Poll: Do I ever arrive in the (ideal) town of Speed?
  - ▶ \* No. I am always an hour away!
  - ▶ \* But I do get extremely close (after several hours)
- ▶ Does anyone remember Zeno's paradox?
  - ▶ \* Don't worry about it, then

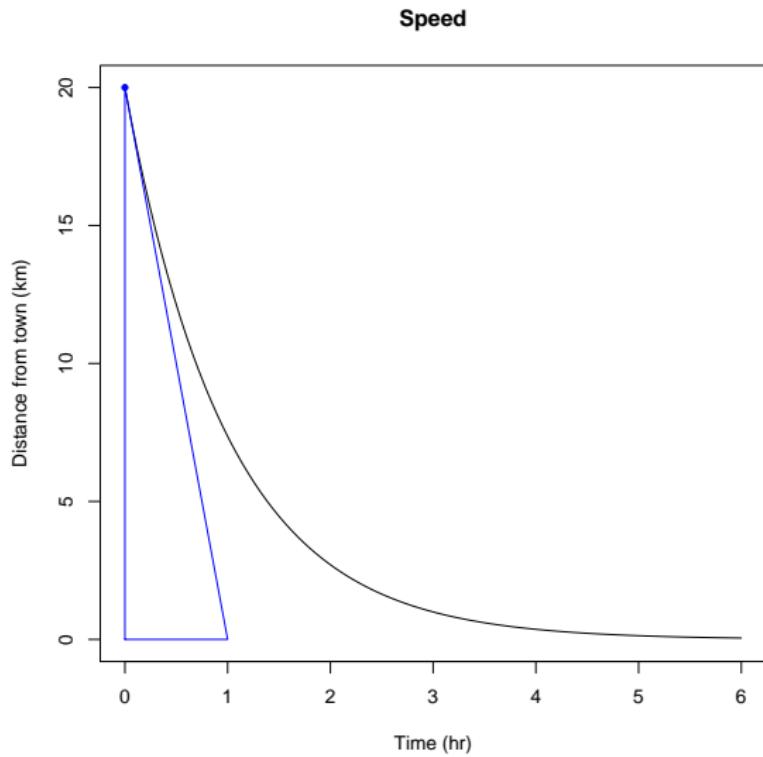
## Characteristic times

- ▶ If something is declining exponentially, the rate of change (units [widgets/time]) is always proportional to the size of the thing ([widgets]).
- ▶ The constant ratio between the rate of change and the thing that is changing is:
  - ▶ the **characteristic time** (something/change), or
  - ▶ the **rate of exponential decline** (change/something)
- ▶ *I'm always 1 hour away from the town of Speed*

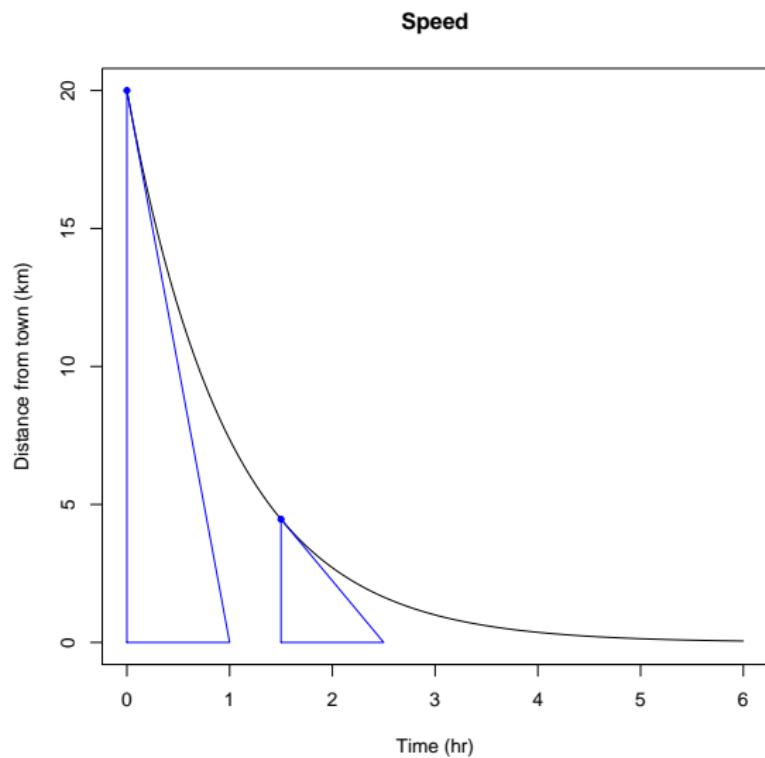
## *Characteristic times*



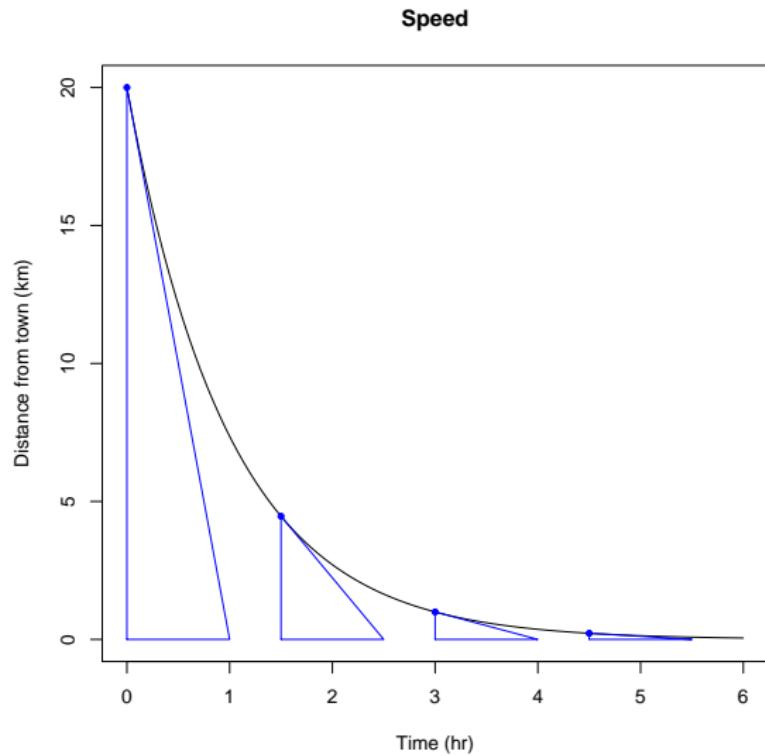
## *Characteristic times*



## Characteristic times



## Characteristic times



## Bacterostasis

- ▶ What if we add an agent to the tank that makes the birth and death rates nearly zero?
- ▶ Now the bacteria are merely washing out at the rate of 0.02/hr
- ▶ If we start with 10 bacteria/ml, how many do we have after:
  - ▶ Poll: 1 hr?
  - ▶ Poll: 1 wk?

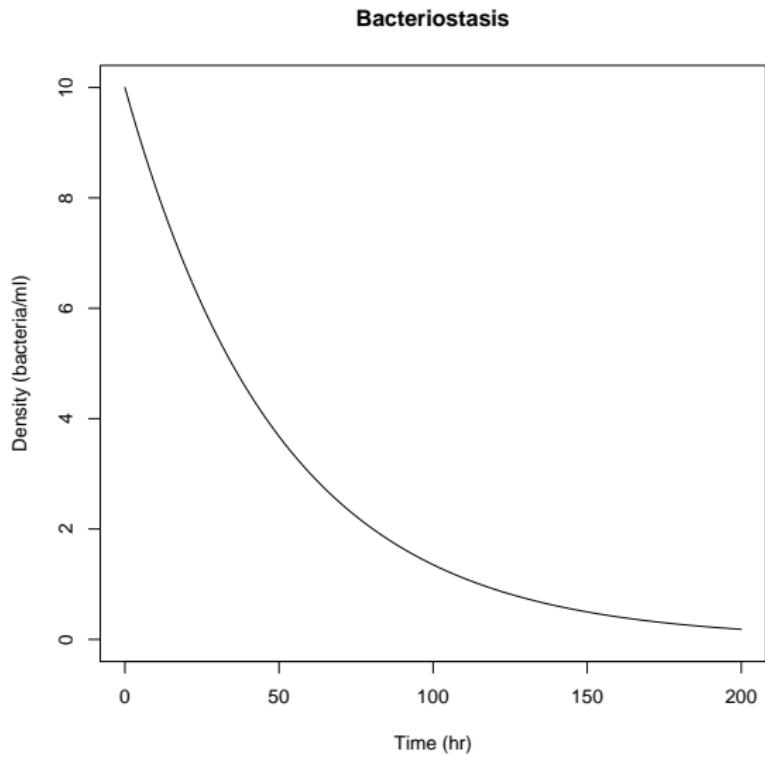
## Bacteriostasis answers

- ▶ Bacteria wash out at the rate of 0.02/hr
  - ▶ \* This can only make sense with concrete units if we think of it as an instantaneous rate – more soon
  - ▶ \*  $N = N_0 \exp(-rt)$
- ▶ Start with 10 bacteria/ml:
  - ▶ \* After one hour, 9.802 bacteria/ml
  - ▶ \* After one week, 0.347 bacteria/ml

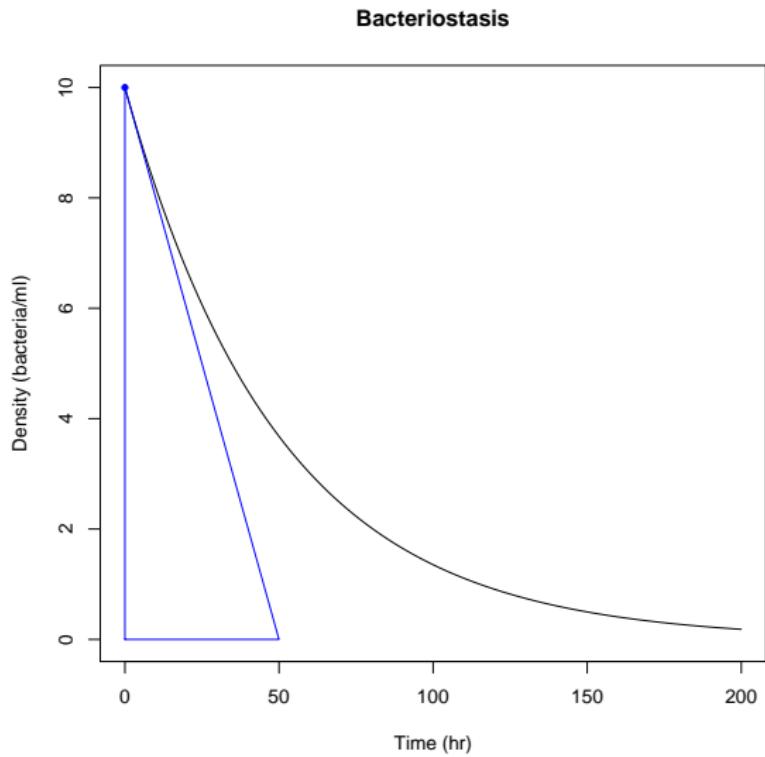
## Bacteriostasis analysis

- ▶ Rate of exponential decline is  $r = 0.02/\text{hr}$
- ▶ Characteristic time is  $T_c = 1/r = 50\text{ hr}$
- ▶ If experiment time  $t \ll T_c$ , then proportional decline  $\approx t/T_c$
- ▶ The answer makes sense for short times and for long times
- ▶ *We will come back to this*

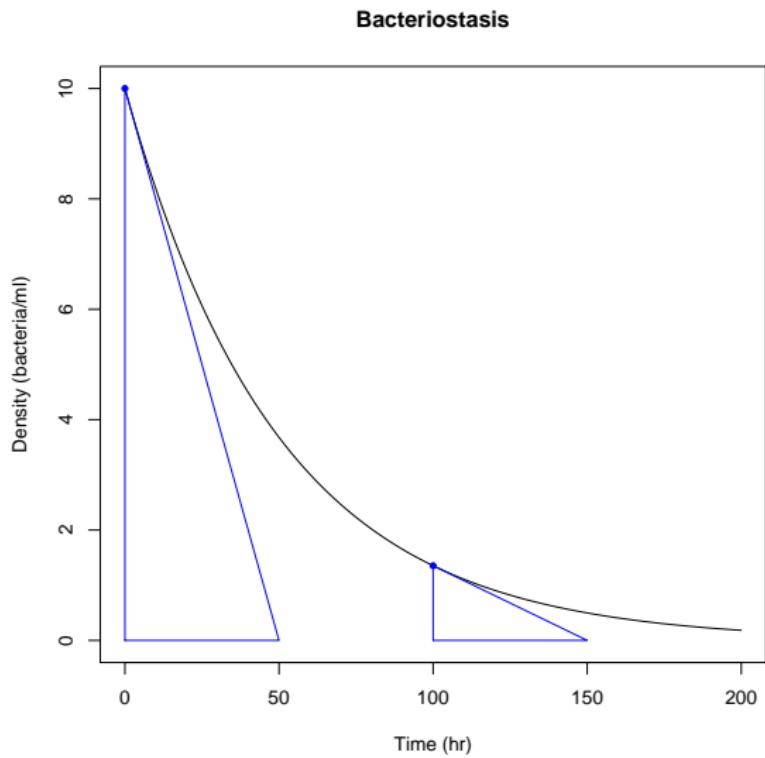
## *Characteristic times*



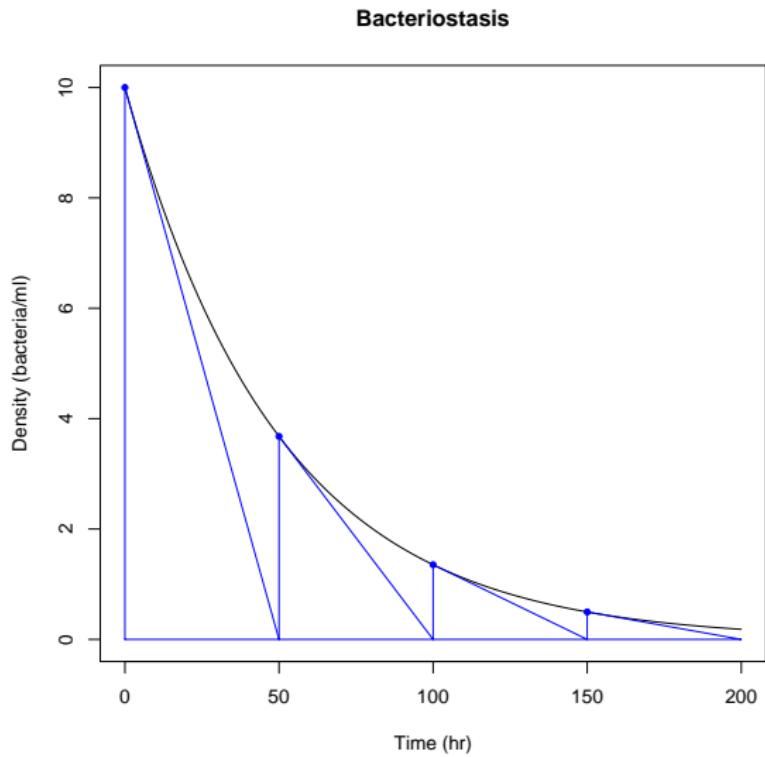
## *Characteristic times*



## Characteristic times



## *Characteristic times*



## Euler's e

- ▶ The reason mathematicians like  $e$  is that it makes this link between instantaneous change and long-term behaviour
- ▶ If I drive for an hour, how much closer do I get to the ideal town of Speed?
  - ▶ \*  $e$  times closer

## Euler's e

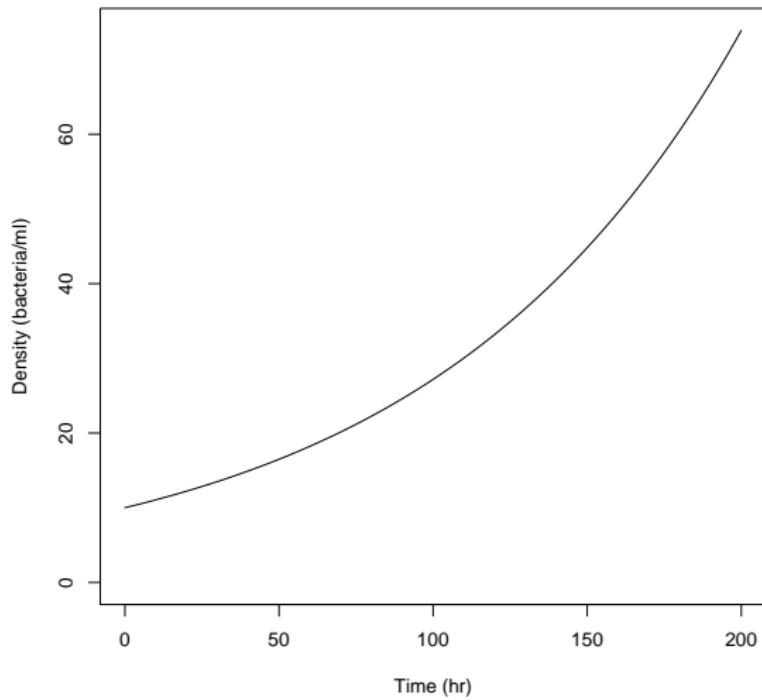
- ▶  $e$  or  $1/e$  is the approximate answer to a lot of questions like this one
  - ▶ If I compound 1%/year interest for 100 years, how much does my money grow?
  - ▶ If two people go deal out two decks of cards simultaneously, what is the probability they will never match cards?
  - ▶ If everyone picks up a backpack at random after a test, what's the probability nobody gets the right backpack?

# Exponential growth

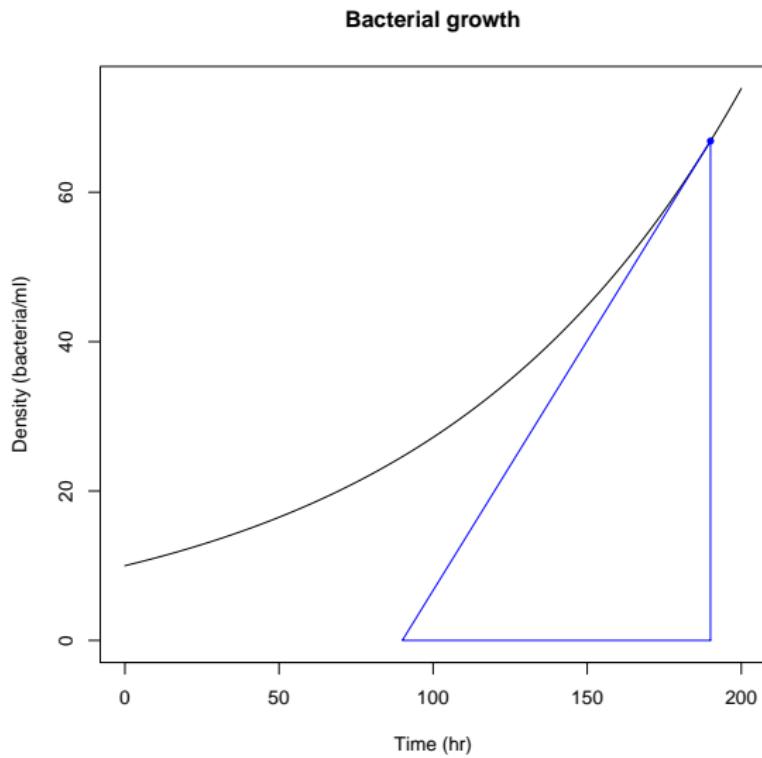
- ▶ We can think about exponential growth the same way as exponential decline:
  - ▶ Things are always changing at a rate that would take a fixed amount of time to get (back) to zero
  - ▶ This is the characteristic time
  - ▶ Exponential growth follows  $N = N_0 \exp(rt) = N_0 \exp(t/T_c)$

## *Characteristic times*

Bacterial growth

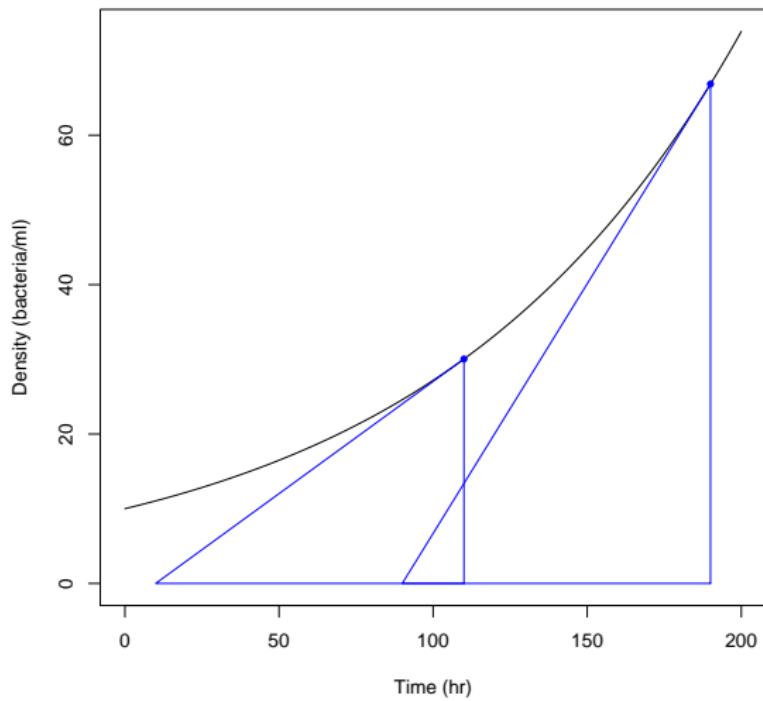


## Characteristic times



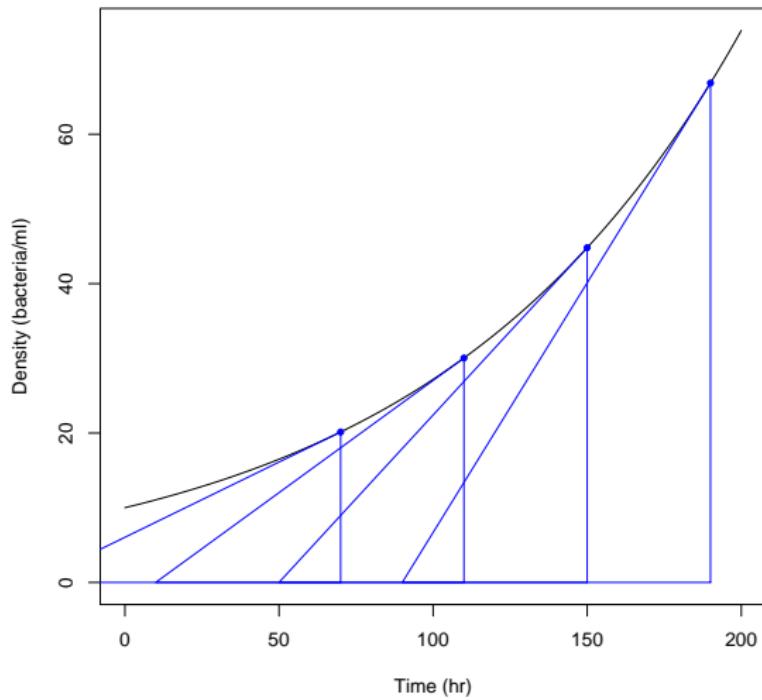
## Characteristic times

Bacterial growth



## Characteristic times

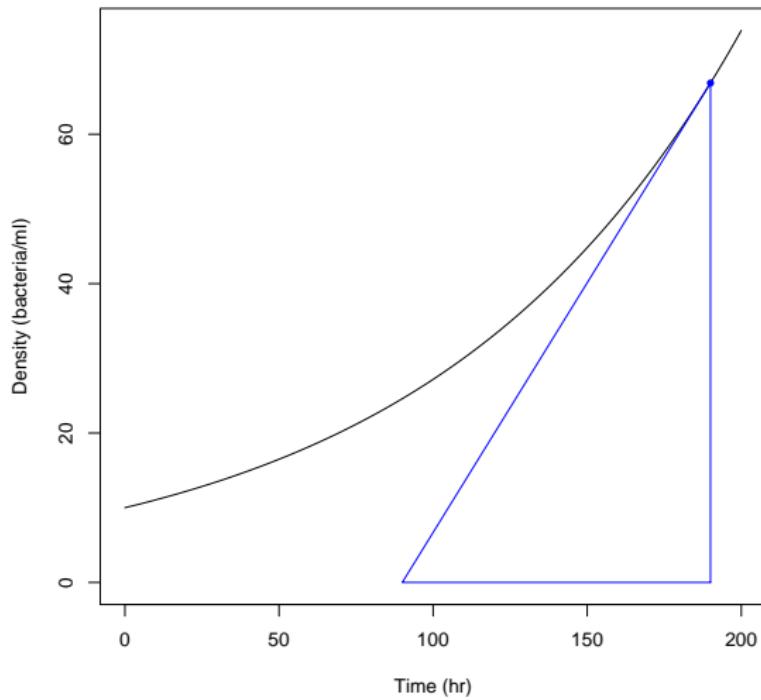
Bacterial growth



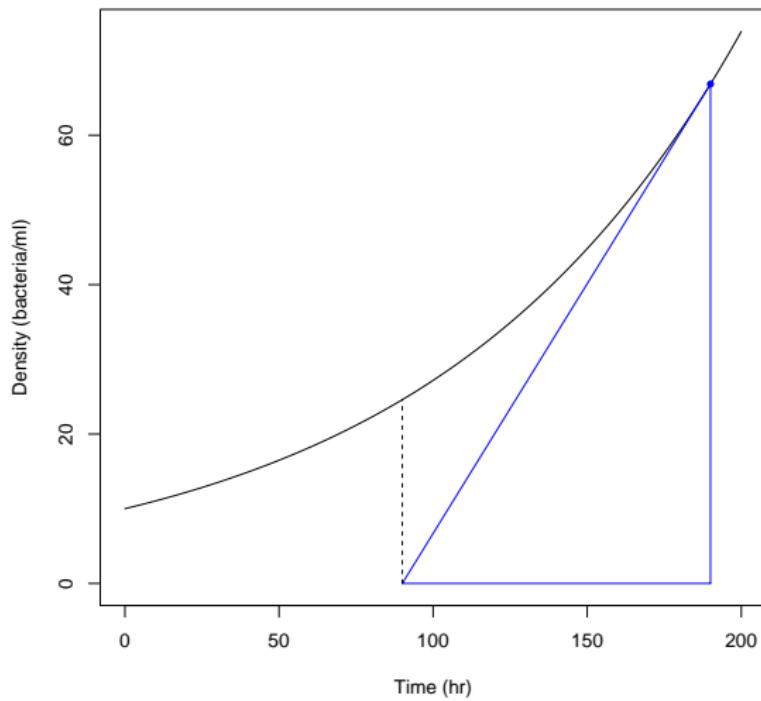
## Doubling time

- ▶ Some people prefer to think about doubling times.
- ▶ These make just as much sense as characteristic times, but don't have the direct link to the instantaneous change.
  - ▶ It takes  $T_c$  time to increase by a factor of  $e$
  - ▶ It takes  $\log_e(2)T_c \approx 0.69 T_c$  to increase by a factor of 2
  - ▶ We can write  $T_d = \log_e(2)T_c$
- ▶ You should be able to do this calculation

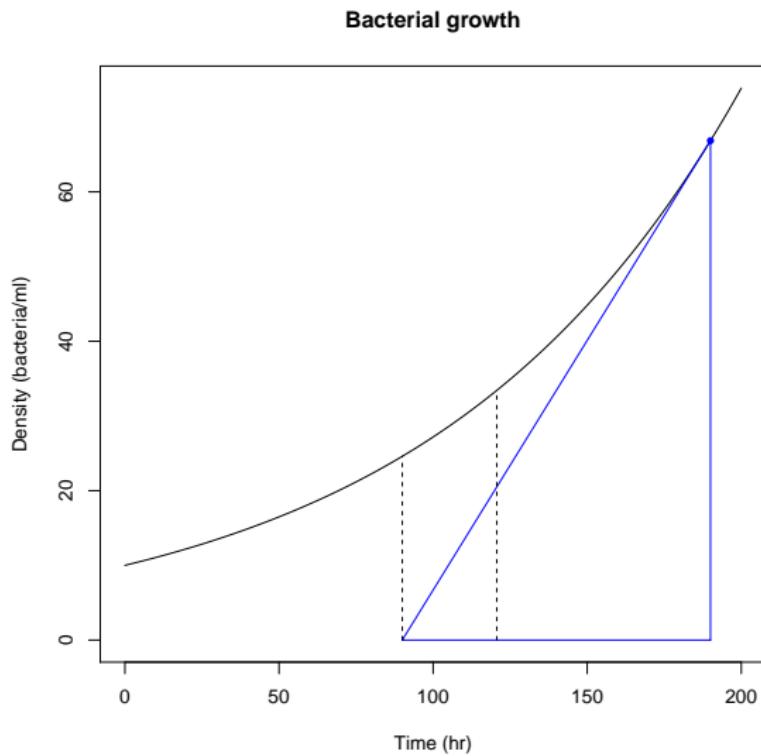
### Bacterial growth



### Bacterial growth



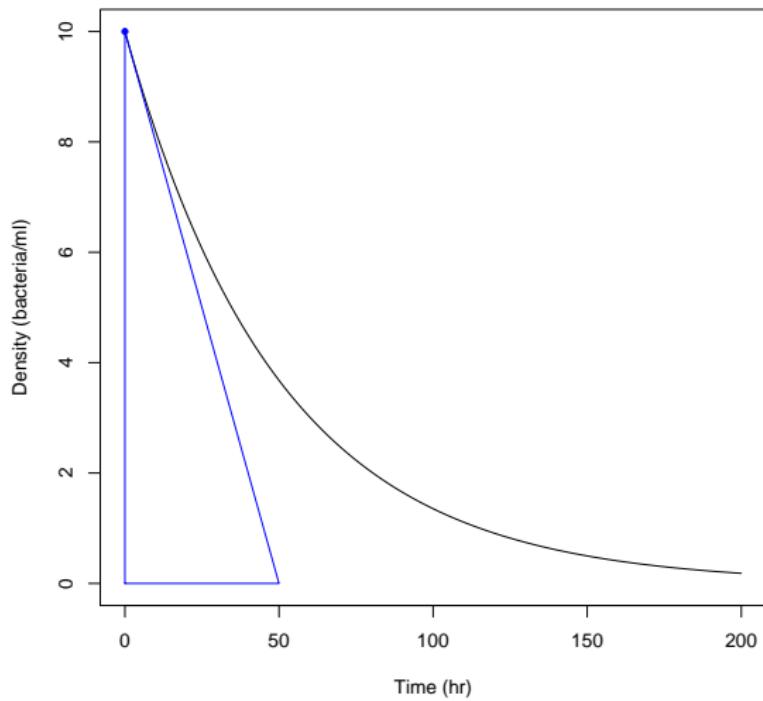
## Characteristic time and doubling time



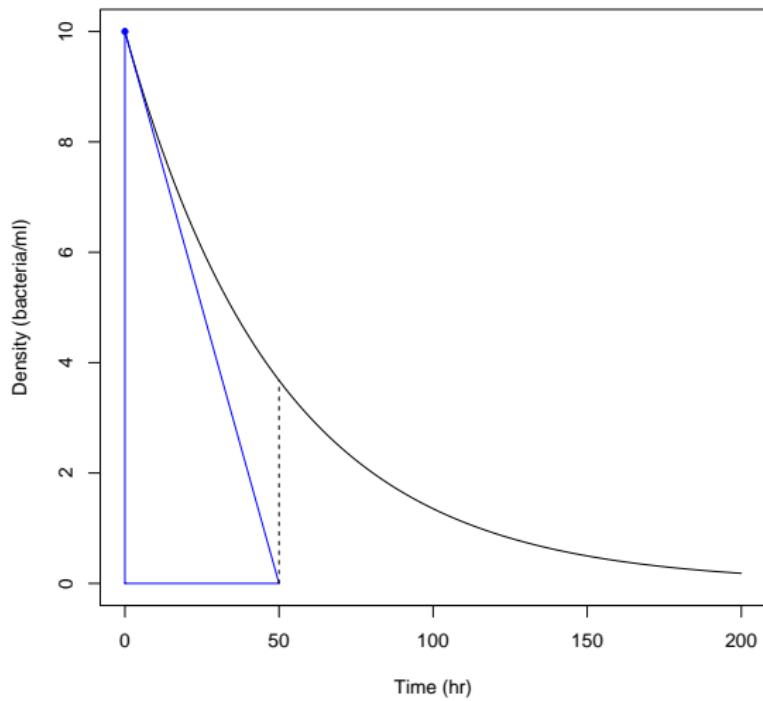
# Half life

- ▶ The half life plays the same role for exponential decline as the doubling time does for exponential growth:
  - ▶  $T_h = \log_e(2) T_c$
  - ▶ It takes  $T_c$  time for a declining population to decrease by a factor of  $e$
  - ▶ It takes  $\log_e(2) T_c \approx 0.69 T_c$  to decrease by a factor of 2
  - ▶ We can write  $T_h = \log_e(2) T_c$

## Bacteriostasis



### Bacteriostasis



## Characteristic time and half life

