UNIT 2: Linear population models

Outline

Outline

Dynamical models

Tools to link scales

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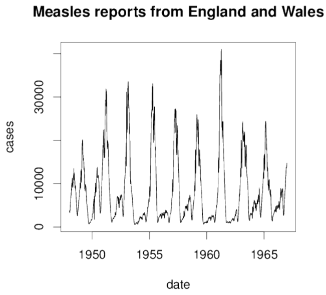

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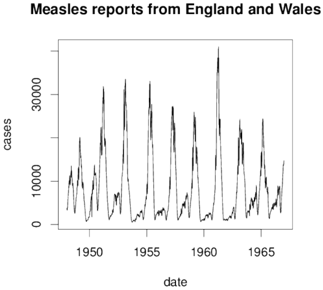

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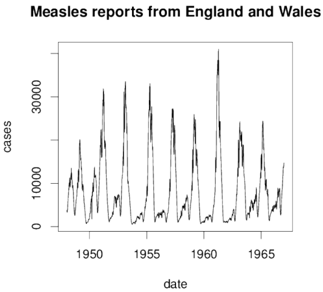

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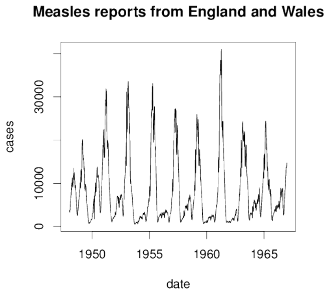

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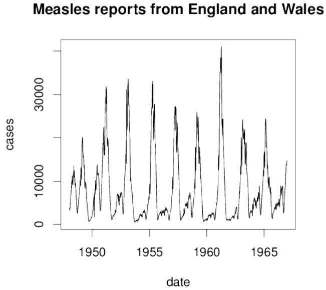

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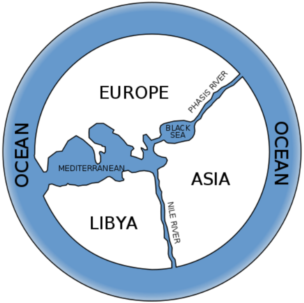

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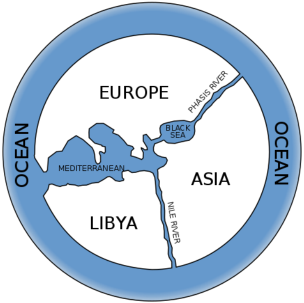

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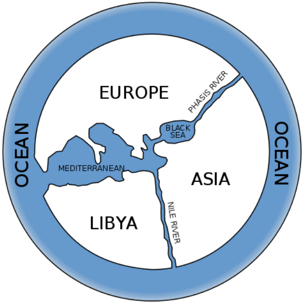

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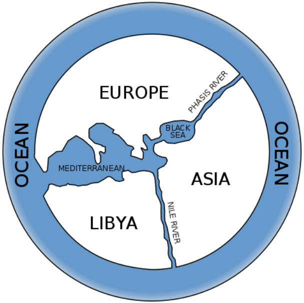

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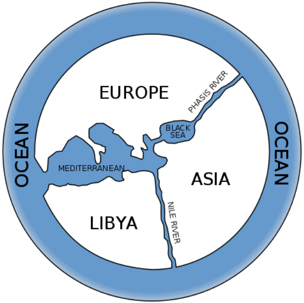

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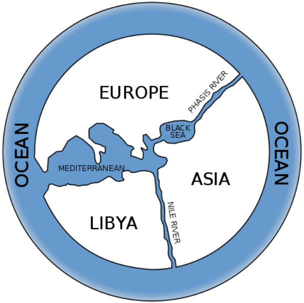

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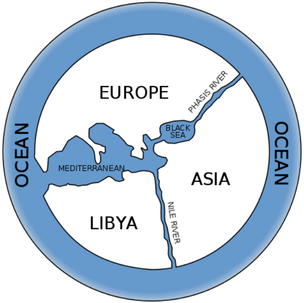

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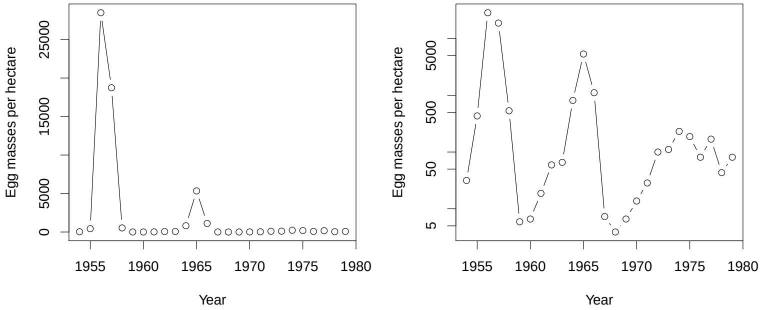

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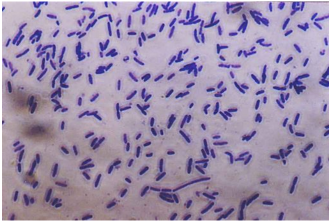

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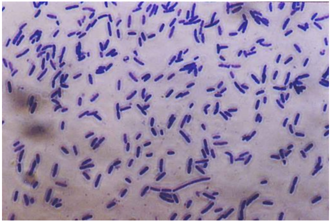

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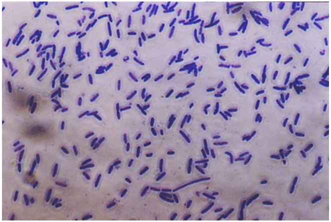

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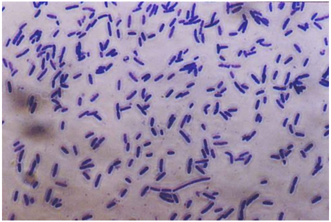

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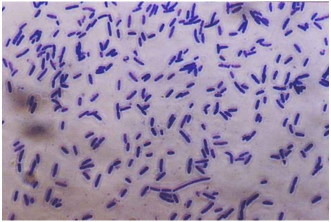

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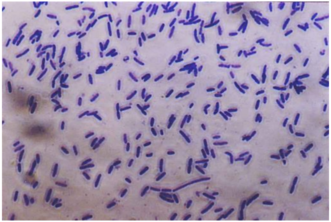

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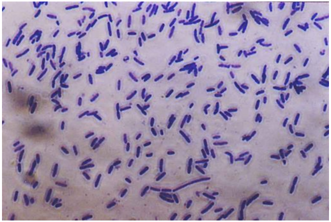

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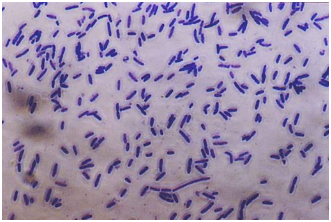

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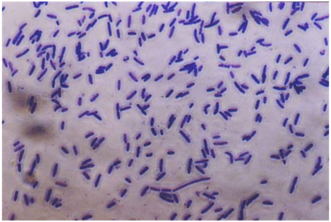

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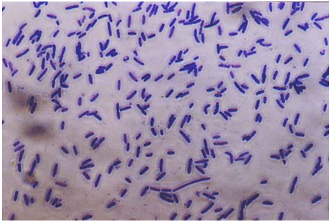

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  - Each individual creates (on average) f new individuals that will be counted at time step $T+1$
- How many individuals do we expect in the next time step?
  - * $N_{T+1} = (pN_T + fN_T) = (p+f)N_T$
- Diagram

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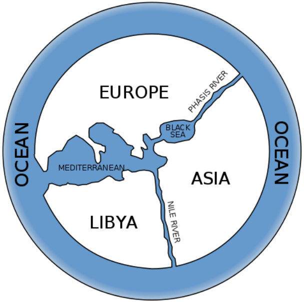

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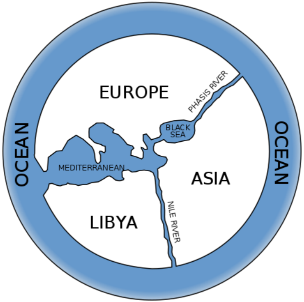

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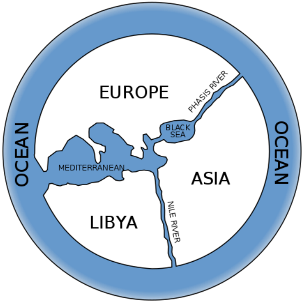

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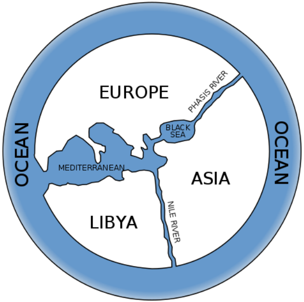

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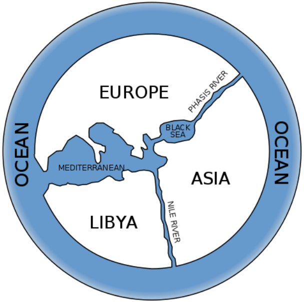

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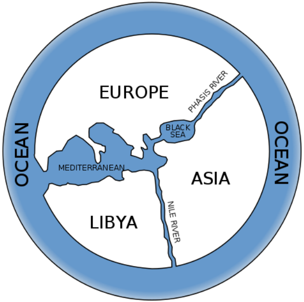

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6	2005-12-21	122 €	134 €	-12 €
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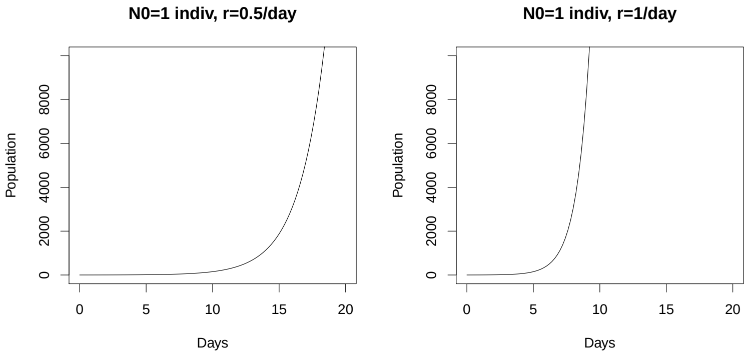
N0=1 indiv, r=0.5/day
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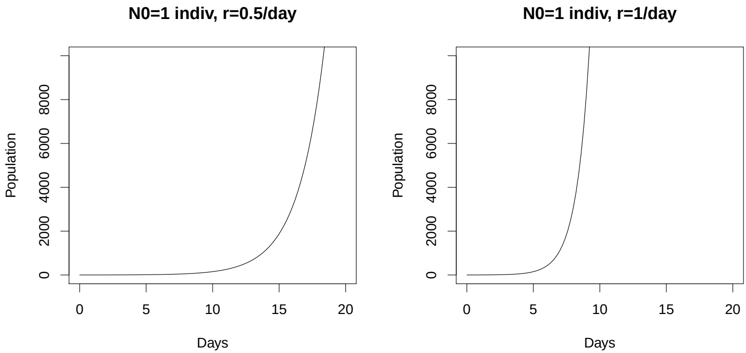
Scaling time in bacteria



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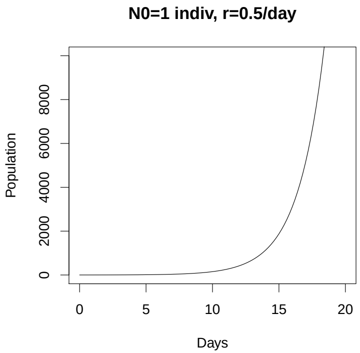
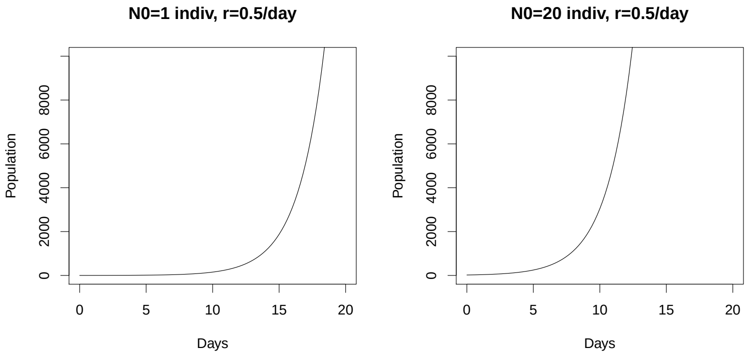


Scaling population



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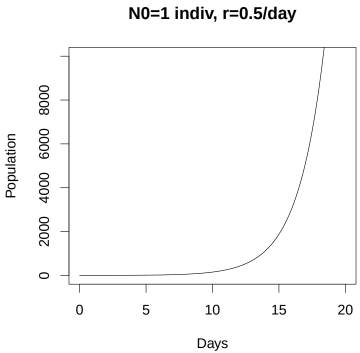

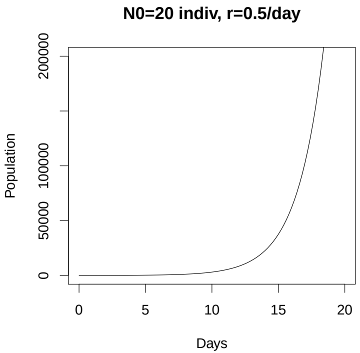

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