

# UNIT 1: Introduction

# Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

Example populations

Dandelions

Gypsy moths

Bacteria

Exponential growth

Log and linear scales

Time scales

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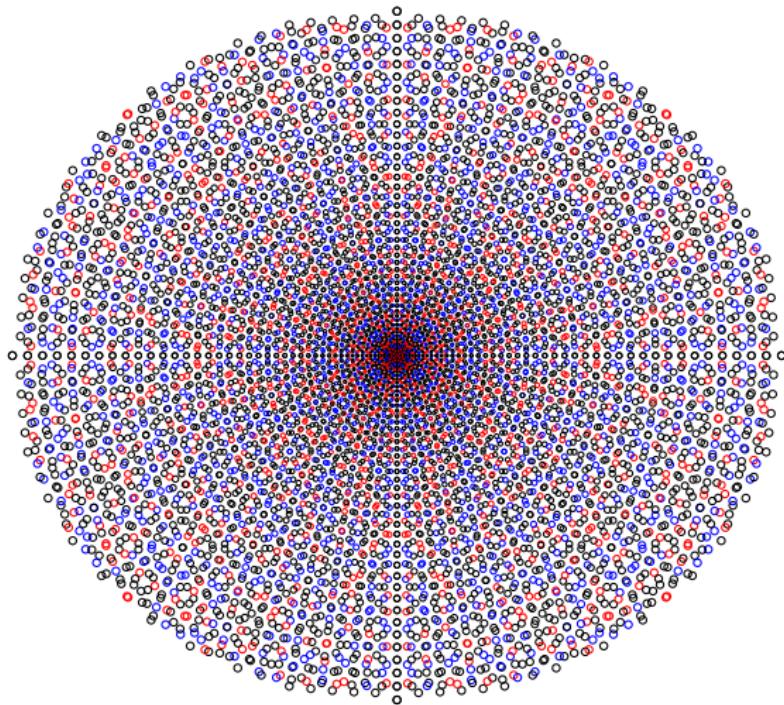
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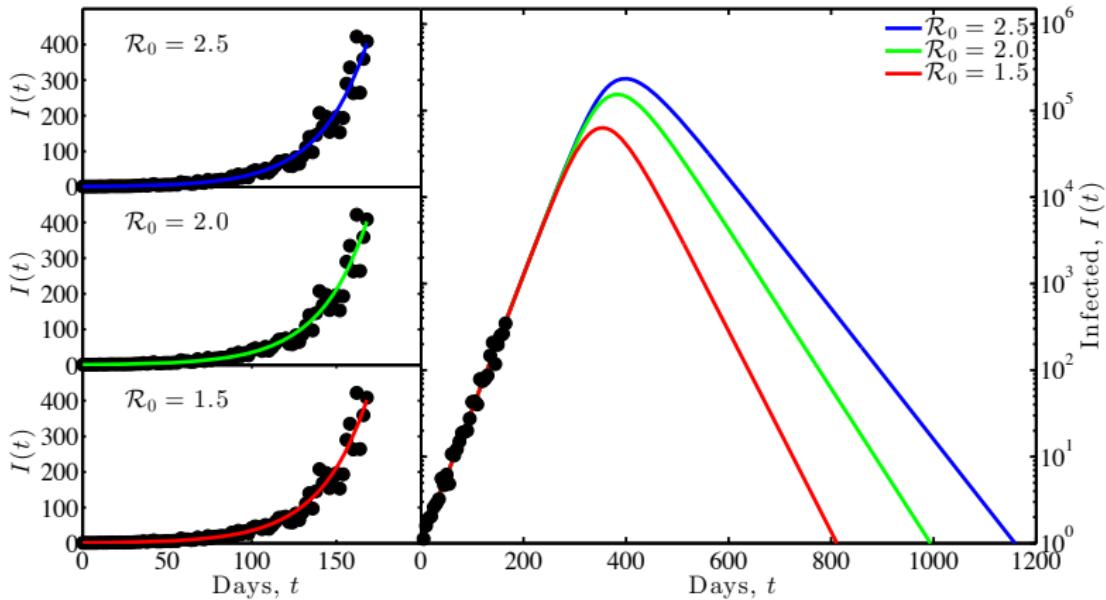
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# Ebola research



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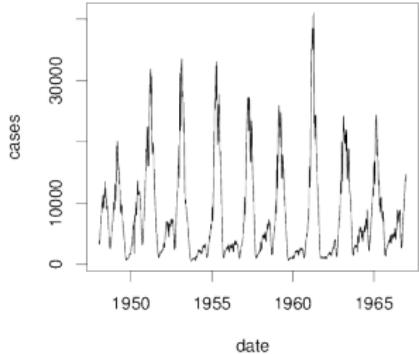
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Measles reports from England and Wales

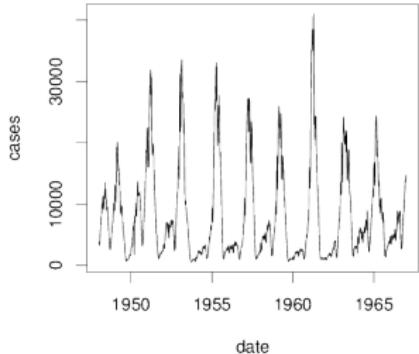


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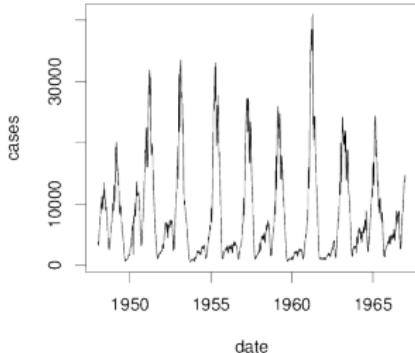


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value

energy | mass | speed of light

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$c^2 = 89,875,517,873,681,800 \text{ m}^2/\text{s}^2$

A diagram illustrating the components of the equation  $E = mc^2$ . The equation is centered at the bottom. Above it, three arrows point from boxes containing "value" and "energy/mass/speed of light" to the  $m$  and  $c^2$  terms. Below the equation, three arrows point from boxes containing "units" and "J/kg/299,792,458 m/s" to the  $E$  and  $c^2$  terms. The entire diagram is contained within a light blue rectangular frame.

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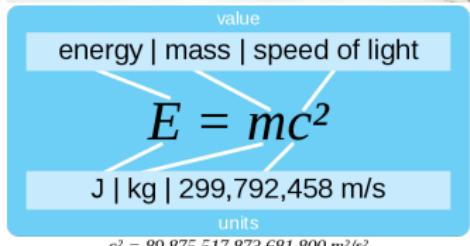
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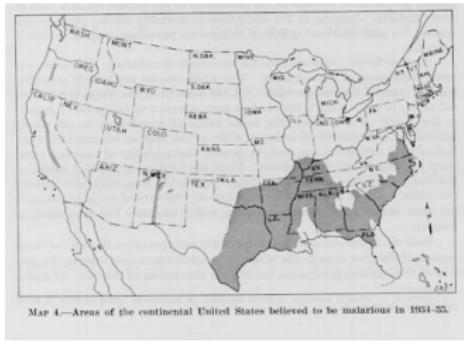
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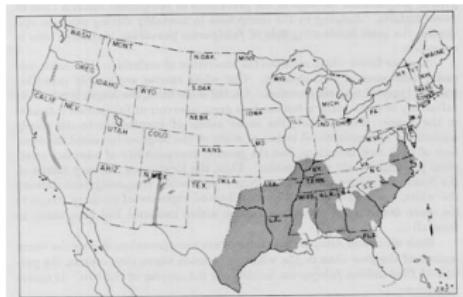
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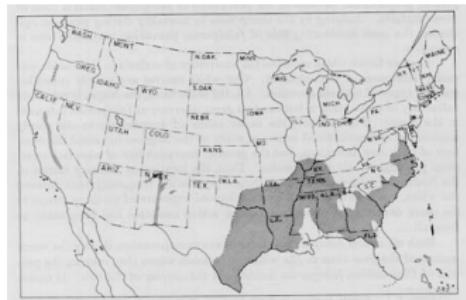
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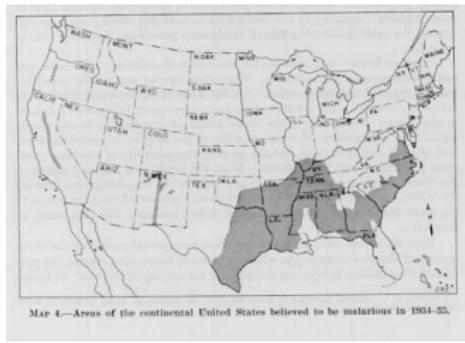
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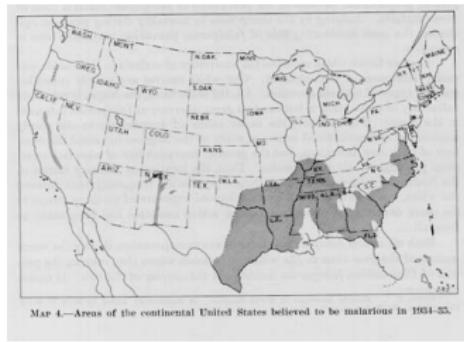
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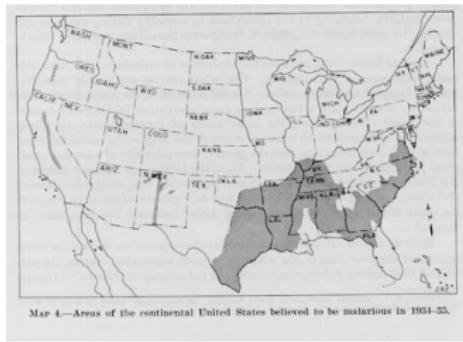
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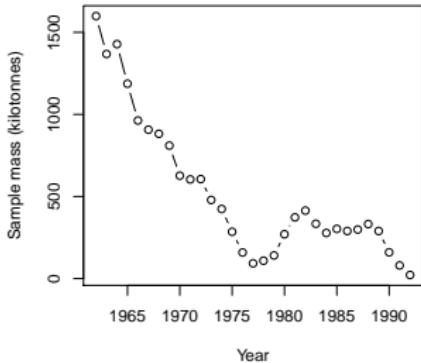
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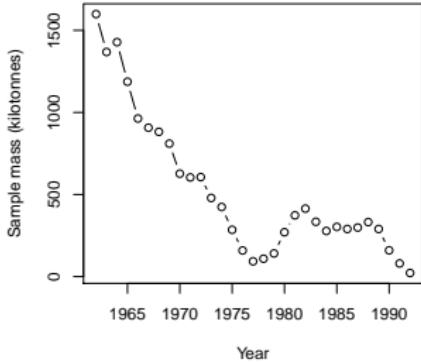
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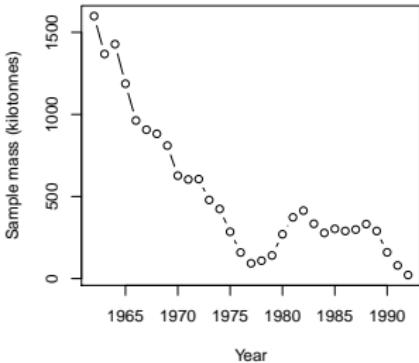
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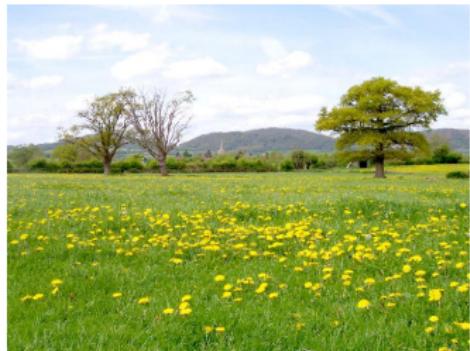
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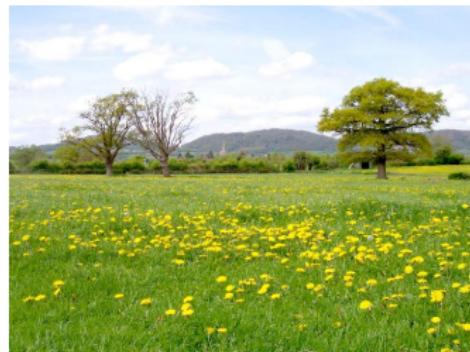
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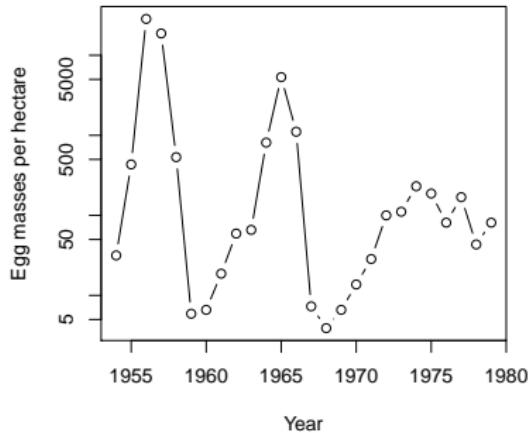
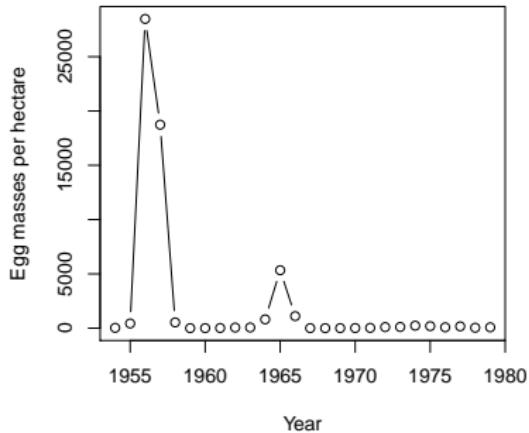


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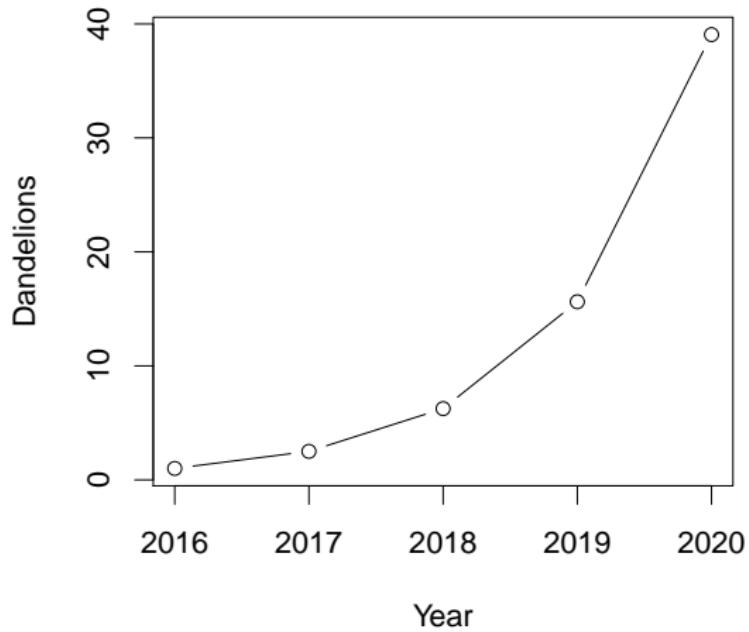
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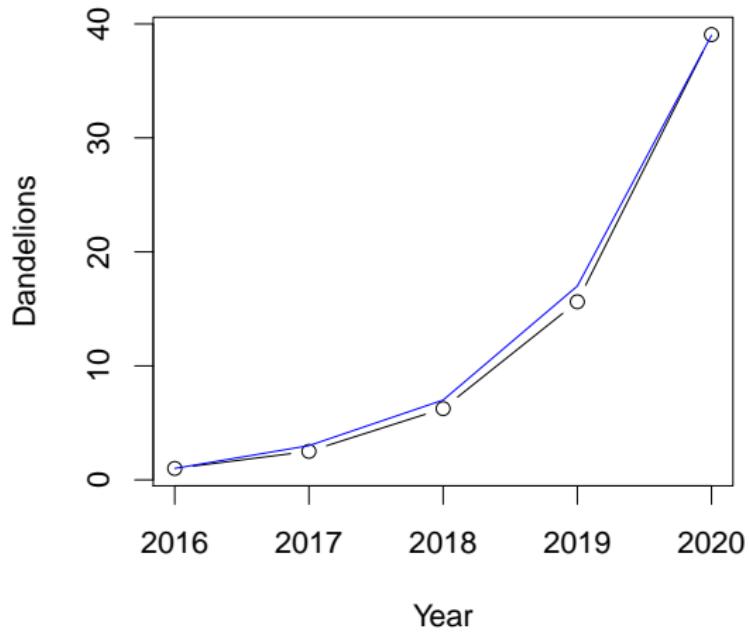
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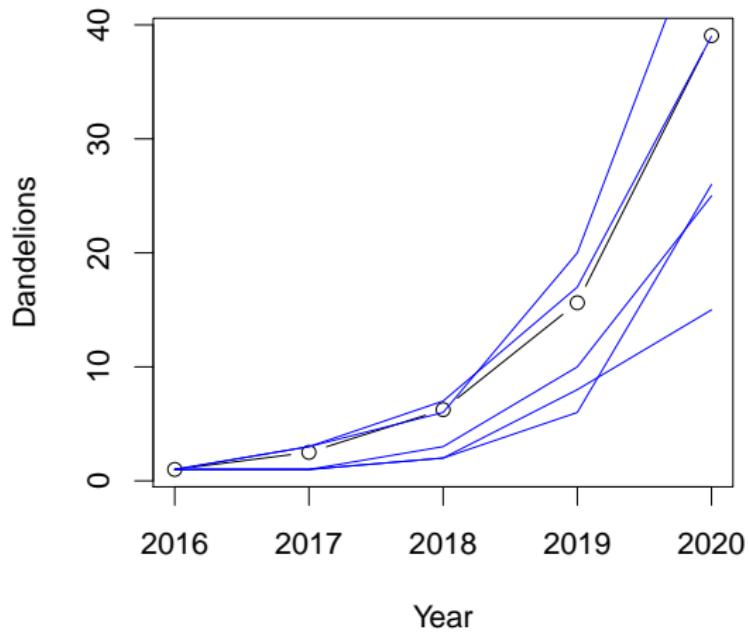
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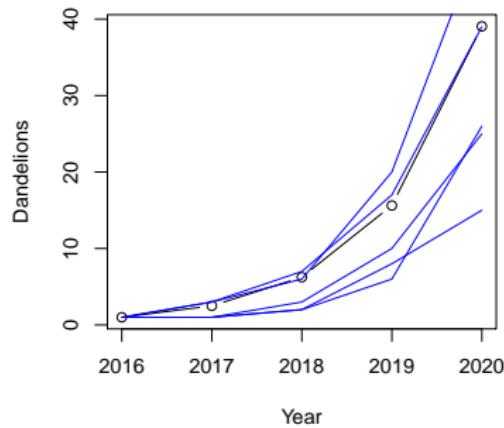
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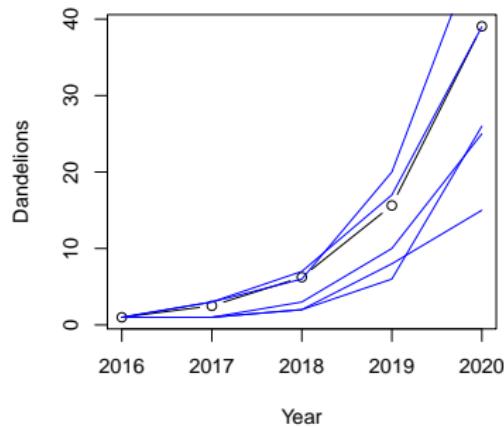


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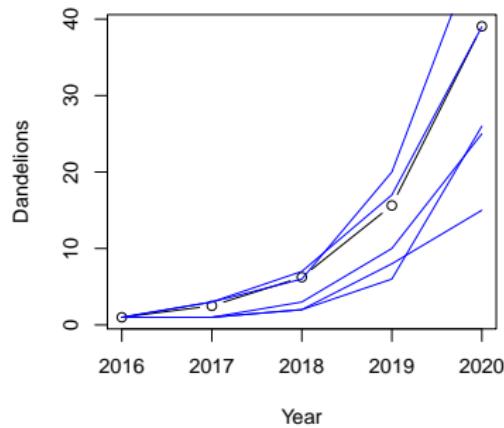
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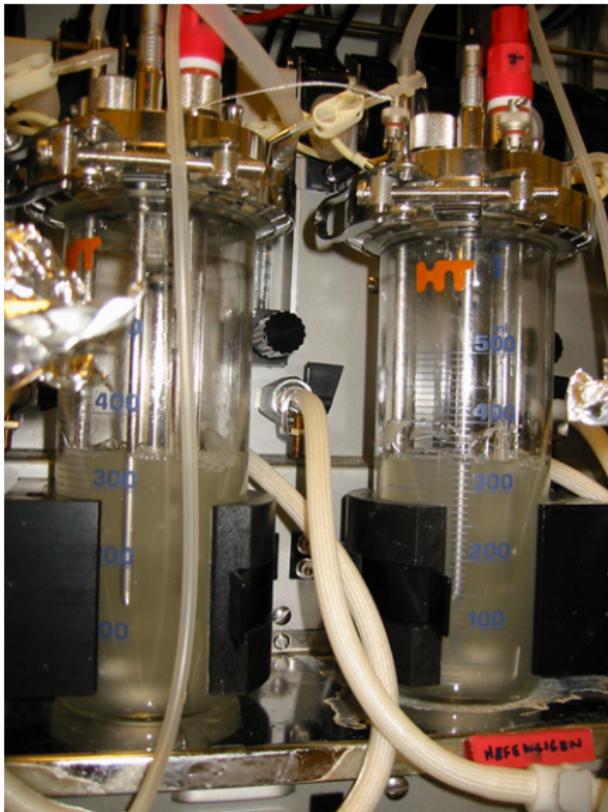
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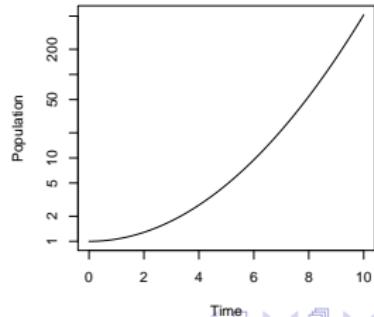
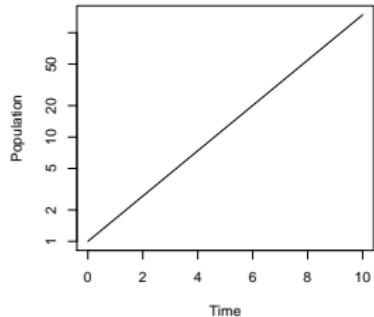
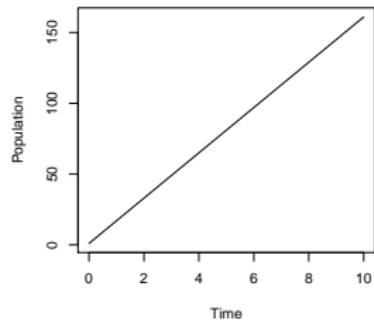
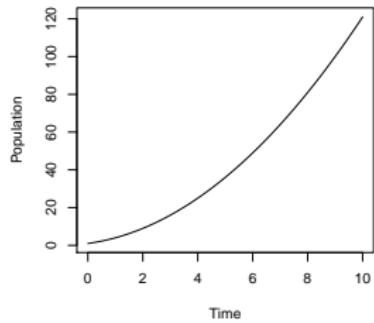
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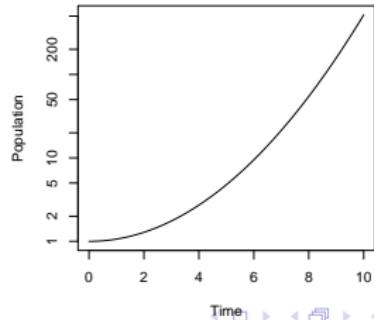
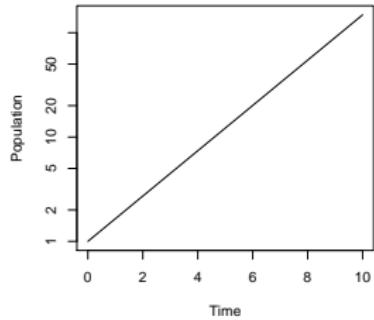
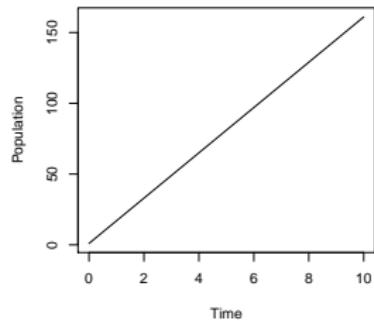
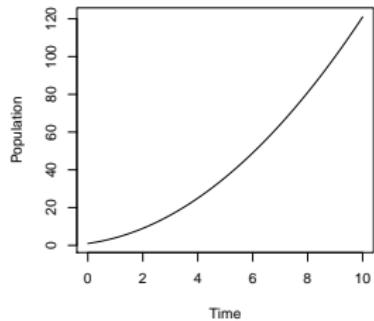
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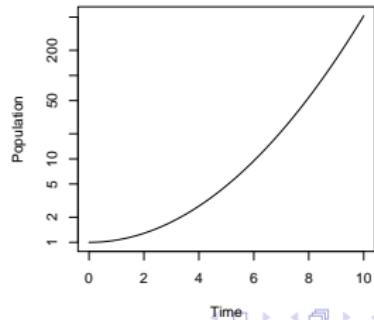
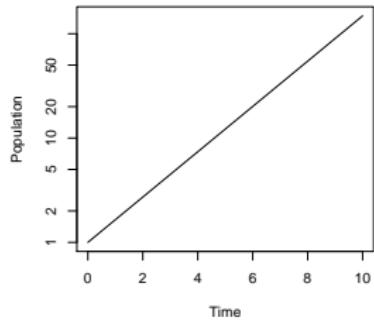
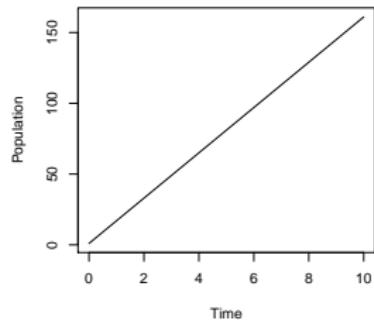
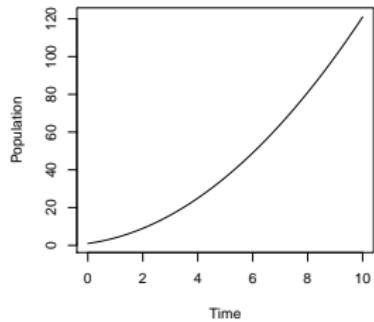
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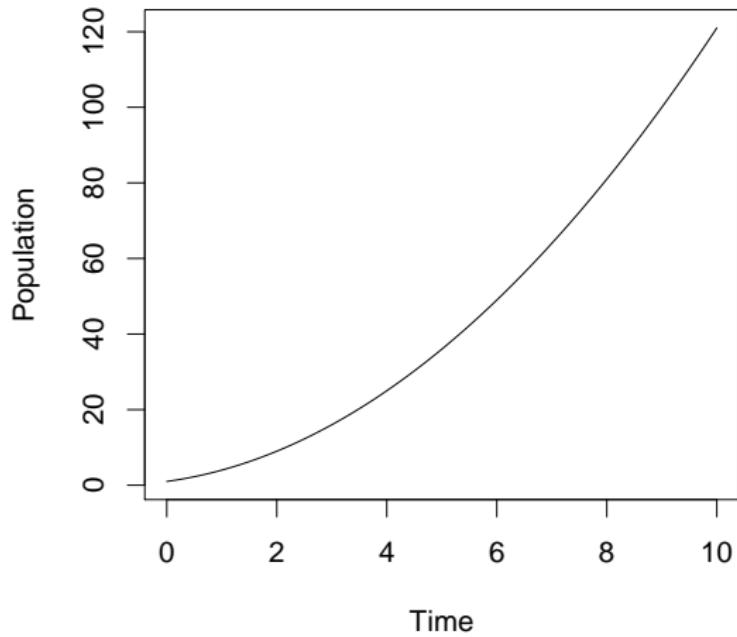


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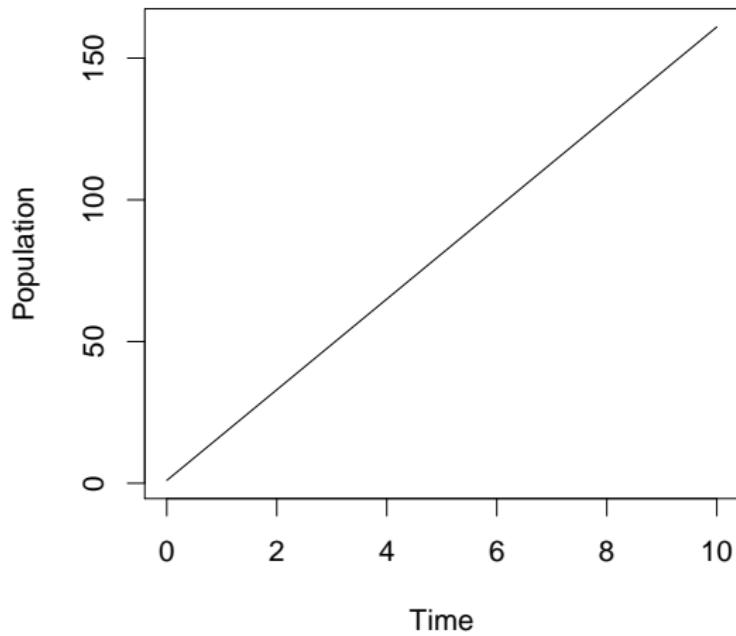
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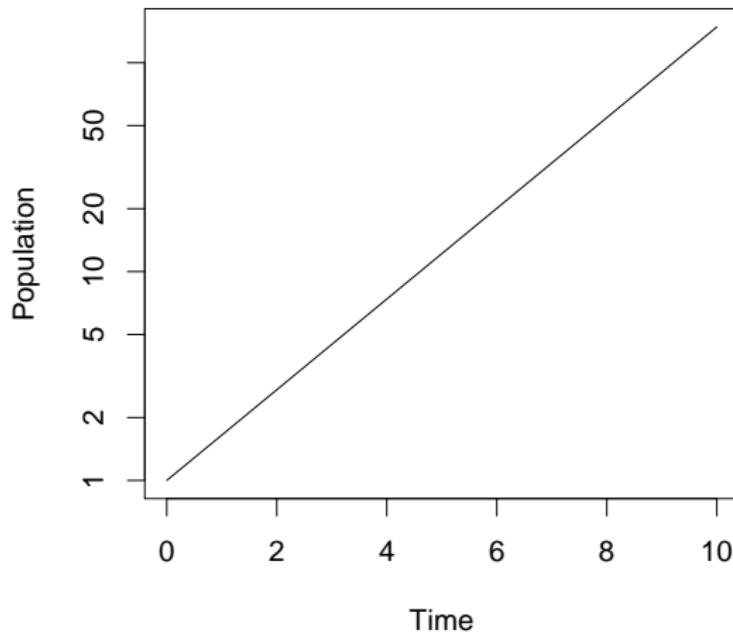
## *A (repeat)*



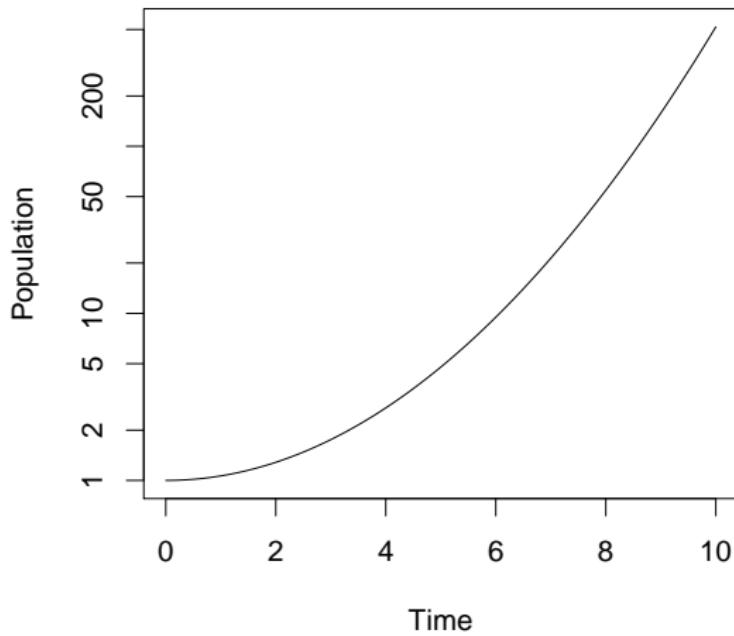
## *B* (repeat)



## *C (repeat)*

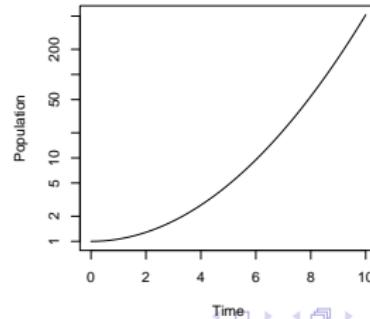
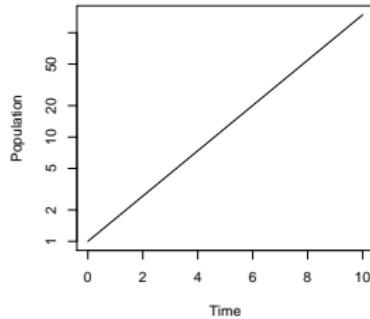
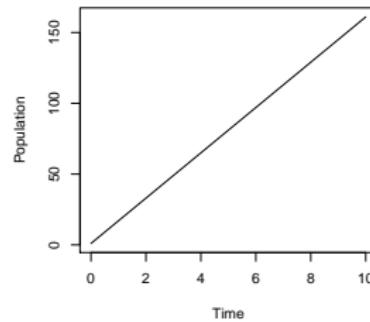
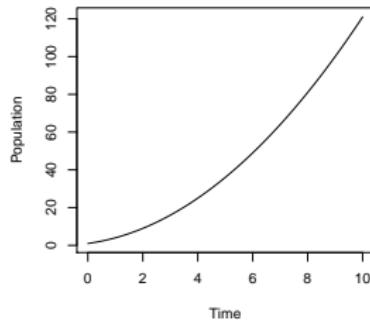


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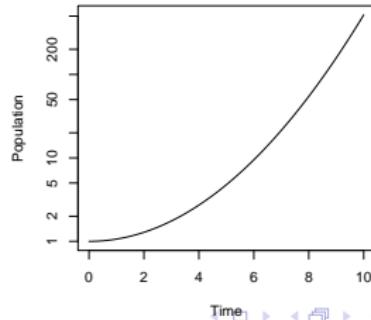
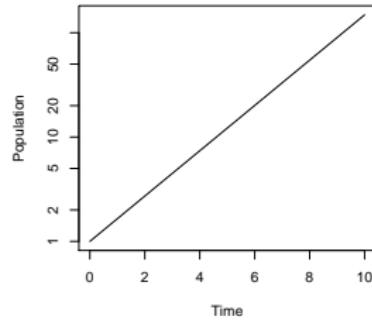
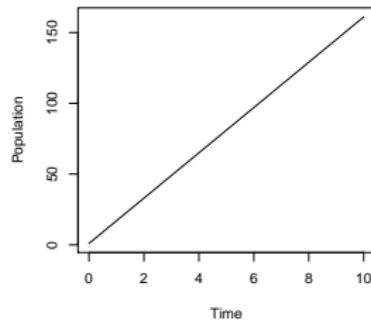
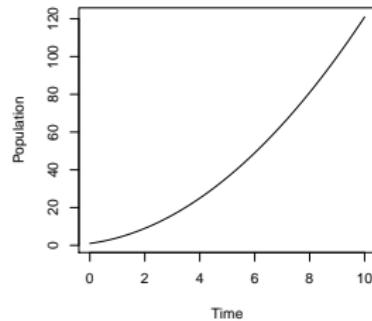
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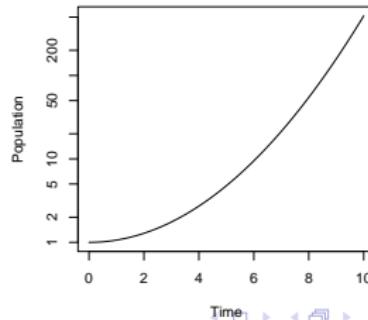
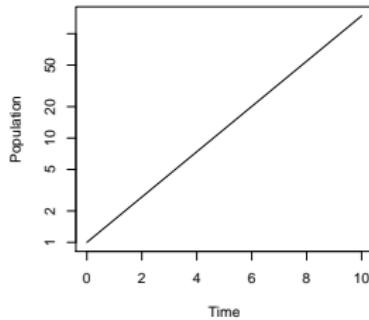
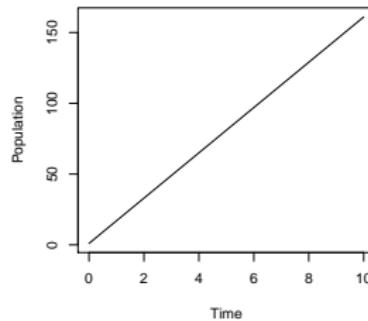
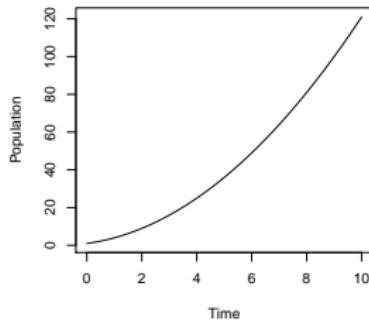
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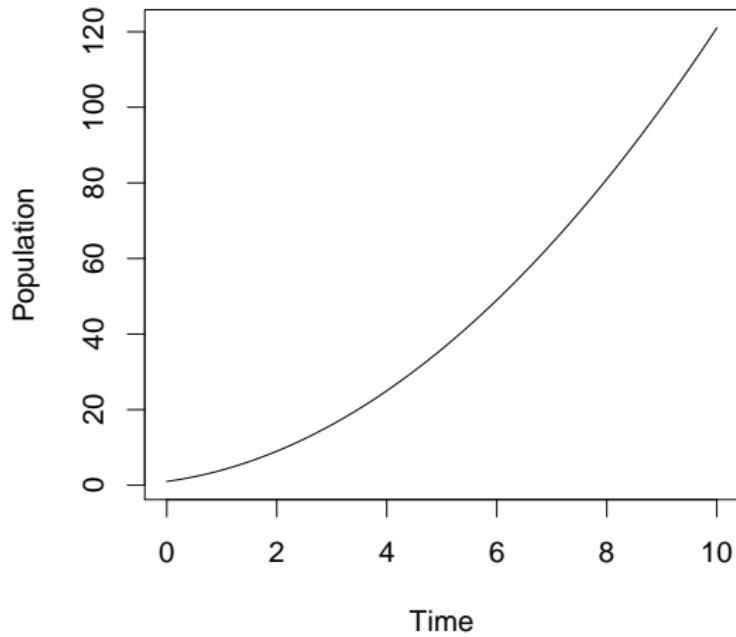


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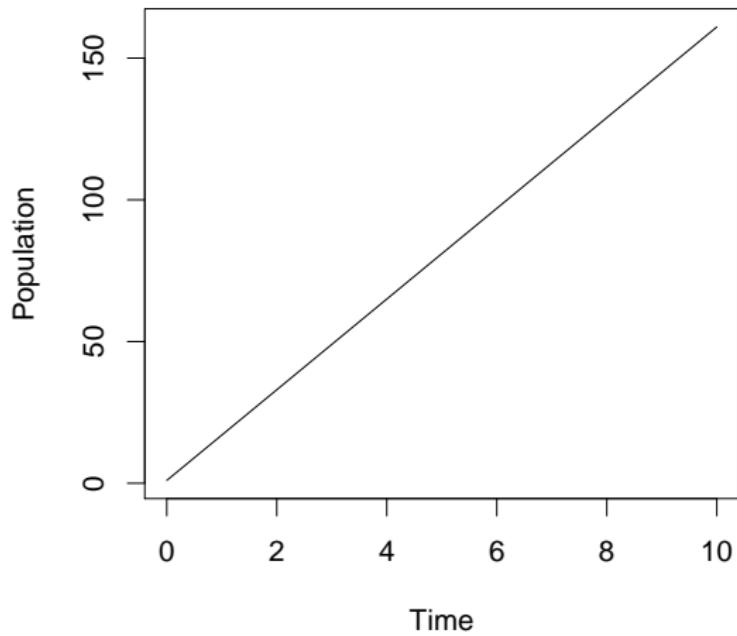
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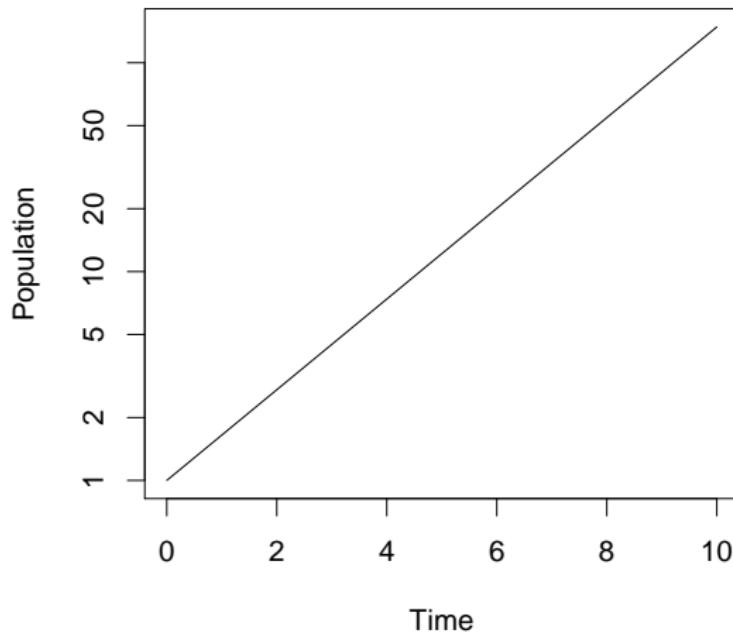
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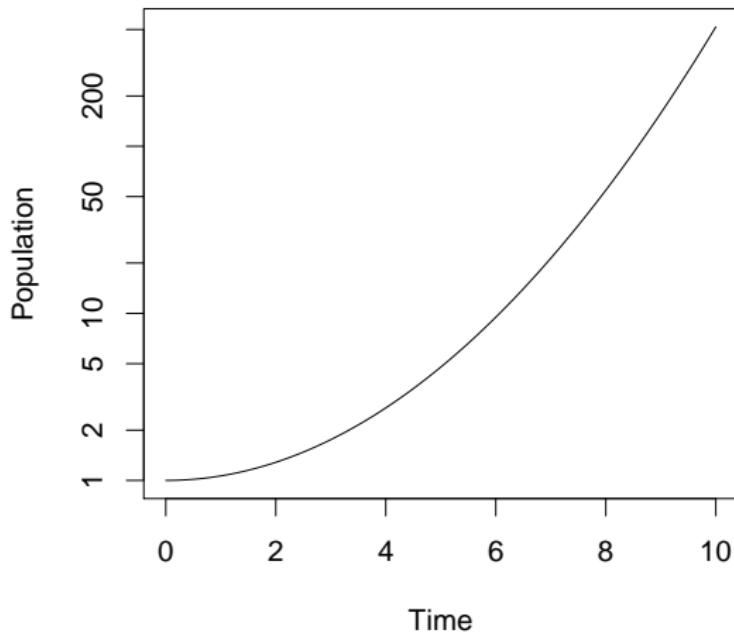
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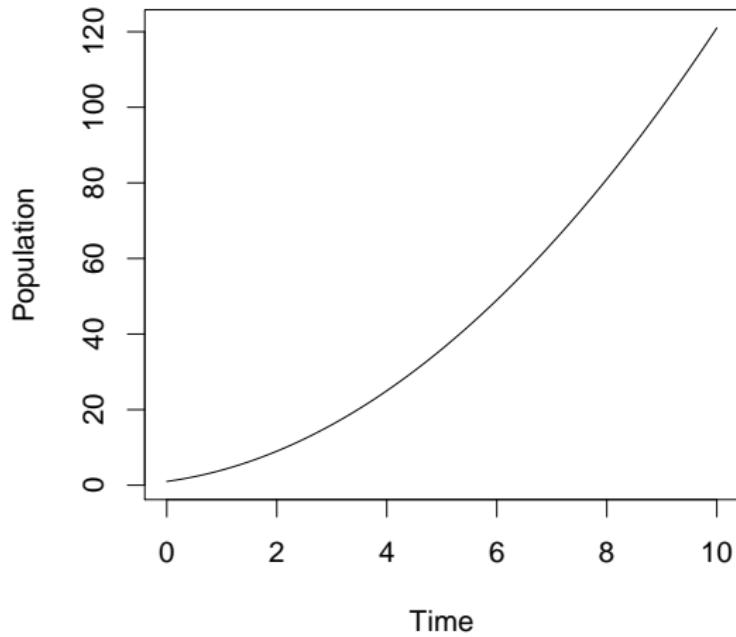
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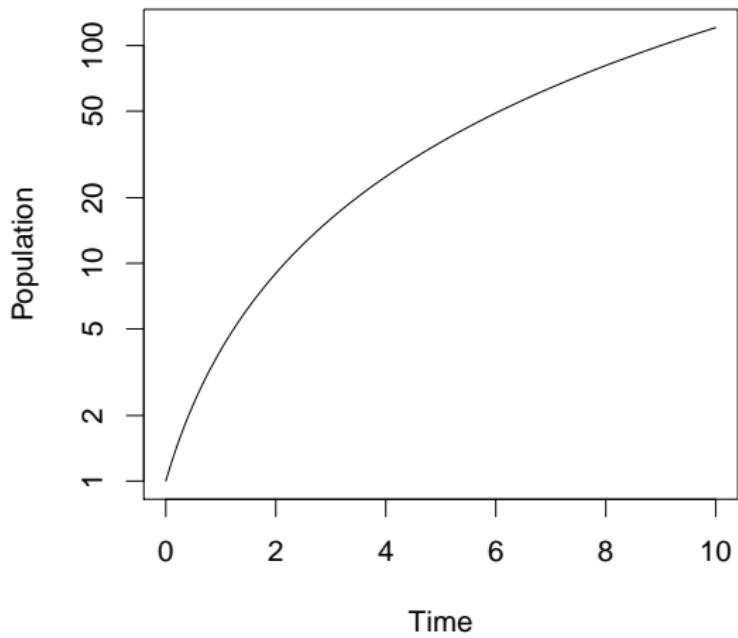
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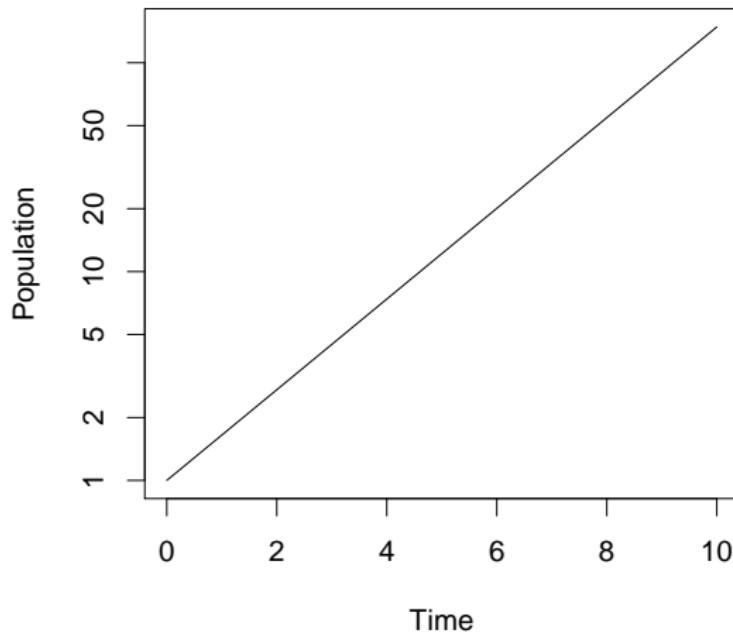
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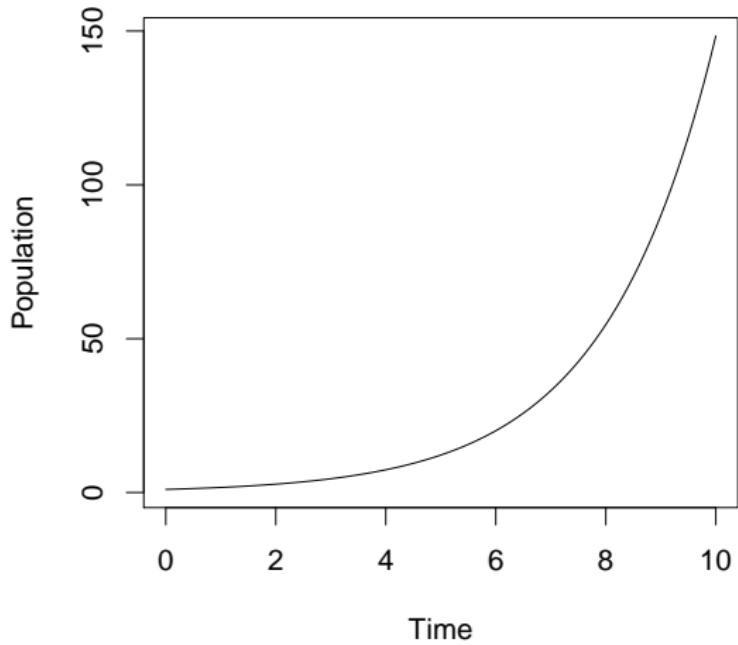
## A on the log scale



## *C (repeat)*



## C on the linear scale



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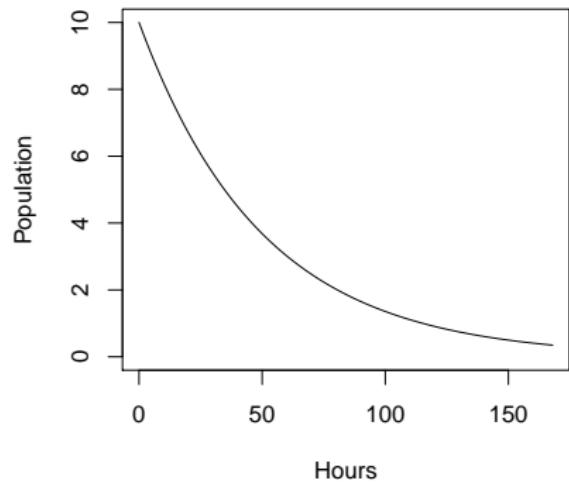
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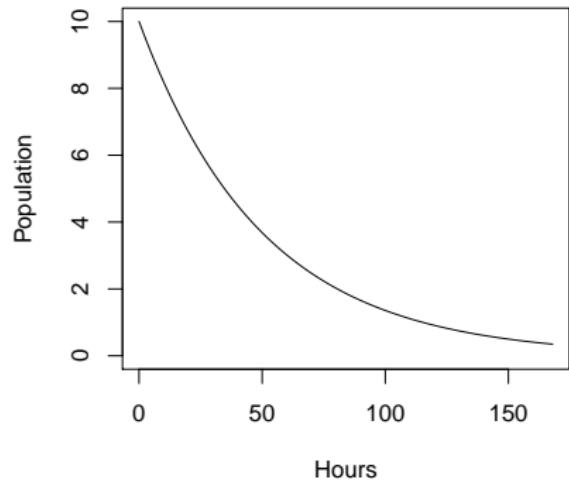
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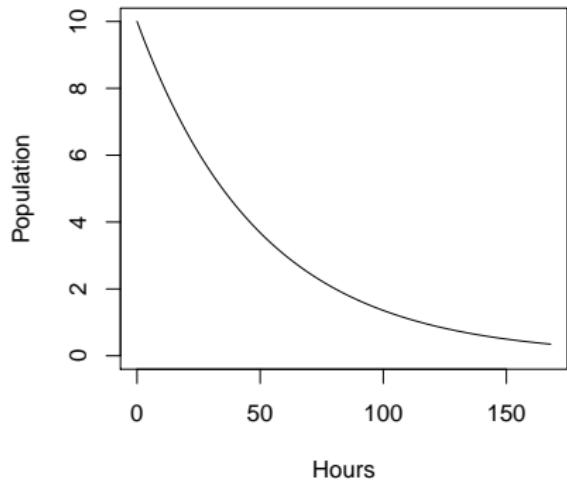
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# Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

Example populations

Dandelions

Gypsy moths

Bacteria

Exponential growth

Log and linear scales

Time scales

# Scales of comparison

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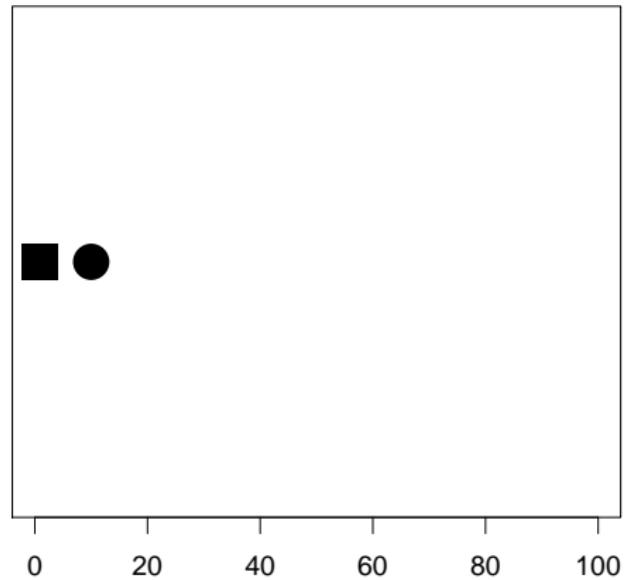
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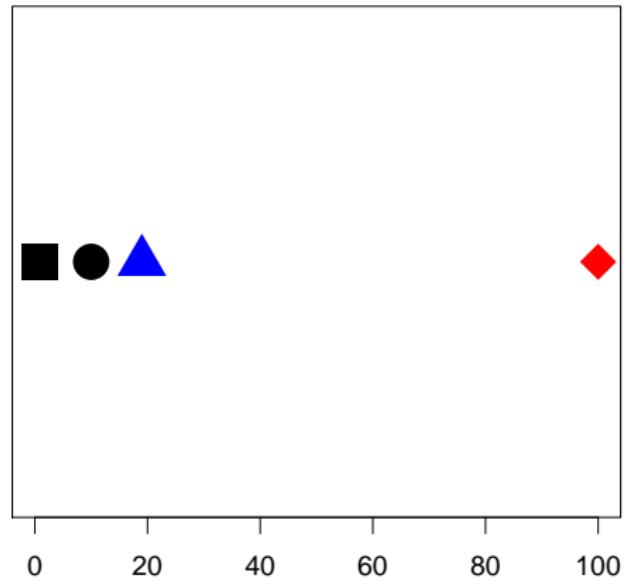
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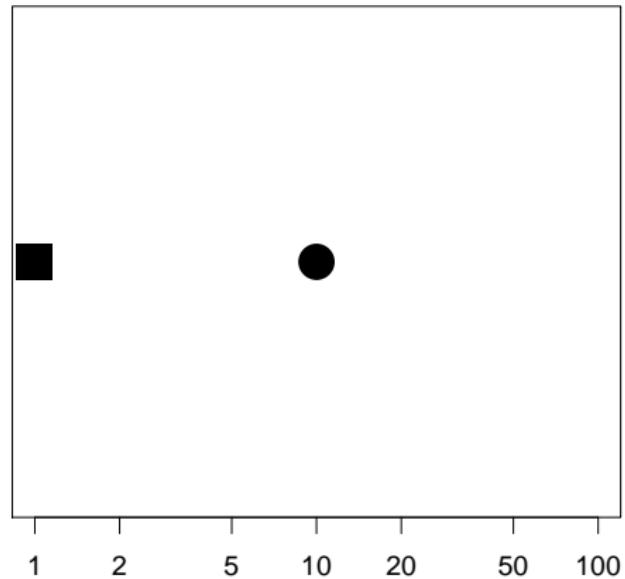
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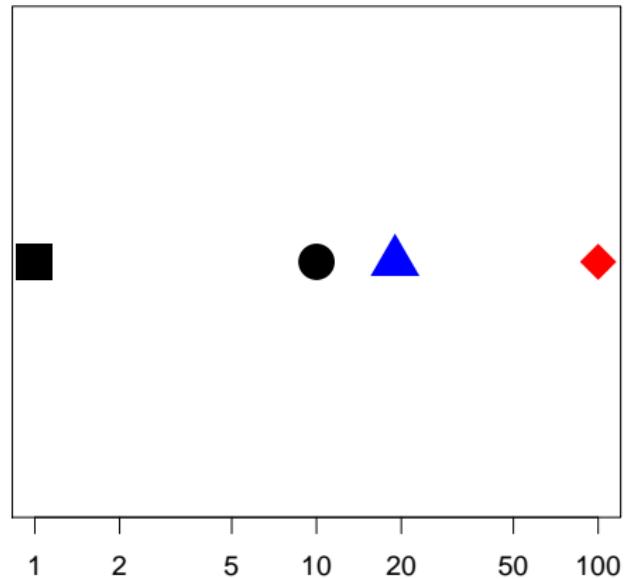
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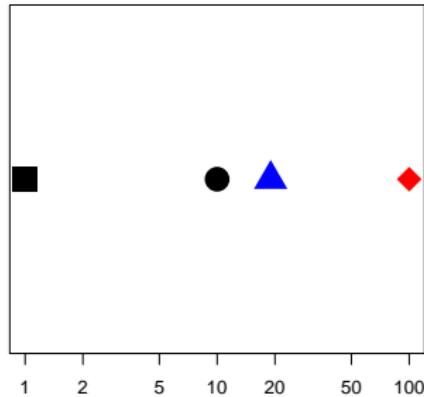
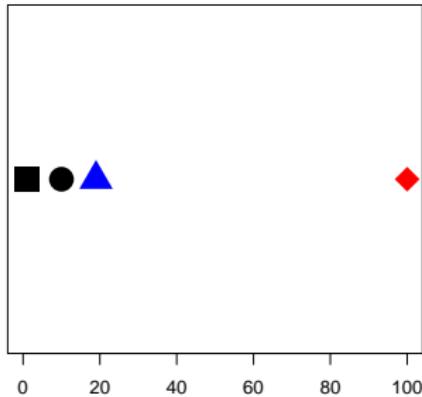
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There is only one log scale; it doesn't matter which base you use!

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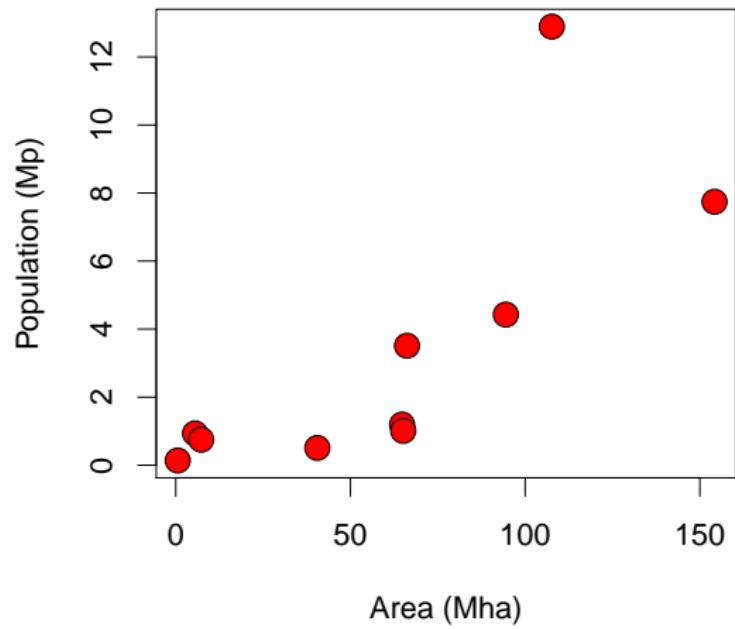
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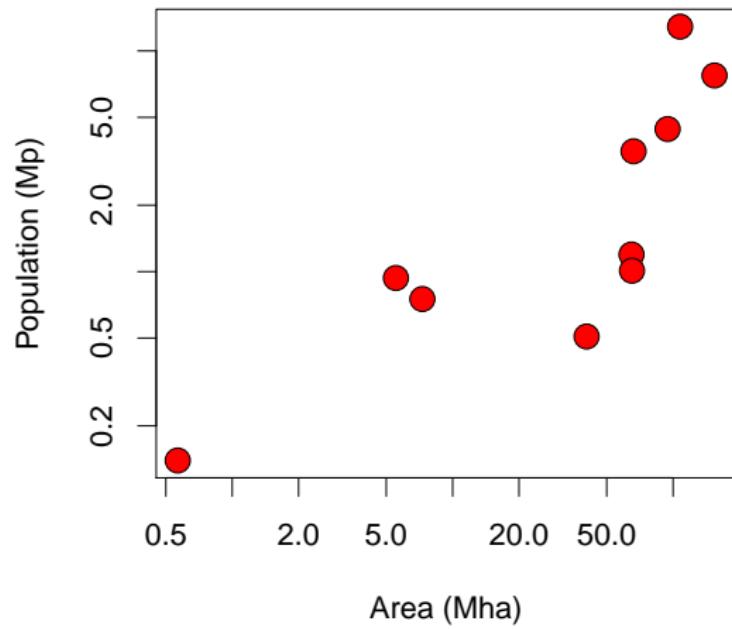
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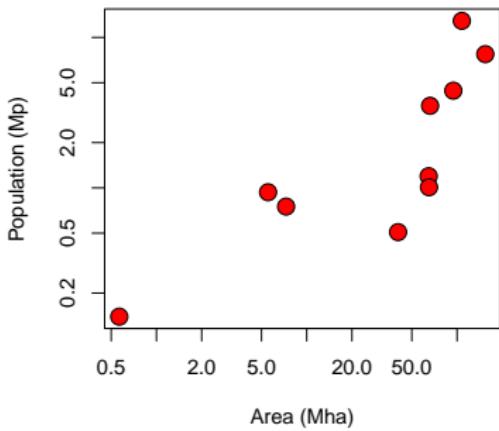
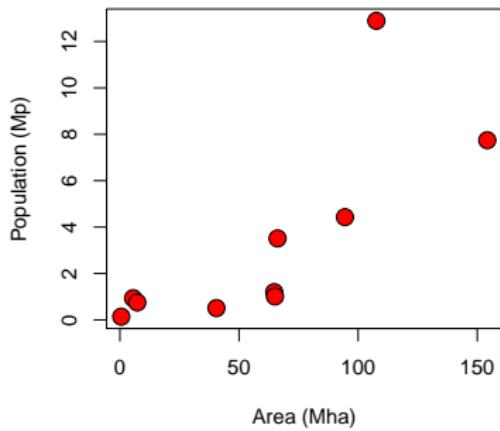
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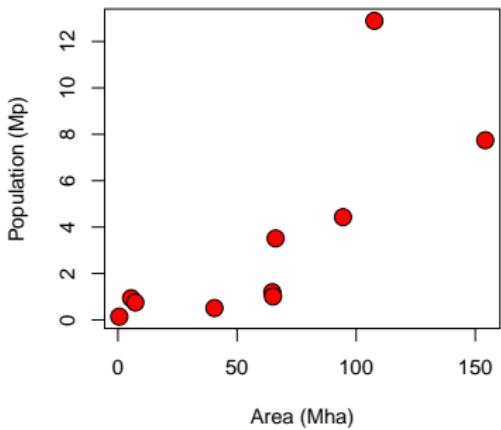
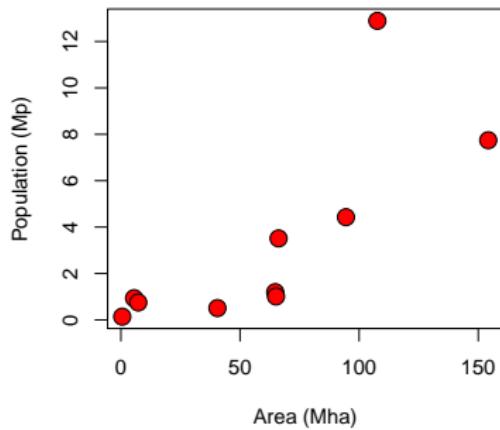
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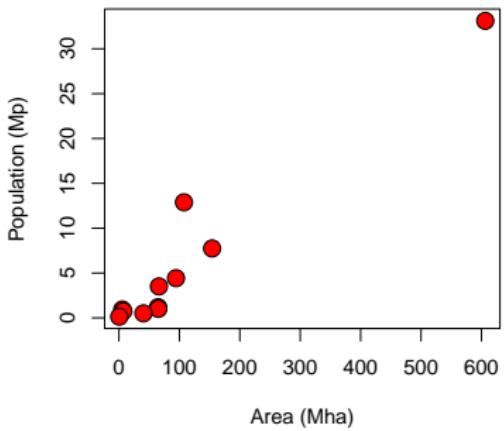
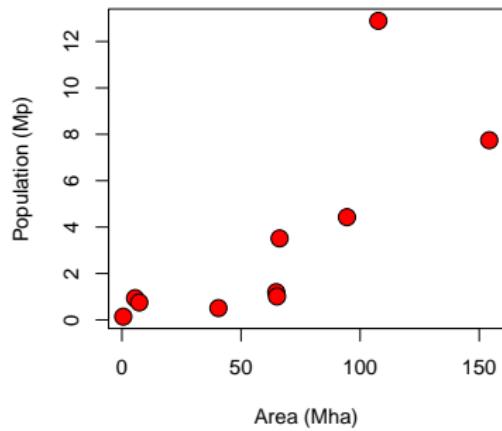
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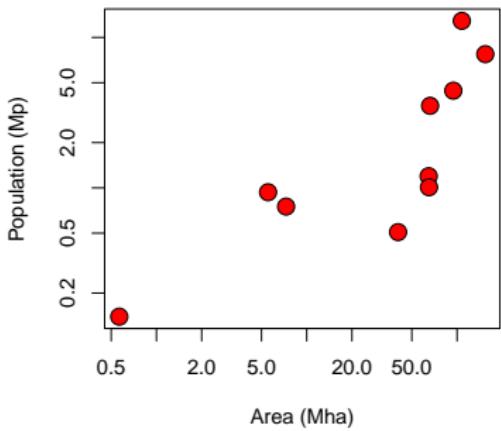
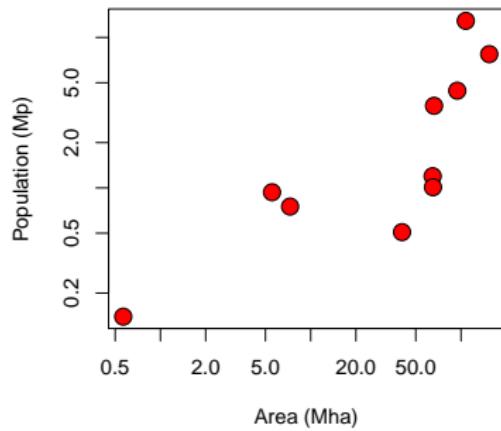
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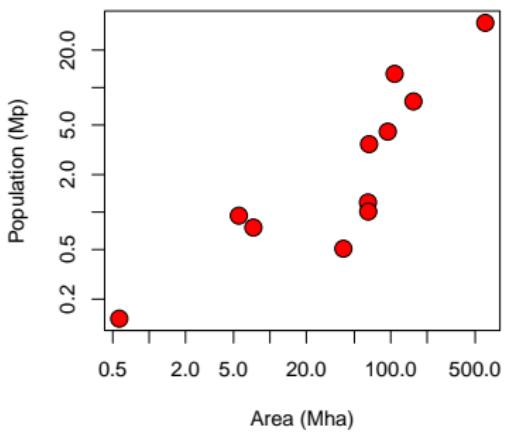
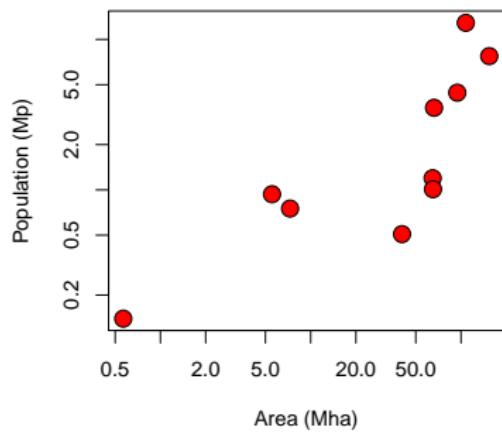
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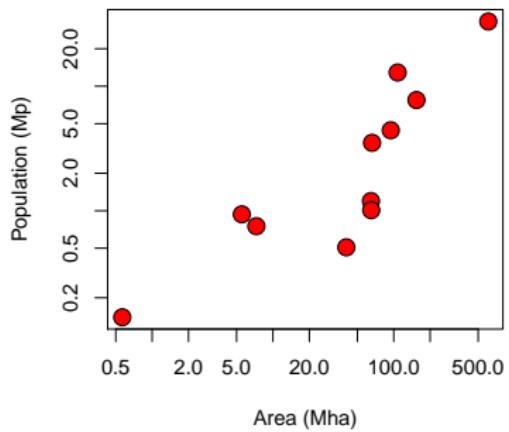
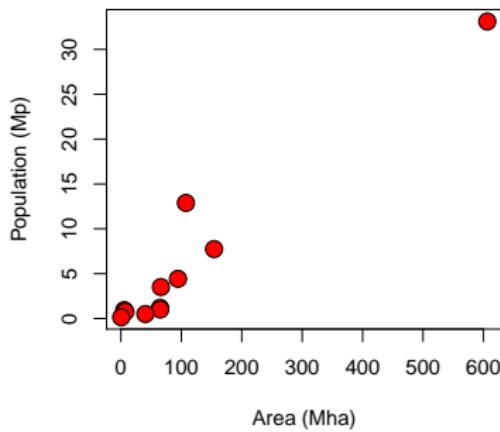
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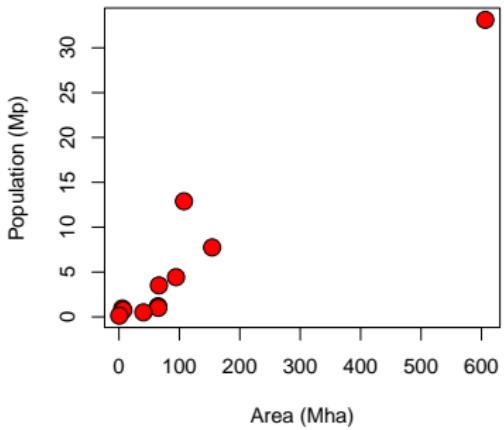
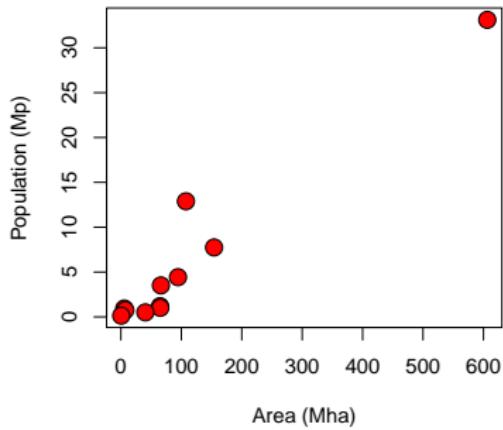
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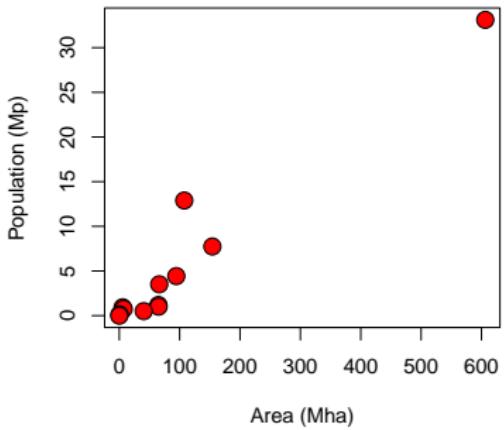
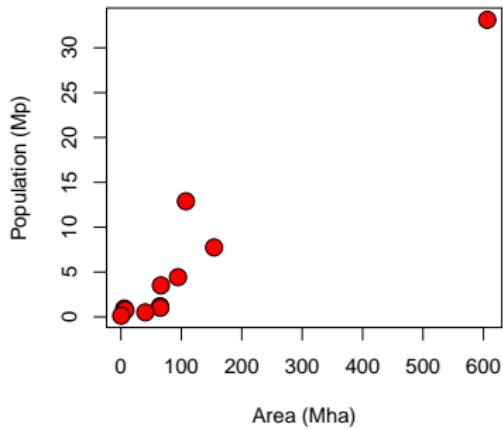
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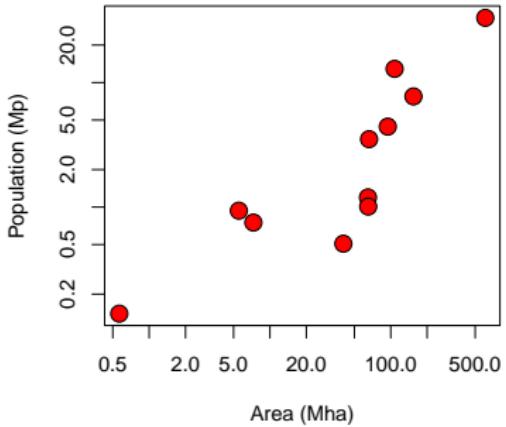
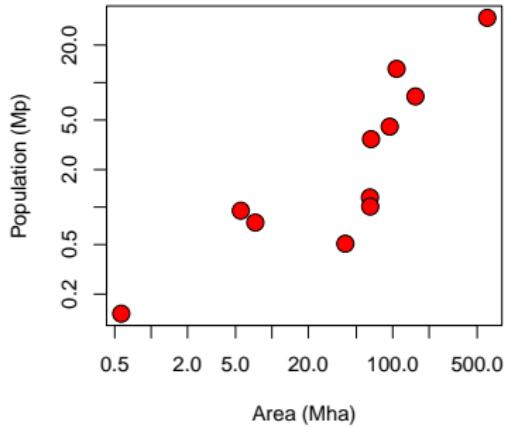
# Canada plus B135



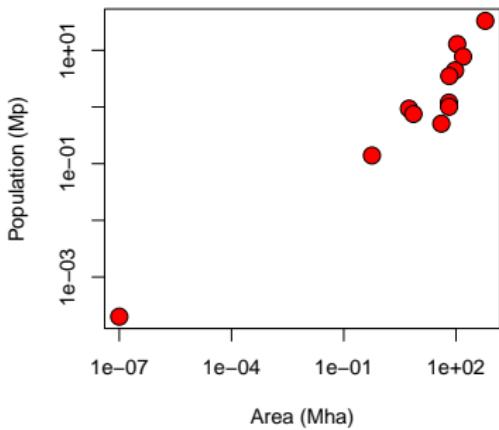
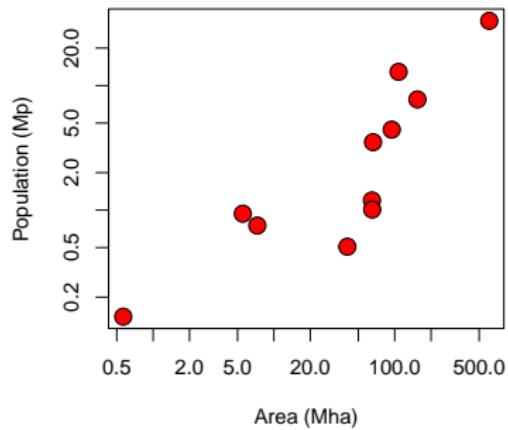
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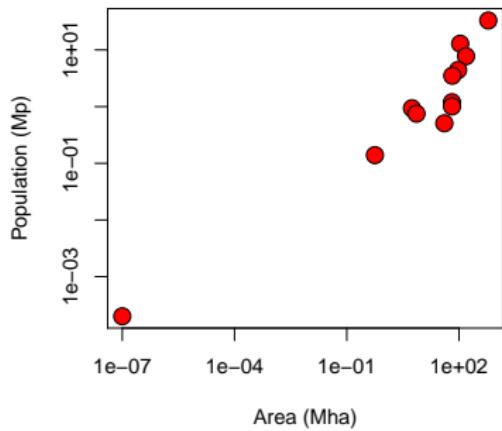
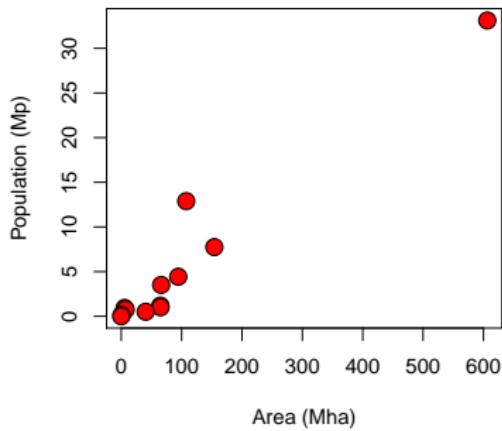
# Canada plus room B135?



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## Predation comparison



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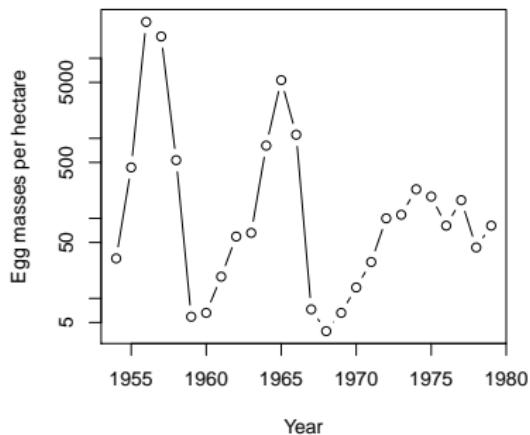
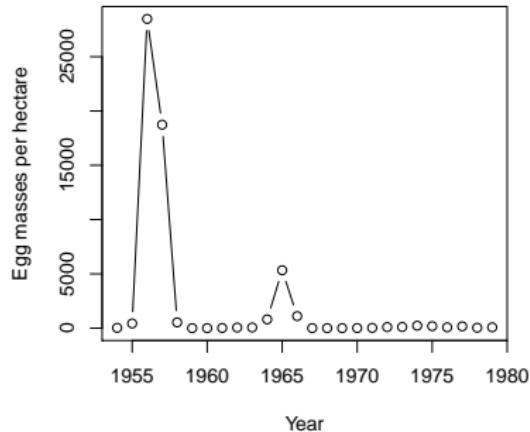
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# Outline

Course overview

Course structure

People

Course content

Learning goals

Examples

Example populations

Dandelions

Gypsy moths

Bacteria

Exponential growth

Log and linear scales

Time scales

# *Speeding in Taiwan*

- A life experience



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- ▶ A life experience
- ▶ Some clarifications



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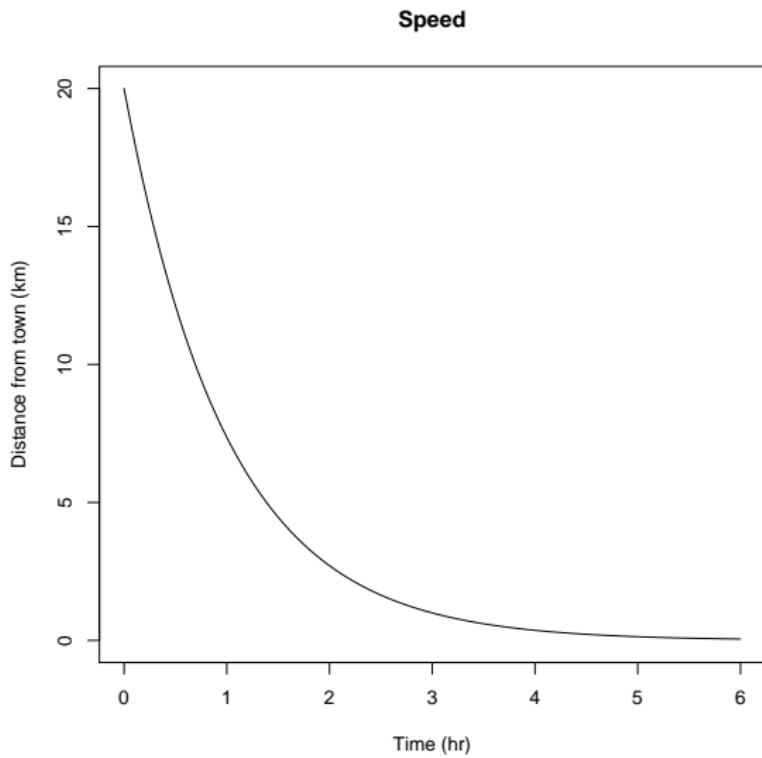
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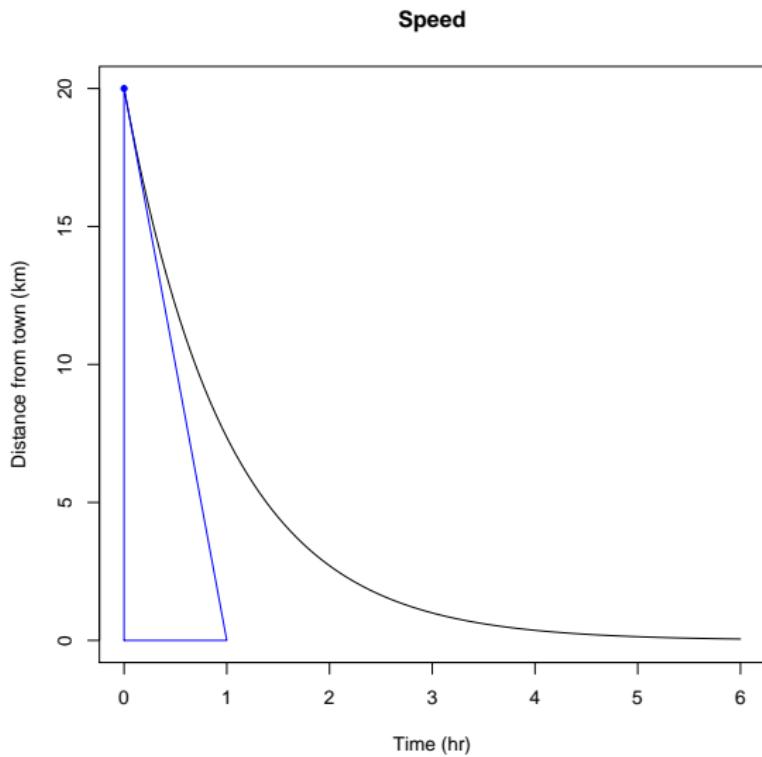
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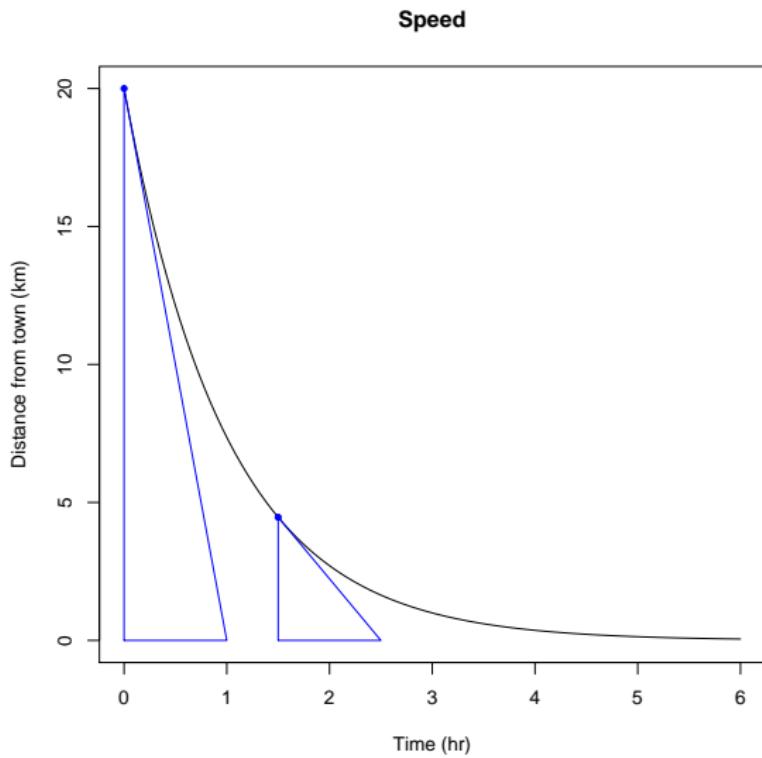
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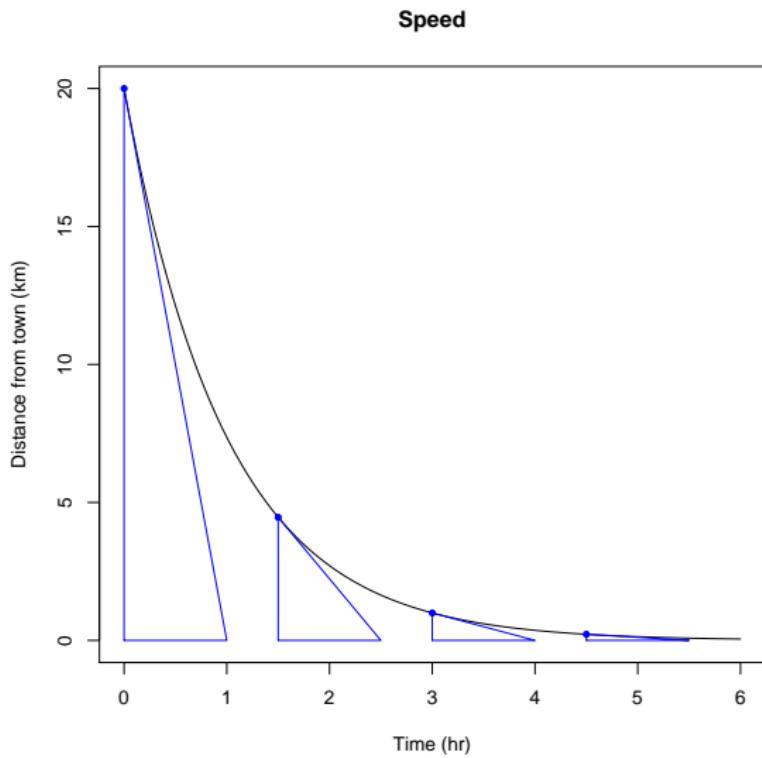
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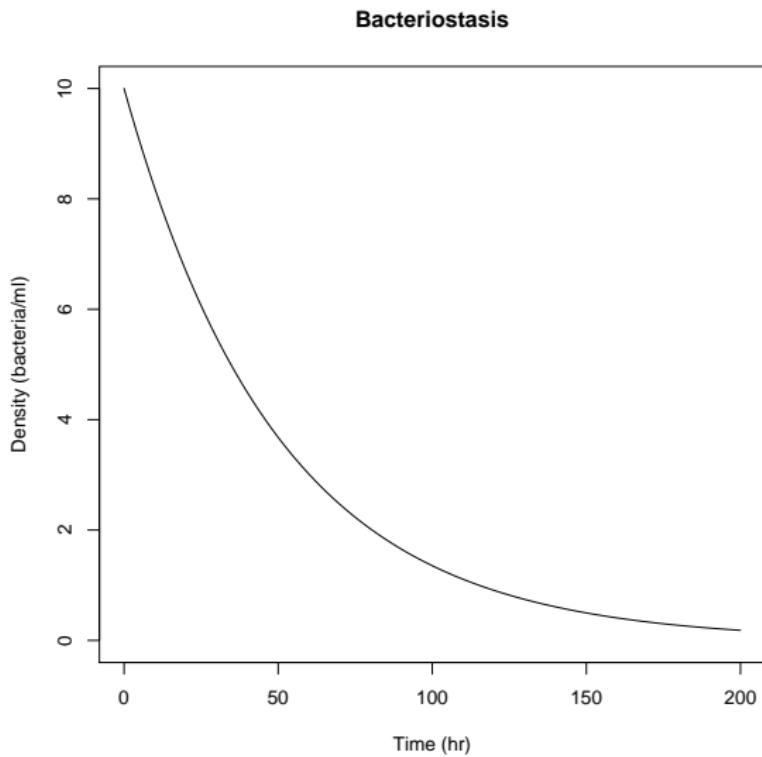
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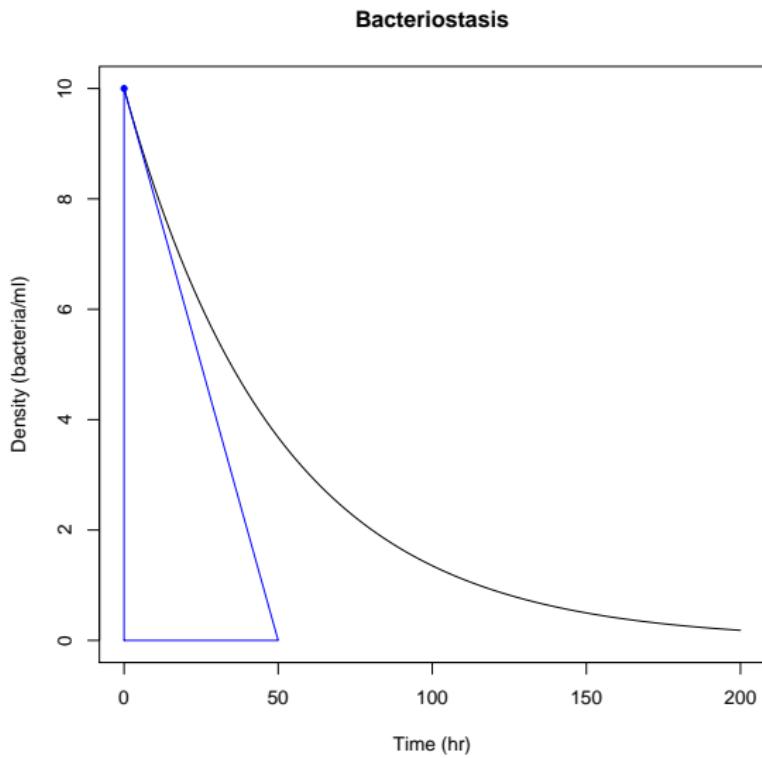
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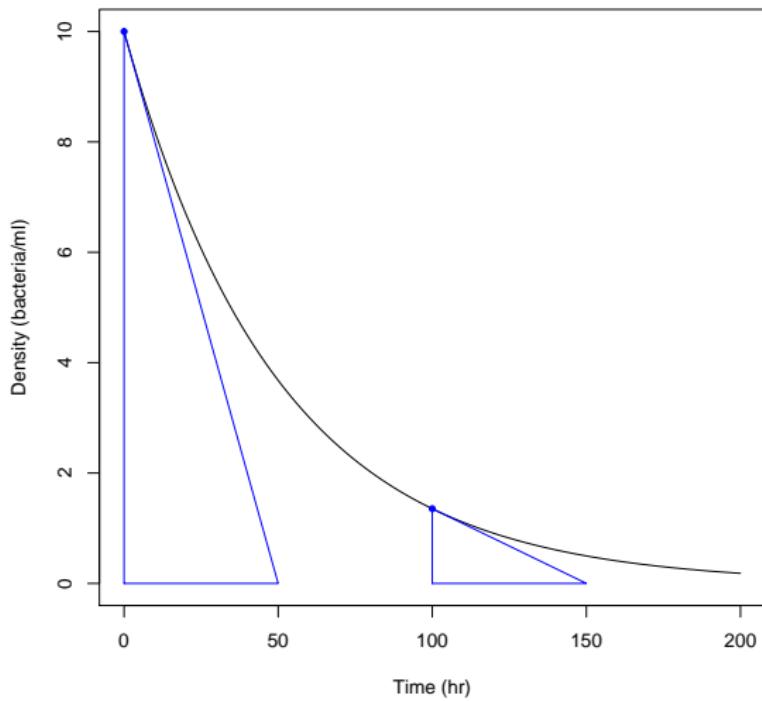


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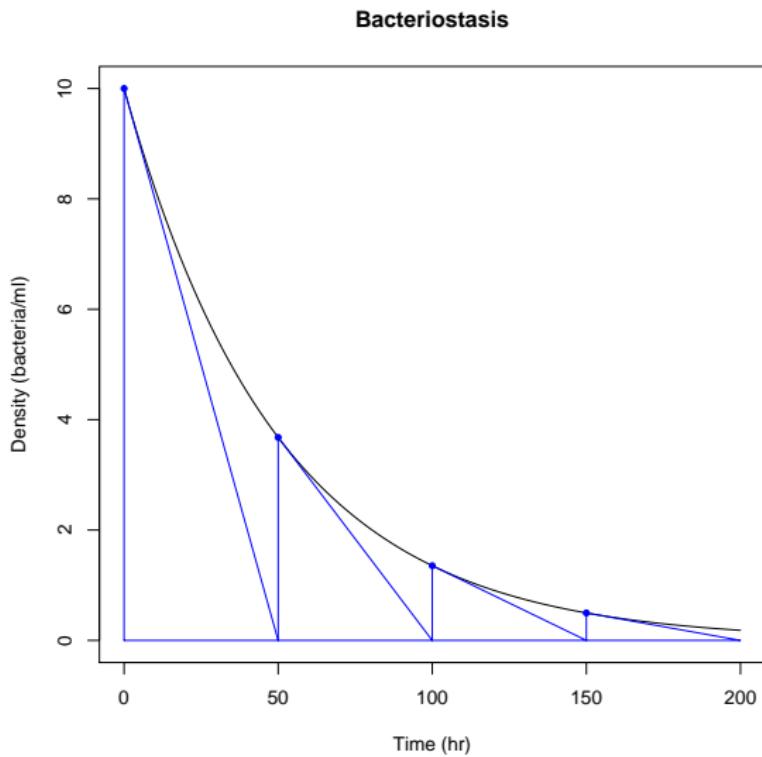


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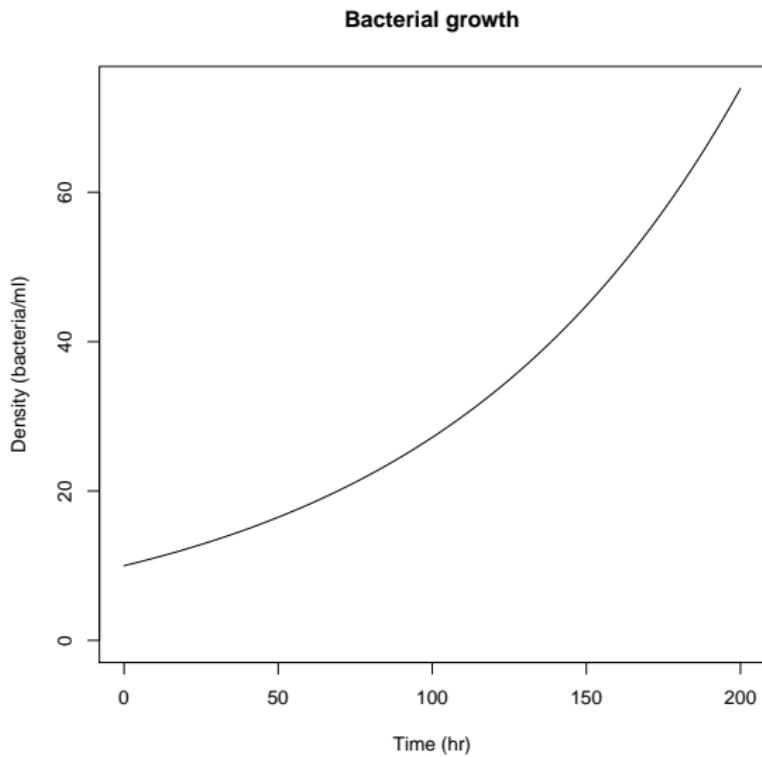
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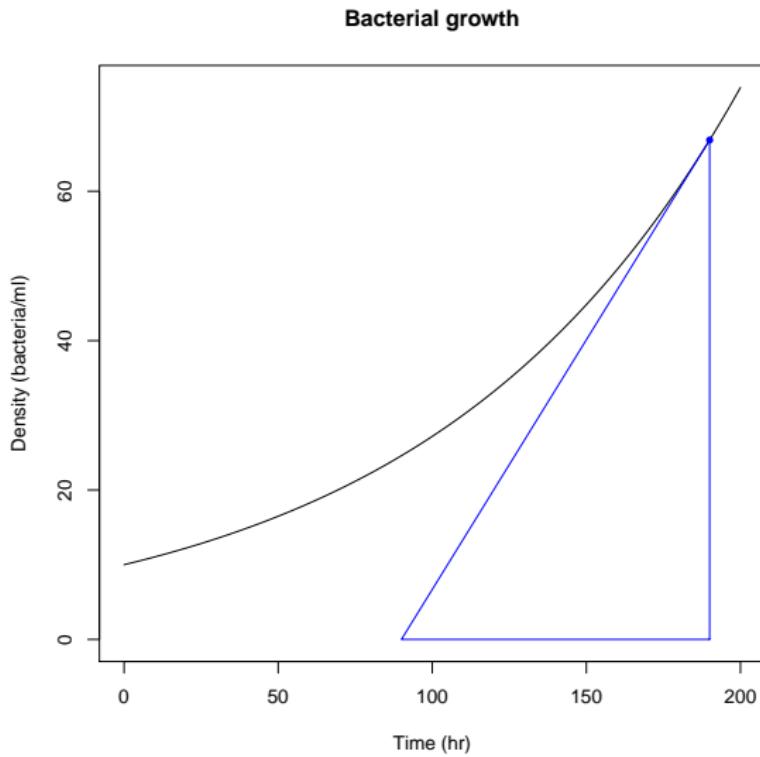
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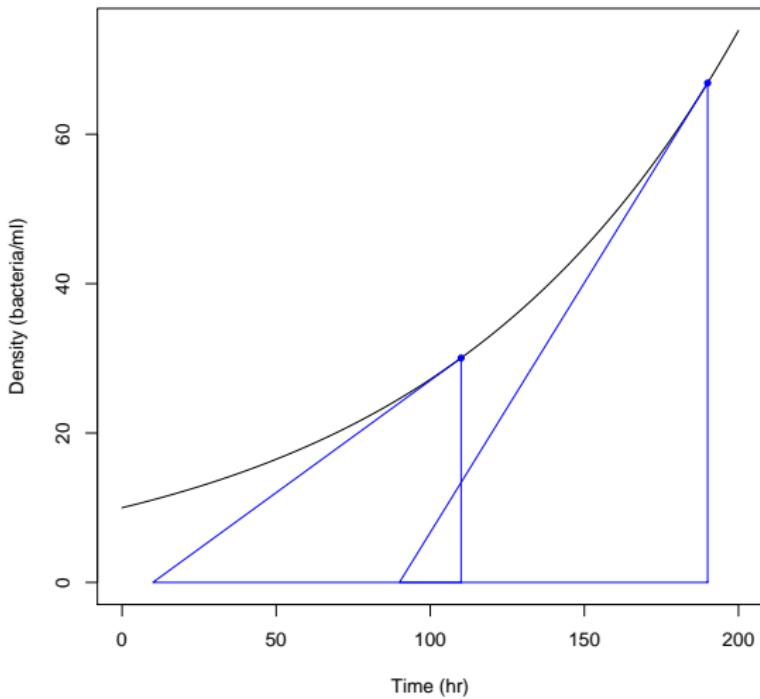


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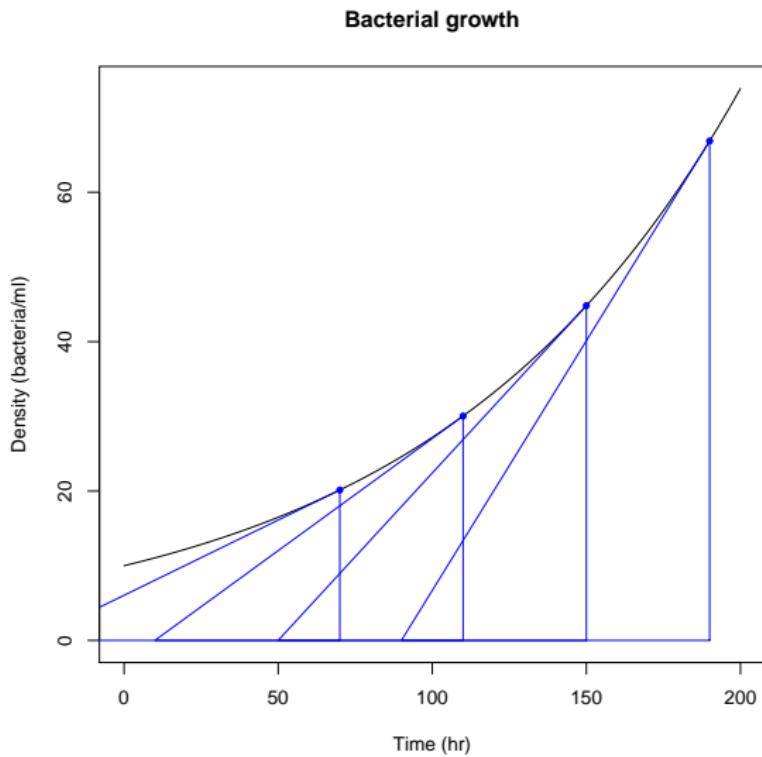


## Characteristic times

Bacterial growth



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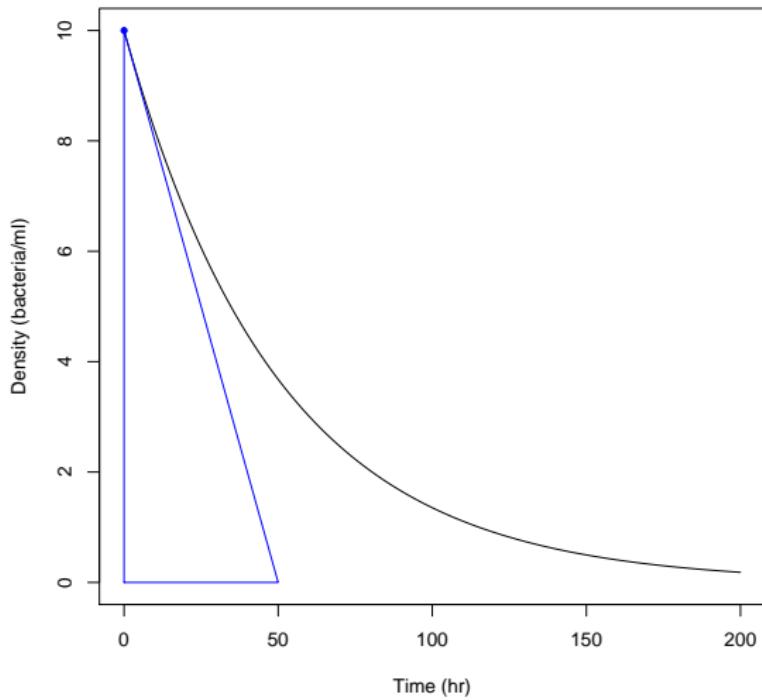
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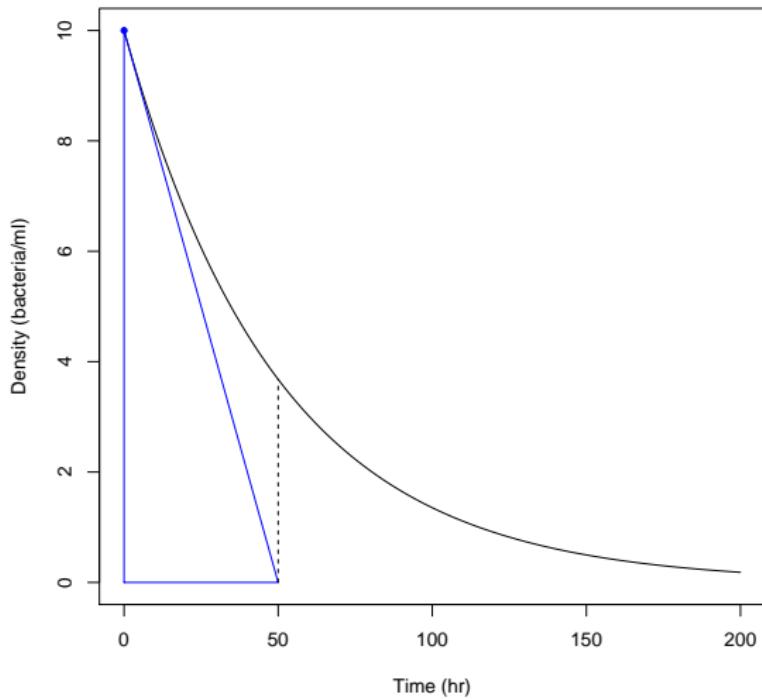
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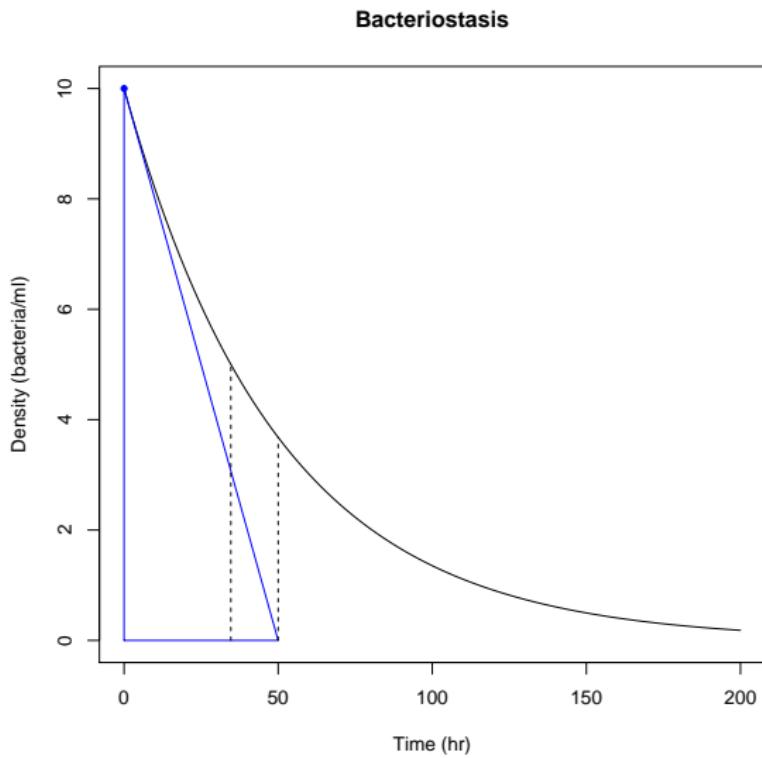
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## Characteristic time and half life



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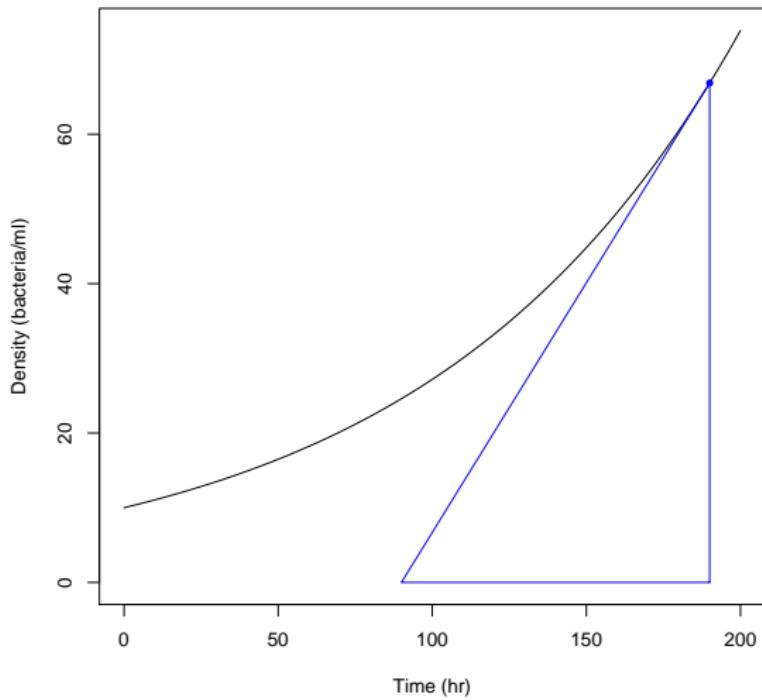
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  - ▶  $T_h = \log_e(2) T_c$
  - ▶ It takes  $T_c$  time for a declining population to decrease by a factor of  $e$
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  - ▶ We can write  $T_h = \log_e(2) T_c$

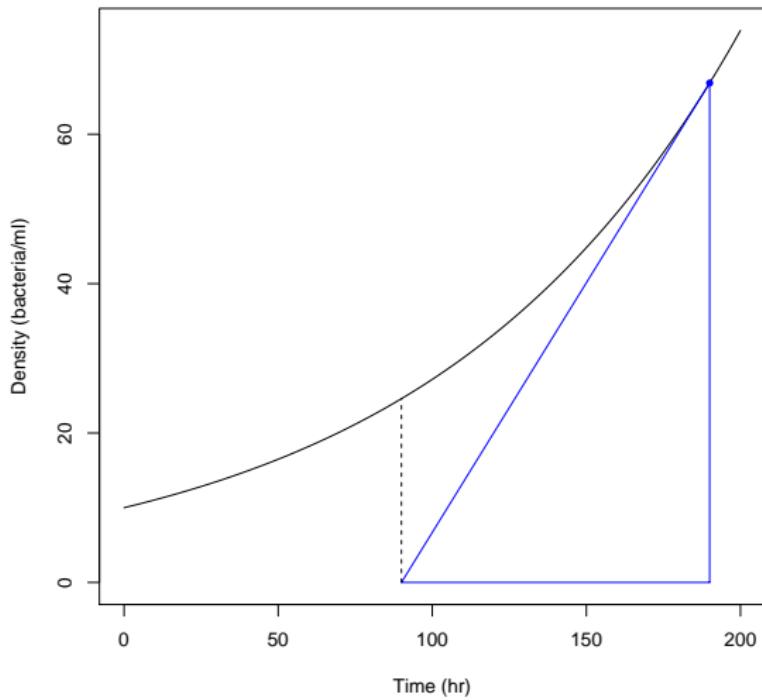
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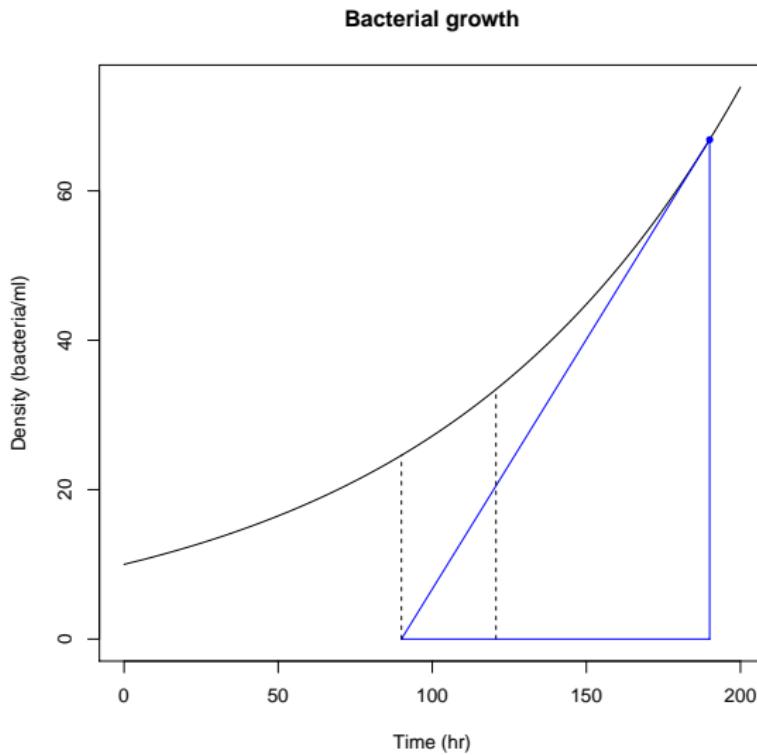
### Bacterial growth



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## Characteristic time and doubling time



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