

# UNIT 8: Infectious disease

# Outline

## Introduction

## Rate of spread

## Single-epidemic model

### Epidemic size

## Recurrent epidemic models

### Dynamics

## Reproductive numbers and risk

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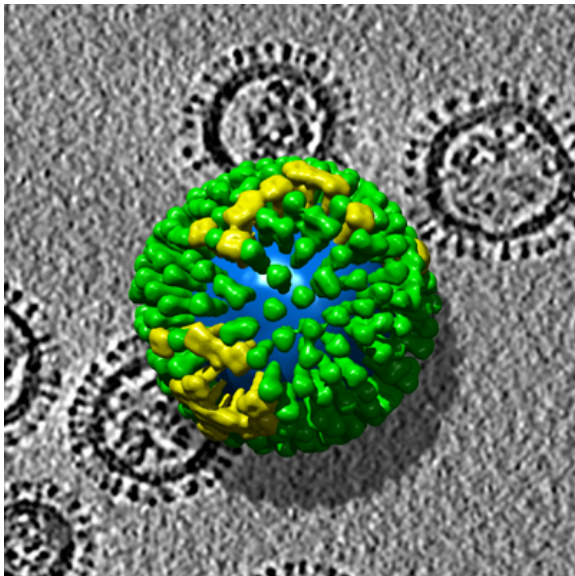
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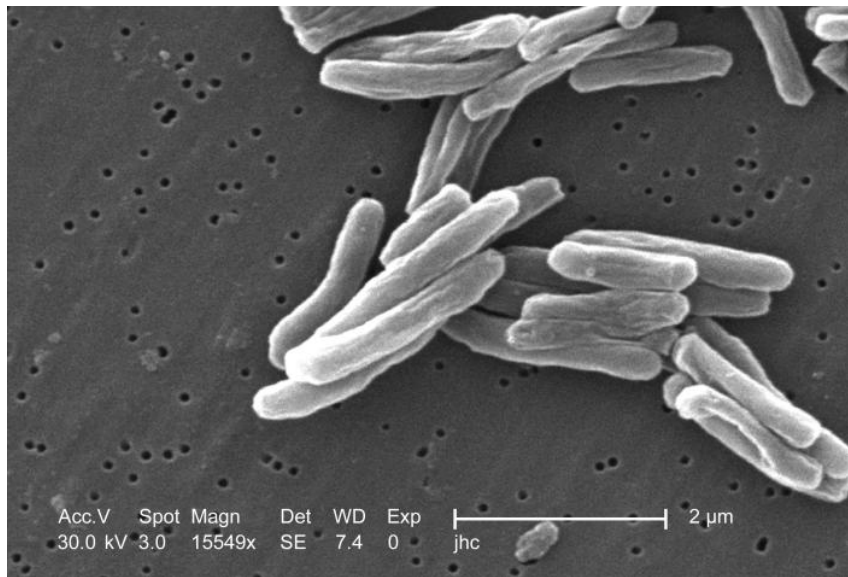
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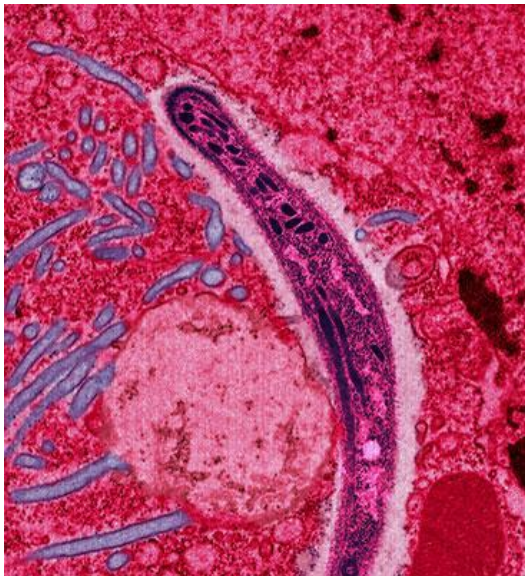
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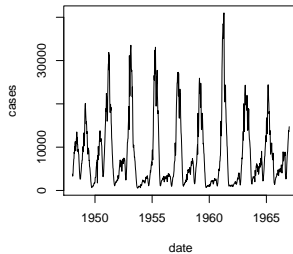
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**Measles reports from England and Wales**

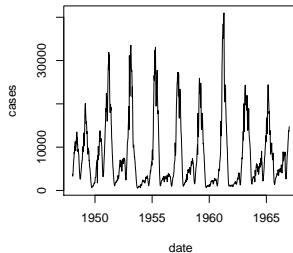


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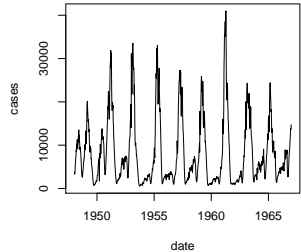


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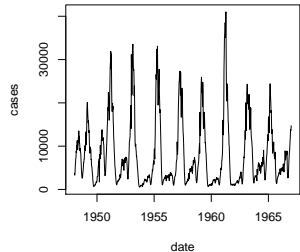


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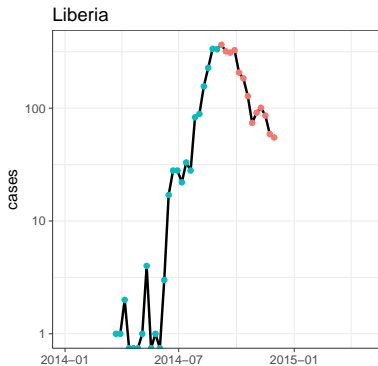
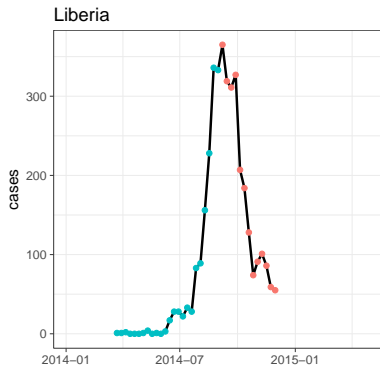
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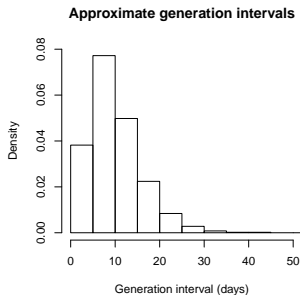
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# Example: the West African Ebola epidemic



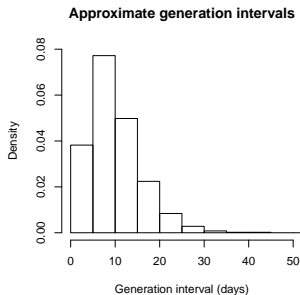
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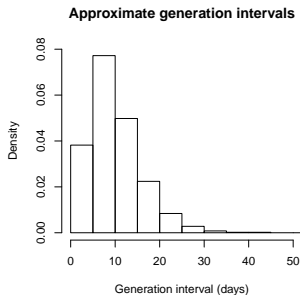
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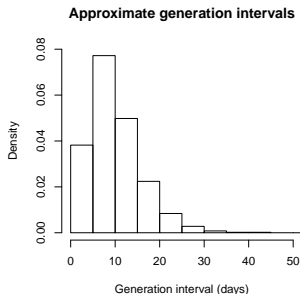
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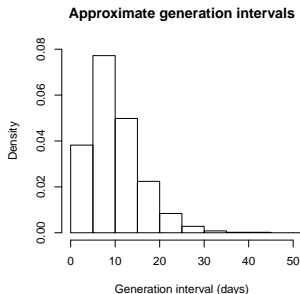
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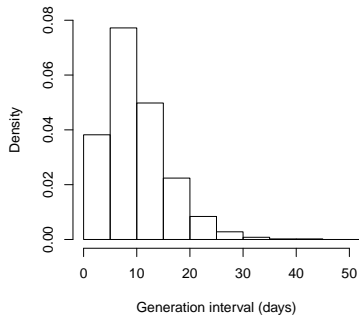
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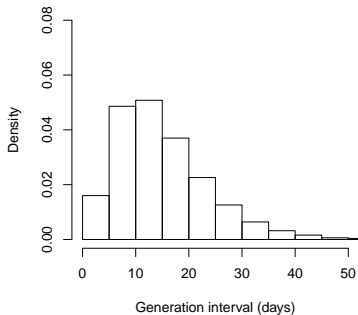


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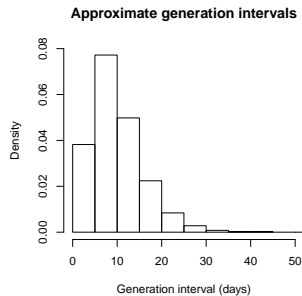


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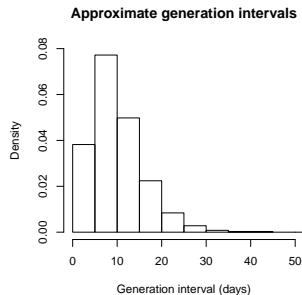
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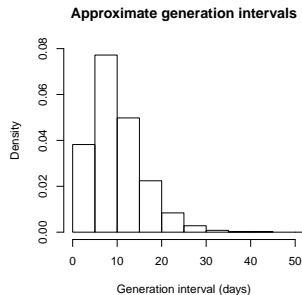
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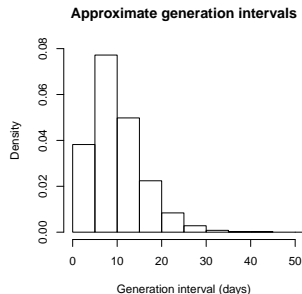
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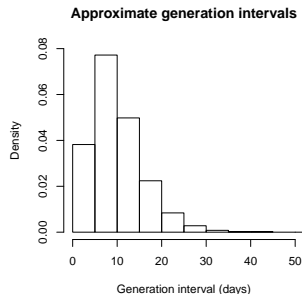
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# Fighting Ebola



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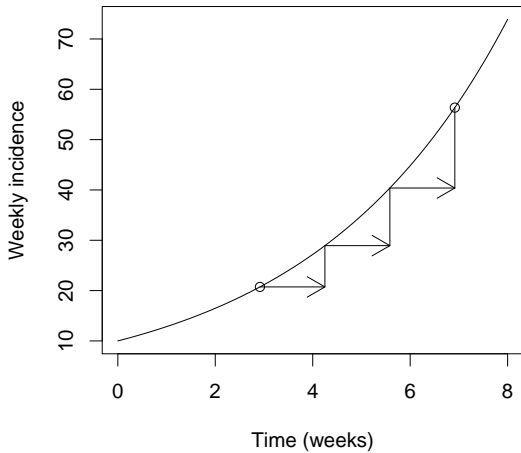
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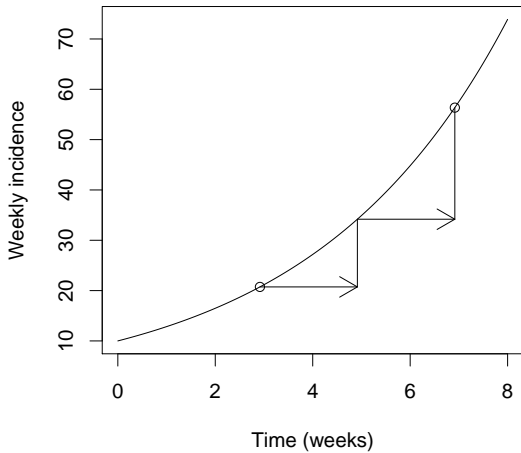
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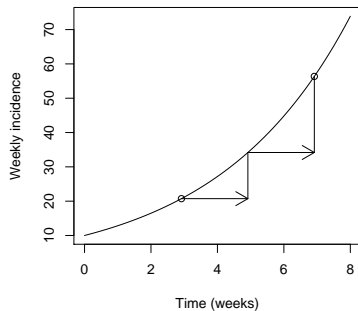
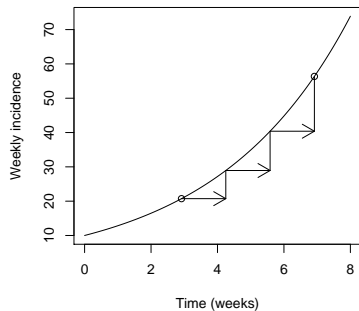
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- ▶ If we know generation speed, then a faster epidemic speed means:

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Single-epidemic model

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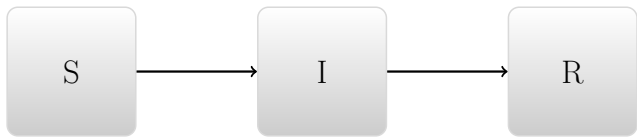
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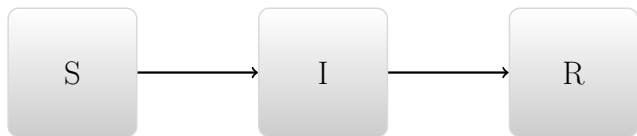


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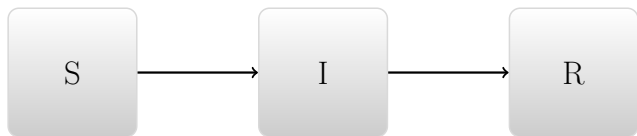
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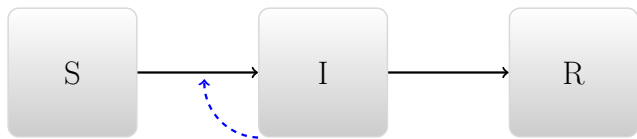
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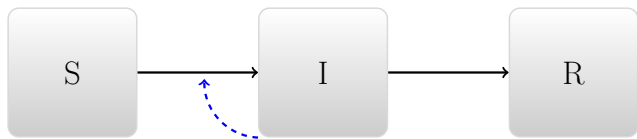
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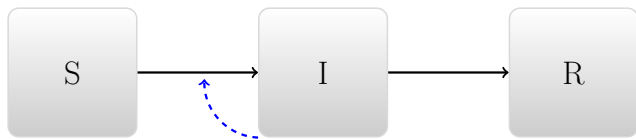
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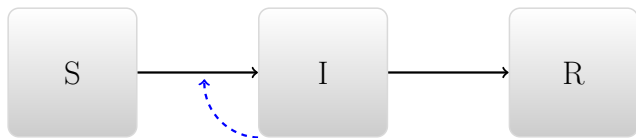
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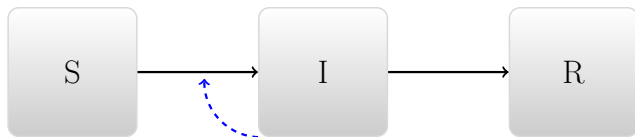
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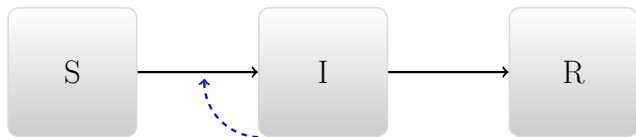
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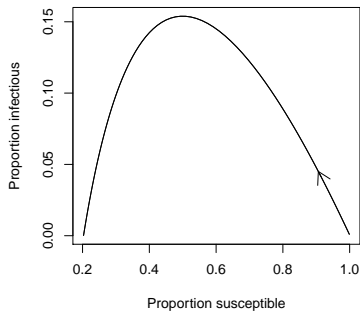
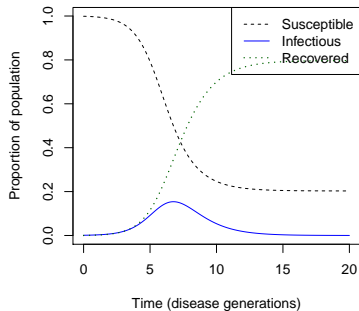
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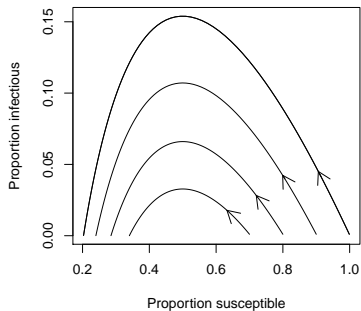
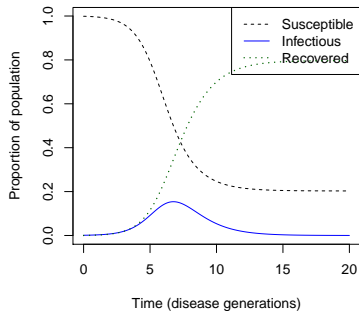
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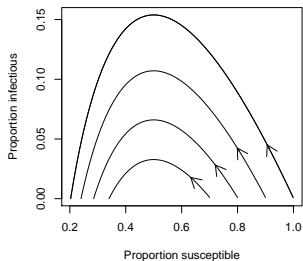
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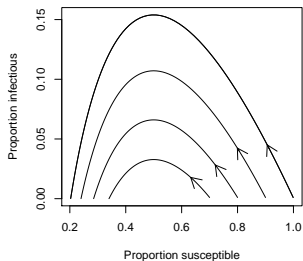
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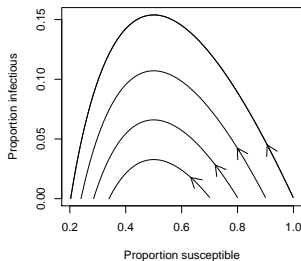
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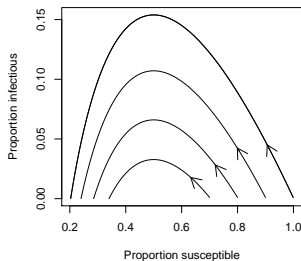


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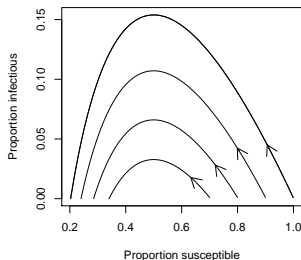
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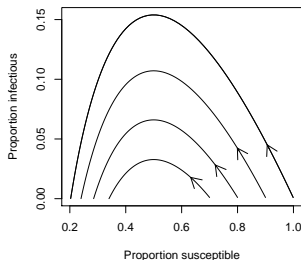
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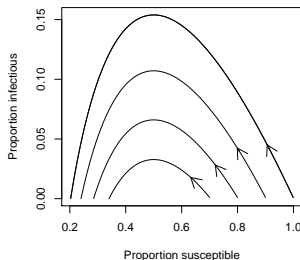
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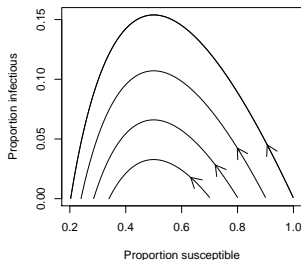
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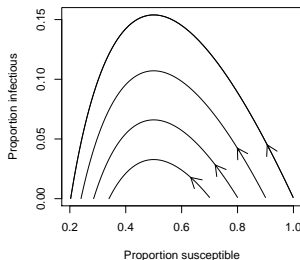
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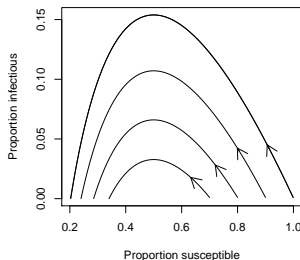
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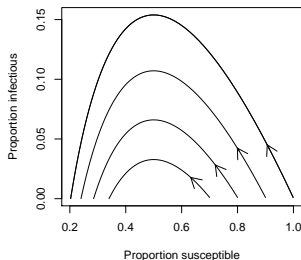
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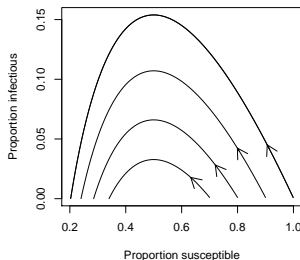
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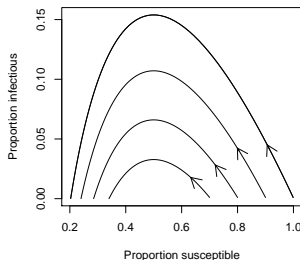
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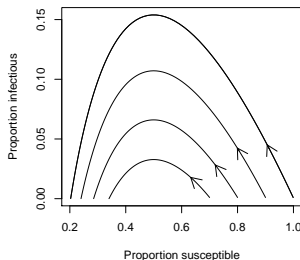
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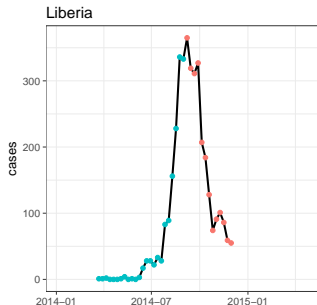
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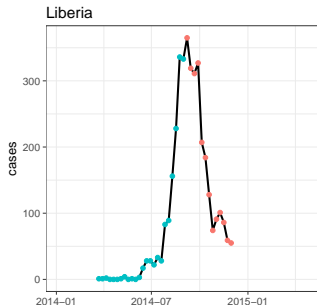
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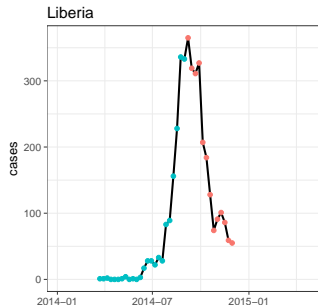
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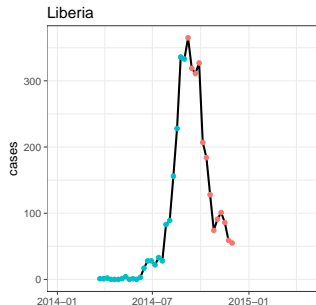
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Rate of spread

Single-epidemic model

Epidemic size

Recurrent epidemic models

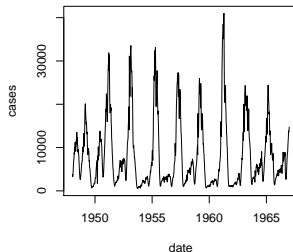
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Measles reports from England and Wales

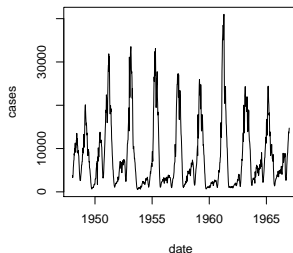


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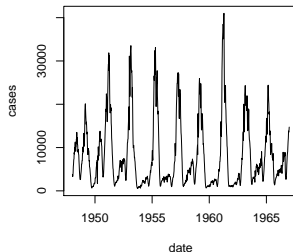
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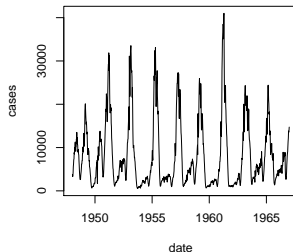
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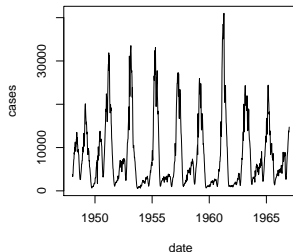
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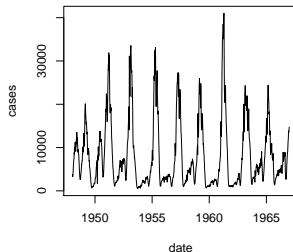




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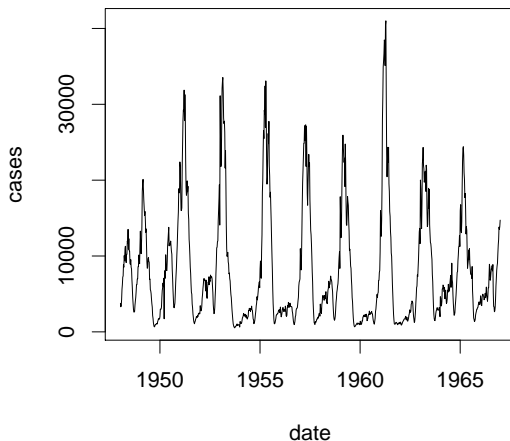
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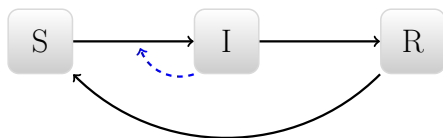


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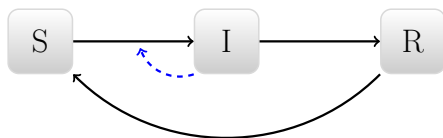
## Measles reports from England and Wales



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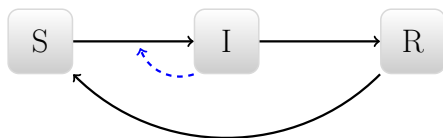


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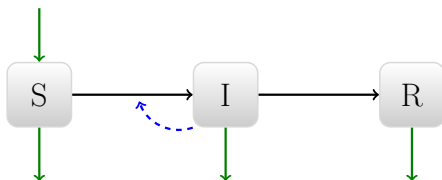
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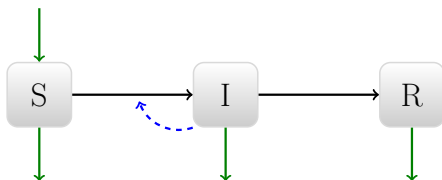


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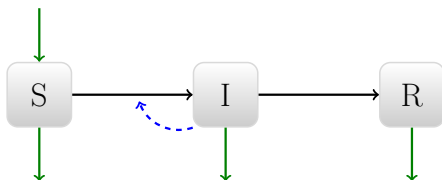


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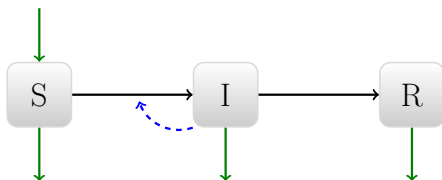


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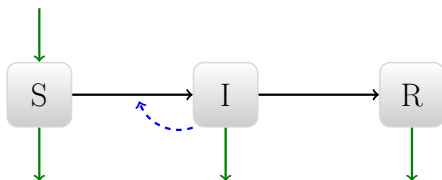


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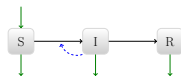


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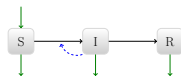
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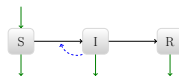
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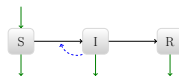
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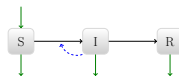
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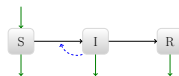


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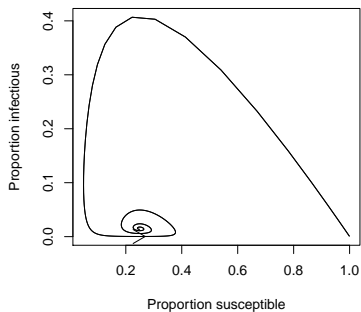
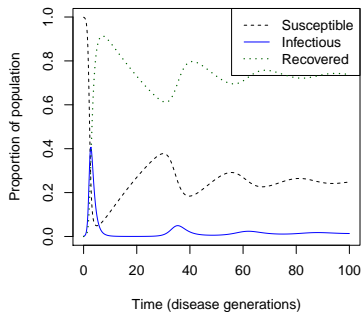
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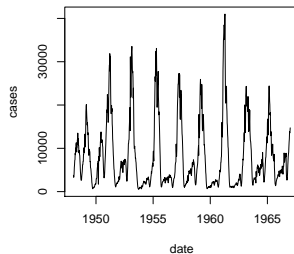
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**Measles reports from England and Wales**

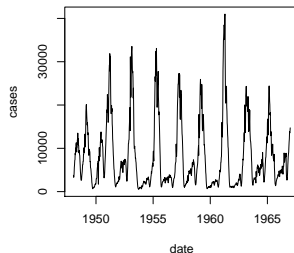


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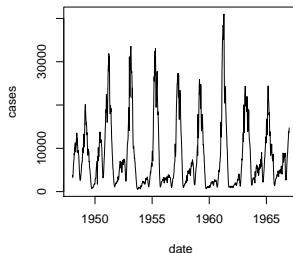


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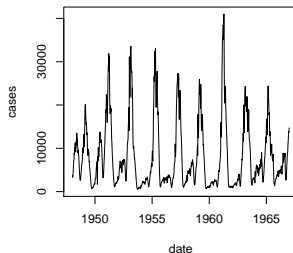
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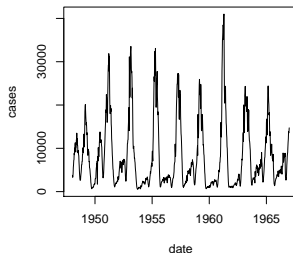
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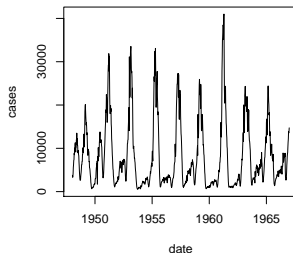


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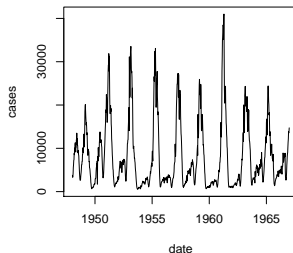




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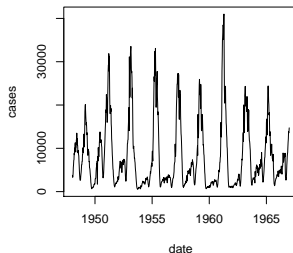
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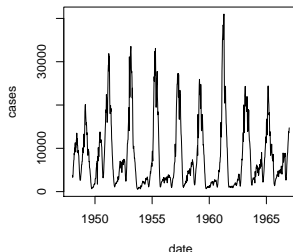


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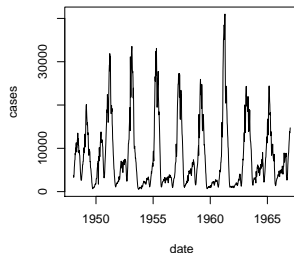


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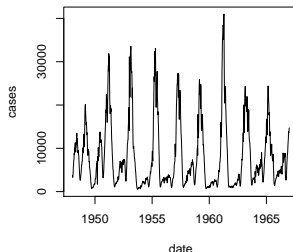
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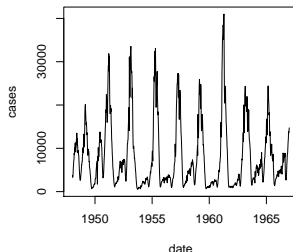


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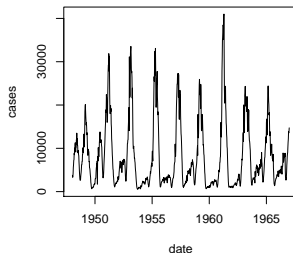
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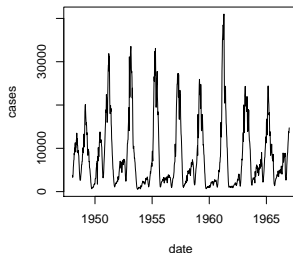
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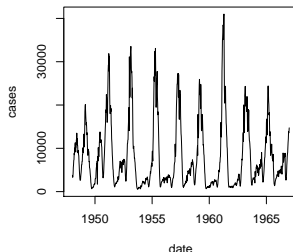




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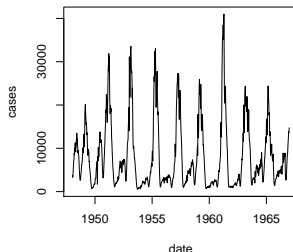
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# Outline

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Rate of spread

Single-epidemic model

Epidemic size

Recurrent epidemic models

Dynamics

Reproductive numbers and risk

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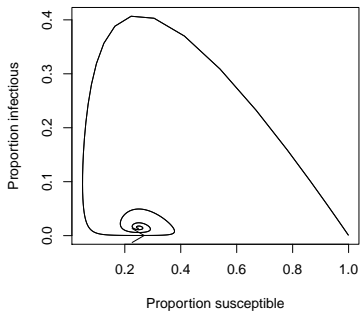
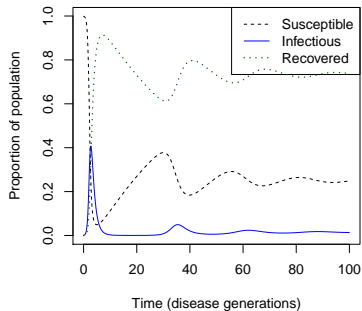
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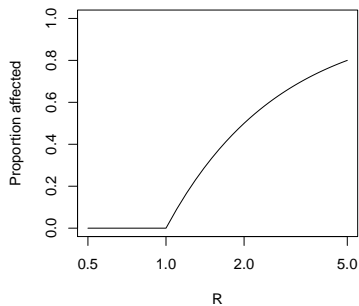
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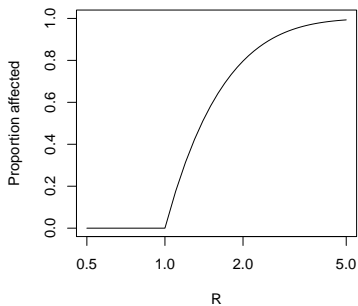


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**Equilibrium**



**Single epidemic**



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- ▶ Polio has an  $\mathcal{R}_0$  of about 5.
- ▶ Poll: What proportion of the population should be vaccinated to eliminate polio?
  - ▶ \* At least  $1 - 1/5 = 80\%$
- ▶ Measles has an  $\mathcal{R}_0$  of about 20. What proportion of the population should be vaccinated to eliminate measles?
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