06/02/2022

Project Reference 1: Hemodynamics Ali Nashmi

Hemodynamics is study of the relationship among physical factors affecting blood flow through the vessels.

Darcy's Law (Blood flow is a function of pressure difference and resistance)

Blood Flow (F)
pressure difference (ΔP)
resistance to blood flow (R) through the vessel

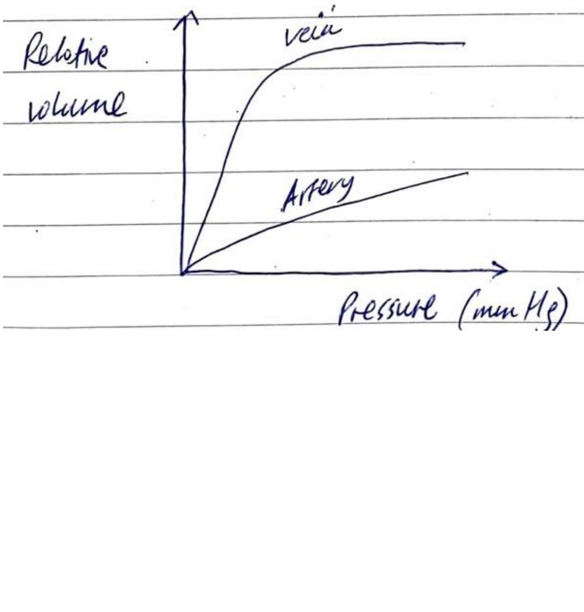
$$\boxed{F = \frac{\Delta P}{R}}$$

known as Darcy's law or Ohm's Law.

Definitions from VFT2 Descriptor

Compliance $\left(C = \frac{dV}{dp}\right)$

Compliance: sometimes called capacitance / distensibility refers to the ability of a vessel to respond to an increase in pressure by distending or swelling, and increase the volume of blood it can hold, or with decreased pressure, a decrease in volume.



Arteries : any of the muscular-walled tubes forming part of the circulation system by which blood (mainly oxygenated) is conveyed from the heart to all parts of the body

Veins : any of the tubes forming part of the blood circulation system of the body carrying (oxygen-depleted blood towards the heart.

Blood vessel wall shear stress : Shear stress is the tangential force of the flowing blood on the endothelial surface of the blood vessel.

- High shear stress (laminar flow) promotes endothelial cell survival and quiescence, alignment in the direction of flow, and secretion of substances that promote vasodilation and anticoagulation.

- Low shear stress (or changing shear stress direction as found in turbulent flow, promotes endothelial proliferation and apoptosis, shape change and secretion of substances that promote vasoconstriction, coagulation and platelet aggregation.

Rigid Walls Blood vessels : Rigidity is a condition when blood vessels lose elasticity. Rigidity causes blood vessels hardening of blood vessels so blood supply decreases.

Only a small amount of stroke volume is used for blood circulation, mostly only arteries are rigid due to hardening and thickening of artery walls.

Hemodynamics Ali Nasimi (cont.d)

Darcy's Law / Ohm's Law

$$F = \frac{\Delta P}{R} \qquad I = \frac{V}{R}$$

F (Flow) is defined as the volume of blood passing each point of the vessel in one unit time

Pressure (P) which is the force that pushes the blood through the vessel is defined as the force exerted on a unit surface of the wall of the tube perpendicular to flow. Pressure is expressed as millimeters of mercury (mmHg). As pressure changes over the course of the blood vessel there is no one value for pressure, so ΔP) change in pressure is used instead.

$$\Delta P = P_1 - P_2$$

Resistance is how difficult it is for blood to flow from point 1 to point 2. The resistance equation is:

$$R = \frac{8 \eta L}{\pi r^4}$$

η = fluid viscosity
L = vessel length
r = inside radius of the vessel.

Poiseulle's law (sub in R equation into Darcy's Law)

$$F = \frac{\Delta P}{R} = \frac{\Delta P}{\left(\frac{8 \eta L}{\pi r^4}\right)} = \boxed{\frac{\pi r^4 \Delta P}{8 \eta L}} \qquad \boxed{F = \frac{\pi \Delta P r^4}{8 \eta L}}$$

Shear stress in blood vessels

Shear stress is the tangential force of the flowing blood on the endothelial surface of the blood vessel.

High shear stress (found in laminar flow) promotes endothelial cell survival and quiescence, alignment in the direction of flow, and secretion of substances that promote vasodilation and anticoagulation,

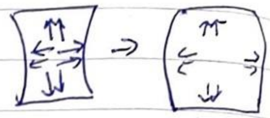
Simplified

endothelial cells form a single cell layer that lines all blood vessels

- Endothelial lining of blood vessels presents a large surface area for exchange of materials between blood and tissue
- Endothelial cells are quiescent meaning that they are proliferating (~~pr~~ proliferation - rapid reproduction of a cell, part or organism)
- High shear stress in blood vessels promote vasodilation (widening of blood vessels as a result of the relaxation of the blood vessel's muscular walls.) & anticoagulation - (prevent the blood from clotting).

• Compliance of blood vessels

Compliance
- Flexible
- Responsive
- Easy moving
- Expand easily without a lot of force
- Stretchability

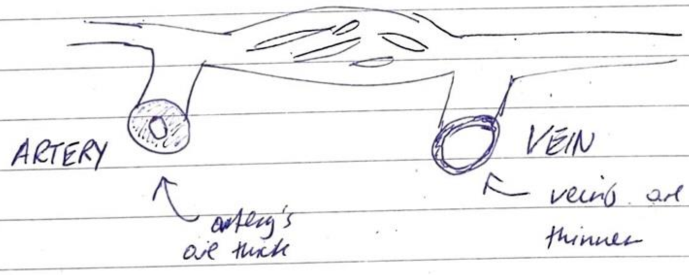


Pressure and Volume

Pressure ↑ Volume ↓
Pressure ↓ Volume ↑

– However in a closed space as Pressure $\boxed{\uparrow}$ so does the Volume $\boxed{\uparrow}$

$$\text{Compliance} = \frac{\text{Change in Volume}}{\text{Change in Pressure}} = \frac{\Delta V}{\Delta P}$$



Arteries: Carry Oxygen-rich blood from your heart to body

Veins: Carry deoxygenated blood from your organs back to your heart.

- Arteries are less compliant
- Veins are more compliant

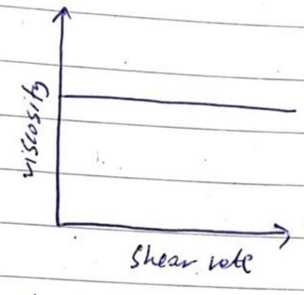
Navier-Stokes Equations

In physics, Navier-Stokes equations are certain partial differential equations which describe the motion of viscous fluid substances.

$$\boxed{\nabla \cdot u = 0}$$ ← First Navier-Stokes Equation

$$\boxed{\rho \frac{du}{dt} = -\nabla p + \mu \nabla^2 u + F}$$ ← Second Navier-Stokes Equation

Newtonial Fluid :



← Newtonian Fluid

Assumptions : Newtonian Fluid
Incompressible
Isothermal

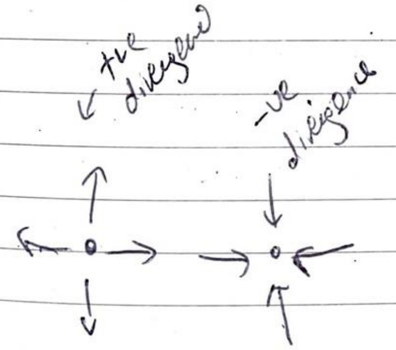
Velocity vector field

$$\underbrace{\nabla \cdot}_{\text{Divergence}} \; \overbrace{u}^{\text{Velocity vector field}} = 0$$

Divergence

Divergence (Mathematical Expression)

$$div \vec{F} = \nabla \cdot \vec{F} = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} \cdot \vec{F}$$



Divergence of vector field indicates how much or how little a point acts as a source of a fluid.

Navier-Stokes Equation

Friction / viscosity

$$\boxed{\rho \frac{du}{dt} = -\nabla p + \mu \nabla^2 u + F}$$ ← 2nd Navier-Stokes Equation

pressure

$$ma = \Sigma F$$

$\rho = \frac{m}{V}$ ← mass, volume

density

$$\downarrow$$

$$\rho a = \Sigma F$$

$$\downarrow$$

$$\rho \frac{du}{dt} = \Sigma F$$

$a = \frac{du}{dt}$

acceleration

pressure

$$\rho \frac{du}{dt} = -\nabla p + \Sigma F$$

$$\boxed{\frac{du}{dt} = \frac{\partial u}{\partial t} + u \cdot \nabla u}$$

$$\rho \frac{du}{dt} = -\nabla p + \boxed{\mu \nabla^2 u} + \Sigma F$$

Friction / viscosity

extend force (usually gravity)

$$\boxed{F = \rho g}$$

gravity / Extend force

Navier-Stokes Equation

$\nabla \cdot \underline{u} = 0$ ← Conservation of Mass

$\rho \frac{du}{dt} = -\nabla p + \mu \nabla^2 u + F$ ← Conservation of momentum. (Newton's 2nd Law) (F = ma)

$\underline{u} = [u, v, w]$ ← vector with components in x, y and z direction

$\nabla \cdot u = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$ ↖ tells us that mass is conserved

Change in x-direction, change in y-direction, change in z-direction

$\underbrace{\rho}_{\text{density}} \frac{\overset{\text{velocity}}{du}}{\underset{\text{time}}{dt}} \rightarrow \text{acceleration} = \underbrace{-\nabla p + \mu \nabla^2 u}_{\text{Internal Force}} + \underbrace{F}_{\text{External Force (gravity)}}$

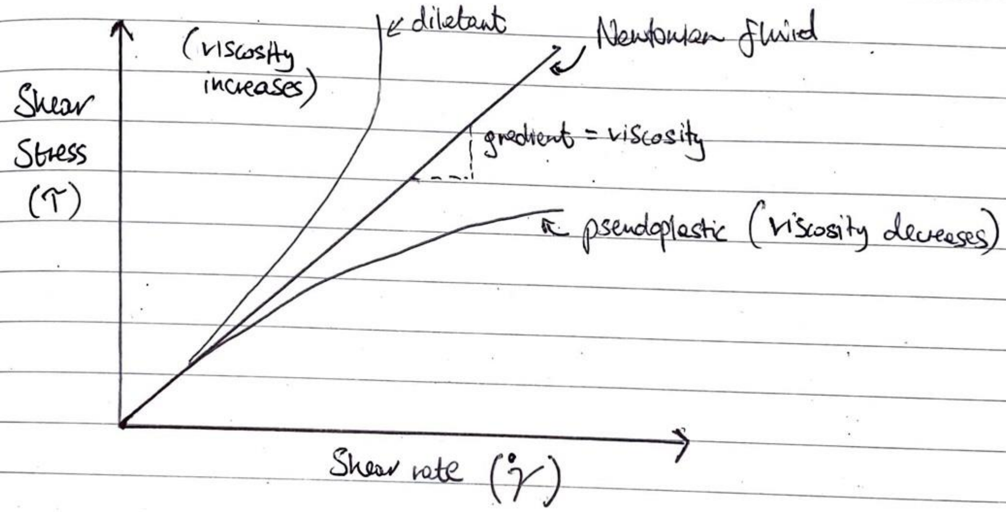
Internal Forces:

$\nabla p = \left(\frac{\partial P}{\partial x}, \frac{\partial P}{\partial y}, \frac{\partial P}{\partial z} \right)$ ← Change in pressure. (Fluid moves from high pressure to low pressure)

$\mu \nabla^2 u$ ← viscosity

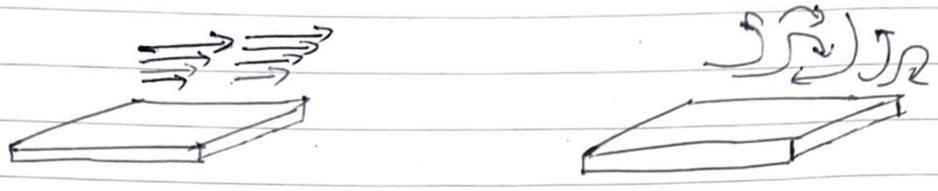
Newtonian Fluids.

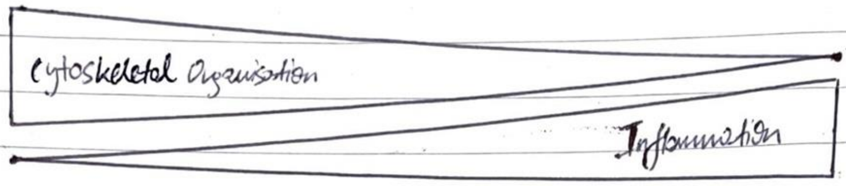
Newtonian Fluids have constant viscosity. Their shear rate is directly related to the shear stress. Viscosity shows the resistance to the deformation of fluid. The more viscous a substance the greater the resistance to the deformation of the fluid. For Newtonian Fluids: in the shear stress, shear rate graph there is a linear relationship. The gredient of this graph is the viscosity. As this is a linear relationship for a Newtonian Fluid the viscosity value is constant. For the purpose of this project the blood flow is assumed to be a Newtonian Fluid.



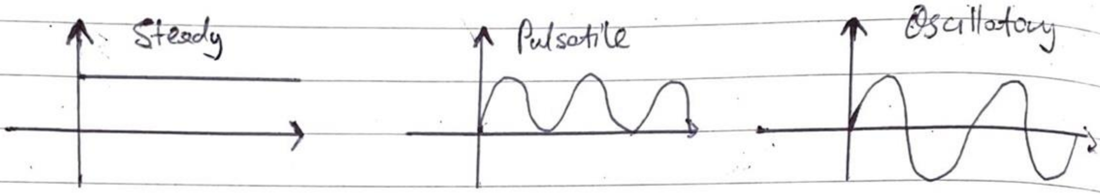
CSK (cytoskeletal)

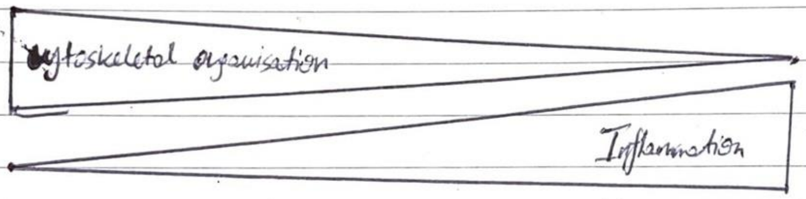
TYPE



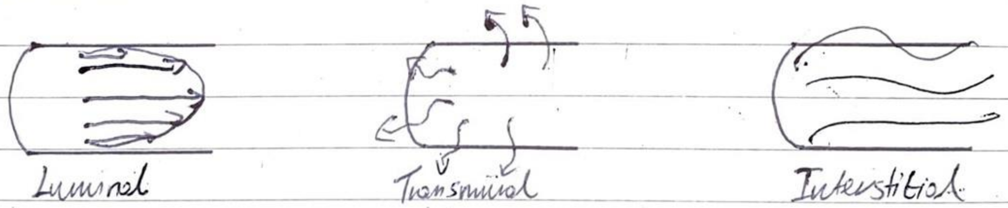


DYNAMICS





DIRECTION



Quiescence →

Angiogenesis

Blood Vessel Compliance

- We saw that pessure differences exist in arteries and veins as a result of a difference in anatomy of the two blood vessels. Blood pressure is not the only measurement that can be used to describe the properties of blood vessels.

Qualitative Measure of Compliance

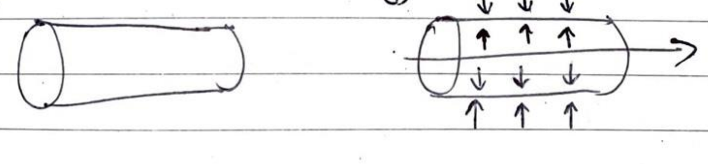
- Compliance basically describes how easy it is to expand a blood vessel.
- If expanding the blood vessel causes it to resist this expansion and recoil back to its normal size, then the blood vessel has a low compliance.

- If the blood vessel remains expanded without much recoil, it has a high compliance.

Arteries (Low compliance)

- Arteries have a thick layer of smooth muscle, which gives them the ability to recoil during expansion. This means that when the blood pushes against the walls of the arteries, they push right back. therefore, when blood fills the large arteries, they only expand by a small amount (10%).

a) b)



* Arteries can withstand high pressures without increasing in volume by too much,

Veins (High Compliance)

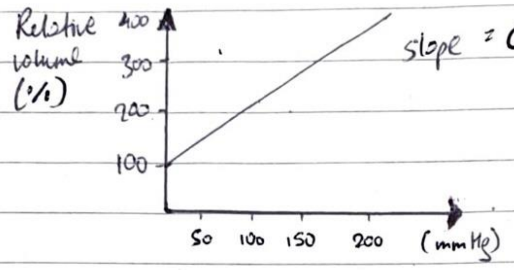
- Veins behave very differently compared to arteries. When they experience blood flow, the blood pushes on the walls but the walls cannot push back as hard. This expands the cross-sectional area and therefore increases the volume of blood that can pass inside the vein. It also ensures there is less built up pressure in the veins. Veins are stretchy therefore no build up in pressure inside them.

Quantitative Measure

$C = \frac{\Delta V}{\Delta P}$

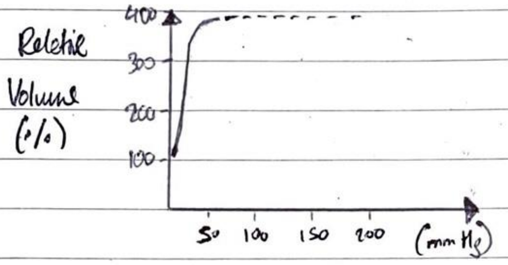
C = compliance value for blood vessel
ΔV = change in volume of blood vessel
ΔP = change in pressure between inside & out.

Arteries (LOW COMPLIANCE)



- For arteries, the slope is small (LOW COMPLIANCE).
- Arteries can resist high pressures without really increasing in volume by that much.

Veins (HIGH COMPLIANCE)

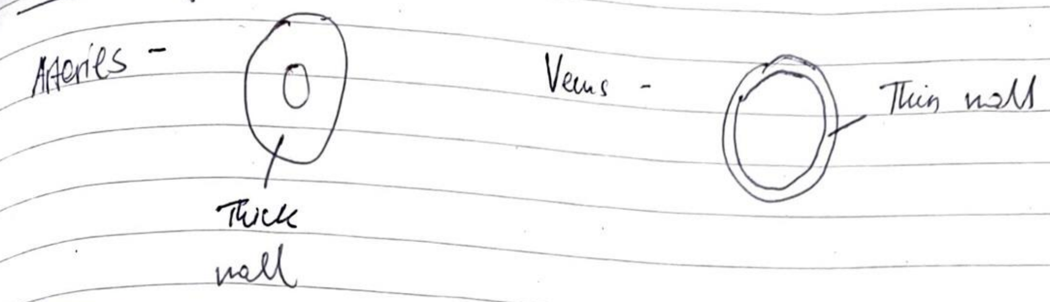


- For Veins, the slope is very large. (HIGH COMPLIANCE)
- Veins change in volume even when there is a tiny increase in pressure.
- Hence veins expand (distend) very well and store large volumes of blood.

Water is Newtonian Fluid
Blood is non-newtonian.

Viscosity of blood = $\boxed{3.5 - 5.5\,cP}$

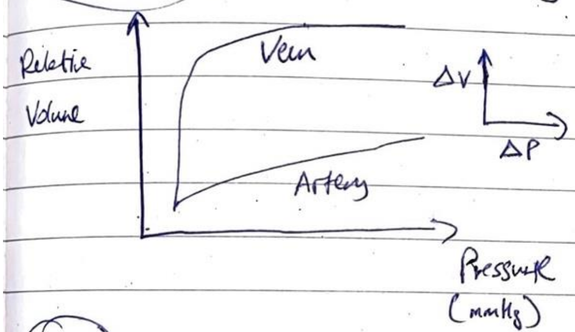
Important Definitions



ARTERIES - Blood vessels that deliver $\boxed{\text{oxygen-rich}}$ blood from heart to the tissues of the body.

VEINS - Blood vessels that returns $\boxed{\text{oxygen-depleted}}$ blood from organs back to heart.

VASCULAR COMPLIANCE - The ability of a blood vessel wall to expand and contract passively with change in pressure



$$\boxed{\text{gradient} = \frac{\Delta V}{\Delta P} = \text{Compliance}}$$

WSS - The force per unit area exerted by a solid boundary on a fluid in motion (and vice-versa) in a direction tangent to flow of the fluid. It is caused by the friction within the fluid and between the fluid and the vessel wall.

Hagen-Poiseuille Equation = $\Delta p = \frac{8 \mu L Q}{\pi R^4}$

L = Length of pipe

Q = Volumetric flow rate

R = pipe radius.

μ = dynamic viscosity

Finite element analysis - predict non fluid flows