UNIT 3 Non-linear population models

Outline

Introduction

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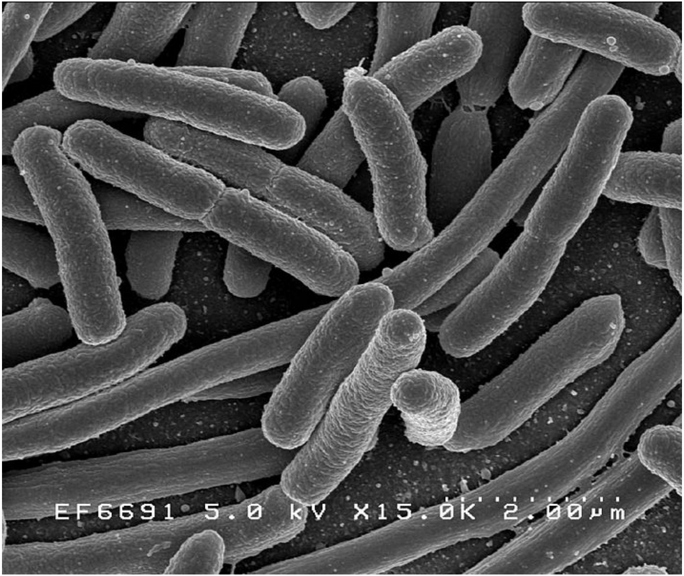
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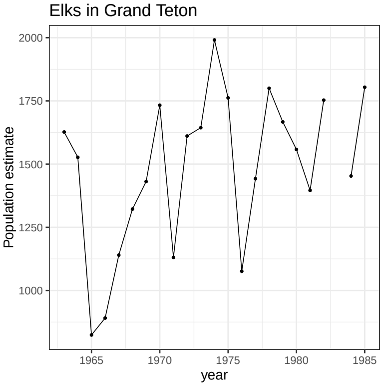
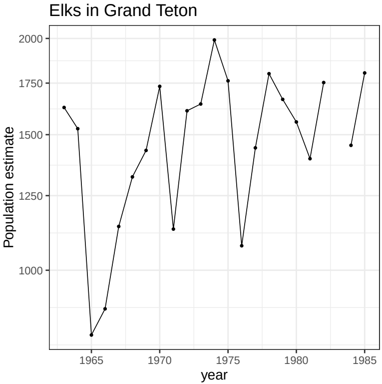
(preview)

Elk


Elks in Grand Teton
Population estimate
2000
1750
1500
1250
1000
1965
1970
1975
1980
1985
year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2000
1750
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1965
1970
1975
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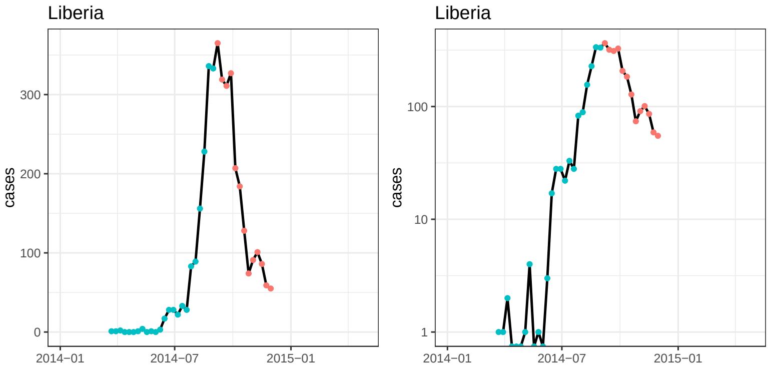
year

Ebola



Liberia
cases
300
200
100
0
2014−01
2014−07
2015−01
Liberia
cases
100
10
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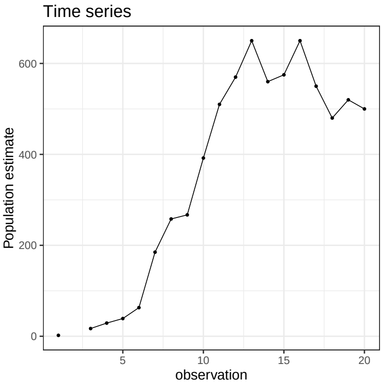
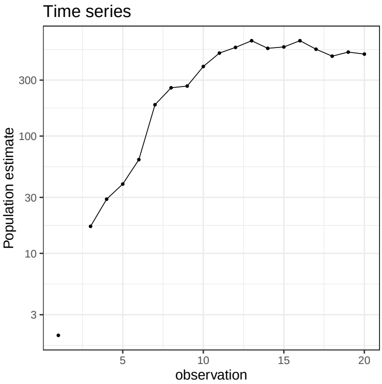
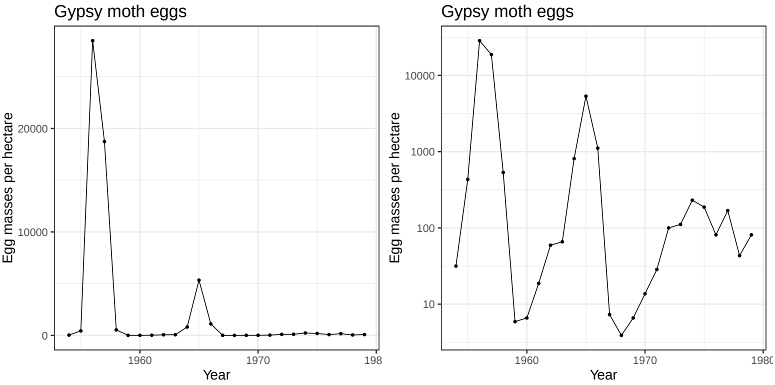
2014−01
2014−07
2015−01

Paramecia (preview)





Gypsy moths (preview)



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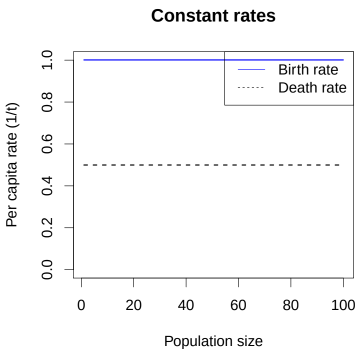

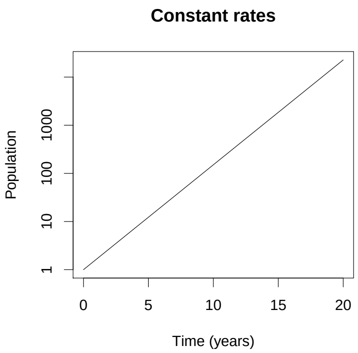

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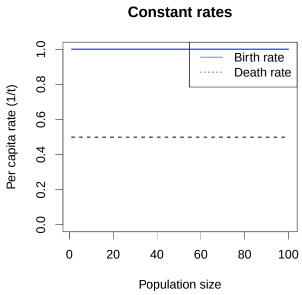

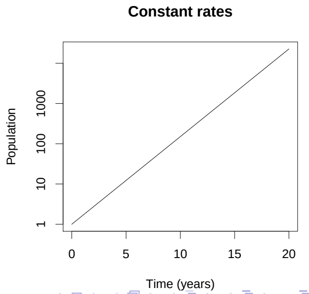

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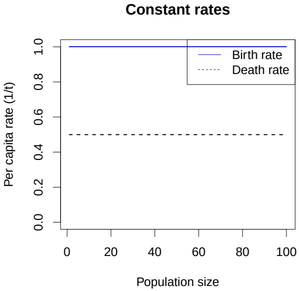

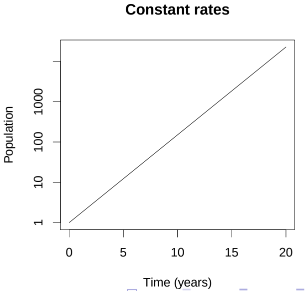

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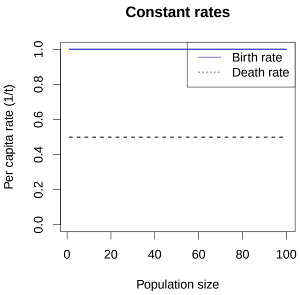

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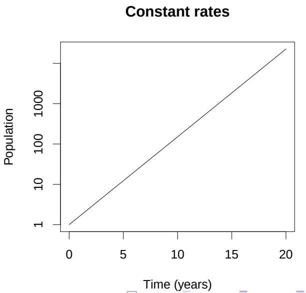

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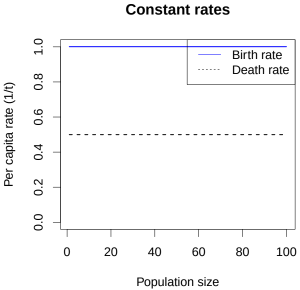

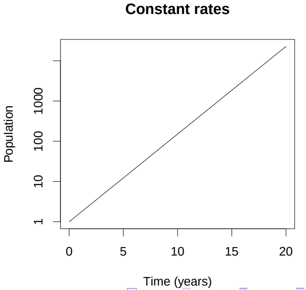

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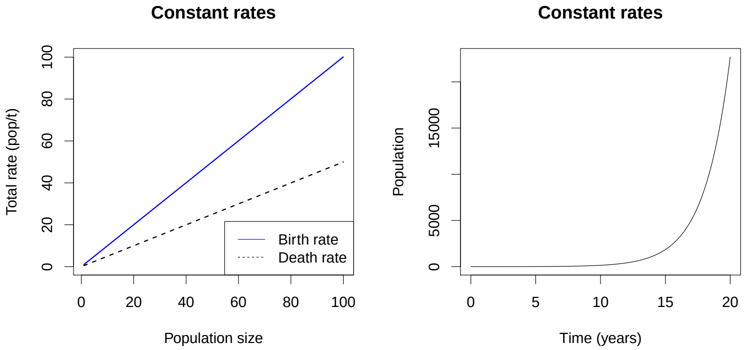

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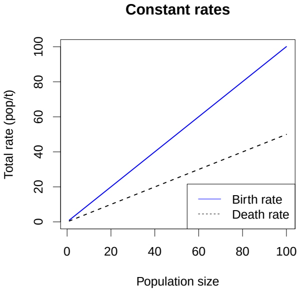

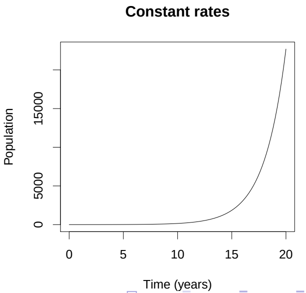

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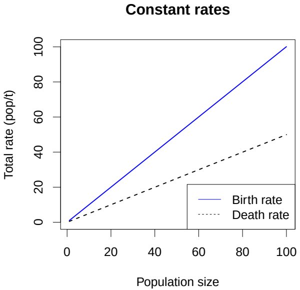

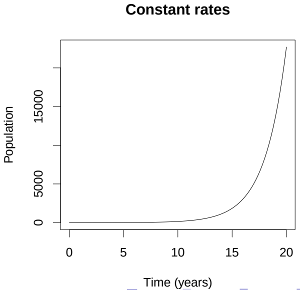

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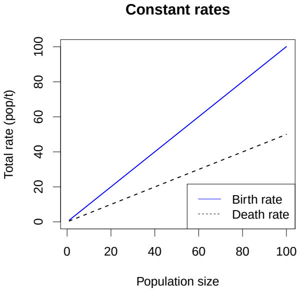

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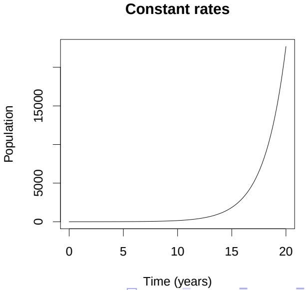

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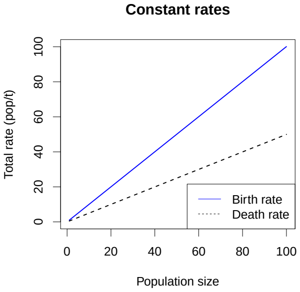

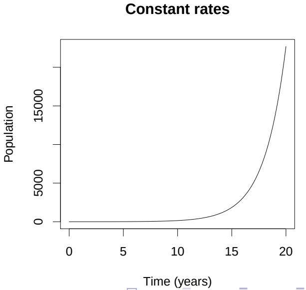

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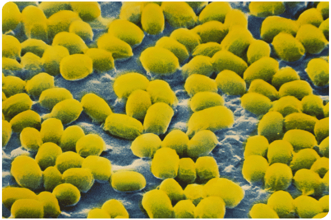

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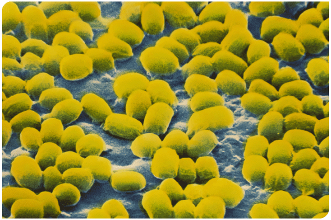

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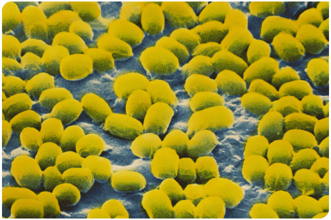

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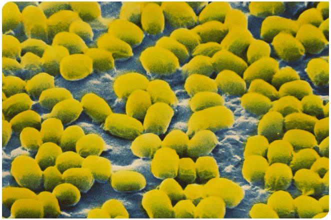

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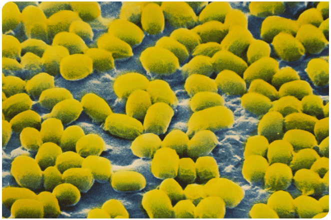

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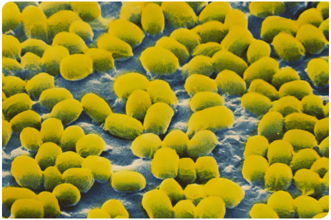

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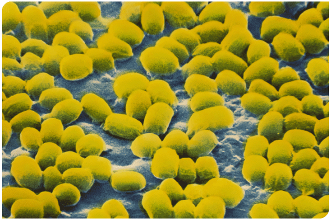

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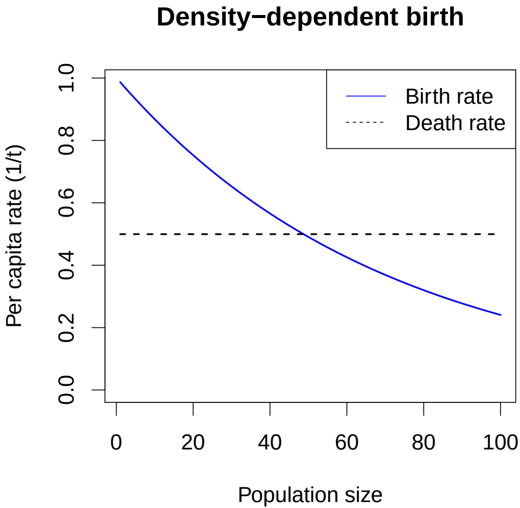

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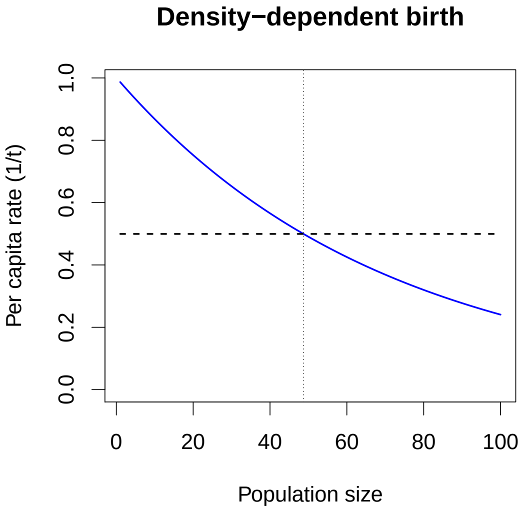

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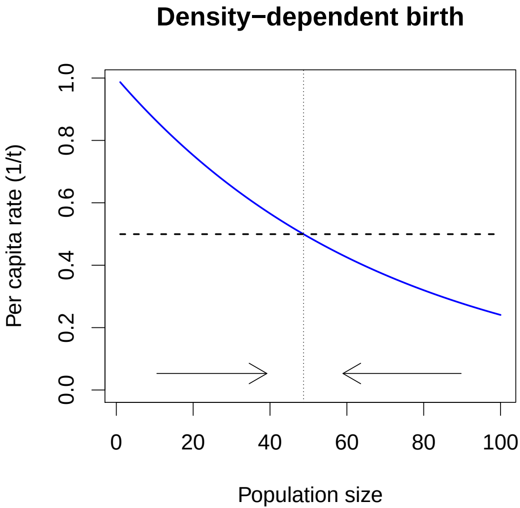

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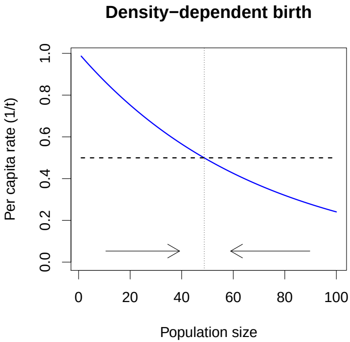

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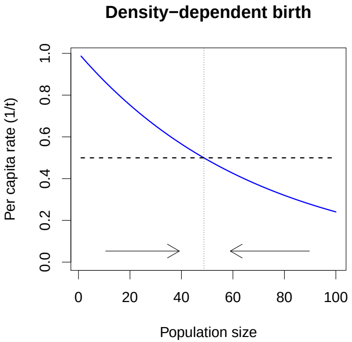

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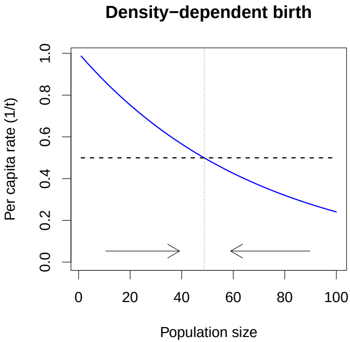

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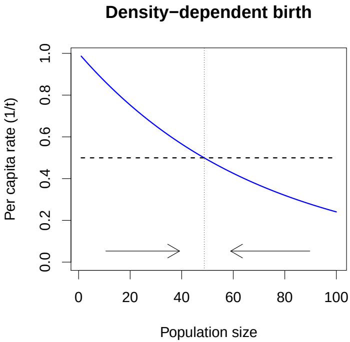

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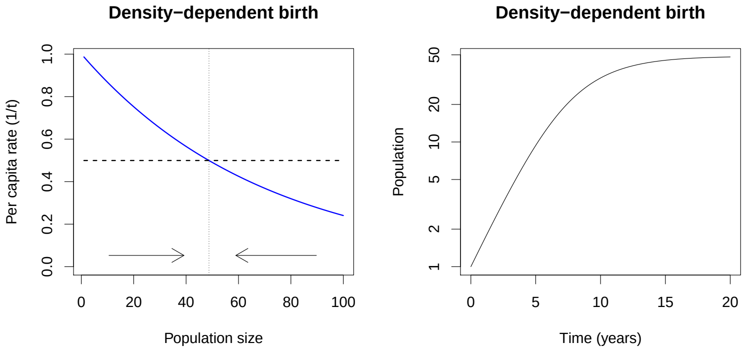

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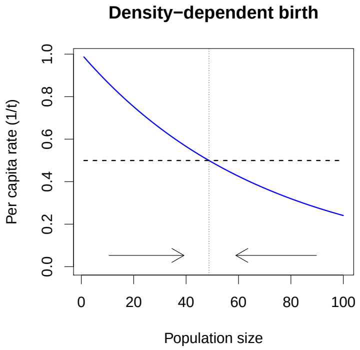

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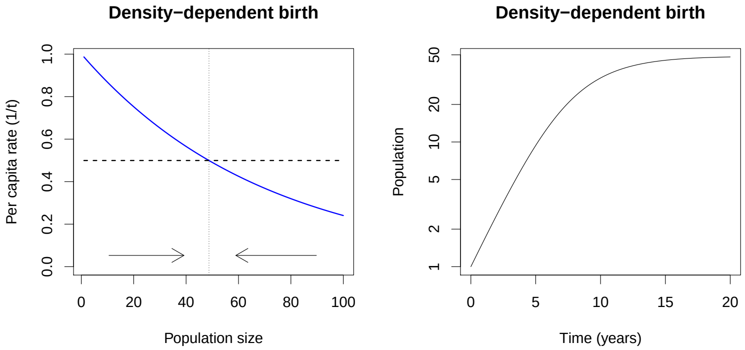
What will this model do? (repeat)



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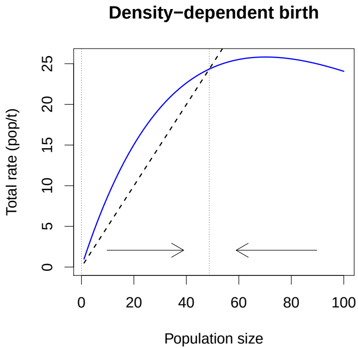

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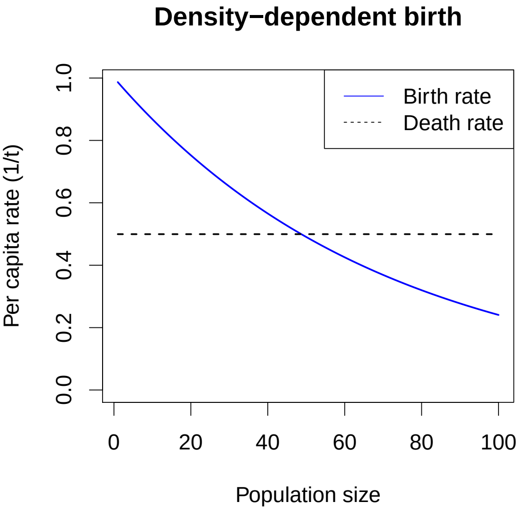

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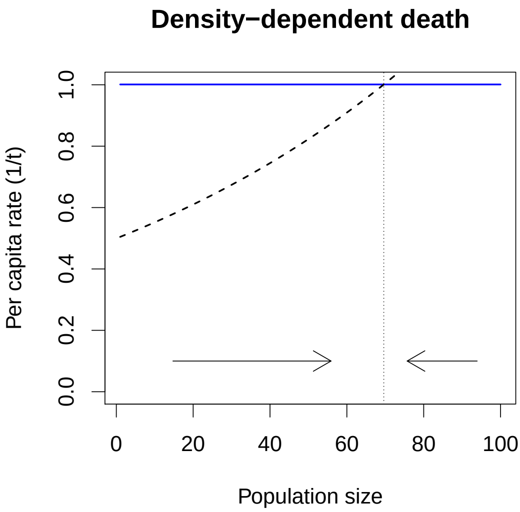

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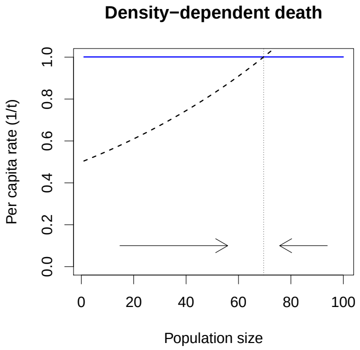

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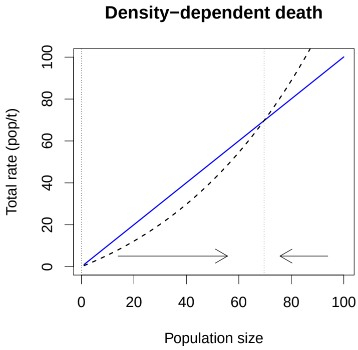

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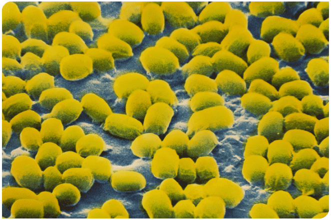

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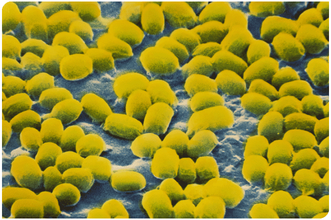

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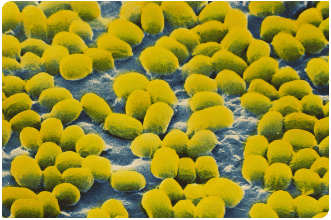

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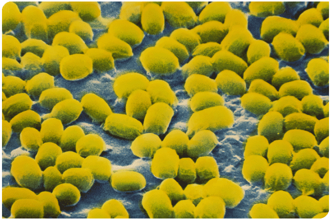

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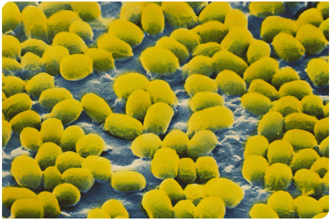

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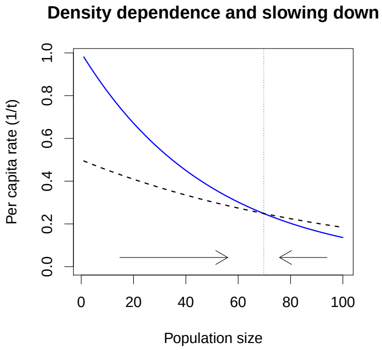

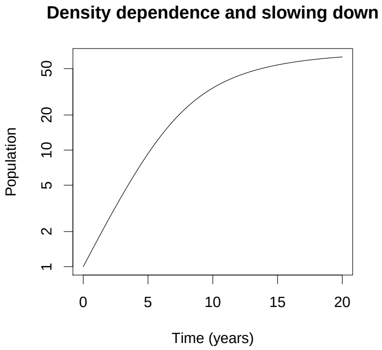

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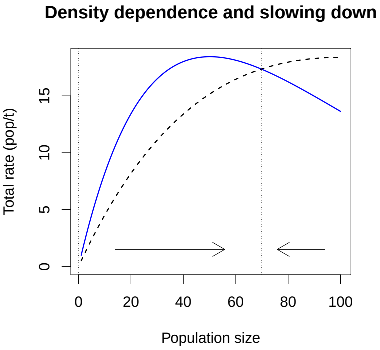

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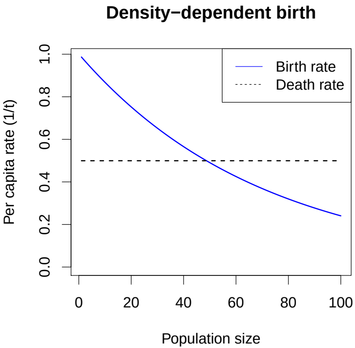

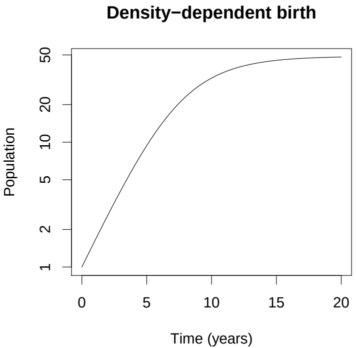

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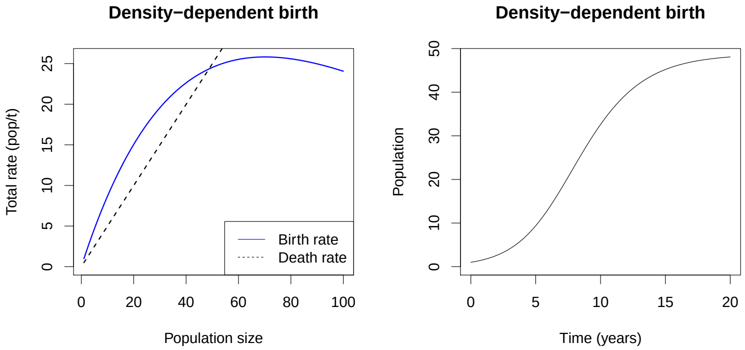

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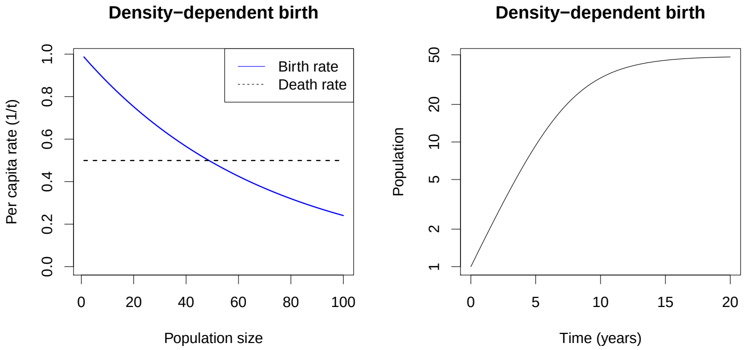

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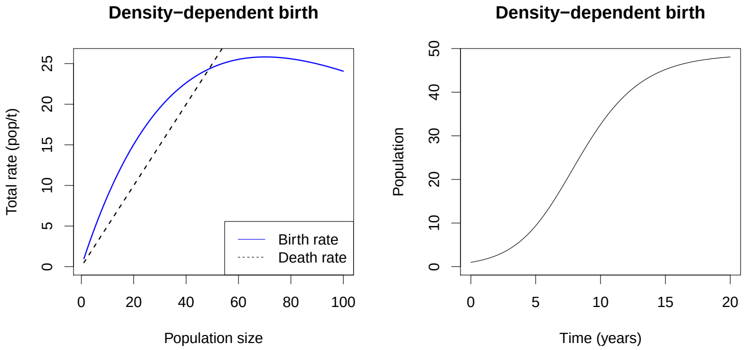

Outline

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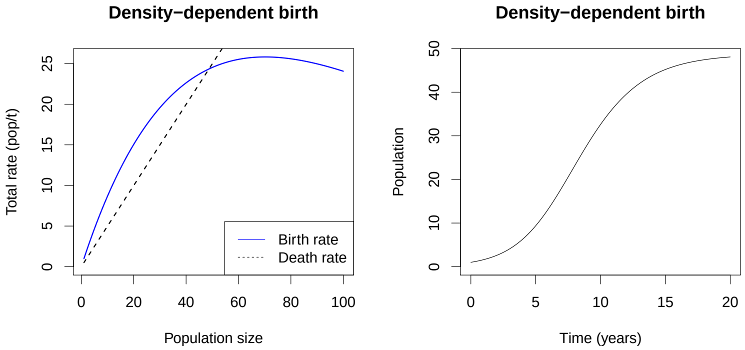

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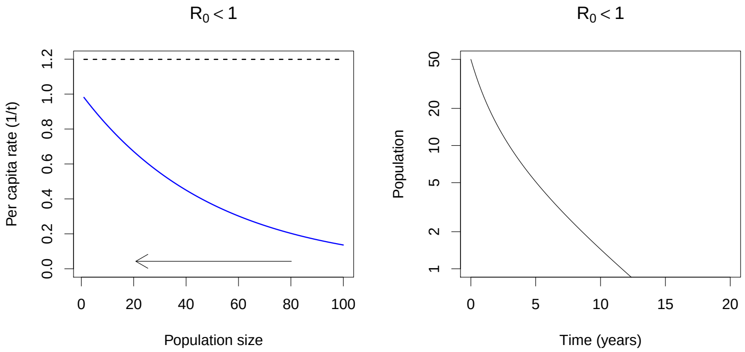

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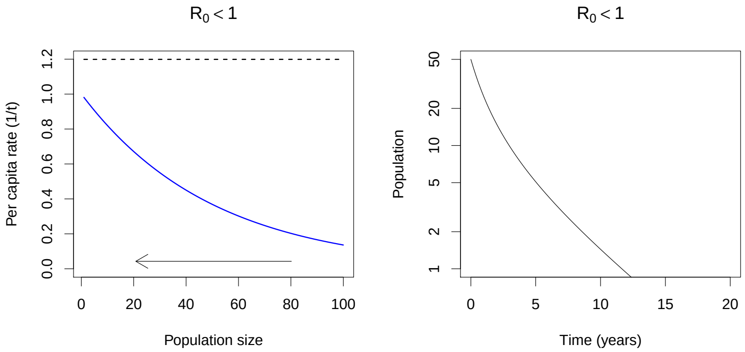

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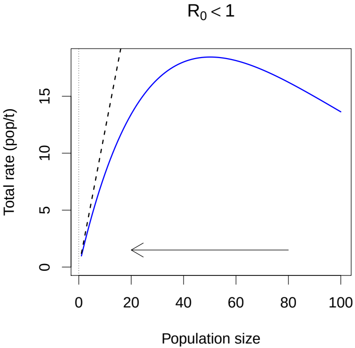

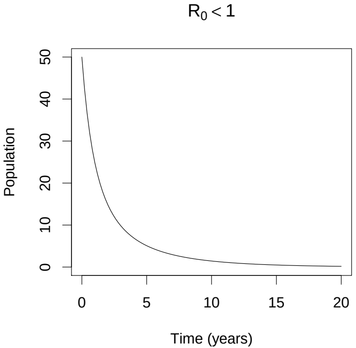

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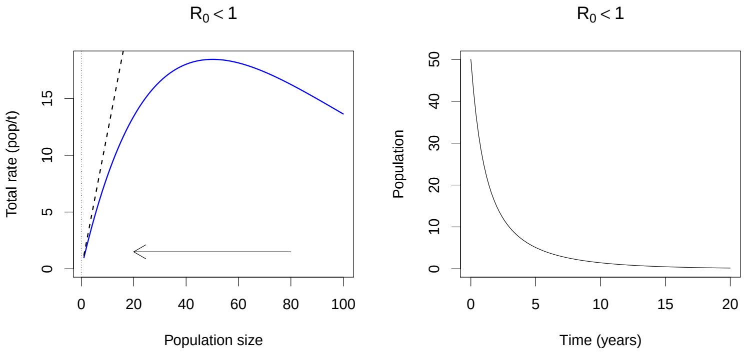

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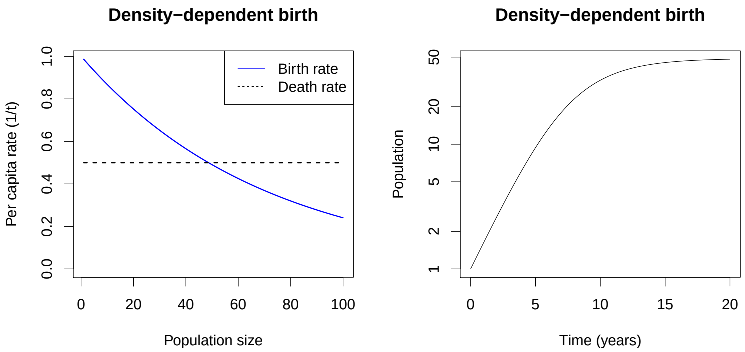

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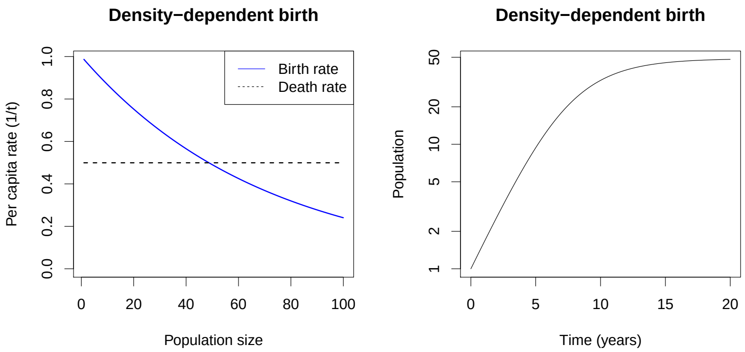

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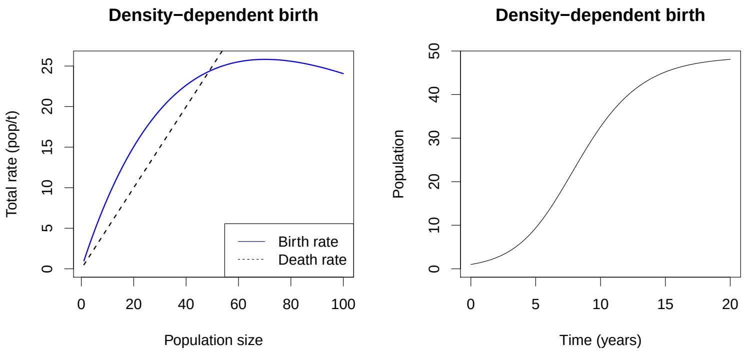

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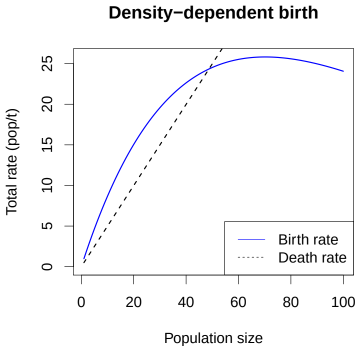

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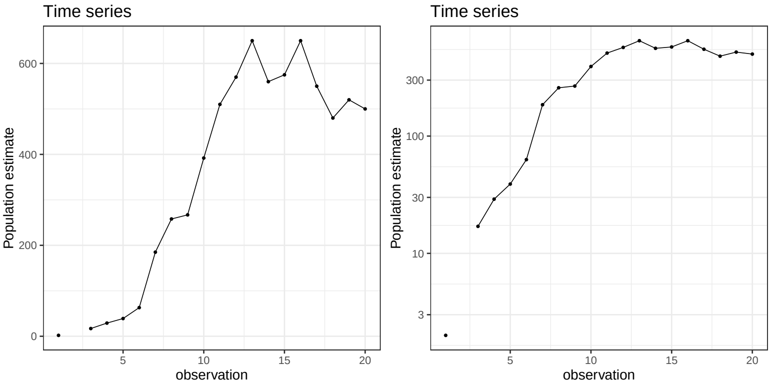
Time series
Population estimate
600
400
200
0
5
10
15
20
observation
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300
100
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10
3
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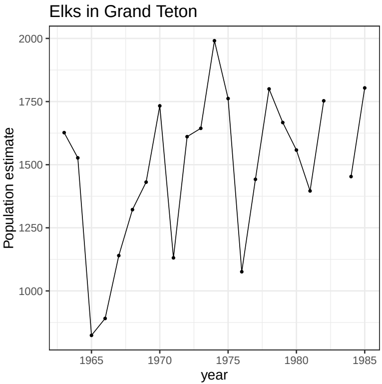
Elk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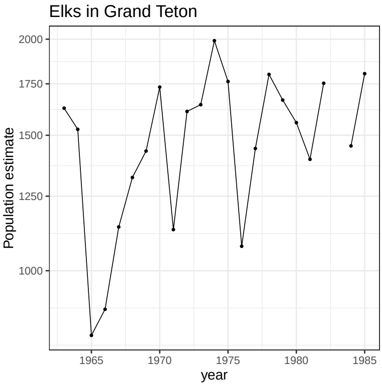
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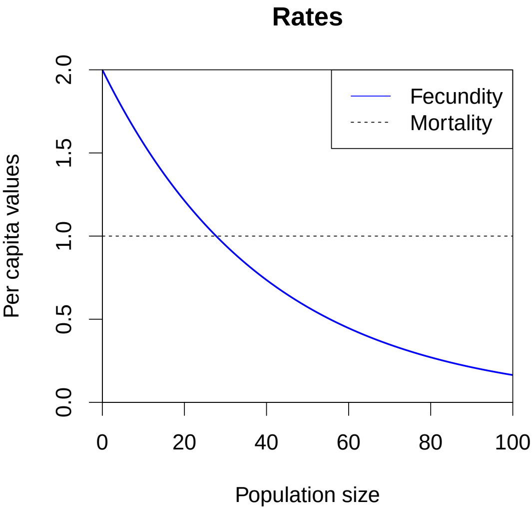

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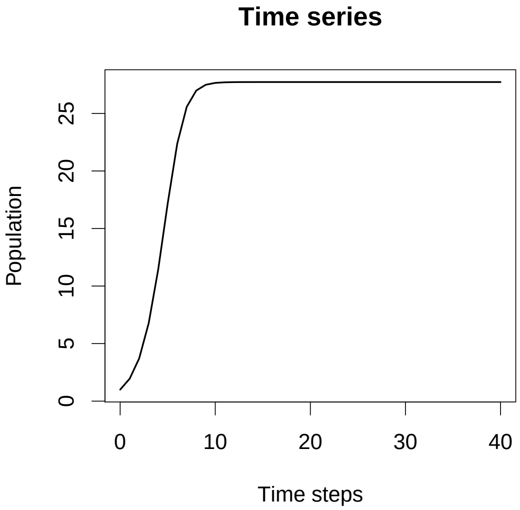

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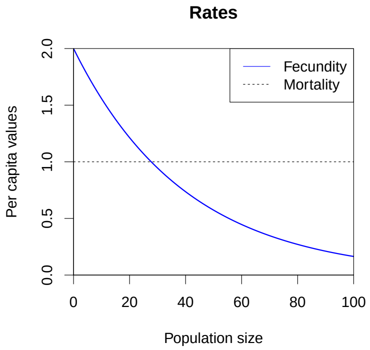
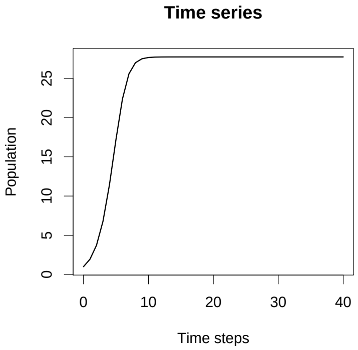
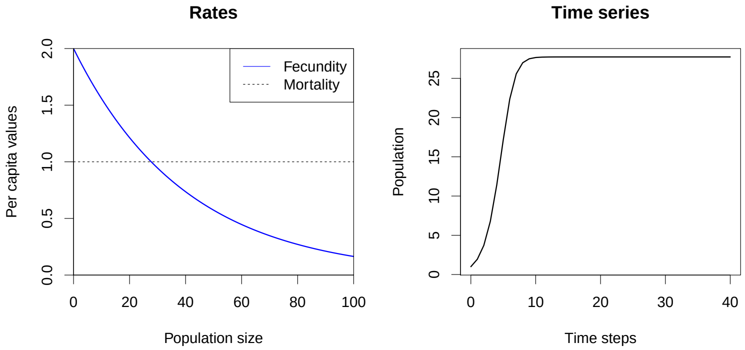


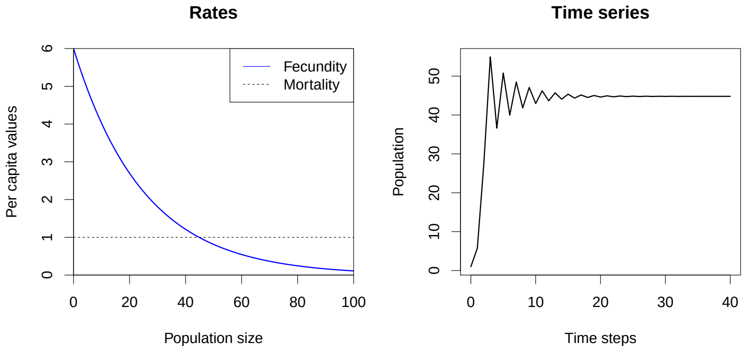


What dynamics do we get?

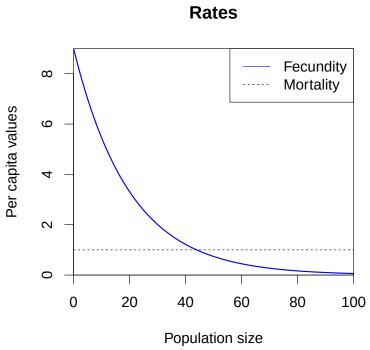
Simple dynamics (repeat)



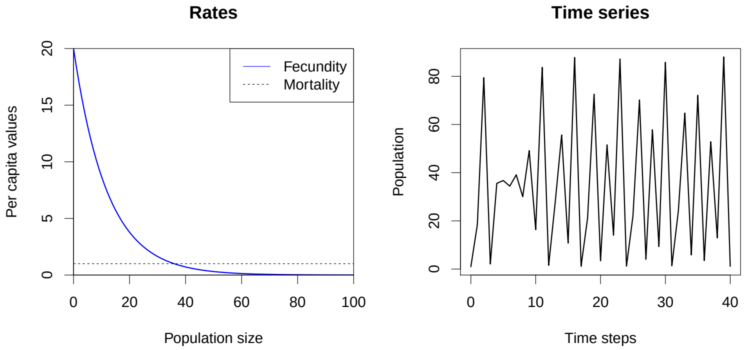
Damped oscillations



Persistent oscillations



Lots of other behaviours



Outline

Interpreting complex behaviour

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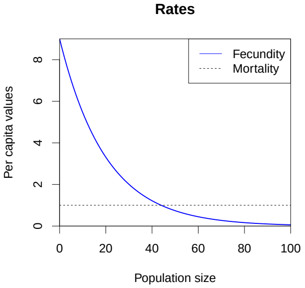

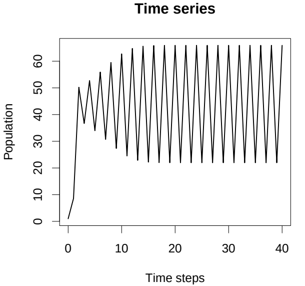

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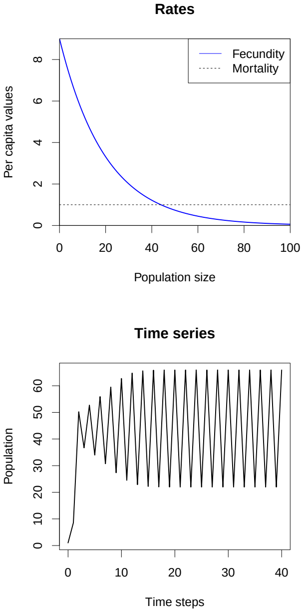

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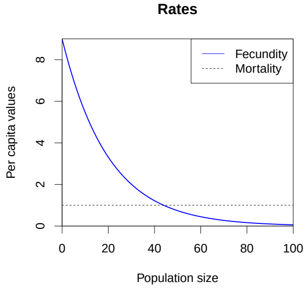

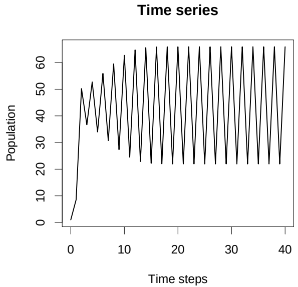

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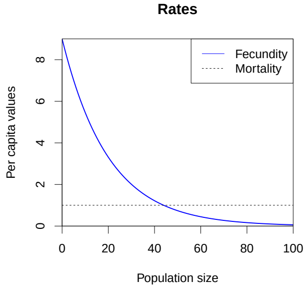
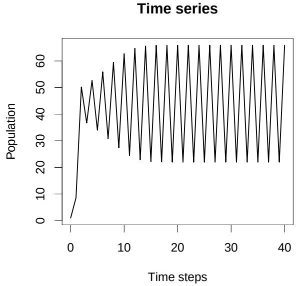
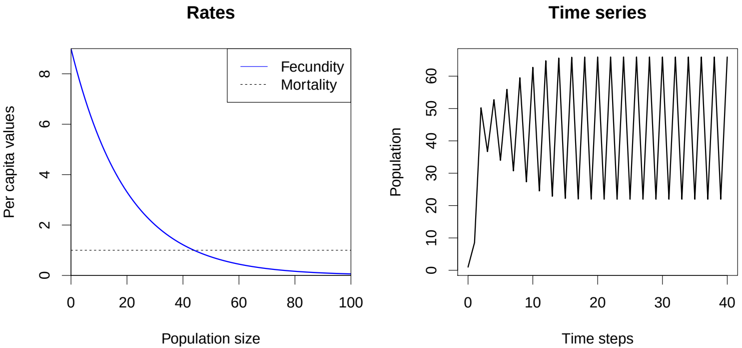




Interpreting complex behaviour (repeat)



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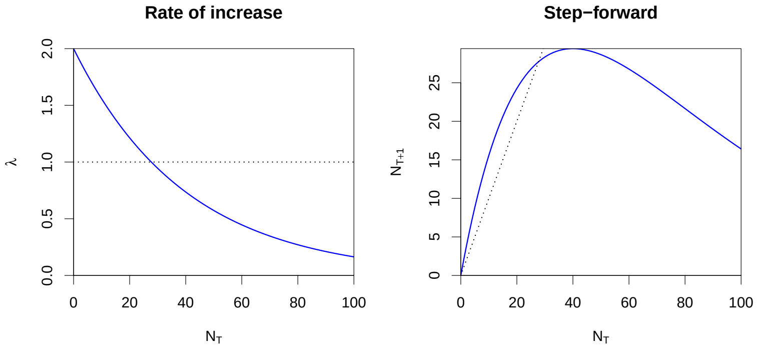

Turnover

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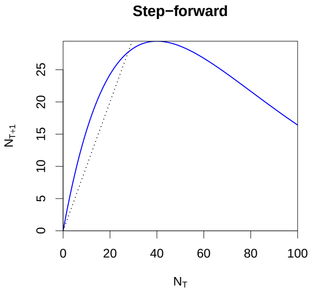

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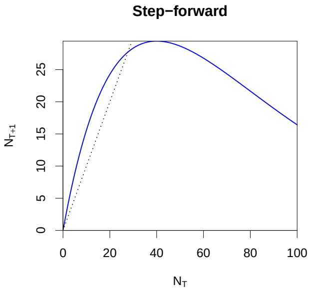

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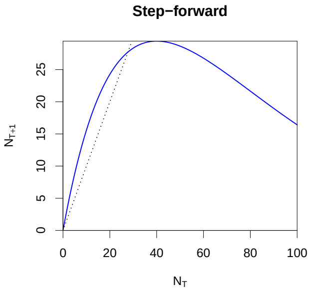

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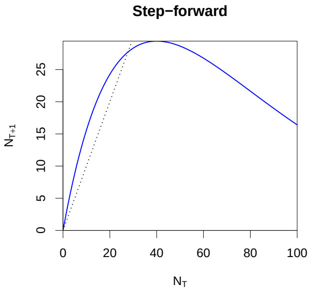

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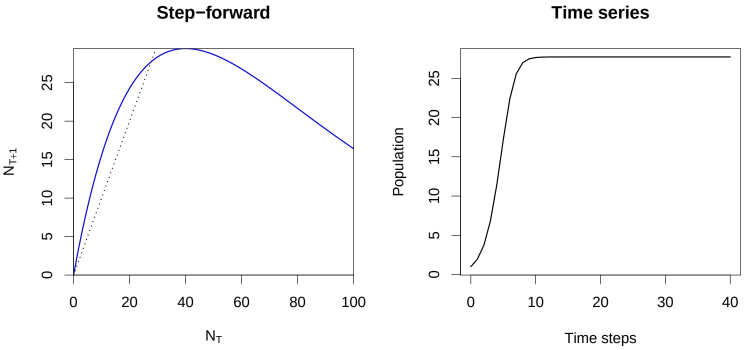
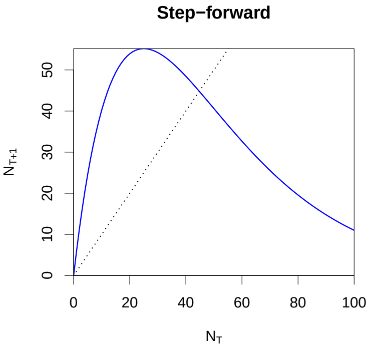


Damped oscillations



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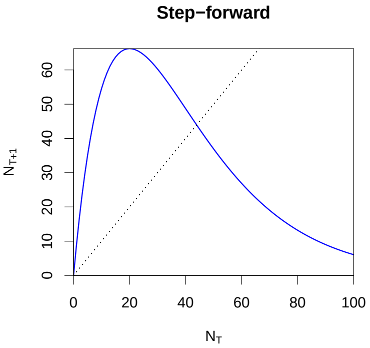

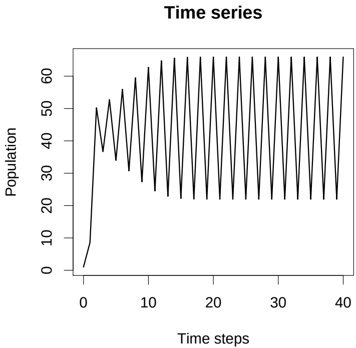

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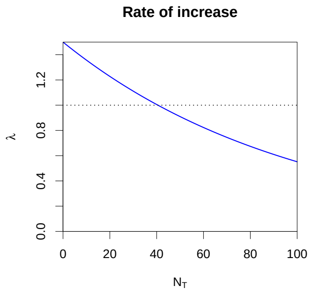

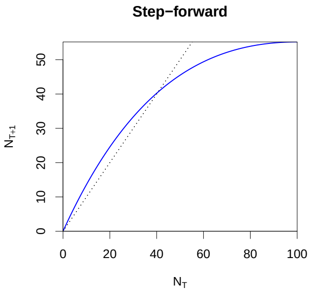

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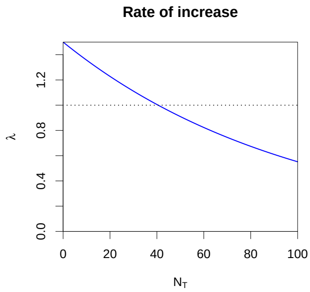

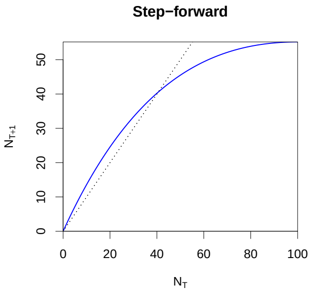

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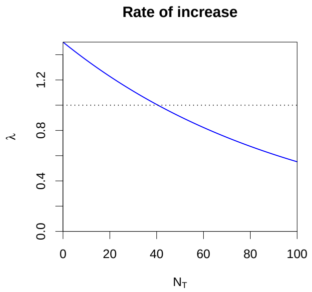

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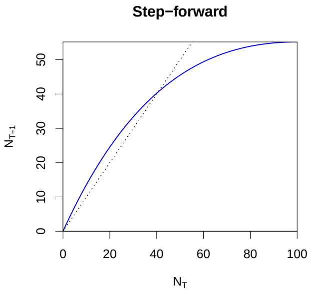

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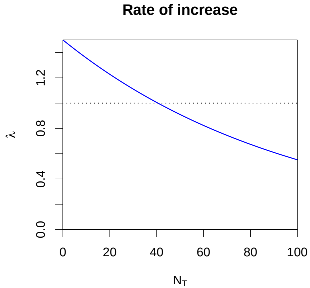

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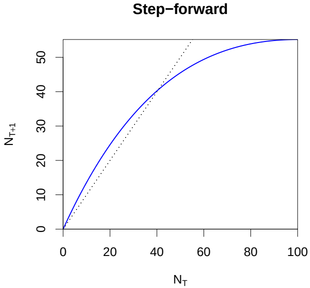
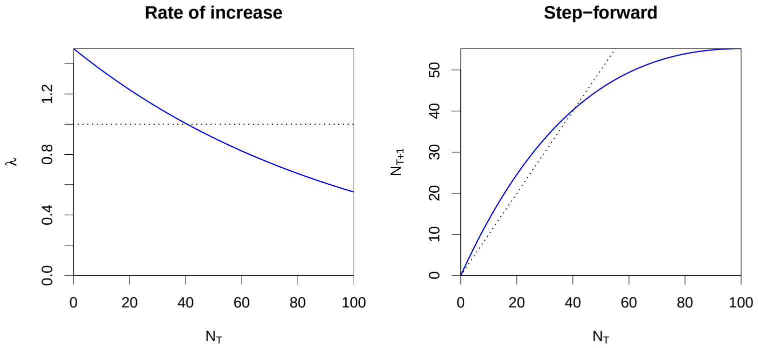


Contest regulation



Songbirds

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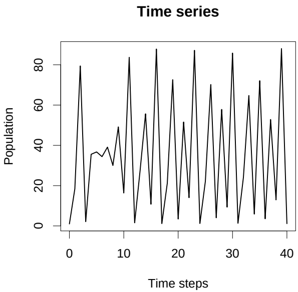

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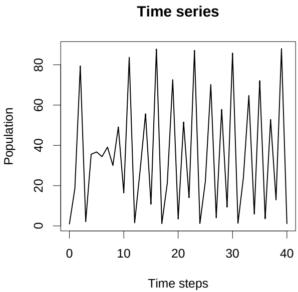

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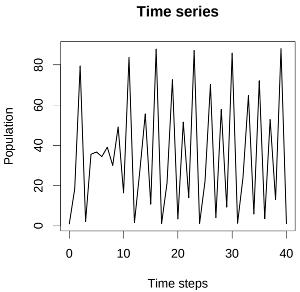

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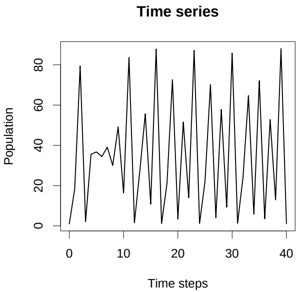

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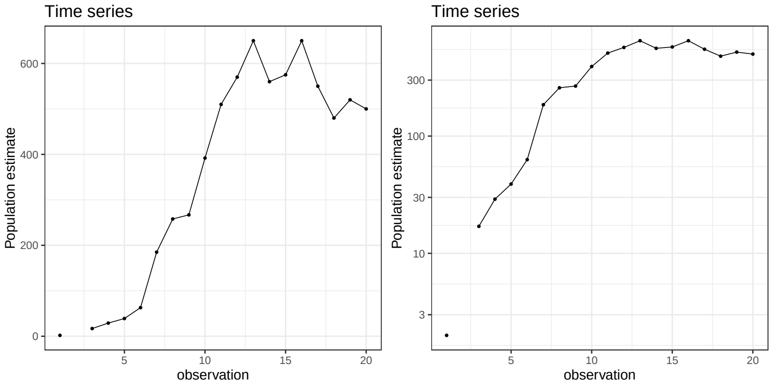
Paramecia (repeat)



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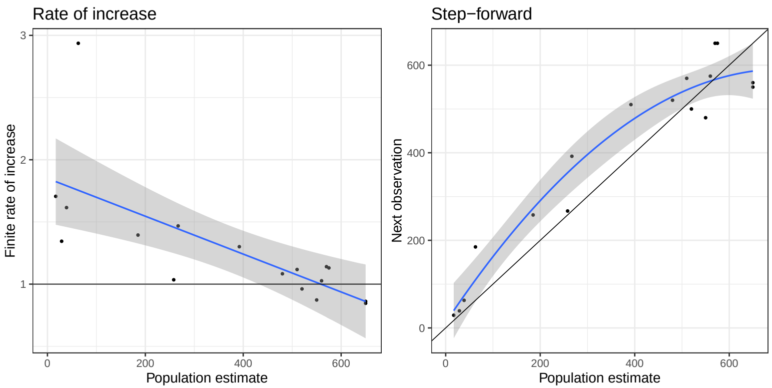
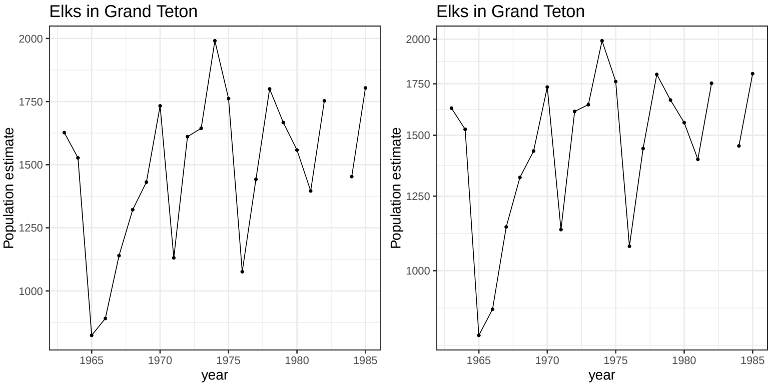


Rate of increase
Finite rate of increase
3
2
1
0
200
400
600
Population estimate
Step−forward
Next observation
600
400
200
0
0
200
400
600
Population estimate

Elk (repeat)



Elks in Grand Teton
Population estimate
2000
1750
1500
1250
1000
1965
1970
1975
1980
1985
year
Elks in Grand Teton
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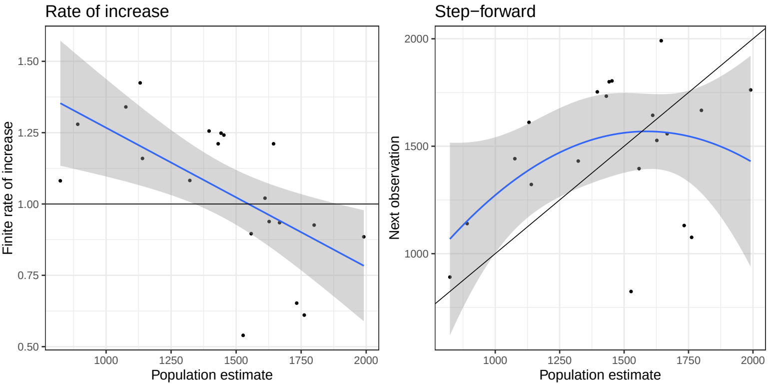
2000
1750
1500
1250
1000
1965
1970
1975
1980
1985
year

Elk


Rate of increase
Finite rate of increase
1.50
1.25
1.00
0.75
0.50
1000
1250
1500
1750
2000
Population estimate
Step−forward
Next observation
2000
1500
1000
1000
1250
1500
1750
2000
Population estimate

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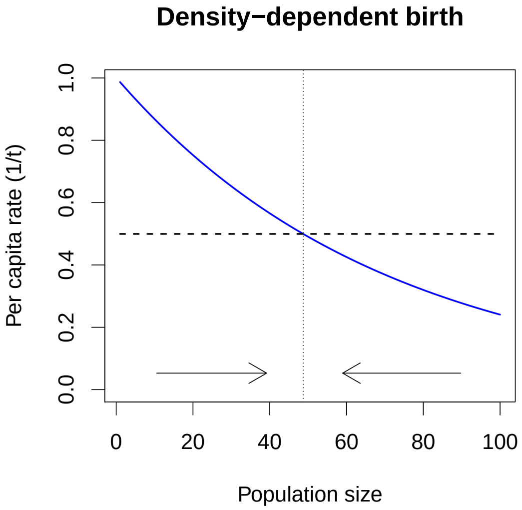
Arrows with time delay



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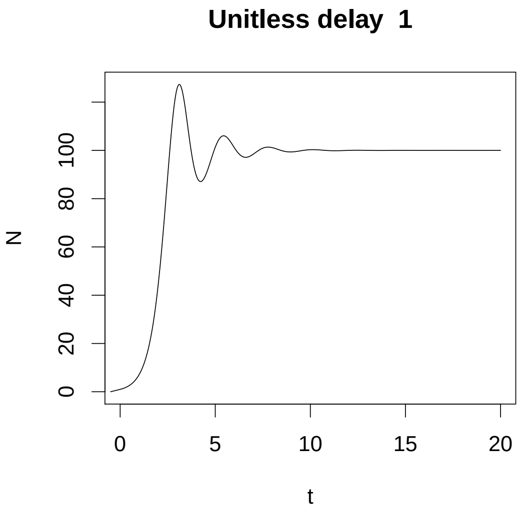

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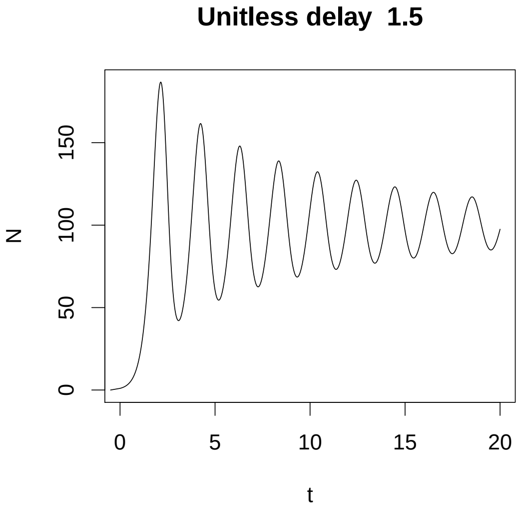

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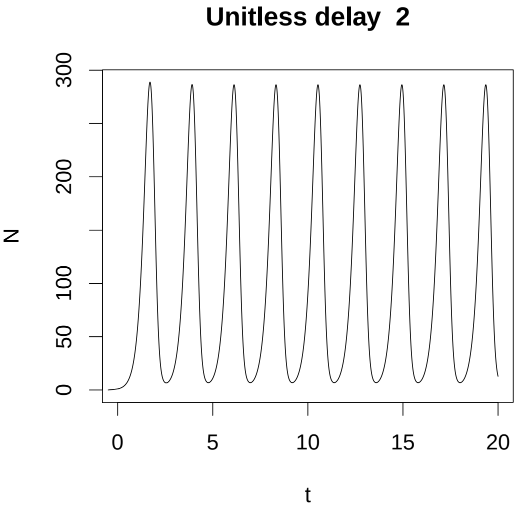

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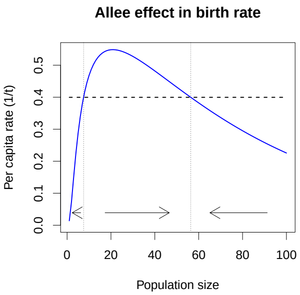

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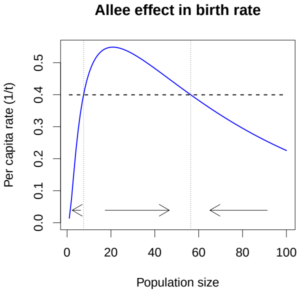

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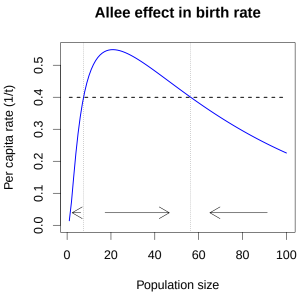

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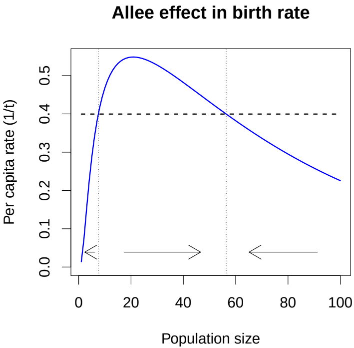

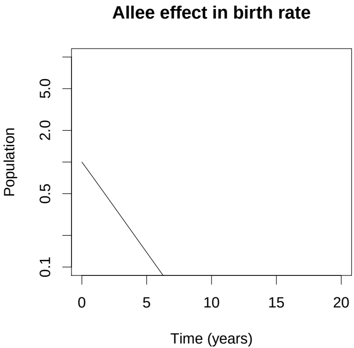

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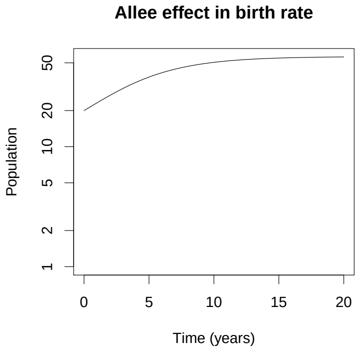

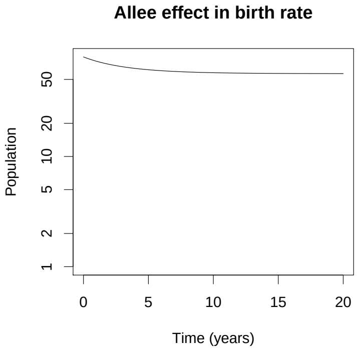

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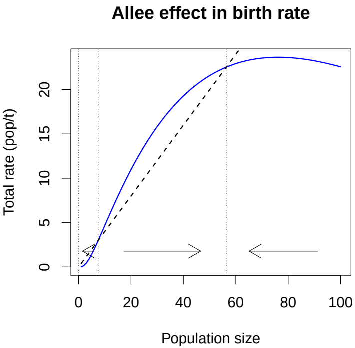

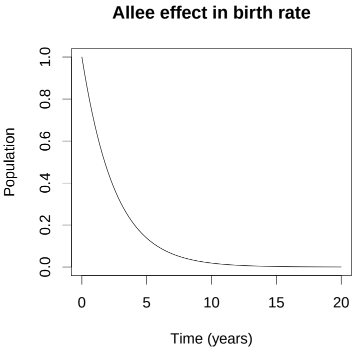

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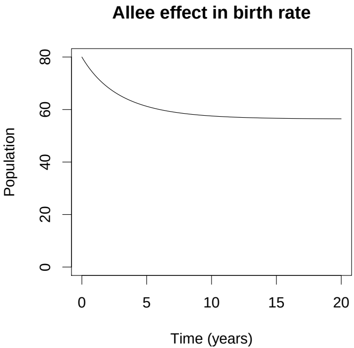
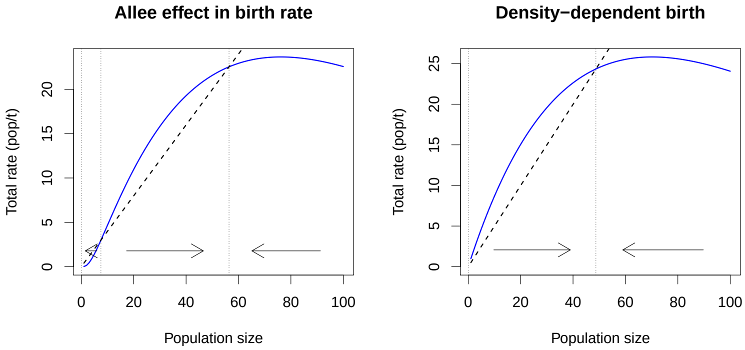


Population comparison (repeat)



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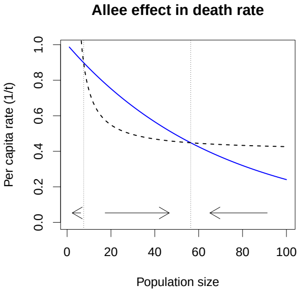

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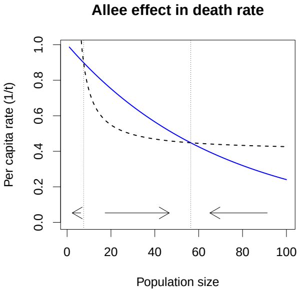

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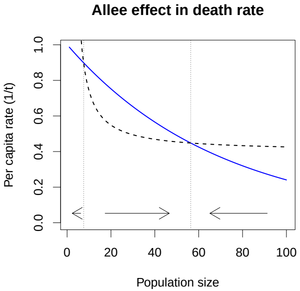

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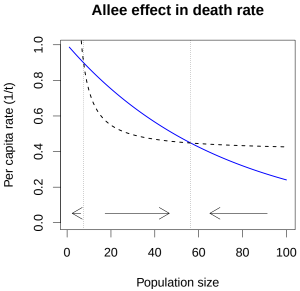

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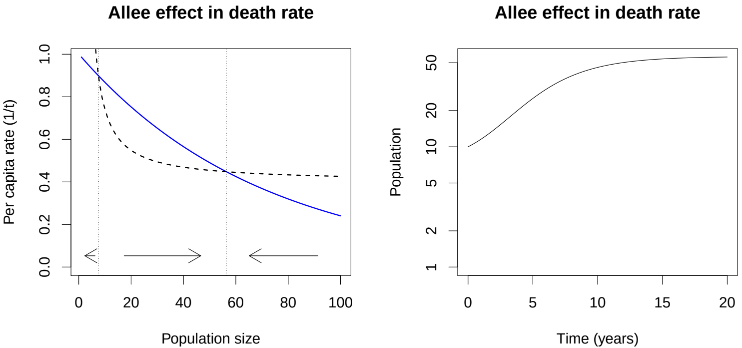

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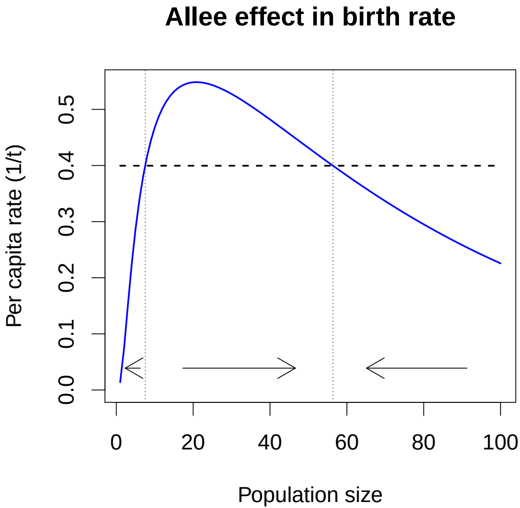
$\mathcal{R}_0$ and $\mathcal{R}_{max}$ (repeat)



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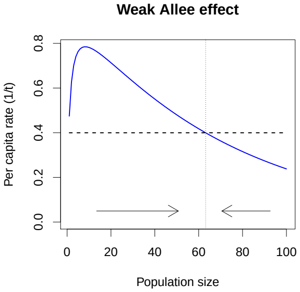

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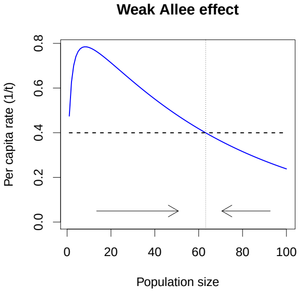

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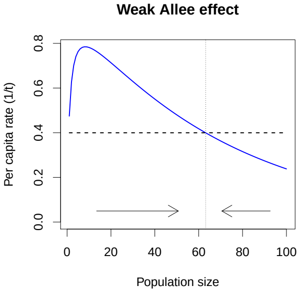

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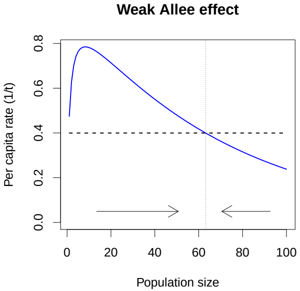

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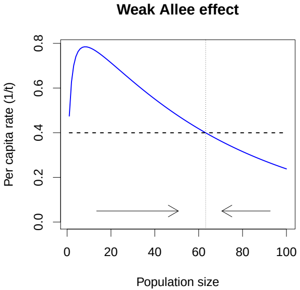

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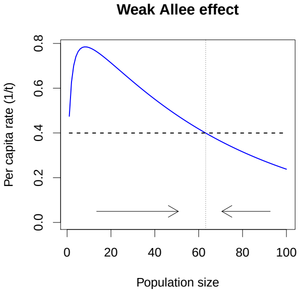
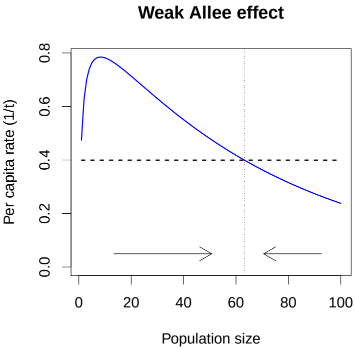
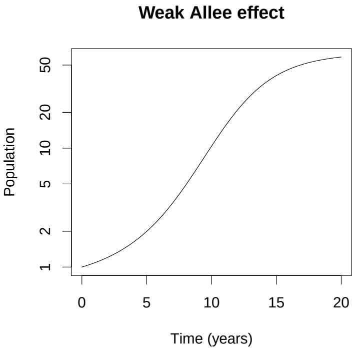


Individual perspective





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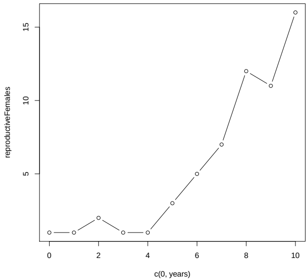

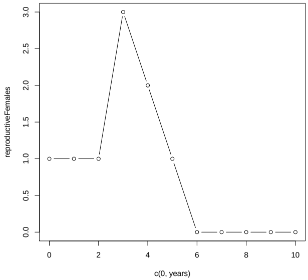

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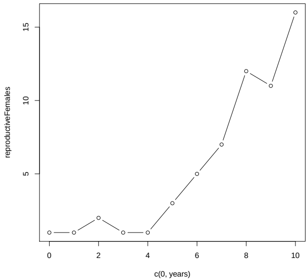

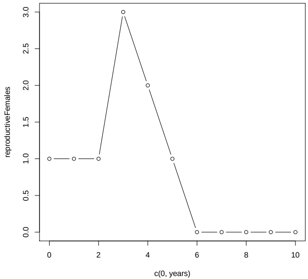

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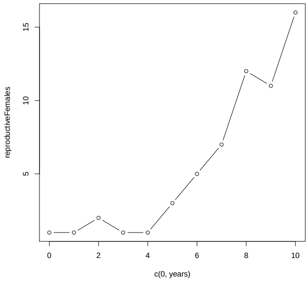

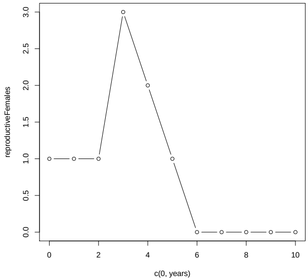

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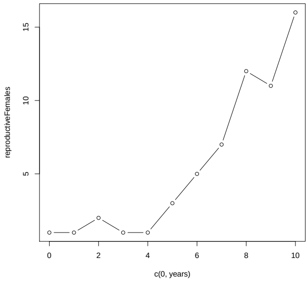

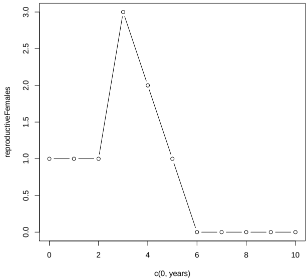

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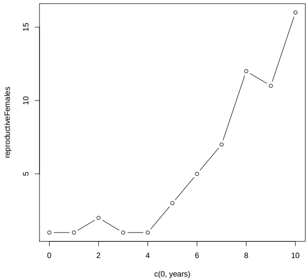

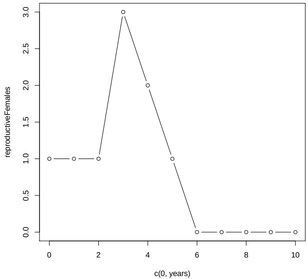

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