UNIT 6: Competition

Outline

Introduction

Population model with competition Balanced competition Unbalanced competition

Population-level interactions Invasion theory

Colonization and co-existence

Niches and coexistence

The competitive exclusion principle

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The confused flour beetle

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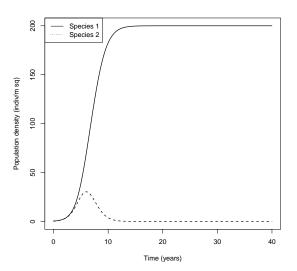
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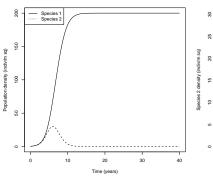
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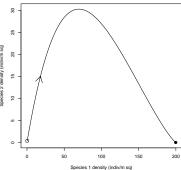
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Dominance time plot

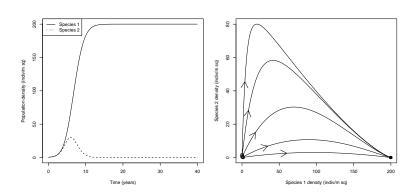


Dominance phase plot

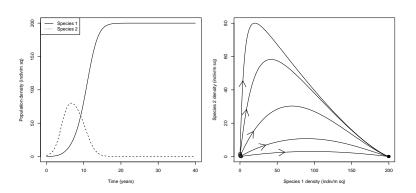




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► Log or linear (per capita vs. total perspective)

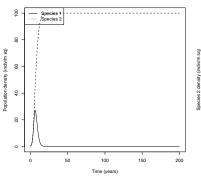
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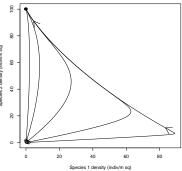
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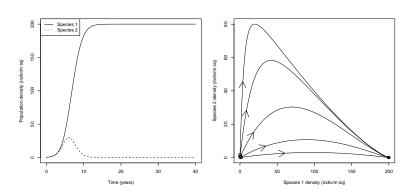
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Dominance again





What's the difference? (repeat)



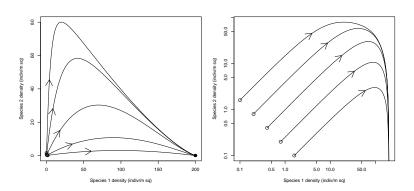
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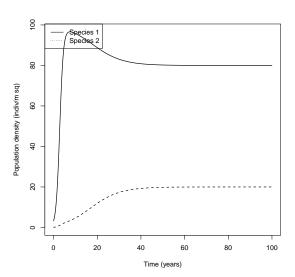
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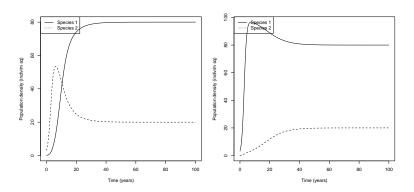
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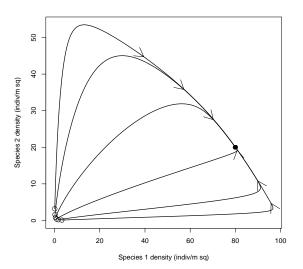




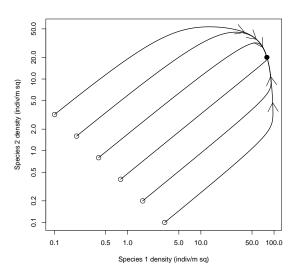




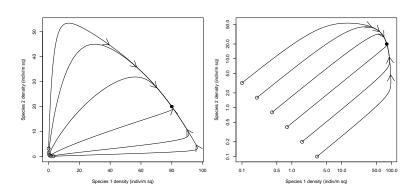
Coexistence phase plot



Coexistence phase plot (log scale)



Coexistence phase plots



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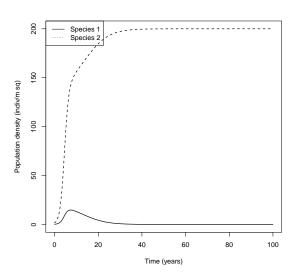


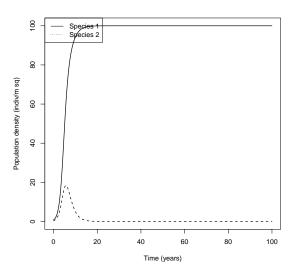
Co-operation

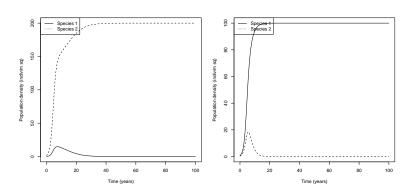




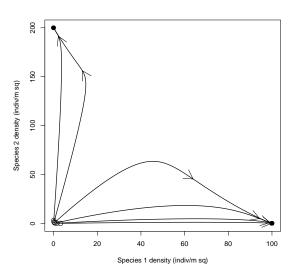
Changing the environment



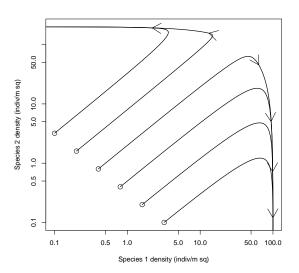




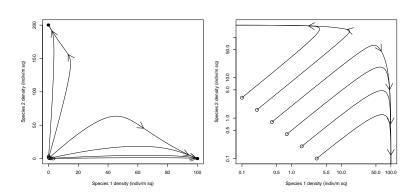
Founder control phase plot



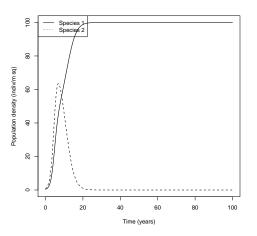
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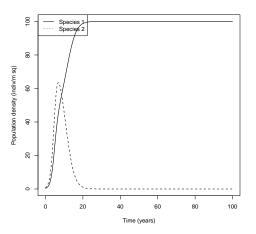


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- ► The *amount* of competition needed for species 1 to be at equilibrium is:

$$\blacktriangleright$$
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 - $\blacktriangleright * \tilde{N}_1 = \alpha_{21}N_2$
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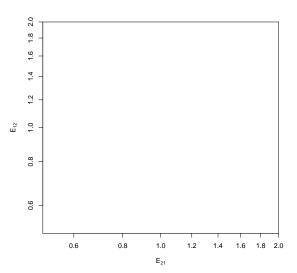
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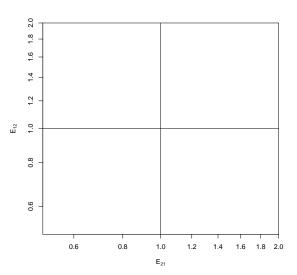
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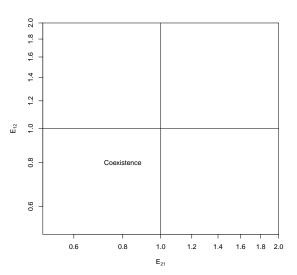
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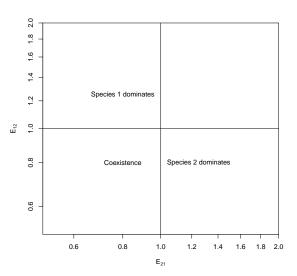
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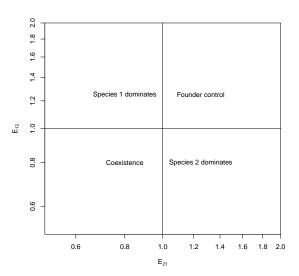
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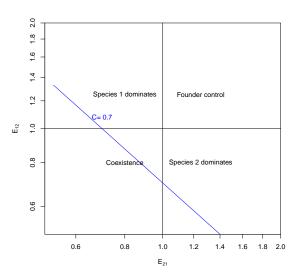


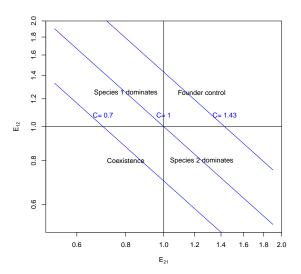












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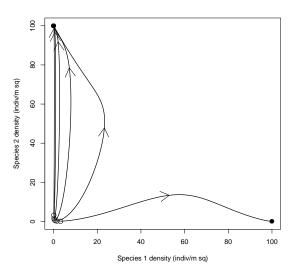
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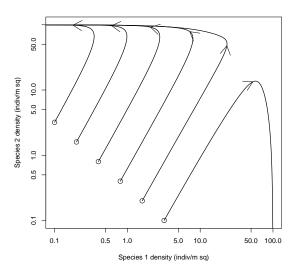
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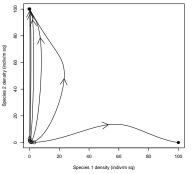
Growth rate and founder control

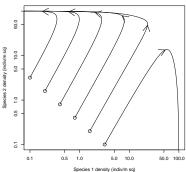


Growth rate and founder control



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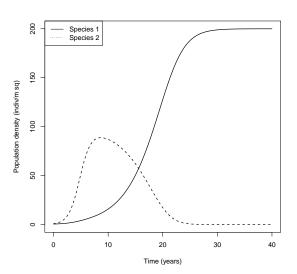
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