UNIT 3 Non-linear population models

Outline

Introduction

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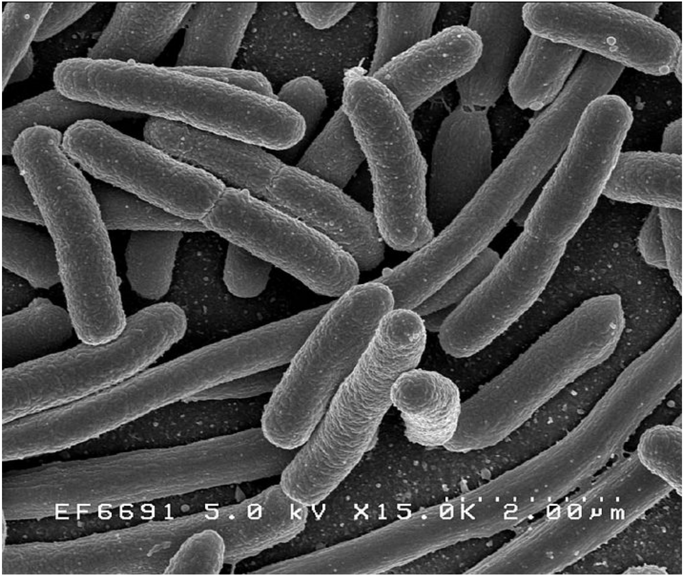
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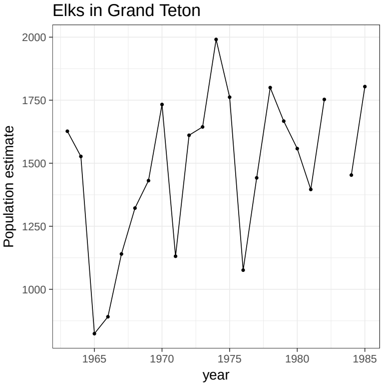
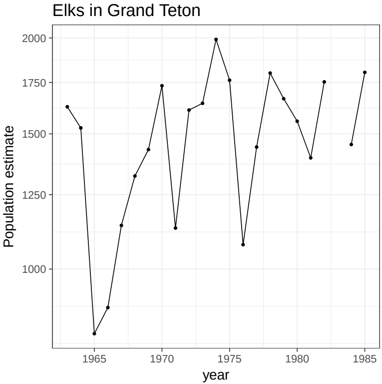
Elk



Elks in Grand Teton
2000
1750
1500
1250
1000
Population estimate
1965
1970
1975
1980
1985
year



Elks in Grand Teton
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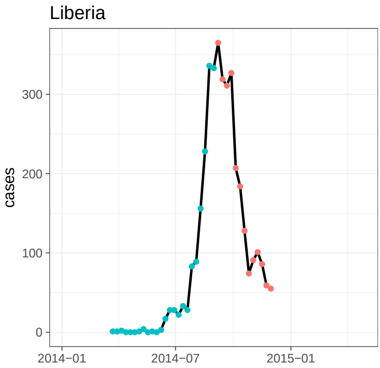
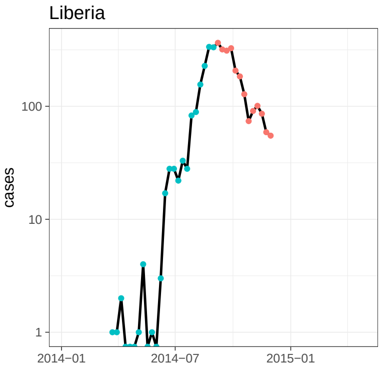
Ebola


Liberia
cases
0
100
200
300
2014−01
2014−07
2015−01


Liberia
cases
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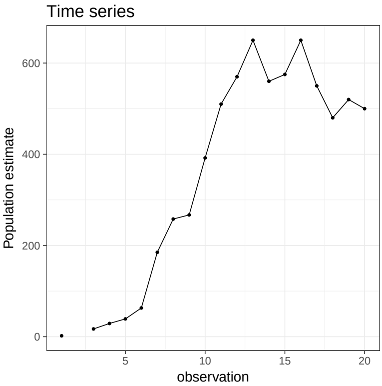
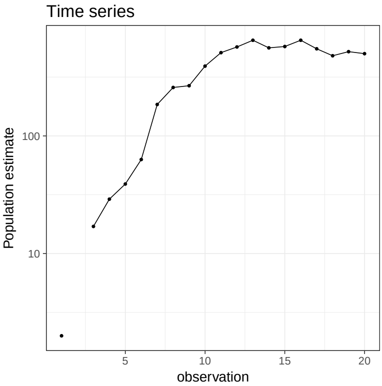
2014−01
2014−07
2015−01

Paramecia


Time series
Population estimate
600
400
200
0
5
10
15
20
observation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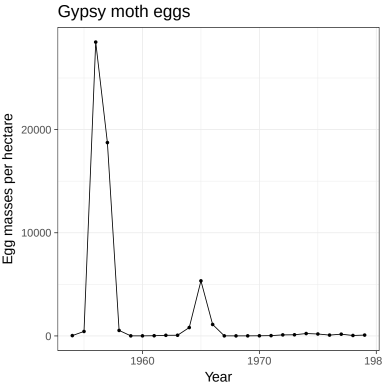
Gypsy moths



Gypsy moth eggs
Egg masses per hectare
20000
10000
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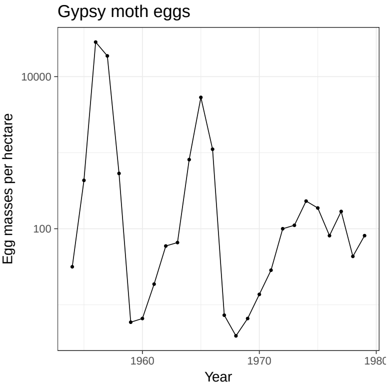
1960
1970
198
Year



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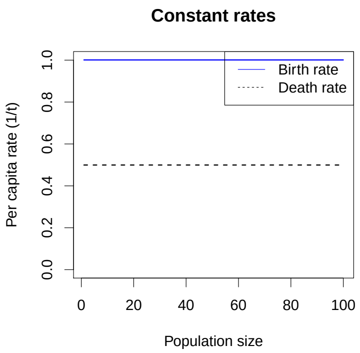

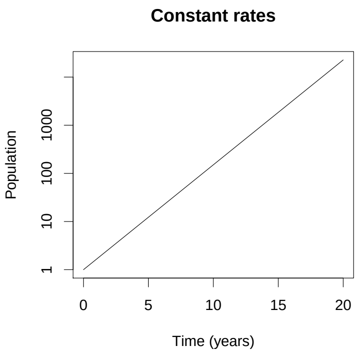

Individual perspective

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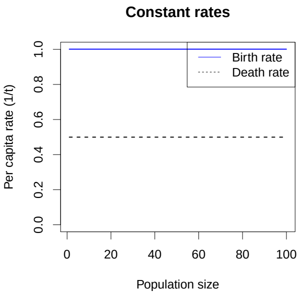

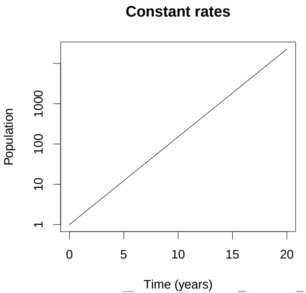

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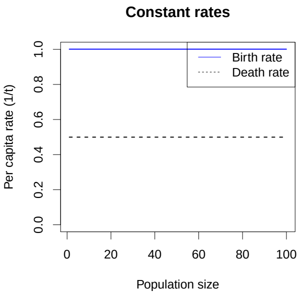

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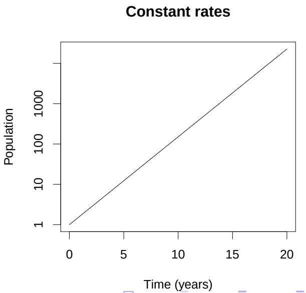

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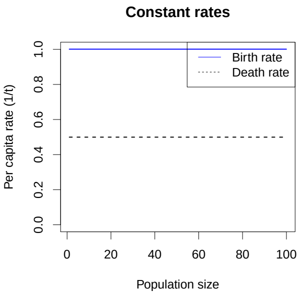

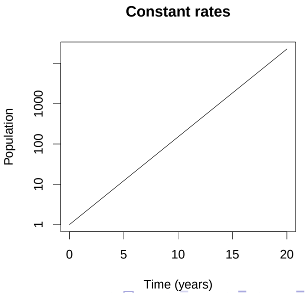

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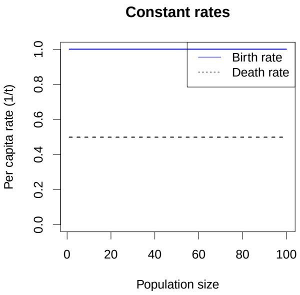

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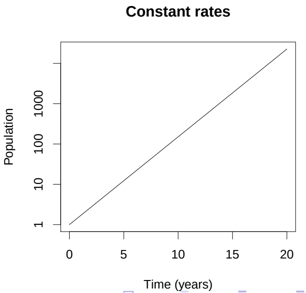

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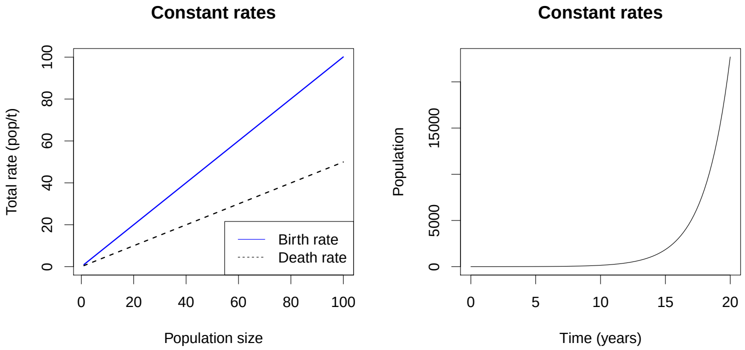

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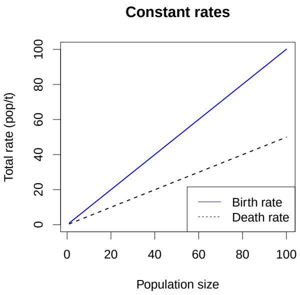

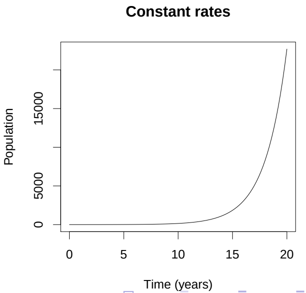

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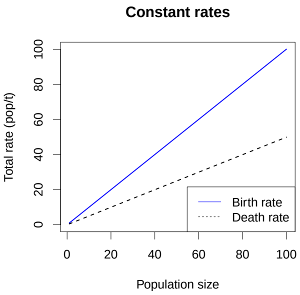

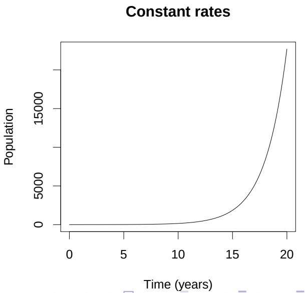

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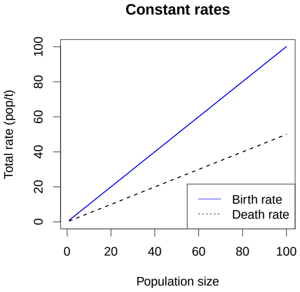

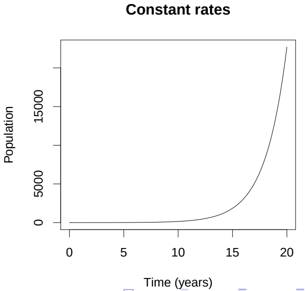

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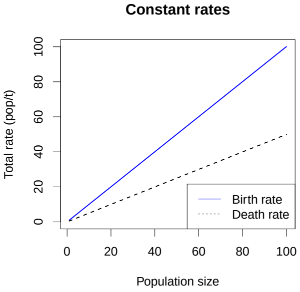

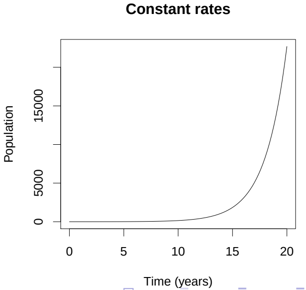

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Recruitment

- **Recruitment** is when an organism moves from one life stage to another:
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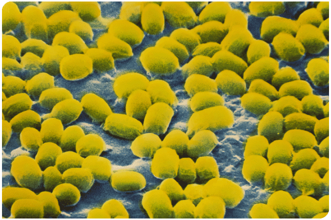

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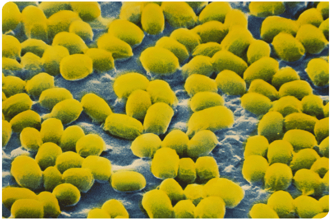

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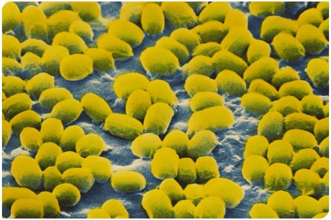

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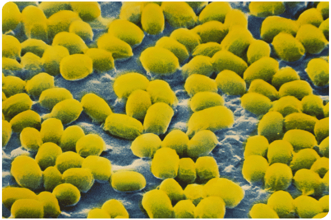

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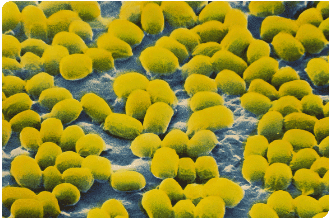

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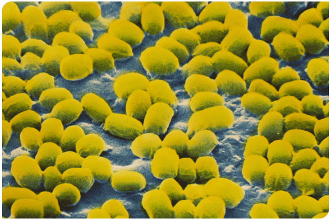

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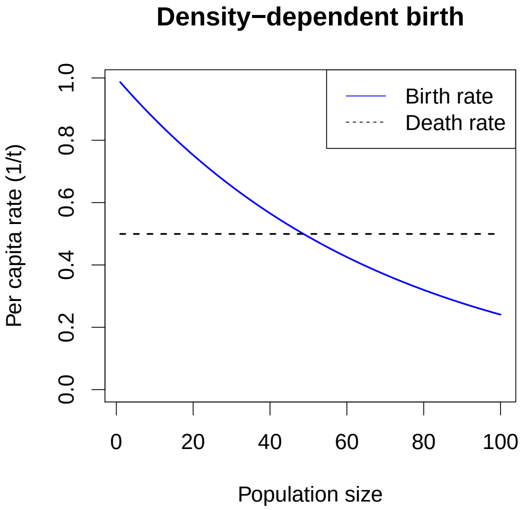

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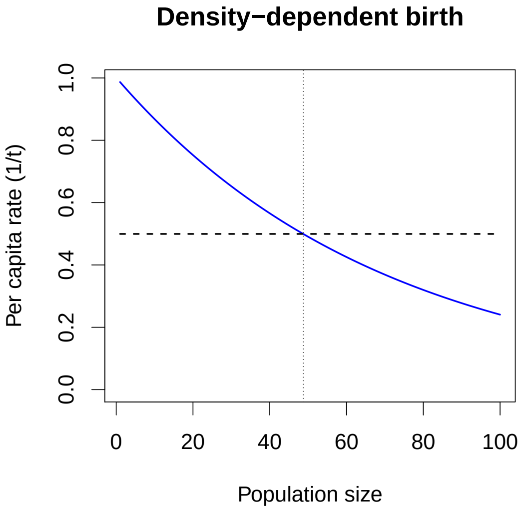

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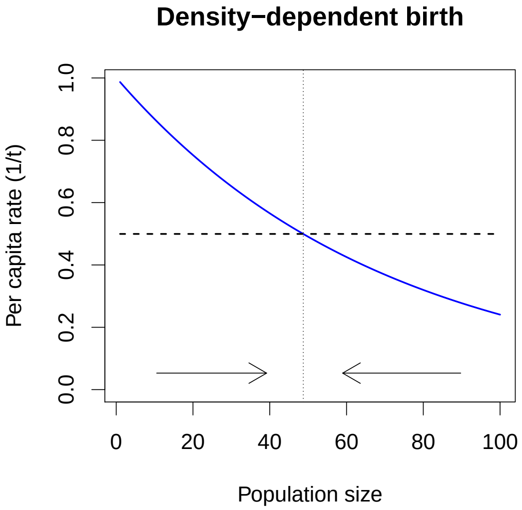

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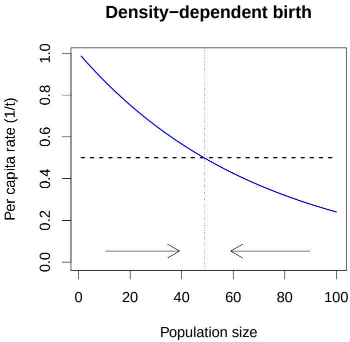

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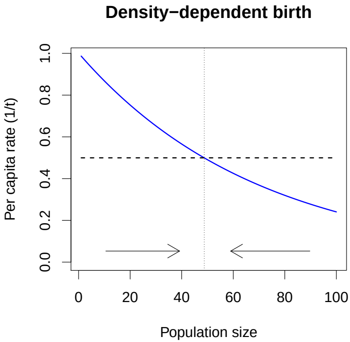

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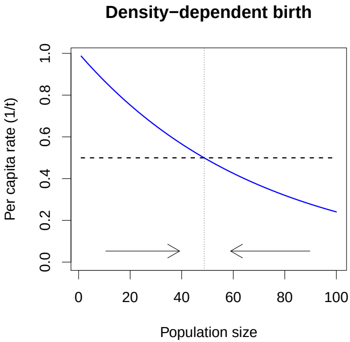

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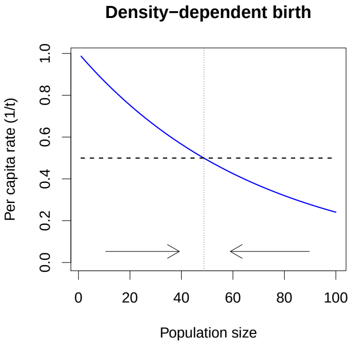

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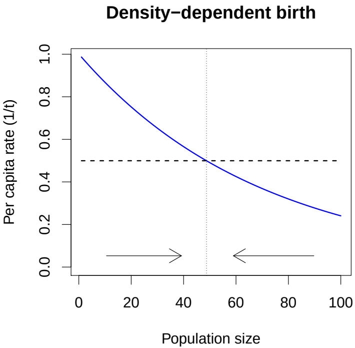

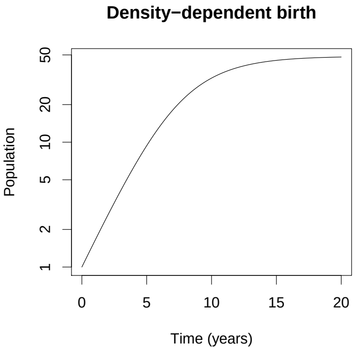

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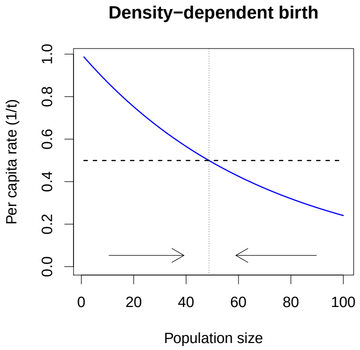

Examples


Density−dependent birth
Population
1
2
5
10
20
50
0
5
10
15
20
Time (years)


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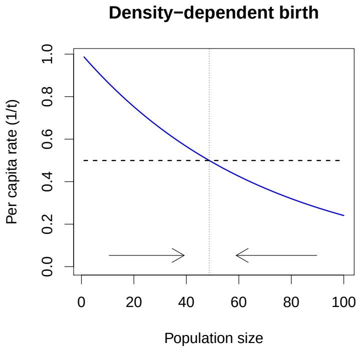

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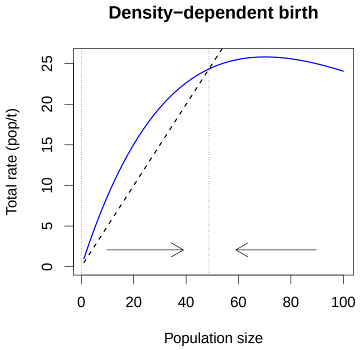

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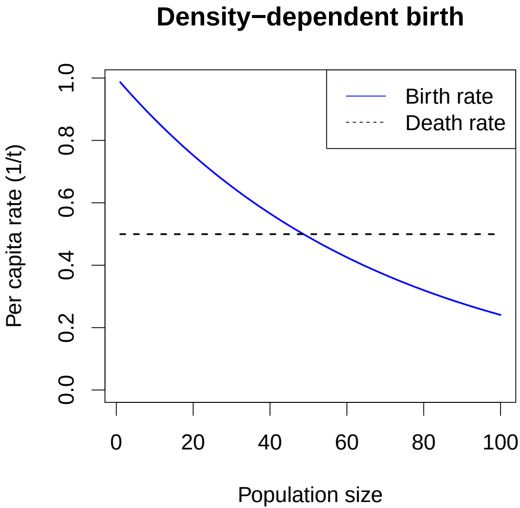

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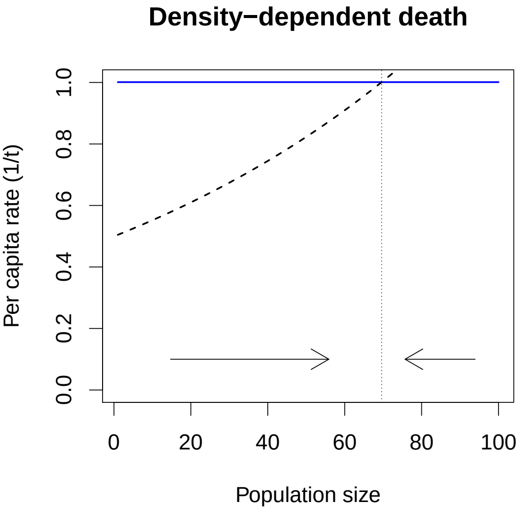

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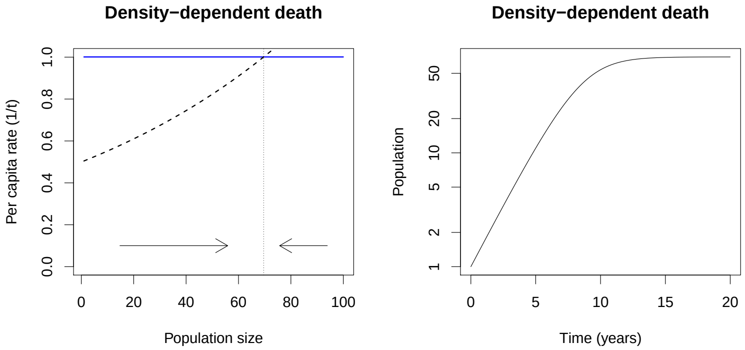

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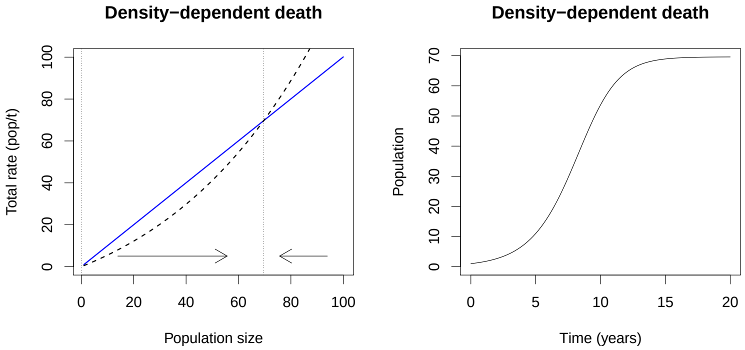

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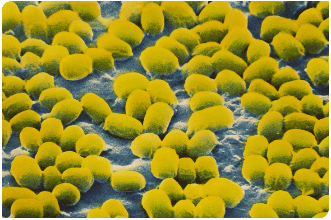

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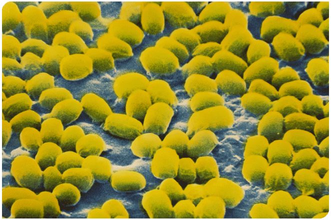

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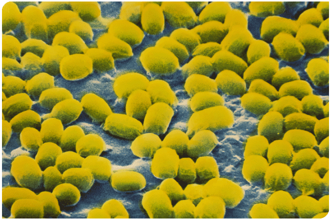

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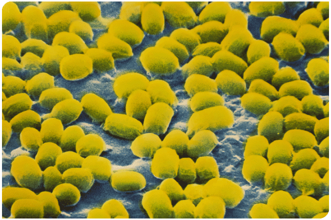

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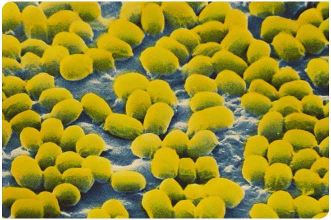

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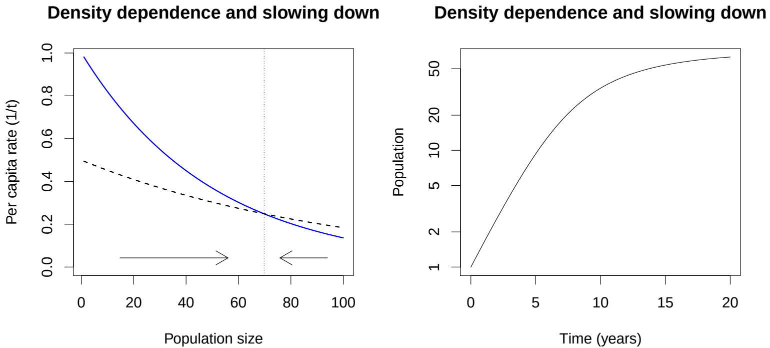

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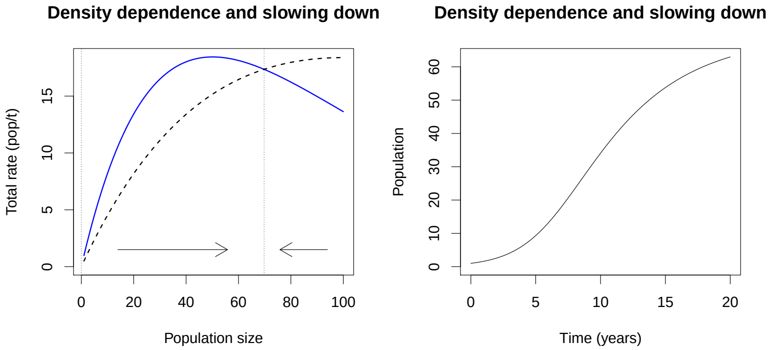

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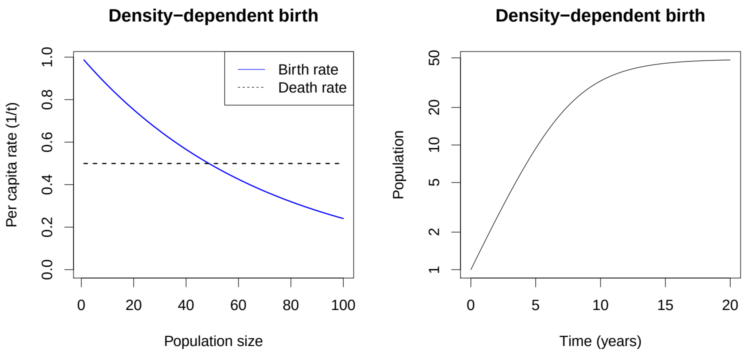

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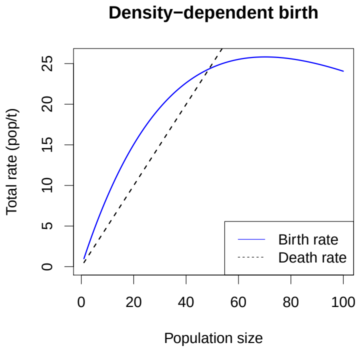

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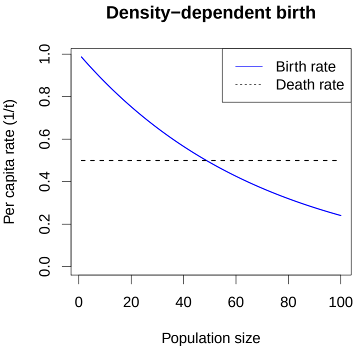

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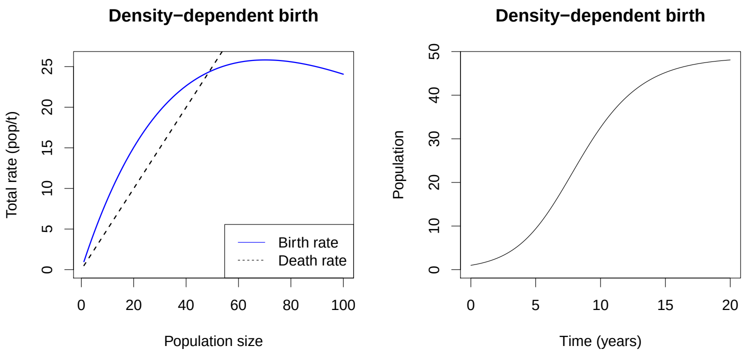

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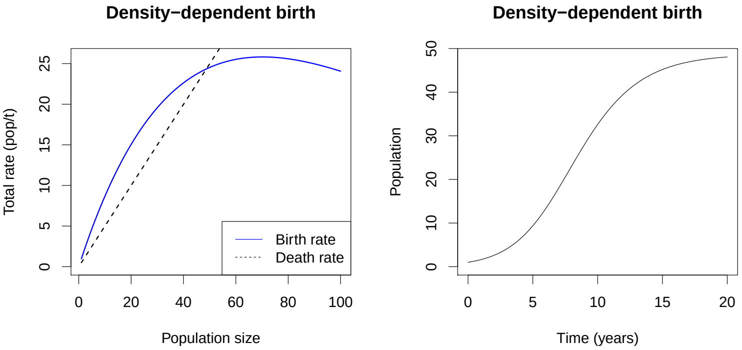

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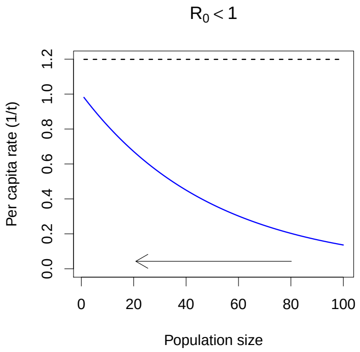

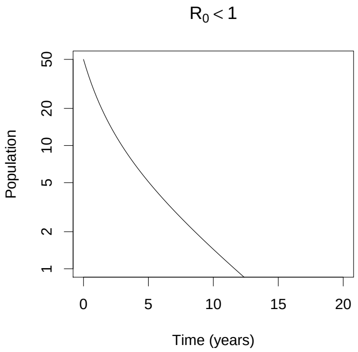

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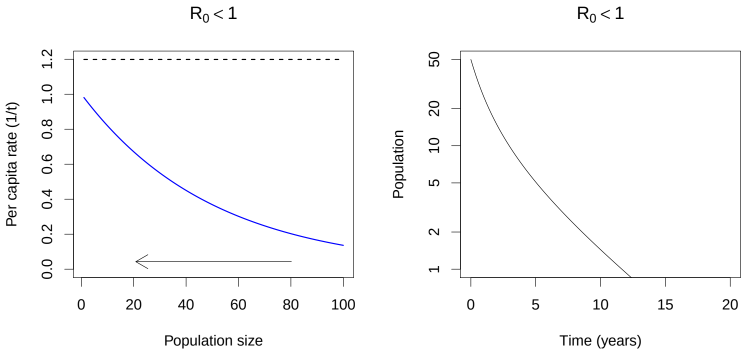

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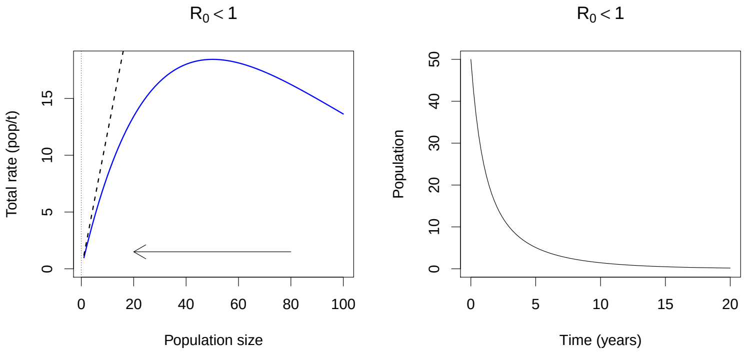

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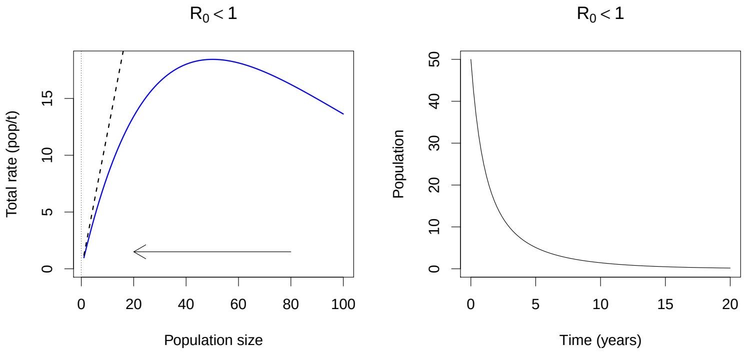

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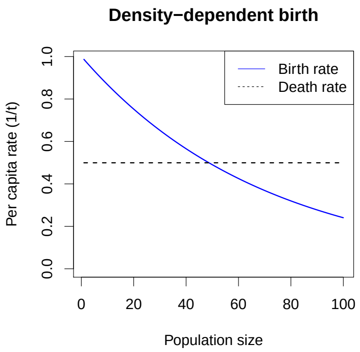

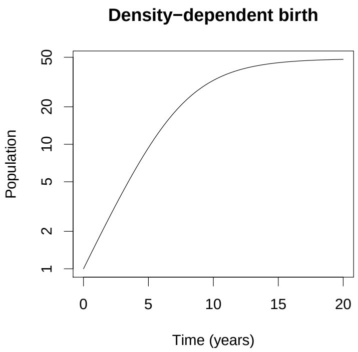

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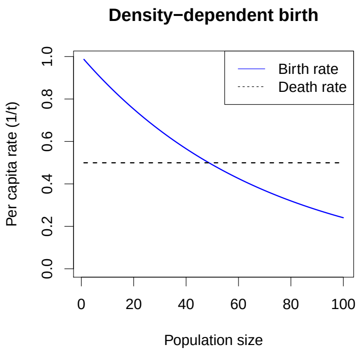

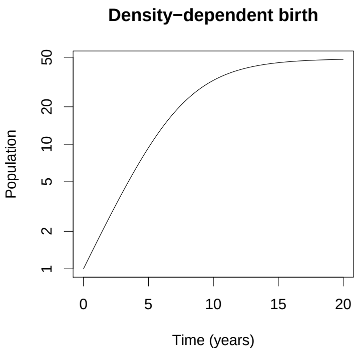

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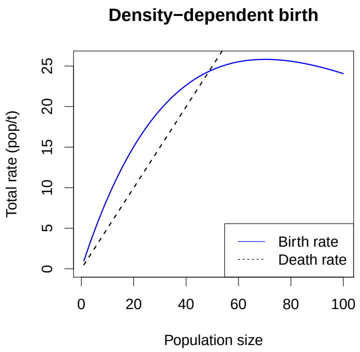

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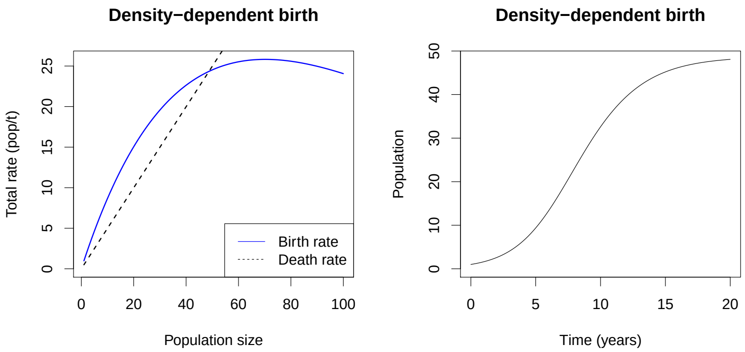

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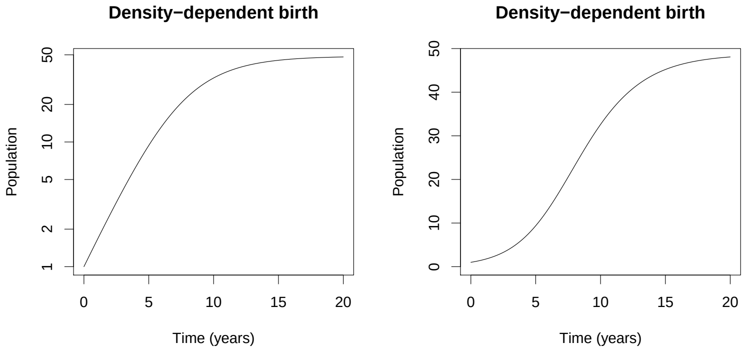

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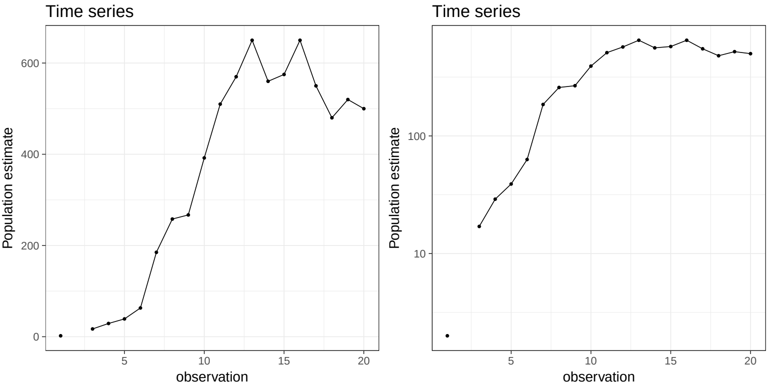
Time series
Population estimate
observation
0
200
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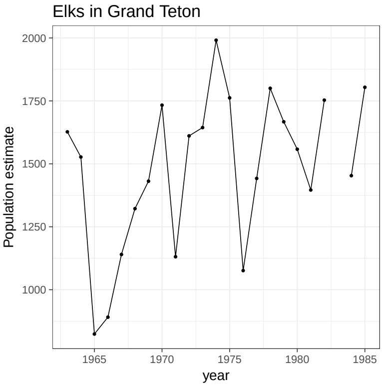
Elk



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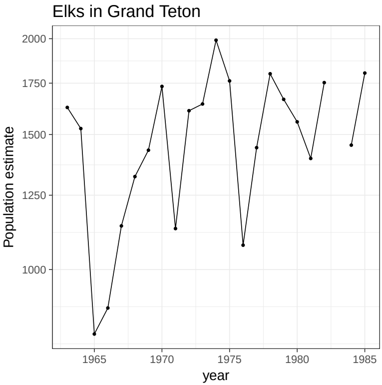

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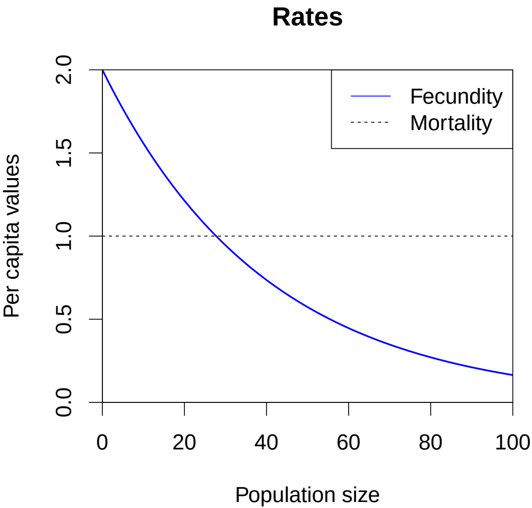

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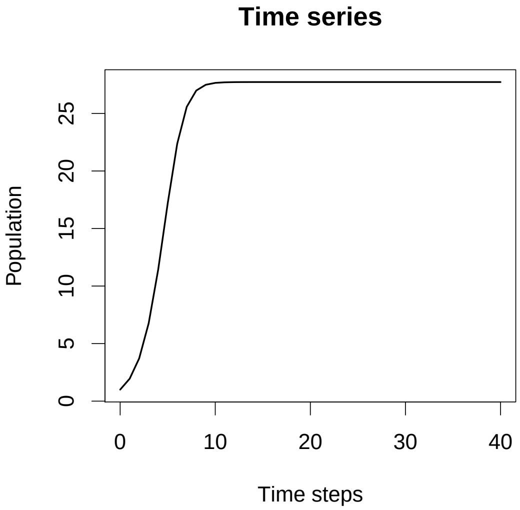

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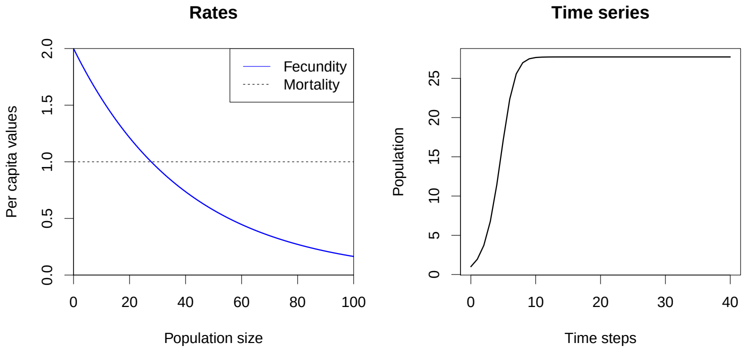

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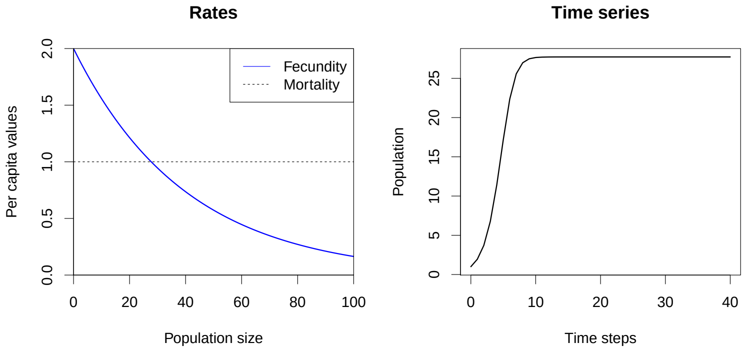

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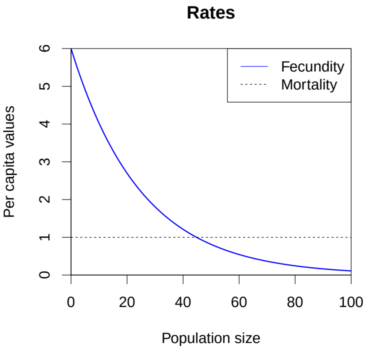
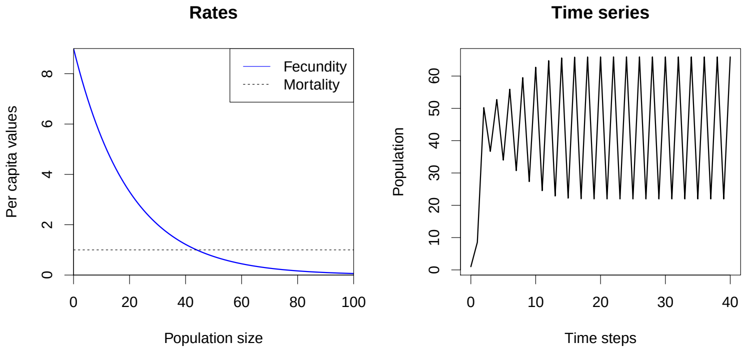
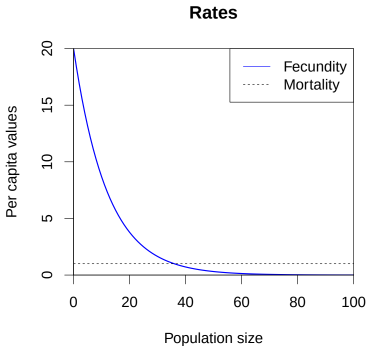
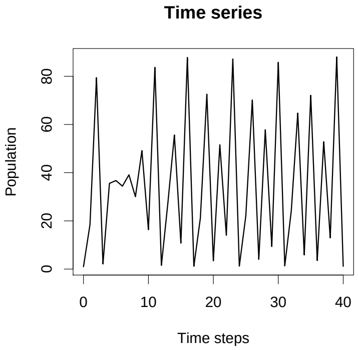


Persistent oscillations



Lots of other behaviours

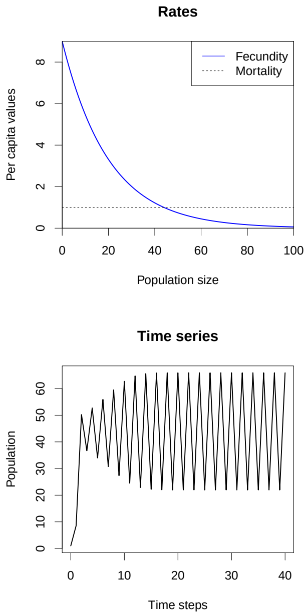




Outline

Interpreting complex behaviour

- In a simple cycle:



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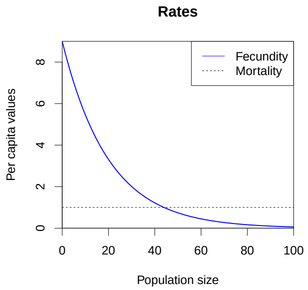

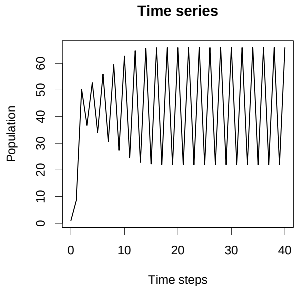

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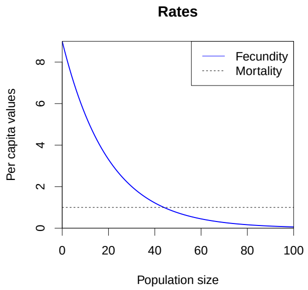

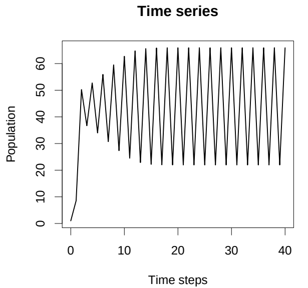

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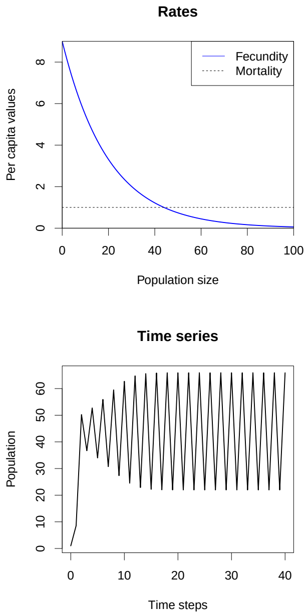

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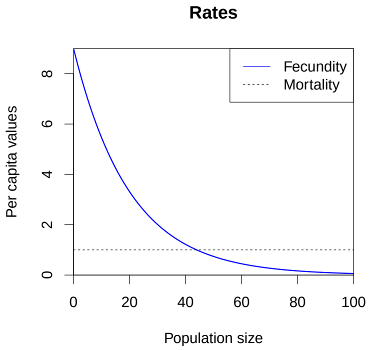

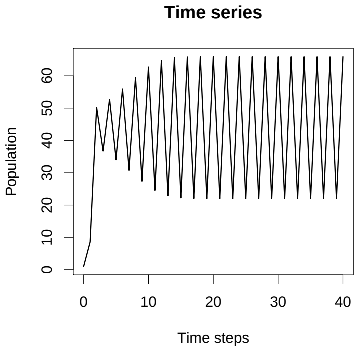

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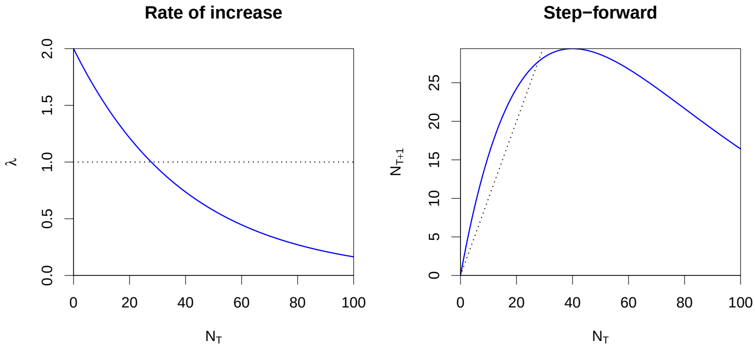

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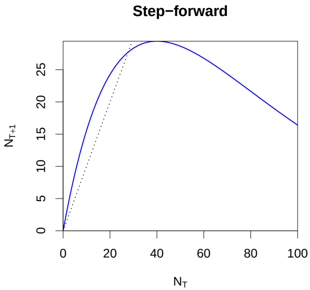

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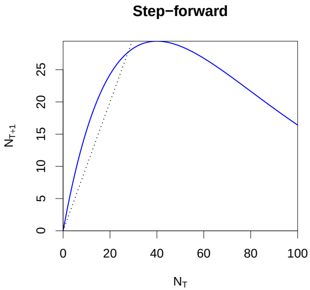

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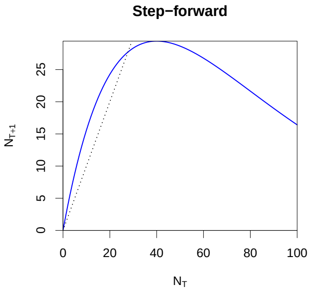

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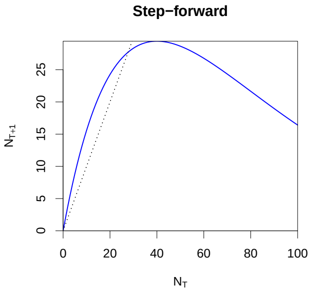

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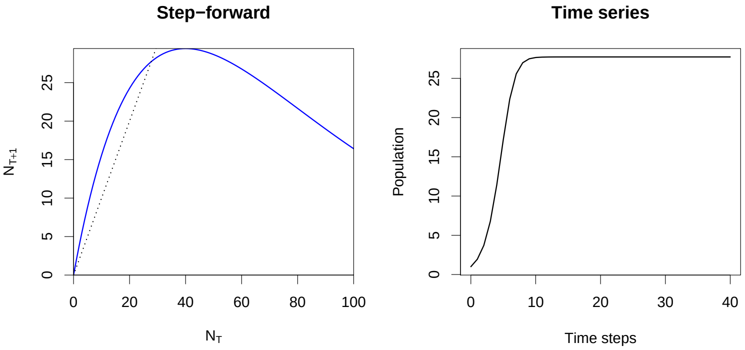

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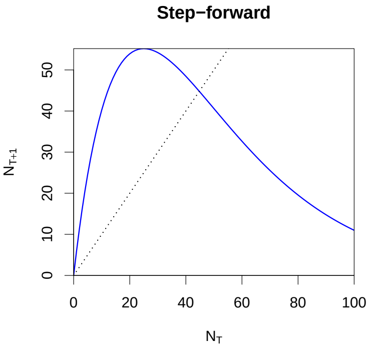

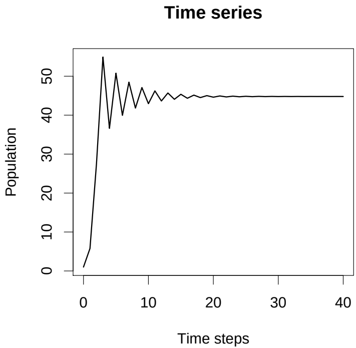

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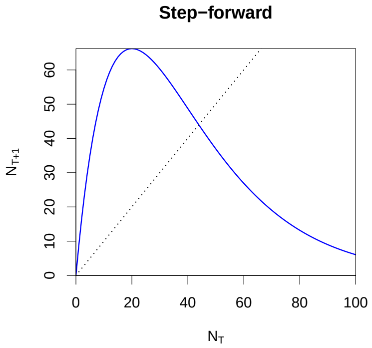

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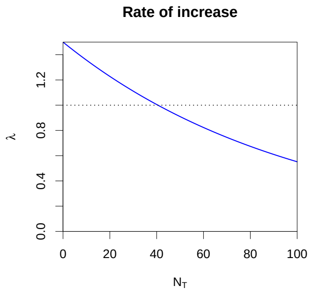

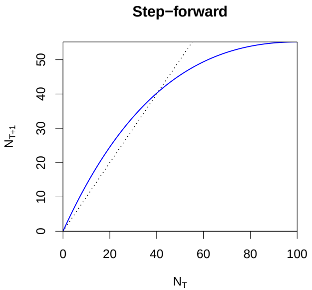

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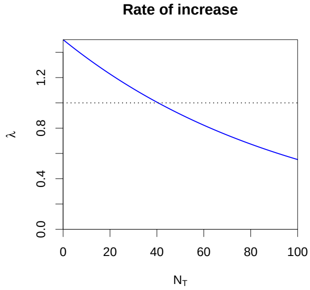

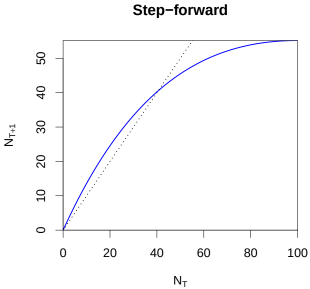

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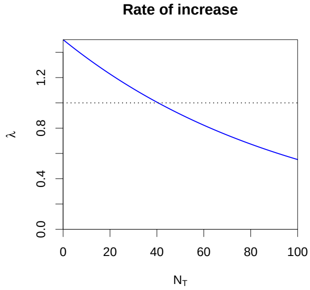

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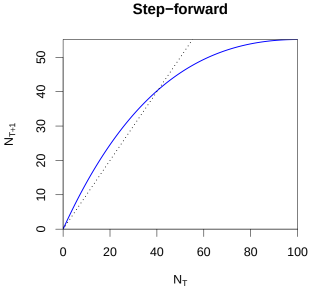

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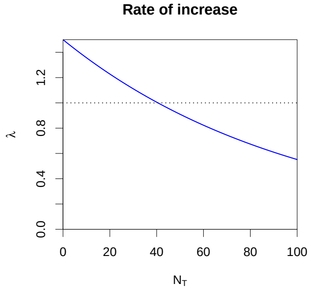
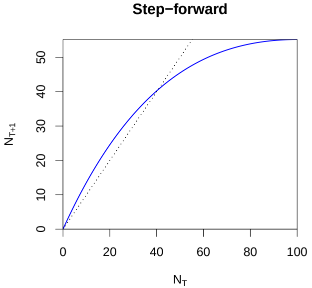
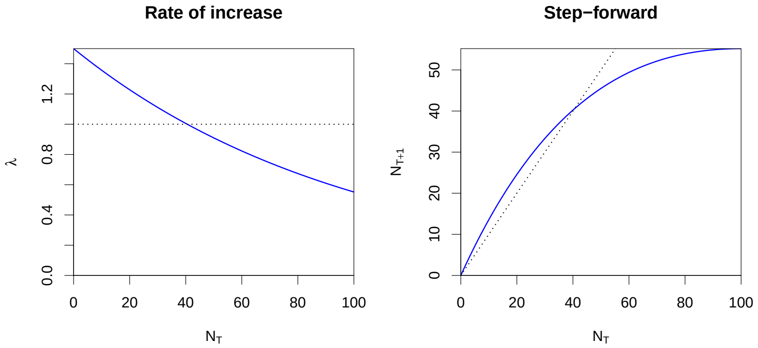




Contest regulation



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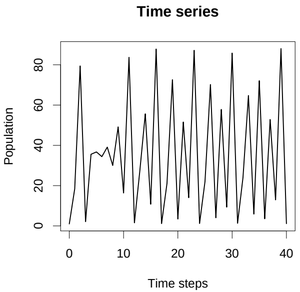

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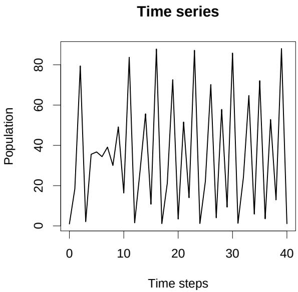

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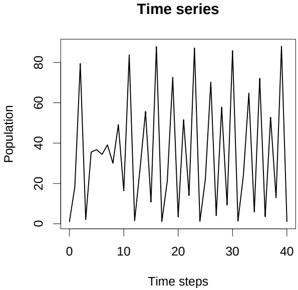

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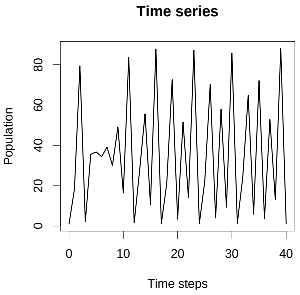

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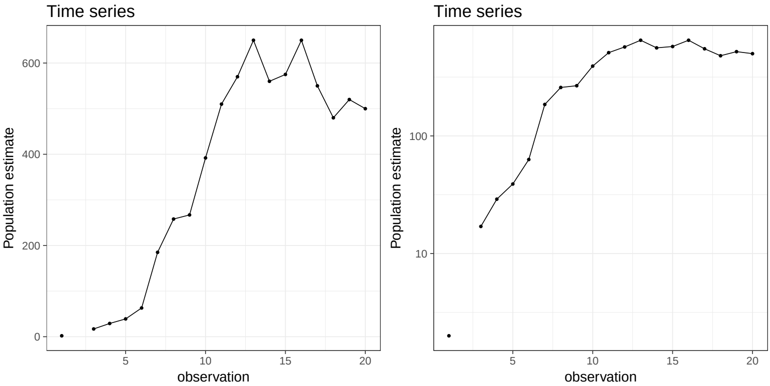
Time series
Population estimate
600
400
200
0
5
10
15
20
observation
Time series
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100
10
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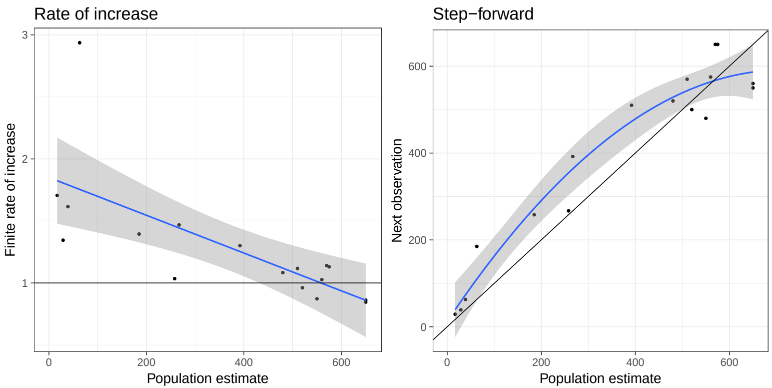
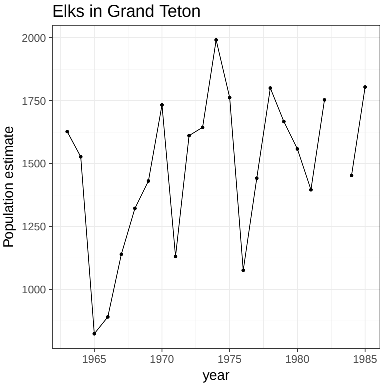
15
20
observation

Paramecia



Rate of increase
Finite rate of increase
3
2
1
0
200
400
600
Population estimate
Step−forward
Next observation
600
400
200
0
0
200
400
600
Population estimate

Elk



Elks in Grand Teton
Population estimate
year
2000
1750
1500
1250
1000
1965
1970
1975
1980
1985
Elks in Grand Teton
Population estimate
year
2000
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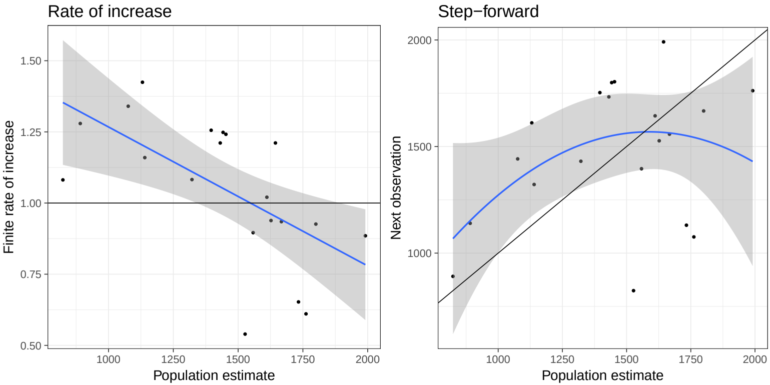

Rate of increase
Finite rate of increase
1.50
1.25
1.00
0.75
0.50
1000
1250
1500
1750
2000
Population estimate
Step−forward
Next observation
2000
1500
1000
1000
1250
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1750
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Population estimate

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Outline

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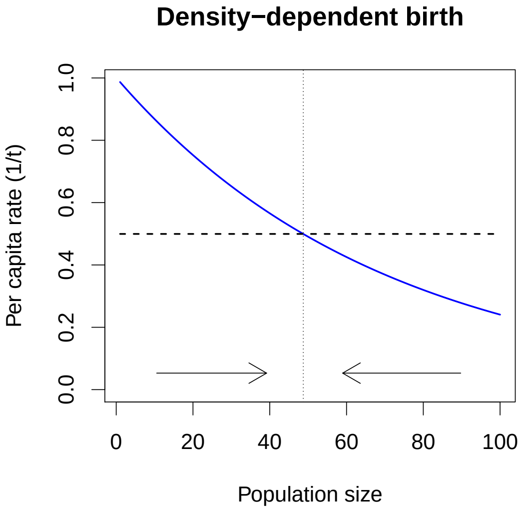
Arrows with time delay



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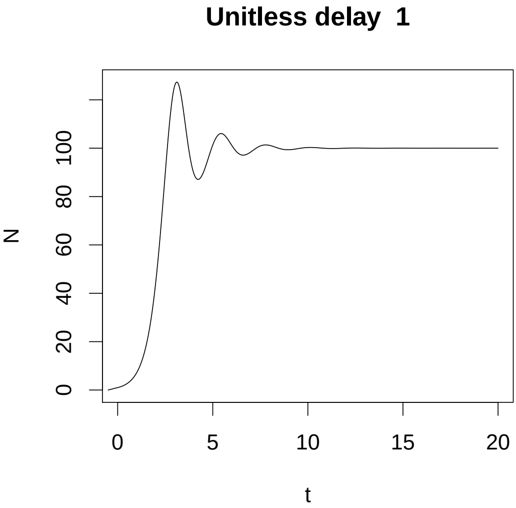

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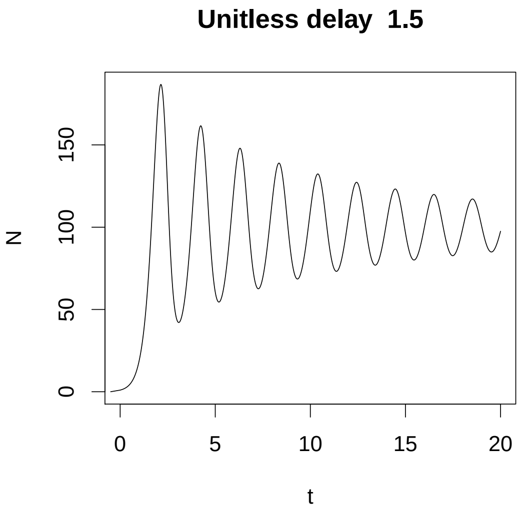

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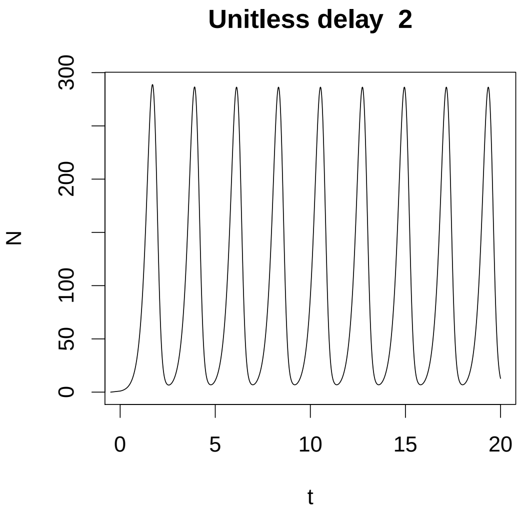

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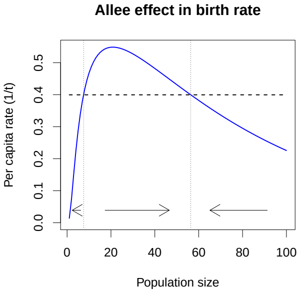

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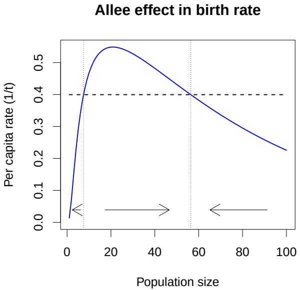

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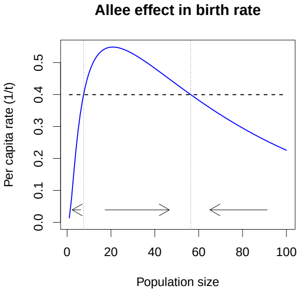

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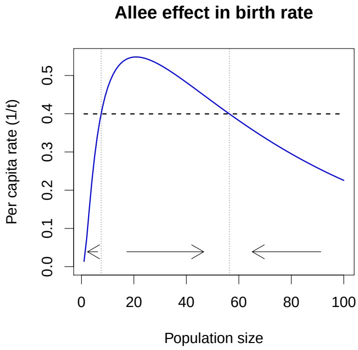

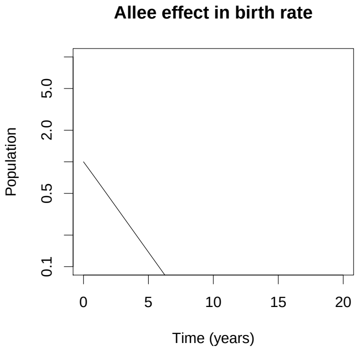

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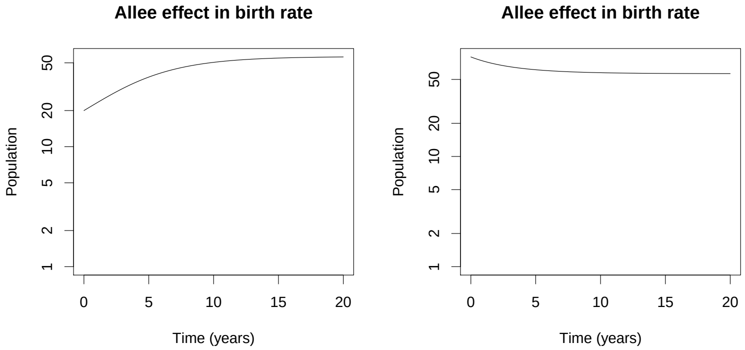

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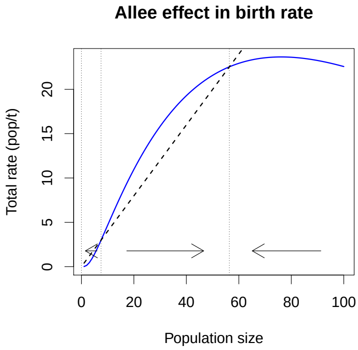

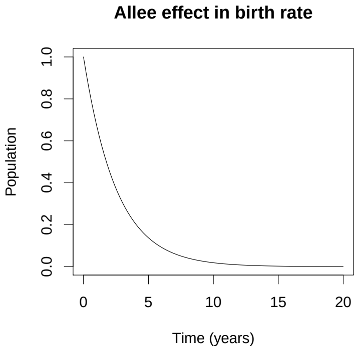

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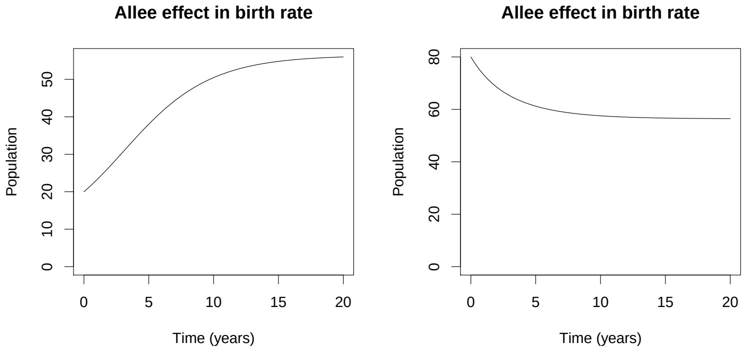
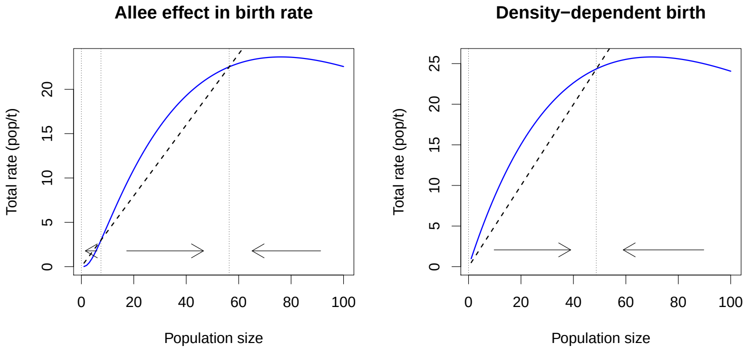


Population comparison



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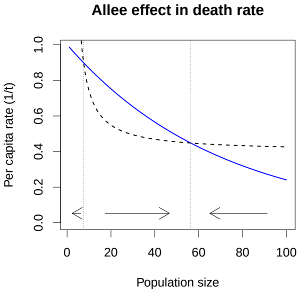

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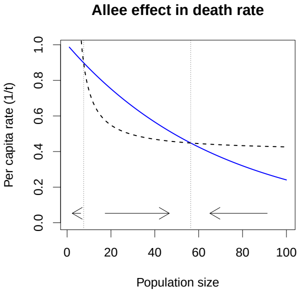

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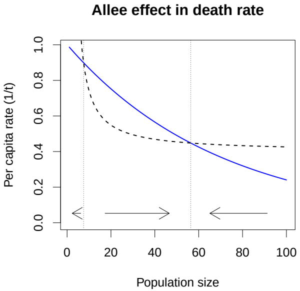

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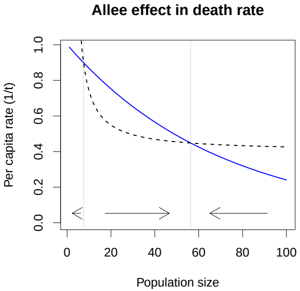

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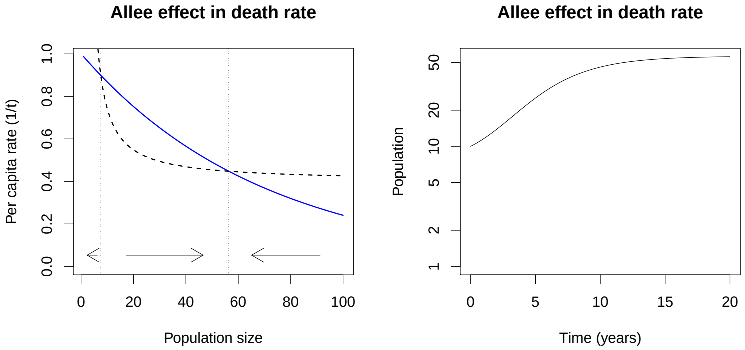

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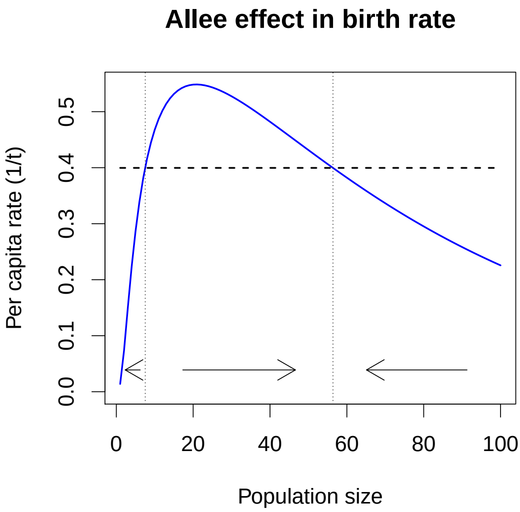

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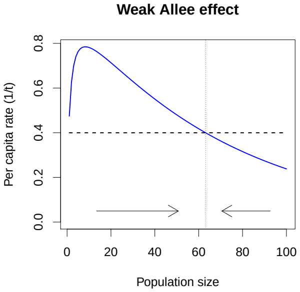

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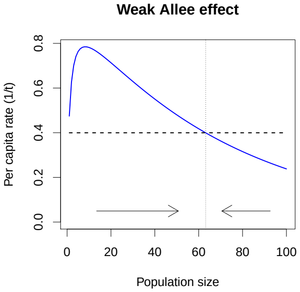

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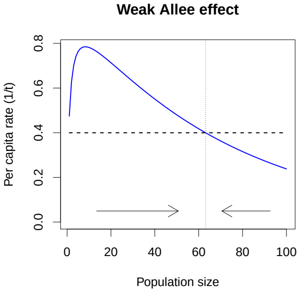

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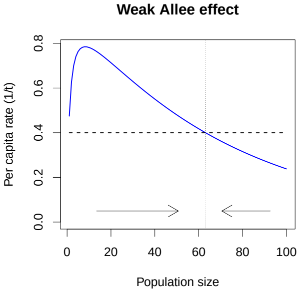

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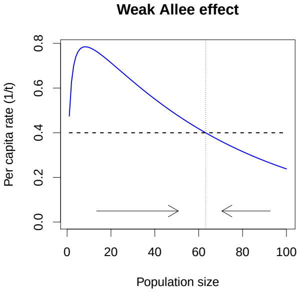

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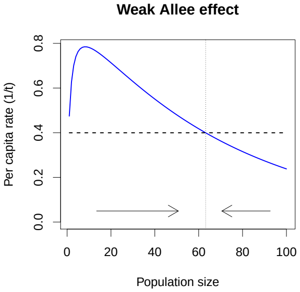
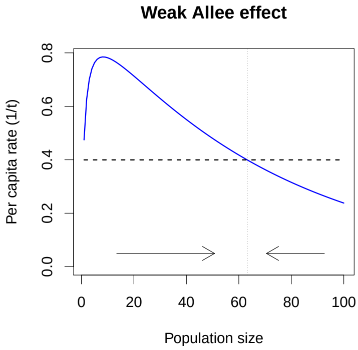
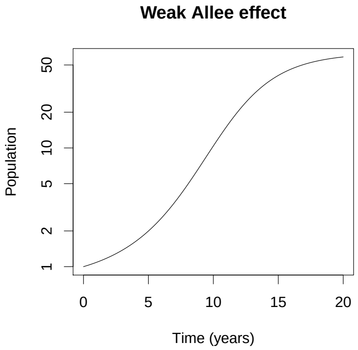


Individual perspective





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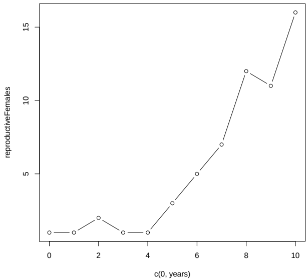

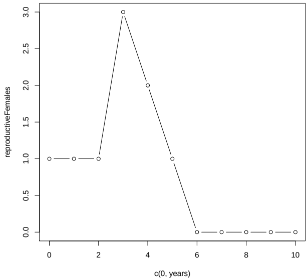

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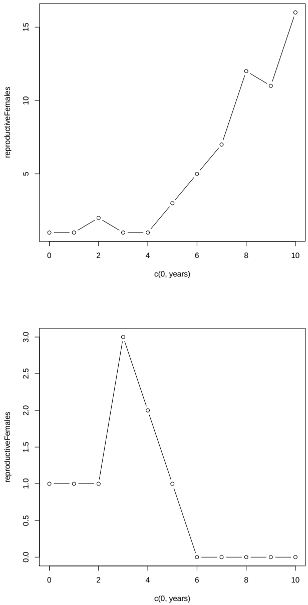

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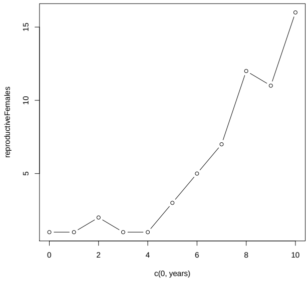

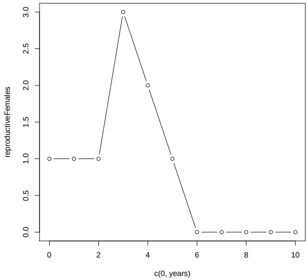

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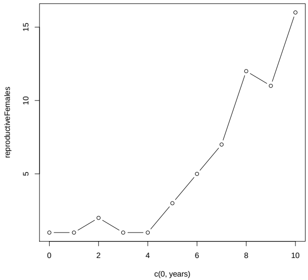

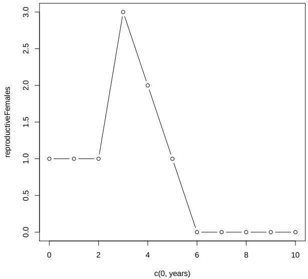

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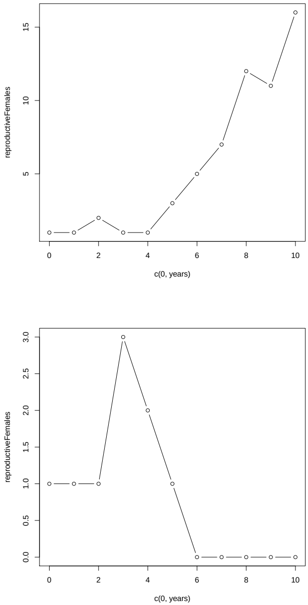

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